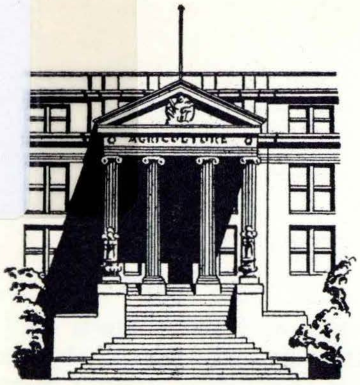

Fertilizer Production Functions in Relation to Weather, Location, Soil and Crop Variables

by John T. Pesek, Jr., Earl O. Heady
and Eduardo Venezian

Departments of Agronomy and Economics
and
Tennessee Valley Authority, cooperating

AGRICULTURE AND HOME ECONOMICS EXPERIMENT STATION
IOWA STATE UNIVERSITY of Science and Technology

RESEARCH BULLETIN 554 . . . AUGUST 1967 . . . AMES, IOWA

CONTENTS

SUMMARY

This study is based on several long-term experiments of crop fertilization at three Iowa locations: Howard County, with Clyde and Cresco soils; Hancock County, with acid and calcareous Webster soils; and Wayne County, with Seymour soil. The trials at these widely separated experimental farms included corn and oats fertilized in a 3-year rotation of corn-oats-meadow. The periods of the trials were: Clyde and Cresco soils, 1945-1960; Webster soils, 1954-1960; Seymour soil, 1949-60. Although meadow was not fertilized, residual nutrients from fertilization of the oats nurse crop were expected to affect hay yield. Applied nutrients included only phosphorus and potassium.

The objectives of the analysis were: (a) to estimate annual production functions for each crop and compare them with average production functions estimated from the several years of data for the same crops, locations and soil types; (b) to analyze the variability or degree of uncertainty involved in such physical and economic relationships as isoquants, isoclines and profit-maximizing nutrient inputs; (c) to estimate weather indexes and their quantitative relationship to fertilizer response; (d) to estimate generalized production functions that incorporate weather, soil nutrients, location and soil into the production function along with the quantities of K and P applied annually.

Numerous algebraic forms were tested in the analysis of the data. A conventional quadratic form was selected as the best function for all years and crops. Since the trials were all based on a 3 x 3 factorial, maximum response was infrequently attained. This design is seldom adequate for estimating fertilizer response functions, although statistically acceptable annual functions were estimated for various crops in particular years. The average functions, derived by pooling the data over all years for each crop at each location, provided somewhat more stable estimates. The values of R^2 for annual corn functions on Clyde soil were all more than 0.996. The range was somewhat greater for annual corn function on Cresco soil; but R^2 for the average function was 0.998, and regression coefficients were all significant at the 0.01 level of probability. Results were somewhat more variable for oats, and especially for hay where inputs were measured as the amount of nutrients applied on oats in the previous year. The best annual function was obtained for oats on Clyde soil where all five regression coefficients were significant at the 0.05 probability level, and the value of R^2 was 0.990. The regression coefficients for the annual hay functions were sometimes unstable with respect to signs. The average functions, however, had signs for regression coefficients consistent with logic and R^2 values of 0.969 for Clyde soil and 0.991 for Cresco soil. Less than half of the individual regression coefficients estimated for hay were significant at the 0.20 level of probability.

Analysis was made to determine the economic efficiency of decisions where expectations might be based on (a) the annual optimum nutrient inputs where it is assumed that the response function for the particular year is known, (b) the function with greatest annual fertilizer response, where it is assumed to prevail in each year, (c) the function with the lowest annual response, where it is assumed to prevail in each year, and (d) the average production function, where it is assumed to prevail in each year. Compared with the annual optimum, the assumption of the lowest response was too conservative. Although it prevented loss in each year, it gave a lower average return per acre over several years than did assumptions using the highest response as the average response. With annual decisions based on the average function, returns would have been slightly higher than for those based on the highest response. However, further and more elaborate experiments are necessary before the efficiency of these various assumptions can be fully tested.

Confidence limits were estimated only from the average functions for corn on Cresco and Clyde soils. On the basis of these, a 70-bushel corn isoquant is predicted, for example, with application of 20.8 pounds of K and from 2.0 to 8.7 pounds of P on Clyde soil. A parallel prediction for Cresco soil is a 75-bushel corn isoquant with 12.5 pounds of K and from 4.8 to 7.4 pounds of P. Confidence limits for isoclines included the predicted point of maximum yield in each case. The confidence regions for profit-maximizing inputs were much smaller for the average corn function on Cresco soil than for similar estimates on Clyde soil.

In preparation for estimating the generalized functions, a combined analysis of variance was per-

formed to assess the relative contribution of soils, weather and fertilizer nutrients to yield variation at each experimental location and generally indicated that weather contributed the largest proportion of yield variance. Soils and fertilizer treatments also contributed significantly to yield variation. Significant interactions between soils and weather, soils and fertilizer treatments, and weather and fertilizer treatments were also present. The weather indexes, estimated from rainfall and temperature data for each crop at each location, explained approximately 57 percent of the variance for corn yield, 60 percent for the oats yield and 70 percent for hay yield.

The R^2 values of the generalized functions were all between 0.552 and 0.928. The over-all regressions were all highly significant. Frequently, the partial regression coefficients of the variables were statistically significant at the 0.01 probability level. Even though isoquants, isoclines and marginal rates of substitution were derived from two of the generalized corn production functions, results for only one of the functions were generally acceptable. Further improvements in the collection of soil test and weather data are needed before the procedure can be feasible for use in practical recommendations. Also, experimental designs including a wider range and larger number of applied nutrient levels are needed for improving estimates over time and in relation to the auxiliary variables studied. With these improvements, however, it is likely that generalized response functions can be estimated both to improve decision making as it relates to fertilization under weather variability and to allow greater information over more soils and locations from given research funds.

Fertilizer Production Functions in Relation to Weather, Location, Soil and Crop Variables[1]

by John T. Pesek, Jr., Earl O. Heady
and Eduardo Venezian

Knowledge of yield response functions is essential for the most profitable use of fertilizer. However, estimation of fertilizer-generated production or yield response functions is complicated by variables relating to soil characteristics, crop rotations, weather and other environmental conditions. The magnitude of these variables differs among locations and over time. An empirical production function estimated from data of one experiment often has little predictional value beyond the specific year and experimental conditions under which it was derived. If information of more general application and greater practical usefulness is to be obtained, fertilizer trials must be extended to cover a wider range of the variables just mentioned.

Previous studies of the economics of fertilizer use ordinarily have been based on experimental data for one location and year and for a single crop within the rotation. Hence, certain aspects of fertilizer use, such as the variability and economic implications of yield response over time, have not been analyzed on the basis of experimental results.

ESTIMATES IN THIS STUDY

This study includes fertilizer production functions estimated for crops grown in a rotation sequence on different soils and over a period of years. Production or fertilizer response functions are estimated separately for corn and oats grown in different years and at different locations in this rotation sequence. Production functions also are estimated for the total value product forthcoming from this rotation on different soils. Attempts are made to generalize the annual production functions to account for yield variation due to location, soil and weather. Optimum rates of phosphorus and potassium fertilizer application are derived from the estimated production functions for various economic conditions, as represented by alternative crop and fertilizer nutrient prices, and for various amounts of capital.

The use of common or fixed-ratio fertilizer mixtures, as compared with use of least-cost nutrient combinations, based on production functions, also is analyzed from the standpoint of profits. Finally, analysis is made of the statistical variability of economically optimum quantities derived from the estimated production functions. The implications of such variability are discussed in terms of their practical importance in recommendations based on empirical production functions.

DATA AND PROCEDURES

Basic data are from five long-term experiments that included applications of phosphorus and potassium fertilizers to a corn-oats-meadow (C-O-M) rotation. Varieties of corn and oats, although not identical in all experiments, were those available and considered best adapted to each location. The meadow, a forage mixture with alfalfa the dominant species, was seeded with oats. Land was fall plowed for corn in the year following meadow.

Experimental designs, 3 x 3 factorials, included phosphorus applied at 0, 20 and 40 pounds per acre per rotation (0, 45 and 90 of P_2O_5) and potassium at 0, 37 and 74 pounds per acre (0, 45 and 90 of K_2O). One-third of the fertilizer for each rotation was applied in the hill or row to the corn and the rest broadcast ahead of the oats.

The three Iowa locations where experiments were conducted were: (a) Howard County Experimental Farm, Howard County; (b) Clarion-Webster Experimental Farm, Hancock County; (c) Seymour-Shelby Experimental Farm, Wayne County. Experiments at the first location were conducted simultaneously on two different soil types, Clyde and Cresco. At the second location, experiments were on a calcareous Webster and on an acid Webster soil. The experiment was on Seymour soil at the third location. In order that yield observations would be available for each year and each crop in the C-O-M rotation cycle at the Clarion-Webster and Seymour-Shelby farms, three land segments within each soil type were used (table 1).

The experimental design for the Howard County Farm fertilizer trials was a randomized block with three replicates per set of treatment combinations. At the Clarion-Webster and Seymour-Shelby farms, the design was of randomized complete blocks, with the blocks replicated twice.

---

[1] Projects 1135, and 1148 Iowa Agricultural and Home Economics Experiment Station.

Table 1. Cropping plan and the time periods over which experiments were conducted at each location.

Year	Soils							
			Seymour			Acid and Calcareous Webster		
			Land Segments			Land Segments		
	Cresco	Clyde	A	B	C	A	B	C
1945	Oats	—	—	—	—	—	—	—
1946	Meadow	—	—	—	—	—	—	—
1947	Corn	—	—	—	—	—	—	—
1948	Oats	Oats	—	—	—	—	—	—
1949	Meadow	Meadow	Corn	—	Oats	—	—	—
1950	Corn	Corn	Oats	—	Meadow	—	—	—
1951	Oats	Oats	Meadow	Oats	Corn	—	—	—
1952	Meadow	Meadow	Corn	****	Oats	—	—	—
1953	Corn	Corn	Oats	Corn	Meadow	—	—	—
1954	Oats	Oats	Meadow	Oats	Corn	Corn	—	—
1955	Meadow	Meadow	Corn	Meadow	Oats	Oats	Corn	—
1956	Corn	Corn	Oats	Corn	Meadow	Meadow	Oats	Corn
1957	Oats	Oats	Meadow	Oats	Corn	Corn	Meadow	Oats
1958	Meadow	Meadow	Corn	Meadow	Oats	Oats	Corn	Meadow
1959	Corn	Corn	****	Corn	Meadow	Meadow	Oats	Corn
1960	Oats	Oats	Meadow	Oats	Corn	Corn	Meadow	Oats

— No crop that year
**** Crop failure

Table 2. Corn on Clyde Soil: Regression coefficients (b_i), standard errors, t-values and coefficients of determination (R^2) during specified years.

Item	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1950	24.15	0.98267	1.74533	−0.05717	−0.39657	0.02881	0.975
Standard error		0.70200	0.36908	0.04877	0.01348	0.01813	
t		1.400	4.729	1.171	2.942	1.590	
Probability level[a]		b	*	c	+	b	
1953	38.71	0.91444	1.89015	−0.13030	−0.04601	0.06991	0.988
Standard error		0.61111	0.32129	0.04247	0.01114	0.01579	
t		1.496	5.883	3.069	3.921	4.430	
Probability level[a]		b	**	+	*	*	
1956	44.17	0.31110	3.39724	−0.04163	−0.08181	0.03743	0.996
Standard error		0.42522	0.22356	0.02956	0.00817	0.01098	
t		0.732	15.196	1.410	10.017	3.407	
Probability level[a]		d	**	b	**	*	
1959	56.90	0.10285	2.06442	−0.08867	−0.05160	0.07927	0.985
Standard error		0.73089	0.38427	0.05076	0.01403	0.01888	
t		0.141	5.372	1.746	3.676	4.198	
Probability level[a]		d	*	++	*	*	
Average function	40.98	0.57789	2.27389	−0.07943	−0.05475	0.05385	0.996
Standard error		0.37365	0.19645	0.02599	0.00718	0.00966	
t		1.547	11.575	3.060	7.630	5.579	
Probability level[a]		b	**	+	**	*	

—No crop sown that year

** $p \leqslant 0.01$
* $0.01 < p \leqslant 0.05$
+ $0.05 < p \leqslant 0.10$
++ $0.10 < p \leqslant 0.20$
b $0.20 < p \leqslant 0.30$
c $0.30 < p \leqslant 0.40$
d $\geqslant 0.40$ (not significant)

Regression Analysis for Annual Data

The first step in evaluation of yield response was an analysis of variance for each set of experimental data. Corn showed a fairly consistent response to both phosphorus and potassium on Cresco and Clyde soils. Response to potassium was dominant in most years on the Clyde soil. On the acid-Webster and calcareous-Webster soils, the corn response was mainly to phosphorus, and generally only a linear component was statistically significant. The yield effect of fertilizer on corn was unstable on Seymour soil. In some years significant response was absent; and, in other years, the linear component of phosphorus was significant. In one year, there were significant responses to both phosphorus and potassium.

Oats gave a strong response to phosphorus on Cresco and Clyde soils. Response also was consistent on Webster soils but generally mostly linear. The effect on oats grown on Seymour soil was variable, response being greater and more frequent for phosphorus.

Meadow showed significant responses to both phosphorus and potassium on Cresco and Clyde soils in the various years. The annual response was mainly to phosphorus on Webster soils, the effect of potassium usually being negligible. Meadow response was variable on Seymour soil since phosphorus had a significant yield effect during most years, and potassium also showed an effect in some years.[2]

Several algebraic forms of production functions were fitted to data of selected crops and years. However, the form selected for general application was the quadratic equation with the second order cross-product term. Regression equations were estimated for each crop, location and year. Results from these individual year estimates at the Howard County Farm are presented in tables 2 through 7. The levels of probability for the corresponding regression coefficients are those indicated in the footnote to table 2. The coefficients and related statistics all are ordered in the tables in conformance with the following form of production function, where C is

$$C = b_0 + b_1P + b_2K + b_3P^2 + b_4K^2 + b_5PK \tag{1}$$

crop yield per acre in bushels for grain and in tons of hay for meadow, P is pounds per acre of P and K is pounds per acre of K.

All functions were fitted with the independent variables coded as −1, 0, +1 for the low, medium and high levels of phosphorus and potassium application. The regression coefficients were decoded to express corn and oats response to fertilizer in pounds per acre applied in the particular year. The hay functions were decoded by the total amount of fertilizer applied to the crop rotation, although the actual nutrients available to the meadow did not correspond to such applied quantities.

The P and K terms were retained in the functions used for analysis, although the estimated regression coefficients were not always significant at the 5-percent level of probability.

In general, the quadratic equation fitted the data well as indicated by the high values of R^2. In several cases, however, the predicted surfaces were not convex due to the presence of positive squared terms. The potassium regression coefficients were unstable for several annual functions with a negative linear coefficient and a concave surface. Consequently, individual functions had to be selected that had the appropriate characteristics for a more detailed economic analysis. Two considerations were relevant in this respect: First, a theoretical consideration required the functions to have negative coefficients for the squared terms; second, the interest of the present analysis was in two-input relationships to allow

Table 3. Corn on Cresco soil: Regression coefficients (b_i), standard errors, t-values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1947	61.01	1.62930	0.84322	−0.09449	−0.00064	0.01656	0.899
Standard error		0.58229	0.30614	0.04047	0.01129	0.01540	
t		2.798	2.754	2.336	0.058	1.101	
Probability level[a]		+	+	++	d	c	
1950	47.38	3.99892	0.70108	−0.20101	−0.02171	0.00367	0.965
Standard error		0.69744	0.36668	0.04845	0.01339	0.01802	
t		5.734	1.912	4.148	1.620	0.204	
Probability level[a]		*	++	*	b	d	
1953	50.74	1.84746	0.73307	−0.13733	−0.02699	0.04079	0.927
Standard error		0.69932	0.36767	0.04861	0.01344	0.01808	
t		2.641	1.994	2.826	2.010	2.258	
Probability level[a]		+	*	b	*	d	
1956	70.74	1.18888	1.29696	−0.05019	−0.03385	0.00491	0.973
Standard error		0.44571	0.23433	0.03097	0.00856	0.01151	
t		2.667	5.535	1.620	3.955	0.426	
Probability level[a]		+	*	b	*	d	
1959	74.66	0.74847	0.31775	−0.03522	−0.01271	0.05777	0.950
Standard error		0.69860	0.36729	0.04856	0.01342	0.01805	
t		1.071	0.865	0.726	0.947	3.202	
Probability level[a]		c	d	d	d	*	
Average function	60.86	1.88337	0.62639	−0.10373	−0.01918	0.02478	0.998
Standard error		0.12164	0.06395	0.00845	0.00234	0.00315	
t		15.482	9.794	12.271	8.212	7.889	
Probability level[a]		**	**	**	**	**	

[a] See table 2 footnote.

---

[2] In general, for all annual production functions, the response was mainly linear. Higher rates of fertilizer application would have been desirable to reach the maximum yield point and the area of diminishing total product.

estimation of substitution relationships and the subsequent economic analysis of optimum input combinations. Thus, estimates were made for those sets of functions where both phosphorus and potassium showed a response and for which the surfaces, as described by signs of the regression coefficients, were convex. Largely, the average production functions at the bottom of tables 2 through 7 meet these requirements.

Data from the Howard County Farm best provided the desired characteristics and the analysis rests mainly on them. Production functions for corn on Cresco and Clyde soils, those for oats on Clyde soil, and some of the production functions for hay on Cresco and Clyde soils were selected for the derivation of technical and economic relationships. Several functions were discarded because of inconsistencies related to these mathematical relationships. A brief discussion of the production functions selected for each crop follows.

CORN

The production functions for corn were consistent, in signs of coefficients, with theory and previous technical knowledge. The R^2-values for individual year functions were between 0.899 and 0.998; F-tests of the over-all regressions were all significant at a level of 5 percent or lower. The partial regression coefficients for the annual production functions were generally acceptable only at large probability levels. The high probability levels were due mainly to the few degrees of freedom. The average corn function for Cresco soil (table 3) had all coefficients significant at the 1 percent level of probability.

The regression coefficients differed considerably from year to year within each soil, possibly as the result of weather variation, meadow growth the previous year and experimental errors. Therefore, interpretation of the individual yearly coefficients of the production functions is obscure.

OATS

The R^2-values for the oats response functions on Clyde soil (table 4) ranged from 0.902 to 0.990 in 3 of the 5 years. The 1948 response was nonsignificant, and the 1960 response had a strong P^2K^2 interaction, reducing the goodness of fit for the quadratic equation used. None of the partial regression coefficients were statistically significant in some years; for 1957, all terms were significant at the 5- or 1-percent levels of probability.

The shape of the oats response surfaces corresponds to theory, as the signs of the regression coefficients indicate. Hence, all production functions for oats on Clyde soil are used to derive the technical and economic relationships presented subsequently.

HAY

The estimated functions for hay were unstable,

Table 4. Oats on Clyde soil: Regression coefficients (b_i), standard errors, t-values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1948	47.90	0.35933	0.12535	−0.00798	−0.00260	0.00069	0.436
Standard error		0.48227	0.25356	0.01675	0.00463	0.00624	
t		0.745	0.494	0.475	0.562	0.111	
Probability level[a]		d	d	d	d	d	
1951	54.53	0.63810	0.21903	−0.01349	−0.00357	0.00254	0.902
Standard error		0.29227	0.15366	0.01013	0.00280	0.00378	
t		2.183	1.425	1.330	1.272	0.670	
Probability level[a]		++	b	b	b	d	
1954	22.55	0.76602	0.33314	−0.01759	−0.00583	0.00629	0.980
Standard error		0.18071	0.09501	0.00630	0.00174	0.00235	
t		4.239	3.506	2.801	3.359	2.693	
Probability level[a]		*	*	+	*	+	
1957	47.27	1.41775	0.24280	−0.04063	−0.00550	0.00582	0.990
Standard error		0.13830	0.07271	0.00483	0.00133	0.00179	
t		10.252	3.339	8.458	4.146	3.263	
Probability level[a]		**	*	**	*	*	
1960	49.87	0.79638	0.33719	−0.01669	−0.00402	0.00028	0.701
Standard error		0.69348	0.36460	0.02410	0.00666	0.00894	
t		1.148	0.925	0.694	0.604	0.032	
Probability level[a]		c	d	d	d	d	
Average	44.34	0.79548	0.25186	0.01927	−0.00431	0.00312	0.959
Standard error		0.20245	0.10644	0.00703	0.00194	0.00262	
t		3.929	2.366	2.740	2.218	1.194	
Probability level[a]		*	+	+	++	c	

[a] See table 2 footnote.

Table 5. Oats on Cresco soil: Regression coefficients (b_i), standard errors, t-values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1945	38.96	1.29201	0.04016	−0.03139	−0.00022	−0.00215	0.934
Standard error		0.28145	0.14797	0.00976	0.00270	0.00364	
t		4.590	0.271	3.211	0.079	0.591	
Probability level[a]		*	d	*	d	d	
1948	54.82	0.78859	−0.28474	−0.01801	0.00430	0.00530	0.962
Standard error		0.21579	0.11345	0.00751	0.00208	0.00279	
t		3.654	2.510	2.399	2.075	1.898	
Probability level[a]		*	+	+	++	++	
1951	39.16	0.87769	0.13553	−0.01218	0.00173	−0.00207	0.969
Standard error		0.25205	0.13252	0.00877	0.00242	0.00326	
t		3.482	1.023	1.388	0.710	0.636	
Probability level[a]		*	c	b	d	d	
1954	26.22	1.30855	−0.00601	−0.02908	0.00075	0.00199	0.989
Standard error		0.15159	0.07970	0.00525	0.00145	0.00196	
t		8.633	0.075	5.204	0.516	1.018	
Probability level[a]		**	d	*	d	c	
1957	45.23	1.40528	0.04084	−0.02819	−0.00174	0.00030	0.981
Standard error		0.21546	0.11328	0.00751	0.00208	0.00279	
t		6.522	0.360	3.766	0.844	0.110	
Probability level[a]		**	d	*	d	d	
1960	35.48	1.45778	0.17102	−0.04767	−0.00332	0.00359	0.811
Standard error		0.52223	0.27456	0.01816	0.00502	0.00673	
t		2.791	0.623	2.628	0.662	0.531	
Probability level[a]		+	d	+	d	d	
Average	39.97	1.18895	0.01649	−0.02777	0.00025	0.00116	0.986
Standard error		0.13981	0.07350	0.00488	0.00135	0.00179	
t		8.503	0.224	5.718	0.180	0.641	
Probability level[a]		**	d	*	d	d	

[a] See table 2 footnote.

Table 6. Hay on Clyde soil: Regression coefficients (b_i), standard errors, t-values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1949	1.95	−0.00493	0.00304	0.00010	−0.00006	0.00025	0.824
Standard error		0.01407	0.00704	0.00031	0.00009	0.00011	
t		0.350	0.410	0.318	0.557	2.026	
Probability level[a]		d	d	d	d	+	
1952	2.73	0.05549	0.01320	−0.00089	−0.00012	0.00019	0.915
Standard error		0.02310	0.01214	0.00052	0.00015	0.00019	
t		2.403	1.088	1.656	0.735	0.926	
Probability level[a]		+	c	++	d	d	
1955	1.64	0.02186	0.00832	−0.00063	−0.00001	0.00011	0.966
Standard error		0.00793	0.00417	0.00021	0.00006	0.00008	
t		2.760	1.999	3.323	0.283	1.450	
Probability level[a]		+	++	*	d	b	
1958	2.31	0.04523	0.00820	−0.00068	−0.00009	0.00019	0.951
Standard error		0.01453	0.00764	0.00031	0.00009	0.00014	
t		3.116	1.075	2.056	0.861	1.472	
Probability level[a]		+	c	++	d	b	
Average	2.16	0.02983	0.00823	−0.00052	−0.00006	0.00017	0.969
Standard error		0.00866	0.00458	0.00021	0.00006	0.00008	
t		3.445	1.808	2.623	1.142	2.331	
Probability level[a]		*	++	+	c	++	

[a] See table 2 footnote.

Table 7. Hay on Cresco soil: Regression coefficients (b_i), standard errors, t-values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1946	1.16	0.03111	−0.00230	−0.00021	0.00004	−0.00003	0.972
Standard error		0.00825	0.00434	0.00021	0.00006	0.00008	
t		3.775	0.530	1.065	0.838	0.432	
Probability level[a]		*	d	c	d	d	
1949	1.18	0.66032	−0.00129	−0.00100	0.00007	−0.00006	0.992
Standard error		0.00582	0.00306	0.00016	0.00004	0.00006	
t		11.350	0.423	7.217	2.117	0.952	
Probability level[a]		**	d	**	++	d	
1952	2.92	0.02541	0.01724	−0.00031	−0.00017	0.00003	0.971
Standard error		0.00706	0.00371	0.00016	0.00004	0.00006	
t		3.597	4.642	1.769	3.908	0.280	
Probability level[a]		*	*	++	**	d	
1955	2.02	0.06090	0.00070	−0.00089	0.00010	−0.00003	0.921
Standard error		0.01998	0.00872	0.00047	0.00013	0.00017	
t		3.048	0.066	1.895	0.738	0.139	
Probability level[a]		+	d	++	d	d	
1958	2.45	0.06269	0.01050	−0.00089	−0.00007	0.00008	0.971
Standard error		0.01221	0.00642	0.00026	0.00007	0.00011	
t		5.138	1.136	3.241	0.902	0.681	
Probability level[a]		*	c	*	d	d	
Average	1.95	0.04892	0.00430	−0.00063	−0.00000	−0.00000	0.991
Standard error		0.00527	0.00277	0.00010	0.00003	−0.00006	
t		9.266	1.553	5.275	0.176	0.075	
Probability level[a]		**	b	**	d	d	

[a] See table 2 footnote.

possibly because response was measured only with respect to residual effects. The availability of nutrients may have been affected by the growth of the oats the previous year. Also, since the meadow was a mixture, some species may have predominated in particular years; differential responses to the applied nutrients arising accordingly. Even though the R^2 's are high (between 0.915 and 0.992 for all the experiments except one), the shape of the production surfaces depicted by the signs of the regression coefficients was not convex in all cases (tables 6 and 7). For purposes of generalization of the production functions, however, all data for each year are used.

COMPARISON OF ECONOMIC OPTIMA UNDER VARIOUS ESTIMATES

A distribution of production functions exists for any crop at one location, corresponding to the distribution of weather and other stochastic variables over time. Farm decision makers can, where data or estimates are available, respond in various ways to this distribution or variability of production functions over time. They can assume an average production function and decide on inputs and fertilization accordingly. They can attempt to measure the otherwise "stochastic variables" and predict variations in the production function in particular years. Or, if they possessed the appropriate information and methods, they might incorporate the environmental variables into the production function, and, on the basis of the generalized production function, they might estimate the production function by year accordingly. Still other farmers, especially those with fewer resources, might use a strategy that supposes that the "worst" (the smallest response to fertilizer) might be expected, with inputs and fertilization planned accordingly.

Many years of data, with experiments planned accordingly, will be needed before these approaches to decisions under variability and uncertainty of the fertilizer response can be appraised. The data used in this study are not ideally suited to an analysis of this type. However, since they are one of the longest time series available and since farmers do have to make decisions in an environment of even less information, comparison is made of returns from fertilizer when different ones of these strategies might be used. Also, measurement is made of variability of economic relationships and optima when certain procedures are used in estimating an average production function. Finally, attempt is made to incorporate the environmental variables into the production function so that uncertainty in applicability of experimental response data might be lessened as the functions are applied to different locations, soil condition and weather factors.

In this section, we examine the optimum amounts of fertilizer when decisions might be made on the basis of four different approaches: (a) the *annual* *optimum,* where the response function of the individual year is considered known, as estimated in tables 2 through 7, and fertilizer application is determined from it; (b) the *highest* response of any one year in the series of annual response functions; (c) the *lowest* response of any one year; and (d) the *average* response functions over the several years (the last row of tables 2 through 7).

In estimating fertilizer quantities and profits from fertilization under each approach, we follow this method: For the *annual optimum,* we suppose that the production function for the particular year is known and that fertilizer quantities to maximize profits are recalculated annually according to this function. For the *highest* approach, we use the production function from the year of greatest response for all years, and the same nutrient rates are specified for all years. Similarly, the same rates are specified for all years by the *lowest* and *average* approaches, where the basic production functions are, respectively, the one giving the lowest response in any one year and the average response function over all years. These means and extremes are based only on the data discussed previously. The variability (confidence limits) of certain estimates for the average production functions will be examined in the next section, and results from fitting a generalized production function will be presented in the concluding section. Analysis is made only for Clyde and Cresco soils. Although it is true that "strong" response was not predicted in each year, exactly the same situation faces farmers as they make decisions in a framework wherein weather is variable. Hence, the great uncertainty surrounding certain of the annual response functions estimated is realistic in relation to the farmer's decision-making environment.

Comparison of Returns under Four Decision Approaches

Comparisons are made for (a) profit-maximizing mixes of nutrients and (b) profit-maximizing amounts of the 0-20-20[3] fertilizer grade. In the profit-maximizing mix, the optimum proportion of each nutrient and the optimum quantity of each are determined, for the production function in question, by solving the two relations in 2 and 3 where the mar-

$$(2) \quad \frac{\partial C}{\partial P} = P_p/P_c \quad \text{and} \quad (3) \quad \frac{\partial C}{\partial K} = P_k/P_c$$

ginal physical products, the partial derivatives of crop yield with respect to the particular nutrients, are set to equal the price ratio, P_p/P_c and P_k/P_c, and where P_p is price per pound of P, P_k is price

Table 8. Corn on Clyde Soil: Net returns to fertilizer if alternative P and K rates of application were used over a period of years.

Rates of application (lbs./A.)			Annual profit ($/A.)				
Function	P	K	1950	1953	1956	1959	Av.
			Profit-maximizing mix				
Annual optimum[a]	11.6	27.1	26.88	28.57	38.81	28.03	30.57
Highest[b]	13.1	26.0	26.88	26.86	38.13	27.33	29.80
Lowest[c]	11.3	23.0	26.49	27.02	38.81	26.65	29.74
Average[d]	10.9	25.6	26.60	28.19	38.21	27.69	30.17
			Fixed grade				
	0-20-20 (lbs)						
Annual optimum[a]	160		26.84	26.47	38.78	27.26	29.84
Highest[b]	170		26.64	25.75	36.74	27.26	29.10
Lowest[c]	140		26.60	26.45	38.78	26.44	29.57
Average[d]	150		26.83	26.47	38.50	26.91	29.68

[a] The optimum each year based on the production function estimated for that year. The amount of nutrients used varies by year depending on the production function of that year. (The nutrient quantities shown are the average for the several years.) In the case of the other three approaches, however, the same nutrient levels would be applied each year because the same production function would be assumed each year.

[b] Based on the production function of year with greatest response.

[c] Based on the production function of year with smallest response.

[d] Based on the average production function of all years (the last section of tables 2 through 7).

Table 9. Corn on Cresco soil: Net returns to fertilizer if alternative P and K rates of application were used over a period of years.

Rates of application (lbs./A.)			Annual profits ($/A.)					
	P	K	1947	1950	1953	1956	1959	Av.
			Profit-maximizing mix					
Annual optimum[a]	10.2	20.7	27.79	22.93	13.47	16.80	18.10	19.82
Highest[b]	13.1	24.9	26.70	18.60	10.97	15.37	18.10	17.95
Lowest[c]	9.5	15.6	19.29	22.93	12.95	16.37	11.26	16.56
Average[d]	10.6	21.6	24.62	21.95	13.02	16.58	14.36	18.11
			Fixed grade					
	0-20-20 (lbs.)							
Annual optimum[a]	130		26.70	22.78	13.32	16.77	18.10	19.53
Highest[b]	150		26.70	18.60	11.00	9.90	18.10	16.68
Lowest[c]	100		20.30	22.80	13.30	5.00	10.90	14.46
Average[d]	120		23.09	22.36	13.09	16.77	13.72	17.81

[a] The optimum each year based on the production function estimated for that year. The amount of nutrients used varies by year depending on the production function of that year. (The nutrient quantities shown are the average for the several years.) In the case of the other three approaches, however, the same nutrient levels would be applied each year because the same production function would be assumed each year.

[b] Based on the production function of year with greatest response.

[c] Based on the production function of year with smallest response.

[d] Based on the average production function of all years (the last section of tables 2 through 7).

---

[3] Oxide basis used in the trade.

per pound of K and P_c is price per unit (bushel or ton) of the respective crops. The actual prices used are those current at the time of this study: corn, \$1 per bushel; oats \$0.55 per bushel; hay, \$15 per ton, P_2O_5, 10 cents per pound; and K_2O, 5 cents per pound. In the case of the given mix, 0-20-20, the profit-maximizing amount was determined through the relationship in 4. In this case, the estimated production functions were converted to those in which P_2O_5 and K_2O were held in a constant 1:1 ratio. The marginal products were then determined as the left member of 4 and were equated to the price ratio formed by dividing the fertilizer price per pound by the crop price per unit. The equation was then solved for the amount of fertilizer to be applied.

$$(4) \qquad C/F = P_f/P_c$$

CORN

Results for corn are presented in table 8 for Clyde soil and in table 9 for Cresco soil. The P and K columns show the amount of each nutrient that would be applied under the respective decision procedures. Other columns show the profit per acre from fertilization by individual years and as an average for the years. (In the case of the fixed grade, the figures under rates of application indicate the quantity of 0-20-20 fertilizer to be applied for the particular decision procedure.)

If *ex ante* knowledge of the yearly production functions for Clyde soil existed and the annual optimum P and K rates were applied each year, average profits of the four years would be \$30.57—the maximum attainable under the given weather and price conditions (table 8). The annual rates of application would average 11.6 pounds of P and 27.1 pounds of K. If the annual P and K rates predicted by the 1950 response function, the *highest* rates, were applied each year, 13.1 pounds of P and 26.0 pounds of K would be used each year, and profits would average \$29.80.

Profits would vary by years since the rates derived from the *highest* production function and the given prices would not be optimum for the weather and function actually realized in that year. Alternatively, if the *lowest* P and K rates predicted by the 1956 production function were applied each year, profit would average \$29.74. Application of the P and K rates estimated by the *average* production function would result in average profit of \$30.17. Rates in the latter case would average 10.9 pounds of P and 25.6 pounds of K.

The effect on profits of alternative choices for corn response on Clyde soil was small. The average annual profits differed by only \$0.83 if the lowest P and K rates, as compared with *optimum* rates for the particular year, had been used in each year. The greatest discrepancy in profits for any one year, \$1.71 in 1953, that would have occurred if the highest rates had been applied instead of the optimum rates. (The rates would have been too high for the actual response realized.)

Similar results would have resulted in use of the 0-20-20 grade. Average profits from use of alternative rates of the 0-20-20 mix were generally less than if the *optimum* nutrient combinations were applied. (Compare the lower and upper sections of table 8.) The differences are small, however. Highest average profit would have come, obviously, from using the annual optimum quantity, depending on the production function of that year. However, the annual optimum, with 160 pounds of fertilizer, would have returned only \$0.27 more than an annual rate based on the *lowest* response function with 140 pounds.

Table 9 presents the results for corn on Cresco soil. The maximum attainable profit from fertilization in the five years averaged \$19.82 from use of 10.2 pounds of P and 20.7 pounds of K under the annual optimum. Use of the fertilizer rates predicted by the average production function resulted in the next highest profits, \$18.11 per year. The *lowest* optimum P and K quantities predicted by the 1950 production function would have returned only \$16.56 per year from fertilization with 9.5 pounds of P and 15.6 pounds of K.

Table 10. Oats on Clyde soil: Net returns to fertilizer if alternative P and K rates of application were used on Clyde soil over a period of years.

Rates of application (lbs/A.)			Annual profits ($/A.)					
	P	K	1948	1951	1954	1957	1960	Av.
			Profit-maximizing mix					
Annual optimum[a]	10.9	20.9	0.01	1.61	3.85	5.29	3.70	2.89
Highest[b]	16.0	28.6	-2.11	1.36	3.85	5.04	3.57	2.34
Lowest[c]	0.0	4.8	0.01	0.29	0.59	0.36	0.63	0.38
Average[d]	12.5	22.0	-1.24	1.60	3.65	5.20	3.55	2.55
			Fixed grade					
	0-20-20 (lbs.)							
Annual optimum[a]	120		0.00	1.61	3.84	5.18	3.65	2.86
Highest[b]	180		-2.15	1.34	3.84	4.99	3.60	2.32
Lowest[c]	0		0.00	0.00	0.00	0.00	0.00	0.00
Average[d]	140		-1.26	1.60	3.67	5.15	3.59	2.55

[a] The optimum each year based on the production function estimated for that year. The amount of nutrients used varies by year depending on the production function of that year. (The nutrient quantities shown are the average for the several years.) In the case of the other three approaches, however, the same nutrient levels would be applied each year because the same production function would be assumed each year.

[b] Based on the production function of year with greatest response.

[c] Based on the production function of year with smallest response.

[d] Based on the average production function of all years (the last section of tables 2 through 7).

Differences were somewhat greater with respect to projected use of the 0-20-20 grade. Under the *annual optimum,* profit would average $19.53 per year from 130 pounds of fertilizer. Under the *lowest,* the annual profit would be only $14.46 from 100 pounds. The *average* rate would be 120 pounds, with an annual profit from fertilization of only $1.72 less than for the optimum. In general, profit reduction from use of the fixed mix rather than the optimum mix, for corresponding decision approaches, would be relatively small.

OATS

Estimates for oats and hay are presented only for Clyde soil. Profits obtained from use of alternative fertilizer rates for oats on Clyde soil are shown in table 10. Use of the *annual optimum* rates in the 5-year period would have resulted in maximum profits of $2.89 per year. If the P and K rates estimated by the average function were used on oats, annual profits would have averaged $2.55. Use of the *lowest* fertilizer rates, zero of P and 4.8 pounds of K estimated by the 1948 production function, would not have resulted in losses; but the annual profit would have averaged only $0.38. Use of either the *highest* or of the *average* P and K would have resulted in losses in 1948 (when there was no response to fertilizer). The lowest fertilizer rates should have been applied only if the decision criterion were one of avoiding losses in any year.

Use of the quantity of 0-20-20 grade predicted by the *average* production function would have resulted in annual profits of $2.55, an amount only $0.31 less than those obtained by use of the P and K rates predicted by the *optimum* function. Again, profit differences were very small in use of the 0-20-20 or fixed grade as compared with the profit-maximizing mix for parallel decision approaches.

HAY

Table 11 shows profit from hay through fertilization over the rotation under the four decision approaches explained earlier. (Figures are provided only for the profit-maximizing mix of nutrients.) In fertilization throughout the rotation, the annual optimum would have returned an average profit above fertilizer costs of $7.52. In contrast, the *lowest* rate of fertilization would have been zero, to conform with nonprofitable response realized in a particular year. Profits also would be zero, but never negative as in the case of the *highest* and *average* approaches. The *average* approach would return annual profits of $2.72 less than the *annual optimum* and only slightly more than the *highest.* A fairly large loss would have occurred in 1949 if rates of *average* or *highest* decision approaches had been used.

JOINT OATS-HAY FERTILIZATION

Since hay was not fertilized separately, response was largely to nutrients applied on oats. Hence, response to fertilizer applied on oats perhaps should be considered as the joint product of oats and hay. Considered accordingly, the profits from fertilization under the four approaches outlined earlier are those in table 12 for the aggregate response of oats and hay. If the optimum rates for each 2-year oats-hay period were applied over the rotation, total profits over the eight years would have averaged

Table 11. Hay: Net returns to fertilizer if alternative P and K rates of application were used for a corn-oats-meadow rotation on Clyde soil over a period of years.

Rates of application (lbs./A./3 yrs.)			Annual profit ($/A.)				
	P	K	1949	1952	1955	1958	Av.
			Profit-maximizing mix				
Annual Optimum[a]	17.7	50.6	0	13.61	7.91	8.57	7.52
Highest[b]	29.7	67.4	-4.69	13.61	1.21	8.53	4.67
Lowest[c]	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Average[d]	26.2	74.7	-5.00	13.29	2.83	8.09	4.80

[a] The optimum each year based on the production function estimated for that year. The amount of nutrients used varies by year depending on the production function of that year. (The nutrient quantities shown are the average for the several years.) In the case of the other three approaches, however, the same nutrient levels would be applied each year because the same production function would be assumed each year.

[b] Based on the production function of year with greatest response.

[c] Based on the production function of year with smallest response.

[d] Based on the average production function of all years (the last section of tables 2 through 7).

Table 12. Oats and hay: Net returns to fertilizer if alternative P and K rates of application were used jointly on Clyde soil over a period of years.

Rates of application (lbs/A./3 yrs.)			Annual profits ($/A.)				
	P	K	1948 -49	1951 -52	1954 -55	1957 -58	Av.
			Profit-maximizing mix				
Annual optimum[a]	22.1	48.3	3.28	25.07	15.90	25.74	8.75
Highest[b]	26.2	49.8	3.28	24.77	13.25	21.42	7.84
Lowest[c]	21.2	39.6	2.28	24.69	14.00	25.74	8.34
Average[d]	22.0	46.6	2.37	25.01	14.71	22.47	8.07

[a] The optimum each year based on the production function estimated for that year. The amount of nutrients used varies by year depending on the production function of that year. (The nutrient quantities shown are the average for the several years.) In the case of the other three approaches, however, the same nutrient levels would be applied each year because the same production function would be assumed each year.

[b] Based on the production function of year with greatest response.

[c] Based on the production function of year with smallest response.

[d] Based on the average production function of all years (the last section of tables 2 through 7).

\$8.75 annually. Application of the fertilizer rates estimated by the *average* approach would have resulted in annual profits averaging \$8.07. The *lowest* approach would have given \$8.34 per year, the second highest average return. Rates based on the *highest* approach would have given lowest average profits. The *highest* rate used in each year would result in too much fertilizer in years of low response, thus causing more profit sacrifice than the more conservative *lowest* approach.

IMPLICATIONS IN ALTERNATIVES

The data just analyzed provide information over more crops and years than generally holds true for fertilizer trials. Even with this extended information, however, data are too few to allow broad conclusions on the approach that might be used where the fertilizer response function varies with weather and other factors associated with time. The *annual optimum* approach gives the highest profit in each individual year as well as the highest average profit. It does so because the rates of application specified by it are based on the production function of each individual year. Hence, it never specifies an *overage* of fertilizer, as in the case of *highest* approach, or an *underage,* as in the case of the *lowest* approach. (The *average* approach results in an *overage* in some years and an *underage* in others.) However, the difficulty with the *annual optimum* approach is that the production function is not known for the individual years. It is a useful approach only to the extent that weather and other variables related to time can be quantified and entered into the production function, allowing a prediction of response in individual years. These predictions must be available before the growing season in order that fertilizer can be applied in appropriate amounts and time. Hence, for this approach, weather variables must be measured, and generalized production functions must be estimated to allow appropriate predictions.

Profit sacrifice, as compared with the annual optimum, in using the *average* or *highest* approach would have been small for all crops and years (see tables 8 through 12) except for corn on Cresco soil and hay on Clyde soil. Profit sacrifice through the *lowest* approach would have been especially large for oats or corn on Cresco soil, oats on Clyde soil and hay on Clyde soil. Of course, the *lowest* approach would never have caused loss in any individual year, a condition of planning that might be desired by beginning farmers or others with severely limited funds.

The results just presented are largely illustrative, even though they provide a broader range of predictions than generally holds true for decisions or recommendations on fertilizer use. However, additional research and information will be necessary before broader inferences can be made even for the particular soil types studied. For soil and weather conditions at other locations, results might lead in quite different directions.

SOME CONFIDENCE LIMITS

Although approaches using different estimates of the production function for individual years have been used, all these production functions also involve uncertainty. They may have high or low standard errors, and the standard error of estimate of an optimum nutrient quantity for any one year may be large. To evaluate these possibilities, we compute confidence intervals for isoquants and isoclines of the production function since these physical relations are basic in determining the ratios and amounts of nutrients that will maximize profits under given levels of price.

Also, we compute confidence limits for the estimated profit-maximizing quantities and ratios of P and K under the prices cited earlier. For these indications of the degree of uncertainty involved in predictions, we use only the production function with (a) the largest coefficients of determination and (b) the smallest standard errors relative to regression coefficients. Hence, these confidence limits are estimated only for the average corn function on Clyde and Cresco soils. (See the last datum lines of tables 2 and 3.) The method used in computing measures of reliability for isoquants, isoclines and maximum profit points is that proposed by Fuller.[4]

Isoquants

The confidence boundaries for isoquants were computed from the following equation:

$$(5) \qquad Y_c = \hat{y}_{(P,K)} \pm st_{(1-\alpha)} \sqrt{\frac{1}{n} - q_0 A^{-1} q_0'}$$

where Y_c are points on the confidence boundary, A denotes the matrix of sums of squares and products of the independent variables in equation 1, q_0 is the vector of deviations of these variables from their respective means for some point (P_0, K_0), $\hat{y}_{(P,K)}$ is the estimated value of Y at point (P_0, K_0), s is the standard error of estimate, $t_{(1-\alpha)}$ is the tabular t-value at probability $(1-\alpha)$ and n is the number of observations. The s^2 values of the average production functions for corn on Clyde and Cresco soils were 2.476 and 0.262, respectively.

The 95-percent confidence limits for the 60 and 70 bushel isoquants of corn on Clyde soil, and for the 70 and 75 bushel isoquants of corn on Cresco soil

---

[4] Wayne A. Fuller. Estimating the reliability of quantities derived from empirical production functions. Jour. Farm Econ. 44:82-99. 1962.

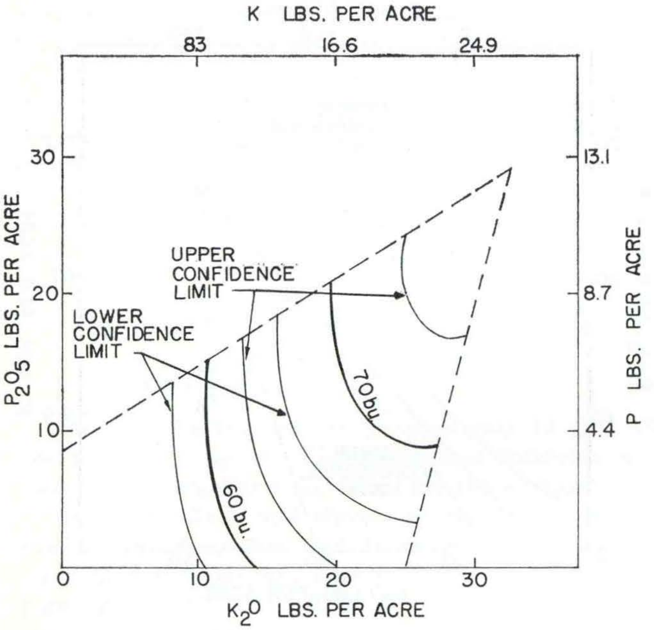

Fig. 1. Upper and lower 95-percent confidence limits for two corn isoquants—average corn response on Clyde soil.

were computed by evaluating equation 5 by successive approximations. The 95-percent confidence boundaries are shown in figs. 1 and 2. The confidence boundaries indicate that, given the conditions underlying the response function, the yields defined by an isoquant are obtained by application of the P and K combinations included within the boundaries, with a probability of 95 percent. For example, given the same environmental conditions and with a 0.95 probability, 70 bushels of corn might be obtained on Clyde soil with application of 20.8 pounds of K in combination with quantities of P varying between 2.0 and 8.7 pounds. The confidence boundaries are wide in fig. 1, especially for the P input. However, that the confidence boundaries for the 60 and 70 bushel isoquants do not overlap indicates that significantly different input combinations are required to produce either yield.

The 95-percent confidence intervals of the corn isoquants on Cresco soil are shown in fig. 2. The confidence regions are narrower than those for corn on Clyde soil. The difference is due mainly to the smaller standard error of estimate of the average production functions for Cresco soil. The confidence boundaries in fig. 2 are wider for the K input. For example, a 75-bushel yield of corn might be produced on Clyde soil by using 12.5 pounds of K in combination with 4.8 to 7.4 pounds of P, with a 0.95 probability. Alternatively, 6.6 pounds of P might be combined with 7.9 to 14.9 pounds of K and produce 75 bushels of corn. The relative width of the confidence limits for each input follows from the magnitude of the t-values for the partial regression coefficients of the average corn production functions. (See tables 2 and 3.)

Isoclines

The confidence boundaries for the isoclines are computed from the following equation:

$$(6) \qquad B^2 (v_1^2 - t^2s^2c_{11}) - 2B (v_1v_2 - t^2s^2c_{12}) + v_2^2 - t^2s^2c_{22} \leq 0$$

where v_1 and v_2 are the denominator and numerators, respectively, in the isoquant equation 7, and where P is expressed as a function of K from the initial

$$(7) \qquad P = \frac{\alpha_0 b_2 - b_1}{2b_3 - \alpha_0 b_5} + \frac{2\alpha_0 b_4 - b_5}{2b_3 - \alpha_0 b_5} K$$

production function in 1. In 6 the s^2c_{ii} are the variances and covariances of the v_i and $B^2 = \frac{v_2}{v_1}$.

The 95-percent confidence intervals were computed only for the isoclines, where the marginal rate of substitution of P for K (the price ratio of the two nutrients, P_p/P_k) is equal to 2.0.

The equation of the 95-percent confidence boundary for the 2.0-isocline of the average function for corn on Clyde soil is:

$$(8) \qquad 1834.0 K^2 - 2780.1 K - 3262.3 PK + 894.0 + 2149.6 P + 1059.9 P^2 = 0$$

The 2.0-isocline and its estimated confidence limits are shown in fig. 3. The confidence region is wide,

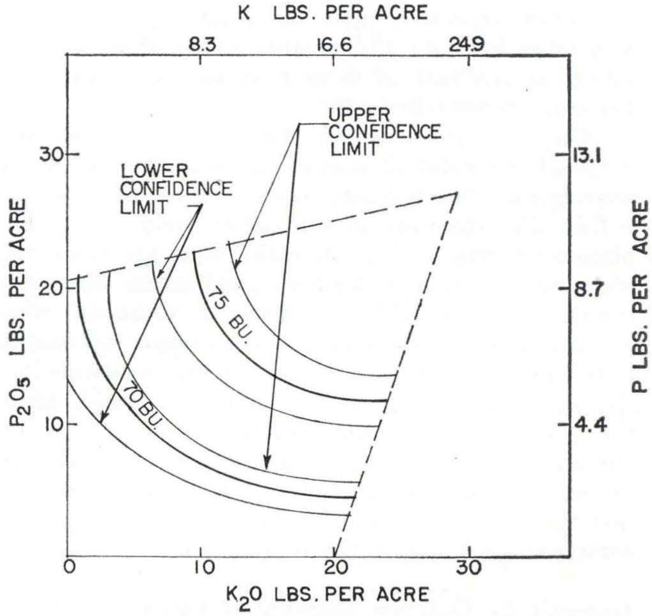

Fig. 2. Upper and lower 95-percent confidence limits for two corn isoquants—average corn response on Cresco soil.

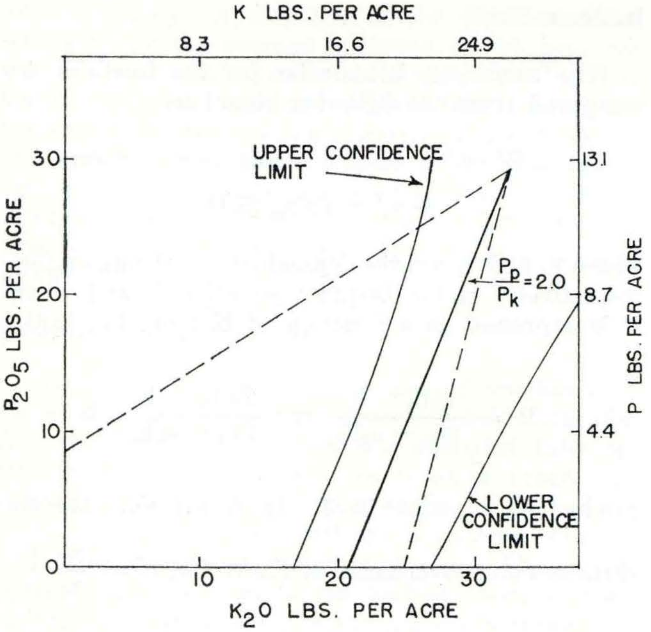

Fig. 3. Upper and lower 95-percent confidence limits for the 2.0-isocline—average corn response on Clyde soil (dotted lines are ridgelines).

especially for P, and includes the lower ridgeline.

The 95-percent confidence interval for the 2.0-isocline of the average function for corn on Cresco soil has the equation:

$$(9) \quad 835.7\,P^2 + 135.2\,P - 986.4\,PK + 3.0 - 76.5\,K + 249.3\,K^2 = 0$$

The isocline and its corresponding confidence boundaries are presented in fig. 4. The confidence region is narrow, especially near the mean levels of P and K application. At 12.5 pounds of K, P substitutes for K at the rate of 2 to 1 in the range between 4.6 and 6.6 pounds of P.

Since the isoclines for the functions fitted converge at the point of maximum yield, the confidence boundaries (for the isoclines) include all isoclines within the ridgelines at the higher yield levels. Replacement rates and price ratios thus are measured with low precision at the high yield levels, where the inputs approach the condition of technical complementarity, in specifying the economic optima in nutrient combinations. Hence, if the economic optimum yield is close to the maximum yield, as in the present experiments, determination of the optimum fertilizer rates is less critical as profit depression from selection of a suboptimum nutrient mix is small. However, this is less true for nutrient combinations lower in the input plane.

Economically Optimum Quantity of Inputs

Approximate confidence regions for selected maximum profit points over the input plane were estimated by the procedure described by Fuller.[5] The 90- and 95-percent confidence limits were computed for the maximum profit points (the quantities of P and K) when corn and fertilizer nutrients are valued at the prices mentioned previously.

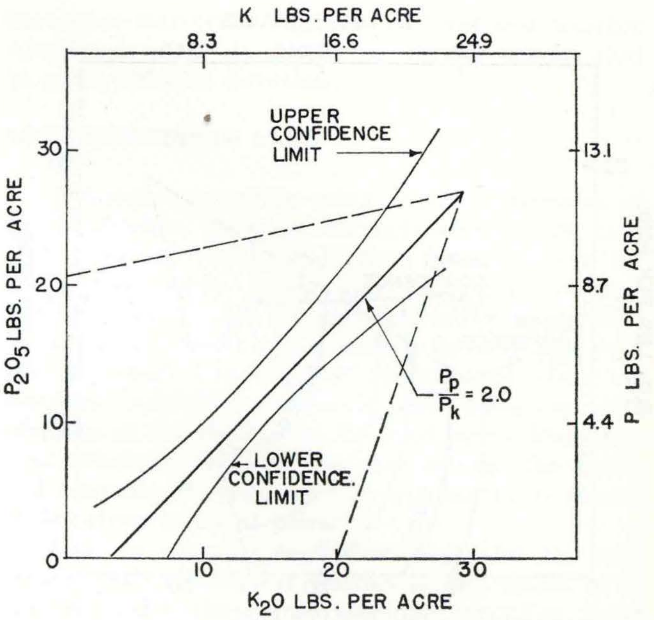

Fig. 4. Upper and lower 95-percent confidence limits for the 2.0-isocline—average corn response on Cresco soil (dotted lines are ridgelines).

The equations of the 90- and 95-percent confidence regions for the optimum level of inputs for the average corn response function on Clyde soil are, respectively:

$$(10) \quad 243.1\,P^2 + 149.2\,P - 331.7\,PK - 258.1\,K + 202.9\,K^2 + 43.8 = 0$$

$$(11) \quad 243.1\,P^2 + 149.2\,P - 331.7\,PK - 258.1\,K + 202.9\,K^2 + 64.0 = 0$$

The confidence regions derived from equations 10 and 11 are shown in fig. 5. The regions are wide, especially in the direction of the P axis. The 90-percent confidence region includes the point of predicted maximum yield and all other optimum levels of inputs that would be obtained under a fairly wide range of price combinations. A general conclusion derived from fig. 5 is: At a 0.90 probability level, at least 17.4 pounds of K should be used in combination with P, or at least 3.3 pounds of P should be used in combination with K to reach the economic optimum level of corn yield. Application of fertilizer quantities smaller than those covered by the confidence region would most likely fail to maxi-

---

[5] *Ibid.*

mize profits, if the average function was used as the basis for decisions on fertilizer application.

The equations of the 90- and 95-percent confidence intervals for the maximum profit point of the average corn response on Cresco soil are, respectively:

$$(12) \quad 278.8\,P^2 - 81.6\,P - 110.1\,PK - 7.7\,K + 30.6\,K^2 + 8.2 = 0$$

$$(13) \quad 278.8\,P^2 - 81.6\,P - 110.1\,PK - 7.7\,K + 30.6\,K^2 + 10.3 = 0$$

The boundaries defined by equations 12 and 13 are shown in fig. 6. The confidence intervals are considerably smaller than those for the average corn response on Clyde soil shown in fig. 5. As in the case of the isoquants and isoclines, the confidence interval for profit-maximizing points on the input plane is wider for the K input on Cresco soil than on Clyde soil. At a 0.90 probability level, the maximum point is obtained by using 17.0 to 25.7 pounds of K in combination with 10.5 pounds of P. Alternatively, if 21.6 pounds of K are applied, the maximum point is reached 90 percent of the time with combined application of 9.2 to 12.0 pounds of P. The 90-percent confidence region for the average corn response on Cresco soil also includes the point of maximum yield and the optimum levels of output obtained under the various price combinations.

Confidence Interval Implications

The computed confidence limits show a relatively high variability with respect to the specified quantities even for the average production functions. The variability of isoquants, isoclines and maximum profit points are even greater for the annual production functions since they have smaller values of R^2 and larger standard errors relative to regression coefficients. The experimental data used in this study were not originally intended for production function analysis. The few degrees of freedom resulting from the experimental design cause the confidence regions to be greater than would otherwise be expected. They do illustrate further research needs if a refined basis is to be provided in decisions on fertilizer use.

The confidence boundaries for the isoquants and isoclines considered did not overlap. Hence, if the average function is used, significantly different P and K combinations can be specified for production of given outputs and for profit-maximizing nutrient combinations. From the average production functions of corn, input combinations can be predicted with 0.95 probability for the 10-bushel yield increases on Clyde soil and for the 5-bushel yield increases on Cresco soil. Hence, optimum input combinations with 0.95 probability can be established, step by step, according to crop-fertilizer price ratios. Given the wide range of the confidence intervals, a precise mathematical specification of optimum nutrient inputs has little to recommend it over a more naive arithmetic and discrete prescription of quantities.

The quadratic equation used has linear isoclines that converge at the maximum yield point. Hence,

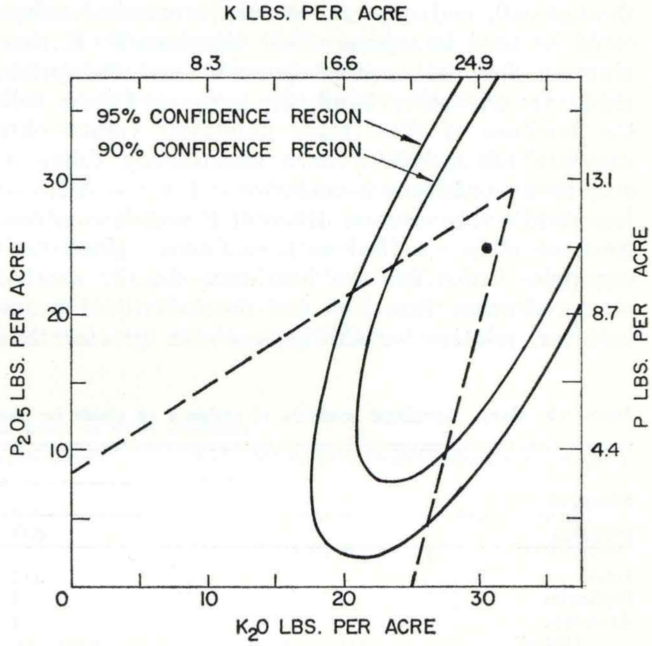

Fig. 5. Confidence regions for the maximum profit point—average corn response on Clyde soil (dotted lines are ridgelines).

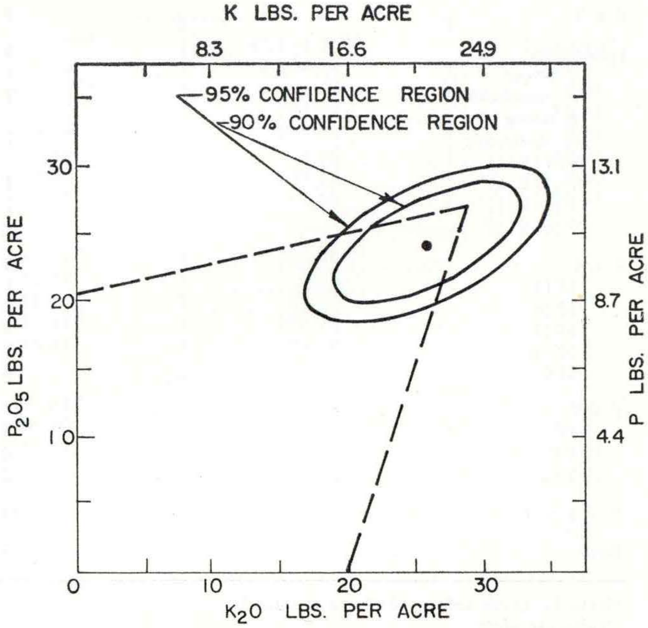

Fig. 6. Confidence regions for the maximum profit point—average corn response on Cresco soil (dotted lines are ridgelines).

the confidence intervals, for the specific numerical isoclines just considered, include, at yields approaching the maximum, the entire family of isoclines falling between the ridgelines (the nutrient combinations of economic relevance). For corn on Clyde soil, the confidence interval for the 2.0-isocline included all isoclines computed over the numerical value range of 1.0 to 5.0, as well as the lower ridgeline. There was not a statistically significant difference among the computed isoclines of the values 1.0 through 5.0, and any one of these numerical values could be used to represent the least-cost P - K combination for the average function and the prices used. On the other hand, for corn on Cresco soil, the isoclines of these same numerical values (1.0, 2.0, 3.0, 4.0 and 5.0) were significantly different, and their confidence boundaries did not overlap at low yield levels. Hence, different P and K combinations could be specified with confidence, given various price ratios for the nutrients. In the case of the production functions for the individual years, however, relative variability was even greater than for the average production function for corn on Clyde soil and the optimum mix of nutrients could not be specified, at high confidence levels, for various price ratios for nutrients and yields of crops.

However, the previous analysis is based on experimental designs and quantities of data insufficient for purposes of specifying optimum input ratios and fertilizer quantities based on annual functions and for average functions other than corn on Cresco soil. Greater certainty in predicting economic optimum nutrient ratios and profit-maximizing fertilization levels is possible through improved designs and on extended time series of experiments. The analysis made emphasizes the need for more elaborate experiments, with respect to both experimental design and time period covered.

GENERALIZED FUNCTIONS

An alternative to selecting a particular annual or average production function as a basis for de-

Table 13. Corn: Combined analyses of variance of yields for the several years at three locations.

Source of variation	Farm					
	Howard Co.[a]		Clarion-Webster		Seymour-Shelby	
	d.f[b]	Mean square[d]	d.f.	Mean square[d]	d.f[c]	Mean square[d]
Total	213		251		187	
Replicates	12		14		11	
Covariate (Stand)	1		9		—	
Soils (S)	1	10,379.19**	1	3,587.40**		
Years (Y)	3	8,924.06**	6	3,393.96**	10	23,839.18**
Weather (W)	2	7,157.70**	2	6,060.40**	2	72,709.51**
S x Y	3	630.44**	6	270.32**	—	
Treatments	8		8		8	
P linear	1	1,751.43**	1	19,990.60**	1	745.98**
P quadratic	1	867.00**	1	222.68*	1	86.06
K linear	1	13,782.75**	1	4.60	1	346.94**
K quadratic	1	1,950.75**	1	107.72	1	0.04
P(1)K(1)	1	1,053.36**	1	103.76	1	29.34
P(1)K(q)	1	64.98	1	298.32*	1	0.04
P(q)K(1)	1	0.09	1	189.28	1	0.02
P(q)K(q)	1	26.46	1	110.52	1	1.16
S x T	8		8		—	
SP(1)	1	800.91**	1	1,377.16**	—	
SP(q)	1	4.20	1	17.00	—	
SK(1)	1	4,349.43**	1	117.00	—	
SK(q)	1	344.55**	1	6.00	—	
SPK	4	46.00	4	105.64	—	
Y x T	24		48		80	
YP	6	31.92	12	414.68**	20	133.35**
YK	6	42.79	12	41.96	20	50.18
YPK	12	16.34	24	40.40	40	25.48
Y x S x T	24	30.69	48	58.68	—	
Error	129	49.46	103	57.12	78	30.86

[a] Data for Clyde soil in 1947 are excluded.

[b] 3 missing plots.

[c] 10 missing plots.

[d] Probability levels are: **0.01
*0.05

cision strategy under weather variability is to predict a generalized production function that includes weather and related variables of the interyear environment. If weather can be introduced into the production function, with response predicted accordingly, a different year or short-run production function then can be specified for the particular season. This step, to the extent that it might be accomplished by predicting the weather in the year ahead and setting the corresponding variable in the production function at this level, would allow specification of an annual optimum of the general nature indicated for tables 8 through 12. A great difficulty exists, of course, in accurate specification of the weather variable *ex ante* to the growth and decision year. It is possible, however, that such measures can be appropriately aggregated from soil moisture content at about planting time and from data from climatological research. Hence, this section is devoted to estimation of generalized production functions from the experimental data explained previously.

For the purposes just discussed, generalized production functions are estimated for the particular crop at the particular locations. However, generalized functions (termed crop functions) also are predicted over locations for two reasons: (a) Data from the several locations give more observations with respect to weather as well as for other variables. (b) If a generalized production function can be predicted across soil types and conditions, given experimental results can have much broader application (and, conversely, a given amount of information for several soil types can be had at a lower total cost). Although the data used are not the most appropriate for these purposes, the data are the most complete and broadest set currently available over time.

Two problems arise in connection with the generalization of the production function: First, the added variables and their hypothetical functional relationships with yields and other inputs must be defined. The second problem deals with aggregation of experimental data and environmental variables and with the incorporation of the new variables

Table 14. Oats: Combined analyses of variance of yields for the several years at three locations.

Source of variation	Farm					
	Howard Co.[a]		Clarion-Webster		Seymour-Shelby	
	d.f.[b]	Mean square[e]	d.f.[c]	Mean square[e]	d.f.[d]	Mean square[e]
Total	268		205		191	
Replicates	15		11		11	
Soils (S)	1	691.20**	1	1,016.60**	—	
Years (Y)	4	4,789.42**	5	9,212.59**	10	15,552.56**
Weather (W)	2	7,377.08**	2	21,324.96**	2	42,687.00**
S x Y	4	1,425.26**	5	288.41**	—	
Treatments	8		8		8	
P linear	1	5,892.75**	1	40,824.20**	1	2,843.95**
P quadratic	1	946.44**	1	2,304.64**	1	302.49**
K linear	1	472.38**	1	4.92	1	0.09
K quadratic	1	90.27	1	293.04	1	11.85
P(1) K(1)	1	78.24	1	319.74	1	0.94
P(1) K(q)	1	0.69	1	4.80	1	13.05
P(q) K(1)	1	12.00	1	259.16	1	7.50
P(q) K(q)	1	108.12	1	10.66	1	36.45
S x T	8		8		—	
SP(1)	1	144.18*	1	781.20**	—	
SP(q)	1	26.52	1	0.02	—	
SK(1)	1	18.45	1	75.40	—	
SK(q)	1	121.41*	1	177.10	—	
SPK	4	18.51	4	98.70	—	
Y x T	32		40		80	
YP	8	63.68*	10	985.34**	20	109.49**
YK	8	49.20	10	103.71	20	22.55
YPK	16	29.74	20	50.26	40	25.75
Y x S x T	32	20.10	40	66.52	—	
Error	164	29.91	87	88.26	82	26.49

[a] Data for Clyde soil in 1945 are excluded.
[b] 1 missing plot.
[c] 1 replicate was lost; also, 1 missing plot.
[d] 6 missing plots.
[e] Probability levels are: **0.01
*0.05

into the production function. These problems are discussed in the following sections.

Analysis of the Combined Experimental Data

The first step toward the generalization of results in the current study was a combined analysis of variance of the experimental data and the environmental variables. The analysis was computed for each crop at each of the three experimental locations. The results are presented in tables 13, 14 and 15, where P(1) and K(1) refer to linear effects of P and K, P(q) and K(q) refer to quadratic effects, Y refers to years, T refers to treatment, S refers to soils and W refers to weather (see later discussion on measure).

Table 16. Bartlett tests of homogeneity of the error variances for corn, oats and hay experiments.

Crop	Location	χ^2	Probability level[a]
Corn	Howard County	14.426	*
	Clarion-Webster	22.311	+
	Seymour-Shelby	17.606	+
Oats	Howard County	43.909	**
	Clarion-Webster	31.972	**
	Seymour-Shelby	17.460	+
Hay	Howard County	388.332	**
	Clarion-Webster	151.887	**
	Seymour-Shelby	81.035	**

[a] Probability levels are: **0.01
*0.05
+0.10

Bartlett's test of homogeneity of variance was made to test the assumption that the experimental error variances were the same in all experiments for each crop at each location. The tests are presented in table 16. In most cases, the χ^2 values were significant at the 0.05 or 0.01 significance level, indicating that the error variances are heterogeneous. The combined analyses that follow were performed, nevertheless, since some information can be gained in spite of the shortcomings represented by the heterogeneity of variance.

In general, the contribution of soils and years (weather) to yield variation was highly significant for all crops at the three locations. At the Howard County Farm and the Clarion-Webster Farm, there

Table 15. Hay: Combined analyses of variance of yields for the several years at three locations.

Source of variation	Howard Co.[a] d.f.	Howard Co.[a] Mean Square[c]	Clarion-Webster d.f.	Clarion-Webster Mean square	Seymour-Shelby d.f.	Seymour-Shelby Mean square[b]
Total	215		179		173	
Replicates	12		10		10	
Soils (S)	1	2.0709**	1	0.8488*	—	
Years (Y)	3	25.1167**	4	4.3941**	9	9.7379**
Weather (W)	2	34.1336**	2	4.4586**	2	32.5962**
S x Y	3	2.9847**	4	1.8658**	—	
Treatments	8		8		8	
P linear	1	21.6924**	1	142.0058**	1	5.6942**
P quadratic	1	2.9454**	1	1.6838**	1	0.6899**
K linear	1	6.8121**	1	0.0530	1	0.2134
K quadratic	1	0.1563	1	0.0024	1	0.0523
P(1)K(1)	1	0.4293	1	0.1712	1	0.0432
P(1)K(q)	1	0.1986	1	0.1392	1	0.0851
P(q)K(1)	1	0.0000	1	0.2420	1	0.0008
P(q)K(q)	1	0.0105	1	0.0066	1	0.0095
S x T	8		8		—	
SP(1)	1	0.9555**	1	1.9304**	—	
SP(q)	1	0.1053	1	0.4202	—	
SK(1)	1	0.2601	1	0.0026	—	
SK(q)	1	0.0471	1	0.2690	—	
SPK	4	0.2479	4	0.1138	—	
Y x T	24		32		72	
YP	6	0.2592*	8	3.5080**	18	0.0866
YK	6	0.1784	8	0.1449	18	0.0315
YPK	12	0.0696	16	0.0521	36	0.0623
Y x S x T	24	0.1853*	32	0.1204	—	
Error	132	0.1103	80	0.1414	74	0.0710

[a] Data for Clyde soil in 1946 are excluded.
[b] 6 missing plots.
[c] Probability levels are: **0.01
*0.05

were significant interactions of soils and years, meaning that the weather factors affected yields differently on each soil type.

The response to applied nutrients varied among locations and crops. (Fertilizer treatment effects can be disregarded, since more precise results are available from the individual analyses of variance discussed previously.) Of special interest are the soils x treatments and years x treatments interactions, which indicate changes in crop response to fertilizer among soils and weather conditions. In all cases the soils x P interaction was statistically significant. Evidently response to phosphorus was conditioned by soil characteristics. Since the quadratic component of P was not similarly affected, the curvature of the surface may be assumed equal for all years and pairs of soils at each location. Soil x K interactions were statistically significant for corn and oats on Cresco and Clyde soils (Howard Co.). Since potassium produced strong yield response only on these soils, interactions were likely to occur only at the Howard County location. Weather affected response to phosphorus at all locations, except for corn at the Howard County Farm as seen from the corresponding years x P interactions.

These interactions are important in the generalization of the production functions since they indicate, not only that yield levels are affected by soil and climatic variables, but also that the response to applied nutrients changes with these environmental variables. Therefore, appropriate variables must be included in the generalized fertilizer response functions if we are to account for variation in yield from fertilizer due to the significant interactions.

Use of the F-ratio under heterogeneity of the interaction variances and the experimental error variances may be open to question. Lack of independence of the individual experiments is a further limitation of the data. Under these conditions, the errors may be autocorrelated. However, the possible gain in estimational precision from using an autoregressive scheme was not considered great enough to warrant the added computations. A complete analysis of variance, with all data pooled was also not deemed advisable, although greater heterogeneity of variances and a greater number of significant interactions might have been expected. However, even with limitations in the data, generalization of the production functions was considered worthwhile. The combined analyses of the data suggest important variables to be included in the generalized production functions.

Quantification of Climatic Factors

Analyses in the previous section indicated considerable yield variation due to weather. Hence, a weather variable was designed to be incorporated in the production function as a means of increasing the R^2-value and reducing deviations from regression. The predictive value of the production functions might thus be increased. In addition, if a probability distribution of weather can be established from climatological records, this information could be greatly useful in establishing the mathematical expectation of optimum fertilizer rates under uncertainty.

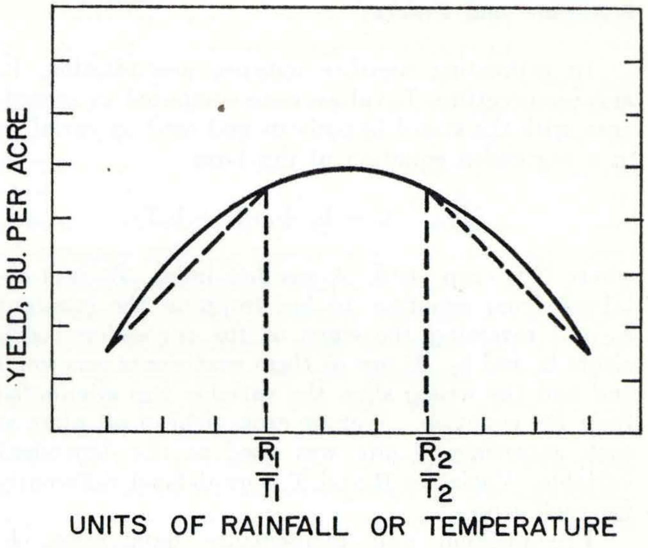

Fig. 7. Hypothetical relationship of rainfall, temperature, or both, with crop yield.

The hypothesis used in measuring weather is that yield has a curvilinear relationship to precipitation and temperature as shown in fig. 7. Yield increases to a maximum as precipitation and temperature increase and is depressed if precipitation or temperature becomes excessive. Several levels of yield curves could be depicted, according to interactions among weather elements. For simplicity, an average function of parabolic shape is assumed.

The yield curve in fig. 7 is flat near the peak, variation in medium values of weather only slightly changing yield. But at the extremes, small variations in precipitation or temperature cause sharp changes in yields. If only the more extreme weather variations are considered, a further simplification can be made. The portions of the yield curve outside the "average or common range" ($\overline{R}_1\overline{R}_2$ or $\overline{T}_1\overline{T}_2$) but sometimes taken only as the mean $\overline{R}$ or $\overline{T}$ are assumed to be of uniform slope, which can be approached by a straight line. With rainfall and temperature measured as absolute deviations from their "average or common range," the hypothesis becomes one of a general negative and linear relationship between yield and rainfall or temperature; the greater the deviation of precipitation, temperature, or both, from an "average range," the greater is the depression on yields.

Procedure and Results

In estimating weather indexes, precipitation, R, and temperature, T, values were computed in accordance with the stated hypothesis and used as variables in a regression equation of the form

$$(14) \quad C = b_0 + b_1R + b_2T$$

where C is crop yield. A weather index, W, was obtained from equation 14 by dropping the constant b_0 and reversing the signs of the regression coefficients b_1 and b_2. If one of these coefficients was small and had the wrong sign, the variable was eliminated from the equation. Average crop yield of all plots at each experimental site was used as the dependent variable. Variables R and T were defined differently for each crop.

Precipitation and temperature data were obtained from the weather stations nearest to the experimental sites, as reported by the Weather Bureau.[6]

CORN

Previous studies in Iowa have shown that weather conditions during July and August are most important in affecting corn yields.[7] Thus, rainfall and temperature during these two months were used to derive the corn weather indexes Daily rainfall during July and August was measured in 10-day periods, and deviations from an average amount, $\overline{R}$, were computed for each period. All negative deviations were added together; if there was no rainfall in any 10-day period, that deviation was given double weight. The sum of the negative deviations over the six 10-day periods was used to express R.

Temperature during July and August was assumed harmful only if it were above a level, $\overline{T}$, specific for each month. Monthly temperature was used to compute the temperature variable, T, by adding the deviations for July and August of temperature above $\overline{T}$.

The $\overline{R}$ and $\overline{T}$ values were established for each experiment from local data. The corn varieties used were those adapted to each location. $\overline{R}$ and $\overline{T}$ and the R and T variables for all locations and years are presented in Appendix D.

Weather indexes for corn, W_c, were computed following the method described previously. The resulting equations and the respective coefficients of determination are:

(15) Howard County:

$$W_c = 4.667R + 0.0T \qquad R^2 = 0.54$$

(16) Clarion - Webster:

$$W_c = 5.706R + 1.207T \qquad R^2 = 0.57$$

(17) Seymour - Shelby:

$$W_c = 0.0R + 9.141T \qquad R^2 = 0.61$$

More than 50 percent of the yield variance is explained by rainfall and temperature (linear) variables at the three locations. The index values derived from equations 15, 16 and 17 are presented in Appendix D.

OATS

Precipitation and temperature over the growing season were used for the oats weather indexes. Rainfall was measured in two periods: from planting to heading time and from heading time to harvest. Shortages or excesses of precipitation were computed as absolute deviations from a minimum $\overline{R}_1$ or a maximum $\overline{R}_2$ precipitation for each of the two periods. These deviations were added to provide the rainfall variable R (see Appendix E). Monthly temperature for April, May and June was used for estimating the temperature variable T. Deviations below a "normal" minimum temperature in April were added to deviations below minimum or above maximum "normal" temperatures in May and June for the temperature variable T.

The equations for the oats weather indexes, W_0, and the corresponding R^2 values, computed following this general procedure, are:

(18) Howard County:

$$W_0 = 3.550R + 1.929T \qquad R^2 = 0.61$$

(19) Clarion - Webster:

$$W_0 = 28.761R + 5.274T \qquad R^2 = 0.92$$

(20) Seymour - Shelby:

$$W_0 = 9.908R + 7.029T \qquad R^2 = 0.55$$

The index values derived from these equations are shown in Appendix E.

MEADOW

Weather variables were estimated for two separate periods within the hay growing season to derive two hay weather indexes corresponding to the two cuttings. For the first cutting, the precipitation variable, $\overline{R}_1$, was measured as the rainfall deviation

---

[6] Precipitation was measured in inches. Temperature was measured in degrees Fahrenheit.

[7] Louis M. Thompson. Weather and technology in the production of corn and soybeans. (Mimeo.) Center for Agricultural and Economic Development, Iowa State University of Science and Technology. CAED Report 17. 1963. 66pp.

below an average amount, $\overline{R}_1$, from initial meadow growth to June harvest. The temperature variable T_1 was computed as the sum of deviations of monthly temperatures above or below the amounts $\overline{T}_1$ and $\overline{T}_2$ for April, May and the portion of June before harvest. The precipitation variable, R_2, for the second cutting was computed as the rainfall deviation below an amount $\overline{R}_2$ for the period between cutting dates. The temperature variable, T_2, was the sum of deviations of the monthly temperatures for part of June, July and part of August from the limits $\overline{T}_1\overline{T}_2$, defining the "optimum" range of temperature for these months. The equations for the hay weather indexes, W_h, were obtained by adding the coefficients of the equations for each cutting.

(21) Howard County:

$$W_h = 0.32R_1 + 0.27T_1 + 0.43R_2 + 0.00T_2$$

$$R^2 = 0.97$$

(22) Clarion - Webster:

$$W_h = 0.00R_1 + 0.26T_1 + 0.00R_2 + 0.12T_2$$

$$R^2 = 0.42$$

(23) Seymour - Shelby:

$$W_h = 1.12R_1 + 0.33T_1 + 0.13R_2 + 0.02T_2$$

$$R^2 = 0.81$$

Dates, standard precipitation ($\overline{R}$) and temperature ($\overline{T}$) values used and the index values derived from equations 21, 22 and 23 are presented in Appendix F.

Available Soil Nutrients

Preliminary analysis of the combined data showed yield response to fertilizer varied with different soils. Several soil characteristics, including the amounts of soil nutrients present and available to plants, are responsible for differential yield responses. Differences in available soil nutrients may produce the following effects: (a) The height of the response surfaces may be changed while their slope and shape remain the same. (No interaction between soil nutrients and fertilizer uptake.) (b) The slope, the shape, or both, as well as the height, of the surfaces may be changed.

Soil test data, reflecting nutrient content of soils, can be used in two ways to generalize the production functions: (a) They can be used to estimate a factor of proportionality and convert soil nutrients and fertilizer nutrients to the same units and estimate the function in terms of total nutrients. (b) Soil nutrients can be considered as independent variables and included separately in the response function. The second procedure is used in this study.

Extending this procedure to more than one soil requires additional variables for soil characteristics other than available nutrients. If other data reflecting these differences are not available, as in the current case, dummy variables can be used to represent different soil types. The combined analyses of variance indicate that only the linear components of phosphorus interact with the soils in most of the experiments. Finally, weather conditions (soil moisture and temperature) may be important in regulating the availability of soil nutrients. Under such conditions the use of soil test data alone in the generalized production functions would be insufficient for our purposes. Soil x weather interactions should be included, as is suggested by the significant soils x years interaction of the combined analyses of variance.

The soil test data used in this study are presented in Table 17. These values were determined from soil samples taken from the checkplots of each experiment in 1958. Only P and K values were used in the generalized functions since (a) great variations in nitrogen available were expected over the rotation because of rotation meadow grown and used as green manure and (b) the soils were mainly deficient in phosphorus and potassium.

This method has a limitation. The soil test data were collected for only 1 year and may not serve as representative of the time series of experiments if the soil nutrients are built up or depleted over years. However, data were not available from all the experimental plots and for all years or on a quantitatively equivalent basis.

Determination of the Generalized Production Functions

Two generalizations of data were used for esti-

Table 17. Soil test level of N, P and K of experimental plots on five Iowa soils.

Soil	N	P	K
Clyde	111	1.6	91
Cresco	103	1.8	109
Acid Webster, segment A	86	1.2	166
Acid Webster, segment B	80	1.6	170
Acid Webster, segment C	73	1.5	156
Calcareous Webster, segment A	98	0.9	156
Calcareous Webster, segment B	76	0.6	144
Calcareous Webster, segment C	81	0.8	136
Seymour, segment A	104	3.0	128
Seymour, segment B	90	2.5	152
Seymour, segment C	102	3.3	134

mating the production functions. First, the experiments at one location (subject to the same weather) were pooled as in the combined analyses of variance. A least-squares regression equation was estimated, over soils, for each crop at each location with two soils. Since there were only two soils at each of these two locations, coding factors of -1 and $+1$ were used to reflect soil variables in the production functions. The weather indexes were used to explain part of the yearly yield variance. The production functions derived in the manner described are termed "location functions."

Next, all the experimental data for each crop were pooled over locations and alternative regression equations were fitted to the data. Weather variables, soil test data, dummy variables for locations and for soil types, and fertilizer terms were used to quantify the generalized response functions. The production functions derived by this method are termed "crop functions."

The mathematical form used to characterize all the generalized response functions was a polynomial. The location, soil and weather terms were entered linearly. (See previous analysis and discussion.) The P and K terms were entered with linear and squared terms. Interactions among these factors also were used as variables. Because of the different procedures used, the two sets of functions are presented separately.

LOCATION FUNCTIONS

The regression coefficients for the location functions were obtained directly from the combined analyses of variance. (They do not include soil nutrients as a variable.) Therefore, the statistical significance of the coefficients is given by the F-tests in tables 13, 14 and 15. The interaction terms included were those suggested by the statistical significance of the corresponding variates in the analyses of variance. The interaction terms involving weather usually explained a small portion of the sums of squares, because of the manner the weather indexes were derived. Therefore, several of the weather interaction terms were eliminated from some of the location functions.

The location functions and their respective R^2's are:

CORN

(24) Clyde - Cresco:

$$\begin{gathered} C = 51.086 + 12.75417S - 1.00W + 0.02630SW \\ + 0.45824P + 0.86096K - 0.099165P^2 - 0.04111K^2 \\ + 0.04063PK + 0.36022SP - 0.87166SK \\ + 0.01728SK^2 \qquad R^2 = 0.742 \end{gathered}$$

(25) Clarion - Webster:

$$\begin{gathered} C = 57.107 + 6.636205 - 0.95554W - 0.55370SW \\ + 2.13056P + 0.15991K - 0.04651P^2 - 0.00894K^2 \\ + 0.01162PK - 0.22992WK - 0.00678WP \\ R^2 = 0.649 \end{gathered}$$

(26) Seymour:

$$\begin{gathered} C = 72.852 - 0.8195W + 0.87923P + 20.137K \\ - 0.03265P^2 - 0.00020K^2 - 0.00715PK \\ - 0.02963WP + 0.00105WP^2 - 0.00251WK \\ R^2 = 0.605 \end{gathered}$$

OATS

(27) Clyde - Cresco:

$$\begin{gathered} C = 38.827 - 0.74500S - 1.02472W + 0.38094SW \\ + 0.98196P + 0.13113K - 0.02315P^2 - 0.00197K^2 \\ + 0.00248PK + 0.06834SP - 0.13331SK \\ + 0.00232SK^2 - 0.01226WP + 0.00068WP^2 \\ R^2 = 0.702 \end{gathered}$$

(28) Clarion - Webster:

$$\begin{gathered} C = 34.643 + 4.50417S - 0.41155W + 2.48235P \\ - 0.13257K - 0.04037P^2 + 0.00396K^2 - 0.00560PK \\ - 0.17789SP - 0.05329WP + 0.00063WP^2 \\ R^2 = 0.875 \end{gathered}$$

(29) Seymour:

$$\begin{gathered} C = 40.587 - 0.81105W + 0.74709P - 0.46891K \\ - 0.01528P^2 - 0.00084K^2 + 0.00033PK \\ - 0.20298WP + 0.00047WP^2 \qquad R^2 = 0.552 \end{gathered}$$

HAY

(30) Clyde - Cresco:

$$\begin{gathered} C = 2.146 + 0.05896S - 0.86343W + 0.05685SW \\ + 0.04259P + 0.00708K - 0.00068P^2 - 0.00004K^2 \\ + 0.00008PK + 0.00415SP - 0.00113SK \\ - 0.00889WP + 0.00010WP^2 \qquad R^2 = 0.811 \end{gathered}$$

(31) Clarion - Webster:

$$\begin{gathered} C = 0.816 + 0.04433S + 0.02216W + 0.25622SW \\ + 0.07394P - 0.00027K - 0.00052P^2 - 0.00000K^2 \\ + 0.00006PK - 0.00399SP + 0.00026SP^2 \\ - 0.02880WP + 0.00121WP^2 \qquad R^2 = 0.825 \end{gathered}$$

(32) Seymour:

$$C = 2.255 - 0.92937W + 0.02564P - 0.00018K - 0.00037P^2 + 0.00003K^2 - 0.00003PK - 0.00532WP + 0.00010WP^2 \qquad R^2 = 0.726$$

The symbols are defined as: C, yield in bushels or tons per acre; S, soil type; W, weather in terms of deviations mean; P, pounds of P per acre; K, pounds of K per acre. In the following, the meaning of the estimated coefficients is discussed briefly.

CORN

Function (24) for corn at the Howard County location (Clyde-Cresco soils), computed with 72 yield observations, had an R^2 of 0.742. The positive sign of the soil coefficient indicates the higher average productivity of Cresco soil (coded as +1). The weather variable had a coefficient of − 1.00, which follows from the definition of the weather index, which says that a negative value is associated with good climatic conditions. Therefore, if weather is favorable, the predicted yield is increased. The SW term was included in the function, although its contribution to the reduction of the unexplained variance was small.

Coefficients for fertilizer terms all had the appropriate signs, representing a convex surface or diminishing marginal products. The regression coefficients were significant at the 1-percent level, as expected, given the average functions for Cresco and Clyde soils.

The positive SP interaction suggests that the response to phosphorus was stronger on Cresco soil, which is in accordance with the results presented in previous tables. Likewise, the negative SK and the positive SK^2 interactions indicate that the response to potassium was stronger on Clyde soil, especially at the low levels of potassium application.

Function 25 for corn (acid soil coded as +1 and calcareous as − 1) at the Clarion-Webster location had an R^2 of 0.649. Only the phosphorus fertilizer terms at the Clarion-Webster location appear relevant. As expected, the K terms were statistically nonsignificant. The SP interaction was significant at the 1-percent level. All other soil x fertilizer interaction terms were negligible and therefore were deleted from the function. The WP variable was included in the function although it was not statistically significant at a high level of probability. The frequent failure of the weather x treatments interaction variables to account for the year x treatments interaction (table 13) may result from the definition of the weather index.

Function 26 for corn at the Seymour location, with an R^2 of 0.605, covers only one soil and, therefore, does not include soil and soil-interaction terms. The WP and WP^2 terms accounted for almost half of the years x phosphorus interaction. The greater weather variation at the Seymour location had a strong effect on the corn response to phosphorus. Therefore, imperfections in the weather index did not have the same adverse effect as for the other functions. The signs of the regression coefficients for the WP and WP^2 terms indicate the yield response to phosphorus to be greater under favorable weather.

OATS

Function 27 for oats at the Clyde-Cresco location, with an R^2 of 0.702, and W and SW variables explaining a significant portion of the yield variance. The regression coefficients of the fertilizer terms all had the appropriate signs, although only the coefficients for P, P^2 and K were significant at low probability levels. The positive coefficient for the SP interaction term indicates that the oats response to phosphorus was greater on Cresco soil. Similarly, the signs of the coefficients for the SK and SK^2 terms indicate that the response to potassium was greater on Clyde soil (coded as − 1). The weather x P interaction terms were retained in function 27 because the signs of the coefficients were appropriate.

Function 28 at the Clarion-Webster location and function 29 at the Seymour location had R^2's of 0.875 and 0.552, respectively. The oats response to fertilizer at these locations was mainly to phosphorus. Hence, the location functions have coefficients for the K terms that are negligible. Interpretation of the interaction terms follows the same reasoning explained for the previous location functions.

HAY

The R^2 values for functions 30, 31 and 32 for hay at the Clyde-Cresco, Clarion-Webster and Seymour locations were 0.811, 0.825 and 0.726, respectively. The high R^2 's for the Clyde-Cresco and Seymour location functions are due to the greater proportion of the yearly yield variation explained by the respective weather indexes. For the Clarion-Webster location, the weather index explained only 42 percent of the yield variance due to years. The greatest proportion of the hay yield variance at the Clarion-Webster location was due to fertilizer treatments, mainly phosphorus, and the fit of the functions was good, even with a "weak" weather index. The coefficients for the P terms were significant for all three functions. The linear term for potassium was statistically significant only for function 30; all other K terms had nonsignificant coefficients.

The interesting feature of the location functions is the nature of the interaction terms. The interaction terms show that response of crops to fertilizer was altered by weather conditions and by soil characteristics. Although the WP and WP^2 terms were statistically significant at only low probability levels in most cases, the consistency of the signs of the respective coefficients lends support to the estimating procedures used. With respect to the soils x treatments interactions, the sign and the magnitude of the coefficients show the relative degree of crop response to the applied nutrients on the particular soils.

Crop Functions

The crop functions were obtained from all the experimental data pooled for each crop. Two alternative methods were used in the estimation of the crop functions. In the first case, the dependent variable was the mean treatment yield from each one of the experiments. The independent variables used were: two dummy variables for locations, d_1 and d_2; P_s, available soil phosphorus; K_s, available soil potassium; W, the weather index in terms of deviations from its 15-year mean; P, pounds P per acre; K, pounds K per acre; and several interactions of these variables. The fertilizer variables were coded as -1, 0, $+1$ for the low, medium and high levels of fertilizer application, respectively.

In the second case, the yields were averaged over the years for each soil test group listed in table 17. The independent variables used were the same as for the location functions, except for the dummy variables and the weather variable. A new set of orthogonal variables, d_i, to represent locations and soil types were defined:

	d_1	d_2	d_3	d_4
Cresco soil	+1	+1	−1	0
Clyde soil	+1	+1	+1	0
Acid Webster soil	−1	+1	0	−1
Calcareous Webster soil	−1	+1	0	+1
Seymour soil	0	−4	0	0

The weather variables were defined as deviations from the mean index values for the years concerned.

In both cases the estimating procedure consisted of fitting a regression equation with the maximum number of variables likely to contribute to the reduction of yield variance. Several variables that appeared unimportant, or were highly correlated to others, were successively deleted from the equations, thus yielding alternative generalized functions. The functions containing fewer terms were preferred if the R^2 values were not greatly changed by the elimination of some variables and if the significance of the partial regression coefficients was increased.

The crop functions, with the corresponding standard errors of the coefficients, values of t, probability levels and R^2's are presented in tables 18, 19 and 20. The corresponding analyses of variance are presented in Appendix G. The probability levels of t were determined on the basis of the degrees of freedom for the sums of squares of deviations from regression. The tests of significance of the coefficients must be interpreted with caution because of the heterogeneity of the experimental error variances and the unequal number of observations on each soil type.

CORN

Crop functions 33 through 36 for corn were estimated by the first method described at the beginning of the previous section. Function 33 contained 30 variables that explained 62.9 percent of the yield variation (table 18). The relatively low R^2 was due mainly to the year-to-year yield variation unexplained by the weather indexes. The dummy variables were defined as: Clyde-Cresco location, $d_1 = 1$; Clarion-Webster location, $d_2 = 1$; otherwise d_1 and d_2 were equal to 0.

The F test for the over-all regression was significant at a probability level smaller than 0.01. However, several of the partial regression coefficients were not significantly different from zero. Several of the higher interaction terms were deleted without greatly affecting the coefficient of determination.

For crop function 34, the variables were reduced to 14 with an R^2 of 0.586. Function 35 contained only 12 variables and had an R^2 of 0.576. The 18 variables eliminated accounted for 5.24 percent of the yield variance. In crop function 36, the two dummy variables were added to the 12 variables of function 35, and the R^2 was increased to 0.612. Therefore, comparison of functions 33 and 36 shows that the 16 interaction terms eliminated from function 33 explained only 1.72 percent of the yield variance. Crop function 36 was decoded to fertilizer units of pounds per acre and was used later to derive the isoquant maps in fig. 8.

The dummy variable for the Clyde-Cresco location, d_1, was significant at the 1-percent level in functions 33 and 36. The difference in the average yield level of the checkplots between this location and the average for all locations was successfully explained by the dummy variable d_1. The variable d_2 was not highly significant, probably because the average check-plot yield at the Clarion-Webster locations was close to the over-all average check-plot yield.

The soil variables P_s, K_s and P_sK_s were effective in explaining part of the variance due to soils. The significance level of the respective coefficients shifted between the functions, depending on whether the dummy variables, as well as certain interaction terms, were included. The positive signs of the P_s

Table 18. Generalized crop functions for corn: Regression coefficients (b_i), standard errors (s_b), value of t, probability levels and coefficients of determination (R^2).

Function number	Variable[a]	b_i	s_b	t	Prob. level[b]
(33)	b_0	-26.52653	—	—	
	d_1	17.88608	5.1607	3.466	**
	d_2	-13.86335	7.8132	1.774	+
	P_s	28.39457	18.1401	1.565	++
	K_s	0.75424	0.2824	2.671	**
	P_sK_s	-0.22019	0.1391	1.582	++
	W	-1.33382	0.6340	2.104	*
	P_sW	-0.14046	0.0850	1.653	+
	K_sW	0.00555	0.0042	1.306	++
	P	9.76422	17.0600	0.572	d
	K_2	35.67125	8.5388	4.178	**
	P_2	-38.01057	65.4483	0.581	d
	K	-26.80085	18.0908	1.481	++
	PK	26.56092	24.3312	1.092	b
	P_sP	-8.87975	2.9666	2.993	**
	P_sK	-1.17836	1.5351	0.768	d
	P_sP^2	2.19843	11.6793	0.188	d
	P_sK^2	2.32167	3.2283	0.719	d
	P_sPK	-3.21121	4.3420	0.740	d
	K_sP	0.15972	0.1088	1.466	++
	K_sK	-0.21914	0.0553	3.961	**
	K_sP^2	0.15954	0.4205	0.379	d
	K_sK^2	0.14463	0.1162	1.244	b
	K_sPK	-0.13162	0.1565	0.842	d
	WP	-0.38891	0.9511	0.409	d
	WK	-0.08574	0.0782	1.096	b
	WP^2	0.11512	0.5848	0.197	d
	WK^2	0.02223	0.1616	0.135	d
	WPK	0.02294	0.2175	0.105	d
	P_sWP	-0.11896	0.2264	0.525	d
	K_sWP	0.00396	0.0071	0.558	d
		$R^2 = 0.629$			
(43)	b_0	68.03827	—	—	—
	P_s	2.26136	1.0926	2.069	*
	K_s	-0.00030	0.0682	0.004	d
	W	-0.44166	0.6177	0.715	d
	WP_s	-0.19426	0.0862	2.253	*
	WK_s	0.00024	0.0042	0.057	d
	P	32.48023	6.1060	5.319	**
	K	32.63188	7.7865	4.191	**
	P^2	-12.26359	10.0966	1.215	b
	K^2	-19.98819	15.8273	1.263	b
	PK	2.82023	3.7536	0.751	d
	P_sP	-9.60114	2.9735	3.229	**
	K_sK	-0.21295	0.0564	3.780	**
	WP	-0.24181	0.1492	1.622	+
	WK	-0.08339	0.0805	1.036	c
		$R^2 = 0.586$			
(35)	b_0	64.71967	—	—	—
	P_s	2.12270	1.0855	1.956	+
	K_s	0.02745	0.0391	0.702	d
	W	-0.95645	0.0544	17.595	**
	P	32.48028	6.1404	5.290	**
	K	32.63188	7.8303	4.167	**
	P^2	-12.26359	10.1533	1.208	b
	K^2	-2.92171	2.8065	1.041	b
	PK	2.82023	3.7745	0.747	d
	P_sP	-9.60114	2.9902	3.211	**
	K_sK	-0.21296	0.0566	3.759	**
	WP	-0.24184	0.1498	1.613	+
	WK	-0.08339	0.0809	1.031	c
		$R^2 = 0.576$			
(36)	b_0	9.41984	—	—	—
	P_s	2.33891	3.1242	0.749	d
	K_s	0.41705	0.0883	4.720	**
	W	-0.97819	0.0527	18.565	**
	d_1	19.12454	5.0432	3.792	**
	d_2	-6.68304	6.7836	0.985	c
	P	32.48010	5.9001	5.505	**

Table 18—(Continued)

Function number	Variable[a]	b_i	s_b	t	Prob. level[b]
	K	32.63188	7.5238	4.337	**
	p^2	-12.26359	9.7559	1.257	b
	k^2	-2.92181	2.6966	1.083	b
	PK	2.82023	3.6269	0.778	d
	P_sP	-9.60111	2.8732	3.341	**
	K_sK	-0.21296	0.0544	3.912	**
	WP	-0.24172	0.1441	1.678	+
	WK	-0.08339	0.0777	1.073	b
		$R^2 = 0.612$			
(37)	b_0	44.57762	—	—	—
	P	-23.90338	29.0778	0.822	d
	K	18.18140	15.2877	1.189	b
	P^2	-13.38065	5.4543	2.453	*
	K^2	-4.15180	1.5077	2.754	**
	PK	2.51597	1.7650	1.425	++
	W	-0.68888	0.1605	4.291	**
	P_s	-3.72007	2.5341	1.468	++
	K_s	0.25185	0.0624	4.033	**
	P_sP	2.23300	7.1110	0.314	d
	K_sP	0.26651	0.1753	1.521	++
	P_sK	-2.71629	3.7386	0.726	d
	K_sK	-0.06481	0.0922	0.703	d
	d_1	8.27361	2.0739	3.989	**
	d_2	1.12666	0.9434	1.194	b
	d_3	-3.11332	2.3393	1.331	b
	d_4	-2.85333	1.7447	1.635	++
	d_1P	-2.16919	4.8344	0.449	d
	d_2P	3.44239	2.3975	1.436	++
	d_3P	-2.07127	3.4066	0.608	d
	d_4P	9.22769	3.6710	2.514	*
	d_1K	4.89228	2.5417	1.925	+
	d_2K	0.17304	1.2605	0.137	d
	d_3K	6.28536	1.7910	3.509	**
	d_4K	-0.43822	1.9300	0.227	d
	d_1P^2	-5.21537	6.4281	0.811	d
	d_2P^2	-2.00871	2.2269	0.902	c
	d_3P^2	2.73856	11.1339	0.246	d
	d_4P^2	2.32589	6.4281	0.362	d
	d_1k^2	-3.02325	1.7768	1.701	+
	d_2K^2	-1.14316	0.6155	1.857	+
	d_3K^2	3.99889	3.0776	1.299	++
	d_4K^2	-0.69329	1.7768	0.390	d
	WP	0.56503	4.5050	1.254	b
	WK	-0.09775	2.3685	0.413	d
		$R^2 = 0.904$			
(38)	b_0	44.57762	—	—	—
	P_s	-3.72007	2.4377	1.526	++
	K_s	0.25183	0.0601	4.192	**
	W	-0.68888	0.1544	4.461	**
	P	-33.90690	23.0632	1.470	++
	K	15.38686	7.7382	1.992	+
	P^2	-13.38065	5.2470	2.550	*
	K^2	-4.15180	1.4503	2.862	**
	PK	2.51597	1.6977	1.482	++
	P_sP	3.79604	6.3778	0.595	d
	K_sP	0.32317	0.1427	2.264	*
	K_sP	-2.22160	0.9943	2.234	*
	K_sK	-0.04980	0.0653	0.762	d
	d_1	8.27361	1.9949	4.147	**
	d_2	-1.12666	0.9075	1.242	b
	d_3	-2.76554	1.7926	1.543	++
	d_4	-2.87647	1.2584	2.286	*
	d_1P	-1.08933	4.3253	0.252	d
	d_2P	4.04262	2.1017	1.923	+
	d_4P	10.27757	3.1160	3.298	**
	d_4K	5.14521	2.1666	2.375	*
	d_3K	6.46991	1.5185	4.260	**
	d_1P^2	-5.21537	6.1835	0.843	d
	d_2P^2	-2.00871	2.1418	0.938	c
	d_1K^2	-3.02325	1.7092	1.768	+
	d_2K^2	-1.14316	0.5920	1.931	+
	d_3K^2	-3.99889	2.9605	1.351	++

Table 18—(Continued)

Function number	Variable[a]	b_i	s_b	t	Prob. level[b]
	WP	0.55053	0.4328	1.272	b
	WK	-0.10423	0.2085	0.450	d
		$R^2 = 0.902$			
(39)	b_0	43.89386	—	—	—
	P_s	-3.72007	2.4576	1.514	++
	K_s	0.25183	0.0606	4.158	**
	W	-0.68888	0.1557	4.425	**
	P	-33.90690	23.2518	1.458	++
	K	14.41268	7.5374	1.912	+
	P^2	-10.75375	4.6044	2.336	*
	K^2	-3.38971	1.4069	2.409	*
	PK	2.51597	1.7117	1.470	++
	P_sP	3.79604	6.4298	0.590	d
	K_sP	0.32317	0.1439	2.246	*
	P_sK	-2.35345	0.9665	2.435	*
	K_sK	-0.04044	0.0631	0.641	d
	d_1	7.17362	1.8347	3.910	**
	d_2	-1.90696	0.8286	2.301	*
	d_3	-2.76554	1.8073	1.530	++
	d_4	-2.87647	1.2687	2.267	*
	d_1P	-1.08933	4.3606	0.250	d
	d_2P	4.04262	2.1189	1.908	
	d_4P	10.27757	3.1417	3.271	**
	d_1K	5.45786	2.0913	2.610	*
	d_3K	6.54104	1.5242	4.291	**
	d_1K^2	-2.07062	1.6499	1.255	b
	d_3K^2	-3.99889	2.9847	1.340	++
	WP	0.55053	0.4362	1.262	b
		$R^2 = 0.895$			

[a] b_0 is the yield intercept.

[b] Probability levels are:

** $p \leqslant 0.01$ + $0.05 < p \leqslant 0.10$ b $0.20 < p \leqslant 0.30$ d $p > 0.40$

* $0.01 < p \leqslant 0.05$ ++ $0.10 < p \leqslant 0.20$ c $0.30 < p \leqslant 0.40$

and K_s coefficients indicate that higher yields were forthcoming from soils with higher initial phosphorus and potassium content. If soil test data covering a wider range of conditions had been available, squared soil terms could have been included in the functions. The negative sign of the P_sK_s coefficient in equation 33 suggests that the effects of the linear soil terms were not additive. That is, there might have been a small degree of substitution between soil phosphorus and soil potassium as in the case of the fertilizer nutrients.

The weather variable was highly significant in functions 34, 35 and 36. The quadratic fertilizer terms were significant at lower probability levels; the PK interaction usually was negligible.

The coefficients for the P_sP and K_sK terms were significant at the 0.01 probability level in functions 34, 35 and 36. Both terms had negative coefficients, indicating that at a higher soil nutrient content, crop response to the applied nutrients was diminished. Notice the effectiveness of soil test data to characterize such a situation. The WP and WK terms were retained in the crop functions under discussion because of the known weather x treatments interactions. Their negative coefficients indicate that crop response to phosphorus and potassium was stronger under favorable weather conditions.

Crop functions 37, 38 and 39 were estimated according to the second method described at the beginning of this section. The main reason for averaging annual yield over soils was to reduce yield variation attributed to weather and thus improve the fit of the functions. The weather variable, however, was included as a weighting factor and to absorb the remaining yield variation due to weather.

Soil test data, as used for functions 37, 38 and 39, explain the within-location yield variance due to soils. Therefore, these soil variables were actually redundant for the Clyde and Cresco experiments, which covered only one segment of soil each, because the dummy variable helped to explain the same differences.

Function 37 was fitted with 34 independent variables and had an R^2 of 0.904. Function 38, obtained by deleting six interaction terms from function 37 had an R^2 of 0.902. For function 39, the variables were reduced to 24, and the R^2 was 0.895, only 0.009 less than the R^2 for function 37. High correlation between several of the independent variables was responsible for only slight changes in the coefficients of determination. Crop function 38 was decoded to fertilizer units of pounds per acre and was used later for derivation of the isoquant maps in fig. 9.

Interpretation of the partial regression coefficients of crop functions 37, 38 and 39 is not readily made because of the definition of the dummy variables and the coding of fertilizer terms. The main interest is in the statistical significance of the regression coefficients.

The number of coefficients with probability levels greater than 0.30 was reduced from 14 in equation 37 to three in equation 39. The results were fairly consistent for the three functions. The coefficients for K_s were significant at the 0.01 probability level in all cases. Soil potassium was a good indicator of soil fertility, at least within the three land segments of acid Webster, calcareous Webster and Seymour soils.

The weather variable was highly significant in crop functions 37, 38 and 39. The regression coefficient for variable d_1 was always significant at the 1-percent probability level. This variable acted as a location variable, explaining the relative difference in average yields at the three experimental sites. The interaction terms d_4P and d_3K were also highly significant. The different response to phosphorous on acid as compared with calcareous Webster soils was established by d_4P; d_3K accounted for the greater response to potassium on Clyde soil.

The linear fertilizer terms were not significant at the usual statistical levels because their effect upon yields was shown through the fertilizer x dummy variables interaction terms. The squared fertilizer terms were significant at the 0.05 or 0.01 probability levels. The explanation is that the quadratic terms were statistically significant in most of the average functions for each soil; therefore, functions 37, 38 and 39, based on average yields, would be expected to show significant P^2 and K^2 terms.

OATS

The two crop functions for oats presented in table 19 were estimated by the second method outlined at the beginning of this section. The same general remarks made for the corn functions 37, 38 and 39 are valid for the two oats functions.

Function 40 was fitted with 34 variables and had an R^2 of 0.916. Function 41 included 15 variables and had an R^2 of 0.876, only 0.040 less than equation 40. That is, the 19 variables that were deleted accounted for only 4 percent of the yield variation. Soil phosphorus had a positive yield effect, as suggested by the highly significant coefficient for P_s. The regression coefficients for the dummy variables d_1 and d_2 were both significant at the 0.01 probability level. They denote the different average yield levels of oats at the three experimental locations. Interpretation of the remaining coefficients follows the same logic as for the corn functions.

HAY

The crop functions for hay, estimated by the same procedure used for functions 37 through 41, are presented in table 20.

Function 42 included 34 variables and had an R^2 of 0.928. Fifteen variables were deleted from function 42 to yield function 43, which had an R^2 of 0.924, only slightly less than 42. Thirteen of the 19 partial regression coefficients of function 43 were significant at either the 0.05 or the 0.01 probability levels.

The coefficients for the two soil nutrient terms, P_s and K_s, were highly significant. The coefficient for the dummy variable d_1, which differentiated among the three experimental sites, was significant beyond the 0.01 probability level. The other three dummy variables were usually significant only at higher probability levels. The contribution of the fertilizer terms to the reduction of yield variance was important, as shown by the statistical significance of the regression coefficients of P and K terms and of their interactions with the dummy variables.

Yield Isoquants and Isoclines from Generalized Functions

Crop functions 36 and 38 for corn were used to derive the technical and economic relationships presented now. Soil test values corresponding to Clyde and Cresco soils and the appropriate values for the dummy and (average) weather variables were substituted into the functions to predict the average yield response of corn to fertilizer on each of the two soils mentioned. Two production functions of the simple form 1, with only P and K variables, were thus obtained from each of the two generalized crop functions. Crop function 36 was transformed into:

Table 19. Generalized crop functions for oats: Regression coefficients (b_i), standard errors (s_b), value of t, probability levels and coefficients of determination (R^2).

Function number	Variable[a]	b_i	s_b	t	Prob. level[b]
(40)	b_0	**39.17936**	—	—	
	P_s	20.25198	3.6301	5.579	**
	K_s	-0.14464	0.1422	1.017	b
	W	-1.16927	0.0936	12.495	**
	P	-35.79530	61.1753	0.585	d
	K	-4.49699	32.1629	0.140	d
	P^2	-25.94295	8.4744	3.061	**
	K^2	0.87805	2.3424	0.375	d
	PK	-2.31651	2.7420	0.845	d
	P_sP	12.67089	10.1864	1.244	b
	K_sP	0.28427	0.3989	0.712	d
	P_sK	2.87580	5.3555	0.537	d
	K_sK	0.00158	0.2097	0.007	d
	d_1	-13.58647	4.0858	3.325	**
	d_2	8.37874	1.4500	5.778	**
	d_3	2.94511	3.7724	0.781	d
	d_4	4.49977	2.9358	1.633	++
	d_1P	-8.88335	10.3017	0.862	d
	d_2P	7.59640	3.6762	2.066	*
	d_3P	1.99073	6.0048	0.331	d
	d_4P	12.20075	6.5219	1.871	+
	d_1K	0.20013	5.4161	0.037	d
	d_2K	1.07472	1.9328	0.556	d
	d_3K	0.81470	3.1570	0.258	d
	d_4K	1.83824	3.4289	0.536	d
	d_1P^2	7.61339	9.9874	0.762	d
	d_2P^2	-2.84817	3.4595	0.823	d
	d_3P^2	3.81908	17.2985	0.221	d
	d_4P^2	0.09481	9.9874	0.009	d
	d_1K^2	-2.70582	2.7606	0.980	c
	d_2K^2	-0.00660	0.9562	0.007	d
	d_3K^2	-2.04962	4.7815	0.429	d
	d_4K^2	-2.77919	2.7606	1.007	c
	WP	-0.52418	0.2626	1.996	+
	WK	-0.03378	0.1380	0.245	d
		$R^2 = 0.916$			
(41)	b_0	20.40688	—	—	
	P_s	20.14268	3.4306	5.871	**
	W	-1.09420	0.0680	16.103	**
	P	56.86355	10.6002	5.364	**
	K	0.17485	1.0439	0.167	d
	P^2	-27.67573	7.8802	3.512	**
	K^2	1.86439	2.1782	0.856	c
	PK	-2.31651	2.9294	0.791	d
	P_sP	-18.33299	6.5595	2.795	**
	d_1	-9.87651	1.4061	7.024	**
	d_2	8.28905	1.1032	7.514	**
	d_4	5.82533	2.0153	2.890	**
	d_2P	-1.78267	2.3815	0.748	d
	d_4P	-0.78716	3.4654	0.227	d
	d_4K^2	-2.77919	2.9493	0.942	c
	WP	-0.57397	0.1895	3.030	**
		$R^2 = 0.876$			

[a] b_0 is the yield intercept.

[b] Probability levels are:

** $p \leqslant 0.01$ + $0.05 < p \leqslant 0.10$ b $0.20 < p \leqslant 0.30$ d $p > 0.40$
* $0.01 < p \leqslant 0.05$ ++ $0.10 < p \leqslant 0.20$ c $0.30 < p \leqslant 0.40$

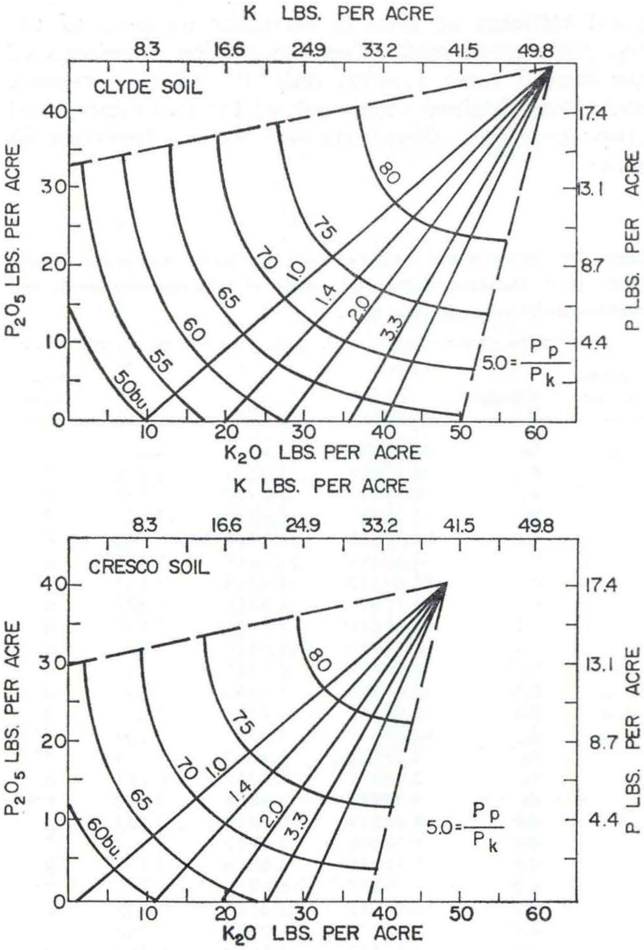

Fig. 8. Yield isoquants, isoclines and ridgelines (dotted lines) for corn on Clyde and Cresco soils. Average response derived from generalized crop function 36.

(44) Clyde soil

$$C = 41.768 + 1.565006P + 1.078700K - 0.054507P^2 - 0.012986K^2 + 0.012533PK$$

(45) Cresco soil

$$C = 53.763 + 1.436993P + 0.823144K - 0.054507P^2 - 0.012986K^2 + 0.012533PK$$

Crop function 38 was transformed into:

(46) Clyde soil

$$C = 36.742 + 1.361729P + 2.551172K - 0.091574P^2 - 0.054743K^2 + 0.011183PK$$

(47) Cresco soil

$$C = 60.559 + 1.800150P + 0.713887K - 0.091574P^2 - 0.019198K^2 + 0.011183PK$$

Yield, C, is expressed in bushels per acre and fertilizer inputs in pounds per acre.

Fig. 8 shows the isoquant maps corresponding to

Table 20. Generalized crop functions for hay: Regression coefficients (b_i), standard errors (s_b), value of t, probability levels and coefficients of determination (R^2).

Function number	Variable[a]	b_i	s_b	t	Prob. level[b]
(42) -----	b_0	-0.42367	—	—	
	P_s	0.38589	0.1666	2.316	*
	K_s	0.01737	0.0046	3.767	**
	W	-0.76236	0.1729	4.409	**
	P	-0.06899	1.9654	0.035	d
	K	-0.27220	1.0333	0.263	d
	P^2	-0.95427	0.3633	2.627	*
	K^2	-0.02725	0.1004	0.271	d
	PK	0.05959	0.1176	0.507	d
	P_sP	1.58116	0.4676	3.381	**
	K_sP	-0.00946	0.0128	0.731	d
	P_sK	-0.02379	0.2459	0.097	d
	K_sK	0.00331	0.0067	0.487	d
	d_1	0.77292	0.1543	5.008	**
	d_2	0.10522	0.0612	1.718	+
	d_3	0.23323	0.1561	1.493	++
	d_4	0.17184	0.1143	1.504	++
	d_1P	-1.48394	0.3755	3.951	**
	d_2P	0.69116	0.1547	4.467	**
	d_3P	-0.10459	0.2287	0.457	d
	d_4P	0.77798	0.2374	3.279	**
	d_1K	0.21144	0.1974	1.071	b
	d_2K	0.01088	0.0813	0.134	d
	d_3K	0.09771	0.1202	0.812	d
	d_2K	0.02968	0.1248	0.238	d
	d_1P^2	-0.16478	0.4278	0.385	d
	d_2P^2	-0.06651	0.1480	0.448	d
	d_3P^2	0.11812	0.7412	0.159	d
	d_4P^2	0.47735	0.4278	0.991	c
	d_1K^2	-0.01532	0.1183	0.129	d
	d_2K^2	-0.02757	0.0409	0.673	d
	d_3K^2	-0.05804	0.2049	0.283	d
	d_4K^2	-0.09915	0.1183	0.838	d
	WP	-2.47605	0.4853	5.104	**
	WK	-0.00320	0.2551	0.012	d
		$R^2 = 0.928$			
(43) -----	b_0	-0.44151	—	—	
	P_s	0.38589	0.1546	2.496	*
	K_s	0.01737	0.0043	4.060	**
	W	-0.76236	0.1604	4.752	**
	P	-0.58613	1.4913	0.393	d
	K	0.10864	0.4300	2.529	*
	P^2	-0.87017	0.2935	2.966	**
	K^2	-0.01165	0.8109	0.144	d
	PK	0.05959	0.1090	0.546	d
	P_sP	1.64164	0.4161	3.945	**
	K_sP	-0.00625	0.0101	0.620	d
	d_1	0.74496	0.1242	5.999	**
	d_2	0.08411	0.0512	1.644	c
	d_3	0.22156	0.0756	2.929	**
	d_4	0.18017	0.0785	2.296	*
	d_1P	-1.41468	0.3189	4.436	**
	d_2P	0.71653	0.1340	5.347	**
	d_4P	0.82799	0.1954	4.237	**
	d_1K	0.10451	0.0504	2.074	+
	WP	-2.44519	0.4459	5.485	**
		$R^2 = 0.924$			

[a] b_0 is yield intercept.

[b] Probability levels are:

** $p \leqslant 0.01$ + $0.05 < p \leqslant 0.10$ b $0.20 < p \leqslant 0.30$ d $p > 0.40$
* $0.01 < p \leqslant 0.05$ ++ $0.10 < p \leqslant 0.20$ c $0.30 < p \leqslant 0.40$

equations 44 and 45. The maximum yield of corn on Clyde soil predicted by function 44 is 85.3 bushels with an application of 20.3 pounds of P and 51.3 pounds of K. These estimates contrast with a maximum yield of 75.5 bushels obtained with application of 12.8 pounds of P and 27.1 pounds of K, predicted by the average function of corn on Clyde soil presented in table 1. Equation 45 predicts a maximum yield of 83.1 bushels of corn on Cresco soil with application of about 17.8 pounds of P and 40.3 pounds of K. The predicted maximum yield is only 3.4 bushels higher than that estimated by the average function for corn on Cresco soil as presented in table 2. But the rates of fertilizer application estimated by crop function 36 are 5.9 pounds of P and 15.9 pounds of K greater than for the average function.

The configuration and position of the isoquants and isoclines, corresponding to equation 36, are similar for the two maps in fig. 8. The configuration is similar since the squared and interaction terms of P and K were identical in functions 44 and 45—the corresponding equations for 36. Only the yield intercept and the linear terms of functions 44 and 45 were altered by the inclusion of dummy and weather variables and of interaction terms in crop function 36. As a consequence, only the slope of the surfaces and the point of maximum yields relative to the input axes were different for the two derived functions 44 and 45.

In contrast, the isoquant maps obtained from equations 46 and 47 and presented in fig. 9 give somewhat different results in comparison with the average function in table 2. The maximum predicted yield, from the generalized function, of corn on Clyde soil is 73.7 bushels with 8.9 pounds of P and 24.2 pounds of K. The yield isoquants are only slightly bent and vertically disposed on the plane. The isoclines are clustered together near the lower ridgeline and intercept only the K axis. These features of the isoquant map denote the stronger response of corn to potassium on Clyde soil, and correspond to the findings described in the previous section.

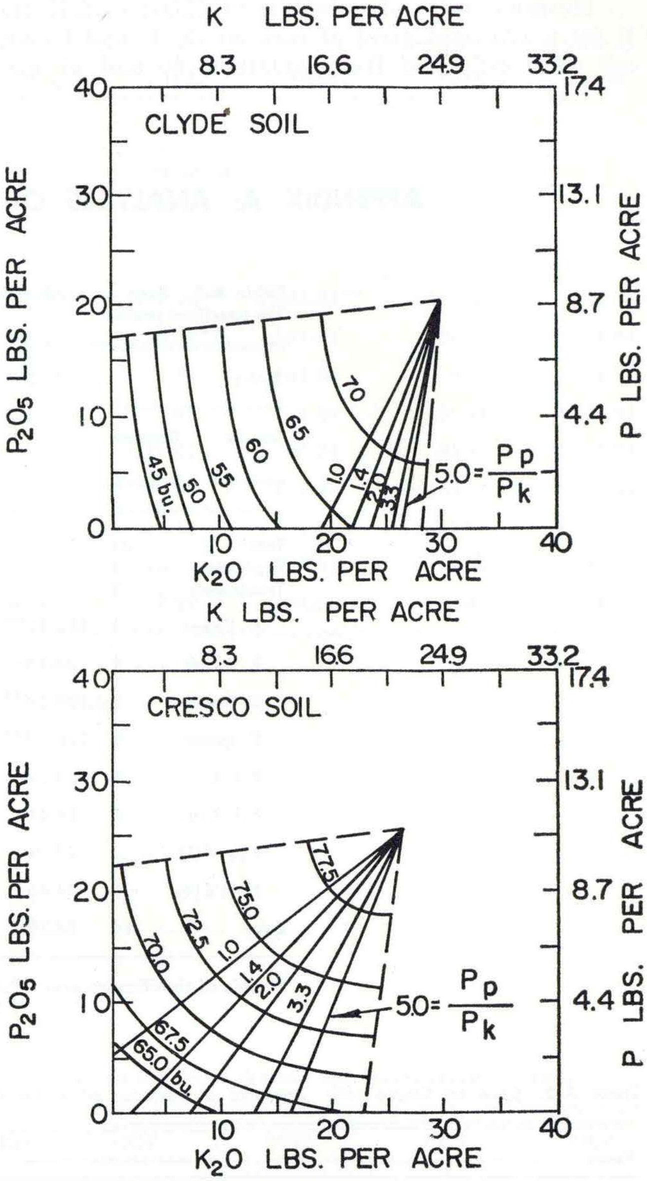

Fig. 9. Yield isoquants, isoclines and ridgelines (dotted lines) for corn on Clyde and Cresco soils. Average response derived from generalized crop function 38.

Table 21. Nutrient combinations and marginal rates of substitution for a 70-bushel yield of corn on Clyde and Cresco soils, estimated from the generalized crop function 38; weather index set at average value.

Clyde (44)[a]			Cresco (45)[a]		
Lbs. of P	Lbs. of K	MRS $\frac{\partial P}{\partial K}$	Lbs. of P	Lbs. of K	MRS $\frac{\partial P}{\partial K}$
8.4	15.9	∞	9.9	0.7	∞
6.0	16.6	3.404	4.8	4.2	1.210
3.9	18.7	1.212	3.0	8.3	0.606
2.9	20.8	0.561	2.0	12.5	0.313
2.6	22.8	0.133	1.6	16.6	0.105
2.5	23.6	0	1.5	19.0	0

[a] Number of "short-run" or derived equation estimated from the generalized crop function 38. Weather index set at average value.

For Cresco soil, the maximum predicted yield of corn is 78.4 bushels, with application of 11.2 pounds of P and 21.8 pounds of K. The yield isoquants are curved and more "symmetrically" placed relative to the input axes. Therefore, the isoclines are spread apart and intercept both the P and K coordinates. The results for corn on Cresco soil also correspond closely with the results obtained from the average function in table 3.

Marginal rates of substitution (MRS) of K for P for a 70-bushel yield of corn on Clyde and Cresco soil were estimated from equations 46 and 47 and are presented in table 21. The range of nutrient substitution is smaller and the replacement rates change much faster for Clyde than for Cresco soil.

APPENDIX A: ANALYSES OF VARIANCE OF ANNUAL YIELDS

Table A-1. Corn on Clyde soil: Analyses of variance of yields during specified years.

Years		1950	1953	1956	1959
Source of variation	Degrees of freedom	Mean squares[a]			
Total	26				
Replicates	2				
Treatments	8				
P linear	1	272.22**	4.70	40.50	3.92
P quadr	1	36.18	188.16*	18.49	86.64
K linear	1	2,499.24**	4,050.00**	7,212.00**	4,704.50**
K quadr	1	226.12**	308.17**	963.51**	384.00
P(l)K(l)	1	66.27	392.16**	110.41*	500.52
P(l)K(q)	1	28.44	0.00	13.69	57.00
P(q)K(l)	1	27.04	0.01	0.49	0.90
P(q)K(q)	1	24.46	59.85	14.81	27.30
Error	16	24.79	31.23	20.61	138.36

[a] Levels of significance are: **:0.01
*:0.05

Table A-2. Corn on Cresco soil: Analyses of variance of yields during specified years.

Years		1947	1950	1953	1956	1959
Source of variation	Degrees of freedom	Mean squares[a]				
Total	26					
Replicates	2					
Treatments	8					
P linear	1	276.12**	1,558.68**	239.80*	271.44*	787.48**
P quadr	1	98.42*	439.76*	207.68*	28.02	13.29
K linear	1	87.12*	92.93	300.12**	658.84**	398.04*
K quadr	1	0.06	65.12	105.00	163.98	22.78
P(l)K(l)	1	21.87	0.52	132.67	1.84	262.21*
P(l)K(q)	1	0.64	0.03	54.02	18.92	0.86
P(q)K(l)	1	47.61	72.53	23.52	12.60	61.32
P(q)K(q)	1	5.88	1.79	0.70	0.32	16.78
Error	16	15.60	46.72	34.05	55.41	48.57

[a] Levels of significance are: **:0.01
*:0.05

Table A-3. Corn on acid Webster soil: Analyses of variance of yields during specified years.

Years		1954	1955	1956	1957	1958	1959	1960
Source of variation	Degrees of freedom				Mean squares[a]			
Total	17							
Replicates	1							
Treatments	8							
P linear	1	71.05	733.20**	302.00	639.05**	2,682.03**	1,689.81**	383.07
P quadr	1	66.15	14.19	94.09	93.90**	129.20	80.40	9.61
K linear	1	20.80	95.20	11.21	0.21	145.60	0.65	2.90
K quadr	1	0.13	2.45	0.16	7.67	9.20	131.48	31.92
P (l) K (l)	1	3.38	33.62	0.18	31.03	57.24	55.12	0.06
P (l) K (q)	1	0.43	44.83	55.21	0.98	118.82	33.61	28.38
P (q) K (l)	1	11.21	49.31	23.21	30.16	4.68	20.17	8.00
P (q) K (q)	1	12.84	122.72	204.02	14.71	5.01	4.40	154.00
Error	8	25.04	48.48	61.86	7.39	103.52	52.47	95.33

[a] Levels of significance are: **:0.01
*:0.05

Table A-4. Corn on calcareous Webster soil: Analyses of variance of corn yields on calcareous Webster soil during specified years.

Years		1954	1955	1956	1957	1958	1959	1960
Source of variation	Degrees of freedom				Mean squares[a]			
Total	17							
Replicates	1							
Treatments	8							
P linear	1	193.60*	788.94**	320.33*	3,468.00**	5,022.52**	4,981.68**	5,357.38**
P quadr	1	26.01	16.40	42.68	7.11	176.45*	1.48	0.01
K linear	1	16.33	0.07	63.48	0.08	211.68*	17.52	65.51
K quadr	1	4.00	21.62	82.20	29.52	11.11	58.01	14.21
P (l) K (l)	1	10.12	125.61	14.58	0.40	49.00	69.62	52.82
P (l) K (q)	1	11.48	34.32	97.61	2.54	1.60	69.36	128.02
P (q) K (l)	1	0.20	4.59	475.26*	8.88	18.38	96.80	18.46
P (q) K (q)	1	8.40	1.53	231.84	112.00	1.03	204.69	18.57
Error	8	30.57	39.68	45.93	99.67	24.64	48.61	120.00

[a] Levels of significance are: **:0.01
*:0.05

Table A-5. Corn on Seymour soil: Analyses of variance of yields during specified years.

Years		1949	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960
Source of variation	Degrees of freedom	Mean squares[a]										
Total	17											
Replicates	1											
Treatments	8											
P linear	1	31.36	1,496.33**	396.75*	110.41	164.28*	34.00	31.69	173.28**	26.11	395.60*	257.61
P quadr	1	0.03	143.20	35.20	0.19	52.32	8.22	1.65	1.07	5.84	3.55	171.61
K linear	1	78.54	0.03	731.64**	34.34	0.12	6.45	11.80	14.30	4.44	235.85	110.41
K quadr	1	1.48	12.48	54.51	2.20	4.13	8.22	1.65	18.63	5.68	8.80	90.25
P (l) K (l)	1	148.78*	6.84	23.12	79.38	9.90	1.20	10.35	4.50	44.18	16.53	2.42
P (l) K (q)	1	17.17	26.04	0.00	26.88	0.09	17.17	18.90	61.44*	36.02	12.76	25.63
P (q) K (l)	1	42.40	54.00	0.04	8.40	10.53	8.52	34.32	2.67	0.03	21.47	3.68
P (q) K (q)	1	6.18	61.98	2.49	6.24	2.61	0.96	0.02	4.30	49.67	14.67	19.84
Error	8	19.86	52.83	33.06	31.72	20.72	30.92	7.65	6.26	22.04	51.86	60.78

[a] Levels of significance are: **:0.01
*:0.05

Table A-6. Oats on Clyde soil: Analyses of variance of yields during specified years.

Years		1948	1951	1954	1957	1960
Source of variation	Degrees of freedom	Mean squares[a]				
Total	26					
Replicates	2					
Treatments	8					
P linear	1	88.44	375.38**	657.64**	774.87**	412.80**
P quadr	1	11.30	31.43	54.40*	292.60**	49.31
K linear	1	0.29	61.98	176.09**	22.89	220.50*
K quadr	1	15.47	29.63	78.96*	70.04	37.50
P (l) K (l)	1	0.56	8.33	50.84*	42.94	0.10
P (l) K (q)	1	102.01	11.56	1.48	2.51	8.51
P (q) K (l)	1	5.92	1.60	0.01	6.33	13.32
P (q) K (q)	1	40.09	39.84	19.34	3.31	285.19**
Error	16	47.30	18.77	10.50	18.35	33.23

[a] Levels of significance are: **:0.01
*:0.05

Table A-7. Oats on Cresco soil: Analyses of variance of yields during specified years.

Years		1945	1948	1951	1954	1957	1960
Source of variation	Degrees of freedom	Mean squares[a]					
Total	26						
Replicates	2						
Treatments	8						
P linear	1	534.64**	621.87**	798.67**	1,101.37**	1,411.58**	274.56
P quadr	1	176.76*	56.02	26.32	147.34**	142.11**	400.71
K linear	1	0.02	0.06	419.53**	36.98	20.27	30.68
K quadr	1	0.07	43.02	6.90	1.18	7.48	25.76
P(l) K(l)	1	5.88	35.02	5.33	4.69	0.08	16.33
P(l) K(q)	1	6.25	27.21	19.07	5.37	1.79	16.27
P(q) K(l)	1	40.96	2.51	19.65	0.00	0.11	0.13
P(q) K(q)	1	3.93	0.73	1.81	9.66	27.60	157.44
Error	16	25.78	21.18	19.92	15.86	13.45	104.74

[a] Levels of significance are: **:0.01
*:0.05

Table A-8. Oats on acid Webster soil: Analyses of variance of yields during specified years.

Years		1955	1956	1957	1958	1960
Source of variation	Degrees of freedom	Mean squares[a]				
Total	17					
Replicates	1					
Treatments	8					
P linear	1	3,356.71**	2,324.08**	12.81	5,034.80**	2,149.36**
P quadr	1	742.56**	705.79**	9.61	284.48*	1.96
K linear	1	7.21	51.67	7.52	85.33	30.40
K quadr	1	28.62	11.45	69.72	4.99	183.60
P(l) K(l)	1	0.21	11.52	139.44	0.32	111.76
P(l) K(q)	1	60.48	12.91	5.80	136.33	15.20
P(q) K(l)	1	6.30	10.94	1.60	66.00	110.51
P(q) K(q)	1	5.61	241.27*	25.20	0.93	155.76
Error	8	42.27	24.03	45.16	52.44	36.02

[a] Levels of significance are: **:0.01
*:0.05

Table A-9. Oats on calcareous Webster soil: Analyses of variance of yields during specified years.

Years		1955	1956	1957	1958	1959	1960
Source of variation	Degrees of freedom	Mean squares[a]					
Total	17						
Replicates	1						
Treatments	8						
P linear	1	6,097.52**	1,875.00*	293.04**	6,519.34**	11,907.00**	4,892.44**
P quadr	1	259.75	417.52	109.20*	364.17	16.81	160.87
K linear	1	161.33	12.20	1.40	9.36	80.60	82.16
K quadr	1	0.28	60.58	4.20	1.78	103.02	49.94
P(l)K(l)	1	150.51	141.12	55.65	32.40	508.80	18.91
P(l)K(q)	1	78.12	7.26	9.50	31.97	65.34	9.50
P(q)K(l)	1	65.67	11.21	0.22	4.25	130.67	3.92
P(q)K(q)	1	139.17	159.61	21.45	64.03	83.20	9.31
Error	8	72.90	204.64	16.34	307.31	105.93	54.85

[a] Levels of significance are: **:0.01
*:0.05

Table A-10. Oats on Seymour soil: Analyses of variance of yields during specified years.

Years		1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1960
Source of variation	Degrees of freedom	Mean squares[a]										
Total	17											
Replicates	1											
Treatments	8											
P linear	1	174.80**	16.57	25.23	461.28**	155.52*	6.02	1,518.75**	66.27*	320.33**	1,399.68**	247.52
P quadr	1	47.15	0.10	8.80	46.69	15.47	9.71	704.02**	33.25	0.13	75.69	3.18
K linear	1	15.64	0.91	5.88	1.20	5.33	10.83	20.80	6.31	45.24	178.64	0.27
K quadr	1	3.67	32.30	70.00	0.05	26.35	22.40	5.92	0.97	8.90	0.72	0.59
P(l)K(l)	1	0.04	1.20	3.51	2.00	5.95	2.20	15.96	5.61	20.16	80.64	19.53
P(1)K(q)	1	10.67	5.70	24.60	52.22	1.35	4.00	6.93	6.30	0.00	103.34	21.47
P(l)K(q)	1	16.67	47.88*	32.43	4.17	0.01	67.34	7.82	1.65	12.76	196.08	41.34
P(q)K(q)	1	32.27	6.18	7.67	0.01	2.38	33.89	109.77	0.45	32.94	0.60	40.35
Error	8	10.05	6.95	15.30	25.92	11.76	50.61	26.09	10.56	21.14	42.06	58.41

[a] Levels of significance are: **:0.01
*:0.05

Table A-11. Hay on Clyde soil: Analyses of variance of yields during specified years.

Years		1949	1952	1955	1958
Source of variation	Degrees of freedom	Mean squares[a]			
Total	26				
Replicates	2				
Treatments	8				
P linear	1	0.4802*	5.3029**	0.0187	4.2827**
P quadr	1	0.0096	0.6823**	0.3392	0.4231*
K linear	1	0.4294*	1.9208**	2.1287**	0.8756**
K quadr	1	0.0294	0.1330	0.0031	0.0733
P (l) K (l)	1	0.3924*	0.2187	0.0616	0.2187
P (l) K (q)	1	0.0506	0.0608	0.0245	0.2336
P (q) K (l)	1	0.0191	0.5625*	0.0044	0.0413
P (q) K (q)	1	0.2160	0.1394	0.0636	0.0307
Error	16	0.0597	0.0706	0.0773	0.0909

[a] Levels of significance are: **:0.01
*:0.05

Table A-12. Hay on Cresco soil: Analyses of variance of yields during specified years.

Years		1946	1949	1952	1955	1958
Source of variation	Degrees of freedom	Mean Squares[a]				
Total	26					
Replicates	2					
Treatments	8					
P linear	1	3.3368**	3.7174**	1.4964**	4.5300**	5.9858**
P quadr	1	0.0358	0.5974**	0.0719	0.6913	0.7397*
K linear	1	0.0029	0.3280**	0.4900*	1.3448*	0.2939
K quadr	1	0.0236	0.0007	0.3733	0.1102	0.0613
P (l) K (l)	1	0.0061	0.0154	0.0019	0.0040	0.0363
P (l) K (q)	1	0.0831	0.0283	0.0012	0.4096	0.0576
P (q) K (l)	1	0.0000	0.0020	0.0702	0.0625	0.1045
P (q) K (q)	1	0.0110	0.0098	0.0000	0.1070	0.0313
Error	16	0.0354	0.0374	0.1015	0.2855	0.1598

[a] Levels of significance are: **:0.01
*:0.05

Table A-13. Hay on acid Webster soil: Analyses of variance of yields during specified years.

Years		1956	1957	1958	1959	1960
Source of variation	Degrees of freedom			Mean Squares[a]		
Total	17					
Replicates	1					
Treatments	8					
P linear	1	4.8641**	9.0100**	3.3180**	7.8894**	47.2430**
P quadr	1	0.0592	1.0700**	0.0210	0.0240	5.2212
K linear	1	0.1121	0.1000	0.0208	0.0133	0.0660
K quadr	1	0.0205	0.3800*	0.1024	0.0001	0.1260
P (l) K (l)	1	0.0045	0.5000*	0.0004	0.0045	0.1035
P (l) K (q)	1	0.2926	0.0400	0.0001	0.2035	0.0442
P (q) K (l)	1	0.1080	0.1700	0.0043	0.1190	0.0925
P (q) K (q)	1	0.0184	0.0700	0.0084	0.0990	0.0781
Error	8	0.2703	0.0700	0.1534	0.0780	0.1212

[a]Levels of significance are: **:0.01
*:0.05

Table A-14. Hay on calcareous Webster soil: Analyses of variance of yields during specified years.

Years		1956	1957	1958	1959	1960
Source of variation	Degrees of freedom			Mean squares[a]		
Total	17					
Replicates	1					
Treatments	8					
P linear	1	9.7741**	13.2300**	13.2510**	16.3100**	43.5103**
P quadr	1	0.2450	0.1100	0.0010	0.0251	1.1413**
K linear	1	0.1474	0.1400	0.0091	0.0114	0.1825*
K quadr	1	0.2070	0.0100	0.0477	0.0191	0.0374
P (l) K (l)	1	0.0021	0.1000	0.0220	0.1352	0.0181
P (l) K (q)	1	0.0273	0.0000	0.0043	0.0241	0.0301
P (q) K (l)	1	0.4082	0.1200	0.1094*	0.0033	0.0782
P (q) K (q)	1	0.0021	0.0700	0.4110**	0.0174	0.0284
Error	8	0.4162	0.0800	0.0106	0.1732	0.0343

[a]Levels of significance are: **:0.01
*:0.05

Table A-15. Hay on Seymour soil: Analyses of variance of yields during specified years.

Years		1950	1951	1953	1954	1955	1956	1957	1958	1959	1960
Source of variation	Degrees of freedom	Mean squares[a]									
Total	17										
Replicates	1										
Treatments	8										
P linear	1	0.6257**	0.6120**	0.4107**	0.4641**	0.7057*	0.3104**	0.5896**	0.0432	0.4408**	2.6040**
P quadr	1	0.0064	0.0002	0.1156*	0.0576	0.2070	0.0001	0.0289	0.1272	0.0625	0.5305**
K linear	1	0.0102	0.0736*	0.0030	0.1365*	0.1323	0.0021	0.2002*	0.0444	0.0588	0.0001
K quadr	1	0.0240	0.0049	0.0006	0.0400	0.0441	0.0000	0.0002	0.0393	0.0016	0.0160
P(l)K(l)	1	0.0028	0.0112	0.0364	0.0120	0.2701	0.0002	0.0098	0.0338	0.0032	0.0021
P(l)K(q)	1	0.0015	0.0641*	0.1441*	0.0057	0.4620	0.0353*	0.0888	0.0038	0.0308	0.0925
P(q)K(l)	1	0.0000	0.0140	0.0561	0.0070	0.0408	0.0113	0.1473	0.1504	0.0006	0.0057
P(q)K(q)	1	0.0000	0.5202**	0.0000	0.0136	0.0036	0.0006	0.0181	0.0624	0.0024	0.0159
Error	8	0.0220	0.0149	0.0273	0.0253	0.1576	0.0093	0.0537	0.2912	0.0213	0.0573

[a] Levels of significance are: **:0.01
*:0.05

APPENDIX B: STATISTICS FOR SQUARE ROOT FUNCTIONS

(The b_1 and b_2 refer to linear terms for P and K, respectively, and b_3 and b_4 refer to square roots of these same two nutrient quantities. The b_5 term refers to the interaction coefficient PK.)

Table B-1. Corn on acid Webster soil: Regression coefficients (b_i), standard errors, t values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1954	64.961	1.51423	0.08325	-0.02734	-0.00090	0.00798	0.868
Standard error		0.48003	0.25238	0.00961	0.00697	0.01239	
t		3.154	0.330	2.844	0.129	0.643	
Probability[a]		+	d	+	d	d	
1955	49.402	1.45670	-0.26475	-0.01268	-0.00383	0.02514	0.802
Standard error		1.42742	0.75047	0.02858	0.02073	0.03687	
t		1.020	0.353	0.443	0.185	0.682	
Probability[a]		c	d	d	d	d	
1956	64.050	-0.69245	0.05756	0.03262	0.00098	-0.00185	0.591
Standard error		1.62918	0.85654	0.03262	0.02366	0.04209	
t		0.425	0.067	1.000	0.041	0.044	
Probability[a]		d	d	c	d	d	
1957	74.033	2.50758	-0.22353	-0.03530	0.00122	0.02821	0.954
Standard error		0.66048	0.34725	0.01322	0.00959	0.01706	
t		3.797	0.644	2.669	0.127	1.654	
Probability[a]		*	d	+	d	++	
1958	74.494	3.61169	-0.73789	-0.03823	0.00739	0.03282	0.959
Standard error		1.09872	0.57765	0.02200	0.01596	0.02837	
t		3·287	1.277	1.738	0.463	1.156	
Probability[a]		*	b	++	d	c	
1959	57.853	3.59590	1.14638	-0.03036	-0.02789	-0.03221	0.971
Standard error		0.75053	0.39459	0.01502	0.01090	0.01938	
t		4.791	2.905	2.021	2.558	1.661	
Probability[a]		*	+	++	+	++	
1960	66.173	1.37243	0.45413	-0.00841	-0.01483	-0.00588	0.778
Standard error		1.16966	0.61483	0.02342	0.01699	0.03019	
t		1.174	0.739	0.359	0.873	0.195	
Probability[a]		c	d	d	d	d	
Average	64.148	1.92779	0.06605	-0.01741	-0.00515	0.00756	0.958
Standard error		0.58139	0.30567	0.01165	0.00845	0.01501	
t		3.316	0.216	1.496	0.609	0.504	
Probability[a]		*	d	b	d	d	

[a] Probability levels for square root functions are same as indicated by symbols on table 2 and Appendix tables.

Table B-2. Corn on calcareous Webster soil: Regression coefficients (b_i), standard errors, t values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1954	56.558	1.22071	-0.15727	-0.01714	0.00487	0.01380	0.926
Standard error		0.43439	0.22838	0.00870	0.00631	0.01123	
t		2.810	0.689	1.971	0.773	1.230	
Probability		+	d	++	d	c	
1955	43.683	1.24488	0.05756	-0.01345	-0.01122	0.04847	0.959
Standard error		0.61833	0.32507	0.01238	0.00898	0.01598	
t		2.013	0.177	1.086	1.249	3.034	
Probability		++	d	d	c	+	
1956	48.237	-0.00204	1.02150	0.02195	-0.02213	-0.01656	0.394
Standard error		2.74999	1.44581	0.05507	0.03994	0.07104	
t		0.0000	0.706	0.399	0.553	0.233	
Probability		d	d	d	d	d	
1957	57.678	2.97015	-0.46083	-0.00897	0.01325	0.00276	0.966
Standard error		1.07705	0.56626	0.02157	0.01565	0.02782	
t		2.758	0.814	0.416	0.846	0.099	
Probability		+	d	d	d	d	
1958	50.947	4.78045	0.40394	-0.04481	-0.00812	0.03066	0.966
Standard error		0.45613	0.23981	0.00914	0.00663	0.01179	
t		10.480	1.684	4.907	1.226	2.603	
Probability		**	++	*	c	+	
1959	34.380	3.45589	0.44235	-0.00908	-0.02090	0.00091	0.885
Standard error		2.46393	1.29541	0.04933	0.03579	0.06365	
t		1.403	0.341	0.184	0.584	0.001	
Probability		b	d	d	d	d	
1960	52.307	2.75989	0.45333	0.00343	-0.01435	0.03152	0.954
Standard error		1.62792	0.85588	0.03260	0.02365	0.04206	
t		1.695	0.530	0.105	0.606	0.750	
Probability		a	d	d	d	d	
Average	50.060	2.34798	0.25203	-0.00974	-0.00839	0.01595	0.956
Standard error		1.00013	0.52582	0.02003	0.01453	0.02583	
t		2.348	0.479	0.487	0.577	0.617	
Probability		+	d	d	d	d	

Table B-3. Corn on Seymour soil: Regression coefficients (b_i), standard errors, t values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1949	71.791	0.00383	-0.29245	-0.00974	0.00220	0.05796	0.621
Standard error		1.25911	0.66198	0.02520	0.01829	0.03251	
t		0.003	0.442	0.387	0.120	1.782	
Probability		d	d	d	d	++	
1951	67.337	3.68171	-0.19515	-0.04024	0.00836	-0.01165	0.921
Standard error		1.15330	0.60635	0.02309	0.01675	0.02978	
t		3.192	0.322	1.753	0.499	0.391	
Probability		*	d	++	d	d	
1952	91.590	2.05445	1.35852	-0.02017	-0.01804	-0.02087	0.998
Standard error		0.12663	0.06658	0.00254	0.00184	0.00328	
t		16.223	20.404	7.955	9.811	6.376	
Probability		**	**	**	**	**	
1953	50.627	1.00888	0.50098	-0.00145	-0.00349	-0.03834	0.839
Standard error		0.63739	0.33511	0.01277	0.00926	0.01645	
t		1.583	1.495	0.113	0.377	2.329	
Probability		b	b	d	d	++	
1954	18.047	-1.50761	-0.06050	0.02445	0.00481	-0.01350	0.944
Standard error		0.35988	0.18921	0.00720	0.00522	0.00930	
t		4.189	0.320	3.392	0.919	1.452	
Probability		*	d	*	d	b	
1955	8.645	0.13161	0.13746	-0.00998	-0.00700	0.00522	0.681
Standard error		0.50892	0.26757	0.01020	0.00740	0.01314	
t		0.259	0.514	0.980	0.947	0.397	
Probability		d	d	c	d	d	
1956	57.208	0.22021	-0.09302	0.00437	0.00317	-0.01380	0.515
Standard error		0.70686	0.37163	0.01413	0.01025	0.01824	
t		0.312	0.250	0.309	0.309	0.756	
Probability		d	d	d	d	d	
1957	77.470	-0.53156	-0.32230	-0.00360	0.01056	0.00919	0.756
Standard error		0.80075	0.42100	0.01604	0.01163	0.02067	
t		0.664	0.765	0.224	0.908	0.445	
Probability		d	d	d	d	d	
1958	94.763	0.95671	-0.05112	-0.00817	0.00578	-0.02881	0.505
Standard error		0.89306	0.46953	0.01788	0.01297	0.02307	
t		1.071	0.109	0.457	0.446	1.250	
Probability		c	d	d	d	b	
1959	109.688	1.37548	0.23061	-0.00616	0.00724	-0.01747	0.932
Standard error		0.67366	0.35418	0.01349	0.00979	0.01739	
t		2.042	0.651	0.456	0.740	1.005	
Probability		a	d	d	d	c	
1960	88.095	2.28279	0.78928	-0.03284	-0.01529	-0.00643	0.946
Standard error		0.52084	0.27383	0.01042	0.00756	0.01344	
t		4.383	2.882	3.150	2.021	0.479	
Probability		*	+	+	++	d	
Average	66.830	0.87895	0.18082	-0.00939	-0.00014	-0.00707	0.997
Standard error		0.03359	0.01766	0.00067	0.00048	-0.00086	
t		26.330	10.106	14.164	0.300	8.140	
Probability		**	**	**	d	**	

Table B-4. Oats on acid Webster soil: Regression coefficients (b_i), standard errors, t values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1955	37.617	3.36678	0.24561	-0.02286	-0.00317	-0.00047	0.983
Standard error		0.41230	0.21677	0.00413	0.00300	0.00533	
t		8.166	1.133	5.539	1.059	0.086	
Probability		**	c	*	c	d	
1956	30.603	3.17943	-0.17079	-0.02230	0.00205	-0.00360	0.907
Standard error		0.85581	0.44994	0.00856	0.00621	0.01104	
t		3.715	0.380	2.603	0.330	0.333	
Probability		*	d	+	d	d	
1957	35.570	0.00101	-0.13599	0.00266	0.00510	-0.01289	0.881
Standard error		0.27710	0.14568	0.00277	0.00201	0.00359	
t		0.004	0.933	0.960	2.535	3.598	
Probability		d	d	d	+	*	
1958	61.500	2.83602	-0.20410	-0.01418	0.00136	0.00061	0.964
Standard error		0.68644	0.36090	0.00687	0.00498	0.00886	
t		4.132	0.566	2.063	0.273	0.069	
Probability		*	d	++	d	d	
1959	62.733	2.46558	-1.48032	-0.01219	0.01854	0.01777	0.948
Standard error		1.01555	0.53393	0.01017	0.00738	0.01311	
t		2.428	2.772	1.199	2.513	1.356	
Probability		+	+	c	+	b	
1960	43.429	1.41450	-0.45792	-0.00118	0.00826	-0.01145	0.905
Standard error		0.78531	0.41288	0.00787	0.00571	0.01016	
t		1.801	1.109	0.150	1.448	1.130	
Probability		a	c	d	b	c	
Average	45.247	2.21011	-0.36727	-0.01166	0.00536	-0.00168	0.984
Standard error		0.33108	0.17406	0.00331	0.00240	0.00428	
t		6.676	2.110	3.519	2.229	0.394	
Probability		**	++	*	++	d	

Table B-5. Oats on calcareous Webster soil: Regression coefficients (b_i), standard errors, t values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1955	31.750	3.28405	0.29780	-0.01352	0.00036	-0.01333	0.959
Standard error		0.81425	0.42809	0.00815	0.00592	0.01052	
t		4.033	0.696	1.660	0.062	1.268	
Probability		*	d	++	d	b	
1956	20.458	2.84045	0.43599	-0.01723	-0.00470	-0.01281	0.933
Standard error		0.64878	0.34109	0.00649	0.00471	0.00839	
t		4.378	1.278	2.653	0.996	1.528	
Probability		*	b	+	c	b	
1957	23.912	1.37511	0.17420	-0.00876	-0.00124	-0.00806	0.937
Standard error		0.27116	0.14256	0.00271	0.00196	0.00351	
t		5.071	1.222	3.230	0.629	2.299	
Probability		*	c	*	d	++	
1958	46.783	3.07914	0.00805	-0.01602	-0.00083	0.00629	0.986
Standard error		0.48583	0.25542	0.00487	0.00353	0.00615	
t		6.338	0.031	3.295	0.236	1.002	
Probability		**	d	*	d	c	
1959	30.981	3.33179	-0.18638	-0.00351	0.00611	-0.02445	0.978
Standard error		0.81186	0.42684	0.00812	0.00589	0.01049	
t		4.104	0.437	0.431	1.038	2.332	
Probability		*	d	d	c	++	
1960	31.957	2.62819	-0.11694	-0.01067	0.00431	-0.00472	0.996
Standard error		0.23118	0.12154	0.00231	0.00168	0.00298	
t		11.368	0.962	4.605	2.566	1.578	
Probability		**	d	*	+	b	
Average	30.977	2.75746	0.10192	-0.01163	0.00067	-0.00949	0.992
Standard error		0.30187	0.15871	0.00303	0.00220	0.00389	
t		9.135	0.642	3.848	0.306	2.439	
Probability		**	d	*	d	+	

Table B-6. Oats on Seymour soil: Regression coefficients (b_i), standard errors, t values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1949	25.236	0.82126	0.03160	-0.00578	-0.00114	-0.00017	0.805
Standard error		0.37113	0.19512	0.00372	0.00270	0.00480	
t		2.213	0.161	1.554	0.423	0.032	
Probability		++	d	b	d	d	
1950	19.172	0.03725	-0.25276	0.00014	0.00370	0.00224	0.494
Standard error		0.40197	0.21134	0.00402	0.00292	0.00519	
t		0.093	1.196	0.035	1.268	0.428	
Probability		d	c	d	b	d	
1951	19.037	-0.06466	-0.28027	0.00247	0.00508	-0.00199	0.635
Standard error		0.38884	0.20443	0.00389	0.00282	0.00502	
t		0.166	1.371	0.633	1.799	0.397	
Probability		d	b	d	++	d	
1952	35.600	1.03104	0.04149	-0.00572	-0.00012	-0.00155	0.903
Standard error		0.35942	0.18897	0.00360	0.00261	0.00464	
t		2.869	0.220	1.589	0.047	0.330	
Probability		+	d	b	d	d	
1953	20.975	0.89421	0.01438	-0.00747	-0.00183	0.00207	0.835
Standard error		0.36299	0.19084	0.00363	0.00263	0.00469	
t		2.463	0.075	2.058	0.694	0.442	
Probability		+	d	++	d	d	
1954	55.583	0.25586	-0.25364	-0.00269	0.00293	0.00168	0.336
Standard error		0.49460	0.26004	0.00495	0.00359	0.00638	
t		0.517	0.975	0.543	0.815	0.264	
Probability		d	d	d	d	d	
1955	92.847	2.77719	-0.01152	-0.02230	-0.00148	0.00431	0.947
Standard error		0.54343	0.28571	0.00545	0.00395	0.00701	
t		5.110	0.040	4.099	0.375	0.612	
Probability		*	d	*	d	d	
1956	13.337	0.67507	0.02624	-0.00486	0.00057	-0.00259	0.933
Standard error		0.13761	0.07235	0.00138	0.00100	0.00177	
t		4.979	0.362	3.523	0.568	1.466	
Probability		*	d	*	d	b	
1957	50.230	0.54734	-0.13384	-0.00033	0.00183	-0.00491	0.894
Standard error		0.33064	0.17384	0.00331	0.00240	0.00428	
t		1.655	0.770	0.146	0.762	1.149	
Probability		a	d	d	d	c	
1958	76.355	1.24909	0.06177	-0.00734	-0.00050	0.00974	0.853
Standard error		0.83954	0.44139	0.00841	0.00610	0.01085	
t		1.488	0.140	0.873	0.083	0.898	
Probability		b	d	d	d	d	
1960	38.883	0.32714	0.08767	0.00151	-0.00048	-0.00475	0.725
Standard error		0.49018	0.25771	0.00490	0.00356	0.00632	
t		0.667	0.340	0.308	0.137	0.751	
Probability		d	d	d	d	d	
Average	40.695	0.77046	-0.06003	-0.00467	0.00078	0.00039	0.980
Standard error		0.11674	0.06137	0.00116	0.00085	0.00152	
t		6.600	0.978	4.004	0.914	0.254	
Probability		**	c	*	d	d	

Table B-7. Hay on acid Webster soil: Regression coefficients (b_i), standard errors, t values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1956	1.292	0.01824	-0.00188	0.00009	0.00004	0.00003	0.923
Standard error		0.02103	0.01106	0.00014	0.00010	0.00019	
t		0.868	0.169	0.674	0.381	0.188	
Probability		d	d	d	d	d	
1957	1.352	0.08189	-0.02477	-0.00039	0.00015	0.00039	0.974
Standard error		0.01723	0.00906	0.00012	0.00009	0.00014	
t		4.755	2.734	3.358	1.834	2.527	
Probability		*	+	*	++	+	
1958	1.389	0.01948	0.00747	-0.00006	-0.00009	-0.00000	0.996
Standard error		0.00373	0.00196	0.00003	0.00002	0.00003	
t		5.208	3.801	2.245	4.640	0.317	
Probability		*	*	++	**	d	
1959	0.716	0.04825	0.00052	-0.00006	-0.00000	0.00003	0.949
Standard error		0.02112	0.01111	0.00014	0.00010	0.00019	
t		2.284	0.046	0.424	0.026	0.169	
Probability		++	d	d	d	d	
1960	0.497	0.21157	-0.01056	-0.00085	0.00010	0.00017	0.998
Standard error		0.01097	0.00577	0.00008	0.00005	0.00008	
t		19.287	1.831	11.659	1.811	1.641	
Probability		**	++	**	++	++	
Average	1.048	0.07595	-0.00581	-0.00023	0.00004	0.00011	0.992
Standard error		0.00971	0.00511	0.00006	0.00004	0.00008	
t		7.812	1.135	3.546	0.852	1.392	
Probability		**	c	*	d	b	

Table B-8. Hay on calcareous Webster soil: Regression coefficients (b_i), standard errors, t values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1956	0.978	0.02021	0.13531	0.00018	-0.00012	0.00003	0.960
Standard error		0.02129	0.01119	0.00014	0.00010	0.00019	
t		0.949	1.327	1.288	1.209	0.112	
Probability		d	b	b	c	d	
1957	0.664	0.07616	-0.00301	-0.00012	0.00003	-0.00017	0.986
Standard error		0.01437	0.00755	0.00009	0.00007	0.00014	
t		5.302	0.399	1.299	0.455	1.240	
Probability		*	d	b	d	c	
1958	0.180	0.05423	0.00655	0.00002	-0.00007	-0.00008	0.963
Standard error		0.02328	0.01221	0.00015	0.00011	0.00019	
t		2.335	0.537	0.088	0.538	0.341	
Probability		++	d	d	d	d	
1959	0.987	0.06046	0.00083	-0.00006	-0.00003	0.00019	0.997
Standard error		0.00699	0.00367	0.00005	0.00003	0.00006	
t		8.642	0.224	1.227	1.067	2.998	
Probability		**	d	c	c	+	
1960	0.108	0.14897	0.00723	-0.00039	-0.00005	0.00006	0.999
Standard error		0.00373	0.00196	0.00003	0.00002	0.00003	
t		39.838	3.677	15.978	2.887	1.957	
Probability		**	*	**	+	++	
Average	0.584	0.07194	0.00530	-0.00008	-0.00004	0.00000	0.997
Standard error		0.00742	0.00390	0.00005	0.00003	0.00006	
t		9.682	1.358	1.544	1.348	0.138	
Probability		**	b	b	b	d	

Table B-9. Hay on Seymour soil: Regression coefficients (b_i), standard errors, t values and coefficients of determination (R^2) during specified years.

	b_0	b_1	b_2	b_3	b_4	b_5	R^2
1950	2.036	0.00394	-0.00136	0.00003	0.00001	0.00019	0.931
Standard error		0.01068	0.00561	0.00008	0.00005	0.00008	
t		0.370	0.242	0.473	0.105	2.132	
Probability		d	d	d	d	a	
1951	2.202	0.00238	-0.00754	0.00005	0.00004	0.00014	0.659
Standard error		0.02348	0.01235	0.00015	0.00011	0.00019	
t		0.101	0.610	0.284	0.356	0.710	
Probability		d	d	d	d	d	
1953	2.393	0.03018	0.00167	-0.00012	0.00001	-0.00008	0.742
Standard error		0.01432	0.00753	0.00009	0.00007	0.00014	
t		2.107	0.222	1.332	0.784	0.741	
Probability		a	d	b	d	d	
1954	1.820	0.02376	0.00170	-0.00009	0.00005	-0.00006	0.961
Standard error		0.00548	0.00288	0.00003	0.00002	0.00006	
t		4.337	0.589	2.366	2.160	1.162	
Probability		*	d	+	a	c	
1955	2.061	0.04502	0.00234	-0.00017	0.00005	-0.00025	0.727
Standard error		0.02305	0.01212	0.00015	0.00011	0.00019	
t		1.954	0.193	1.114	0.484	1.252	
Probability		a	d	c	d	b	
1956	0.928	0.00788	0.00005	0.00000	-0.00000	-0.00000	0.860
Standard error		0.00724	0.00381	0.00005	0.00003	0.00006	
t		1.091	0.012	0.077	0.077	0.107	
Probability		c	d	d	d	d	
1957	1.933	0.01812	0.00230	-0.00006	0.00000	0.00006	0.762
Standard error		0.01624	0.00854	0.00011	0.00008	0.00014	
t		1.116	0.269	0.583	0.034	0.315	
Probability		c	d	d	d	d	
1958	3.065	0.02461	-0.00173	-0.00014	0.00005	-0.00008	0.564
Standard error		0.01512	0.00795	0.00011	0.00008	0.00014	
t		1.626	0.219	1.327	0.700	0.683	
Probability		b	d	b	d	d	
1959	3.263	0.02138	0.00053	-0.00009	0.00001	0.00003	0.948
Standard error		0.00566	0.00299	0.00005	0.00007	0.00006	
t		3.770	0.179	2.469	0.296	0.549	
Probability		*	d	+	d	d	
1960	2.891	0.05943	-0.00404	-0.00027	0.00003	0.00003	0.963
Standard error		0.01116	0.00587	0.00008	0.00005	0.00011	
t		5.323	0.688	3.628	0.655	0.247	
Probability		*	d	*	d	d	
Average	2.252	0.02410	-0.00076	-0.00009	0.00002	-0.00000	0.993
Standard error		0.00238	0.00125	0.00002	0.00001	0.00003	
t		10.149	0.608	5.654	2.210	0.333	
Probability		**	d	*	a	d	

APPENDIX C: WEATHER STATIONS

Table C-1. Weather stations from which precipitation and temperature data were obtained, by years.

Experimental farm	Weather station location and years	
	Precipitation data	Temperature data
Howard County	Cresco (1946-48) Saratoga (1949-60)	Cresco (1946-51) Saratoga (1952-60)
Clarion-Webster	Britt (1946-49) Kanawha (1950-60)	Britt (1946-60)
Seymour-Shelby	Millerton (1946-50) Corydon (1951-60)	Millerton (1946-50) Corydon (1951-60)

APPENDIX D: CORN WEATHER INDEXES

The precipitation, R, and temperature, T, variables used to compute the weather indexes for corn were defined as:

$$\text{(D.1)} \qquad \bar{R} = \Sigma\,(r - \bar{R}) + (2R, \text{if } r = -\bar{R})$$

$$\text{(D.2)} \qquad T = (t_{Jy} - \bar{T}_{Jy}) + (t_{Ag} - \bar{T}_{Ag})$$

where R is the estimated normal rainfall in inches for 10-day periods in July and August; r is the rainfall in 10-day periods in July and August, smaller than $\bar{R}$; $\bar{T}$ is the estimated normal average maximum temperature in July in degrees Farenheit; $\bar{T}_{Ag}$ is the estimated normal average maximum temperature in August; t_{Jy} is the average maximum temperature in July $> \bar{T}_{Jy}$; and t_{Ag} is the average maximum temperature in August $> \bar{T}_{Ag}$.

The estimated normal rainfall and temperatures for each experimental location are shown in table D.1.

The R and T variables and the weather indexes for corn estimated by equations 14, 15 and 16 are presented in table D-2.

To test the validity of the procedure, the weather indexes were used to explain the variation in the average corn yield of Howard, Hancock and Wayne counties over the period 1946-1960. A time variable t was included in the regression to account for the increase in county yields, Y, due to technological advancement.

The estimating equations with the corresponding R^2 values are:

(D.3) Howard County

$$Y = 56.16 - 0.437W + 1.124t \qquad R^2 = 0.80$$

(D.4) Hancock County

$$Y = 68.17 - 0.534W + 0.822t \qquad R^2 = 0.87$$

(D.5) Wayne County

$$Y = 47.93 - 0.260W + 0.847t \qquad R^2 = 0.82$$

Table D-1. Estimated normal rainfall for 10-day periods in July and August and estimated normal temperature in July and August at three Iowa locations.

Locations	$\bar{R}$ (inches)	$\bar{T}_{Jy}$ (°F)	$\bar{T}_{Ag}$ (°F)
Howard County	1.4	83	82
Clarion-Webster	1.1	86	85
Seymour-Shelby	1.3	87	86

Table D-2. Precipitation (R) and temperature (T) variables and weather indexes (W) for corn at three Iowa locations.

	Locations								
	Howard County			Clarion-Webster			Seymour-Shelby		
Year	R	T	W	R	T	W	R	T	W
1946	2.0	0	9.3	3.2	0	18.3	2.7	0	0
1947	8.4	8.9	39.2	7.9	9.7	56.2	6.1	9.0	82.3
1948	5.5	7.3	25.7	1.6	4.4	14.4	4.0	2.5	22.8
1949	5.4	5.4	25.2	5.8	3.1	36.8	1.7	0	0
1950	7.1	0	33.1	3.9	0	22.2	2.9	0	0
1951[a]	1.0	0	4.7	0.5	0	2.8	1.6	0	0
1952	1.2	0	5.6	1.6	0	9.1	4.0	0	0
1953	6.3	0	29.4	2.2	0	12.6	7.4	3.9	35.6
1954	2.8	0.3	13.1	2.6	1.2	16.3	4.2	6.1	55.8
1955	9.0	9.5	42.0	4.1	9.2	34.5	7.5	8.8	80.4
1956	2.5	0	11.7	2.5	0	14.3	2.2	0	0
1957	1.5	1.0	7.0	1.8	3.5	14.5	7.3	4.7	43.0
1958	2.7	0	12.6	2.4	0.9	14.8	2.6	0	0
1959	5.7	0.2	26.6	3.7	1.7	23.2	4.8	1.6	14.6
1960	2.2	0	10.7	3.3	0	18.8	0.7	0	0

[a] Early frost.

APPENDIX D: CORN WEATHER INDEXES

The precipitation, R, and temperature, T, variables used to compute the weather indexes for corn were defined as:

$$\text{(D.1)} \qquad \bar{R} = \Sigma\,(r - \bar{R}) + (2R, \text{if } r = -\bar{R})$$

$$\text{(D.2)} \qquad T = (t_{Jy} - \bar{T}_{Jy}) + (t_{Ag} - \bar{T}_{Ag})$$

where R is the estimated normal rainfall in inches for 10-day periods in July and August; r is the rainfall in 10-day periods in July and August, smaller than $\bar{R}$; $\bar{T}$ is the estimated normal average maximum temperature in July in degrees Farenheit; $\bar{T}_{Ag}$ is the estimated normal average maximum temperature in August; t_{Jy} is the average maximum temperature in July $>\bar{T}_{Jy}$; and t_{Ag} is the average maximum temperature in August $>\bar{T}_{Ag}$.

The estimated normal rainfall and temperatures for each experimental location are shown in table D.1.

The R and T variables and the weather indexes for corn estimated by equations 14, 15 and 16 are presented in table D-2.

To test the validity of the procedure, the weather indexes were used to explain the variation in the average corn yield of Howard, Hancock and Wayne counties over the period 1946-1960. A time variable t was included in the regression to account for the increase in county yields, Y, due to technological advancement.

The estimating equations with the corresponding R^2 values are:

(D.3) Howard County

$$Y = 56.16 - 0.437W + 1.124t \qquad R^2 = 0.80$$

(D.4) Hancock County

$$Y = 68.17 - 0.534W + 0.822t \qquad R^2 = 0.87$$

(D.5) Wayne County

$$Y = 47.93 - 0.260W + 0.847t \qquad R^2 = 0.82$$

Table D-1. Estimated normal rainfall for 10-day periods in July and August and estimated normal temperature in July and August at three Iowa locations.

Locations	$\bar{R}$ (inches)	$\bar{T}_{Jy}$ (°F)	$\bar{T}_{Ag}$ (°F)
Howard County	1.4	83	82
Clarion-Webster	1.1	86	85
Seymour-Shelby	1.3	87	86

Table D-2. Precipitation (R) and temperature (T) variables and weather indexes (W) for corn at three Iowa locations.

	Locations								
	Howard County			Clarion-Webster			Seymour-Shelby		
Year	R	T	W	R	T	W	R	T	W
1946	2.0	0	9.3	3.2	0	18.3	2.7	0	0
1947	8.4	8.9	39.2	7.9	9.7	56.2	6.1	9.0	82.3
1948	5.5	7.3	25.7	1.6	4.4	14.4	4.0	2.5	22.8
1949	5.4	5.4	25.2	5.8	3.1	36.8	1.7	0	0
1950	7.1	0	33.1	3.9	0	22.2	2.9	0	0
1951[a]	1.0	0	4.7	0.5	0	2.8	1.6	0	0
1952	1.2	0	5.6	1.6	0	9.1	4.0	0	0
1953	6.3	0	29.4	2.2	0	12.6	7.4	3.9	35.6
1954	2.8	0.3	13.1	2.6	1.2	16.3	4.2	6.1	55.8
1955	9.0	9.5	42.0	4.1	9.2	34.5	7.5	8.8	80.4
1956	2.5	0	11.7	2.5	0	14.3	2.2	0	0
1957	1.5	1.0	7.0	1.8	3.5	14.5	7.3	4.7	43.0
1958	2.7	0	12.6	2.4	0.9	14.8	2.6	0	0
1959	5.7	0.2	26.6	3.7	1.7	23.2	4.8	1.6	14.6
1960	2.2	0	10.7	3.3	0	18.8	0.7	0	0

[a] Early frost.

APPENDIX E: OATS WEATHER INDEXES

The precipitation, R, and temperature, T, variables used to compute the weather indexes for oats were defined as:

$$(E.1) \qquad R = r_A + r_J$$

$$(E.2) \qquad T = t_A + t_M + t_J$$

where r_A is the precipitation from April 11 to June 10, $\leqslant \overline{R}_{A1}$ or $\geqslant \overline{R}_{A2}$ (the estimated low and high normal rainfall in the period mentioned); r_J is precipitation from June 11 to July 20, $\leqslant \overline{R}_{J1}$ or $\geqslant \overline{R}_{J2}$ (the estimated low and high normal rainfall in the period mentioned); t_A is the average maximum temperature in April, $\leqslant \overline{T}_A$ (the estimated low normal temperature in April); t_M is the average maximum temperature in May, $\leqslant \overline{T}_{M1}$ or $\geqslant \overline{T}_{M2}$ (the estimated low and high normal temperature in May); t_J is the average maximum temperature in June, $\leqslant \overline{T}_{J1}$ or $\geqslant \overline{T}_{J2}$ (the estimated low and high normal temperature in June).

The estimated limits of normal precipitation and temperature at the locations under study are shown in tables E-1 and E-2.

The R and T variables and the weather indexes estimated by equations 17, 18 and 19 are shown in table E-3.

The weather indexes and a time variable, t, were used to explain the variation of the average oats yields in Howard, Hancock and Wayne counties over the period 1946-1960. The regression equations and the corresponding R^2's are:

(E.3) Howard County

$$Y = 36.37 - 0.181W + 0.081t \qquad R^2 = 0.07$$

Table E-1. Estimated limits of normal rainfall (in inches) for specified periods of time at three Iowa locations.

Locations	$\overline{R}_{A1}$	$\overline{R}_{A2}$	$\overline{R}_{J1}$	$\overline{R}_{J2}$
Howard County	4	10	4	7
Clarion-Webster	4	9	3	7
Seymour-Shelby	4	8	3	6

(E.4) Hancock County

$$Y = 44.96 - 0.060W + 0.569t \qquad R^2 = 0.41$$

(E.5) Wayne County

$$Y = 41.94 - 0.314W - 0.442t \qquad R^2 = 0.51$$

Table E-2. Estimated limits of normal temperature (°F) for specified periods of time at three Iowa locations.

Locations	$\overline{T}_A$	$\overline{T}_{M1}$	$\overline{T}_{M2}$	$\overline{T}_{J1}$	$\overline{T}_{J2}$
Howard County	50	66	72	72	77
Clarion-Webster	55	70	76	75	81
Seymour-Shelby	60	68	76	77	83

Table E-3. Precipitation (R) and temperature (T) variables and weather indexes (W) for oats at three Iowa locations.

	Locations								
	Howard County			Clarion-Webster			Seymour-Shelby		
Year	R	T	W	R	T	W	R	T	W
1945	1.9	4.5	15.4	—	—	—	—	—	—
1946	0	0	0	0	4.2	22.2	8.0	0.8	84.9
1947	6.0	4.5	30.0	3.0	1.7	95.2	7.0	0.7	74.3
1948	1.1	0.6	5.1	0	0.8	4.2	0	0.8	5.6
1949	0	5.4	10.4	0	4.0	21.1	4.7	0	46.6
1950	5.8	0.5	21.6	0	11.6	58.0	4.0	1.4	49.5
1951	0	1.2	4.3	4.0	5.4	143.5	1.8	3.8	44.5
1952	0	2.6	5.0	6.1	5.0	201.8	2.4	4.1	52.6
1953	3.7	2.7	18.3	0	7.5	39.6	0	7.8	54.8
1954	1.8	4.0	14.1	5.5	3.3	175.6	4.0	2.3	55.8
1955	1.6	1.7	9.0	0	1.7	9.0	0	0	0
1956	0	5.0	9.6	2.7	0	1.7	9.0	0	0
1957	0	0.3	0.6	1.3	1.2	43.7	0.6	0	5.9
1958	0.4	0	1.4	0	0.5	2.6	2.1	1.1	28.5
1959	0	1.3	2.5	0	2.8	14.8	5.6	0	55.5
1960	0.3	0	1.1	0.8	0	23.0	2.8	0	27.7

APPENDIX F: HAY WEATHER INDEXES

The precipitation, R_i, and temperature, T_i, variables used to derive the weather indexes for the meadow were defined as:

First cutting—

$$R_1 = r_1 - \bar{R}_1 \tag{F.1}$$

$$T_1 = t_A + t_M + t_{J1} \tag{F.2}$$

Second cutting—

$$R_2 = r_2 - \bar{R}_2 \tag{F.3}$$

$$T_2 = t_{J2} + t_{Jy} + t_{Ag} \tag{F.4}$$

where:

r_1 = precipitation from April 11 to nearest 5-day period before cutting, $\leqslant \bar{R}_1$ (estimated required rainfall for the period); r_2 = precipitation between 1st and 2nd cuttings, approximated in 5-day periods and $\leqslant \bar{R}_2$ (estimated required rainfall for the period).

Table F-1. Estimated required rainfall for two cuttings of hay at three Iowa locations.

	Estimated rainfall (inches)	
Locations	$\bar{R}_1$	$\bar{R}_2$
Howard County	5.0	5.0
Clarion-Webster	5.0	3.5
Seymour-Shelby	5.0	4.5

Table F-2. Estimated limits of normal temperature for hay for specified periods of time at three Iowa locations.

	Estimated average maximum temperature (°F)											
Locations	$\bar{T}_{A1}$	$\bar{T}_{A2}$	$\bar{T}_{M1}$	$\bar{T}_{M2}$	$\bar{T}_{J11}$	$\bar{T}_{J12}$	$\bar{T}_{J21}$	$\bar{T}_{J22}$	$\bar{T}_{Jy1}$	$\bar{T}_{Jy2}$	$\bar{T}_{Ag1}$	$\bar{T}_{Ag2}$
Howard County	55	65	64	74	72	82	74	84	76	86	76	86
Clarion-Webster	56	66	65	75	75	85	77	87	79	89	79	89
Seymour-Shelby	58	68	68	78	75	85	77	87	80	90	80	90

Table F-3. Precipitation variables (R_i) and temperature variables (T_i) and weather (W) for hay at three Iowa locations.

	Locations														
	Howard County					Clarion-Webster					Seymour-Shelby				
Year	R_1	T_1	R_2	T_2	W	R_1	T_1	R_2	T_2	W	R_1	T_1	R_2	T_2	W
1946	0.0	0.0	0.0	0.0	0.000	0.0	2.2	0.0	0.0	0.572	0.5	0.7	0.0	0.0	0.791
1947	0.0	2.5	1.7	4.9	1.406	0.2	0.0	0.0	1.5	0.180	0.0	0.0	0.0	1.1	0.022
1948	0.0	0.0	2.1	0.6	0.903	0.4	2.1	0.0	0.0	0.546	0.9	0.0	0.0	0.0	1.008
1949	0.0	0.0	0.0	0.0	0.000	0.9	0.0	0.0	0.0	0.000	0.8	0.0	0.0	0.0	0.896
1950	0.0	0.0	0.0	0.0	0.000	0.0	2.4	0.0	0.0	0.624	0.0	0.0	0.8	0.6	0.937
1951	0.0	0.1	0.0	1.5	0.027	0.0	0.0	0.0	0.0	0.000	0.0	1.0	0.0	1.5	0.360
1952	0.0	0.0	0.0	0.0	0.000	0.2	0.0	0.0	0.0	0.000	0.0	0.0	0.0	1.6	0.032
1953	0.0	0.0	0.0	0.0	0.000	0.0	0.0	0.0	0.8	0.096	0.0	1.8	0.2	2.2	0.664
1954	0.0	0.1	0.3	0.0	0.156	0.0	1.3	0.0	0.0	0.338	0.0	1.1	2.8	7.1	0.869
1955	0.0	1.0	2.1	2.2	1.173	0.7	0.0	0.0	0.7	0.084	0.0	1.8	0.0	1.0	0.614
1956	0.0	0.0	0.0	0.0	0.000	0.0	0.1	0.0	2.6	0.338	1.7	0.0	0.0	4.2	1.988
1957	0.0	0.0	0.0	0.0	0.000	0.0	0.0	0.0	0.5	0.060	0.0	0.0	2.0	0.7	0.274
1958	0.6	1.0	0.0	0.0	0.462	0.0	2.2	0.0	0.0	0.572	0.0	0.0	0.0	0.0	0.000
1959	0.0	0.0	2.0	0.0	0.860	0.0	0.0	0.0	1.4	0.168	0.0	0.0	0.0	0.0	0.000
1960	0.0	0.0	0.0	0.0	0.000	0.0	0.0	1.9	0.0	0.000	0.0	0.0	0.0	0.0	0.000

t_A = average maximum temperature from April 11 to 30, $\leqslant \bar{T}_{A1}$ or $\geqslant \bar{T}_{A2}$.

t_M = average maximum temperature during May $\leqslant \bar{T}_{M1}$ or $\geqslant \bar{T}_{M2}$.

t_{J1} = average maximum temperature from June 1 to approximate cutting date, $\leqslant \bar{T}_{J11}$ or $\geqslant \bar{T}_{J12}$.

t_{J2} = average maximum temperature from approximate cutting date to June 30, $\leqslant \bar{T}_{J21}$ or $\geqslant \bar{T}_{J12}$.

t_{Jy} = average maximum temperature during July, $\leqslant \bar{T}_{Jy1}$ or $\geqslant \bar{T}_{Jy2}$.

t_{Ag} = average maximum temperature from August 1st to approximate 2nd cutting dat $\leqslant \bar{T}_{Ag1}$ or $\geqslant \bar{T}_{Ag2}$.

The $\bar{T}$ quantities are the estimated low and high temperatures for the periods concerned.

The $\bar{R}$ and $\bar{T}$ values for the three locations studied are presented in tables F-1 and F-2.

The R_i and T_i variable defined in the manner described and the weather indexes derived from equations 20, 21 and 22 are presented in table F-3.

APPENDIX G: ANALYSES OF VARIANCE FOR THE GENERALIZED CROP FUNCTIONS

Analyses of variance were performed according to each generalized crop function for corn, oats and hay to estimate the contribution of the regression variables to the reduction of sums of squares of deviations. The analyses are presented in table G-1.

Table G-1. Analyses of variance for the generalized crop functions for corn, oats and hay.

Function no.	Source of variance	Degrees of freedom	Sums of squares	Mean squares	F[a]
(33)	Total	305	176,029.27		
	Regression	30	110,725.10	3,690.84	
	Deviations	275	65,304.17	237.47	15.542**
(34)	Total	305	176,029.27		
	Regression	15	103,083.49	68,972.23	
	Deviations	290	72,945.78	251.54	27.321**
(35)	Total	305	176,029.27		
	Regression	12	101,497.62	8,458.13	
	Deviations	293	74,531.65	254.37	33.251**
(36)	Total	305	176,029.27		
	Regression	14	107,686.33	7,691.88	
	Deviations	291	68,342.94	234.86	32.751**
(37)	Total	98	11,944.79		
	Regression	34	10,793.21	317.45	
	Deviations	64	1,151.58	17.99	17.642**
(38)	Total	98	11,944.79		
	Regression	28	10,779.31	384.98	
	Deviations	70	1,165.48	16.65	23.122**
(39)	Total	98	11,944.79		
	Regression	24	10,692.48	445.52	
	Deviations	74	1,252.30	16.92	26.333**
(40)	Total	98	33,315.26		
	Regression	34	30,535.58	898.10	
	Deviations	64	2,779.68	43.43	20.679**
(41)	Total	98	33,315.26		
	Regression	15	29,200.91	1,946.73	
	Deviations	83	4,114.35	49.57	39.270**
(42)	Total	98	71.1093		
	Regression	34	66,0031	1.9413	
	Deviations	64	5.1062	0.0798	24.327**
(43)	Total	98	71,109.268		
	Regression	19	65,683·224	3,457.012	
	Deviations	79	5,426.044	68.684	50.332**

[a] See previous tables for significance levels.