Iowa Elementary Teachers Handbook, Arithmetic

Iowa Dept. of Public Instruction

ARITHMETIC

VOLUME VIII

IOWA ELEMENTARY TEACHERS HANDBOOK

Iowa Elementary Teachers Handbook

VOLUME VIII

ARITHMETIC

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JESSIE M. PARKER, Superintendent
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FOREWORD

In keeping with the universal trend now prevalent in the teaching of arithmetic, the committees responsible for the preparation of this handbook have placed major emphasis on *understanding* as the method of approach throughout the course. In contrast to the prevailing philosophy at the beginning of the present century, when major emphasis was given to the importance of drill (on the assumption that mastery of the fundamentals acquired in this way would readily transfer to practical situations), present philosophy stresses the importance of meaning or understanding as essential requisites in the teaching of arithmetic.

Lest the foregoing statement be misunderstood it should be mentioned that this does not mean that there is no place for drill in the present concept of arithmetic teaching. On the contrary, the need for drill in mastering the intricacies of arithmetic is just as essential as ever, but the nature of the drill situation used is quite different from the old concept of drill. While formerly much stress was given to the importance of isolated drill with abstract number combinations, drill now takes on the character of much repetition of these elements associated with meaning. So important is meaning, in fact, that the use of so-called crutches formerly frowned upon as an inhibiting factor is now encouraged as an aid to better understanding.

The emphasis given to the social usefulness of arithmetic is another characteristic of the course which aids in furthering the emphasis on the meaningful or understanding approval. The importance of this practice is stressed especially during the period of introduction to new ideas in the child's number experiences and is an essential condition for thorough understanding.

In dealing with the problem of individual differences the teacher will find material aid in the use of the diagnostic tests provided throughout the course. Through the use of these tests the difficulties peculiar to each child can be determined, and practice sheets built to aid in correcting his difficulties can be prepared for him. This procedure avoids a common error of the past—that of exposing to much unnecessary practice those individuals in the group who have no need for such practice.

As is true of other subjects in the elementary course of study, an effort has been made by the committee to suggest means of correlating the work in arithmetic with other subjects in the curriculum. The area which best lends itself to this procedure is the social studies. Helpful suggestions in this connection will be found in this course in the last section, "Suggestions for Activities Which Tend to Correlate Arithmetic With Other Subject Matter Areas," as well as in the Social Studies course, Volume VI. In addition, Volume I, the *Manual for Use of Iowa Elementary Teachers Handbooks,* will be largely devoted to suggestions of this kind in all areas.

This volume, like those previously issued in the elementary course of study series, has been made possible through the generous donation of time and services by Iowa educators. Grateful acknowledgment is made to members of the Central committee—Mr. H. K. Bennett, Dr. Ernest Horn, Dr. Barton Morgan, Mr. Paul B. Norris, and Dr. Elmer L. Ritter; and to members of the special Arithmetic committee—Dr. H. F. Spitzer, Dr. H. Van Engen, Miss Adelaide Lloyd, Mr. J. F. Tracy, Sister M. Regis, and Mr. Theodore Johnson.

JESSIE M. PARKER
Superintendent of Public Instruction

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INTRODUCTION

Arithmetic is the one subject which has as its major objectives the development of the ability (1) to think in terms of quantity, (2) to read quantitative materials, (3) to express quantitative relations intelligently, and (4) to discover relationships between quantities ordinarily found in daily life situations. This being granted, it follows as a corollary that the arithmetic program of the school must, in order to achieve its goal, emphasize the development of mathematical concepts, meanings, number imagery, reading, oral and written expression of quantitative ideas. In addition, (1) arithmetic should be expected to provide pupils with a tool for organizing data in such a way that they will simplify the more complex situations of daily life; (2) arithmetic should provide enough knowledge of mathematical and business procedures to enable the child to solve the ordinary quantitative problems of everyday life; and (3) arithmetic should furnish knowledge of the development of number and number processes as a basis for better understanding of our civilization.

To attain these objectives economically the instructional program must give emphasis to the major characteristics of the number system (decimal plan, significance of grouping, place value, ordinal and cardinal concepts of number, interrelationships between various processes, and the true nature of the processes).

Understanding and an appreciation of the social usefulness of arithmetic must be the chief goals of the arithmetic program throughout all grades, especially during the period in which a new idea is being introduced to the child. Therefore, much attention should be given to the use of concrete materials—even in the upper grades. Above all else, actual life situations involving arithmetic ideas should be reproduced as nearly as possible in the schoolroom. Children should actually handle and arrange sticks and blocks, make marks, diagrams and models, use fingers, cut paper to represent fractions, draw diagrams to illustrate a division or multiplication problem, use the tape to measure the width of the schoolhouse, etc.

It will require a great deal of teaching time to obtain understanding and mastery of the fundamental processes, especially since longer procedures are frequently employed by the children when first attempting a new process and when emphasis is placed upon the relationships between processes. In so far as possible, the instructional program should be so directed that the mathematical rules, concepts, and relationships will be discovered by the child himself. (See Morton: *Teaching Arithmetic in the Elementary School,* Vol. I, pp. 78, 91—Vol. II, p. 352; *Sixteenth Yearbook* of National Council of Teachers of Mathematics, pp. 13, 46, 55, 58, 71, 72, 103, 284.)

However, the discovery of these relationships should not be left to chance. Problems and number situations should be constantly guiding the child in the direction of the truth or the best procedure. For example, children can work so-called multiplication problems by adding, since work of that kind will either lead them to the discovery of multiplication or will give the experience which will enable them to see the economy of the multiplication process.

In problem solving the task before the child and the point of emphasis by the teacher should be the understanding, the discovery of, and an accurate expression of the relationships (quantitative and otherwise) that exist in the situation portrayed. In no case should the emphasis be on the obtaining of an answer. The correct answer is merely an indication of the child's having perceived the correct relationships as expressed in the problem. In the first stages of problem solving the teacher should tolerate and even encourage long, cumbersome procedures if they make for understanding. So-called crutches, if they aid understanding, are to be used.

As a check of the child's understanding, frequent proof or showing of the child's thinking will be required. Pencil and paper and blackboard work will then be a means of observing "how the child thought" in solving a number situation. In the more difficult cases individual conferences will frequently indicate gross errors made by the pupil in expressing and analyzing number relationships. This requiring the child to exhibit his method of thinking through diagrams, pictures, conferences, etc., will also permit the teacher to ascertain the child's level of accomplishment. Since all good teaching begins at the child's level of accomplishment, the above procedure is very important and should be used frequently. If the child does not have a simple (though often long) solution to a problem, it is not likely that he will understand the shorter, more direct adult method. For example, if the child does not see that two pictures could be made to represent three plates and four plates, and that these could be counted as one group, he is not likely to profit from saying, "Four plus three make seven." In such cases simpler work such as counting exercises should be used. The final goal of all instruction is the child's understanding and mastery of the best processes. After time has been given to building concepts and exploring the various ways, there will be direct systematic instruction and drill in the use of the accepted procedures.

As was indicated in the preceding paragraphs, long and uneconomical procedures are to be permitted during the initial learning period if they make for understanding. As a means of checking understanding and as a means of illustrating parts of procedures that are not clear, these longer procedures may also be used after the period of initial instruction. It should be clearly recognized, however, that such procedures are merely used as a means of securing better understanding of the commonly accepted adult procedures. In other words, the child should in the end employ the short, economical procedures of arithmetic that are advocated by modern textbooks. To illustrate the above, the following description of two phases of a child's achievement is offered: When first dealing with the addition of 6 and 5, it is quite proper for the child to take 6 objects and 5 objects, combine into one group, and then to count the total, or to change the two groups mentally into two 5's and a 1. Later, however, when the child is confronted with the addition of 6 and 5 he should respond automatically with 11. No mental visualizing or rearrangement of objects or numbers should be employed.

The point of view of the committee on the purposes of arithmetic and on methods of procedures has been stated in broad terms in the above discussion. In order to make clear the committee's position on points mentioned or implied, the following sections are included in this Introduction:

1. The Place of Drill

Drill or practice has a very important place in the arithmetic program. After an understanding of a fact or process has been attained, and if that fact or process is one that is frequently used, then practice should be used to fix and maintain the fact or process. Through possession of automatic mastery of frequently used and well understood facts and processes, the child's mind is relieved of some thinking and is therefore free to devote more attention to other aspects of the quantitative situation.

In addition to understanding, another prerequisite of drill is that the child see a need for possessing automatic mastery of a fact. If the conditions mentioned above have been met, then any of the well-known drill techniques may be employed. In general, the more direct (free from extraneous factors such as reading or following the rules of a game) the exercise, the better it is for drill purposes. Flash cards, practice examples and tables would, therefore, be considered good exercises for the drill part of the program. For the use of flash cards the following procedure is suggested: "Try to give the answers to the facts presented on one side of the card. If you do not know the fact immediately, turn the card over and look at the complete statement of the fact. Repeat the complete statement (e.g., 6 and 7 equal 13) trying to visualize the numbers as you do so. In case you cannot repeat the fact without hesitation look again at the complete statement. This process should be repeated until you are sure you can give the fact without hesitation. Put this card aside, and after you have gone through the pack try to give the fact for this card just as you tried the first time you went through the pack."

2. The Use of Tests

The major portion of the tests used in arithmetic are those informal tests made by the teachers or supervisors. Testing in the field of arithmetic is composed of tests made by classroom teachers, tests taken from the textbooks and workbooks, tests prepared by administrative and supervisory officers, and standardized tests.

Because of the more exact nature of arithmetic, almost every teacher feels that she is well qualified to construct tests in that field. For this reason an unnecessary proportion of the total arithmetic time is used for testing. Quite frequently, too, the tests are of inferior quality. In order to avoid the two criticisms mentioned above, these principles should govern testing in the field of arithmetic:

a. Tests should be used for the purpose of ascertaining a child's achievement or status with the idea that this information will be of use in future instruction. A minor part of finding the child's achievement is related to the giving of grades.

b. General tests should be varied in content, not just over one phase of arithmetic. However, diagnostic tests can be of a more limited nature since they are used as a teaching instrument. It is frequently convenient to have a diagnostic test which covers only a limited area, such as addition in fractions, or multiplication of decimals, in order to determine whether the pupil has mastered a specific area. Diagnostic testing will be covered in greater detail in the section entitled "Using Diagnostic Tests."

c. The amount of time given to tests (this includes those assignments which require a child to work problems and examples merely for the sake of working them) should be only a small fraction of the total instructional time.
   The inventory tests suggested for the early grades are not concerned primarily with finding out what a child knows about the work outlined for that year, but are intended to ascertain how much a child knows of the facts and processes with which he has had opportunity to become acquainted in his previous experiences. These inventory tests are really very general and make no attempt to give the teacher a complete picture of what the child knows.

3. Arithmetic in the Content Subjects

As was implied in the statement of objectives, arithmetic plays a major part in every area of life where quantity is involved. Since much of the work in the content fields (history, geography, science, etc.) involves quantity, it is obvious that arithmetic is needed to handle efficiently the work in these areas. This statement should not be interpreted as an argument for a longer or more intensive arithmetic period. Rather, it should be a clear indication to the teacher that much arithmetic can be taught as a part of the content subjects. Through such a procedure not only will arithmetic be more meaningful, but there will result an increased understanding of the materials of the content subjects. Throughout this course of study several opportunities to integrate arithmetic with geography, science, etc. have been worked out. It is hoped that the teachers will work out other units as opportunity arises. Only by relating arithmetic to science, health, etc., is it possible to develop a "real life" use of arithmetic. For the adult real, quantitative experiences occur in the context of other subject areas or while engaging in other activities; for example, while reading the newspaper, studying the market, paying taxes, etc. For the pupil these real uses must be looked for in his activities on the playground, his reading in other books, his interests in planes, cars, dresses, etc. The good teacher of arithmetic will take advantage of every opportunity to teach arithmetic during the geography period or geography during the arithmetic period if this opportunity presents itself in such a way as to lead to a natural correlation.

The teacher should take advantage of every opportunity to develop a critical or questioning attitude toward statements of quantity found in other subjects. For example, if a child makes a statement that Des Moines is larger than Waterloo, the teacher should sometimes ask for a clarification of the term *larger*. Obviously the statement that Des Moines is three times as large as Waterloo will in most cases be a more meaningful statement than the word *larger* is. Teachers and children should become conscious of the language of quantity.

4. Reading, Oral and Written Expression, AND ARITHMETIC

It is the function of the teacher in any school system to see the child and his needs, as well as society and its needs, as a whole. Granted this is a postulate on which to base a philosophy of teaching, it follows that the teacher of arithmetic is also responsible for the development of certain general abilities which heretofore have usually been considered the function of some special teacher. The attitude taken by many teachers of arithmetic as regards the development of reading skills, oral and written expression, and work-study habits serves to illustrate the point under discussion.

As a rule teachers of arithmetic are not too conscientious about assisting the child to learn to express quantitative ideas clearly and concisely. This is true not only of language (English) used in the classes but also as regards the statements in the language of arithmetic. It is a common practice for teachers and pupils to work the following problems as shown below:

Five per cent of the planes raiding town A failed to return. If 25 planes failed to return, how many planes raided the town?

The work.

$$5\% = 25$$
$$1\% = 5$$
$$100\% = 500$$

Now consider what is being said in the above three lines. The first line (in the English language) says: five per cent equals 25. This is not so. Five per cent equals .05. What is really meant is that five per cent of the raiding planes equals 25. The same error is made in the remaining two statements shown above. The point is not a mere "quibble" about an obscure point in mathematics. It involves expressing ideas clearly in the language of arithmetic. Whenever the idea is expressed clearly in arithmetic it can be translated into good English. This is not true of the statement $5\% = 25$. This statement translates into a false statement in the English because it is false as a mathematical statement. Pupils should be able to "translate" the arithmetical statement into good English; e.g., $n + 2 = 13$ says: some number and two are thirteen.

Also consider the type of response one so frequently hears in the arithmetic class. Do they not frequently consist of single words and incomplete sentences? "Johnny, what do you do to get the answer?" "Divide." This reply is accepted in spite of the fact that when two numbers are given in the problem, there are two different division problems possible. Johnny may be, and frequently is, thinking of the wrong division possibility.

The above illustrations will serve to convey the general idea of the intent of the course with reference to the development of the ability to express quantitative ideas clearly. The use of the English language to express concisely and clearly the ideas of quantity in an arithmetic class is an important element in the total instructional program. This, of course, is true of both oral and written work. Have the pupils express the ideas of arithmetic in clear-cut statements that make sense. In a world of quantity this is important.

Developing the ability to read is also a difficult problem. However, the attack on this problem by teachers of arithmetic is often not well planned. There should be a constant effort on the part of the teacher to have the pupil read his arithmetic textbook—not only the verbal problems, but also those parts of the text explaining processes which are new to the pupil. It is true that many arithmetic texts are too difficult for the child to read. However, every text has some sections that the youngster can read. Make use of these sections. Before explaining a process in arithmetic, have the pupils study the materials in the text introducing this process. Pupils learn to read only by reading. Find other quantitative materials for the pupils to read. One can find some reading material which involves quantitative ideas in almost any subject area. In the study of geography, health, and science, the teacher should constantly watch to see if the pupil understands the quantitative ideas contained in his reading assignment. Only in this way can the ability to understand the use of quantity, as it is normally used in the life of an adult, be adequately developed for the children in the elementary school.

USING DIAGNOSTIC TESTS

The diagnostic test is a very useful teaching instrument. It enables the teacher who understands its use to locate many of the more common difficulties the elementary school child may have with the processes of arithmetic. With a diagnostic test teaching becomes a cyclical operation, the cycles being as follows: test, teach, practice, re-test, re-teach, practice, and so on. In order to illustrate briefly what is meant by the cyclical operation of teaching and the use of the diagnostic test, the test on subtraction found on page 52 will be discussed in detail as to its use.

In the teaching of subtraction the first few difficulties encountered by the child will be, in all probability, steps one to five. Suppose a class has been taught these five steps as found in the diagnostic test. The teacher can mimeograph or ditto the problems as found in the test or make problems involving the same difficulties illustrated by these five steps. The class can then be given a test consisting of five steps in subtraction. If Johnnie, for example, misses two or three problems in step four and probably all those in step five, the teacher would know immediately that Johnnie is having trouble with borrowing in the first place and other places as well. If Johnnie succeeds in working all the problems of step four and misses some of the problems in step five, the teacher would know that he has trouble with borrowing. The same situation would apply to steps one, two, and three. Hence, the teacher will know that Johnnie needs to be drilled in problems similar to those found in the step in which the errors occur. Another child may need some individual work in bridging as illustrated in step three, and so on throughout the class.

By this procedure the teacher is enabled to individualize the work to a great extent and at the same time keep each class together as a group. It might be necessary for the teacher to work out practice sheets illustrating one particular type of difficulty such as those found in step eight of the subtraction test which involves zero in the subtraction processes (in one place only). After the individual difficulties have been located, practice sheets on *this particular difficulty* can be passed out to the pupils having trouble with, say, borrowing. However, after having located a weakness, the teacher should first reteach the process, then give the pupils the opportunity to practice. After the pupil has had time to practice, the testing process can be repeated.

There is nothing sacred about using the first five steps in the subtraction test. Of course, any number of steps could be included in a diagnostic test. However, the teacher should guard against making the test too long inasmuch as this would bring in the fatigue element of the testing process. Pupils should be allowed a considerable period of time to work the test. Diagnostic tests should have flexible time limits if they are to be used strictly as a diagnostic instrument. Obviously, if the youngster does not have sufficient time to work the problems of a test, the teacher learns nothing about the difficulties which may show up in those problems omitted. All diagnostic tests included in this course of study can be used in the manner illustrated above. Furthermore, the teacher should feel free to reproduce the tests included in this course as well as to build tests of similar nature for their own use if the gradation of difficulties does not fit the textbook being used.

CONTENT AND PROCEDURES BY GRADES

PRIMER—FIRST GRADE

There is ample evidence to show that children entering school do possess some knowledge of number and that they can profit from instruction in the field of numbers. In the kindergarten-primary program it is recommended that emphasis be placed on the building of concepts rather than the systematic teaching of facts. No specific recommendation in the way of required content is therefore made for the primer and first grade. The suggestions listed should not under any conditions be considered as the content to be mastered. In counties where a primer grade exists some of the suggested procedures will be used for that grade and others reserved for the first grade. In counties where beginners are in first grade, all suggestions listed below may be used.

It is important for the first grade teacher to realize that many number experiences occur at other times during the day than the designated number period. She should be constantly alert to notice these and take advantage of them. For example, a child can get enough chairs for a certain class, or drawing paper or boxes of crayons, or tell how many children are present or absent, etc.

Inventory

This inventory of the child's number achievement will have to be on an oral and an individual basis. It may well constitute some of the first two weeks of teaching. It should include exercises or tests similar to the following:

a. "I want to know how far you can count. Let me hear you count." If the child hesitates, say, "Begin like this: 1, 2, 3. Now go on from there."

b. (1) Have the child count the number of objects in a group.
   (2) Have the child count from a large group a certain specified number.

c. Which is the biggest? Longest? Which is low? High? (Use appropriate references such as blocks, objects on the wall, etc.)

d. Which has more? Fewer?

e. Which is at the bottom? Far away?

f. Which is in the right-hand corner?

g. How many days in a week?

h. Can you tell what time it is?

i. About how long is a foot?

j. Can you tell me how big a quart is?

k. Tell me the names of these coins. Use penny, nickel, dime, quarter.

l. Show me which of the drawings shows a half. Give me half of the blocks.

m. Tell me how many two pencils and three more pencils would be.

Content and Recommended Procedures

The first number experiences of the child should be very concrete. If, after a few weeks' work, the teacher finds that the child has already had lots of experience with concrete numbers, the next steps in semi-concrete or abstract number experiences may be provided. The outline of number experiences given below will indicate the successive stages through which the number concept of the first grade child is quite readily developed. Some children will need more of the concrete and semi-concrete number work than others before beginning on the more abstract number experiences outlined below. In every case, understanding will be the factor which determines the rate at which the child is introduced to the various difficulties given below.

I. Counting

A. Concrete experiences. These involve such activities as rhymes, skipping, bouncing balls, tapping, listening to other pupils tap fingers, counting in all possible number situations, counting the number of pencils needed, counting the days before holidays, counting chairs, papers, etc. In fact, the possibilities of this kind of concrete work involving counting are almost innumerable. The "wide-awake" teacher will make use of these situations to assist her in developing the number concepts of the first grade pupils.

These concrete experiences involve not only counting but the recognition of groups of two, three, four, and five without counting. Of course this ability can only be developed in easy stages.

B. Semi-concrete experiences involving counting and recognition of groups. These involve counting the number of boys in a picture, counting dots on dominoes, marks made on the board, squares, circles, and any other configuration which may or may not have any special meaning. Work in recognition of groups of dots, marks, circles, etc. should be provided.

C. Abstract number experiences. These involve simple situations which may be almost semi-concrete. In fact, the first stages should go back to the concrete work. Problems of this kind might involve Mary's finding out how many papers will be needed for the class. If she did not take enough, the teacher can ask, How many more will you need, Mary? This stage of number development will involve using the daily experiences of the children a great deal. If John brought two apples to school for his lunch and Mary brought one apple, how many apples did John and Mary bring to school? The more abstract experiences in this stage will involve the recognition of the symbols *two*, *2*, etc., and answering questions such as: One and two are *?*. This sequence of experiences with quantity should eventually develop the ability to:

Count and enumerate by ones to 100

Count by tens to 100

In teaching the order of number names, the teacher may rely chiefly upon imitation. Some use of games and rhymes may be made especially for purposes of motivation. Among the most popular of the rhymes are *One Two Buckle My Shoe*, *Ten Little Indians*, and *Here Is the Bee Hive Where Are the Bees.* Other teaching and motivating devices are to ask the children to skip a certain number of times, to bounce a ball a certain number of times, etc. Still another exercise is to make a mark for every tap the teacher makes on the desk and then count the taps.

Along with the counting (ordinal even though cardinal names are used) some concept of quantity (cardinal) develops. To encourage this development have the children show 7 fingers, 8 fingers, etc., when they have counted to 7 or 8.

A need for the reading of number names comes early in school work. Teaching the reading of numbers may, therefore, be combined with the counting exercises. Children should be asked to show the numbers from 1 through 9 in many ways. Dots, fingers, small objects, chairs, and children themselves are probably the best things to use. Specific procedures to use appear later in the outline.

In writing numbers, the accepted cursive (1 2 3 4 5 6 7 8 9 0) form, not the manuscript, should be used. Since the number words are used so frequently in early reading, the work as well as the symbol and the concrete and semi-concrete representation of the quantity should be used; e.g., the pupil should recognize that the columns below are all special instances of the numbers *four, three,* and *five,* respectively.

four	three	five
4	3	5
1 111	1 11	111 11
/ ///	111	.. ...
// //	...	† † † ††
.... .	. ..	11 11 1
.. ..	111	

A common device is to tell children to show in another way what you have written.

As a device to aid in teaching the reading of numbers and in teaching counting, a number chart similar to this should be made:

	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

This chart may be changed so that the sequence is vertical rather than horizontal. The chart has many uses; e.g., it shows the tens so that a child in counting by 10's has only to look at the left-hand column. To aid in counting, children can be asked to count the numbers in a row or column or to say the names from 1 to 99, pointing to each in turn.

The role of zero can be taught by asking what the difference is between the 1 in the second and third rows. (In the third row the *1* means 1 *ten,* while in the second it means 1 *one.*) The teacher may frequently use the chart by asking for the numbers that all show 7 tens and some ones; or the number that is 10 more than 23; or 10 more than 3 tens and 4 ones.

In teaching children to count by 10's after counting by 1's to 20 has been learned, heavy emphasis should be put on the fact that 30 is the third 10 and 40 is the 4th ten, etc. In this way the teacher is helping the child to use what he has already learned. In other words, advantage is taken of the number system.

Since counting is so basic to advanced work in arithmetic, it can hardly be overdone. In addition to those already mentioned, the following are types of exercises that have been found useful:

a. (1) Have all pupils guess at the number of beans in a bottle and then have children in need of counting count the beans. After their first count ask that they arrange the beans so that the count can be easily checked. (Grouping into 10's with subgroups of 2 or 5 is best.)

(2) Finding the ear of corn with the most grains. This may become a contest. Finding the ear with the greatest number of rows of grains may be another part of the contest. Have children check each other in counting rows or grains. Record the number of rows on different ears. Point to the fact that each is an even number.

b. (1) Have children put in order a pack of number cards.
(2) Take out some of the cards and have children put remainder in good order. (Cards might then be in this order: 1, 2, 4, 6, 7, 9, 14, 18, etc.)

c. Counting dots or marks on 10's square. In this square 100 marks or dots are arranged in 10 rows and 10 columns. By means of paper markers children are asked to show 2 rows; then, count the dots in the rows. This procedure is followed by asking how many dots in 4 rows or columns and by asking the children to show with markers 25, 40, 47, etc.

d. Ask children what page number 16 in a book means. Have them count to find out that 15 pages precede it and that it is therefore the 16th page.

e. Have children count the number of days until a holiday like Thanksgiving or Christmas; count the number of children present; the absences in a month.

f. Conduct writing exercises in which children write as many numbers as they can.

g. Require children to tell frequently what the numerals in two-figure numbers mean; e.g., What does the 2 in 32 mean? The 3?

h. Ask children to show numbers like 17 in other ways. (The word seventeen, 10 and 7, and other combinations as well as regrouping of actual objects, are the most serviceable ways of showing 17.)

i. In showing numbers in different ways, the Roman numerals as used on the clock may be used.

II. Recognition and Reproduction of Groups

1. Exercises which require the child to recognize the number of objects in a group

a. Bring the box with four blocks.
b. Bring the stack of three books.
c. Play the domino that has 5 dots on one end of it.
d. Draw a ring around the box with 6 dots in it.

2. Exercise in which the child reproduces groups
   a. Make four marks on your paper.
   b. Make a mark for each tap made on the desk.
   c. Make as many marks in this box as there are in the other box.
   d. Show five with your fingers. Show six.

III. Quantitative Comparisons

For each of the suggestions it is assumed that appropriate objects such as blocks, sticks, boxes, lines, or drawings will be used.

1. Which box has more sticks in it?
2. Bring the box with the fewest pieces of chalk in it.
3. Which line is long? Short?
4. Make a line shorter than this one: longer than this one.
5. Get a block two times as big as this one; one half as big.
6. Which is the largest?
7. Put red marks on all the big ones.
8. Which child has fewest blocks? Most?
9. Which child is the tallest?

IV. Recognizing and Counting Coins

1. Tell me the name of this. Show penny, nickel, dime, and quarter.
2. Count this money.
3. Which of these will buy more? (Show nickel and 5 pennies, etc.)
4. Put your finger on the penny; nickel; dime; quarter.

V. Telling Time

An attempt to get the child to see that time pieces tell us when to do certain things is the major goal. Assign to different children the job of deciding when it is time to go home, to play, or to have luncheon.

VI. Oral Problems Involving Number

1. Jack has two books and Jane has three. How many books do the two children together have?
2. Henry had 4 marbles but lost 1 of them. How many marbles does he now have?

VII. Playing Games Involving Number

1. Dominoes first in which the children merely match, the object being to see who can play all his dominoes first. Later the counting domino game may be introduced.
2. Playing pegity
   In playing this game use a pegity board or a board about eight inches square with holes in it at intervals of one-half inch. Each player has some small colored pegs to fit in these holes. In playing the game, children take turns putting pegs in holes. The object of the game is to get a certain number (3, 4, or 5) of pegs in a straight line. The other players try to prevent anyone from getting the number in a straight row by putting one of their pegs in front of his row. This game is valuable in helping to teach grouping.
3. Playing various race games in which the distance that a player's marker may be moved is determined by chance; e.g., drawing of a numbered card or a card having only dots on it.
4. Rhymes were mentioned earlier.

VIII. Weighing and Measuring

In order to build the proper concepts for this work, the children must work a great deal with cups, pints, quarts, the ruler, and the scales (25-pound vegetable scale is ideal).

1. (a) Which of these is heavier? Weigh them to find out.
   (b) How much does this book weigh?
   (c) For a standard reference for one pound find a book that weighs one pound and make frequent reference to it.
2. Measuring. Will this desk go through the door? How can we find out? (Until children get the idea that a stick or string can be used as a substitute for the width of the desk it is foolish to talk in terms of feet and inches.) After much measuring with sticks, arms, fingers, etc., the standard foot can be introduced.
3. Have children measure frequently with cup, pint, and quart. Use either sand or water.

IX. Reading of symbols denoting U. S. money may be developed through the use of these symbols on bulletin boards which give cost of articles. Extensive use can be made of the toy and game sections of old mail order catalogs.

Summary

For the convenience of the teachers and supervisors of the State of Iowa the following abbreviated outline of content for the first grade is included. However, the teacher must remember that the heart of the program lies in the method and point of view expressed in the previous pages. The addition combinations given below are important but only of secondary importance. Of what value is it to have the child give the correct response to $3 + 4$ if he cannot think in terms of $3 + 4$? (That is, if the mechanism of the response is much the same as that of an adult faced with the memorization of a sequence of nonsense syllables such as:

rxtlad — eebrate = shsht
talste — caldkd = prprt etc.)

The foundational ideas built at this time are of utmost importance. Take time to develop concepts. Take time at this stage to have the pupil do all kinds of counting, drawing of pictures, etc., as suggested on the previous pages. A good foundation of understanding built at this time will pay big dividends when the pupil advances to the upper grades. To repeat—Remember that the important things in an education are the ideas, the understandings, the concepts which you, as a teacher, convey to the child. The ideas and concepts of arithmetic are important. The combinations are necessary, but of secondary importance.

It is important that the child feel at home with number and quantity. He must work with quantity, play with quantity, and above all, think in terms of quantity. The teacher who can succeed in getting a child to think in terms of quantity will have no trouble teaching the basic number facts.

Again, the number facts are important, of course, but too often teachers take it to be the heart of the arithmetic program. This cannot be so; it must not be so. Responses to addition or subtraction combinations which are not understood make arithmetic a "dead load" to the pupil.

Content*

No teaching order is implied in listing the content of the first grade.

1. Counting by one to 100
2. Counting by tens to 100
3. Counting by twos and fives to 50
4. Addition combinations with sums less than 10

*Not considered to be exhaustive but merely suggestive.

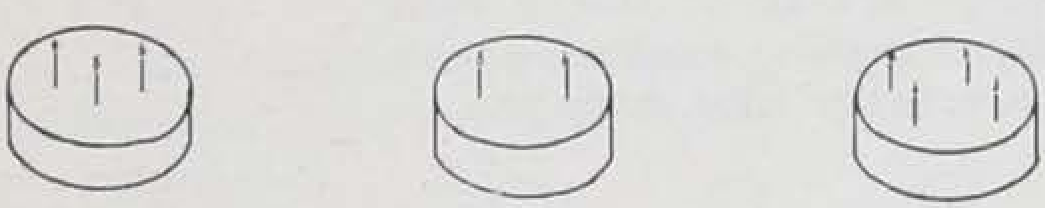
5. The inverses of these addition combinations (The inverse of $3 + 4 = 7$ is $7 - 4 = 3$.)
6. The measures *pint, quart,* and *cup*
7. The foot and the yard and inch
8. The pound
9. The fraction 1/2 of a whole and 1/2 of an even group
10. Recognizing the penny, nickel, dime, and quarter, and which will buy more
11. The ordinal numbers through 10
12. Reading and writing numbers through 100

Throughout this course of study it is suggested that the teachers and supervisors consider whether the above load is too heavy or too light for their particular school or schools. The course of study is flexible enough to allow some topics to be shifted by as much as six months.

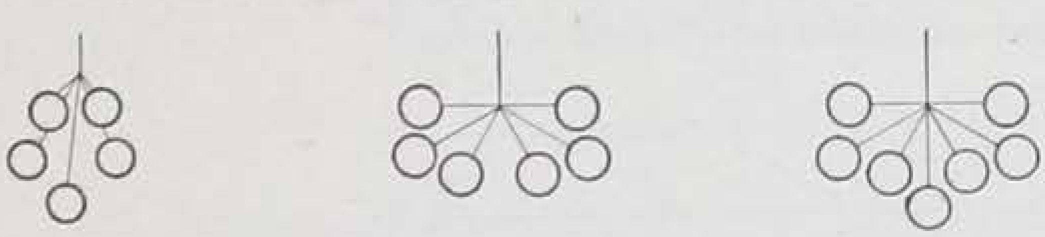
Testing for Some Basic Number Ideas
(Semi-diagnostic)

All instructions are read to the pupils. The exercises given below are merely suggestive. The teacher will be able to think of many other kinds of exercises testing the abilities mentioned in the right-hand column.

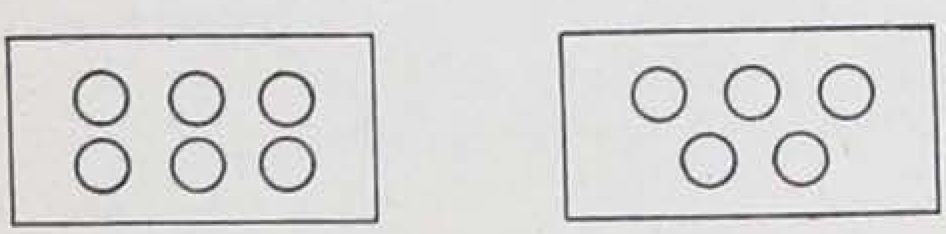
1. Draw a line under the cake which has the most candles.



2. Which bunch has the most cherries? Draw a line under that bunch.

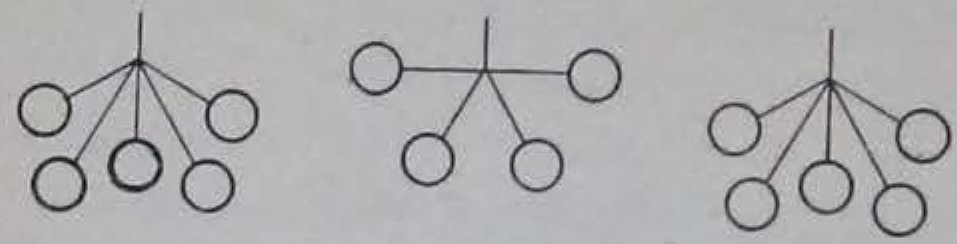


3. Below you will see two tables with cookies. Place one more cookie on the table having the fewer cookies.

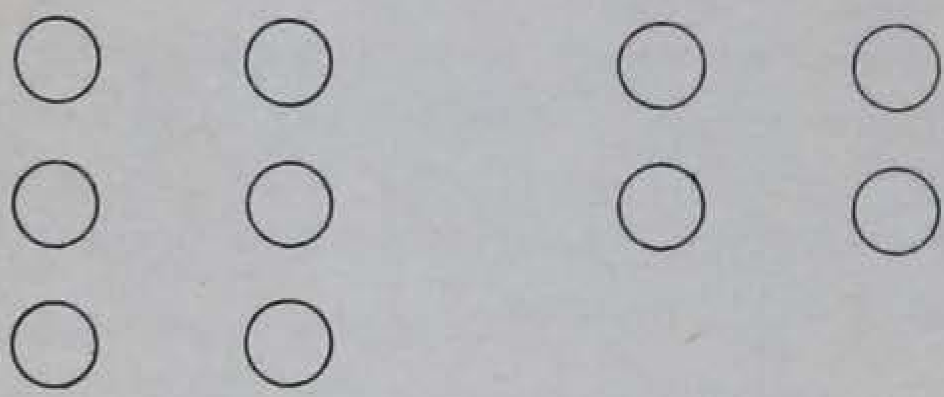


Ability to recognize that one group has more or less than some other group. Also the ability to recognize key quantitative terms such as *more, fewer, less,* etc.

4. Find two bunches having the same number of cherries.

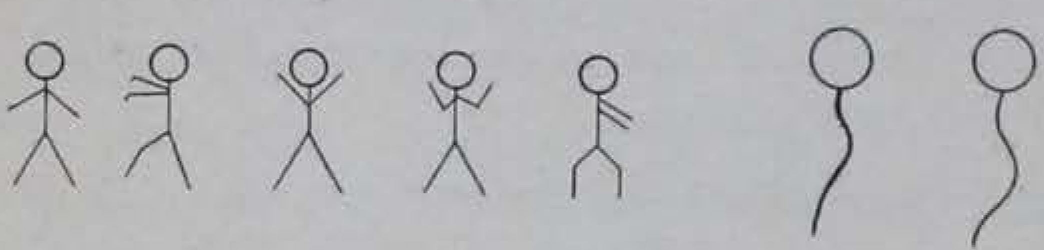


5. Here are two groups of balls. Place balls in one group so that there are the same number of balls in each group.



Terminology and ability to recognize equal groups

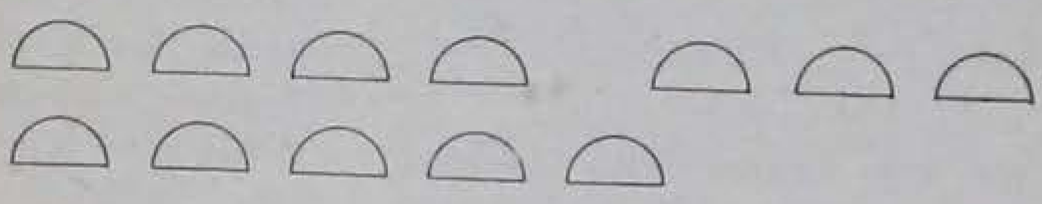
6. Here are some children and balloons. Draw more balloons so that each child will have a balloon.



7. Mary has five balls. Are all of Mary's balls in the picture? If not draw in some more balls in order to show how many balls Mary has.

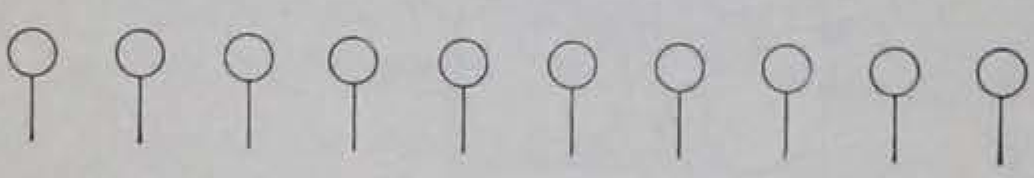
Recognition of number names

8. Johnny has three chocolate drops. Draw a ring around the number of chocolate drops Johnny has.

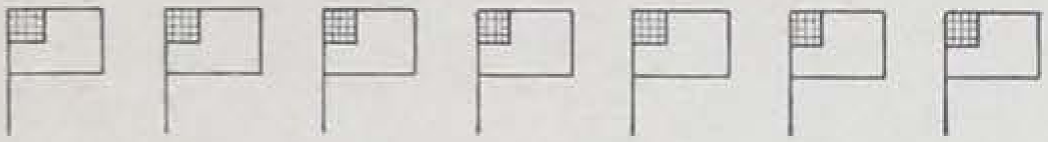


9. Place a ring around seven of the balloons shown in the picture.

Ability to do rational counting



10. Draw a line under five of the flags you see in this picture.



11. Mary had a small bunch of 9 grapes. She ate some of them and then had this bunch left. Place some more grapes in the bunch to show how many Mary had before she ate the grapes.

The following may be given orally or in written form depending upon the ability of the class:

12. Which number is one more than 6?
13. What number is one greater than 4?
14. What number is one less than 7?
15. Mary had six cookies and John had one more than Mary. How many cookies did John have?
16. Which of these numbers is the smaller? 4; 6
17. Which of these numbers is the larger? 7; 8

Recognition of *more, less, greater,* as well as comparative values of numbers

Problems of type 16 and 17 may be increased to three and four number choices if the group is sufficiently mature to make the choices.

SECOND GRADE

Inventory Test

This inventory test should be administered the second or third week of school. Results of the test should show the child's achievement of the skills practiced in grade one. (Directions should be given orally. Children write answers.)

The first week may be spent in doing activities similar to those used in the first grade, in doing some project which requires measurement, or in going over the review part of your text.

1. Count by 1's. The marks have been counted by ones. Will you recount them and put the number you count above each mark that is not already numbered?

1 2 __ 4 __ __ 7 8 __ 10 11 __ 13 14

15 16 __ __ 19 __ __

As an addition to this form of counting test have children count objects; e.g., grains of corn in a pile.

2. Counting by 5's. Each of these marks stands for five. Will you recount them, placing the correct number above each mark?

5 __ __ 20 __ 30 __ 40 __ __

10 15 __ __ 30 __ __ 45 50 __

3. Use similar exercises for counting by 10's, 2's, and 3's.

4. Make enough rings (0) to tell these numbers:

10	
12	

Draw a circle around the number which tells how many rings:

0 0 0 0 0 0 0 4 6 5 7	0 0 0 0 6 4 3 2	0 0 0 0 0 0 0 0 0 7 9 8 6

5. Draw a circle around *Yes* or *No*:

Does 6 come before 9? Yes No

When you count 2's, does 16 come before 14? Yes No

Can you write:

How many fingers you have? ________

How many on one hand? ________

What time you go to bed? ________

What time school begins? ________

6. Add or subtract. Write the answers. Teacher reads, using "and" for plus and "take away" for minus.

a. $\begin{array}{r}4\\+3\\\hline\end{array}$ $\begin{array}{r}2\\+4\\\hline\end{array}$ $\begin{array}{r}2\\+2\\\hline\end{array}$ $\begin{array}{r}5\\+2\\\hline\end{array}$ $\begin{array}{r}2\\+6\\\hline\end{array}$ $\begin{array}{r}6\\+4\\\hline\end{array}$

b. $\begin{array}{r}7\\-4\\\hline\end{array}$ $\begin{array}{r}7\\-5\\\hline\end{array}$ $\begin{array}{r}7\\-6\\\hline\end{array}$ $\begin{array}{r}8\\-2\\\hline\end{array}$ $\begin{array}{r}8\\-3\\\hline\end{array}$ $\begin{array}{r}9\\-2\\\hline\end{array}$

7. Which number is more? Draw a ring:

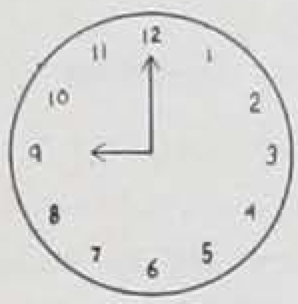
    30 or 20
    15 or 18
    70 or 80
    60 or 90
    18 or 81
    46 or 58

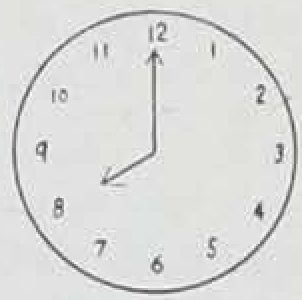
Which number is less? Draw a ring:

    60 or 40
    15 or 19
    50 or 70
    45 or 37

8. Draw a line under the correct time:

12:00
9:00
11:00





10:00
12:00
8:00

9. Draw a line under the correct answer:
   Which is worth more—a dime, a penny?
   Which is larger—one, one hundred?
   Which is smaller—a pair of buttons, a dozen buttons?

10. The boys were planning a marble tournament. Tom brought 3 boys. Bobby brought 1 boy. Did Tom bring more boys than Bobby? How many more?

11. John went to the store. He saw an apple that cost 5 cents. John had 3 cents. How many more cents did he need to buy the apple?

Content and Recommended Procedures

I. Exercises to give the child experiences in breaking a group (number) into two or more smaller numbers, and in making large numbers out of small numbers. The experiences listed are for the purpose of illustration. To the resourceful teacher innumerable opportunities not mentioned here present themselves for worth-while number experiences.

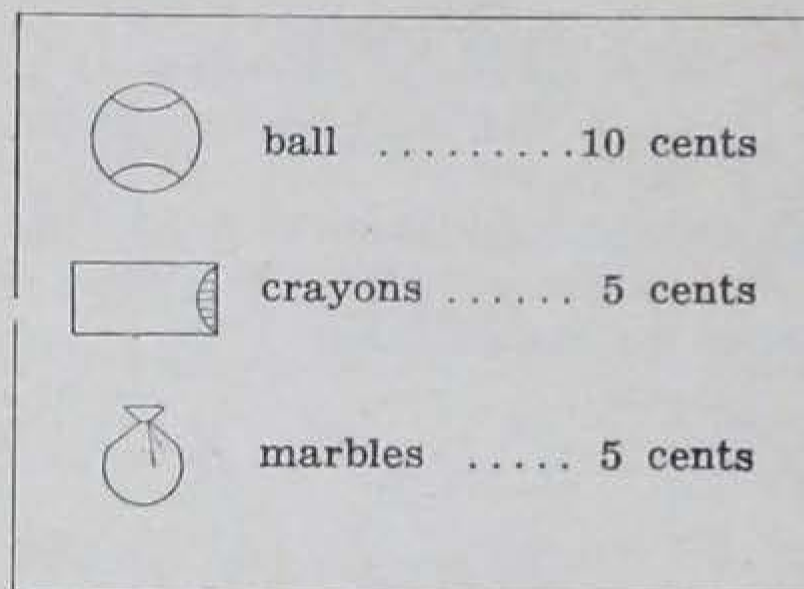
   1. A boy had 8 marbles. He made 3 rings on the ground. He put 3 in one ring, 2 in another, and 3 in another. Draw the rings showing the number of marbles in each. Show with drawings a number of other ways that he might have put the marbles.

   2. Jim cut out 2 kites and Jack cut out 4. Show how many kites each cut out and how many there were in all.

3. John earned 30 cents running errands. He spent 5 cents for ice cream, 10 cents for marbles, and 15 cents for a game. Did John spend all his money?

4. To play a game 10 children are needed. Four were needed on one side. How many were left for the other side?

5. Make posters: Things I can buy for 20 cents:



6. List articles for sale, the entire lot to sell for 25 cents (in connection with grocery store unit).

7. Make number stories in which two or three numbers are combined to make larger numbers:

   a. I picked 6 violets; Margaret picked 10. We put them in a vase. How many were there in the vase?

   b. The second grade made a bird chart. We had 10 robins on one side and 12 bluejays on the other side. How many birds were on the chart?
   See also *Games,* page 33.

8. Have children show frequently with marks or objects what such symbols as 6, 8, and 9 really stand for.

9. Ask for meaning of 11, 12, 22, etc. (1 ten and 1 one, 1 ten and 2 ones, 2 tens and 2 ones).

10. What is another way of saying thirty? forty? (3 tens, 4 tens)

11. What does the 6 in sixty tell you? What does the 0 in sixty tell you?

12. Show a number on the number chart that is 10 more than 3 tens.

13. Count by 10's to 100.

14. Count by 100's to 1000.

15. Seat work consisting of this type:
    In 26 the 2 means __________ and the 6 means __________.
    In 120 the 1 means __________ and the 2 means __________
    and the 0 means __________.

16. Experiences in buying in connection with Toy Store Unit:

   a. John wants to buy a top that costs 25 cents. In how many ways could he pay for it?

      1 nickel and __________ dimes

      __________ pennies, *1* nickel, and __________ dime

      2 dimes and __________ pennies

      20 pennies, __________ nickel, and __________ dime

17. Activity: Cutting three bricks of ice cream so as to serve 12 children. Into how many parts shall one brick be cut? Then if each of the three bricks is cut into four pieces how many servings will there be?

18. Arranging cans on the shelves of a grocery store (Grocery Store Unit). Taking a group of 10 or 12 cans and arranging them in equal groups on two of the shelves.

19. Selecting pictures from a box so that each child in a row may have four or six, etc. Finding out how many two children have together, how many three have together, etc.

20. Painting a certain number of Easter eggs to place in baskets. Finding out how many eggs are needed for two baskets, four baskets, etc.

II. Comparison

1. Compare rows of desks, number of children in grades, size of books on reading table, etc.

   a. Which rows have two more desks than the middle row of desks?

   b. Which grade has three less pupils in it than the fourth grade?

2. Playing number games

   Each child thinks of a number story for the others to guess. The children take turns in going to the front of the room and giving their stories. Each child tries to see how many he can guess.

   Examples:

   a. I am thinking of two numbers that make 8. Of what numbers am I thinking?

   b. I am thinking of a milk bottle twice as large as a pint bottle. How many quarts does it hold?

   c. I am thinking of a number six smaller than 14. Of what number am I thinking?

3. Draw a picture of some objects on the board in a group. Have children draw on their paper a group with two more in it; 3 less in it; 5 less in it; etc.

4. Group objects such as sticks, blocks, pencils into groups of 10, 3, 6, 8, 4. Have children take a pointer and point to largest group and then to the one that has 4 less in it; 7 less in it; 6 less in it; 2 less in it. Vary the order frequently.

5. Have children put out books on the reading table. If there are 10 children in school, ask them to put 3 more than the number in school; 2 less than the number, etc.

6. Ask children to color objects. Directions such as the following may be given: Color two less than the whole number of kites. Color four more marbles for Jack than for Billy.

7. To help visualize the meaning of more and less than a certain number, drawing may be used to advantage.

   John has three boats. Richard has five boats.

   Draw enough boats for John so that he will have 5 more boats than Richard.

8. Reading total number of days for each month on a calendar.

   Telling how many less days February has than March. How many more days September has than February, etc.

9. Listing articles that can be bought for 1¢, 2¢, 3¢, 5¢, 10¢, 25¢. Telling how many more cents or less cents one article costs than the other.

10. Problem solving

    a. John has 6 marbles. Billy has 4 marbles. Sam has 10 marbles. Which boy has 2 more marbles than Billy? Which boy has 4 less marbles than Sam? Show by drawing.

    b. Mary picked 12 violets. Rose picked 5 violets. They met Alice, who had picked 7 violets. Rose had 7 less than which girl?

III. Reading and Writing Numbers

1. Fill in the missing numbers:

   156, 157, ________, ________, ________, ________, 162
   435, 436, ________, ________, ________, ________, 441

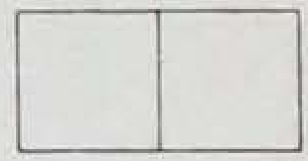
2. Fill in the missing numbers (e.g., 156 as 1 hundred, 5 tens, and 6 ones).

   578 means ________ hundreds ________ tens ________ ones.
   709 means ________ hundreds ________ tens ________ ones.
   960 means ________ hundreds ________ tens ________ ones.

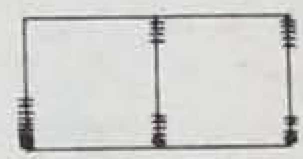
3. Put the right numbers on the lines:

   3 hundreds, 6 tens, and 2 ones are __________.
   Eight hundred fifty-four __________.
   9 hundreds, 0 tens, and 5 ones are __________.
   6 hundred eight __________.

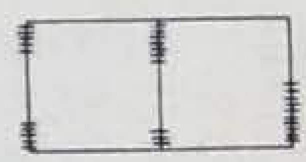
4. Show numbers like 16, 45, 118, and 306 on the abacus. An abacus may be made from a wire coat hanger and some wooden beads from a ten cent store. Bend the wire to the shape indicated in figure at the right and put ten beads on each line, five of one color and five of another. The wire at the right stands for the *ones* column, the middle wire for the *tens* column and the wire to the left for the *hundreds*.



Thus 45 would look like this:



and 570 like this:



5. Tell how much money is in each group:

a.

$.25, $.25, $.50	

Dollar sign, decimal point, cent sign

$.50, $1.00, $20.00	

How to write dollars, cents; dollars and cents

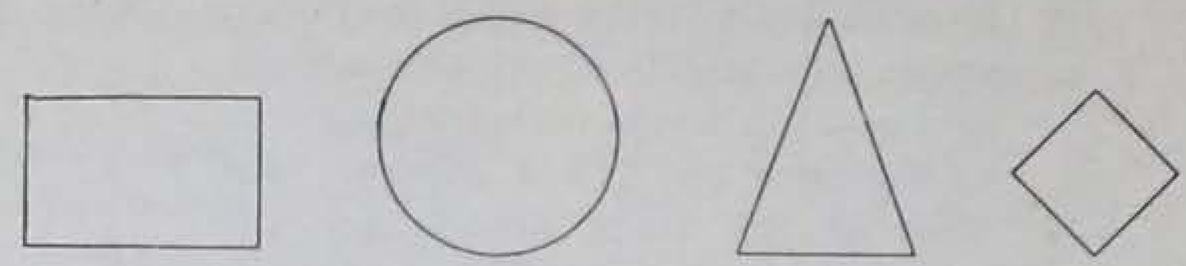
6. Show how Mother's shopping list might look:

Shoes ....$ 4.00
Suit for Baby.......... 2.50
Cap for Brother....... .50

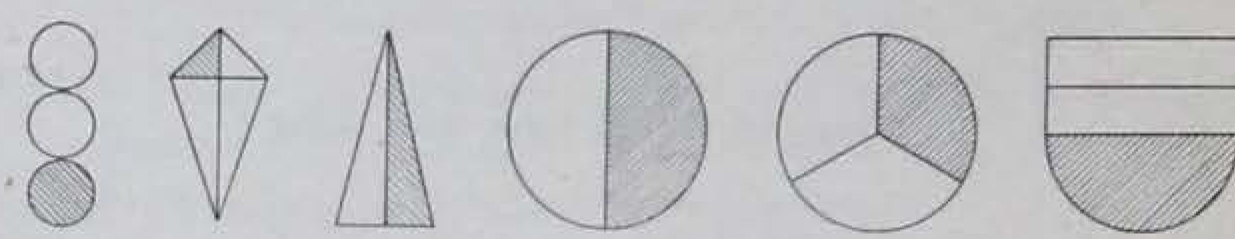
7. Cut out automobiles with prices (this in connection with a unit on Transportation). Write the prices of automobiles, arranging in order from highest to lowest price; e.g.,

Cadillac ....$ 1,195
Chrysler 995
Lincoln-Zephyr 950
Chevrolet 665
Plymouth 645

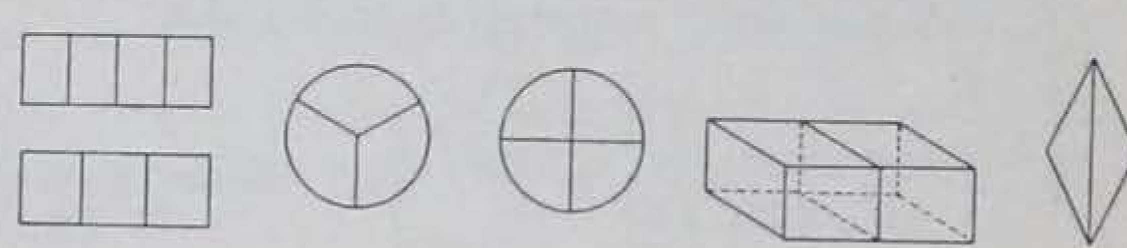
8. a. Draw a line to show one half (1/2) of these figures. (Emphasize that there must be two *equal* parts.)



b. Put a + on pictures that show thirds.

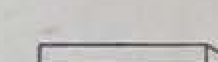


c. Write the number one half on every drawing which shows halves, one third on every drawing which shows thirds, one fourth on every drawing which shows fourths.



d. Color one half the apple red.

Color one third of the pencil blue.



Color one fourth of the kite orange

e. Oral directions may be given as follows:

Place one half the books on the table. Take one fourth of the pencils from the box.
Place one third of the chairs at the table.

IV. Any problems or examples involving the desired facts may be used. Questions should be direct; e.g., How much are 6 pencils and 7 pencils? Or, How much are

32 and 21? In adding tens call attention to similarity between tens and ones.

V. Problems Involving Basic Addition and Subtraction Facts (In order not to handicap the poor readers a good many of these should be given orally.)

1. Mary has 10 flowers. Jane has 3. How many have they together?
2. John has 16 marbles. He gave Fred 10. How many did John have then?
3. Billy has 6 little white chicks. He has 7 little black chicks. How many chicks has Billy?
4. Father has 9 big fish. John has 3 little fish. Father has how many more fish than John?

VI. Column Addition. This phase of arithmetic is introduced incidentally in connection with the exercises which show that a number is made up of two or more smaller numbers. (See exercise in section 2, putting the proper number of marbles in rings.) The introductory exercises should, of course, be easy. The first four of the following list are examples:

1	1	1	1	2	2	2	2
1	1	1	2	8	6	6	6
5	7	2	3	9	7	6	9

In two-column addition much reliance can be placed on adding by endings. The program should, therefore, bring out through frequent use the association between 4 and 3 and other decade combinations involving 4 and 3; e.g., 14 and 3, 24 and 3, etc., up to 94 and 3.

VII. Notation. Rote counting by tens helps pupils in understanding the number system to 100.

VIII. Systematic Instruction and Drill on Basic Addition and Subtraction Facts

1. After children have had much experience with exercises similar to those suggested in sections above, give a test involving the basic addition and subtraction facts. (See Morton: "Teaching Arithmetic in the Elementary School.")
2. In order to be sure that children understand the processes and facts, have children demonstrate the truth of their answers by using marks or objects.

3. For those facts missed by the children and for which they know a longer or indirect solution assign direct study. Any or all of the following may be used:
   a. Write fact in accepted form; e.g., $\begin{array}{r} 3 \\ +\ 4 \\ \hline \end{array}$ and then draw dots or marks to represent each number and the total. Look at the fact. Say it to yourself giving the total; e.g., 3 and 4 are 7. Close your eyes, try to see the fact, and then say it to yourself. Look at the paper again to see if you are right. Repeat this process until you know the fact.
   b. Make cards with the facts on one side and answers on the other. Study these by going through your list trying to give the answer to each fact. When you miss one, turn the card over and look at the answer. Then repeat the whole statement (3 and 4 are 7) several times.
   c. Write the combinations and the answers. Start by adding every other number to the number you choose. Do the same for subtraction except that you start with 18 and subtract only the one-figure numbers.
4. In learning the harder addition facts (sums above 10) take advantage of the making of tens. In others make 10's and 1's. $8 + 7$ becomes $8 + 2 + 5$. Regrouping objects or dots into 10's and 1's will aid in establishing this procedure. Advantage may also be taken of the fact that the doubles are easy. Other facts can then be related to these; e.g., $8 + 7$ then becomes one more than $7 + 7$ or one less than $8 + 8$. However, the teacher should remember that the goal is automatic responses.
5. Very brief exercises in which the class works with flash cards may also be used for purposes of motivating learning of the basic facts. Timed written tests serve the same purpose but should be used with care.

IX. Standard References

(1) Six feet or the height of an average man
(2) Thirty feet or the length of the schoolroom
(3) Ten pounds

The distance equal to the length of the schoolroom is suggested because it can be referred to frequently. Each child should measure this distance and many references should be made to the distance; e.g., How does the length of the truck compare with the length of the room? Is that distance longer or shorter than our room?

A similar procedure should be followed with the other standard references.

X. Additional work in weights and measures. All work in weights and measures should be accompanied by activities as suggested below in order to develop sense of length and weight. Too many children (and adults) can do a certain amount of the mechanics required in these grades but have little or no concept of length. Hence the emphasis on standard references as in IX. Children should learn to estimate lengths and weights as well as the mechanical manipulations.

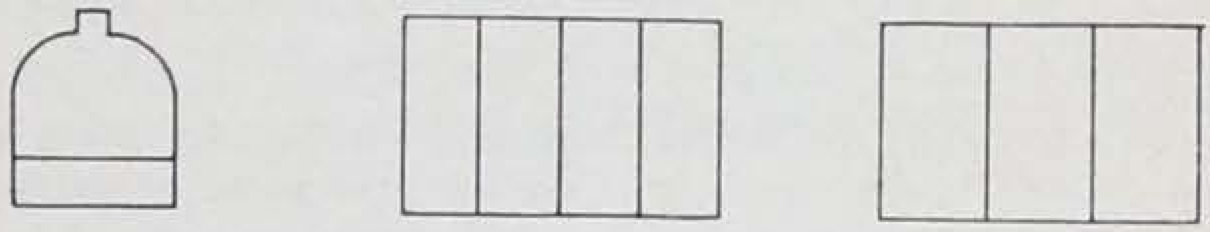
1. Children measure each other's heights and record (feet, inches).
2. They measure how high they can reach (feet, inches).
3. Measuring is used in constructing objects in connection with Unit work; for example, wheelbarrow in Transportation (feet, inches).
4. Weighing articles sold in the Grocery Store Unit (pounds, ounces).
5. Weighing each other, pupils become familiar with pounds and ounces.
6. Working examples such as the following:
   a. Dick can carry 40 pounds. A big box of tools weighs 60 pounds. Can Dick carry the box?
   b. Alice has 20 pounds of sand. Jerry's wagon will hold 30 pounds. If Alice puts her sand in Jerry's wagon, how many more pounds will be needed to fill it?

XI. Further Use of Fractions in Measuring and Weighing

1. Which is more:

   1/2 pound or 1/4 pound of butter? 1/4 of a dozen eggs or 1/2 of a dozen eggs?

2. a. Draw a ring to show the fraction each part is:



one-third one-fourth one-half | one-third one-fourth one-third
one-half | one-fourth | one-half

   b. Making a syrup for popcorn balls:
   1 1/4 cups of syrup, 1/2 cup of sugar, 1 1/2 tablespoons of butter, 1/4 teaspoon of vinegar

   c. Weighing candy and cookies to sell by the 1/2 pound, 1/4 pound, etc.

XII. Games Involving Number

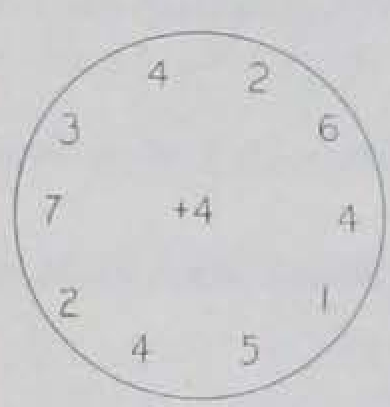
1. Children have cards numbered from 1 to 9. One child comes to the front of the room with his card and calls for the card of some child. This card is placed beside his and builds a new number. The first one of the two on the floor to read the new number correctly gets to be "it" and calls for a card. Example:

   : 1 :   : 9 :   : 1 : 9 :

2. Children stand in two lines and are given numbers from 1 to 9, odd numbers on one side and even numbers on the other. The leader of the odd side calls 48, or any other number. The child holding 4 and the child holding 8 stand out before their line holding their cards close together. The leader calls numbers until a mistake is made; for example, if 36 is called and 6 steps out first followed by 3, the cards held together now read 63 instead of 36. Larger numbers, such as 462, might be called. When the even number side makes a mistake, the leader on the even side calls for cards on the odd side, as 71, etc.

	1	(2)	leader
	3	4	
	5	6	
	7	8	
leader	(9)		

3. Race Track Game: Numbers from 1 to 8 are written around a circle in no definite order. The number of the combination to be drilled upon is placed in the center. The pupil begins at the top of the circle and repeats the combinations as rapidly as possible until he has completed the circle. Time is recorded. If a combination is given incorrectly the pony has stumbled and the pupil must study the combination before he can continue the race.



4. Whirl the Wheel: Numbers from 1 to 12 are placed around a circular piece of cardboard. A second circular cardboard with a V-shaped notch is placed over the first cardboard. The second one is whirled. Wherever it stops the child must give the combination.

5. Children stand in line holding numbers from 1 to 9. The child who is "it" stands in front of the line holding a number of cards. He selects one, for example 4. He then calls on

different children in the line to answer the sum of his card and theirs. For example, when he holds up 4 and calls on the child holding 9, that child must answer 13. If he holds up 6 and calls on the child holding 9, that child must answer 15, etc. The one who is "it" should call on children here and there in the line and change his own card frequently. If a child fails to answer correctly, the one who is "it" must answer. If he fails, the one in the line who recognizes the error first may become "it."

6. Who Is First: Pupils are arranged in a circle. Combination cards are flashed. Each pupil in turn answers. The first one that misses steps in the center of the circle. He may take his place any time in the circle if he answers for a combination that someone has missed.

Summary of Content for Second Grade*

No teaching order is implied in listing the content of the second grade.

1. Basic ideas of addition and subtraction (See I, page 31)
2. Counting by 10's to 100
3. Counting by 100's to 1000
4. Counting to 1000
5. Positional number notation (See I, 15, page 32 and III, 2, page 34)
6. Basic ideas of comparison
7. Dollar and cent signs; monetary notation
8. 100 addition and subtraction facts (taught simultaneously)
9. Fraction, 1/4, 1/3 as part of a whole and part of a group
10. The ordinals

THIRD GRADE

Inventory Test

A. Counting

1. By ones to see how far the child can go
2. Counting objects in room (not beyond 201)
3. Counting small objects such as toothpicks or paste sticks, and putting them in groups of ten (to facilitate checking and to make for accuracy)
4. Sampling as counting from 150 to 200 and from 300 to 350 (Begin at 150 instead of at 1.)
5. Counting by 2's to 50, by 5's to 100, by 10's to 100
6. Pointing out third girl in row, 5th book on shelf, etc.

*The summary for the first grade given on page 25 clearly expresses the use which should be made of the items listed in the summary on this page.

7. Exercises in finding pages in book. Questions such as "If you open your book to page 90, which way will you turn to find page 75?" "Is page 29 near the front, middle, or back of your book?" (Since this part of the test must be administered to individual pupils, the other pupils of the school should be given an assignment such as working at a construction project or doing an example test like the one required in I-D-4 below.)

B. Write with words: $1.23.

Write with numbers: five dollars and sixteen cents.

Compare value of various coins; e.g., 50 cents is equal to how many nickels?

C. Fractions

Coloring one half of a square or circle

Similar exercise with other fractions learned during the second year

D. Addition and subtraction facts

Give a test of all facts either by writing them on the board or by presenting them on a sheet of paper. Put easy facts first.

Content and Recommended Procedures

I. Systematic Instruction in Addition and Subtraction

A. Meaning of addition and subtraction. These terms do not mean "more" or "less" but a rearrangement of groups into a different pattern that is more easily understood; that is, $6 + 9$ is regrouped into one 10 and 5 (15).

B. Re-introduce each fact by means of a problem and have children demonstrate understanding of the fact as illustrated in the problem by drawing diagrams and by using objects; e.g., If John has 9 marbles and buys 6 more, how many will he have then?

```
              000                       000
John has 000 marbles.  He bought 000.  He then has 15
              000
  marbles.
```

C. See Second Grade outline for suggestions on direct teaching.

D. Drills and devices useful in teaching addition facts

1. Giving answers to facts on flash cards (See Second Grade outline.)
2. Using circle game (See Morton, *Teaching Arithmetic in the Elementary School,* Vol. I.)
3. Games such as bean bag board or target games in which children add their scores

4. Give the facts in different forms on hectographed sheets.

$8 + 6 =$ $\quad \begin{array}{r} 8 \\ +6 \\ \hline \end{array} \quad \begin{array}{r} n \\ +6 \\ \hline 14 \end{array} \quad \begin{array}{r} 8 \\ +n \\ \hline 14 \end{array}$

$8 + n = 14$

In the above examples *n* (for number) is used instead of the conventional question mark.

The examples in which *n* is used instead of the conventional dash or question mark should grow out of the earlier practice of using number symbols to express a quantitative situation; e.g., ... (four dots) and ... (three dots) are .... (seven dots) is changed to read $4 + 3 = 7$. Similarly the problem: "John saw 8 ducks in one flock and 6 in another. How many ducks did he see altogether?" may be stated in this manner: "He saw a certain number. That number is the number that we get when we put together the 8 and the 7, or 8 and $7 = n$." Third grade pupils are familiar with the common practice of writing initials for the names of individuals. The use of *n*, as above, is easily introduced by relating it to initials for names. After discussing this idea with the class, the statement "8 and what number make 14?" is abbreviated to "$8 + n = 14$." In like manner the following will be introduced at the proper time: $n + 6 = 9$; $n - 8 = 4$; $8 - n = 2$.

5. Timed tests over 81 facts. Hectographed sheets are useful here since child need write only answers. Allow three minutes. Discuss with child who makes a poor showing the economy of learning the fact instead of depending upon counting.

E. Further work in adding by endings as preparation for column addition and multiplication with carrying (First work to be oral)

1. Without bridging; e.g., $14 + 4$. The 14 should be thought of as one ten and four ones. Therefore, $14 + 4$ becomes 1 ten and 8 ones since 4 ones and 4 ones make 8 ones. Children rework the above example with bundles of sticks, comparing $\begin{array}{r} 4 \\ +4 \\ \hline \end{array}$ and $\begin{array}{r} 14 \\ +4 \\ \hline \end{array}$

   Continue with $\begin{array}{r} 24 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 34 \\ +4 \\ \hline \end{array}$ etc.

2. With bridging; e.g., $5 + 6$, $15 + 6$, $25 + 6$, $35 + 6$, etc. Have children note similarity of units figures in the above examples.
   Ask "How does the tens figure change in each example?" Illustrate with sticks tied in bundles of ten and loose sticks.

F. Column addition

1. Illustrative example:

3		111	
4	Children show	1111	Regrouping into 1 ten and
7		1111111	4 ones 1111111111 1111
14			

(See also Second Grade outline and E above.)

2. Suggestions

a. Show economy of thinking, "Three and four are seven and seven are fourteen."

b. Add downward and place answer below line. Check by adding upward and placing answer above problem.

```
14
 3
 4
 7
14
```

G. Addition with carrying

1. Underlying ideas. In our number system we refer to ones only in groups up to 10. For economy of thought and for easy grasp, amounts larger than 9 are thought of as 10's or 10's and 1's. Therefore when numbers are combined in which the units parts total more than nine the units are changed to 10's and these are added or carried to the 10's part of the numbers being combined. Thus 25 + 37 is really 2 tens and 5 ones plus 3 tens and 7 ones. 5 ones + 7 ones = 12 ones. Since it is easier to think of 12 ones as 1 ten and 2 ones the 2 ones are placed in the ones column and the one ten is added or carried to the tens. Have children demonstrate with bundles of sticks, two groups of ten and five single sticks plus three groups of tens and seven single sticks.

Teach carrying from tens to hundreds like carrying from units to tens; i.e., 12 tens is one hundred and 2 tens. Teacher demonstrates why we use 1 ten and 2 ones rather than 12 ones by putting the two before the children and asking, "In which is it easier to see the amount that is represented?"

H. In order to make the processes of addition and subtraction meaningful, it is suggested that the teacher encourage the pupil to draw diagrams or play with bundles of sticks until he can explain to the teacher the meaning of carrying in addition or borrowing in subtraction. Of course, the teacher will have to help some pupils with this basic idea more than others. In any case, any or all of the following might easily be developed by third graders who have grasped the meaning of borrowing and carrying and who have worked out some device in order that they can more easily explain it to the teacher.

e.g., a. 34
+62

Other ways of solving 34 +62 which make the first steps in addition more meaningful:

(1) 3 tens + 4 ones
6 tens + 2 ones
9 tens + 6 ones = 96

(2) 11111 11111   11111 11111   11111 11111   1111
11111 11111   11111 11111   11111 11111
11111 11111   11111 11111   11111 11111   11

9 tens   6 ones

e.g., b. 26
+38
64

Other ways of solving to develop meaning:

(1)
1: 11111 11111   2: 11111 11111   6: 111111
3: 11111 11111   4: 11111 11111   5: 11111 11111   1111   1111

6 tens   and 4 ones

Note that the process of addition involves grouping by tens.

(2) Children show on abacus how these numbers are added and subtracted. (See second grade for explanation of abacus.)

(3) Children show with bundles of sticks.

(4) 26
38
14
5
64

(5) 2 tens 6 ones
3 tens 8 ones
5 tens 14 ones
6 tens 4 ones = 64

e.g., c. 36
—14
22

Other ways of solving:

(1) 11111 11111   11111 11111   11111 11111   11   11 11
11111 11111   11 11

2 tens   2 ones left

The boxed marks represent the subtraction, or those marks which could be erased to leave a remainder of 22.

(2) 3 tens 6 ones
1 ten 4 ones
2 tens 2 ones = 22

(3) Repeat b (3) and b (4).

e.g., d. 32
—17
15

Other ways of solving:

(1) 11111 11111 11111
1 ten and 5 ones left

11111 11111
11111 11111

11111 11
1111 111

(2) Repeat c (2) and c (3).

I. Use problems and examples involving two and three-digit numbers.

J. Subtraction with two- and three-digit numbers. The decomposition (take away) method is recommended because it is easier to rationalize. (The additive or other methods of subtraction may be used. The county superintendent should make this decision.)

1. Underlying ideas based on example 38-19: 38 = 3 tens and 8 ones; 19 = one ten and nine ones. We cannot take 9 ones from 8 ones so we borrow (change) one ten from three tens into 10 ones. Ten ones plus 8 ones equals 18. Subtract 9 ones. Subtract 1 ten from 2 tens. Have children demonstrate with sticks—three bundles of ten and eight loose sticks.

2. To check subtraction draw two lines under answer (to distinguish it easily from checking answer) and add.

38
—19
19
38

II. Reading and Writing Numbers to 10,000, Telling Time, Fractions, Roman Numbers, the Calendar

A. Notation

1. What is the largest number that can be written with three figures? (999) "What does each digit in this number represent? Add one to this number. What does the number now equal or become?" (Ten hundreds or one thousand)

Write the largest number that can be written with these numbers: 3, 2, 6. Write the smallest number that can be written with these numbers.

2. Practice in reading and writing numbers

   a. Emphasize   10 ones = 1 ten
10 tens = 1 hundred
10 hundreds = 1 thousand

   b. Dollars and cents

   c. Years and house numbers. 1492 is sometimes read 14 hundred 92. In reading years and house numbers *hundred* is left out.

   d. Telephone numbers. 4510 is read 4-5-1-0 (0 as in *no*).

   e. Roman numerals to 30

      (1) Fundamental learnings

         (a) I, V, X and their meanings

         (b) Repeating a letter doubles its value (V is never repeated.)

         (c) If a letter of smaller value comes first, subtract: IV = 4.

         (d) If a letter of lesser value follows one of greater value, add: VI = 6.

      (2) Uses of Roman numerals—clocks, monuments, buildings, book chapters, book prefaces, etc.

B. Telling time

1. Fundamental learnings

   a. The long hand tells the minutes; the short hand the hours.

   b. It takes the long hand five minutes to go from one number to the next.

   c. It takes the short hand one hour to go from one number to the next.

2. Time is read in different ways; e.g., 5:15 may be a quarter after five, five fifteen, 15 after five, or 45 minutes to 6.

3. Activities

   a. Draw clock faces showing time you get up, dinner time, bedtime, etc.

   b. Find out and report about ways of telling time in olden days as the hourglass, the water clock, the time candle, etc.

   c. Make and use a clock dial with movable hands.

C. The calendar

1. Learnings

   a. Months and seasons

   b. Number of days in months, days in week, etc.

   c. Finding special days on calendar

D. Fractions

1. Stress that the number or object divided is divided into *equal* parts.

2. Exercises in identifying fractional parts as half an apple, one third of a candy bar

3. Exercises in comparing fractions as "Would you rather have 1/2 or 1/3 of a candy bar? 1/8 or 1/4 of an orange?"

4. Exercises in finding parts (1/2, 1/3, 1/4, 1/5) of numbers

III. Multiplication and Division Facts Through 5's (The suggestion that the facts through 5's be taught is only a suggestion. The county superintendent may elect to teach fewer facts at this level.)

A. Use introduction of text. To aid in the development of meaning of multiplication and division have children draw diagrams to illustrate problems given in the text. In these initial experiences the two ideas of multiplication should be emphasized.

1. A short form of addition when addends are equal, as

$$\begin{array}{r} 5 \\ 5 \\ 5 \\ 5 \\ \hline 20 \end{array}$$

2. A regrouping of a number of equal groups into one group consisting of tens or tens and ones (when product is ten or more)

11	11	11	11	
111	111	111	111	4 × 5 = 20 (2 tens)
5	5	5	5	

B. Teach corresponding division fact with each multiplication fact. (If your text does not follow this procedure you will have to use two different sections of the book simultaneously.)

1. Have children demonstrate with concrete objects or with marks the problems they solve; e.g., "If the bag contains 18 marbles and each boy is given 3 marbles, how many boys will get marbles?" Have a child take 3 marbles out of the bag and give to boys until supply is exhausted. Writing 18 marks and then circling 3's and finally counting circles is a semi-concrete method of showing the same thing. Serial subtraction (18 — 3 = 15; 15 — 3 = 12; 12 — 3 = 9; 9 — 3 = 6; 6 — 3 = 3; 3 — 3 = 0.) This should be used also to show how this type problem can be solved.

2. Have children continually ask the question, "What am I trying to find?" In answering this question the two ideas of division partition (finding the size of a part when a collection is divided) and measurement (finding how many parts of a given size are in a collection) will be contrasted.

3. When ready to teach the symbols for division be sure to teach the symbols ÷ and $\overline{)\quad}$. Do not use the short division symbol. The symbol for short division may be introduced in the fifth or sixth grade (fourth if it seems necessary) and short division taught as a shortcut to long division.

C. Many of the devices suggested for addition drill can be used for drill in multiplication and division. Among the various forms taught be sure to teach this type: $4 \times n = 24$, $7 \times 3 = n$.

Exercises of the type $4 \times n = 24$ serve to emphasize the relationship between multiplication and division. That is, if 4 times some number is 24, then the number must be 24 divided by 4 or 6. It is important to emphasize this relationship between the two processes of multiplication and division. (Of course the relationship between addition and subtraction should not be overlooked.)

Other exercises which can be used to show the pupil that multiplication and division are related are:

> If 9 is divided by 3, and that answer is multiplied by 3, what is the final number?
>
> If 6 is multiplied by 5, and that answer is divided by 5, what is the answer?

After several exercises of this nature the pupil should see that he invariably arrives at the first number; that is, the number on which he has been operating. Do not overlook these relationships.

The following problem shows one way of introducing this phase of arithmetic: "If one box contains 4 blocks and there are 6 boxes, how many blocks are there altogether?" The answer to the essential question of how many blocks is obviously a number, or in the thinking of the child, there are as many blocks as six 4's. In the symbol of arithmetic the problem then becomes $6 \times 4 = n$. Similar problems to illustrate $6 \times n = 24$, $n \times 4 = 24$ should be used. The fact that these short statements are arithmetical ways of recording how the pupils thought should be continually emphasized.

This kind of thinking is the foundation for percentage and fractions and should not be overlooked in the third, fourth and fifth grades.

IV. Measurement

A. Practice in measuring in inches, feet, and yards

1. Length and width of tablet
2. Height of desk
3. Width of blackboard

B. Practice in estimating as, "Which of these lines is nearer an inch?" "How tall do you think Mary is?"

C. Use of principles learned in art work and in building and construction in connection with social studies

V. Standard References

Use comparisons familiar to children; i.e., mile—distance from schoolhouse corner to next crossroads, acre—about size of school grounds or certain field.

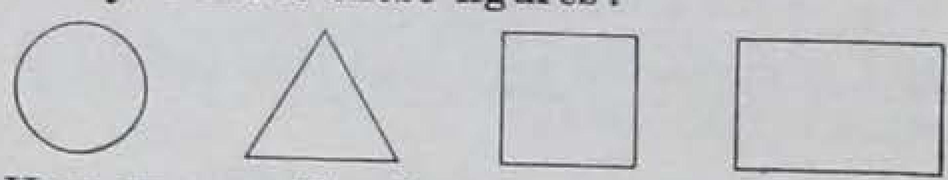
VI. Oral and Mental Work

Use problems and examples in text. Ask direct questions as, How much are 32 and 20; or 64 — 30? Make comparisons in social studies or science where measures are used; e.g., compare the amount of land plowed in an hour with the standard reference used.

VII. Geometric Forms

A. Types of questions

1. Can you name these figures?



2. How many sides has the triangle? The square? The rectangle? How many corners?
3. What is the difference between the square and the rectangle?
4. Is your arithmetic book square? Why not?
5. Name something that has the shape of a circle.

VIII. History and Development of Number. Magic square. (See Wheat, *The Psychology and Teaching of Arithmetic*, p. 105.)

Summary*

1. Multiplication and division facts through fives
2. Addition and subtraction

*The summary for the first grade given on page 25 clearly expresses the use which should be made of the items listed in the summary on this page.

3. Column addition
4. Upper decade addition facts
5. Roman numerals
6. Place value of Hindu number system
7. Telling time
8. Months, seasons, weeks, days of week, etc.
9. Fractions, 1/2, 1/3, 2/3, 1/4, 3/4, as part of group and whole; also comparison of size of these fractions
10. Further practice with linear measure; estimating lengths, widths, heights, etc.
11. Recognizing the circle, triangle, square, and rectangle

Diagnostic Test

(Mechanics of Addition)

	Examples			Difficulty Introduced
1.	3 1 5	7 1 1	5 2 1	Single column addition, no bridging
2.	5 8 2	9 2 3	4 8 9	Single column with bridging
3.	92 14 31	142 916 731	164 214 821	Two- and three-digit numbers, no carrying; bridging in last column
4.	32 60 91	104 201 842	301 440 906	Zero difficulties
5.	14 26 13	344 926 101	567 200 419	Carrying in right column only
6.	98 46 27	426 931 382	406 392 721	Carrying in second column
7.	932 46 208	20 342 826	4 21 938	Irregular left margin; involving first six skills
8.	492 284 199	366 520 848	821 706 497	Two successive cases of carrying

9.	384 + 56 + 928 406 + 3 + 28 982 + 81 + 8						Proper placement of numbers for addition
10.	4282 964 1384 21	9848 421 15282 1004	4608 26009 9872	462 9821 308 1080 29 362	529 2018 492 426 37 552	1034 2642 528 9764 92 16421 242	Increase of attention span, including all of above difficulties

Diagnostic Test

(Mechanics of Subtraction)

	Examples				Difficulty Introduced
1.	58 22	821 411	39 15		Two and three place numbers with no borrowing
2.	426 204	546 320			Zeros and no borrowing
3.	28 9	47 8	15 9	25 7	Bridging
4.	86 29	72 14	448 219		One place borrowing, first column
5.	463 271	928 562	355 291		One place borrowing, second column
6.	582 91	149 57	384 91		Borrowing and ragged left margin
7.	5362 1295	3621 2588	742 98		Two place borrowing
8.	580 239	406 292	428 309		Zeros in one place
9.	5006 2849	400 188	6002 908		Two successive zeros and borrowing
10.	14392 5829	40422 9291	1492 528		Alternate borrowing
11.	5020 2942	18040 9219	40604 29105		Alternate zero; two and three place borrowing
12.	5321 — 421 40920 — 16282 4280 — 38				Placement of number for subtraction

Suggested Examination Questions*

(Addition and Subtraction)

1. What is the sum of 16 and 39? The difference?

2. How much larger is 421 than 162?

3. How much smaller is 29 than 491?

4. By how much must 32 be increased in order to make 97?

5. Write in short form:

   Some number added to 6 equals 29.
   32 minus some number equals 14.

6. Start with the number 38. Add 18 and then subtract 18 from this answer. What is the final result?

7. If 36 is subtracted from some number and then 36 added to this result, what would be the final result?

8. Subtract: 364 (a) Why is the 9 placed under the 4?
   29 (b) In taking 9 from 4, why may we say 9 from 14?

9. How would you find out how many 16's in 76?

10. What is a "sum"? Difference?

11. What are "addends"?

12. Increase 36 by 426.

13. 426 is decreased by 132. What is the result?

14. Show by using marks that $38 + 42 = 80$.

15. By use of marks show what happens when 36 and 59 are added. Especially show why the 1 is carried when 6 and 9 are added in this example.

16. Which is the largest number? 3642, 3593, 3641, 3692

17. $N + 36 = 98$ $42 - N = 20$ $116 = N - 49$ $144 = N - 94$
    $N =$ $N =$ $N =$ $N =$

18. Add: 36 and 142; 9 and 1463.

19. Subtract 382 from 691.

20. Take 38 from 92.

21. Find the difference between 982 and 466.

---

*Although verbal problems are not included, they should be stressed in almost every test.

FOURTH GRADE

Inventory Test

1. Oral exercises

   Write only the answers to these:

   a. 5 and 6

   b. 11 take away 8

   c. 7 and 8

   d. 18 take away 9

   e. 20 and 30

   f. 61 and 20

   g. 40 take away 18, etc.

2. Do these exercises:

27	35	53	32	8	16
+14	+76	+26	+17	7	32
				3	14
				+ 7	+ 7

3. How much are 5 3's, 4 4's, 8 2's, etc.?

4. How many 4's in 12, 5's in 20, etc.?

5. About how large is our school lot? (Answer in acres.)

6. About how far is it from our school to __________? (Use a place about 1 mile away.)

7. Draw a circle on your paper.
   Draw a triangle on your paper.
   Draw a square on your paper.

8. Give other problems of the type used in third grade.

Content and Recommended Procedures

I. Addition and Subtraction

A. Review and practice, including carrying and borrowing. Children should be required to demonstrate that they understand each process. (See Grade Three.) Problems involving the following types of number operations should be used in this review:

(1) 249	(2) 208	(3) 300	(4) 124	(5) 428	(6) 226
+164	+534	+328	+ 29	—314	—117

(7) 843	(8) 301	(9) 702	(10) Column addition up to 9,
—358	—255	—507	2 or 3 digit addends

B. Oral work emphasizing the rounding of numbers; e.g.,

1. Two trucks, one weighing 4,246 lbs. and the other 2,098 lbs., entered the station. How do the trucks compare in size? What can you do to make such comparisons easy? What do you call this process? (In rounding numbers emphasize the fact that only the most significant numbers are dealt with.)

2. In rounding numbers such as 42 to the nearest 10, what are you really doing? (Dropping ones and thinking only of nearest tens. The same principle should be brought out in rounding to nearest 100.)

3. Round these numbers to nearest tens: 42, 208, 71, 1,006, 43,896, etc.

4. Round these numbers to nearest hundreds: 420, 692, 1,583.

II. Multiplication and Division (major share of year's work in this area)

A. Facts not developed and mastered in grade three (Usually involves 6's, 7's, 8's, and 9's.)

1. Introduce multiplication facts through problems presented in text.

   a. Have children work problem several ways; e.g.,

      (1) Addition is frequently the plan used in text. For the fact six 7's equal 42, the book usually shows that when six 7's are added the sum is 42.

      (2) Since multiplication is really regrouping into 10's and 1's, have child secure 6 groups of 7 objects each and regroup objects into 10's and 1's.

      |1111111 111|1111 111111|1 1111111 11|11111 11111|11

      Later, marks instead of objects may be used to show the above. To show the relationship between counting and multiplication, children should also count the objects in the 6 groups. (This exercise shows the six groups which are not nearly so evident in the usual way of indicating the operation. See (3) below.)

      (3) $\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$ $\begin{array}{r} 7 \\ \times 5 \\ \hline \end{array}$ then $6 \times 7 = 35 + 7 = 42$.

      (This emphasizes relationships between facts.)

      (4) After all facts have been introduced and the facts presented in several ways, bring together the various facts in table form.

(5) After time for the above steps has been taken, and after the teacher is reasonably sure that the children understand the processes, a timed test or exercise should be given. Use the data obtained from such an exercise to show the necessity for knowing the facts instead of having to count or to add. Intensive study of the facts using tables, flash cards, sets of examples and other methods of presentation should follow.

(6) Use problems of text to illustrate facts. Have children illustrate frequently to show that they know what they are doing.

2. Introduce division facts through problems presented in text or problems of a similar nature which pupils may have been encountering outside of the arithmetic class. (The latter is the preferable method.)

   a. Follow much the same plan as in multiplication; e.g., John has 15 cents. If apples cost 3 cents apiece, how many apples can he buy?

```
                 1      2      3      4      5    apples
John's money  | 111 | 11  1 | 111 | 1  11 | 111 |
```

   Hence there are 5 threes in 15, or five apples that John can buy.

   Have the child repeat using other problems. Eventually introduce the symbols $3\overline{)\;15}$ and $15 \div 3$.

   b. Follow much the same procedure when division with remainders is introduced. Write remainders as follows:

$$\begin{array}{r} 2\text{—}2R \\ 3\overline{)\;8\quad} \end{array}$$

   Since the fraction concept is none too well developed at this time, it would seem useless and meaningless to the child to write quotients as 2 2/3 except with very small divisors. The notation 2—2R is more meaningful for the youngster. Later on the other way of writing the quotient must be taught.

B. Multiplication and division (multiplicands and dividends larger than 10)

1. Present problem in which children have to multiply orally 2×20, 3×20, etc. Ask children another way of stating their answers (4 tens, 6 tens, etc.).

2. Develop the generalization that tens are multiplied just as ones are. (See Wheat, *The Psychology and Teaching of Arithmetic*, pp. 315 ff.)

3. Present many problems involving tens and ones.

(1)		(2)	(3)
21	2 tens 1 one	x x 1	21
× 3	× 3 ones	x x 1	21
	6 tens 3 ones	x x 1	21
		xxxxxx 111 = 63	63

4. Present many problems involving carrying.

26	2 tens 6 ones
× 3	× 3
	6 tens 18 ones = 7 tens and 8 ones = 78.

5. Repeat steps 1, 2, 3, and 4, except that division is used instead of multiplication.

C. Consult text for problems to be used in introducing multiplication with two- and three-digit multipliers.

In the development of multiplication have children demonstrate the reason for putting products in specified columns.

$$\begin{array}{r} 10 \\ \times 10 \\ \hline 100 \end{array}$$

The product is written in the *hundreds* column because 10 tens are equal to 100. Children should be questioned frequently concerning their reason for placing numbers in certain columns. Working through examples like the following will help to make this clear:

$$\begin{array}{r} 32 \\ \times 21 \\ \hline \end{array}$$

	3 tens	2 ones
	2 tens	1 one
	3 tens	2 ones
6 hundreds	4 tens	
6 hundreds	7 tens	2 ones

To show that 2 tens × 3 tens is equal to 600 have children write 30 in a column until they have 2 tens or twenty 30's and then let them add.

D. After much work in multiplication with two- and three-digit multipliers, considerable work should be devoted to the short method of multiplying 10 and 100. This should be arrived at by the pupils as a generalization from several exercises of the type: 14 × 10, 156 × 10, 59 × 10 and 100 × 78, 987 × 100, etc. The pupil will see that multiplying by 10 simply means adding a zero to the multiplicand in order to obtain the product. To multiply by 100 one must add two zeros to the multiplicand.

III. Reading and Writing Numbers to 1,000,000

A. For review, present numbers like 642 and ask what the 2 indicates; the 4; the 6. Do the same for 1,047 and 1,760. Emphasize the idea that the zero indicates that no hundreds or no ones are in this particular number and that the zero is holding that place.

B. Use numbers like 10,385, 115,396 and 1,000,000 from the news or geography books for further work in reading numbers.

IV. Fractions

A. In comparison, as in comparing sizes of cities, states, farms, distances, etc., use half, third and fourth. If concept needs clarification use measuring cup, ruler, or cardboard circles to show relation between whole and fractional parts. This work has for its purpose the building and fixing of concepts. No computation with fractional number symbols such as 1/2—1/4 should be taught directly in this grade.

B. In division, remainders may be written in two ways—fraction and remainder form. Through a problem like "16 sticks of candy were divided equally among 3 boys. How much did each boy receive?" the idea that the left-over stick can also be divided can be demonstrated for the simpler divisors. However, see page 56 for method of writing remainders.

C. Emphasize the relationship between the process of taking a unit fractional part of a number and dividing by the reciprocal of the fraction. For example: 1/3 of 24 (one third of 24) gives the same answer as 24 divided by 3 $(3\overline{)24})$.

V. Measurement

Before work with standard measures is begun children should have had considerable experience measuring and estimating the height of desks and chairs with books, sticks, etc. In this way the value of standard measures is demonstrated. For some of those measurement problems in the text you are using, have children do the actual measuring. (See textbook.)

VI. Graphs

The work should be concerned primarily with the reading of simple line and bar graphs. The entire work on graphs is optional at this time but should be introduced in fifth grade at the latest. Consult your superintendent.

First lesson will probably arise out of work in other subjects; for example, a spelling chart which shows the weekly spelling record of children or temperature charts. Attendance records for the school may also be used. The construction of simple line and bar graphs to show some geographic facts is an added step that may be taken in learning to read and to use graphs.

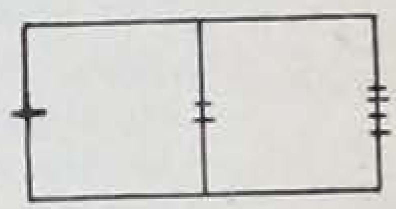
VII. History

The exercise below illustrates how the history may be presented to children.

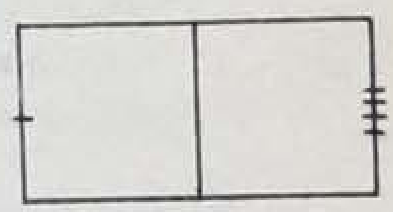
The Development of Zero

Have you ever thought about the difference between zero and the other numerals? You know that it is a place holder. It does not stand for a number of objects as does 2 or 5. It merely fills a space when no quantity is to be represented. If that is all zero does, you may wonder why it is given an important place in our number system. To answer that question you need only to try writing large numbers with Roman numerals. Not only did the Romans use clumsy numbers because they did not have a zero or place holder, but many other people did the same. The discovery of this simple but important idea was probably accidental. No one knows who discovered zero. It is generally believed that the use of zero came as a result of written records of number quantities represented on the abacus. (See second grade outline.) At first, when people made a record of sales or of taxes paid, the entire abacus was drawn on the clay tablet.

If, for example, the amount were 124, this appear on the tablet:



If it were 104, the abacus would appear thus:



Later, in order to save time, only marks and no lines were drawn; 124 then appeared — — —, and 104 — —. Since there

— —   —
—   —
—   —

were no printing presses at that time, all copy work had to be done by hand. If the copyist were careless and did not leave the proper space between the hundreds and ones, the 104 might be read as 14 or 1004. Obviously, something in the way of a mark was needed to hold the empty space. Many different marks have been used. The Arabs used a dot (.) and gave it the name *sifr*, meaning empty or blank. When our forefathers learned about this place holder from the Arabs, they used the Arab name *sifr*, but after a long time the word was changed to our word *zero*. Our forefathers also changed the mark from a dot to a circle so that it would be about the size of the other numerals.

VIII. Problems

Although many problems may be furnished by other activities of the children, the chief source of problems will be the textbook. It should be remembered that the chief use of problems at this level is to illustrate procedures. Proof should be frequently required and should be of the type described under *addition, subtraction, multiplication,* and *division* in an earlier section. If your text uses the formal analysis plan, that procedure should be followed part of the time, but not to the extent that diagrams and illustrations are excluded. Illustrations should also be used to show that the child has a grasp of the situation.

IX. Standard References

All references listed for the preceding grades plus (1) the area of state and county, and (2) the distance in miles between two well known points—one for 10-15 miles and one where hundreds of miles are needed

A. Introduce the new references (state, county; 10-15 miles, and 100-200 miles) to be learned through oral problems requiring the making of comparisons. For example, in social studies when a statement like "The area of California is 158,297 square miles" is made, such questions as the following should be asked: "How big is that?" "How does it compare with Iowa?" (The standard reference to be used.) Similar situations for the two distances should be used. Select for these references the distance to some convenient or well known places.

B. For every standard reference build up as many associations as possible; e.g., the time required to cover the distance, to go around the area, etc.

Summary*

1. Review of addition and subtraction
2. Complete multiplication and division combinations
3. Multiplication with two- and three-digit multipliers
4. Division with single digit divisors; long division method
5. Rounding numbers to nearest 10's, 100's, and 1000's
6. Reading and writing numbers to 1,000,000
7. Fractions 2/3 and 3/4 as parts of a whole and parts of a group
8. Reading simple line and bar graphs
9. The generalization that a fractional part of a number and division are related; e.g., 1/4 of 16 is the same as $4\overline{)16}$.

*The summary for the first grade given on page 25 clearly expresses the use which should be made of the items listed in the summary on this page.

Diagnostic Test
(Mechanics of Multiplication)

	Examples			Difficulty Introduced
1.	14 2	12 4	23 3	Two place numbers multiplied by 2, 3, or 4—no carrying
2.	46 2	28 3	23 4	Introducing carrying
3.	81 3	52 4	81 2	Last partial product greater than 10
4.	Steps 1, 2, and 3 with multipliers 5, 6, and 7			
5.	321 3	122 4	212 3	Three place numbers — no carrying
6.	420 2	303 3	400 2	Zeros in multiplicand
7.	328 2	116 7	128 3	Carrying in the first partial product only
8.	351 2	181 5	191 7	Carrying in the second partial product
9.	340 5	306 7	500 9	Zeros and carrying
10.	129 5	416 6	924 3	Two cases of carrying
11.	4214 5	5280 6	6008 7	Four place number and zero difficulties
12.	Extension of steps one to nine to include multipliers of eight and nine			
13.	24 20	36 50	49 40	Two place numbers multiplied by 10, 20, 30 90
14.	51 16	23 22	14 21	Two place number multiplied by two place number — no carrying
15.	28 49	32 46	80 29	Carrying

16.	326 21	502 48	465 90	Three place multiplicand, two place multiplier — zeros in both
17.	3004 26	4030 50	5080 92	More extensive use of zeros
18.	4280 206	3921 508	9462 700	Introducing middle zero and two zeros in multiplier
19.	4282 4006	1821 1369	5804 9090	Two successive zeros and alternate zeros
20.	42 10	463 100	96 100	Multiplication by 10, 100 and 1,000. Pupils should be able to write answers without involved mechanical forms.
21.	45 × 10	36 × 100	128 × 1000	

Suggested Examination Questions

(Multiplication)

1. Eight eights are __________.
2. Multiply seven by eight.
3. Multiply two by four.
4. Find the product of 16 and 12.
5. Twenty-four times 16 is __________.
6. What number is six times larger than 24?
7. What is the short way to work this problem?
   28 + 28 + 28 + 28 + 28 + 28
8. By making marks or pictures on your paper show that five sevens are 35.
9. If there are 8 cabbage plants in each of 7 rows, how many cabbage plants are there in all the rows?
10. Six times seven plus 9 = ?
11.
```
 38
 24
---
152
76
---
912
```
In multiplying these two numbers,

(a) Why is the six placed under the 5?
(b) Why did we carry the 3 and add it to 12 when multiplying by 4?
(c) 2 tens × 3 tens = 6 ________.
(d) 2 tens × 4 ones = 8 ________.
(e) 3 ones × 5 ones = 1 ________ and 5 ________.

12. If Mary has $1.20 and John has three times as much as Mary, how much does John have?

13. (a) If one multiplies 16 by 2 and then takes 1/2 of the product, the result is __________.

(b) Teacher repeats using other numbers until pupils see the generalization; i.e., taking 1/2 the product will always give the original factor other than 2.

(c) If one finds the product of 3 and 7, then takes 1/3 of that answer, what is the result?

14. Multiply

$$\begin{array}{r} 36 \\ 2 \\ \hline 72 \end{array}$$

(a) Which number is the product?
(b) Which number is the multiplier?
(c) Which number is the multiplicand?

15. Are two threes and six threes the same as eight threes?

16. What is the shorthand way of writing

(a) Three times some number equals 15?

(b) Some number × 18 equals 72?

17. Six times some number is 147. What is the number?

18. What is the short way of multiplying a number by 100? 10? Illustrate.

19. $5 \times N = 35$ $\quad$ $18 \times N = 165$
$N = ?$ $\quad$ $N = ?$

To the Teacher: Include also a variety of verbal problems in almost every test.

FIFTH GRADE

Inventory Test

1. Multiplication and Division

a. If each child contributes 8 cents to the party what is the total amount that 9 children will give?

b. How much does the average Scottie dog weigh if 8 of them weigh 136 lbs.?

c. How many oranges are in 26 boxes if each box contains 84 oranges?

2. Write with words:

a. 709

b. 2,116

c. 800,415

3. Write with numbers:

a. Five thousand one hundred eight

b. Two hundred fifty-six thousand sixteen

4. From the spelling graph on the board tell how many words Jack spelled on Friday.

5. Round these numbers to nearest 10's; to nearest hundreds:
   a. 103
   b. 16,898
   c. 40,311

6. How does the population of city A compare with that of city B if the population of A is 10,118 and the population of B is 19,674?

7. Work these examples:

Add	117	Add	$7.18	Sub.	7008	Sub.	9814
	982		.60		516		8929
	409		1.74				
	360		3.19				
Mul.	206	Mul.	975	Div.	6/ 671	Div.	8/ 988
	21		207				

8. a. The area of Kansas is 82,158 sq. mi. How does Kansas compare in size with Iowa?
   b. The distance from Council Bluffs to Sioux City is 98 miles. How does this compare with the distance between two cities that you know? (Teacher: Choose two cities in your vicinity that are known to the children.)

9. Which of these numbers is about 1/4 of 2000?
   a. 705 b. 654 c. 510 d. 418

10. Draw an abacus which will show the number 525; draw another to show 116; another to show 301. (See second and fourth grade outlines for use of abacus. Also Wheat, *The Psychology and Teaching of Arithmetic*, pp. 40-44.)

Content and Recommended Procedures

I. For some of the problems and exercises in multiplication and division of the inventory, proof is required similar to that suggested for fourth grade.

Emphasize particularly during this review the similarity between multiplication and division of ones and tens. Later include hundreds and thousands on the same basis. Include with the exercises in which proof is required oral exercises in which children have to round numbers if work is to be done efficiently; e.g., "If there are 192 lbs. in a bbl. of flour, about how much do 20 bbls. weigh?" or "What is the average weight of the fifth grade children if all of them weigh 1042 lbs.?"

II. Division Using Two- and Three-Digit Divisors

The basic cycle of operations has already been set up in the work of the fourth grade while using one-digit divisors. Some time should be spent in doing remedial work in division with one-digit divisors before using two-digit divisors. Furthermore, before starting division the teacher should ascertain whether the pupil is sufficiently proficient in multiplication and subtraction since these two processes are involved in the division process. Any attempt to teach long division before the pupil is sufficiently proficient in subtraction and multiplication is sure to lead to failure and will unduly discourage the pupil.

The pupil should understand and be able to work simple verbal problems involving division before he is asked to do long division. Present verbal problems daily illustrating the uses of the fundamental operations. These should be given both orally and in written form, the latter to develop the ability to read quantitative materials.

In general, the best procedure in teaching long division is to follow the procedure used in the text.

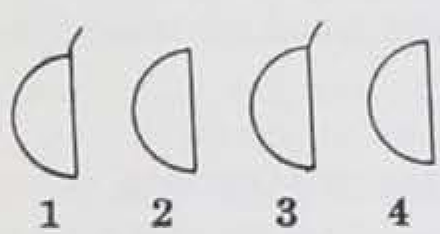
Sometime during the pupil's study of long division he should arrive at the generalization that numbers ending in zero may be divided by 10 dropping the last zero. A similar generalization should be arrived at for division by 100. This understanding will be used later in percentage and should not be overlooked.

III. Addition and Subtraction of Common Fractions

For the first problems have children draw fractional circles or rectangles to represent the values involved; e.g., "If John gave half an apple to each of 4 friends, how many apples did he use?"

Work this problem by drawing diagrams.

a.



1/2 for each friend

b.

or 2 apples

Have children construct cardboard circles of equal diameter and then divide several circles into halves, thirds, fourths, and eighths. These fractional parts should be used to show what is obtained when fractions are added or subtracted.

In the teaching of fractions—

A. Teach that the denominator tells in how many equal parts the whole (or group) has been divided and that the numerator tells how many of these parts are under consideration.

B. The major share of the work with fractions in this grade will be concerned with addition and subtraction. Considerable time should be taken to introduce this work through the use of diagrams and models mentioned in the paragraphs above. Eventually, however, the children should use the more economical adult procedure of working with fractions. The gradual approach through paper cutting, diagrams, etc. insures a foundation of understanding upon which to build the work in fractions. Throughout the work the children should be asked to demonstrate their understanding of the processes by illustrations or models.

C. Teach reduction of fractions as an application of the following principle: If both the numerator and denominator of a fraction are multiplied (divided) by the same number the value of the fraction is not changed.

D. The following form for addition is recommended for the work in fractions:

$$\begin{array}{rcll} 3\,3/4 & = & 3\,6/8 & \\ \underline{5\,7/8} & = & \underline{5\,7/8} & \\ & & 8\,13/8 & = 9\,5/8 \end{array}$$

Writing the 8 13/8 may be omitted if the pupil so desires. The 3/4 in the above problem should be changed to 6/8 by means of multiplying numerator and denominator by 2. Do not teach the pupils to say: 4 into 8 is 2; 2 times 3 is 6. Some time should be spent adding proper fractions in the following form:

$$1/2 + 2/3 = 3/6 + 4/6 = 7/6 = 1\,1/6$$

This serves as a means to emphasize meaning as well as develop an ability which is used in later work and everyday life.

E. The recommended form for subtraction is:

$$\begin{array}{rcl} 3\,1/4 & = & 2\,10/8 \\ 1\,7/8 & = & \underline{1\;\;7/8} \\ & & 1\;\;3/8 \end{array} \qquad \text{or} \qquad \begin{array}{rcrcl} 3\,1/4 & = & 3\,2/8 & = & 2\,10/8 \\ 1\,7/8 & = & 1\,7/8 & = & \underline{1\;\;7/8} \\ & & & & 1\;\;3/8 \end{array}$$

Almost any other form for the addition and subtraction of fractions involves too much "machinery" or involves errors in expression. For proper fractions also teach

$$5/6 - 1/2 = 5/6 - 3/6 = 2/6 = 1/3$$

F. Knowing that the value of the fraction is increased if the numerator is increased or that the fraction is decreased if the numerator is decreased, while the denominator is constant, is a valuable understanding for pupils in the fifth grade. This understanding may be attained by exercises of the following nature:

(a) Which of the following fractions is the largest: 1/9, 7/9, 4/9, 6/9?

(b) Pick the smallest number: 8/3, 5/3, 9/3, 2/3.

(c) Pick the largest number: 11/5, 11/12, 11/15, 11/21. etc.

G. It is well to remember that if the pupil cannot illustrate by means of diagrams, paper cutting, etc. the simple processes of addition and subtraction of fractions, in all probability he is not ready for the more formal work in addition and subtraction of fractions.

IV. Decimals

The work with decimals at this level is optional. Consult your superintendent. Principal work with decimals is to follow the work with common fractions. Follow the approach used by the textbook* but continually emphasize the relationship to common fractions:

(a) 1/10 = .1 (b) 7/100 = .07

Show through study exercises on such numbers as 444.44, relation between decimals and positional value. Each number to the right is 1/10 of the preceding number. (See Morton, "Teaching Arithmetic in the Elementary School," Vol. II, pp. 320-321.) The main purpose of work with decimals at this level is to give children the idea or meaning of decimal fractions and how to read them.

V. Percentage

Like the work in decimals, the main purpose is to help children to read books in social studies, health, science, etc. where it is necessary to make comparisons in order to establish an important point in the minds of the pupils. Therefore, the elementary idea of percentage can and should come into the arithmetic course during the latter part of the fifth grade; that is, after the pupil has learned to read simple decimal fractions.

Percentage should be taught as another way of writing hundredths. That is, 5% = 5/100 = .05; 25% = 25/100 = .25 = 1/4; 50% = 50/100 = 1/2 = .50 = .5, etc.

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*The most natural way of introducing decimal fractions is to present them as a new way of writing common fractions whose denominators are 10, 100, etc. See page 79, VI, 1.

In the news and in social studies such statement as "50% of the farmer's grain is fed at home" is found. This should be explained as meaning 50/100 or 1/2 of the grain is fed at home. Since children know what 1/2 of any quantity means, the term 50% should be easily understood. Similar statements may be found in other subjects. For example, seed corn has a germination test of 90%. This means that 90/100 or 9/10 of the seed corn will grow. Many other examples can be found, and should be used, in other areas. The continual use of percentage in other subjects will help drive home the percentage idea and get the pupil ready for more formal working percentage in the seventh grade.

Nothing more than suggested above should be done with percentage in the fifth and sixth grades, except under unusual circumstances.

VI. Measurement

Primarily area (floor space in sq. ft. and land in acres and sq. mi.)

In order to be sure that children know what measurement of area means, have them apply a square foot to areas like a portion of the room floor, table tops, etc., to see how many such square feet are needed to cover the surface. Then have children draw to reduced scale the surface measured showing the square feet needed to cover it. Make scale drawings of school grounds placing building correctly.

VII. Standard References

All references for preceding grades plus (1) height of a door, (2) 100 ft. both horizontal and vertical, (3) a ton, (4) population of cities, one for 5,000—15,000 and one for 100,000—200,000.

The height of a door (7 ft.) is taken as standard reference because in discussing any problem where a length or height from 4 to 20 feet is involved, comparison can be made with the classroom door.

For each standard reference many experiences with the measure involved should be used. In case of distance the children should actually measure the distance and frequent reference should be made to it in oral exercises and in discussions where numbers involving similar measures are used. For other measures such as ton, population of city, etc., use any available comparison to help child get a good concept.

VIII. Reading and writing numbers to a billion with Hindu-Arabic numerals and Roman numerals to two thousand

Emphasize writing the number of years B.C. and A.D. in Roman numerals.

IX. Graphs

Line and bar graphs plus pictographs with emphasis still primarily on the reading of graphs. The graphs should be taken from social studies, science, daily papers, etc.

X. Problems

Although most of the problems at this level will be used for purposes of illustration, some attempt is made to teach pupils how to solve the types of problems they will meet in life outside the school. Proof should be required frequently. For type of proof needed see fourth grade outline, and section 3 of the fifth grade outline. While the textbook will be the source of most problems, many good problems, especially those for oral exercises, should be supplied by the teacher from news and content subjects that children are studying. For a thorough discussion of problem solving the teacher should consult Morton, *Teaching Arithmetic in the Elementary School,* Vol. II, pp. 434-494, and Wheat, *The Psychology and Teaching of Arithmetic,* pp. 110F-212F.

Summary*

1. Remedial work in multiplication and division
2. Complete long division with two- and three-digit divisors. Toward the end of year, quotients will be written as 126 5/14.
3. Addition and subtraction of common fractions
4. Introduction of decimals (optional)
5. Introduction of percentage. If decimals are not introduced, teach that 15% = 15/100 = 3/20; otherwise teach in the order 15% = 15/100 = .15 = 3/20.
6. Measurement of area (rectangles only)
7. Standard references of area, height, etc.
8. Reading and writing large numbers
9. Roman numbers to include dates
10. Reading line and bar graphs
11. An abundance of verbal problems involving above processes and ideas

*The summary for the first grade given on page 25 clearly expresses the use which should be made of the items listed in the summary on this page.

Diagnostic Test

(Mechanics of Division)

	Examples			Learning Difficulty
1.	2/ 48	2/ 24	2/ 86	Division by two — Placing quotient properly
2.	2/ 64	2/ 34	2/ 98	Introducing carrying but no remainders
3.	3/ 36	3/ 72	3/ 57	Division by three with and without carrying
4.	4/ 84	5/ 80	4/ 76	New divisors 4 and 5
5.	5/ 480	4/7544	3/4596	Three and four place dividends
6.	3/ 261	4/ 512	5/1265	Proper placing of quotients
7.	3/ 390	2/9460	5/5600	End zeros in quotient
8.	6/ 360	9/ 144	8/9200	New divisors 6, 7, 8, and 9
9.	2/ 141	8/ 904	7/3462	Remainders
10.	8/ 814	7/3567	5/1613	Middle zeros, remainder and carrying

Two Place Divisors

1.	10/ 30	20/ 460	60/ 120	Placing quotient, two place divisor
2.	21/ 45	39/ 82	71/ 163	Two place divisor, one place quotient
3.	31/ 628	52/1536	88/ 964	With remainders 10, 20, 30, 40, ... 90; two place quotients; first trial divisor correct
4.	43/ 860	89/3568	23/1850	Terminal zero in quotient with and without remainder
5.	43/9073	58/54708	21/13025	Three place quotient figures with and without remainder and terminal zeros

	Examples			Difficulty Introduced
6.	51/10306	79/71586	19/11419	Middle zero in quotient
7.	46/2376	53/3084	21/ 625	True quotient figure one less or one more than trial quotient figure
8.	95/8465	25/1526	35/3642	Trial divisor may be either of two figures

Three Place Divisors

1.	400/4362	300/16421	900/38428	Proper place of quotient figure and estimating trial divisor, end zero in division
2.	482/16384	926/38498	794/94628	
3.	456/173385	902/838876	392/22486	
4.	444/267786	739/666688	132/14000	Middle zeros

Diagnostic Test

(Mechanics of Fractions)

Addition

1.	1/3 + 1/3	1/5 + 2/5	1/7 + 3/7		Denominations alike
2.	3/5 4/5	2/3 2/3	8/9 7/9		
3.	1/4 1/4	3/8 1/8	5/10 3/10	5/7 2/7	Sum proper fraction but can be reduced
4.	1/4 + 1/4 = 1/8 + 5/8 = 7/10 + 1/10 =				Horizontal form
5.	1/3 + 5/6 =	1/2 3/8	2/3 4/9		One denominator a multiple of the other
6.	1/2 + 1/3	2/3 + 1/4	2/7 + 4/5		Denominators prime to one another
7.	1/6 + 5/12	7/12 9/10	5/8 7/12		Denominators with common factor
8.	2 1 1/2	6 4 2/3	19 1/3 3		Mixed numbers and integers

	Examples			Difficulty Introduced
9.	3 1/5 5 1/10	13 5/8 21 1/6	142 1/2 368 1/3	Two mixed numbers sum of fractional parts proper
10.	21 3/4 14 1/2	146 1/3 21 2/3	4 7/8 29 3/4	Any two mixed numbers sum of fractional parts proper

Diagnostic Test

(Mechanics of Subtraction)

1.	4/5 — 1/5 = 9/10 — 5/10 =	7/8 — 3/8 =		Like denominators
2.	7/8 — 1/4 =	9/10 1/2	3/4 2/3	Unlike denominators
3.	26 1/2 4	14 2/3 7	21 7/8 13	Minuend mixed number, subtrahend integer
4.	4 1/2 3 1/4	419 7/8 62 3/4	21 3/8 — 14 1/4	Mixed numbers, no borrowing
5.	48 21 1/2	32 14 3/4	46 8 7/8	Minuend an integer, borrowing
6.	36 1/2 14 3/4	29 2/3 16 5/6	445 1/4 31 3/8	Borrowing and two mixed numbers
7.	81 1/3 29 4/5	65 3/4 19 7/8	40 1/2 29 3/4	Borrowing in both fractional part and integral part of mixed number

Suggested Examination Questions

(Division)

1. How many eights in 64?
2. Sixty-four divided by 6 equals __________.
3. Show by means of marks that there are 8 fours in 32.
4. What is the shortest way to find how many 7's in 84? What other way could be used?
5. N ÷ 6 = 12   18 ÷ N = 3
   N = ?   N = ?
6. How would one check the answer obtained in division?
7. 38 = N ÷ 2   55 = N ÷ 5
8. If 16 is divided by two and this result multiplied by two, what is the final result?
9. Multiplication and division are called "inverse processes." Can you name two other processes which are "inverses"? Illustrate.

10. How many halves in 6?

11. Take a number like 36. Divide it by 4 and then multiply that answer by 4. What is the final result? Reverse the operations. That is, multiply 36 by 4 and divide the product by 4. What is the final quotient? Will this same thing be true if one uses numbers other than 36?

12. Eight 9's = 72. Then 72 ÷ 8 = n. n = ? Also 72 ÷ 9 = ?

13. If one multiplies 15 by some number and then divides this product by the same number, what is the quotient?

14. $65\overline{)4587}^{\,N}$ then 65 times n = ?

15. What is the difference between 1/4 of 8 and 8 divided by four? (Only so far as the numerical answer is concerned.)

16. How many facts can you state using the numbers 3, 8 and 24 and the signs +, =, —, ÷, and ×.

17. In problems 11, 12 and 13 it has been pointed out that multiplication "undoes" division and division "undoes" multiplication. Are there two other operations that are so related?

18. John bought 4 apples at 4 cents each. He gave the clerk a quarter. How much change should he receive? (Don't forget the sales tax.)
    a. How many apples did John buy?
    b. How much did each apple cost?
    c. How much money did he give the clerk?
    d. How much did John pay for all four apples?
    e. How much would the tax be on this amount of sales (money)?
    f. So the apples and the tax amounted to __________ cents and John gave the clerk __________ cents. Then he should receive __________ in change.

Suggested Examination Questions

(Basic Concept of Fractions)

1. By means of diagrams show what is meant by:
    a. 2/3 of an apple
    b. 1/4 of a sheet of paper
    c. 3/5 of a quart of water
    d. 1/3 of a half apple
    e. How many minutes in 1/4 of an hour?
    f. Which is the larger number, 1/2 or 1/3?
    g. How many thirds are there in a cake?
    h. How many two-thirds are there in two cakes?
    i. Is there any difference between 2/4 and 1/2 of a pie?

2. How many fifths does it take to make a whole? Make a drawing which will show that your answer is right.

3. Write in words: 7/8, 2/3, 5/6.

4. Write in numbers: one third, five eighths, nine tenths.

5. What is one third of 27? What is 27 divided by 3? Multiplying a number by 1/3 is the same as dividing the number by ________.

6. Dividing a number by 5 is the same as multiplying the number by ________.

7. Consider the fraction 1/2. If the numerator and denominator are multiplied by 2, does the value of the fraction change? If the value of the fraction is not to be changed and the denominator is multiplied by 3, what must one do to the numerator?

8. Consider the fraction 4/6. If the numerator is divided by two, what must one do to the denominator if the value of the fraction is to remain unchanged? (To the teacher: The basic ideas brought out in questions 5, 6, 7, and 8 are very important.)

9. Draw a diagram which would show a fourth grader how to get this problem: In a circus train one car contained 15 small tigers in 5 different cages. If there were the same number in each cage, how many tigers were there in each cage? How many cages would the circus need if no more than two tigers were to be placed in a cage?

10. Show what would be the short way of getting the answer to problem 9.

11. Why must one change fractions so that they have the same denominator before adding or subtracting the fractions?

12. By means of a drawing or using pieces of paper, show that 5/2 = 2 1/2.

13. Dividing a number by 5/8 is the same as multiplying the number by ________.

14. $\frac{5}{6} N = 10$. $N = ?$ (This should be taught as a generalization of problems of the type $3N = 9$.)

15. Pupils should be taught to translate the terminology of the problem into the short statements of arithmetic. For example, John knew that 1/3 of the boys in the fifth grade were going to be assigned to go on an excursion with the sixth grade. If five boys were on hand when the bus was ready to start then there must be at least ________ boys in the fifth grade.

16. This problem really says: 1/3 of the number of boys is 5.

$$\text{or } 1/3 \text{ of } N = 5$$

$$\text{or } \frac{1}{3} N = 5$$

$$N = 15$$

SIXTH GRADE

Inventory Test

1. Show by means of diagrams how much 1/2 — 1/4 equals.
2. The grocer sold 1 1/2 lbs. of his 7 lbs. of butter. How much did he have left? Show by number and by diagram.
3. What does the denominator of a fraction tell you?
4. Which of these fractions is larger, 1/3 or 1/4? Show by diagram.
5. Write one tenth as a common fraction. As a decimal fraction.
6. Write these numbers with words: 1.24, 2.04, 41.2.
7. A man lost 25% of his sheep. What fractional part did he lose?
8. How does the area of New York State, 49,204 sq. mi., compare with Iowa?
9. A ditch is 20 feet deep. How would you tell the class how deep 20 ft. is?
10. The population of a city is 450,000. How does that compare with some city you know?
11. Write these numbers with words: 1,263,051 64,000,512.
12. Write these numbers with figures: ten million, one hundred seven thousand, sixty-four; fifty-two million, six hundred twenty-four thousand, eight hundred two.
13. Read these dates: 1807 A.D., 1919 A.D., 440 B.C., Jan. 6, 1945.
14. Solve these examples:

64/ 2324	71/ 1568	307 ×109	700 ×126
1/2 + 1/3	1/4 + 3/4		1/2 + 3/4
3/4 — 1/2	2/4 — 1/2		

15. Problems of the type used in fourth and fifth grades.

Content and Recommended Procedures

I. The inventory or diagnostic test materials should be used as the first part of the review program. Children should be required to show that they understand multiplication and division by working problems in several different ways; e.g., 12 × 114 may be worked by addition:

(a)
```
1140
 114
 114
----
1368
```

(b) 114 + 114 + — — — (12 addends of 114) = 1368

(c)
```
 114
  12
 ---
 228
114
----
1368
```
etc.

By breaking the multiplier into parts and then adding partial products, by rounding 48 to 50 and then subtracting 228 from the product of 50×114. Similarly $812 + 96$ may be worked by serial subtraction (see Third Grade) by calling 96 to 100 and **finding how many 100's in 800.**

Equally searching tests of understanding should be demanded for other work covered in the inventory test. If children do not understand the processes, simple problems should be given and proof required. The solutions of the simplest problem should include actual manipulation of objects.

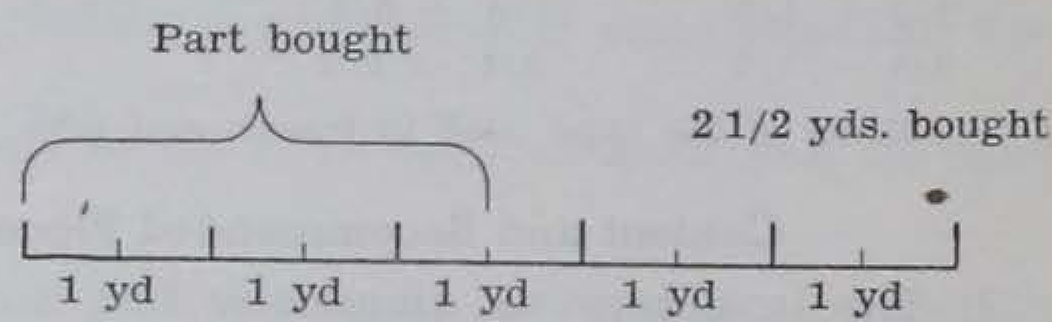
II. Fractions

Systematic instruction in addition, subtraction, multiplication, and division of common fractions in picture form or diagrams.

After the review of addition and subtraction of fractions, give several weeks to work in addition and subtraction of fractions in which numbers (not words or diagrams only) are used. Your text will furnish examples. Use both fractions and mixed numbers. Every few days ask the child to work some problem or example using drawing, diagrams or objects.

Introduce multiplication and division through problems, but ask child to solve by drawing first. This may necessitate copying problems from the text because the text will give the solution with numbers. After solution with diagrams ask for solution with numbers; e.g., Jennie bought 1/2 of a roll of ribbon. If there are 5 yards on a roll, how much ribbon did she buy?

Diagram:



Number: 1/2 of 5 or $1/2 \times 5 = \frac{5}{2} = 2\,1/2$ yds.

After children understand how to solve problems by diagram and understand number process, give much practice with numbers. Your text will give problems and examples. A few days afterwards ask children to prove solution to one problem by making a diagram.

Division with a fraction as a divisor should be introduced by a problem situation which involves dividing an integer by a unit fraction; e.g., $6 \div 1/3$. This should be solved much as recommended for multiplication. (See above.) The meaning of division will be reinforced if the teacher will remind the class that in the lower grades the problem 24 ÷ 6 was read, "How many six's in 24?" Similarly the problem 6 ÷ 1/3 is read, "How many one thirds in 24?" This "tie-up" between division of integers and division of fractions should not be forgotten.

Some verbal problems in fractions are always difficult to teach. The discussion below may help to unify the terminology and methods of solving certain kinds of problems for the pupil. Note how abilities developed in previous grades assist in solving the problems of teaching fractions in the fifth and sixth grades.

The problem: Harry raised 18 chickens and sold 12 of them; what part of his chickens did he sell? should be worked somewhat as follows: First, the pupils should be reminded that the term "part of" means a fraction times a number. That is, $1/2 \times 6$ means the 1/2 part of 6. Now when one reads the question in the problem it is evident that the problem is asking, what part of 18 is 12 or $n \times 18 = 12$. This is solved in the same way that $n \times 6 = 12$ is solved; that is, by dividing 12 by 6. In the problem above the pupil would write $n = 12/18 = 2/3$.

It is readily seen that all problems of the type:

2/3 of what number equals 12?
What part of 18 is 12?
What number times 3/4 is 12?

can be solved by the one method; namely, by letting "n" be the initial of the number and then translating the above three exercises as follows:

$2/3\ n = 12$
$n \times 18 = 12$
$n \times 3/4 = 12$

III. Reading and Writing Large Numbers

The large numbers should be taken from social studies work, science, health, daily papers, magazines, etc., rather than from the arithmetic book. This part of the arithmetic program will be more functional if teachers look upon this as a necessary skill to convey basic ideas in other areas and spend some time on it wherever the needed skill arises. Of course this same thing can be said of all arithmetic skills.

IV. Measurement of Areas and Volumes

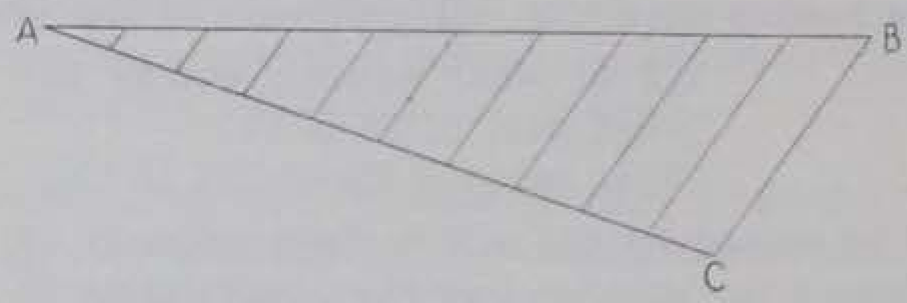
To include area of rectangles measured in sq. ft., sq. in., sq. yd., sq. mi., and sq. rd. Volume of a rectangular parallelepiped (box) only. Cubic units to include cu. ft., cu. in., and cu. yd. only unless special occasions call for the introduction of other units.

To develop the meaning of area, see the fifth grade outline. For cubic measure, be sure to fit a cube of one cubic inch volume into a larger box, letting the children experiment and generalize as to how to find the volume.

On linear measure, some practice should be provided in making measurements to the nearest tenths of an inch, tenths of a foot, and tenths of a yard.

This probably should not extend beyond the study of a yardstick and a foot rule to see the relative size of 1/4 yd. and 2/10 or 3/10 of a yd.; 1/2 yd. and 5/10 yd.; 1/8 in. and 1/10 in.; etc. This kind of work will assist in fixing the concept of decimal fractions and show the relationship (in terms of size) between common fractions of a unit and the decimal fractions of a unit.

One way to approach the work on measure as outlined above would be to divide (teacher doing the division previous to class discussion) a yardstick into ten equal parts. This can be done as follows:



Draw a line one yard long on the board. Call this line AB. Then, beginning at A draw any other line such as AC which is 30 inches long. Mark off AC in 3-inch intervals. Connect C and B. Then, through each of the 3-inch division marks on AC draw a line parallel to BC. The lines can be made parallel to BC by making the angles at the division points equal to the angle ACB by means of a protractor. (See almost any seventh or eighth grade text on arithmetic.) These parallel lines divide the yard AB into tenths of a yard. These divisions can be transferred to the yardstick and then the pupils told that the marks divide the yardstick into ten equal parts or tenths of a yard. The yardstick should now be used to measure various distances in yards and tenths of a yard. This same procedure can be followed for the foot rule.

It is very important that considerable practice be given in making measurements of various distances during the fourth, to, and including, the eighth grade. Very few understandings are more important in daily life than measurement. Furthermore, the problems of measurement lead into so many good situations which require the pupil to do some computing with various kinds of units, thereby providing a more nearly normal daily life use of the arithmetic operations.

Pupils in this grade as well as in previous grades should be given practice in estimating distances all the way from a number of inches to four hundred or five hundred feet.

V. Denominate Numbers

The work with denominate numbers should include only that with which children are likely to be familiar, like oz., lb., pint, qt., gal., days, weeks, months, yrs. Consult your textbook for work with such problems. (See Morton: *Teaching Arithmetic in the Elementary School,* pp. 411-426, for good discussion of denominate numbers.)

VI. Decimals

1. The work with decimals in the sixth grade should re-emphasize the relationship between decimal fractions and common fractions. Decimals should be taught as a new way of writing common fractions whose denominators are powers of 10; i.e., fractions whose denominators are 10, 100, 1000, etc. The pupil is told that 7/10 = .7, the explanation being that it is only a different way of writing 7/10 and that this new way of writing 7/10 has been found to be more convenient by people who are employed in industry, by people who study science, and by everyone else. Of course, decimal notation should be related to the way of writing U. S. money.

2. With decimal numbers larger than one, the decimal point should be read as "and." Thus, 345.86 is read "three hundred forty-five *and* eighty-six hundredths." This convention helps to distinguish between the following two numbers when read to the pupil: 200.036 and .236.

3. As an aid to understanding the place value of decimals, as well as their relationship to common fractions, it is suggested that some work of the following type be assigned and discussed in class until the pupil understands decimal notation as related to common fractions:

   $$3/10 + 7/100 = 30/100 + 7/100 = 37/100 = .37$$

   Also work on the inverse of the above example; that is,

   $$.56 = 5/10 + 6/100 = .5 + .06$$

   Then prove the result as shown above.

4. Exercises in picking largest and smallest number should be provided at frequent intervals.

5. In teaching the addition and subtraction of decimal fractions the emphasis should be placed upon the addition of like

quantities instead of keeping the decimal points in a vertical line. This serves to bring out the meaning of the processes involved as well as bringing to the attention of the pupil an important idea; that is, in addition and subtraction, only like quantities may be used.

6. Rounding off decimal numbers to the nearest tenths and hundredths and the application of this ability in measurement

7. Work in counting by tenths from 4 to 5, or counting by hundredths from 10 to 11, or from 9.1 to 9.2 will help fix the idea of decimals and provide a foundation for rounding off decimals to nearest tenths, hundredths, etc. To illustrate: Counting from 4 to 5 by tenths, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 5.0. Questions such as, Is 4.6 nearer 5 or nearer 4? Is 4.2 nearer 4 or nearer 5? will help the pupil visualize the meaning of rounding off numbers to nearest tenths.

VII. Charts and Graphs

Continued emphasis on reading with some work on construction of graphs and some discussion of the ways of presenting information graphically.

VIII. Problems

Greater emphasis will be given to solution of problems as preparation for the type of problems met in life and less attention will be given to illustrative use of problems.

IX. Standard References

All references listed for previous grades plus (1) size and speed of ships, (2) mountain (altitude), (3) railroads in a state or country, (4) rainfall in a certain area.

X. Percentage (Same as fifth grade)

XI. Oral Work

The program of oral work started in the earlier grades should be continued in this grade. Although the oral work need not be as frequent, it should be given at least once a week. Problems and examples involving both whole numbers and fractions should be used. The following problems are examples of the two types:

What is the combined area of Iowa, Illinois, and Indiana, if Iowa has 56,147, Illinois 56,665, and Indiana 36,358 square miles? (135,000 to 160,000 accepted as correct) A man planted one half of his 300-acre farm in wheat. How many acres of wheat did he plant?

In order to motivate and to administer the oral program, children should often write the answers. In this way the oral work of several grades can proceed at once. The answers on the papers can be checked at a later time.

XII. Story of the Development of Fractions

This exercise illustrates how historical material may be presented to the children.

The adding or subtracting of fractions is very easy if the denominators are alike. In other words if fractions are the same size they can be added just like tons can be added to tons and lbs. to lbs. Ancient man had just as much trouble adding unlike fractions as does the modern schoolboy. In fact, he probably had more trouble because he had no text and often no teacher. In order to make addition and subtraction easier, ancient man tried to make fractions all the same size. The Babylonians tried to make fractions all *sixtieths*. When a fraction did not come out to be exactly a sixtieth, they made another part which they called a little sixtieth. Thus, an hour was first divided into 60 parts and then each sixtieth divided into 60 more parts. Of course, you can see where we get the 60 minutes in an hour that we use, and the 60 seconds in a minute. You don't even think of them as fractions but they really are fractions of an hour. The Romans found that a sixtieth was too small a fraction for some things. They divided their foot into 12 parts. From that we get our inch as a twelfth of a foot. None of these schemes of using fractions of the same size worked for all things; e.g., if a pie is to be divided among 8 people, each will get an eighth and it is difficult to use tenths, twelfths, or sixtieths in dividing it so that each of the eight people will get his share. Even though ancient man's scheme did not work, we still use some things which resulted from his attempts to make all fractions the same size. Can you name three of the things we use that are the result of ancient man's attempt to simplify fractions? (For further discussion of this topic, see Wheat: *The Psychology and Teaching of Arithmetic,* pp. 82-90; *The Story of Numbers,* American Council on Education; or any good history of arithmetic.)

Summary*

1. Review of processes taught in previous grades
2. Multiplication and Division with fractional quantities
3. Measurements to nearest tenths of a foot and yard
4. Addition and subtraction of decimals
5. Continued use of percentage idea
6. Continued use of charts and graphs

Diagnostic Test

(Mechanics of Fractions)

Multiplication

	Examples			Difficulty Introduced
1.	1/2 × 1/2	2/3 × 1/4	3/8 × 2/3	Fraction multiplied by a fraction with and without reduction
2.	4 × 4/5	2 × 1/3	5 × 3/10	Integer multiplied by fraction
3.	3/8 × 2	5/9 × 3	4/5 × 2	Fraction multiplied by whole numbers
4.	1 1/2 × 4	3 1/4 × 8	4 2/3 × 6	Mixed number times integer
5.	3 × 2 1/3	14 × 8 1/2	21 × 7 2/3	Integer times mixed number
6.	2 1/2 × 2/3 1 1/3 × 3/5 4/5 × 2 1/10			Products of mixed numbers and fractions
7.	3 1/2 × 4 2/3	1 1/2 × 8 1/2	4 5/6 × 2 1/2	Products of mixed numbers

Division

1.	1 ÷ 1/2	2 ÷ 1/3	6 ÷ 1/8	Integer divided by unit fraction
2.	3 ÷ 2/3	9 ÷ 2/7	6 ÷ 3/8	Integer divided by multiple fraction
3.	2/3 ÷ 2	1/2 ÷ 4	2/5 ÷ 3	Fraction divided by integer

*The summary for the first grade given on page 25 clearly expresses the use which should be made of the items listed in the summary on this page.

Examples			Difficulty Introduced
4. 2/3 ÷ 1/3	2/5 ÷ 3/4	3/5 ÷ 3/4	Fraction divided by fraction
5. 1 1/2 ÷ 2	3 3/4 ÷ 5	14 2/3 ÷ 6	Mixed number divided by integer
6. 16 ÷ 1 1/2	24 ÷ 3 3/4	38 ÷ 4 2/3	Integer divided by mixed number
7. 6 1/2 ÷ 3 1/2	4 1/2 ÷ 1 1/3	6 2/3 ÷ 2 3/4	Mixed number divided by mixed number

SEVENTH GRADE

Inventory Test

1. Using only the decimal point and the numbers 3, 6, and 2, write (a) the smallest number possible, (b) the largest number, (c) a number between 10 and 100, (d) the largest (three places) decimal.

2. (a) $\begin{array}{r} 407 \\ \times\ 80 \\ \hline \end{array}$ (b) $\begin{array}{r} 600 \\ \times\ 800 \\ \hline \end{array}$

3. Multiply and divide each of these numbers by 10 and 100:

(a) 2.6	(c) 362	(e) 19.32
(b) 34	(d) 462.21	(f) .623

4. 5/8 + 1/2 + 1/6 =

5. $92\overline{)2746}$

6. How many two-thirds in 8?

7. Count from 2 to 3 by (a) tenths, (b) hundredths.

8. 93.08 + 2.29 + 82.078 =

9. (a) 3 1/2 ÷ 2 (b) 14 1/3 × 3/2 (c) 1/4 ÷ 3/8

10. $\begin{array}{r} 13\ 5/6 \\ 4\ 3/4 \\ \hline \end{array}$ Subtract

11. Round the following:

(a) .65	3.421	52.86	To nearest tenths.
(b) 4632	521.4	634.82	To nearest tens.
(c) 44,642	1,634,214		To nearest thousands.
(d) 42,463,258	8,943,632		To nearest millions.

12. Find (a) 7/8 of 92.
(b) 3/4 of 15.

13. 18 is what part of 30?

14. (a) What number multiplied by 28 gives 7?
    (b) What number multiplied by .8 gives .2?
    (c) What number multiplied by .6 gives .02?

Content and Recommended Procedures

I. Methods of Review

It is very important for the elementary teacher to realize that individualized remedial work should be a continuous process; e.g., it should be distributed throughout the entire school year. According to this view a teacher cannot teach a given topic, say percentage, and then move on to the next topic and forget all about percentage. If the teacher does this she will find that the pupil will also forget about percentage. Percentage, decimals, fractions, etc., once taught must be brought back to the minds of the pupils by means of little tests and exercises scattered throughout the year. What has been said about percentage in the above discussion can be said about the other topics in arithmetic as well.

II. Multiplication and Division of Decimal Fractions

A. The multiplication of decimal fractions is best approached through the multiplication of common fractions whose denominators are 10 or 100. For example: $.2 \times 3 = 2/10 \times 3 = 6/10 = .6$. If the pupil works examples of this nature he will soon see that he can write the answer immediately without doing all the intermediate work. Then follow with $.3 \times 12$, etc. Let the pupil make the inductive generalizations rather than teach him by rule. After working several examples of the nature suggested above the pupils can easily make the necessary generalizations. Once the pupil has observed (for himself) the short way of working the above problems the multiplication of two or more decimal fractions will cause no difficulty. Emphasize the short way of multiplying by 10 and 100. Approach the idea inductively.

B. The division of decimal fractions should be approached inductively by means of a series of examples of the following type:

$\begin{array}{r}4\\2\overline{)8}\end{array} \qquad \begin{array}{r}4\\20\overline{)80}\end{array}$ (Multiply both dividend and divisor by 10.)

$\begin{array}{r}2\\3\overline{)6}\end{array} \qquad \begin{array}{r}2\\30\overline{)60}\end{array}$; $\begin{array}{r}2\\9\overline{)18}\end{array} \qquad \begin{array}{r}2\\90\overline{)180}\end{array}$ In each of the above pairs, both dividend and divisor have been multiplied by 10 and the quotient has not been changed. Hence, $.2\overline{)9} = 2\overline{)90}$.

This transition can be made readily by the pupil's being sure that he knows how to multiply by 10 without any extra work.

At first the problems can be written in the form: $.2\overline{)8} = \begin{array}{r} 40 \\ 2\overline{)80.} \end{array}$

However eventually the method: $\begin{array}{r} 40 \\ .2\overline{)8.0} \end{array}$ may be taught.

Those examples involving divisors such as .36, 4.26, etc. should be approached inductively just as outlined above for the divisors, such as, .3, .8.

Emphasize division by 10 and 100 as this will be a necessary skill in percentage. This may be taught by giving a large number of exercises of the following type:

$100\overline{)34.6}$ $100\overline{)421.}$ $100\overline{)1362.}$ $100\overline{)4.29}$ etc.

The better pupil will soon look for a short way "out" and discover that relationship between divisor and quotient is a very simple one, thereby enabling him to write the quotients immediately.

III. Graphs

A. Principally the bar, line and pictorial graph are to be taught during the first part of the seventh grade. The circle graph may be introduced in the latter part of the seventh or in the early eighth grade.

The students should know that in general the bar graph is used to compare two or more quantities with one another, the line graphs to show trends, and that the circle graph is used to show the relation of the part to the whole. Given any meaningful situation the student should be able to decide which of the three graphs could best be utilized in order to get across the particular point brought out in the original data.

The teacher should keep in mind that the main objective in teaching graphs is to teach pupils to read graphs. The graphs found in the daily papers and magazines, materials of the social studies and health should be studied critically for the comparison and the relationships which are presented by the graph. However, the student should construct a number of neat, well-balanced graphs as this will assist in developing meanings as well as give experience in choice of the type of graph to use and artistic arrangement. In teaching graphs it is not enough to say that the graph shows "such and such" but several pertinent questions concerning what the graph does (and does not) show should be asked to see if the pupils really understand what the graph is trying to show.

B. Suggested graphs

1. Show by means of a graph the relative lengths of the five largest rivers in the world.
2. Have pupils find approximates to daily average temperature and keep a graph of results. (Correlate with science work.)
3. Variation in food prices
4. Variation in market prices (bonds or livestock)
5. Comparison of size of states, productive capacity, waste lands, etc.
6. Number of books read
7. Height of mountains
8. Average rainfall, etc.
9. Health data, etc.
10. Facts gleaned from the daily papers
11. Facts about coal, corn, oats, farm tractors, etc.
12. Temperature variations, weekly, monthly, and yearly

Remember: (1) let the pupil decide what kind of graph would best represent the data; (2) correlate the work with science, health, etc.; (3) also have the pupil find his own data for some of his graphs; (4) discuss in class and make comparisons in every way possible by such questions as (a) What state is twice as large as Iowa? (b) When was the temperature 30°? (c) What does the "steepness" of the line indicate? (d) Why make a line graph instead of a bar graph? Also discuss general appearance, neatness, and the visual appeal, etc., of each graph made by the class.

IV. Ways of Comparing Quantities

The mathematics of the elementary school provides several ways of making comparisons between quantities. Ratio is only one of the several ways of making comparisons. Below are listed some of the ways of comparing numbers which are taught in the grades below the seventh grade.

A. The different ways of comparing quantities taught in the previous grades

1. How much larger, taller, heavier, etc.? (subtraction)
   Ex. How much taller is John than Mary?
2. How many more? (subtraction)
   Ex. How many more apples are there in this basket?
3. How many times greater (smaller)? How many times as many, etc.? (division) (multiplication)
   Ex. John has five times as much money as Mary.

4. What part of? (division)
   Ex. 6 is what part of 12?

B. The methods of comparing quantities to be taught in the seventh grade

   1. Percentage
      Ex. 21% of all accidents happen in the home. See Section V for the discussion of percentage.

   2. Ratio
      It is probably best to teach ratio in connection with graphing.

Center the pupils' attention on the idea of comparisons; i.e., if in certain recipes the ratio of sugar to flour is 1 to 6, it means that 6 times as much flour as sugar is required. Recipes furnish many good choices of using the ratio idea.

Emphasize the two ways of writing ratio:
(a) 2 to 3 and (b) 2/3.

Situations for class use:

1. Recipes of all kinds.
2. Mixtures.
3. Have pupils find the ratio of the length of a room to its width.
4. Same as (3) for picture frame, rectangles, square, etc.
5. One pile contains three arithmetic books and one 5 arithmetic books. What is the ratio of their weights, heights, etc.?
6. Science and social science furnish many illustrations of the ratio idea. Town A is twice as large as town B, etc.
7. Draw lines on the board and have pupils find the ratios of their lengths.
8. Name numbers whose ratio is 3/4, 2 to 3, etc.
9. Find cities whose populations are approximately in the ratio of 1 to 2, etc.
10. Find ratios of areas of rectangles.
11. Scale drawings of all kinds (very important).

V. Percentage

This is a very important topic and its ideas should be well mastered because thinking in terms of percentage is so common in everyday life. Percentage is merely a new name for an old idea. The student should be able to relate the percentage terms with the common fraction and with the decimal fraction; i.e., 6% = 6/100 = .06.

Given any one of the three equivalent forms the student should be able to give the other two. And teach it so that they know what it means. Percentage is not difficult unless the teacher makes it difficult.

Begin by emphasizing that

$$6\% = 6/100$$

In other words that per cent is merely a new name for hundredths. If this idea has been emphasized during the fifth and sixth grades and *if* the pupil has been taught to multiply and divide by 10 and 100, then the next step will be very simple, in fact, obvious to the pupil. Now

$$6\% = 6/100 = .06$$

Work for a considerable time with the very simple cases.

The more difficult cases in percentage can be taught somewhat as follows:

$$1/4\% = .25\% = .25/100 = .0025$$
$$300\% = 300/100 = 3$$

Percentage should not be taught as three cases. Instead the cases should be taught as slight variations of one general idea. The general idea is that $2 \times n = 6$, then one must divide 6 by 2 in order to find *n*. This is a fact which the work in multiplication in the third and fourth grade should have taught the children. This can also be generalized by saying that the product can be divided by one of its factors to find the other factor. As an example: In working the problem, Mr. Brown allowed his son Bob $800 for his first year of college. Of this amount Bob had to spend $150 for tuition. What per cent of his allowance did he spend for tuition? This problem should be discussed long enough so that the student sees that the essential question is: "$150 is what % of $800?" Now by easy steps this last statement can be changed to $150 = n \times 800$.

Note that this last step is just a short way of writing the essential question. Of course, it has been put into a positive statement and the symbol *n* has been used for the unknown "per cent." The solution now is quite similar to that used by the third grade child in solving the problem: "Rabbit A weighs 12 lbs. while rabbit B weighs only 3 lbs. How many times as heavy as B is rabbit A?" The essential question then becomes, "12 is how many times 3," or in brief arithmetical symbols $12 = n \times 3$. In solving such a problem 12 is divided by 3. The same plan is followed in solving $150 = n \times 800$. From previous work the student should know that in order to find *n*, he must divide 150 by 800. This gives the answer in terms of decimals. The last step is to change the decimal fraction to per cent.

The other "hard case" in percentage is of the type: 60% of what number is 42 (commonly known as Case III). This can be abbreviated to $.60 \times n = 42$. Again use the fact that if the product and one of the factors are known, one divides the product by the known factor to find the unknown factor. Therefore, $n = 42/.60 = 70$.

This type of thinking has been used by the child since he was in the third grade. To demonstrate the simplicity of the procedure use a third grade problem like the following: "How many walnuts can Jack give to each of four boys if each is to receive the same number and he has only 20 walnuts?" The essential question becomes: 4 times what number is 20? In arithmetical language this statement is written: $4 \times n = 20$.

Since case II and case III have been illustrated perhaps consideration of case I would be worth while even if it is much simpler than the other two. In case I problems the procedure is to find a per cent of a number; e.g., 6% of 24. This becomes $.06 \times 24 = n$. The following is a third grade problem illustrating the procedures followed in the solution of such problems. "How many cakes are there in 4 packages if each package contains 6 cakes?" Changed to a positive statement this problem is, "How many are 4 times 6?" Changed to a positive statement this becomes 4 times 6 is some number. In arithmetical symbols this statement is written $4 \times 6 = n$.

Note that it makes no difference whether the problem is a Case I, II, or III in percentage. They all fall under this one general idea.

VI. Intuitive Geometry

In the seventh grade the teacher should place more detailed emphasis on the refined use of the ruler, yard, square foot, etc. Seventh graders should be able to use these instruments intelligently. Seventh graders should also be given considerable practice in estimating lengths, heights, weights, areas, and other standard units found in the outline. The use of the protractor, compass and the meter stick are the only new instruments which are introduced in the seventh grade.

In the field of geometry the teacher has an excellent chance to teach her pupils to discover things for themselves. Do not tell the pupil that the diagonals are equal. Ask the pupil if he can discover something that is true about the diagonals. Then have him do the same with squares, parallelograms, etc. Tell as a last resort. Good teaching is not a continuous stream of "telling." Furthermore, the children should be allowed to measure geometric figures in every conceivable situation. Have the class draw to scale the school grounds, construct baseball diamonds, make models of buildings, draw farms to scale, etc.

A. Measurement

1. Linear (English and Metric)
    a. Standard units of measure
    b. Errors and their causes
    c. The ability to measure to nearest
        (a) ft. (m)
        (b) inch (cm)
        (c) quarter or eighth of an inch
        (d) tenth of a cm.
    d. History of the metric system
        (a) Reasons for the development of the system
        (b) Why the system is used by industry and science
        (c) Arguments for and against adopting the system in the U. S.

2. Angular
    a. Define an angle as the amount of turning about a point. Have the pupils illustrate by use of rulers, arms, angles found in the room, etc.
    b. Kinds of angles: acute, right, straight
    c. Units of measure: degree, min., sec.
    d. Historical backgrounds for units of angular measure
    e. Use of the protractor

B. General Principles

These principles should be taught by a discovery method, the teacher giving hints whenever necessary.

To illustrate the method of inductively approaching the facts of geometry (grades 6, 7, 8, and 9), an outline of a possible way of teaching the important relationship $C = \pi D$ is presented below.

The question of the relationship of the circumference to the diameter of a circle can be brought up in class through some problem situation. Of course the question might also arise as to whether there is a relationship between the circumference and diameter of a circle and whether this relationship is the same for all circles.

This question leads to an investigation, the methods of investigating the relationship to be planned by the class with the help of the teacher. In any case the class and teacher would eventually decide that one way of shedding some light on the problem would be to measure the circumference and diameter of a number of circles. For this purpose pupils may take tape measure, rulers, string, etc., and go about the schoolhouse and school grounds measuring a number of circles (tops of wastepaper baskets, jars, cans, bicycle wheels, spare tires on automobiles, lid to drinking fountain, etc.). These data are brought in and organized in tabular form, in some such manner as follows:

	Object Measured				
	Tire	Wastebasket	Wheel	Tin Can	Etc.
C	______	______	______	______	______
D	______	______	______	______	______

The question now arises as to how to compare the circumferences and the diameters. Comparison might involve subtraction or it might involve finding out how many times larger the circumference is than the diameter. If the latter is the method which seems most promising to the class of producing results, they then carry out the necessary divisions and will then have a third row of figures to add to the table given above. This row will be the ratio C/D.

If the class has been careful in making the measurements, each of the numbers C/D will all be somewhat larger than 3. This is readily observed by the pupils. However, the variations will also be observed and the question now is: Does it seem that the ratio C/D is the same for all circles regardless of size? This question will also lead into a discussion of errors in measurement. The class is now ready to consider a method of finding a more reliable number to represent the ratio C/D, and the problem of how to do this is brought up. After some discussion and with the help of the teacher they will eventually arrive at finding the average of all the ratios obtained in their previous work. If the measuring has been done with care and if 10 to 20 ratios have been obtained, the result will be a very close approximation to the value of *pi*, the ratio of circumference to diameter. Further remarks by the teacher will lead the pupils to accept readily the value 3.14 for the ratio of circumference to diameter.

The following general mathematical principles are readily taught by a method very similar to that outlined above for the relationship between the circumference and diameter of the circle.

1. The sum of the angles of a triangle
2. The sum of the angles of a quadrilateral
3. The areas of rectangles, parallelograms, squares, circles, and trapezoids
4. The relationship between the angles formed by parallel lines and a transversal

C. Construction with the ruler and compass
   1. Right angle
   2. Perpendiculars
   3. Rectangles, squares, and parallelograms
   4. Parallel lines

5. Regular hexagons
6. Equilateral and isosceles triangles
7. Designs

D. Terms

The pupils should be able to define and illustrate: circle, circumference, cube, degree, diagonal, diameter, equilateral triangle, isosceles triangle, minute, octagon, parallel lines, parallelogram, perpendicular, perpendicular bisector, pi, protractor, congruent triangles, quadrilateral, radius, rectangle, right angle, right triangle, sphere, similar triangle, square, trapezoid. The pupil should be able to illustrate acute triangles, amount of turning, arc, area, cone, obtuse triangle, prism, pyramid.

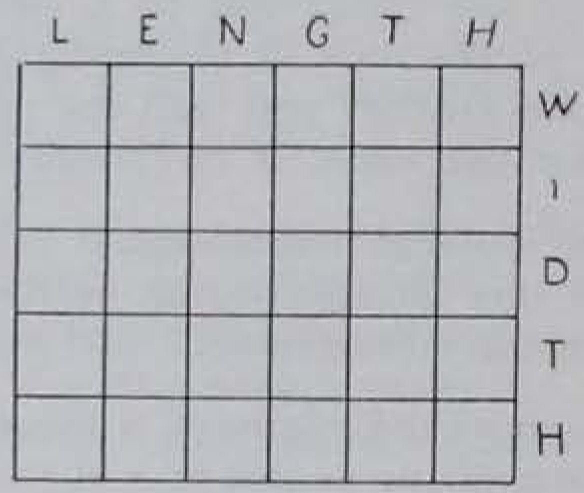
VII. Formulas

Understand and be able to use the following:

$I = prt$	$A = bh$	$C = \pi d$
$A = bh/2$		
	$A = S^2$	$d = rt$

Oftentimes it is difficult for the student to change from verbalism to symbolism. Therefore, it is necessary for the teacher to make sure the students have an understanding of the formula by changing from verbalism to symbolism, by easy steps. Following are some hints.

For example, in the case of the formula for finding the area of a rectangle:



A. Use the words *length* and *width* rather than *l* and *w*.

B. From their discussion and study the students will see that—the area of a rectangle is equal to the length times the width.

C. The students will know the symbols for *equal* and *times*. Then substitute these symbols, thus — The area of a rectangle = the length × the width.

D. Next, so the expression is not so long we write — Area = length × width.

E. Then, ask how it could possibly be shortened because it still is too long. What letter of your name do you use for your initial? How are the following abbreviated: rural route? free on board? cash on delivery? works progress administration? Now, how would you abbreviate area, length, and width? We get this statement, $A = l \times w$.

F. After you have taught the formula, have the students state it in verbalized form.

G. Be sure that pupils understand that the formula is a "shorthand way" of writing a complete sentence.

VIII. Social Uses

The following are topics which are usually discussed in other areas of the elementary curriculum to which mathematics can make a definite contribution. It is suggested that these topics be studied whenever they will correlate with the work in the other areas. Only in this way can this work be made to function as it should.

A. Mail service (air, train, etc.)

B. Roads

C. Health and thrift

D. Food values

E. Electricity in the home

F. Owning an automobile

G. Money and banking
   1. Simple interest
   2. Compound notes
   3. Promissory notes
   4. Savings accounts
   5. Checking

H. Buying in quantities

I. Profit, loss and margin

J. Commission and discount

K. Bus, train, and airline time tables

L. Sending money
   1. Postal money order
   2. Telegraph
   3. Personal checks
   4. Sending cash through mails

IX. Historical Materials

No fixed time or no full periods are devoted to the history of the ideas of quantity. This material is to be introduced as the occasion arises and as interesting sidelights of the processes and ideas of arithmetic.

Some good references are:

Wheat. "The Psychology and Teaching of Arithmetic"
Smith, D. E. "The Wonderful Wonders of One, Two, Three"
Smith, D. E. "Stories of Long, Long Ago"
Smith and Ginsburg. "Numbers and Numerals"
Parker, Bertha. "The Story of Numbers"
Parker, Bertha. "The Story of Measurement"
Parker, Bertha. "The Story of the Calendar"

The second and last three references are written for the elementary school child.

X. The elementary school child should be taught how to estimate answers to problems

If the problem involves multiplying 42 by 19 the pupil should be taught to think $42 \times 20 = 840$, therefore 42×19 should equal about 840. If the problem involved the multiplying of 6 5/6 by 4 the pupil should think $7 \times 4 = 28$, therefore the answer should be about 28. Etc.

This type of check is extremely useful in everyday life and also very common. If one buys a number of articles, one for $1.49, another for $.27, and another for $2.10, he is apt to check the clerk's figures mentally by saying $1.50 + $.25 + $2.00 (or $2.10) = $3.75 ($3.85). Hence if the figure deviates from $3.75 by more than a few cents the clerk's figures are wrong.

Diagnostic Test
(Mechanics of Decimals)

Addition and Subtraction

Add				Difficulty Introduced
1.	3.8 14.2 263.4	.03 .46 1.42	46.2 28.3 19.4 21.8	
2.	3.42 + 46.4 + 214.63 .049 + .98 + .483 462.9 + .03 + 384			Placing of numbers
3.	Subtract 462 36.4	 92.14 1.82	 43.6 39	
4.	16.1 — 4.9; 34.8 — 14.9; 961.38 — .042			

Multiplication

1.	.4 × 5	2.1 × 9	9.85 5	Mixed decimal by integer
2.	36. × .4	482 × .42	96 × 4.5	Product of integer by mixed decimal
3.	38.4 2.4	3.62 4.1	4.62 .034	Mixed decimal by mixed decimal
4.	3.6 × 10	4.289 × 100	.0434 × 1000	Short cut in multiplication by 10, 100, and 1000

Division

1.	4/ .84	9/ 3.86	41/ 96.42	Decimals and mixed decimals by integers
2.	36/ .0342	142/ 1.4936	962/ .003	Harder cases
3.	.8/ 36	.84/ 362	1.4/ 14.6	Divisor a decimal
4.	19/ 12	142/ 26	92/ 8	Integer divided by integer
5.	36.4 ÷ 10	4.34 ÷ 10	942 ÷ 100	Division by 10, 100, and 1000 as short cut

Common Fractions to Decimals

1.	5/8 =	9/13 =	15/22 =	Answer to two decimal places
2.	36/43 =	9/42 =	18/37 =	Answer to *nearest* tenths
3.	15/29 =	36/84 =	192/235 =	Answer to *nearest* hundredths
4.	9 3/5 =	18 2/19 =	34 21/34 =	Mixed numbers to mixed decimal; answer to nearest tenth or hundredth

Suggested Examination Questions

(Decimal Fractions)

1. Which is the largest number: 3.1, 2.99, 3.15, 3.01?
2. Count by tenths from 1 to 2. Count by hundredths from 1.1 to 1.3.
3. 44.4. The first *four* on the left represents a value which is ________ of the middle *four*. The middle *four* represents a value which is ________ of the *four* on the right.

4. Why are the decimal points in line when decimal fractions are added?

5. By means of diagrams show that .3 plus .6 = .9.

6. Remembering that decimal fractions are only a new way of writing common fractions whose denominators are 10, 100, 1000, etc., write the following as decimal fractions: 5/10; 15/1000; 7/100; 56/100.

7. Write the following as common fractions and reduce to lowest terms: .15; .45; 2.5; .156; .98.

8. Eight-tenths times some number gives 16 as a product. Find the number.

9. By changing to common fractions show why .12 times 4 equals .48.

10. .5 equals how many hundredths?

11. By means of a diagram find how many .3 there are in 4.

12. Draw a line six inches long and place on it the marks corresponding to 1 inch, 2 inches, 3 inches, 4 inches, 5 inches. Label these points. Now place points on the line which correspond to the following numbers: 3.5 inches; 2.75 inches; 4.6 inches; .75 inch. What point is half way between 3.4 and 3.5 inches? What number would correspond to this point? What number is half way between 4.8 and 4.9 inches?

13. 4.5 is how much larger than 3? 4.5 is how many times larger than 3?

14. If an automobile averages 1.24 miles per minute, what would be its average speed in miles per hour?

    a. How many minutes in an hour?

    b. If the automobile averaged 2 miles per minute, what would be its average hourly speed?

    c. If it averaged 1.24 miles per minute, what would be its average hourly speed?

15. What is the short way of stating that: Harry has 33 cents. This is .3 of the money that John has. How much money does John have? (Note: The problem says that .3 of John's money equals 33 cents

    or .3 times N = 33.)

    Now to find what number times .3 will give 33 as an answer, one divided 33 by .3.

EIGHTH GRADE

Introductory Remarks

Social Uses. Approximately one half of the work in the eighth grade deals with topics which are also covered in other areas of the elementary curriculum. For example, the general subjects of insurance and taxation are covered in the social studies. A prevalent practice is to have the arithmetic teacher discuss taxation when it is convenient, and the social studies teacher is guided by the same rule. The result is that taxation in arithmetic becomes a problem unit without the necessary social understandings to make the topic meaningful and significant; and taxation in the social studies avoids almost all quantitative expressions and relationships, thus making the work of this area deal in rather vague generalities, lacking the concreteness and definiteness which are necessary for well defined concepts and understandings. It is strongly recommended that the elementary teacher use her own initiative in seeing that the work in these areas is correlated at least to the extent that they are treated simultaneously or that the work in arithmetic follows shortly after the corresponding unit in the social studies has been completed. Correlating the work in mathematics in this manner will get away from the "problem course" which is bound to lead to considerable artificiality and which tends to cause a lack of interest on the part of the pupil. Mathematics cannot function as it should without occasionally borrowing background material from other areas and the other areas need the language of quantity developed in the mathematics class.

Use of Tables. Tables provide an opportunity to teach pupils to observe relationships between quantities. Some of these relationships are of vital importance and can be brought out only by detailed analysis of the table. For example, in life insurance doubling the age of the insured may or may not double the premium, or an increase in the age of the insured means, in general, an increase in the premium paid. The implications of the relationship existing between the amount of the premium and the age of the insured are important to anyone planning an insurance program and should be discussed. In the same way compound and simple interest tables can be used to show how interest and amounts vary as the time and the rate of interest vary. Many other relationships can be brought to light in any class discussion of any given table. Even tables of square root and squares of numbers have value in this respect.

The following are some of the tables with which the pupils should be familiar:

> compound interest, American experience mortality tables, tables of premiums on life insurance, table of squares and square roots of numbers, railway or bus or airplane time tables, tables of simple interest, budget tables as well as other tables usually found in the social sciences and the sciences as offered in the elementary school.

Formulas. In this grade several new formulas are presented. See grade seven for limits on presentation.

Inventory Test

1. Measure this line to (a) nearest foot, (b) nearest inch, (c) nearest meter, (d) nearest centimeter. (Teacher: Draw a line about four feet long on the board.)
2. Without using letters in the sentence express in words what the following formulas say: (a) $A = bh$ (b) $C = \pi d$ (c) $A = \left(\frac{b + b'}{2}\right)h$
3. Make the most appropriate graph of the following data:

Noonday Temperatures

Nov.	1	2	3	4	5	6	7	8	9	10	11	12
Temp.	60°	71°	76°	74°	50°	45°	52°	58°	55°	56°	73°	78°

4. With reference to the graph above answer the following questions:
   a. Does the graph show that the month of November was a cold month?
   b. When did the most rapid decrease in noonday temperature occur?
   c. What was the temperature on the evening of the fourth day? Are you sure? Why?
   d. Between what dates did the temperature rise?
   e. How does the graph show increases?
5. Find 60% of 34.4.
6. Dan bought a tennis racket for $3.50, which was 25% below the regular price. Find the regular price.
7. Using your compass and rule draw (a) a square, (b) a parallelogram.

8. Find Mr. Wilson's margin on eggs bought at 28¢ a dozen and sold for 35¢ a dozen. Did he make a profit of 7¢?

9. 3 is what per cent of 10?

10. To the Teacher: Find a suitable graph in the paper, magazine, or in one of the books used in health, social studies, science, etc. Ask several questions bringing out the pertinent facts shown (or not shown) by the graph. This will show whether the pupil can read graphs and whether they read "too much into" a graph. (e.g. See question 4a of this test.)

11. a. What will be the effect upon the area of a triangle if the base is doubled? (Altitude remains constant.)
    b. Doubling the side of a square changes the area to __________.
    c. If the diameter of a circle is increased by 1 inch, the circumference is ____________ by __________.

12. If you had forgotten the value of *pi*, how could you find its approximate value? (Assuming no books are available.)

Content and Recommended Procedures

I. Expressing Relationships

A. Considerable emphasis should be put upon the different methods of expressing the relationships between quantities which vary; namely, the

1. Verbal statement
2. Table
3. Graph
4. Formula

In order to insure understanding the class should discuss advantages and disadvantages of each method of expressing relationships and become familiar with their use in various situations. See grade seven for method of presenting formula.

B. The equation is another way of expressing relationship between certain kinds of quantities. If percentage has been taught as suggested by this course of study, the student will already have had an introduction to this method of expressing relationships between quantities, and little need be said to justify a further study of equations of the type

1. $x + 4 = 6$
2. $2x = 42$
3. $x/3 = 9$
4. $x - 5 = 1$

At this time the student should be introduced to the four principles of addition, subtraction, multiplication, and division used in solving equations.

Since mathematics is the language of quantity, some emphasis should be placed upon translation exercises of the kind: A certain number less 5 is 8. Find the number. Translated in symbols this says $n - 5 = 8$.

The topic of proportion can also be very conveniently taught in connection with relationships between quantities. Recipes and mixtures of all types furnish excellent material for a study of proportion and the use of equations of the type $x/2 = 3/4$.

II. Insurance (Follow the textbook.)

A. Life Insurance
   1. Kinds of policies
      a. term
      b. ordinary
      c. limited life
      d. endowment
   2. American Experience Mortality Tables
   3. Terms to be taught: policyholder, insured, beneficiary, policy, premiums, expected lifetime

B. Fire Insurance

C. Automobile Insurance
   1. Kinds of protection obtainable
      a. property damage
      b. liability-property
      c. liability-personal
      d. fire-hail-wind, etc.
      e. theft
      f. collision
   2. Terms to be taught: face of policy, premiums

D. Other kinds of Insurance
   1. Old age
   2. Farm crop
   3. Bank deposit
   4. Health
   5. Unemployment
   6. Employer's liability
   7. Accident

III. Taxation (Follow the textbook.)

A. Why pay taxes?

B. What taxes do we pay?
   1. School
   2. City

3. County
4. State
5. Federal
6. Income tax
7. Sales
8. Inheritance
9. Licenses
10. Property
    a. real
    b. personal
11. Social Security
12. Internal Revenue
13. Customs

C. Terms to be taught: tax, assessment, rate, assessed valuation, levy, assessor, assessed value

IV. Investments. (Follow the textbook.)

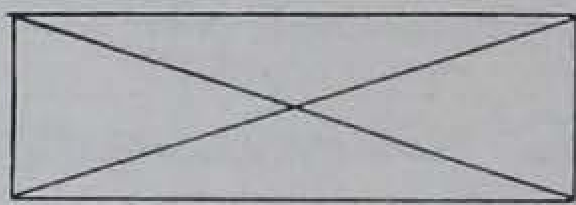
A. Way to invest money
1. Savings bank
    a. Simple interest
    b. Compound interest
2. Real estate
3. Mortgages
4. Bonds
    a. U. S. bonds
    b. Mortgage bonds
    c. City bonds
    d. Road bonds
5. Stocks
    a. Common
    b. Preferred
6. Annuities
7. Promissory notes
8. Building and Loan Associations

B. Factors to consider in making an investment
1. Safety
2. Yield
3. Marketability

C. Terms to be taught: simple interest, compound interest, mortgage, foreclosure, corporation, par value, dividend, market value, stock exchange, brokers certificate, brokerage, yield, share, board of directors, stockholder, face value, common and preferred stock, buying on margin

V. Measurement

If a pupil is given a pair of dividers (compasses), a ruler and a protractor, he can discover many vitally important relationships between the various parts of simple geometric figures. It is important that the pupil discover these relationships himself, or at least assist in their discovery. Relationships discovered for oneself are remembered longer and, from a teaching point of view, the method of discovery has even greater significance. To discover a relationship means that the pupil must measure, draw and study. In other words, he must become familiar with the use of the ruler, protractor and compass. This is very important. There is yet another important feature of the method of discovery. There is no better way to find out whether the pupil has mastered certain elementary concepts and techniques of measurement than to let him demonstrate his ability to discover other relationships. For example, consider the multitude of relationships that can be discovered by the pupil in the figure below.



There are numerous equal angles to give some practice in measuring angles. The fact that the diagonals are equal and bisect one another, and the fact that the sides are equal furnish a number of situations for use of the compasses as dividers, and even the construction of the rectangle itself is educationally worth while from the standpoint of use of instruments, manipulative skills with instruments, and geometric facts needed for the construction.

In the same way it would be possible to approach the idea that the base angles of an isosceles triangle are equal. Have the pupil construct a number of triangles having two sides equal. Then the problem for the pupil is to find some relationship that is characteristic of triangles having two equal sides. This method of procedure can do much toward vitalizing the teaching. To the pupil who is discovering, mathematics is fun; but to the pupil who is told to remember a series of facts given by a teacher or the book, mathematics is too apt to become drudgery. Above all have the seventh and eighth graders use the techniques taught them in construction of baseball diamonds, finding areas of playgrounds, etc. This can easily be done during school hours and need not take the time away from the playing time of the pupil. Below is an outline of material that lends itself to teaching by the discovery method.

A. Direct measurement

Be able to experimentally verify and discover each of the following:

1. Parallelograms and rectangles
    a. Diagonals of a parallelogram or rectangle bisect one another.

b. Diagonals of a rectangle are equal.
c. Diagonals of a parallelogram bisect the parallelogram.
d. A tabulation of all the equal angles of a parallelogram and rectangle.

2. Isosceles and equilateral triangles
   a. The base angles of an isosceles triangle are equal.
   b. All the angles of an equilateral triangle are equal.
   c. The altitudes of equilateral triangles bisect the base.

3. Triangles and quadrilaterals in general
   a. The sum of the angles of a triangle equals 180°.
   b. The sum of the angles of a quadrilateral equals 360°.

4. A study of regular figures
   a. Triangles
   b. Quadrilaterals
   c. Pentagons
   d. Hexagons

5. Parallel lines
   a. If two parallel lines are cut by a transversal, the corresponding angles are equal.
   b. Two lines perpendicular to the same line are parallel.
   c. Recognition of parallel lines involved in drawing various geometric figures.

6. Areas (Be sure to develop the idea of a square unit as outlined in grade five.) Furthermore, the foundation idea here is the idea involved in finding the area of a rectangle. See seventh grade outline, page 92. Formulas are to be taught in connection with the work on areas.
   a. The area of a parallelogram
      (1) The pupils may cut parallelograms out of paper and then make rectangles out of a number of the parallelograms. This kind of exercise would demonstrate that the area of a parallelogram equals the base times the height. What other figures have the same formula for finding the area?
      (2) Exercises of the type: The area of a parallelogram is 48 square inches and the base is 12 inches. What is the height? Note how the method of working percentage problems helps in solving this problem.
   b. The area of a triangle
      Base the development of this idea upon the previously taught fact that the diagonal of a parallelogram bisects the parallelogram. Paper cutting may help get the idea across.
   c. The area of a trapezoid
      Two congruent trapezoids can be so placed that they form a parallelogram. From this fact the formula

can be derived. Have the pupils cut out two trapezoids to demonstrate that the desired formula is: $A = 1/2\ (b + b')\ h$.

d. The area of a circle

7. Volumes

Develop the idea of a cubic inch (ft.) by having the pupils construct a cube one inch (ft.) on each edge.

a. Prisms

(1) Rectangular: The formula for the volume can be nicely developed by means of placing a number of cubes one inch on each edge in a box whose dimensions are approximately $3 \times 4 \times 5$. As in all formulas, stress the fact that the formula is a shorthand way of writing the rule, the volume of a rectangular prism is equal to the length times the width times the height.

(2) Triangular

b. Pyramid

(1) Base a square

(2) Base a triangle

c. Cylinder

8. Surface

a. Prisms

b. Cylinder

B. Indirect measurement

Indirect measurement furnishes abundant opportunities for the intuitive study of similarity and symmetry. Ruler, compass, and protractor are, as in direct measurement, the only instruments needed for this work. Emphasize the use of these instruments for experimental work on the study of the geometry of this section.

1. What characteristics do similar things possess?

2. Similar triangles (outgrowth of B, 1)

a. Let the pupils experiment by drawing triangles which "look like" a triangle which the teacher has drawn on the board. Discuss the characteristics that the two triangles must possess in order to "look alike"; i.e., be similar.

b. As a special case of similarity discuss the conditions that must prevail in order that the triangles not only "look alike" but are in addition the same size; i.e., congruent triangles.

3. Ratios of corresponding sides of similar triangles are equal. Develop the idea of ratio. (See Seventh Grade outline, page 87.) Start with a triangle having sides of 5 in., 4 in., and 6 in. Have the pupils draw a similar

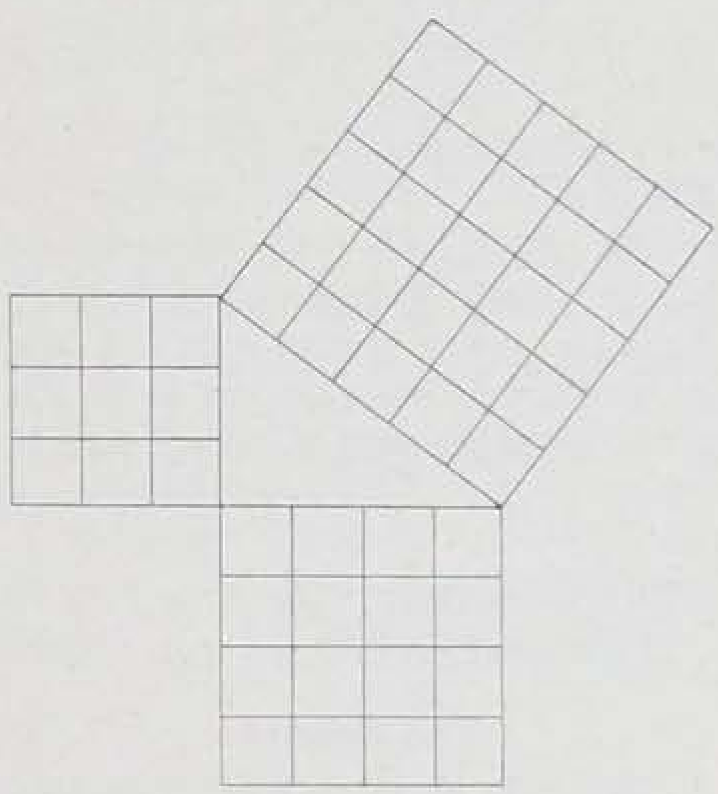
triangle to the above, using as the base line a 3-inch line which corresponds to the base of 6 inches in the triangle. What relationship, if any, exists between the sides of these two triangles?

4. Similar triangles provide another instance in which use can be made of the simple equation, such as $n/6 = 4/8$.
5. Use of similar triangles to find the height of trees, buildings, smokestacks, distance across rivers, etc. See Boy Scout and Girl Scout manuals for problems of interest to boys and girls.
6. Triangles are similar if corresponding sides have the same ratio. Most notable use of this idea is in the construction of scale drawings. The construction of scale drawings of rooms, farms, school grounds, etc., is of value. Furthermore, teach the pupils to read maps drawn to scale. In many cases county officials will furnish maps drawn to scale. The ability to read scales on maps, blueprints, etc., should be developed.
7. Study the symmetry of plants, flowers, leaves, designs, etc. Pupils should be able to point out axis of symmetry and show the use of symmetry in making designs.

VI. Study of the Right Triangle

A. The sum of the two acute angles of a right triangle equals 90°.

B. The Pythagorean Theorem
Have the pupil draw a right triangle whose sides are 3 inches, 4 inches, and 5 inches. Then have them construct squares on these sides obtaining a figure as shown below.



Leading questions will help the pupil to discover that the area of the square on the hypotenuse equals the sum of the areas of the other two squares. Repeat this for other triangles having sides (1) 6, 8, 10; (2) 5, 12, 13; etc.

C. Square root

The study of the Pythagorean theorem requires that the pupil get acquainted with a table of square roots. The general technique of using tables and the value obtained from the use of tables are discussed under the heading of Introductory Remarks for the eighth grade outline.

D. Use the Pythagorean theorem to
   1. Find diagonal of the schoolroom.
   2. Find the distance from one corner of the schoolhouse to the opposite corner.
   3. Find how the carpenter uses the Pythagorean theorem to check whether he is making a "square" house.
   4. Check the "squareness" of the baseball diamond, the tennis courts and the football field.

Supplementary Reading Materials for Grades 7 and 8

Below are listed several references written for the elementary school child. Many of these books can be found in juvenile libraries. Others could probably be purchased through library funds.

General References

1. Rugg, *Man at Work: His Arts and Crafts*, pp. 423-446
2. Linnell, *Behind the Battlements*, pp. 58-60
3. American Council on Education, *The Story of Numbers*
4. Smith, *Number Stories of Long Ago*
5. Smith, *Wonderful Wonders of One, Two, Three*
6. Smith and Ginsburg, *Numbers and Numerals*

Stories About Time

1. Bragdon, *Tell Me the Time Please* (suitable for third and fourth grades)
2. Flynn-Lund, *Tick-Tock—A Story of Time*
3. Ilin, *What Time Is It?*
4. American Council on Education, *The Story of Time*
5. Rugg, *Man at Work: His Arts and Crafts*, pp. 483-551; 68-74; 423-446
6. Beauchamp, *Discovering Our World*, Book I, pp. 156-163; 166-171

The Metric System and Weights and Measures

1. Rugg, *Man at Work: His Arts and Crafts*, pp. 464-465; 468-472; 72-74; 87; 421-480
2. American Council on Education, *The Story of Weights and Measures*
3. Schorling-Clark, *Mathematics in Life*, pp. 17050

The Calendar

1. Rugg, *Man at Work: His Arts and Crafts*, pp. 532-550
2. Lansing, Man's Long Climb, pp. 50-55
3. American Council on Education, *Story of the Calendar*

SUGGESTED ACTIVITIES WHICH TEND TO CORRELATE ARITHMETIC WITH OTHER SUBJECTS IN OTHER AREAS

(Grade Placement Rather Flexible)

Some Activities That May Be Carried On in Connection With a Gardening Unit

1. In connection with the study of the method of plant reproduction, or when planning a garden, the discussion may include something about the germination of seeds of the various kinds of plants. Such striking facts as the following will always arouse the curiosity of the pupils:

   a. The seed of the mulberry must be eaten by birds before it will germinate.

   b. The seeds of the sweet pea may be placed in boiling water for an hour or more without injuring them; in fact, this practice will make the seeds germinate more quickly.

   c. Seeds germinate faster in the darkness.

   d. Exposure to the light of an electric light bulb affects the period required for germination in a good many cases.

   The pupils will be interested in methods of proving that such statements as given above are correct. For this purpose an experiment will be described below from which pupils of various degrees of ability can derive something educational and worth while which can be carried out over a wide range. Of course, the upper grade pupils would be required to do more extensive experiments and analyze the data more thoroughly than the lower grade pupils.

   Children living in the country will undoubtedly know that soaking seeds in water hastens germination. However, the effect of light and chemicals on the rate of germination may be new to them and may furnish a very interesting problem for investigation. Select the seed of some garden plant that is slow to germinate, such as lettuce or parsnip. Place the seeds in a solution of household ammonia (one or two drops to a glassful of water) and set in sunlight for a time interval as outlined in the table below. After washing the seeds thoroughly, sow them in the garden recording the date sown. Then watch to see when the first seedlings appear.

Lettuce Seeds Sown May 1

Group	Time in Solution	Date of Appearance of First Seedling	Percentage of Germination
Control	0		
Exp. A	15 min.		
Exp. B	30 min.		
Exp. C	45 min.		
Exp. D	60 min.		

The class should thoroughly understand the need for a control group in this experiment. In the table above, Exp. A means experimental group A. The percentage of germination is determined by counting the number of seeds sown and the number of seedlings to appear. Members of the class could divide in groups using different concentrations of the solution to see if they could determine the optimum concentration—one group using 2 drops per glass, another 3 drops per glass, etc.

The effect of the light from an electric light bulb may be demonstrated by the same method outlined above.

2. An experiment may be conducted to show the effect of both the light and the ammonia solution. Place the seeds in the ammonia solution as before, and after washing them in clear water expose them to the light of an electric light bulb (about 40 watts) for 30 minutes. Vary the length of time during which the seeds are in the ammonia solution, but keep the time of exposure to the light constant, as well as the concentration of the solution. Have the class develop a method of recording data. Determine as before how long it takes for the seedling to appear and the per cent of seeds germinating. (The lower grades would say that 10 out of 50 seeds germinated instead of using the percentage terminology.)

3. Proceed as outlined above but keep the concentration of the solution constant as well as the time in solution, varying the time of exposure to the light.

   From the above experiments, the pupils should see that one factor is always allowed to vary but that the other two factors are held constant. After having collected the above data, determine the best and quickest way of raising lettuce or parsnip seeds. Did the parsnip seeds react in the same way that the lettuce seeds reacted?

4. Make a scale drawing of the school (or home) garden, showing how far apart the various vegetables should be sown. Write to the agricultural college at Ames, Iowa, or look in the various gardening books or catalogs.

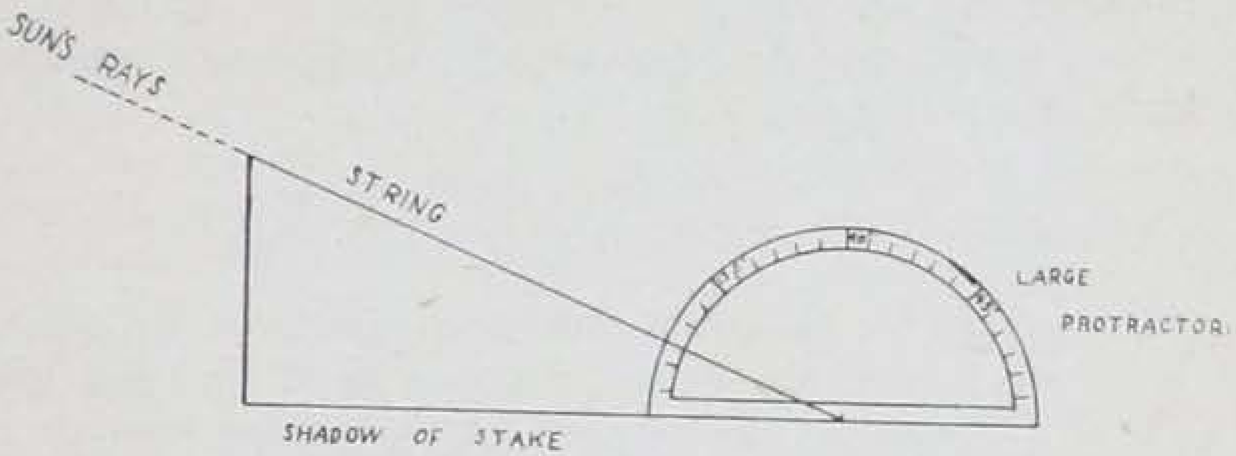
5. Numerous problems will suggest themselves to the teacher, for example:
   a. If cabbages are to be set 18 inches to 24 inches apart, how many cabbages can be set in a 30-foot row? In a space 20 ft. × 15 ft.?
   b. Plants used enormous quantities of water. If one pint of water is needed daily for the maturing of a plant, find how many gallons of water are necessary for a garden of 100 plants if they mature in six weeks.

Arithmetic Activities Related to Geography

1. Seasonal Variation of the Sun's Altitude
   The variation of the altitude of the sun causes the seasons of the year. This is an important fact to remember as it affects the lives

of the people inhabiting any given region of the earth's surface. In order to impress this fact upon the fifth, sixth, seventh, and eighth graders, the following activity may be carried out in connection with their study of the year.

Drive a stake in the ground and locate it so that the sun will shine on the stake at high noon. The location should be on fairly level ground so that the ground will make right angles with the stake. When the sun is at the highest point in the sky, measure the length of the shadow. Also measure the angle of elevation as shown in the diagram.



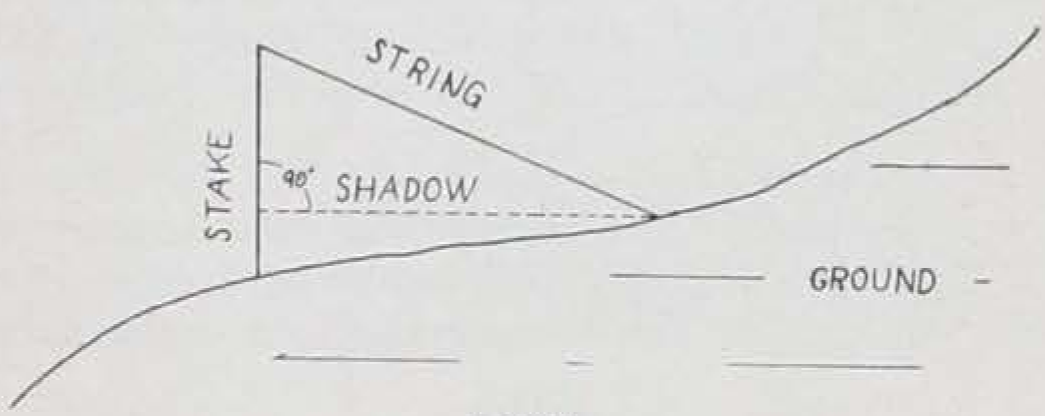
The large protractor may be made of plywood or several layers of stiff cardboard. The side of a large paper box would serve very well. (The project of making the protractor is worth while in itself.)

Some permanent place for keeping records should be selected, and the method of organizing the data should be discussed as well as the kind of data one would have to collect to show that the altitude of the sun did change as the season changed. A table such as shown below is probably sufficient, although the pupils should be allowed to modify this to suit their needs.

Date	Length of Shadow	Angle of Elevation of the Sun
September 10	2 ft.	75°
September 17	1 ft. 11 1/2 in.	73°
etc.	etc.	etc.

As soon as sufficient data have been accumulated the class can discuss the type of graph to be made in order to give a graphic picture of the data accumulated. This project may be continued throughout the school year, taking reading once or twice a week.

In case the stake is not situated on level ground the length of the shadow should be measured as shown below:



The teacher should refer to geography and science texts for source materials so that the class can study the relationship between the amount of heat transmitted to the earth's surface and the altitude of the sun. Excellent background materials will also be found in social science texts relating to the living conditions in the different degrees of latitude.

This method of studying shadows can easily be carried further so that the pupil is introduced to the study of the similar triangle and ratios. This part of the project would probably be reserved for the seventh and eighth graders.

If a yardstick casts a shadow 1 ft. long, then a tree whose shadow is 10 ft. long will be 10 yds. high or 30 ft. high; that is, the following proportion always holds:

$$\frac{\text{Length of stick}}{\text{Length of shadow}} = \frac{\text{Height of tree}}{\text{Length of tree's shadow}}$$

Have the pupils determine the height of various trees, buildings, windmills, etc. Discuss whether the triangles need be right triangles. Also find out if the pupils can discover a way to draw similar triangles on paper. What about the angles of similar triangles? The teacher will be able to supply more materials relating to similar triangles, or any seventh or eighth grade text on arithmetic will supply additional materials.

2. Teaching Scales on Maps and Globes

The importance of this work cannot be overestimated. The concept of representing distances by scale drawings on maps and globes is vital to any real understanding of many science, economic, and geographic facts.

Discuss methods of measuring amount of rotation of objects about a point such as yardsticks, wheels, etc. The pupils will have heard of an engine making "so many" revolutions per minute, or revolutions per second. Then what about a wheel that does not make a complete revolution in one second? The discussion is finally lead around to dividing one revolution (the circle) into 360 parts called degrees. Each degree is divided into 60 equal parts called minutes. These units are used to measure angles as well as distance along an arc that is a part of the circumference of a circle. Be sure to have the pupils decide on a definition for angles. The following definition can be used as a guide: An angle represents an amount of turning of a line about a point. (See arithmetic text.) Give considerable practice measuring angles and arcs of circles by means of protractors.

The problem to be studied is that of finding the distance between two points on the globe. The points selected should be of interest to the pupils. If they have been studying about London it may be Des Moines and London. If you were going to London by plane, what would be the shortest way? This question leads to the concept of great circles on the earth's surface as being the shortest distance. How could we measure this distance? There are a number of ways of measuring distances on a globe. One method would be to tell the class that the circumference of the earth is about 24,900 miles, to the nearest hundred miles. (What does nearest one hundred miles mean?) Then how would one find how many miles 1° on the earth's surface would represent? After doing the necessary division the class, led by the teacher, may decide that they will use 1° of arc = 69 miles. But the question still remains: How to find distances in miles on a globe? Stretch a string between London and Des Moines. Be sure the string is taut. Then, taking hold of the string at Des Moines and London, transfer it to the equator, placing one end on the Prime Meridian. The position of the other end will read the number of degrees of the great circle between Des Moines and London. But one degree represents 69 miles. Then the distance between Des Moines and London is 69 × the number of degrees of the great circle between these two cities. Repeat for other points of interest.

Navigators (aerial and marine) do not measure distance in terms of miles as we do. They prefer to use nautical miles because 60 nautical miles equal 60°; 1 nautical mile represents 1° of arc. How many statute miles (our miles) equal one nautical mile? How many minutes of arc between Des Moines and London? How many nautical miles? How many statute miles? Why do navigators use nautical miles (6,080 ft.) instead of statute miles (5,280 ft.)? City A has a latitude of 46° N., longitude 76° W. City B has latitude 20° N., longitude 76° W. How many nautical miles from city B to city A? How many statute miles?

3. Teaching the Meaning of Great Circles

Why is the path of ocean-going vessels represented on most maps by a curved line? On most maps (great circle charts are the exceptions) the great circle path on the globe which is the shortest distance between two points is represented by a curved line. Select two points separated by a considerable distance, such as New York City and Calcutta, India. Lay a yardstick across the map and read the latitude and longitude of several points on the straight line between New York and Calcutta. Plot these points on the globe as accurately as possible. Now stretch a string between New York and Calcutta on the globe. This string represents the great circle route and the shortest distance between the two cities. Does it follow the points plotted from the straight line route?

Now select two points on the globe and transfer points of the great circle route to corresponding points on the map. Does the shortest distance transfer to a straight line on the map?

Repeat the above for points in the southern hemisphere. On what side of the straight line does the great circle route lie in each case?

How Machines Aid Man in Doing Work

(See *Science and Nature Study, Iowa Elementary Teachers Handbook,* or consult index in your science text.)

In almost every science book used in the elementary school the topic of using machines to help do our work is discussed at length. Discussions and grade placement of this topic differ in different series of science texts. However, this need not involve any difficulty in so far as it concerns the use of quantitative language in learning how to use the machines. Below is described an experiment which can be carried out in any elementary school and any rural school. The only equipment needed is the yardstick and some known weights such as 1-pound, 2-pound, 3-pound, 4-pound, and 10-pound, or any other combination of weights. The primary purpose of this work would be to teach youngsters to collect and organize data in such a way that they can arrive at certain generalizations. The emphasis should be placed on the youngster doing the experimental work and arriving at the generalizations. The teacher should help only by means of suggestions and leading questions, if necessary.

Have the children balance a 2-pound and 1-pound weight on each side of the fulcrum (balancing point) and note the distance of the 2-pound weight from the fulcrum and the distance of the 1-pound weight from the fulcrum. Have the children change the weights to different positions, each time recording the distance somewhat as follows:

Distance of the 2-pound weight from the fulcrum (first weight)	Distance of the 1-pound weight from the fulcrum (second weight)
2½ in.	5 in.
6 in.	12 in.
etc.	etc.

Altogether the youngsters should have 6 to 10 pairs of numbers in the table which has been started above. The question for class discussion involves the relationship between the two distances given above. For example, what relationship is there between the 2½ inch and the 5 inch distances in the first reading of our experiment? For the lower grades the generalization which you would like to have the pupil arrive at is that in this case the 1-pound weight is twice as far away from the balancing point as is the 2-pound weight. Then the question arises: Is this true of all the other distances?

If any pupil has a reading which varies a great deal from this rule it might be well for the class to discuss the reasons for this variation and eventually suggest that maybe the reading should be retaken. For the upper grades the answer to the above question would involve the terminology of ratios. The upper grades would say that the ratios of the distances are 2 to 1. Seventh and eighth graders will be able to make the general statement:

Distance of first weight from fulcrum x no. lbs. = distance of second weight from fulcrum x no. lbs., or

$$W_2D_2 = W_1D_1$$

where the little twos and ones mean weights and distances of the 2-pound and 1-pound weights, respectively.

The above experiment can be repeated for various combinations of weights, such as 4-pound and 1-pound, 2-pound and 5-pound. In every case the pupil would be asked to find the relationship between the two distances, thus bringing in the idea of the comparison of lengths by means of ratios in both the lower and upper grades. However, note that the terminology used will differ depending upon the grade in which this experiment is performed. Note again the values obtained from an experimental procedure of this kind:

1. The pupil must collect his own data.
2. He must organize the data.
3. He must arrive at a certain generalization if at all possible.

The teacher should observe that "How Machines Aid Man in Doing Work" can be integrated with the work in social studies as well as science and mathematics. The effect of the machine age upon an economic and social system is discussed in social studies series written for the elementary schools.

Signs of Fall

A unit on "Signs of Fall" illustrates how readily materials are tied together for the elementary school pupil. Here one finds numerous opportunities for art, science, and language skills and arithmetic to work together in developing educational ideas relating to some firsthand experiences. Work of the nature outlined below should stimulate the pupils to become more observant and to become cognizant of what takes place in the months of September, October, and November.

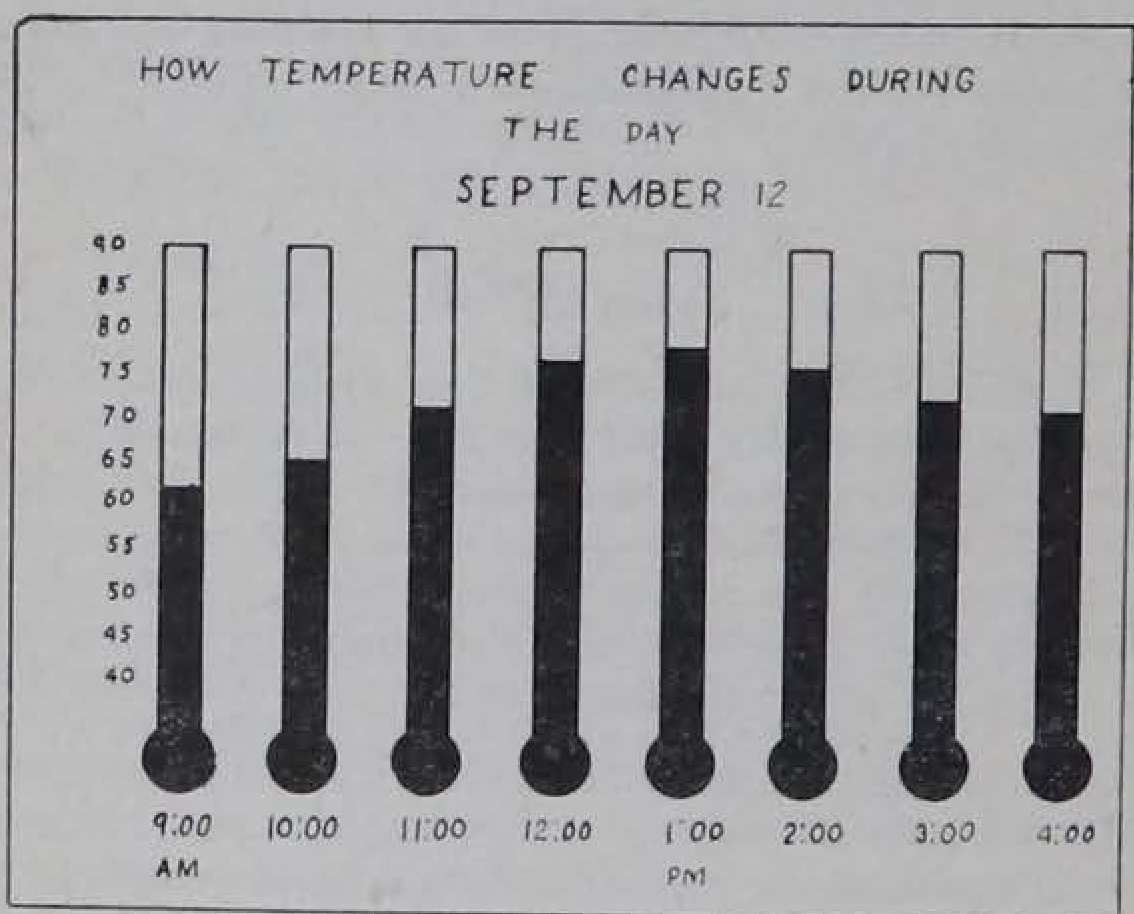
The science work centers around the coloring and falling of the leaves. It also deals with what happens to the animals and birds, how they change their living habits or habitat. Work centering around the change in living habits of people, what people do to prepare for winter, and many other topics readily found in science and social science series should be included in this unit. The resourceful teacher can easily correlate the art and language work with materials dealing with signs of approaching winter.

It is our purpose to go into greater detail as to the quantitative aspects of this unit. We suggest, therefore, that the teacher use this material in presenting this unit in order to make more understandable valuable scientific and social concepts.

A concept of enormous importance to an adequate understanding of what takes place in the months September, October, and November centers around the variations in temperature. Placing a dime store thermometer outside the window of the schoolroom affords an excellent opportunity for second and third graders to keep records of temperature variations.

For example, have the class make a table of daily temperatures taken each hour of the day. Probably from 9 A.M. to 4 P.M. would be the most convenient hours. However, if a member of the class can take the readings at 7 and 8 o'clock in the morning, it would be well to include these readings. Having the data in tabular form, the class can make a graph of temperature variations for the 5 days of the week as illustrated below. Having followed this for 2 or 3 weeks, they should arrive at the generalization that the temperature in general is higher during the hours from 12 to 2 P.M. They would note, for instance, that during sunny days the daily temperature variation is greater than during the cloudy days, and that on some days the temperature may decrease during the day rather than increase.

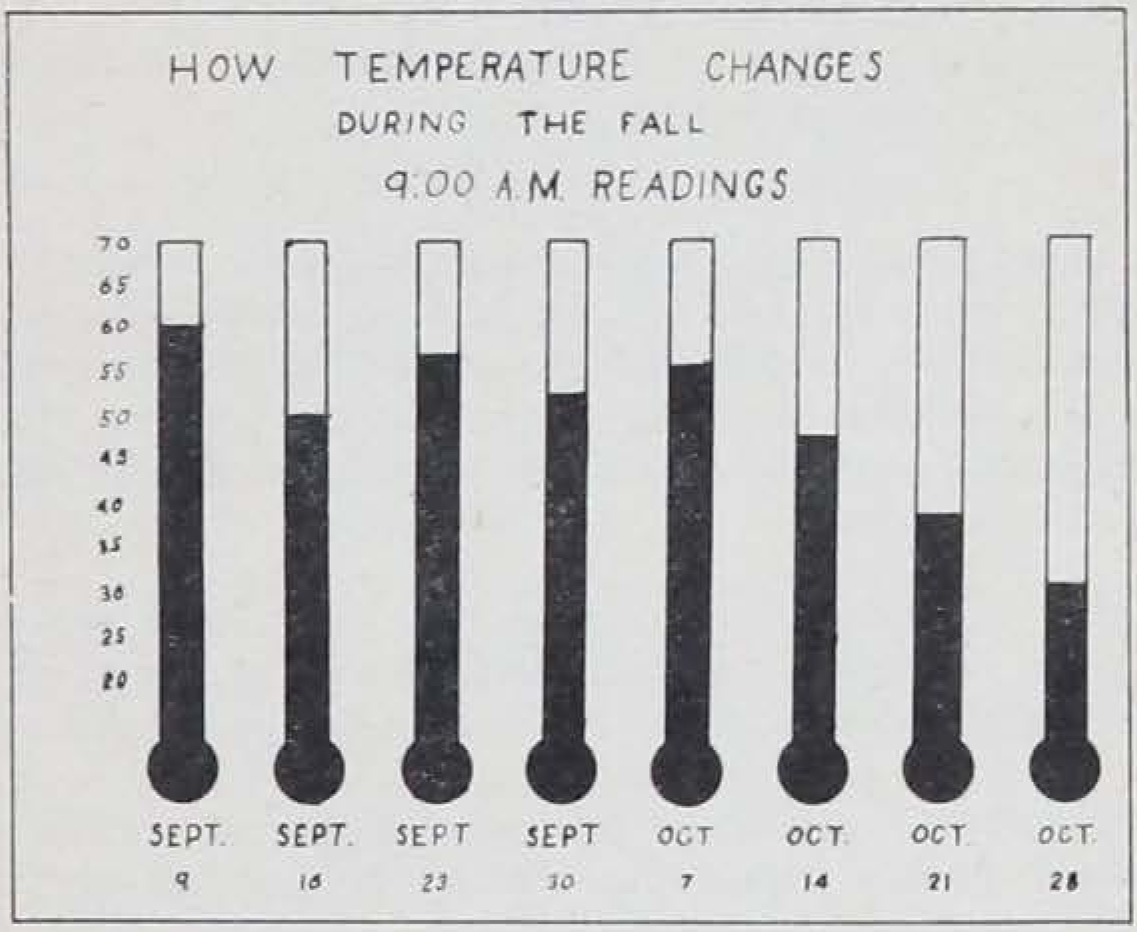
How Temperature Changes During the Day
September 12



Discuss the change in temperature shown by the graph and its causes. How much warmer was it at noon on September 12 than it was at noon on September 13? Compare the temperature readings taken at 9 o'clock and 2 o'clock on September 20.

The above figure illustrates the variations in daily temperature changes. It is also important to show the seasonal temperature changes. Take the temperature reading of one day of each week for six or eight consecutive weeks. Make a thermometer graph of these readings. For example, the figure below will illustrate a seasonal temperature change. Be sure to discuss the advantages of making a graph (picture) of the temperature changes.

How Temperature Changes During the Fall
9:00 A.M. Readings



The data for this graph may be taken from the daily readings taken during previous weeks. (Of course the teacher must see that the pupils save both the data and the graphs.) By having some children take the 9 o'clock Monday readings, others the 12 o'clock Monday reading, and others the 2 o'clock Tuesday reading, etc., a number of graphs using different data are available. Have the children compare graphs as to neatness, etc. Also discuss whether each graph tells the story as to seasonal changes.

This graph showing weekly temperature changes and the record of the temperatures for its worth may be made by the class. In fact the resourceful teacher will find numerous opportunities to develop quantitative thinking in a work of this nature.

Having prepared the graphs as illustrated in the two figures above, the teacher can ask questions about variations in temperature. For example:

1. During last week, on which day did we have the highest temperature?
2. What was the lowest temperature during the week?
3. On what days was there an increase in temperature, and on what days did the temperature decrease?

4. What was the increase in temperature between Monday and Tuesday?
5. What days show the greatest change in temperature?
6. What weeks show the greatest change in temperature?
7. What does the graph show about the temperature when each successive thermometer reads a little higher?
8. What happens when the thermometer reads at 32° F.?
9. What would the thermometer read if we placed it in boiling water?
10. What is the difference in reading between the temperatures of boiling water and freezing water?

Etc.

It will be readily noted that there are many opportunities in a unit of this kind to give the child an insight into the need for learning to read and write numbers. It will also enable the teacher to bring in some useful problem material involving subtraction and addition. The keeping of neat, readable records is a valuable learning technique which can be developed at this stage.

A unit of this kind should not be thought of as an arithmetic unit. The above material outlines the quantitative aspects of a unit of "Signs of Fall." The basic understandings in the areas of social science, art, and languages should be presented along with the quantitative aspects. It will be noted that a procedure for presenting quantitative aspects as outlined above gives a more vivid picture of seasonal and daily variations of temperatures than can possibly be given by any other method. It is not sufficient to tell the youngster that it gets colder in the fall or warmer during the noon hour period, inasmuch as *colder* and *warmer* are relative terms. By bringing in graphical and tabular forms, the pupils see just how much variation and what factors affect the variation in temperatures, both seasonal and daily.

In order to bring in some work on measurements it is instructive for second and third grade pupils to measure the length of the shadow of a yardstick over the period of a number of weeks. Care should be taken that the stick is always vertical and that the time at which the measurement is taken is constant; that is, taken at the same hour of the day every day. The noon hour is probably the preferable hour for measuring the length of the shadow. Having kept a record of the length of the shadow and having made a graph of the data, the second and third grader gets some quantitative idea as to what happens to the sun during the autumn months.

This variation in the length of the shadow can easily be illustrated by using a flashlight to represent the sun and casting a shadow of the yardstick on the floor of the schoolroom. However, this illustration should be given to the youngsters after they have collected the data and discovered what actually happens to the length of the shadow in the autumn.

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