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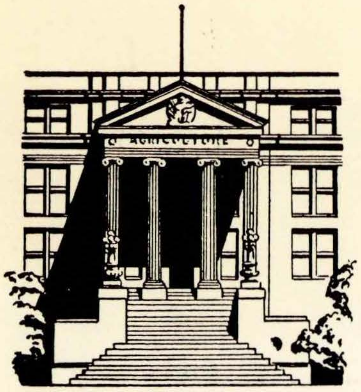

ANALYSIS OF CROP-ROTATION EXPERIMENTS, WITH APPLICATION TO THE IOWA CARRINGTON-CLYDE ROTATION-FERTILITY EXPERIMENTS

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CONTENTS

SUMMARY

This report presents a model by which several crop rotations are compared, and optimal fertilization and rotation practices determined. The model is developed with specific applicability to the rotation-fertility experiments at the Carrington-Clyde Experimental Farm near Independence, Iowa. The substitutability of legume meadow and chemical nitrogen fertilizer and the effect of carry-over of applied nitrogen from crop to crop are incorporated into the analysis. The split-plot nature of the rotation-fertility trials is noted, and a transformation of the yield data is employed to create nearly uncorrelated observations. Response functions are estimated for each crop in each rotation. Optimal fertilizer rates and rotations are determined on the basis of average annual return. Variance of return arising from yield variability over years is estimated. Continuous corn yielded the largest net income for the prices considered in the study. The net income per acre decreased with the introduction of oats and an increasing number of years of meadow. Variability of annual net return, however, was largest with continuous corn and decreased as the number of years of meadow in the rotations increased.

Analysis of Crop-Rotation Experiments, With Application to the Iowa Carrington-Clyde Rotation-Fertility Experiments[1]

by G.E. Battese, W.A. Fuller, and W.D. Shrader[2]

The practice of rotating the crops grown on a particular piece of land is rooted in antiquity. The principal reasons for rotation cropping have been control of weeds, insects, diseases, and soil erosion, and the improvement of soil structure and fertility.

In determining the optimal rotation, questions requiring consideration are: Which crops should be used in the rotation? (In this report, continuous cropping is considered a "rotation.") What should be the cropping sequence? How much fertilizer should be applied to each crop in the rotation sequence? These questions can be answered satisfactorily only after carrying out appropriate analyses of rotation experiments conducted over several years. Practical considerations limit one to comparing only a few alternative rotations.

LITERATURE REVIEW

Agronomists have been conducting long-term rotation experiments for many years. The early experiments generally involved the investigation of the effects of treatments on the crops of a single rotation. For example, such experiments were conducted at the Rothamsted Experimental Station in the mid-nineteenth century. Cochran (1939) presents a numerical example from a Woburn rotation experiment for 1886-1897. Scientifically designed crop-rotation experiments were started after the development of experimental-design techniques by Fisher and his associates in the 1920's.

The literature on the analysis of rotation experiments began to appear in the 1930's. Cochran (1939) outlined the statistical principles governing the design of some important types of long-term agricultural experiments and suggested approaches useful in analyzing the experimental data. Crowther and Cochran (1942) discussed the analysis of two experiments carried out in the Sudan. These experiments were probably the first of modern design to compare a number of rotations. Yates (1949) discussed in detail the design of rotation experiments. Both Cochran (1939) and Yates (1949) stressed the importance of having observations for all phases of the rotations in each year. Yates (1952) suggested how some useful results could be salvaged from a poorly designed Brazilian experiment comparing continuous corn, continuous cotton, and the 2-year rotation of corn and cotton. Yates (1954) outlined much of the terminology used to describe experiments involving a comparison of several rotations. He suggested analysis-of-variance procedures for investigating sources of yield variation in the presence of various error structures.

Patterson (1953, 1959) analyzed fixed-rotation types of experiments. He investigated particular treatment effects by using mean yields over years and the regression of yields on time. The main statistical problem receiving attention was the estimation of experimental error in the presence of correlations arising from common-plot effects. Stevens (1956) considered the design and proposed analysis for Brazilian experiments suggested after Yates' visit to Brazil in 1952. Patterson (1964) reviewed some statistical problems arising in the design and analysis of crop-rotation experiments comparing different rotations. He considered in some detail the error structure of such experiments and suggested procedures for handling the correlated errors that may result from observing the same plots year after year.

Fuller and Cady (1965) studied the corn yields from a rotation experiment in Iowa and investigated the appropriateness of an exponential model to estimate limiting yields and rates of approach to the limiting yields. A transformation of the yields was employed to reduce correlations between yields from the same rotation. Shrader, Fuller, and Cady (1966) tested (and accepted) the hypothesis that corn yields from different rotations could be fitted

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to one common function relating yield to nitrogen. Shih (1966) outlined the nature of rotation experiments and presented a brief review of the literature. Employing the Fuller and Cady (1965) and Shrader, Fuller, and Cady (1966) techniques, Shih analyzed certain corn data from rotation experiments in Iowa. Patterson and Lowe (1970) considered the effect of positive serial-plot correlations in investigating yields from the same plot over a period of years.

The literature on the economic analysis of crop-rotation experiments is limited. The classic marginal tools that may be used in selecting the optimal rotation are given by Heady (1952), who also discussed the several dimensions of rotation cropping. Heady, McAlexander, and Shrader (1956) used a linear-programming approach to examine the effects of different levels of labor, amounts of operating capital, soil types, and other relevant factors in determining optimal rotation and fertilizer schemes. Since that time, the inclusion of several rotation and fertilizer possibilities in linear-programming models for the farm business has been a common practice, but model formulation and estimation specific to the rotation problem are less common. Abraham and Agarwal (1967) and Agarwal (1968) employed the statistical principles outlined by Patterson (1953) and Yates (1954) to analyze rotation experiments in India. Consideration was given to the economics of crop rotations, although the response-function approach was not used to determine economic optima.

ROTATION MODEL

We present the crop-rotation model with reference to the Carrington-Clyde rotation experiment near Independence in northeastern Iowa. A somewhat more general presentation is found in Battese and Fuller (1972).

Description of the Carrington-Clyde Experiment

The rotation experiment was initiated in 1952 and was primarily designed to investigate the long-term performance of corn when grown in rotations with or without oats and meadow. The crop rotations were continuous corn, CCO, CCOM, CCOMM, and CCOMMM. In the rotation sequence, corn is denoted by C, oats by O, and meadow by M. Thus, CCOMM denotes the 5-year rotation with 2 years of corn, followed by 1 year of oats, followed by 2 years of meadow.

Nitrogen was applied to the corn at three different levels. In terms of actual elemental nitrogen, the rates of application (in pounds per acre) were 40, 80, and 160 for continuous corn; 0, 20, and 40 for first-year corn after meadow; and 0, 40, and 80 for first-year corn after oats and for second-year corn. In actual elemental nutrients, all corn plots annually received 13 pounds per acre of P and 25 pounds per acre of K. No nitrogen was applied to oats and meadow. P and K were applied to oats and meadow at the following rates per acre: 22 pounds of P and 42 pounds of K to oats in CCO, 35 pounds of P and 67 pounds of K to oats in the meadow rotations, 13 pounds of P and 25 pounds of K to second-year meadow in CCOMM, 26 pounds of P and 50 pounds of K to second-year meadow in CCOMMM.

The crop-rotation experiments were of a split-plot type, with whole-plots in a randomized-complete-block design. The number of whole-plots per block was 19, the sum of the number of rotation-positions for each crop in the experiments. The term *rotation-position* is used with reference to a *particular crop,* and it distinguishes the position (phase) in which the crop is grown in the different crop rotations. For example, for corn in the rotation CCOM, we say that first-year corn, denoted by *C*COM, is a different rotation-position from second-year corn, denoted by C*C*OM. Thus, for the 5 rotations considered here, there are 9 rotation-positions for corn, 4 for oats, and 6 for meadow. The experiment was initiated such that each rotation-position occurred on a whole-plot. For each whole-plot, there were 3 subplots, and the subplot treatments were different levels of applied nitrogen. There were two replications of the design.

Response-Function Models

Central to the analysis of the experimental data is the estimation of yield functions in terms of levels of nitrogen applied to the crops in the rotations. Response functions are estimated for each of the crops in the experiment.

In the presentation of the response functions for each of the crops in the rotations, the crop yields are denoted by the letter y followed by 4 subscripts. The first subscript denotes the crop involved. That is, c refers to corn, o, to oats, and h, to hay. The second subscript i denotes the year in which the yield is observed. The third subscript j denotes the rotation-position from which the yield is obtained for the given crop. The fourth subscript k distinguishes the level of nitrogen applied to the given plot. The yields for each crop are assumed to depend on the year in which they are observed, the rotation-position from which they come, and the level of nitrogen fertilization in current and past years. Some of the parameters of the response functions are permitted to display year-to-year variation.

The response functions used are defined by equations 1, 2, and 3.

For corn

$$y_{cijk} = \alpha_{ci} + \gamma_{cij} \exp(\beta_c N_{ijk}) + u_{cijk}, \quad j=1, 2, ..., 9 \qquad [1]$$

for oats

$$y_{oijk} = \alpha_{oi} + \gamma_{oi} (\beta_{oj} - N_{ijk})^2 + u_{oijk}, \quad j=1, 2, 3, 4 \qquad [2]$$

for hay

$$y_{hijk} = \alpha_{hip} + \gamma_{hi} N_{ijk} + u_{hijk}, \quad j=1, 2, ..., 6; p=1, 2, 3 \qquad [3]$$

where y_{mijk} is the average yield over replications for the k-th applied nitrogen level (k=1, 2, 3) of the j-th rotation-position for the m-th crop (m=c, o, h) in the i-th year; N_{ijk} is the quantity of fertilizer nitrogen from present and previous applications available at the j-th rotation-position (of the crop involved) in year i at the k-th level of application; u_{mijk} is the random effect associated with the yield y_{mijk}.

For the corn-response function (equation 1), α_{ci} is the asymptote for corn yield in year i, and γ_{cij} is the rotation-position parameter. For $\gamma_{cij} < 0$ for all i and j, the rotation-position parameter determines the amount by which the corn yield from rotation-position j falls short of the asymptotic yield in year i for the given level of nitrogen. Shrader, Fuller, and Cady (1966) formulated the corn-response function (equation 1) in investigating the hypothesis that fertilizer nitrogen could be considered a substitute for meadow in growing corn in the Midwest. They showed that the exponential corn-response function with exponential parameter constant over years adequately fitted corn yields from different rotations at two Iowa experimental sites.

In the quadratic oat-response function (equation 2), the parameter β_{oj} represents the level of total nitrogen (applied plus carry-over) required for oat rotation-position j to obtain the maximum oat yield, α_{oi}, in year i. The quadratic oat function is suggested because of the incidence of lodging (and, hence, lower harvested yields) of oats from plots with a high level of carry-over nitrogen available.

The parameter α_{hip} in (equation 3) denotes the hay yield obtained in the i-th year if zero nitrogen is available as carry-over from applications to the preceding crops and if the meadow is in its p-th year, p=1, 2, 3. For example, p=2 for CCOM*M* and CCOM*M*M. It was hypothesized that, if other things were equal, yield differences in hay would be due to the number of years the meadow was down and to the different nitrogen applications to the corn preceding the meadow in the rotations. Thus, we consider the hay-response function(equation 3), which specifies different intercepts for first-, second-, and third-year meadow, but a common coefficient for the available carry-over nitrogen.

Nitrogen Carry-Over Model

To estimate the effect of current applications of nitrogen on the yield of crops in subsequent years, a nitrogen carry-over function is required. In practice, the carry-over is complicated, the level depending on fertilizer applications in previous years, yield of previous crops, weather, and soil conditions. In considering an explicit algebraic form for carry-over, it is hypothesized that the proportion of current nitrogen carried over to the next year increases at a decreasing rate, ultimately reaching a maximum, as the current nitrogen level increases. Expressing nitrogen in year i as the sum of the application in year i and the carry-over from previous applications, $N_i = N^a_i + N^c_i$, the carry-over model considered is

$$N^c_i = N_{i-1} [\phi_0 - \phi_1 \exp(-\phi_2 N_{i-1})] \qquad [4]$$

where ϕ_0, ϕ_1, and ϕ_2 are parameters to be determined.

N^c_i / N_{i-1} asymptotically approaches ϕ_0 from below, given that ϕ_i, i=0, 1, 2, are positive numbers.

Model for Response-Function Errors

The choice of an efficient method for estimation of the parameters in the response functions in equations 1, 2, and 3 depends upon the distributional properties of the random errors, u_{mijk}. It is assumed that these errors are the sum of two random components, one associated with the whole-plot, and the other associated with the subplot on which the yields occur. Thus, the errors are expressed

$$u_{mijk} = w_{mij} + s_{mijk}, \quad m = c, o, h \qquad [5]$$

where w_{mij} is the random (whole-plot) effect for the j-th rotation-position for the crop m in the year i and s_{mijk} is the random (subplot) effect for nitrogen level k of the j-th rotation-position for crop m in year i.

Since the response functions (equations 1, 2, and 3) are defined for the average over replicates, w_{mij} and s_{mijk} also are averages over replicates. We assume that w_{mij} and s_{mijk} are independently distributed as normal random variables with zero means and variances $\sigma_{mw}{}^2$ and $\sigma_{ms}{}^2$, respectively. Thus, the errors u_{mijk} have the covariance properties given in equation 6.

$$\mathrm{Cov}(u_{mijk}, u_{mpqr}) = \begin{cases} \sigma_{mw}{}^2 + \sigma_{ms}{}^2, & \text{if } i=p,\ j=q,\ k=r \\ \sigma_{mw}{}^2 & \text{if } i=p,\ j=q,\ k\neq r \\ 0 & \text{, otherwise} \end{cases} \qquad [6]$$

Note that there is no yearly component in the variability of the error terms. Yearly variation enters the response functions through the parameters subscripted with the index i. The goodness-of-fit tests for the estimated response functions support the claim that the response functions in equations 1, 2, and 3 adequately account for year effects.

The error model given by equations 5 and 6 assumes that there is no correlation between the errors from year to year. This assumption of zero plot effects was investigated in a related earlier study by Fuller and Cady (1965). This study considered the error for the r-th plot of nitrogen level k on corn rotation-position j in year i, u_{cijkr}, to be the sum of two error components, one a plot component and, the other, a plot-within-year component.

$$u_{cijkr} = v_{cjkr} + e_{cijkr} \qquad [7]$$

where v_{cjkr} is the random plot effect for the r-th plot of nitrogen level k on corn rotation-position j and e_{cijkr} is the random plot-by-year effect for the r-th plot of nitrogen level k on corn rotation-position j in year i. The errors v_{cjkr} and e_{cijkr} were assumed distributed independently, with variances $\sigma_{cv}{}^2$ and $\sigma_{ce}{}^2$, respectively.

An analysis was conducted on the whole-plots to estimate the relative magnitude of the two components of variance. Because the experiment studied contained two replicates, it was possible to construct simple analyses of variance for the corn yields from each pair of plots receiving the same treatments. The data studied in Fuller and Cady (1965) were the 1952 through 1962 data on the Carrington-Clyde rotations, CCO, CCOM, CCOMM, and CCOMMM. Thus, for example, the rotation CCOMMM had a pair of whole-plots with first-year corn in 1952 followed by second-year corn in 1953, first-year corn in 1958, and second-year corn in 1959. The analysis of variance for these data is presented in table 1.

Table 1. An analysis of variance on a pair of plots observed in 4 years

Source	df	E(M.S.)
Years	3	
Plots	1	$\sigma_{ce}{}^2 + 2\sigma_{cv}{}^2 + 2K_b{}^2$
Plots by years	3	$\sigma_{ce}{}^2$

In the expected mean square for plots in table 1, K_b denotes the difference between the two replicates. The difference K_b was estimated by the average differences for the period of the experiment.

Since the experiment had observations on each rotation-position in each year, there were 18 pairs of whole-plots for the 4 rotations (3 for CCO, 4 for CCOM, 5 for CCOMM, and 6 for CCOMMM). Pooling the sums of squares for the 18 analyses of variance, and adjusting for replicates, Fuller and Cady (1965) obtained the analysis of variance given in table 2, in which the degrees of freedom for plots are 17 because 1 degree of freedom is used in the adjustment for replicate effects.

Table 2. The Fuller and Cady[a] (Table 3) plot analysis of variance

Source	df	M.S.
Plots	17	146
Plots by years	70	161

[a] From: W.A. Fuller and F.B. Cady. 1965. Estimation of asymptotic rotation and nitrogen effects. Agron. J. 57:299-302.

From this table, a negative estimate for the variance component associated with plots, $\sigma_{cv}{}^2$, is obtained. Therefore, the plot effect is taken to be zero for the analysis in this report. Battese and Fuller (1972) present an error model with both whole-plot and subplot effects. With plot effects, the yearly response functions are estimated simultaneously. If there are no plot effects, however, considerable simplification occurs because the response functions can be estimated year by year.

Given the error structure (equations 5 and 6) for our crop-yield data, a split-plot analysis of variance could be obtained for each crop in the experiment. For example, the corn data for 1958-65 gives the split-plot analysis of variance in table 3.

Note that the expected mean squares for whole-plot and subplot error have a multiple of 2, the number of replications, since the variance components $\sigma_{mw}{}^2$ and $\sigma_{ms}{}^2$ are defined in equation 5 for

Table 3. Analysis of variance for corn yields, Carrigton-Clyde Experiment, 1958-65

Source	df	E(M.S.)
Replications (R)	1	
Rotation-positions (P)	8	
Years (Y)	7	
PxY	56	
Whole-plot error[a]	71	$2(\sigma_{cs}^2 + 3\sigma_{cw}^2)$
Nitrogen (N)	2	
NxP	16	
NxY	14	
NxPxY	112	
Subplot error[b]	144	$2\,\sigma_{cs}^2$

[a]Whole-plot error is the sum of the interaction effects RxP, RxY, and RxPxY.

[b]Subplot error is the sum of the interaction effects RxN, RxNxP, RxNxY, RxNxPxY.

averages over replicates. The split-plot analyses of variance on the crop yields thus furnish unbiased estimators for the variance components involved in the response-function models.

Transformation for Obtaining Uncorrelated Yield Data

The presence of the positive covariance, σ_{mw}^2, between crop yields from the same whole-plot, but different subplots, (equation 6) suggests that estimation of the parameters in the response functions should employ generalized least-squares. After performing a relatively simple transformation to obtain uncorrelated errors, the response-function parameters are estimated by ordinary least squares. The transformation involved was suggested by Fuller in 1965 and presented by Shih (1966, pp. 94-96).

Given the error structure specified by equation 6, the transformed errors, ϵ_{mijk}, where

$$\epsilon_{mijk} = u_{mijk} - T_m \sum_{k=1}^{3} u_{mijk}/3 \quad [8]$$

and

$$T_m = 1 - [\sigma_{ms}^2/(\sigma_{ms}^2 + 3\,\sigma_{mw}^2)]^{1/2}, \quad [9]$$

are uncorrelated with variance σ_{ms}^2. Proof of this result is presented in the theorem in the appendix. For the error model with whole-plot and subplot effects, a similar transformation giving uncorrelated errors is presented in Battese and Fuller (1972).

Table 3 shows that, for the m-th crop, m = c, o, h, the expected value of the error mean square of the subplot is $2\,\sigma_{ms}^2$, and, for the whole-plot, is $2(\sigma_{ms}^2 + 3\,\sigma_{mw}^2)$. The transformation factor T_m, defined in equation 9, is estimated by

$$\text{est. } (T_m) = 1 - (\text{Error } B_m/\text{Error } A_m)^{1/2} \quad [10]$$

where Error A_m is the whole-plot error mean square and Error B_m is the subplot error mean square, as computed from the split-plot analysis of variance for the m-th crop in the rotations, m = c, o, and h.

Using the estimates for the transformation factors to transform the variables in the respective response functions in equations 1, 2, and 3 and applying ordinary least squares to the transformed data is asymptotically as efficient as applying generalized least squares (with the variance components known) for estimation of the parameters in the response functions. A more detailed discussion of the transformation and an alternative method of estimation of the variance components involved are found in Fuller and Battese (1972).

Net-Return Functions

In the analysis of the Carrington-Clyde experiments, there are 3 applied nitrogen variables for each rotation, excluding continuous corn; namely, the nitrogen applied to first-year corn, second-year corn, and oats. For convenience of notation, these levels of applied nitrogen are denoted N_1, N_2, and N_3, respectively, in the net-return functions. Obviously, there is only one nitrogen application for continuous corn. Although the oats in the rotations did not receive nitrogen fertilizer in the experiment, nitrogen applied to the oats is permitted in the determination of economic optima.

Net return for a complete cycle of a crop rotation is given as a function of the applied nitrogen levels and is the sum of returns over crops and positions within a rotation, less the variable and fixed costs. For the r-th crop rotation (r = 1, 2, ..., 5), the net-return function is of the form

$$\pi_r(N) = \sum_m \sum_j y_{mj(r)}(N)\, p_m - (\sum_{k=1}^{3} N_k) p_N - F_r \quad [11]$$

where $y_{mj(r)}(N)$ is the average (over-years) response function for the m-th crop at the j-th rotation-position within rotation r and is a function of all nitrogen applications because of carry-over, p_m is the net price per unit of the m-th crop, p_N is the price per unit of elemental nitrogen, and F_r is the fixed cost per cycle for the r-th crop rotation. The price p_m is the market price per unit, less the flat, per-unit harvesting and handling costs. In general, F_r includes all fixed costs relevant for comparing the r - th crop rotation with the other 4 rotations.

In the net - return function (equation 11), the summations over m and j are different for different crop - rotations. For example, for the rotation CCO, the sum of the response functions for *C*CO and C*C*O represents the summation over j for m= c, but for m=o, j assumes only one value and represents the response function for CC*O*.

No attempt is made to introduce discounting into the net - return function (equation 11) since the method of initiating a rotation is arbitrary.

The yield functions $y_{mj(r)}(N)$ are given as the average over years of the response functions in equations 1, 2, and 3. These yields are functions of total applied nitrogen available to the given crop in the particular rotation - position for the crop rotation involved. Since carry - over is itself a function of applied nitrogen, by repeated substitution, the yields are expressed as functions of N_1, N_2, and N_3. In this report, we define the optimal applied nitrogen levels for the r - th crop rotation as those that maximize the net - return function, and the optimal crop rotation is defined as the rotation that gives the maximum net return per year for the optimal applied nitrogen levels.

Model for Yearly Variability in Response-Function Parameters

In the response - function models (equations 1, 2, and 3), some parameters are permitted to vary from year to year. Such parameters are denoted by the symbols α and γ, including an i subscript. The estimates for these yearly parameters are expressed as the sum of an average - year effect, a random error associated with the given year, and a random error associated with estimation of the true value of the yearly parameter. This is expressed in equation 12

$$\begin{aligned} \text{est.}(\theta_{mij}) &= \theta_{mij} + d_{mij} \\ &= (\theta_{m.j} + \lambda_{mij}) + d_{mij} \end{aligned} \quad [12]$$

where θ_{mij} represents one of the response - function parameters, which are permitted to vary over years (Note: For fixed m and j, θ_{mij} is assumed random.); d_{mij} is the random error associated with estimation of θ_{mij}; $\theta_{m.j}$ denotes the expectation (over years) of θ_{mij}; and λ_{mij} is the random error associated with yearly variation in the parameter θ_{mij}.

The errors d_{mij} are assumed distributed independently of the errors λ_{mij}. Further, the covariance matrices for λ_{mij}, m = c, o, h, are assumed constant over years. Estimates for the covariance matrices for λ_{mij}, m = c, o, h, are sought to estimate the variance of the return function associated with the net return for each of the crop rotations (see equation 11).

For the response functions in equations 1, 2, and 3, the covariance matrix for the λ_{cij} is of order 10, for λ_{oij} is of order 2, and for λ_{hij} is of order 4. That is, for m=c, the parameters permitted to vary with years are α_{ci} and γ_{cij}, j=1, 2, ..., 9; for m=o, the yearly parameters are α_{oi} and γ_{oi}; and for m=h, the yearly parameters are α_{hip}, p=1, 2, 3, and γ_{hi}. The covariance matrix for the yearly variation in the parameters for the m - th response function is estimated by equation 13

$$\text{est.}\Sigma(\lambda_m) = \text{est.}\Sigma(\text{est.}\theta_m) - \text{est.}\Sigma(d_m) \quad [13]$$

where $\text{est.}\Sigma(\lambda_m)$ denotes the estimated covariance matrix for the λ_{mij} parameters in each year; $\text{est.}\Sigma(\text{est.}\theta_m)$ denotes the sum of squares and cross products for the estimated yearly parameters, in the m - th response function, divided by the number of years less one; and $\text{est.}\Sigma(d_m)$ denotes the estimated covariance matrix for the sampling errors involved in estimation of the θ_{mij} parameters in the m - th response function.

For example, if the parameter α_{ci} for the corn-response function (equation 1) is listed first in the θ_c vector, the first diagonal element of the matrix $\text{est.}\Sigma(\lambda_c)$ is

$$\text{est.Var}(\lambda_{ci1}) = \sum_{i=1}^{T} [\text{est.}(\alpha_{ci}) - \text{est.}(\alpha_{c.})]^2/(T-1) - \text{est.Var}(d_{ci1}) \quad [14]$$

where T is the number of yearly observations on α_{ci} and $\text{est.}(\alpha_{c.})$ is the average of the T yearly estimates on α_{ci}.

The estimate for the variance of the sampling variability in the estimates for α_{ci}, denoted $\text{est.Var}(d_{ci1})$, is obtained from the regression analysis involved in estimating α_{ci}. In equation 14, the mean square for the observed variation in the estimates for α_{ci} denoted

$$\sum_{i=1}^{T} [\text{est.}(\alpha_{ci}) - \text{est.}(\alpha_{c.})]^2/(T-1)$$

estimates the sum of the variance arising from plot - to - plot variability and the yearly variance in response. Subtracting from the computed mean square, an unbiased estimator for the sampling variance, an unbiased estimator for the yearly variance in response is obtained in equation 14.

Given the covariance matrices, $\text{est.}\Sigma(\lambda_m)$, m=c, o, h, the variance of the net-return function (equation 11) is estimated as a function of the applied nitrogen levels for each crop rotation. We estimate the standard deviation of the yearly return for the optimal applied nitrogen rates for each crop rotation. Given the net-return functions and the associated functions of variance of yearly return, however, alternative criteria for optimization are: maximize net return subject to a constraint for variance of yearly return, or minimize variance of yearly return subject to a net-return constraint.

Although est.Σ(est.θ_m) and est.$\Sigma(d_m)$ are estimated covariance matrices with the property of positive-definiteness, the matrix est.$\Sigma(\lambda_m)$, obtained by equation 13, does not necessarily possess this property. If the matrix est.$\Sigma(\lambda_m)$ is not positive-definite, one of two alternatives could be followed.

(a) If prior information is available on the nature of the variability of the parameters, the matrix could be modified to ensure positive-definiteness. An example of such a modification is to assume that the correlations are of the form

$$\rho(\theta_{mir}, \theta_{mis}) = \rho_m, \text{ for r not equal to s.}$$

The problem of estimating a positive-definite matrix then reduces to the estimation of a single parameter ρ_m subject to the restriction $-1/(v_m-1) < \rho_m < 1$, where v_m is the dimension of the covariance matrix $\Sigma(\lambda_m)$.

(b) The matrices, est.$\Sigma(\lambda_m)$, m = c, o, h, could be used directly in forming the variance-of-return functions, subject to the restriction that negative estimates for the variance of return be set equal to zero.

The procedure adopted depends on the information available and the optimization criterion used.

ESTIMATION AND EMPIRICAL ANALYSIS

Although the Carrington-Clyde experiment was started in 1952, only data from 1958-65 are considered in this study. The longest rotation, CCOMMM, required from 1952 until 1957 to complete one cycle, and we suggest that data be included in an analysis only after every rotation has completed at least one full cycle.

The significance of the different factors, which contribute to yield variability, can be investigated from the split-plot analyses of variance for each crop. Table 4 presents the mean squares and degrees of freedom for the different sources of variation for each of the 3 crops. Comparing the whole-plot mean square with the mean squares for rotation-positions and years suggests that significantly different yields are obtained over years and also from different rotation-positions. The level of applied nitrogen also contributes to significantly different crop yields. The latter result is evident from a comparison of the nitrogen-effect mean squares with the split-plot error mean square.

The analyses of variance in table 4 provide the data required to estimate the transformation factors by equation 10. For corn, the transformation factor is estimated by est.$(T_c) = 1 - [76.21/191.08]^{1/2} = 0.37$, where 76.21 is the split-plot error mean square and 191.08 is the whole-plot error mean square, from the analysis of variance for corn.

Similarly, the transformation factor for oats is estimated by 0.52, and, for hay, by 0.58. These estimates are used in equation 8 to transform the variables in the response functions. Estimates for the parameters in the response functions are obtained by using these transformed variables.

Table 4. Analyses of variance for yields of corn, oats, and hay for 1958-65

Source of variation	Corn (bu/acre) df	Corn (bu/acre) M.S.	Oats (bu/acre) df	Oats (bu/acre) M.S.	Hay (tons/acre) df	Hay (tons/acre) M.S.
Replications	1	61.36	1	43.32	1	0.1152
Rotation-positions (R)	7[a]	5,509.78	3	6,256.47	5	1.7966
Years (Y)	7	9,351.38	7	6,726.54	7	10.5576
RxY	49	296.12	21	640.88	35	0.5756
Whole-plot error	63	191.08	31	257.00	47	0.4134
Nitrogen (N)	2	16,610.14	2	2,679.24	2	0.2438
NxR	14	2,164.70	6	85.30	10	0.1110
NxY	14	381.40	14	160.81	14	0.1513
NxRxY	98	103.95	42	69.89	70	0.0703
Split-plot error	128	76.21	64	58.94	96	0.0723

[a] Continuous-corn data were not included in the analysis of variance.

Nitrogen Carry-Over Functions

In equations 1, 2, and 3, the independent variables in the response functions are functions of the total nitrogen from fertilizer. A suitable nitrogen carry-over function is thus required to estimate the total nitrogen from applied sources before estimation of the parameters in the response functions is undertaken. Because no data on carry-over nitrogen were available from the Carrington-Clyde experiment, the carry-over function

$$N^c_i = N_{i-1}\,[0.325 - 0.25\ \exp.(-0.81\ N_{i-1})] \quad [15]$$

was used in a preliminary analysis. This function is presented in Fuller (1965, p. 108) and was obtained from data on nitrogen carry-over provided by John Pesek of the Department of Agronomy at Iowa State University.

In equation 15 and in all subsequent functions involving nitrogen, measurements are in terms of 40-pound units of elemental nitrogen.

Preliminary response-function estimation suggested that the carry-over function in equation 15 was not satisfactory for all rotations and that it specified too much carry-over from low fertilizer rates on the rotations without meadow. Hence, a second carry-over function, with less carry-over than that in equation 15, was obtained for rotations without meadow.

The oat yields were used to estimate a carry-over function of the same form as equation 4. The oat crops in the rotation experiments did not receive applied nitrogen. To estimate the oat-response function (equation 2), we thus consider nitrogen carry-over only. Averaging over years, the average oat-response function is denoted

$$y_{o.jk} = \alpha_{o.} + \gamma_{o.}\,(\beta_{oj} - N_{.jk})^2 + u_{o.jk} \quad [16]$$

The parameters $\alpha_{o.}$, $\gamma_{o.}$, and the β_{oj} corresponding to the rotation-positions CC*O*M, CC*O*MM, and CC*O*MMM were estimated by the Gauss-Newton procedure [e.g., see Hartley (1961)], using the average (over-years) oat yields for the 3 oat rotations with meadow. The estimated carry-over nitrogen, $N_{.jk}$ entering equation 16, was obtained from equation 15. The variables in equation 16 were transformed by using est. (T_o) = 0.52 and equation 8.

The parameter β_{o1}, corresponding to the rotation-position CC*O*, was estimated by substituting the Gauss-Newton estimates for $\alpha_{o.}$ and $\gamma_{o.}$ into equation 17

$$y_{o.11} = \text{est.}(\alpha_{o.}) + \text{est.}(\gamma_{o.})(\beta_{o1})^2 \quad [17]$$

where $y_{o.11}$ is the experimental average yield (over years and replicates) for CC*O* with zero nitrogen on the preceding corn crops, and est.$(\alpha_{o.})$ and est.$(\gamma_{o.})$ are the estimates obtained from equation 16 and the oat data from the meadow rotations.

Estimates for nitrogen carry-over to CC*O* were obtained from the plots receiving applied nitrogen by solving equation 18 for $N_{.1k}$, k = 2, 3,

$$y_{o.1k} = \text{est.}(\alpha_{o.}) + \text{est.}(\gamma_{o.})[\text{est.}(\beta_{o1}) - N_{.1k}]^2 \quad [18]$$

where $y_{o.1k}$ denotes the experimental average yield (over years and replicates) for CC*O* with (k-1) 40-pound units of nitrogen applied to the preceding corn crops. The two estimated nitrogen carry-over quantities obtained from equation 18 were used to estimate the carry-over function for the rotations without meadow. The function

$$N^c_i = 0.325\ N_{i-1}\ [1 - \exp.(-0.25\ N_{i-1})] \quad [19]$$

gave a satisfactory graphic fit to the data points.

Thus, equation 15 was used to estimate carry-over for rotations with meadow, and equation 19 was used to estimate carry-over for rotations without meadow.

Oat-Response Functions

In the oat-response function (equation 2)

$$y_{oijk} = \alpha_{oi} + \gamma_{oi}(\beta_{oj} - N_{ijk})^2 + u_{oijk}$$

the rotation-position parameters, β_{oj}, j = 1, 2, 3, 4, were estimated from the average (over-years) function, defined by equation 16. With the estimated carry-over nitrogen, and the estimates for $\alpha_{o.}$, $\gamma_{o.}$, and β_{oj}, j = 1, 2, 3, 4, obtained in the process of obtaining the carry-over function of equation 19, the Gauss-Newton procedure was used to obtain final estimates for the parameters. The Gauss-Newton estimates obtained by applying ordinary least squares to the transformed variables in equation 16 were est. $(\alpha_{o.})$ = 81.1373, est.$(\gamma_{o.})$ = —15.9744, est.(β_{o1}) = 1.4994, est.(β_{o2}) = 1.1131, est.(β_{o3}) = 1.0148, and est.(β_{o4}) = 0.7477. The standard errors for these estimates were 2.59, 5.36, 0.28, 0.25, 0.24, and 0.20, respectively.

To obtain estimates for the yearly parameters α_{oi} and γ_{oi} in the oat-response function (equation 2), the Gauss-Newton estimates for β_{oj} were used, and the yearly parameters estimated from the regression model

$$y_{oijk} = \alpha_{oi} + \gamma_{oi}\,[\text{est.}(\beta_{oj}) - N_{ijk}]^2 + u_{oijk} \quad [20]$$

The estimated oat-response functions, together with goodness-of-fit tests, are presented in table 5. The goodness-of-fit statistics are the ratio of the residual mean squares for the yearly regressions on the transformed variables in equation 20 to 58.94/2, the split-plot, analysis-of-variance estimator for the subplot error variance for oats,

Table 5. Estimates for parameters in the oat-response model

$$y_{oijk} = \alpha_{oi} + \gamma_{oi} (\beta_{oj} - N_{ijk})^2 + u_{oijk}$$

Estimates for β_{oj}: est.(β_{o1}) = 1.499, est.(β_{o2}) = 1.113, est.(β_{o3}) = 1.015, est.(β_{o4}) = 0.748

Yearly parameters	1958	1959	1960	1961	1962	1963	1964	1965	1958 - 65
α_{oi}	112.5	94.9	69.9	86.4	45.7	48.7	84.7	106.1	81.1
γ_{oi}	—25.0	—17.7	—12.7	—19.2	—0.9	—5.7	—14.6	—31.9	—16.0
Test-statistics[a]	0.28	2.85	0.87	1.19	1.11	0.81	0.74	0.96	0.80

[a]The critical values for the test - statistics for 1958 through 1965 are from the F(10, 95) distribution, whereas the critical values for the test - statistic for the average response function are from the F(6, 95) distribution.

σ_{os}^2. The critical values for the test - statistics are approximated from the F(10, 95) distribution. Ninety - five is the sum of the degrees of freedom for the whole - plot and subplot error mean squares of oats from table 4. The test - statistics are only approximately distributed as Snedecor's F because the transformation factor T_o (defined in equation 8) was estimated, and nonlinear regression was used to estimate the parameters in the oat - response function (equation 2). In table 5, all but one of the "F - values" in the goodness - of - fit tests are less than the 5 - percent significance value for F(10, 95). These results indicate that the estimated oat - response functions adequately represent the experimental yields.

Corn-Response Functions

The corn yields for only 1961 - 65 were used for initial estimation because of a change in the time of fertilization of continuous corn in 1961. This change and the associated change in the yield data for continuous corn led us to conclude that nitrogen carry - over on this rotation was different during the two periods 1958 - 61 and 1961 - 65.

In computing the nitrogen carry - over, we assumed that there was carry - over from the preceding corn and from the oats in CCO, but we assumed that there was no nitrogen carry - over after the meadow in the rotations containing meadow.

By applying the Gauss - Newton procedure to the transformed variables in the average corn - response function for 1961 - 65, β_c was estimated by —0.84, with standard error 0.11. By using β_c = —0.84 and the estimated N^c_{ijk}, the parameters in the corn - response function

$$y_{cijk} = \alpha_{ci} + \gamma_{cij} \exp.(-0.84 N_{ijk}) + u_{cijk} \quad [21]$$

were estimated for 1958 - 65, for all rotations containing oats and for continuous corn for 1961 - 65. The rotation - position parameter estimates for continuous corn were highly correlated with those for corn in the CCO rotation for the period 1961 - 65. The regression of the continuous corn rotation - position parameter estimates on the parameter estimates for corn in CCO was computed and used to obtain parameter estimates for continuous corn for the years 1958 - 61.

The estimates for the parameters in the corn - response function (equation 21) are given in table 6. The goodness - of - fit tests are calculated in the manner described for the oat - response function. The goodness - of - fit test - statistics for 1958 - 60 have approximate F(15, 191) distributions, whereas the test - statistics for 1961 - 65 have approximate F(17, 191) distributions. In table 6, some of the rotation - position parameters, which are postulated to be negative, are estimated to have positive signs. Random variation in the data seem the reason for this occurrence. The test - statistics indicate that the postulated corn - response function (equation 1) is accepted by the experimental data.

Hay-Response Functions

Ordinary least - squares were applied to the transformed variables in equation 3 to estimate the parameters in the hay - response function. The estimated nitrogen available to the oats preceding meadow in the rotations was used for the N_{ijk} in estimation of the hay - response function (equation 3). The estimated coefficients and goodness - of - fit tests for the hay function are presented in table 7. The test - statistics calculated indicate that the postulated model is accepted by the data on hay yields.

Table 6. Estimates for parameters in the corn-response model

$$y_{cijk} = \alpha_{ci} + \gamma_{cij} \exp(-0.84\ N_{ijk}) + u_{cijk}$$

Yearly parameters[a]	1958	1959	1960	1961	1962	1963	1964	1965	1958 - 65[b]
α_{ci}	106.9	134.0	104.3	147.0	106.0	139.8	102.5	108.2	118.6
γ_{ci1}	−110.5[c]	−161.0[c]	−87.8[c]	−194.6	−102.9	−144.9	−59.4	−125.1	−123.3
γ_{ci2}	− 34.8	− 69.9	−49.4	− 89.8	− 41.8	− 51.1	−22.0	− 49.4	− 51.0
γ_{ci3}	− 71.4	− 88.9	−58.5	−101.1	− 66.6	− 84.4	−49.9	− 75.9	− 74.6
γ_{ci4}	+ 10.5	− 9.8	−10.1	− 17.6	− 9.5	− 4.5	−14.6	− 5.8	− 7.7
γ_{ci5}	− 13.9	34.6	−33.0	− 43.5	− 53.1	− 24.7	−15.5	− 21.7	− 30.0
γ_{ci6}	+ 6.4	− 14.1	− 4.2	− 20.0	− 35.2	− 16.3	−13.6	+ 1.6	− 11.9
γ_{ci7}	− 0.6	− 19.8	−25.1	− 27.2	− 55.4	− 56.1	− 1.9	+ 3.0	− 22.9
γ_{ci8}	+ 8.7	12.1	− 5.7	− 15.8	− 18.6	− 35.4	−17.6	+ 0.9	− 12.0
γ_{ci9}	− 9.8	− 17.7	−11.8	− 32.5	− 32.7	− 46.5	+ 3.6	− 11.5	− 19.9
Test − statistics[d]	0.56	0.59	0.34	1.70	0.79	1.65	1.00	1.40	

[a] The parameters γ_{cij} for j = 1, 2, ..., 9 correspond to continuous corn, *C*CO, C*C*O, *C*COM, C*C*OM, *C*COMM, C*C*OMM, *C*COMMM, and C*C*OMMM, respectively.

[b] These are the averages of the estimates for the individual years 1958 through 1965.

[c] Estimated from the 1961 - 65 regression relationship between the estimates for γ_{ci1} and those for α_{ci}, γ_{ci2}, and γ_{ci3}.

[d] The critical value for the test - statistics for 1958 - 60 is from the F(15, 191) distribution; for 1961 - 65 the F(17, 191) distribution is used.

Table 7. Estimates for parameters in the hay-response model

$$E(y_{hijk}) = \alpha_{hip} + \gamma_{hi} N_{ijk},\ p = 1, 2, 3$$

Yearly parameters	1958	1959	1960	1961	1962	1963	1964	1965	1958 - 65
α_{hi1}	3.2859	3.6062	4.3923	4.0675	3.4877	3.1149	3.9767	2.7183	3.5810
α_{hi2}	3.2544	3.8975	4.4083	4.2913	3.4802	3.6876	4.0893	3.4229	3.8165
α_{hi3}	2.7211	4.1086	3.7658	3.7453	3.4401	2.4297	4.6460	1.8551	3.3395
γ_{hi}	+.1661	+.1215	−.0769	−.2422	−.2243	−.3476	−.3603	−.3091	−.1588
Test − statistics[a]	0.46	1.14	1.16	1.35	0.99	1.39	0.28	1.32	1.32

[a] The critical values for the test - statistics are from the F(14, 143) distribution.

Net-Return Functions

Given the estimated average crop-response functions for the different rotation-positions for each crop, the net-return functions (equation 11) are estimated for each crop-rotation. For continuous corn, the total soil nitrogen from applied sources, as a function of the yearly rate of application, denoted N_O, was approximated by assuming a 25% carry-over rate. This was obtained by comparing the proportion of carry-over specified by equations 15 and 19 for rates of application of 160 pounds of N. The average net-return function used for continuous corn (rotation 1) was

$$\pi_1(N_O) = [118.6 - 123.3 \exp(-0.84\, N_O/0.75)] p_c - N_O\, p_N - F_1 \quad [22]$$

For the rotation CCO (rotation 2), carry-over of applied nitrogen from oats to first-year corn was estimated. Thus, the total soil nitrogen from applied sources for a given crop in the rotation was approximated in terms of the yearly application rates to first- and second-year corn, and oats. For this rotation, the average net-return function used was

$$\begin{aligned}\pi_2(N_1, N_2, N_3) = (&118.6 - 51.0 \exp. [-0.84(1.006N_1 + 0.025N_2 + 0.101N_3)] \\ &+ 118.6 - 74.6 \exp. [-0.84(0.252N_1 + 1.006N_2 + 0.025N_3)])p_c \\ &+ [81.1 - 16.0\,(1.499 - 0.063N_1 - 0.252N_2 - 1.006N_3)^2]p_o \\ &- (N_1 + N_2 + N_3)p_N - F_2 \quad [23]\end{aligned}$$

For the rotations with meadow, carry-over of applied nitrogen through the meadow phase to the corn was assumed to be zero. The net-return function used for the rotation CCOM (rotation 3) was

$$\begin{aligned}\pi_3(N_1,N_2,N_3) = (&118.6 - 7.7 \exp. (-0.84N_1) + 118.6 - 30.0 \exp. [-0.84 (N_2 + C_2)])p_c \\ &+ [81.1 - 16.0(1.113 - N_3 - C_3)^2]p_o \\ &+ [3.581 - 0.159(N_3 + C_3)]p_h \\ &- (N_1 + N_2 + N_3)p_N - F_3 \quad [24]\end{aligned}$$

where $C_2 = N_1[0.325 - 0.25 \exp.(-0.81N_1)]$, and $C_3 = (N_2 + C_2)\,[0.325 - 0.25 \exp(-0.81[N_2 + C_2])]$.

In similar notation, the net-return functions used for CCOMM (rotation 4) and CCOMMM (rotation 5) are given by equations 25 and 26, respectively,

$$\begin{aligned}\pi_4(N_1, N_2, N_3) = (&118.6 - 11.9 \exp. (-0.84N_1) + 118.6 - 22.9 \exp.[-0.84 (N_2 + C_2)])p_c \\ &+ [81.1 - 16.0(1.015 - N_3 - C_3)^2]p_o \quad [25] \\ &+ [3.581 + 3.817 - 0.159 (N_3 + C_3)2]p_h - (N_1 + N_2 + N_3)p_N - F_4\end{aligned}$$

$$\begin{aligned}\pi_5(N_1, N_2, N_3) = (&118.6 - 12.0 \exp. (-0.84N_1) + 118.6 - 19.9 \exp.[-0.84 (N_2 + C_2)])p_c \\ &+ [81.1 - 16.0(0.748 - N_3 - C_3)^2]p_o \\ &+ [3.581 + 3.817 + 3.340 - 0.159(N_3 + C_3)3]p_h \\ &- (N_1 + N_2 + N_3)p_N - F_5 \quad [26]\end{aligned}$$

The average net-return functions, given by equations 22 through 26, give net return for the full cycle of the given rotation. To obtain the average annual net return, these functions are divided by the number of years required to complete a cycle for the given rotation.

Functions of Variance of Yearly Return

To obtain the functions of variance of yearly return for each crop rotation, the covariance matrices est.$\Sigma(\lambda_m)$, m = c, o, h, defined in equation 11, are first calculated. The estimated covariance matrices for the coefficients of the oat-, corn-, and hay-response functions are obtained from the sums of squares and cross products of the estimated coefficients in tables 5, 6, and 7, and the covariance matrices involved in the respective regressions. For example, the estimated covariance matrix for the coefficients in the oat-response function, denoted by est.$\Sigma(\lambda_o)$, was calculated in the following steps:

(a) From the oat regressions based on equation 20, the estimated covariance matrix for sampling variability of parameters α_{oi} and γ_{oi} is obtained.

$$\text{est.}\Sigma(d_o) = (X'X)^{-1}\,\text{est.}\sigma_{os}^2 = \begin{pmatrix} 0.68 & -0.36 \\ -0.36 & 0.41 \end{pmatrix} 58.94/2 = \begin{pmatrix} 20 & -11 \\ -11 & 12 \end{pmatrix}$$

where est.σ_{os}^2 = 58.94/2 is the subplot error mean square for oat yields, divided by the number of replicates.

(b) By dividing the sum of squares and cross products for the eight yearly estimates for α_{oi} and γ_{oi} by 7, the estimated covariance matrix

$$\text{est.}\Sigma(\text{est.}\theta_o) = \begin{pmatrix} 609 & -232 \\ -232 & 99 \end{pmatrix}$$

is obtained.

(c) The estimated covariance matrix for the yearly parameters in the oat-response function is obtained by the difference between these matrices,

$$\text{est.}\Sigma(\lambda_o) = \text{est.}\Sigma(\text{est.}\theta_o) - \text{est.}\Sigma(d_o) = \begin{pmatrix} 589 & -221 \\ -221 & 87 \end{pmatrix}$$

The estimated covariance matrices for the yearly variation in the parameters in the corn- and hay-response functions are similarly obtained. For corn, the estimated covariance matrix calculated by equation 11 was modified to ensure that all estimated correlations were less than one in absolute value. The estimated covariance matrices used to obtain the estimated functions of variance of yearly return are presented in tables 8, 9, and 10.

By use of the data in table 8, the function of variance of yearly return for continuous corn is estimated by

$$\text{est. Var.}[\pi_1(N_0)] = [328.65 - 1393.88 \exp.(-0.84\, N_0/0.75) + 1534.60 \exp.(-1.68\, N_0/0.75)](p_c)^2 \qquad [27]$$

The variance-of-return function associated with the net-return function in equation 23 for CCO is estimated by use of data from the estimated covariance matrices for corn (table 8) and oats (table 9),

$$\begin{aligned}\text{est. Var.}\ [\pi_2(N_1, N_2, N_3)] &= (1314.60 - 576.88 \exp.[-0.84(1.006 N_1 + 0.025 N_2 + 0.101 N_3)] \\ &+ 296.51 \exp.[-1.68(1.006N_1 + 0.025N_2 + 0.101N_3)] \\ &- 576.88 \exp.[-0.84(0.252N_1 + 1.006N_2 + 0.025N_3)] \\ &+ 296.51 \exp.[-1.68(0.252N_1 + 1.006N_2 + 0.025N_3)] \\ &+ 577.08 \exp.[-0.84(1.258N_1 + 1.032N_2 + 0.126N_3)])(p_c)^2 \\ &+ [589.28 - 441.80(1.499 - 0.063N_1 - 0.252N_2 - 1.006N_3)^2 \\ &+ 87.23(1.499 - 0.063N_1 - 0.252N_2 - 1.006N_3)^4](p_o)^2 \qquad [28]\end{aligned}$$

The variance-of-return function associated with the net-return function in equation 24 for CCOM is estimated by use of the appropriate variance and covariance estimates from tables 8, 9, and 10,

Table 9. Estimated covariance matrix for yearly variation in the oat-response-function parameters.

	α	γ
α	589.28	−220.90
γ	−220.90	87.23

Table 10. Estimated covariance matrix for yearly variation in the hay-response-function parameters.

	α_1	α_2	α_3	γ
α_1	0.2761	0.2000	0.4200	0.0202
α_2	0.2000	0.2100	0.4088	−0.0057
α_3	0.4200	0.4088	0.7961	0.0288
γ	0.0202	−0.0057	0.0288	0.0129

Table 8. Estimated covariance matrix for yearly variation in the corn-response-function parameters[a]

	α	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8	γ_9
α	328.65									
γ_1	−696.94	1534.60								
γ_2	−288.44	756.51	296.51							
γ_3	−288.44	756.51	288.54	296.51						
γ_4	−72.21	68.12	35.46	35.46	110.46					
γ_5	−142.63	288.60	124.79	124.79	108.45	269.27				
γ_6	−72.21	68.12	35.46	35.46	73.91	108.45	110.46			
γ_7	−142.63	288.60	124.79	124.79	108.45	214.73	108.45	269.27		
γ_8	−72.21	68.12	35.46	35.46	73.91	108.45	73.91	108.45	110.46	
γ_9	−142.63	288.60	124.79	124.79	108.45	214.73	108.45	214.73	108.45	269.27

[a]Only the lower triangle of the symmetric covariance matrix is presented.

$$\text{est. Var.}[\pi_3(N_1, N_2, N_3)]$$
$$= (1314.60 - 144.42 \exp.(-0.84N_1) + 110.46 \exp.(-1.68N_1)$$
$$-285.26 \exp.[-0.84(N_2 + C_2)] + 216.90 \exp.[-0.84(N_1 + N_2 + C_2)]$$
$$+ 269.27 \exp.[-1.68(N_2 + C_2)](p_c)^2$$
$$+ (589.28 - 441.80[1.113 - (N_3 + C_3)]^2 + 87.23[1.113 - (N_3 + C_3)]^4)(p_o)^2$$
$$+ (0.2761 + 0.0404[N_3 + C_3] + 0.0129[N_3 + C_3]^2)(p_h)^2 \quad [29]$$

The variance-of-return function associated with the net-return function (equation 25) for CCOMM is estimated by

$$\text{est. Var.}[\pi_4(N_1, N_2, N_3)]$$
$$= (1314.60 - 144.42 \exp.(-0.84N_1) + 110.46 \exp.(-1.68N_1)$$
$$-285.26 \exp.[-0.84(N_2 + C_2)] + 216.90 \exp.[-0.84(N_1 + N_2 + C_2)]$$
$$+ 269.27 \exp.[-1.68(N_2 + C_2)])(p_c)^2$$
$$+ (589.28 - 441.80[1.015 - (N_3 + C_3)]^2 + 87.23[1.015 - (N_3 + C_3)]^4)(p_o)^2$$
$$+ (0.8861 + 0.0290[N_3 + C_3] + 0.0516[N_3 + C_3]^2)(p_h)^2 \quad [30]$$

The variance-of-return function associated with the net-return function (equation 26) for CCOMMM is estimated by

$$\text{est. Var.}[\pi_5(N_1, N_2, N_3)]$$
$$= (1314.60 - 144.42 \exp.(-0.84N_1) + 110.46 \exp.(-1.68N_1)$$
$$-285.26 \exp.[-0.84(N_2 + C_2)] + 216.90 \exp.[-0.84(N_1 + N_2 + C_2)]$$
$$+ 269.27 \exp.[-1.68(N_2 + C_2)])(p_c)^2$$
$$+ (589.28 - 441.80[0.748 - (N_3 + C_3)]^2 + 87.23[0.748 - (N_3 + C_3)]^4)(p_o)^2$$
$$+ (3.3398 + 0.0866[N_3 + C_3] + 0.1161[N_3 + C_3]^2)(p_h)^2 \quad [31]$$

The estimated functions of variance of return in equations 28 through 31 are given for a rotation cycle. The estimated function of variance of yearly return for a given rotation is obtained by dividing the estimated function of variance of return by the square of the number of years to complete a cycle of the crop rotation.

Determination of Economic Optima

Net-prices of $1.01 for corn, $0.67 for oats, and $15.76 for hay were used in the net-return functions and variance-of-return functions. These prices are the 1967 average Iowa prices as reported by James (1968), less variable costs. The variable costs subtracted from the crop price quotations included shelling, drying, and transportation charges for corn ($0.12 per bushel), transportation and drying charges for oats ($0.02 per bushel), and baling costs for meadow ($4.00 per ton). James (1968) presents 1967 central Iowa prices per pound of N as 11.5 cents from dry bulk sources, 6.2 cents from anhydrous ammonia, and 7.5 cents from aqua ammonia. For the profit analysis, the prices of N per pound of 3, 5, 7, and 9 cents were considered. By using data from James (1968), fixed costs were calculated as $34.59 for continuous corn, $90.74 for CCO, $112.85 for CCOM, $129.97 for CCOMM, and $145.48 for CCOMMM per acre per rotation cycle. Custom rates for the basic farming operations involved for the different rotations were included in the fixed costs. Such costs as rent on land, depreciation of buildings, and management charges were not included.

Because the net-return functions (equations 22-26) are relatively complicated, the optimal applied nitrogen levels were obtained by enumeration. Net return was calculated for combinations of values of N_1, N_2, and N_3. The optimal nitrogen applications were determined from the resulting values for net return. The optimal rates of applied N are presented in table 11, together with the associated crop yields and average annual net returns. The rates of N for first- and second-year corn were approximated to the nearest 4-pound unit, and the rates for oats approximated to the nearest 5-pound unit. The variances of yearly return were estimated from equations 27 through 31 for the optimal nitrogen applications for each rotation. The estimated standard deviations of yearly return are presented in table 11.

DISCUSSION

The optimal nitrogen applications for continuous corn are within (or slightly larger than) the experimental range for the product prices considered. Rates on corn in other rotations generally exceed the experimental rates. For the nitrogen price of 5 cents per pound (see table 11), the optimal rates of nitrogen (pounds per acre) are estimated to be 152 for continuous corn, 160 for *C*CO, 80 for *C*COM, 100 for *C*COMM, and 100 for *C*COMMM. The nitrogen rates for *C*CO exceed those for continuous corn. This is associated with the relatively small nitrogen application to oats and, hence, relatively small carry-over. The estimated optimal nitrogen rate for first-year

Table 11. Estimated crop yields, net-return, and standard deviation of yearly return for optimal applied nitrogen rates[a]

Crop rotation	Item and unit[b]	Rotation-position	Price of nitrogen (cents per pound) at 3	5	7	9
Continuous corn	Nitrogen		168	152	140	132
(fixed cost per	Corn yield		117.5	116.8	116.1	115.5
year, $34.59	Net return		79.00	75.81	72.90	70.20
per acre)	Std. dev., return		17.96	17.76	17.54	17.35
CCO	Nitrogen	*C*CO	184	160	144	128
		C*C*O	160	140	124	120
		CC*O*	5	10	15	15
(fixed cost per	Yield	*C*CO	117.9	117.4	116.9	116.5
year, $30.25		C*C*O	117.2	116.3	115.3	114.0
per acre)		CC*O*	81.0	80.9	80.9	80.7
	Net return		63.51	61.34	59.39	57.57
	Std. dev., return		13.28	13.22	13.16	13.10
CCOM	Nitrogen	*C*COM	104	80	60	48
		C*C*OM	104	104	104	100
		CC*O*M	0	0	0	0
(fixed cost per	Yield	*C*COM	117.7	117.1	116.4	115.8
year, $28.21		C*C*OM	116.8	116.4	116.1	115.7
per acre)		CC*O*M	81.1	80.8	80.4	79.9
		CCO*M*	3.42	3.43	3.44	3.45
	Net return		56.46	55.48	54.62	53.83
	Std. dev., return		10.18	10.13	10.08	10.04
CCOMM	Nitrogen	*C*COMM	124	100	84	68
		C*C*OMM	76	76	76	76
		CC*O*MM	0	0	0	0
(fixed cost per	Yield	*C*COMM	117.7	117.1	116.5	115.7
year, $25.99		C*C*OMM	116.5	116.1	115.7	115.4
per acre)		CC*O*MM	80.7	80.2	79.8	79.3
		CCO*M*M	3.45	3.46	3.47	3.47
		CCOM*M*	3.68	3.69	3.70	3.71
	Net return		53.37	52.62	51.94	51.33
	Std. dev., return		8.49	8.45	8.42	8.38
CCOMMM	Nitrogen	*C*COMMM	124	100	84	72
		C*C*OMMM	48	52	52	48
		CC*O*MMM	0	0	0	0
(fixed cost per	Yield	*C*COMMM	117.7	117.1	116.5	115.9
year, $24.25		C*C*OMMM	115.3	115.0	114.5	113.7
per acre)		CC*O*MMM	80.8	80.6	80.3	79.6
		CCO*M*MM	3.49	3.49	3.50	3.51
		CCOM*M*M	3.72	3.73	3.74	3.75
		CCOMM*M*	3.24	3.25	3.26	3.27
	Net return		50.57	50.03	49.56	49.13
	Std. dev., return		7.06	7.04	7.01	6.98

[a] Prices of corn, oats, and meadow are held constant at $1.01, $0.67, and $15.76, respectively.

[b] Yields are in bushels per acre for corn and oats, and tons per acre for hay. Net-return figures are in terms of dollars per acre annually. Applied nitrogen rates are in pounds per acre of elemental N.

corn is less for the rotation with 1 year of meadow than for the rotations with 2 or 3 years of meadow. The main reason for this is that the estimate for the rotation-position parameter for *C*COM was smaller in absolute value than those for *C*COMM and *C*COMMM. The opposite result is more consistent with agronomic expectations. The estimated optimal nitrogen rates for second-year corn (140 for C*C*O, 104 for C*C*OM, 76 for C*C*OMM, and 52 for C*C*OMMM) reflect the additional nitrogen supplied by the meadow crops.

For the CCO rotation, increasing the price of nitrogen results in decreases in the optimal nitrogen for first- and second-year corn, but slight increases in the applied nitrogen for oats. The slight increase for oats is reasonable since the reductions in nitrogen applied to corn result in less carry-over nitrogen for oats. With increases in nitrogen price, the changes in the nitrogen rates for second-year corn in the meadow rotations are small relative to those for first-year corn. Marked reductions in N_2 not only reduce yields of second-year corn, but also reduce oat yields.

Zero nitrogen is applied to oats in the meadow rotations at all prices investigated. Less applied nitrogen is required in the meadow rotations, and nearly maximum oat yields are obtained from carry-over of nitrogen from the corn. The carry-over nitrogen available to the oats in all rotations at all prices, however, is less than the applied nitrogen levels necessary for maximum oat yields. Since high oat yields have a deleterious effect on yields of hay after oats in the rotation, applied nitrogen rates in the meadow rotations are lower than those required to maximize returns to the oats alone.

It is clear that maximization of the net-return function gives relatively high rates of applied nitrogen in comparison with the experimental rates. The variance of the yearly return, however, increases with increases in nitrogen applications. Thus, the high nitrogen rates may be considered unsatisfactory if a lower variance of yearly return is desired.

Continuous corn displayed the highest net return per year for all prices of nitrogen considered. For the nitrogen price at 5 cents per pound, the estimated net return for continuous corn is $75.81 per acre. Net return per acre annually for continuous corn is about $15 greater than that for CCO, the nearest competitor. The difference between the average profit per year for CCO and CCOM decreases from $7.05 per acre for nitrogen at 3 cents per pound to $3.74 per acre for nitrogen at 9 cents per pound. For nitrogen at 5 cents per pound, increasing the length of the meadow phase from 1 to 2 years reduces the average profit per year by $2.86, and increasing the meadow phase to a third year reduces the average profit per year by $2.59. For any given rotation, as the price of nitrogen increases, the optimal applied nitrogen levels decrease, causing a decrease in the average profit per year and a slight decrease in the variance of profit per year. For a fixed nitrogen price, the average profit per year and variance of profit per year decrease as the proportion of corn in the rotation decreases.

Within any rotation, the profit curve is relatively flat in the neighborhood of the optimal applied nitrogen rate. For example, for the rotation CCO, with nitrogen at 5 cents per pound, the optimal applied nitrogen rates are 160 pounds for first-year corn, 140 pounds for second-year corn, and 10 pounds for oats. If the rate of application of N to oats is fixed at 10 pounds, however, decreasing the rates to first- and second-year corn by 40 pounds results in a decrease in average annual profit of only $1.10 per acre.

The optimal rates of applied nitrogen and the optimal rotation to adopt depend on the attitude of the farmer toward yearly return variability as well as other factors, such as labor availability, market for forage, etc. In this study, we assumed that the objective was to maximize net return from the crop enterprise, and the set of alternatives consists of the 5 crop rotations involved in the Carrington-Clyde experiments. To determine the optimal combination of crop and livestock enterprises for a given farm would require data on livestock enterprises and that the pricing of products be altered accordingly. This report, however, provides a framework in which appropriate statistical and economic analyses of crop-rotation experiments can be carried out.

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APPENDIX

Theorem.

If the random errors ϵ_{mijk} are defined by

$$\epsilon_{mijk} = u_{mijk} - T_m \sum_{k=1}^{K} u_{mijk}/K;$$

$$T_m = 1 - [\sigma_{ms}^2/(\sigma_{ms}^2 + K\sigma_{mw}^2)]^{1/2}; \text{ and}$$

$$\begin{aligned} Cov(u_{mijk}, u_{mpqr}) &= \sigma_{mw}^2 + \sigma_{ms}^2, && \text{if } i=p, j=q, k=r \\ &= \sigma_{mw}^2 && \text{, if } i=p, j=q, k\neq r \\ &= 0 && \text{, otherwise} \end{aligned}$$

$$\begin{aligned} \text{then } Cov(\epsilon_{mijk}, \epsilon_{mpqr}) &= \sigma_{ms}^2 && \text{, if } i=p, j=q, k=r \\ &= 0 && \text{, otherwise.} \end{aligned}$$

Proof: Because there exist only nonzero covariances between the u_{mijk} and u_{mpqr} where $i=p$ and $j=q$, we only require proof that $Cov(\epsilon_{mijk}, \epsilon_{mijr}) = \sigma_{ms}^2 \psi_{kr}$,

$$\begin{aligned} \text{where } \psi_{kr} &= 1 && \text{, if } k=r \\ &= 0 && \text{, otherwise.} \end{aligned}$$

Now $Cov(\epsilon_{mijk}, \epsilon_{mijr})$

$$= Cov(u_{mijk} - T_m \sum_{k=1}^{K} u_{mijk}/K, u_{mijr} - T_m \sum_{k=1}^{K} u_{mijk}/K)$$

$$= Cov(u_{mijk}, u_{mijr}) - 2T_m Cov(u_{mijk}, \sum_{k=1}^{K} u_{mijk}/K) + T_m^2 Var \sum_{k=1}^{K} u_{mijk}/K)$$

$$= (\sigma_{mw}^2 + \sigma_{ms}^2 \psi_{kr}) - 2T_m [(\sigma_{mw}^2 + \sigma_{ms}^2) + (K-1)\sigma_{mw}^2]/K + T_m^2 [(\sigma_{mw}^2 + \sigma_{ms}^2) + (K-1)\sigma_{mw}^2]/K$$

$$= (\sigma_{mw}^2 + \sigma_{ms}^2 \psi_{kr}) + (\sigma_{ms}^2 + K\sigma_{mw}^2)(T_m^2 - 2T_m)/K$$

$$= (\sigma_{mw}^2 + \sigma_{ms}^2 \psi_{kr}) + (\sigma_{ms}^2 + K\sigma_{mw}^2)(1 - T_m)^2/K - (\sigma_{ms}^2 + K\sigma_{mw}^2)/K$$

From the definition of T_m, it is seen that

$$(1 - T_m)^2 = \sigma_{ms}^2/(\sigma_{ms}^2 + K\sigma_{mw}^2).$$

$$\therefore Cov(\epsilon_{mijk}, \epsilon_{mijr}) = (\sigma_{mw}^2 + \sigma_{ms}^2 \psi_{kr}) + \sigma_{ms}^2/K - (\sigma_{ms}^2 + K\sigma_{mw}^2)/K = \sigma_{ms}^2 \psi_{kr}$$

$$\begin{aligned} \text{Thus, } Cov(\epsilon_{mijk}, \epsilon_{mpqr}) &= \sigma_{ms}^2 && \text{, if } i=p, j=q, k=r \\ &= 0 && \text{, otherwise.} \end{aligned}$$

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