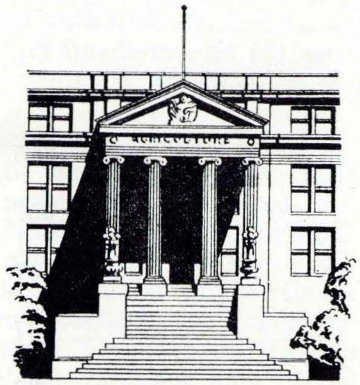

# Fertilizer Production Functions From Experimental Data With Associated Supply and Demand Relationships

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## CONTENTS

## SUMMARY

This study includes (1) estimation of physical production functions from fertilizer experiments designed for this purpose, (2) prediction of corresponding marginal products, isoquants and isoclines, (3) estimation of economic optima under various price situations, (4) prediction of confidence regions for isoquants, isoclines and economic optima, (5) derivation of static corn supply and fertilizer demand functions and (6) estimation of gross marginal rates of substitution of fertilizer for land.

The analysis of this study is based on two sets of experiments conducted in 1959 and 1960 on corn. The experiments, conducted on Clarion, Nicollet and Webster soil series, included nitrogen (N), phosphorus (P) and potassium (K) as variable nutrients. A central composite design was used for each experiment. A quadratic function with crossproduct terms of the three nutrients was fitted to the combined data of both years. However, K was fixed at zero level for the purpose of this study. The derived production function is given by equation a, where Y denotes predicted corn yield in bushels per acre and N, P and K denote the pounds per acre of nitrogen, phosphorus and potassium, respectively.

$$\text{(a)}\quad K = 0;\ Y = 78.0596 + 0.2616N + 0.1102P - 0.001168N^2 - 0.001449P^2 + 0.000782NP$$

As fertilizer applications increase, the surface appears relatively flat. Decreasing total products are predicted for both N and P.

The production function equation a then was used in deriving (1) single nutrient input-output or response curves, (2) marginal response coefficients, (3) yield isoquants, (4) marginal replacement ratios and (5) nutrient isoclines. As examples, equations b and c give isoquant and isocline equations. The $\theta$ in equation c represents the N/P price ratio.

$$\text{(b)}\quad K = 0;\ N = 111.9902 + 0.3348P \pm (79373.6439 - 856.1644Y + 169.3701 - 1.1285P)^{1/2}$$

$$\text{(c)}\quad -(0.002336 + 0.000782\theta)N + (0.000782 + 0.002898\theta)P + (0.261609 - 0.110248\theta) = 0$$

The nutrient isoclines, which show equal replacement ratios of nutrients at different yields, are linear. They converge at the maximum yield, indicating no substitution of nutrients at this point.

Profit-maximizing nutrient combinations were derived for various nutrient and corn prices. These combinations included, in general, large applications of N and relatively small applications of P.

The 95-percent confidence regions for 85-, 90- and 95-bushel isoquants for corn are illustrated graphically in the text. Equation d gives the 95-percent confidence region for the isocline when $\theta = 0.55$.

$$\text{(d)}\quad 0.0727N^2 - (10.2922 + 0.1375P)N - 313.1355 + 12.0526P + 0.0275P^2 \leqslant 0$$

The 95-percent confidence region for the economic optima when the prices of N and P are 12.5 and 23.0 cents per pound, respectively, and that of corn is $1.00 per bushel, is given by equation e.

$$\text{(e)}\quad 0.0644N^2 - 7.5373N + 1.9810P - 0.0339NP + 0.0130P^2 + 188.4656 \leqslant 0$$

The regions represented by equations d and e also are shown graphically in the text. The optimum of N can be predicted with greater certainty than the level of P.

The short- and long-run static supply functions for corn and the demand functions for the nutrients were derived from the production function a. The long-run static supply functions for corn and its elasticity, $E_s$, when the prices of N and P are 12.5 and 23.0 cents, respectively, are given by equations f and g

$$\text{(f)}\quad Y = 100.131 - 17.361P_c^{-2}$$

$$\text{(g)}\quad E_s = \frac{34.722}{100.131\,P_c^2 - 17.361}$$

where $P_c$ denotes the price per bushel of corn. The predicted supply quantity is 83 bushels per acre when corn price is $1.00 per bushel and nutrients have the prices just given. The static corn supply is inelastic (i.e., less than unity) when the price of corn is over 73 cents per bushel. At the price of $1.00 per bushel of corn, the price elasticity of static supply is predicted to be 0.42. Supply elasticity for corn is very low for the relevant prices of corn (e. g., those of the last decade), and it increases as the price of corn falls.

The long-run static demand for N, its price elasticity $E_d(N)$, the long-run static demand for P and its price elasticity $E_d(P)$ are given by equations h, i, j and k, respectively, when the price of corn is $1.00 per bushel.

$$\text{(h)}\quad N = 166.317 - 470.592P_n$$

$$\text{(i)}\quad E_d(N) = \frac{470.592P_n}{470.592P_n - 166.317}$$

$$\text{(j)}\quad P = 90.914 - 379.331P_p$$

$$(k)\ E_d(P) = \frac{379.331P_p}{379.331P_p - 90.914}$$

$P_n$ and $P_p$ denote the prices of 1 pound of N and P, respectively. With corn at $1.00 per bushel, the demand quantity for N is predicted to be 107.5 pounds per acre at the price of 12.5 cents per pound. At the same prices, the elasticity of demand for N is 0.55. However, the price elasticity for N is greater than unity when the price of N is more than 17.5 cents per pound and corn price is $1.00 per bushel. The predicted demand quantity for P is 2.7 pounds per acre at the price of 23 cents per pound of P. The price elasticity of demand for P cannot be predicted at this price. However, the elasticity is greater than unity when the price of P is more than 12 cents per pound.

The study shows, based on the experimental data used, that static nutrient demand tends to be more elastic than static corn supply. Also, the demand for P is predicted to be more elastic than the demand for N.

By suitably incorporating land into production function a, the "gross" marginal rates of substitution (MRS) between land (A) and fertilizer (F) ($F = N + P = rN$) were computed for varying values of r and for different isoquants. Results in the summary table are presented for $r = 1/3$ and for 85- and 95-bushel isoquants.

**Summary table. Isoquants and marginal rates of substitution.**

| F | 85 bushels | | 95 bushels | |
|---|---|---|---|---|
| (pounds per acre) | A | MRS | A | MRS |
| 0 | 1.0889 | −0.002867 | 1.2170 | −0.002867 |
| 20 | 1.0346 | −0.002551 | 1.1623 | −0.002596 |
| 40 | 0.9867 | −0.002214 | 1.1134 | −0.002291 |
| 80 | 0.9135 | −0.001433 | 1.0353 | −0.001603 |
| 120 | 0.8721 | −0.000652 | 0.9855 | −0.000980 |

An 85-bushel output of corn can be predicted with 20 pounds of fertilizer and 1.0346 acres of land. Hence, 1 pound of fertilizer substitutes for 0.002551 acre of land; i. e., 1 ton of fertilizer substitutes for 5.102 acres of land. With 80 pounds of fertilizer and 0.9135 acre of land to produce the same amount of corn, 1 ton of fertilizer substitutes for 2.866 acres of land. But with 20 pounds of fertilizer and 1.1623 acres of land to produce 95 bushels of corn, a ton of fertilizer substitutes for 5.192 acres of land. With 80 pounds of fertilizer and 1.0353 acres of land to produce the same amount of corn, a ton of fertilizer substitutes for 3.206 acres of land.

# Fertilizer Production Functions From Experimental Data With Associated Supply and Demand Relationships[1]

by Earl O. Heady, John T. Pesek and V. Y. Rao

This study represents a continuation of agronomic-economic analysis of fertilizer response functions. It is fifth in a series of studies designed to derive corn response surfaces, yield isoquants and economic optima in fertilizer use.[2] The present study is based upon experiments specifically designed to derive certain auxiliary physical and economic relationships that rest on production functions and are relevant both to decision processes by farmers and to certain structural problems relating to product supply and resource demand of agriculture.

The major emphasis in this study is the derivation of (a) static corn supply functions and fertilizer demand equations and (b) substitution relationships between land and fertilizer. These several relationships can be derived directly from fertilizer response functions estimated experimentally. However, in addition to the estimation of these basic relationships, which have important implications in the future structure of agriculture, analysis is also made of the conventional economic quantities relating to the profit-maximizing levels and combinations of fertilizer nutrients represented by the particular experiments.

## OBJECTIVES

Stated specifically, the objectives of this study are:

1. To estimate physical production functions based on experiments designed for this purpose.
2. To derive the auxiliary relationships represented by response surfaces, marginal quantities, isoquants and isoclines.
3. To estimate the profit-maximizing ratio and level of nutrients under a range of price situations.
4. To derive the confidence regions for isoquants, isoclines and optimum quantity of inputs.
5. To derive static corn supply functions and static fertilizer demand functions as these relate to physical production function and alternative price situations.
6. To incorporate land into the experimentally estimated production function and estimate the marginal rates of substitution of fertilizer for land under different yield levels and nutrient combinations.

## EXPERIMENTS, DATA AND ANALYSIS

The analysis of this study is based on data from two sets of experiments conducted in 1959 and 1960 on corn. The purpose of the experiments was to provide data allowing improved estimation of fertilizer response and the economic application of the resulting relationships and functions.

Of course, a single experiment cannot provide conclusive estimates of the numerous quantities involved. The various relationships will differ depending on (a) weather conditions and past management for given soils, (b) soil types, (c) crops grown and (d) other variables. However, the data presented and the analysis completed provide both predictions for specific environmental conditions and indications of concepts for further substantiation of the static supply-demand relationships that rest on physical production functions and the potential marginal rates of substitution between fertilizer nutrients and the land on which they are used.

The 1959 data included six multi-rate N-P-K fertilizer experiments for corn conducted on fields

[1] Projects 1530 and 1135 of the Iowa Agricultural and Home Economics Experiment Station, Tennessee Valley Authority cooperating.

[2] Past studies are: Earl O. Heady, John T. Pesek and William G. Brown. Corn response surfaces and economic optima in fertilizer use. Iowa Agr. Exp. Sta. Res. Bul. 424. 1955; William G. Brown, Earl O. Heady, John T. Pesek and Joseph A. Stritzel. Production functions, isoquants, isoclines and economic optima in corn fertilization for experiments with two and three variable nutrients. Iowa Agr. Exp. Sta. Res. Bul. 441. 1957; John P. Doll, Earl O. Heady and John T. Pesek. Fertilizer production functions for corn and oats; including an analysis of irrigated and residual responses. Iowa Agr. and Home Econ. Exp. Sta. Res. Bul. 463. 1958; and John T. Pesek, Earl O. Heady, John P. Doll and R. P. Nicholson. Production surfaces and economic optima for corn yields with respect to stand and nitrogen levels. Iowa Agr. and Home Econ. Exp. Sta. Res. Bul. 472. 1959.

Earl O. Heady is professor of economics and the Executive Director of the Center for Agricultural and Economic Development; John T. Pesek is professor and head, Department of Agronomy; and V. Y. Rao was a graduate student working with Dr. Heady during the time of the study.

of cooperating farmers. The 1960 data consisted of 12 such experiments. The names and locations of the cooperators are presented in table A-1 of Appendix A.

The 18 experimental sites were located on Clarion, Nicolett and Webster soil series in central and north-central Iowa. Requirements in selection of the sites were as follows: second-year corn, no fertilization (i.e., no manure or commercial fertilizer following the crop of the previous year), a soil test indicating low or very low phosphorus or potassium, and a site with uniform slope and drainage.

The fertilizer sources were ammonium nitrate for N, concentrated superphosphate for P and muriate of potash for K. The fertilizer was hand-spread on plots that were 13⅓ feet by 40 feet. Plots of this size and shape allowed four rows spaced at distances of 40 inches. Fertilizer, spread on the corn stubble from the previous year, was plowed under. The cooperator's normal cultural practices (preparation of seedbed, cultivation, planting data, selection of hybrid and planting rate) were allowed throughout the rest of the crop season. Cooperators were encouraged to plant at least 16,000 kernels per acre.

Aid was given to the cooperator in control of weeds and corn borers. Granular DDT was applied to control first-brood corn borers. Hand pulling and hoeing were used to help control weeds.

Corn yields were estimated by hand harvesting —the yield from each plot being weighed from the center two rows of approximately 35 feet. Shelled corn samples from individual plots were weighed before and after drying to determine the moisture content. By use of a standard conversion table, yields were then calculated in bushels of shelled corn per acre at a common 15.5-percent moisture level (No. 2 corn).

Temperature during the seasons of 1959 and 1960 did not deviate appreciably from normal seasonal temperatures. It exceeded 90 degrees Fahrenheit in very few days, and there were no exceptionally cool periods.

A central composite design plus a check plot, not part of the design, was used for each experiment. This design requires fewer treatment combinations than other designs for estimating a yield function. The treatment rates and combinations used in 1959 and 1960, presented in table 1, indicate the 5 x 5 x 5 N, P and K composite design used. The tabular levels of P and K are not exactly equally spaced because of conversion of the nutrients from fertilizer materials originally applied. These differences are negligible, however, and the levels of P and K are considered equally spaced (for P as multiples of 17.4 pounds and K as multiples of 20.8 pounds) to facilitate the fitting of the production function.

**Table 1. Treatment number with corresponding fertilizer rates (in pounds per acre) and combinations.**

| 1959 | | | | 1960 | | | |
|---|---|---|---|---|---|---|---|
| Treatment | Fertilizer rate | | | Treatment | Fertilizer rate | | |
| number | N | P | K | number | N | P | K |
| 1 | 40 | 17.4 | 20.8 | 1 | 40 | 17.4 | 20.8 |
| 2 | 40 | 17.4 | 62.2 | 2 | 40 | 17.4 | 62.2 |
| 3 | 40 | 52.4 | 20.8 | 3 | 40 | 52.4 | 20.8 |
| 4 | 40 | 52.4 | 62.2 | 4 | 40 | 52.4 | 62.2 |
| 5 | 120 | 17.4 | 20.8 | 5 | 120 | 17.4 | 20.8 |
| 6 | 120 | 17.4 | 62.2 | 6 | 120 | 17.4 | 62.2 |
| 7 | 120 | 52.4 | 20.8 | 7 | 120 | 52.4 | 20.8 |
| 8 | 120 | 52.4 | 62.2 | 8 | 120 | 52.4 | 62.2 |
| 9 | 80 | 35.0 | 41.6 | 9 | 80 | 35.0 | 41.6 |
| 10 | 0 | 35.0 | 41.6 | 10 | 0 | 35.0 | 41.6 |
| 11 | 160 | 35.0 | 41.6 | 11 | 160 | 35.0 | 41.6 |
| 12 | 80 | 0.0 | 41.6 | 12 | 80 | 0.0 | 41.6 |
| 13 | 80 | 69.8 | 41.6 | 13 | 80 | 69.8 | 41.6 |
| 14 | 80 | 35.0 | 0.0 | 14 | 80 | 35.0 | 0.0 |
| 15 | 80 | 35.0 | 83.0 | 15 | 80 | 35.0 | 83.0 |
| 16 | 0 | 69.8 | 83.0 | 16 | 0 | 0.0 | 83.0 |
| 17 | 160 | 0.0 | 83.0 | 17 | 0 | 69.8 | 0.0 |
| 18 | 160 | 69.8 | 0.0 | 18 | 0 | 69.8 | 83.0 |
| 19 | 0 | 0.0 | 0.0 | 19 | 160 | 0.0 | 0.0 |
| | | | | 20 | 160 | 0.0 | 83.0 |
| | | | | 21 | 160 | 69.8 | 0.0 |
| | | | | 22 | 160 | 69.8 | 83.0 |
| | | | | 23 | 0 | 0.0 | 0.0 |

Table 2 shows additional details of the treatment combinations used for both years. The four treatment combinations not in the 1959 design were added to the 1960 experiments. Their addition helped to remove intercorrelation between the linear regression terms and interaction terms containing like variables.

Each experiment was replicated twice and analyzed individually. A combined analysis of the experiments for each year was carried out.[3] The mean yields of the treatments for 1959 and 1960 are presented in table 3.

### Regression Analysis

A quadratic production function with cross-product terms was fitted to the combined data of 1959 and 1960, since there was no appreciable variation in weather between these two seasons. (The production functions for individual years and the economic quantities derived from them are presented in Appendix B.)

Equation 1 is the fitted quadratic function where Y is the predicted corn yield expressed as bushels per acre, and N, P and K are the pounds per acre of nitrogen, phosphorus and potassium, respectively. D is the dummy variable introduced as the number of treatments differed from 1959 to 1960. The variable takes a value of $-1$ in 1959 and $+1$ in 1960. When $D = 0$, equation 1 represents the average production function over the 2 years.

[3] For details see: Regis D. Voss. Yield and foliar composition of corn as affected by fertilizer rates and environmental factors. Unpublished Ph. D. thesis. Iowa State University Library, Ames, Iowa. 1962.

(1) $Y = 78.0750 + 0.2615N + 0.1115P + 0.0541K - 0.001170N^2 - 0.001458P^2 + 0.000020K^2 + 0.000782NP + 0.000162NK - 0.000195PK - 2.8101D + 0.010450DN - 0.002121DP - 0.029716DK$

The higher order interaction terms with D (such as $DN^2$, DNP, etc.) are not included in equation 1. The value of $R^2$ for this equation is 0.95520. Probability levels of the t values for each coefficient are given in table 4.

Probability levels of the terms $K^2$ and DP are very high. For these terms, a t value as large or larger might have been obtained at least nine out of ten times in samples drawn at random from the t distribution. It is possible, however, that when an input variable is included in several regression terms, none of these terms will be significant even if the total effect of the input variable is significant. Hence, reduction in sums of squares was examined for the terms $K^2$ and DP. When these two terms were deleted from equation 1, the $R^2$ value was lowered from 0.95520 to 0.95516. In the analysis of variance of the reduction in sums of squares due to regression (table 5), the F value is not significant.

Hence, equation 2 resulted when $K^2$ and DP terms were removed from the original equation.

Table 2. Fertilizer rates (in pounds per acre) and combinations used, 1959 and 1960 experiments.

| P | K | N 0 | 40 | 80 | 120 | 160 |
|---|---|---|---|---|---|---|
| 0 | 0 | X | | | | a |
| | 20.8 | | | | | |
| | 41.6 | | | X | | |
| | 62.4 | | | | | |
| | 83.2 | a | | | | X |
| 17.4 | 0 | | | | | |
| | 20.8 | | X | | X | |
| | 41.6 | | | | | |
| | 62.4 | | X | | X | |
| | 83.2 | | | | | |
| 34.8 | 0 | | | X | | |
| | 20.8 | | | | | |
| | 41.6 | X | | X | | X |
| | 62.4 | | | | | |
| | 83.2 | | | X | | |
| 52.2 | 0 | | | | | |
| | 20.8 | | X | | X | |
| | 41.6 | | | | | |
| | 62.4 | | X | | X | |
| | 83.2 | | | | | |
| 69.6 | 0 | a | | | | X |
| | 20.8 | | | | | |
| | 41.6 | | | X | | |
| | 62.4 | | | | | |
| | 83.2 | X | | | | a |

[a]Denotes treatment combinations not used in 1959 but added for 1960. The X's denote common treatments in 1959 and 1960.

(2) $Y = 78.0596 + 0.2616N + 0.1102P + 0.0565K - 0.001168N^2 - 0.001449P^2 + 0.000782NP + 0.000153NK - 0.000195PK - 2.8835D + 0.010450DN - 0.29716DK$

The probability levels of the t values for the coefficients in equation 2 are given in table 6. The check plot yield differs from 1959 to 1960 at 0.01 probability level. The linear effects of N and K

Table 3. Observed mean yields for corn (bushels per acre) for the individual treatments.

| 1959 Treatment number | Mean yield | 1960 Treatment number | Mean yield |
|---|---|---|---|
| 1 | 95.31 | 1 | 85.88 |
| 2 | 95.79 | 2 | 89.51 |
| 3 | 91.62 | 3 | 89.64 |
| 4 | 99.42 | 4 | 92.93 |
| 5 | 101.31 | 5 | 97.64 |
| 6 | 106.37 | 6 | 95.60 |
| 7 | 105.12 | 7 | 97.56 |
| 8 | 105.19 | 8 | 96.60 |
| 9 | 102.12 | 9 | 95.18 |
| 10 | 83.94 | 10 | 75.61 |
| 11 | 99.81 | 11 | 100.47 |
| 12 | 96.81 | 12 | 89.97 |
| 13 | 102.19 | 13 | 93.67 |
| 14 | 94.43 | 14 | 92.89 |
| 15 | 104.44 | 15 | 97.15 |
| 16 | 88.99 | 16 | 77.38 |
| 17 | 99.12 | 17 | 77.64 |
| 18 | 102.00 | 18 | 75.14 |
| 19 | 82.13 | 19 | 87.58 |
| | | 20 | 93.32 |
| | | 21 | 97.25 |
| | | 22 | 101.79 |
| | | 23 | 75.31 |

Table 4. Values of t for the coefficients of equation 1.

| Coefficient | Value of t | Probability level[a] |
|---|---|---|
| N | 9.44 | 0.001 |
| P | 1.76 | 0.10 |
| K | 1.01 | 0.35 |
| $N^2$ | 7.75 | 0.001 |
| $P^2$ | 1.84 | 0.10 |
| $K^2$ | 0.04 | 0.99 |
| NP | 3.27 | 0.01 |
| NK | 0.81 | 0.45 |
| PK | 0.42 | 0.65 |
| D | 2.85 | 0.01 |
| DN | 1.57 | 0.15 |
| DP | 0.14 | 0.90 |
| DK | 2.32 | 0.05 |

[a]Probability of drawing a t value as large or larger by chance, given the null hypothesis.

Table 5. Analysis of variance of deletion of $K^2$ and DP terms from equation 1.

| Source of variation | Degrees of freedom | Sum of squares | Mean square | F |
|---|---|---|---|---|
| Total | 41 | 3,064.80 | | |
| Due to regression on all variables | 13 | 2,927.48 | | |
| Due to regression on all variables except $K^2$ and DP | 11 | 2,927.37 | | |
| Difference | 2 | 0.11 | 0.05 | 0.01 |
| Deviation from regression | 28 | 137.32 | 4.90 | |

Table 6. Values of t for the coefficients of equation 2.

| Coefficient | Value of t | Probability level[a] |
|---|---|---|
| N | 10.21 | 0.001 |
| P | 1.87 | 0.10 |
| K | 2.28 | 0.05 |
| $N^2$ | 8.44 | 0.001 |
| $P^2$ | 1.99 | 0.10 |
| NP | 3.38 | 0.01 |
| NK | 0.83 | 0.45 |
| PK | 0.44 | 0.65 |
| D | 3.60 | 0.01 |
| DN | 1.62 | 0.15 |
| DK | 2.40 | 0.05 |

[a]Probability of drawing a t value as large or larger, by chance, given the null hypothesis.

Table 7. Analysis of variance for equation 2.

| Source of variation | Degrees of freedom | Sum of squares | Mean square | F |
|---|---|---|---|---|
| Total | 41 | 3,064.80 | | |
| Due to regression | 11 | 2,927.37 | 266.12 | 58.09 |
| Deviation from regression | 30 | 137.42 | 4.58 | |

differ from 1959 to 1960 at 0.15 and 0.05 probability levels, respectively. The analysis of variance of regression for equation 2 is presented in table 7. The F value is highly significant.

Since equation 2 does not contain a quadratic term for K, the economic analysis must be carried out only after specifying an alternate value for K. Fixing K (in pounds per acre) at some reasonable value, the economic analysis can be made. For the purpose of this study, K is held constant at zero in all computations. Also the value of D is kept at zero so that the derived production function represents the combined data of 1959 and 1960. Hence, keeping K at zero level, the production function for the pooled data of 1959 and 1960 is given by equation 3.

$$(3)\quad K = 0; Y = 78.0596 + 0.2616N + 0.1102P - 0.001168N^2 - 0.001449P^2 + 0.000782NP$$

## PRODUCTION SURFACE AND PREDICTED YIELDS

Because of economic derivations and applications presented later in this study, we provide certain detail on response curves to indicate the nature of the underlying production surface. Table 8 shows the predicted yields for corn for various combinations of fertilizer inputs, based on equation 3. The highest predicted yield in table 8 is 99.8 bushels per acre, about 21.7 bushels more than the lowest predicted yield. Decreasing total returns are evidenced for N as well as for P.

Figure 1 illustrates geometrically the production surface predicted by equation 3 for varying rates of N and P. This production surface illustrates the high marginal products for the first 40 pounds of N. Beyond 80 pounds, however, the predicted corn yield response to N flattens out and diminishes slightly at the highest input level. The highest marginal response for P comes with the first 17-pound input, compared with higher inputs. After the 35-pound input of P, total yield starts decreasing at a very slow rate.

Table 8. Predicted corn yields (bushels per acre) when K is held constant at zero.

| Pounds of P per acre | Pounds of N per acre | | | | |
|---|---|---|---|---|---|
| | 0 | 40 | 80 | 120 | 160 |
| 0 | 78.06 | 86.66 | 91.51 | 92.63 | 90.02 |
| 17.4 | 79.54 | 88.68 | 94.08 | 95.74 | 93.67 |
| 34.8 | 80.14 | 89.82 | 95.77 | 97.98 | 96.45 |
| 52.2 | 79.87 | 90.09 | 96.58 | 99.34 | 98.35 |
| 69.6 | 78.71 | 89.49 | 96.52 | 99.82 | 99.38 |

Again for the production surface in fig. 1, if one fertilizer input is held constant while the other is varied, a single input-output curve is obtained. Input-output curves of this nature are shown for N and P in figs. 2 and 3, respectively.

In fig. 2, the two response curves for N, when P is held constant at 34.8 and 69.6 pounds per acre, intersect at about 60 pounds of N. In fig. 3, the two response curves for P when N is constant at 80 and 160 pounds per acre also intersect and are much above the response curve for zero N. This phenomenon again is due to positive interaction between the nutrients. A high marginal productivity of N, when its input is below 80 pounds per acre, is thus indicated.

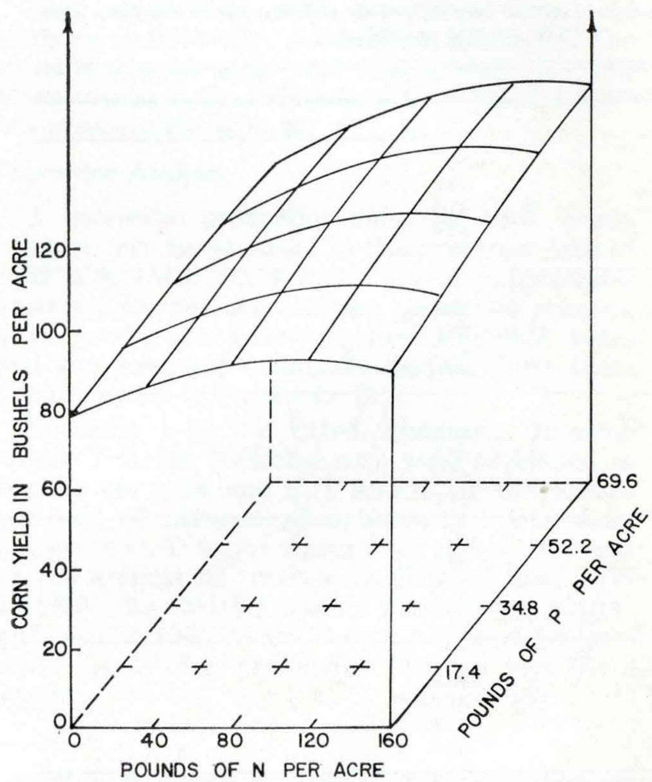


Fig. 1. Corn yield response to N and P (K held constant at zero).

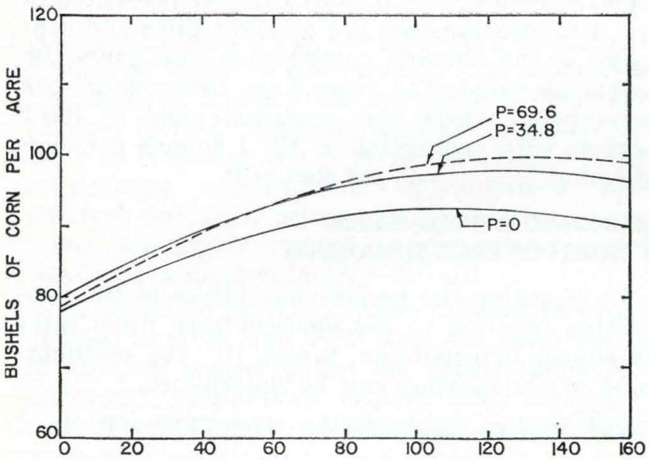


**Fig. 2. Corn yield response to N (P held constant at three levels) when K is zero.**

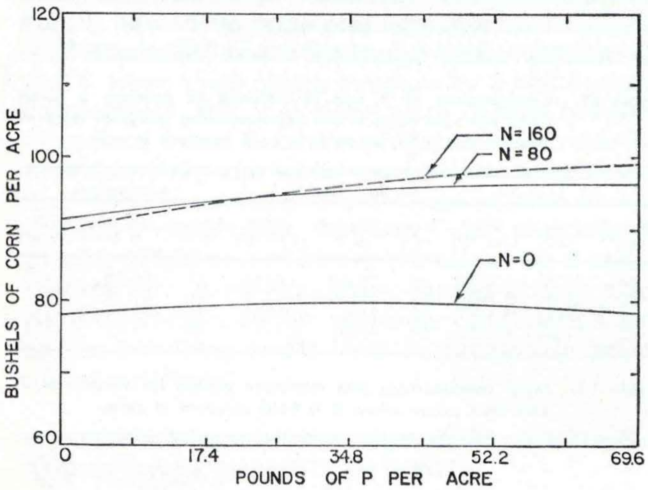


**Fig. 3. Corn yield response to P (N held constant at three levels) when K is zero.**

## Marginal Physical Products

The marginal physical products of the two inputs, when K is held constant at zero, are predicted from the two partial derivative equations 4 and 5, derived from equation 3. These quantities are important in the land/fertilizer substitution ratios derived later.

$$(4)\ \partial Y/\partial N = 0.2616 - 0.002336N + 0.000782P$$

$$(5)\ \partial Y/\partial P = 0.1102 - 0.002898P + 0.000782N$$

Tables 9 and 10 provide the marginal physical products obtained by substituting the input values into these equations.

When P is held constant at zero, the marginal physical product of N becomes negative when its level is above 112 pounds per acre. Similarly, when N is held constant at zero, the marginal physical product of P becomes negative when its level is above 39 pounds per acre. The marginal physical product of either N or P at any given level increases as the level of the other factor is increased.

**Table 9. Marginal physical products of N at different levels of P when K is held constant at zero.**

| Pounds of P | Pounds of N per acre | | | | |
|---|---|---|---|---|---|
| per acre | 0 | 40 | 80 | 120 | 160 |
| 0 | 0.2616 | 0.1682 | 0.0747 | −0.0187 | −0.1122 |
| 17.4 | 0.2752 | 0.1816 | 0.0883 | −0.0051 | −0.0985 |
| 34.8 | 0.2888 | 0.1952 | 0.1019 | 0.0085 | −0.0849 |
| 52.2 | 0.3024 | 0.2088 | 0.1155 | 0.0221 | −0.0713 |
| 69.6 | 0.3160 | 0.2224 | 0.1292 | 0.0357 | −0.0577 |

**Table 10. Marginal physical products of P at different levels of N when K is held constant at zero.**

| Pounds of N | Pounds of P per acre | | | | |
|---|---|---|---|---|---|
| per acre | 0 | 17.4 | 34.8 | 52.2 | 69.6 |
| 0 | 0.1102 | 0.0598 | 0.0094 | −0.0410 | −0.0914 |
| 40 | 0.1415 | 0.0911 | 0.0407 | −0.0097 | −0.0602 |
| 80 | 0.1728 | 0.1224 | 0.0720 | 0.0215 | −0.0289 |
| 120 | 0.2041 | 0.1537 | 0.1032 | 0.0528 | 0.0024 |
| 160 | 0.2354 | 0.1849 | 0.1345 | 0.0841 | 0.0337 |

With the marginal products or the first partial derivatives at zero, the maximum yield and its corresponding nutrient quantities can be derived. The predicted maximum yield is 100.1 bushels per acre of corn with 137.1 pounds per acre of N and 75.0 pounds per acre of P. However, an application of 112.0 pounds per acre of N results in a maximum yield of 92.7 bushels per acre of corn when P is held constant at zero. Similarly, when N is held constant at zero, a maximum yield of 80.2 bushels per acre of corn can be attained with 38.0 pounds per acre of P.

## Yield Isoquants

Equation 6 is the isoquant equation expressing N as a function of P when K is held constant at zero.

$$(6)\ N = 110.9902 + 0.3348P \pm (79373.6439 - 856.1644Y + 169.3701P - 1.1285P^2)^{1/2}$$

where Y denotes the level of the isoquant in bushels per acre.

The marginal rates of substitution between the two inputs, when K is held constant at zero, can be predicted by equation 7.

$$(7)\ \frac{\partial Y/\partial P}{\partial Y/\partial N} = \frac{\partial N}{\partial P} = \frac{0.1102 - 0.002898P + 0.000782N}{0.2616 - 0.002336N + 0.000782P}$$

The isoquants for 80, 85, 90 and 95 bushels per acre of corn yield are illustrated in fig. 4. Since they are convex to the origin, decreasing marginal rates of substitution hold true for the two nutrient inputs. The ridge lines indicate the range in

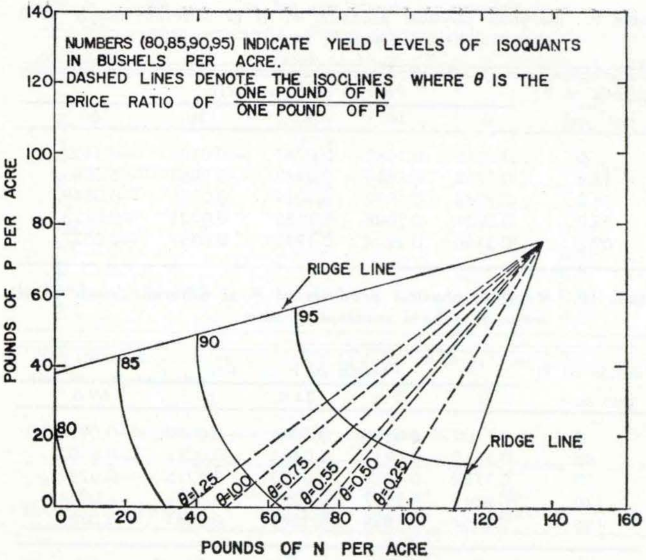


Fig. 4. Corn yield isoquants and isoclines for N and P (K held constant at zero).

the nutrient plan in which substitution can take place. These ridge lines fall on the isoquants at nutrient combinations where the marginal physical products of the nutrients are zero. This relationship holds true because the marginal rate of substitution between two resources is the ratio of the marginal physical products. Isoquant schedules and marginal rates of substitution of P for N for three yield levels are given in table 11. The marginal rates of substitution given by equation 7 are suggestive of nutrient ratios that will allow attainment of a given yield with minimum fertilizer costs.

The nutrient ratio giving minimum fertilizer costs for a particular yield is defined by equating the marginal rate of substitution with the inverse price ratio. The present cost of purchasing and applying the nutrients is approximately 12.5 cents per pound of N and 23.0 cents per pound of P, which gives P/N price ratio equal to 1.84. The optimum ratio for 95 bushels per acre of corn is approximately 90.0 pounds per acre of N and 18.1 pounds per acre of P. The extent to which nutrient ratios other than the optimum mix increase costs for a given yield depends on the curvature of the isoquants and the slope of the isoclines.

### Nutrient Isoclines

Equation 8 provides the isocline family. The $\Theta$ represents a constant substitution or price ratio of N/P. The isocline represents the least-cost expansion path for any given fertilizer price ratio for a specified value of $\Theta$.

$$(8)\quad -(0.002336 + 0.000782\Theta)N + (0.000782 + 0.002898\Theta)P + (0.261609 - 0.110248\Theta) = 0$$

The isoclines for different values of $\Theta$(N/P price ratio), varying from 0.25 to 1.25, are presented in fig. 4. These isoclines are straight lines and converge at the nutrient combination that gives the maximum yield. The ridge lines converge at the same point where the predicted yield is 100.1 bushels with application of 137.1 pounds per acre of N and 75.0 pounds per acre of P.

## ECONOMIC OPTIMA UNDER A RANGE OF PRICE SITUATIONS

By equating the partial derivatives of the production function to the nutrient/corn price ratio as shown in equations 9 and 10, the optimum level of fertilization can be determined.

$$(9)\quad 0.2616 - 0.002336N + 0.000782P = P_n/P_c$$

$$(10)\quad 0.1102 - 0.002898P + 0.000782N = P_p/P_c$$

Table 12 lists profit-maximizing combinations of nutrients for alternate corn and fertilizer prices. The required amounts of N and P are derived by using equations 9 and 10, where $P_c$ is

Table 11. Combinations of P and N required to produce a given yield per acre of corn and corresponding marginal rates of substitution (MRS) when K is held constant at zero.

| 85 bushels | | | 90 bushels | | | 95 bushels | | |
|---|---|---|---|---|---|---|---|---|
| Pounds of P | Pounds of N | MRS ($\partial N/\partial P$) | Pounds of P | Pounds of N | MRS ($\partial N/\partial P$) | Pounds of P | Pounds of N | MRS ($\partial N/\partial P$) |
| 10 | 24.89 | 0.4768 | 10 | 52.89 | 0.8406 | 15 | 98.99 | 3.4250 |
| 20 | 21.03 | 0.3013 | 20 | 46.20 | 0.5221 | 20 | 87.48 | 1.6557 |
| 30 | 18.76 | 0.1574 | 30 | 42.13 | 0.3014 | 30 | 76.17 | 0.7735 |
| 40 | 17.82 | 0.0329 | 40 | 40.01 | 0.1284 | 40 | 70.54 | 0.3863 |

Table 12. Input combinations that maximize profits for various N, P and corn prices when K is held constant at zero.

| Situation number | Price per unit: Corn per bushel | N per pound | P per pound | Optimum inputs in pounds per acre: N | P | Predicted corn yield in bushels per acre | Profit from use of optimum inputs[a] |
|---|---|---|---|---|---|---|---|
| 1 | $1.60 | $0.17 | $0.23 | 68.86 | 7.02 | 91.62 | $ 8.37 |
| 2 | 1.60 | 0.17 | 0.18 | 72.82 | 18.87 | 93.56 | 9.02 |
| 3 | 1.60 | 0.17 | 0.13 | 76.79 | 30.73 | 95.13 | 10.26 |
| 4 | 1.30 | 0.17 | 0.18 | 57.99 | 5.91 | 90.17 | 4.82 |
| 5 | 1.30 | 0.17 | 0.13 | 62.87 | 20.50 | 92.55 | 5.48 |
| 6 | 1.00 | 0.17 | 0.13 | 40.60 | 4.14 | 87.32 | 1.82 |
| 7 | 1.60 | 0.15 | 0.23 | 74.74 | 8.61 | 92.43 | 9.80 |
| 8 | 1.60 | 0.15 | 0.18 | 78.71 | 20.46 | 94.32 | 10.53 |
| 9 | 1.60 | 0.15 | 0.13 | 82.68 | 32.32 | 95.84 | 11.85 |
| 10 | 1.30 | 0.15 | 0.18 | 65.23 | 7.87 | 91.33 | 6.06 |
| 11 | 1.30 | 0.15 | 0.13 | 70.11 | 22.46 | 93.64 | 6.82 |
| 12 | 1.00 | 0.15 | 0.13 | 50.01 | 6.68 | 89.16 | 2.73 |
| 13 | 1.60 | 0.13 | 0.23 | 80.62 | 10.19 | 93.18 | 11.36 |
| 14 | 1.60 | 0.13 | 0.18 | 84.59 | 22.05 | 95.02 | 12.16 |
| 15 | 1.60 | 0.13 | 0.13 | 88.56 | 33.90 | 96.49 | 13.56 |
| 16 | 1.30 | 0.13 | 0.18 | 72.47 | 9.82 | 92.38 | 7.43 |
| 17 | 1.30 | 0.13 | 0.13 | 77.35 | 24.41 | 94.61 | 8.29 |
| 18 | 1.00 | 0.13 | 0.13 | 59.42 | 9.22 | 90.80 | 3.82 |
| 19 | 1.60 | 0.11 | 0.23 | 86.50 | 11.78 | 93.84 | 13.03 |
| 20 | 1.60 | 0.11 | 0.18 | 90.47 | 23.64 | 95.64 | 13.92 |
| 21 | 1.60 | 0.11 | 0.13 | 94.44 | 35.49 | 97.06 | 15.39 |
| 22 | 1.30 | 0.11 | 0.18 | 79.71 | 11.77 | 93.32 | 8.95 |
| 23 | 1.30 | 0.11 | 0.13 | 84.59 | 23.36 | 95.48 | 9.91 |
| 24 | 1.00 | 0.13 | 0.13 | 68.84 | 11.76 | 92.26 | 5.10 |

[a]Profit is computed for the increase in yield over 78.06 bushels per acre. (See equation 2).

the price of 1 bushel of corn, $P_n$ is the price of 1 pound of N and $P_p$ is the price of 1 pound of P. Profits in table 12 are computed by subtracting the value of inputs from the value of the added product from fertilization. The prices in table 12, used to illustrate the possible changes in profit-maximizing combinations of nutrients when nutrient and corn prices change relative to each other, are highly related to the static supply functions presented later.

As would be expected from examination of production surface and individual response curves, N has the greater effect on yield. Hence, the optimum combinations require more N than P even when the price of N is high relative to the price of P.

Small shifts in nutrient prices (2 cents and 5 cents per pound in N and P prices, respectively) cause small shifts in the predicted optimum yields. Inputs of N and P decrease by approximately 10 and 13 pounds, respectively, between situations 15 and 8 when their prices increase by 2 and 5 cents, respectively. For a similiar price change between situations 8 and 1, inputs of N and P decrease by approximately the same quantities, respectively. The profit resulting from these situations decreases considerably, however, and amounts to \$13.56, \$10.53 and \$8.37 for situations 15, 8 and 1, respectively. A given relative change in corn price has more effect on the optimum combination and on the resulting profit than do changes in prices of the nutrients.

## CONFIDENCE REGIONS FOR ISOQUANTS, ISOCLINES AND OPTIMUM QUANTITY OF INPUTS

The general method followed in computing the confidence intervals for isoquants, isoclines and the optimum quantity of inputs is that proposed by Fuller.[4]

Figure 5 indicates the size and position of the 95-percent confidence regions for the 85-bushel, 90-bushel and 95-bushel corn isoquants. The lower limit of the confidence region for a given corn yield is represented by $L_i$, and the corresponding upper limit is represented by $U_i$. The regions so defined indicate, under the conditions of the production function, the varying combinations of the nutrients for a specified yield which fall within 95 percent of the limits. For example, under similar experimental conditions, in the 95-percent confidence region, it is predicted that 90 bushels per acre of corn can be obtained with an application of 20.0 pounds per acre of P in combination with quantities of N varying between 39.0 and 54.0 pounds per acre.

[4] Wayne A. Fuller. Estimating the reliability of quantities derived from empirical production functions. Jour. Farm Econ. 44: 82-99. 1962.

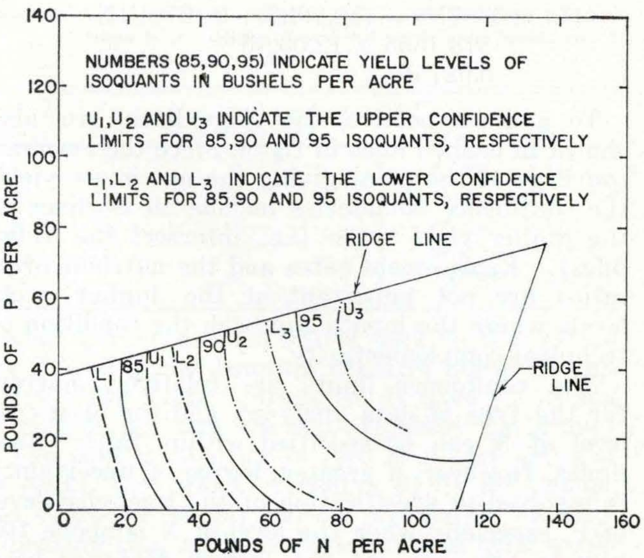


Fig. 5. Upper and lower 95-percent confidence boundaries for 85, 90 and 95 (bushels per acre of corn) isoquants as estimated from equation 3.

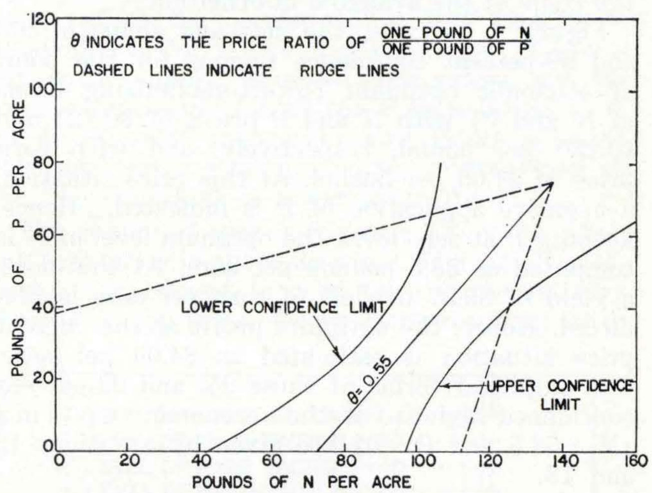


Fig. 6. Upper and lower 95-percent confidence limits for isocline, $\Theta = 0.55$, as estimated from equation 3.

The confidence limits for N are relatively narrow throughout the range of N. But the confidence limits for P extend widely, the upper limit sometimes being beyond a finite quantity. For example, 90 bushels per acre of corn might be produced, at the 95-percent confidence level, by 15.0 pounds per acre of P in combination with any quantity between 42.0 and 57.5 pounds per acre of N. But, at the same level of probability, 90 bushels per acre of corn could also be produced by using 50 pounds per acre of N in combination with any quantity between 3.0 and 27.5 pounds per acre of P.

Figure 6 indicates the position and magnitude of 95-percent confidence limits for the isocline whose marginal rate of substitution of N for P is equal to 0.55. The empirical form of 95-percent confidence region is given by equation 11.

$$(11)\quad 0.0727N^2 - (10.2922 + 0.1375P)N - 313.1355 + 12.0526P + 0.0275P^2 \leqslant 0$$

To add perspective, the ridge lines are also shown in dashed lines in fig. 6. Since the isoclines converge at the point giving the maximum yield, the confidence boundaries include all isoclines at the higher yield levels (i.e., intersect the ridge lines). Replacement rates and the nutrient price ratios are not important at the higher yield levels where the inputs approach the condition of technical complementarity.

The confidence limits are relatively narrow for the type of data analyzed, and the least-cost level of N can be specified within fairly close limits. However, a greater degree of uncertainty is involved in specification of the least-cost level of P, especially when the level of N is above 105 pounds per acre. At the higher doses of both nutrients, the exact height and slope of this isocline cannot be ascertained with certainty on the basis of the available information.

Figure 7 indicates the size and shape of 95- and 99-percent confidence regions for the point of economic optimum (profit-maximizing levels of N and P) with N and P prices of \$0.125 and \$0.230 per pound, respectively, and with corn price of \$1.00 per bushel. At this price situation, a negative application of P is indicated. Hence, keeping P at zero level, the optimum level of N is computed as 58.5 pounds per acre. At this dose, a yield of 89.37 bushels of corn per acre is predicted. Hence, the optimum profit at the current price situation is estimated as \$4.00 per acre. The empirical forms of these 95- and 99-percent confidence regions of the economic optima (N = 58.5 and P = 0) are given by equations 12 and 13.

$$(12)\quad 0.0644N^2 - 7.5373N + 1.9810P - 0.0339NP + 0.0130P^2 + 188.4656 \leqslant 0$$

$$(13)\quad -0.0644N^2 + 7.5373N + 1.9810P - 0.0339NP + 0.0130P^2 + 171.0098 \leqslant 0$$

The 95- and 99-percent confidence boundaries are narrow. However, these confidence regions are extended "in the P direction," indicating, as in the case of isocline $\Theta = 0.55$, that the optimum level of N can be predicted with greater certainty than the economic optimum level of P. Under the price level at the time of the study, it can be predicted, at the 95-percent confidence level, that the economic optimum level of N is between 37 and 81 pounds per acre and the economic optimum level of P is between 0 (or some negative quantity) and 52 pounds; the range being 44 pounds for N and more than 52 pounds for P (also see table 13).

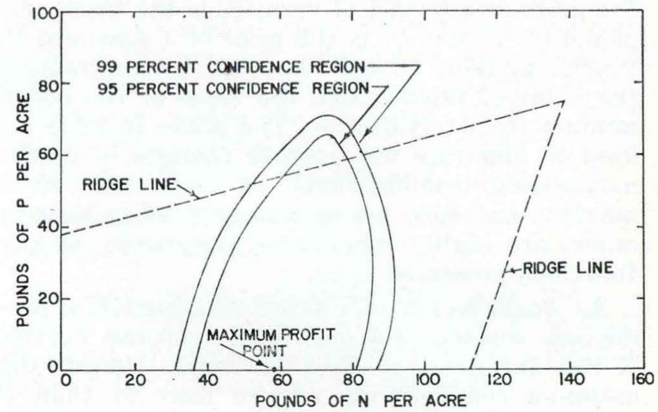


Fig. 7. **95- and 99-percent confidence regions for the maximum profit point as estimated from equation 3.**

Point estimates of inputs for optimum yield levels are associated with rather wide confidence limits. Hence, more specific predictions might be attained in the future (i.e., the confidence regions can be made smaller) by reducing the error mean square or the residual mean square of the production function. The reduction in the residual variance of the production function can be accomplished by (a) increasing the number of observations in the experiment, (b) introducing in the production function additional variables having significant effects on the output and (c) using more suitable designs for estimating the production function.

## STATIC CORN SUPPLY AND FERTILIZER DEMAND FUNCTIONS

This section deals with static-normative supply and demand relationships for an extremely short-run period for a single product and for a restricted set of resources. More specifically, it provides estimates of normative supply functions for corn and normative demand functions for fertilizer. The "length of run" considered supposes land and other resources to be fixed, while only fertilizer is considered variable. The general purpose in deriving the corn supply and fertilizer demand functions is to determine, based on the underlying technological conditions, whether response in pro-

**Table 13. The 95-percent interval estimates for N and P as obtained from the confidence region for the maximum profit point with N and P prices of 12.5 and 23.0 cents per pound, respectively, and with corn price of \$1.00 per bushel.**

| N (lbs./A.) | Confidence limits for P (lbs./A.) Lower | Upper | P (lbs./A.) | Confidence limits for N (lbs./A.) Lower | Upper |
|---|---|---|---|---|---|
| 40 | 0[a] | 15.0 | 0 | 36.5 | 81.0 |
| 50 | 0[a] | 38.0 | 10 | 39.0 | 84.0 |
| 60 | 0[a] | 53.5 | 20 | 41.5 | 85.5 |
| 70 | 0[a] | 64.0 | 30 | 46.0 | 86.0 |
| 80 | 0[a] | 56.0 | 40 | 51.0 | 85.0 |

[a]The actual estimated quantities here are negative. As the negative input quantities have little meaning, they are presented as zero.

duction of corn and use of fertilizer might be large or small in relation to price changes. For example, if the supply and demand functions are highly elastic, then a lower corn price would be expected to have a great effect in causing corn output and fertilizer use to be restricted. On the other hand, if these functions have low elasticity, a considerable drop in corn price might have only a small effect in reducing output and fertilizer use. Possibilities of this nature are examined in the analysis that follows.

The static corn supply functions and fertilizer demand functions, and their associated elasticities, are derived from an experimentally estimated production function (equation 3). The term "static" is used because it is supposed that the corn and fertilizer prices and the production functions are known with certainty. The analysis may be termed normative, since the functions indicate what the supply and demand would be, based on the production function derived from the fertilizer experiments, if farmers maximized profits under conditions where capital, institutional and behavioral restraints are unimportant. Such normative concepts are referred to simply as "static supply" and "static demand."

The term short-run, as used in this study, indicates that a single nutrient (either N or P) is variable. The term long-run indicates that the two fertilizer nutrients (both N and P) are variable. (Both concepts are short run in the usual terminology, since inputs other than the fertilizer would be variable in the conventional meaning of long run.)

### Short-Run Static Corn Supply

The short-run static supply functions, fixing N and P alternately, are derived from the quadratic production function (equation 3). It is impractical to present a complete family of short-run supply curves for all values of the fixed factor and input or product prices when two nutrients are included in the production function. Hence, the magnitude of the fixed resources is set nearly at those levels used in the experiments. The prices used (except for the relevant variable) are those current at the time (1964) of the analysis; namely, 12.5 and 23.0 cents per pound, respectively, for N and P and $1.00 per bushel for corn.

With N as the only variable input, the estimates of the static supply functions are presented in table 14, where $P_c$ denotes the price of corn per bushel and P is fixed at several levels.

The supply functions presented in table 14 are shown graphically in fig. 8. (The supply curves are extended until they have zero elasticity with respect to corn price—at about $1.40 per bushel.) The supply curve shifts to the right as the level of the fixed factor, P, increases from 0 to 80

Table 14. Equations of the short-run static supply functions for corn when N is variable, P is fixed at different levels and K is held constant at zero.

| Situation number | Level of fixed factor, P (pounds per acre) | Supply equation |
|---|---|---|
| 1 | 0 | $Y = 92.709 - 3.344\ P_c^{-2}$ |
| 2 | 20 | $Y = 96.137 - 3.344\ P_c^{-2}$ |
| 3 | 40 | $Y = 98.512 - 3.344\ P_c^{-2}$ |
| 4 | 60 | $Y = 99.832 - 3.344\ P_c^{-2}$ |
| 5 | 80 | $Y = 100.099 - 3.344\ P_c^{-2}$ |

pounds per acre; then shifts back to the left as the P level is one denoting negative marginal products. (The supply curves for P fixed at 60 and 80 pounds per acre are approximately the same.)

The supply quantity is zero at a corn price of 18 cents. For prices exceeding $1.10 per bushel, the short-run supply curves are nearly vertical, indicating that an increase in price of corn would result in negligible changes in supply quantity.

The steep slopes of the static supply curves in fig. 8, for prices exceeding 80 cents per bushel of corn, reflect the corresponding low elasticities illustrated in fig. 9. The price elasticities of short-run static supply, at given corn prices, are approximately equal for all the P fixed levels of 0, 20, 40, 60 or 80 pounds per acre. Hence, a single "average" price elasticity curve, representing all the five situations, is presented in fig. 9.

The supply of corn is inelastic ($E_s < 1$) when the corn price is above 35 cents. The supply elasticity with respect to corn price decreases very sharply as the price of corn increases from 25 to

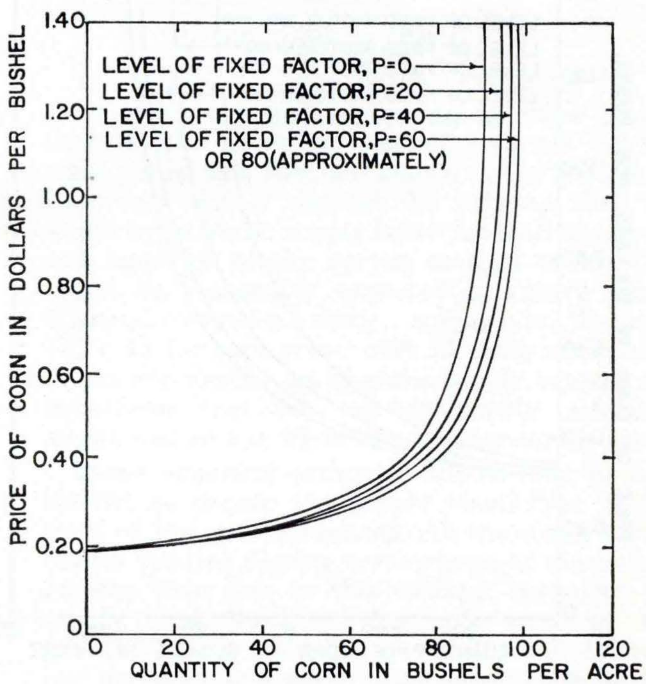


Fig. 8. Short-run static supply curves for corn, as estimated from equation 3. The price of N, the variable factor, is 12.5 cents per pound.

35 cents per bushel. When the price of corn increases from 60 cents to $1.40 per bushel, the static supply elasticity declines from 0.22 to 0.44.

The static supply functions with P as the only variable factor are presented in table 15 for various fixed levels of N.

Figure 10 depicts graphically the static corn supply equations given in table 15. (The supply curve for N fixed at 160 pounds per acre is not shown in fig. 10, since it nearly coincides with the curve for N fixed at 120 pounds.) Although the supply curve shifts to the right as the fixed level of N moves to 120 pounds per acre, it moves back to the left for higher fixed levels of N.

The supply curves with N fixed at 0, 40, 80 and 120 pounds are more widely dispersed than the derived supply curves when P is fixed at different levels (fig. 8). This is because nitrogen has a greater corn yield response than P. The maximum supply quantity of corn ranges from 75.5 to 95.2 bushels per acre as the fixed level of N extends from 0 to 120 pounds per acre. Hence, the greatest difference among the maximum supply quantities when N is fixed at different levels is 19.7 bushels, as compared with 7.4 bushels when P is fixed at the same levels.

The associated elasticities of supply curves with P as the variable input are illustrated in fig. 11. (Curves for N fixed at 40 and 80 pounds are not shown, since they nearly coincide with the curve for N fixed at 120 pounds per acre.) The elasticities of supply with respect to corn price differ very little when N is fixed at various levels. The elasticities are less than unity when corn price is more than 60 cents per bushel. Again the elasticities with respect to corn price are low, for static supply curves with P as the variable resource, throughout the range of recent and prospective prices for corn.

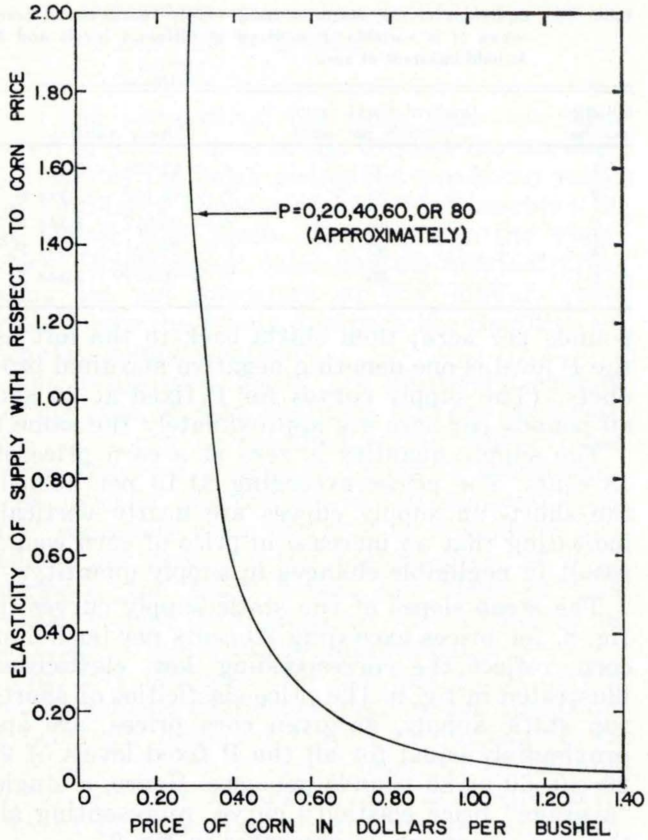


**Fig. 9.** **Price elasticities of short-run static supply curves for corn, illustrated in fig. 8.**

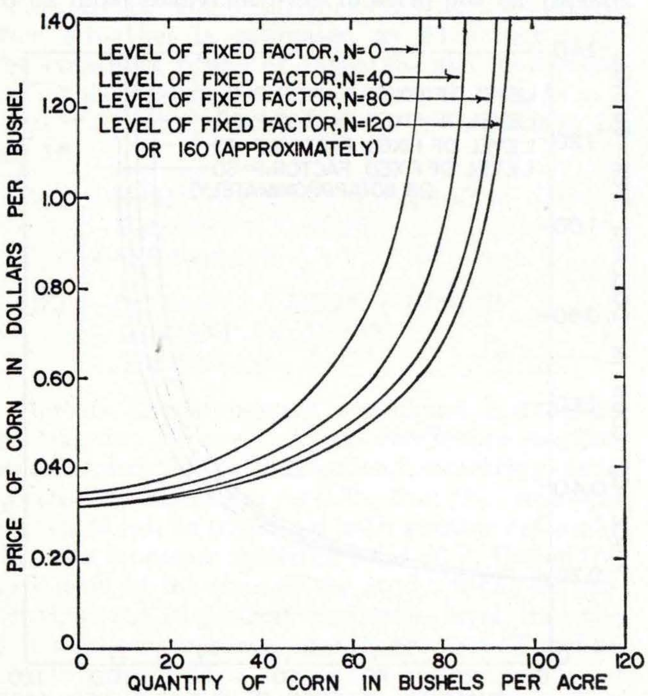


**Fig. 10.** **Short-run static supply curves for corn, as estimated from equation 3. The price of P, the variable factor, is 23 cents per pound.**

## Long-Run Static Corn Supply

The foregoing analysis deals with the short-run static supply curves when one nutrient is variable and the other is fixed at specified levels. However, it is quite unlikely that either N or P would be varied alone, as combinations are sought to maximize profits. Hence, long-run static supply

**Table 15.** **Equations of the short-run static supply functions for corn when P is variable, nitrogen is fixed at different levels and K is held constant at zero.**

| Situation number | Level of fixed factor, N (pounds per acre) | Supply equation |
|---|---|---|
| 1 | 0 | $Y = 80.157 - 9.127\ P_c^{-2}$ |
| 2 | 40 | $Y = 90.110 - 9.127\ P_c^{-2}$ |
| 3 | 80 | $Y = 96.665 - 9.127\ P_c^{-2}$ |
| 4 | 120 | $Y = 99.819 - 9.127\ P_c^{-2}$ |
| 5 | 160 | $Y = 99.574 - 9.127\ P_c^{-2}$ |

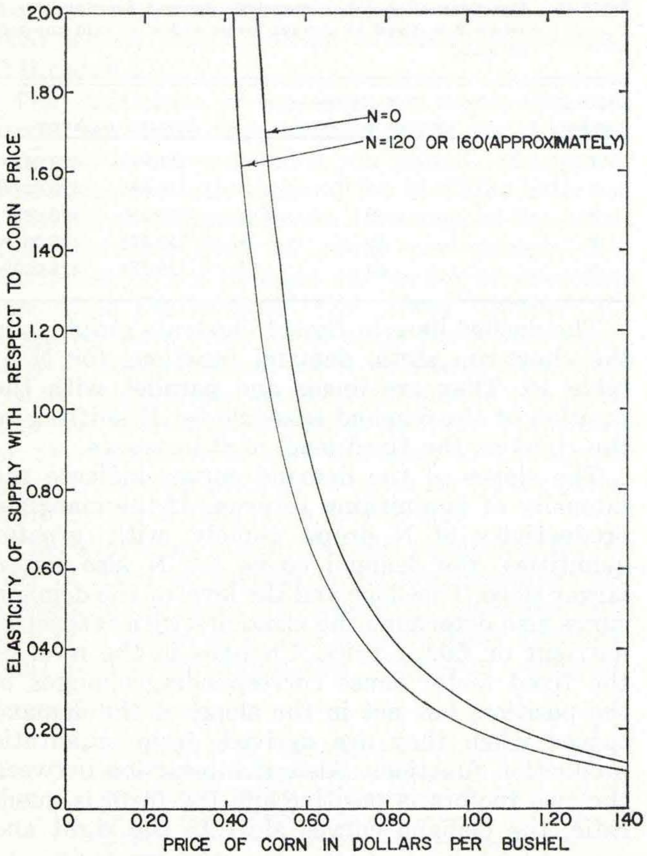


**Fig. 11. Price elasticities of short-run static supply curves for corn, illustrated in fig. 10.**

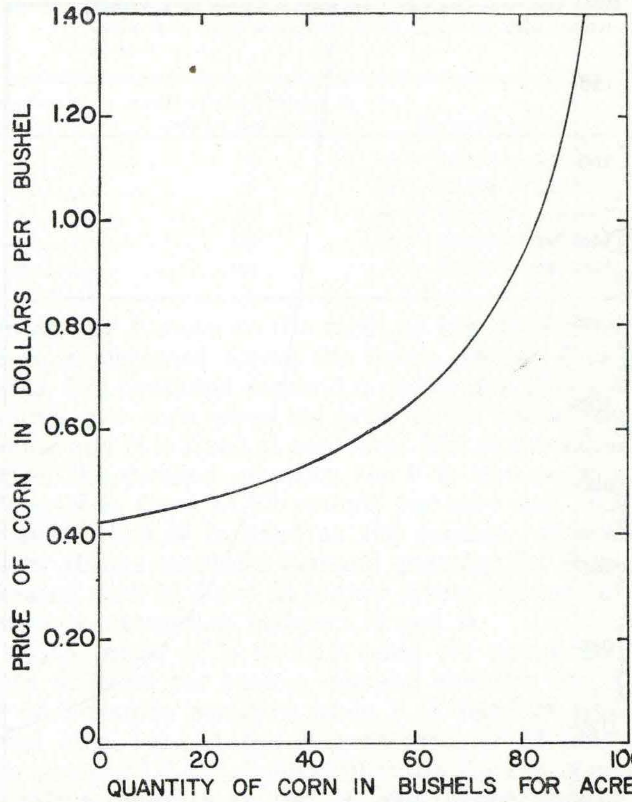


**Fig. 12. The long-run static supply curve for corn, as estimated from equation 3. Both N and P are variable. Their prices are 12.5 and 23.0 cents per pound, respectively.**

functions, with both N and P variable, are estimated in this section.

The long-run static supply function, derived from production function 3 with the prices of N and P at 12.5 and 23.0 cents, respectively, is defined by equation 14 which also is illustrated in fig. 12. It has less slope than the derived short-run supply curves for either N or P as variable factors. (The elasticity of the long-run supply curve is always greater than for the short-run curves derived from a given production function.) Under the long-run function, supply quantity of corn is specified to be zero when the price of corn is 42 cents per bushel and lower; the quantity rises quite rapidly for corn prices between 42 and 80 cents per bushel. For increases in corn price over $1.10 per bushel, increments in the supply quantity of corn are indicated to be negligible.

$$(14)\quad K = 0;\ Y = 100.131 - 17.361P_c^{-2}$$

The price elasticity of the long-run static supply ($E_s$) is indicated in equation 15 and is shown graphically in fig. 13. While of greater elasticity than the short-run curves, the long-run static supply curve has an elasticity of less than unity when the price of corn is over 73 cents per bushel. Its elasticity is less than 0.50 when the price of corn is over 93 cents and less than 0.20 when the price of corn is over $1.37 per bushel. Its elasticity is 0.42 at a price of $1.00 per bushel of corn.

$$(15)\quad E_s = \frac{34.722}{100.131P_c^2 - 17.361}$$

The long-run supply elasticity curve of fig. 13 gives a more realistic estimate of static supply than do the short-run elasticity curves for the same production function shown in figs. 9 and 11. However, figs. 8 through 13 indicate that the elasticity of static supply is low for both the short- and long-run supply curves at corn prices that might be reasonably expected in future years. Without exception, static supply is inelastic ($E_s < 1$) for corn prices over 73 cents per bushel. These low elasticities of static supply support the hypothesis that the market supply elasticity might well be low when the corn acreage is given.

These empirical estimates should best be considered to denote the supply elasticities at the start of the growing season. All the static supply curves studied display some range of elasticities greater than zero in this context. However, the supply elasticity does approach zero as the growing season ends. The possibilities of increasing or decreasing corn yield, in response to the price changes, diminish steadily as the growing season advances.

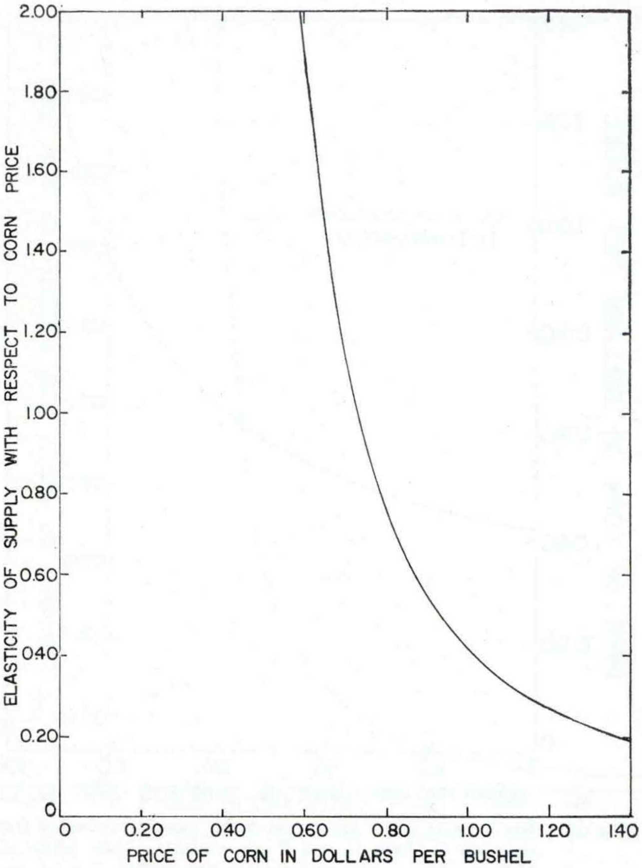


Fig. 13. Price elasticity of long-run static supply curve for corn, illustrated in fig. 12.

## Short-Run Static Factor Demand

Static demand functions for a factor also may be computed in either a short-run or a long-run context. The term short run again is used here to mean that the level of one factor or nutrient, in production function 3, is fixed. The term long run means that the levels of both nutrients are variable and substitution of one factor for another is possible, depending on change in prices and the nature of interaction between the factors.

The static demand functions that follow are derived with the price of corn fixed at \$1.00 per bushel. However, the derivations can be used to generalize for other corn prices by considering the fertilizer-corn price ratio (since the demand quantity is more a function of this ratio than of corn price alone).

A family of short-run static demand schedules can be generated from a given production function for different levels of the fixed resource. For the demand function analysis that follows, the fixed resource or nutrient is set at the same levels as in the previous short-run supply analysis. Since the production function is quadratic, the derived static demand functions are linear. Fixing P at various levels, the short-run static demand equations for N are those presented in table 16, where $P_n$ denotes the price of 1 pound of N.

Table 16. Equations of the short-run static demand functions for N when P is fixed at various levels and K is held constant at zero.

| Situation number | Level of fixed factor, P (pounds per acre) | Demand equation |
|---|---|---|
| 1 | 0 | $N = 111.990 - 428.082P_n$ |
| 2 | 20 | $N = 118.685 - 428.082P_n$ |
| 3 | 40 | $N = 125.380 - 428.082P_n$ |
| 4 | 60 | $N = 132.076 - 428.082P_n$ |
| 5 | 80 | $N = 138.771 - 428.082P_n$ |

The dashed lines in fig. 14 illustrate graphically the short-run static demand functions for N of table 16. They are linear and parallel, with the position of the demand schedule for N shifting to the right as the fixed level of P increases.

The slopes of the demand curves indicate the intensity of diminishing returns.[5] If the marginal productivity of N drops rapidly with greater quantities, the demand curve for N also has a larger slope. The slope and the level of the demand curve also determine the elasticity with respect to nutrient or factor price. Changes in the level of the fixed factor cause corresponding changes in the position, but not in the slope, of the demand curves when they are derived from quadratic production functions. Also, if interaction between the two factors is positive and the form is quadratic, the demand curves shift to the right and

[5] The computation of static demand is independent of the "control yield." Hence, static demand is not directly affected by the initial nutrients in the soil. However, the initial nutrient influences the level of demand indirectly, since a high level of nitrogen demand reflects a large response of corn yield to additional inputs of nitrogen.

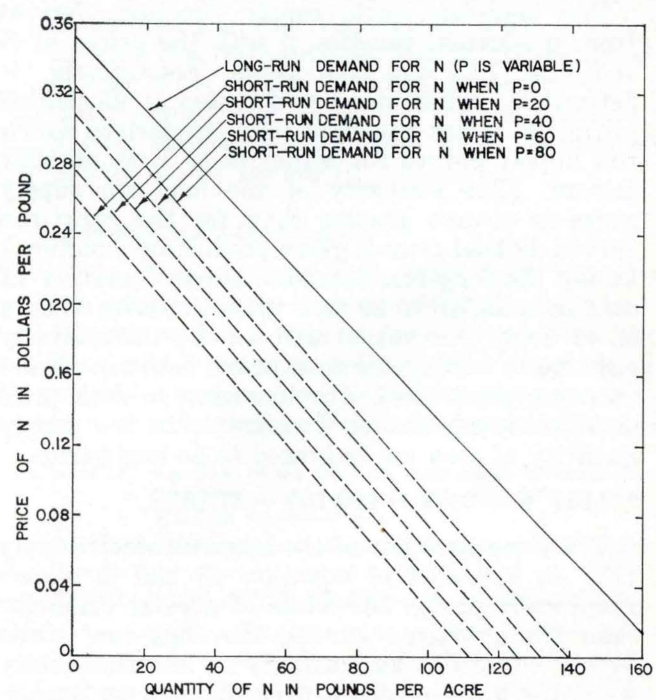


Fig. 14. Short- and long-run static demand schedules for N as estimated from equation 3. The price of corn is \$1.00 per bushel, and the price of P is \$0.23 per pound.

the elasticity decreases with higher levels of the fixed factor. This condition is illustrated in figs. 14 through 17.

The elasticities of the short-run static demand curves for N are the dashed lines in fig. 15. For prices of N below 8 cents per pound, the price elasticities of static demand for N differ little as P is fixed at different levels. However, as the price of N increases over 14 cents per pound, the elasticities of the demand curves for N decrease sharply as the level of the fixed factor, P, increases. At a price of 12 cents for N, the elasticity of static demand for N decreases from 0.85 to 0.59, respectively, as the fixed factor (P) level increases from 0 to 80 pounds per acre. However, for the same levels and increases in the fixed factor level, with price of 15 cents for N, the price elasticity for N falls from 1.34 to 0.86.

The short-run static demand equations for P with N fixed at various levels are presented algebraically in table 17, where $P_p$ indicates the price of 1 pound of P, and geometrically (dashed lines) in fig. 16. Again the short-run demand

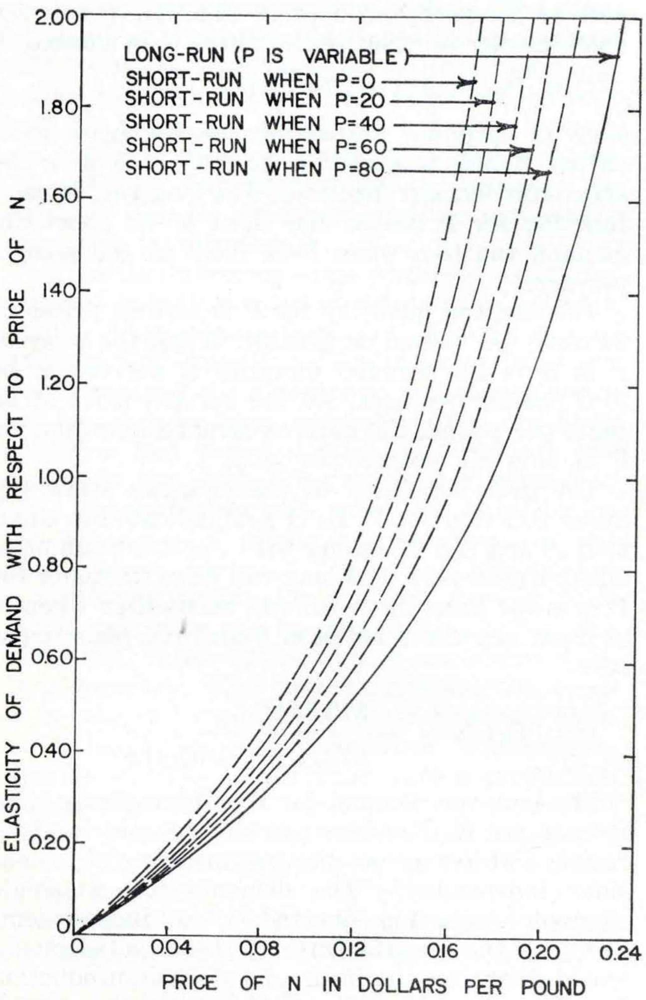


Fig. 15. Price elasticities of static demand functions for N, illustrated in fig. 14.

Table 17. Equations of the short-run static demand functions for P when N is fixed at various levels and K is held constant at zero.

| Situation number | Level of fixed factor, N (pounds per acre) | Demand equation |
|---|---|---|
| 1 | 0 | $P = 38.043 - 345.066P_p$ |
| 2 | 40 | $P = 48.836 - 345.066P_p$ |
| 3 | 80 | $P = 59.630 - 345.066P_p$ |
| 4 | 120 | $P = 70.424 - 345.066P_p$ |
| 5 | 160 | $P = 81.217 - 345.066P_p$ |

curves for P move to the right as the fixed level of N is increased. Given the linear demand functions, the predicted demand quantity for P is 3.5 pounds per acre when its price is 10 cents per pound and N is fixed at zero level. But at the same price, the demand quantity for P is 25.0 pounds when N is fixed at 80 pounds per acre and 46.5 pounds when N is fixed at 160 pounds. Hence, other things constant, demand quantity for P increases with N fixed at higher levels, because of positive interaction between N and P.

With prices of N at 12.5 cents per pound and corn at $1.00 per bushel, demand quantity for N is 58.5 pounds per acre when P is fixed at zero level (fig. 14). At the current price of 23 cents per pound of P, the demand quantity for P is some negative quantity in fig. 16 and, hence, can be taken as zero when N is fixed at zero level. Thus, under parallel situations, the demand for N is much higher than the demand for P, because N has larger marginal productivities. (These points are evident from the figs. 14 and 16.) The derived short-run demand functions for N are much farther from the origin, when compared with the derived short-run demand functions for P.

The price elasticities of short-run demand functions for P are the dashed lines in fig. 17.

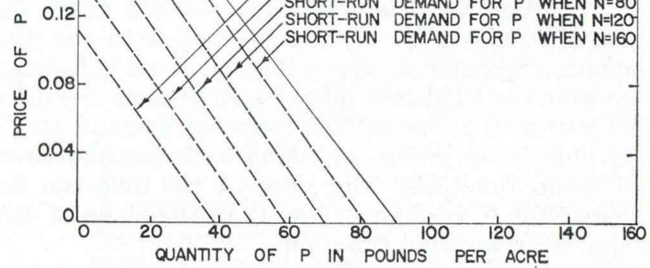


Fig. 16. Short- and long-run static demand schedules for P, as estimated from equation 3. The price of corn is $1.00 per bushel, and the price of N is $0.125 per pound.

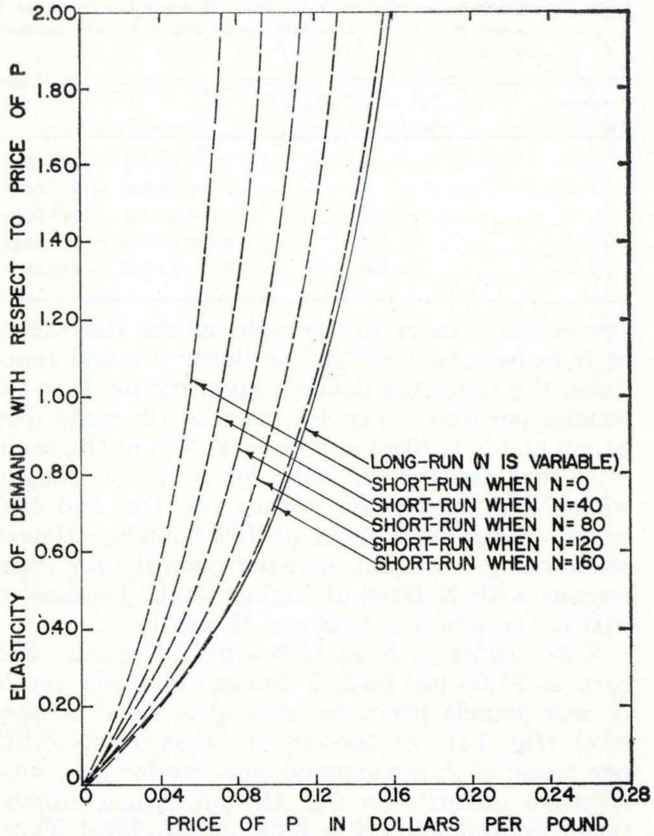


**Fig. 17. Price elasticities of static demand functions for P illustrated in fig. 16.**

Again, for any fixed price of P, the price elasticities of short-run demand for P becomes smaller as the fixed level of N increases. They vary greatly when the price of P is above 4 cents per pound. For example, at a price of 10 cents per pound of P, the price elasticity of demand for P diminishes from 9.80 (not shown in fig. 17) to 0.74 as the level of the fixed factor (N) increases from 0 to 160 pounds per acre. The elasticity of short-run demand function for P when N is fixed at 160 pounds per acre is greater than 1.0 for all prices of P above 12 cents per pound. However, at the current price of 23 cents per pound of P, the elasticities for short-run demand functions cannot be determined.

**Long-Run Static Factor Demand**

Equation 16 represents the long-run static demand equation for N, also derived from the production function 3, where P is not fixed but varies to give the least-cost mix of nutrients as the price of nitrogen changes. The long-run demand for N is illustrated in fig. 14, along with the short-run demand functions. The slope of the long-run demand for N is slightly less than the slope of any short-run demand functions.

(16) $N = 166.317 - 470.592P_n$

The predicted demand quantity for N is 166 pounds per acre at a zero price (not shown in fig. 14). At the other extreme, the demand quantity for N would be zero when its price is at 35.5 cents per pound. At the current price of 12.5 cents per pound of N, the demand quantity for N is predicted to be 107.5 pounds per acre.

The price elasticity of long-run static demand for N, $E_d(N)$, is indicated in equation 17 and fig. 15. The long-run elasticity is lower than the corresponding short-run elasticities when P is fixed anywhere between 0 and 80 pounds per acre. The long-run elasticity for N is less than unity when the price of N is below 17.5 cents per pound and is greater than unity when the price of N is over 17.5 cents per pound. The long-run elasticity curve for N increases rapidly in slope as the price of N increases above 20 cents. At the price of 12.5 cents per pound of N, the price elasticity of long-run static demand is predicted to be about 0.55.

$$(17)\quad E_d(N) = \frac{470.592P_n}{470.592P_n - 166.317}$$

The long-run static demand function for P derived from the production function 3 is given algebraically in equation 18 where N is allowed to

$$(18)\quad P = 90.914 - 379.331P_p$$

vary in optimum proportions as the price of P varies. It also is shown in fig. 16, along with the short-run demand functions. The long-run demand function for P lies to the right of its short-run demand function when N is fixed at 160 pounds per acre.

The demand quantity for P is zero at prices of 24 cents per pound or greater. When the price of P is zero, the demand quantity is derived to be 91.0 pounds per acre. At the current price of 23 cents per pound, the derived demand quantity for P is only 2.7 pounds per acre.

The price elasticity of the long-run static demand function for P, $E_d$ (P), is indicated in equation 19 and fig. 17 (along with the short-run price elasticity curves). The long-run price elasticity for P is lower than the short-run elasticities when N is fixed anywhere between 0 and 160 pounds per acre.

$$(19)\quad E_d(P) = \frac{379.331P_p}{379.331P_p - 90.914}$$

The long-run demand for P is more elastic than is that for N. Fertilizers are often sold in fixed ratios, and it may not be very meaningful to consider independently the demand for a single element. Assuming demand to be independent, however, the manufacturer of these two elements would likely, on the basis of the basic production functions in this study, find the purchase of P more responsive than that of N for prices reduced by the same percentage. The demand curve for a fertilizer mixture, with N and P held in some fixed ratio, would fall to the right of the demand curve for any one element. Also, the demand for a fixed ratio of these two elements would be less elastic than the demand for either element alone.

The analysis of this section indicates fertilizer to have greater elasticity of demand with respect to its own price than does corn supply with respect to its own price. Because of diminishing returns, increases in fertilizer will add smaller and smaller increments to corn output. Fertilizer consumption also must increase (decrease) by a larger percentage than corn output in response to a favorable (unfavorable) change in corn price. Thus, a change in the price of corn should have a greater impact on fertilizer use than on corn production.

## MARGINAL RATES OF SUBSTITUTION OF FERTILIZER FOR LAND

Fertilizer is an effective substitute for land. It substitutes for land in the sense that a given product can be produced with less land if fertilizer is used on the remaining acreage. The marginal rates at which fertilizer substitutes for land would be useful knowledge for national planning. Knowledge of these marginal rates not only is useful in developed economies but also is even more useful in developing countries where output must increase but land supply is severely restricted or can be increased only through costly reclamation investments.

This section presents some empirical estimates of "gross" marginal rates of substitution between fertilizer in aggregate form (i.e., a given mix of nutrients) and land. These substitution rates are "gross" because the machinery and other capital items, as well as labor associated with fertilizer application and per-acre yield increase, are not included in the study. The method of derivation of the "gross" marginal rates of substitution from experimental production functions is not detailed here.[6] However, the central idea is that the particular production function derived from a fixed land area can be extended to reflect the substitution rates where land is considered variable in quantity. The particular production function is related initially to fertilizer response. However, through some mathematical conversions, it can be suitably transformed into a production function that includes land and has "constant returns to scale" for the two factors considered alone, so that a doubling of both land and fertilizer will double output.

The conversion used in transforming production function 3 into a suitable form is as follows where r units of P are always used for each one of N, or $P = rN$ to produce one unit of F or fertilizer. Hence, a given quantity of fertilizer is composed as $F = N + P = (1 + r)N$ where F, N and P are all measured in pounds per acre. Using this relation, the "gross" marginal rates of substitution of fertilizer for land are computed from the estimated production function 3. These substitution rates are derived when r, the ratio P to N, takes values 1/4, 1/3 and 1/2. Though r can take any positive finite value, these three values are selected for r in the light of the information regarding the optimum fertilizer combinations given in table 12. Also, for convenience, only three isoquants, each representing a yield level attainable on a single acre, are considered. These isoquants represent 85, 90 and 95 bushels of corn, respectively. The same isoquants were used in an earlier section for estimating confidence regions. The corresponding isoquants and marginal rates of substitution for fertilizer and land are given in tables 18, 19 and 20 when $r = 1/4$, $1/3$ and $1/2$, respectively.

As expected, the substitution rates of fertilizer for land increase in absolute value with the higher yield isoquants for any value of r, the ratio P to N. Also, for any selected isoquant level and for any given value of r, the marginal rates of substitution decrease as the fertilizer (F) rate increases.

For lower rates of fertilization, the marginal rates of substitution of fertilizer for land decrease in absolute value as the value of r increases. However, the substitution rates increase with greater values of r at higher rates of fertilization. This point is illustrated quantitatively in fig. 18 for the 95-bushel land-fertilizer isoquant. As r increases from 1/4 to 1/2, the marginal rates of substitution decrease up to 40 pounds per acre

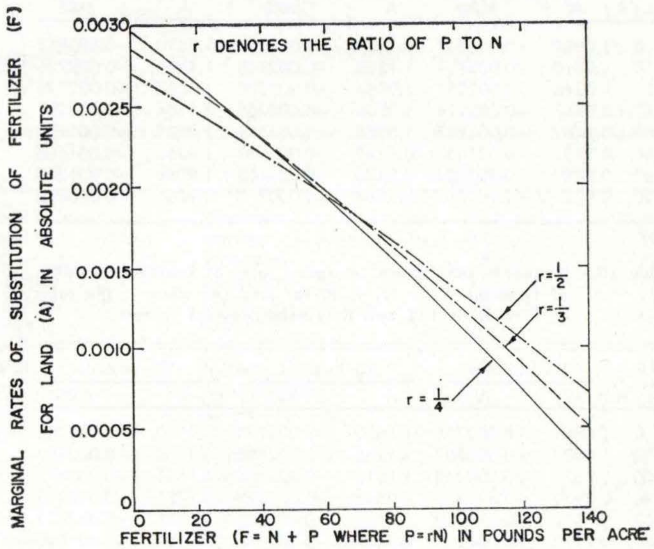


Fig. 18. Marginal rates of substitution of fertilizer (F) for land (A) for different values of r for the isoquant of 95 bushels per acre.

[6] See: Earl O. Heady. Marginal rates of substitution between technology land and labor. Jour. Farm Econ. 45: 137-145. 1963.

of fertilizer. But over 70 pounds of fertilizer, the substitution rates increase with greater values of r.

Hence, to have greater substitution rates (of fertilizer for land) a smaller proportion of P is required at lower fertilizer rates, and a greater proportion is needed at higher fertilizer rates. The substitution rates become zero for some value of fertilization. For example, when r is 1/4, the substitution rate of fertilizer for land is zero for 153 pounds of fertilizer. When r is 1/2, the substitution rate is zero for 175 pounds of fertilizer.

When r = 1/4 (table 18), a predicted yield of 85 bushels of corn is forthcoming with 1.0889 acres of land and no fertilizer; with 1.0330 acres of land and 20 pounds of fertilizer; with 0.9129 acre of land and 80 pounds of fertilizer; etc.

With a combination of 20 pounds of fertilizer and 1.0330 acres of land for 85 bushels of corn, a pound of fertilizer substitutes for 0.002618 acre of

**Table 18.** Isoquant and "gross" marginal rates of substitution (MRS) of fertilizer (F = N + P) for land (A) when r, the ratio of P to N, is 1/4 and K is held constant at zero.

| F | 85 bushels | | 90 bushels | | 95 bushels | |
|---|---|---|---|---|---|---|
| (lbs./A.) | A | MRS | A | MRS | A | MRS |
| 0 | 1.0889 | −0.002964 | 1.1530 | −0.002964 | 1.2170 | −0.002964 |
| 10 | 1.0601 | −0.002797 | 1.1241 | −0.002807 | 1.1881 | −0.002815 |
| 20 | 1.0330 | −0.002618 | 1.0969 | −0.002638 | 1.1607 | −0.002656 |
| 40 | 0.9845 | −0.002224 | 1.0477 | −0.002270 | 1.1110 | −0.002310 |
| 60 | 0.9443 | −0.001794 | 1.0063 | −0.001867 | 1.0686 | −0.001932 |
| 80 | 0.9129 | −0.001347 | 0.9732 | −0.001446 | 1.0339 | −0.001535 |
| 100 | 0.8904 | −0.000907 | 0.9484 | −0.001027 | 1.0072 | −0.001136 |
| 120 | 0.8764 | −0.000497 | 0.9319 | −0.000629 | 0.9883 | −0.000751 |

**Table 19.** Isoquants and "gross" marginal rates of substitution (MRS) of fertilizer (F = N + P) for land (A) when r, the ratio of P to N, is 1/3 and K is held constant at zero.

| F | 85 bushels | | 90 bushels | | 95 bushels | |
|---|---|---|---|---|---|---|
| (lbs./A.) | A | MRS | A | MRS | A | MRS |
| 0 | 1.0889 | −0.002867 | 1.1530 | −0.002867 | 1.2170 | −0.002867 |
| 10 | 1.0610 | −0.002719 | 1.1250 | −0.002728 | 1.1890 | −0.002736 |
| 20 | 1.0346 | −0.002551 | 1.0984 | −0.002579 | 1.1623 | −0.002596 |
| 40 | 0.9867 | −0.002214 | 1.0500 | −0.002255 | 1.1134 | −0.002291 |
| 60 | 0.9462 | −0.001833 | 1.0085 | −0.001899 | 1.0709 | −0.001957 |
| 80 | 0.9135 | −0.001433 | 0.9742 | −0.001523 | 1.0353 | −0.001603 |
| 100 | 0.8889 | −0.001034 | 0.9476 | −0.001123 | 1.0068 | −0.001243 |
| 120 | 0.8721 | −0.000652 | 0.9284 | −0.000776 | 0.9855 | −0.000890 |

**Table 20.** Isoquants and "gross" marginal rates of substitution (MRS) of fertilizer (F = N + P) for land (A) when r, the ratio of P to N, is 1/2 and K is held constant at zero.

| F | 85 bushels | | 90 bushels | | 95 bushels | |
|---|---|---|---|---|---|---|
| (lbs./A.) | A | MRS | A | MRS | A | MRS |
| 0 | 1.0889 | −0.002705 | 1.1530 | −0.002705 | 1.2170 | −0.002705 |
| 10 | 1.0625 | −0.002582 | 1.1265 | −0.002590 | 1.1905 | −0.002594 |
| 20 | 1.0373 | −0.002449 | 1.1012 | −0.002464 | 1.1651 | −0.002477 |
| 40 | 0.9912 | −0.002159 | 1.0546 | −0.002193 | 1.1181 | −0.002223 |
| 60 | 0.9512 | −0.001840 | 1.0137 | −0.001895 | 1.0764 | −0.001943 |
| 80 | 0.9178 | −0.001501 | 0.9789 | −0.001577 | 1.0405 | −0.001645 |
| 100 | 0.8912 | −0.001156 | 0.9506 | −0.001252 | 1.0106 | −0.001337 |
| 120 | 0.8715 | −0.000819 | 0.9289 | −0.000930 | 0.9870 | −0.001030 |

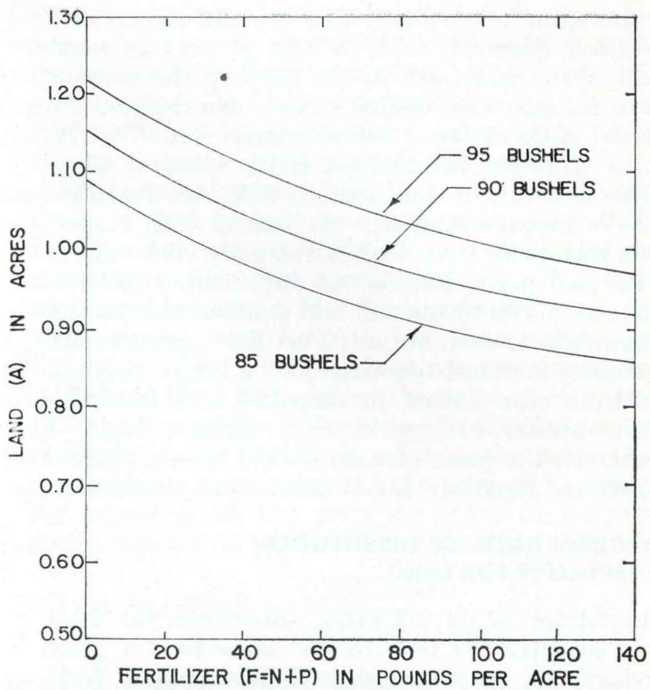


**Fig. 19.** Land-fertilizer isoquants for corn when P = 1/3 N.

land. Hence, a ton of fertilizer spread similarly over more acres is estimated to substitute for 5.236 acres of land (i.e., 2,000 x 0.002618). With 80 pounds of fertilizer and 0.9129 acre of land to produce the same amount of corn, a ton of fertilizer substitutes for 2.694 acres of land. For the same value of r, with 20 pounds of fertilizer and 1.1607 acres of land to produce 95 bushels of corn, a ton of fertilizer substitutes for 5.312 acres of land. With 80 pounds of fertilizer and 1.0339 acres of land to produce 95 bushels of corn, a ton of fertilizer substitutes for 3.070 acres of land.

When r is equal to 1/3 (table 19), with 20 pounds of fertilizer and 1.0346 acres of land to produce 85 bushels of corn, a ton of fertilizer substitutes for 5.102 acres of land. With 20 pounds of fertilizer and 1.1623 acres of land to produce 95 bushels of corn, a ton of fertilizer substitutes for 5.1926 acres of land. With 80 pounds of fertilizer and 0.9135 acre of land to produce 85 bushels of corn, a ton of fertilizer can substitute for 2.866 acres of land. With 80 pounds of fertilizer and 1.0353 acres of land to produce 95 bushels of corn, a ton of fertilizer substitutes for 3.206 acres of land.

When r is equal to 1/2 (table 20), with a combination of 20 pounds of fertilizer and 1.0373 acres of land, to produce 85 bushels, a ton of fertilizer substitutes for 4.898 acres of land. With 20 pounds of fertilizer and 1.1651 acres of land to produce 95 bushels of corn, a ton of fertilizer substitutes for 4.954 acres of land. With 80 pounds of fertilizer and 0.9178 acre of land to produce 85 bushels of corn, a ton of fertilizer substitutes for 3.002 acres of land. But with the same amount of fertilizer and 1.0405 acres of land to produce 95 bushels of corn, a ton of fertilizer substitutes for 3.290 acres of land.

The land-fertilizer isoquants for corn are illustrated in fig. 19, when r, the ratio P to N, is ⅓. The slope of these isoquants defines the gross marginal rates of substitution between fertilizer and land. The slopes of these three isoquants, representing 85, 90 and 95 bushels per acre, differ very little. Also, the slopes of these isoquants decline steadily as the fertilizer rate increases. The 85-, 90- and 95-bushel isoquants will be parallel to the fertilizer axis at about 155, 165 and 175 pounds of fertilizer, respectively. Hence, at these fertilizer rates, the slopes of the corresponding isoquants are zero, indicating that no land can be replaced by fertilizer.

As mentioned earlier, the marginal rates of substitution are "gross" in the sense that resources that complement fertilizer and land are also involved. For example, a given quantity of fertilizer that replaces certain acres in maintaining a fixed level of production, would also involve less machinery, less labor, less pesticides, etc., to be used on a smaller acreage. Hence, a single major factor is rarely substituted for another single major factor in agriculture. However, the "gross" marginal rates of substitution of fertilizer for land are important. Given a favorable supply price for those resources that complement both fertilizer and land, agricultural policy administrators can be concerned with the rate at which a major resource such as fertilizer can substitute for a restricting resource such as land.

## APPENDIX A

**Table A-1. Cooperator, location and year of 18 corn experiments.**

| Experiment number | Cooperator | County | Township | Section | Year |
|---|---|---|---|---|---|
| 1 | Fuller | Humboldt | Wacousta | 33 | 1959 |
| 2 | Korslund | Humboldt | Norway | 12 | 1959 |
| 3 | Thomason | Wright | Woolstock | 8 | 1959 |
| 4 | Webster | Cerro Gordo | Dougherty | 18 | 1959 |
| 5 | Clark | Cerro Gordo | Owen | 32 | 1959 |
| 6 | Newton | Blackhawk | Cedar | 7 | 1959 |
| 1 | Gugisberg | Humboldt | Lake | 1 | 1960 |
| 2 | Kellem | Humboldt | Norway | 26 | 1960 |
| 3 | Huddleston | Hamilton | Independence | 7 | 1960 |
| 4 | Olson | Hamilton | Ellsworth | 35 | 1960 |
| 5 | Boten | Hamilton | Scott | 25 | 1960 |
| 6 | Henderson | Story | Lincoln | 18 | 1960 |
| 7 | Carolus | Wright | Lincoln | 13 | 1960 |
| 8 | Hill | Wright | Blaine | 1 | 1960 |
| 9 | Nodland | Wright | Belmond | 14 | 1960 |
| 10 | Dunton | Cerro Gordo | Dougherty | 12 | 1960 |
| 11 | Crone | Hancock | Bingham | 15 | 1960 |
| 12 | Millar | Story | Indian Creek | 7 | 1960 |

## APPENDIX B

The production functions for individual years are presented in summary form. In the equations that follow, Y refers to bushels per acre of corn, while N, P and K refer, respectively, to pounds per acre of nitrogen, phosphorus and potassium. $P_n$, $P_p$ and $P_k$ refer to the prices of 1 pound of N, P and K, respectively. The current prices of N, P and K are taken to be 12.5, 23.0 and 5.0 cents per pound, respectively, and the price of corn is taken to be $1.00 per bushel.

**Results of 1959 Experiments**[7]

The production function is $K = 41.6$;

$$Y = 81.7501 + 0.3096N + 0.1132P - 0.001256N^2 - 0.000343P^2 - 0.000308NP$$

The value of $R^2$ for this equation is 0.9740.

Table B-1 indicates probability levels of regression coefficients, and table B-2 provides related statistics. Tables B-3 through B-6 include data relating to marginal productivities and optimum use of fertilizer. Tables B-7 through B-10 include static supply and demand functions, while tables B-11 through B-13 relate to substitution relations between fertilizer and land for the 1959 experiments.

The yield isoquant is

$$K = 41.6;\ N = 123.2603 - 0.1226P \pm (80280.7463S) - 796.1783\gamma + 59.8877P - 0.2580P^2)^{1/2}$$

The yield isocline is

$$K = 41.6;\ (0.000308\theta - 0.002512)N + (0.000686\theta - 0.000308)P + (0.309630 - 0.113203\theta) = 0$$

where $\theta$ denotes the price ratio of N to P.

The long-run static supply function for corn when both N and P are variable and K is held constant at 41.6 pounds per acre is

$$Y = 105.199 - 38.656\ P_c^{-2}$$

The elasticity of this long-run supply for corn ($E_s$) is

$$E_s = \frac{77.312}{105.199P_c^2 - 38.656}$$

The long-run static demand function for N when P is variable and K is held constant at 41.6 pounds per acre is

$$N = 152.533 - 421.281\ P_n$$

The elasticity of this long-run demand for N, $E_d$ (N) is

$$E_d(N) = \frac{421.281P_n}{421.281P_n - 152.533}$$

[7] The estimates of 1959 experiments are biased as a result of the procedure adopted in pooling the data over the factor potassium (holding constant at 41.6 pounds per acre).

The long-run static demand function for P when N is variable and K is held constant at 41.6 pounds per acre is

$$P = 139.710 - 1542.649P_p$$

The elasticity of this long-run demand for P, $E_d$ (P) is

$$E_d(P) = \frac{1542.649P_p}{1542.649P_p - 139.710}$$

These equations are not presented in tables.

**Results of 1960 Experiments**

Production function: $K = 0$; $Y = 74.8754 + 0.2526N + 0.1991P - 0.001063N^2 - 0.002495P^2 + 0.000696NP$

The value of $R^2$ for this equation is 0.9593.

Tables B-14 and B-15 provide basic probability statistics for the 1960 data. Tables B-16 through B-19 relate to marginal productivities and profit-optimizing quantities of fertilizer. Tables B-20 through B-23 include static supply and demand curves. Tables B-24 through B-26 relate to substitution relations between fertilizer and land for the 1960 data.

The yield isoquant is

$$K = 0;\ N = 118.7935 + 0.3274P \pm (84549.7181 - 940.7337Y + 263.0793P - 2.2400P^2)^{1/2}$$

The yield isocline is

$$K = 0;\ -(0.002126 + 0.000696\theta)N + (0.000696 + 0.004990\theta)P + (0.252550 - 1.990991\theta) = 0$$

where $\theta$ denotes the price ratio of N to P.

The long-run static supply function for corn when both N and P are variable and K is held constant at zero is

$$Y = 98.213 - 11.381\ P_c^{-2}$$

The elasticity of the above long-run supply for corn ($E_s$) is

$$E_s = \frac{22.762}{98.213P_c^{-2} - 11.381}$$

The long-run static demand function for N when P is variable and K is held constant at zero is

$$N = 153.976 - 492.872P_n$$

The elasticity of the above long-run demand for N, $E_d$ (N) is

$$E_d\ (N) = \frac{492.872P_n}{492.872\ P_n - 153.976}$$

The long-run static demand function for P when N is variable and K is held constant at zero is

$$P = 67.764 - 209.989P_p$$

The elasticity of the above long-run demand for P, $E_d$ (P) is

$$E_d\,(P) = \frac{209.989P_p}{209.989P_p - 67.764}$$

The above equations are not presented in tables.

**Table B-1. Values of t for the coefficients of the production function from 1959 experiments.**

| Coefficient | Value of t | Probability level[a] |
|---|---|---|
| N | 11.14 | 0.001 |
| P | 1.77 | 0.15 |
| $N^2$ | 8.01 | 0.001 |
| $P^2$ | 0.41 | 0.70 |
| NP | 1.14 | 0.30 |

[a]Probability of drawing a t value as large or larger by chance, given the null hypothesis.

**Table B-2. Analysis of variance, 1959 experiments.**

| Source of variation | Degrees of freedom | Sum of squares | Mean square | F[a] |
|---|---|---|---|---|
| Total | 12 | 648.88 | | |
| Due to regression | 5 | 632.04 | 126.41 | 52.46 |
| Deviation from regression | 7 | 16.84 | 2.41 | |

[a]The F value is highly significant.

**Table B-3. Marginal physical products of N at different levels of P when K is held constant at 41.6 pounds per acre, 1959 experiments.**

| Pounds of P per acre | Pounds of N per acre: 0 | 40 | 80 | 120 | 160 |
|---|---|---|---|---|---|
| 0 | 0.3096 | 0.2092 | 0.1087 | 0.0082 | −0.0923 |
| 17.4 | 0.3043 | 0.2038 | 0.1033 | 0.0028 | −0.0976 |
| 34.8 | 0.2989 | 0.1984 | 0.0980 | −0.0025 | −0.1030 |
| 52.2 | 0.2936 | 0.1931 | 0.0926 | −0.0079 | −0.1084 |
| 69.6 | 0.2882 | 0.1877 | 0.0872 | −0.0132 | −0.1137 |

**Table B-4. Marginal physical products of P at different levels of N when K is held constant at 41.6 pounds per acre, 1959 experiments.**

| Pounds of N per acre | Pounds of P per acre: 0 | 17.4 | 34.8 | 52.2 | 69.6 |
|---|---|---|---|---|---|
| 0 | 0.1132 | 0.1013 | 0.0893 | 0.0774 | 0.0654 |
| 40 | 0.1009 | 0.0889 | 0.0770 | 0.0651 | 0.0531 |
| 80 | 0.0886 | 0.0766 | 0.0647 | 0.0528 | 0.0408 |
| 120 | 0.0762 | 0.0643 | 0.0524 | 0.0404 | 0.0285 |
| 160 | 0.0639 | 0.0520 | 0.0400 | 0.0281 | 0.0162 |

**Table B-5. Combinations of P and N required to produce a given yield of corn and corresponding marginal rates of substitution (MRS) when K is held constant at 41.6 pounds per acre, 1959 experiments.**

| 95 bushels | | | 100 bushels | | |
|---|---|---|---|---|---|
| Pounds of P | Pounds of N | MRS ($\partial N/\partial P$) | Pounds of P | Pounds of N | MRS ($\partial N/\partial P$) |
| 0 | 55.12 | 0.5622 | 0 | 97.51 | 1.2853 |
| 20 | 45.06 | 0.4499 | 20 | 78.90 | 0.7140 |
| 40 | 36.95 | 0.3637 | 40 | 66.92 | 0.5042 |
| 60 | 30.42 | 0.2919 | 60 | 58.22 | 0.3734 |
| 80 | 25.23 | 0.2281 | 80 | 51.79 | 0.2735 |
| 100 | 21.27 | 0.1688 | 100 | 47.19 | 0.1876 |

**Table B-6. Input combinations that maximize profits for various N, P and corn prices when K is held constant at 41.6 pounds per acre, 1959 experiments.**

| Situation number | Price per unit: Corn per bushel | N per pound | P per pound | Optimum inputs in pounds per acre: N | P | Predicted corn yield in bushels per acre | Profit from use of optimum inputs[a] |
|---|---|---|---|---|---|---|---|
| 1 | $1.60 | $0.17 | $0.13 | 79.64 | 10.82 | 99.36 | $11.15 |
| 2 | 1.60 | 0.15 | 0.13 | 84.90 | 8.46 | 99.70 | 12.80 |
| 3 | 1.60 | 0.13 | 0.13 | 90.17 | 6.09 | 99.96 | 14.55 |
| 4 | 1.60 | 0.11 | 0.13 | 95.43 | 3.73 | 100.17 | 16.40 |

[a]Profit is computed for the increase in yield over 81.75 bushels per acre. Also, the cost of 41.6 pounds of K, the fixed factor, is accounted for.

**Table B-7. Equations of the short-run static supply functions for corn when N is variable, P is fixed at different levels and K is held constant at 41.6 pounds per acre, 1959 experiments.**

| Situation number | Level of fixed factor, P (pounds per acre) | Supply equation |
|---|---|---|
| 1 | 0 | $Y = 100.833 - 3.110\ P_c^{-2}$ |
| 2 | 20 | $Y = 102.218 - 3.110\ P_c^{-2}$ |
| 3 | 40 | $Y = 103.324 - 3.110\ P_c^{-2}$ |
| 4 | 60 | $Y = 104.180 - 3.110\ P_c^{-2}$ |
| 5 | 80 | $Y = 104.777 - 3.110\ P_c^{-2}$ |

**Table B-8. Equations of the short-run static supply functions for corn when P is variable, N is fixed at different levels and K is held constant at 41.6 pounds per acre, 1959 experiments.**

| Situation number | Level of fixed factor, N (pounds per acre) | Supply equation |
|---|---|---|
| 1 | 0 | $Y = 91.090 - 38.557P_c^{-2}$ |
| 2 | 40 | $Y = 99.544 - 38.557P_c^{-2}$ |
| 3 | 80 | $Y = 104.199 - 38.557P_c^{-2}$ |
| 4 | 120 | $Y = 105.056 - 38.557P_c^{-2}$ |
| 5 | 160 | $Y = 102.115 - 38.557P_c^{-2}$ |

**Table B-9. Equations of the short-run static demand functions for N when P is fixed at various levels and K is held constant at 41.6 pounds per acre, 1959 experiments.**

| Situation number | Level of fixed factor, P (pounds per acre) | Demand equation |
|---|---|---|
| 1 | 0 | $N = 123.260 - 398.089P_n$ |
| 2 | 20 | $N = 120.808 - 398.089P_n$ |
| 3 | 40 | $N = 118.356 - 398.089P_n$ |
| 4 | 60 | $N = 115.904 - 398.089P_n$ |
| 5 | 80 | $N = 113.455 - 398.089P_n$ |

**Table B-10. Equations of the short-run static demand functions for P when N is fixed at various levels and K is held constant at 41.6 pounds per acre, 1959 experiments.**

| Situation number | Level of fixed factor, N (pounds per acre) | Demand equation |
|---|---|---|
| 1 | 0 | $P = 165.019 - 1457.772P_p$ |
| 2 | 40 | $P = 147.060 - 1457.772P_p$ |
| 3 | 80 | $P = 129.100 - 1457.772P_p$ |
| 4 | 120 | $P = 111.141 - 1457.772P_p$ |
| 5 | 160 | $P = 93.182 - 1457.772P_p$ |

**Table B-11. Isoquants and "gross" marginal rates of substitution (MRS) of fertilizer (F = N + P) for land (A) when r, the ratio of P to N, is zero and K is held constant at 41.6 pounds per acre, 1959 experiments.**

| F (pounds per acre) | 95 bushels A | 95 bushels MRS | 100 bushels A | 100 bushels MRS |
|---|---|---|---|---|
| 0 | 1.1621 | −0.003788 | 1.2232 | −0.003788 |
| 10 | 1.1256 | −0.003510 | 1.1867 | −0.003521 |
| 20 | 1.0920 | −0.003208 | 1.1528 | −0.003239 |
| 40 | 1.0343 | −0.002541 | 1.0942 | −0.002611 |
| 60 | 0.9906 | −0.001824 | 1.0487 | −0.001932 |
| 80 | 0.9614 | −0.001112 | 1.0169 | −0.001251 |
| 100 | 0.9458 | −0.000460 | 0.9984 | −0.000615 |
| 120 | 0.9424 | .............. | 0.9918 | −0.000057 |

**Table B-12. Isoquants and "gross" marginal rates of substitution (MRS) of fertilizer (F = N + P) for land (A) when r, the ratio P to N, is 1/4 and K is held constant at 41.6 pounds per acre, 1959 experiments.**

| F (pounds per acre) | 95 bushels A | 95 bushels MRS | 100 bushels A | 100 bushels MRS |
|---|---|---|---|---|
| 0 | 1.1621 | −0.003307 | 1.2232 | −0.003307 |
| 10 | 1.1299 | −0.003177 | 1.1911 | −0.003126 |
| 20 | 1.0998 | −0.002911 | 1.1600 | −0.002932 |
| 40 | 1.0460 | −0.002458 | 1.1063 | −0.002505 |
| 60 | 1.0018 | −0.001962 | 1.0608 | −0.002038 |
| 80 | 0.9677 | −0.001448 | 1.0249 | −0.001557 |
| 100 | 0.9438 | −0.000947 | 0.9987 | −0.001069 |
| 120 | 0.9295 | −0.000483 | 0.9820 | −0.000617 |

**Table B-13. Isoquants and "gross" marginal rates of substitution (MRS) of fertilizer (F = N + P) for land (A) when r, the ratio of P to N, is 1/2 and K is held constant at 41.6 pounds per acre, 1959 experiments.**

| F (pounds per acre) | 95 bushels A | 95 bushels MRS | 100 bushels A | 100 bushels MRS |
|---|---|---|---|---|
| 0 | 1.1621 | −0.002987 | 1.2232 | −0.002987 |
| 10 | 1.1329 | −0.002841 | 1.1940 | −0.002849 |
| 20 | 1.1053 | −0.002685 | 1.1663 | −0.002701 |
| 40 | 1.0550 | −0.002342 | 1.1154 | −0.002378 |
| 60 | 1.0118 | −0.001966 | 1.1012 | −0.002051 |
| 80 | 0.9765 | −0.001568 | 1.0346 | −0.001648 |
| 100 | 0.9491 | −0.001167 | 1.0012 | −0.001250 |
| 120 | 0.9297 | −0.000781 | 0.9839 | −0.000984 |

**Table B-14. Values of t for the coefficients of the production function from 1960 experiments.**

| Coefficient | Value of t | Probability level[a] |
|---|---|---|
| N | 6.74 | 0.001 |
| P | 2.31 | 0.04 |
| $N^2$ | 5.00 | 0.001 |
| $P^2$ | 2.22 | 0.05 |
| NP | 2.53 | 0.03 |

[a]Probability of drawing a t value as large or larger by chance, given the null hypothesis.

**Table B-15. Analysis of variance, 1960 experiments.**

| Source of variation | Degrees of freedom | Sum of squares | Mean squares | F[a] |
|---|---|---|---|---|
| Total | 22 | 1,588.97 | | |
| Due to regression | 9 | 1,524.32 | 169.37 | 34.08 |
| Deviation from regression | 13 | 64.65 | 4.97 | |

[a]The F value is highly significant.

**Table B-16. Marginal physical products of N at different levels of P when K is held constant at zero, 1960 experiments.**

| Pounds of P per acre | Pounds of N per acre: 0 | 40 | 80 | 120 | 160 |
|---|---|---|---|---|---|
| 0 | 0.2526 | 0.1675 | 0.0825 | −0.0026 | −0.0876 |
| 17.4 | 0.2647 | 0.1796 | 0.0946 | 0.0095 | −0.0755 |
| 34.8 | 0.2768 | 0.1917 | 0.1067 | 0.0216 | −0.0634 |
| 52.2 | 0.2889 | 0.2038 | 0.1188 | 0.0338 | −0.0513 |
| 69.6 | 0.3010 | 0.2160 | 0.1309 | 0.0459 | −0.0392 |

**Table B-17. Marginal physical products of P at different levels of N, when K is held constant at zero, 1960 experiments.**

| Pounds of N per acre | Pounds of P per acre: 0 | 17.4 | 34.8 | 52.2 | 69.6 |
|---|---|---|---|---|---|
| 0 | 0.1991 | 0.1123 | 0.0254 | −0.0614 | −0.1483 |
| 40 | 0.2269 | 0.1401 | 0.0533 | −0.0336 | −0.1204 |
| 80 | 0.2548 | 0.1680 | 0.0811 | −0.0057 | −0.0925 |
| 120 | 0.2826 | 0.1958 | 0.1090 | 0.0221 | −0.0647 |
| 160 | 0.3104 | 0.2236 | 0.1368 | 0.0500 | −0.0369 |

**Table B-18. Combinations of P and nitrogen required to produce a given yield of corn and corresponding marginal rates of substitution (MRS) when K is held constant at zero, 1960 experiments.**

| 85 bushels: Pounds of P | Pounds of N | MRS ($\partial N/\partial P$) | 90 bushels: Pounds of P | Pounds of N | MRS ($\partial N/\partial P$) |
|---|---|---|---|---|---|
| 0 | 51.06 | 1.6277 | 10 | 74.00 | 1.9640 |
| 10 | 37.72 | 0.9784 | 20 | 59.85 | 1.0123 |
| 20 | 29.84 | 0.5914 | 30 | 52.32 | 0.5291 |
| 30 | 25.43 | 0.3059 | 40 | 48.80 | 0.1895 |
| 40 | 23.92 | 0.0703 | | | |

**Table B-19. Input combinations that maximize profits for various N, P and corn prices when K is held constant at zero, 1960 experiments.**

| Situation number | Price per unit: Corn per bushel | N per pound | P per pound | Optimum inputs in pounds per acre: N | P | Predicted corn yield in bushels per acre | Profit from use of optimum inputs[a] |
|---|---|---|---|---|---|---|---|
| 1 | $1.60 | $0.17 | $0.28 | 73.77 | 15.12 | 90.94 | $ 8.92 |
| 2 | 1.60 | 0.17 | 0.23 | 75.91 | 21.68 | 92.21 | 9.84 |
| 3 | 1.60 | 0.17 | 0.18 | 78.06 | 28.24 | 93.28 | 11.09 |
| 4 | 1.60 | 0.17 | 0.13 | 80.21 | 34.80 | 94.14 | 12.67 |
| 5 | 1.30 | 0.17 | 0.28 | 58.90 | 4.95 | 87.19 | 4.61 |
| 6 | 1.30 | 0.17 | 0.23 | 61.55 | 13.03 | 89.12 | 5.06 |
| 7 | 1.30 | 0.17 | 0.18 | 64.19 | 21.10 | 90.74 | 5.91 |
| 8 | 1.30 | 0.17 | 0.13 | 66.84 | 29.18 | 92.05 | 7.17 |
| 9 | 1.00 | 0.17 | 0.18 | 42.00 | 9.68 | 85.58 | 1.83 |
| 10 | 1.00 | 0.17 | 0.13 | 45.44 | 20.18 | 87.80 | 2.57 |
| 11 | 1.60 | 0.15 | 0.28 | 79.93 | 15.98 | 91.70 | 10.46 |
| 12 | 1.60 | 0.15 | 0.23 | 82.08 | 22.54 | 92.95 | 11.42 |
| 13 | 1.60 | 0.15 | 0.18 | 84.22 | 29.10 | 93.99 | 12.72 |
| 14 | 1.60 | 0.15 | 0.13 | 86.37 | 35.66 | 94.83 | 14.34 |
| 15 | 1.30 | 0.15 | 0.28 | 66.49 | 6.01 | 88.35 | 5.86 |
| 16 | 1.30 | 0.15 | 0.23 | 69.13 | 14.09 | 90.24 | 6.37 |
| 17 | 1.30 | 0.15 | 0.18 | 71.78 | 22.16 | 91.82 | 7.27 |
| 18 | 1.30 | 0.15 | 0.13 | 74.42 | 30.24 | 93.09 | 8.58 |
| 19 | 1.00 | 0.15 | 0.23 | 48.42 | 0.56 | 84.74 | 2.47 |
| 20 | 1.00 | 0.15 | 0.18 | 51.86 | 11.06 | 87.41 | 2.76 |
| 21 | 1.00 | 0.15 | 0.13 | 55.30 | 21.56 | 89.55 | 3.58 |
| 22 | 1.60 | 0.13 | 0.28 | 86.09 | 16.84 | 92.39 | 12.12 |
| 23 | 1.60 | 0.13 | 0.23 | 88.24 | 23.40 | 93.61 | 13.13 |
| 24 | 1.60 | 0.13 | 0.18 | 90.38 | 29.96 | 94.63 | 14.46 |
| 25 | 1.60 | 0.13 | 0.13 | 92.53 | 36.52 | 95.44 | 16.12 |
| 26 | 1.30 | 0.13 | 0.28 | 74.07 | 7.07 | 89.40 | 7.27 |
| 27 | 1.30 | 0.13 | 0.23 | 76.71 | 15.14 | 91.24 | 7.82 |
| 28 | 1.30 | 0.13 | 0.18 | 79.36 | 23.22 | 92.78 | 8.78 |
| 29 | 1.30 | 0.13 | 0.13 | 82.00 | 31.30 | 94.01 | 10.15 |
| 30 | 1.00 | 0.13 | 0.23 | 58.28 | 1.94 | 86.44 | 3.54 |
| 31 | 1.00 | 0.13 | 0.18 | 61.72 | 12.44 | 89.04 | 3.90 |
| 32 | 1.00 | 0.13 | 0.13 | 65.15 | 22.94 | 91.11 | 4.78 |
| 33 | 1.60 | 0.11 | 0.28 | 92.25 | 17.70 | 93.00 | 13.90 |
| 34 | 1.60 | 0.11 | 0.23 | 94.40 | 24.26 | 94.20 | 14.95 |
| 35 | 1.60 | 0.11 | 0.18 | 96.54 | 30.82 | 95.19 | 16.33 |
| 36 | 1.60 | 0.11 | 0.13 | 98.69 | 37.38 | 95.97 | 18.04 |
| 37 | 1.30 | 0.11 | 0.28 | 81.65 | 8.12 | 90.32 | 8.83 |
| 38 | 1.30 | 0.11 | 0.23 | 84.30 | 16.20 | 92.13 | 9.44 |
| 39 | 1.30 | 0.11 | 0.18 | 86.94 | 24.28 | 93.63 | 10.45 |
| 40 | 1.30 | 0.11 | 0.13 | 89.58 | 32.35 | 94.82 | 11.86 |
| 41 | 1.00 | 0.11 | 0.23 | 68.14 | 3.31 | 87.94 | 4.80 |
| 42 | 1.00 | 0.11 | 0.18 | 71.57 | 13.81 | 90.47 | 5.23 |
| 43 | 1.00 | 0.11 | 0.13 | 75.01 | 24.31 | 92.47 | 6.19 |

[a] Profit is computed for the increase in yield over 74.88 bushels per acre.

**Table B-20. Equations of the short-run static supply functions for corn when N is variable, P is fixed at different levels and K is held constant at zero, 1960 experiments.**

| Situation number | Level of fixed factor, P (pounds per acre) | Supply equation |
|---|---|---|
| 1 | 0 | $Y = 89.876 - 3.675\ P_c^{-2}$ |
| 2 | 20 | $Y = 94.560 - 3.675\ P_c^{-2}$ |
| 3 | 40 | $Y = 97.338 - 3.675\ P_c^{-2}$ |
| 4 | 60 | $Y = 98.211 - 3.675\ P_c^{-2}$ |
| 5 | 80 | $Y = 97.180 - 3.675\ P_c^{-2}$ |

**Table B-21. Equations of the short-run static supply functions for corn when P is variable, N is fixed at different levels and K is held constant at zero, 1960 experiments.**

| Situation number | Level of fixed factor, N (pounds per acre) | Supply equation |
|---|---|---|
| 1 | 0 | $Y = 78.847 - 5.301\ P_c^{-2}$ |
| 2 | 40 | $Y = 88.437 - 5.301\ P_c^{-2}$ |
| 3 | 80 | $Y = 94.781 - 5.301\ P_c^{-2}$ |
| 4 | 120 | $Y = 97.878 - 5.301\ P_c^{-2}$ |
| 5 | 160 | $Y = 97.729 - 5.301\ P_c^{-2}$ |

**Table B-22. Equations of the short-run static demand functions for N when P is fixed at various levels and K is held constant at zero, 1960 experiments.**

| Situation number | Level of fixed factor, P (pounds per acre) | Demand equations |
|---|---|---|
| 1 | 0 | $N = 118.793 - 470.367P_n$ |
| 2 | 20 | $N = 125.341 - 470.367P_n$ |
| 3 | 40 | $N = 131.888 - 470.367P_n$ |
| 4 | 60 | $N = 138.436 - 470.367P_n$ |
| 5 | 80 | $N = 144.983 - 470.367P_n$ |

**Table B-23. Equations of the short-run static demand functions for P when N is fixed at various levels and K is held constant at zero, 1960 experiments.**

| Situation number | Level of fixed factor, N (pounds per acre) | Demand equations |
|---|---|---|
| 1 | 0 | $P = 39.900 - 200.401P_p$ |
| 2 | 40 | $P = 45.479 - 200.401P_p$ |
| 3 | 80 | $P = 51.058 - 200.401P_p$ |
| 4 | 120 | $P = 56.637 - 200.401P_p$ |
| 5 | 160 | $P = 62.216 - 200.401P_p$ |

**Table B-24. Isoquants and "gross" marginal rates of substitution (MRS) of fertilizer (F = N + P) for land (A) when r, the ratio of P to N, is 1/4 and K is held constant at zero, 1960 experiments.**

| F (pounds per acre) | 85 bushels: A | MRS | 67 bushels: A | MRS |
|---|---|---|---|---|
| 0 | 1.1352 | −0.003230 | 1.2020 | −0.003230 |
| 10 | 1.1037 | −0.003066 | 1.1704 | −0.003076 |
| 20 | 1.0739 | −0.002888 | 1.1405 | −0.002909 |
| 40 | 1.0200 | −0.002495 | 1.0860 | −0.002541 |
| 60 | 0.9744 | −0.002060 | 1.0391 | −0.002131 |
| 80 | 0.9378 | −0.001602 | 1.0007 | −0.001704 |
| 100 | 0.9103 | −0.001144 | 0.9710 | −0.001270 |
| 120 | 0.8919 | −0.000711 | 0.9498 | −0.000851 |

Table B-25. Isoquants and "gross" marginal rates of substitution (MRS) of fertilizer (F = N + P) for land (A) when r, the ratio of P to N, is 1/3 and K is held constant at zero, 1960 experiments.

| F (pounds per acre) | 85 bushels A | 85 bushels MRS | 90 bushels A | 90 bushels MRS |
|---|---|---|---|---|
| 0 | 1.1352 | −0.003194 | 1.2020 | −0.003194 |
| 10 | 1.1040 | −0.003042 | 1.1708 | −0.003050 |
| 20 | 1.0744 | −0.002876 | 1.1410 | −0.002895 |
| 40 | 1.0205 | −0.002510 | 1.0865 | −0.002553 |
| 60 | 0.9743 | −0.002103 | 1.0392 | −0.002173 |
| 80 | 0.9365 | −0.001671 | 0.9997 | −0.001769 |
| 100 | 0.9074 | −0.001236 | 0.9684 | −0.001356 |
| 120 | 0.8870 | −0.000818 | 0.9439 | −0.001049 |

Table B-26. Isoquants and "gross" marginal rates of substitution (MRS) of fertilizer (F = N + P) for land (A) when r, the ratio of P to N, to nitrogen is 1/2 and K is held constant at zero, 1960 experiments.

| F (pounds per acre) | 85 bushels A | 85 bushels MRS | 90 bushels A | 90 bushels MRS |
|---|---|---|---|---|
| 0 | 1.1352 | −0.003135 | 1.2020 | −0.003135 |
| 10 | 1.1046 | −0.002989 | 1.1713 | −0.002997 |
| 20 | 1.0755 | −0.002832 | 1.1421 | −0.002850 |
| 40 | 1.0222 | −0.002483 | 1.0883 | −0.002524 |
| 60 | 0.9764 | −0.002096 | 1.0414 | −0.002162 |
| 80 | 0.9386 | −0.001683 | 1.0020 | −0.001776 |
| 100 | 0.9091 | −0.001265 | 0.9794 | −0.001381 |
| 120 | 0.8879 | −0.000862 | 0.9467 | −0.000994 |