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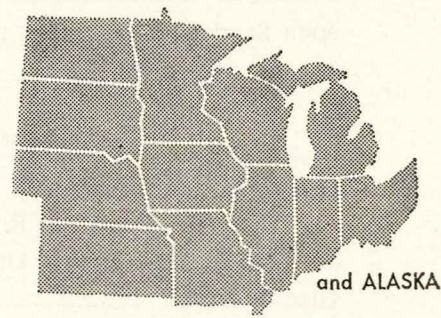
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EVALUATION OF A FIRM MODEL IN ESTIMATING AGGREGATE SUPPLY RESPONSE

Agricultural Experiment Stations of Alaska, Illinois, Indiana, Iowa, Kansas, Michigan, Minnesota, Missouri, Nebraska, North Dakota, Ohio, South Dakota, and Wisconsin, and the U. S. Department of Agriculture cooperating.

AGRICULTURE AND HOME ECONOMICS EXPERIMENT STATION, IOWA STATE UNIVERSITY of Science and Technology, Ames, Iowa RESEARCH BULLETIN 558



PREFACE

This is one of two North Central Regional publications from regional research project NC-54, "Supply Response and Adjustments for Hog and Beef Cattle Production." The regional research results are reported in a publication by Colyer and Irwin (4).

Now that the research has been completed, it is possible to evaluate some of the key decisions made throughout the research. This evaluation is made to help those doing similar research in the future. The purpose of this publication is to evaluate one of the several parts (Phase I) of the NC-54 research. The

approach used in Phase I was similar to the approach used in the regional supply adjustment studies in the Lake States, the Northeast, the South, the Great Plains and the West. Thus, many of the conclusions in this report are also applicable to the models used in the other regional adjustment studies.

Although many shortcomings of the Phase I model and of the Phase I results are presented in this report, useful conclusions are, nevertheless, derived from the NC-54 study, which included several other analytical procedures that complemented the Phase I analysis.

Regional Research Committee NC-54

Supply Response and Adjustments for Hog and Beef Cattle Production

Sponsored by the agricultural experiment stations of Illinois, Indiana, Iowa, Kansas, Michigan, Minnesota, Missouri, Nebraska, North Dakota, Ohio, South Dakota and Wisconsin, and the Farm Production Economics Division, Economic Research Service, U. S. Department of Agriculture.

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SUMMARY

This paper has two main purposes, to evaluate the Phase I model of NC-54 and to examine alternative models.

In economic literature several concepts of industry (regional) supply are presented. Among these are the "supply relation" and the "response relation." External effects (external to the firm) are held constant in the first—or classical—case, but they are incorporated into the response relation. Clearly, external effects exist in the Corn Belt. But in the Phase I model external effects are assumed not to exist. Thus, it is not surprising to find that the Phase I results show unrealistic hog and cattle supply functions for the Corn Belt. This does not necessarily mean, however, that the Phase I model—or any representative farm (no regional restraints) model—is worthless. This type of model is useful under conditions where (1) the region is so small that external effects can be ignored or (2) emphasis is placed on the farm management implications of representative farm results, but the aggregate implications of these individual farm adjustments are desired. The NC-54 committee chose the representative farm (no regional restraints) model primarily because of the latter condition.* Some might argue that the compromises forced on the model by this condition made any regional analysis *per se* meaningless, but others could defend the analysis by arguing that the farm management information was valuable and that the additional cost of aggregating was very small.

A Phase I type of model will probably play a less important role in future regional research. The choice between a Phase I type of model and any of the regional models suggested in the last section of this report depends upon the objectives of the study. It appears that the objectives of regional research are shifting from a micro (farm management) analysis to a macro analysis. Another reason for abandoning the Phase I type of model is the constant improvement in computer technology. The types of models discussed in the last section of this report are not just conceptual; they are operational.

*Another reason for the choice of the representative farm (no regional restraints) model was that the procedural and computational problems associated with a representative farm model containing regional restraints had not been worked out.

Regardless of the type of model used or relative weight given different objectives, there are several considerations that should be made in model development. First, the model should take into account external effects. Since in the Phase I model external effects were held constant, only those solutions that showed regional production near (less than, say, two times recent levels) recent levels of actual production were realistic. But only one of the 27 sets of regional results even comes close to these levels of production.

Second, the model should meet the defined conditions of internal consistency. We believe the Phase I model does not do a very good job of meeting conditions of internal consistency. Enterprise levels on many representative farms at the acceptable levels of regional production were far removed from the enterprise levels that were assumed when the model was constructed.

Third, the assumption about the level of applied technology has a critical bearing on the costs of production and on the aggregate results.

Fourth, other assumptions that affect the cost of production need to be considered. They are: (1) charging a reservation price for family labor, (2) charging depreciation and other ownership costs for existing buildings or machinery and (3) including discount rates for risk. Failure to recognize these cost components has seriously affected the level of the aggregate supply curves.

Fifth, a potential weakness with models based upon representative farms can be found in the method of farm stratification. The conventional methods of stratification often do not isolate those factors that cause some farmers to specialize in hogs, some to specialize in beef and others to raise only cash crops. The NC-54 model incorporated the best stratification techniques available at that time, but the results show that these techniques are far from adequate. Factors such as farmers' preferences, differences in levels of management, etc., may be more important bases for stratification. Further research is badly needed in this area.

Finally, considerations of economic time are important here, as in all economic research. Studies of the rates of change in size of farm, resource ratios and technology would insure greater consistency with respect to time between the various parts of the model.

Evaluation Of A Firm Model In Estimating Aggregate Supply Response

by Jerry A. Sharples, Thomas A. Miller
and Lee M. Day¹

The North Central Regional Research Project NC-54, "Supply Response and Adjustments for Hog and Beef Cattle Production," was started in 1961. The project statement lists these objectives:

- (1) To estimate farm resource use and supply response of hogs and beef cattle in representative farm situations.
- (2) To estimate total production of hogs and beef cattle and patterns of resource use for states in the North Central Region and for the nation.
- (3) To determine the production situations and the areas in which a specified output of hogs and beef cattle would or could be produced most efficiently under various projected levels of demand and prices and at a given level of technology representing that now known but not yet generally adopted.

Linear-programming, time-series analysis, production function analysis and "outlook" research were used in the study. The linear-programming research was divided into two phases. Phase I involved (a) estimating the optimum organization and production for representative farms at various prices for hogs, cattle and feed grains and (b) aggregating these results to give estimates of regional production. The purpose of Phase II was to examine the effects of permitting acquisition and disposal of factors of production assumed fixed in the Phase I model. This was accomplished by including purchase and sale activities for fixed assets at predetermined prices. Insofar as the purchases and sales were not conducted within a framework of regional constraints and because an appropriate weighting scheme was not readily available, no aggregation of the Phase II results was made. Time-series analysis, production function analysis and "outlook" analysis were used to complement the programming analysis.

Phase I was developed as a major part of the overall project, but was not intended to stand alone as a predictor of supply conditions. Nevertheless, in the analysis that follows, the approach is to evaluate the usefulness of a Phase I type model in achieving the

three objectives of NC-54. The reader is cautioned that the criticisms in this publication are not directed at the entire NC-54 research, but only at the Phase I portion.

The purpose of this report is to evaluate the methods used in Phase I of the NC-54 project. The specific objectives are to (1) review the assumptions of the NC-54 (Phase I) model, (2) evaluate the role of specific features of this model in contributing to the results and (3) use this experience to suggest alternative formulations that may provide a more adequate representation of the workings of the feed-livestock economy of a region.

ASSUMPTIONS AND RESULTS OF THE PHASE I STUDY

Twelve states participated in the NC-54 project.² The geographic region covered in the study approximately coincides with the Corn Belt. Representative farms were defined in each of the states participating in Phase I. Twenty-seven optimal organizations were then computed for each of the representative farms—one solution for each combination of three hog prices, three cattle prices and three feed grain prices. The programming results were aggregated to give state and Corn Belt totals for each price combination.

Attempts to maintain comparability among states

Differences between the program solutions of any two representative farms may generally result from two sources: (1) real differences in the economic potentials and (2) differences in the economic models applied to the two farms. The state participants in NC-54 wanted the results of their research to reflect differences in the potential competitive position of the various states in the production of hogs, cattle and feed grains rather than to reflect differences in the analytical procedures. This required that all participants use the same general assumptions and follow the same set of procedures in developing coefficients for their own contributing models.

The technical committee adopted a set of Phase I

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²The study area includes western Ohio, southern Michigan, Indiana, Illinois, Iowa, northern Missouri, eastern Kansas, Nebraska, South Dakota, North Dakota and southern Minnesota. Wisconsin contributed to the NC-54 research, but did not contribute to Phase I.

assumptions that were used by all the states. These assumptions are summarized as follows:

- (1) All prices except the prices of corn, hogs and beef cattle are set at levels expected to exist on the average from 1963 to 1970.
- (2) By 1970, all farmers are assumed using the best of the commercially acceptable techniques known in 1961.
- (3) Depreciation is not charged on existing buildings and machinery.
- (4) Investment in new buildings and machinery is allowed, but depreciation is charged on this investment.
- (5) Methods of obtaining credit and costs of financing approximate those used in the Corn Belt in 1961 and are kept unchanged through 1970.
- (6) Farm capital can be invested off the farm.
- (7) Corn acreage on each farm is limited to its 1961 corn acreage, and wheat acreage per farm is limited either to the 1961 wheat allotment or to 15 acres.
- (8) Corn can be purchased in unlimited quantities at 10 cents per bushel above the sale price.
- (9) Labor available to each representative farm approximates that used at survey time.³
- (10) Land available to each representative farm is the same as at survey time.⁴

The technical committee developed a set of procedures to be followed by every participant. The objective was to have the results reflect only real differences among states. No interstate differences were allowed in:

- (1) the alternative methods of purchasing, fattening and selling cattle and hogs,
- (2) livestock-feed efficiency,
- (3) labor efficiency for a given size of livestock enterprise,
- (4) most of the itemized costs of livestock production (except protein cost) and
- (5) methods and costs of obtaining credit.

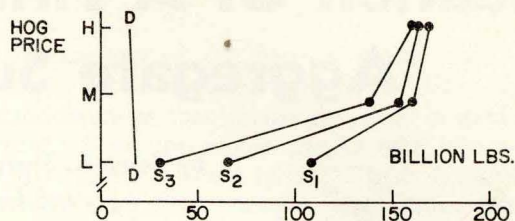
Each area program within a state could have a unique set of crop activities. Crop yields were to reflect the natural environment and agronomic practices consistent with known and commercially feasible technology. Each programming model could also include other competitive livestock enterprises such as dairying or laying flocks.

Optimum solutions were developed for each of 27 price combinations. It was important for aggregation purposes that every participant use equivalent prices; i.e., the same prices adjusted for area location differentials. The maximum corn price was set at the 1961 price support level for each state. The medium and low corn prices were 20 cents per bushel and 40 cents per bushel, respectively, below the maximum price.

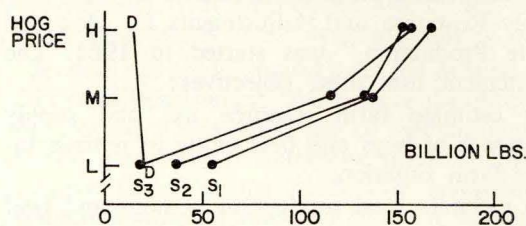
³A survey of farms was taken to identify representative farms. The date of the survey varied among the states from 1959 to 1962.

⁴In Phase II, land purchase, rental and sale were allowed.

A. WITH LOW BEEF PRICES



B. WITH MEDIUM BEEF PRICES



C. WITH HIGH BEEF PRICES

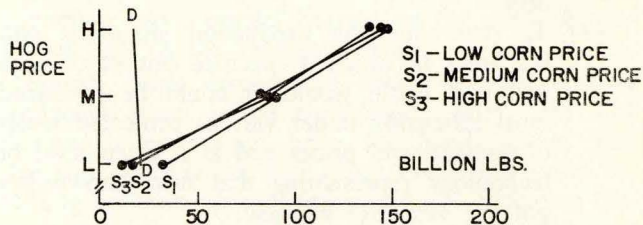


Fig. 1. Hog supply and demand in the Corn Belt.

The U.S. average support price for corn in 1961 was \$1.20 per bushel.

Three hog prices (\$17.76, \$14.80 and \$11.84) were computed for Chicago.⁵ A hog price differential was determined for each state by using Chicago as a base. This differential was added to each of the three price levels. The programming model included the sale of hogs four times during the year; so historical quarterly price differentials were also added to the hog prices.

Three choice slaughter steer prices (\$24.96, \$20.80 and \$16.64) were computed for Chicago in the same manner as the three hog prices.⁶ Geographical price differentials were also added to each of the three steer price levels. A constant dollar margin was assumed between the purchase price of calves and the sale price of cattle. A constant dollar margin was also assumed for purchased yearlings.

Results

The aggregate results are presented graphically in figs. 1, 2 and 3. Actual optimal linear-programming supply curves are stepped, but since only three observa-

⁵The hog prices were computed by multiplying the average corn prices of \$1.20, \$1.00 and \$0.80 by the average annual 1955-1960 hog-corn price ratio of 14.8:1. The hog-corn price ratio was computed by dividing the average annual 1955-1960 Chicago price for barrows and gilts (\$16.87) by the average annual U.S. corn price for the same period (\$1.14).

⁶The steer and corn prices were averaged over 1950-1960 to get a steer-corn price ratio of 20.8:1.

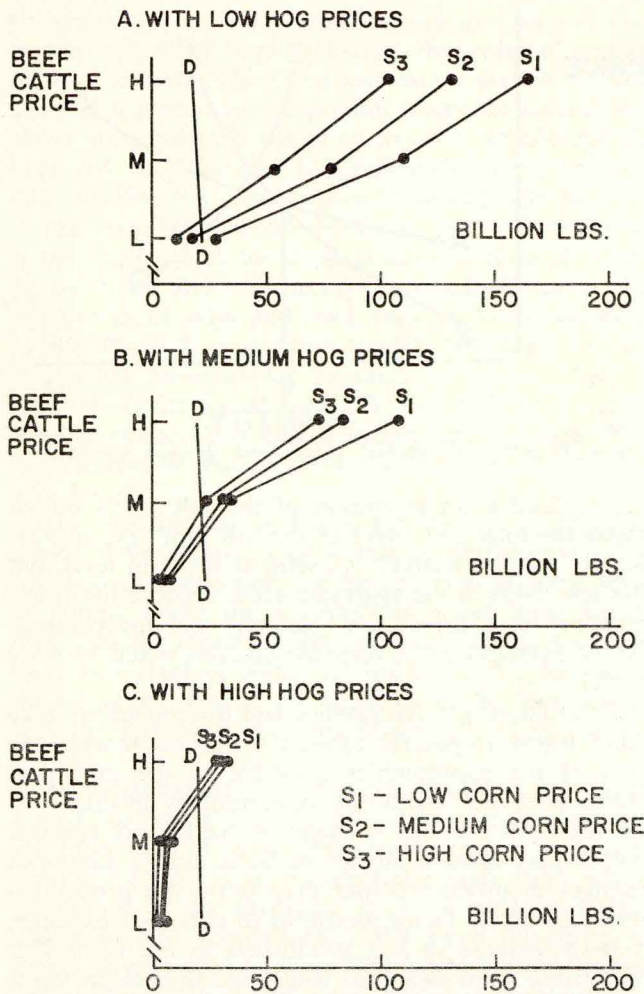


Fig. 2. Beef cattle supply and demand in the Corn Belt.

tions are made on each curve, the true shapes of the curves are not known. The dots on each supply curve represent the quantity-price locations of the solutions. The dots are connected by line segments to show the general shape of the supply relationship. Figs. 1 and 2 also show demand projections for the Corn Belt for 1970.⁷

The supply curves shown in figs. 1 and 2 show the effect of an increase in product price on quantity. Since the purchase of livestock facilities is allowed in the Phase I model, the supply curves are not reversible in an intermediate-run context. For example, assume that the high price of hogs prevailed and farmers expanded their hog facilities to produce the corresponding quantity shown in fig. 1. If the price was then to drop, the optimal response of farmers would give a curve less elastic than shown in fig. 1.

Hog sales range from 11 billion pounds to 169 billion pounds, and beef cattle sales range from 2 billion pounds to 165 billion pounds (figs. 1 and 2).

⁷A complete explanation of the estimates of supply, demand and equilibrium conditions in the Corn Belt is included in the NC-54 research report by Colyer and Irwin (4).

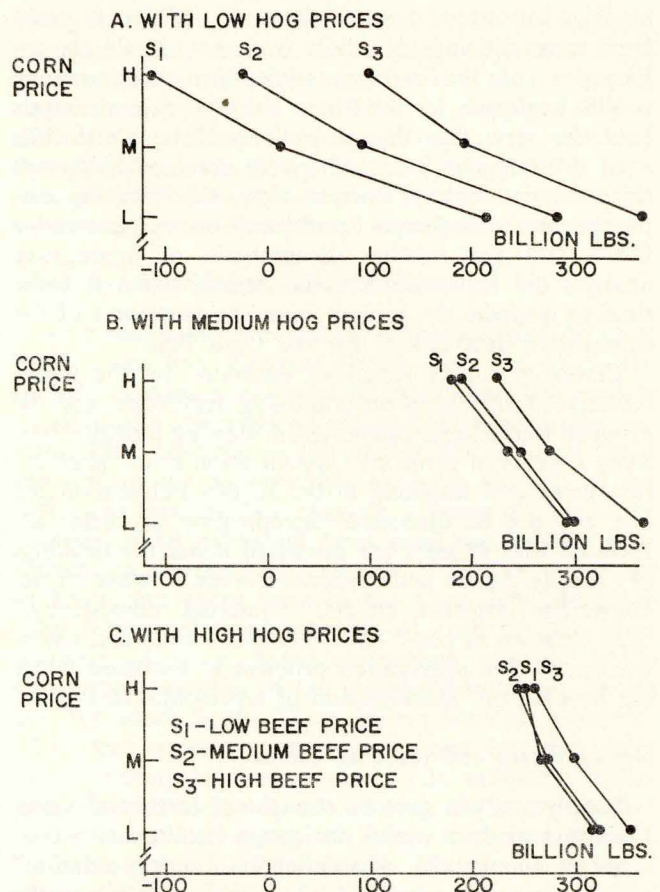


Fig. 3. Net feed grain purchases in the Corn Belt.

In comparison, 14 billion pounds of hogs and 22 billion pounds of cattle and calves were sold in the 11-state area in 1965. Figs. 1 and 2 indicate that a joint equilibrium exists at approximately \$11.75-hogs and \$17-cattle. At higher prices for hogs and cattle, the Corn Belt would be a net importer of corn (fig. 3). Corn importing is consistent with the large quantities of cattle and hogs produced in the model.

The aggregate supply functions do not appear reasonable in view of current 1970 expectations. If livestock production over the last 5 years is to guide realism, most of the prices programmed in this study are far too high. On the other hand, if livestock prices over the last 5 years are to guide realism, the cattle and hog production estimates obtained from the model are far too high. The next section will evaluate the role of various assumptions in contributing to these unrealistic results.

AN EVALUATION OF THE PHASE I MODEL ASSUMPTIONS

The theoretical and technical problems of defining a linear-programming model for a representative farm are well known, but additional problems are encountered when representative farm results are aggregated to give regional supply functions. When the NC-54

model was conceived, some of these "additional" problems were anticipated; others were not. A simple aggregation of the representative farm programming results was made for the Phase I study.⁸ Some analysts held the view that the project would be worthwhile even if no aggregative results were obtained. Although this was no doubt a correct view, the resulting emphasis on management problems on representative farms and the relative de-emphasis of aggregative analysis did leave much to be desired when it came time to evaluate the project from the viewpoint of the aggregative implications for the Corn Belt.⁹

Errors in supply functions estimated by the representative farm linear-programming technique can be grouped into several categories. Following Stovall (16), three sources of error are: specification error, aggregation error and sampling error. In this publication the last will not be discussed. Specification problems associated with Phase I are discussed under the headings of "supply curves and response curves," "ways to acknowledge external effects," "internal consistency," "specification of costs" and "economic time and calendar time." The aggregation problem is discussed under the heading of "identification of representative farms."

Supply curves and response curves

Supply analysis may be thought of in several ways. Cochrane made a useful distinction between two concepts of supply (3). He defined a "supply relation" and a "response relation." The supply relation is the traditional *ceteris paribus* supply concept. It indicates the responsiveness of a commodity to a change in its price with everything else held constant. Cochrane defined the response relation as a change in quantity associated with a change in price with all other factors allowed to interact. Cochrane pointed out that a study of the response relation was a "study of the shifters of supply."

A regional supply relation is analogous to a short-run industry supply curve under perfect competition. The latter is defined by Liebhafsky as ". . . the horizontal sum of the relevant ranges of the various marginal cost curves of the individual firms" (8, p. 232). The level of output of one firm often affects the costs of another firm. These effects are called external economies and diseconomies.¹⁰ Henderson and Quandt define external economies and diseconomies by examining two random firms in the industry: "External economies are realized if an expansion of the *j*-th firm's output lowers the total cost of the *i*-th firm. External diseconomies

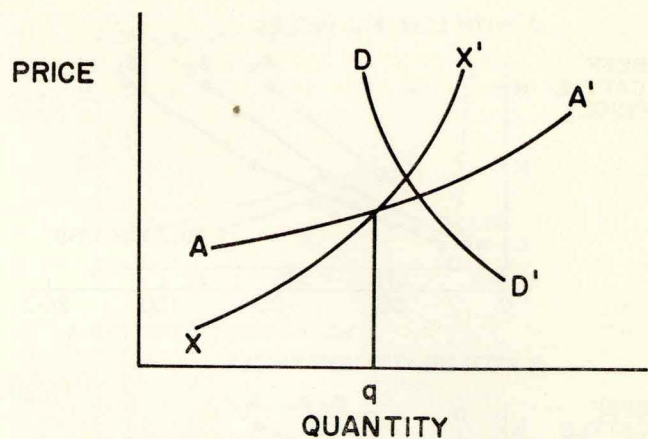


Fig. 4 Hypothetical regional supply and demand curves.

are realized if an expansion of the *j*-th firm's output raises the total cost curve of the *i*-th firm" (6, p. 92). Supply relation analysis is useful at the firm level, but has less value in the aggregate sense because the firms' marginal cost curves are generally not independent. At the aggregate level, response analysis would be more useful.

External effects (economies and diseconomies) take many forms. In general, external effects occur when the sum of the opportunities available to the individual farms does not equal the opportunities available to the region as a whole. A change in the level of regional output may cause a change in factor prices. Long-run changes in product output may cause the production functions of the factor industries to shift. For example, a large increase in hog production in the Corn Belt may cause Corn Belt feed companies to enlarge, causing a shift in their production functions. The end result could be lower feed costs. The concept of supply that incorporates external economies and diseconomies is analogous to Cochrane's "response relation."

The following example shows the differences between the two concepts of regional supply. Suppose that a research project were set up to estimate the supply function of hogs in the Corn Belt. Two approaches were used. One was to ignore external effects—assume that they did not exist—and the other was to incorporate them into the model. Assume that the analyses were short-run and that the only external effect considered was the effect of the level of hog production in the region on the price of inputs. All other assumptions were the same. The results would probably look like those in fig. 4. Line AA' represents the sum of the supply curves over all the firms in the region with the assumption that external effects did not exist. Perfectly elastic regional supply curves for the inputs are implied. The line XX' is constructed the same as AA', except that external effects are recognized.¹¹ In this

⁸Although a simple aggregation of Phase I programming results was made, the committee realized that this would yield inadequate results. Thus, the plans of NC-54 included use of alternative approaches such as time-series, Cobb-Douglas and "outlook" analysis to enable the committee to approach an estimate of probable aggregate production by a variety of techniques.

⁹Allowing the size of the representative farms to vary in Phase II precluded the use of farm numbers as weights in an aggregation. However, it might have been fruitful for NC-54 to have put more emphasis on the problem of handling external effects and aggregation problems for a Phase II type model and less emphasis on the Phase I model.

¹⁰Throughout the text external effects refer to effects that are external to the firm.

¹¹It would be virtually impossible to include all external effects in a regional model. Only a general equilibrium model containing all consumers and all producers in the world could incorporate all external effects. Input prices are used in this example because the effect of the level of product production on input prices is one of the most important external effects in the short run.

particular presentation, diseconomies are shown; i.e., as the output of hogs increases, the costs of inputs increase causing XX' to be less elastic than AA' . The quantity q represents the amount where the assumption about input prices is the same for both supply curves. Line AA' is analogous to Cochrane's supply relation (3), and line XX' is analogous to his response relation.

The response function definition, line XX' in fig. 4, is the most useful for application to regions as large as the Corn Belt. Obviously, external effects in the production of hogs and beef exist in the Corn Belt. In the Phase I aggregation model, however, external effects are assumed not to exist. Thus, the Phase I regional results are of type AA' in fig. 4. In a region the size of the Corn Belt, a supply curve of type AA' is unacceptable except at output levels approximating q in fig. 4.

The best that can be hoped for from an aggregation model of the Phase I type is that the intraregional distribution of production can be observed at some level of output where the assumptions about input prices built into the representative farm coefficients are realistic. This level of output is equivalent to q in fig. 4. As a practical matter, the accuracy of the data is such that the results in the "neighborhood" of q are acceptable.

As stated earlier, the input prices used in the Phase I model are set at levels expected to exist on the average from 1963 to 1970. Thus, input prices in the model generally do not differ substantially from 1965 levels. Therefore, at an aggregate level of production that would require approximately the same quantities of inputs as were used on Corn Belt farms in 1965, output would be consistent with input prices (equivalent to q in fig. 4).¹² But at most production levels shown in figs. 1 and 2, the quantity of inputs demanded would be substantially above 1965 levels. In only one of the 27 solutions (\$1.20-corn, \$12-hogs and \$17-cattle) do both hog and cattle production "approach" 1965 production levels. Even in this case, hog production is about twice actual production. One can argue that even this solution is not consistent with the assumed external conditions. If in the model the low hog price had been lowered to about \$11.50, the regional production of hogs and cattle would have been closer to actual production levels.

One further observation can be made from fig. 4. Assume that DD' is the expected demand curve for the region. The point where XX' crosses DD' is the regional equilibrium. But for two exceptions, little significance can be attached to the point where DD' crosses AA' . The two exceptions are (1) DD' might cross AA' at output level q , or (2) the two supply curves, AA' and XX' , might not be significantly different; i.e., external effects might be realistically ignored. The first exception is unlikely. The second exception is not reasonable for a region as large as the Corn

Belt. In the NC-54 project—and in all the regional supply studies—one of the implicit objectives is to derive demand functions and determine a regional equilibrium. But "with the kind of supply function generated by Phase I, it seems that it is stretching the results too far to try to make any but the most general equilibrium statements. There is no justification for any elaborate demand and equilibrium analysis.

Ways to acknowledge external effects

As pointed out previously, external effects do exist in the Corn Belt. External effects—most of which are thought diseconomies—may be incorporated in models as regional constraints on production. Clearly the opportunities available to the region are different from the sum of the opportunities available to each of the individual farmers. For example, a linear programming analysis of an individual farm need not contain constraints on the purchase and sale of feed or the hiring of labor, but every farm in the Corn Belt obviously cannot buy unlimited amounts of corn or hire large amounts of labor. Some assumption must be made about the allocation of the regional constraints among the individual farms. Three assumptions are possible: (1) Open-ended purchases and sales by each farm (i.e., ignoring external effects), (2) arbitrary allocation of purchases and sales among the representative farms and (3) changing the entire format of Phase I by constructing a single regional model or a series of regional submodels.

In the Phase I model, corn sales and purchases were handled by the first method. Fig. 3 shows that, at all but two solutions, corn was imported into the Corn Belt. Hired labor constraints were generally defined by the second method. The historical amount of labor hired on the various representative farms was used as a basis for determining the hired labor constraints.

Given that external effects exist, the third alternative might have been best.¹³ The first alternative is based upon the assumption that the supply of inputs is perfectly elastic. In the second alternative the elasticity of input supply to a firm is perfectly elastic to a point and then perfectly inelastic. No competition for the limited amount of inputs is allowed among the representative farms. The advantage of the third alternative is that a stepped supply function for inputs could be incorporated into the model and the representative farms (if used in the model) could compete for the regional supply of inputs.¹⁴ The third alternative could be pursued several ways. In the regional model(s) the concept of the representative farm could be disregarded entirely. Aggregate regional constraints could be used as a substitute. Another alternative would be to build the representative farms into a regional model by including resource constraints and activities of each

¹²Assuming there will be no shift in the supply functions for inputs over the period of analysis.

¹³However, at the time the Phase I analysis was made, a model that contained both the detail of representative farms and regional constraints was not computationally feasible.

¹⁴For further comments, see page 58.

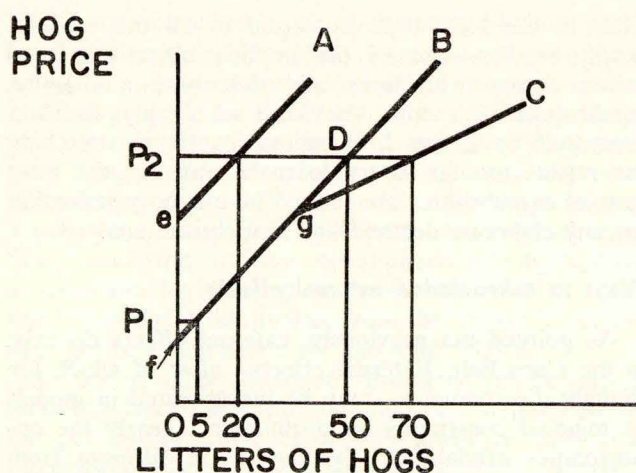


Fig. 5. Hog production on a representative farm.

representative farm within the region in addition to the aggregate regional constraints.

Internal consistency

The problems of externalities affect the Phase I type of aggregation model where no regional restraints are included. Other problems, however, are encountered that apply to all linear programming representative farm models. One is the problem of internal consistency. Internal consistency relates to the traditional problem of trying to estimate a curvilinear production function with a linear model. A point on a supply curve is internally consistent if the optimal solution at that point agrees with the assumptions about enterprise size built into that activity. A point on an aggregate supply curve is internally consistent if the corresponding points on all the representative farms' supply curves are internally consistent.

Internal consistency may be described most easily by examining an individual representative farm. Assume that the program solutions are obtained on the farm for every price of hogs from zero to \$30. Let line OfB in fig. 5 be the supply curve for hogs on this farm. Linear supply curves are used in fig. 5 rather than the normal linear-programming (stepped) supply curves to simplify the presentation. The coefficients in the hog activities (variable cost, use of equipment, labor coefficients, etc.) are based upon a specific size of hog enterprise, say 50 liters. The production function is assumed linear in the linear-programming model, but the actual production function is somewhat curvilinear. Thus, solutions that deviate from 50 liters will lack internal consistency. Point D on the optimum supply curve is internally consistent. If price P_1 were used in the model, production of hogs on farm B would be 5 liters. This solution differs substantially from the assumptions about size of enterprise built into the coefficients. Internal consistency is more of a problem as the production function of an enterprise becomes more nonlinear.

When the supply curves of several representative farms are aggregated, the problem of internal consistency becomes further complicated. Consider a hypothetical region consisting of two farms. Assume that the linear-programming model used on both farms contained identical hog activities; i.e., the coefficients in the hog enterprise for both farms are based upon a 50-litter hog enterprise. Let the optimum supply functions for these two farms be OeA and OfB in fig. 5. The regional supply curve would be OefA. At price P_2 , farm A would produce 20 liters and farm B, 50 liters, giving a regional total of 70 liters. Hog production from farm B is consistent with assumptions underlying the hog coefficients, but the production from farm A is not. Indeed, no point on the aggregate supply curve OfgC where positive quantities are produced is internally consistent.

Two conclusions can be drawn from the hypothetical example. First, farm A and farm B ideally should have different coefficients in their respective hog activities. Second, internal consistency becomes a more critical factor as the difference between the assumed sizes of the enterprise and the optimal size increases.

An attempt was made by each participant in the NC-54 project to make his model internally consistent at certain price combinations. But there was no reason to believe that this was done at the same price combinations by all participants. By test-programming several representative farms, one could estimate the levels the activities would come into the optimum solution for each set of price combinations. However, the optimum solutions would differ drastically over the 27 price combinations used in this project. Only one set of activities was included in the linear programming model for each representative farm. At most only a few of the solutions could be made internally consistent. The researcher would then have to choose which price combination should exhibit the most internal consistency.

As an example, consider Iowa representative farm number 3. The coefficients in the hog and beef activities for this representative farm are based upon the assumption that there will be approximately 40 liters in the hog enterprise and approximately 100 calves in the beef-feeding enterprise.

Table 1 shows 9 of the 27 optimal solutions for this farm. At all price combinations the programmed size of enterprise differs substantially from the size of enterprise assumed when the coefficients were developed. Thus, none of the solutions is internally consistent. In solution 5, for example, 162 liters of hogs are to be farrowed and 13 calves are to be fattened. Both figures differ substantially from the assumptions made when the coefficients were developed. Of course, the interesting question is whether the data in table 1 would be substantially altered by using technical coefficients that are correct for the size of enterprise.

Another approach would be to approximate the increasing returns portion of the production function

Table 1. Optimal hog and beef production on Iowa representative farm number 3.

Corn	Price levels		Litters farrowed	Calves and yearlings fed
	Hogs	Beef		
M	L	L	68	37
M	L	M	16	195
M	L	H	0	309
M	M	L	166	0
M	M	M	162	13
M	M	H	77	204
M	H	L	168	0
M	H	M	168	0
M	H	H	153	59

with a mixed-integer programming model. This is discussed by Musgrave (11) and others, but will not be presented here.

To the extent that the relationship between inputs and the size of the enterprise is linear in the totals but starts with a positive fixed cost, the problem is greatly simplified. In such cases it would only be necessary to predict—test program—whether or not the representative farm would have a beef, dairy or hog enterprise. If this prediction is correct, then points on the supply function will be internally consistent.

To summarize, because of external effects, only the Phase I results near 1965 production levels were acceptable. It would have been desirable that these solutions would have also been internally consistent. Nothing was done, however, to assure this internal consistency.

Specification of costs

The shape and location of the regional supply curves are primarily determined by production costs. Thus, the specification of costs is vitally important. This section evaluates several of the more critical assumptions that affected costs of production.

The assumption about level of technology is a critical factor in the determination of production costs. A change in the level of technology will change the factor-product transformations and cause the costs of production to shift. A high level of applied technology is assumed in the Phase I model. If a lower level had been assumed, the production costs would have been increased. This is pointed out to emphasize the critical nature of this assumption and not to criticize the specific level of technology used in the Phase I model. In addition the assumption of technology level becomes more critical as greater emphasis is placed on the aggregate aspects of the model.

The effect of the level of technology on the Phase I model was studied at Iowa State University (15). It was found that if average 1961 technology were assumed for input-output coefficients in the model, the model results would approximate 1965 levels of production of hogs and cattle in Iowa at about \$12.50 per hundredweight and \$22.50 per hundredweight, respectively. With the high level of applied technology

assumed in the model, the same levels of output of hogs and cattle were achieved at about \$10.75 and \$16 per hundredweight, respectively. In both cases the sale price of corn was \$1, and all other assumptions were held constant.

A second assumption in the Phase I model that has an important effect upon costs of production is: "Fixed costs are not charged on *existing* buildings and machinery."¹⁵ If fixed costs had been charged, the supply curves would have been altered—especially at quantities near historical levels. Admittedly, there are some critical decisions to make in the process of supply curve estimation regarding which inputs to hold fixed in the linear programming model and which to allow to vary. It would seem that the 1970 target date assumed for this model would warrant varying *all* machinery costs. A further discussion of length-of-run problems is presented in the next section on "economic time and calendar time."¹⁶

In addition, a means might be devised to include the pure overhead costs into the model; e.g., such things as the farm share of the auto and farm utility bills. One possible solution is to make an *ex post* adjustment in the programmed prices that would raise them enough to cover these fixed costs. Thus, the supply functions would be raised by the amount necessary to cover these costs.

One of the results of the study was to demonstrate which farms in the region have a competitive advantage in the production of hogs and cattle. Theoretically, if two farms have identical variable costs for producing a given output but the second farm has twice the fixed costs of the first, the first farm will have a competitive advantage—except in the short run. This difference will not show up in the Phase I results because fixed costs are not included in the model. Of course, the problem of what to vary and what to hold fixed will have different solutions depending on the main objectives of the study.

It is also assumed that family labor is of value only in farm production. In many areas of the Corn Belt nonfarm employment opportunities are plentiful. But in some parts, nonfarm demands for farm labor are limited. In either case there is still a strong justification for charging a reservation price for labor. If no reservation price is charged, the value of the marginal product of labor can be driven to zero. If any reservation price is charged for labor, the aggregate supply curves will shift upward.

A similar argument could be developed for charging a discount rate for risk. However, the problem of choosing the proper discount rate or the proper reservation price for resources is somewhat arbitrary. Either a discount rate or a reservation price might bring re-

¹⁵"Fixed costs" are charged in the model on purchases of buildings and equipment.

¹⁶In the Phase II model, the upper limit on opportunity costs of resources was acquisition cost with appropriate attention to depreciation, interest, repairs and taxes and the lower limit was disposal value with appropriate attention to savings on those items. In the Phase II model, acquisition costs exceed disposal value by the amount of transfer costs.

gional livestock production and prices into reasonable bounds, but the appropriate combination of the two is a difficult question—perhaps one that cannot be researched.

A fourth assumption that had a less-important effect on costs was that real estate assets, as well as chattel assets, could be mortgaged to obtain funds to meet operating expenses. Since interest charges were lower on real estate credit, it would be the first source of credit used for operating expenses. But this practice is not generally found in actual farming situations. If real estate credit had not been allowed, the cost of financing large enterprises (especially beef feeders) would have been higher. However, removing the option to use real estate credit for operating expenses would not have made capital limiting on most representative farms at 1965 production levels.

Economic time and calendar time

Our concepts of economic time are borrowed from firm or micro theory. But the economic time concept is nebulous when used in aggregative or macro analysis. In the aggregate, long-run, intermediate-run and short-run decisions are being made by farmers at any point in calendar time. In aggregative analysis it would seem appropriate to allow everything to vary, but by a limited amount. With respect to Phase I, does it seem appropriate in a study with a target date a decade away from the initiation date to assume a fixed structure of agriculture (no change in the number and size of farms and no change in land/labor or labor/capital availability ratios) and at the same time assume continued adoption of new practices and production techniques? To assume away some of the major adjustment opportunities available to the agriculture of a region does not seem an appropriate procedure. An alternative procedure might be to allow farm size, the land/labor and labor/capital availability ratios and the technology to vary, but by a limited amount.¹⁷

Studies could be made of the historical rate of change in variables such as these. The magnitude of admissible change could then become a means of translating between economic and calendar time. These studies would insure greater agreement with respect to time between the various parts of the model—for example, the assumptions regarding a fixed quantity of land per farm but a greatly improved technology. If studies of the rate of change of the variables used in the adjustment studies were initiated, the results would facilitate the improvement of intertemporal models of the year-to-year dynamic or recursive variety as well as comparative static models. But we repeat, the farm is by no means the only relevant level at which to study the rates of change. Rates of change in macro variables, such as the resources available to a region or imports and exports of the region, are equally important because

rates of change among representative farms must take place within the framework of the changes in resources available to the region.

Identification of representative farms

In the Phase I model, aggregation error arises from linear-programming representative farms to estimate area supply functions rather than linear-programming all the individual farms in the area. The aggregation problem may be one of the more critical problems of a programming project like NC-54. At the same time it is often passed over quite lightly by the researcher who is in a hurry to attack what at first appear more pressing or concrete problems.

The typical procedure of identifying representative farms for Phase I type studies is: (1) Collect data on relevant resources such as cropland, pasture, labor and capital for a large sample of farms in the population of interest; (2) array the farms by two or three of the most important factors thought to affect production response and develop a two- or three-way frequency distribution or stratification; and (3) identify a typical or representative farm for each cell of this stratification containing a significant number of farms.

The objective of the stratification is to divide farms into groups so that farms within each group will have similar response patterns—similar optimal farm plans. Then each group can be represented by a “representative farm” and the aggregation error can be held to a minimum. The researcher faces two opposing goals in setting up and following such a stratification procedure. On one hand, there is the cost of analysis or computing and the pressure to reduce the number of representative farms analyzed. On the other hand, there is the desire for reliability and the notion that the aggregate estimate will be more accurate as the number of representative farms studied is increased. Reaching a satisfactory compromise between these two opposing objectives is no easy task, especially when the relationship between accuracy and the number of representative farms is not clear. Often the researcher merely stratifies his farms into the number of cells for which he has funds and time to analyze.

A procedure similar to the three-step procedure just outlined was followed to set up the representative farms for NC-54¹⁸. In all states except Iowa, the data were obtained from farm surveys. In Iowa, selected data were obtained for a 5 percent sample of individual farms from the U.S. Bureau of Census. These basic data were supplemented in Iowa with a mailed questionnaire to bankers and county agents. The costs were substantially less than a survey and the results seemed adequate.

In most of the states, the farms were stratified on the basis of farm size, geographical area (or soil type)

¹⁷For an application of this—and a good bibliography of other work in this area—see Schaller and Dean (14).

¹⁸The NC-54 committee discussed at length the procedure to be used for farm stratification. The weaknesses of the method used were recognized, but it was still considered the best procedure available.

and type of farm. Information available at the time of stratification suggested that these three factors were important in determining response patterns.

One representative farm was delineated to represent each cell of this stratification. Here, three alternative methods were used. Some states defined the representative farms as the average farm in the cell. In other states, each important cell of the stratification was represented by a typical rather than an average farm. In other words, the modal farm from each cell was chosen to represent the cell. The third method was a combination of the two listed: an average for easily divisible inputs like acres and a mode for "lumpy" inputs like silos.

The aggregation coefficient for the modal farm was defined as the total cropland in the cell divided by total cropland on the modal farm, rather than the alternative method (when the average farm was used to represent a cell) of defining the aggregation coefficient as simply the total number of farms in the cell. There is no simple and rigorous way to evaluate these two methods of classification. It would appear that the "modal method" may have an advantage when looking at the effect of the aggregate adjustments on a typical real world farm, but the "average method" may give the better aggregate estimate since its resources reflect the variance of resources of all farms found in the cell. The best method to use would depend upon the area of emphasis in the over-all project.

Several shortcomings can be seen in the procedure used to develop the representative farms for NC-54. In general, for a given set of prices the linear programming solutions are similar among the representative farms. In many areas the magnitudes of the results were nearly proportional to the sizes of the representative farms. Thus, in these specific areas, stratification by farm size added little to the analysis. In other areas the same comment could be made about the farm-type stratification. The restraints and activities included in Phase I do not capture those elements that cause some farms to specialize in grain production, some in beef production and others in hog production. The number of representative farms could have been reduced substantially without having a significant effect upon the aggregate results. Stated another way, the same results could have been obtained with fewer representative farms at considerable savings in programming costs without a significant buildup of aggregation error.

This raises the question of possible alternative stratifications of farms that would have the potential of reducing aggregation error in the estimated supply functions.¹⁹ To answer this question, consider the variety of different factors that may affect supply response of individual farms:

- (1) physical environment, such as climate and topography,

- (2) institutional restrictions, such as markets and government regulations,
- (3) motivational forces, including risk aversion and demand for leisure,
- (4) management ability,
- (5) technology and
- (6) resource endowments.

All these factors cause different individual farms to react differently to a given economic stimuli and, hence, become possible sources of aggregation error if they are neglected during the stratification and delineation of representative farms. The NC-54 stratification of farms was based on differences in factors 1 and 6. Differences in factors 2 through 5 were generally assumed away.

This framework suggests two possible approaches for improvement in the stratification procedure. The first is re-stratification on the previously used factors, 1 and 6, to better reflect differences among individual farms. For example, delineation of representative farms to account for the extremes of resource ratios may increase differences among the representative farms and thereby decrease aggregation error. This would require a significant increase in the number of farms programmed. The original NC-54 stratification could probably not be significantly improved as long as the same factors are used.

A second approach would appear more promising. This is stratification according to some of the factors listed in categories 2 through 5. Failure to recognize these factors is one of the shortcomings of the NC-54 work. Of course, it may be argued that ignoring these factors agreed with the rest of the assumptions of NC-54; that is, in the normative framework of NC-54, differences in preferences, management ability and institutional factors are not relevant. However, consideration of such factors should add both realism and accuracy to the resulting estimates.

Another shortcoming is that the aggregation coefficients and representative farms derived for NC-54 are based on farm numbers and sizes in the early sixties. It would have been more realistic to project these data to 1970, as was done with most of the coefficients used in the study. This projection should provide a more realistic resource base for the NC-54 work.

In the last few years, an increasing amount of theoretical and empirical work has been aimed at the problems of representative farms and aggregation bias. A fairly complete and up-to-date review of contributions in this area has been made by Sheehy and McAlexander (12). Empirical results have also been reported in the Sheehy and McAlexander article and in the work of Frick and Andrews (5), Barker and Stanton (2) and Miller (9). Work on the theory of aggregation error has been done by Miller (9, 10) and Lee (7).

Research people interested in representative farm specification will want to review these ideas. In gen-

¹⁹In this discussion the specification of the Phase I model is "given" and aggregation error is examined.

eral, it is desirable to define a representative farm for every separate group of real world farms with "significantly different production characteristics." The more accurate one is in defining what constitutes "significantly different production characteristics," the more reliable will be the aggregate estimate. The definition changes, however, depending on (1) the farming area of interest, (2) the type or types of aggregate estimates required and (3) the degree of accuracy required. As was mentioned before, the amount of research resources available for the analysis also is a factor. Since research resources are generally limited, some compromise between aggregation error and computing costs will have to be made.

The factors just mentioned usually vary among research projects. This leads to a unique answer to the representative farm problem for each specific research project. Current empirical studies of the representative farm identification problem in somewhat diverse problem areas are a step in this direction.

ALTERNATIVE FORMULATIONS

Two ways are considered to explore the objective of comparing the competitive position of different areas and types of farm organization: (1) Retain the Phase I approach and ignore the problem of external effects or (2) reformulate the model to include regional restraints. The first alternative is useful where the region is so small that external effects can be ignored or where emphasis is placed on the farm management implications of representative farm results, but the aggregate implication of these individual farm adjustments is also desired. The problems associated with the first method of formulation were discussed in the previous section. The second alternative is worthy of further discussion.

At this stage in the development of supply analysis, it may be too much to expect any formal model to be a good predictor of agricultural production and prices 5 or 10 years in the future. However, it is possible that a ranking of the benefits of alternative policy measures would be quite valuable even if the estimates of absolute benefits were quite wide of the mark. More explicitly with respect to regional adjustment studies such as NC-54, we believe that an analysis of the effects on the competitive position of areas and farms resulting from different levels of instrument variables would be of considerable value. Some examples of instrument variables are the provisions of the feed grain program, the level of technology and the admissible beef and pork production based on changes in aggregate production or changes in the region's share of the national market.

Economic models of regions or sectors are recognized as simplifications of the real world. Yet, they are expected to capture the significant operating characteristics of the sector under study and thus are often subjected to a "realism" test. Basically, this tests

whether or not the model yields results consistent with other theoretical or empirical observations. Tests of realism may be applied to many variables. Examples are: (1) product and factor prices at the national, regional and farm levels; (2) imports and exports of products and factors by the nation, region or farm; (3) the rate of change in the number and size of farms; and (4) the compatibility of the level of input-output coefficients with a timespan of the model and the level of production indicated by the model solution. The first three in this list can be handled more realistically with a regional model. The fourth is no more difficult (and no easier) on a regional level than on an individual farm level. One cannot reasonably expect a model to meet all tests of realism, but the analyst does have considerable freedom in the construction of a model to insure that the more relevant tests are met.

The analyst has some *a priori* knowledge about the relevant variables in the region he is analyzing. He knows that for the model to be realistic, certain relationships among the variables must hold. It does not seem reasonable, for example, for the Corn Belt to be importing corn or for one state in the Corn Belt to produce enough beef for the whole country.

All available *a priori* knowledge should be used in the construction of the model rather than to ignore some of that knowledge and hope that the results will be realistic anyway. Thus, if external effects exist (i.e., if the sum of the opportunities available to the firms do not equal the opportunities available to the region) a model including regional constraints would be superior to a representative farm (no regional restraints) model.

W. Neill Schaller developed a chart that may help us to gain some perspective of the alternative approaches to the analysis of aggregate supply (13). We have made some changes to adapt that chart to the purposes of this report (fig. 6).

The NC-54 (Phase I) project and other regional adjustment projects can be said to have used approach A of fig. 6, which can be characterized as the micro approach. In this approach, data on individual farms are collected from farm surveys and other sources. A micro model is constructed for each of a series of representative farms. The results are summed to give estimates of supply. The demand analysis, although not usually conducted by production economists participating in the regional projects, follows the approach outlined in the center line of fig. 6. The processing, marketing and transportation block is largely short-circuited by transforming the demand functions to the farm level and ignoring the possibility of changed flows of products resulting in changed locational differentials of product prices.

Approach B may be characterized as a micro-macro approach. The distinguishing characteristic of this approach is that the model is constructed at a higher level of aggregation than that of the representative farm, thus facilitating the use of macro constraints. These constraints limit the opportunities available to the region

Approach A: Micro Approach

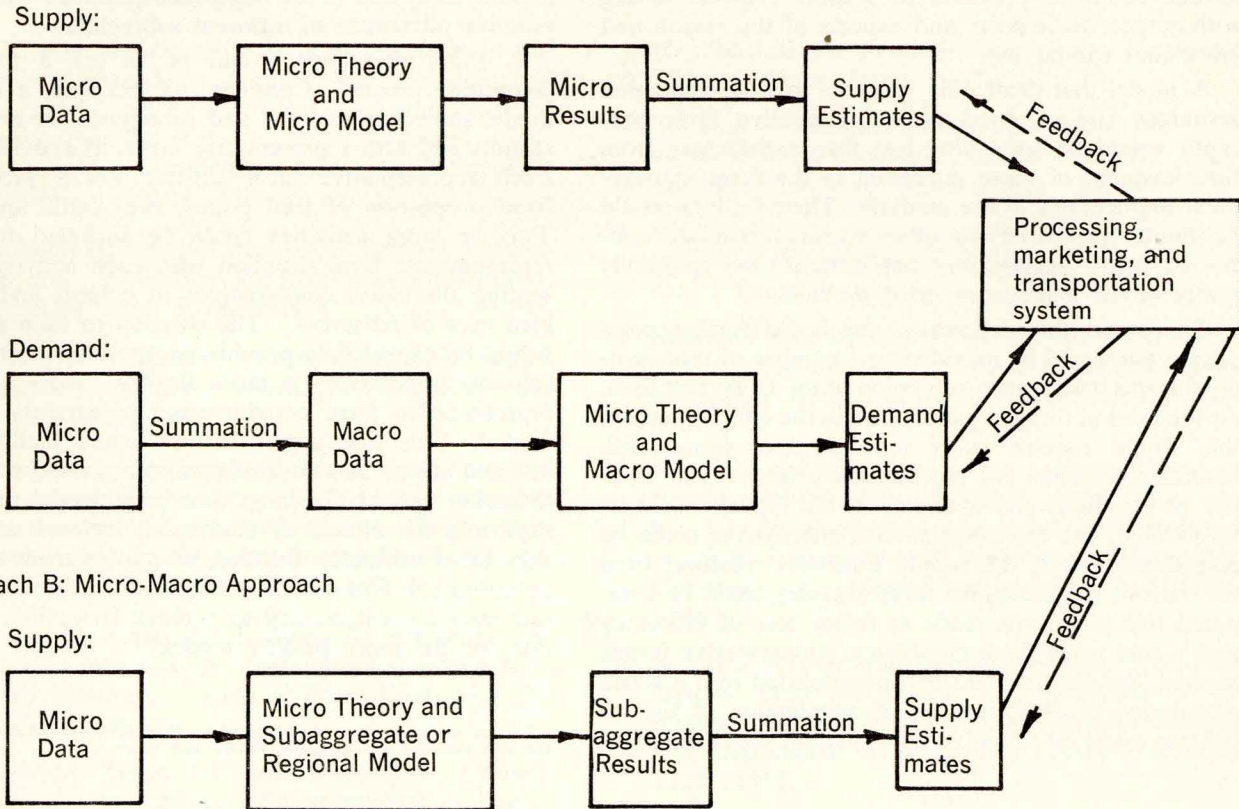


Fig. 6. Alternative approaches to the analysis of aggregate supply.

to something less than the sum of the opportunities available to the individual farms. The macro constraints derived from macro data make it possible to keep production, resource use and prices within the bounds of reasonableness defined from recent experience.²⁰

There are, of course, a large variety of possible models that would fall within the category of micro-macro models. They might differ from one another on one or more of the following features: (1) the treatment of demand (fixed or functional form), (2) the degree of detail incorporated in the processing and transportation sector, (3) the number of resource constraints (single resource such as land or a complete listing of resources), (4) the degree of aggregation at which the production or supply model is developed, (5) the use of representative farm or resource situations and (6) the timespan of the analysis. We will concentrate our discussion on the production sector of the over-all model.

Consider the question of the adequacy of a region

as a unit of analysis. Clearly, the Corn Belt is not an isolated unit. Rather it depends upon the western United States for many of its feeder cattle and upon the entire country as a market for its products. Conceptually, at least, the region is something less than ideal as a unit of analysis. Yet in the foreseeable future, models that are national in scope are not likely to provide information at a point anywhere near the level of disaggregation exemplified by the NC-54 project. Obviously, there is no clear-cut solution to the dilemma. Yet, we believe a regional model is worthy of further investigation. A regional model could take any one of several forms, ranging from one extreme of a near national model including the Corn Belt in a relatively disaggregated form and the rest of the United States depicted by one or more highly aggregated regions to another extreme with the region (Corn Belt) assumed a closed but not necessarily self-sufficient economy. In the former case, production would compete in national markets with regional and national production and prices determined in the model. In the latter case, regional imports and exports and prices would be determined outside the model. Even in the latter case, it seems probable that the model could provide much useful information about the competitive position of different areas within the region and different types of farm organizations within areas. Further,

²⁰Purdue University and the Farm Production Economics Division, U.S. Department of Agriculture, have a cooperative project as a part of NC-54 to build a regional model containing macro constraints. John Berry is in charge of the project. Gaylord Worden is working on a similar model as part of a cooperative project between Farm Production Economics Division, U.S. Department of Agriculture, and Iowa State University.

in contrast to the current Phase I approach, the information could be provided in a more realistic setting with respect to imports and exports of the region and labor and capital use.

A model that dealt only with the area or subregion resources (i.e., ignored the representative farm concept) would be something less than satisfactory from the viewpoint of those interested in the farm management implications of the analysis. Their feelings would no doubt be shared by other researchers more concerned with aggregative implications but painfully aware of the aggregation-error problem.

To correct this deficiency, one could further complicate the model by introducing a number of representative farms within each subregion of the Corn Belt area. At the level of disaggregation used in the NC-54 project, this would require many representative farms with several constraints per representative farm. The number of equations and activities in the model would be formidable. At this point several alternatives could be considered. First, the supply functions resulting from the various representative farm analyses could be compared and judgments made as to the loss of efficiency that would result from combining representative farms. Second, the model could be disaggregated into a series of submodels. Although the disaggregation of the resources might be rather straightforward, the disaggrega-

tion of demand on a share of the market basis would assume away one of the important questions—the comparative advantage of different subregions.

A third alternative would be to use a two-stage computing process. Consider, for example, a regional model subject to regional and subregional or area constraints and with representative farms as activities (1). Each representative farm activity would produce a fixed proportion of feed grains, beef cattle and hogs. Two or more activities could be included for each representative farm situation with each activity representing alternative combinations of outputs and associated uses of resources. The solution to such a model would be expected to provide an approximation of the solution achieved by a more flexible model in which representative farm activities and constraints are included. This first approximation would facilitate the external effects and internal consistency checks referred to earlier and at the same time provide the means of exploring the effects of alternative regional and subregional constraints. Further, an approximation of the optimum solution should greatly reduce the computing time and costs necessary to achieve an optimum solution for the more flexible model.²¹

²¹For an example of a two-stage computing procedure where land is the only regional restraint, see Varley and Tolley (17).

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