Milk Production Functions in Relation to Feed Inputs, Cow Characteristics and Environmental Conditions

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SUMMARY

The main emphasis of this study is on milk production functions as they relate to various levels and proportions of grain and hay in a ration restricted to these two feeds. However, certain auxiliary variables representing cow characteristics and environmental conditions are also incorporated into the production function. The auxiliary variables include: stage of lactation, milk-producing ability, temperature, age, maturity, body weight and a coefficient of inbreeding. These auxiliary variables are fixed at different levels to allow expression of milk production functions when the characteristics of cows are set at different magnitudes.

This study considers the economic optima in ration specification for dairy cows and represents a cooperative interdisciplinary effort. The estimates allow derivation of production surfaces, milk isoclines and isoquants, marginal rates of feed substitution and profit-maximizing rations for cows of different characteristics (maturity, ability, inbreeding, weight) producing under specified environmental conditions.

A total of 450 regressions was computed in estimating milk production functions in relation to feed inputs and auxiliary variables. The main regression equation selected for the analysis is a quadratic form with 26 terms, based on weekly observations of feed input and milk output for 72 lactations of Holstein cows. In this regression equation, 21 of the regression coefficients were significant at the 0.01 level of probability, five at the 0.05 level of probability, and one at the 0.10 level of probability. The value of R^2 was 0.836. Such variables as age, maturity and coefficient of inbreeding can be incorporated successfully into the production function. Future research, therefore, may be able to integrate the scientific aspects of dairy cattle breeding and nutrition along with other environmental characteristics into a single mathematical formulation of milk production, animal productivity and feed evaluation.

The isoquants and isoclines computed show the substitution of grain and forage to be at diminishing marginal rates. Consequently, the optimum ration varies with the prices of the two types of feed. Similarly, since the isoclines derived are not linear through the origin of the feed plane, the optimum ration also changes as the price of milk increases or decreases and as feed prices are constant relative to each other. Selected cow characteristics also are predicted to substitute for feed and for each other at diminishing marginal rates in attaining a given level of milk production. Although the step is not taken in this study, these measurements allow quantification of the relative economic importance in increasing the milk output per cow through improved breeding, alternative rations and other adjustments.

Confidence regions for several representative quantities were estimated. In general, the magnitude of the confidence regions indicates that some input-output relationships and quantities can be predicted with a fairly high degree of precision, whereas others can be predicted only within wide confidence bounds. The use of additional observations and of more refined techniques in the experimental design and analysis can further reduce the sizes of the confidence regions, thus increasing the precision and accuracy of the input-output relationships computed from the estimated milk production function.

Milk Production Functions in Relation to Feed Inputs, Cow Characteristics and Environmental Conditions[1]

by Earl O. Heady, N. L. Jacobson, J. Patrick Madden and A. E. Freeman

This study provides estimates of milk production functions as they relate to levels and proportions of grain and hay (forage) feeding. The over-all purposes of this study were to develop certain mathematical concepts relative to dairy-cow nutrition, to provide estimates of marginal rates of substitution among feeds and to determine other relationships basic to the evaluation of feeds and feeding standards. It had the auxiliary objectives of relating these milk-feed relationships to certain cow and environmental characteristics.

This is the second of a series of studies that consider the economic optima in ration specification for dairy cows.[2] Details on the basic concepts and on the alternatives involved in the practical application of such concepts have been provided in earlier reports and will not be repeated here.[3]

The basic data for this study came from two experiments conducted by dairy scientists in the Department of Animal Science, Iowa State University. Certain nutritional, physiological and economic aspects of these experiments were analyzed by Bloom,[4] Hotchkiss[5] and Madden.[6] Both experiments represent an interdisciplinary analysis, and they both have the same general design and purpose. Three levels of feeding were used in each of four forage-concentrate ratios that included 15, 35, 55 and 75 percent of estimated net energy intake from alfalfa hay. This design provided a spacing of points under the production surface, as suggested in the experimental details presented later.

The economic aspects of milk production are complex, involving many resources or variables. Feeds represent only one class of inputs. Another class includes labor, management, land, buildings, machinery and other forms of capital. These resources are assumed "fixed" and are not measured in this study.

Several other variables—including cow characteristics, temperature and stage of lactation—also affect milk production. These variables were treated as stochastic, or random, disturbances in the initial experimental design, but, in the current study, methods were devised for measuring several of these variables and incorporating them into the production function as auxiliary independent variables. The variables so treated include: stage of lactation, maturity, age, inbreeding, body weight, milk-producing ability and temperature. However, the main purpose of this study was the estimation of the milk production function, particularly as it related to forage and grain consumption.

Numerous algebraic equations (including quadratic, square root, linear, power and exponential) were used in estimating milk production functions. Each algebraic form represents a model with specific assumptions. In this study, the quadratic form, with weekly observations, is used for the predictions. The physiological and economic implications of this model are discussed later, along with the role of auxiliary variables in modifying the height and slope of the milk production surface.

Isoquants, isoclines, marginal rates of substitution, marginal physical products, ridge lines and other relevant quantities of production surfaces have been predicted. These derived quantities are presented along with the least-cost rations and profit-maximizing levels of production for the numerous combinations of feed and milk prices. The confidence regions of the various derived quantities also are presented.

Since several auxiliary variables appear in the "best" milk production function, it is possible to approximate various production conditions by "setting" these variables at different levels. By systematically changing the stage-of-lactation variable, it is possible to examine the derived quantities at different stages of the lactation. By a similar alteration of the temperature variable, it is possible to predict how the derived quantities might change over seasons of the year. By setting the cow-characteristic variables at different levels, we can derive production functions for cows of varying maturity, ability, inbreeding and weight.

[1] Project 1135 of the Iowa Agricultural and Home Economics Experiment Station.

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[2] For the first analysis, see: Earl O. Heady, N. L. Jacobson, J. L. Schnittker and Solomon Bloom. Milk production functions, hay/grain substitution rates and economic optima in dairy cow rations. Iowa Agr. Exp. Sta. Res. Bul. 444. 1956. For other details on the two experiments, see: Solomon Bloom. Effects of various dietary hay-concentrate ratios on nutrient utilization and production responses of dairy cows. Unpublished Ph.D. thesis. Iowa State University Library. Ames, Iowa. 1955; S. Bloom, N. L. Jacobson, L. D. McGilliard, P. G. Homeyer, and E. O. Heady. Effects of various hay-concentrate ratios on nutrient utilization and production responses of dairy cows. I. Relationships among feeding level, predicted producing ability, and milk production. Jour. Dairy Sci. 40:81-94. 1957; Donald Keith Hotchkiss. Effect of various dietaries on milk composition and efficiency of production of dairy cows. Unpublished Ph.D. thesis. Iowa State University Library. Ames, Iowa. 1960.

[3] Heady, Jacobson, Schnittker and Bloom, op. cit.

[4] Bloom, op. cit.

[5] Hotchkiss, op. cit.

[6] J. Patrick Madden. Multiple variable milk production functions with point and interval estimates of derived quantities. Unpublished Ph.D. thesis. Iowa State University Library. Ames, Iowa. 1962.

EXPERIMENTS AND DATA

Initial Experiment

The first experiment, conducted from March 1953 to September 1954, included 36 Holstein cows.[7] Cows were started on the experiment according to a predetermined schedule. Immediately after calving and for a 14-day adjustment period, cows were fed alfalfa hay and grain in a ratio of 7:4. By the end of the second week, each cow had adjusted to a full feeding level. This ratio and level of feeding was continued for each cow for the next 50 days (the preliminary period). Production during the preliminary period was used for extrapolating to an 8-month lactation, with the latter providing the basis for dividing cows into groups of high-, medium- and low-producing ability. Ability ranges for the cows, in terms of 4-percent fat-corrected milk (FCM) over the extrapolated 8-month production period, were as follows: high = 10,500 pounds and more; medium = 9,000 to 10,499 pounds; and low = less than 9,000 pounds.

Four hay-grain ratios were fed: 15, 35, 55 and 75 percent of estimated net energy intake from alfalfa hay. Each of the four ratios was fed at three levels, and one cow from each of the three groups (high-, medium- and low-producing ability) was randomly assigned to each treatment. For each ratio, cows were fed, not according to milk produced, but at three fixed levels arbitrarily based on the amount of feed energy required to produce 13,000, 11,000, and 9,000 pounds of 4-percent FCM. The daily quantity of feed was reduced as the lactation period progressed, but the hay-grain ratios were maintained. The experiments were conducted in a stall barn.

Milk production of each cow was recorded at each milking. Morning and afternoon milk samples, collected every week, were composited for each cow to compute the level of FCM for each week. Several first-calf heifers were used in this first experiment.

The daily weights of hay and grain (concentrate mixture) fed to each cow were recorded. Feed refused was weighed back. Weekly hay and grain consumption was then computed from these daily observations.

Freshening dates of cows extended over several months in the first experiment. Hence, some cows were observed during hot summer months, whereas others were observed during the cooler months. It was believed that the heat of the summer months adversely affected the milk production of some of the cows, especially those on high hay rations.

Second Experiment

The second experiment extended from October 1956 through April 1959. In many respects, the design was identical to that of the first experiment. The same four ratios and three levels of feeding were used. Measurement of feed input and milk output also was the same. In contrast to the first experiment, allocation by production ability was not used. Since this procedural difference was considered negligible, however, predictions were made by pooling the observations from both experiments.

The second experiment included 36 lactations from Holstein cows. Although some of the animals were used for 2 or more years, they were never in the same position of the design more than once.

One complete replication of the 12 ratio-level treatments was attempted during each of the 3 years of the second experiment. One cow was dropped from the experiment late in the second year because of prolonged illness. Therefore, during the following year, two cows were allotted to this ratio-level position. Thus, three complete replications were observed, as in the first experiment, for this treatment.

Other differences between the two experiments relate to: restrictions placed upon starting dates, length of preliminary and experimental periods, age of cows and contents of the concentrate mixtures. Modifications of starting dates and lengths of the preliminary and experimental periods were adopted in the second experiment to avoid the high temperatures that prevailed during part of the first experiment. As far as possible, only cows freshening during the fall were used. As a general management policy, a relatively small portion of the herd freshens in the fall; therefore, it was necessary to extend the experiment over a 3-year period, with a third of the cows observed each year.

The length of the experimental period was only 12 weeks in the second experiment, compared with 26 weeks in the first. Using a 12-week period made it possible to complete the experiment each year by spring or early summer, before environmental temperature became extremely high. In the analysis that follows, the parallel observations for a 12-week period are used from the first experiment. The rations, methods and timing used in the adjustment and preliminary periods were the same as those of the first experiment, except for the preliminary period which extended for only 49 days instead of 50 days.

Another important difference between the two experiments was the age of cows. The first experiment included five first-calf heifers (which use a substantial amount of the energy consumed for body growth), whereas the second experiment excluded them. Since maturity is used as a variable in the prediction model, observations on first-calf heifers and other immature cows from the first experiment are not expected to confound results.

Feeds used in the experiments are compared in tables 1 and 2 (page 189). Both experiments included good-quality, second-cutting alfalfa hay but differed somewhat in concentrate mixtures. The concentrate mixture used in the second experiment contained a larger proportion of corn and wheat bran, a smaller proportion of oats and no linseed oil. These small dif-

[7] Bloom, Jacobson, McGilliard, Homeyer and Heady, op. cit.

ferences in the concentrates were not expected to confound the hay-grain substitution rates.

Variables for Equations

Estimated regression equations include hay and grain consumption and milk production; each week of the 12-week experimental period is used as a separate observation or measurement of milk output and feed input. The feed and milk variables, with measurement after initiation of the experimental period, are defined as follows:

H: alfalfa hay, measured as pounds consumed by a cow during 1 week.

G: grain, as explained in table 2 and measured as pounds consumed by a cow during 1 week.

M: milk, measured as pounds of 4-percent FCM produced by a cow during 1 week.

Values also were formed for 4-week intervals or the entire 12 weeks by summing consecutive weekly observations for the specified length of time. Numerous equations were estimated with feed and milk measured for 4 and 12 weeks and are reported by Madden.[8]

The auxiliary variables are defined as follows:

T: stage of lactation, measured as the ordinal number of the week, with T = 1 for the first experimental week.

A: index of ability, measured as total 4-percent FCM produced during the 50-day preliminary period.[9]

K: coefficient of inbreeding, [10] measured in percentage. (Cows with unrelated parents for many past generations have an inbreeding percentage of zero.)

W: body weight, measured in pounds at the beginning of the experimental period.

F: outside temperature, measured weekly to correspond with weekly input-output data and computed as the arithmetic mean of daily high temperature readings, in degrees Fahrenheit, as recorded at the Iowa State University Agronomy Farm.[11] High temperatures were used, since evidence indicates that feed consumption is reduced during severely high temperatures.[12]

J: index of maturity, measured in months from time of birth but with an upper value of J = 66 for mature cows. The maturity index is truncated at 66 months, because Holstein population studies[13] indicate that cows mature at about that age, with milk production approaching a plateau or a mathematical limit.

Data Limitations

Since the experiments were restricted in funds, cows, barn space and other facilities, the study has the following limitations: (1) A limited range of types and qualities of feeds was considered. The rations do not include an extremely high or low proportion of hay. (Analysis of different types of forage and concentrates, as well as pasture feeding, is beyond the scope of the experiments.) (2) The experimental data were obtained from a select group of Holstein cows. (3) The number of observations is too small for great precision in estimating the milk production function, as reflected by the confidence regions about the derived quantities. (4) Each cow was kept on a fixed ration throughout the experimental period; therefore, the experiments provide no information about the effects of changing rations. (5) Temperatures were measured outside, whereas barn temperatures would have been more appropriate.

These limitations can be overcome as additional resources and facilities become available for more extensive research. However, the results derived from the present data increase the knowledge available on milk production functions.

WEEKLY MILK PRODUCTION FUNCTIONS FOR 72 HOLSTEIN COWS

Equations

Several different algebraic forms were used in estimating the milk production function. The quadratic form was used for deriving the relevant physical and economic quantities that follow. Other forms used included Cobb-Douglas, Spillman, linear and square root. A total of 450 regression equations was computed. Each equation was evaluated according to conventional statistical criteria. Only a few of the equations are presented in this report. The equation that appeared most satisfactory, on the basis of several criteria, is equation 1 which refers to weekly feed input and milk output.

$$(1)\quad M = 248.42 + 1.8358\,G + 1.4117\,H - 0.00505\,G^2 - 0.00109\,H^2 - 0.00352\,GH - 0.00557\,GT + 0.00069\,WG - 0.00015\,HA + 0.0749\,A + 1.0060\,F + 3.1619\,J - 5.4269\,K + 0.3694\,W + 0.09091\,T^2 - 0.00398\,F^2 + 15.3569\,K^{1/2} - 27.0461\,W^{1/2} - 0.00164\,AT - 0.00023\,AF + 0.00065\,WF - 0.00187\,WJ + 0.00164\,KA + 0.03865\,KT - 0.02967\,KF - 0.03864\,JT - 0.01454\,JF.$$

In general, the sign of each term in equation 1 is consistent with established principles and facts in dairy nutrition, animal breeding and production economics, even though the magnitude of the coefficient may differ from other studies. This difference in magnitude is not

[8] Madden, op. cit.

[9] The two experiments had different lengths of preliminary periods: 50 days in the first, 49 days in the second. To put these on a comparable basis, a correction factor was devised. One-seventh of the milk produced during the final week of the preliminary period (days 43 through 49) was added to the 49-day total obtained in the second experiment. This gives an ability index based on a 50-day preliminary period.

[10] S. Wright. Coefficients of inbreeding and relationships. Amer. Nat. 56:330-338. 1922.

[11] U. S. Department of Commerce, Weather Bureau. Climatological data, Iowa. Vols. 64-70. 1953-59.

[12] Harold D. Johnson, A. C. Ragsdale and Chu Shan Cheng. Comparison of the effects of environmental temperature on rabbits and cattle. Mo. Agr. Exp. Sta. Res. Bul. 646. 1957.

[13] Jay L. Lush and Robert R. Shrode. Changes in milk production with age and milking frequency. Jour. Dairy Sci. 33: 338-357. 1950.

surprising, particularly for the auxiliary variables, because of the relatively small amount of data available for these characteristics. The 27 t values for the regression coefficients and the constant are included in table 3, along with their corresponding regression coefficients. Of these coefficients, 21 are acceptable at the 0.01 level of probability, 19 at the 0.001 level of probability, and 5 at the 0.05 level. The one remaining coefficient, with a t value of 1.8, is significantly different from zero at the 0.1 level of probability. As shown in table 3, equation 1 explains 83.6 percent of the variance in milk production for the pooled weekly observations from the 72 Holstein lactations involved in the two experiments.

Other equations predicted include 2 and 3, with basic data presented in table 3. Regression coefficients, t values and R^2 for each of these equations are included for comparative purposes with equation 1. Equations 2 and 3 were derived as intermediate steps in obtaining equation 1. Equations 2 and 3 were first thought most satisfactory for predictions; however, each was subsequently dropped as additional variables were included in the regression equations. Initially, only the auxiliary variables A, T and F were used to predict several simple quadratic equations. A set of regressions was then computed with equation 2 as the basic equation and with J and K as the additional auxiliary variables. Linear and square-root terms were included for each of these new variables. Terms denoting interaction between K and each other variable were formed. A similar set of interaction terms with J also was formed. Many other individual terms and combinations of terms were compared in the same manner.

Using equation 2 as the basic equation, we computed another set of regressions with W included as an additional auxiliary variable. Linear, squared and square-root weight terms were included in this set of regressions, along with interaction terms G, H, F and J. Again statistical criteria were used in evaluating different equations. Regression coefficients, t values and R^2 computed for three trial equations 4, 5 and 6 are presented in table 4.

Equations 4 and 5 provide a comparison of the two terms, $W^{1/2}$ and W^2. Coefficients of both equations are acceptable, in signs and significance, except for GJ. At the 0.15 level of probability, the sign of GJ is not significantly different from zero in either equation (hence, this term was excluded from subsequent equations). Equation 5 has a higher R^2 than equation 4, suggesting that $W^{1/2}$ is superior to W^2 in explaining variance in milk production.

Equation 6 differs from equation 5 in two respects: (a) the GJ term was deleted because it was not significantly different from zero in previous equations, and (b) the HW term was substituted for HA so that these two parallel terms could be compared. When the t values for these two terms are compared in their respective equations, we find that HA, rather than HW, should be included in the milk production function.

Equation 1, formed by exchanging HA for HW in equation 6, appears to be the most satisfactory equation using all the auxiliary variables. Figure 1 (figures are grouped on pages 182-188) shows the milk production surface predicted from equation 1 when T = 11 and the other auxiliary variables are set at their mean levels. In terms of grain and hay alone, the production function upon which fig. 1 is computed is:

$$(7)\quad M = -25.9304 + 2.5563\,G + 1.0465\,H - 0.005047\,G^2 - 0.001088\,H^2 - 0.003521\,GH.$$

Data for selected isoquants from equation 1 are presented in table 5. Tables 6 and 7 provide parallel data for equations 2 and 3. Tables 8, 9 and 10 contain data for surfaces represented, respectively, by equations 1, 2 and 3. In each table, the marginal physical products and marginal rate of substitution are indicated for the feed points listed. Some of the production surface and isoproduct quantities are derived for feed combinations falling outside the range of the experiment, but are included for illustrative purposes. (Further experimentation is needed to better define the slope of relationships falling outside the range of the experiments reported.)

Marginal Quantities

The derived marginal quantities show diminishing productivity of feed as a given diet is fed at a higher level. They also show a decline in the rate at which one feed replaces the other as the ration is changed but as the milk output is retained at a given level. In table 5, for example, the level of grain and the corresponding levels of hay shown for a given milk output are the data representing a milk isoquant. These data indicate the various combinations of feeds predicted to produce the given milk level. Combinations such as 60 pounds of grain and 131 pounds of hay, or 90 pounds of grain and 55 pounds of hay, are predicted to produce 200 pounds of milk per week when the auxiliary variables are set at their mean values. With 60 pounds of grain and 131 pounds of hay producing 200 pounds of milk, an additional pound of grain is predicted to produce 1.49 pounds of milk, $\partial M/\partial G$, and an additional pound of hay is predicted to produce 0.55 pound of milk. At this feed combination, 1 pound of grain is predicted to substitute for 2.71 pounds of hay dH/dG. However, with 90 pounds of grain and 55 pounds of hay producing 200 pounds of milk, the substitution rate, dH/dG, declines to 2.38.[14]

While the step is not taken in this report, the marginal productivity of the various cow characteristics and the environmental conditions also can be predicted. Similarly, the marginal rates of substitution of these characteristics for each other and for feed variables can be derived.

DERIVED QUANTITIES FOR MILK PRODUCTION FUNCTION

The ultimate purpose in estimating milk production functions is to provide a basis for predicting milk pro-

[14] For added detail on the meaning of the marginal quantities in feed evaluation, see Heady, Jacobson, Schnittker and Bloom, op. cit.

duction from alternative rations and for estimating economic optima such as the least-cost ration and the profit-maximizing level of feeding for various grain-hay-milk price combinations. Economic optima, based on the marginal products and marginal rates of substitution as derived in previous equations, are presented in this section.[15] First, equations 1, 2 and 3 are compared with respect to their implied derived quantities of economic optima when all auxiliary variables are set at their respective mean levels. Second, derived quantities, as represented by equation 1, are computed to illustrate the predicted effect of differences in ability, weight, inbreeding and other conditions on the milk production function.

Comparison of Equations 1, 2 and 3

Average experimental conditions can be approximated by setting auxiliary variables at their mean levels, except for T which is set at 11. The approximate mean values of auxiliary variables, with T = 11, are as follows:

$\overline{T}$ = 11, the eleventh week of the experiment or the twentieth week of the lactation.

$\overline{F}$ = 52°, the average of daily high temperature readings (taken outside).

$\overline{A}$ = 2,459 pounds of milk produced during a 50-day preliminary period on full feeding.

$\overline{W}$ = 1,129 pounds, the body weight at the beginning of the experimental period.

$\overline{J}$ = 54, index of maturity based on 66 for a mature cow.

$\overline{K}$ = a coefficient of inbreeding of 9.

Basic data for the isoquants and surfaces are presented in tables 5 through 10. Figures 2, 3 and 4 show the derived quantities for equations 1, 2 and 3, respectively.[16] Economic optimum data for these figures are presented in tables 11, 12 and 13.

Different grain-hay price ratios were achieved by keeping the grain price constant at $3 per cwt. and by varying the hay price from $0.75 per cwt. to $1.75 per cwt. The isocline for each feed price ratio (figs. 2, 3 and 4) is shown by a dotted line labeled with an encircled fraction to indicate the relevant grain-hay price ratio. For each grain-hay price ratio, three different milk prices are used to represent low, medium and high price levels. Each triangle represents an economic optimum for a milk price of $3 per cwt.; circles and squares represent economic optima for milk prices of $4 and $5, respectively. Short-run profit is shown for each economic optimum. For instance, in fig. 2 the expected profit is $3.84 when prices per cwt. are $3, $0.75 and $3 for grain, hay and milk, respectively. This profit is defined as return above feed cost per cow per week. Ridge lines, isoclines and milk isoquants are also shown in the derived quantity graphs. (Symbols used in representing each of the derived quantities are indicated in the legend of fig. 2.) Maximum milk production per cow for the period is represented by the point of convergence of isoclines.

Table 14 provides a comparison of equations 1, 2 and 3 with respect to their implied economic optima. Corresponding economic optima estimated from each of the three equations are somewhat similar.

Derived Quantities for Equation 1 With Variation in Auxiliary Variables

With the auxiliary variables set at selected levels, equation 1 is used to approximate different relevant production conditions or cow characteristics. Quantities are derived for different stages of the lactation, for different seasons of the year and for cow characteristics set at a range of combinations.

Stage of Lactation

Figure 5 illustrates the derived quantities of milk in relation to feed for the first week of the experiment (T = 1), or the tenth week of the lactation.[17] All other auxiliary variables are set at their respective mean levels. Figure 6 represents the derived quantities for the fifth week of the experiment (T = 5). Economic optimum data for these derived quantity graphs are presented in tables 15 and 16.

Similarly, data for the isoquants of T = 1 and T = 5 are given in tables 17 and 18. Tables 19 and 20 indicate the data for the corresponding production surfaces.

When the surfaces for the different stages of lactation are compared, it is obvious that a given quantity of feed will produce a progressively smaller quantity of milk as the lactation progresses. This decline is consistent with well-known biological conditions.[18] Algebraically, this decline is affected by the negative coefficient for the GT interaction term in equation 1.

Figure 7 illustrates changes in the economic optima over the lactation. For a given grain-hay milk price combination, the proportion of grain in the economic optimum ration declines as T increases. This result shows that, as T increases, the marginal product of grain—hence the marginal rate of substitution of grain for hay—decreases. As the stage of lactation progresses, the isoclines shift toward the hay axis. Table 21 contains a comparison, including profit or return over feed cost, of the predicted economic optima for various stages of the lactation.

Temperature

When the temperature variable (F) is set at dif-

[15] There are several instances in which the computed economic optimum ration includes a zero or near-zero level of hay feeding. Since it has been established that a ruminant generally requires a minimum amount of roughage, it is understood that the physiological minimum level of hay should be fed when the derived economic optimum ration indicates little or no hay.

[16] Economic optima points indicated on the grain axis are used to imply that the physiological minimum level of hay should be fed.

[17] Again, as for the following figures, economic optima indicated on the grain axis imply that the physiological minimum level of hay should be fed.

[18] For example, see previous estimates of production functions in: Earl O. Heady and John L. Dillon. Agricultural production functions. Iowa State University Press. Ames. 1961.

ferent levels, it is possible to approximate the milk production function for different seasons of the year. Derived quantities for low temperatures, represented by F = 10, are illustrated in fig. 8. (Economic optima falling on the grain axis imply that the physiological minimum level of hay should be fed.) Figure 9, with F set at 90, is a similar representation of a high temperature. All other auxiliary variables are set at their mean values, with T = 11. Tables 22 and 23 contain the economic optimum data; tables 24 and 25 contain isoquant data for low and high temperatures, respectively. The predicted economic optimum rations did not vary with outside temperature; F does not appear in any feed interaction term in equation 1. Existing knowledge suggests that there should be an interaction of temperature with hay or grain; apparently, our data were not comprehensive enough to show such an interaction. However, since equation 1 contains several nonfeed terms involving F, the height of the milk production surface (milk output from a given ration) does vary as temperature changes. Only the profit-maximizing level of milk output and, consequently, the profit are subject to change as temperature varies, given the predictions of the production function used in this study.

Cow Characteristics J, K, M and A

Other defined cow characteristics are considered in the predictions made in this section. We realize that additional research, with variables representing cow characteristics, is needed to provide more precise estimates than those from this relatively small study. However, since the regression coefficients associated with these variables are significant at normally acceptable levels of probability, certain physical and economic quantities are predicted from them. This section is intended to illustrate the type of useful derivations that can be forthcoming from the kind of analyses reported here.

In the preceding section, stage of lactation (T) and temperature (F) were varied separately; this procedure was possible because T and F are independent. However, some variables for cow characteristics are mutually dependent, and a major change in one is accompanied by a simultaneous change in others; e.g. as cows mature, they normally gain weight. Similarly, ability (as measured in the present study) is expected to increase with maturity. Hence, when a change in the age variable is considered, a corresponding change in weight and ability should also be made. Although the coefficient of inbreeding remains constant throughout a cow's life, inbreeding normally lowers milk production and reduces body weight, particularly at early ages.[19]

To quantify the interrelationship among the cow characteristics, two simultaneous equations were computed by using pooled data from both experiments. When J and K were set at different combinations, this system of simultaneous equations was solved, and realistic values for W and A were found (table 26).

By substituting these values of the cow characteristics into equation 1, we can synthesize different production functions, each representing a different hypothetical cow. Derived quantities can then be computed for each production function. Figures 10, 11, 12 and 13 indicate the economic optima and isoquants for four combinations of cow characteristics.[20] In each situation, T = 11, and the temperature is set at the mean level, or F = 52. Data for economic optima, milk isoquants and specified quantities paralleling these figures are presented in tables 27 through 34.

Figure 10 represents non-inbred heifers since the coefficient of inbreeding and maturity are set at K = 0 and J = 26, respectively. Accordingly, ability and weight are adjusted to A = 2,255 and W = 984.

Although mature cows are considered (J = 66) in fig. 11, the coefficient of inbreeding is zero, and ability and weight are adjusted to A = 2,711 and W = 1,250. All three economic optimum points for a hay price of $1.75 per cwt. lie on the grain axis and imply that the physiological minimum level of hay should be fed.

Variation of K with J kept constant. Figure 14 compares heifers with no inbreeding with inbred heifers: The coefficient of inbreeding, K is set first at 0 percent and then at 25 percent. Maturity index, J, is kept at 26 to represent 26-month-old heifers. Ability and weight are adjusted as indicated in table 35. Two facts are emphasized in the data of fig. 14. First, heifers with no inbreeding are predicted to be more profitable than inbred heifers. Second, the least-cost rations for heifers with no inbreeding contain a smaller proportion of hay than corresponding least-cost rations for inbred heifers. On this basis, as higher hay prices are considered, the profit differential between non-inbred and inbred heifers is predicted to increase.

Figure 15 compares cows with no inbreeding and inbred cows at maturity (J = 66), with weight and ability again adjusted as indicated in table 35. As with heifers, mature cows with no inbreeding have a smaller proportion of hay in the least-cost rations than do the mature inbred cows. However, fig. 15 indicates that mature cows with no inbreeding are not consistently more profitable than inbred mature cows. In fact, when the hay price is very low relative to the grain price, inbred cows are predicted, on the basis of the data available from the production function, to be more profitable. This difference is due to the higher proportion of hay specified for least-cost rations in the two cases. Thus, according to the economic optima estimated from equation 1, inbreeding depresses profits more for heifers than for mature cows. Previous research also indicates that inbred

[19] R. H. Nelson and J. L. Lush. The effects of milk inbreeding on a herd of Holstein-Friesian cattle. Jour. Dairy Sci. 33:186-193. 1950.

[20] When the economic optima fall on the grain axis, it is supposed that the ration actually represents the physiological minimum level of hay.

cows mature later than cows with no inbreeding but that, at maturity both achieve nearly the same level of production.[21]

The results of the preceding analysis on inbreeding suggest some of the technological detail that may be incorporated into well-designed production-function experiments. Much additional research will be necessary before final quantitative values can be given to the coefficients relating to inbreeding[22] or to other variables included in this study.

Variation of J with K constant. Weight and ability, which increase with maturity, are included in feed interaction terms of equation 1. As J is changed, both the height and the slope of the milk production surface change. Hence, feed input for a cow's economic optimum ration is related to her age. Since immature cows convert a portion of their feed to body growth, profit earnings should increase with cow maturity. Maturation is predicted to raise the weekly profit per cow more for inbred cows than for cows with no inbreeding (table 36).

Figure 16 shows the differences in profit and in composition of least-cost rations under various prices for cow maturity indexes of 26 to 66. Non-inbred cows ($K = 0$) are considered in this graph. Figure 17 shows the same comparisons for highly inbred cows ($K = 25$ percent). For both groups, weight and ability were adjusted as indicated earlier. (Background data for figs. 16 and 17 are presented in table 35.) The differences due to age, shown in figs. 16 and 17, seem excessively high. Further studies are needed to provide a clearer assessment.

When additional data become available, we will be able to make more reliable estimates of the milk production function. Nevertheless, our present results can provide the basis for further research which may clearly establish the interdependency of certain variables affecting milk production. The confidence regions are presented in the following section as an indication of the degree of certainty of our estimates.

CONFIDENCE REGIONS FOR PHYSICAL AND ECONOMIC QUANTITIES

The general procedure used in computing the confidence regions from equation 1 is that outlined by Fuller.[23] Since his applications involved a relatively simple quadratic production function with two variable resources, the method has been expanded for equation 1 with the numerous auxiliary variables.

Isoclines and Isoquants

Confidence regions were computed for the 4.0 and 1.7 isoclines of fig. 2. The isoclines denote least-cost rations for specified levels of milk output when the grain:hay price ratio is 3:0.75 and 3:1.75, respectively. Figures 18 and 19 indicate the position and magnitude of the confidence regions for the two selected isoclines. The wide region surrounding the 4.0 isocline indicates that we do not have enough data to accurately estimate its height and slope. In contrast, the confidence region for the 1.7 isocline is much closer. Because of this difference between the confidence regions, the least-cost level of grain feeding cannot be defined as precisely as can the optimum level of hay feeding.

Confidence regions also were computed for 200-pound and 300-pound milk isoquants (fig. 2). Figure 20 indicates the size and position of these confidence regions.[24]

The isoquant confidence regions in fig. 20 are narrow and imply that equation 1 provides a fairly reliable estimate of the quantity of hay required in combination with a fixed amount of grain. As expected, each confidence region is narrowest near the mean point of the observed values of grain and hay consumption (103 pounds of grain and 145 pounds of hay).

Economic Optimum Levels of Hay and Grain

Figure 21 indicates the size and shape of confidence regions for points of economic optima that denote the profit-maximizing levels of grain and hay feeding. The confidence regions are for grain and milk, each priced at $3 per cwt., and for hay priced in a range from $0.75 to $1.75 per cwt. The elongated confidence regions suggest, as do the isoclines, that the optimum level of feeding can be predicted with greater certainty for grain than for hay.

Again, the confidence boundaries for the economic optima are wider as the distance from the means increases. It can be predicted with 0.95 probability that the economic optimum level of grain feeding for the grain-hay-milk price ratio of 3:1.00:3 lies between 75 and 110 pounds, or a range of 35 pounds. On the other hand, the same confidence range for the price 3:0.75:3 is twice as wide, extending from 25 to 95 pounds of grain.

Reducing Confidence Boundaries and Other Needs in Experiments

Point estimates for input-output relationships and economic optimum levels of inputs predicted from the estimated production function are accompanied by

[21] R. H. Nelson and J. L. Lush. The effects of milk inbreeding on a herd of Holstein-Friesian cattle. Jour. Dairy Sci. 33:186-193. 1950.

[22] Equation 1 has a nonlinear term for the coefficient of inbreeding. Other analyses have estimated only a linear relationship. For example, see: C. M. Von Krosigk. Effect of inbreeding on production in Holsteins. Unpublished M.S. thesis. Iowa State University Library. Ames, Iowa. 1956. He obtained an intra-sire linear regression of -54 lbs. of milk per 1 percent increase in inbreeding. Using data from 502 cows by 45 sires, he noted no evidence of a deviation from linear regression within the range of inbreeding from 0 to 34 percent. From other observations, we might expect this relationship to become curvilinear as the range in inbreeding increased.

[23] Wayne A. Fuller. Estimating the reliability of quantities derived from empirical production functions. Jour. Farm Econ. 44:82-99. 1962.

[24] The feed quantities are extended to levels and combinations exceeding measurements in the experiment to better illustrate the curvature of the confidence boundaries.

rather wide confidence limits. More specific predictions (reduction of the confidence regions) can be made by lessening (a) the error mean square or residual variance of milk production and (b) the variances and covariances for the regression coefficients of the feed terms in the equation. Reduction in the error mean square, or unexplained variance in milk production, can be accomplished in three ways: (1) The number of observations can be increased through additional research. (2) The model may be improved by the inclusion of additional or different auxiliary variables in the model. (3) More refined experimental design techniques can be used.

A limiting factor in this type of research is the cost of obtaining large amounts of nutritional data. Under the best conditions, nutritional observations are likely to be available on only a few hundred cows. Research in the field of dairy cattle breeding shows that large amounts of data are needed for precise estimates of the relationship of inbreeding, age, body weight and stage of lactation to milk production. It would be impractical to have the well-controlled nutritional experiment large enough to yield these estimates with sufficient precision. However, since such estimates are available in published reports, a practical approach might be to use those estimates from the literature. They could be incorporated into an equation such as 1, while the nutritional data would be derived from the relatively small number of animals feasibly included in nutritional experiments.

The results presented here are encouraging and useful—encouraging, because of the relative success of our predictions from equation 1, and useful, because the input-output relationship and the economic optimum comparisons may serve as guides for future research.

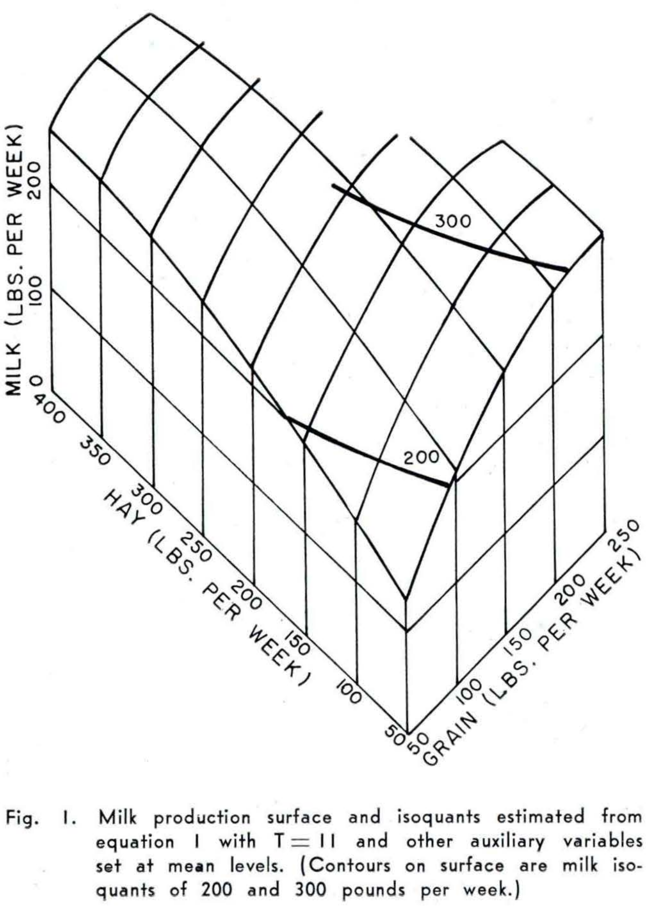

Fig. 1. Milk production surface and isoquants estimated from equation 1 with T = 11 and other auxiliary variables set at mean levels. (Contours on surface are milk isoquants of 200 and 300 pounds per week.)

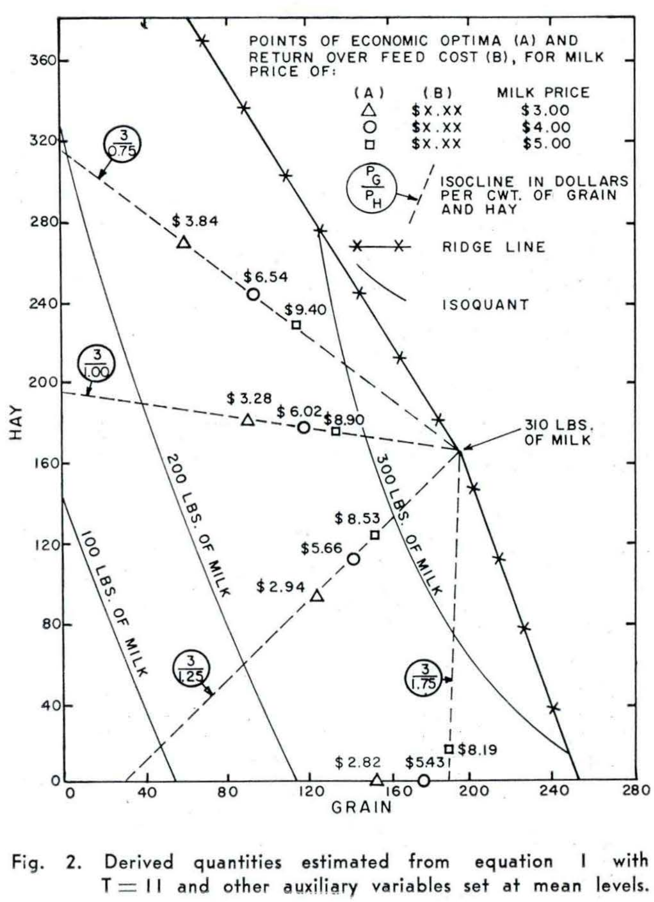

Fig. 2. Derived quantities estimated from equation 1 with T = 11 and other auxiliary variables set at mean levels.

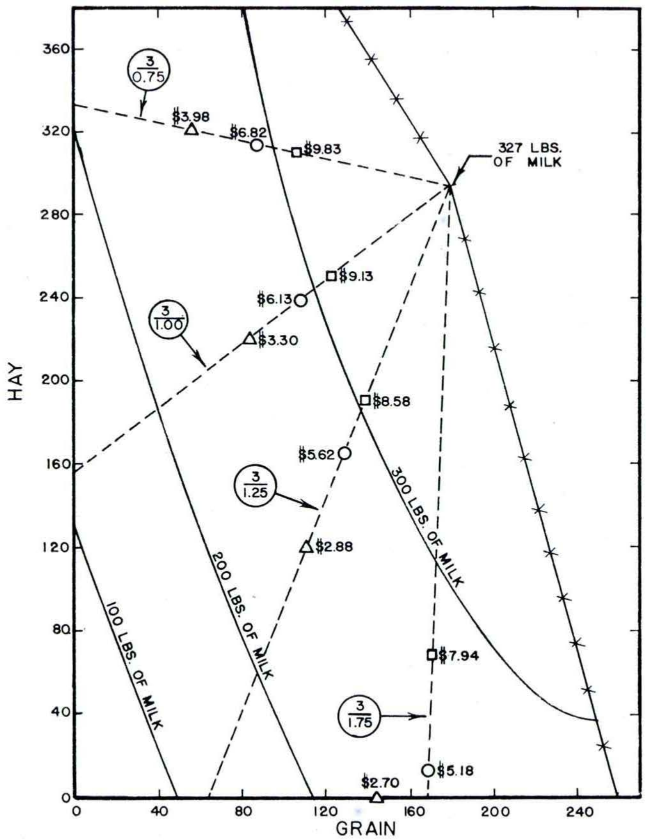

Fig. 3. Derived quantities estimated from equation 2 with T = 11 and other auxiliary variables set at mean levels. (See fig. 2 for legend.)

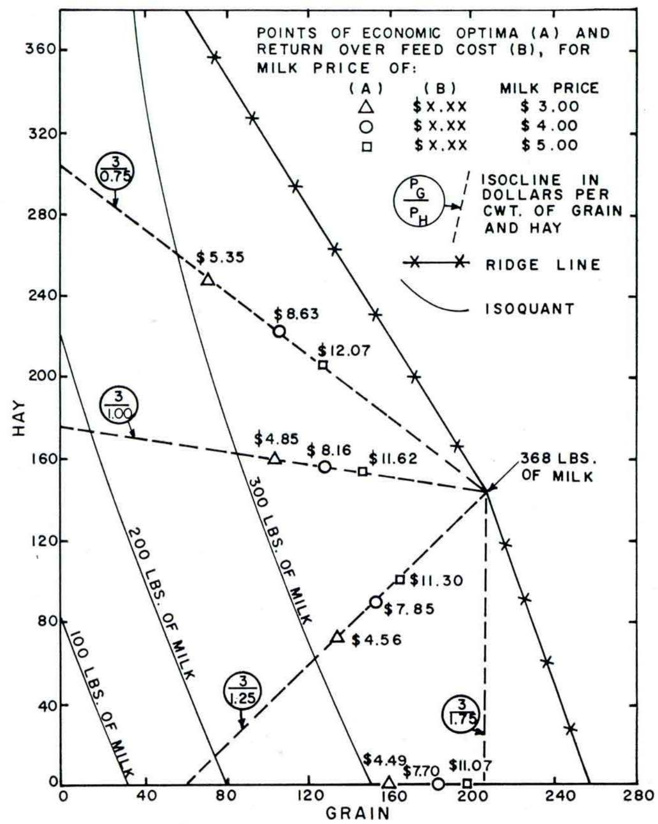

Fig. 5. Derived quantities estimated from equation 1 for first experimental week (T = 1); other auxiliary variables set at mean levels.

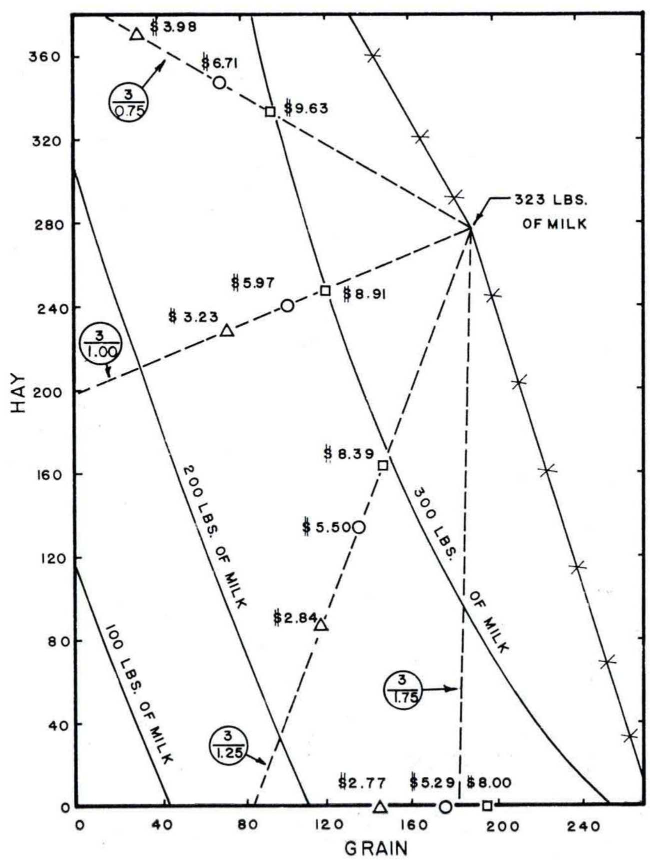

Fig. 4. Derived quantities from equation 3 with T = 11 and other auxiliary variables set at mean levels. (See fig. 2 for legend.)

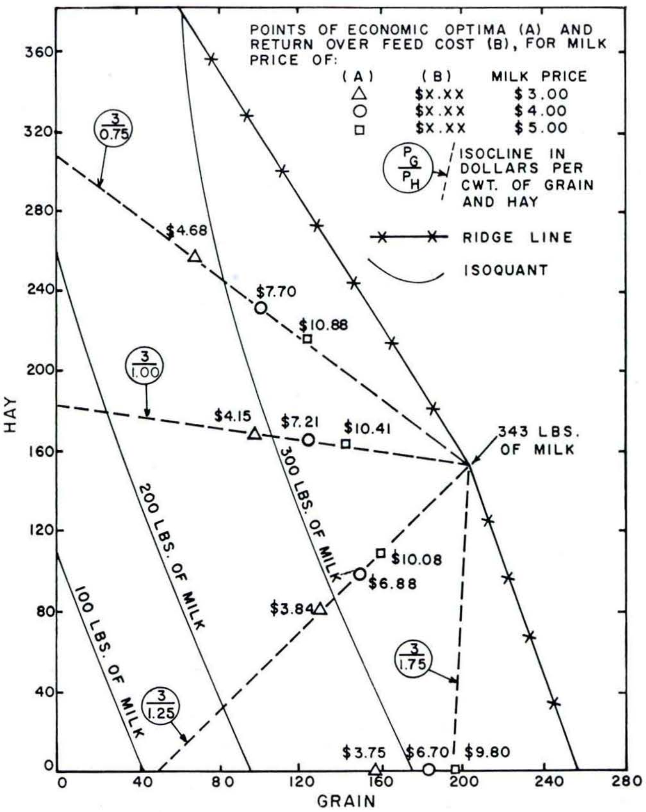

Fig. 6. Derived quantities estimated from equation I for fifth experimental week (T = 5); other auxiliary variables set at mean levels.

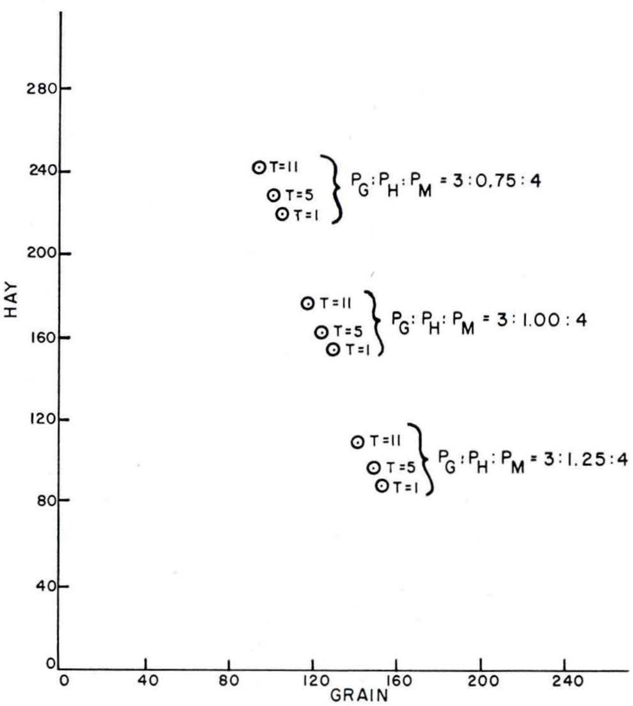

Fig. 7. Changes in profit-maximizing ration over the lactation for various price ratios, estimated from equation I with other auxiliary variables set at mean levels.

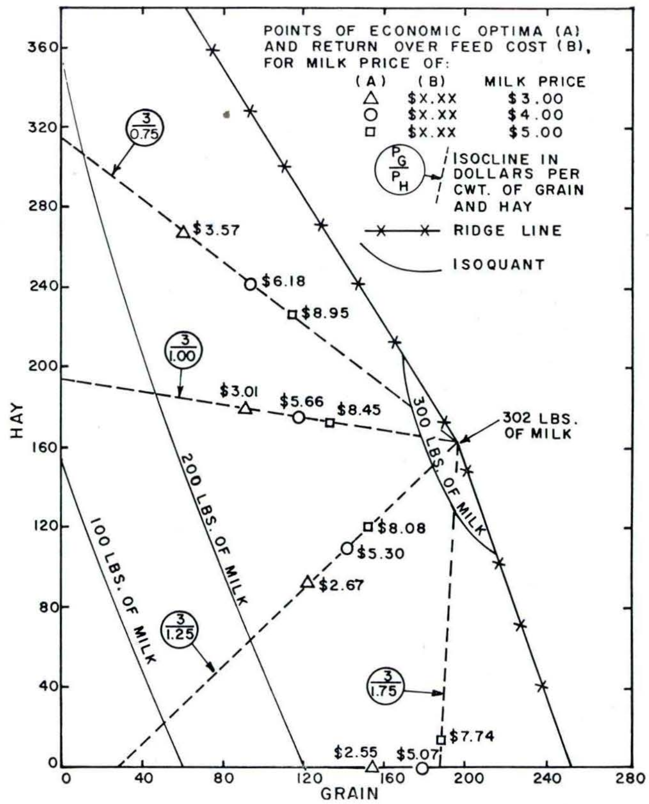

Fig. 8. Derived quantities estimated from equation I for low temperatures (F = 10); T = 11 and other auxiliary variables set at mean levels.

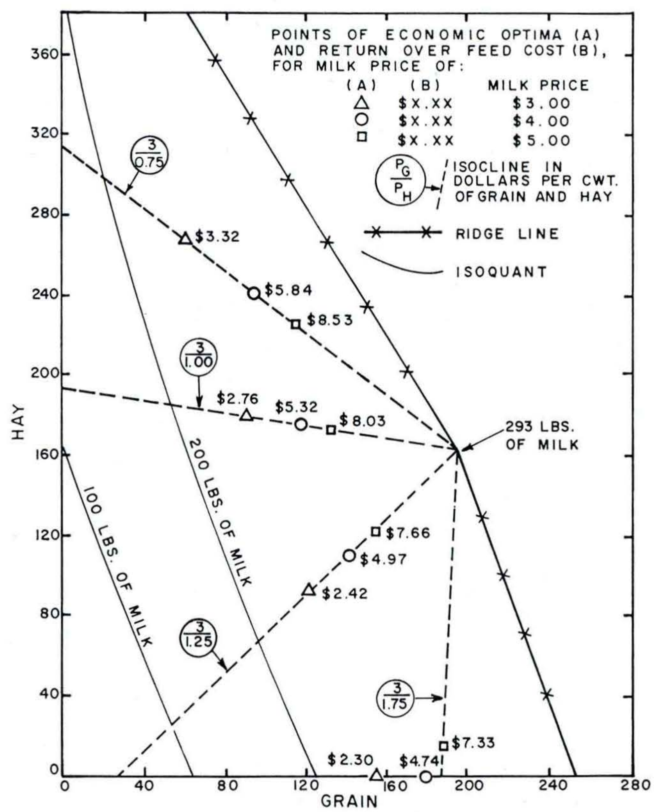

Fig. 9. Derived quantities estimated from equation I for high temperatures (F=90); T=11 and other auxiliary variables set at mean levels.

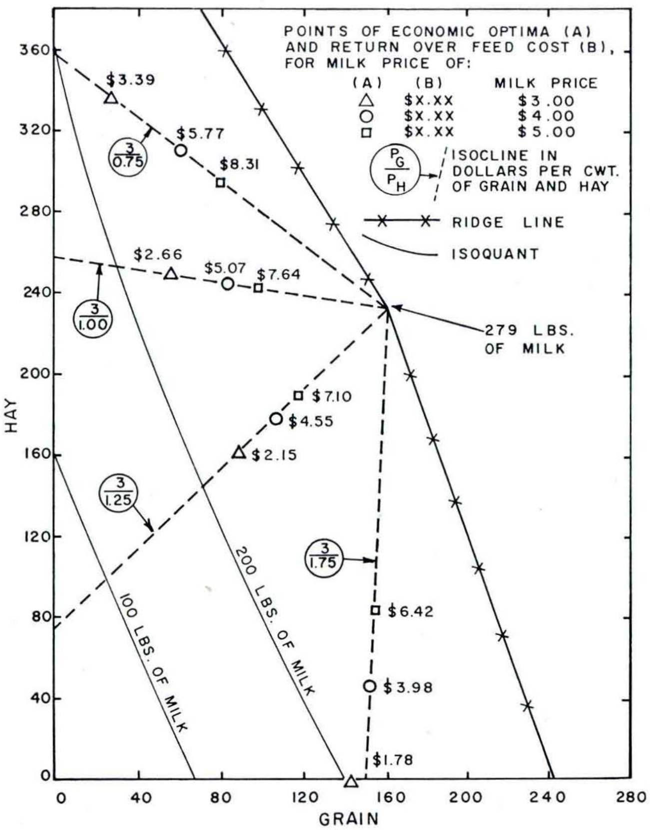

Fig. 10. Derived quantities estimated from equation I for heifers with no inbreeding (J = 26, K = 0; A = 2,255; W = 984), T = 11 and other auxiliary variables set at mean levels.

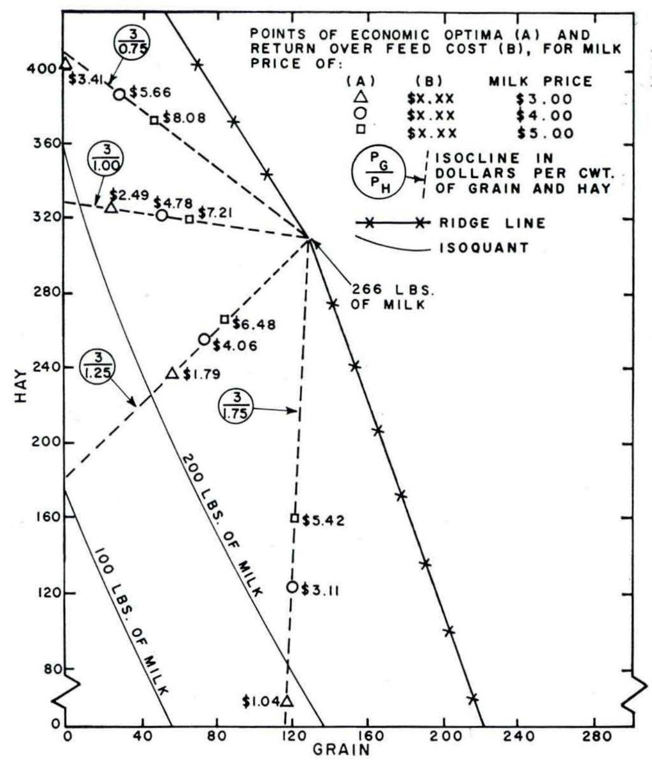

Fig. 12. Derived quantities estimated from equation I for inbred heifers (J = 26; K = 25; A = 1,908; W = 898), T = 11 and other auxiliary variables set at mean levels.

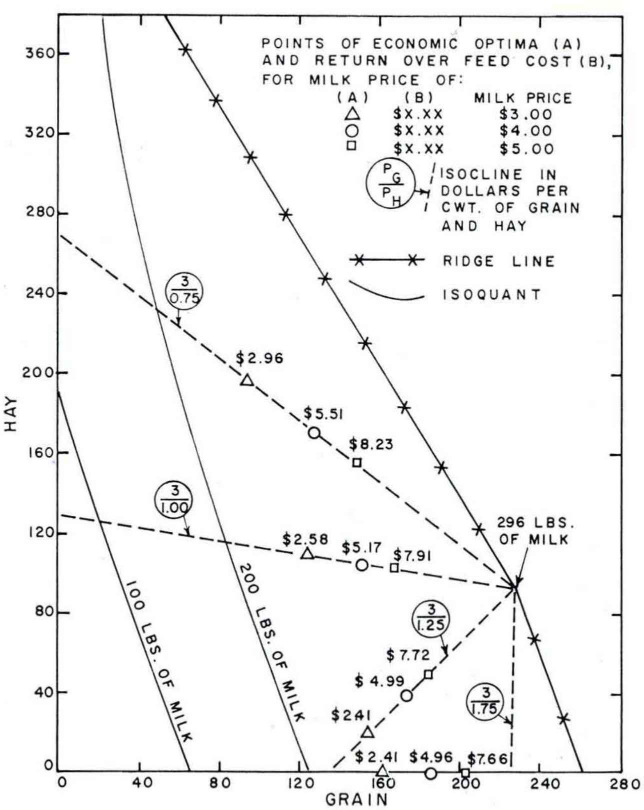

Fig. 11. Derived quantities estimated from equation I for mature cows with no inbreeding (J = 66; K = 0; A = 2,711; W = 1,250), T = 11 and other auxiliary variables set at mean levels.

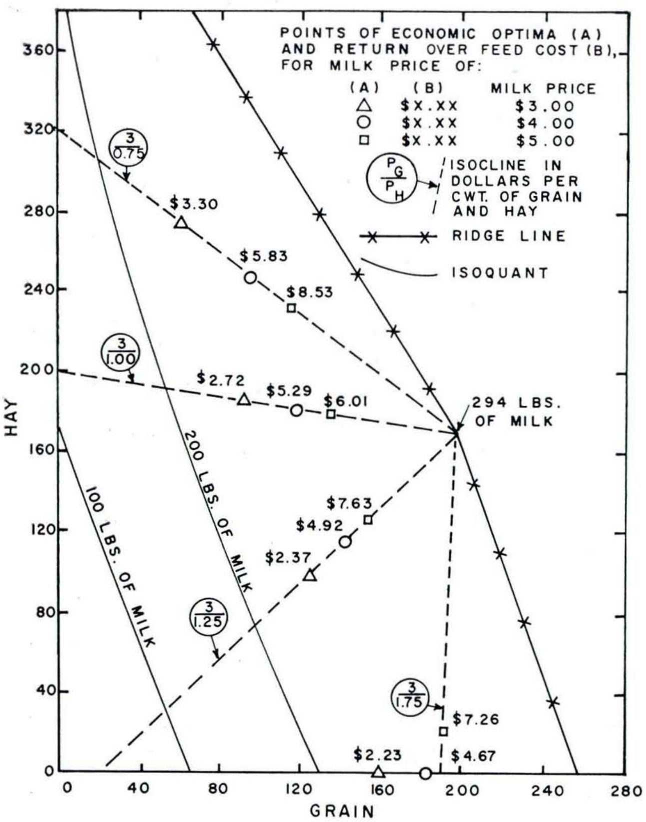

Fig. 13. Derived quantities estimated from equation I for inbred mature cows (J = 66; K = 25; A = 2,364; W = 1,164), T = 11 and other auxiliary variables set at mean levels.

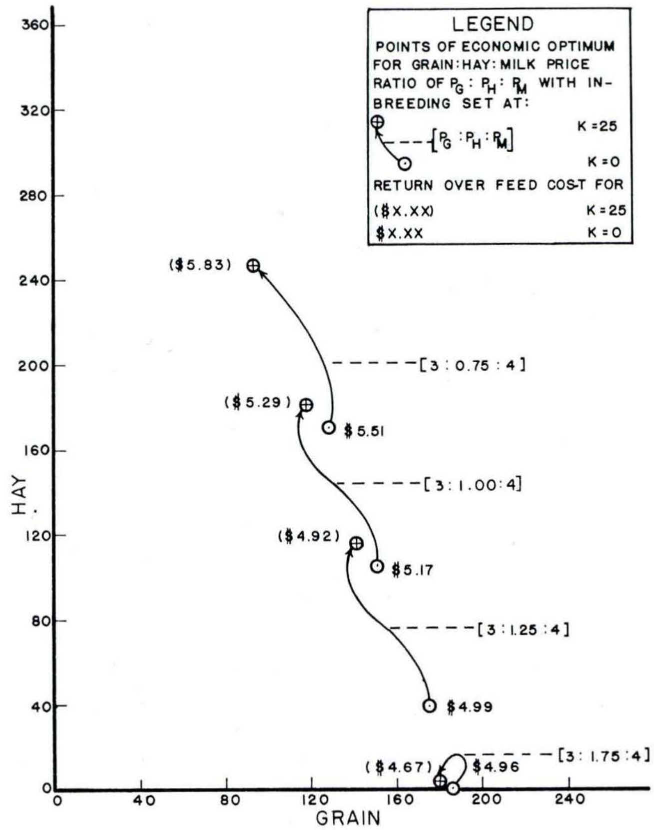

Fig. 15. Comparison of least-cost rations and returns over feed cost for non-inbred (K = 0) versus inbred (K = 25) mature cows (J = 66), with weight and ability set as indicated in table 35, temperature set at mean and T = 11. The circled dot and circled cross represent economic optima for cows with no inbreeding and inbred cows, respectively.

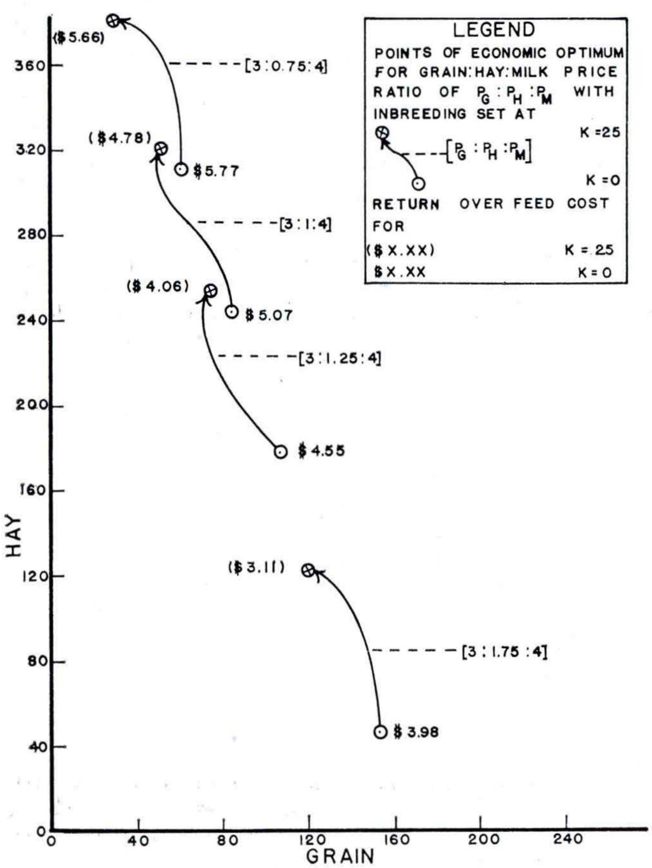

Fig. 14. Comparison of least-cost rations and returns over feed cost for heifers with no inbreeding (K=0) versus inbred (K = 25) heifers (J = 26) with weight and ability set as indicated in table 35, temperature set at mean and T = 11. The circled dot and circled cross represent economic optima for the coefficient of inbred heifers, for K = 0 and K – 25, respectively.

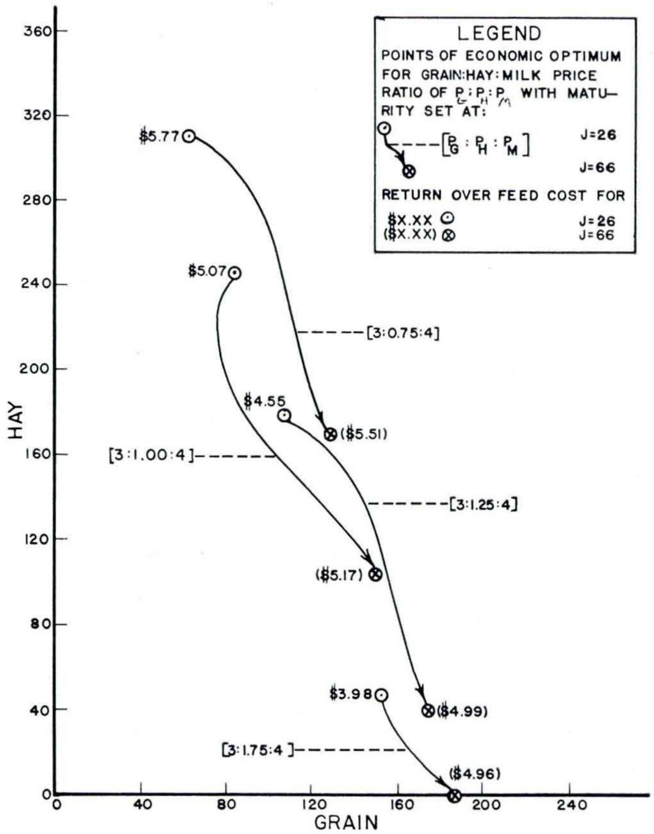

Fig. 16. Comparison of least-cost ration and returns over feed cost for non-inbred (K = 0) heifers (J = 26) versus mature cows (J = 66), with weight and ability set as indicated in table 35, temperature set at means and T = 11. The circled dot and circled cross represent economic optima for non-inbred heifers and non-inbred mature cows, respectively.

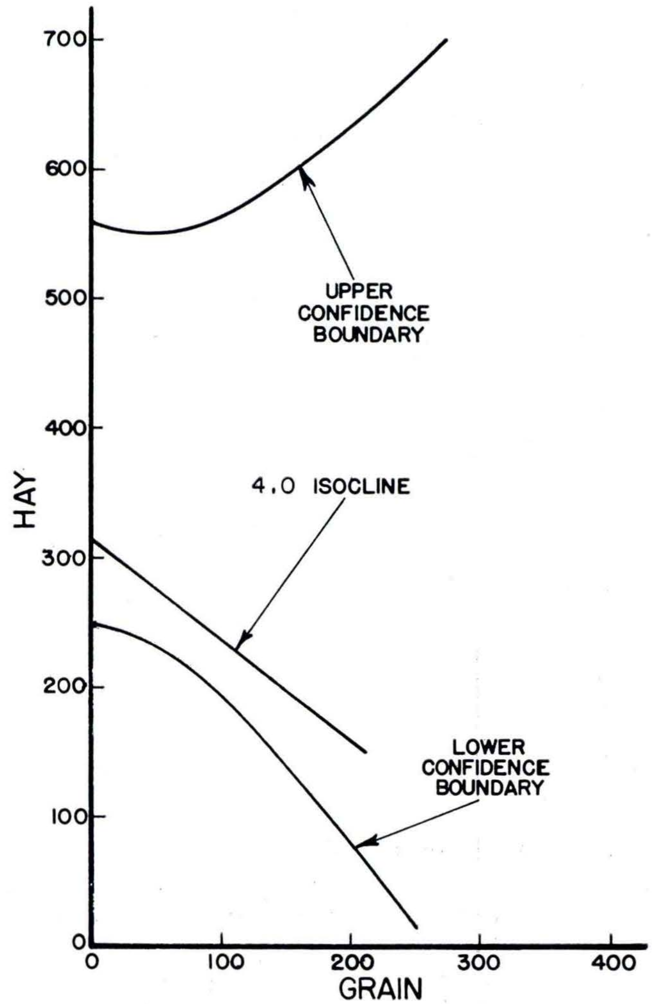

Fig. 18. The 95-percent confidence region for the 4.0 isocline, as estimated from equation 1 with T = 11 and other auxiliary variables set at mean levels.

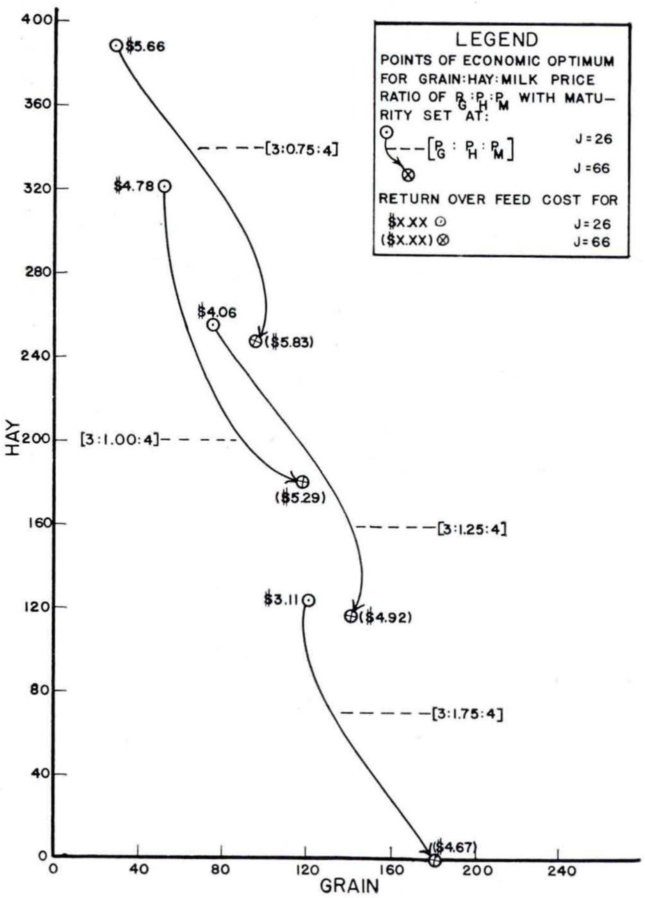

Fig. 17. Comparison of least-cost rations and returns over feed cost for highly inbred (K = 25) heifers (J = 26) versus highly inbred mature cows (J=66), with weight and ability set as indicated in table 35; temperature set at means at T = 11. The circled dot and circled cross refer, respectively, to J=26 and J=66.

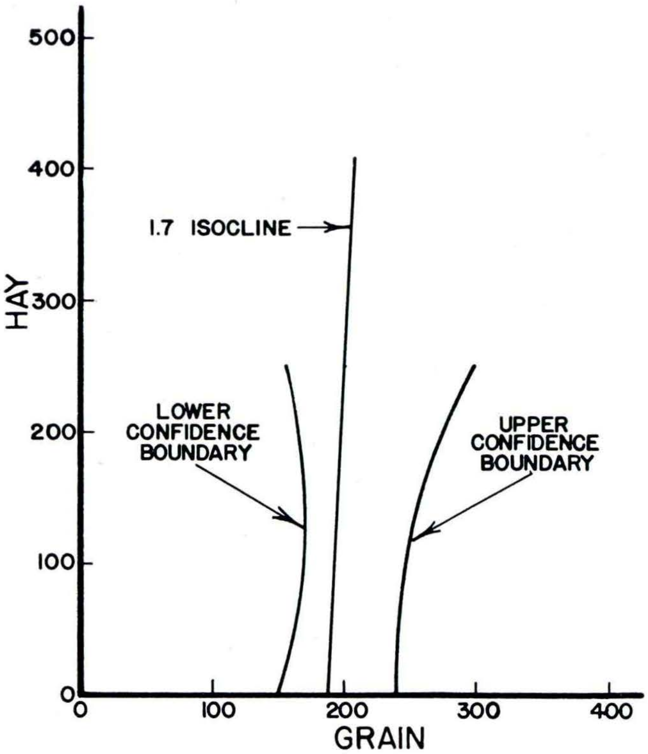

Fig. 19. The 95-percent confidence region for the 1.7 isocline, as estimated from equation I with T = 11 and other auxiliary variables set at mean levels.

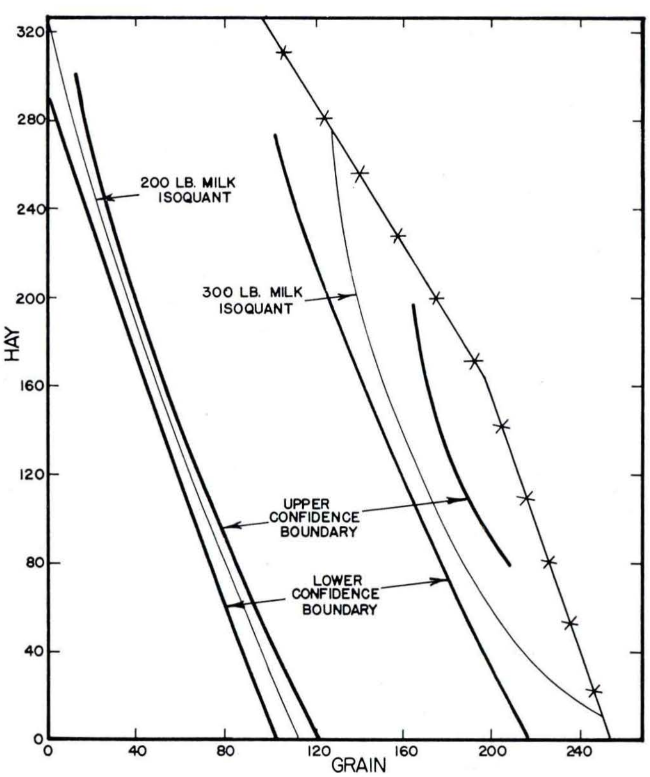

Fig. 20. The 95-percent confidence boundaries for the 200- and 300-pound milk isoquants, as estimated from equation I with T = 11 and other auxiliary variables set at mean levels.

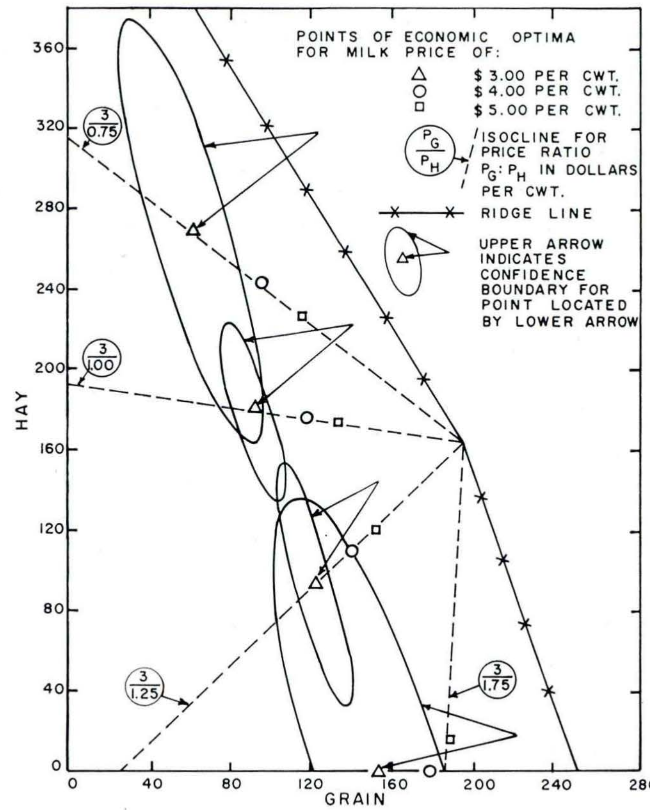

Fig. 21. The 95-percent confidence regions for economic optima as estimated from equation I with T = 11 and other auxiliary variables set at mean levels. As in previous graphs, triangular, round and square symbols represent milk prices per cwt. of $3.00, $4.00 and $5.00, respectively, with indicated feed prices.

Table 1. Average composition of feeds used during the experimental period.

	Protein[a]	EE[b]	CF[c]	Ash[d]	NFE[e]
	(Percent dry matter basis)				
Grain					
First experiment	19.5	2.9	8.5	6.3	62.8
Second experiment	19.1	3.4	8.0	6.5	63.0
Hay					
First experiment	15.4	1.6	31.5	7.5	44.0
Second experiment	16.2	1.6	35.2	7.1	39.9

[a] Protein: crude protein (nitrogen x 6.25).
[b] EE: ether extract.
[c] CF: crude fiber.
[d] Ash: mineral content.
[e] NFE: nitrogen-free extract.

Table 2. Composition of concentrate mixtures in the two experiments.

	First	Second	
	1953-54	1956-57	1957-59
	(Pounds)		
Ground yellow corn	400	500	500
Oats	500	300	300
Wheat bran	200	400	400
Soybean meal	200	200	100
Linseed meal	100	0	0
Steamed bonemeal	30	14	14
$CaCO_3$	0	14	14
Salt	15	14	14

Table 3. Regression coefficients (b's), t values and R^2 for equations using weekly observations from 72 Holstein cow lactations.

Independent variable	Equation 1 b	Equation 1 t	Equation 2 b	Equation 2 t	Equation 3 b	Equation 3 t
Constant	248.4190	3.9**	−183.35371	4.6**	−140.74272	3.7**
G	1.83582	5.3**	2.18515	6.3**	2.15982	6.2**
H	1.41166	6.3**	1.12002	4.9**	0.88438	3.8**
G^2	−0.00505	5.3**	−0.00433	4.5**	−0.00382	4.1**
H^2	−0.00109	2.9**	−0.00074	1.9	−0.00065	1.6
GH	−0.00352	3.1**	−0.00236	2.1*	−0.00233	2.1*
GJ	—	—	0.00262	1.8	—	—
GT	−0.00557	2.5*	−0.00731	3.2**	−0.00592	2.5*
GW	0.00069	4.9**	—	—	—	—
HA	−0.00015	6.2**	−0.00011	4.2**	−0.00008	3.4**
HF	—	—	—	—	0.00228	4.4**
A	0.07493	11.3**	0.06990	10.1**	0.09460	16.9**
F	1.00634	2.1*	1.21706	3.8**	0.48906	1.7
J	3.16193	4.8**	0.39307	1.6	—	—
K	−5.42694	6.6**	−5.24802	6.3**	—	—
W	0.36939	7.2**	—	—	—	—
$K^{1/2}$	15.35695	4.3**	19.78151	5.8**	—	—
$W^{1/2}$	−27.04613	10.0**	—	—	—	—
F^2	−0.00398	2.3*	−0.00443	2.6**	−0.00451	2.4*
T^2	0.09091	7.4**	0.09045	7.0**	0.05454	4.0**
AF	−0.00024	2.4*	−0.00015	1.5	−0.00034	4.0**
AT	−0.00164	8.0**	−0.00162	7.7**	−0.00242	14.3**
JF	−0.01454	3.5**	−0.00795	2.4*	—	—
JT	−0.03864	4.5**	−0.03683	4.1**	—	—
KA	0.00164	5.2**	0.00139	4.3	—	—
KF	−0.02967	4.1**	−0.02349	3.3**	—	—
KT	0.03865	2.4*	0.04069	2.4*	—	—
TF	—	—	—	—	0.09205	3.7**
WF	0.00065	1.8	—	—	—	—
WJ	−0.00187	3.8**	—	—	—	—
R^2	0.836		0.822		0.808	

* Acceptable at the 0.05 level of probability.
** Acceptable at the 0.01 (or lower) level of probability.

Table 4. Regression coefficients (b's), t values and R^2 for intermediate equations using weekly observations from 72 Holstein cow lactations.

Independent variable	Equation 4 b	Equation 4 t	Equation 5 b	Equation 5 t	Equation 6 b	Equation 6 t
Constant	−156.20149	2.7*	253.13930	3.9**	245.16500	3.6**
G	1.84091	5.0**	1.86369	5.2**	1.65765	4.6**
H	1.17574	5.2**	1.40741	6.3**	1.05195	4.5**
G2	−0.00460	4.6**	−0.00510	5.3**	−0.00463	4.6**
H2	−0.00070	1.8	−0.00109	2.9**	−0.00123	2.8**
GH	−0.00265	2.3*	−0.00353	3.1**	−0.00366	2.9**
GJ	−0.00160	0.8	0.00069	0.3	—	—
GT	−0.00636	2.8**	−0.00566	2.5*	−0.00418	1.9
CW	0.00061	3.0**	0.00065	3.3**	0.00078	3.2**
HA	−0.00012	4.4**	−0.00015	5.8**	—	—
HW	—	—	—	—	0.00004	0.2
A	0.07084	10.2**	0.07449	11.1**	0.06485	10.0**
F	0.69421	1.4	1.00063	2.1*	0.09060	1.9
J	2.55262	2.6*	3.02899	4.0**	3.26576	4.6**
K	−5.52127	6.5**	−5.42047	6.6**	4.48739	5.5**
W	−0.08437	1.0	0.37084	7.2**	0.33091	5.1**
$K^{1/2}$	17.77267	4.7**	15.56920	4.3**	16.24582	4.5**
$W^{1/2}$	—	—	27.15626	9.9**	24.55833	9.0**
F^2	−0.00337	1.9	−0.00040	2.3*	−0.00401	2.3*
T^2	0.09035	7.1**	0.09086	7.4**	0.08804	7.0**
W^2	0.00003	0.6	—	—	—	—
AF	−0.00015	1.5	−0.00023	2.4*	−0.00031	3.2**
AT	−0.00167	7.9**	−0.00163	8.0**	−0.00159	7.6**
JF	−0.01294	3.0**	−0.01464	3.5**	−0.01216	2.9**
JT	−0.03630	4.0**	−0.03852	4.5**	−0.04062	4.6**
KA	0.00139	4.2**	0.00164	5.2**	0.00113	3.6**
KF	−0.01604	2.1*	−0.03019	4.1**	−0.02806	3.8**
KT	0.04106	2.5*	0.03837	2.4*	0.03915	2.4*
WF	0.00053	1.4	0.00066	1.8	0.00075	2.1*
WJ	−0.00133	1.8	−0.00180	3.4**	−0.00212	3.9**
R^2	0.824		0.836		0.831	

* Acceptable at the 0.05 level of probability.

** Acceptable at the 0.01 (or lower) level of probability.

Table 5. Milk isoquants, marginal products and marginal rates of substitution based on equation 1, with T = 11 and other auxiliary variables set at mean levels.

Level of grain (lbs.)	Pounds of hay required to maintain milk output of: 100 lbs.	200 lbs.	300 lbs.	Derived quantities[a] along indicated milk isoquants (lbs.): 100 $\frac{\partial M}{\partial G}$	100 $\frac{\partial M}{\partial H}$	100 $\frac{dH}{dG}$	200 $\frac{\partial M}{\partial G}$	200 $\frac{\partial M}{\partial H}$	200 $\frac{dH}{dG}$	300 $\frac{\partial M}{\partial G}$	300 $\frac{\partial M}{\partial H}$	300 $\frac{dH}{dG}$
10	114			2.06	0.76	2.69						
20	87			2.05	0.79	2.60						
30	62			2.04	0.81	2.52						
40	37			2.02	0.83	2.45						
50	13			2.01	0.84	2.38						
60		131					1.49	0.55	2.71			
70		104					1.48	0.57	2.59			
80		79					1.47	0.59	2.48			
90		55					1.45	0.61	2.38			
100		31					1.44	0.63	2.29			
140			198							0.45	0.12	3.63
150			166							0.46	0.16	2.91
160			139							0.45	0.18	2.50
170			115							0.43	0.20	2.21
180			95							0.41	0.21	1.96
190			76							0.37	0.21	1.75
200			60							0.33	0.21	1.54
210			45							0.28	0.21	1.33
220			33							0.22	0.20	1.10
230			23							0.15	0.19	0.82

[a] Derived quantities are defined as follows:

$\partial M/\partial G$ = marginal product of grain: pounds of milk resulting from feeding 1 additional pound of grain, hay being kept constant.

$\partial M/\partial H$ = marginal product of hay: pounds of milk resulting from feeding 1 additional pound of hay, grain being kept constant.

dH/dG = marginal rate of substitution: pounds of hay required to replace 1 additional pound of grain. This quantity is always negative in the relevant range because of the negative slope of the isoquant. Hence, the absolute value of the marginal rate of substitution is given.

Table 6. Milk isoquants, marginal products and marginal rates of substitution based on equation 2, with T = 11 and other auxiliary variables set at mean levels.

Level of grain (lbs.)	Pounds of hay required to maintain milk output of: 100 lbs.	200 lbs.	300 lbs.	Derived quantities[a] along indicated milk isoquants (lbs.): 100 $\frac{\partial M}{\partial G}$	100 $\frac{\partial M}{\partial H}$	100 $\frac{dH}{dG}$	200 $\frac{\partial M}{\partial G}$	200 $\frac{\partial M}{\partial H}$	200 $\frac{dH}{dG}$	300 $\frac{\partial M}{\partial G}$	300 $\frac{\partial M}{\partial H}$	300 $\frac{dH}{dG}$
10	103			1.92	0.68	2.82						
20	75			1.90	0.70	2.72						
30	48			1.87	0.71	2.62						
40	23			1.85	0.73	2.54						
60		130					1.42	0.52	2.72			
70		104					1.40	0.54	2.59			
80		78					1.37	0.55	2.48			
90		54					1.34	0.56	2.38			
100		31					1.31	0.57	2.28			
110			259							0.68	0.21	3.18
120			229							0.67	0.24	2.83
130			202							0.64	0.25	2.56
140			178							0.61	0.26	2.33
150			155							0.58	0.27	2.13
160			135							0.54	0.28	1.94
170			117							0.50	0.28	1.76
180			100							0.45	0.28	1.59
190			85							0.40	0.28	1.42
200			72							0.34	0.28	1.24
210			60							0.29	0.27	1.05
220			51							0.22	0.26	0.84
230			43							0.15	0.25	0.61
240			39							0.08	0.23	0.33

[a] See footnote for table 5.

Table 7. Milk isoquants, marginal products and marginal rates of substitution based on equation 3, with T = 11 and other auxiliary variables set at mean levels.

Level of grain (lbs.)	Pounds of hay required to maintain milk output of: 100 lbs.	200 lbs.	300 lbs.	Derived quantities[a] along indicated milk isoquants (lbs.): 100 $\frac{\partial M}{\partial G}$	100 $\frac{\partial M}{\partial H}$	100 $\frac{dH}{dG}$	200 $\frac{\partial M}{\partial G}$	200 $\frac{\partial M}{\partial H}$	200 $\frac{dH}{dG}$	300 $\frac{\partial M}{\partial G}$	300 $\frac{\partial M}{\partial H}$	300 $\frac{dH}{dG}$
10	91	276		1.81	0.66	2.74	1.37	0.42	3.27			
20	64	244		1.79	0.67	2.67	1.37	0.44	3.13			
30	38	214		1.78	0.68	2.60	1.37	0.45	3.01			
40	12	184		1.76	0.69	2.54	1.36	0.47	2.90			
50		156					1.35	0.48	2.79			
60		128					1.34	0.50	2.70			
70		102					1.32	0.51	2.61			
80		76					1.31	0.52	2.53			
90		51					1.29	0.53	2.45			
100			316							0.59	0.16	3.75
110			281							0.60	0.18	3.32
120			249							0.60	0.20	3.01
130			221							0.59	0.21	2.76
140			194							0.57	0.22	2.56
150			169							0.55	0.23	2.38
160			146							0.53	0.24	2.22
170			125							0.50	0.24	2.07
180			105							0.47	0.25	1.93
190			86							0.44	0.25	1.79
200			69							0.40	0.25	1.65
210			53							0.36	0.24	1.50
220			39							0.32	0.24	1.35
230			26							0.27	0.23	1.19
240			15							0.22	0.22	1.01

[a] See footnote for table 5.

Table 8. Milk production marginal products and marginal rates of substitution[a] for specified levels of hay and grain feeding, estimated from equation 1 with T = 11 and other auxiliary variables set at mean levels.

Level of grain (lbs.)		Level of hay (lbs.): 50	100	150	200	250	300	350
50	Milk (lbs.)	130	165	195	220	239	253	261
	$\partial M / \partial G$	1.88	1.70	1.52	1.35	1.17	1.00	0.82
	$\partial M / \partial H$	0.76	0.65	0.54	0.44	0.33	0.22	0.11
	dH / dG	2.46	2.60	2.80	3.09	3.59	4.57	7.51
100	Milk (lbs.)	211	238	259	275	285	290	289
	$\partial M / \partial G$	1.37	1.19	1.02	0.84	0.67	0.49	—
	$\partial M / \partial H$	0.59	0.48	0.37	0.26	0.15	0.04	—
	dH / dG	2.34	2.51	2.77	3.25	4.43	11.73	—
150	Milk (lbs.)	267	285	297	304	306	302	292
	$\partial M / \partial G$	0.87	0.69	0.51	0.34	—	—	—
	$\partial M / \partial H$	0.41	0.30	0.19	0.08	—	—	—
	dH / dG	2.12	2.29	2.68	4.06	—	—	—
200	Milk (lbs.)	298	307	310	308	301	288	270
	$\partial M / \partial G$	0.36	0.19	0.01	—	—	—	—
	$\partial M / \partial H$	0.23	0.12	0.02	—	—	—	—
	dH / dG	1.55	1.49	0.59	—	—	—	—

[a] See footnote for table 5.

Table 9. **Milk production, marginal products and marginal rates of substitution[a] for specified levels of hay and grain feeding, estimated from equation 2 with T = 11 and other auxiliary variables set at mean levels.**

Level of grain (lbs.)		Level of hay (lbs.)						
		50	100	150	200	250	300	350
50	Milk (lbs.)	137	163	196	220	240	257	270
	$\partial M / \partial G$	1.70	1.58	1.46	1.34	1.22	1.11	0.99
	$\partial M / \partial H$	0.66	0.59	0.52	0.44	0.37	0.33	0.22
	dH / dG	2.55	2.67	2.82	3.03	3.31	3.73	4.44
100	Milk (lbs.)	211	236	258	276	291	301	308
	$\partial M / \partial G$	1.26	1.14	1.03	0.91	0.79	0.67	0.55
	$\partial M / \partial H$	0.55	0.47	0.40	0.33	0.25	0.18	0.10
	dH / dG	2.31	2.42	2.57	2.79	3.14	3.77	5.31
150	Milk (lbs.)	263	283	298	311	319	324	325
	$\partial M / \partial G$	0.83	0.71	0.59	0.47	0.36	0.24	0.12
	$\partial M / \partial H$	0.43	0.35	0.28	0.21	0.13	0.06	0.00
	dH / dG	1.94	2.01	2.11	2.29	2.67	3.98	—
200	Milk (lbs.)	294	307	317	324	326	325	320
	$\partial M / \partial G$	0.40	0.28	0.16	0.04	—	—	—
	$\partial M / \partial H$	0.31	0.24	0.16	0.09	—	—	—
	dH / dG	1.28	1.17	0.98	0.47	—	—	—

[a] See footnote for table 5.

Table 10. **Milk production, marginal products and marginal rates of substitution[a] for specified levels of hay and grain feeding, estimated from equation 3 with T = 11 and other auxiliary variables set at mean levels.**

Level of grain (lbs.)		Level of hay (lbs.)						
		50	100	150	200	250	300	350
50	Milk (lbs.)	142	171	197	220	240	256	269
	$\partial M / \partial G$	1.60	1.48	1.36	1.25	1.13	1.01	0.90
	$\partial M / \partial H$	0.62	0.56	0.49	0.43	0.36	0.30	0.23
	dH / dG	2.57	2.67	2.78	2.93	3.13	3.43	3.88
100	Milk (lbs.)	212	236	256	273	287	297	305
	$\partial M / \partial G$	1.21	1.10	0.98	0.86	0.75	0.63	0.51
	$\partial M / \partial H$	0.50	0.44	0.37	0.31	0.24	0.18	0.11
	dH / dG	2.41	2.50	2.62	2.80	3.06	3.52	4.50
150	Milk (lbs.)	263	281	295	307	315	319	321
	$\partial M / \partial G$	0.83	0.71	0.60	0.48	0.36	0.25	—
	$\partial M / \partial H$	0.39	0.32	0.26	0.19	0.13	0.06	—
	dH / dG	2.15	2.22	2.33	2.51	2.87	3.98	—
200	Milk (lbs.)	295	307	316	321	323	322	318
	$\partial M / \partial G$	0.45	0.33	0.22	0.10	—	—	—
	$\partial M / \partial H$	0.27	0.21	0.14	0.08	—	—	—
	dH / dG	1.66	1.62	1.54	1.31	—	—	—

[a] See footnote for table 5.

Table 11. Economic optima and return over feed cost estimated from equation 1, with T = 11 and other auxiliary variables set at mean levels.

Price per cwt. for:			Milk[a]	Least-cost ration (lbs.)		Profit[b]
Grain	Hay	Milk	(lbs.)	Grain	Hay	
$3.00	$0.75	$3.00	256	61	268	$3.84
3.00	1.00	3.00	261	91	180	3.28
3.00	1.25	3.00	258	122	92	2.94
3.00	1.75	3.00	248	154	0[c]	2.82
3.00	0.75	4.00	280	95	242	6.54
3.00	1.00	4.00	282	118	176	6.02
3.00	1.25	4.00	281	141	110	5.66
3.00	1.75	4.00	270	179	0[c]	5.43
3.00	0.75	5.00	291	115	226	9.40
3.00	1.00	5.00	293	133	173	8.90
3.00	1.25	5.00	292	152	120	8.53
3.00	1.75	5.00	282	189	15[c]	8.19

[a] Profit-maximizing level of milk output.
[b] Return over feed cost per cow per week.
[c] The physiological minimum level of hay should be fed.

Table 12. Economic optima and return over feed cost estimated from equation 2, with T = 11 and other auxiliary variables set at mean levels.

Price per cwt. for:			Milk[a]	Least-cost ration (lbs.)		Profit[b]
Grain	Hay	Milk	(lbs.)	Grain	Hay	
$3.00	$0.75	$3.00	269	56	321	$3.98
3.00	1.00	3.00	267	84	220	3.30
3.00	1.25	3.00	257	111	120	2.88
3.00	1.75	3.00	234	144	0[c]	2.70
3.00	$0.75	4.00	295	87	314	6.82
3.00	1.00	4.00	294	108	239	6.13
3.00	1.25	4.00	288	128	163	5.62
3.00	1.75	4.00	262	169	13[c]	5.18
3.00	0.75	5.00	306	106	310	9.83
3.00	1.00	5.00	306	122	250	9.13
3.00	1.25	5.00	302	138	190	8 58
3.00	1.75	5.00	286	171	69	7.94

[a] Profit-maximizing level of milk output.
[b] Return over feed cost per cow per week.
[c] The physiological minimum level of hay should be fed.

Table 13. Economic optima and return over feed cost estimated from equation 3, with T=11 and other auxiliary variables set at mean levels.

Price per cwt. for:			Milk[a]	Least-cost ration (lbs.)		Profit[b]
Grain	Hay	Milk	(lbs.)	Grain	Hay	
$3.00	$0.75	$3.00	255	30	371	$3.98
3.00	1.00	3.00	257	73	229	3.23
3.00	1.25	3.00	247	117	87	2.84
3.00	1.75	3.00	235	143	0[c]	2.77
3.00	0.75	4.00	285	70	348	6.71
3.00	1.00	4.00	286	102	241	5.97
3.00	1.25	4.00	281	135	134	5.50
3.00	1.75	4.00	264	176	0[c]	5.29
3.00	0.75	5.00	299	94	334	9.63
3.00	1.00	5.00	300	120	248	8.91
3.00	1.25	5.00	296	146	163	8.39
3.00	1.75	5.00	277	195	0[c]	8.00

[a] Profit-maximizing level of milk output.
[b] Return over feed cost per cow per week.
[c] The physiological minimum level of hay should be fed.

Table 14. Comparison of equations 1, 2 and 3 with respect to estimated economic optimum inputs and profit, with T = 11 and other auxiliary variables set at mean levels.

Price per cwt. for: Grain	Hay	Milk		Economic optima (lbs.) profit for equation 1	2	3
$3.00	$0.75	$4.00	Grain (lbs.)	95	87	70
			Hay (lbs.)	242	314	348
			Milk (lbs.)	280	295	285
			Profit ($)	6.54	6.82	6.71
3.00	1.00	4.00	Grain (lbs.)	118	108	102
			Hay (lbs.)	176	239	241
			Milk (lbs.)	283	294	286
			Profit ($)	6.02	6.13	5.97
3.00	1.25	4.00	Grain (lbs.)	141	128	135
			Hay (lbs.)	110	163	134
			Milk (lbs.)	281	288	281
			Profit ($)	5.66	5.62	5.50
3.00	1.75	4.00	Grain (lbs.)	179	169	176
			Hay (lbs.)	0[a]	13[a]	0[a]
			Milk (lbs.)	270	262	264
			Profit ($)	5.43	5.18	5.29

[a] The physiological minimum level of hay should be fed.

Table 15. Economic optima and return over feed cost estimated from equation 1 for the first experimental week (T = 1); all other auxiliary variables set at mean levels.

Price per cwt. for: Grain	Hay	Milk	Milk[a] (lbs.)	Least-cost ration (lbs.) Grain	Hay	Profit[b]
$3.00	$0.75	$3.00	269	73	247	$5.35
3.00	1.00	3.00	319	104	159	4.85
3.00	1.25	3.00	316	135	71	4.56
3.00	1.75	3.00	309	160	0[c]	4.49
3.00	0.75	4.00	338	107	221	8.63
3.00	1.00	4.00	340	130	155	8.16
3.00	1.25	4.00	339	153	89	7.85
3.00	1.75	4.00	331	184	0[c]	7.70
3.00	0.75	5.00	349	128	206	12.07
3.00	1.00	5.00	351	146	153	11.62
3.00	1.25	5.00	350	164	100	11.30
3.00	1.75	5.00	341	199	0[c]	11.07

[a] Profit-maximizing level of milk output.
[b] Return over feed cost per cow per week.
[c] The physiological minimum level of hay should be fed.

Table 16. Economic optima and return over feed cost estimated from equation 1 for the fifth experimental week (T = 5); all other auxiliary variables set at mean levels.

Price per cwt. for: Grain	Hay	Milk	Milk[a] (lbs.)	Least-cost ration (lbs.) Grain	Hay	Profit[b]
$3.00	$0.75	$3.00	288	68	256	$4.68
3.00	1.00	3.00	293	99	167	4.15
3.00	1.25	3.00	291	130	79	3.84
3.00	1.75	3.00	283	158	0[c]	3.75
3.00	0.75	4.00	312	102	229	7.70
3.00	1.00	4.00	315	125	163	7.21
3.00	1.25	4.00	314	148	97	6.88
3.00	1.75	4.00	304	182	0[c]	6.70
3.00	0.75	5.00	323	123	214	10.88
3.00	1.00	5.00	325	141	161	10.41
3.00	1.25	5.00	324	159	108	10.08
3.00	1.75	5.00	315	196	2[c]	9.80

[a] Profit-maximizing level of milk output.
[b] Return over feed per cow per week.
[c] The physiological minimum level of hay should be fed.

Table 17. Milk isoquants, marginal products and marginal rates of substitution based on equation I, for first experimental week (T = 1); all other auxiliary variables set at mean levels.

Level of grain (lbs.)	Pounds of hay required to maintain milk output of: 200 lbs.	300 lbs.	Derived quantities[a] along indicated milk isoquants (lbs.): 200 $\partial M/\partial G$	200 $\partial M/\partial H$	200 dH/dG	300 $\partial M/\partial G$	300 $\partial M/\partial H$	300 dH/dG
30	132		1.84	0.65	2.82			
40	105	344	1.84	0.68	2.71	1.00	0.16	6.32
50	78	291	1.83	0.70	2.62	1.08	0.24	4.56
60	52	249	1.82	0.72	2.53	1.13	0.29	3.85
70	27	213	1.81	0.74	2.44	1.16	0.34	3.43
80	3	180	1.79	0.76	2.37	1.17	0.37	3.14
90		150				1.18	0.40	2.92
100		122				1.17	0.43	2.73
110		95				1.17	0.45	2.58
120		70				1.15	0.47	2.45
130		46				1.14	0.49	2.33

[a] See footnote for table 5.

Table 18. Milk isoquants, marginal products and marginal rates of substitution based on equation I, for fifth experimental week (T = 5); all other auxiliary variables set at mean levels.

Level of grain (lbs.)	Pounds of hay required to maintain milk output of: 200 lbs.	300 lbs.	Derived quantities[a] along indicated milk isoquants (lbs.): 200 $\partial M/\partial G$	200 $\partial M/\partial H$	200 dH/dG	300 $\partial M/\partial G$	300 $\partial M/\partial H$	300 dH/dG
30	167		1.90	0.58	2.94			
40	138		1.70	0.60	2.81			
50	111		1.69	0.63	2.69			
60	85		1.69	0.65	2.59			
70	59	310	1.67	0.67	2.49	0.79	0.12	6.32
80	35	259	1.66	0.69	2.41	0.87	0.20	4.33
90	11	220	1.64	0.71	2.33	0.91	0.25	3.61
100		186				0.92	0.29	3.20
110		156				0.93	0.32	2.91
120		128				0.93	0.35	2.68
130		102				0.92	0.37	2.50
140		78				0.90	0.38	2.35
150		55				0.88	0.40	2.21

[a] See footnote for table 5.

Table 19. Milk production, marginal products and marginal rates of substitution[a] for specified levels of hay and grain feeding during first experimental week, estimated from equation I with other auxiliary variables set at mean levels.

Level of Grain (lbs.)		Level of hay (lbs.): 50	100	150	200	250	300
50	Milk (lbs.)	180	215	245	269	288	302
	$\partial M / \partial G$	1.93	1.76	1.58	1.40	1.23	1.05
	$\partial M / \partial H$	0.76	0.65	0.54	0.44	0.33	0.22
	dH / dG	2.54	2.69	2.90	3.22	3.76	4.82
100	Milk (lbs.)	264	290	311	327	337	342
	$\partial M / \partial G$	1.43	1.25	1.07	0.90	0.72	0.55
	$\partial M / \partial H$	0.59	0.48	0.37	0.26	0.15	0.04
	dH / dG	2.44	2.62	2.92	3.46	4.80	13.06
150	Milk (lbs.)	322	340	352	359	361	357
	$\partial M / \partial G$	0.92	0.75	0.57	0.39	—	—
	$\partial M / \partial H$	0.41	0.30	0.19	0.08	—	—
	dH / dG	2.25	2.48	2.97	4.73	—	—
200	Milk (lbs.)	356	365	368	366	359	346
	$\partial M / \partial G$	0.42	0.24	0.07	—	—	—
	$\partial M / \partial H$	0.23	0.12	0.02	—	—	—
	dH / dG	1.79	1.93	4.08	—	—	—

[a] See footnote for table 5.

Table 20. Milk production, marginal products and marginal rates of substitution[a] for specified levels of hay and grain feeding during the 5th experimental week, estimated from equation I with other auxiliary variables set at mean levels.

Level of Grain (lbs.)		Level of hay (lbs.) 50	100	150	200	250	300
50	Milk (lbs.)	158	193	223	247	266	280
	$\partial M / \partial G$	1.91	1.73	1.56	1.38	1.20	1.03
	$\partial M / \partial H$	0.76	0.65	0.54	0.44	0.33	0.22
	dH / dG	2.51	2.65	2.86	3.17	3.69	4.72
100	Milk (lbs.)	240	267	288	304	314	319
	$\partial M / \partial G$	1.40	1.23	1.05	0.88	0.70	0.52
	$\partial M / \partial H$	0.59	0.48	0.37	0.26	0.15	0.04
	dH / dG	2.40	2.58	2.86	3.38	4.65	12.53
150	Milk (lbs.)	298	316	328	335	336	332
	$\partial M / \partial G$	0.90	0.72	0.55	0.37	—	—
	$\partial M / \partial H$	0.41	0.30	0.19	0.08	—	—
	dH / dG	2.20	2.41	2.85	4.46	—	—
200	Milk (lbs.)	330	339	343	341	334	321
	$\partial M / \partial G$	0.40	0.22	0.04	—	—	—
	$\partial M / \partial H$	0.23	0.12	0.02	—	—	—
	dH / dG	1.69	1.76	2.69	—	—	—

[a] See footnote for table 5.

Table 21. Comparison of economic optima at different stages of lactation with various price combinations; temperature set at mean; cow characteristics set at mean for equation I.

Price per cwt. for: Grain	Hay	Milk		Week of experiment 1	5	11
$3.00	$0.75	$4.00	Grain (lbs.)	107	102	95
			Hay (lbs.)	221	229	242
			Milk (lbs.)	338	312	280
			Profit ($)	8.63	7.70	6.54
3.00	1.00	4.00	Grain (lbs.)	130	125	118
			Hay (lbs.)	155	163	176
			Milk (lbs.)	340	315	283
			Profit ($)	8.16	7.21	6.02
3.00	1.25	4.00	Grain (lbs.)	153	148	141
			Hay (lbs.)	89	97	110
			Milk (lbs.)	339	314	281
			Profit ($)	7.85	6.88	5.66
3.00	1.75	4.00	Grain (lbs.)	184	182	179
			Hay (lbs.)	0[a]	0[a]	0[a]
			Milk (lbs.)	331	304	270
			Profit ($)	7.70	6.70	5.43

[a] The physiological minimum level of hay should be fed.

Table 22. Economic optima and return over feed cost estimated from equation I for low temperatures (F = 10); all other auxiliary variables set at mean levels.

Price per cwt. for: Grain	Hay	Milk	Milk[a] (lbs.)	Least-cost ration (lbs.) Grain	Hay	Profit[b]
$3.00	$0.75	$3.00	247	61	268	$3.57
3.00	1.00	3.00	252	91	180	3.01
3.00	1.25	3.00	250	122	92	2.67
3.00	1.75	3.00	239	154	0[c]	2.55
3.00	0.75	4.00	271	95	242	6.18
3.00	1.00	4.00	274	118	176	5.66
3.00	1.25	4.00	272	141	110	5.30
3.00	1.75	4.00	261	179	0[c]	5.07
3.00	0.75	5.00	282	115	226	8.95
3.00	1.0C	5.00	284	133	173	8.45
3.00	1.25	5.00	283	152	120	8.08
3.00	1.75	5.00	273	189	15[c]	7.74

[a] Profit-maximizing level of milk output.
[b] Return over feed cost per cow per week.
[c] The physiological minimum level of hay should be fed.

Table 23. Economic optima and return over feed cost estimated from equation I for high temperatures (F = 90); T = 11 and other auxiliary variables set at mean levels.

Price per cwt. for: Grain	Hay	Milk	Milk[a] (lbs.)	Least-cost ration (lbs.) Grain	Hay	Profit[b]
$3.00	$0.75	$3.00	238	61	268	$3.32
3.00	1.00	3.00	244	91	180	2.76
3.00	1.25	3.00	241	122	92	2.42
3.00	1.75	3.00	231	154	0[c]	2.30
3.00	0.75	4.00	262	95	242	5.84
3.00	1.00	4.00	265	118	176	5.32
3.00	1.25	4.00	264	141	110	4.97
3.00	1.75	4.00	253	179	0[c]	4.74
3.00	0.75	5.00	273	115	226	8.53
3.00	1.00	5.00	275	133	173	8.03
3.00	1.25	5.00	274	152	120	7.66
3.00	1.75	5.00	265	189	15[c]	7.33

[a] Profit-maximizing level of milk output.
[b] Return over feed per cow per week.
[c] The physiological minimum level of hay should be fed.

Table 24. Milk isoquants, marginal products and marginal rates of substitution based on equation I, for low temperature (F = 10); T = 11 and other auxiliary variables set at mean levels.

Level of grain (lbs.)	Pounds of hay required to maintain milk output of: 100 lbs.	200 lbs.	Derived quantities[a] along indicated milk isoquants (lbs.) 100: $\frac{\partial M}{\partial G}$	100: $\frac{\partial M}{\partial H}$	100: $\frac{dH}{dG}$	200: $\frac{\partial M}{\partial G}$	200: $\frac{\partial M}{\partial H}$	200: $\frac{dH}{dG}$
20	99	274	2.01	0.76	2.64	1.39	0.38	3.65
30	73	239	2.00	0.78	2.55	1.41	0.42	3.35
40	48	207	1.98	0.80	2.47	1.43	0.46	3.12
50	23	176	1.97	0.82	2.40	1.43	0.49	2.94
60		148				1.43	0.51	2.78
70		121				1.43	0.54	2.65
80		95				1.42	0.56	2.53
90		70				1.40	0.58	2.43
100		46				1.38	0.59	2.33

[a] See footnote for table 5.

Table 25. Milk isoquants, marginal products and marginal rates of substitution based on equation I, for high temperature (F = 90); T = 11 and other auxiliary variables set at mean levels.

Level of grain (lbs.)	Pounds of hay required to maintain milk output of: 100 lbs.	200 lbs.	Derived quantities[a] along indicated milk isoquants (lbs.) 100: $\frac{\partial M}{\partial G}$	100: $\frac{\partial M}{\partial H}$	100: $\frac{dH}{dG}$	200: $\frac{\partial M}{\partial G}$	200: $\frac{\partial M}{\partial H}$	200: $\frac{dH}{dG}$
20	110	297	1.97	0.74	2.67	1.31	0.33	3.97
30	84	260	1.96	0.76	2.58	1.34	0.38	3.56
40	58	226	1.95	0.78	2.50	1.36	0.41	3.27
50	34	194	1.93	0.80	2.42	1.37	0.45	3.05
60	10	164	1.92	0.81	2.35	1.37	0.48	2.87
70		137				1.37	0.50	2.72
80		110				1.36	0.53	2.59
90		85				1.35	0.55	2.47
100		60				1.33	0.56	2.37

[a] See footnote for table 5.

Table 26. Body weights and ability indexes with maturity and the coefficient of inbreeding (K) set at different levels.

Maturity (Age, months)	Body weights (lbs.) for: K=0	Body weights (lbs.) for: K=25	Ability index for: K=0	Ability index for: K=25
26	1,075	975	2,794	2,400
66	1,359	1,259	3,396	3,002

Table 27. Economic optima and return over feed cost estimated from equation I for heifers with no inbreeding (K=0)[a]; T=11 and all other auxiliary variables set at mean levels.

Price per cwt. for: Grain	Hay	Milk	Milk[b] (lbs.)	Least-cost ration (lbs.) Grain	Hay	Profit[c]
$3.00	$0.75	$3.00	224	27	337	$3.39
3.00	1.00	3.00	229	57	249	2.66
3.00	1.25	3.00	227	88	161	2.15
3.00	1.75	3.00	204	144	0[d]	1.78
3.00	0.75	4.00	248	61	311	5.77
3.00	1.00	4.00	251	84	245	5.07
3.00	1.25	4.00	249	107	179	4.55
3.00	1.75	4.00	235	153	47	3.98
3.00	0.75	5.00	259	81	295	8.31
3.00	1.00	5.00	261	99	242	7.64
3.00	1.25	5.00	260	118	189	7.10
3.00	1.75	5.00	250	155	84	6.42

[a] See fig. 10 for magnitudes of auxiliary variables.
[b] Profit-maximizing level of milk output.
[c] Return over feed cost per cow per week.
[d] The physiological minimum level of hay should be fed.

Table 28. Economic optima and return over feed cost estimated from equation I for mature cows with no inbreeding (K=0)[a]; T=11 and other auxiliary variables set at mean levels.

Price per cwt. for: Grain	Hay	Milk	Milk[b] (lbs.)	Least-cost ration (lbs.) Grain	Hay	Profit[c]
$3.00	$0.75	$3.00	242	94	197	$2.96
3.00	1.00	3.00	247	124	109	2.58
3.00	1.25	3.00	244	155	21[d]	2.41
3.00	1.75	3.00	243	162	0[d]	2.41
3.00	0.75	4.00	266	128	171	5.51
3.00	1.00	4.00	268	151	105	5.17
3.00	1.25	4.00	267	174	39	4.99
3.00	1.75	4.00	264	187	0[d]	4.96
3.00	0.75	5.00	277	148	156	8.23
3.00	1.00	5.00	278	166	103	7.91
3.00	1.25	5.00	278	185	50	7.72
3.00	1.75	5.00	274	202	0[d]	7.66

[a] See fig. 11 for magnitudes of auxiliary variables.
[b] Profit-maximizing level of milk output.
[c] Return over feed cost per cow per week.
[d] The physiological minimum level of hay should be fed.

Table 29. Economic optima and return over feed cost estimated from equation I for inbred heifers (K=25)[a]; T=11 and other auxiliary variables set at mean levels.

Price per cwt. for: Grain	Hay	Milk	Milk[b] (lbs.)	Least-cost ration (lbs.) Grain	Hay	Profit[c]
$3.00	$0.75	$3.00	215	0	404	$3.41
3.00	1.00	3.00	216	25	325	2.49
3.00	1.25	3.00	214	56	237	1.79
3.00	1.75	3.00	187	117	61	1.04
3.00	0.75	4.00	235	28	387	5.66
3.00	1.00	4.00	238	51	321	4.78
3.00	1.25	4.00	237	74	255	4.06
3.00	1.75	4.00	222	120	123	3.11
3.00	0.75	5.00	246	48	371	8.08
3.00	1.00	5.00	248	67	319	7.21
3.00	1.25	5.00	247	85	266	6.48
3.00	1.75	5.00	238	122	160	5.42

[a] See fig. 12 for magnitudes of auxiliary variables.
[b] Profit-maximizing level of milk output.
[c] Return over feed cost per cow per week.

Table 30. Economic optima and return over feed cost estimated from equation I for inbred mature cows (K=25)[a]; T=11 and other auxiliary variables set at mean levels.

Price per cwt. for: Grain	Hay	Milk	Milk[b] (lbs.)	Least-cost ration (lbs.) Grain	Hay	Profit[c]
$3.00	$0.75	$3.00	239	61	274	$3.30
3.00	1.00	3.00	245	92	186	2.72
3.00	1.25	3.00	242	123	98	2.37
3.00	1.75	3.00	231	157	0[d]	2.23
3.00	0.75	4.00	263	95	248	5.83
3.00	1.00	4.00	266	118	182	5.29
3.00	1.25	4.00	265	141	116	4.92
3.00	1.75	4.00	253	181	0[d]	4.67
3.00	0.75	5.00	274	115	232	8.53
3.00	1.00	5.00	276	134	179	8.01
3.00	1.25	5.00	275	152	126	7.63
3.00	1.75	5.00	266	189	21[d]	7.26

[a] See fig. 13 for magnitudes of auxiliary variables.
[b] Profit-maximizing level of milk output.
[c] Return over feed cost per cow per week.
[d] The physiological minimum level of hay should be fed.

Table 31. Milk production, marginal products and marginal rates of substitution[a] for specified levels of hay and grain feeding for heifers with no inbreeding[b]; T=11 and other auxiliary variables set at mean levels; estimated from equation 1.

Level of Grain (lbs.)		Level of hay (lbs.) 50	100	150	200	250	300
50	Milk (lbs.)	107	144	175	201	222	237
	∂M / ∂G	1.78	1.68	1.42	1.25	1.07	0.89
	∂M / ∂H	0.79	0.68	0.57	0.47	0.36	0.25
	dH / dG	2.24	2.34	2.48	2.68	3.00	3.61
100	Milk (lbs.)	183	211	234	251	263	269
	∂M / ∂G	1.27	1.09	0.92	0.74	0.57	0.39
	∂M / ∂H	0.62	0.51	0.40	0.29	0.18	0.07
	dH / dG	2.06	2.16	2.31	2.56	3.13	5.41
150	Milk (lbs.)	234	253	267	275		
	∂M / ∂G	0.77	0.59	0.41	0.24		
	∂M / ∂H	0.44	0.33	0.22	0.11		
	dH / dG	1.74	1.78	1.86	2.09		
200	Milk (lbs.)	260	270				
	∂M / ∂G	0.26	0.09				
	∂M / ∂H	0.26	0.16				
	dH / dG	0.99	0.55				

[a] See footnote for table 5.
[b] See fig. 10.

Table 32. Milk production, marginal products and marginal rates of substitution[a] for specified levels of hay and grain feeding for mature cows with no inbreeding[b]; T=11 and other auxiliary variables set at mean levels; estimated from equation 1.

Level of Grain (lbs.)		Level of hay (lbs.) 50	100	150	200	250	300
50	Milk (lbs.)	105	139	167	189	207	218
	∂M / ∂G	1.96	1.78	1.61	1.43	1.26	1.08
	∂M / ∂H	0.72	0.62	0.51	0.40	0.29	0.18
	dH / dG	2.71	2.90	3.17	3.60	4.34	5.98
100	Milk (lbs.)	191	215	235	248	257	
	∂M / ∂G	1.45	1.28	1.10	0.93	0.75	
	∂M / ∂H	0.55	0.44	0.33	0.22	0.11	
	dH / dG	2.65	2.91	3.33	4.18	6.63	
150	Milk (lbs.)	251	267	277	282		
	∂M / ∂G	0.95	0.77	0.60	0.42		
	∂M / ∂H	0.37	0.26	0.15	0.05		
	dH / dG	2.55	2.94	3.87	9.20		
200	Milk (lbs.)	286	293				
	∂M / ∂G	0.45	0.27				
	∂M / ∂H	0.20	0.09				
	dH / dG	2.27	3.09				

[a] See footnote for table 5.
[b] See fig. 11.

Table 33. Milk production, marginal products and marginal rates of substitution[a] for specified levels of hay and grain feeding for inbred heifers[b]; T=11 and other auxiliary variables set at mean levels; estimated from equation I.

Level of Grain (lbs.)		Level of hay (lbs.) 50	100	150	200	250	300
50	Milk (lbs.)	88	128	162	190	214	231
	$\partial M / \partial G$	1.72	1.54	1.36	1.19	1.01	0.84
	$\partial M / \partial H$	0.84	0.73	0.63	0.52	0.41	0.30
	dH / dG	2.03	2.10	2.18	2.30	2.48	2.79
100	Milk (lbs.)	162	192	217	237	252	260
	$\partial M / \partial G$	1.21	1.03	0.36	0.68	0.51	0.33
	$\partial M / \partial H$	0.67	0.56	0.45	0.34	0.23	0.12
	dH / dG	1.81	1.85	1.91	2.00	2.18	2.67
150	Milk (lbs.)	210	231	248	259		
	$\partial M / \partial G$	0.71	0.53	0.35	0.18		
	$\partial M / \partial H$	0.49	0.38	0.27	0.17		
	dH / dG	1.44	1.39	1.29	1.08		
200	Milk (lbs.)	232	245				
	$\partial M / \partial G$	0.20	0.03				
	$\partial M / \partial H$	0.32	0.21				
	dH / dG	0.64	0.12				

[a] See footnote for table 5.

[b] See fig. 12.

Table 34. Milk production, marginal products and marginal rates of substitution[a] for specified levels of hay and grain feeding for inbred mature cows[b]; T=11 and other auxiliary variables set at mean levels; estimated from equation I.

Level of Grain (lbs.)		Level of hay (lbs.) 50	100	150	200	250	300
50	Milk (lbs.)	109	145	175	201	220	235
	$\partial M / \partial G$	1.90	1.72	1.55	1.37	1.20	1.02
	$\partial M / \partial H$	0.78	0.67	0.56	0.45	0.34	0.23
	dH / dG	2.45	2.58	2.77	3.05	3.51	4.39
100	Milk (lbs.)	191	218	240	256	267	273
	$\partial M / \partial G$	1.40	1.22	1.04	0.87	0.69	0.51
	$\partial M / \partial H$	0.60	0.49	0.38	0.27	0.16	0.06
	dH / dG	2.33	2.48	2.73	3.17	4.20	9.20
150	Milk (lbs.)	248	267	280	287		
	$\partial M / \partial G$	0.89	0.71	0.54	0.36		
	$\partial M / \partial H$	0.42	0.31	0.21	0.10		
	dH / dG	2.10	2.27	2.61	3.72		
200	Milk (lbs.)	280	290	294			
	$\partial M / \partial G$	0.39	0.21	0.03			
	$\partial M / \partial H$	0.25	0.14	0.03			
	dH / dG	1.56	1.51	1.12			

[a] See footnote for table 5.

[b] See fig. 13.

Table 35. **Estimated economic optima for maturity and coefficient of inbreeding each at two levels, with weight and ability adjusted[a]; temperature set at experimental means and T = 11.**

Prices per cwt. for:			Item	K = O		K = 25	
Grain	Hay	Milk		J=26 984[b] 2,255[c]	J=66 1,250[b] 2,711[c]	J=26 898[b] 1,908[c]	J=66 1,164[b] 2,364[c]
$3.00	$0.75	$4.00	Grain (lbs.)	61	128	28	95
			Hay (lbs.)	311	171	387	248
			Milk (lbs.)	248	266	235	263
			Profit ($)	5.77	5.51	5.66	5.83
3.00	1.00	4.00	Grain (lbs.)	84	151	51	118
			Hay (lbs.)	245	105	321	182
			Milk (lbs.)	251	268	238	266
			Profit ($)	5.07	5.17	4.78	5.29
3.00	1.25	4.00	Grain (lbs.)	107	174	74	141
			Hay (lbs.)	179	39	255	116
			Milk (lbs.)	249	267	237	265
			Profit ($)	4.55	4.99	4.06	4.92
3.00	1.75	4.00	Grain (lbs.)	153	187	120	181
			Hay (lbs.)	47	0[d]	123	0[d]
			Milk (lbs.)	235	264	222	253
			Profit ($)	3.98	4.96	3.11	4.67

[a] Weight and ability are adjusted for maturity and inbreeding as indicated in table 26.
[b] Body weight.
[c] Ability index.
[d] The physiological minimum level of hay should be fed.

Table 36. **Profit[a] increase[b] with maturity for outbred cow versus highly inbred cow under various price ratios; temperature set at experimental means and T = 11; ability and weight adjusted for inbreeding and age, as in table 35.**

Price per cwt. for:			Increase in profit with increase in age from 26 to 66 months	
Grain	Hay	Milk	K = 0	K = 25
$3.00	$0.75	$4.00	$0.26	$0.17
3.00	1.00	4.00	0.31	0.51
3.00	1.25	4.00	0.44	0.86
3.00	1.75	4.00	0.98	1.56

[a] Profit, as defined here, is return over feed cost per week per cow.
[b] Profit increase is defined as profit for mature cow minus profit for heifer, at economic optima with indicated price ratios.