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Analysis of Aggregation Error in Supply Functions Based on Farm-Programming Models

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CONTENTS

SUMMARY

The over-all purpose of this study is to investigate problems of aggregation error in supply estimates. Specific elements of the analysis include an exploration of the theoretical aspects of developing error-free or minimum-error aggregates, the development of empirical supply estimates for Iowa pork and beef (based on different stratifications of representative farms), determination of the relative magnitude of the aggregation error and possible factors contributing to it, and recommending practical research procedures for controlling aggregation error.

The grouping implied by Theorem I developed in the study extends beyond the practice of grouping farms by restrictive resources. Although grouping by restrictive resources is important, it does not reflect differences in the response patterns of individual farms. The requirements of Theorem I provide a basic principle to follow in controlling aggregation error.

A representative farm model was developed to estimate aggregate supply functions for pork and beef for the population of commercial farms in Iowa. Four stratifications of representative farms were made within this population, involving successively smaller numbers of representative farms. The four groups of representative farms were: (a) 36 representative farms classified by 10 soil areas, three sizes of farms, and high and low hired-labor availabilities; (b) 10 representative farms, one for each of the 10 soil areas; (c) three representative farms, one for each of the three sizes of farms common in Iowa; and (d) one representative farm to represent the entire population.

Programming results from the four groups of representative farms were aggregated into four sets of population supply functions for beef and four sets for pork. Differences among the estimated population supply functions are due to aggregation error. The four sets of estimated supply functions are generally quite similar. The over-all slopes or elasticities of the functions are in agreement, and differences between them are small. The difference between the 36-farm estimate and the estimates determined from the three smaller groups of representative farms is measured for one of the beef-supply functions and one of the pork-supply functions.

The programmed supply functions also are in agreement with the hypothesized lack of predominant direction of aggregation error. When compared with the 36-farm aggregate estimate, the three smaller groups of representative farms overestimate production about as often as they underestimate production. Thus, the programming results do not indicate any significant bias in aggregation error.

Different delineations of representative farms may be required for estimating supply functions for different products. The empirical work reveals large differences in the amounts of aggregation error found in different product-supply estimates. The theory developed shows that, in some instances, exact aggregation is more easily achieved for estimates of the supply of one product than for simultaneous estimates for several products. Hence, unique stratification factors may be best for each specific research project.

Analysis of Aggregation Error in Supply Functions Based on Farm-Programming Models[1]

by Thomas A. Miller and Earl O. Heady[2]

Knowledge of regional and national supply relationships is important for efficient guidance of adjustments in agriculture. Supply relationships underlie the adaptation of agriculture to market demand conditions, changing resource supplies, and new technologies. An understanding of both supply relationships and the agricultural adjustment process is important for decision makers in agriculture. This understanding enables farmers to plan the best use of their resources and to realize greater incomes, can help farm-input suppliers to more accurately predict the demand for their products, and can provide policy makers with better insights of prospective changes in agriculture and of the corresponding farm program needs.

This is a technical study concerning a single aspect of agricultural supply analysis. It deals with the problem of aggregation error in representative-farm, linear-programming models designed to estimate supply responses.

OBJECTIVES

The objectives of the study are:

(1) To explore and extend the theoretical aspects of error-free or minimum-error aggregates.

(2) To compare empirical supply estimates of Iowa pork and beef resulting from different stratifications of representative farms as a means of determining the relative magnitude of the aggregation error and possible factors that contribute to it.

(3) To utilize results of the first two objectives in recommending practical procedures for controlling aggregation error.

A brief review of previous work relating to aggregation error in representative-farm, linear-programming supply estimation is included as background for the analysis. The theory of aggregation error is then discussed in relation to procedures that may be useful in controlling aggregation error. Empirical supply estimates then are developed for pork and beef in Iowa. These results are interpreted within the framework of practical research procedures directed toward holding aggregation error to reasonable levels.

[1]Project 1849, Iowa Agriculture and Home Economics Experiment Station, Center for Agricultural and Economic Development, and the Farm Production Economics Division, Economic Research Service, U.S. Department of Agriculture, cooperating; contributing project to North Central Regional Research Project NC-54, "Supply Response and Adjustments for Hog and Beef Cattle Production."

[2]Respectively: Farm Production Economics Division, Economic Research Service, U.S. Department of Agriculture; Department of Economics and Center for Agricultural and Economic Development, Iowa State University.

The Study of Supply

Most agricultural supply studies have been based on regression analysis of time-series data drawn from observations of the past. Regression analysis is useful to determine supply relationships at a relatively high level of aggregation. It requires adequate time-series data for all important independent variables postulated to affect the quantity of a commodity supplied. The effect of each independent variable comes directly from the estimated regression equation. The supply function thus estimated expresses price-quantity relationships as they have existed, with the other independent variables in the equation used as shifters of the function. The effect of past changes in the structure of agriculture and in technology may be expressed through dummy variables and time trends. If these changes are gradual and continue at historical rates, regression estimates can serve for future extrapolations. Regression models are less adequate in appraising anticipated changes when historical data are unavailable relative to the change being considered.

In an attempt to meet the limitations of supply analysis, linear-programming models have been applied in recent years to better express potential changes in farm structure and output. Linear-programming estimation of supply rests on variable-pricing models of producing units. The producing unit may be either a farm or a region. The variable-pricing process generates a synthetic supply curve for this unit, subject to the usual assumptions of linear programming. The programmed supply curves of individual producing units are then summed into area, regional, or industry aggregate estimates, depending on the definition of the original producing unit and the estimate desired. This technique is particularly useful in determining the production potential of the agricultural industry. These models simulate the decision-making process of the producing unit under study and have potential for improving forecasts of supply relationships when agriculture is changing rapidly.

Representative-Farm Supply Models

The unit of analysis in linear-programming models can be a region, such as the Corn Belt; a smaller area, such as northeastern Iowa; a group of farms , such as Iowa cash-grain farms; or the individual farm itself. Models of the type used by Egbert and Heady (5), Egbert, Heady, and Brokken (6), and Heady and Whittlesey (12) use the producing region as the unit of analysis. Alternatively, other recent adjustment studies (8, 14, and 23) have used the individual, or representative, farm as the unit of analysis.

Use of the representative farm as the unit of analysis has several advantages. It permits analysis of the impact of aggregate changes at the individual farm level, thus relating macro and micro variables and conditions. It simulates the response decisions made by the managerial units actually involved. Compared with area models, it allows restrictions on resource mobility among farms.

The estimation of supply through the individual-farm, linear-programming model is accomplished as follows: (a) Data are collected from a farm sample for the resources, costs, outputs, and other items of concern in the population of relevance. (b) Sampled farms are stratified into a smaller number of groups, and a representative (typical or benchmark) farm is selected for each group. (c) A linear-programming model is developed for each representative farm, and its supply function is estimated by variable pricing (parametric) techniques. (d) The supply functions of the representative farms are expanded to estimates of the supply functions for each group of sample farms. (e) The supply functions of these groups are summed horizontally to obtain supply functions for the over-all population represented by the sample.

The Problem of Aggregation Error

Aggregation error is one of three possible sources of error in representative-farm, linear-programming supply estimates. These three sources or types of error, reviewed by Stovall (31), are:

1. *Specification error* arises because the programming model fails to reflect accurately the conditions actually facing the farm firm for a given length of run. Specification error may include errors in the technical coefficients, the resource restrictions or product, and the input prices.[3]

2. *Sampling error* arises when the distribution of the model's parameters over all firms in the population is not known, but is estimated by sampling techniques.

3. *Aggregation error* as defined by Frick and Andrews (7) is "the difference between the area supply function as developed from the summation of linear-programming solutions for each individual farm in the area and summation from a smaller number of typical or benchmark farms."

Each type of error can be related to the previously outlined five steps in estimation. Specification error is associated primarily with the third step in which the individual-farm, linear-programming models are developed and supply functions for each are estimated. Sampling error arises in the first and fifth steps in obtaining sample data and estimating population totals from sample estimates. Aggregation error arises in the second and fourth steps as sample farms are stratified into groups, representative farms are delineated, and representative-farm supply functions are aggregated into group supply estimates.

[3]This definition does not agree completely with the traditional concept of specification error. Failure to incorporate appropriate activities and restraints and incorrect specification of the objective function are obviously specification errors. In contrast, errors in estimating technical coefficients and product and input prices seem more a problem of sampling than of specification.

PREVIOUS ANALYSES

Thiel (32) and others (13, 17) discussed aggregation problems in estimating macro-parameters from their micro counterparts. Their analyses relate largely to estimation based on regression models. With the increased use of normative supply estimations, research workers have become aware of the problems of aggregating farm supply functions into aggregate supply functions. This concept of the firm aggregation problem is one of four types listed by Heady (10). Nerlove and Bachman (20) list it as one of the main problems in agricultural supply analysis by farm-programming methods.

Initially, research workers such as Mighell and Black (18) and Christensen and Mighell (2) believed that solutions to the farm-supply aggregation problem could be solved by following correct procedures in delineating the farms to be programmed. Plaxico (21) tested programming of a small number of farms, fitting regression lines through the programmed points, and testing the regression coefficients for significance. He suggested that characteristics with no effect or a linear effect on supply need not be considered as weighting factors in aggregation. Thompson (33) indicated that a group of farms with similar resource ratios will not necessarily allocate their resources in the manner of a single farm with resource ratios similar to the average of the group. Also, he indicated that the ways in which farms of the group allocate their resources will not necessarily offset each other.

Day (3) suggested that the possibilities for completely eliminating aggregation error or bias were unlikely within a manageable number of representative farms. Carter (1) indicated that sufficient refinements could be made in farm selection if it were possible to isolate the primary characteristics of farms and farmers that dominate or strongly influence particular decisions. Plaxico and Tweeten (22), discussing the potential application of representative farms in public-policy evaluation, observed that increasing the number of representative farms programmed reduces within-group variance.

SUGGESTED SOLUTIONS TO THE AGGREGATION PROBLEM

Hartley (9) suggested a solution to the aggregation problem. If the original stratification of farms is based on land and buildings, while supply is dependent on a third factor, such as labor, a three-way classification including the omitted factor could be avoided by variable-factor programming on the omitted factor. Then, a statistical adjustment of the programmed supply functions could be made to incorporate the influence of the third factor. It is not clear, however, if the variable-factor programming and statistical correction require less computation than the original stratification of the farms by the third factor.

Day (4) stated the initial conditions necessary to insure exact aggregation. He showed that a linear-programming model for the aggregate is equivalent to summing the solutions of individual firms if there is proportional variation in the sense that resource and net return vectors of farms are scalar multiples and that technical coefficients are the same. Frick and Andrews (7) evaluated five methods of grouping 51 farms to estimate an aggregate milk-supply function. Grouping farms by the most limiting resource gave the least aggregation error, but the method had the disadvantages of ignoring size of farm, using farm classes hard to project to the future, and being complex for more than one product. The method did, however, provide a workable solution for holding aggregation error at reasonable levels.

Sheehy (27) and Sheehy and McAlexander (28) compared aggregate output estimates developed under two methods of selecting representative or "benchmark" farms: the "conventional method," where farms were classified on the basis of absolute levels of certain resources, and the "homogeneous-restriction method," analogous to the "most limiting resource method" used by Frick and Andrews (7). Four sets of representative farms were delineated and programmed under the conventional method. Since more detail was used in the stratification under the conventional method, the estimated supply functions moved to the left. The estimated supply function for the homogeneous-restriction method, however, was even further to the left, but Sheehy and McAlexander (28) expressed belief that this supply function was virtually unbiased. They concluded that selection of representative farms on estimated restrictions of the commodity in question, rather than on the absolute level of resources, reduced aggregation error.

BASIC THEORETICAL CONDITIONS FOR ERROR-FREE AGGREGATION

We now provide a detailed definition of the aggregation problem as a foundation for the analysis that follows. Consider the linear-programming model representing the g-th farm of n farms. The problem is to select a vector of production activity levels, X_g, such that profit

$$[1]\quad \pi_g = Z_g X_g$$

is a maximum, subject to

$$[2]\quad B_g X_g = C_g$$

and

$$[3]\quad X_g \geqslant 0$$

where π_g = total net returns to the g-th farm, Z_g = the vector of activity net returns, X_g = the vector of activity levels to be chosen, B_g = the matrix of input output coefficients, and C_g = the vector of available resources of the g-th farm. This standard programming form is outlined by Heady and Candler (11) in application to farm management problems under variable pricing conditions. The end result desired is the total production of the n farms in the set. If n farms are programmed separately and their production is summed, the aggregate solution for the n farms is

$$\sum_{g=1}^{n} X_g.$$

This quantity is *free* of aggregation error because it is the exact sum of the solution vectors for all farms in the set and can serve as the standard for judging other procedures.

The number of farms in the set usually makes it impractical to obtain optimum solutions for all of them. Abstraction is necessary to make the problem computationally feasible. Hence, after the representative farm within the set is defined and its optimum production is computed by linear programming, production of the entire set is estimated by the appropriate weight applied to the representative-farm solution. If the representative farm is defined as the average farm in the set (total resources of the set divided by n), the appropriate weighting factor becomes n, the total number of farms. Alternatively, we may define the representative farm as the sum of the set, omitting the weighting step, and select a vector of aggregate area production levels, X, such that

$$[4]\quad \pi = ZX$$

is a maximum, subject to

$$[5]\quad BX = C$$

and

$$[6]\quad X \geqslant 0$$

where X is a $1 \times m$ vector of all activities.

The lack of subscript indicates the entire set of farms, while the subscripts in equations 1-3 represent individual farms. The dimensions of the matrices are the same in both cases. Because resources of individual farms are summed to obtain the resources of the aggregate set,

$$[7]\quad C = \sum_{g=1}^{n} C_g$$

Exact aggregation then can be defined: The levels of activities in equation 5 are exactly the sum of those for each individual farm programmed separately, or

$$[8]\quad X = \sum_{g=1}^{n} X_g$$

Conversely, aggregation error is defined if

$$[9]\quad X \neq \sum_{g=1}^{n} X_g$$

This definition of aggregation error is general since it encompasses all m outputs or elements of vector X. It may, of course, be possible to achieve equality for one or more of the m outputs while aggregation error remains for other outputs.

Exact Aggregation Over All Products

Given the set of n farms and the aggregation problem specified in the previous section, what conditions are sufficient among the set of farms to achieve exact aggregation?

Proportional heterogeneity

The sufficient conditions for exact aggregation in terms of "proportional heterogeneity" (4) are

[10] $B_1 = B_2 = \ldots = B_n = B$

[11] $Z_g = \gamma_g Z$

where γ_g is a scalar greater than zero for all g, and

[12] $C_g = \lambda_g C$

where λ_g, a scalar greater than zero and less than one for all g, represents the proportion of the set's resources possessed by the g-th farm. Equation 10 specifies identical input-output matrices for all farms; equation 11 indicates that all farms must have proportional net-return expectations; equation 12 states that the farms have proportional variation in resource constraint vectors.

Under these conditions, exact aggregation is attained. One representative farm in the set of n may be programmed, and the weighted solution will exactly equal the sum of the n individual farm solutions. Then

[13] $R = (1/n) \left(\sum_{g=1}^{n} R_g \right)$

also prevails where R is the "average marginal net revenue productivities" of the resources in the set, representing the solution of the dual linear-programming problem, and the R_g are vectors of marginal net resource productivities for the individual farms.

The requirement of proportional heterogeneity is a tight restraint and involves dividing a large number of individual farms into groups meeting the requirements of equations 10-12 so that each group can be represented without error by a representative farm. A very large number of representative farms would be required. Exact aggregation thus seems extremely difficult because of computational burdens and costs. Since proportional heterogeneity is only a sufficient condition for exact aggregation and not a necessary condition, we may develop less restricting sufficient conditions.

Qualitatively homogeneous output vectors, QHOV

A less binding sufficient condition is the concept of qualitatively homogeneous output vectors (QHOV). We define this concept, first intuitively and then more rigorously, by a theorem and proof of the sufficiency requirements.

Assume that farms in a set under consideration are similar to the extent that all optimum farm solutions include identical sets (but not quantities) of activities. This set of farms may vary in both resource and net-return vectors, as long as all farms in the set have the same activities. The variation in resource vectors among farms will, of course, cause them to differ in optimum activity levels. We only require that the identity of activities in the optimum solutions is the same for all farms. A set of farms with this similarity is defined intuitively as having *qualitatively homogeneous output vectors.*

For a more rigorous specification of this condition, consider the optimum solution for the g-th farm, which may be expressed as a column vector

$$X_g = \begin{bmatrix} x_{1g} \\ x_{2g} \\ \cdot \\ \cdot \\ \cdot \\ x_{mg} \end{bmatrix}$$

In notation, m is the number of production processes considered by the farm plus the number of slack vectors equal to the number of resources, and k is the number of resources or constraints; m>k, since k is the number of slack vectors included in m to achieve equality in the restraints. For each optimum solution, X_g is made up of, at most, k activities greater than zero and, at least, m minus k activities equal to zero.[4]

For each farm, we can express an abbreviated or partitioned output vector as

$$X'_g = \begin{bmatrix} x'_{1g} \\ x'_{2g} \\ \cdot \\ \cdot \\ \cdot \\ x'_{kg} \end{bmatrix}$$

by omitting the m minus k zero activities common to each farm. The X'_g (abbreviated output vector) for farms having QHOV has the same k activities. Accordingly, all farms have the same limiting resources, the same resources at nonzero levels in disposal activities, and the same real processes in their final solution vectors.

Theorem I. Sufficient conditions for exact aggregation are that all farms (a) have identical coefficient matrices or $B = B_g$ for all g and (b) have QHOV.

Proof. The original programming problem for farms meeting conditions of the theorem may be reduced to the trivial one of solving a set of k equations in k unknowns

[14] $B'X'_g = C_g$

where $B' = B'_g$ is a k by k order coefficient matrix corresponding to the k activities in X'_g. This reduced equation set is then equivalent to the original constraint set, equation 2, with the unused activities (columns) of the coefficient matrix and

[4]These k activities are often called the basic variables in the literature, and the remaining activities are called nonbasic variables. The theorem generally developed is that an optimum solution involves, at most, k unknowns at nonzero values (where k equals the number of equations).

the zero elements of X_g omitted. In other words, if the identity of the final basic activities were known, the linear-programming problem could be solved simply as a set of simultaneous equations.

Similarly, the solution to the aggregate farm may be determined by the relation

$$[15]\quad B'X' = C$$

which is developed in a fashion parallel to the reduced equations for the n individual farms.

Summing equation 14 over all n farms gives

$$[16]\quad B' \sum_{g=1}^{n} X'_g = \sum_{g=1}^{n} C_g$$

Since $\sum_{g=1}^{n} C_g = C$ by definition, it is obvious from equations 15 and 16 that $X' = \sum_{g=1}^{n} X'_g$.

Including the m minus k zero level elements in both vectors, we have $X = \sum_{g=1}^{n} X_g$. The conditions of the theorem are thus sufficient conditions for exact aggregation and are general with respect to the price or revenue vectors. Hence, the theorem covers variable-price programming since the consideration of different prices merely has the effect of further restricting the groups of farms that meet the requirement of QHOV (assuming that all farms have identical input-output coefficient matrices) for every set of prices. The conditions are substantially less binding than proportional heterogeneity. A range of resources and net-return situations can be covered without aggregation error. There is no restriction on the type of variation in resource vectors among farms as long as they all have both QHOV and identical input-output matrices.

On the negative side, the conditions of Theorem I are a requirement of the solutions for the individual farms rather than a requirement of the farms themselves. They thus provide a less-than-ideal solution in the problem of delineating representative farms to eliminate aggregation error. A translation of these conditions into requirements on the data of the individual farms is needed to make the process truly operational. This translation may be made as follows.

An alternative statement

Interpretations of Theorem I into requirements of the coefficients, rather than the solutions, of the individual farms requires a close examination of the dual linear-programming solutions of farms. Define R and R_g (for $g = 1 \ldots n$) as m by 1 vectors representing the dual solutions for the linear-programming problems of the aggregate farm and the n individual farms, respectively. Each dual-solution vector has a 1:1 correspondence with the m activities (including slack and disposal activities) in the respective primal linear-programming problem. The values in the dual-solution vectors represent the marginal decrease in the functional value, π, that results from a marginal increase of one unit of the activity in question. Accordingly, the values are all nonnegative for an optimum solution.

In developing Theorem I, Miller (19) observed that a parallel argument could be developed for aggregation of the dual solutions over the same set of n farms. Under the conditions of Theorem I,

$$C = \sum_{g=1}^{n} C_g \text{ leads to } X = \sum_{g=1}^{n} X_g$$

For the dual solutions, a similar argument can be developed to show that if

$$Z = (1/n)\left(\sum_{g=1}^{n} Z_g\right), \text{ then } R = (1/n)\left(\sum_{g=1}^{n} R_g\right)$$

Lee (15,16) observed that the statement for aggregation of the dual solutions could be modified slightly to show that, if $Z_g = Z$ for all g, then $R_g = R$ for all g. If all individual farms in a set meeting the conditions of Theorem I have identical net-return vectors, they will all have identical dual-solution vectors. With use of this relationship, the observed ranges of the resource ratios can be used as criteria for grouping individual farms on the basis of observable characteristics. Thus, an additional aggregation theorem, hereafter referred to as *Theorem II,* was explained by Lee (15) as:

> Sufficient conditions for exact aggregation are (a) that all farms have identical coefficient matrices, (b) that all farms have the same net-return expectations, and (c) that the range of resource ratios be such that the dual-solution vector is the same for all farms. . . .This would delineate sets of farms identical to those delineated by the original theorem. It may be more useful, however, since it lends itself to interpretation in terms of observable characteristics. The link between theorem and application is the empirical task of determining the exact ranges of resource ratios over which the marginal revenue product is constant.

The theorem also can be explained as follows: Consider the dual-solution vectors $R = R_g$. They correspond to the m activities in the primal solution vectors X and X_g; when an activity is in the optimal primal basis, its value in R is zero; when an activity is not in the primal basis, it has a nonzero value. From this correspondence, $R = R_g$ for all g, and the QHOV is implied directly. Then, by our Theorem I, QHOV and identical coefficient matrices imply exact aggregation.

This proof does not require condition b of Theorem II that all farms have identical net-return expectations. The remaining two conditions are actually sufficient conditions for exact aggregation, and the theorem is true without condition b. The groups of farms delineated by Theorem II are identical to those delineated by Theorem I only when all individual farms have identical net-return vectors. A given group of individual farms meeting the conditions of Theorem I may still have different net-return vectors and need further subdividing to meet requirements of Theorem II.

Although the requirement of identical net-return vectors can force a larger number of groups, it makes the conditions of the theorem observable in individual farm data and eliminates the need to determine the optimum solution before individual farms can be grouped accurately. With Theorem II, farms can be classified and grouped solely on the basis of resource vectors and coefficient matrices.

Lee (15), using fig. 1, shows this method of classification. He supposes a group of farms each with resources C and L, processes A_1, A_2, and A_3, and identical technical coefficients and net income expectations. With all possible resource ratios represented by points on bar C_1 C'_1 resulting as various amounts of resource L are added to C_1, a fixed amount of resource C. With the isorevenue lines represented by the solid lines, net-return expectations are indicated. He indicates that farms with C_1 of C and none of L will have zero revenue. As resource L is increased from L_0 to L_1, resource L remains the limiting resource, and net revenue increases in proportion to the amount of L. Accordingly, when the shadow price of L is constant over the range L_0 L_1, only A activity will be produced. All farms with resources C and L in ratios between C_1/L_0 and C_1/L_1 can be aggregated without error since all requirements of Theorem II are met. For farms with other resource ratios (15):

> With resource combination C_1L_1 (denoted by point P_1), both resources are exactly used by activity vector A_1. With further increases in resource L (beyond L_1), both resources C and L are limiting. However, the full amount of both resources can be utilized and net revenue maximized by combinations of activities A_1 and A_2 (for example, L_0 b_1 of A_2 and b_1 a_1 = d_1 P_1 of A_1 in figure 1). The locus of resource combinations, P_1, P_2, is also the path of net revenue expansion as resource L is increased. This expansion path intersects the iso-net revenue field at constant angles (i.e., as L increases, the net revenue from A_2 substitutes for net revenue from A_1 at constant rates). Thus, between L_1 and L_2, the marginal revenue product of L is constant and the conditions of. . .Theorem II are again met. Note that the MVP of L between L_1 and L_2, while constant, is less than the constant MVP of L between L_0 and L_1. The reason is that as L is increased it becomes less scarce relative to resource C. This is reflected in the flatter slope of the iso-revenue curve. Obviously, farms with resource ratios between C_1/L_1 and C_1/L_2 can be aggregated with bias. . . .As resource L is increased from L_2 to L_3, its MVP is again constant, though lower than previously. Farms with resource ratios between C_1/L_2 and C_1/L_3 meet the conditions for exact aggregation. Beyond L_3 amounts of resource L, C becomes the only limiting resource; A_3 is the only activity in the solution, and the MVP of L is constant at zero. Thus, all farms with resource ratios of C_1/L_3 or less can be aggregated without bias. . . .With resource C fixed at C_1, the line $L_0P_1C'_1$ represents the maximum efficiency net revenue expansion path as L is increased from L_0 to infinity.

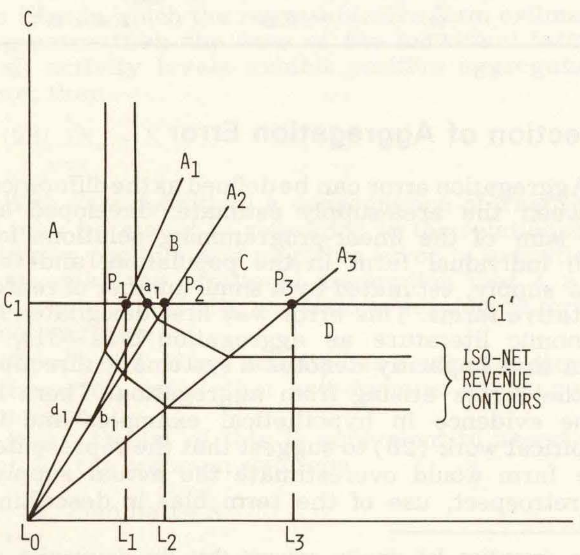

Fig. 1. Diagram of a linear-programming model of two resources and three activities for farms with varying labor-capital ratios. From: John E. Lee, Jr., Exact aggregation: A discussion of Miller's theorem. Agr. Econ. Res. 18:58-61. 1966.

The maximum number of groups of farms required to eliminate aggregation error is four in this case. For the net-revenue vectors reflected in the iso-revenue contours, only four dual solutions are possible. The activity vectors A_1, A_2, and A_3 themselves, along with the axis, form the dividing lines that separate the different dual solutions. All combinations of C and L falling between two such vectors will have the same dual solutions and may be aggregated without error.

An extension may be made to all possible net-revenue vectors (as would be encountered in the generation of supply functions by variable pricing) as long as all farms in the set have identical net-revenue vectors at each point aggregated. If these three activities are the only ones available to the farms, the four groups of farms A, B, C, and D are the maximum number of groups required for exact aggregation, regardless of the number of individual farms in the original group. There is, of course, the possibility that, with a single set of net-revenue vectors, the number of groups may be reduced, since two or more groups may have the same dual solutions.

Fig. 1 allows us to conclude that the sufficient conditions for exact aggregation expressed by Theorem I are much less restrictive than is the condition of proportional heterogeneity. To meet the condition of proportional heterogeneity, a separate representative farm is needed for all points on line $C_1 C_1'$, since each point on this line contains a new resource ratio. Thus, to achieve proportional heterogeneity, a group of farms in fig. 1 must be situated on a straight line extending from L_0. An example is a group of farms with the C/L ratio expressed by a line such as A_3. Since an extremely large number of such lines may be required, conditions of proportional heterogeneity are considerably more restrictive than the conditions of Theorem I.

The value of Theorem II and the fig. 1 analysis is now clear. The boundaries of the groups of farms having different shadow prices are formed by the technical coefficients themselves. These input-output coefficients contain the information needed to determine the range of resource ratios that may be included in a group of farms that can still be aggregated without error. The individual linear programs need not be solved to determine groups of farms meeting the requirements of Theorem I.

Number of representative farms required

The boundaries of the various groups may be ascertained from information contained in the coefficient (B) matrix common to all farms. The method is to divide the first row of the B matrix by the second. This step gives all the different ratios in which the activities use resources c_1 and c_2. When arrayed from smallest to largest, these ratios become the critical boundaries of farms classified by the ratio of resource 1 to resource 2.

With more than two resources, this process is repeated for every possible resource ratio, subdividing the previous groups for every additional ratio considered. Thus, with three resources and three activities, the farms are divided into four groups on the c_1/c_2 ratios; each of these groups is subdivided into four more groups on the c_1/c_3 ratios; finally, each of these 16 groups is subdivided into four more on the basis of the c_2/c_3 ratios. Thus, if the B matrix is 3 by 3, 64 groups of farms would be the maximum ever required to achieve exact aggregation.

In general, the maximum number of groups of farms, N, required for exact aggregation with a B matrix of k rows and p real activities is

$$[17] \quad N = (p + 1)^r$$

where r is k raised to the power C_2 and

$$r = k!/2!(k-2)! = k(k-1)/2$$

This is the maximum number of groups required for the situation in which all the elements of B were nonzero and all the critical B ratios were different. Defining d as the probability of a nonzero item in a particular location of the B matrix, then d^2 becomes the probability of a coefficient ratio composed of two nonzero elements and allows correction of equation 17 for d, which is also the density of the B matrix, resulting in the equation

$$[18] \quad N' = (pd^2 + 1)^{k(k-1)/2}$$

where N' is the expected number of groups required.[5]

The value of N' grows rapidly for increasing values of k, p, and d (table 1). The number of groups required to eliminate aggregation error would, of course, never exceed the number of individual farms in the set to be analyzed. The high N' values for larger coefficient matrices indicate that to assure exact aggregation, the number of representative farms needed may approach the number of individual farms. The number of representative farms required by Theorem I approaches the number under the proportional heterogeneity condition only when the number of activities reaches infinity (15). Actually, the number of resources (rows) is a more important criterion, increasing N' as an exponential. As a result, both criteria become unmanageable for eliminating aggregation error when large B matrices are involved. For small B matrices, the Theorem I requirements offer considerable improvement over the proportional heterogeneity condition.

On an intuitive level, Theorem I provides some guidelines to the problem of grouping farms to minimize aggregation error. It suggests that individual farms should be grouped into homogeneous groups on the basis of their coefficient matrices and then subdivided, so that each subgroup will have the same optimum set of production activities. This idea will be developed more completely in later sections.

Table 1. Expected number of representative farms required to eliminate aggregation error for different sizes and densities of matrices.

Rows k	Columns p	Density d	Approximate number required N'
3	3	1	64
10	20	0.25	3,325
30	60	0.25	2.68×10^{321}

Direction of Aggregation Error

Aggregation error can be defined as the difference between the area-supply estimate, developed as the sum of the linear-programming solutions for each individual farm in the population, and the area supply, estimated by a small number of representative farms. This error was first designated in economic literature as aggregation bias (31), a term that implicitly denotes a systematic direction in the errors arising from aggregation. There is some evidence in hypothetical examples and in empirical work (28) to suggest that the representative farm would overestimate the actual supply. In retrospect, use of the term bias in describing

[5]The correction for density assumes that the distribution of zero value in each row of the B matrix is independent of the distribution of zero values in the other rows. If this assumption is not met, serious errors could arise from using this formula for low-density matrices.

errors in representative-farm, linear-programming supply estimates may have contributed greatly to questions concerning the validity of the procedure.

It is possible to treat some aspects of the direction of aggregation error in a somewhat rigorous manner. The first relationship established is the direction of error in the estimate of the total maximum net returns compared with the actual maximum net returns. Under the assumptions that $Z_g = Z$ and $B_g = B$ for all g = 1...n, the following theorem holds:

Theorem III. The representative (aggregate) farm estimate of total maximum returns for the set of farms is at least as great as the value found by summation of the individual farms; that is

$$[19]\quad \pi \geqslant \sum_{g=1}^{n} \pi_g$$

Proof. For all individual farms, the optimal feasible solutions are such that

$$[20]\quad B\,X_g = C_g$$

Summing over n farms

$$[21]\quad B\sum_{g=1}^{n} X_g = \sum_{g=1}^{n} C_g = C$$

Therefore, $\sum_{g=1}^{n} X_g$ is feasible for the aggregate problem (equation 5), but it is not necessarily optimum. Therefore

$$[22]\quad \pi \geqslant Z\sum_{g=1}^{n} X_g = \sum_{g=1}^{n} ZX_g = \sum_{g=1}^{n} \pi_g$$

proving that, under the stated conditions, the total maximum net returns estimated by the representative (aggregate) farm is at least as great as the summation of the values for the individual farms. Such an estimate is biased in the statistical sense—in this case, the expected value of the estimate is greater than the parameter to be estimated.

Positive aggregation error may be defined as the case in which the representative farm estimate is greater than the sum of the individual farms. If all activity levels exhibit positive aggregation error, then

$$[23]\quad \sum_{g=1}^{n} X_g < X$$

With this definition, a consequence of Theorem III may be stated in reference to the relationship between positive and negative error in activities. First, make the additional assumption that $Z \geqslant 0$ (the vector of activity net returns serving all farms is not negative).

Corollary I. If one activity has a negative aggregation error, some other activity must have a positive error.

Proof. Denial of this is equivalent to asserting that, for the optimum solutions

$$\sum_{g=1}^{n} X_g \text{ dominates } X$$

That is, at least one activity level in the optimum $\sum_{g=1}^{n} X_g$ is higher than it is in X, while no activity is strictly less. Therefore

$$[24]\quad Z\sum_{g=1}^{n} X_g > ZX \text{ and } \sum_{g=1}^{n} \pi_g > \pi$$

contradicting the primary theorem.

Thus, under the condition of nonnegative net-return vectors, no activity can have negative aggregation error without at least one activity having positive error. The number of activities with negative errors can exceed the number of activities with positive errors. But there must be at least one with a positive error if there is one with a negative error.

The effect of the assumption that $Z \geqslant 0$ now can be seen. Existence of activities with negative net returns would allow activity estimates with negative error to exist without being offset by any other activity estimates with positive error. Thus, in the general case, these results are not suggestive of any consistent direction or bias in aggregation error in the individual product estimates. Both positive and negative errors may exist, and, when the Z vectors are unrestricted as to sign, no relationship is indicated between the errors in either direction.

Situations and Factors Causing Aggregation Error

One cause of aggregation error can be intuitively explained in terms of resource redundancies. If a resource is redundant on one farm, it must be redundant on all other farms in the group before exact aggregation can be achieved. One of the goals of farm stratification should be grouping farms such that all farms within a set have the same limiting and redundant resources.

Absence of QHOV

Resource redundancy is a special case of a more general situation causing aggregation error: the absence of qualitatively homogeneous output vectors. Theorem I states that the condition of QHOV is a sufficient condition for exact aggregation. As we now illustrate, QHOV is also a necessary condition for exact aggregation under certain conditions.

Theorem IV. When all elements of the optimal basic vectors are not zero, the condition of exact aggregation can be achieved only if all farms in the set to be aggregated have QHOV. Restated, QHOV is a necessary condition for exact aggregation under this condition.

Proof. Assume that the theorem is false and that exact aggregation is accomplished without QHOV. Then n-l of the n farms in the group may have a common set of k basic nonzero variables in their optimum solutions, but at least one farm must have a new basic nonzero variable replacing one of the k common to the rest. Under this condition $\sum_{g=1}^{n} X_g$

will have the k basic nonzero variables common to the set of n−1 farms plus the one additional nonzero variable for the unhomogeneous farm, a total of k + 1 nonzero elements.

But, X has only k basic nonzero variables. Therefore

$$[25]\ X \neq \sum_{g=1}^{n} X_g$$

contradicting the basic premise.

Theorem IV and its proof are dependent upon the requirement of nonzero optimal basic vectors. This requirement is necessary because of a technicality of linear programming that has little counterpart in real-world production problems. Optimal basic variables of zero level are quite common in linear-programming problems of the type being considered, and, although they are technically a form of degeneracy, their occurrence presents no problem in economic interpretations. If, for example, corn is in the optimal solution at a zero level, it is merely ignored in the interpretation of the results. The occurrence of optimal basic variables at zero levels, however, allows exact aggregation to occur without QHOV being met, as long as the unhomogeneous activities are all in their respective solutions at zero levels. Farms not meeting the requirement of QHOV may be aggregated without error if all basic activities outside the set of basic activities common to all farms are at zero levels. This problem arises from QHOV being strictly defined as a requirement of optimal basic vectors and not merely one of the nonzero activity levels.

Variation in net returns

Another cause of aggregation error is variation in net-return vectors among individual farms. Such variation becomes a problem when it destroys the condition of QHOV. Variation in net returns within the range in which QHOV is still achieved does not lead to aggregation error. Variation in net returns for a farm changes the slope of its isoprofit lines. Any farm may have a different net-return vector, and as long as the slope of its isorevenue line is not changed to the extent that would move the solution to a new corner point, farms still meet the QHOV requirement and may be aggregated without error. Thus, Theorem I covers the problem of aggregation of farms with different net-return vectors. Theorem I does not include a condition on net-return vectors simply because the requirement of QHOV overrides such a condition. As long as QHOV is met, there is no need to be concerned about variation in net returns. When net returns vary outside this range, additional stratification must be made to eliminate aggregation error.

Variation in coefficient matrices

Exact aggregation for a set of farms with different coefficient matrices seems uncommon, but is not impossible. For farms that have different coefficient matrices but still meet the requirement of QHOV, there are at least two instances in which exact aggregation is possible. The first is the type of coefficient variance equivalent to row scaling. For any linear-programming problem, a given row (including the value in the resource vector for that row) can be multiplied by a constant without affecting the solution. Thus, variation that similarly affects all elements of a row can occur in the coefficient matrices of individual farms without leading to aggregation error. If one coefficient in a set of farms differs, then all rows containing that coefficient in the set of farms must be scalar multiples of each other. Such variation may occur in actual data where larger amounts of a resource are offset by decreased productivity.

A second type of variation that could occur in coefficient matrices without causing aggregation error is more likely, but of little concern from a practical standpoint. This situation includes variation in coefficients of activities not in the optimum solutions of any farms and of rows not restrictive on any farms. Such variation does not lead to aggregation error simply because it does not enter any of the solutions.

These two pure types of variation would be improbable in actual data. Generally, other types of variation would be expected among the coefficient matrices of individual farms and would lead to aggregation error. As a result, stratification of farms into groups with similar coefficient matrices would be the first step in representative-farm identification to assure exact aggregation. This step would be followed by definition of substrata for all groups until all farms within each substratum meet the requirement of QHOV.

PROGRAMMED SUPPLY FUNCTIONS FROM DIFFERENT STRATIFICATIONS

The theoretical aspects summarized and developed to this point leave unanswered questions concerning (a) the magnitude of the aggregation error in actual models and (b) the relative importance of different factors contributing to aggregation error. These are essentially empirical questions, with the answers depending upon the area, type of agriculture, and type of supply estimates desired.

A model was developed to answer these questions concerning aggregation error for an existing research project involving supply estimation by representative-farm, linear-programming models. This section discusses the development of the model and presents supply functions based on four different groupings of representative farms. The aggregation error and the possible factors that contribute to it are then analyzed and interpreted to develop stratifications that reduce aggregation error. The organization of this section parallels somewhat the steps outlined previously in supply estimation from representative-farm, programming models.

The Population

Basic data from Iowa's contributing project to North Central regional research project NC-54 were used to investigate aggregation error. The phase of the regional project reported in this bulletin was conducted cooperatively by the Iowa Agriculture and Home Economics Experiment Station and the Economic Research Service, U.S. Department of Agriculture. Project NC-54 had the objective of supply estimation for pork and beef, based on linear programming models of representative farms.

Supply estimates were desired for a specified population of commercial farms in Iowa. Specifically, the population was defined by using Census of Agriculture definitions and data (25) and included all livestock, general, and cash-grain farms in economic classes I through V in Iowa. The 1959 census listed 136,331 farms, or 78.0 percent of all Iowa farms, in the defined population. These farms accounted for 89.4 percent of the total farmland in the state and a similarly high percentage of other resources and major products.

The Sampling Procedure and Data

The sampling procedure was developed, and the data in Iowa were collected, to meet the requirements of the NC-54 project (24). No additional data were collected for the study of aggregation error. A 5-percent sample of 1959 Census of Agriculture data was used as the basis for information on farm resources. Data on costs, returns, and input-output coefficients on farms were collected from secondary sources. Most of the data for livestock coefficients were developed by the regional NC-54 committee as a means of obtaining comparability among all states participating in the project. The data-gathering techniques are all described in detail by Sharples (25).

Information obtained on individual farms in the 5-percent sample of census data included all farm characteristics found in the published form of the Census of Agriculture. Major sections of information were farm size, land use, tenure, land value, type of farm, labor used, cash expenditures, conservation practices, amounts of crop and livestock products produced, machinery inventory, livestock programs, fertilizer use, and certain miscellaneous information. This core of data provided the basic information on resources for individual farms.

The Stratification Procedure

The exact measurement of aggregation error is expensive and often impossible because it implies programming every farm in the population. The alternative used for this study was to make four stratifications involving successively smaller numbers of representative farms. Included were a basic stratification, which resulted in 36 representative farms, and three less-detailed stratifications involving 10, three and one representative farms. Differences among the state supply-function estimates based on these four groups of representative farms were, for purposes of this study, then considered aggregation error. This error was analyzed to determine how it is affected by the number of representative farms programmed and by the method of stratification. This procedure does not give an exact measurement of aggregation error.[6]

Basic stratification into 36 representative farms

The stratification for the NC-54 project was made on the basis of (a) 10 soil-association areas of the state, (b) three sizes of farms, in acres, and (c) two types of farms (livestock farms and cash-grain, or general, farms on the basis of the Census of Agriculture definitions). This procedure resulted in a total of 63 representative farms for the NC-54 project. But, the NC-54 programming results revealed no significant differences in activities in the optimal solutions caused by the third stratification factor, type-of-farm.

Hence, given Theorem I and the NC-54 programming results, type-of-farm stratification adds no accuracy in the estimates, leaving soil area and size of farm as the two main stratification factors for the study of aggregation error. Previous NC-54 results (26) and knowledge of Iowa agriculture suggested that these two factors were important in determining the optimal organization of individual farms. The sample of 6,800 individual farms was stratified by these two factors to delineate strata of individual farms approximating the conditions of Theorem I, farms that might have similar coefficient matrices and approach qualitatively homogeneous output vectors to minimize aggregation error for the basic group of 36 representative farms.

Soil classification specialists in the Agronomy Department of Iowa State University provided the guides for the stratification into 10 soil-association areas. The areas used, following county lines because of the availability of most other data at the county level, are shown in fig. 2. Since land quality, yields, and fertilization practices vary among these areas, 10 sets of crop-yield coefficients and fertilizer costs were developed based on the work of Shrader and others (29, 30).

[6]Rather, it shows how aggregation error accumulates as smaller numbers of representative farms are used.

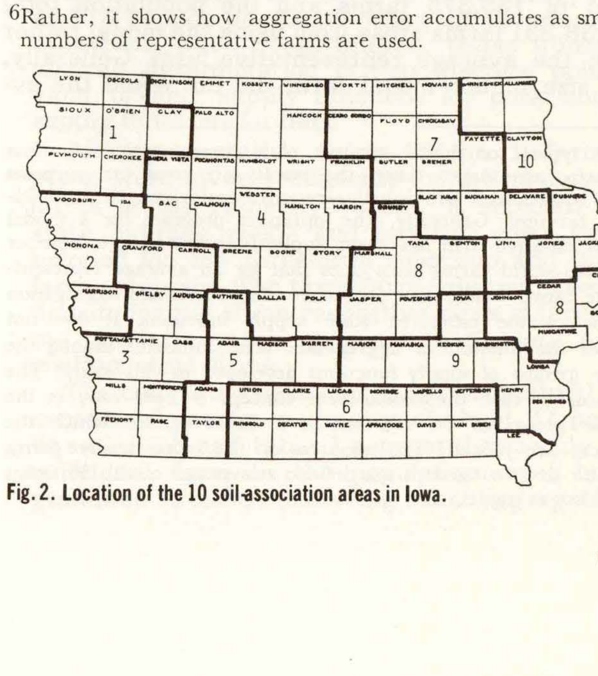

Fig. 2. Location of the 10 soil-association areas in Iowa.

Sample farms in each of the 10 areas then were divided into three size strata. The size strata, based on total farmland, were: less than 140 acres (the A group in table 2), 140-239 acres (the B group in table 2) and 240 or more acres (the C group in table 2) and correspond with three farm sizes common in Iowa; namely, 80 acres, 160 acres, and 320 acres. Stratification by size was designed to separate farms into groups with similar coefficient and production-response characteristics.

The 6,800 sample farms were divided into 30 strata through classification by soil and size. Representative farms were delineated for each of the 30 strata as follows: First, one representative farm was defined for each of the small- and medium-farm strata in each of the 10 soil areas shown in fig. 2. In areas 1, 3, 5, and 7, one representative farm was defined for the large-farm stratum, with two representative farms being defined for the large-farm stratum in the remainder of the areas. These additional farms were selected to represent sample farms with significantly different hired-labor availabilities. These steps resulted in 36 representative farms in the basic stratification.

The 36 farms were developed to represent the typical, rather than the average, bundle of resources of the strata. This procedure was followed for all NC-54 work, and the secondary data on farm resources also were compiled for typical, rather than average, farms.[7] The size and location of these representative farms are shown in table 2. The first part of the farm-number identification tells the area (as shown in fig. 2) in which the farm is located; the capital letter denotes the size stratum; and, where used, the lower-case letter denotes the two levels of hired labor available on the farm. As an example, farm number 2Ca designates the large representative farm (over 239 acres) in area 2 (see fig. 2) with a lower availability of hired labor (one man year or less).

The aggregation coefficients in table 2 represent the factors necessary to (a) aggregate the 36 representative farm results to the sample total of 6,800 farms and (b) estimate the state response from the 6,800 farm sample in a single step. The discrepancy between the aggregation coefficient total of 135,375 farms and the population total of 136,331 farms arose from using the modal rather than the average representative farm. Generally, the size of the modal farm did not equal the average size of farm in a particular stratum. As a result, the aggregation coefficients were defined as the total cropland acreage in each stratum divided by the cropland acreage of the respective representative farm. This procedure assured that the total cropland figure for the population would be equaled by the aggregation of the representative farms, a desirable characteristic in view of the primary importance of cropland in determining the amount of production for the representative farms.

The basic group of 36 representative farms was developed to provide population-supply estimates relatively free of aggregation error. The stratification of sample farms was carried out with the objective of approximating the conditions of Theorem I. As discussed previously, the number of representative farms required to assure exact aggregation is extremely large for a model of the size being considered; it seems possible, however, to achieve reasonably accurate aggregation with much smaller numbers of representative farms.

Table 2. Average size and aggregation coefficients for the group of 36 representative farms.

Farm number[a]	Cropland	Total farmland	Aggregation coefficient
	(acres)	(acres)	
1A	75.4	87.9	2,104
1B	151.7	177.0	5,521
1C	282.4	329.4	4,134
2A	58.0	77.0	598
2B	134.0	169.0	2,571
2Ca	275.5	364.8	2,372
2Cb	474.0	513.0	256
3A	73.8	93.1	2,775
3B	149.0	177.8	5,118
3C	303.7	374.3	5,731
4A	76.0	93.0	3,108
4B	156.7	176.1	14,650
4Ca	268.9	311.2	7,771
4Cb	316.0	362.2	4,608
5A	58.0	83.4	1,544
5B	124.2	175.7	2,767
5C	245.1	401.3	2,855
6A	58.0	97.0	1,491
6B	122.9	191.7	3,621
6Ca	204.2	365.6	4,716
6Cb	256.0	410.0	996
7A	70.5	90.7	4,683
7B	148.6	178.3	7,847
7C	286.4	348.2	4,926
8A	76.8	95.4	4,250
8B	153.0	178.4	8,474
8Ca	244.8	317.0	3,763
8Cb	279.0	355.0	1,320
9A	33.8	57.5	4,917
9B	137.4	180.8	4,201
9Ca	259.7	355.8	3,319
9Cb	310.0	423.0	328
10A	62.0	94.0	1,815
10B	121.7	178.0	4,042
10Ca	202.0	342.0	1,469
10Cb	265.0	378.0	714
			135,375

[a] The number at the first indicates the soil area as identified in fig. 2. The middle or capital letter indicates the farm size group (A = 0 - 139 acres, B = 140 - 239 acres and C = 240 and over acres). The lower case letter at the end denotes hired labor amount (a = one hired man equivalent or less, b = more than one hired man).

Substratifications

After delineation of the 36 representative farms, the three smaller groups of representative farms (groups of 10, three, and one farms) were developed by using the weighted averages of resources from the original 36 farms. Data for these

[7] The typical or modal concept of a representative farm has certain advantages when the results are used for purposes not mentioned previously—recommendations to individual farmers. Generally, the optimum program for a modal representative farm has exact applicability for a larger number of real-world farms than does that for an average representative farm. While the procedure might increase aggregation error in the estimated state supply functions, it does not affect the amount of aggregation error indicated among the four groups of supply functions developed in this study. The reason is that the modal-farm concept is used only in the basic group of 36 representative farms against which the others are judged. The resources on the representative farms in the three subgroups are defined as averages of the 36 farms and not as modes.

three subgroups of representative farms are presented in table 3. The first subgroup (10 representative farms) consisted of an average farm in each of the 10 soil areas. For example, farm 1BB, the average farm in area 1, is a weighted average of resources on farms 1A, 1B, and 1C, with the aggregation coefficients used as the weights. The aggregation coefficient for farm 1BB is then the sum of the coefficients of the other three farms. As a result, the aggregation coefficients for the 10 representative farms also are the ones necessary to obtain population estimates for the state by using these 10 farms.

A second subgroup (three representative farms) was delineated to represent the small, medium, and large farms in the population. These three representative farms, StA, StB, and StC in table 3, were determined by weighted averages of resources on the 10 small farms, the 10 medium farms, and the 16 large farms. Again, the aggregation coefficients sum to the state total and may be used to obtain estimates of the total population based on this group of three representative farms.

The final stratification considered (one representative farm), StBB, was to be used alone in estimating production for the population of interest. Farm StBB was the weighted average of resources on all the 36 basic representative farms; its aggregation coefficient being the total of the 36 basic aggregation coefficients.

Because of the manner in which (a) the four groups of representative farms were developed and (b) their aggregation coefficients were determined, all groups represent exactly the same total amounts of resources for the over-all population. For example, labor available in the 36-farm group

Table 3. Average size and aggregation coefficients for the three subgroups of representative farms

Farm number	Cropland	Total farmland	Aggregation coefficient
	(acres)	(acres)	
Ten-farm subgroup[a]			
1BB	184.0	214.6	11,759
2BB	199.1	254.4	5,797
3BB	198.8	243.2	13,624
4BB	201.7	230.8	30,137
5BB	158.1	245.7	7,166
6BB	161.6	274.5	10,824
7BB	166.6	202.7	17,456
8BB	163.6	201.0	17,807
9BB	133.7	185.0	12,765
10BB	135.6	206.8	8,040
			135,375
Three-farm subgroup[b]			
StA	64.0	85.6	27,285
StB	145.6	178.0	58,812
StC	269.8	351.8	49,278
			135,375
One-farm subgroup			
StBB[c]	174.4	222.7	135,375

[a]The notation BB indicates an average (weighted) farm size in the region denoted by the digit at the front. Thus 1BB denotes an average (weighted) for the three farm sizes in soil area 1 (see fig. 2).

[b]The notation St refers to the state total and the A, B or C refer to the three size groups. Thus StA is the "aggregated" small farm for the state, StB is the "aggregated" medium sized farm and StC is the "aggregated" large farm for the state.

[c]StBB is the average (weighted) farm for the state.

multiplied by the respective aggregation coefficients sums to the same total as the labor available in the three groups multiplied by their respective aggregation coefficients. As a result, the four sets of supply estimates developed are free from differences that would arise if different amounts of resources were used or designated for the over-all population.

The Representative-Farm Models

A separate linear-programming model was developed for each of the 50 representative farms (36 original representative farms, plus the 14 indicated in table 3). The models for all farms had the same number of restrictions and activities; the value of many coefficients, however, varied among farms. The 36 restrictions used in each farm model are identified in Appendix table A-1, along with the quantities of resources available on the medium-sized representative farm in area 9, farm 9B. Appendix table A-2 summarizes the 73 activities considered in the linear-programming models of the representative farms. Appendix A also contains an explanation of the purpose of the different restrictions and activities in the models since theoretical work suggests that the structure and complexity of the model affect aggregation error.

The representative-farm supply functions were generated by varying the price of pork and beef over a range of 16 discrete price combinations and recording the quantities produced at each price. Pork prices programmed were $10.50, $11, $12, and $13 per hundredweight; beef prices programmed were $14, $15.50, $17, and $19 per hundredweight. The quantities of pork and beef produced on the representative farms were expressed on a liveweight basis and were determined as follows: For pork, the levels of the selling activity P_1 (Appendix table A-2) were used directly. For beef, the quantity produced was defined as net beef produced on the farm and computed as the level of the selling activity P_2, plus the level of activity P_6 at 430 pounds per head, minus the level of activity P_3 at 440 pounds per head, and minus the level of activities P_4 and P_5 at 715 pounds per head. When all 16 price combinations were programmed, these quantities allowed determination of four discrete points on each of four supply functions for pork and four supply functions for beef.

All representative farms in the 10-farm subgroup were given coefficients for the medium-sized farms in the respective areas. A separate set of state average yields and fertilizer costs was developed for the three-farm subgroup and the one-farm subgroup. The three-farm subgroup used coefficients for the three respective sizes of farms, and the one-farm subgroup used coefficients for the medium-sized farms.

The complete coefficient matrix contained 2,628 nonzero elements, a density of about 27 percent. Equation 18 and table 1 suggest that nearly every individual farm in the population must be programmed to assure achieving exact aggregation for all activity levels with a coefficient matrix of this size and density. This relation emphasizes the difficulty of obtaining exact aggregation in the estimates of a research project of typical scope. The same difficulty is, of course, encountered in obtaining an exact measurement of aggregation error in such estimates.

Resulting Supply Functions

Optimum linear-programming solutions were obtained for all 50 of the defined representative farms under each of the 16 price combinations previously discussed. These representative-farm optimal solutions were then used to obtain four different supply estimates of the population by use of the aggregation coefficients presented in tables 2 and 3. The optimal solutions for the basic group of 36 representative farms were aggregated to obtain one set of supply estimates of the population; the same procedure being repeated with the optimal solutions for each of the three smaller groups of representative farms—the 10-farm, three-farm, and one-farm groups.

The four sets of estimated supply functions for beef are presented in fig. 3 and, for pork, in fig. 4. Four functions are included in each graph because changes in the price of pork cause shifts in the beef-supply functions; similarly, changes in the price of beef cause shifts in the pork-supply functions. These shifts are a result of the familiar cross-elasticity relationships of supply. Each of the supply functions is drawn from estimation of four discrete points.[8] The numerical production estimates used as a basis for figs. 3 and 4 are presented in tables 4 and 5.

Aside from a few points on those curves involving the three and one representative farms, the supply curves are similar as to location and elasticities. The differences are indeed small in comparison with differences in computational costs involved in obtaining the different estimates. The computational costs of the four different estimates were roughly proportional to the number of representative farms involved. Programming and aggregation costs on the IBM 7074 computer for this study were about $41 per farm. Thus, the computing cost difference was about $1,066 between the 36 and 10 representative-farm estimates and $369 between the 10 and one representative-farm estimates. These do not include other normal research costs involved in getting different numbers of farms prepared for computations.

[8]This is in contrast to the more usual "stepped" supply functions, estimated by linear programming, that result when the price is continuously varied within a given range. In the two-product case, varying two prices continuously within even a small range results in a multitude of different solutions; for this reason, only 16 discrete price combinations were programmed.

COMPARISON OF RESULTS FROM PROGRAMMED SUPPLY FUNCTIONS

The beef- and pork-supply aggregation errors are evaluated in this section, and state estimates for other major farm products derived from the four groups of representative farms are presented and appraised. Optimum solutions for representative farms of one soil area also are presented in detail to provide an understanding of the complexity of the relation between representative-farm data, optimum solutions, and aggregation error.

State Supply Estimates

Differences among beef- and pork-supply estimates

Figs. 3 and 4 and the data in tables 4 and 5 provide bases for a detailed analysis of aggregation error among the different estimated beef- and pork-supply functions for the state. Aggregation error can be analyzed at all 16 price combinations. Since not all 16 sets of results are equally realistic by real-world standards, however, price combinations resulting in Iowa beef and pork production in the neighborhood of present and historic production levels are used for the analysis.

Iowa produced 2.8 billion pounds of beef and 4.4 billion pounds of pork in 1965 (34). Reference to fig. 4 reveals that this level of pork production is most consistent with an $11 pork price. With a pork price of $11, reference to the beef-supply curves in fig. 3 shows that recent levels of beef production are achieved in the model at approximately a $15.50 beef price. This $15.50 beef price also is consistent with the $11 pork price in obtaining near-current levels of pork production in fig.4. Hence, the analysis of aggregation error is based primarily on beef-supply functions estimated with the price of pork held at $11 and pork-supply functions estimated with the price of beef held at $15.50. The four such functions for beef are superimposed in fig. 5. Similarly, the four pork-supply functions estimated with a beef price of $15.50 are superimposed in fig. 6.

In fig. 5, the four Iowa beef-supply functions estimated by the four different groups of representative farms are very similar. The estimate for the one representative farm is relatively higher at the $15.50 price, but the rest of the estimated points are quite similar. The over-all slopes of the functions are in agreement, and differences between them are small and nearly disappear at beef prices of $17 and $19. As mentioned previously, these functions are strikingly uniform relative to the large differences in computational costs.

The four Iowa pork-supply functions in fig. 6 exhibit somewhat larger differences, especially at the two highest pork prices. Pork production at these highest prices is 3-5 times greater than recent production levels. Linear programming is expected to provide a poor simulation of actual farm conditions at these production levels (26). Thus,

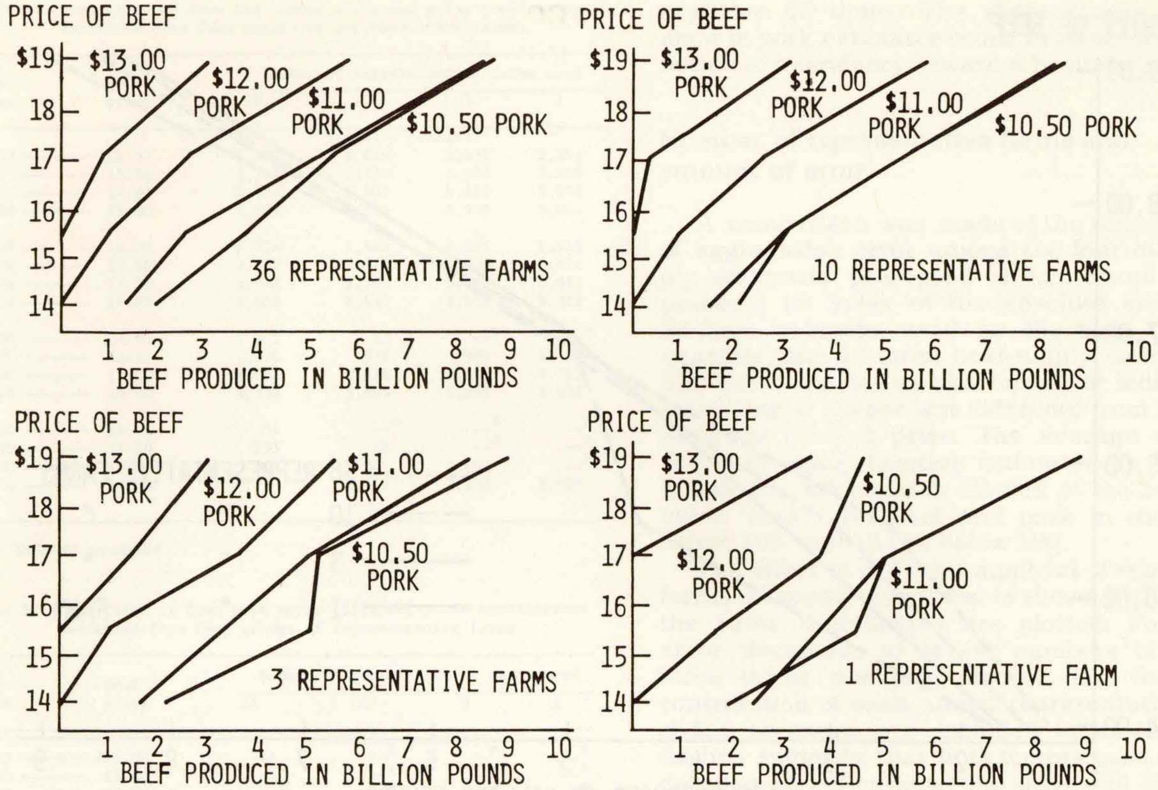

Fig. 3. Iowa beef-supply functions estimated from four groups of representative farms.

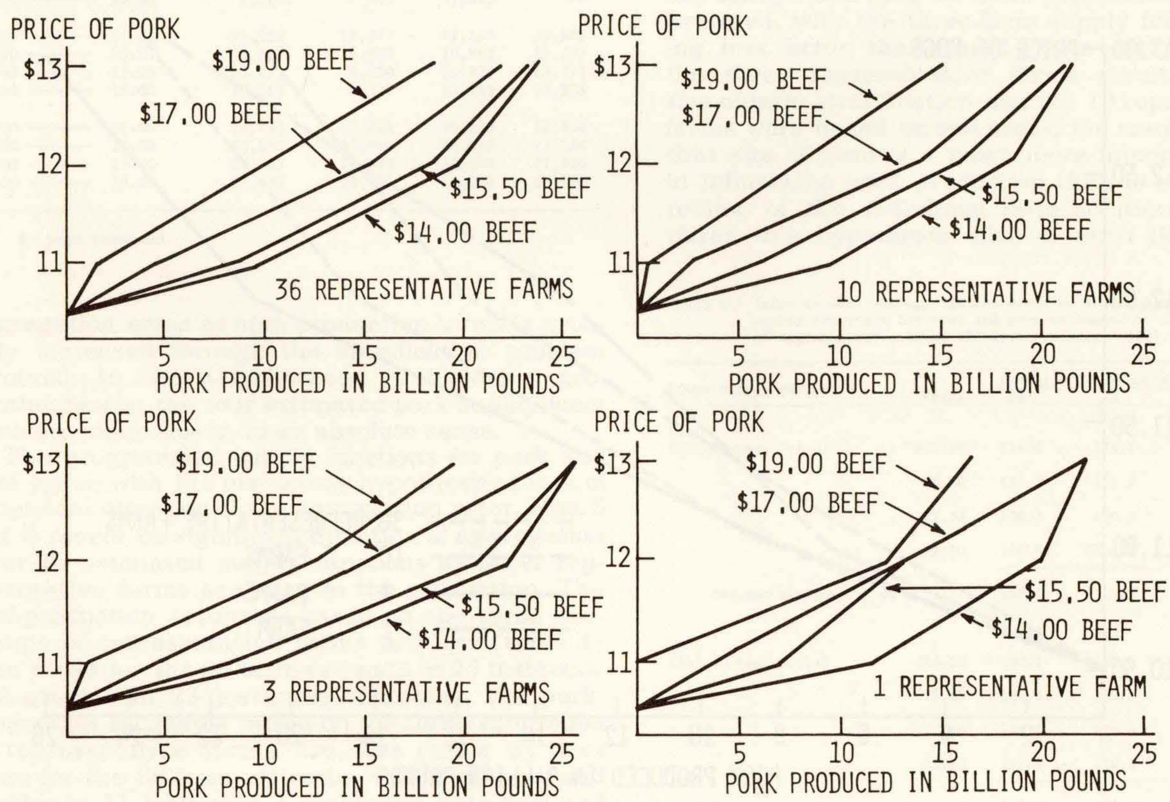

Fig. 4. Iowa pork-supply functions estimated from four groups of representative farms.

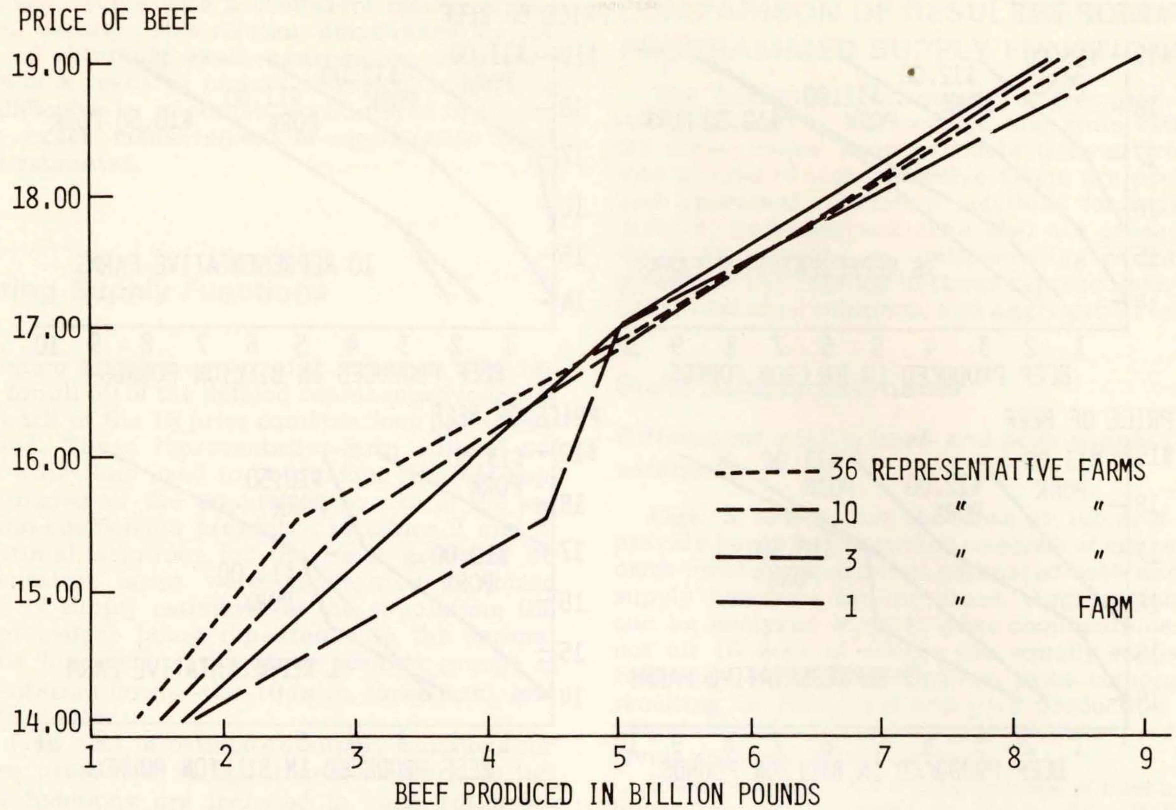

Fig. 5. Iowa beef-supply functions at the $11.00 pork price.

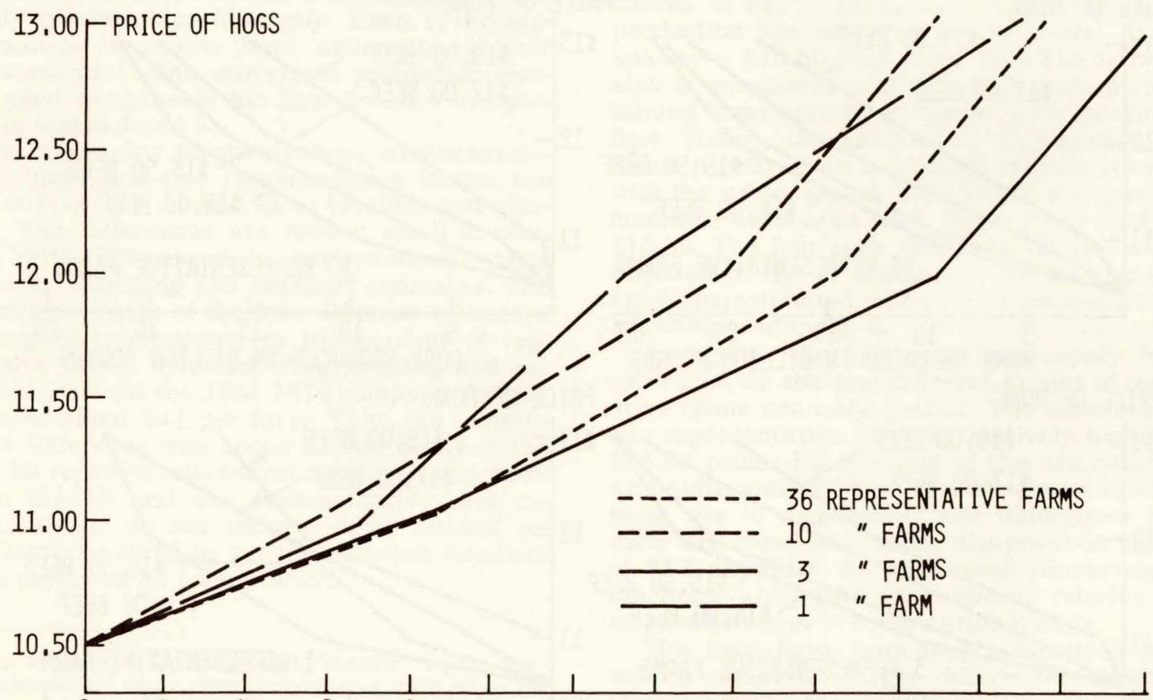

Fig. 6. Iowa pork-supply functions at the $15.50 beef price.

Table 4. Production of Iowa beef under different price combinations estimated from four groups of representative farms.

Pork price	Beef price	Number of representative farms used 36	10	3	1
		(million pounds)			
$10.50 -------	$14.00	1,909	2,028	2,171	2,354
10.50 -------	15.50	3,762	3,030	5,093	3,750
10.50 -------	17.00	5,454	5,405	5,213	5,093
10.50 -------	19.00	8,802	8,508	8,939	9,012
11.00 -------	14.00	1,316	1,508	1,686	1,766
11.00 -------	15.50	2,542	2,978	3,335	4,431
11.00 -------	17.00	5,343	5,375	5,103	5,093
11.00 -------	19.00	8,625	8,461	8,439	9,012
12.00 -------	14.00	195	57	--[a]	--[a]
12.00 -------	15.50	1,026	936	981	1,725
12.00 -------	17.00	2,566	2,626	3,613	3,712
12.00 -------	19.00	5,752	5,855	5,667	4,638
13.00 -------	14.00	54	--[a]	--[a]	--[a]
13.00 -------	15.50	155	18	--[a]	--[a]
13.00 -------	17.00	845	365	1,458	--[a]
13.00 -------	19.00	3,110	3,247	3,130	3,589

--[a] No beef produced

Table 5. Production of Iowa pork under different price combinations estimated from four groups of representative farms.

Pork price	Beef price	Number of representative farms used 36	10	3	1
		(million pounds)			
$10.50 -------	$14.00	344	--[a]	--[a]	--[a]
10.50 -------	15.50	--[a]	--[a]	--[a]	--[a]
10.50 -------	17.00	--[a]	--[a]	--[a]	--[a]
10.50 -------	19.00	--[a]	--[a]	--[a]	--[a]
11.00 -------	14.00	9,923	9,659	11,041	10,726
11.00 -------	15.50	8,473	5,817	8,461	6,884
11.00 -------	17.00	4,623	2,837	4,496	3,769
11.00 -------	19.00	1,567	413	1,043	--[a]
12.00 -------	14.00	20,055	18,277	22,118	19,981
12.00 -------	15.50	18,365	15,838	20,942	13,235
12.00 -------	17.00	15,577	14,220	15,539	13,398
12.00 -------	19.00	10,269	8,387	12,285	12,535
13.00 -------	14.00	23,743	20,824	26,119	22,520
13.00 -------	15.50	23,582	20,803	26,119	22,520
13.00 -------	17.00	22,057	19,672	24,301	22,520
13.00 -------	19.00	18,634	15,294	19,868	16,560

--[a] No pork produced.

aggregation error at high production levels is probably increased through the specification problem involved. In the neighborhood of historical production levels, the four estimated pork-supply functions are much closer, in an absolute sense.

The programmed supply functions for pork and beef agree with the previously hypothesized lack of consistent direction of the aggregation error. Figs. 5 and 6 reveal no significant direction of aggregation error in estimated supply functions as fewer representative farms are used in the estimation. The beef-production estimates based on the three subgroups of representative farms are less (table 4) than those for the 36-farm estimate in 25 instances and greater in 23 instances. Likewise, the pork-production estimates based on the three subgroups of representative farms are less (table 5) than those for the 36-farm estimate in 28 instances and greater in 11 instances. Considering both beef and pork, the aggregation error is positive 34 times and negative 53 times. The slight excess of negative error in pork estimates tends to refute the historical notion of a tendency toward a positive bias.

Number of representative farms and amount of error

A comparison was made of the relative amounts of aggregation error among the four different supply estimates presented in figs. 5 and 6. Table 6 presents an index of the absolute error with the 36-farm estimate used as the base of 100. For example, the 10-farm, beef-supply estimate at the $19 beef price has an absolute error index of 101.9, indicating a 1.9-percent difference from the 36-farm estimate at that price. The average error along the beef-supply function estimated by the 10-farm function is 8.6 percent. Eleven of the 24 individual index values for beef and pork in the table are below 105, and 18 are below 120.

The effect of different numbers of representative farms on supply estimates is shown by fig. 7, where the table 6 averages are plotted. For beef, the error decreases as larger numbers of representative farms are programmed, and the marginal contribution of each added representative farm declines as more are included in the model. This decline suggests that both factors used in the more detailed stratifications, soil area and size of farm, were useful in increasing accuracy of beef-supply estimates.

For the pork-supply estimates, the error does not always decrease as more representative farms are used, with the three-farm supply function having less error than the 10-farm estimate. Since the three representative farms resulted from a size-of-farm stratification and the 10 representative farms were based on soil areas, the results suggest that size of farm is a much more important factor in influencing pork production than is soil type. A review of the individual farm solutions substantiates this hypothesis. Size of farm is quite im-

Table 6. Index of absolute aggregation error of points along state supply functions for beef and pork estimated by three subgroups of representative farms (36-farm estimate = 100).

State supply estimates	Price	Number of representative farms 10	3	1
Beef ($11.00 pork price)	$14.00	114.6	128.1	134.2
	15.50	117.2	131.2	174.3
	17.00	100.6	104.5	104.7
	19.00	101.9	102.2	104.5
Average absolute error		108.6	116.5	129.4
Pork ($15.50 beef price)	$10.50	100.0	100.0	100.0
	11.00	131.3	100.1	118.8
	12.00	113.8	114.0	127.9
	13.00	111.8	110.8	104.5
Average absolute error		114.2	106.2	112.8

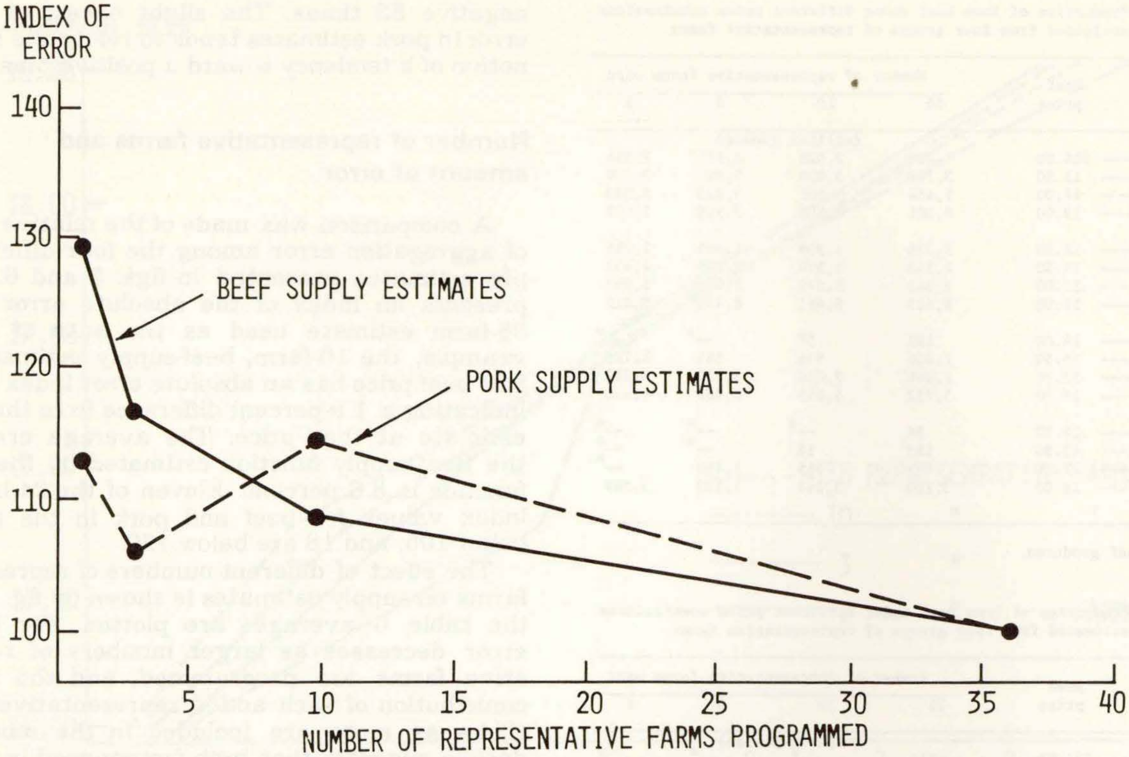

Fig. 7. Effect of programming different numbers of representative farms on aggregation error in state beef- and pork-supply estimates.

portant in influencing whether or not a farm produces pork; area of the state is relatively less important. Thus, it seems that omitting the size-of-farm classification results in more error in the pork estimates when 10 representative farms were used, even though the number of representative farms programmed is increased from three to 10.

The relationships of fig. 7 substantiate our previous theory that: (a) more detailed stratification increases accuracy only when it recognizes factors that influence the existence of certain enterprises on the farms and (b) different stratification factors may be required in the estimation of different products. When a factor, such as soil area, has only a small effect on the existence of pork production enterprises on farms, additional stratification based on it provides relatively small gains in accuracy. Likewise, overlooking a factor that does affect the existence of the hog enterprise on the farms results in a substantial increase in error. Finally, a factor useful for controlling error in the estimation of one product may not have the same effect on estimates of another product.

Error in elasticity estimates

Many questions of agricultural policy have answers depending on the price elasticities of supply for the products in question. The representative-farm, linear-programming technique may provide more accurate estimates for supply elasticity than for the actual level of supply. For example, overestimating all resources on a farm would affect the level of the supply estimates, but not necessarily the elasticity. Thus, elasticity estimates may have usefulness even when the actual level of the estimated supplies is inaccurate.

The estimated elasticities and cross-elasticities of supply between the prices of beef and pork and the quantities of beef and pork produced are presented in tables 7 and 8. These elasticities are computed by the equation

$$e = [(Q_2 - Q_1)/(Q_2 + Q_1)]/[(P_2 - P_1)/(P_2 + P_1)]$$

and represent the arc elasticities since they are averages between the two points programmed on the supply functions.

A few generalizations can be made about the estimated price elasticities of supply. In table 8, which expresses the effect of a change in the price of beef, the 36-farm and 10-farm estimates generally agree. This comparability is especially high at the $11.00 pork price. In table 7, the effect of a change in the price of pork is generally quite similar, as estimated by each of the four groups of representative farms. Even the estimates developed by programming one representative farm provide a reasonably accurate estimate of the changes in the quantities of pork and beef produced resulting from changes in pork prices.

Error in other major estimates

The severity of the aggregation error problem and the number and type of representative farms required are extremely dependent upon the particular estimate desired. Table 9 shows the amount of aggregation error present in various state estimates developed by using the four groups of

Table 7. Elasticities and cross-elasticities between price of beef and quantities of beef and pork estimated by four groups of representative farms.

		Percentage change in:							
		Beef production				Pork production			
Pork price	Arc range of beef price	36 farms	10 farms	3 farms	1 farm	36 farms	10 farms	3 farms	1 farm
$10.50									
	$14.00-15.50	6.43	3.90	7.91	4.50	--[b]	--[b]	--[b]	--[b]
	15.50-17.00	3.98	6.10	0.25	3.29	--[b]	--[b]	--[b]	--[b]
	17.00-19.00	4.23	4.01	4.74	5.00	--[b]	--[b]	--[b]	--[b]
$11.00									
	$14.00-15.50	6.25	6.44	6.46	8.46	-1.55	-4.88	-2.60	-4.29
	15.50-17.00	7.70	6.22	4.54	1.51	-6.35	-7.46	-6.63	-6.34
	17.00-19.00	4.23	4.01	4.44	5.00	-8.89	-13.43	-11.22	--[b]
$12.00									
	$14.00-15.50	13.38	17.41	19.67	19.67	-0.87	-1.41	-0.54	-3.99
	15.50-17.00	9.29	10.28	12.41	7.92	-1.78	-1.17	-3.21	0.13
	17.00-19.00	6.89	6.85	3.98	2.00	-3.70	-4.64	-2.11	-0.60
$13.00									
	$14.00-15.50	9.46	19.67	--[a]	--[a]	-0.07	-0.01	--[b]	--[b]
	15.50-17.00	14.99	19.63	21.67	--[a]	-0.72	-0.61	-0.78	--[b]
	17.00-19.00	10.31	14.36	6.56	18.00	-1.51	-2.25	-1.81	-2.75

--[a] No beef produced.
--[b] No pork produced.

Table 8. Elasticities and cross-elasticities between price of pork and quantities of pork and beef estimated by four groups of representative farms.

		Percentage change in:							
		Pork production				Beef production			
Beef price	Arc range of pork price	36 farms	10 farms	3 farms	1 farm	36 farms	10 farms	3 farms	1 farm
$14.00									
	$10.50-11.00	40.12	43.00	43.00	43.00	-7.91	-6.32	-5.41	-6.14
	11.00-12.00	7.77	7.10	7.68	6.93	-17.06	-21.32	-23.00	-23.00
	12.00-13.00	2.11	1.63	2.07	1.49	-14.16	-25.00	--[a]	--[a]
$15.50									
	$10.50-11.00	43.00	43.00	43.00	43.00	-8.32	-0.37	-8.97	3.58
	11.00-12.00	8.48	10.64	9.76	7.26	-9.77	-12.00	-12.54	-10.11
	12.00-13.00	3.11	3.39	2.75	6.49	-18.47	-24.06	-25.00	-25.00
$17.00									
	$10.50-11.00	43.00	43.00	43.00	43.00	-0.44	-0.12	-0.46	--[a]
	11.00-12.00	12.47	15.35	12.68	12.90	-8.08	-7.90	-3.93	-3.61
	12.00-13.00	4.30	4.02	5.50	6.35	-12.61	-18.90	-10.62	-25.00
$19.00									
	$10.50-11.00	43.00	43.00	43.00	43.00	-0.45	-0.12	-1.24	--[a]
	11.00-12.00	16.91	20.85	19.40	23.00	-4.60	-4.19	-4.52	-7.37
	12.00-13.00	7.24	7.29	5.90	3.46	-7.45	-7.16	-7.21	-3.19

--[a] No beef produced.

Table 9. Comparison of major state estimates developed by using four groups of representative farms ($15.50 beef price and $11.00 pork price).

		Number of representative farms						
		36	10		3		1	
State estimate	Unit	Estimate	Estimate	Percent error	Estimate	Percent error	Estimate	Percent error
Litters farrowed-------	1,000 litters	4,475.2	3,063.7	-31.5	4,456.4	-0.4	3,614.9	-19.2
Total pork produced----	mil. lb.	8,472.8	5,817.0	-31.3	8,461.3	-0.1	6,884.2	-18.8
Beef cows--------------	1,000 head	1,273.3	1,382.7	+ 8.6	865.4	-32.7	37.9	-97.0
Net feeder calves purchased-----------	1,000 head	2,009.1	2,510.8	+25.0	3,800.8	+89.2	6,658.9	+231.4
Calves fed-------------	1,000 head	3,015.1	3,603.2	+19.5	4,484.5	+48.7	6,688.8	+121.8
Total beef produced----	mil. lb.	2,542.1	2,977.7	+17.2	3,335.0	+31.2	4,431.1	+ 74.3
Corn acreage-----------	1,000 acres	9,007.1	8,935.9	- 0.8	8,748.5	-2.9	8,748.5	- 2.9
Corn production--------	mil. bu.	772.2	771.8	- 0.1	755.8	-2.1	755.8	- 2.1
Net corn sold in state-	mil. bu.	123.5	246.4	+99.5	24.3	-80.3	--	-100.0
Oat acreage------------	1,000 acres	1,218.8	1,054.3	-13.5	997.6	-18.1	997.6	-18.1
Oat production (in corn equivalent)---------	mil. bu.	32.3	28.8	-10.8	27.5	-14.9	27.5	-14.9
Soybean acreage--------	1,000 acres	9,702.1	9,615.3	- 0.9	9,746.1	+0.5	9,746.1	+0.5
Soybean production-----	mil. bu	318.9	316.0	- 0.9	320.6	+0.5	320.6	+0.5
Meadow acres (cropland)	1,000 acres	3,674.8	3,997.1	+ 8.8	4,110.5	+11.9	4,110.5	+11.9
Hay production---------	1,000 ton	3,930.1	4,488.2	+15.0	4,302.7	+10.2	4,538.4	+16.3
Total labor hired------	1,000 hr.	12,032.9	8,040.0	-33.2	9,346.3	-22.3	6,856.5	-43.0
Maximum net returns----	mil. $	930,755	853,773	- 8.3	944,389	+1.5	868,282	- 6.7

representative farms. These errors for beef and pork production are the same as the ones in table 6 for the appropriate price combination. Other estimates shown in table 9 are hog and cattle numbers, major crop acreages and production, hired labor use, and the maximum net returns (linear-programming functional value) for the state.

Table 9 shows a wide range in the amount of aggregation error in the different estimates. Estimated net feeder-calf purchases and sales and net corn sales have the greatest amounts of aggregation error. Estimates of corn and soybean acreage and production have low errors. If an estimate of net feeder-calf purchases for the state is desired, a considerable number of representative farms of the types developed in this study would be required. For estimates of corn and soybean acreages, however, much smaller numbers of representative farms would suffice. One representative farm for the entire state has an aggregation error of only 0.5 percent for soybeans and 2.9 percent for corn. This finding has relatively important implications for models designed solely for estimating Iowa crop acreages and production when there is no desire for livestock production estimates.

The value of the optimal functional (maximum net returns) for the state from the 10- and one-farm groups is less than the optimal functional estimated by the 36 representative farms, as shown on the last line of table 9. On the surface, this seems a contradiction of Theorem III. There is no inconsistency in these results, however, since Theorem III specifies farms with identical cost and coefficient matrices, a requirement not met by the groups of farms programmed for developing the state estimates. Thus, the estimate of the functional as shown in table 9 may have negative error rather than the positive error implied by Theorem III.

Area 3 in Detail

The aggregation error found at one price combination in one of the 10 soil areas was analyzed in more detail in an attempt to ascertain (a) the individual farm characteristics that lead to aggregation error and (b) the interrelationships among the errors in the various estimates that may be derived from the programming results. The analysis is based on comparison of the optimal solutions of farms 3A, 3B, and 3C (table 2) with the optimal solutions of farm 3BB (table 3).

Soil area 3, composed of 10 counties in southwestern Iowa (fig. 2), was chosen for detailed analysis primarily because relations and solutions in this area are less complex than those in other areas. At the same time, data and results of this area portray many of the characteristics found throughout the state. Only one solution was analyzed in detail for area 3; the solution obtained from the $11.00 pork and the $15.50 beef price combination.

Three farms from the 36 representative-farm group, 3A, 3B, and 3C, representing the small, medium, and large farms in the area, were used to represent area 3. For comparison, farm 3BB from the group of 10 representative farms was used to represent this area. The resources for farm 3BB (table 3) were the weighted averages of resources available on farms 3A, 3B, and 3C (table 2). The rest of the net-return and input-output coefficients for farm 3BB were the same as the coefficients for farm 3B since both farms were in the medium-sized range in the same area.

The first step in analyzing the aggregation error in area 3 is to review the four optimum representative-farm, linear-programming solutions in some detail. Table 10 shows amounts of resources available on each of these four representative farms and identifies resources restrictive under the optimal solutions. All livestock facility resources are included in the table because of their primary importance in determining the optimum solutions at the $11.00 pork and $15.50 beef price combination. Other nonrestrictive resources play no part in influencing the final solutions and are not shown in table 10. Restrictive resources at this price combination are cropland, pasture, selected hog and beef facilities, and April, October, and November operator and family labor, as shown in table 10. Generally, more resources become restrictive as farm size increases, with farm 3A having only four restrictive resources and farm 3C having 10 restrictive resources. This increase in number of restrictive resources is a result of more complex solutions, including greater numbers of activities, on the larger farms.

A summary of the optimum solutions for each of the four representative farms in area 3 is presented in table 11. The optimum solution for farm 3A includes two crop rotations (corn-soybeans and CSSOMM), meadow on the low-capability cropland, 26.8 purchased calves fed to utilize the pasture available, and sale of the surplus corn produced. The solution for the medium-sized farm, farm 3B, has the same cropping pattern as farm 3A. This solution, however, also includes one- and four-litter hog systems and beef cows. These three enterprises and additional purchased feeder calves are combined to utilize all the corn, pasture, portable farrowing facilities, and April operator and family labor available on the farm. No corn is sold, and no labor is hired. Beef-housing or beef-feeding facilities are not fully utilized by this combination of enterprises. The solution for farm 3C in table 11 is more complex than the other solutions. One additional rotation, COMM, is added to the crop system on part of the low-capability cropland. One-, two-, and four-litter hog systems, beef cows, and a feeding enterprise for farm-raised beef calves compose the livestock enterprises. These enterprises utilize all corn, pasture, portable hog farrowing and feeding facilities, and beef housing

Table 10. Available resources and the extent of their use on representative farms in area 3 ($15.50 beef price and $11.00 hog price).

Resource[a]	Unit	Representative farm number 3A	3B	3C	3BB
Number of farms represented	No.	2,775	5,118	5,731	13,624
Land with 25% row-crop capability	acre	6.9 R[b]	14.0 R	28.5 R	18.7 R
Land with 50% row-crop capability	acre	34.6 R	69.9 R	142.4 R	93.2 R
Land with 100% row-crop capability	acre	32.3 R	65.1 R	132.7 R	86.9 R
Total Cropland	acre	73.8	149.0	303.6	198.8
Pasture (not tillable)	ton AHY[c]	15.1 R	19.7 R	54.4 R	33.4 R
Central farrowing facilities	sow	-- N	9.6 N	12.1 N	8.7 N
Portable farrowing facilities	sow	14.1 N	1.8 R	9.4 R	7.5 R
Portable feeding facilities	pig	92.6 N	156.7 N	225.7 R	172.7 N
Beef housing -- both periods	a.u.[d]	23.1 N	28.6 N	39.8 R	32.2 R
Low beef mechanization -- both periods	head	48.3 N	11.4 N	19.2 N	22.2 N
High beef mechanization -- both periods	head	-- N	89.5 N	127.9 N	87.4 N
April operator and family labor	hour	227.7 N	246.6 R	285.4 P	259.1 P
October operator and family labor	hour	227.7 N	246.6 N	285.4 P	259.1 N
November operator and family labor	hour	202.7 N	221.6 N	255.6 P	232.1 N

[a]Resources not listed had no effect on determination of optimal solutions

[b]The letters R, P and N indicate whether the responce was restrictive:
R = restrictive in optimal solution
P = restrictive in optimal solution and additional quantities purchased or hired
N = not restrictive

[c]Tons of anticipated hay yield

[d]Animal units

available on the farm. No corn is sold, and additional labor is hired for the months of April, October, and November.

The average representative farm for area 3, farm 3BB, has a solution much like medium-sized farm 3B. The same cropping pattern and livestock enterprises are present. On farm 3BB, however, beef housing is restrictive, and April labor is hired.

Aggregation errors for activity levels in area 3 are shown in table 12. At the $15.50 beef price and $11.00 pork price, one representative farm underestimates area pork production by 21.1 percent (line 12) and overestimates beef production by 35.1 percent (line 17). The magnitude of these errors is comparable to the size of errors found in the state supply estimates of pork and beef (see table 6).

The levels of the corn-soybean and CSSOMM rotations are the only two activities estimated without error by farm 3BB. Since these two rotations produce all the soybeans found in the optimal solutions, soybean acreage and production also are estimated without error (line 7). Table 12 shows three items estimated without error, seven items estimated with positive error, and 13 items estimated with negative error.

Relations Between Solutions and Aggregation Error

It is possible to explain the aggregation error in some estimates in table 12 by the makeup of the optimal solutions for the representative farms summarized in table 11 and by the amounts of the resources available as shown in table 10.

First, exact aggregation is achieved in three of the estimates listed in table 12. Comparing the acres of the corn-soybeans and CSSOMM rotations in table 11 with the amounts of land available in the 50- and 100-percent, row-crop capability classes in table 10 shows that these two resources are each completely exhausted on all farms by the same two respective rotations. When a given resource is exhausted by a given activity on all representative farms in the three-farm group, it is exhausted by that activity on the one representative farm. No error results in the estimates based on these activity levels. Hence, corn-soybean rotation acres, CSSOMM rotation acres, and soybean acres (since they are a function only of these two activities) are estimated without error by farm 3BB.

Next, it is possible to identify some characteristics of the optimum solutions that lead to error in the estimates. In table 11, farms 3A, 3B, and 3BB use all the 25-percent row-crop capability land for meadow, whereas farm 3C uses 22.1 acres of it for a COMM rotation and only 6.4 acres of it for meadow. Hence, farm 3BB overestimates meadow and underestimates COMM rotation acres—which, in turn, leads to an underestimation of corn production. This difference in land use accounts for all crop-production aggregation errors found in table 12.

From this point on, it becomes extremely difficult to determine the individual solution characteristics that lead to aggregation error. Within the linear-programming models, subsets of activities and resources interact as simultaneous-equation systems. The optimum level of the activities within these systems equals the solution to the system of simultaneous equations. Thus, several characteristics of an optimal solution interact to cause aggregation error in several activity levels. Differences in solutions are generally due to (a) differences in resource availabilities, (b) differences in costs and returns, and (c) differences in input requirements.[9] It is difficult to assign the responsibility for a particular aggregation error because these three factors vary simultaneously among the four representative farms in area 3. It may be possible to isolate the origin of the error (for example, the corn-production aggregation error), but impossible to determine what data characteristics cause it.

STRATIFICATIONS TO REDUCE ERROR

The empirical results provide information for more efficient control of aggregation error. The potentials for (a) reducing the number of representative farms programmed with a minimum generation of aggregation error and (b) achieving further reduction in aggregation error are now explored. The optimum solutions of the 36 representative farms may be analyzed to identify farm combinations consistent with the theory developed in early sections of this report.

Postprogramming Analysis of the 36 Farms

The three preprogramming groupings of the original 36 representative farms (into groups of 10, three, and one representative farms) were based primarily on data characteristics of the original 36 farms. Optimal solutions for the original 36 farms may be used to determine groupings implied by the optimal solutions themselves and relationships that exist between the optimal-solution groups and the preprogramming groups.

[9]The coefficient matrices of the representative farms in the three subgroups were not averages of coefficient matrices of the original 36 representative farms. Only three sets of coefficients were developed for different sizes of farms, and these were used for all representative farms in the three respective size groups. Thus, in area 3, farm 3BB had the same coefficient matrix as farm 3B, even though it was 50 acres larger. Possibly the use of average or weighted average coefficients would have decreased aggregation error.

Table 11. Optimum solutions on four representative farms in area 3 ($15.50 beef and $11.00 hogs).

Activity	Unit	3A	3B	3C	3BB
		Representative farm number			
Meadow on 25% row-crop capability land	acre	6.9	14.0	6.4	18.7
COMM rotation on 25% row-crop capability land	acre	0	0	22.1	0
CSSOMM rotation on 50% row-crop capability land	acre	34.6	69.9	142.4	93.2
Corn-soybean rotation on 100% row-crop capability land	acre	32.3	65.1	132.7	86.9
Corn harvested as grain	bu.	1,615	3,260	6,730	4,348.8
Corn sold, not fed	bu.	238.5	0	0	0
Hay meadow	ton	18.0	26.5	53	38.0
1-litter sow system (portable farrow and feed)	litter	0	1.8	3.9	7.5
2-litter sow system (portable farrow and feed)	litter	0	0	11.1	0
4-litter sow system (central farrow and portable feed)	litter	0	21.0	37.6	16.5
Pork produced	lb.	0	43,100	99,400	45,800
Calves on pasture (low-mechanization feeding)	head	26.8	0	0	0
Calves on pasture (high-mechanization feeding)	head	0	15.5	20.8	33.7
Beef cows	head	0	10.7	26.3	10.3
Purchased beef calves	head	26.8	7.1	0	25.6
Beef produced	lb.	17,700	14,900	25,100	26,700
Hired April labor	hour	0	0	130.5	56.2
Hired October labor	hour	0	0	76.7	0
Hired November labor	hour	0	0	14.3	0
Cash invested off farm	$	7,069	6,625	15,585	9,693

Table 12. Aggregation error in activity levels and production estimates for area 3 ($11.00 pork price and $15.00 beef price).

Activity	Unit	Three-farm estimate	Farm 3BB estimate	Amount of error	Percent error
Meadow on 25% row-crop capability land	1,000 acre	217.6	254.5	+36.9	+17.0
COMM rotation	1,000 acre	36.9	0	-36.9	-100.0
CSSOMM rotation	1,000 acre	1,270.0	1,270.0	0	0
Corn-soybean rotation	1,000 acre	1,183.3	1,183.3	0	0
Corn harvested as grain	1,000 bu.	59,737	59,239	-498	-0.8
Corn sold, not fed	1,000 bu.	661.8	0	-661.8	-100.0
Soybean production[a]	1,000 bu.	30,344	30,344	0	0
Hay harvested on meadow	1,000 ton	492	518	+26	+5.3
1-litter sow system (portable farrow and feed)	1,000 litter	31.4	102.2	+70.8	+225.5
2-litter sow system (portable farrow and feed)	1,000 litter	63.5	0	-63.5	-100.0
4-litter sow system (central farrow and portable feed)	1,000 litter	322.9	224.9	-98.0	-30.3
Pork produced	mil. lb.	790.5	623.6	-166.9	-21.1
Calves on pasture, no silage (low-mechanization feeding)	1,000 head	74.5	0	-74.5	-100.0
Calves on pasture, no silage (high-mechanization feeding)	1,000 head	198.6	459.5	+260.9	+131.4
Beef cows, no silage	1,000 head	205.7	140.0	-65.7	-31.9
Purchased beef calves	1,000 head	110.5	348.9	+238.4	+215.7
Beef produced	mil. lb.	269.4	364.4	+94.6	+35.1
Hired April labor	1,000 hour	748.2	765.9	+17.7	+2.4
Hired October labor	1,000 hour	439.7	0	-439.7	-100.0
Hired November labor	1,000 hour	81.8	0	-81.8	-100.0
Total labor hired[a]	1,000 hour	1,269.7	765.9	-503.8	-39.7
Cash surplus and invested off farm	mil. $	142.8	132.1	-10.7	-7.5
Maximum net returns (functional value)[a]	mil. $	88.1	76.1	-12.0	-13.6

[a] A function of activity levels.

Grouping by response patterns

To the extent that the data are available, grouping farms by response patterns (or more specifically, stratification so that farms meet the QHOV requirement of Theorem I) should yield better estimates than grouping by soil area or size. Fig. 8 shows the original 36 farms grouped by major response patterns. Thirteen of the cells in fig. 8 contain one or more representative farms. These 13 groups represent the major different types of response patterns of farm organizations found on the original 36 farms. Programming new representative farms for each of these 13 groups should yield supply estimates nearly as accurate as the original 36-farm estimates.

Three pairs of farms of the 36 farms meet the strict QHOV requirements of Theorem I. These pairs, denoted by * in fig. 8, are 4A and 8A, 1A and 7A, and 5A and 9A. These three pairs could be combined, reducing the number of representative farms to 33, with little loss of accuracy in the aggregate estimates.[10]

[10] These pairs of farms do not meet the identical coefficient-matrix requirements of Theorem I because of differences in crop yields and costs between areas. Thus, there is no guarantee that exact aggregation could be achieved if the pairs were combined.

Grouping areas or sizes

The combinations of representative farms suggested by response patterns often stretch across both size and area classifications. Because of differences in yields, costs, and input-output relations, however, some of these combinations may be impractical because of the coefficient problems involved. For example, it may be difficult to determine the correct crop yields for the representative farm covering farms 6Ca and 10B in fig. 8. Hence, for some research projects, it may be meaningful to consider mainly combinations of strata within the existing classification, rather than the completely new classifications of fig. 8.

The central question then is one of identifying areas and (or) sizes of the original 36-cell stratification that may be combined without large increases in aggregation error. Fig. 8 suggests some answers to this question. Considering combinations of areas, areas 4 and 8 might be combined with less aggregation error than can the other areas. The small farms in these areas, 4A and 8A, could be combined with almost no aggregation error since they have qualitatively homogeneous output vectors, as discussed previously. In fig. 8, farms 4B and 8B and farms 4Cb and 8Cb also are in the same group. Thus, all pairs of farm sizes except 4Ca and 8Ca in these two areas are grouped together by the procedure. These results imply

		SELL FED BEEF							
		SELL CORN				FEED OWN CORN			
		BUY CALVES		FEED OWN CALVES		BUY CALVES		FEED OWN CALVES	
		BUY FACTORS	DON'T BUY FACTORS	BUY FACTORS	DON'T BUY FACTORS	BUY FACTORS	DON'T BUY FACTORS	BUY FACTORS	DON'T BUY FACTORS
SELL HOGS	HIRE LABOR		4B 4Ca 4Cb 7B 7C 8B 8Cb	2Cb 9Cb		2Ca 5C 6Cb 10Ca 10Cb	1C 8Ca 9B 9Ca	6Ca 10B	3C
	NO HIRED LABOR		1B 4A* 8A*		10A		3B 5A* 9A*	6B	2B 5B 6A
NO HOGS	NO HIRED LABOR		1A* 3A 7A*				2A		

Fig. 8. Stratification of the 36 representative farms by response patterns at the $11.00 pork price and $15.50 beef price.

that areas 4 and 8 could be combined with a minimum of aggregation error.

Possibilities for size combinations within areas may be approached similarly, although the problem is less complex. Different sizes of farms in the same area generally are not grouped together in fig. 8. Thus, in terms of response error, the three original sizes of representative farms are a meaningful classification for purposes of reducing aggregation error.

New Delineations of Representative Farms

The empirical work provides little information on the effect of new stratification factors on aggregation error. Such additional factors could help isolate other differences in response patterns among representative farms, thereby reducing aggregation error in supply estimates. The list of such additional factors obviously is long because of the complexity and variety of the relations and factors that relate to realization of conditions under Theorem I. Possible additional stratification factors for Iowa farms would include: (a) physical factors, such as climate and topography, which affect crop yields and production costs; (b) institutional restrictions, such as tenure and government regulations, when these factors have an unequal effect on individual farm response; (c) motivational forces, including risk aversion, demand for leisure, age, and preferences for certain enterprises; (d) management ability; and (e) technology, which may have an unequal impact on different farms. Resource endowments (f), such as specific buildings, are not considered in this study.

The first five classes of factors (a-e) generally affect the coefficient matrices of the linear-programming models, while the resource vectors of the models are primarily determined by the sixth class (f). The choice of stratification factors to use in a particular research project is always difficult. It requires a thorough knowledge of the agriculture in the population and the main relationships that determine the optimum solutions of the linear-programming models being used.

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APPENDIX: BASIC STRUCTURE OF THE LINEAR-PROGRAMMING MODELS

The resource restrictions (table A-1)

All the restrictions compose upper bounds on the activities, with the exception of the first three (which are strict equalities). The first three restrictions, which reflect agronomic restraints, represent the three classes of cropland available and sum to the total cropland on the farm. Restriction 4 represents nontillable pasture available for grazing on the farms, and restriction 5 represents cropland planted to alfalfa and grass, which may be used for hay or grazing. Restriction 6 is an accounting row for the intermediate product, unharvested corn, produced by the different crop rotations.

Hog facilities are represented by the next four restrictions, 7 through 10. These are central and portable farrowing facilities and confinement and portable feeding facilities. The next six restrictions are on beef facilities. These are beef-housing capacity and beef-feeding facilities involving a low and a high level of mechanization; each divided into two use periods of the year, November through April (period 1) and May through October (period 2). These two periods are defined so that enterprises using the facilities at different times in the year will not compete for the same facilities. The same facility is represented in each of the two periods; thus, the amount available for both periods in the resource vector is the same.

Restrictions 17, 18, and 19 are accounting rows for intermediate livestock feed products. The corn-equivalent row collects corn purchased and corn and oats harvested as grain from the rotations in corn-equivalent units. It is available for feed to both the hog and the beef enterprises or for sale. The corn-silage and hay-equivalent rows make feed available to the beef enterprises. The next three restrictions make purchased and farm-raised feeder calves and yearlings available to the beef-feeding enterprises.

Restriction 23 is the operating capital available on the farm in $10 units. The amount available in the resource vector includes cash on hand and the farm value of feed and livestock inventories, less short-term liabilities, at the beginning of the year. Feed and livestock inventories were converted to cash, and, as a result, were not included elsewhere in the resources available.

Restriction 24 limits chattel credit that may be obtained without providing additional collateral. The amount available in the resource vector represents 50 percent of the owned-machinery inventory, less current intermediate-term liabilities. Only 15 percent of the collateral required for livestock loans was required to come from this source; the remainder being provided by the livestock.

The next two restrictions, numbers 25 and 26, accumulate all pork and beef produced by the respective enterprises and allow their sale through two selling activities. The number of selling activities was minimized to accommodate the variable pricing technique used with the model.

Restrictions 27 through 35 are operator and family-labor restrictions. Restriction 27 is the total annual operator and family labor. The total amount of 27 is less than the sum of the amounts for the individual periods of the year (restrictions 28 through 35) because the fixed labor requirements for the farm (which may be performed at any slack time during the year) have been deducted. Restrictions 28 through 35 represent eight potentially restrictive labor periods of the year. The last restriction, 36, sets a limit on the number of hours of labor that may be hired. Each representative farm's labor hiring was restricted to its historical level to prevent aggregate labor hiring in the state from exceeding the amounts of farm labor available.

Table A-1. Identification of restrictions and a typical resource vector for the linear-programming model.

Row number	Item	Unit	Amounts available farm 9B[a]
c_j ------	Net returns over variable costs	$	(maximized)
1 ------	Land with 25% row-crop capability	acre	20.7
2 ------	Land with 50% row-crop capability	acre	51.2
3 ------	Land with 100% row-crop capability	acre	65.4
4 ------	Pasture (not tillable)	ton AHY[b]	50.5
5 ------	Meadow to be harvested	ton	0
6 ------	Corn to be harvested	bu.	0
7 ------	Central farrowing facilities	sow	17.6
8 ------	Portable farrowing facilities	sow	0
9 ------	Confinement feeding facilities	pig	0
10 ------	Portable feeding facilities	pig	166.0
11 ------	Beef housing -- period 1[c]	a.u.[d]	26.4
12 ------	Beef housing -- period 2	a.u.	26.4
13 ------	Low beef mechanization -- period 1	head	8.6
14 ------	Low beef mechanization -- period 2	head	8.6
15 ------	High beef mechanization -- period 1	head	42.4
16 ------	High beef mechanization -- period 2	head	42.4
17 ------	Corn equivalents	cwt.	0
18 ------	Corn silage	cwt.	0
19 ------	Hay equivalents	cwt.	0
20 ------	Purchased yearlings -- either period	head	0
21 ------	Purchased yearlings -- both periods	head	0
22 ------	Beef calves	head	0
23 ------	Cash account	$10	1,448.4
24 ------	Chattel mortgage	$10	346.5
25 ------	Beef for sale	cwt.	0
26 ------	Hogs for sale	cwt.	0
27 ------	Total operator and family labor	hour	2,368.0
28 ------	Dec., Jan., Feb., March labor	hour	786.4
29 ------	April labor	hour	246.6
30 ------	May labor	hour	271.6
31 ------	June labor	hour	321.6
32 ------	July labor	hour	321.6
33 ------	August, September labor	hour	568.2
34 ------	October labor	hour	246.6
35 ------	November labor	hour	221.6
36 ------	Total hired labor limit	hour	124.4

[a] This is the resource vector for the medium-sized representative farm in area 9.

[b] Tons of anticipated hay yield.

[c] The year is divided into two use periods so that enterprises using the facilities at different times in the year will not compete for the same facilities. Period 1 is November through April, and period 2 is May through October.

[d] Animal units.

The activities considered (table A-2)

Activities P_1 through P_6 are the variable-pricing section of the model. The next section of the model, composed of activities P_7 through P_{17}, contains the alternative crop rotations considered. These were divided into three groups on the basis of the three qualities of cropland considered; the more intensive rotations were limited to the higher-quality land.

Activities P_{18} through P_{21} allow for the considerations of alternative harvesting methods. P_{18} and P_{19} allow harvesting corn produced and accumulated in row 6 either as grain or as silage. P_{20} and P_{21} allow either harvesting cropland meadow (row 5) as hay or transferring it into row 4, where it may be utilized as pasture by the livestock enterprises.

The eight hog enterprises considered are activities P_{22} through P_{29}. These consist of selected combinations of litters per year and feeding and farrowing facilities. The unit of these activities is the size required to utilize one unit of farrowing capacity at any one time. Thus, P_{28}, with three sows each producing two litters per year, provides for six nonoverlapping farrowings and requires the same farrowing capacity as P_{24}, with one sow and two litters.

Activities P_{30} through P_{49} are the beef-feeding enterprises considered. These alternative enterprises involve (a) feeding hay or feeding a combination of hay and silage, (b) use of a low or a high level of mechanization in feeding, (c) feeding calves or yearlings, (d) feeding the calves on drylot or on drylot with summer pasture, and (e) the choice of two different periods of the year for yearling feeding. The information given in table A-2 is self-explanatory, with the possible exception of the two-period yearling feeding enterprises. These were developed because of the lower annual capital requirements per head when yearlings are fed in two nonoverlapping feeding periods. The beef-cow enterprises are activities P_{50} and P_{51}; the first with hay as the only roughage, and the second utilizing a combination of hay and silage. These activities include all the requirements and returns of the cow and calf up to weaning time, plus the cow's share of replacement heifers and bull cost.

The chattel-credit borrowing activity is P_{52}. It requires chattel credit capacity or collateral from row 24 and makes the money available for expenses in the model in row 23 at a cost of 7-percent annual interest. Activity P_{53} allows investment of unused operating capital off the farm at t-percent interest (t = the interest rate a farmer can obtain on various types of other investments, t varying with the type of investment). A reservation price on operator and family labor of $0.50 per hour is included as activity P_{54}. Activities P_{55} and P_{56} are the corn selling and buying activities. The selling price of corn was $0.85 per bushel, and the purchase price was $1 per bushel. Investment in additional hog and beef facilities is allowed by activities P_{57} through P_{65}. Annual costs are included in the functional (C_j), and capital for the investment is drawn from available or borrowed capital.

Activities P_{66} through P_{73} allow labor hiring in each of the eight labor periods. As previously stated, the total labor hired in these activities was limited by row 36 to the historical labor-hiring practices of the particular representative farm.

Table A-2. Identification of activities for the linear-programming models

Activity number	Explanation	Unit
P_1	---- Sell pork[a]	1,000 lb.
P_2	---- Sell beef[a]	cwt.
P_3	---- Buy feeder calves[a]	head
P_4	---- Purchase yearlings -- one-period feeding program only[ab]	head
P_5	---- Purchase yearlings -- two-period feeding program[ab]	2 head
P_6	---- Sell calves[a]	head
	Rotations for land with 100% row-crop capability	
P_7	------ Corn	acre
P_8	------ Corn-soybeans	acre
P_9	------ CCOM$_1$	acre
P_{10}	----- CSOM$_1$	acre
P_{11}	----- CSSOM$_1$	acre
	Rotations for land with 50% row-crop capability	
P_{12}	----- CCOM$_2$	acre
P_{13}	----- CSOM$_2$	acre
P_{14}	----- CSSOMM	acre
P_{15}	----- Meadow$_2$	acre
	Rotations for land with 25% row-crop capability	
P_{16}	----- COMM	acre
P_{17}	----- Meadow$_3$	acre
	Harvest corn	
P_{18}	----- As grain	10 bu.
P_{19}	----- As silage	ton
	Harvest meadow	
P_{20}	----- As hay	ton
P_{21}	----- By grazing	ton AHY[c]
	Hog enterprises	
P_{22}	----- One sow, one litter, portable farrow and feed	1 litter
P_{23}	----- One sow, two litters, portable farrow and feed	2 litters
P_{24}	----- One sow, two litters, central farrow and portable feed	2 litters
P_{25}	----- One sow, two litters, central farrow and feed	2 litters
P_{26}	----- Two sows, four litters, central farrow and portable feed	4 litters
P_{27}	----- Two sows, four litters, central farrow and feed	4 litters
P_{28}	----- Three sows, six litters, central farrow and portable feed	6 litters
P_{29}	----- Three sows, six litters, central farrow and feed	6 litters
	Beef-feeding enterprises	
	No silage, low-mechanization feeding	
	Calves	
P_{30}	--------- Drylot	head
P_{31}	--------- Pasture	head
	Yearlings	
P_{32}	--------- Period 1[d]	head
P_{33}	--------- Period 2[e]	head
P_{34}	--------- Periods 1 and 2[f]	2 head
	No silage, high-mechanization feeding	
	Calves	
P_{35}	--------- Drylot	head
P_{36}	--------- Pasture	head

Table A-2. (Continued).

Activity number	Explanation	Unit
	Yearlings	
P_{37}	--------- Period 1	head
P_{38}	--------- Period 2	head
P_{39}	--------- Periods 1 and 2	2 head
	Silage, low-mechanization feeding	
	Calves	
P_{40}	--------- Drylot	head
P_{41}	--------- Pasture	head
	Yearlings	
P_{42}	--------- Period 1	head
P_{43}	--------- Period 2	head
P_{44}	--------- Periods 1 and 2	2 head
	Silage, high-mechanization feeding	
	Calves	
P_{45}	--------- Drylot	head
P_{46}	--------- Pasture	head
	Yearlings	
P_{47}	--------- Period 1	head
P_{48}	--------- Period 2	head
P_{49}	--------- Periods 1 and 2	2 head
	Beef cows	
P_{50}	------- Hay	1 cow unit
P_{51}	------- Silage	1 cow unit
P_{52}	--- Borrow chattel credit	\$100
P_{53}	--- Invest cash off farm	\$100
P_{54}	--- Reservation price on labor	hr.
P_{55}	--- Sell corn	cwt.
P_{56}	--- Buy corn	cwt.
	Investment in additional hog facilities	
P_{57}	------ Central farrowing	1 litter
P_{58}	------ Portable farrowing	1 litter
P_{59}	------ Confinement feeding	1 litter
P_{60}	------ Portable feeding	1 litter
P_{61}	------ Central farrowing and feeding	1 litter
	Investment in beef facilities	
P_{62}	------ Housing	a.u.[g]
P_{63}	------ Low mechanization feeding equipment	head
P_{64}	------ High mechanization feeding equipment	head
P_{65}	------ Conversion of low mechanization to high mechanization feeding equipment	head
	Labor hiring activities	
P_{66}	------ Dec., Jan., Feb., March	hr.
P_{67}	------ April	hr.
P_{68}	------ May	hr.
P_{69}	------ June	hr.
P_{70}	------ July	hr.
P_{71}	------ August, September	hr.
P_{72}	------ October	hr.
P_{73}	------ November	hr.

a These activities were variably priced to generate the supply curves.

b One-period feeding program refers to either (1) buying feeders in October and selling in April, or (2) buying in April of one year and selling in October of the same year. Two-period feeding program refers to inclusion of both feeding systems or periods in the farm organization.

c Tons of anticipated hay yield.

d Fed October to April (bought in October and sold in the next April).

e Fed April to October (bought in April and sold in the next October).

f One steer fed October to April and another steer fed April to October.

g Animal units.

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