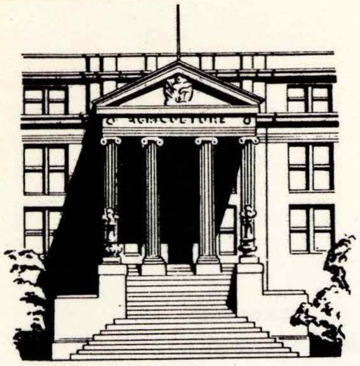

ECONOMETRIC AND PROGRAMMING ANALYSES OF THE BEEF-PORK MARKETING SECTOR

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SUMMARY

In this study we: a) estimated quarterly demand functions for red meats and broilers, b) estimated quarterly farm-wholesale and wholesale-retail marketing-margin equations for red meats, c) estimated red-meat quarterly inventory-investment equations, d) developed a complete econometric model of quarterly behavior of the beef- and pork-marketing sector, and e) applied quadratic programming to determine quarterly and annual levels of farm marketings of hogs and cattle that would maximize farmers' annual cash receipts from hogs and cattle.

Linear and logarithmic (constant elasticity) quarterly demand functions (with price as a dependent variable) were estimated for beef, pork, lamb, and broilers to measure quarterly variations in demand. Results from both functional forms indicated that, in beef and pork demand functions, intercepts vary quarterly, but slopes do not, whereas neither slopes nor intercepts vary quarterly in lamb demand equations. Logarithmic versions of the broiler demand function found quarterly variation in intercept, but not in slopes, whereas linear versions generally found no significant quarterly variation in slopes or intercept.

The main purposes of the study of marketing margins were to determine effects of factor prices and labor productivity on margins and to determine interrelations among margins on various meats. In general, the addition of factor-price and labor-productivity variables to margin equations containing lagged margins, farm marketings, farm or wholesale price, and seasonal dummy variables resulted in statistically significant, but small, increases in the value of R^2. It was found that beef and pork margins are not independent.

An econometric model of the beef- and pork-marketing sectors of the United States economy was estimated by using quarterly data for the first quarter of 1954 through the fourth quarter of 1968. The model contained 12 stochastic equations: two retailers' demand equations, two inventory equations, two consumer demand equations, two wholesale-retail margin equations, two farm-wholesale margin equations, and two farm price equations. The model also contained a number of identities.

Final-form equations for farm prices of cattle and hogs were derived from the model. These equations state the values of farm prices in quarter q (q = 1, 2, 3, 4) of year y as a linear function of values of endogenous variables in the last quarters of year y—1, and of current and lagged exogenous variables. After testing the goodness of fit of these equations, we selected 4 past years for quadratic-programming analyses to determine quarterly or annual farm marketings that would maximize cash receipts from hogs and cattle, and we also projected levels of marketings that would maximize cash receipts in 1972, 1973, and 1974.

Given the actual historic levels of annual cattle and hog marketings, farmers' annual cash receipts from cattle, from hogs, and from both could not have been increased by altering the quarterly distributions of cattle and hog marketings.

Given the actual historic levels of quarterly hog marketings, annual cash receipts from cattle could not have been increased by changing levels of cattle marketings, but net revenue from cattle could have been increased by reducing annual cattle marketings.

Given the actual historic levels of quarterly cattle marketings, annual cash receipts from hogs could have been increased by 13 percent by reducing annual marketings of hogs by one-fourth.

If annual marketings of cattle and hogs had been reduced by 16 and 24 percent, respectively, annual cash receipts from cattle could have been increased by 3 percent, and annual cash receipts from hogs could have been increased by 19 percent.

Comparison of the econometric model constructed in this study with findings of previous studies suggests that systematic changes have occurred in the seasonal patterns of behavior of consumers and of firms involved in beef and pork marketing.

Econometric and Programming Analyses of the Beef-Pork Marketing Sector[1]

by George W. Ladd and Georg Karg

The objective of the research reported here was to determine quarterly levels and (or) annual levels of cattle and hog marketings that would maximize net farm income from cattle and hogs. Because farm-level demands for cattle and hogs vary quarterly, determination of quarterly levels of marketings can be formulated as a problem of price discrimination with eight interdependent markets: four quarterly markets for cattle and four quarterly markets for hogs. Analysis of price-discrimination problems requires knowledge of demand functions. In this study, farm-level demand functions were obtained from a quarterly econometric model of the beef- and pork-marketing sectors of the economy.

The econometric model was constructed in four stages. The purpose of the first three stages was to provide basic information for use in constructing the complete model.

One stage investigated the structure of quarterly demand equations for beef and pork and also for lamb and mutton and broilers (1). Another stage investigated the influence of factor prices and labor productivity on farm-wholesale and wholesale-retail marketing margins for red meats (9). The third stage investigated inventories of red meats. The fourth stage used results of the three previous stages to develop the quarterly econometric model. Results of this stage are reported by Karg (9).

We also investigated quarterly cattle and hog production costs. Within the limits of the available resources, we were not able to develop reliable measures of quarterly variation in costs of production of cattle and hogs. Hence, we were unable to accomplish our prime objective. We did, however, determine quarterly levels of cattle and hog marketings to maximize farmers' annual cash receipts from cattle and hogs. Results of this work also are reported by Karg (9).

QUARTERLY RED-MEAT AND BROILER DEMAND

Quarterly farm-level demands for cattle and hogs are derived from consumer demands for beef and pork. In 1962, Logan and Boles (14) used quarterly data for 1948-59 to analyze quarterly variation in linear consumer demand equations for beef, pork, broilers, and lamb and mutton. To update and extend this study, Buttimer (1) used quarterly data for the period from the third calendar quarter of 1953 (1953-III) through the fourth calendar quarter of 1966 (1966-IV) to analyze quarterly variation in linear and logarithmic demand equations.

The problem of investigating seasonal variation in consumer demand equations can be stated as a problem of choosing among alternative models or as a problem in hypothesis testing. Let

DRP_{iqy} = retail price of i-th product in q-th quarter of year y deflated by contemporaneous consumer price index;

PCC_{iqy} = per-capita civilian commercial consumption (i.e., consumption from commercial sources) of product i in q-th quarter of year y;

Y_{qy} = per-capita personal disposable income deflated by consumer price index for quarter q of year y; and

T_{qy} = linear time trend.

Also let i = 1 denote beef, i = 2 denote pork, i = 3 denote lamb and mutton, and i = 4 denote broilers. The linear consumer demand equation for the i-th product in the q-th quarter can be written as

$$DRP_{iqy} = \beta_{iq0} + \sum_{j=1}^{4} \beta_{iqj} PCC_{jqy} + \beta_{iq5} Y_{qy} + \beta_{iq6} T_{qy} + \epsilon_{iqy}$$

Let null hypothesis H(CS,CI)i for product i consist of the seven expressions

$$\beta_{i10} = \beta_{i20} = \beta_{i30} = \beta_{i40}$$
$$\beta_{i11} = \beta_{i21} = \beta_{i31} = \beta_{i41}$$
$$\beta_{i12} = \beta_{i22} = \beta_{i32} = \beta_{i42}$$
$$\vdots$$
$$\beta_{i16} = \beta_{i26} = \beta_{i36} = \beta_{i46}$$

The first expression states that the intercept in the (price-dependent) demand equation for the i-th meat does not vary among quarters. The second expression states that the coefficient of beef consumption (j=1) in the demand equation for the i-th meat does not vary among quarters. Other expressions state that other slope coefficients in the demand equation for the i-th meat remain constant over the calendar quarters. A test of hypothesis H(CS,CI) i is a test of the hypothesis that the intercepts (β_{iq0}) and the slopes (β_{iqh} for h = 1, 2, ..., 6) remain constant between quarters. Let the null hypothesis H(CS,VI)i for product i consist of all but the first of these seven expressions. A test of this hypothesis is a test of the

[1]Project 1705 of the Iowa Agriculture and Home Economics Experiment Station. The authors are grateful to J.M. Skadberg for help provided on this research.

hypothesis that slopes remain constant between quarters with intercepts unspecified. If H(CS,CI)i is accepted, we will write that Model CS, CI (constant slopes and intercepts) is the appropriate linear model for product i. If H(CS,CI)i is rejected, but H(CS,VI)i is accepted, we will write that Model CS, VI (constant slopes, but varying intercepts) is the appropriate linear model for product i.

If the variables in the demand equation are in logarithmic form rather than in natural number form, H(CS,VI)i states that the price flexibilities are constant over the quarters for product i, and H(CS,CI)i states that the flexibilities and intercept are constant over the quarters for product i.

Buttimer (1) tested H(CS,CI)i and H(CS,VI)i for beef, pork, lamb and mutton, and broilers for each of eight different formulations of the demand functions, four linear equations and four logarithmic equations. With each functional form, he estimated two equations with per-capita civilian commercial consumption as the consumption variable and two equations with total per-capita civilian consumption. With each choice of consumption variable, two different income variables were used, deflated per-capita disposable personal income and deviations of this income variable from the linear trend value of deflated per-capita disposable personal income. This latter formulation was used because of the high intercorrelation between income and time. Results on choices of models are summarized in table 1.

The different formulations of the beef and pork demand equations unanimously indicated Model CS,VI to be appropriate. That is, H(CS,CI) was rejected, but H(CS,VI) was accepted. These results agree with the earlier findings of Logan and Boles (14) and Stanton (17). All formulations of the lamb and mutton demand equations selected Model CS,CI. These results differed from results of Logan and Boles, who found seasonal variation in slopes and intercepts in the lamb and mutton equations. Different formulations of the broiler demand equations lead to choices of different models. All logarithmic equations selected Model CS,VI, but three of the four linear equations selected Model CS,CI.

Table 2 presents selected statistics from linear and logarithmic formulations of the demand equations for lamb and mutton and for broilers. (Demand equations for beef and pork, estimated from a more recent sample, will be included in a later section, which presents a complete econometric model of the beef- and pork-marketing sectors of the economy.) In this table and in all subsequent statistical results, an * indicates significance of a coefficient at the 10-percent level; ** indicates significance at the 5-percent level; *** indicates significance at the 1-percent level. In text, coefficients significant at the 10-percent, 5-percent, and 1-percent levels will be referred to as lowly significant, significant, and highly significant.

In comparing the linear equation on broiler price in table 2 with results of Logan and Boles (14), we find: a) The two disagree on the sign of the beef-consumption coefficient, but agree on its nonsignificance. Note, however, that the logarithmic broiler price equation in table 2 has a highly significant negative coefficient. But both studies found a lower sum of squares of deviations of estimated prices from actual prices for linear equations than for logarithmic equations, and hence, concluded that the linear equation was more appropriate. b) Both have a negative coefficient of pork consumption in the linear broiler price equation, but Logan and Boles' coefficient was not significant. c) Both have a negative and highly significant coefficient of broiler consumption. d) Both have a positive coefficient of lamb consumption. This coefficient was lowly significant in the Logan and Boles study. e) Logan and Boles found a negative and nonsignificant coefficient for income. The coefficient of income deviations in table 2 is positive and highly significant.

A brief but accurate comparison of price equations for lamb in table 2 with Logan and Boles' lamb price equations is difficult because Logan and Boles found Model VS,VI to be appropriate for analysis of lamb price, and Buttimer found

Table 1. Summary of results of tests of hypotheses of constancy of slopes and intercepts in beef, pork, lamb and mutton, and broiler quarterly demand equations, 1953-III to 1966-IV: Appropriate linear and logarithmic models.[a]

Commodity	Commercial Consumption				Total Consumption			
	Linear equation		Logarithmic equation		Linear equation		Logarithmic equation	
	Income	Income deviations	Income	Income deviations	Income	Income deviations	Income	Income deviations
Beef	CS, VI	CS, VI	CS, VI	CS, VI	CS, VI	CS, VI	CS, VI	CS, VI
Pork	CS, VI	CS, VI	CS, VI	CS, VI	CS, VI	CS, VI	CS, VI	CS, VI
Lamb and mutton	CS, CI	CS, CI	CS, CI	CS, CI	CS, CI	CS, CI	CS, CI	CS, CI
Broilers	CS, CI	CS, VI	CS, VI	CS, VI	CS, CI	CS, CI	CS, VI	CS, VI

[a] Model CS, CI: Constant slopes and intercepts

Model CS, VI: Constant slopes but varying intercepts

Table 2. Selected statistics from lamb and mutton and broiler demand equations, 1953-III to 1966-IV[a]

Dependent variable	1	D_1[b]	D_2[b]	D_3[b]	Per-capita civilian commercial consumption: Beef	Pork	Lamb	Broilers	Income deviations	Time	R^2
DRP_L[c]	124.98				-1.057	-1.600	-13.990	-0.060	0.026	0.206	0.682
	11.20***				0.467**	0.353***	3.986***	0.701	0.010**	0.084*	
$\log DRP_L$[d]	0.184				-0.135	-0.241	-0.225	0.088	0.628	-0.033	0.637
	0.97				0.121	0.092**	(0.071***	0.064	(0.312)**	0.018	
DRP_C[e]	66.70	0.718	3.576	4.299	0.210	-1.071	9.083	-3.826	0.036	-0.108	0.949
	10,39***	0.936	1.272***	1.503***	0.474	0.331***	3.680**	0.846***	0.009***	0.111	
$\log DRP_C$[d]	-2.010	0.016	0.039	0.050	-0.398	-0.204	0.091	-0.381	1.424	-0.021	0.956
	1.093	0.009*	0.010**	0.010***	0.138***	0.116	0.088	0.0078***	0.359***	0.022	

[a] Coefficients are presented on the top line for each equation; standard errors, on the second line.

[b] D_q is a dummy variable that equals unity in the q-th quarter of each year and equals zero in all other quarters.

[c] DRP_L = Deflated retail price of lamb and mutton.

[d] Independent variables were in logarithmic form in these equations.

[e] DRP_C = Deflated retail price of broilers.

Model CS,CI to be appropriate. Buttimer's findings concerning consumption and income-deviations variables differed little between models CS,CI, and CS, VI. If we compare the linear Model CS,VI lamb price equations from the two studies, we find: a) Both have negative highly significant coefficients of beef and pork consumption. b) Both have negative significant coefficients of lamb consumption. c) Both have negative nonsignificant coefficients of broiler consumption. d) Logan and Boles found a negative nonsignificant coefficient of income. Buttimer found a positive highly significant coefficient of income deviations.

QUARTERLY RED-MEAT MARKETING MARGINS

With given consumer demand equations for beef and pork, derived farm-level demand equations for cattle and hogs are affected by marketing margins. Our study of quarterly marketing margins for red meat used dynamic models of margins presented earlier by Fuller (3) and by Fuller and Ladd (4) and included measures of factor prices similar to the measures used by Manchester (15). Among the measures of factor prices used were:

$$CN_t = 0.3C_t + 0.7N_t$$

where

C_t = container and packaging materials price index in quarter t, 1957—59 = 100

N_t = new plant and equipment price index in quarter t, 1957—59 = 100

$$CNW_t = 0.1C_t + 0.2N_t + 0.7IHW_t$$

where

IHW_t = index of average hourly earnings per production worker in meat-products industry, SIC 201, in quarter t, 1957-59 = 100

HW_t = average hourly earnings per production worker in meat-products industry, SIC 201, in quarter t

$$CNR_t = 0.1C_t + 0.2N_t + 0.7IHR_t$$

where

IHR_t = index of average hourly earnings of nonsupervisory workers in grocery, meat, and vegetable stores in quarter t, 1957-59 = 100

HR_t = average hourly earnings of nonsupervisory workers in grocery, meat, and vegetable stores in quarter t

CN_t is an index of prices of containers and packaging materials and prices of new plant and equipment. CNW_t and CNR_t contain these same prices. In addition, CNW_t contains wage rates in the meat-products industry, whereas CNR_t contains wage rates in food retailing. Two indexes of labor productivity in marketing also were used

$$LP1_t = HLC_t / ULC_t$$

and

$LP2_t$ = index of output per man-hour in manufacturing meat products, 1957-59 = 100

where

HLC_t = hourly labor cost in marketing farm-food products, quarter t

ULC_t = unit labor cost in marketing farm-food products, quarter t

HLC equals total labor cost (TLC) divided by total man–hours (TMH); unit labor cost (ULC) equals TLC divided by the volume of food marketed (VFM). Hence

$$HLC/ULC = (TLC/TMH)/(TLC/VFM) = VFM/TMH$$

$LP1_t$ measures the volume of food marketed per man-hour and hence is a measure of labor productivity in all farm-food marketing. Data for C_t, N_t, HLC_t, ULC_t and $LP2_t$ were taken from various issues of the Marketing and Transportation Situation. Data for HW_t and HR_t were taken from publications of the Bureau of Labor Statistics. The following deflated cost variables were used:

$$CN1_t = CN_t/LP1_t$$
$$CNW1_t = CNW_t/LP1_t$$
$$CNR1_t = CNR_t/LP1_t$$
$$HW1_t = HW_t/LP1_t$$
$$HR1_t = HR_t/LP1_t$$
$$CN2_t = CN_t/LP2_t$$
$$CNW2_t = CNW_t/LP2_t$$
$$CNR2_t = CNR_t/LP2_t$$
$$HW2_t = HW_t/LP2_t$$
$$HR2_t = HR_t/LP2_t$$

The sample period for the study of quarterly marketing margins was from 1954-I through 1967-IV. The general procedure followed was to add factor-price or labor-productivity variables, or both, to equations containing lagged margins, farm marketings, seasonal dummy variables and, sometimes, farm or wholesale prices and to use an F-ratio to test the significance of the added variables. Tables 3 and 4 summarize some of the results by showing some of the sets of factor-price and labor-productivity variables causing significant increases in the values of R^2. Equation 3.3 in table 3, for example, shows that addition of the variables CN_t, HW_t and $LP2_t$ to a pork farm-wholesale margin equation resulted in a significant increase in the value of R^2; and the coefficient of CN_t was positive and significant at the 1-percent level, the coefficient of HW_t was negative and nonsignificant, and the coefficient of $LP2_t$ was positive and nonsignificant. Not all the signs, even of significant coefficients, are as expected. For example, one would expect the sign of CN_t to be positive; it is negative in equations 3.4 and 3.5 in table 3. Other combinations, not shown in these tables, caused significant increases in the values of R^2, but contained unexpected signs. Although the combinations in tables 4 and 5, and others, made significant increases in the values of R^2, the increases usually were small.

Interrelations between margins also were analyzed. We found that: a) Farm-wholesale pork margins were not influenced by current farm-wholesale beef margins; b) Farm-wholesale beef margins were affected by farm-wholesale pork margins; c) Wholesale-retail pork margins were influenced by wholesale-retail beef margins, and vice versa. At neither level of the marketing channel did beef or pork margins affect lamb and mutton margins. The study also tested for effects of inventory changes on margins. In every case, the null hypothesis of no effect of inventory change on marketing margin of the same meat was accepted at the 5-percent level.

All equations in this study were estimated by classical least squares. Because of the possibility of simultaneous determination of margins and some of the explanatory variables—especially farm and wholesale prices—several equations were re-estimated by two-stage least squares. Differences between the least-squares and two-stage, least-squares results were negligible. It also seemed likely that errors in equations for different margins would be correlated and, consequently, that Zellner's method of seemingly unrelated regressions (20) would be appropriate. To check on this possibility, residuals were computed for selected equations, and simple correlation coefficients between residuals for pairs of equations were computed. None of the simple correlation coefficients was significant. Hence, Zellner's method was not used.

Tables 5 and 6 present selected margin equations for lamb and mutton. (Beef and pork margin equations will be presented later.) Symbols in tables 5 and 6 not previously defined are:

MW_{Lt} = farm-wholesale margin on lamb and mutton, current quarter;
WP_{Lt} = wholesale price of lamb in quarter t;
$\Delta WP_{Lt} = WP_{Lt} - WP_{Lt-1}$;
CP_{Lt} = farm marketings of lamb in quarter t;
$\Delta CP_{Lt} = CP_{Lt} - CP_{Lt-1}$;
D_{qt} = 1 in q-th quarter of year;
= 0 in all other quarters; and
MR_{Lt} = wholesale-retail margin on lamb in quarter t.

QUARTERLY RED-MEAT INVENTORIES

Quarterly demands for livestock from farmers are affected by demand for meat for inventory holdings. An earlier study (10) of end-of-quarter inventories of beef and pork that used 1949-III to 1960-IV data found that some 95 percent of the variance in inventories could be explained by lagged inventories, changes in farm livestock marketings, and seasonal dummy variables. This earlier study found no evidence that changes in meat inventories were affected by levels or changes in meat sales or by levels or changes in wholesale meat prices.

In a study using data for 1954-I through 1967-IV we also found lagged inventories, livestock marketings, and seasonal dummy variables to be important determinants of inventory change. We also found significant evidence, however, of relations

Table 3. Factor price and labor productivity variables making significant additions to value of R^2 in quarterly farm-wholesale margin equations, 1954-I to 1967-IV[a/]

Equation no.	Commodity	Labor productivity measure	CN	CNW	HW	LP1 or LP2	CNW1	HW1
3.1	Pork	LP1		+1		-1		
3.2	"	LP1					+1	
3.3		LP2	+1		-	+		
3.4	Beef	LP1	-1		+1	-1		
3.5	"	LP2	-1		+1	-1		
3.6	Lamb	LP1		+1		-		
3.7	"	LP1					+1	
3.8	"	LP1						+1

a/ Entries in each row indicate variables added in that equation. First element in each entry indicates sign of coefficient of variable, second entry indicates percentage level of significance of variable. Absence of second entry indicates nonsignificant coefficient.

Table 4. Factor price and labor productivity variables making significant additions to value of R^2 in quarterly wholesale-retail margin equations, 1954-I to 1967-IV[a/]

Equation no.	Commodity	Labor-productivity measure	CN	CNW	CNR	HR	LP1 or LP2	CN1 or CN2	HW1 or HW2	HR1
4.1	Pork	LP1	+1			-	+			
4.2	"	LP1		+1			-10			
4.3	"	LP1			+1					
4.4	"	LP1						-1	+1	
4.5	"	LP1						-		+1
4.6	"	LP1		+1		-	+			
4.7	"	LP2		+1			-			
4.8	Beef	LP1	+1			-	+5			
4.9	"	LP1						-1	+1	
4.10	"	LP2		+1						
4.11	"	LP2						-1	+1	
4.12	Lamb	LP1	+			+1	-10			
	"	LP1						-5		+1

a/ Entries in each row indicate variables added in that equation. First element in each entry indicates sign of coefficient of variable, second entry indicates percentage level of significance of variable. Absence of second entry indicates nonsignificant coefficient.

Table 5. Selected statistical results from quarterly farm-wholesale lamb and mutton marketing margin equations, 1954-I to 1967-IV.

Equation no.	Dependent variable	1	ΔWP_{Lt}	MW_{Lt-1}	ΔCP_{Lt}	$CNW1_t$	$HW2_t$	D_{1t}	D_{2t}	D_{3t}	R^2
5.1	MW_{Lt}	-0.918	0.331	0.591	0.024	0.066		-1.393	-0.132	-0.232	0.727
		2.870	0.050***	0.095***	0.011**	0.030**		0.324***	0.370	0.294	
5.2	MW_{Lt}	-1.166	0.319	0.485	0.025		3.761	-1.373	-0.107	-0.106	0.753
		2.162	0.048***	0.101***	0.010**		1.176***	0.307***	0.352	0.285	

Table 6. Selected statistical results from quarterly wholesale-retail lamb and mutton marketing margin equations, 1954-I to 1967-IV.

Equation no.	Dependent variable	1	WP_{Lt}	ΔWP_{Lt}	MR_{Lt-1}	CN_t	CNR_t	HR_t	$LP1_t$	D_{1t}	D_{2t}	D_{3t}	R^2
6.1	MR_{Lt}	-5.359	-0.033	-0.405	0.343		0.275	-9.163		0.852	0.386	0.877	0.883
		4.241	0.108	0.104***	0.121***		0.136**	11.703		0.683	0.838	0.693	
6.2	MR_{Lt}	2.022	-0.160	-0.348	0.238	0.086		20.620	-23.368	1.015	0.668	1.005	0.893
		7.052	0.121	0.102***	0.127*	0.066		7.459***	13.091*	0.661	0.820	0.665	

between pork sales and pork inventory, between beef sales and beef inventory, and between lamb and mutton price and inventory. (Results of a still more recent study on beef and pork inventories will be presented later.) The following equations illustrate the 1954-I to 1967-IV results on lamb and mutton inventories

$$
\begin{aligned}
\Delta I_{Lt} = & -0.191 I_{Lt-2} + 0.003\Delta CP_{Bt} + 0.009\Delta CP_{Lt} \\
& \quad 0.072^{**} \qquad 0.003 \qquad\quad 0.050 \\
& + 0.092\Delta CP_{Lt-1} - 0.435\Delta WP_{Lt} + 0.464\Delta WP_{Lt-1} \\
& \quad 0.042^{**} \qquad\quad 0.209^{**} \qquad\quad 0.183^{**} \\
& + 1.714 D_{1t} + 0.620 D_{2t} - 1.588 D_{3t} + 2.519 \\
& \quad 0.501^{***} \quad 0.745 \qquad 0.711^{**} \quad 0.988^{**}
\end{aligned}
$$

$R^2 = 0.557$

Variables not previously defined are

I_{Lt} = inventories of lamb and mutton at the end of quarter t;

$\Delta I_{Lt} = I_{Lt} - I_{Lt-1}$; and

CP_{Bt} = farm marketings of cattle in quarter t.

COMPLETE QUARTERLY MODEL OF BEEF- AND PORK-MARKETING SECTOR

Quarterly data for 1954-I through 1968-IV were used to construct a complete econometric model of the beef- and pork-marketing sectors of the United States economy. Quarterly farm-level demand equations for cattle and hogs were then derived from this model. Construction of this model drew heavily on the three previously summarized studies of demand, margins, and inventories. The model contained 12 behavioral equations: 2 retailers'-demand equations, 2 inventory-investment equations, 2 retail-price equations, 2 wholesale-retail margin equations, 2 farm-wholesale margin equations, and 2 farm-price equations. The model also contained 18 identities. In the presentation of the model, endogenous variables will be represented by capital letters; exogenous variables by lower case letters. Each symbol will be defined as it is introduced.

Every behavioral equation was estimated in more than one way. Methods of estimation are identified as: OLS indicates least squares; ALS indicates autoregressive least squares, a method that assumes the presence of autocorrelated errors and only one endogenous variable in an equation; 2SLS indicates two-stage least squares, a method that assumes the presence of more than one endogenous variable and temporal independence of errors; A2SLS indicates autoregressive, two-stage, least squares, a method that assumes both autocorrelated errors and more than one endogenous variable.

Every behavioral equation contained three seasonal dummy variables defined as follows:

d_{1t} = 1 in 1st quarter of each year,
= 0 in all other quarters;

d_{2t} = 1 in 2nd quarter of each year,
= 0 in all other quarters; and

d_{3t} = 1 in 3rd quarter of each year,
= 0 in all other quarters.

Coefficients, and their standard errors, of these variables are presented in table 7, along with intercept terms, autoregressive error coefficients, values of R_A^2, and Durbin-Watson d statistics.

Table 7. Estimation method used, intercept, coefficients of seasonal dummy variables and of autoregression in errors, R_A^2 and Durbin-Watson d-statistics.

Equation no.	Dependent variable	Estimation method	I	d_{1t}	d_{2t}	d_{3t}	U_{it-1}	R_A^2	d
1'	RD_{Bt}	OLS	-40.65	54.929	67.185	106.333		0.993	0.741
			156.06	21.470**	22.111***	21.111***			
1	RD_{Bt}	A2SLS	167.58	53.379	75.983	105.069	0.579	0.985	1.843
			70.631	11.918***	13.846***	11.745	0.098***		
2'	RD_{Pt}	OLS	-188.75	12.330	130.722	221.407		0.971	1.762
			110.02*	21.076	24.185***	25.336***			
2	RD_{Pt}	2SLS	-221.62	12.807	131.956	221.544		0.972	1.699
			109.16	20.679	23.735***	24.857***			
3	ΔI_{Bt}	OLS	8.171	-58.834	-41.109	-34.823		0.879	2.098
			16.986	11.524***	13.321***	11.491***			
4'	ΔI_{Pt}	OLS	-78.205	1.681	11.248	-62.468		0.889	2.213
			66.790	21.881	26.689	27.047**			
4	ΔI_{Pt}	2SLS	-76.344	2.623	12.482	-60.434		0.887	2.293
			67.354	22.132	27.018	27.649**			
5'	RP_{BCt}	OLS	130.36	-2.185	-1.702	1.610		0.806	1.305
			9.22***	0.722***	1.020	1.204			
5	RP_{BCt}	A2SLS	123.93	-2.385	-1.803	1.284	0.330	0.678	1.792
			6.666***	0.746***	1.134	1.325	0.124**		
6'	RP_{Pt}	OLS	134.15	-6.090	-8.666	-5.880		0.833	0.964
			10.76***	0.996***	1.040***	1.139***			
6	RP_{Pt}	A2SLS	143.20	-3.781	-5.823	-3.472	0.484	0.760	2.074
			6.495***	0.674***	0.806***	0.937***	0.109***		
7'	MR_{BCt}	OLS	-1.395	-0.906	-0.336	-0.338		0.954	1.904
			2.848	0.311***	0.301	0.313			
7	MR_{BCt}	2SLS	-1.322	-0.939	-0.353	-0.434		0.913	1.767
			3.966	0.432**	0.417	0.437			
8'	MR_{Pt}	OLS	3.182	-0.093	-0.598	0.642		0.901	1.992
			5.086	0.329	0.321*	0.326*			
8	MR_{Pt}	2SLS	2.772	-0.010	-0.511	0.728		0.903	1.729
			5.054	0.332	0.325	0.330**			
9'	MW_{BCt}	OLS	2.707	0.731	0.339	0.011		0.766	1.835
			1.489*	0.268***	0.284	0.238			
9	MW_{BCt}	2SLS	2.586	0.759	0.386	0.063		0.755	1.860
			1.555*	0.278***	0.300	0.256			
10'	MW_{Pt}	OLS	2.668	-0.408	-0.855	-0.196		0.831	2.231
			0.869***	0.214*	0.259***	0.276			
10	MW_{Pt}	2SLS	2.835	-0.376	-0.787	-0.093		0.766	2.273
			1.031***	0.255	0.309**	0.333			
11'	FP_{Bt}	OLS	-0.767	1.282	1.341	0.305		0.956	1.202
			1.033	0.196***	0.188***	0.198			
11	FP_{Bt}	A2SLS	-0.874	1.197	1.322	0.345	0.387	0.929	1.827
			0.775	0.165***	0.166***	0.166**	0.120***		
12'	FP_{Pt}	OLS	0.702	0.081	-0.336	-0.510		0.993	1.810
			0.179***	0.091	0.093***	0.094***			
12	FP_{Pt}	2SLS	0.695	0.080	-0.336	-0.511		0.958	2.262
			0.479	0.241	0.245	0.249***			

R_A^2 is the coefficient of multiple determination adjusted for the number of regressors. This table also identifies methods of estimation used. Results other than those in table 7 are presented in the text. Standard errors are given below each coefficient. The numbers of the different equations are written with or without prime; e.g., (1) ' denotes a preliminary estimate of this equation, and (1) denotes the final estimate of this equation to be used later.

Retailers' Demand

Throughout the rest of this report, subscript B identifies beef, BC identifies choice-grade beef, and P identifies pork. Subscript t identifies current values; one- and two-period lags are identified by t-1 and t-2. Variables in retailers' demand equations are defined as:

RD_{it} = retailers' demand for i-th meat = CC_{it};
CC_{it} = civilian consumption of commercially produced i-th meat (in mil. lb. carcass-weight equivalent), i = B,P;
VW_{it} = wholesale value of quantity of i-th carcass equivalent to 1 lb. of retail cuts (in cents/lb. retail cuts), i = BC, P;
mt_{it} = military takings of commercially produced i-th meat (in mil. lb. carcass-weight equivalent), i = B, P;
cp_{it} = domestic production of commercially produced i—th meat (in mil. lb. carcass-weight equivalent), i = B, P; and
t_{2t} = linear time trend between 1954-I and 1962-IV and constant thereafter.

VW_i is computed by multiplying wholesale price by a conversion factor representing the number of pounds of carcass required to produce 1 pound of retail cuts. The conversion ratio for beef increased gradually from 1.34 in 1954 to 1.41 in 1962 and remained constant thereafter. Variable t_{2t} was included in the retailers' beef demand equation to allow for this change. OLS estimation yielded:

$$(1)'\quad RD_{Bt} = \underset{2.015}{2.858}\, VW_{BCt} - \underset{0.550}{0.064}\, mt_{Bt} + \underset{0.039^{***}}{0.910}\, cp_{Bt} + \underset{1.495^{***}}{11.026}\, t_{2t}$$

$$(2)'\quad RD_{Pt} = \underset{1.464^{***}}{3.641}\, VW_{Pt} - \underset{0.817^{***}}{1.607}\, mt_{Pt} + \underset{0.027^{***}}{0.993}\, cp_{Pt}$$

The signs of the coefficients in equations 1' and 2' are as expected. Because d indicated positive serial correlation in equation 1', this equation was re-estimated with A2SLS, and equation 2' was re-estimated with 2SLS, yielding:

$$(1)\quad RD_{Bt} = \underset{2.198^{*}}{3.760}\, VW_{BCt} - \underset{0.430^{***}}{1.110}\, mt_{Bt} + \underset{0.034^{***}}{0.952}\, cp_{Bt} + \underset{1.908^{***}}{11.718}\, t_{2t}$$

$$(2)\quad RD_{Pt} = \underset{1.472^{***}}{4.288}\, VW_{Pt} - \underset{0.807^{**}}{1.784}\, mt_{Pt} + \underset{0.026^{***}}{0.998}\, cp_{Pt}$$

A2SLS increased the significance of coefficients of VW_{BCt} and mt_{Bt}. In equations 1 and 2, the hypothesis of serial independence of residuals is accepted.

Inventories

Variables in inventory equations not previously defined are:

I_{it} = ending stocks of commercially produced i-th meat (mil. lb. carcass weight) i = B, P;
ΔI_{it} = $I_{it} - I_{it-1}$;
S_{it} = sales of meat i, i = B, P,
= $cp_{it} - \Delta I_{it}$;
ΔS_{it} = $S_{it} - S_{it-1}$;
Δcp_{it} = $cp_{it} - cp_{it-1}$;
$\Delta' cp_{Bt}$ = Δcp_{Bt} in first, second, and third quarters,
= $-3\Delta cp_{Bt}$ in fourth quarter; and
Δf_t = 0 in second and third quarters,
= spring pig crop of current year, minus fall pig crop of current year in fourth quarter of current year and first quarter of subsequent year (in 10,000's of head of pigs saved).

OLS yielded

$$(3)\quad \Delta I_{Bt} = \underset{0.005^{***}}{0.026}\, S_{Bt-1} - \underset{0.069^{***}}{0.433}\, I_{Bt-2} + \underset{0.018^{***}}{0.155}\, \Delta cp_{Bt} + \underset{0.012^{***}}{0.038}\, \Delta cp_{Pt} - \underset{0.013^{***}}{0.048}\, \Delta' cp_{Bt}$$

$$(4)'\quad \Delta I_{Pt} = \underset{0.037^{**}}{-0.078}\, \Delta S_{Pt} - \underset{0.046^{***}}{0.161}\, S_{Pt-1} - \underset{0.069^{***}}{0.243}\, I_{Pt-2} + \underset{0.042^{***}}{0.204}\, cp_{Pt} + \underset{0.013^{***}}{0.064}\, \Delta f_t$$

Since d is biased in equations 3 and 4', rho—the coefficient of autocorrelation in the errors—was computed by A2SLS. This confirmed the d-statistics in accepting the null hypotheses of serial independence of the errors. Equation 4' was re-estimated with 2SLS.

$$(4)\quad \Delta I_{Pt} = \underset{0.046^{*}}{-0.089}\, \Delta S_{Pt} - \underset{0.053^{***}}{0.172}\, S_{Pt-1} - \underset{0.070^{***}}{0.240}\, I_{Pt-2} + \underset{0.048^{***}}{0.214}\, cp_{Pt} + \underset{0.013^{***}}{0.063}\, \Delta f_t$$

The most noteworthy differences between equations 3, 4', and 4 and the earlier results by Ladd (10) are in the sales variables. In equations 3, 4', and 4, S_{Bt-1}, ΔS_{Pt}, and S_{Pt-1} have significant coefficients. In analyses of inventory investment with data for 1954-I through 1967-IV, we also found significant evidence of a relation between sales and inventories. In a study covering 1949-III to 1960-IV, Ladd (10) found no significant relation between sales variables and inventory changes.

One appealing explanation of the difference between the study published in 1963 (10) and the recent studies is that packers have recently become more adept at forecasting levels of meat sales, and they now consider expected or forecast levels of meat sales in determining levels of meat inventories, whereas, previously, they were unable to predict meat sales accurately and consequently did not consider expected sales in determining levels of inventories.

Retail Prices

Variables in the retail-price equations not previously defined are:

- RP_{it} = retail price of i-th meat (cents/lb. retail cuts) i = BC, P;
- icp_t = index of consumer prices of all items, 1957-59 = 100;
- $DRP_{it} = RP_{it}/icp_t$;
- PCC_{it} = per-capita civilian consumption of commercially produced i-th meat (lb. carcass-weight equivalent per person), i = B, P;
- pcc_{it} = per-capita civilian consumption of commercially produced i-th meat, i = L denotes lamb (lb. carcass-weight equivalent per person), i = C denotes broilers (lbs. ready-to-cook per person);
- dy_t = per-capita disposable personal income (dollars) seasonally adjusted, deflated by icp_t, computed as a deviation from a linear trend of income on time; and
- t_{1t} = linear time trend between 1954-I and 1968-IV.

The Buttimer (1) and Logan and Boles (14) studies found quarterly variation in intercepts, but not in slopes, of the beef and pork retail-price equations. Hence Model CS,VI, as defined earlier in the section on quarterly red-meats and broiler demand, was used here. OLS estimation yielded:

$$(5)'\ DRP_{BCt} = \underset{0.381^{***}}{-3.824}\ PCC_{Bt} - \underset{0.273}{0.093}\ PCC_{Pt} + \underset{3.543^{***}}{9.449}\ pcc_{Lt} + \underset{0.697^{**}}{1.432}\ pcc_{Ct} + \underset{0.006^{**}}{0.016}\ dy_t - \underset{0.00007^{***}}{0.00028}\ dy^2_t + \underset{0.092^{***}}{0.434}\ t_{1t}$$

$$(6)'\ DRP_{Pt} = \underset{0.484^{*}}{-0.936}\ PCC_{Bt} - \underset{0.350^{***}}{4.947}\ PCC_{Pt} + \underset{4.584^{***}}{16.215}\ pcc_{Lt} + \underset{0.008^{***}}{0.047}\ dy_t - \underset{0.0001^{**}}{0.0002}\ dy^2_t + \underset{0.075^{***}}{0.247}\ t_{1t}$$

In both equations, the hypothesis of serial independence of the residuals was rejected. Therefore, equations 5' and 6' were re-estimated by using A2SLS and obtaining:

$$(5)\ DRP_{BCt} = \underset{0.460^{***}}{-3.477}\ PCC_{Bt} - \underset{0.363}{0.178}\ PCC_{Pt} + \underset{3.687^{***}}{10.945}\ pcc_{Lt} + \underset{0.755^{*}}{1.276}\ pcc_{Ct} + \underset{0.008^{*}}{0.015}\ dy_t - \underset{0.00009^{***}}{0.00026}\ dy^2_t + \underset{0.102^{***}}{0.407}\ t_{1t}$$

$$(6)\ DRP_{Pt} = \underset{0.475^{*}}{-0.892}\ PCC_{Bt} - \underset{0.359^{***}}{3.842}\ PCC_{Pt} + \underset{3.607^{***}}{9.262}\ pcc_{Lt} + \underset{0.009^{***}}{0.040}\ dy_t - \underset{0.00010}{0.00010}\ dy^2_t$$

These results agree with the previous results of Logan and Boles (14) and Buttimer (1) in several important respects. a) Beef-consumption coefficients in beef price equations are negative and highly significant in all three studies. b) Pork-consumption coefficients in pork price equations are negative and highly significant in all three studies. c) Pork-consumption coefficients in beef price equations are negative, but nonsignificant. d) All coefficients of beef consumption are negative in the pork price equations; although this coefficient was highly significant in the Logan and Boles study, it was not significant in the Buttimer study and is lowly significant in equation 6. e) All coefficients of lamb consumption in beef price equations are positive; only in the Buttimer study is this coefficient not significant. f) The coefficients of lamb consumption in the pork price equations are all positive and significant, although only lowly significant in the Logan and Boles study. g) Coefficients of broiler consumption in the beef price equations are all positive; this coefficient is lowly significant in equation 5 and in Buttimer's study and is highly significant in the Logan and Boles study. h) None of the studies found a significant relation between broiler consumption and pork price.

The results on income are difficult to compare because Logan and Boles used income (y), Buttimer used dy, and equations 5 and 6 contain dy and dy^2. In the equation for beef price, Logan and Boles obtained a positive and not significant coefficient for income, and Buttimer obtained a positive lowly significant coefficient for dy. In the equation for pork price, Logan and Boles obtained a negative significant coefficient for income; Buttimer obtained a positive highly significant coefficient for dy.

Wholesale-Retail Margins

Variables in wholesale-retail margin equations not previously defined are:

MR_{it} = wholesale-retail margin of meat i (cents/lb. retail cut), i = BC, P;

cn_t = $0.3c_t + 0.7n_t$;

c_t = index of prices of containers and packaging material, 1957—59 = 100;

n_t = index of prices of new plant and equipment, 1957-59 = 100;

hr_t = average hourly earnings of nonsupervisory workers in grocery, meat, and vegetable stores divided by $wp_t/100$; and

Wp_t = index of output per man-hour in manufacturing meat products, 1957—59 = 100. (This variable was labeled $LP2_t$ in table 4.)

Because of the availability of several indexes of costs and labor productivity in retailing, these margin equations were estimated with stepwise regression, and the following equations were selected:

$$(7)'\ MR_{BCt} = -0.391\,\Delta VW_{BCt} + 0.444\,MR_{BCt-1} + 0.109\,cn_t + 0.095\,t_{2t}$$
$$\qquad\quad 0.034^{***} \qquad 0.068^{***} \qquad 0.032^{***} \qquad 0.028^{***}$$

$$(8)'\ MR_{Pt} = -0.195\,\Delta VW_{Pt} + 0.150\,MR_{Pt-1} + 0.191\,cn_t - 7.417\,hr_t$$
$$\qquad\quad 0.031^{***} \qquad 0.100 \qquad 0.026^{***} \qquad 2.719^{***}$$

The time trend t_{2t} was included in equation 7' because of the nature of VW_{Bt}, as explained in connection with equation 1. Since d is biased in equations 7' and 8', the coefficient of autocorrelation in errors was computed for both equations. Neither autocorrelation coefficient was significant, confirming the hypothesis of serial independence of the residuals in equations 7' and 8'. Therefore, 2SLS was applied to equations 7' and 8', yielding:

$$(7)\ MR_{BCt} = -0.349\,\Delta VW_{BCt} + 0.436\,MR_{BCt-1} + 0.103\,cn_t + 0.101\,t_{2t}$$
$$\qquad\quad 0.053^{***} \qquad 0.094^{***} \qquad 0.045^{***} \qquad 0.039^{**}$$

$$(8)\ MR_{Pt} = -0.212\,\Delta VW_{Pt} + 0.138\,MR_{Pt-1} + 0.195\,cn_t - 7.349\,hr_t$$
$$\qquad\quad 0.033^{***} \qquad 0.100 \qquad 0.026^{***} \qquad 2.689^{***}$$

These results, obtained by using data for 1954-I through 1968-IV, agree with results obtained with data for 1954-I through 1967-IV, and summarized in table 4, in finding positive highly significant relations between cn_t and the two margins. (Variable CN in table 4 is the same as cn_t.) The results from the two sample periods differ in the estimated effects of hr_t. (Variable HR1 in table 4 is similar to hr_t in equation 8.) The coefficient of hr_t has an unexpected sign.

Farm-Wholesale Margins

Variables in farm-wholesale margin equations not previously defined are:

MW_{it} = farm-wholesale margin on meat i (cents/lb. retail cuts), i = BC, P;

ch_t = prime and choice steer sales as a percentage of all steer grades sold at Chicago, Omaha, and Sioux City;

ΔFP_{it} = $FP_{it} - FP_{it-1}$; and

FP_{it} = average price received by farmers for beef cattle (i = B) or for hogs (i = P), ($/cwt. liveweight).

As with retail margins, several indexes on costs and labor productivity in the meat-packing industry were available. Therefore, stepwise regression was applied, and the following equations were selected:

$$(9)'\ MW_{BCt} = 0.198\,MW_{Pt} + 0.041\,\Delta VW_{BCt} + 0.443\,MW_{BCt-1} + 0.0020\,\Delta cp_{Bt} - 0.049\,wp_t + 0.043\,ch_t + 0.028\,t_{2t}$$
$$\qquad\quad 0.067^{***} \qquad 0.022^{*} \qquad 0.105^{***} \qquad 0.0006^{***} \qquad 0.014^{***} \qquad 0.016^{**} \qquad 0.019$$

$$(10)'\ MW_{Pt} = 0.540\,\Delta VW_{Pt} - 1.108\,\Delta FP_{Pt} + 0.681\,MW_{Pt-1} + 0.00094\,cp_{Pt}$$
$$\qquad\quad 0.084^{***} \qquad 0.157^{***} \qquad 0.077^{***} \qquad 0.00031^{***}$$

Again, t_{2t} was included in equation 9' because of the nature of VW_{BCt} as explained in connection with equation 1. The signs of the coefficients agree with previous results and are as expected. The coefficient of autocorrelation in residuals was computed, but was not significant in either equation. Re-estimation of equations 9' and 10' with 2SLS yielded:

$$\text{(9)}\quad MW_{BCt} = \underset{0.075^{***}}{0.212}\, MW_{Pt} + \underset{0.026}{0.033}\, \Delta VW_{BCt} + \underset{0.108^{***}}{0.434}\, MW_{BCt-1} + \underset{0.0006^{***}}{0.0019}\, \Delta cp_{Bt} - \underset{0.014^{***}}{0.050}\, wp_t + \underset{0.016^{**}}{0.044}\, ch_t + \underset{0.020}{0.029}\, t_{2t}$$

$$\text{(10)}\quad MW_{Pt} = \underset{0.113^{***}}{0.467}\, \Delta VW_{Pt} - \underset{0.212^{***}}{0.979}\, \Delta FP_{Pt} + \underset{0.092^{***}}{0.668}\, MW_{Pt-1} + \underset{0.00036^{**}}{0.00094}\, cp_{Pt}$$

The only factor-price or labor-productivity variable in these equations is the labor-productivity variable wp_t, having an expected negative coefficient in equations 9' and 9.

Farm Prices

Variables in farm price equations that have not been previously defined are:

VF_{it} = net farm value of quantity of choice-grade beef cattle ($i = B$) or hogs ($i = P$) equivalent to 1 lb. of retail cut, minus byproduct allowance, in cents per lb. of retail cut.

OLS estimation yielded

$$\text{(11)'}\quad FP_{Bt} = \underset{0.022^{***}}{0.229}\, VF_{BCt} - \underset{0.032^{***}}{0.135}\, VW_{BCt-1} + \underset{0.064^{***}}{0.821}\, FP_{Bt-1} - \underset{0.0089}{0.0013}\, t_{2t}$$

$$\text{(12)'}\quad FP_{Pt} = \underset{0.005^{***}}{0.520}\, VF_{Pt} + \underset{0.0018^{***}}{0.0073}\, t_{1t}$$

The variable ch_t was included in preliminary versions of equation 11', but its coefficient was not significant. Residuals obtained by A2SLS from a version of equation 11', not including VW_{BCt-1} and FP_{Bt-1}, suggested a second-order, autoregressive-error scheme. Inclusion of these two variables reduced the second-order to a first-order autoregressive scheme. No such problems were encountered in the farm price equation for hogs, which was re-estimated with 2SLS.

$$\text{(11)}\quad FP_{Bt} = \underset{0.023^{***}}{0.244}\, VF_{BCt} - \underset{0.039^{***}}{0.114}\, VW_{BCt-1} + \underset{0.105^{***}}{0.708}\, FP_{Bt-1} + \underset{0.015}{0.011}\, t_{2t}$$

$$\text{(12)}\quad FP_{Pt} = \underset{0.014^{***}}{0.520}\, VF_{Pt} + \underset{0.0049}{0.0072}\, t_{1t}$$

Quantity Identities

The endogenous variables PCC_{it} and CC_{it} are exactly related by the identity $PCC_{it} = CC_{it}/p_t$ where

p_t = civilian resident population (tens of thousands of persons).

To keep the complete model linear, these two nonlinear identities were replaced by the linear approximations obtained by OLS.

$$\text{(13)}\quad PCC_{Bt} = \underset{0.472^{***}}{16.942} + \underset{0.00006^{***}}{0.0049}\, CC_{Bt} - \underset{0.00003^{***}}{0.0008}\, p_t \qquad R^2 = 0.998 \quad d = 0.943$$

$$\text{(14)}\quad PCC_{Pt} = \underset{0.229^{***}}{13.904} + \underset{0.00006^{***}}{0.0055}\, CC_{Pt} - \underset{0.00001^{***}}{0.0007}\, p_t \qquad R_A^2 = 0.992 \quad d = 0.825$$

To complete the model, several other linear identities pertaining to quantities also are needed.

$$\text{(15)}\quad IM_{Bt} - EX_{Bt} = mt_{Bt} + RD_{Bt} + \Delta I_{Bt} - cp_{Bt}$$

$$\text{(16)}\quad IM_{Pt} - EX_{Pt} = mt_{Pt} + RD_{Pt} + \Delta I_{Pt} - cp_{Pt}$$

Here

$IM_{it} - EX_{it}$ = net foreign trade of commercially produced i-th meat (mil. lb. carcass-weight equivalents).

$$\text{(17)}\quad \Delta I_{Bt} = I_{Bt} - I_{Bt-1}$$

$$\text{(18)}\quad \Delta I_{Pt} = I_{Pt} - I_{Pt-1}$$

$$\text{(19)}\quad S_{Bt} = cp_{Bt} - \Delta I_{Bt}$$

$$\text{(20)}\quad S_{Pt} = cp_{Pt} - \Delta I_{Pt}$$

$$\text{(21)}\quad \Delta S_{Pt} = S_{Pt} - S_{Pt-1}$$

Price Identities

The endogenous variables RP_{it} and DRP_{it} are exactly related by $RP_{it} = DRP_{it}(icp_t)$. These were replaced by the OLS linear approximations.

$$\text{(22)}\quad RP_{BCt} = -\underset{0.469^{***}}{72.370} + \underset{0.005^{***}}{1.014}\, DRP_{BCt} + \underset{0.002^{***}}{0.714}\, icp_t \qquad R^2 = 0.999 \quad d = 0.844$$

$$\text{(23)}\quad RP_{Pt} = -\underset{0.992^{***}}{60.224} + \underset{0.010^{***}}{1.028}\, DRP_{Pt} + \underset{0.006^{***}}{0.584}\, icp_t \qquad R^2 = 0.996 \quad d = 0.402$$

Several other price identities complete the model.

$$\text{(24)}\quad VW_{BCt} = RP_{BCt} - MR_{BCt}$$

$$\text{(25)}\quad VW_{Pt} = RP_{Pt} - MR_{Pt}$$

(26) $VF_{BCt} = VW_{BCt} - MW_{BCt}$
(27) $VF_{Pt} = VW_{Pt} - MW_{Pt}$
(28) $\Delta VW_{BCt} = VW_{BCt} - VW_{BCt-1}$
(29) $\Delta VW_{Pt} = VW_{Pt} - VW_{Pt-1}$
(30) $\Delta FP_{Pt} = FP_{Pt} - FP_{Pt-1}$

Equations 1 through 30 constitute a complete econometric model of the beef– and pork–marketing sector of the economy.

Final Forms of Farm Price Equations

From equations 1 through 30, we can obtain the reduced form of the system. Each reduced-form equation states one endogenous variable as a linear function of exogenous variables, lagged endogenous variables, and residuals. From the reduced form of the system, we can obtain the final form of the system. Each final-form equation states the value of one endogenous variable in quarter q (q = 1, 2, 3, 4) of year y as a linear function of values of endogenous variables in the last quarters of year y−1, of current and lagged exogenous variables, and residuals. The difference between a reduced-form equation and a final-form equation for, say, quarter 3 of year y is this: The right-hand side of the reduced-form equation will contain values of endogenous variables for quarters 1 and 2 of year y. The final-form equation will not contain these lagged endogenous variables but will contain values of endogenous variables in quarters 2, 3, and 4 of year y−1. The remainder of this analysis makes use of final-form equations for farm prices of cattle and hogs.

Let FP_{iq} be the farm price of animal i in quarter q of year y and cp_{iq} be commercial production of i-th meat in q-th quarter of year y. Define the vectors Y_B, Y_P, Q_B, and Q_P as

$$\begin{pmatrix} FP_{B1} \\ FP_{B2} \\ FP_{B3} \\ FP_{B4} \end{pmatrix} = Y_B, \qquad \begin{pmatrix} FP_{P1} \\ FP_{P2} \\ FP_{P3} \\ FP_{P4} \end{pmatrix} = Y_P$$

$$\begin{pmatrix} cp_{B1} \\ cp_{B2} \\ cp_{B3} \\ cp_{B4} \end{pmatrix} = Q_B, \qquad \begin{pmatrix} cp_{P1} \\ cp_{P2} \\ cp_{P3} \\ cp_{P4} \end{pmatrix} = Q_P$$

Letting U_B and U_P denote vectors of residuals, we can then write for farm cattle prices

(31) $Y_B = C_B + D_1 Q_B + D_3 Q_P + U_B$

and for farm hog prices

(32) $Y_P = C_P + D_2 Q_B + D_4 Q_P + U_P$

Each element of C_B and C_P is a function of current and lagged exogenous variables and of endogenous variables of the preceding year. Hence, these vectors change in value from year to year, but the D_j matrices do not. Table 8 presents the elements of C_B and C_P for selected years. Table 9 presents the D_j matrices.

Table 8. Elements of vectors C_B and C_P in 1957, 1960, 1967, and 1970

Year y	Quarter q	C_B	C_P
1957	I	37.44	64.20
	II	38.52	49.23
	III	40.91	50.74
	IV	41.36	56.95
1960	I	42.51	67.16
	II	43.52	51.57
	III	45.37	51.89
	IV	44.68	58.05
1967	I	52.78	79.32
	II	54.05	63.24
	III	56.91	64.67
	IV	57.63	71.35
1970	I	59.94	85.17
	II	61.56	70.25
	III	63.65	69.17
	IV	65.96	77.91

Table 9. Coefficient matrices D_1, D_2, D_3, and D_4

D_1

-0.00593804			
0.00010047	-0.00593804		
-0.00030928	0.00010047	-0.00593804	
-0.00024652	-0.00030928	0.00010047	-0.00593804

D_2

-0.00250358			
0.00058586	-0.00250358		
-0.00001370	0.00058586	-0.00250358	
0.00001002	-0.00001370	0.00058586	-0.00250358

D_3

-0.0005072			
-0.00017331	-0.0005072		
-0.00015870	-0.00017331	-0.0005072	
-0.00012360	-0.00015870	-0.00017331	-0.0005072

D_4

-0.01504462			
0.00334511	-0.01504462		
-0.00021996	0.00334511	-0.01504462	
0.00001937	-0.00021996	0.00334511	-0.01504462

The final–form equations 31 and 32, with $U_B = U_P = 0$, were used to estimate farm prices and first differences in farm prices for each quarter in the period 1955–III through 1970–IV. Two different measures of accuracy of estimates were computed. One version of Theil's inequality coefficient U (18) is defined as

$$U = [(1/n)\,\Sigma\,(P_t - A_t)^2]^{1/2} \div ([(1/n)\,\Sigma\,P_t^2]^{1/2} + [(1/n)\,\Sigma\,A_t^2]^{1/2})$$

where

P_t = predicted change in value of variable between periods t–1 and t,

A_t = actual change in value of variable between periods t–1 and t.

U is not less than zero and not greater than one. If U = 0, the forecasts agree completely with the actual observations. If U = 1, the forecasts disagree completely with the actual observations. A small value of U indicates a more accurate forecasting equation than a large value of U. For equations 31 and 32 we obtained:

U = 0.322 for the cattle price equation,
U = 0.396 for the hog price equation.

Theil (18, pp. 88–89, 119, 121, 150–153, 186, 304, 314) used U to measure the forecasting accuracy of about 420 equations. Some 62 percent of the values of U were less than 0.30; 25 percent were between 0.31 and 0.40; and 13 percent exceeded 0.41.

Another measure of accuracy refers to the ability of an equation to predict turning points. An actual turning point occurs if two successive changes have opposite signs; i.e., if sign $A_t \neq$ sign A_{t-1}. A predicted turning point occurs if two successive predicted changes have opposite signs; i.e., if sign $P_t \neq$ sign P_{t-1}. An actual turning point occurs and is correctly predicted if these three conditions are satisfied:

sign $A_t \neq$ sign A_{t-1};
sign P_t = sign A_t; and
sign P_{t-1} = sign A_{t-1}

Table 10 presents a 2 x 2 contingency table summarizing performance of final-form equations in predicting turning points. The top row of the table shows that 30 turning points did occur in actual farm prices for cattle; of these, 20 were correctly predicted by the final-form equation for farm-cattle prices, and 10 were not correctly predicted. Both values of chi-square are significant, indicating that the number of correct predictions of turning points is greater than the number that would be expected by chance alone.

A test of the predictive ability of final-form equations is a stricter test than a test of the predictive ability of reduced-form equations because the final-form equations use less prior information. For example, in predicting farm price of beef in quarter 3 from the reduced-form equation, we use information on commercial production of beef and pork in the second and third quarters of *that same* year. In predicting farm price of beef in quarter 3 from the final-form equations, however, we only use information on commercial production in the last quarters of the *preceding* year.

Table 10. Number of actual and predicted turning points for 62 observations on prices of cattle and hogs (1955-III to 1970-IV)

	Actual turning point correctly predicted	Actual turning point not correctly predicted	chi-square
	Prices of cattle		
Actual: Turning point	20	10	
Actual: No turning point	10	22	7.763***
	Prices of hogs		
Actual: Turning point	21	10	
Actual: No turning point	12	19	5.246**

OTHER RECENT STUDIES

Several studies of beef or pork sectors of the United States economy have been published in recent years.

Myers, Havlicek, and Henderson (16) published a monthly model of the hog-pork sector. The major conceptual differences between their model and the hog-pork part of our model are these: a) In their model, slaughter hog supply and slaughter cattle supply were endogenous, and inventory changes were exogenous. In our model, the classification is reversed: Supplies of slaughter livestock are exogenous and inventory changes are endogenous. b) They had one marketing margin (farm-retail); our study contains two marketing margins for each product.

The quarterly model developed by Crom (2) also differs from our model in its treatment of farm supplies. His model contains detailed analysis of factors affecting marketings and commercial production. Among his endogenous variables are: number of fed cattle marketed; number of nonfed cattle marketed; average weights of fed and nonfed cattle; commercial hog slaughter; dressing percentages for nonfed cattle and for hogs; feeder steer prices; gross price margin between choice steers and feeders; sows farrowing; placements of cattle on feed; commercial beef-cow slaughter; and Jan. 1 inventories of beef cows, of beef cattle, and of 1-2 year old beef heifers. Crom's model also contains two equations that explain beef exports and imports as functions of lagged price of commercial cow-beef and lagged per-capita supply of nonfed beef. His model contains two equations that explain pork exports and imports as functions of lagged wholesale pork price, lagged per-capita pork supply, and a time trend. Since the main focus of his study is the livestock economy, Crom's treatment of consumer demand and marketing margins is cursory.

A third study published in 1970 (7) contained 5 structural equations and 5 endogenous variables: Monthly cattle and hog prices, monthly commercial slaughter supplies of beef and pork, and pork inventory change. In a fourth study published in 1970, Trierweiler and Hassler (19) used monthly data to analyze retail beef and pork prices, wholesale beef prices, slaughter-steer prices, feeder-steer prices, and slaughter-hog prices. They also analyzed differences between wholesale prices of beef carcasses of various weights and grades; differences between prices of slaughter cattle of various weights, grades, and classes; and differences between prices of slaughter hogs of various weights at different locations. Farm supplies were determined normatively: by a linear-programming, time- and-form, equilibrium-allocation model.

Gruber and Heady (5) used annual data to study the cattle cycle. They divided the cattle cycle into three cycles: inventory cycle, price and income cycle, and the slaughter and import cycle. They analyzed forces generating each cycle and relations among the three cycles.

Other studies published within the last decade include: the study by Leuthold (13) of daily supply and demand for barrows and gilts; a simultaneous-equations study of beef demand, supply and price (12); and a quarterly recursive model of price and supply of hogs and pork (6).

OPTIMAL MARKETINGS

Equations 31 and 32 relate farm prices (measured in dollars per 100 pounds liveweight) to commercial production (measured in millions of pounds carcass-weight equivalent). To obtain relations between farm prices and farm marketings (measured in hundreds of pounds liveweight), define D_{iq} as the dressing yield of livestock i in quarter q of year y multiplied by 10^4, and define fm_{iq} as farm marketings of i-th livestock in quarter q of year y in hundreds of pounds liveweight. Then $cp_{iq} = D_{iq} fm_{iq}$. Define X_B, X_P and D_i as

$$X_B = \begin{pmatrix} fm_{B1} \\ fm_{B2} \\ fm_{B3} \\ fm_{B4} \end{pmatrix}$$

$$X_P = \begin{pmatrix} fm_{P1} \\ fm_{P2} \\ fm_{P3} \\ fm_{P4} \end{pmatrix}$$

$$D_i = \begin{pmatrix} D_{i1} & 0 & 0 & 0 \\ 0 & D_{i2} & 0 & 0 \\ 0 & 0 & D_{i3} & 0 \\ 0 & 0 & 0 & D_{i4} \end{pmatrix} i = B,P$$

Then $Q_B = D_B X_B$, $Q_P = D_P X_P$. Using these last two relations, and defining

$$D_{1B} = D_1 D_B \qquad D_{2B} = D_2 D_B$$
$$D_{3P} = D_3 P_P \qquad D_{4P} = D_4 D_P$$

equations 31 and 32 can be rewritten as

$$(33)\ Y_B = C_B + D_{1B} X_B + D_{3P} X_P + U_B$$
$$(34)\ Y_P = C_P + D_{2B} X_B + D_{4P} X_P + U_P$$

Let CR(B) and CR(P) denote cash receipts from sale of cattle and hogs, respectively, in year y, and let CR(B + P) denote the sum of cash receipts from the two. With the notation defined previously (in the section Final Forms of Farm Price Equations),

$$CR(B) = Y'_B X_B$$
$$CR(P) = Y'_P X_P$$
$$CR(B + P) = (Y'_B, Y'_P) \begin{pmatrix} X_B \\ X_P \end{pmatrix}$$

Substituting equations 33 and 34 into these expressions yields

$$CR(B) = (C_B + D_{3P} X_P)' X_B + X'_B D_{1B} X_B + U'_B X_B$$
$$CR(P) = (C_P + D_{2B} X_B)' X_P + X'_P D_{4P} X_P + U'_P X_P$$
$$CR(B + P) = (C'_B, C'_P) \begin{pmatrix} X_B \\ X_P \end{pmatrix}$$
$$+ (X'_B, X'_P) \begin{pmatrix} D_{1B}, D_{3P} \\ D_{2B}, D_{4P} \end{pmatrix} \begin{pmatrix} X_B \\ X_P \end{pmatrix} + (U'_B, U'_P) \begin{pmatrix} X_B \\ X_P \end{pmatrix}$$
$$= C'X + X'DX + U'X$$

If we drop the stochastic terms $U'_B X_B$, $U'_P X_P$, and U'X from these three expressions, the remaining right-hand sides are expected cash receipts ECR(B), ECR(P), and ECR(B + P).

Let fm_{iqA} be the actual historical level of farm marketings of i-th meat in q-th quarter of year y, and let X_{iA} be the four-element column vector of fm_{iqA}; i.e., the vector of actual quarterly farm marketings. Also, define the row-sum vector σ' = (1,1,1,1). Then define the six quadratic maximization problems for year y:

$$(1)\ \text{Maximize } ECR(B) = (C_B + D_{3P} X_{PA})' X_B + X'_B D_{1B} X_B$$
$$\text{subject to } \sigma' X_B = \sigma' X_{BA}, \quad X_B \geq 0$$

$$(2)\ \text{Maximize } ECR(P) = (C_P + D_{2B} X_{BA})' X_P + X'_P D_{4P} X_P$$
$$\text{subject to } \sigma' X_P = \sigma' X_{PA}, \quad X_P \geq 0$$

$$(3)\ \text{Maximize } ECR(B + P) = C'X + X'DX$$
$$\text{subject to } \sigma' X_B = \sigma' X_{BA}, \quad \sigma' X_P = \sigma' X_{PA}, \quad X_B, X_P \geq 0$$

$$(4)\ \text{Maximize } ECR(B) = (C_B + D_{3P} X_{PA})' X_B + X'_B D_{1B} X_B$$
$$\text{subject to } X_B \geq 0$$

$$(5)\ \text{Maximize } ECR(P) = (C_P + D_{2B} X_{BA})' X_P + X'_P D_{4P} X_P$$
$$\text{subject to } X_P \geq 0$$

$$(6)\ \text{Maximize } ECR(B + P) = C'X + X'DX$$
$$\text{subject to } X \geq 0$$

Economic interpretations of these problems are given in table 11. Problem 1 fixes quarterly and annual levels of hog marketings and annual level of cattle marketings and calls for allocating the fixed annual cattle marketings among the four quarters in such a way that cash receipts from cattle are maximized. Problem 2 is similar to problem 1, but maximizes cash receipts from hogs. Problem 3 fixes annual levels of cattle and hog marketings and determines how to allocate these annual levels among quarters to maximize cash receipts from cattle and hogs. Problem 4 is like problem 1, except that optimal quarterly and annual levels of cattle marketings are to be determined. Problem 5 is like 2, except that quarterly and annual levels of hog marketings are to be determined in problem 5. Problem 6 calls for determining annual and quarterly levels of cattle and hog marketings to maximize cash receipts from cattle and hogs. The solutions to problems 1 and 4 provide values of vector X_B; let this solution vector be X_{BO}. Solutions to problems 2 and 5 provide solution vectors X_{PO} (optimal values of X_P); solutions to problems 3 and 6 provide solution vectors X_O (optimal values of X). The original objective of this research project was to determine levels and quarterly distributions of marketings to maximize net farm income from cattle or hogs or both. As indicated earlier, however, we were not able to develop reliable estimates of quarterly farm-production costs.

Problems 1 through 6 were solved by quadratic programming for the years 1957, 1960, 1967, and 1970. Results are summarized in tables 12 through 14. For comparative purposes, these tables also summarize results from a similar study that covered the years 1950 through 1961 (11). The first two columns in the body of table 12 contain ratios of $100fm_{BqO}/fm_{BqA}$, ratios of solution values of optimal cattle marketings to actual values of cattle marketings.

If U_B is dropped from equation 33 and actual cattle and hog marketings are inserted, we have estimates of actual farm cattle prices, est Y_{BA}

$$\text{est } Y_{BA} = C_B + D_{1B}X_{BA} + D_{3P}X_{PA}$$

If X_{BA} is replaced by optimal marketings (X_{BO}), obtained by solving the quadratic program, we have estimates of optimal farm cattle prices, est Y_{BO}

$$\text{est } Y_{BO} = C_B + D_{1B}X_{BO} + D_{3P}X_{PA}$$

The second pair of columns in the body of table 12 contains ratios of elements of est Y_{BO} to corresponding elements of est Y_{BA}. These entries measure the effect on farm cattle prices of changing cattle marketings from actual levels (X_{BA}) to optimal levels (X_{BO}). Estimated actual cash receipts from cattle are

$$\text{est } CR(B)_A = (\text{est } Y_{BA})\,'X_{BA}$$

Estimated optimal cash receipts from cattle are

$$\text{est } CR(B)_O = (\text{est } Y_{BO})\,'X_{BO}$$

The third pair of columns in table 12 contains ratios of elements of est $CR(B)_O$ to est $CR(B)_A$. These entries measure the effect on cash receipts from cattle of changing cattle marketings from actual levels to optimal levels. If U_P is dropped from equation 34 and actual cattle and hog marketings are inserted, we obtain estimated farm hog prices

$$\text{est } Y_{PA} = C_4 + D_{2B}X_{BA} + D_{4P}X_{PA}$$

Estimated cash receipts from hogs are now

$$\text{est } CR(P)_A = (\text{est } Y_{PA})\,'X_{PA}$$

If X_{BA} is now replaced by X_{BO}, we estimate adjusted farm hog prices (est Y_{Pa}),

$$\text{est } Y_{Pa} = C_P + D_{2B}X_{BO} + D_{4P}X_{PA}$$

Adjusted cash receipts from hogs are computed as

$$\text{est } CR(P)_a = (\text{est } Y_{Pa})\,'X_{PA}$$

The last pair of columns in table 12 contains ratios of elements of est CR $(P)_a$ to corresponding elements of est $CR(P)_A$. These entries show the effects, on cash receipts from hogs, of changing cattle marketings from actual levels to optimal levels.

Referring to the first row in the body of table 12, for example, we see that the average of the

Table 11. Fixed and variable farm marketings in six optimization problems.

Problem	Objective function	Annual cattle marketings	Quarterly cattle marketings	Annual hog marketings	Quarterly hog marketings
1	ECR(B)	F[a]	V[b]	F	F
2	ECR(P)	F	F	F	V
3	ECR(B+P)	F	V	F	V
4	ECR(B)	V	V	F	F
5	ECR(P)	F	F	V	V
6	ECR(B+P)	V	V	V	V

a/ F = Fixed at actual level.

b/ V = Variable; level to be determined by maximization.

Table 12. Means and ranges of ratios of solution values to actual values for problems 1 and 4 in percentages.

Problem	Years	Period	Ratio of optimal to actual cattle marketings		Ratio of optimal to actual farm cattle prices[a]		Ratio of optimal to actual cash receipts from cattle[b]		Ratio of adjusted to actual cash receipts from hogs[c]	
			Mean	Range	Mean	Range	Mean	Range	Mean	Range
1.	1957, '60, '67, '70	Q I	96	93- 99	105	101-109	101	99-101	102	101-103
		Q II	101	97-103	99	97-103	100	99-100	99	98-101
		Q III	102	99-104	98	95-101	100	99-100	99	97-101
		Q IV	101	98-103	99	96-101	100	99-100	100	98-101
		Year	100	F[d]	N.C.[e]	N.C.	100	N.V.[f]	100	N.V.
	1950-61[g]	Q I	102	98-106	96	86-103	98	97-106	98	95-103
		Q II	116	112-120	73	66- 84	84	76-100	87	81- 92
		Q III	84	82- 87	140	118-159	117	100-130	103	101-106
		Q IV	100	96-104	104	95-110	104	98-110	88	82- 95
		Year	100	F			101	100-101	94	91- 96
4.	1957, '60, '67, '70	Q I	88	83- 95	116	105-123	101	100-102	107	103-111
		Q II	92	85- 94	110	101-120	101	100-102	103	100-107
		Q III	93	91- 96	109	105-112	101	101-102	103	102-103
		Q IV	93	89- 95	110	106-115	102	101-103	103	102-105
		Year	91	87- 96	N.C.	N.C.	101	100-102	104	102-107
	1950-61[g]	Q I	68	63- 72	184	158-227	125	115-144	131	118-148
		Q II	79	74- 84	166	143-196	131	119-146	131	118-147
		Q III	69	65- 73	165	149-190	114	108-122	148	126-165
		Q IV	67	65- 70	186	166-203	124	116-132	142	128-154
		Year	71	N.C.	N.C.	N.C.	123	117-132	138	122-150

[a] Optimal beef price is price that would have existed if actual cattle marketings had equalled optimal cattle marketings obtained from solution of quadratic program.

[b] Optimal cash receipts from cattle is cash receipts that would have been received if actual cattle marketings had equalled optimal marketings. These are ratios of estimated optimal to estimated actual cash receipts.

[c] Adjusted cash receipts from hogs is amount of cash receipts that would have been received from sale of hogs if actual cattle marketings had equalled optimal cattle marketings. These are ratios of estimated adjusted to estimated actual cash receipts.

[d] F = fixed. Fixed at 100 percent in statement of quadratic program.

[e] N.C. = not computed.

[f] N.V. = no variation.

[g] Source: George W. Ladd and Harvey Kuang. Optimal beef and pork marketings. J.Farm Econ. 48:209-224. 1966.

four solutions to problem 1 called for reducing farm marketings of cattle by 4 percent in the first calendar quarter. This would have increased first-quarter farm cattle prices by an average of 5 percent and first-quarter cash receipts from cattle by an average of 1 percent. It also would have increased first-quarter cash receipts from hogs by 2 percent, on the average.

Problems 2 and 5 are obtained from problems 1 and 4 by interchanging the roles of cattle and hogs. Table 13, which follows the same format as table 12, summarizes results of analyzing problems 2 and 5.

Table 14 summarizes results from analyses of problems 3 and 6. Ladd and Kuang (11) used classical optimization rather than quadratic programming. Some of their solutions to problems 3 and 6 were not feasible, requiring negative marketings in some quarters. They did estimate a solution to problem 6 by using X_{BO} from problem 4 and X_{PO} from problem 5.

Tables 15 and 16 provide additional information on seasonality of marketings. Perhaps the most striking contrasts in these tables are the differences between actual 1950-61 and optimal 1950-61 patterns of hog marketings and the differences between 1950-61 optimal patterns and 1957, 1960, 1967, 1970 optimal patterns of hog marketings.

In tables 12 through 16, the differences between the earlier results of Ladd and Kuang (11) and the present results are much more pronounced than the similarities. In general, the optimal solutions in their study called for greater changes in seasonal patterns of marketings than do our solutions and called for greater reductions in annual marketings than do our solutions. The solutions in their study also resulted in greater increases in cash receipts than do our solutions.

Table 13. Means and ranges of ratios of solution values to actual values for problems 2 and 5 in percentages.

Problem	Years	Period	Ratio of optimal to actual hog marketings		Ratio of optimal to actual farm hog price[a]		Ratio of optimal to actual cash receipts from hogs[b]		Ratio of adjusted to actual cash receipts from cattle[c]	
			Mean	Range	Mean	Range	Mean	Range	Mean	Range
2	1957, '60, '67, '70	Q I	101	95-111	100	81-113	100	90-109	100	99-100
		Q II	105	102-107	89	84- 92	94	90- 96	100	99-100
		Q III	108	104-111	85	77- 94	92	86- 98	99	99
		Q IV	88	82- 93	136	122-157	120	114-130	100	100-101
		Year	100	F[d]	N.C.[e]	N.C.	102	101-103	100	100
	1950-61[f]	Q I	112	104-122	79	60- 95	88	71- 98	100	100-101
		Q II	64	58- 71	157	117-200	100	83-122	98	95-100
		Q III	87	71-101	182	142-242	157	115-196	109	106-115
		Q IV	128	121-139	87	75-115	110	87-142	107	103-111
		Year	100	F	N.C.	N.C.	113	107-122	104	102-105
5	1957, '60, '67, '70	Q I	79	72- 87	155	123-192	121	107-138	102	101-102
		Q II	79	72- 82	136	125-157	107	102-113	102	101-102
		Q III	81	78- 86	131	117-139	106	101-110	102	102-103
		Q IV	67	64- 70	178	167-202	120	116-129	104	103-104
		Year	76	72- 79	N.C.	N.C.	113	109-118	102	102-103
	1950-61[f]	Q I	61	51- 79	177	122-238	106	96-122	100	99-100
		Q II	47	40- 56	261	169-349	121	95-151	111	106-119
		Q III	66	43- 81	266	187-387	173	115-228	117	112-126
		Q IV	88	83- 94	208	148-305	183	138-260	110	105-115
		Year	68	N.C.	N.C.	N.C.	146	121-178	110	106-113

a/ Optimal farm hog price is price that would have been received if actual hog marketings had equalled optimal hog marketings obtained from solution of quadratic program.

b/ Optimal cash receipts from hogs is cash receipts that would have been received if actual hog marketings had equalled optimal hog marketings. These are ratios of estimated optimal to estimated actual cash receipts.

c/ Adjusted cash receipts from cattle is cash receipts that would have been received from sale of cattle if actual hog marketings had equalled optimal hog marketings. These are ratios of estimated adjusted to estimated actual cash receipts.

d/ F = fixed. Fixed at 100 percent in statement of quadratic program.

e/ N.C. = not computed.

f/ Source: George W. Ladd and Harvey Kuang. Optimal beef and pork marketings. J.Farm Econ. 48:209-224. 1966.

The two sets of solutions to problem 1 are in substantial agreement. Both indicate that, with annual levels of cattle and hog marketings and quarterly levels of hog marketings fixed at historical levels, annual cash receipts from cattle marketings could not be improved by changing the quarterly pattern of cattle marketings. Whereas the Ladd and Kuang (11) solution to problem 4 found that reducing annual cattle marketings by 29 percent would have increased annual cash receipts from cattle by 23 percent (with quarterly and annual levels of hog marketings fixed), our study finds that reducing annual levels of cattle marketings by 9 percent would have little effect on cash receipts, and reducing annual levels of marketings by more than this amount would reduce cash receipts.

The solutions to problem 2 are quite different. Ladd and Kuang (11) found that changing the seasonal pattern of hog marketings while annual marketings of cattle and hogs and quarterly marketings of cattle remained fixed would have increased annual cash receipts from hogs by 13 percent. Our study finds that changing only the quarterly pattern of hog marketings would have little effect on cash receipts from hogs. In the solution to problem 5, Ladd and Kuang found that reducing annual hog marketings by 32 percent would have increased cash receipts by 46 percent. Our study finds that reducing annual levels of hog marketings by 24 percent would increase annual cash receipts by 13 percent.

Our solution to problem 3 is an extension of the solutions to problems 1 and 2 because it shows that changing quarterly patterns of marketings with annual levels fixed would have negligible impact on cash receipts. In their solution to problem 6, Ladd and Kuang (11) found that annual cash receipts from cattle, hogs, and from the two combined could have been increased by 31, 65, and 45 percent, respectively, by reducing annual marketings of cattle and hogs by 29 and 32 percent,

Table 14. Means and ranges of ratios of solution values to actual values for problems 3 and 6 in percentages.

Prob-lem	Years	Period	Ratio of optimal to actual cattle marketings		Ratio of optimal to actual hog marketings		Ratio of optimal to actual farm cattle price[a/]		Ratio of optimal to actual farm hog price[a/]		Ratio of optimal to actual cash receipts from cattle[b/]		Ratio of optimal to actual cash receipts from hogs[b/]		Ratio of optimal to actual cash receipts from cattle and hogs[b/]	
			Mean	Range	Mean	Range	Mean	Range	Mean	Range	Mean	Range	Mean	Range	Mean	Range
3	1957, '60, '67, '70	Q I	97	93- 99	100	95-110	104	101-109	104	84-117	101	100-102	103	92-111	102	97-104
		Q II	102	98-104	104	101-106	98	96-102	89	86- 92	100	99-100	93	91- 96	97	97- 98
		Q III	102	100-105	108	104-111	97	94- 99	84	76- 91	99	98-100	91	85- 96	96	95- 98
		Q IV	99	96-102	89	84- 95	101	99-104	134	120-154	101	100-101	119	114-129	107	106-110
											100	100-101	102	101-103	100	100-101
6	1957, '60, '67, '70	Q I	80	75- 87	77	69- 84	126	116-136	171	137-209	101	100-103	130	115-145	112	105-116
		Q II	85	79- 92	76	69- 80	119	110-130	147	134-168	101	100-103	111	107-115	105	103-106
		Q III	87	85- 90	78	75- 84	119	115-122	141	125-149	103	103-104	110	106-113	105	104-106
		Q IV	85	83- 86	66	63- 69	124	120-128	187	174-212	105	104-106	124	119-134	112	110-113
		Year	84	81- 89	74	70- 78					103	102-104	119	115-122	108	107-109
6	1950-'61[c/]	Q I	68	63- 72	61	51- 79	184	158-226	201	140-287	125	114-143	120	101-147	125	113-145
		Q II	79	74- 84	47	40- 56	178	151-211	284	188-389	140	127-159	132	103-168	139	120-162
		Q III	69	65- 73	66	43- 81	138	162-216	303	204-441	127	118-139	196	134-269	152	126-178
		Q IV	67	65- 70	88	83- 94	198	178-218	240	196-356	132	125-142	211	145-304	165	141-187
		Year	71		68						131	123-141	165	131-204	145	129-159

[a/] Optimal farm price is price that would have existed if actual cattle and hog marketings had equalled optimal cattle and hog marketings obtained from solutions of quadratic programs.

[b/] Optimal cash receipts is cash receipts that would have been received if actual levels of cattle and hog marketings had equalled optimal levels of marketings. These are ratios of estimated optimal to estimated actual cash receipts.

[c/] Source: George W. Ladd and Harvey Kuang. Optimal beef and pork marketings. J. Farm Econ. 48:209-224 1966. These results obtained by assuming optimal marketings in solution to problem 6 equal optimal marketings in solutions to problems 4 and 5.

Table 15. Quarterly marketings of cattle as percentages of annual marketings.

Quarter	1950-61 average	1957, '60, '67, '70 average	Solution 1 1950-'61 average	Solution 1 1957, '60, '67, '70 average	Solution 4 1950-'61 average	Solution 4 1957, '60, '67, '70 average
I	24.1	24.8	24.5	23.8	22.7	23.7
II	24.4	24.6	28.2	24.9	26.6	24.8
III	25.8	25.5	21.6	25.9	24.6	26.0
IV	25.7	25.1	25.7	25.4	26.1	25.5

Table 16. Quarterly marketings of hogs as percentages of annual marketings.

Quarter	1950-61 average	1957, '60, '67, '70 average	Solution 2 1950-'61 average	Solution 2 1957, '60, '67, '70 average	Solution 5 1950-'61 average	Solution 5 1957, '60, '67, '70 average
1	26.6	25.7	29.7	25.9	24.3	26.6
2	22.9	23.8	14.6	25.1	16.1	24.6
3	21.7	22.9	18.8	24.8	21.5	24.4
4	28.8	27.6	36.9	24.2	38.1	24.4

Table 17. Effects of optimal levels of marketings on total cash receipts and total production costs in percentages.

Problem	Years	Reduction in cost of cattle production	Increase in cash receipts from cattle	Reduction in cost of hog production	Increase in cash receipts from hogs
4	1957,'60, '67, '70	9	1	--	4
	1950-61	29	23	--	38
5	1957, '60 '67, '70	--	2	24	13
	1950-61	--	10	32	46
6	1957, '60 '67, '70	16	3	26	19
	1950-61	29	31	32	65

respectively. We find somewhat smaller reductions in marketings to be optimal and find substantially smaller increases in cash receipts.

In summary: Our solutions to problems 1, 2, and 3 indicate that changing quarterly patterns of marketings while leaving total annual levels of marketings unchanged would have negligible impact on annual cash receipts. Solutions to problems 4 and 6 indicate that annual cash receipts from cattle would be increased only slightly by reducing annual levels of cattle marketings. Solutions to problems 5 and 6 indicate that reducing annual levels of hog marketings could substantially increase annual cash receipts from hogs.

Table 17 summarizes the effects of adjusting actual marketings to optimal marketings on total cash receipts. Because we were unable to estimate quarterly variations in farm costs of production of cattle and hogs, we cannot estimate the effects of these adjustments in marketings on net farm income. Table 17 presents rough estimates of the effects of these adjustments on costs of production. Effects on costs of production are estimated by assuming no quarterly variation in average cost and assuming total cost to be proportional to quantity marketed so that a 10-percent decrease in quantity marketed reduces total production cost by 10 percent. Both the Ladd and Kuang (11) and our solutions to problems 5 and 6 indicate substantial room for improving net income from hogs by reducing annual marketings. Both sets of solutions to problems 4 and 6 also indicate room for increasing net income from cattle by reducing cattle marketings.

Problems 1 through 6 also were solved for the years 1972 through 1974 to obtain predicted optima. To solve problems 1 through 5, it was necessary to make some predictions of actual marketings. No such predictions were needed for solving problem 6. Solutions to problem 6 are summarized in table 18.

CHANGES IN QUARTERLY INTERCEPTS

What, aside from sampling or random errors, can explain the differences between the earlier solutions and the present solutions? One likely explanation of the differences is changes in seasonal patterns of behavior of consumers and marketing agents. The equations used by Ladd and Kuang (11) were not saved; hence, it is not possible to compare their set of structural equations with equations 1 through 30. Some inferences, however, can be drawn from studies covering nearly the same sample period as Ladd and Kuang used. Table 19 reproduces coefficients from table 7 and coefficients from previous studies. The other studies cited in table 19 did not use the same set of quarterly dummy variables that we did. Each behavioral equation in the econometric model in this report can be written

$$V_t = \alpha_0 + \alpha_1 d_{1t} + \alpha_2 d_{2t} + \alpha_3 d_{3t} + \text{other terms}$$

From these, the values of the intercepts for the four calendar quarters are obtained as:

$$\begin{aligned} &\text{Q-I} &&\text{intercept} = \alpha_0 + \alpha_1 \\ &\text{Q-II} &&\text{intercept} = \alpha_0 + \alpha_2 \\ &\text{Q-III} &&\text{intercept} = \alpha_0 + \alpha_3 \\ &\text{Q-IV} &&\text{intercept} = \alpha_0 \end{aligned}$$

Table 19 contains the transformed coefficients from other studies and ranks of the quarterly intercepts. For example, the first row shows that, for the beef retail-price equation in our study, the intercept is largest in quarter III, second largest in quarter IV, third largest in quarter II, and smallest in quarter I.

Our study differs from all three of the previous studies cited in its ranking of intercepts for quar-

Table 18. Predicted optimal marketings of cattle and hogs, optimal farm prices and optimal cash receipts, 1972, 1973, 1974 from solution to problem 6.

Period	Cattle marketings (mill. lbs. live wt.)	Hog marketings (mill. lbs. live wt.)	Cattle prices ($/cwt. live)	Hog prices ($/cwt. live)	Cash receipts from cattle (billions of $)	Cash receipts from hogs (billions of $)
1972-I	8,000	3,944	36.07	36.17	2.886	1.427
II	8,371	3,773	35.21	33.76	2.947	1.274
III	8,674	3,791	34.62	32.82	3.003	1.244
IV	8,346	3,836	35.36	40.66	2.916	1.560
1972-Total	33,391	15,344			11.752	5.504
1973-I	8,186	3,960	36.11	36.66	2.956	1.452
II	8,543	3,805	36.07	34.35	3.081	1.307
III	8,848	3,826	35.46	33.44	3.138	1.279
IV	8,518	3,863	35.85	41.21	3.054	1.592
1973-Total	34,095	15,454			12.229	5.630
1974-I	8,366	3,979	37.00	37.23	3.095	1.481
II	8,727	3,832	36.96	35.01	3.225	1.342
III	9,030	3,859	36.32	34.13	3.280	1.317
IV	8,701	3,883	36.71	42.10	3.194	1.635
1974-Total	34,824	15,553			12.795	5.775

Table 19. Quarterly intercepts from various studies.

Dependent variable	Sample period	Study	1	d_{1t}	d_{2t}	d_{3t}	Ranks of quarterly intercepts Q-I	Q-II	Q-III	Q-IV
RP_{BCt}	1954-I to 1970-IV	Present	124	-2.38	-1.80	1.28	4	3	1	2
	1949-I to 1958-III	Fuller[a/]	91	-2.84	-3.78	-0.21	3	4	2	1
	1949-III to 1960-I	Fuller and Ladd[b/]	123	-2.15	-2.28	0.72	3	4	1	2
	1949-I to 1959-IV	Logan and Boles[c/]	106	-4.44	-4.47	-0.09	3	4	2	1
RP_{Pt}	1954-I to 1970-IV	Present	143	-3.78	-5.82	-3.47	3	4	2	1
	1949-I to 1958-III	Fuller[a/]	79	-3.02	-7.46	-6.71	2	4	3	1
	1949-III to 1960-I	Fuller and Ladd[b/]	129	-2.17	-5.99	-7.04	2	3	4	1
	1949-I to 1959-IV	Logan and Boles[c/]	151	-4.97	-7.56	-6.46	2	4	3	1
ΔI_{Bt}	1954-I to 1970-IV	Present	8	-59	-41	-35	4	3	2	1
	1949-I to 1958-III	Fuller[a/]	74	-48	-50	-48	2.5	4	2.5	1
	1949-III to 1960-I	Fuller and Ladd[b/]	69	-40	-33	-50	3	2	4	1
	1949-III to 1960-IV	Ladd[d/]	68	-41	-33	-34	4	2	3	1
ΔI_{Pt}	1954-I to 1970-IV	Present	-76	3	12	-60	2	1	4	3
	1949-I to 1958-III	Fuller[a/]	52	168	-23	-75	1	3	4	2
	1949-III to 1960-IV	Ladd[d/]	-36	177	149	86	1	2	3	4
MR_{BCt}	1954-I to 1970-IV	Present	-1.32	-0.94	-0.35	-0.43	4	2	3	1
	1949-I to 1958-III	Fuller[a/]	-4.16	-0.83	-0.71	-0.50	4	3	2	1
	1949-III to 1960-I	Fuller and Ladd[b/]	0.16	0.34	0.10	0.11	1	3	2	4
MR_{Pt}	1954-I to 1970-IV	Present	2.77	-0.01	-0.51	0.73	3	4	1	2
	1949-I to 1958-III	Fuller[a/]	4.04	-0.59	-0.77	-0 21	3	4	2	1
	1949-III to 1960-I	Fuller and Ladd[b/]	2.98	-0.59	-0.44	1.84	4	3	1	2
MW_{BCt}	1954-I to 1970-IV	Present	2.59	0.76	0.39	0.06	1	2	3	4
	1949-I to 1958-III	Fuller[a/]	0.82	-0.12	-0.17	0.15	3	4	1	2
	1949-I to 1958-III	Fuller[a/]	0.77	-0.04	-0.31	-0.17	2	4	3	1
	1949-III to 1960-I	Fuller and Ladd[b/]	-20	-1.45	-1.30	-0.81	4	3	2	1
MW_{Pt}	1954-I to 1970-IV	Present	2.84	-0.38	-0.79	-0.09	3	4	2	1
	1949-I to 1958-III	Fuller[a/]	2.75	0.75	0.62	1.05	2	3	1	4
	1949-I to 1958-III	Fuller[a/]	2.52	0.60	0.71	0.94	3	2	1	4
	1949-III to 1960-I	Fuller and Ladd[b/]	14.41	-0.40	-0.35	0.16	4	3	1	2

[a/] Wayne A. Fuller. A non-static model of the beef and pork economy. Ph.D. thesis. Iowa State University, Ames. 1959.

[b/] Wayne A. Fuller and George W. Ladd. A dynamic model of the beef and pork economy. J. Farm Econ. 43:797-812. 1961.

[c/] Samuel H. Logan and James N. Boles. Quarterly fluctuations in retail prices of meat. J. Farm Econ. 44: 1050-1060. 1962.

[d/] George W. Ladd. Distributed lag inventory analyses. Iowa Agr. and Home Econ. Exp. Sta. Res. Bul. 515. 1963.

ters I and II in the equation for RP_{BCt} and differs from two of the three studies in its ranking of intercepts for quarters III and IV. Comparison of our study with previous studies indicates that the rank of the intercept for quarter I in the equation for RP_{Pt} has declined and that rank of the intercept for quarter III has risen during recent years. No consistent patterns of change are found in the ranks of intercepts in the equation for ΔI_{Bt}. Equations for ΔI_{Pt} suggest that the rank of the intercept for quarter I has fallen and that the rank of the intercept for quarter II has risen. Equations for MR_{BCt} indicate that the ranks of the intercepts for quarters II and III have changed over the years. No consistent pattern of change is suggested by the ranks of intercepts in the equation for MR_{Pt}. Results for MW_{BCt} suggest that the ranks of the first and second quarter intercepts have risen, whereas the rank of the fourth quarter intercept has fallen. Results for MW_{Pt} indicate that the ranks of the intercepts for quarters II and III have fallen and that the rank of the intercept for quarter IV has risen.

In summary, these comparisons do strongly suggest that changes have occurred in seasonal patterns of behavior of consumers and marketing agents. (One could use F-ratios to test the hypotheses that such changes have occurred. We have not done this.) These changes would affect quarterly intercepts in final-form equations for farm prices and would affect optimal quarterly patterns of marketings. In addition, the earlier study of inventories (10) found no relation between inventory changes and sales of meat, whereas this study does find such a relation. This means that changes in farm marketings have relatively less influence on inventory now than formerly and that meat sales have relatively more influence. This change would affect seasonal behavior of meat inventory demand and would affect demand at the farm level.

Another possible reason for the differences between our results and the results by Ladd and Kuang (11) is a decline in price flexibilities at the farm level. From the 1950-61 solutions to problem 4, the farm-level price flexibilities for cattle for the four quarters were -2.6, -3.1, -2.1, and -2.6; from the more recent solutions, the farm price flexibilities are -1.3, -1.2, -1.3, and -1.4. From the 1950-61 solutions to problem 5, the quarterly farm-level price flexibilities for hogs were -2.0, -3.0, -4.9, and -9.0; from the more recent solutions, the flexibilities are -2.6, -1.7, -1.6, and -2.4. Thus, a 1-percent reduction in farm marketings of cattle or hogs raises farm price of the same commodity by a smaller percentage now than formerly, and a 1-percent increase in farm marketings depresses farm price by a smaller proportion now than it did.

REFERENCES

1. Buttimer, Andreas. Quarterly fluctuations in meat and demand functions. M.S. thesis. Iowa State University Library, Ames, Iowa. 1968.

2. Crom, Richard. A dynamic price-output model of the beef and pork sectors. U.S. Dept. Agr. Tech. Bul. 1426. 1970.

3. Fuller, Wayne A. A non-static model of the beef and pork economy. Ph.D. thesis. Iowa State University, Ames, Iowa. 1959.

4. Fuller, Wayne A., and George W. Ladd. A dynamic quarterly model of the beef and pork economy. J. Farm Econ. 43:797-812. 1961.

5. Gruber, Joseph, and Earl O. Heady. Econometric analysis of the cattle cycle in the United States. Iowa Agr. and Home Econ. Exp. Sta. Res. Bul. 564. 1968.

6. Harlow, Arthur A. Factors affecting the price and supply of hogs. U.S. Dept. Agr. Tech. Bul. 1274. 1962.

7. Hayenga, Marvin L., and Duane Hacklander. Monthly supply-demand relationships for fed cattle and hogs. Amer. J. Agr. Econ. 52:535-544. 1970.

8. Karg, Georg. Quarterly marketing margin equations for beef, pork and lamb. M.S. thesis. Iowa State University Library, Ames, Iowa. 1969.

9. Karg, Georg. Optimal time paths of cattle and hog marketings. Ph.D. thesis. Iowa State University, Ames, Iowa. 1971.

10. Ladd, George W. Distributed lag inventory analyses. Iowa Agr. and Home Econ. Exp. Sta. Res. Bul. 515. 1963.

11. Ladd, George W., and Harvey Kuang. Optimal beef and pork marketings. J. Farm Econ. 48:209-224. 1966.

12. Langemeier, Larry, and Russell G. Thompson. Demand, supply and price relationships for the beef sector, post-World War II period. J. Farm Econ. 49:169-183. 1969.

13. Leuthold, Raymond M. An analysis of daily fluctuations in the hog economy. Amer. J. Agr. Econ. 51:849-865. 1969.

14. Logan, Samuel H., and James N. Boles. Quarterly fluctuations in retail prices of meat. J. Farm Econ. 44:1050-1060. 1962.

15. Manchester, A.C. Prices, costs and marketing margins for fresh fruits and vegetables. J. Farm Econ. 44:888-894. 1962.

16. Myers, L.H., Joseph Havlicek, Jr., and P.L. Henderson. Short-term price structure of the hog-pork sector of the United States. Indiana [Purdue Univ.] Agr. Exp. Sta. Res. Bul. 855. 1970.

17. Stanton, B.F. Seasonal demand for beef, pork and broilers. Agr. Econ. Res. 13:1-14. 1961.

18. Theil, H. Economic forecasts and policy. North-Holland Publishing Co., Amsterdam, The Netherlands. 1965.

19. Trierweiler, John E., and James B. Hassler. Orderly production and marketing in the beef-pork sector. Nebr. Agr. Exp. Sta. Res. Bul. 240. 1970.

20. Zellner, A. An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias. J. Amer. Stat. Assoc. 57:348-368. 1962.

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