Application of Distributed Lag and Autocorrelated Error Models to Short-Run Demand Analysis

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CONTENTS

SUMMARY

The objective of the research reported here was to investigate the usefulness of distributed lag economic models and autocorrelated error statistical models for analysis of monthly and quarterly food demand. Distributed lags are a way of incorporating dynamic considerations into econometric models of consumer demand. In the distributed lag model used here, current consumption is the dependent variable, and lagged consumption is one explanatory variable. Testing the significance of the coefficient of lagged consumption tests the hypothesis of a lag in consumer adjustment to conditions affecting demand.

The presence of autocorrelated errors can have serious effects on least squares (L.S.) estimates of coefficients. Autocorrelated errors may frequently occur in equations fitted to monthly and quarterly data. Therefore, equations were estimated by autoregressive least squares (A.L.S.) as well as by least squares. A.L.S.-1 assumes the errors u_t to follow a first order autoregressive scheme, $u_t = \beta_1 u_{t-1} + e_t$. It provides simultaneous estimates of β_1 and of the coefficients in the demand equation. A.L.S.-2 assumes the errors to be generated by a second order autoregressive process, $u_t = \beta_1 u_{t-1} + \beta_2 u_{t-2} + e_t$. It provides simultaneous estimates of β_1, β_2 and the coefficients in the demand equation.

Data from the Michigan State University consumer panel were used. Static and dynamic equations for beef, pork, fryers and total fresh red meat were estimated with monthly and quarterly data by L.S. and by A.L.S. Static and dynamic demand equations for cheese, eggs and fresh milk were estimated with quarterly data by L.S. and A.L.S.

Distributed lag models appear to be sufficiently useful in the analysis of monthly and quarterly data to justify their regular use in such analyses. There was strong evidence (significant at the 5- or 1-percent level) of a lag in consumer adjustment in monthly pork and fryer demand and quarterly cheese demand. There was weak evidence (significant at the 10-percent level) of a lag in consumer reaction in monthly total meat demand and in quarterly fryer and fresh milk demand. For the commodities for which there was a lag in adjustment, the total adjustment was substantially completed within 1 year of the price or income change.

Our results suggest that the econometrician using monthly or quarterly data would be wise to assume autocorrelated errors and to estimate his equations accordingly. There was significant evidence of autocorrelated errors in half the equations estimated. Estimation with A.L.S. resulted in important differences in magnitudes and levels of significance of coefficients. One-fifth of the coefficients in equations with autocorrelated errors were significant by L.S., but nonsignificant by A.L.S., or vice versa. In nearly all of the equations in which A.L.S. indicated the presence of autocorrelated errors, half or more of the A.L.S. coefficients differed from the corresponding L.S. coefficients by 20 percent or more.

It appears that second-order error models are rarely so useful as first-order error models.

It has been argued that one reason for the existence of autocorrelated errors is the omission of relevant variables and, specifically, that the absence of lagged values of the dependent variable is one cause of autocorrelated errors. The results of this study suggest that the addition of relevant variables may introduce autocorrelation about as frequently as it eliminates autocorrelation. This holds for lagged values of the dependent variable as well as for other variables.

Hildreth and Lu have proposed a method for obtaining estimates under the assumption of first-order autoregressive errors. The A.L.S. estimation procedure was compared with the Hildreth and Lu method. Differences between the coefficients were negligible. The von Neumann-Hart ratio and the Durbin-Watson d statistic appear to be weak tests for autocorrelation in errors. A.L.S. estimation appears to furnish a more powerful test for autocorrelation, especially in equations containing the lagged dependent variable.

More work is needed, either theoretical or Monte Carlo or both, so that we may develop improved tests for autocorrelation in errors and may learn more about the properties of A.L.S. estimates and of other procedures that may be used in the presence of autocorrelated errors.

For the food items studied extensively in this research, the significant economic determinants of demand were:

(a) Beef—beef price, pork price, income;

(b) Pork—beef price, pork price, fryer price;

(c) Fryers—fryer price, price of meats other than beef and pork, income;

(d) Total fresh red meat—fryer price, meat price, income;

(e) Cheese—cheese price, income;

(f) Eggs—egg price, fryer price;

(g) Fluid milk—dried milk price.

Temperature has a significant effect on food demand. There are, however, seasonal variations in demand that are not adequately explained by temperature. Results obtained by the use of seasonal dummy variables ($D_{it} = 1$ in i-th season; $D_{it} = 0$ in all other seasons) were superior to those obtained by the use of temperature.

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Much effort has been expended in analyzing time series data to determine characteristics of consumer demand for food. Usually, the year has been used in these analyses as the unit observation period, and the analyses have been based on static economic theory. In recent years, dynamic economic theory has been used in food-demand analyses through the introduction of the concept of distributed lags. In most studies, however, the year still is used as the unit observation period. Recently an increasing number of studies have been made with the quarter or the month as the unit observation period.

It is quite possible that, at least for some commodities, dynamic influences are important determinants of monthly or quarterly demand, whereas static considerations are sufficient to explain annual demand. There may be a lag in consumers' reactions to changes in determinants of demand, and the lags may be more than 1 month or 1 quarter, but less than 1 year, in duration. Conditions would be expected to be more favorable to the validity of distributed lag models in monthly or quarterly data than in annual data.

The concepts of distributed lags might be fruitfully used in analyses of monthly or quarterly data. Problems arising from autocorrelated errors, however, may be serious with the use of these short unit observation periods. Since the presence of autocorrelated errors can lead to inefficient estimates when the least squares method is used, and even to biased estimates in distributed lag models, it is desirable to investigate the autocorrelation properties of the errors when using short unit observation periods.

In this study, Michigan State University consumer panel data were analyzed by using the quarter and then 4 weeks as the observation period. The objective of the study was to determine the usefulness of distributed lag models and autocorrelated error models in the analysis of monthly and quarterly consumer demand.

DYNAMICS OF DEMAND

Stigler (27, pp. 93-95), Friedman (4), Duesenberry (2), Modigliani (22), Katona (15, p. 43), Bilkey (1, p. 150), Nerlove (23), Ladd and Tedford (19) and others have argued that there may be lags in consumer responses to new situations. Several writers have used the dichotomy of short-run and long-run elasticities to describe one type of lag.

In the theory of the firm, the distinction between the short run and the long run traditionally has been related to the distinction between fixed and variable inputs. In the short run, the available volume of some inputs is fixed; in the long run, the volume of all inputs is variable.

The distinction between short-run and long-run consumer demand elasticities may be clarified by expressing a model of consumer demand as a maximization problem, with an objective (utility) function and constraints. If the only constraint is the budget constraint, we have formulated a model of long-run behavior. In the decisions a consumer makes this month, however, there are other operational constraints. The number (more precisely the area) of paintings, murals or reproductions purchased may be limited by the number of square feet of wall space available. The amount of frozen food purchased may be limited by the volume of freezer space available in the refrigerator or home freezer. The amount of clothing purchased may be restricted by closet and dresser drawer space available. A model containing these or similar constraints, as well as the income constraint, is a model of short-run behavior. Long-run elasticities are obtained from the first model; short-run elasticities, from the second model.

It can be proven that a long-run elasticity of supply can be no smaller than the short-run

[1] Project 1355 of the Iowa Agricultural and Home Economics Experiment Station. The research reported here was partially financed by a grant from the National Science Foundation. The authors are grateful to Professor James D. Shaffer of the Michigan State University Department of Agricultural Economics for furnishing data from the operation of the Michigan State University consumer panel.

elasticity. The same conclusion would apply to the demand elasticities derived from these models. This distinction between short-run and long-run demand elasticities is analogous to the distinction between short run and long run in the theory of the firm.

Unfortunately this distinction is not comprehensive. Among the other bases advanced for expecting differences between immediate (short-run) and delayed (long-run) responses are imperfect knowledge and habit. Consumers may not be immediately aware of price changes. Even if they are aware, habit may delay their reaction. Time and experimentation may be required to discover the new optimum plan. With frequently purchased staples, such as meats, imperfect knowledge may be of little importance in producing lagged effects. Habit, on the other hand, may be an important factor in producing these lagged effects with frequently purchased items. Small changes in the price of a frequently purchased item may not result in a re-evaluation of the consumer's consumption pattern. However, if these small changes persist in the same direction over a period of time, the price differential may become large enough to warrant the consumer's re-evaluation of his consumption pattern.

Koyck Model

Koyck has presented one model that may be used to measure contemporaneous and lagged effects (17). Suppose current demand, y_t, can reasonably be stated as a linear function of current and past prices, x_{1t} and x_{1t-i}, and current and past income, x_{2t} and x_{2t-i}. Ignoring constant terms, current demand is

$$(1.1)\quad y_t = \Sigma b_{1i}\, x_{1t-i} + \Sigma b_{2i}\, x_{2t-i} + u_t,\quad i = 0, 1, \ldots, n.$$

If $b_{1i} = b_{2i} = 0$ for $i \geqslant 1$, this reduces to a conventional static linear demand equation.

Assume

$$(1.2)\quad \frac{b_{1i}}{b_{1i-1}} = \frac{b_{2i}}{b_{2i-1}} = \lambda;\ i \geqslant 1;\ -1 < \lambda < 1.$$

Substituting equation 1.2 into equation 1.1:

$$(1.3)\quad y_t = b_{10}\, \Sigma\lambda^i x_{1t-i} + b_{20}\Sigma\lambda^i x_{2t-i} + u_t.$$

Multiplying equation 1.3 lagged one period by λ and subtracting from equation 1.1 yields

$$(1.4)\quad y_t = b_{10}x_{1t} + b_{20}\, x_{2t} + \lambda\, y_{t-1} + u_t - \lambda\, u_{t-1}.$$

Assume that, up to time period zero, x_{1t} and x_{2t} have been zero and that, between time periods zero and one, x_{1t} rises to 1 and then remains constant. Assume $u_{t-j} = 0$ for all j. After this once-for-all change in x_{1t}, actual demand has achieved the new equilibrium level of demand when $y_t = y_{t-1}$. Denote this equilibrium level by $(y)_t$.

$$(1.5)\quad (y)_t = \frac{b_{10}}{1-\lambda}\ .$$

Then $b_{10}\, \bar{x}_1/(1-\lambda)\bar{y}$ is the long-run elasticity. Actual consumption at any time will be

$$(1.6)\quad y_t = b_{10}\, \frac{1-\lambda^t}{1-\lambda}\ .$$

Since

$$(1.7)\quad y_t - y_{t-1} = b_{10}\, \lambda^{t-1}$$

and

$$(1.8)\quad (y)_t - y_{t-1} = \frac{b_{10}\, \lambda^{\,t-1}}{1-\lambda}$$

it follows that

$$(1.9)\quad y_t - y_{t-1} = (1-\lambda)\ [(y)_t - y_{t-1}]$$

and also that

$$(1.10)\quad \frac{y_t}{(y)_t} = 1-\lambda^t\ .$$

This indicates that, at the end of each period t, the proportion $1-\lambda^t$ of the total adjustment will have taken place.

From equation 1.9

$$(1.11)\quad (y)_t = \frac{y_t - \lambda y_{t-1}}{1-\lambda}\ .$$

Substituting 1.4

$$(1.12\quad (y)_t = \frac{b_{10}}{1-\lambda} x_{1t} + \frac{b_{20}}{1-\lambda} x_{2t} + \frac{u_t - \lambda u_{t-1}}{1-\lambda}\ .$$

Nerlove Model

Koyck assumes equations 1.1 and 1.2. All the other equations follow from these assumptions. Nerlove has presented an alternative argument (23). Writing down his assumptions, replacing $1 - \lambda$ by γ as he does, we have.

$$(1.9')\quad y_t - y_{t-1} = \gamma\ [(y)_t - y_{t-1}] + w_t$$

$$(1.12')\quad (y)_t = a_{10}x_{1t} + a_{20}x_{2t} + v_t$$

where $a_{10} = b_{10}/\ (1 - \lambda)$ and $a_{20} = b_{20}/\ (1 - \lambda)$. Note that $-1 < \lambda < 1$ implies $0 < \gamma < 2$.

Except for the stochastic terms, Nerlove's assumptions are conclusions in Koyck's analysis. The difference equation, 1.9', can be solved as

$$(1.13)\quad y_t = \gamma\ \Sigma(1-\gamma)^i\ (y)_{t-i} + \Sigma(1-\gamma)^i w_{t-i}\quad i = 0, 1, \ldots, n\ .$$

Substituting equation 1.12' and lagged versions of equation 1.12' into equation 1.13,

$$(1.14) \quad y_t = a_{10}\gamma \ \Sigma(1-\gamma)^i x_{1t-i} + a_{20}\gamma \ \Sigma(1-\gamma)^i x_{2t-i} + \gamma \ \Sigma(1-\gamma)^i v_{t-i} + \Sigma(1-\gamma)^i w_{t-i} \ .$$

Now $a_{i0}\gamma = b_{i0}$. Except for the stochastic terms, equation 1.14 is the same as equation 1.3. Hence, we can say that, in their economic content, the two procedures simply exchange assumptions and conclusions.

The reduced equation in the Nerlove model (obtained by substituting equation 1.12' into 1.9') is:

$$(1.4') \quad y_t = a_{10}\gamma \ x_{1t} + a_{20}\gamma \ x_{2t} + (1-\gamma) y_{t-1} + \gamma \ v_t + w_t \ .$$

The Koyck and Nerlove models permit three types of adjustment of actual consumption to a new equilibrium consumption level after a change in price or income. The hypothetical situations in fig. 1 are drawn to represent a situation where price has remained constant for all periods for $t \leqslant 0$. Price, the solid lines P_t, is then assumed to decrease one unit between periods zero and one and to remain constant at the lower level. The equilibrium level of consumption, the solid lines $(Q)_t$, changes two units simultaneously with the price change. If consumers immediately adjust actual consumption to the new equilibrium consumption level, actual consumption coincides with equilibrium consumption; i.e., there is no lagged effect, and λ and $1-\gamma$ in equations 1.4 and 1.4' will be zero.

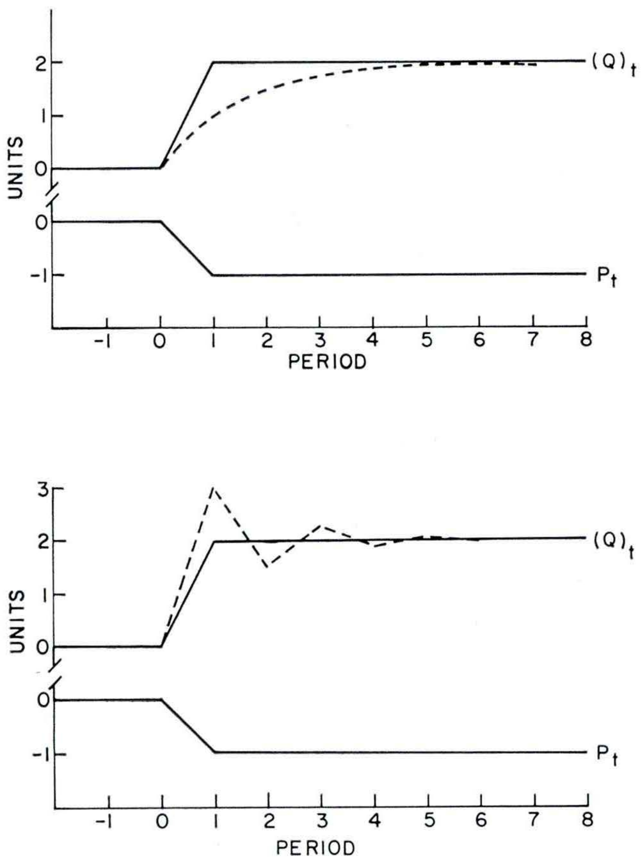

Fig. 1. Hypothetical examples of the types of adjustment to equilibrium that are possible with the Koyck and Nerlove distributed lag models (underadjustment, top; overadjustment, bottom).

In the top portion of fig. 1, λ and $1-\gamma$ are assumed to be 0.5. Actual consumption, the dashed line, moves toward the new equilibrium consumption level by half the distance between the current equilibrium level and the actual level the last period. This type of adjustment may be considered an underadjustment.

In the lower portion of fig. 1, λ and $1-\gamma$ are assumed to be -0.5. Actual consumption, the dashed line, moves toward the new equilibrium consumption level by overshooting the equilibrium level by an amount equal to half the distance between the equilibrium level and the actual level last period. This type of adjustment may be considered an overadjustment of actual consumption. The overadjustment of actual consumption could be the result of inaccurate anticipation or expectations on the part of the consumer. For example, if the price of a commodity decreases between periods zero and one, the consumer with inelastic expectations may anticipate a higher price between periods one and two. As a result of these expectations, the quantity of the commodity purchased might exceed the new equilibrium level. "Special sales" for short periods could produce consumer behavior of this type. This type of overadjustment may also be the result of the consumer's imperfect knowledge of the optimum product mix in the new situation.

The Duesenberry, Katona and Bilkey hypotheses that the consumer continuously acquires new attitudes and motives are inconsistent with either of the types of adjustment depicted in fig. 1. Essentially these hypotheses constitute a relaxation of the assumption of fixed preferences, and "the" equilibrium quantity is undefined. The equilibrium quantity varies.

Tests of the hypotheses underlying fig. 1 cannot be performed until several statistical assumptions have been made. We next discuss estimation procedures.

STATISTICAL CONSIDERATIONS

The least squares (L.S.) approach and a modification of this approach, autoregressive least squares (A.L.S.), are employed in this study. The properties of these two methods of estimation will be considered in detail since most of the equations selected for investigation in the empirical sections will be estimated by each of these methods. The use of both estimation techniques makes possible statistical comparisons of the estimates obtained when the same economic relationship is fitted under different error assumptions.

A regression equation may be written in matrix notation as:

(2.1) $Y = Xb + u$

where Y is a column vector of T observations on the dependent variable, b is a column vector of r unknown coefficients, X is a matrix of independent variables, x_{it} $(i = 1 \ldots r)$, and u is a column vector of errors in the equation. The L.S. estimate of b is

(2.2) $\text{est } b = [X'X]^{-1} X'Y.$

If it is assumed that:

(2.3.1)	$E[u_t] = 0$	all t
(2.3.2)	$E[u_t^2] = \sigma^2$	$< \infty$
(2.3.3)	$E[u_t u_s] = 0$	$s \neq t$
(2.3.4)	$E[x_{it} u_t] = 0$	all i

and that the x_{it} are constants measured without error, the L.S. estimates of b are unbiased and best; i.e., possess the smallest variances among all linear unbiased estimates. If, in addition, the u_t are assumed to be normally distributed, the L.S. coefficients possess the maximum likelihood properties of sufficiency and consistency and are normally distributed. It is, therefore, possible to construct confidence intervals about them.

According to the Markoff theorem (29, p. 83), only assumption 2.3.3 is necessary for est b to remain the best linear unbiased estimates of b. Wold (35, p. 280) demonstrates that est b remains unbiased when 2.3.3 is relaxed, the x_{it} are autocorrelated and intercorrelated and where some x_{it} are lagged values of other x_{it}'s. However, relaxing assumption 2.3.3 causes est b to lose its efficiency and causes the t and F tests to be biased.

If the u_t are known to follow an autoregressive scheme, the Markoff theorem again applies, provided the true autocorrelation coefficients are known and the appropriate linear transformation is made in the data before the L.S. procedure is applied (9, p. 219). Seldom, however, will the true autocorrelation coefficients be known. In practice, the order of the autoregressive scheme will also be unknown.

If, as is the case in many distributed lag models (e.g., the Koyck and Nerlove models), the matrix of independent variables contains a vector of lagged values of the dependent variable (e.g., $x_{rt} = y_{t-1}$), additional statistical problems may be encountered. Hurwicz (14, p. 365) has demonstrated that the L.S. estimate of the coefficient of the lagged dependent variable, est b_r, will be biased in small samples. Hence, the L.S. estimation of equations resulting from a Koyck or Nerlove type reduction will produce biased estimates of the b_i in small samples even when the errors are independent. When the errors are not independent, L.S. estimates will be biased even in large samples. When the errors are not independent, L.S. estimates will be inefficient even though y_{t-1} is not one of the independent variables (33, 34).

It was previously pointed out that, in their economic content, the Koyck and Nerlove procedures are interchangeable. In their statistical content they are different, however. Compare the error terms in equations 1.4 and 1.4'. The very logic of the Koyck argument leads to the conclusion of autocorrelated errors in the estimation equation. In equation 1.4 the error term r_t $(= u_t - \lambda u_{t-1})$ is correlated with r_{t-1} unless $u_t = \lambda u_{t-1} + e_t$, e_t independent of u_{t-i}. No such implication of autocorrelated errors is a necessary conclusion from the Nerlove model.

Accordingly, Koyck developed and applied an estimation procedure that would yield (conditionally) consistent estimates in the presence of autocorrelated errors. Nerlove applied least squares. Since the properties of estimates of equations 1.4 and 1.4' do depend on the error structure, it is worthwhile to investigate the properties of the errors.

The two simplest error assumptions, other than the assumption of independence, are the assumptions that the errors follow the first-order autoregressive scheme:

(2.4) $u_t = \beta_1 u_{t-1} + e_t \qquad -1 < \beta < 1$

or the second-order autoregressive scheme:

(2.5) $u_t = \beta_1 u_{t-1} + \beta_2 u_{t-2} + e_t$
roots x of $x^2 = \beta_1 x + \beta_2$ less than unity in absolute value.

β_1 is the first-order autoregression coefficient, β_2 is the second-order autoregression coefficient, and the e_t have a mean of zero, are of constant variance and are uncorrelated with the independent variables in the model. Griliches (8, p. 65) and Fuller and Ladd (5) have shown that the L.S. estimates of the b_i are biased even in large samples if equation 2.4 represents the true error structure.

Consequently, dynamic models containing the lagged dependent variable as an indepedent variable will result in biased tests of the dynamic hypothesis if the errors are correlated. The autoregressive least squares (A.L.S.) estimation procedure used in this study is one method of obtaining estimates of the parameters of a lag model when errors are assumed to follow an autoregressive scheme, such as equations 2.4 and 2.5. A.L.S. simultaneously yields estimates of the autoregressive coefficients.

A.L.S., a version of the modified Gauss-Newton nonlinear regression procedure (11), was developed by Fuller and Martin (6, 7). A discussion of the procedure and the A.L.S. IBM program used in this study can be found in Martin (21). Hartley (11) has shown that the modified Gauss-Newton estimates that yield the absolute minimum of the residual sum of squares are least squares estimates. Under the proper error assumptions, they are maximum likelihood estimates and are consistent and asymptotically efficient. Hartley and Booker (12) have also discussed the consistency and asymptotic efficiency of modified Gauss-Newton estimates.

Hildreth and Lu (13) developed an alternative procedure for obtaining maximum likelihood estimates and proved that their estimates are consistent. Their proof (13, pp. 52-54) also proves the consistency of A.L.S. estimates. The Hildreth and Lu procedure will be discussed in more detail later in comparisons between it and A.L.S.

There are various other procedures one can apply if the errors are autocorrelated, some more simple and some more complex than A.L.S. Since L.S. coefficients are unbiased, if disturbances are autocorrelated, but their standard errors are biased, one might apply Wold's method (35) for obtaining consistent estimates of the standard errors in a single equation. Zellner (36) recently generalized this to finding consistent estimates of standard errors of L.S. coefficients in systems of equations. His method can be applied to the reduced forms or to two-stage least squares estimates. This is one of the simplest procedures available. Its weakness is that it does nothing to improve the efficiency of the estimates of the coefficients.

Another relatively economical procedure is suggested by Theil and Nagar (28): (a) Compute L.S. estimates of the coefficients and compute the residuals. (b) Estimate β_1 from the residuals. (c) Transform all variables $X_{it} - (\text{est } \beta_1) X_{it-1}$, and use these transformed variables to obtain L.S. estimates of the coefficients. Theil and Nagar point out two weaknesses of their procedure. First, it ignores the sampling variability in est β_1. Second, it requires the use of the same data for three successive steps: (a) to test residuals for independence, (b) to estimate β_1 if the hypothesis is rejected and (c) to recompute the equations. Since the results of the last two steps are conditional upon the results of the first, the tabulated probabilities are not appropriate for testing the results in step c.

Klein (16) has suggested a maximum likelihood procedure. Unfortunately, it is cumbersome and tedious. Suppose we want to estimate $y_t = \alpha x_t + u_t$ subject to equation 2.4. It is necessary to solve a fifth-degree polynomial in β_1 to obtain the maximum likelihood estimates. For each additional independent variable the degree of the polynomial to be solved rises by four. With four independent variables, the polynomial is of 17th degree.

Approximate confidence intervals were computed for selected estimates of the long-run price and income elasticities. The method used to compute these confidence intervals is discussed in Fuller and Martin (6).

Even though the tests used and the confidence intervals computed are approximations, they furnish considerable information about the reliability of the estimates. Since exact statistical tests are not available, the reader would probably like to see such tests performed before evaluating the empirical results. In the test, statistics that differ from the null hypothesis by at least the 10-percent level and 1-percent level when tested by the usual methods will be termed, respectively, "significant" and "highly significant."

The Durbin-Watson d (3, p. 1591):

$$(2.6) \quad d = \frac{\Sigma(u_t - u_{t-1})^2}{\Sigma u_t^2}$$

is computed from the residuals of all of the static regressions and for selected dynamic regressions that are fitted by L.S. Durbin and Watson point out that a test of the d statistic is inappropriate and can only be considered an approximation when the regression contains a lagged dependent variable. This does not imply that the statistic is without value. However, one should be extremely cautious in interpreting the results of a lagged equation, even in the light of a nonsignificant d statistic.

The Hart-von Neumann ratio (10, 31), which equals $dT/(T-1)$, was also computed to test for autocorrelation. This ratio is not shown in the tables but is discussed in the summary. Thus, results from equations fitted by A.L.S. may be compared with the two more widely used tests for nonindependent residuals.

No tests of the residuals from equations fitted by A.L.S. were made, because asymptotic estimates of the autocorrelation coefficients are obtained directly. However, in several cases where the first-order autocorrelation coefficient was large and highly significant, the equation was re-estimated under the assumption that the errors follow a second-order autoregressive scheme.

DATA

This study utilizes data collected by the Agricultural Economics Department at Michigan State University during the operation of the M.S.U. Consumer Panel.[2] This panel consisted of a group of consumers who were selected as representative of the population of Lansing, Michigan. It was in operation from February 1951 through December 1958. Since the data were obtained from a sample, extrapolation of any conclusions to the city of Lansing or to consumers in general depends upon representativeness of the sample for the popula-

[2]Quackenbush and Shaffer (24) present a detailed discussion of the M.S.U. Consumer Panel, the sampling problem and the reliability of the data.

tion under consideration. Therefore, the analyses performed with the data are limited to hypothesized explanations of the consumer demand for selected groups of food items for the M.S.U. Consumer Panel.

The panel data used consist of average price, per-capita quantity, per-capita income after federal income taxes, and temperature for 13-week and 4-week periods. The periods covered by the analyses begin with the first 7-day period of 1952 for the 13-week observations and the twenty-fifth 7-day period of 1951 for the 4-week observations.

The original panel data used include weekly observations on the entire panel of the per-capita quantity (in pounds) of beef, pork, lamb and mutton, veal, broilers and fryers (hereafter referred to simply as fryers) and cheese actually purchased. In addition, the weekly per-capita purchases of the entire panel of the quantity of eggs (in dozens) and fresh milk (in quarts) were used. Thus, each observation of the per-capita quantity of beef, pork, fryers, cheese, eggs and fresh milk used in the analyses is the sum of the weekly quantities purchased of that food item for consecutive 13-week and 4-week periods.

To obtain the per-capita quantity of total fresh red meat (hereafter referred to simply as total meat), the sum of the quantities purchased of beef, pork, lamb and mutton and veal was computed for consecutive 13-week and 4-week periods.

Average price indexes with a base of 1955-57=100 for beef, pork, eggs, and other meat and meat mixtures (hereafter referred to simply as other meats) for 4-week periods were obtained directly from Wang (32). To obtain price indexes for these meat items and for eggs for a 13-week period, weighted averages of the 4-week average price indexes given by Wang (32) were computed. Average prices for total meat were computed by dividing the total per-capita expenditures on beef, pork, lamb and mutton and veal for the 13-week and 4-week periods by the total per-capita quantities of beef, pork, lamb and mutton and veal purchased during the respective periods. A similar procedure was used to compute average price indexes for fryers. The total per-capita expenditures for the 13-week and 4-week periods were divided by the total per-capita quantities of fryers purchased during the respective periods. Then, these series of prices were: (a) deflated by weighted averages of the Detroit Bureau of Labor Statistics Consumer Price Index and (b) converted to a base of 1955-57 equals 100.

The price indexes for cheese, fresh milk, cream, canned milk and dried milk for the 13-week periods were obtained by dividing the total per-capita expenditures on each of these items by the total per-capita quantities purchased of that item during the 13-week period. Next, each series of prices was: (a) deflated by weighted averages of the Detroit Bureau of Labor Statistics Consumer Price Index and (b) converted to a base of 1955-57 equals 100.

The income data used in the analyses were obtained from the panel data of weekly observations of per-capita disposable income after federal income taxes.[3] These data were computed by summing the weekly panel data for the 13-week and 4-week periods. The 13-week and 4-week income series were then deflated by weighted averages of the Detroit Bureau of Labor Statistics Consumer Price Index.

Riley (25) found that temperature was significantly related to the meat purchases of the M.S.U. Consumer Panel. Therefore, 13-week and 4-week mean daily temperature variables were computed. Mean temperatures for Lansing were computed from the daily temperatures (30).

ANALYSES USING 13-WEEK OBSERVATIONS

Static analyses of demand for beef, pork and fryers

Results obtained from the estimation of static demand equations are presented in table 1. The variables are defined as:

Q_{it} = per-capita purchase of i-th food
P_{it} = retail price of i-th food
i = B, beef
= P, pork
= F, fryers
P_{ot} = retail price of other red meats
Y_t = per-capita income
T_t = temperature
D_{it} = 1 in the i-th quarter, $i = 1, \ldots 4$
= 0 all other quarters.

Temperature and seasonal dummy variables were used as alternative shift variables to identify seasonal shifts which may exist in demand.

In the tables of regression results, for all variables except D_{it}, a triple asterisk beside a coefficient indicates significance at the 1-percent level; double asterisk, the 5-percent level and single asterisk, the 10-percent level. A superscript e beside a coefficient indicates that the coefficient exceeds its standard error in absolute value but is not significant at the 10-percent level. Inconclusive and significant values of d are noted in footnotes. The significance status of coefficients of D_{it} is discussed in the text.

Equations 1 through 4 in table 1 present results of the static analyses of the demand for beef. Equations 1 and 2 were obtained by using temperature as the shift variable. The equations were estimated, respectively, by L.S. under the assump-

[3]Income from wages, salaries, commissions, pensions, interest and dividends, annuities, profit from business and professional services, profit from rent, government payments, gifts and other sources minus federal income taxes on such income.

Table 1. Selected statistics from regression estimates of static consumer demand equations for beef, pork and fryers for a 13-week observation period.

Equation number	Dependent variable	Regression coefficients and standard errors												Quarterly intercepts				Constant
		P_{Bt}	P_{Pt}	P_{Ft}	P_{Ot}	Y_t	T_t	SD_t	β_1	β_2	R^2	d	Method of estimation	D_{1t}	D_{2t}	D_{3t}	D_{4t}	
1	Q_{Bt}	—0.0932***	0.0307[e]	0.0043	—0.0183	—0.0005	—0.0339***				0.8456	2.821[a]	L.S.					23.518
2	Q_{Bt}	—0.0777***	0.0464***	0.0045	—0.0331[e]	0.0103[e]	—0.0355***		—0.5811***		0.8919	—	A.L.S.-1					19.701
3	Q_{Bt}	—0.0750***	0.0462**	—0.0016	—0.0392	0.0078					0.9109	2.680[a]	L.S.	18.840	17.184	17.210	17.405	
4	Q_{Bt}	—0.0706***	0.0517***	0.0012	—0.0403[e]	0.0129*			—0.4087*		0.9238	—	A.L.S.-1	15.581	13.909	13.919	14.091	
5	Q_{Pt}	0.0848***	—0.0419**	0.0152**	—0.0761**	0.0145**	—0.0374***				0.8729	1.987	L.S.					6.788
6	Q_{Pt}	0.0862***	—0.0432**	0.0138*	—0.0707*	0.0153*	—0.0385***		0.1769		0.8772		A.L.S.-1					6.643
7	Q_{Pt}	0.0599***	—0.0637***	0.0218***	—0.0419[e]	0.0042		0.0093[e]			0.9224	—	L.S.	9.357	8.355	10.324	11.640	
8	Q_{Pt}	0.0594***	—0.0645***	0.0217***	—0.0391[e]	0.0043		0.0095[e]	0.0589		0.9229	—	A.L.S.-1	9.210	8.187	10.189	11.478	
9	Q_{Ft}	0.0017	—0.0003	—0.0347***	—0.0372[e]	0.0071[e]	0.0179***				0.8643	1.987	L.S.					—1.762
10	Q_{Ft}	0.0193	0.0205[e]	—0.0116*	—0.0274	—0.0045[e]	0.0102***		0.9191***		0.9167	—	A.L.S.-1					2.529
11	Q_{Ft}	0.0140	0.0125[e]	—0.0124**	—0.0191	—0.0025	0.0126***		0.4206*	0.5375**	0.9328	—	A.L.S.-2					2.380
12	Q_{Ft}	0.0163[e]	0.0122[e]	—0.0391***	0.0203[e]	0.0139***					0.9339	1.619[b]	L.S.	—4.398	—4.053	—3.942	—4.815	
13	Q_{Ft}	0.0121	0.0093	—0.0385***	0.0233[e]	0.0112**			0.2519		0.9360	—	A.L.S.-1	—2.928	—2.571	—2.442	—3.288	

[a] Inconclusive test for negative autocorrelation of residuals at 5-percent level.

[b] Inconclusive test for positive autocorrelation of residuals at 5-percent level.

tion of independent errors and A.L.S.-1 under the assumption of first-order autoregressive errors.

Except for the nonsignificant coefficients of income and the price of other meat, the signs of the coefficients in equation 1 agree with **a priori** expectations. In agreement with Riley's findings (25), the temperature variable is highly significant in the L.S. fit in equation 1. The significant coefficient of temperature indicates that a 1-percent increase in temperature will reduce beef consumption by approximately 12 percent. The d statistic for this equation indicates possible negative autocorrelation in the residuals, and the t and F tests may be biased. The coefficients are consistent, though perhaps inefficient estimates of the true parameters.

The results obtained by re-estimating equation 1 by A.L.S.-1, equation 2, substantiate the Durbin-Watson test, since the estimate of β_1 is negative and highly significant. The F ratio for the additional contribution of β_1 was highly significant. Thus, A.L.S.-1 estimation significantly improved the fit of the equation as compared with the L.S. fit, resulted in the expected positive coefficient for income and produced a highly significant coefficient of the price of pork and a noticeable reduction in the size of all asymptotic standard errors. Equation 2 was also estimated by A.L.S.-2. The F for the additional contribution of the second-order autocorrelation coefficient, β_2, was less than 1.

Equation 3 was estimated by L.S., with seasonal dummy variables substituted for the temperature variable of equation 1. Dummy variables are poor explanatory variables in an economic sense, but the R^2 of equation 3 indicates an improved fit of the data as compared with the fit of equation 1. The dummy variables also contributed significantly to a regression that included temperature. The value of F for the additional contribution of the dummy variables to an equation containing temperature was highly significant. Temperature did not contribute significantly to a regression that included the dummy variables. Temperature, therefore, does not appear to be a complete explanation of seasonal shifts in the demand for beef.

The quarterly intercepts of equation 3 are consistent with the negative sign of the temperature coefficients in the first two equations. Each equation indicates that, during the first and fourth quarters, the consumer's demand for beef tends to increase (shift to the right) relative to the demand of the second and third quarters. Tests of the differences between the intercepts indicated that only the first intercept is significantly different from the second intercept. The negative though nonsignificant coefficient of the price of fryers does not agree with **a priori** expectations. Stanton (26) found this coefficient to be negative and significant in most of his demand relationships for beef.

The significant negative estimate of β_1 in equation 4 is consistent with the value of d in equation 3. The F ratio for the additional contribution of β_1 to the L.S. fit was significant at the 5-percent level. A.L.S.-1 again produced a noticeable reduction in the size of the asymptotic standard errors. A.L.S.-1 increased the coefficient of income and made it significant. The seasonal shifts in the demand relation are consistent with the previous estimates with respect to direction and significance.

To determine whether the A.L.S. error scheme would substitute as a shift variable, one equation was estimated by A.L.S.-1 without temperature or seasonal dummy variables. This procedure resulted in a significantly poorer fit than any of the previous equations. The estimate of β_1 became nonsignificant. This suggests that there are situations in which nonindependent errors result not only from the omission of autocorrelated variables, but also from the use of such variables.

Equations 5 through 8 in table 1 present the results of the static analyses of the demand for pork. Equations 7 and 8 contain seasonal dummy variables and the slope dummy SD_t.

$SD_t = P_{Pt}$ for the first and second quarters,

$SD_t = -P_{Pt}$ for the third and fourth quarters.

This variable was added to the pork equation to determine whether there is a significant seasonal change in the slope of the demand relation. The coefficients of SD_t were nonsignificant. Equation 7 also was estimated with P_{Pt}^2. Its coefficient also was nonsignificant.

Here, as with beef, the use of temperature yielded a smaller value of R^2 than did the use of four seasonal variables. The addition of the four seasonal variables to an equation containing temperature increased the value of R^2 by an amount significant at the 5-percent level. The addition of temperature to an equation containing the four dummy variables did not significantly affect the value of R^2.

Comparisons of the intercepts of equation 7 revealed that only the first intercept differed significantly from the second intercept, even though intercepts three and four are larger than the first intercept. This result is consistent with the larger deviations observed about the demand relations for the third and fourth quarters in simple scatter diagrams.

In equation 8, A.L.S.-1 was employed to re-estimate equation 7. Comparisons of the estimated parameters of the two equations reveal negligible differences.

The A.L.S.-1 fit of the equation obtained from 8 by omitting temperature, the seasonal dummy and slope dummy variables, resulted in a much poorer over-all fit of the equation. This equation indicated, as do all previous equations, the existence of a strong substitution relationship between pork and fryers, as well as between pork and beef.

Equations 9 through 13 of table 1 present the results of the static analyses of the demand for fryers. Equations 9, 10 and 11 were obtained by using temperature. The L.S. fit, equation 9, indicates a significantly stronger demand for fryers during the summer months than during the winter months. The d statistic does not reject the hypothesis of independent residuals.

Nevertheless, the application of A.L.S.-1, equation 10, resulted in a highly significant estimate of β_1 and suggested positive autocorrelation of the residuals of equation 9. The F for the additional contribution of β_1 to the regression was highly significant. The coefficient of the price of fryers is only one-third the size of the coefficient in equation 9.

Because of the magnitude of the estimate of β_1 in equation 10, A.L.S.-2 was used in the estimation of equation 11. The estimates of both β_1 and β_2 are significant in equation 11. The F for the contribution of β_2 after β_1 was significant at the 5-percent level. None of the signs of the coefficients of equation 11 differs from those of equation 10, but the A.L.S. estimates of the coefficient of the price of fryers suggest a more inelastic demand relationship than the coefficient in equation 9.

In equation 12, seasonal dummy variables were substituted for the temperature variable, and the equation was estimated by L.S. All coefficients in this equation possess the expected signs. As is indicated by the R^2, the over-all fit of the equation is improved when dummy variables are used as shift variables. The F for the additional contribution of the dummy variables to an equation containing temperature was highly significant. The use of seasonal dummies also eliminates the autocorrelation in the residuals, produces a highly significant coefficient of the income variable and increases the absolute size of the coefficient of P_{Ft}.

A comparison of the intercepts of equations 12 and 13 revealed that intercepts one, two and three were significantly larger than intercept four. This weakening of the demand for fryers during the fourth quarter can be attributed partly to a substitution of beef and pork into the diet during the winter months. The demand for turkey and roasting fowl is also stronger during Thanksgiving and Christmas. This may also contribute to a weakening of the demand for fryers during the fourth quarter.

Re-estimation of equation 12 by A.L.S.-1, equation 13, had little effect on the coefficients, the standard errors or the R^2. β_1 contributed little to the L.S. regression. The use of seasonal dummies eliminated the autocorrelation in the errors, whereas the use of temperature did not.

In one equation the shift variables and temperature were deleted and A.L.S.-1 was applied. Again the estimate of β_1 was highly significant.

Dynamic analyses of demand for beef, pork and fryers

Results are presented in table 2.

The basic dynamic equations which are assumed to represent the demand for beef, pork and fryers are of the form:

$$(3.1) \quad Q_{it} = a_0 + \Sigma a_j P_{jt} + a_4 P_{0t} + a_5 Y_t + a_6 Qi_{t-1}$$

where a significant coefficient of lagged quantity would not reject the hypothesis of a distributed lag in demand due to price or income changes. The assumption is made in equation 3.1 that the distribution of the lag is identical for all variables.

Versions of these equations containing temperature were estimated by L.S. and A.L.S. As with the static equations, the use of temperature yielded results that were inferior to the results obtained by the use of seasonal dummies. The equations containing temperature, therefore, are not presented here.

Equations 1 and 2 present results for beef. Here, as with the static beef demand equations, the estimate of β_1 was nonsignificant in the equation excluding temperature and seasonal dummies but was highly significant in the equation containing temperature. Again, the addition of an autocorrelated variable into an equation with independent errors resulted in the errors in the equation becoming autocorrelated. The addition of the dummy variables did not seem to affect the independence of the error structure of the basic dynamic equation. The addition of β_1 reduced the estimated standard errors. The coefficients of Q_{Bt-1} were not significant. Hence, presence of autocorrelation in equation 1 of table 1 is not due to the exclusion of Q_{Bt-1}. Although the estimate of β_1 in the equation containing temperature was highly significant, the d statistic from the L.S. fit was nonsignificant. This also happened in other equations. This is indicative of low power of the d statistic when applied to residuals in an equation containing the lagged dependent variable as an independent variable.

A.L.S.-2 was also applied to the equation containing temperature to test the hypothesis of independent residuals. The estimate of β_2 was nonsignificant.

In equations 1 and 2 the temperature variable was replaced by seasonal dummy variables. The

Table 2. Selected statistics from regression estimates of dynamic consumer demand equations for beef, pork and fryers for a 13-week observation period.

Equation number	Dependent variable	Regression coefficients and standard errors							R^2	d	Method of estimation	Quarterly intercepts				Lagged dependent variable
		P_{Bt}	P_{Pt}	P_{Ft}	P_{Ot}	Y_t	SD_t	β_1				D_{1t}	D_{2t}	D_{3t}	D_{4t}	
1	Q_{Bt}	−0.0974***	0.0632***	−0.0061	−0.0503[c]	0.0131[c]			0.9203	2.342[a]	L.S.	23.166	22.000	21.499	21.589	−0.3225[c]
2	Q_{Bt}	−0.0822***	0.0586***	−0.0015	−0.0451[c]	0.0142*		−0.3428[c]	0.9260	—	A.L.S.-1	18.079	16.657	16.427	16.551	−0.1516
3	Q_{Pt}	0.0646***	−0.0568***	0.0202***	−0.0511[c]	0.0063			0.9159	1.898	L.S.	9.169	8.178	8.053	9.433	0.0253
4	Q_{Pt}	0.0555*	−0.0628***	0.0212**	−0.0347	0.0040	0.0098[c]	0.0118	0.9232	—	A.L.S.-1	8.665	7.638	9.758	11.083	0.0456
5	Q_{Ft}	0.0139[c]	0.0103[c]	−0.0325***	0.0136	0.0108**			0.9370	1.651[b]	L.S.	−3.156	−2.767	−2.692	−3.558	0.1933
6	Q_{Ft}	0.0219[c]	0.0205*	−0.0203**	−0.0138	0.0016		0.9446***	0.9466	—	A.L.S.-1	3.003	3.146	3.436	3.038	−0.3570*

[a]Inconclusive test for negative autocorrelation of residuals at 5-percent level.

[b]Inconclusive test for positive autocorrelation of residuals at 5-percent level.

substitution of the dummy variables for the temperature variable improved the L.S. fit of the dynamic equation. The intercepts for equations 1 and 2 indicate a stronger demand for beef during the first and second quarters than during the third and fourth quarters. However, only the first intercepts are significantly different from the second intercepts.

Equation 1 was re-estimated by A.L.S.-1, equation 2. Comparisons of the results for equations 1 and 2 indicate little change due to A.L.S.-1 estimation. The estimate of β_1 is nonsignificant, the coefficients are of the same sign, and the standard errors are roughly of the same magnitude as the standard errors of equation 1. In equation 2, the coefficient of the income variable is significant and of the same order of magnitude as previous significant estimates. This, plus the nonsignificance of the coefficients of Q_{Bt-1}, suggests that the coefficients of prices and income in equations 1 and 2 may be considered estimates of long-run changes.

Equations 3 and 4 present results for pork. In both equations the coefficient of lagged quantity is nonsignificant and the hypothesis of a lag in adjustment in pork consumption is rejected. The coefficients of the prices of beef, pork and fryers are significant and possess the expected signs. Comparisons of the intercepts of equation 3 indicated that intercepts one and four were significantly larger than the second intercept.

In equation 4 the slope dummy, SD_t, was included with the seasonal dummy variables, and A.L.S.-1 was applied. The coefficient of the slope dummy has the expected positive sign. As in the static equations in table 2, the coefficient is nonsignificant. The hypothesis of equal slopes is not rejected.

In equation 4 the first intercept differed significantly from the second intercept. Equations 3 and 4 suggest a strong substitution relationship between pork and fryers. The estimate of β_1 is nonsignificant. When temperature and the seasonal dummy variables were deleted from the equation, a significantly poorer fit resulted.

Equations 5 and 6 present results for fryers. The addition of Q_{Ft-1} to static equations containing temperature did not reduce or eliminate the autocorrelation in the residuals. Nor did it result in a significant coefficient of Q_{Ft-1}. The three equations estimated with temperature rejected the hypothesis of a lag in the adjustment of consumption to price and income changes.

In equation 5 seasonal dummy variables were substituted for the temperature variables. Comparisons of the intercepts of equation 5 indicated that the fourth intercept was significantly lower than the first three intercepts.

Re-estimation of equation 5 by A.L.S.-1, equation 6, resulted in a large and highly significant estimate of β_1. The F for the additional contribution of β_1 to the L.S. regression was highly significant. As a result of this improved fit, the coefficients of the price of pork and lagged quantity became significant. The negative sign of the coefficient of lagged quantity suggests an overadjustment of consumption to price and income changes. Here the addition of the lagged dependent variable introduced autocorrelation into the equation. The estimate of β_1 in the static form of equation 5 (equation 13 of table 1) was nonsignificant. Further, the addition of β_1 changed the coefficient of Q_{Ft-1} from positive and nonsignificant in equation 5 to negative and significant in equation 6. The second and third intercepts differ significantly from the fourth intercept.

Equation 6 was re-estimated by A.L.S.-2. This equation is not presented. The coefficient of lagged quantity became nonsignificant, but est β_2 was also nonsignificant. An F test of the additional contribution of the lagged coefficient of equation 6 compared with the fit of the static equation, equation 13 of table 1, was 3.377 with 1 and 17 degrees of freedom. With these degrees of freedom, F must exceed 4.45 to be significant at the 5-percent level. An F test to compare equation 6 with equation 12 of table 1 was only 2.05, far below the 5-percent significance level of 3.59. These results indicate that the overadjustment suggested by the coefficient of lagged quantity in equation 6 is questionable for a period of 13 weeks and that the more valid equation is the static equation 12 in table 1.

When all shift and temperature variables were deleted from the fryer equation and A.L.S.-1 was applied, the fit was significantly poorer. Thus, additional evidence was obtained that indicated the importance of the seasonal shifts in the demand relationship.

Static analyses of demand for total meat, cheese and eggs

Results are presented in table 3. The various subscripts have the following meanings:

M = total fresh red meat,
F = fryers,
O = other meats,
C = cheese,
E = eggs.

The temperature and seasonal dummy variables were used as alternative shift variables in the equation for total meat. In equations for cheese and eggs, however, only seasonal dummy variables were used as shift variables.

Equations 1 to 4 deal with total meat. Equa-

Table 3. Selected statistics from regression estimates of static consumer demand equations for total fresh red meat, cheese and eggs for a 13-week observation period.

Equation number	Dependent variable	Regression coefficients and standard errors										Method of estimation	Quarterly intercepts				
		P_{Ft}	P_{Mt}	P_{Ot}	P_{Ct}	P_{Et}	Y_t	T_t	β_1	R^2	d		D_{1t}	D_{2t}	D_{3t}	D_{4t}	Constant
1	Q_{Mt}	0.0193[c]	—0.0834[c]	—0.0399			0.0097	—0.0574***		0.8509	2.525[a]	L.S.					33.266
2	Q_{Mt}	0.0237***	—0.0352	—0.0829[c]			0.0185**	—0.0661***	—0.4108*	0.8720	—	A.L.S.-1					31.369
3	Q_{Mt}	0.0187*	—0.0631	—0.0596			0.0119[c]			0.8793	2.327	L.S.	30.957	28.611	28.556	29.857	—
4	Q_{Mt}	0.0221**	—0.0421	—0.0772[c]			0.0164*		—0.2632[c]	0.8860	—	A.L.S.-1	28.427	25.998	25.893	27.293	—
5	Q_{Ct}	—0.0030*	0.0069**		—0.0118***	—0.0007	0.0027**			0.8878	—	L.S.	1.629	1.453	1.366	1.482	—
6	Q_{Ct}	—0.0025[c]	0.0026		—0.0100**	—0.0012	0.0026**		0.5331***	0.9189	—	A.L.S.-1	0.943	0.804	0.743	0.828	—
7	Q_{Et}	0.0062*	—0.0070[c]		0.0031	—0.0090**	—0.0028[c]			0.7255	—	L.S.	7.406	7.493	7.243	7.520	—
8	Q_{Et}	0.0074**	—0.0085[c]		0.0051	—0.0113***	—0.0030[c]		—0.0885	0.7274	—	A.L.S.-1	8.227	8.319	8.070	8.345	—

[a]Inconclusive test for negative autocorrelation of residuals at 5-percent level.

tions 1 and 2 in table 3 were obtained from the 13-week observations with temperature used as the shift variable. In equation 1, only the coefficient of temperature is significant. The R^2 indicates that a fairly large percentage of the variation is explained by the equation. However, the d statistic is inconclusive.

In the A.L.S.-1 fit of the equation containing temperature, the estimate of β_1 is negative and significant. The F for the additional contribution of β_1 to the L.S. equation was nonsignificant at the 5-percent level. As a result of A.L.S.-1 estimation, the coefficients of the price of fryers, income and temperature variables became significant. That all of the standard errors of equation 2 are less than the comparable standard errors of equation 1 indicates the inefficiency of the L.S. estimates.

In equation 3, a L.S. fit, the seasonal dummy variables were substituted for the temperature variable of equations 1 and 2. The only significant coefficient in the equation is the coefficient of the price of fryers. The first and fourth intercepts were significantly higher than the second intercept. The R^2 of equation 3 also indicates an improvement of the L.S. fit of the equation as compared with the equation containing temperature. A comparison of the dummy variables with temperature, however, revealed that the apparent improvement resulting from the dummy variables was not significant. The F for the additional contribution of the dummy variables after the temperature variable was 1.814 with 3 and 19 degrees of freedom. Reversing the test, the F for the additional contribution of temperature after the dummy variables gave an F of only 0.771 with 1 and 19 degrees of freedom. Thus, temperature cannot be considered the best explanation of seasonal shifts in the demand for total meat.

Here, as in several previous cases, the dummies remove autocorrelation in the residuals more effectively than does temperature.

In equation 4, A.L.S.-1 was applied to the equation containing the dummy variables. The F for the additional contribution of β_1 to the L.S. fit, equation 3, was nonsignificant. Again, the standard errors of equation 4 are smaller than the comparable standard errors of equation 3. However, only the coefficients of the price of fryers and income variables are significant. In equation 4, the first and fourth intercepts are significantly higher than the second intercept.

An equation was estimated by A.L.S.-1 after the deletion of the shift variables and temperature. The fit of the equation was poorer than any of the previous equations. Except for the coefficient of the income variable, however, all coefficients possessed the **a priori** expected signs. The coefficients of the prices of the total meats and other meats were significant, but the coefficient of the price of fryers was nonsignificant.

These equations indicate that the own-price elasticity for total meat is not significantly different from zero for a 13-week period. An increase in the price of total meat may reduce consumption for a shorter period of time; but, before 13 weeks have elapsed, the average consumer may prefer to resume the previous level of consumption of meat and make substitutions between grades and types of meat such that total expenditures and consumption remain fairly constant. One such substitution could be a reduction in the services purchased with the meat items.

Equations 5 and 6 in table 3 present the results of the analyses of the demand for cheese. Both include seasonal dummy variables. The intercepts of both equations indicate that the demand for cheese is stronger during the first and fourth quarters. Except for the coefficient of the price of eggs, all coefficients in equation 5 are significant. However, unless the errors are in fact independent, the t tests are biased. A.L.S.-1 rejects the hypothesis of independent errors. The estimate of β_1 in equation 6 is highly significant. Only the price of cheese and income coefficients are significant.

Equations 7 and 8 in table 3 present results obtained for demand for eggs. In the L.S. fit, only the coefficients of the prices of fryers and eggs are significant. The A.L.S.-1 fit of the equation had little effect on the coefficients, standard errors or R^2. The intercepts of both suggest the demand for eggs to be slightly stronger during the fourth quarter than during any other quarter.

Dynamic analyses of demand for total meat, cheese and eggs

Results are presented in table 4. Again the assumption is made that the distribution of the lag is identical for all variables. In equation 1 the hypothesis of a 13-week lag in the adjustment of consumption to price and income changes is rejected, just as it was in equations containing temperature. The use of dummy variables produced a better fit of the equation than did the use of temperature. The F for the additional contribution of the dummy variables after temperature was nonsignificant, however.

In equation 1, only the first intercept is significantly larger than the second intercept. Thus, in the first quarter the demand for total meat appears to be stronger than during any other period of the year. Re-estimation of equation 1 by A.L.S.-1 had very little effect on the R^2, the coefficients or the standard errors. Again, the first intercept is the only intercept significantly larger than the second intercept.

One equation was estimated by A.L.S.-1 after the deletion of the temperature and dummy variables. The fit of the equation was poorer than

Table 4. Selected statistics from regression estimates of dynamic consumer demand equations for total fresh red meat, cheese and eggs for a 13-week observation period.

Equation number	Dependent variable	Regression coefficients							Quarterly intercepts				Lagged dependent variable	R^2	Method of estimation
		P_{Ft}	P_{Mt}	P_{Ot}	P_{Ct}	P_{Et}	Y_t	β_1	D_{1t}	D_{2t}	D_{3t}	D_{4t}			
1	Q_{Mt}	0.0190*	—0.0568	—0.0950[e]			0.0163[e]		37.637	35.683	34.982	36.140	—0.2296[e]	0.8806	L.S.
2	Q_{Mt}	0.0209**	—0.0462	—0.0874[e]			0.0167[e]	—0.1979	38.530	36.564	35.764	36.987	—0.0991	0.8831	A.L.S.-1
3	Q_{Ct}	—0.0019[e]	0.0036[e]		—0.0069*	—0.0004	0.0018*		0.957	0.789	0.762	0.892	0.4005***	0.9197	L.S.
4	Q_{Ct}	—0.0019[e]	0.0036[e]		—0.0070[e]	—0.0004	0.0018*	0.0185	0.954	0.786	0.758	0.887	0.3908*	0.9197	A.L.S.-1
5	Q_{Et}	0.0074**	—0.0085[e]		0.0051	—0.0113***	0.0031[e]		8.851	8.959	8.769	8.956	—0.2309[e]	0.7436	L.S.
6	Q_{Et}	0.0077**	—0.0073[e]		0.0058	—0.0120***	0.0028[e]	0.2017	7.268	7.374	7.199	7.354	—0.3298[e]	0.7495	A.L.S.-1

in any of the previous equations, but the coefficients of the price of red meats and the price of other meats were significant. The coefficient of lagged quantity in equation 2 is nonsignificant, and the coefficients differ little from the coefficients of the comparable static equation (equation 5 in table 3). Thus, the hypothesis that a lag of 13 weeks exists in the adjustment of actual consumption to price and income changes is rejected.

Equations 3 and 4 refer to cheese demand. In the L.S. fit, equation 3, the coefficients of the price of cheese, income and lagged quantity are significant and possess the **a priori** expected signs. The significant coefficient of lagged quantity rejects the hypothesis of complete adjustment in the consumption of cheese within 13 weeks after a price or income change.

The application of A.L.S.-1 resulted in small changes in the coefficients, standard errors and R^2. In agreement with the static analysis, the demand for cheese appears to be stronger during the first and fourth quarters than during the second and third quarters.

In the static cheese demand equations, the addition of β_1 reduced several coefficients to nonsignificance. The addition of Q_{Ct-1} to obtain dynamic demand equations also reduces several coefficients, including the estimate of β_1, to nonsignificance. These same phenomena happen in annual analyses of aggregate national demand for cheese covering long periods of time. The reason appears to be the existence of a strong trend in cheese consumption. The variable Q_{Ct-1} may be a proxy for time trend.

Equations 5 and 6 refer to egg demand. In the L.S. fit, equation 5, the coefficients of the price of fryers and the price of eggs are significant. The application of A.L.S.-1 to equation 5 did not significantly improve the fit of the equation. As a result, the coefficients, standard errors and R^2 of equation 6 differ little from the comparable statistics in equation 5. Comparisons of the intercepts of equations 5 and 6 indicate a relatively stable demand for eggs throughout the year. The intercepts of the cheese and egg demand equations were not tested for significance of differences.

Demand for fresh milk

Static analyses. In table 5, P_{FMt}, P_{CRt} and P_{DMt} represent, respectively, the price of fresh milk, price of cream and price of dried milk. Y_t is, again, the income variable.

Seasonal dummy variables were used as shift variables. Their coefficients were not tested for significance. The L.S. fit, equation 1 in table 5, resulted in a R^2 of 0.8348. However, only the coefficient of the price of dried milk is significant. The coefficient of the price of fresh milk has the expected negative sign, but, apparently, the relatively stable price of milk at the retail level contributes to the nonsignificance of the coefficient. Re-estimation of equation 1 by A.L.S.-1 did not significantly improve the fit of the equation.

Dynamic analyses. Seasonal dummy variables were used as shift variables in the L.S. fit of the equation (equation 1 of table 6). Only the coefficients of the price of dried milk and lagged quantity are significant. This result is probably the result of only small variations in the price of fresh milk during the operation of the panel. The F for the additional contribution of lagged quantity to the static equation, equation 1 of table 5, was significant at the 6-percent level. However, unless the errors of equation 1 are, in fact, independent, these results are probably biased and inefficient.

An investigation of the errors in equation 1 was made by re-estimation of equation 1 by A.L.S.-1, equation 2. Again, only the coefficients of the price of dried milk and lagged quantity were significant. The estimate of β_1 was not significant. Thus, there appears to be a definite lagged relationship between changes in prices and income and the consumption of fresh milk. In equations 1 and 2, the demand for fresh milk appears to be stronger during the first and fourth quarters than during the second and third quarters.

Although Q_{FMt-1} is significant, at the 5- and 10-percent levels, its exclusion from the static demand equation does not cause autocorrelation in residuals.

ANALYSES USING 4-WEEK OBSERVATIONS

Demand equations for beef, pork, fryers and total meat were estimated by using a 4-week observation period. These equations contained the same variables as the 13-week analyses, with one exception. Thirteen seasonal dummies are used, rather than four.

$$D_{it} = 1 \text{ in i-th 4-week period.} \quad i = 1, 2, \ldots, 13$$
$$= 0 \text{ all other 4-week periods.}$$

Results are presented in tables 7, 8 and 9. Equations containing temperature were estimated but are not presented, since they were inferior to equations containing seasonal dummy variables.

Beef

In the beef demand equation containing temperature, estimated β_1 was nonsignificant. This contrasts with the 13-week data in which estimated β_1 was significant. Because of this and the nonsignificant value of d in equation 1 of table 7, A.L.S.-1 was not applied to the equation containing seasonal dummy variables. Of the intercepts presented in table 8 and fig. 2, intercepts one and

Table 5. Selected statistics from regression estimates of static consumer demand equations for fresh milk for a 13-week observation period.

Equation number	Dependent variable	Regression coefficients						Quarterly intercepts				R^2	Method of estimation
		P_{FMt}	P_{CRt}	P_{CMt}	P_{DMt}	Y_t	β_1	D_{1t}	D_{2t}	D_{3t}	D_{4t}		
1	Q_{FMt}	—0.0440[e]	—0.0275	—0.0152	0.0404***	0.0058		42.549	42.049	40.265	42.701	0.8348	L.S.
2	Q_{FMt}	—0.0295	—0.0297	—0.0130	0.0378***	0.0063	0.3072[e]	28.696	28.226	26.429	28.824	0.8492	A.L.S.-1

Table 6. Selected statistics from regression estimates of dynamic consumer demand equations for fresh milk for a 13-week observation period.

Equation number	Dependent variable	Regression coefficients						Quarterly intercepts				Lagged dependent variable	R^2	Method of estimation
		P_{FMt}	P_{CRt}	P_{CMt}	P_{DMt}	Y_t	β_1	D_{1t}	D_{2t}	D_{3t}	D_{4t}			
1	Q_{FMt}	—0.0481[e]	0.0042	—0.0139	0.0311***	—0.0008		26.470	25.932	24.507	27.443	0.4178*	0.8628	L.S.
2	Q_{FMt}	—0.0526[e]	0.0185	—0.0146	0.0284**	—0.0043	—0.2403	25.304	24.762	23.489	26.664	0.5870**	0.8672	A.L.S.-1

Table 7. Selected statistics from regression estimates of consumer demand equations for beef, pork and fryers and a 4-week observation period.[a]

Equation number	Dependent variable	Regression coefficients							R^2	d	Method of estimation
		P_{Bt}	P_{Pt}	P_{Ft}	P_{Ot}	Y_t	Lagged dependent variable	β_1			
1	Q_{Bt}	—0.0227***	0.0164***	—0.0005	—0.0135[e]	0.0120**			0.8149	2.188	L.S.
2	Q_{Pt}	0.0190***	—0.0172***	0.0065***	—0.0146***	0.0064*			0.8606	2.194	L.S.
3	Q_{Pt}	0.0190***	—0.0173***	0.0064***	—0.0148***	0.0062*		—0.0920	0.8618		A.L.S.-1
4	Q_{Ft}	0.0005	0.0002	—0.0129***	0.0110***	0.0069***			0.8775	1.472[b]	L.S.
5	Q_{Ft}	—0.0001	—0.0001	—0.0126***	0.0123***	0.0066***		0.0391	0.8786		A.L.S.-1
6	Q_{Bt}	—0.0265***	0.0183***	—0.0007	—0.0145	0.0131**	—0.1474[e]		0.8192		L.S.
7	Q_{Pt}	0.0196***	—0.0176***	0.0066***	—0.0153**	0.0065*	—0.0305		0.8608		L.S.
8	Q_{Pt}	0.0121***	—0.0126***	0.0046***	—0.0081*	0.0039[e]	0.3271***	—0.4430***	0.8827		A.L.S.-1
9	Q_{Ft}	0.0003	0.0002	—0.0126***	0.0112***	0.0068***	0.0140		0.8777		L.S.
10	Q_{Ft}	—0.0025[e]	—0.0021[e]	—0.0161***	0.0190***	0.0050**	—0.3573***	0.4145***	0.8890		A.L.S.-1

[a]The thirteen intercepts for 4-week periods are persented in table 8.

[b]Significant test for positive autocorrelation in residuals at 5-percent level.

Table 8. Four-week intercepts for static and dynamic consumer demand equations for beef, pork, fryers and total meat.

Commodity	For other results see		D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}
	Table	Equation													
Beef	7	1	5.609	5.059	4.894	4.718	4.667	4.664	4.621	4.641	4.595	4.659	4.776	4.790	4.590
Beef	7	6	6.382	5.963	5.718	5.543	5.444	5.444	5.415	5.414	5.372	5.417	5.566	5.582	5.372
Pork	7	2	3.062	2.718	2.650	2.969	2.417	2.368	2.265	2.266	2.517	2.688	2.771	2.787	3.107
Pork	7	3	3.119	2.777	2.710	3.027	2.476	2.426	2.326	2.325	2.576	2.746	2.828	2.845	3.167
Pork	7	7	3.176	2.831	2.753	3.070	2.528	2.463	2.356	2.354	2.606	2.785	2.869	2.890	3.210
Pork	7	8	2.069	1.747	1.784	2.112	1.461	1.582	1.537	1.550	1.796	1.871	1.940	1.912	2.246
Fryers	7	4	—0.0718	—0.0638	—0.0441	0.0221	0.0603	0.0841	0.1401	0.1353	0.106	0.0933	0.0132	—0.2111	—0.2981
Fryers	7	5	—0.1046	—0.0963	—0.0763	—0.0110	0.0319	0.0604	0.1157	0.1119	0.0827	0.0724	—0.0104	—0.2409	—0.3229
Fryers	7	9	—0.0927	—0.0872	—0.0667	—0.0005	0.0384	0.0628	0.1181	0.1129	0.0838	0.0730	—0.0083	—0.2337	—0.3135
Fryers	7	10	0.4010	0.4688	0.4774	0.5307	0.6115	0.6699	0.7372	0.7598	0.7205	0.6970	0.6087	0.3501	0.1921
Total meat	9	1	9.570	8.808	8.662	8.809	8.326	8.290	8.207	8.240	8.382	8.646	8.756	8.590	8.662
Total meat	9	2	11.141	10.534	10.241	10.391	9.902	9.791	9.696	9.689	9.837	10.127	10.281	10.133	10.182

Table 9. Selected statistics from regression estimates of consumer demand equations for total red meat with a 4-week observation period.

Equation number	Dependent variable	Regression coefficients					R^2	d	Method of estimation
		P_{Ft}	P_{Mt}	P_{Ot}	Y_t	Q_{Mt-1}			
1	Q_{Mt}	0.0063**	—0.0419***	0.0095	0.0112**		0.8223	2.002	L.S.
2	Q_{Mt}	0.0067***	—0.0438***	0.0051	0.0133**	—0.1648*	0.8287		L.S.

two are significantly larger than intercept five. None of the other intercepts differs significantly from the fifth intercept. In equation 6 and other dynamic beef demand equations, the coefficient of Q_{Bt-1} is nonsignificant. Adjustments in beef consumption to changes in prices and income are evidently completed within 4 weeks.

Pork

Equations 2 and 3 of tables 7 and 8 contain results for static pork demand equations. The intercepts from equation 2 are also plotted in fig. 3. All intercepts except six, seven, eight and nine are significantly higher than intercept five. These results indicate a significantly stronger demand for pork from September through April than from April to September. Easter occurs during the fourth 4-week period, when per-capita demand rises by about 0.3 pound over the previous period. Beginning around Sept. 1, with intercept nine, demand rises until intercept 11. Demand remains constant through Thanksgiving, intercept 12, and rises during the last 4-week period of the year. The shift during December may be due to increased ham consumption during the Christmas season.

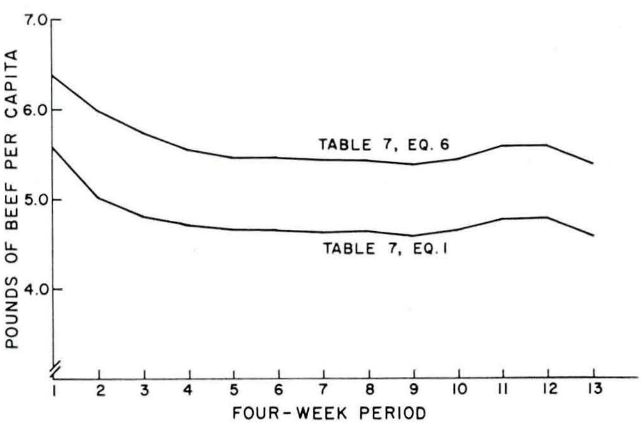

Fig. 2. Four-week intercepts for consumer demand equations for beef.

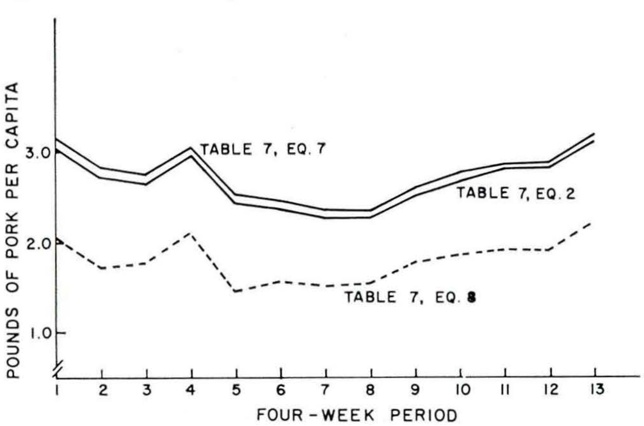

Fig. 3. Four-week intercepts for consumer demand equations for pork.

Equations 7 and 8 of table 7 and 8 present results for dynamic pork demand equations. Intercepts are plotted in fig. 3. All intercepts except six, seven, eight and nine are significantly larger than intercept five. One expects the addition of a significant variable to eliminate or reduce autocorrelation. Here, the addition of a significant variable introduces autocorrelation (equations 3 and 8). In this study, the introduction of a significant variable introduced autocorrelation about as frequently as it eliminated autocorrelation. The coefficient of Q_{Pt-1} is significant only when A.L.S. is used. The coefficients in equation 7 are substantially different from those in equation 8—the standard errors in equation 8 are, with the exception of Q_{Pt-1}, smaller than those in equation 7. The value of F for the additional contribution of β_1 is highly significant.

Fryers

Equations 4 and 5 of tables 7 and 8 present static demand for fryers. Fig. 4 shows the intercepts. The first three and last two intercepts are significantly lower than the fifth intercept. None of the others differs significantly from the fifth intercept. The peak demand for fryers occurs during the seventh 4-week period—the period containing the Fourth of July. The largest single shift (a decrease of about one-fourth pound per capita) occurs between periods 11 and 12; i.e., during November. This may be related to an increased demand for turkeys. Other meats appear to be a substitute for fryers. This relation between other meats and fryers was not significant with the 13-week observations. Although the d statistic in equation 4 is significant, the estimate of β_1 in equation 5 is nonsignificant.

Equations 9 and 10 and fig. 5 present the dynamic demand for fryers. Here, as with pork, the introduction of the lagged dependent variable introduced autocorrelation into the residuals, and the coefficient of the lagged dependent variable is only significant in the equation estimated by A.L.S. Note that it is negative. The value of F for the additional contribution of β_1 is highly significant. Here the use of autoregressive estimation increases the size of the standard errors.

Total meat

Tables 8 and 9 present results for total meat. Intercepts are plotted in fig. 6. In equation 1, the first, second, third, fourth, tenth, eleventh and thirteenth intercepts are significantly higher than the fifth intercept. The others are not significantly different from the fifth intercept. The results for equation 2 are the same except that intercepts 10 and 13 are not significantly higher than the fifth intercept. Since the d statistic in equation 1 is so near 2.0 and since β_1 was nonsignificant in equations containing temperature, we believed little additional information would be obtained from A.L.S. estimation. Here, as with fryers, the significant negative coefficient of lagged quantity leads to acceptance of the hypothesis of an overadjustment of actual consumption to price and income changes within a 4-week period.

SUMMARY OF 4-WEEK AND 13-WEEK RESULTS

Demand elasticities

One way to compare results from 4- and 13-week data is to compute demand elasticities. Estimated elasticities, computed at mean values, are presented in tables 10 through 14. In these tables, the asterisks indicate the level of significance of the coefficient upon which the elasticity is based.

The coefficient of Q_{Bt-1} was nonsignificant in every equation. Table 10 presents beef demand elasticities computed from selected static equations only.

The 4-week and the 13-week analyses lead to the conclusion that temperature, seasonal effects, beef price and pork price are significant determinants of beef demand. The 4-week analyses show that income also is a factor affecting beef demand. The evidence that income is a factor is somewhat weaker in the 13-week analyses.

Since coefficients of Q_{Pt-1} were nonsignificant when 13-week data were used, table 11 presents only results from static equations for 13-week observations. Results from both static and dynamic equations are presented for 4-week data. The long-run elasticities computed from the dynamic 4-week equation do not differ appreciably from the elasticities computed from the static 4-week equation. Nor do these long-run elasticities differ appreciably from the elasticities estimated from the static 13-week equation. This is consistent with the size of the adjustment coefficient,

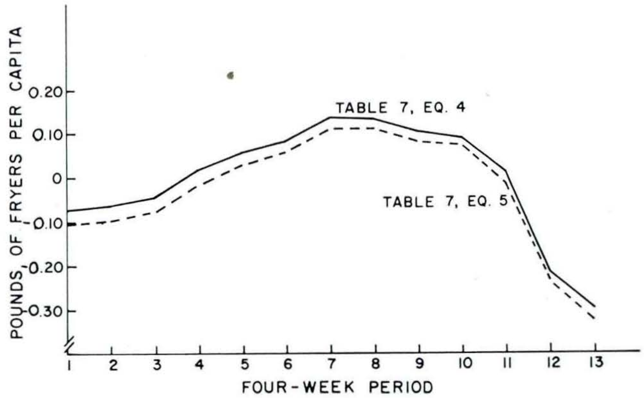

Fig. 4. Four-week intercepts for static consumer demand equations for fryers.

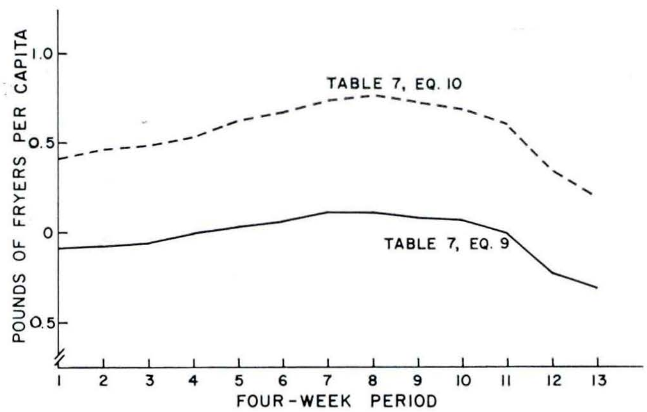

Fig. 5. Four-week intercepts for dynamic consumer demand equations for fryers.

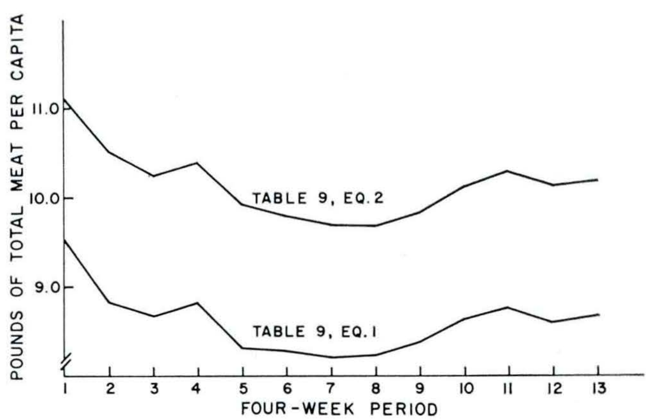

Fig. 6. Four-week intercepts for consumer demand equations for total meat.

which shows that 95 percent of the total adjustment takes place within 8 to 12 weeks.

Long-run elasticity is the ratio of two coefficients. The denominator is one minus the coefficient of lagged quantity. This coefficient is highly significant in equation 8 of table 7, as are the coefficients of P_{Bt}, P_{Pt} and P_{Ft}. Even though both numerator and denominator are highly significant, the confidence intervals for the long-run

Table 10. Selected elasticities for beef.

Table number	Equation number	Elasticities of beef with respect to: P_{Bt}	P_{Pt}	P_{Ft}	P_{Ot}	Y_t	Method of estimation	Length of observation period (weeks)
1	4	—0.57***	0.42***	0.01	—0.31	0.38*	A.L.S.-1	13
7	1	—0.62***	0.45***	—0.01	—0.35	0.36**	L.S.	4

Table 11. Selected elasticities for pork.

Table number	Equation number	Elasticities of pork with respect to: P_{Bt}	P_{Pt}	P_{Ft}	P_{Ot}	Y_t	Method of estimation	Length of observation period (weeks)
1	7	0.72***	—0.77***	0.26***	—0.49	0.19	L.S.	13
7	2	0.75***	—0.68***	0.25***	—0.55**	0.28*	L.S.	4
7	8[a]	0.48***	—0.50***	0.18***	—0.30*	0.17	A.L.S.-1	4
		0.71	—0.74	0.26	—0.45	0.25		
		(—∞ < e < ∞)	(—1.70 < e < 0.63)	(—∞ < e < ∞)	(—1.54 < e < 0.72)	(—∞ < e < ∞)		

[a]First line shows short-run elasticities; second line, long-run elasticities and third line, approximate 95-percent confidence intervals for long-run elasticities.

Table 12. Selected elasticities for fryers.

Table number	Equation number	Elasticities of fryer demand with respect to: P_{Bt}	P_{Pt}	P_{Ft}	P_{Ot}	Y_t	Method of estimation	Length of observation period (weeks)
1	12	0.76	0.57	—1.79***	0.91	2.40***	L.S.	13
2	6[a]	1.02	0.96*	—0.93**	—0.62	0.28	A.L.S.-1	13
		0.75	0.71	—0.69	—0.45	0.21		
		(—0.53 < e < 2.18)	(—0.11 < e < 1.68)	(—1.99 < e < 0.45)	(—3.08 < e < 2.09)	(—0.92 < e < 1.29)		
7	4	0.08	0.03	—2.00***	1.66***	1.21***	L.S.	4
7	10[a]	—0.40	—0.33	—2.49***	2.87***	0.88**	A.L.S.-1	4
		—0.29	—0.24	—1.84	2.12	0.65		
		(—1.02 < e < 0.48)	(—0.88 < e < 0.43)	(—3.96 < e < 0.36)	(0.08 < e < 4.04)	(0.11 < e < 1.18)		

[a]First line presents short-run elasticities; second line long-run elasticities; third line, approximate 95-percent confidence intervals for long-run elasticities.

Table 13. Selected elasticities for total meat.

Table number	Equation number	Elasticities of total meat demand with respect to: P_{Ft}	P_{Mt}	P_{Ot}	Y_t	Method of estimation	Length of observation period (weeks)
3	3	0.09*	—0.29	—0.27	0.21	L.S.	13
9	1	0.10**	—0.64***	0.14	0.19**	L.S.	4
9	2[a]	0.10***	—0.67***	0.08	0.23**	L.S.	4
		0.09	—0.58	0.07	0.20		
		(0.08 < e < 0.21)	(—1.56 < e < 0.38)	(—0.18 < e < 0.33)	(—∞ < e < ∞)		

[a]First line shows short-run elasticities; second line, long-run elasticities and third line, approximate 95-percent confidence intervals for long-run elasticities.

Table 14. Selected elasticities for cheese, eggs and fluid milk.

Table number	Equation number	Dependent variable	Elasticities of demand for dependent variable with respect to: P_{Ft}	P_{Mt}	P_{Ct}	P_{Et}	P_{FMt}	P_{CRt}	P_{CMt}	P_{DMt}	Y_t	Method of estimation
3	6	Q_{Ct}	−0.16	0.16	−0.60**	−0.07					0.61**	L.S.
4	4[a]	Q_{Ct}	−0.12	0.23	−0.42	−0.25					0.42*	A.L.S.-1
			−0.28	0.88	−0.68	−0.04					0.70	
3	7	Q_{Et}	0.12*	−0.14	0.06	−0.17**					−0.20	L.S.
5	1	Q_{FMt}					−0.11	0.07	−0.04	0.10**	0.06	L.S
6	2[a]	Q_{FMt}					−0.13	0.04	−0.04	0.07**	−0.04	A.L.S.-1
							−0.32	0.11	−0.09	0.17	−0.11	

[a]First line shows short-run elasticities; second line, long-run elasticities.

elasticities of pork demand with respect to beef price and fryer price range from plus to minus infinity. The 99-percent confidence interval for the short-run elasticity with respect to pork price does not include zero — the 95-percent confidence interval for the long-run elasticity does. In later tables also, the confidence intervals for the long-run elasticities frequently cover a wide range.

Both the 4- and 13-week analyses show that beef price, pork price, fryer price, temperature and seasonal effects are significant determinants of pork demand. In both cases, there is some evidence that income and prices of other meats also are relevant. But in both cases, the equation yielding the best fit indicates that income is not significant. The best fitting 13-week equation indicates that price of other meats is not a significant factor. In the best fitting 4-week equation, the coefficient of P_{0t} is significant at only the 10-percent level. The coefficient of P_{0t} is invariably negative, whereas we would expect it to be positive. (The best fitting 13-week equation is the static L.S. equation with seasonal dummies. The best fitting 4-week equation is the dynamic A.L.S.-1 equation with seasonal dummies.)

Selected elasticities of demand for fryers are presented in table 12. Some of the long-run elasticities are smaller and some are larger than the elasticities computed from the static equations. There is no consistency. The long-run elasticities are less than the short-run elasticities from the dynamic equations, since the coefficients of Q_{Ft-1} are negative.

The 4-week equations demonstrate that fryer price, price of other meats, income, temperature and seasonal variation are significant determinants of fryer demand. The 13-week equations also show fryer price, temperature and seasonal variation as important determinants. The 13-week equations indicate income to be significant also.

Elasticities of demand for total meat are presented in table 13. The long-run elasticities estimated from dynamic 4-week equations are not appreciably different from elasticities computed from static 4-week equations. Because of the negative coefficients of Q_{Mt-1}, the long-run elasticities are smaller than the short-run elasticities.

The analysis of 4-week data shows that fryer price, red meat price, income, temperature and seasonal effects are significant determinants of total red meat demand. The analysis of 13-week data shows temperature and seasonal effects to be significant, but is ambiguous concerning the effects of fryer price and income and shows meat price to be nonsignificant.

Over-all, elasticities estimated from 4-week and 13-week data are usually in close agreement for variables with significant coefficients. For three of the four commodities studied, long-run elasticities estimated from dynamic equations are not substantially different from elasticities estimated from static equations.

A slightly larger proportion of the coefficients are significant in the equations estimated from 4-week data than in those estimated from 13-week data.

Lags

Varying the length of the observation period has the expected effect on coefficients of lagged consumption. Out of 18 possible comparisons among pairs of equations (many using equations not presented here) coefficients of lagged quantity were: nonsignificant in both 4- and 13-week equations in 13 cases, significant in both in one case and significant in the 4-week equation and nonsignificant in the 13-week equation in four cases. Evidence of a lag is more common in the shorter observation period.

Varying the length of the observation period does not have quite the expected effect on autocorrelation in the errors. Out of 24 possible comparisons among pairs of equations, estimates of β_1 were: nonsignificant in both 4- and 13-week equations in 12 cases, significant in both in five cases, significant in the 4-week equation and nonsignificant in the 13-week equation in three cases and nonsignificant in the 4-week equation and significant in the 13-week equation in four cases. Further, in the five cases in which both were significant, the ratio of β_1 estimated from 13-week data to β_1 estimated from 4-week data was approximately 1.5, 2.0, 2.5, 7.0 and 10.0. These results indicate that the use of the longer unit observation period is likely to increase the autocorrelation in errors rather than to reduce it. The reason for this unexpected finding has not been investigated. It may be due to the nature of the method used in aggregating over time to construct the 13-week data.

Demands for seven commodities were estimated using a unit observation period of 13 weeks—four of these were also studied using a unit observation period of 4 weeks.

There was strong evidence that the adjustment period for pork and fryer demand exceeds 4 weeks. There was weak evidence that the adjustment period for total meat demand exceeds 4 weeks. There was no evidence that the adjustment period for beef exceeds 4 weeks.

There was no evidence that the adjustment periods for beef, pork, total meat and eggs exceed 13 weeks. There was some evidence that the adjustment periods for fryers and milk exceed 13 weeks. There was strong evidence that the adjustment period for cheese exceeds 13 weeks. For these latter three commodities, the magnitude of the adjustment coefficients was such that 87 to 97 percent of the total adjustment for fluid milk and 97 percent of the total adjustment for fryers and cheese would take place within 1 year.

These results do not support the argument that dynamic demand models are needed for an adequate understanding of annual demand.

It has been argued that autocorrelated errors are frequently found in static demand equations because a static equation contains specification bias, which can be eliminated by adding the lagged dependent variable as an independent variable. One advantage of adding a lagged dependent variable is claimed to the reduction or elimination of autocorrelation in the disturbances. This view finds little support in this study. The addition of lagged consumption as an independent variable introduced autocorrelation into the errors as frequently as it eliminated autocorrelation from the errors.

In two cases (pork and fryer demand with 4-week data), there was no autocorrelation in the errors in the static equations. The L.S. coefficient of lagged consumption was nonsignificant in the dynamic equation. These results lead to the conclusion of no lag in behavior and no autocorrelation in the disturbances. The A.L.S.-1 estimation of the dynamic equation, however, yielded highly significant coefficients of lagged consumption and highly significant autocorrelation coefficients. The long-run elasticities of pork demand estimated from the dynamic A.L.S.-1 equation did not differ appreciably from the elasticities estimated from the static L.S. equation.

In equations containing seasonal dummy variables used in the 13-week analysis of fryer demand, again the addition of lagged consumption introduced autocorrelation into the errors. In the static equations, there was no evidence of autocorrelation. In the dynamic L.S. equation, lagged consumption was nonsignificant. In the dynamic A.L.S.-1 equation, lagged consumption was significant at the 10-percent level and estimated β_1 was highly significant.

In two cases (13-week analyses of beef and total meat), the addition of lagged consumption reduced the autocorrelation in the disturbances. However, the coefficients of lagged consumption were nonsignificant, and the estimates of β_1 were significant at only the 10-percent level in the static equations.

Only in the case of cheese was highly significant autocorrelation in the errors of the static equation eliminated by the addition of lagged consumption, which was also highly significant. In the A.L.S.-1 dynamic equation, however, the estimate of β_1 was nonsignificant, and lagged consumption was significant at only the 10-percent level. Even in this case, then, the only sound conclusion appears to be that there is either a lag or an autocorrelated error.

Absence of a relevant variable — lagged consumption or any other — may be a source of autocorrelation in the disturbances. Presence of a relevant variable also appears to be a cause of autocorrelation. In one case (13-week static beef equation), addition of a significant variable (temperature) introduced highly significant autocorrelation into the errors. In two other cases, the addition of significant variables (temperature or seasonal dummies) raised the estimate of β_1 from nonsignificance to significance at the 10-percent level.

If a static elasticity computed from a 4-week observation period is less than the corresponding elasticity computed from a 13-week observation period, this suggests an adjustment period of more than 4 weeks and underadjustment in the short-run. This situation is illustrated in the top part of fig. 1, if each time period is defined as 4 weeks. If a static elasticity estimated from 4-week data exceeds the corresponding elasticity from 13-week data, this suggests an initial overadjustment as illustrated in the bottom part of fig. 1.

In this study, the comparisons of the static elasticities from 13-week and 4-week observations indicated a different type of lag distribution for own-price and income changes in every equation. When the own-price elasticities indicated an initial underadjustment to price changes, the income elasticities always indicated an initial overadjustment and vice versa. Of the lag distributions that were significant in the dynamic analyses, the type of adjustment, as shown by the coefficient of lagged consumption, was always the kind suggested by the static comparisons of the own-price elasticities.

This suggests that the assumption made in the Koyck-Nerlove model, that the distribution of lag is the same for every variable, may not be valid. An assumption of different lag distributions for different variables can be incorporated into a variant of the Koyck model. Consider equation 1.1 and replace equation 1.2 by

$$(1.2.a) \quad b_{1i} = \lambda\, b_{1i-1}, \qquad -1 < \lambda < 1$$
$$b_{2i} = \mu\, b_{2i-1} \qquad -1 < \mu < 1$$

From equations 1.1 and 1.2.a we derive[4]

$$(1.4.a) \quad y_t = b_{10}x_{1t} - b_{11}\mu\, x_{1t-1} + b_{20}x_{2t} - b_{20}\lambda\, x_{2t-1} + (\mu + \lambda)y_{t-1} - \mu\,\lambda y_{t-2}.$$

MEASURES OF AUTOCORRELATION IN ERRORS

Different tests for the presence of autocorrelation in the disturbances were used in this study: A.L.S. estimation, d statistic and von Neumann-Hart ratio. They were not all used for each equation, but all were used for 18 equations. Comparative results are presented in table 15.

A disadvantage of the Durbin-Watson tables of the d statistic is the inconclusive range — the range in which the test does not permit either the conclusion of significance or of nonsignificance. Theil and Nagar (28) have presented a table of significance levels for testing the null hypothesis against the alternative hypothesis of positive serial correlation in the disturbances. Their test is derived on the assumption that the first and second differences of the independent variables are small in absolute value compared with the range of the actual variables. Their significance levels are close to the upper bound (d_u) of Durbin-Watson. Their test leads to rejection of the null hypothesis whenever the Durbin-Watson test does and in almost every case in which the Durbin-Watson test is inconclusive. The Theil-Nagar table provides only a one-tail test against positive serial correlation. An approximate test against negative serial correlation was obtained by assuming symmetry about 2.0. In every case in which the Durbin-Watson test was inconclusive, the Theil-Nagar test was significant.

It is known that the von Neumann-Hart ratio accepts the null hypothesis of serial independence too frequently. One Monte Carlo study indicated that the Durbin-Watson d statistic also accepts the null hypothesis too frequently (18). The results here confirm these findings and also suggest that the Theil-Nagar d may reject it too frequently.

The lagged value of the dependent variable appears in five of the 18 equations. In three of the five, the Durbin-Watson d was inconclusive, whereas the estimate of β_1 was significant. This might be expected, since the Durbin-Watson test is not appropriate for equations containing lagged values of the dependent variable among the independent variables.

In eight different cases where it appeared that a second-order autoregressive error would be appropriate, the A.L.S.-2 procedure was applied. In all eight cases, estimated β_1 from the A.L.S.-1 procedure was highly significant. In four of the eight A.L.S.-2 equations, estimated β_1 was significant and estimated β_2 was nonsignificant. In three A.L.S.-2 equations, estimated β_1 dropped to nonsignificance and estimated β_2 was significant. In only one A.L.S.-2 equation were both estimated β_1 and β_2 significant. The estimates (other than the autocorrelation coefficients) obtained by A.L.S.-2 usually differed little from those obtained by A.L.S.-1 even when the second-order autocorrelation coefficient was significant. This suggests that econometricians may not go far wrong in arguing that a first-order error model is adequate.

[4]The derivation is presented in (21). Martin also presents an A. L. S. procedure for estimating this model under the assumption of autocorrelated disturbances.

Table 15. Number of equations in which Durbin-Watson d, estimated β_1 and von Neumann-Hart ratio are significant and nonsignificant by two-tailed test.[a]

Status of estimated β_1 at 10-percent level	Status of Durbin-Watson d at 10-percent level			
	Significant	Nonsignificant	Inconclusive	Total
Significant	1[b]	1	6[c d]	8
Nonsignificant	1	5	4[e]	10
Total	2	6	10	18

[a]von Neumann-Hart ratio was nonsignificant in all but the three equations noted in footnotes b and c.

[b]von Neumann-Hart test was significant for this equation.

[c]von Neumann-Hart ratio was significant for two of these equations.

[d]Lagged dependent variable appears in three of these equations. In all three estimated β_1 was significant at the 1-percent level.

[e]Lagged dependent variable appears in two of these equations.

EFFECT OF METHOD OF ESTIMATION

In their study of autocorrelated disturbances, Hildreth and Lu (13) classified each equation according to the difference in estimates between L.S. coefficients and coefficients estimated from a model with first-order autoregressive errors. They used three classes:

I. Negligible difference. None of the re-estimated coefficients differ from the corresponding L.S. estimates by as much as 20 percent.

II. Noticeable difference. Some, but fewer than half, of the coefficients change by 20 percent or more.

III. Substantial difference. Half or more of the coefficients change by at least 20 percent.

Out of 17 equations, they placed seven in class I, five in class II and five in class III.

Applying this classification scheme to the results of this study yields the results in table 16. In nearly half — 15 — of these equations, there was evidence of autocorrelation in the errors. And 13 of these 15 equations are in class III. As would be expected, A.L.S. estimation makes less difference in the coefficients when estimated β_1 is nonsignificant.

The number of equations compared here differs from the number compared in the section on Measures of Autocorrelation in Errors, because d statistics were not computed for all equations. Another way of looking at the difference a first-order autoregressive error model makes is to compare levels of significance. These results are summarized in table 17. This table uses 10 percent as the critical level. In 80 percent of the cases, both methods resulted in the same conclusion concerning a coefficient's significance. In 20 percent, however, they yielded different conclusions.

A.L.S. estimation reduced standard errors more often than it increased them. Considering only those cases where estimated β_1 was significant at or beyond the 10-percent level and only those coefficients which were significant at the 5-percent level in either the L.S. estimates or the A.L.S. estimate or both: the A.L.S. standard error was smaller than the L.S. standard error in 25 cases, larger in 14 cases and equal in 2 cases. In 10 of the 14 cases in which the A.L.S. standard error was larger, the equation contained a lagged consumption variable whose coefficient was significant in the A.L.S. equation and nonsignificant in the L.S. equation. These results are suggestive of the inefficiency of L.S. estimates in equations with autocorrelated errors.

Tables 16 and 17 and the Hildreth and Lu results suggest one weakness of the Wold (35) and Zellner (36) procedures. While correcting for the effects of autocorrelated disturbances on estimated standard errors, these procedures ignore the effects of autocorrelated disturbances on the estimated coefficients. These latter effects may be substantial.

In summary, autocorrelated errors appear to be rather common phenomena in equations fitted to monthly or quarterly data. The use of a first-order error model often leads to different conclusions concerning size of coefficient or level of significance.

Table 16. Equations classified according to the percent difference between L.S. and A.L.S.-1 coefficients.

	Number of equations			
	I	II	III	Total
β_1 significant at 10 percent	0	2	13	15
β_1 nonsignificant at 10 percent	6	9	3	18
Total	6	11	16	33

Table 17. Effect of significant autocorrelation as evidenced by estimated β_1, significant at 10-percent level.

	Number of L.S. coefficients		
	Significant at 5 percent	Nonsignificant at 5 percent	Total
Number of A.L.S.-1 coefficients significant at 5 percent	27	7	34
Number of A.L.S.-1 coefficients nonsignificant at 5 percent	7	30	37
Total	34	37	71

COMPARISONS OF A.L.S. WITH HILDRETH AND LU PROCEDURE

Hildreth and Lu (13) have estimated several equations that are assumed to possess first-order autoregressive errors by a procedure similar to the A.L.S.-1 procedure used in this study. Therefore, to provide a basis for comparing the two estimation procedures, three of the equations which Hildreth and Lu considered have been re-estimated by A.L.S.-1. The estimation procedure suggested by Hildreth and Lu, the advantages and disadvantages of their procedure compared with the A.L.S. procedure and comparisons of the results of the two estimation procedures shall now be considered.

Estimation procedure of Hildreth and Lu

Suppose that the parameters of an equation of the form:

$$(4.1) \quad Y_t = a_0 + \sum_{i=1}^{K} a_i x_{it} + u_t \qquad t = 1 \ldots n$$

are to be estimated under the assumption

$$(4.2) \quad u_t = \beta u_{t-1} + \varepsilon_t \qquad -1 < \beta < 1$$

where Y_t is the dependent variable, the X_{it} are K independent variables, u_t is the error in equation 4.1 which is assumed to follow the first-order autoregressive scheme, equation 4.2, with autocorrelation coefficient β, and ε_t, is normally and independently distributed with zero mean and constant variance. Equations 4.1 and 4.2 may be reduced to the form:

$$(4.3) \quad Y_t - \beta Y_{t-1} = a_0(1-\beta) + \sum_{i=1}^{K} a_i(X_{it} - \beta X_{it-1}) + \varepsilon_t.$$

The a_i may now be estimated for any given value of β, β^*, by running the usual L.S. regression of $(y_t - \beta^* y_{t-1})$ on the $(x_{it} - \beta^* x_{it-1})$ where the small letters y_{t-j} and x_{it-j}, represent deviations from the respective means, $\bar{Y}_{t-j}$ and $\bar{X}_{it-j}$. The estimate of the constant term in equation 4.3 is obtained in the usual manner. Therefore, the estimate of a_0 may be obtained by multiplying this constant by $\frac{1}{1-\beta^*}$.

The procedure suggested by Hildreth and Lu consists of selecting several values of β between -1 and 1, transforming either the original data or the moment matrices for each of the selected values of β and obtaining the L.S. fit of equation 4.3 for each of the selected values. Next, select that L.S. fit that resulted in the smallest residual sum of squares. Then, select several new values for β that are slightly larger and slightly smaller than the value which resulted in the smallest residual sum of squares, transform the data by the new estimates of β, refit the equation of L.S. and continue this process until the desired accuracy in the estimation of β is obtained. The final set of estimates for the a_i and β will be maximum likelihood estimates and will be consistent.

Demand for summer lemons[5]

The first equation selected for comparison was Hildreth and Lu's equation for the demand for summer lemons, which is of the form:

$$(4.4) \quad X_{1t} = a_0 + a_1X_{2t} + a_2X_{3t} + a_3X_{4t} + a_4T + a_5T^2 + u_t$$

where X_{1t} is the price of summer lemons, X_{2t} is the United States supply of summer lemons, X_{3t} is the United States nonagricultural income payments, X_{4t} is the average monthly temperature and T is time. This equation, originally estimated by Hoos and Seltzer, was selected for further investigation because of the inconclusive test of the Durbin-Watson statistic reported by Hildreth and Lu.

Equations 1 and 2 in table 18 present, respectively, the L.S. fits obtained by Hildreth and Lu and by us after we reconstructed the data. Although slight differences appear between the L.S. fits, these differences are small and should not invalidate the comparison.

Equations 3 and 4 present, respectively, the results obtained by Hildreth and Lu and by A.L.S.-1 after six iterations. As one would expect, the results of the two estimation procedures give coefficients that are quite comparable; i.e., to two decimal places, the coefficients are identical. After six iterations, the A.L.S.-1 estimates were accurate to eight decimal places; i.e., the computer could find no other set of coefficients that would reduce the residual sum of squares. In addition, the estimate of the autocorrelation coefficient, -0.30, was not significant. Therefore, the hypothesis that the errors in the L.S. equation are independent was not rejected. The apparent contradiction between the negative estimate of β, which would indicate negative autocorrelation of the errors, and the Durbin-Watson statistic of 1.088 reported by Hildreth and Lu, which would indicate positive autocorrelation of the errors, may be partially resolved since the estimate of β is not significant. The use of A.L.S.-1, however, did result in significant coefficients of T and T^2 primarily because of a reduction in the magnitude of their respective standard errors.

Demand for green peppers

The second equation selected for comparison was Hildreth and Lu's equation for the demand for green peppers, which is of the form:

$$(4.5) \quad x_{1t} = a_0 + a_1x_{2t} + a_2x_{3t} + u_t$$

[5] Our Hildreth and Lu procedure estimates were obtained by using the same I.B.M. program that was used to obtain A.L.S. estimates.

Table 18. Selected statistics from regression estimates of the demand for summer lemons.

Equation	X_{2t}	X_{3t}	X_{4t}	T	T^2	β_1	R_{ss}	R^2	Method of estimation	Number of iterations
1	—0.1237	0.0244	0.2876	—0.0754	—0.0065	0	4.0203	—	L.S.	—
2	—0.1233***	0.0244***	0.2877***	—0.0759e	—0.0066e	0	4.0325	0.8604	L.S.	—
3.	—0.1140	0.0241	0.2334	—0.0721	—0.0075	—0.30	3.7894	—	H.L.	—
4.	—0.1139***	0.0241***	0.2328***	—0.0790***	—0.0075**	—0.3047e	3.7800	0.8692	A.L.S.-1	6

Table 19. Selected statistics from regression estimates of the demand for green peppers.

Equation	X_2	X_3	β_1	β_2	R_{ss}	R^2	Method of estimation	Number of iterations
1	0.7043	—0.000019	0	0	8.098	—	L.S.	—
2	0.6297***	—0.000034	0	0	7.5743	0.7221	L.S.	—
3	—0.0383	0.000048	0.88	0	3.705	—	H.L.	27
4	0.0156	0.000049	0.8409***	0	3.4296	0.8742	A.L.S.-1	6
5	0.0153	0.000049	0.8533***	—0.0119	3.4290	0.8742	A.L.S.-2	8

where x_{1t} is the price of green peppers at Clinton, N. C., x_{2t} is the price of North Carolina green peppers on the New York wholesale market and x_{3t} is the supply of green peppers on the Clinton market. This equation, originally estimated by Linstrom and King, was selected because the d statistic rejected the hypothesis of independent errors and because Hildreth and Lu suggested the possibility that the error structure might actually follow an autoregressive scheme of higher order.

Equations 1 and 2 in table 19 present the L.S. fits obtained, respectively, by Hildreth and Lu and the authors. The differences between the coefficients of equations 1 and 2 are due to the use of one less observation in equation 2. To investigate a second-order autoregressive error scheme and to compare the estimates of L.S., A.L.S.-1 and A.L.S.-2, two observations were omitted in the L.S. fit and one observation was omitted in the A.L.S.-1 fit. Thus, results of equation 1 are based on one more observation than the results of equation 2.

Equations 3 and 4 present, respectively, the results obtained by the Hildreth and Lu procedure and A.L.S.-1 after six iterations. Again, there is one less observation in the A.L.S.-1 equation. However, there are no significant differences between the coefficients of equations 3 and 4.

In agreement with the d statistic computed by Hildreth and Lu, the estimate of β_1 is highly significant. Thus, the estimates obtained by L.S. are probably inefficient. The inefficiency of the L.S. estimates is illustrated further by the fact that the coefficient of X_2 is significant in the L.S. fit, equation 2, but is not significant in the A.L.S.-1 fit, equation 4. The F for the additional contribution of β_1 to the L.S. fit, equation 2, was 20 545 with 1 and 17 degrees of freedom.

Equation 5 presents the results of A.L.S.-2 estimation. After eight iterations, the computer could find no other set of coefficients that would reduce the residual sum of squares, R_{ss}. Since the estimate of β_2 is not significant, the hypothesis of independent errors in equation 4 is not rejected. The possibility remains, however, that the error structure follows other than an autoregressive scheme of first or second order.

Demand for ice cream

The form of the original equation is:

$$(4.6) \qquad x_{1t} = a_0 + a_1x_{2t} + a_2x_{3t} + a_3x_{4t} + u_t$$

where x_{1t} is the per-capita consumption of ice cream, x_{2t} is the price of ice cream, x_{3t} is weekly average family income and x_{4t} is mean temperature.

Equations 1 and 2 in table 20 present, respectively, the L.S. fits obtained by Hildreth and Lu and by us after we reconstructed the data. Notice that the estimated coefficients of equations 1 and 2 are identical. In addition, each coefficient is highly significant.

Equations 3 and 4 present the results obtained by Hildreth and Lu and A.L.S.-1 after 10 iterations, respectively. In agreement with Hildreth and Lu's findings, temperature appears to be the most important factor affecting the per-capita quantity of ice cream purchased. Also in agreement with the test of the d statistic computed by Hildreth and Lu, the estimate of β_1 is significant and positive.

This set of equations illustrates how erroneous conclusions may result from the application of L.S. if, in fact, the errors are not independent.

Table 20. Selected statistics from regression estimates of the demand for ice cream.

Equation	X_2	X_3	X_4	β_1	R_{ss}	R^2	Method of estimation	Number of iterations
1	.—0.7378	0.00398	0.00364	0	0.029521	—	L.S.	—
2	—0.7378***	0.00398***	0.00364***	0	0.029521	0.7634	L.S.	—
3	—0.8967	0.00316	0.00355	0.41	0.025452	—	H.L.	24
4	—0.9223[e]	0.00265[e]	0.00353***	0.4116*	0.026261	0.7896	A.L.S.-1	10

Advantages and disadvantages

As was expected, the estimation procedure used by Hildreth and Lu and the A.L.S.-1 procedure resulted in similar estimates for all three of the equations investigated. We anticipate that the two procedures will almost invariably yield substantially the same estimates.

An important element in determining which procedure is superior is the frequency of occurrence of multiple maxima. So long as we assume $-1 \leqslant \beta \leqslant 1$, around 25 iterations suffices for the Hildreth and Lu procedure. If the likelihood function has one maximum, A.L.S.-1 usually requires from four to 10 iterations. Using A.L.S.-1, one can search for local maxima by using more than one initial start vector. The total number of iterations increases accordingly. In work published elsewhere (6, p. 78) and in some unpublished work, we have found cases of multiple maxima, but only rarely. Hildreth and Lu found no examples of multiple maxima in the 17 equations they recalculated. The evidence to date indicates that multiplicity of maximum is a rare phenomenon.

The Hildreth and Lu procedure becomes cumbersome in the estimation of models containing second-order autoregressive errors because of the possible large number of paired combinations of coefficients. The estimation of such models by A.L.S. requires little more work than the estimation of first-order models.

The A.L.S. procedure provides estimates of the large sample variances and covariances of the estimates of a_i, β_1 and β_2. With the Hildreth and Lu procedure, one could obtain conditional variances and covariances of the estimated a_i for each value of β.

Computation for either procedure is relatively simple, involving only transformation of the data and application of L.S. to the transformed data to obtain estimates of the parameters.

Additional work needs to be done in nonlinear estimation using various estimation techniques before one procedure can be recommended as being superior to another.

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