Soybean Yields and Plant Composition as Affected by Phosphorus and Potassium Fertilizers

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SUMMARY

The primary objective of this study was to determine if soybean grain yields could be predicted by the P and K content of the growing plant. A supporting objective was to find what plant parts should be taken, and at what stage of growth, to give the best relationship. Multiple curvilinear regression analysis was used to determine this relationship from data collected in four field experiments.

Yields, chemical composition of soybean plant parts and other data were available from four similar fertilizer experiments conducted at various locations in Iowa in 1958. A randomized block 9 x 9 central composite design, containing various combinations of P and K fertilizer rates, was used in all experiments. Soybean plant samples from each plot, taken in three different growth stages, were separated into various plant parts and chemically analyzed for total P and K contents. (Growth stages used were: Stage 5—Nine to 10 trifoliate leaves unrolled with stem branching evident; full bloom with withered flowers in lower leaf axils. Stage 7—Pods plainly evident in plant tops; lower pods nearly full length with beans developing; flowering ceased. Stage 9—Bottom leaves beginning to yellow; top pods almost fully developed with beans nearing "green bean" stage.)

Data were first examined graphically to determine the simple relationships between yield and percent P and percent K in the plant parts. Linear regression equations and simple correlation coefficients were calculated for some of these relationships. The results of these procedures were used to help specify the nature of subsequent regression analyses.

Multiple regression statistics were calculated for relationships between yield and percent P and percent K of the various plant parts at the selected stages of growth. The two forms of the polynominal function used to express the curvilinear relationships were (a) the two-variable quadratic equation with a linear $\times$ linear interaction term and (b) a square-root transformation of the two-variable quadratic equation with a square root $\times$ square root interaction term. The multiple regression equations, standard errors of the partial regression coefficients and values of R^2 of this relationship for the various soybean plant parts sampled in different stages of growth for individual experiments also were calculated for each form of equation. Yield was a curvilinear function of percent P and percent K in some plant parts, but this curvilinear relationship was not consistent over all experiments.

The highest R^2 values for the regression of yield on P and K content of the various plant parts occurred most often in growth stages 5 and 7; growth stage 9, therefore, was omitted in subsequent multiple regression analyses. A large amount of variability of R^2 was found for the regression equations based on the different plant parts in different growth stages and among experiments. Chemical composition of any one plant part or one form of regression equation did not show a consistently better relationship with yield than any other in this part of the study.

The data for three experiments were combined for multiple regression analyses to determine if the relationship between yield and chemical composition could be expressed better by the wider range of data. The values of R^2 for the square-root equations were somewhat higher than those for the quadratic equations for all plant parts in growth stage 5 and, for the lower petioles, in growth stage 7. There was no apparent difference between the two forms of the equations for the other plant parts in growth stage 7. The values of R^2 were substantially higher in growth stage 7 than in stage 5.

Estimated yields at the critical points of the equations obtained from the regression equations for the combined data of the experiments were all maximum yields except one. The most reliable yield estimates in this study were those associated with the critical points with respect to percent P and percent K which fell within the range of the observed experimental values.

The relationships between percent P and percent K in the upper leaves and upper petioles sampled in growth stage 7 at different estimated yield levels were determined by the isoquant equation calculated from the quadratic form of the regression equation. In this study, only a relatively small portion of the yield isoquants for the upper leaves were within the range of data, whereas a much larger portion of the yield isoquants for the upper petioles in the same growth stage were within the range of observed data.

Only small differences were found between the R^2 values for the quadratic and the square-root forms of the equations and among the lower and upper leaves and upper petioles in growth stage 7. With all things considered, the upper leaves appear to be the most convenient plant parts to use in this type of study. It was possible to account for 73 percent of the variation in soybean yields by the multiple regression equations containing only percent P and percent K as the two independent variables.

Soybean Yields and Plant Composition as Affected by Phosphorus and Potassium Fertilizers[1]

by R. J. Miller, J. T. Pesek, J. J. Hanway and L. C. Dumenil[2]

One of the principal objectives of agronomic research has been the collection of information on crop yield responses to fertilizers under different climatic and soil conditions. Much recent interest in this area of research has been in the determination of yield and yield response equations so that economic analyses could be applied to these data. From many of these analyses, optimum fertilizer rates and ratios can be determined for specific nutrient:nutrient and fertilizer:crop price ratios.

Various methods have been used to estimate the availability of essential nutrient elements in the soil so that yields and yield responses to fertilizers can be predicted more precisely. Of these methods, the use of chemical composition of the crop to estimate nutrient availability and to aid in predicting yields and yield responses has met with some success. Chemical analysis of the whole plant or a suitable plant part to determine its composition is the basis of this approach.

Yield of grain-producing crops might be expected to reflect mineral composition of the plant before and during grain formation because: (a) A change in mineral composition is usually associated with a change in vegetative growth; thus, a larger or smaller photosynthate producing unit is formed with a corresponding capacity for producing grain. (b) Other things equal, the total supply of mineral elements available in the plant for transfer into the developing seed changes with a changing mineral composition. There is evidence that the amount of grain that can be produced by some crops is limited to some extent by the total mineral composition, because the plants cannot or do not absorb the quantities of minerals contained in the grain during the period of grain development. Hammond et al. (6) found that N, P, K, Ca and Mg were transferred from the vegetative plant parts to seed of soybeans. Similar observations for corn were made by Sayre (13) and Hanway (7).

The relationships between yield and the chemical composition of many crops have been studied with regard to the effects of various fertilizers on yield and chemical composition of the plants or of selected plant parts. Although some work of this type has been conducted with soybeans, the results have not furnished sufficient information to clarify adequately the relationships between yield and chemical composition of this crop.

Thus, the principal objectives of this study were (a) to determine if grain yield of soybeans was related to the chemical composition of the growing plant, (b) to determine the growth stage and plant part in which chemical composition best correlated with soybean yield and (c) to determine the effects of P and K fertilizers on yield and soybean plant composition. The method of multiple curvilinear regression analysis applied to the results of four field fertilizer experiments was used in the attempt to attain these objectives.

REVIEW OF LITERATURE

The relationship between crop yield and plant composition is largely influenced by soil nutrient availability, nutrient absorption and nutrient utilization following absorption by the plant roots. Crop yield responses to fertilizers have been recognized as the end result of a number of interacting factors, and these interactions may be affected by any of the three nutrient factors just named. This is exemplified by the fact that the concentration of a nutrient within a plant often reflects the available supply of that nutrient as affected by the supply of other nutrients in the external medium.

The results of investigations dealing with the effects of various fertilizers on yields and on the chemical composition of leaves of plants, particularly corn, have been reported by a number of workers. Tyner (16) and Tyner and Webb (17) found that corn yields correlated well with the chemical composition of the sixth leaf sampled from corn plants during full silk and tassel with pollen shedding. Tyner determined the critical levels of N, P and K in the corn leaves as 3.1, 0.315 and 1.4 percent on an oven-dry basis. Spies[3] reported somewhat lower values for critical percent N under drier conditions.

[1]Project 1189 of the Iowa Agricultural and Home Economics Experiment Station.

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[3]Clifford D. Spies. Relationships of corn yields, leaf composition and fertilizer treatments on southwestern Iowa bottomland soils. Unpublished M.S. thesis, Iowa State University Library, Ames, Iowa. 1956.

Bennett et al. (2) found that sampling procedures of Tyner and Webb were adequate when used under Iowa conditions. In studying the effect of P, applied as a hill fertilizer, on the percent P in corn leaves sampled at early silking stage, Webb and Pesek (18) reported that, although the P fertilizer increased corn yields, the percent P in the leaves increased only slightly.

Investigations with pea plants reported by Tremblay and Baur (15) showed that the greatest differences in percent P in the plants due to P fertilizer were found in the early growth stages. They also found that heavy applications of P fertilizer caused a significant decrease in the K content in the leaves and leaf petioles.

Nelson et al. (11) studied the effects of K and Mg fertilizers on the chemical composition of soybean leaves and petioles and found that K applications increased the K content but decreased the Mg, Ca and P contents, while Mg applications decreased Ca, K and P content of the same plant parts. The K applications increased yields fourfold, while Mg increased yields only slightly.

All studies mentioned considered the influence of the concentration of a single element in the crop upon yields or the effect of applying a nutrient upon the content of this and other elements in plants. A recent comprehensive coverage of relationships between corn yields and the N and P contents of corn leaves was reported by Dumenil (4). This study dealt with the yield of corn as a function of the joint effect of N and P percentages in corn leaves. The soybean study reported[4] here was concerned with investigating, in part, the joint effect of P and K concentration in soybean plant parts on soybean grain yields.

EXPERIMENTAL PLANS AND PROCEDURES

Sites and field procedures

Since the effects of P and K fertilizers on the chemical composition and yield of soybeans were of major interest, four similar experiments with various rates of P and K fertilizers were conducted in 1958. Three experiments were located in northeastern Iowa and one in north-central Iowa.

Each experiment was located on a uniform soil area testing low to very low in available P or K, or both. Soil samples were taken from the plow-layer before the fertilizer applications, whereas subsoil samples to a depth of 24 inches, in 6-inch increments, were taken from each replication at a later date. Each surface and subsoil sample contained from 15 to 20 composited borings. Tests for pH and available P and K were made on all soil samples by the Iowa State University Soil Testing Laboratory according to the methods described by Hanway and Heidel (8). The soil test results are given in table A-1 of the Appendix.

Experiment 1 was located on a Dickinson fine sandy loam that had a pH of 6.6 and tested low and very low in available P and K. Experiment 2 was located on a Floyd silt loam having a pH of 5.9 and testing low and low-medium in available P and K, respectively. Experiment 3 also was located on a Floyd silt loam with a pH of 5.3 and which tested low in both available P and K. Experiment 4 was located on a Nicollet loam that had a pH of 6.3 and tested low and medium in available P and K.

The experimental design was a 9 x 9 central composite type with 22 different fertilizer treatments included (table 1). The treatments consisted of selected combinations of nine P and K fertilizer levels and were replicated twice at each location in randomized blocks.

The individual plots were six rows wide and ranged from 16⅔ by 24 feet to 20 by 30 feet in area. The required amounts of fertilizer were carefully broadcast on each plot by hand and disked in on Experiment 2 but plowed under on the other three. The sources of fertilizers were concentrated superphosphate (20-percent P) and muriate of potash (50-percent K).

The experimental areas were prepared, the seed was inoculated and planted, and the crop was cultivated in the same manner as the rest of the respective farmers' fields. Soybean varieties used in the expriments were: Experiment 1, Harosoy; experiments 2 and 3, Chippewa; and Experiment 4, Hawkeye. All varieties were well adapted to their locations and seeded at about 75 pounds of seed per acre. The soybean experiments were planted along with the rest of the fields by the farmer-cooperators on May 15 and 16, except for Experiment 3 which was planted on May 28. Except for a cold spring at all locations and a dry period during August at the site of Experiment 4, climatic conditions were favorable for soybean production.

Weeds, found in all experiments, were most prevalent

Table 1. The effect of P and K fertilizers on the grain yield of soybeans in experiments at four locations in Iowa in 1958.

Trt. No.	Fertilizer treatment[a] P	Fertilizer treatment[a] K	Av. yield (bu./A.) at 13% H_2O Exp. 1	Exp. 2	Exp. 3	Exp. 4
1	0	0	16.4	26.0	15.6	23.1
2	0	53	23.2	25.5	18.8	24.4
3	1.8	3.3	16.9	26.4	17.5	24.6
4	1.8	163	28.8	26.0	17.7	26.6
5	7.0	13	18.5	25.3	17.9	26.4
6	7.0	53	22.0	27.5	20.0	25.0
7	7.0	119	28.8	27.9	17.1	26.8
8	16	30	18.5	26.7	19.8	26.2
9	16	83	25.7	25.0	20.4	27.9
10	28	0	5.3	26.2	20.3	27.0
11	28	13	17.9	25.3	19.5	26.8
12	28	53	23.0	25.4	19.3	28.3
13	28	179	27.3	27.8	21.1	26.8
14	28	212	30.4	29.3	19.3	25.8
15	44	30	16.3	27.4	22.0	25.5
16	44	83	25.3	27.7	19.0	28.1
17	63	13	14.0	27.1	19.8	27.1
18	63	53	21.7	28.1	20.6	26.2
19	63	119	29.4	27.2	23.6	27.1
20	85	3.3	7.9	26.7	21.6	26.9
21	85	163	35.3	30.7	17.4	28.6
22	112	53	25.1	27.5	20.3	30.6
LSD (0.05)[b]			7.4 bu.	2.9 bu.	3.2 bu.	4.8 bu.
Coefficient of variation:			16.4%	5.1%	8.0%	8.7%

[a]Rates of P and K in pounds per acre.

[b]Least significant difference at the 0.05 significance probability level.

[4]Robert J. Miller. Soybean responses and plant composition as affected by phosphorus and potassium fertilizers. Unpublished Ph.D. dissertation, Iowa State University Library, Ames, Iowa. 1960.

in plots receiving moderate to high rates of P and K fertilizers. Two hand-weedings early in the season were necessary to prevent serious weed competition from limiting soybean yields at all locations.

Lodging of plants resulting from high rates of P and K fertilizers was observed in early June and became more severe as the plants became larger. Lodging scores[5] for all experiments are given in table A-2 in the Appendix. Lodging was most severe in Experiment 1 and may have had some adverse effect on yields. When severe lodging occurred during pod formation and seed set, prolonged contact of pods with the soil surface appeared to depress seed set.

Soybean yields were estimated by hand harvesting and weighing the soybean seed from two harvest rows 16 feet long. The harvested soybean plants were allowed to air dry before threshing and weighing, and a subsample of soybeans was weighed before and after drying at 63°C. for 48 hours to determine the moisture content. By use of the field weight of the soybeans and their moisture content, yields were calculated for each plot in bushels per acre at 13-percent moisture content.

Procedures used in plant sampling and chemical analysis

Plant samples from each plot were taken at growth stages[6] 5, 7 and 9, and the number of plants taken at each sampling was 20, 10 and 10. These plant samples were removed from two rows adjacent to those designated for grain harvest. The experiments were not sampled on the same dates, but they were sampled at the same growth stages. Not all plots reached a specified growth stage at the same time, however, and this was more noticeable as the plants neared maturity. This difference in growth rate was related to the level of fertilization and varied among experiments, so plots with the most advanced plant growth, generally the well fertilized ones, were used as an arbitrary guide in determining the time of sampling.

All whole plants used in this study were immediately separated into upper and lower halves and were promptly dried in a forced hot-air dryer at 65°C. to stop enzymatic action or deterioration of the plant material. After removal from the dryer, the samples were further subdivided into leaves, petioles, stems and pods when present. The plant parts were later redried, weighed, ground in a Wiley mill and stored in glass bottles for chemical analysis.

Total P and K in the various plant parts of the three samplings were determined in the soil fertility laboratory. Before the chemical analyses, the samples were dried in an oven at 65°C. for 24 hours. Each 0.50 gram sample was digested in concentrated H_2SO_4 with Cu as a catalyst until 1 hour after the solution became colorless. After the solution was brought to volume by adding NH_3-free water, the P was determined on an aliquot in a colorimeter in the presence of added vanadomolybdate solution. A flame photometer was used to determine K. All results were reported as percentages of the total P and K in the plant parts on an oven-dry basis.

Statistical methods

The soybean grain yields of all experiments were analyzed by analysis of variance according to procedures described by Snedecor (14). The yield and P and K contents of the plant parts, except the pods, from samplings of all four experiments were used in preliminary linear regression studies and in the multiple regression analyses. There were 44 observations included in the preliminary analyses. When data from the different experiments were combined, 132 or 176 observations were included, depending upon the number of experiments used in the analyses.

The yields of beans and the percentages of P and K in each leaf, petiole and stem sample of the second sampling of each experiment were used in the preliminary multiple curvilinear analyses. The upper and lower stem samples were omitted in subsequent analyses of the first and third samplings. The yields and P and K contents of similar plant parts of the four experiments were combined for each sampling period and used in multiple regression analyses. Similar analyses were run with data from only three experiments combined.

The data for each individual plot were punched on cards. Most of the computations were done by the Iowa State University Statistical Laboratory. In the initial calculations, the sums of squares, cross products, correlation coefficients, totals and means were calculated by the computer. The corrected sums of squares and cross products of the selected variables were punched on new cards, and the matrix was inverted. The sample partial regression coefficients, their standard errors and the *t*-tests of the regression coefficients were also calculated. The tests of significance of the reduction in the residual error due to regression were calculated according to the methods given in Anderson and Bancroft (1). The final procedure was the determination of the regression equations relating grain yields to the percentage of P and K in the various plant parts studied.

RESULTS AND DISCUSSION

Fertilizer effects on growth and yields

Growth responses to P and K fertilizers were observed in all experiments by late June but varied among sites. The greatest growth responses due to fertilizer were found in Experiment 1, located on a Dickinson fine sandy loam deficient in available P and K.

[5]Weber, C. R. Soybean lodging score. Iowa Agr. and Home Econ. Exp. Sta., Ames, Iowa. Private communication. 1958.

[6]Growth stages, as described by Kalton et al. (10), are as follows: Stage 5—Nine to 10 trifoliate leaves unrolled, with stem branching evident; full-bloom stage with withered flowers in lower leaf axils. Stage 7—Pods plainly evident in plant tops; lower pods nearly full length with beans developing; flowering ceased. Stage 9—Bottom leaves beginning to yellow; top pods almost fully developed with beans approaching "green bean" stage.

Visual K-deficiency symptoms were observed on the leaves of plants in the control plots and in the plots receiving high rates of P and no or low rates of K in experiments 1 and 2. High rates of P alone or with low rates of K accentuated the K-deficiency symptoms in both experiments 1 and 2 but much more markedly in the former than in the latter. The depressive effect of high P and low K treatments on plant growth in Experiment 1 became greater as the growing season progressed. At the end of the growing season, plots receiving moderate to high rates of both P and K fertilizers in Experiment 1 were at least 1 week ahead, in maturity, of the control and other K-deficient plots.

The effects of P and K fertilizers on soybean yields (table 1) were much greater in Experiment 1 than in the other experiments. The adverse effects of high rates of P fertilizer on soybean yields are shown by treatments 10 and 20 in Experiment 1. The K supply in the soil was initially lower in Experiment 1 than in the other three experiments and is shown in table A-1 in the Appendix. It appeared that, when the external K supply was very low, moderate to high rates of P fertilizer depressed soybean yields. Since the soybean variety (Harosoy) planted in Experiment 1 was not used in the other experiments, any differential yield response to P and K fertilizers due to variety could not be determined. The low yield level of Experiment 3 (table 1) was probably due to the low soil pH, late planting, or both.

Analysis of variance was run on the grain yields from each experiment, and the LSD (0.05 probability level) and the coefficient of variation of each are given in table 1. The LSD values were 7.4, 2.9, 3.2 and 4.8 bushels per acre in Experiments 1, 2, 3 and 4, respectively. Experiment 1 showed the highest LSD (7.4 bushels per acre) and the highest coefficient of variation (16.5 percent). The coefficients of variation for the other experiments were about half or less of that found in Experiment 1, the most responsive of the four.

Relationship between yield and percent P

The simple relationships between soybean yields and the percentages of P and K in the various plant parts in each experiment were investigated before formulating the mathematical models for multiple regression analyses. The approximate relationships were determined by the method of "successive group means"[7] according to Ezekiel (5). The simple relationships between percent P in the upper leaves sampled in growth stage 7 and soybean yield for the individual experiments are presented in fig. 1. Only data from the upper leaves of growth stage 7 are presented, because the relationship with other plant parts and at other growth stages investigated appeared to follow similar trends.

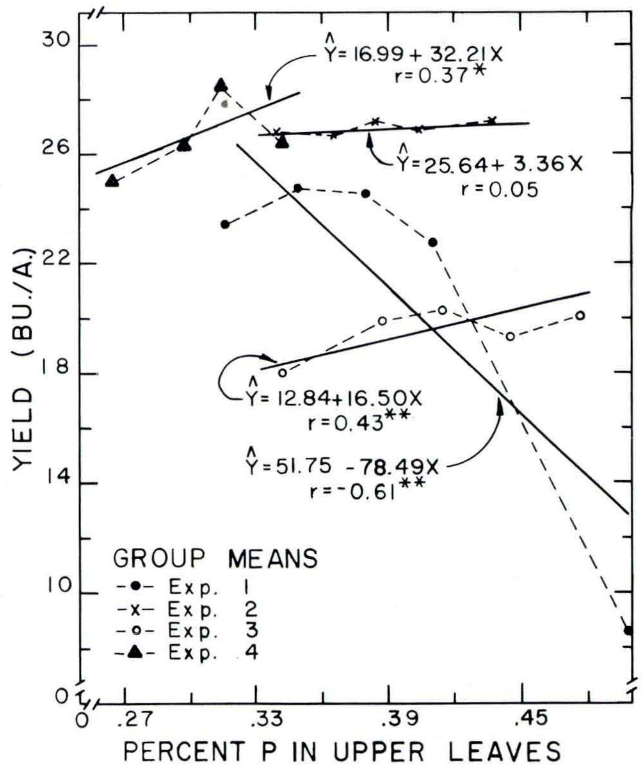

Fig. 1. Correlations and regressions of soybean grain yield ($\hat{Y}$) on percent P(X) in the upper leaves sampled in growth stage 7 from each experiment (44 observations per experiment).

These relationships were not the same in all experiments, and this indicates the presence of other factors that influenced yields. The curvilinear effect of percent P on yield was more apparent in Experiment 1 than it was in the other experiments. The curve shown for Experiment 1 indicated that yield increased with an increase in percent P in the upper leaves until approximately 0.35 percent P was reached; then the yield began to decrease with further increases in percent P. The calculated simple regression equation for Experiment 1 showed a high b_0 (intercept) value and a large negative regression coefficient. The simple correlation coefficient, r, was —0.61** and highly significant.[8]

There was no relationship between yield and percent P in Experiment 2 (r=0.05), and the group means indicated little deviation from linearity.

A highly significant relationship between yield and percent P was found in Experiment 3, but the linear regression shows only a small positive slope. There appeared to be only a slight deviation from linearity in this experiment.

A significant linear relationship between yield and percent P was found in Experiment 4, and the group

[7]The range of the individual observations of the X variable was arbitrarily divided into successive groups or subranges. From the observations within each of the groups, the means of the X variable and the associated Y variable (yield) were calculated. The primary purpose of this simple method was to estimate the deviations from linearity, although it might also have served as a basis for determining whether a square root or a quadratic form of the multiple regression equation better fitted the data. In the interpretation of these group means, it must be remembered that the selection of the subranges may influence the apparent shape of the curve and that unequal frequencies of the observations within the groups, particularly at the extremes, may cause apparent lack of agreement with subsequent regression analyses.

[8]Hereinafter, the 0.05 and 0.01 significance probability level, Snedecor (14), will be referred to as the 5-percent and 1-percent levels. The terms "significant" and "highly significant" also refer to the 5-percent and 1-percent levels. For numbers in tables, figures and text, these levels of probability are designated by an * and **, respectively.

means indicated some curvilinearity. The range of percent P values for the upper leaves was rather narrow, and the values were low relative to those found in the other experiments.

There appeared to be a definite curvilinear relationship between yield and percent P in the upper leaves of plants in Experiment 1, a slight curvilinear effect in experiments 3 and 4 but none in Experiment 2. Since the relationships between yield and percent P in the individual experiments were quite variable, no definite conclusions could be made about the relationship between yield and percent P without considering other factors affecting yields.

Relationship between yield and percent K

The simple relationships between yield and percent K in the upper leaves sampled in growth stage 7 are shown in fig. 2 for individual experiments. It appeared that Experiment 3 belongs to a population different from the other three experiments. This will be discussed later.

The relationship between yield and percent K in Experiment 1 was highly significant. The group means indicated only a slight deviation from linearity. The simple relationship between yield and percent K in the upper leaves (fig. 2) differed greatly from that found between yield and percent P (fig. 1). The lowest yields were associated with the lowest levels of percent K but

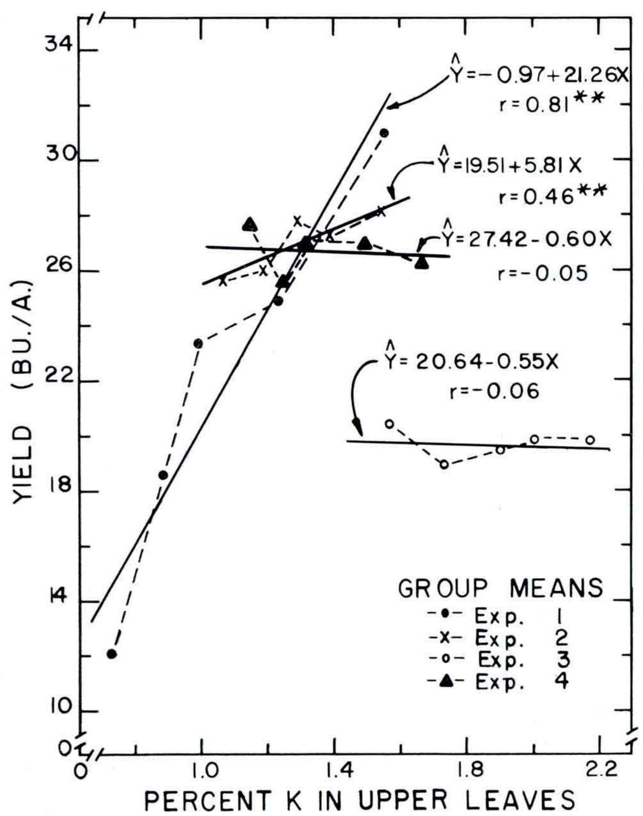

Fig. 2. Correlations and regressions of soybean grain yield ($\hat{Y}$) on percent K (X) in the upper leaves sampled in growth stage 7 from each experiment (44 observations per experiment).

with the highest levels of percent P as the result of applying high rates of P fertilizer and no K or of applying only low rates of K fertilizer.

In Experiment 2, the relationship between yield and percent K was highly significant but showed little deviation from linearity. There was a closer correlation between yield and percent K than between yield and percent P in this experiment.

There appeared to be little relationship between yield and the percent K in the upper leaves in Experiment 3 (r=—0.06). The high values of percent K found in this experiment may have reflected the effect of late planting. Because time of sampling was based primarily on the development of the reproductive organs of the plants, Experiment 3 was sampled at essentially the same calendar time as the other experiments. Because the plants in Experiment 3 had not made as much growth as the plants in the other experiments, the nutrients in the plants in this experiment had not been diluted as much by growth as had the nutrients in the other experiments. This could account for the higher concentrations of K found in plants in Experiment 3 than in plants in the other experiments.

Very little relationship between yield and percent K was found in Experiment 4 (r=—0.05). The regression coefficient was slightly negative as in Experiment 3, but the yields and percent K levels differed markedly from those of Experiment 3, being more-or-less in the same range as experiments 1 and 2. In the cases in which the relationship between yield and percent K in the leaves from plants in the experiments was significant, the yield increased with a corresponding increase in the percent K in the upper leaves. The average percent K in the upper leaves apparently never reached a level high enough to depress yield. The relationships shown in figs. 1 and 2 indicate that yields were not the same at all levels of percent P and percent K in the upper leaves.

Multiple regression statistics for individual experiments

Curvilinear regression equations of soybean yield on the percent P and percent K levels of four soybean plant parts sampled in growth stages 5, 7 and 9 were calculated for each experiment. Both the square-root form,

$$Y = b_0 + b_1 p^{1/2} + b_2 p + b_3 k^{1/2} + b_4 k + b_5 p^{1/2} k^{1/2}, \quad (1)$$

and quadratic form,

$$Y = b_0 + b_1 p + b_2 p^2 + b_3 k + b_4 k^2 + b_5 pk, \quad (2)$$

where the variates[9] p and k represent percent P and percent K, were calculated for all plant parts and growth stages used.

The regression equations, standard errors of the partial regression coefficients and R^2 values for the various

[9]The term "variate" refers to a single term included in the multiple regression model. The term "variable" refers to a factor under study whose effect in the regression model and analysis can be shown as a function of one or more variates.

plant parts sampled in growth stage 5 for each experiment are given in tables 2 and 3 for equations 1 and 2. When the calculated *t* at 38 degrees of freedom exceeded the tabular values of 2.025 or 2.712, the regression coefficients were considered significant at the 5- or 1-percent level, respectively.

The values of R^2 for equation 1 in Experiment 1 ranged from 0.76 to 0.85 and from 0.72 to 0.84 for equation 2. The square-root form of the equations fitted the data a little better (higher R^2 values) than did the quadratic form. The best relationship between yield and percent P and K levels with both forms of the equations was associated with the upper petioles, but the R^2 values were nearly as high for the equations with upper leaves. The percent K level had a greater and more consistent effect upon yield than did the percent P in both forms of the regression equations in Experiment 1.

The R^2 values in Experiment 2 ranged from 0.92 to 0.94 for both the square-root form and the quadratic form of the equations. There was little difference between the corresponding R^2 values of the two forms of equations used. The R^2 values for equations involving the lower petioles were slightly larger than those for the other plant parts studied, although differences among the R^2 values for all parts were small (tables 2 and 3). None of the partial regression coefficients was significant at the 5-percent level.

In Experiment 3, the R^2 values ranged from 0.22 to 0.27 for the square-root form of the equations and, from 0.20 to 0.37, for the quadratic form. The quadratic form fitted the data involving upper leaves much better (higher R^2 values) than did the square-root form. The best relationship between yield and percent P and percent K levels was with the upper leaves in both forms of the equations.

In Experiment 4 (tables 2 and 3), the R^2 values ranged from 0.13 to 0.35 for the square-root form of the equations and, from 0.12 to 0.35, for the quadratic form. Values of R^2 were almost identical for the two equations when corresponding plant parts were considered. The best relationship between yield and percent P and percent K was with the upper leaves in both forms of the equations. The R^2 values indicate that equations with the upper leaves explained only 3 or 4 percent more variation in yield than did those for lower leaves but were substantially higher than for the other plant parts. In this experiment, no partial regression coefficients were significant in either table 2 or 3.

Based on the values of R^2 for the regression of yield on composition of plant parts in all experiments at growth stage 5, the percent P and percent K levels in the upper leaves gave the best prediction of yield. For estimating soybean yields at this stage by a multiple regression equation, the upper leaves, therefore, would be a suitable plant part to use.

The regression equations, standard errors of the partial regression coefficients and R^2 values for various plant parts sampled in growth stage 7 for each experiment are given in tables 4 and 5 for equations 1 and 2 (square-root and quadratic forms).

The R^2 values in Experiment 1 ranged from 0.76 to 0.84 for the square-root equations (table 4) and, from 0.71 to 0.83, for the quadratic equations (table 5). In comparing the R^2 values, the square-root form of the equations fitted the data somewhat better only for the lower petioles. The best relationship between yield and percent P and percent K levels in both forms appeared associated with the upper petioles, but the R^2 values were nearly as high for the equations with the lower and upper leaves. Several of the partial regression coefficients were significant or highly significant in both forms of the equations.

In Experiment 2 the R^2 values ranged from 0.93 to 0.95 for the square-root form of the equations (table 4) and, from 0.93 to 0.94, for the quadratic form (table 5). There was very little difference between the two forms in fitting the data since the R^2 values were essentially the same. The relationship between yield and percent P and percent K levels in both forms of the equations was only slightly better with the lower leaves than with the other plant parts. None of the partial regression coefficients was significant for this experiment.

The R^2 values in Experiment 3 ranged from 0.21 to 0.45 for the square-root form of the equations (table 4) and, from 0.23 to 0.47, for the quadratic form (table 5) of the regression equations. The quadratic form of the equations fitted the data better than did the square-root form. The best relationship between yield and percent P and percent K levels was with the lower leaves in the quadratic form, although values of R^2 for both sets of petioles in this form of equation and for lower petioles in the square-root equation were almost as high. Many of the partial regression coefficients were significant in both forms of the equation.

The R^2 values in Experiment 4 ranged from 0.22 to 0.42 for the square-root form of the equations and, from 0.19 to 0.38, for the quadratic form. In comparing the R^2 values, the square-root form of the equations fitted the data better for data from leaves, but the quadratic form gave a better fit for petiole data. The best relationship between yield and percent P and percent K levels in both forms was associated with the lower leaves. None of the partial regression coefficients was significant in the square-root equations.

If the highest R^2 values of the regression equations for the various plant parts in growth stage 7 were used as the sole criterion in selecting the most suitable plant part to analyze for estimating yield, the lower leaves would have to be selected. Except for the lower leaves and upper petioles in Experiment 3, very little difference was found between the R^2 values of the square-root and quadratic forms of the equations for corresponding parts of plants in this growth stage.

The regression equations, standard errors of the partial regression coefficients and R^2 values for the various plant parts sampled in growth stage 9 for each experi-

Table 2. Multiple regression statistics b_0, b_i, $s(b_i)$ and R^2 values for the square-root equations of estimated yield ($\hat{Y}$) on the X variates for four plant parts sampled during growth stage 5 from individual experiments.

Exp. No.	Plant part[b]	b_0[c]	b_i[a] and $s(b_i)$ for the variates: $p^{1/2}$	p[d]	$k^{1/2}$	k[d]	$p^{1/2}k^{1/2}$	R^2
1	LL	64.04	−164.96	67.91	−37.08	−2.07	145.84*	0.76**
			192.54	195.28	32.46	10.37	60.75	
	UL	4.64	53.69	−208.74	−20.10	−23.54*	169.74*	0.84**
			198.24	140.95	49.36	11.34	62.44	
	LP	4.10	19.26	−108.68	15.56	−15.50**	82.00*	0.79**
			116.31	115.73	15.71	4.72	35.16	
	UP	41.94	−92.18	−28.90	0.26	−6.18*	63.85	0.85**
			185.13	134.95	21.64	2.85	34.34	
2	LL	30.66	−20.99	18.09	−10.57	7.16	17.88	0.92**
			119.32	140.97	32.78	17.76	49.55	
	UL	57.21	−34.88	−69.50	−44.07	−1.56	99.11	0.92**
			157.23	101.29	65.81	22.07	82.96	
	LP	56.20	−121.90	107.93	−17.48	5.22	35.01	0.94**
			83.20	97.35	18.46	4.65	40.42	
	UP	39.58	−59.23	33.52	−0.89	−0.72	15.31	0.92**
			110.56	123.17	18.92	6.69	41.23	
3	LL	−0.51	51.46	−5.82	16.73	−0.48	−39.19	0.24*
			48.24	53.46	30.73	15.58	35.72	
	UL	18.87	44.18	−19.59	−25.41	10.48	−3.72	0.27*
			87.90	78.00	28.48	10.33	9.77	
	LP	−10.73	58.76	48.01	26.59	−0.71	−65.14	0.22*
			75.96	95.23	14.94	1.94	41.61	
	UP	−60.39	151.82	8.43	38.95	0.28	−77.03*	0.23*
			103.90	86.38	26.44	5.64	35.84	
4	LL	42.34	−96.40	108.27	−4.26	−3.73	30.23	0.32**
			110.04	83.72	67.14	23.89	78.38	
	UL	26.71	−20.34	40.76	−5.20	1.25	2.12	0.35**
			111.31	78.28	49.55	11.12	65.10	
	LP	38.23	−76.23	90.58	−0.77	−1.97	16.64	0.16
			124.39	127.67	22.85	5.59	35.60	
	UP	42.06	−15.35	−28.55	−16.22	−0.89	34.30	0.13
			147.27	95.91	38.90	11.86	67.60	

[a] b_i and $s(b_i)$ values are the upper and lower figures, respectively.
[b] Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles and UP is upper petioles.
[c] Regression constant.
[d] Values used in calculations for the lower and upper leaves were coded by subtracting 0.10 percent P and 0.30 percent K from the observed values in growth stage 5.

Table 3. Multiple regression statistics b_0, b_i, $s(b_i)$ and R^2 value for the quadratic equations of estimated yield ($\hat{Y}$) on the X variates for four plant parts sampled during growth stage 5 from individual experiments.

Exp. No.	Plant part[b]	b_0[c]	b_i[a] and $s(b_i)$ for the variates: p[d]	p^2	k[d]	k^2	pk	R^2
1	LL	−4.70	82.88	−184.95	51.95**	−25.93**	1.16	0.79**
			102.78	235.85	7.81	6.53	12.45	
	UL	−27.81	198.91	−331.77	9.21	−3.19	33.90	0.82**
			159.70	173.05	13.37	1.79	30.47	
	LP	5.21	86.53	−227.35	14.51**	−3.44**	16.29	0.72**
			75.82	174.35	3.36	1.12	13.56	
	UP	6.39	29.00	−133.65	6.83	−1.02**	12.38	0.84**
			132.14	182.62	4.02	0.31	11.13	
2	LL	23.49	33.69	−162.83	−3.48	2.23	36.27	0.92**
			120.86	363.10	11.46	7.24	47.32	
	UL	31.86	−30.85	−30.22	−3.21	−1.86	36.82	0.92**
			147.32	199.61	16.69	5.35	30.94	
	LP	37.43	−152.56	415.04	−3.07	0.76	32.39	0.94**
			106.85	376.13	4.71	1.03	28.63	
	UP	31.94	−81.80	160.46	3.36	−0.25	−2.00	0.92**
			92.89	181.10	3.38	.066	6.01	
3	LL	11.44	57.74	−61.80	5.86	−0.83	−27.02	0.24*
			35.56	94.26	9.35	5.00	26.31	
	UL	−17.51	165.30	−146.05	9.67	0.11	−31.40*	0.37**
			60.06	80.53	7.66	1.51	13.23	
	LP	6.23	93.09	−88.63	7.26	−0.89	−30.45	0.20
			58.04	177.08	5.10	1.04	23.83	
	UP	−7.10	96.15	−17.58	6.64	−0.17	−19.99	0.24*
			63.76	123.64	3.60	0.41	10.13	
4	LL	26.49	−24.98	95.55	−2.60	−0.22	19.07	0.31**
			79.54	158.77	15.76	5.38	41.35	
	UL	23.25	7.04	50.54	−2.90	1.53	−9.30	0.35**
			65.30	112.42	11.76	3.18	26.52	
	LP	19.66	51.28	−95.07	0.85	−0.29	2.90	0.15
			102.64	254.73	4.16	0.71	16.79	
	UP	24.84	12.49	−43.14	−1.02	−0.24	8.36	0.12
			84.46	132.92	5.42	0.99	18.81	

[a] b_i and $s(b_i)$ values are the upper and lower figures, respectively.
[b] Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles and UP is upper petioles.
[c] Regression constant.
[d] See table 2 for coding.

ment are given in tables 6 and 7 for equations 1 and 2. In Experiment 1, the R^2 values ranged from 0.71 to 0.78 for the square-root form of the equations (table 6) and, from 0.66 to 0.80, for the quadratic form (table 7). Except for the lower R^2 values in the quadratic form for the lower petioles, there was little difference between the two forms in fitting the data. A number of partial regression coefficients in each set of equations was significant. The best relationship between yield and percent P and percent K levels in both forms was associated with the upper leaves.

The R^2 values in Experiment 2 ranged from 0.32 to

Table 4. Multiple regression statistics b_0, b_i, $s(b_i)$ and R^2 value for the square-root equations of estimated yield ($\hat{Y}$) on the X variates for four plant parts sampled during growth stage 7 from individual experiments.

Exp. No.	Plant part[b]	b_0[c]	b_i[a] and $s(b_i)$ for the variates: $p^{1/2}$	p[d]	$k^{1/2}$	k	$p^{1/2}k^{1/2}$	R^2
1	LL	—105.56	33.24	—87.20	213.80**	—99.44**	52.35	0.81**
			146.96	125.71	56.48	25.97	89.85	
	UL	—70.20	159.43	—301.08*	71.10	—41.56	118.02	0.81**
			206.39	118.57	111.54	38.68	109.69	
	LP	—4.87	20.55	—113.30	41.49*	—26.72**	78.74*	0.76**
			71.96	76.03	17.39	7.23	35.93	
	UP	—20.95	125.72	—272.56*	18.75	—15.91*	89.63	0.84**
			126.90	107.76	26.11	7.16	44.41	
2	LL	9.69	—85.04	69.43	41.10	—21.83	55.49	0.95**
			72.35	77.88	58.85	25.67	49.03	
	UL	87.24	208.77	83.67	—38.15	—3.05	124.80	0.93**
			166.91	136.10	86.84	32.27	74.65	
	LP	11.90	28.64	—11.81	13.08	—1.49	—12.47	0.93**
			119.74	168.11	17.53	6.43	38.58	
	UP	43.90	—160.07	165.28	15.01	—8.04	29.96	0.93**
			106.46	111.42	26.68	8.29	42.31	
3	LL	—46.33	191.13	—118.76	48.56	—8.99	—75.91	0.40**
			77.72	67.02	39.44	12.78	41.99	
	UL	64.40	—23.96	30.02	—67.36	24.37	8.58	0.21
			163.05	79.22	114.48	31.45	83.52	
	LP	—29.67	124.54*	—61.75	46.65**	—11.02**	—60.71**	0.45**
			54.77	69.55	13.17	3.99	19.72	
	UP	—80.62	140.58	—10.82	88.28*	—18.78*	—73.58	0.34**
			113.88	97.20	34.48	9.03	40.83	
4	LL	—42.34	142.30	—120.57	64.68	—22.88	—23.40	0.42**
			101.41	90.01	49.48	17.61	59.62	
	UL	—92.82	41.72	26.38	181.19	—72.53	—28.24	0.22*
			144.45	120.86	103.95	40.51	101.11	
	LP	8.83	63.55	—55.12	8.23	—2.25	—7.82	0.28*
			58.72	78.04	18.58	6.04	26.28	
	UP	—0.11	86.17	—63.21	11.42	—2.71	—13.90	0.22*
			63.59	78.04	30.31	11.91	35.19	

[a]b_i and $s(b_i)$ values are the upper and lower figures, respectively.
[b]Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles and UP is upper petioles.
[c]Regression constant.
[d]The percent P values used in calculations for all plant parts were coded by subtracting 0.15 percent P from the observed values in the lower and upper leaves and 0.04 percent P from the lower and upper petioles.

Table 5. Multiple regression statistics b_0, b_i, $s(b_i)$ and R^2 values for the quadratic equations of estimated yield ($\hat{Y}$) on the X variates for four plant parts sampled during growth stage 7 from individual experiments.

Exp. No.	Plant part[b]	b_0[c]	b_i[a] and $s(b_i)$ for the variates: p[d]	p^2	k	k^2	pk	R^2
1	LL	—26.44	17.13	—215.34	84.60**	—40.10**	89.49	0.82**
			112.98	242.23	16.72	7.83	68.08	
	UL	—8.48	73.11	—322.88	24.61	—9.41	70.96	0.81**
			145.24	170.55	28.52	7.92	68.14	
	LP	11.82	14.17	—179.67	19.15**	—7.77**	62.20	0.71**
			61.79	153.53	6.01	2.09	34.74	
	UP	7.65	53.45	—327.88*	11.72	—4.16**	54.07*	0.83**
			81.47	147.42	6.56	1.32	26.30	
2	LL	13.29	—33.53	106.61	19.98	—7.64	32.87	0.94**
			81.69	271.08	17.82	6.89	48.85	
	UL	—13.49	22.28	78.07	50.29	—13.59	—36.28	0.93**
			150.41	244.55	30.14	8.23	56.25	
	LP	24.91	—61.78	334.64	4.83	—1.51	19.55	0.93**
			117.84	513.92	4.56	1.42	35.73	
	UP	12.17	65.46	—163.92	8.96	—1.81	—0.90	0.93**
			103.15	286.52	5.21	1.47	11.08	
3	LL	—22.24	231.84**	—337.68*	36.31*	—8.95	—85.02**	0.47**
			62.06	143.39	16.03	4.83	27.38	
	UL	—13.14	129.19	—100.07	14.40	—1.40	—32.84	0.23*
			98.84	112.66	33.06	7.42	33.01	
	LP	3.85	161.16**	—367.41	9.33**	—1.49**	—42.62**	0.44**
			55.21	219.39	2.60	0.54	14.02	
	UP	—27.25	269.13**	—384.65*	19.54**	—2.33*	—49.28**	0.44**
			85.09	179.80	5.42	0.92	17.69	
4	LL	8.15	132.35	—343.39	10.86	—3.51	—1.19	0.38**
			106.13	323.46	13.72	4.15	34.97	
	UL	—8.24	193.11	—365.47	27.84	—8.29	—38.16	0.19
			126.40	320.25	32.40	10.28	68.33	
	LP	17.66	114.43*	—326.21	2.21	—0.41	—7.16	0.30**
			56.21	230.95	4.15	0.99	18.18	
	UP	16.35	139.23**	—337.55	—1.54	0.76	—8.21	0.28*
			50.19	181.18	6.10	1.76	19.08	

[a]b_i and $s(b_i)$ values are the upper and lower figures, respectively.
[b]Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles and UP is upper petioles.
[c]Regression constant.
[d]The percent P values used in calculations for all plant parts were coded by subtracting 0.15 percent from the observed values in the lower and upper leaves and 0.04 percent from the lower and upper petioles.

0.37 for the square-root form of the equations (table 6) and, from 0.29 to 0.37, for the quadratic form (table 7). The square-root form of the equations appeared to fit the data slightly better than did the quadratic form. The best relationship between yield and percent P and percent K levels in both forms was with the lower leaves. Only one partial regression coefficient was significant in the square-root equations, whereas none was significant in the quadratic equations.

In Experiment 3, the R^2 values ranged from 0.29 to 0.41 for the square-root form of the equations (table 6) and, from 0.29 to 0.43, for the quadratic form (table 7).

Table 6. Multiple regression statistics b_0, b_i, $s(b_i)$ and R^2 values for the square-root equations of estimated yield ($\hat{Y}$) on the X variates for four plant parts sampled during growth stage 9 from individual experiments.

Exp. No.	Plant part[b]	b_0[c]	b_i[a] and $s(b_i)$ for the variates: $p^{1/2}$	p^d	$k^{1/2}$	k^d	$p^{1/2}k^{1/2}$	R^2
1	LL	49.54	—69.97*	—73.68**	—38.05	—23.51	196.56**	0.73**
			26.35	20.10	42.20	22.42	69.71	
	UL	—32.37	263.58	—346.33**	—12.96	8.82	48.48	0.78**
			153.54	116.42	53.92	16.17	103.62	
	LP	17.36	—86.95	—12.71	40.73	—43.76**	138.63**	0.71**
			91.20	89.43	28.27	13.12	48.89	
	UP	—83.35	353.07**	—412.75**	80.25*	—26.59*	—38.39	0.75**
			120.70	109.79	37.79	12.91	56.89	
2	LL	38.87	—45.48	—2.94	—23.56	—2.71	89.92	0.37**
			111.05	121.90	35.74	13.14	57.29	
	UL	54.82	—54.83	—26.27	—72.34	6.84	158.58*	0.35**
			117.32	108.55	59.11	26.14	71.32	
	LP	32.01	—55.18	66.56	—2.57	—2.30	36.32	0.33**
			98.68	136.39	17.11	6.35	42.02	
	UP	5.72	71.91	—137.66	7.06	—13.45	59.45	0.32**
			116.01	139.58	32.55	12.59	55.62	
3	LL	—50.61	161.81	—71.75	61.23	—12.46	—81.27	0.33**
			106.50	87.00	40.70	11.82	48.03	
	UL	—19.26	164.21	—99.15	—13.85	18.60	—47.06	0.29**
			129.91	90.31	66.22	20.35	67.28	
	LP	—19.07	156.71**	—136.47*	16.72	—1.64	—39.27	0.41**
			56.18	66.07	13.72	3.91	23.37	
	UP	—33.56	173.56	—115.12	32.29	—3.12	—66.29*	0.30**
			90.59	118.91	30.39	10.55	31.84	
4	LL	—2.00	—43.76	180.29	61.67	—20.65	—56.91	0.32**
			109.21	127.57	40.34	13.77	52.32	
	UL	2.54	67.97	13.42	17.01	2.54	—62.81	0.18
			190.99	171.67	81.63	23.54	145.98	
	LP	—20.19	224.43**	—245.40*	14.20	—0.75	—38.61	0.30**
			67.01	91.07	19.91	6.56	30.15	
	UP	1.75	168.62	—154.23	—10.84	11.50	—43.92	0.22*
			134.37	142.64	43.28	12.16	86.25	

[a] b_i and $s(b_i)$ values are the upper and lower figures, respectively.
[b] Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles and UP is upper petioles.
[c] Regression constant.
[d] Values used in calculations for the lower and upper leaves were coded by subtracting 0.05 percent P and 0.30 percent K from the observed values in growth stage 9.

Table 7. Multiple regression statistics b_0, b_i, $s(b_i)$ and R^2 values for the quadratic equations of estimated yield ($\hat{Y}$) on the X variates for four plant parts sampled during growth stage 9 from individual experiments.

Exp. No.	Plant part[b]	b_0[c]	b_i[a] and $s(b_i)$ for the variates: p^d	p^2	k^d	k^2	pk	R^2
1	LL	13.46	53.69	—258.47	10.86	—24.98	141.64*	0.74**
			105.72	183.77	18.29	14.14	61.86	
	UL	6.88	140.76	—389.05**	7.98	2.34	20.83	0.80**
			89.31	135.39	19.68	3.86	77.87	
	LP	20.40	—71.18	11.07	19.62	—15.50**	123.78**	0.66**
			65.39	136.83	9.96	4.83	44.98	
	UP	10.17	65.10	—316.87*	16.94	—7.94*	58.25	0.75**
			81.04	145.81	12.85	3.61	58.31	
2	LL	23.97	8.18	—79.16	—0.07	—4.66	70.00	0.37**
			95.33	256.73	13.86	6.78	54.67	
	UL	29.43	—33.11	14.40	—12.37	—5.63	117.45	0.33**
			97.45	190.91	22.89	13.50	65.51	
	LP	23.28	—4.81	69.59	4.37	—2.27	24.31	0.33**
			150.81	637.15	4.62	2.00	34.83	
	UP	31.15	—104.53	282.97	2.90	—4.84	63.89	0.29**
			134.89	415.84	10.38	4.56	49.13	
3	LL	—17.35	224.35	—313.32*	25.55*	—4.95	—77.19**	0.43**
			68.03	124.15	9.48	2.66	25.90	
	UL	—3.27	132.55	—164.57	9.00	—0.63	—32.97	0.29**
			67.78	119.84	13.84	4.76	23.28	
	LP	6.07	144.32*	—316.14	5.58	—0.72	—29.59	0.37**
			58.73	186.73	3.15	0.70	16.55	
	UP	—1.24	172.83*	—305.23	11.48	—1.61	—50.75*	0.32**
			82.99	249.54	6.04	1.83	19.25	
4	LL	—1.74	282.02**	—652.76*	10.00	—1.41	—47.28	0.37**
			101.95	309.00	9.59	3.46	31.23	
	UL	—14.24	448.93	1,152.30	19.58	—2.47	—119.15	0.24*
			222.38	611.49	29.91	10.99	126.74	
	LP	6.80	311.29**	—1,068.05**	2.88	—0.10	—24.80	0.32**
			79.05	319.91	4.38	1.17	19.00	
	UP	15.02	270.51	—1.078.85	—9.29	3.52	9.93	0.21
			187.56	849.46	6.64	2.93	21.44	

[a] b_i and $s(b_i)$ values are the upper and lower figures, respectively.
[b] Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles and UP is upper petioles.
[c] Regression constant.
[d] Values used in calculations for the lower and upper leaves were coded by subtracting 0.05 percent P and 0.30 percent K from the observed values in growth stage 9.

The quadratic form of the equations fitted the data a little better than did the square-root form for all cases except lower petioles. The best relationships between yield and percent P and percent K levels were with the lower petioles in the square-root form and, with the lower leaves, in the quadratic form. This is indicated by the significance of the partial regression coefficients in tables 6 and 7.

The values of R^2 in Experiment 4 ranged from 0.18 to 0.32 for the square-root form of the equations (table 6) and, from 0.21 to 0.37, for the quadratic form (table 7). The quadratic form of the equations fitted the data better than did the square-root form. The best relationship between yield and percent P and percent K levels in both forms was with the lower leaves. Only one partial regression coefficient was highly significant in the square-root equations while four were significant in the quadratic equations.

When the R^2 values obtained by both forms of the regression equations for the various plant parts sampled in growth stage 9 were compared, the percent P and percent K levels in the lower leaves were the best predictors of yield in three of the four experiments studied.

Before combining experiments and calculating new regression equations, it was decided to determine if the R^2 values were sufficiently low for the various plant parts in any one growth stage to justify the deletion of at least one growth stage from further statistical analyses. To determine which two growth stages should be retained, the R^2 values in all growth stages for each plant part in each experiment were compared (table 8).

In Experiment 1, the growth stages at which there was the best relationship between yields and percent P and percent K were stage 7 for the lower leaves and, stage 5, for the rest of the plant parts. Although most of the highest R^2 values were found in growth stage 5, the R^2 values for the corresponding plant parts were nearly as high in growth stage 7 but not in growth stage 9.

In Experiment 2, the growth stages at which the highest R^2 values for equations were obtained were stage 5 for the lower petioles and, stage 7, for the rest of the plant parts. The R^2 values in growth stages 5 and 7 were nearly the same and substantially higher than those in growth stage 9.

In Experiment 3, the R^2 values were highest for equations in growth stage 5 for the upper leaves and, in growth stage 7, for the rest of the plant parts. In some cases, the R^2 values in growth stage 9 were nearly as high or were comparable to the corresponding R^2 values in growth stage 5 or 7.

In Experiment 4, the R^2 values for equations were highest in growth stage 7 for the lower leaves and upper petioles; in growth stage 5 for the upper leaves; and in growth stage 9 for the lower petioles. There was a large amount of variability in the R^2 values among the three growth stages in this experiment.

The growth stages with the highest R^2 values for the various plant parts in experiments 1, 2, 3 and 4, respectively, were: stages 7, 7, 7 and 7 for the lower leaves; stages 5, 7, 5 and 5 for the upper leaves; stages 5, 5, 7 and 9 for the lower petioles; and stages 5, 7, 7 and 7 for the upper petioles. Since the highest R^2 values for each plant part occurred most often in growth stages 5 and 7, data from growth stage 9 were omitted in the combined analyses. The variability among the R^2 values for the various plant parts and for the square-root and quadratic forms of the equations precludes any conclusion that one plant part or one form of the regression equation could be expected to show a consistently better relationship between chemical composition and yield on the basis of these data.

Estimated maximum yields and associated percent P and percent K values

One of the uses of yield-plant composition relationships is to calculate the estimated maximum yield and the percentage of the nutrient or nutrients associated with this maximum yield (4). The maximum yield is determined by the critical point of the regression sur-

Table 8. Coefficients of multiple determination, R^2, for regressions of soybean yield on percent P and percent K in plant parts expressed by the square-root and quadratic forms of equations for four plant parts sampled at three growth stages from individual experiments.

Exp. No.	Plant part[b]	Values of R^2					
		Growth stage 5[a]		Growth stage 7		Growth stage 9	
		Square root	Quadratic	Square root	Quadratic	Square root	Quadratic
1	LL	0.76[c]	0.79	0.81	0.82	0.73	0.74
	UL	0.84	0.82	0.81	0.81	0.78	0.80
	LP	0.79	0.72	0.76	0.71	0.71	0.66
	UP	0.85	0.84	0.84	0.83	0.75	0.75
2	LL	0.92	0.92	0.95	0.94	0.37	0.37
	UL	0.92	0.92	0.93	0.93	0.35	0.33
	LP	0.94	0.94	0.93	0.93	0.33	0.33
	UP	0.92	0.92	0.93	0.93	0.32	0.29
3	LL	0.24	0.24	0.40	0.47	0.33	0.43
	UL	0.27	0.37	0.21	0.23	0.29	0.29
	LP	0.22	0.20	0.45	0.44	0.41	0.37
	UP	0.23	0.24	0.34	0.44	0.30	0.32
4	LL	0.32	0.31	0.42	0.38	0.32	0.37
	UL	0.35	0.35	0.22	0.19	0.18	0.24
	LP	0.16	0.15	0.28	0.30	0.30	0.32
	UP	0.13	0.12	0.22	0.28	0.22	0.21

[a]Sampling dates for growth stages 5, 7 and 9 were approximately July 30, Aug. 19 and Sept. 3, 1958, respectively.
[b]Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles and UP is upper petioles.
[c]The 0.05 and 0.01 significance probability levels for R^2 with 5 variables and 38 degrees of freedom are approximately 0.22 and 0.29, respectively.

faces, generated by equations 1 and 2, exhibiting a unique maximum value. The percentages of P and K associated with this critical point are calculated from these equations by the simultaneous solution of the first partial derivatives of yield with respect to p and k equated to zero. These calculated percentages are then substituted into the original equation, and the Y at the critical point is determined.[10] For functions in the form of equation 2, these partial derivatives are:

$$\frac{\partial Y}{\partial p} = b_1 + 2b_2p + b_5k \qquad (3)$$

and

$$\frac{\partial K}{\partial Y} = b_3 + 2b_4k + b_5p. \qquad (4)$$

The estimated yields and percentages of P and K at the critical point for all growth stages, plant parts and experiments for equations 3 and 4 are given in table 9.

The hypothesis that the fitted empirical equations provide reliable estimates of maximum yields is logically restricted to those solutions within the range of observations. When the 48 critical points with respect to percent P (table 9) were compared with the observed values, 13 were below and 10 were above the range of observed values for percent P. Eleven critical points with respect to percent K were below, and 13 were above the range of observed values for percent K. The number of critical points with respect to percent P and percent K which were below or above the range of observed values varied among the plant parts and experiments. The estimated yields, associated with the critical points with respect to percent P and percent K, that were outside the range of observations are extrapolated values and are less reliable estimates than those that were interpolated. Reliability of the yields depends on the degree of extrapolation and on how well the empirical equation describes the "actual biological" relationship between yield and plant composition.

The estimated yields obtained by substituting the critical points with respect to percent P and percent K into the original regression equations were as follows: 32 maximum yields, 2 minimum yields and 14 yields at a minimax or a saddle point. Because of the wide range in observed values for percent P and percent K in the plant parts in Experiment 1, 10 of the 32 estimated maximum yields were obtained from the regression equations associated with this experiment. The quadratic equations for equations based on data from growth stage 7 estimated a few more maximum yields than did the equations in growth stage 5 or 9.

There was no good explanation for the failure to obtain maximum yield values from the quadratic equations. There are, however, a number of factors which may be partially responsible for some of the erratic yield estimates shown in table 9. These factors are: (a) There was a limited range of yields in most of the experiments. (b) The initial levels of percent P and percent K in the various plant parts were relatively high in most experiments, and the fertilizer rates had little influence on the ranges of percent P and percent K, particularly in Experiment 3. (c) The number of observations over the entire yield response surface was limited. (d) The errors were high in most of the experiments as indicated by the standard errors associated with the regression coefficients.

It appears that methods of reducing the standard errors in soybean studies of this type need to be investigated. In many of the equations examined, the b_i values for most of the variates were not significant (5- or 1-percent levels), and some were even smaller than their respective standard errors; therefore, the confidence intervals of the b_i values included both positive and negative values. In most of the regression equations showing a minimum or a minimax, the sign of the coefficient of one of the squared terms was positive. Negative coefficients for the squared terms within the confidence inter-

[10]As the critical points of surfaces such as these may be minimum, maximum or minimax points, they must be tested by standard calculus methods.

Table 9. Estimated soybean yields ($\hat{Y}$) calculated from the quadratic form of the regression equations at the point where the first partial derivative of yield with respect to percent P and percent K in the plant parts equals zero.

Exp. No.	Growth stage	Plant part[a]	Critical point values for % P[b]	% K[b]	$\hat{Y}$
1	5	LL	0.33	1.31	30.88[c]
		UL	0.61	4.47	42.43[c]
		LP	0.23	2.67	34.70[c]
		UP	0.37	5.59	30.78[c]
2	5	LL	0.20	0.27	25.23[d]
		UL	0.31	1.48	27.54[c]
		LP	0.61	—11.00	7.58[e]
		UP	0.29	5.63	29.52[d]
3	5	LL	0.13	1.87	19.50[c]
		UL	0.42	0.98	20.37[d]
		LP	0.09	2.53	19.64[c]
		UP	0.26	4.35	19.80[c]
4	5	LL	0.24	0.25	24.86[d]
		UL	0.12	1.32	21.85[e]
		LP	0.32	3.01	29.03[c]
		UP	0.09	—0.59	25.69[c]
1	7	LL	0.49	1.43	36.96[c]
		UL	0.59	2.96	43.98[c]
		LP	0.86	4.53	61.01[c]
		UP	0.47	4.18	43.46[c]
2	7	LL	0.12	1.24	26.18[d]
		UL	0.37	1.56	28.10[d]
		LP	0.08	1.85	28.02[d]
		UP	0.18	2.12	26.31[c]
3	7	LL	0.37	0.99	21.07[c]
		UL	0.36	2.63	19.60[c]
		LP	0.26	—0.03	21.46[c]
		UP	0.29	1.53	22.68[c]
4	7	LL	0.31	1.52	28.60[c]
		UL	0.35	1.22	28.09[c]
		LP	0.19	1.41	28.26[c]
		UP	0.22	2.00	27.47[d]
1	9	LL	0.78	2.59	45.59[c]
		UL	0.17	—1.94	6.48[d]
		LP	0.01	0.58	27.57[d]
		UP	0.30	2.18	38.40[c]
2	9	LL	0.03	0.14	23.89[c]
		UL	0.18	0.55	25.74[d]
		LP	0.07	0.59	26.07[d]
		UP	0.09	0.87	27.90[d]
3	9	LL	1.07	—5.11	27.97[c]
		UL	0.24	2.41	18.91[c]
		LP	1.11	—18.91	33.09[e]
		UP	0.04	2.88	13.74[c]
4	9	LL	0.27	0.11	28.78[c]
		UL	0.09	3.26	24.11[c]
		LP	0.05	8.43	26.36[c]
		UP	0.13	1.14	27.41[d]

[a]Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles, and UP is upper petioles.
[b]Percent P and percent K are decoded values.
[c]Estimated maximum yield and percent P and percent K associated with this maximum yield.
[d]Estimated yield at the minimax or saddle point and associated percent P and percent K at this point.
[e]Critical point is a minimum, and these are the yields, percent P and percent K at this critical point.

val give estimated maximum yields, provided that the coefficient for the interaction term is not too large relative to the coefficients of the negative squared terms.

Regression analyses of combined experiments

Previous graphs and regression analyses showed that the results from Experiment 3 differed markedly from the other three experiments. Whether this was due to variety, date of planting or some other site-controlled factor is not known. Nevertheless, the combined analysis, including all four experiments, was conducted and is presented in tables A-3 and A-4 in the appendix. As expected, the combined analysis yielded multiple regression equations with relatively low R^2 values. Experiment 3 was omitted from further analyses because the data collected did not permit an evaluation of the reasons for the difference, and, therefore, an adjustment of the regression equation was not possible.

The regression statistics of the square-root and quadratic equations are given in tables 10 and 11, respectively, for the combined data from experiments 1, 2 and 4. By omitting Experiment 3 from the combined regression analyses, the R^2 values for the regression equations for all plant parts were increased greatly (compare with tables A-3 and A-4). The R^2 values for the various plant parts ranged from 0.65 to 0.74 and from 0.55 to 0.73 for the square-root and quadratic equations, respectively. The R^2 values were substantially higher in growth stage 7 than in growth stage 5. Since the yields were estimated with less precision (smaller R^2 values) for the various plant parts in growth stage 5 than in growth stage 7, the yield-chemical composition data from growth stage 5 were omitted in subsequent investigations.

In the square-root equations (table 10), the partial regression coefficients of the following variates, in their respective plant parts sampled in growth stage 7, were found significant or highly significant: p in the upper leaves and in the upper and lower petioles; $k^{1/2}$ in the lower leaves and lower petioles; and k and $p^{1/2} \times k^{1/2}$ in all plant parts. In the quadratic equations (table 11), the partial regression coefficients of the following variates, in their respective plant parts sampled in growth

Table 10. Multiple regression statistics b_0, b_i, $s(b_i)$ and R^2 values for the square-root form of estimated yield ($\hat{Y}$) on the X variates for four plant parts sampled during two growth stages from experiments 1, 2 and 4.

Growth stage	Plant part[b]	b_0[c]	b_i[a] and $s(b_i)$ for the variates: $p^{1/2}$	p[d]	$k^{1/2}$	k[d]	$p^{1/2}k^{1/2}$	R^2
5	LL	56.86	—157.57*	53.34	—1.60	—27.34**	133.25**	0.67**
			62.21	63.71	14.96	4.92	24.88	
	UL	49.40	—117.27	—93.91	12.18	—43.32**	179.21**	0.70**
			90.79	69.14	21.63	5.37	28.83	
	LP	39.14	—102.66	17.38	8.19	—13.12**	74.83**	0.65**
			61.78	63.80	8.51	2.25	16.45	
	UP	53.42	—83.67	—98.28	—7.85	—11.53**	102.10**	0.66**
			82.96	69.36	11.01	2.19	17.10	
7	LL	—48.12	—82.96	—8.10	146.88**	—79.40**	107.31**	0.74**
			62.55	55.92	25.34	8.40	32.80	
	UL	—13.45	—75.45	—122.02*	78.99	—59.65**	170.27**	0.73**
			83.32	50.52	53.90	17.79	45.01	
	LP	—2.83	36.59	—110.11*	36.72**	—20.35**	44.05**	0.67**
			41.58	45.56	8.26	2.73	16.14	
	UP	2.08	44.25	—195.17**	14.99	—18.03**	98.04**	0.73**
			59.59	52.97	14.44	4.26	21.22	

[a] b_i and $s(b_i)$ values are the upper and lower figures, respectively.
[b] Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles and UP is upper petioles.
[c] Regression constant.
[d] Values used in calculations for the lower and upper leaves were coded by subtracting 0.10 percent P and 0.30 percent K from the observed values in growth stage 5 and by subtracting 0.15 percent P from the observed values for lower and upper leaves and 0.04 percent P from the lower and upper petioles in growth stage 7.

Table 11. Multiple regression statistics b_0, b_i, $s(b_i)$ and R^2 values for the quadratic form of estimated yield ($\hat{Y}$) on the X variates for four plant parts sampled during two growth stages from experiments 1, 2 and 4.

Growth stage	Plant part[b]	b_0[c]	b_i[a] and $s(b_i)$ for the variates: p[d]	p^2	k[d]	k^2	pk	R^2
5	LL	9.13	11.99	—65.62	39.01**	—20.73**	8.19	0.61**
			41.52	105.42	3.93	2.42	9.98	
	UL	21.53	—6.72	—142.75	4.52	—7.07**	64.84**	0.65**
			61.46	78.39	4.77	1.02	10.65	
	LP	20.21	—11.83	—90.84	8.43**	—2.55**	19.69*	0.55**
			41.48	100.27	1.87	0.47	8.24	
	UP	15.51	36.16	—199.07*	3.55	—1.15**	20.16**	0.59**
			55.76	89.69	2.03	0.24	5.10	
7	LL	—4.21	—28.18	—72.47	46.45**	—20.08**	72.76**	0.69**
			59.72	134.66	7.81	2.25	27.22	
	UL	1.46	—27.04	—165.37*	32.92*	—15.55**	84.03**	0.73**
			60.75	73.30	15.15	4.19	28.75	
	LP	15.13	29.15	—226.88*	12.38**	—4.66**	36.58**	0.60**
			35.82	95.18	2.55	0.68	13.74	
	UP	4.57	124.31**	—459.82**	11.09**	—3.23**	24.00*	0.72**
			38.19	77.63	3.19	0.74	10.34	

[a] b_i and $s(b_i)$ values are the upper and lower figures, respectively.
[b] Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles and UP is upper petioles.
[c] Regression constant.
[d] Values used in calculations for the lower and upper leaves were coded by subtracting 0.10 percent P and 0.30 percent K from the observed values in growth stage 5 and by subtracting 0.15 percent P from the observed values for lower and upper leaves and 0.04 percent P from the lower and upper petioles in growth stage 7.

stage 7, were significant or highly significant: p in the upper petioles; p^2 in the upper leaves and in the lower and upper petioles; and k, k^2 and pk in all plant parts.

Values of R^2 obtained from the multiple regression equations for growth stage 7 given in tables 10 and 11 were of similar magnitude except for the R^2 values for equations including the lower petioles. The R^2 values for the square-root equations were higher than those for the quadratic equations for the lower leaves and lower petioles, but there was little or no difference between the two forms of the equations for the upper leaves and upper petioles.

Estimated yields

As previously mentioned, one important purpose in determining yield-plant composition relationships is to calculate the critical points of the equations with respect to nutrient percentages and the estimated yields associated with these critical points. It is desirable that the estimated yield at the critical point be a maximum rather than a minimum or minimax (saddle point) value for these yield-plant composition relationships. Therefore, the square-root and quadratic equations of yield on percent P and percent K in the four plant parts sampled in growth stage 7 were used to study their relative suitability in estimating these points for the three combined experiments. The estimated values for the two forms of the regression equations for the various plant parts are given in table 12.

Table 12. Estimated soybean yields ($\hat{Y}$), percent P and percent K for combined data from experiments 1, 2 and 4 determined from the regression equations at the point where the first partial derivative of yield with respect to percent P and percent K equals zero (growth stage 7).

Form of equations	Plant part	Critical point values			R^2
		% P[a]	% K	$\hat{Y}$[b]	
Square root	Lower leaves	0.23	0.53	14.73	0.74**
	Upper leaves	1,418.35	2,961.35	29.32	0.73**
	Lower petioles	0.24	1.91	30.58	0.67**
	Upper petioles	0.51	5.22	34.41	0.73**
Quadratic	Lower leaves	4.41	8.88	141.97	0.69**
	Upper leaves	0.75	2.67	37.38	0.73**
	Lower petioles	0.29	2.31	33.10	0.60**
	Upper petioles	0.24	2.46	30.57	0.72**

[a]Decoded values.
[b]Yield in bushels per acre.

To be useful in studying many aspects of nutrient balance, the yield-plant composition relationships, as expressed by regression equations, should predict reasonable estimated maximum yield values when the critical points with respect to nutrient percentages are within the range of observations. When the critical points with respect to percent P and percent K for both forms of the equations were compared with their respective observed values, only those derived from the lower leaves and lower petioles (square-root equations) and the lower and upper petioles (quadratic equations) were within the range of observed values. The critical points for equations derived from the other plant parts were above the range of observed values; thus, these extrapolated values are less reliable estimates than the interpolated values for the other plant parts.

The estimated yields obtained by substituting the critical points with respect to percent P and percent K into the original regression equations were 7 maximum yields and 1 yield (square-root equation for the lower leaves) at a minimax. The most reliable estimates of maximum yields in this study are associated with the lower petioles in the square-root equations and with the lower and upper petioles in the quadratic equations. Of these three, the equation involving the upper petioles has the highest value of R^2 (0.72). (Although the quadratic equation relating yield to composition of the upper leaves has an R^2 value of 0.73, the critical point is at an extrapolated distance beyond the data.)

Yield isoquants

The relationships between percent P and percent K in the upper leaves sampled in growth stage 7 at different estimated yield levels (fig. 3) were determined from the isoquant equation calculated from the quadratic form of the regression equation for the three combined experiments. These relationships are similar to those found between percent P and percent N in corn leaves by Dumenil (4). This figure may be considered analogous in many respects to the "contour maps" of the fertilizer-crop response relationships presented by Heady et al. (9). The isoquants (lines connecting points of equal yields) for yield levels below the maximum show that the same yield can occur over varying levels of percent P and percent K in the upper leaves. The isoquant

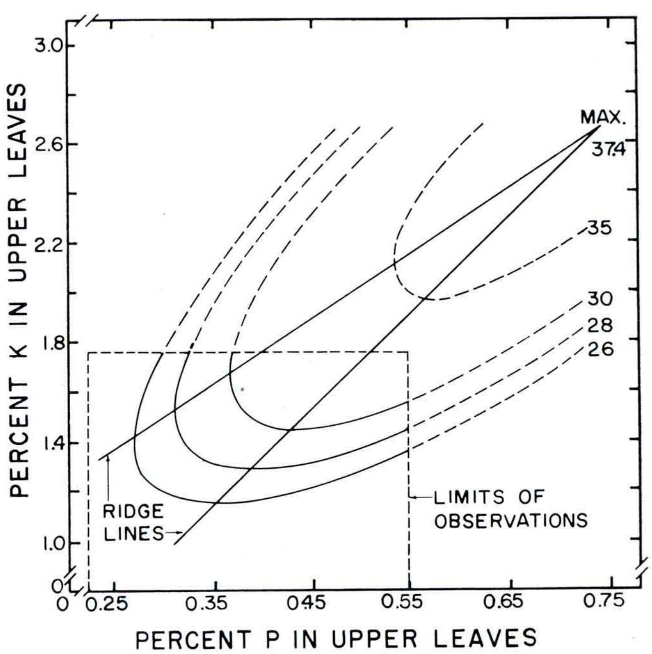

Fig. 3. Yield isoquants calculated from the quadratic equation, relating yield to percent P and percent K in the upper leaves in growth stage 7 from the combined data of experiments 1, 2 and 4 at specified estimated yield levels. (Numbers at ends of isoquants are bushels of grain per acre.)

at the estimated maximum yield reduces to a point. This is interpreted to mean that, only at this maximum yield level, do the associated levels of percent P and percent K at a given yield become single-valued. Along any isoquant, the rate of substitution of percent P for percent K occurs at a diminishing rate within the ridge lines which connect points on the isoquants having zero or infinite rates of substitution. When a positive interaction occurs between the two nutrients, the ridge lines form an angle of less than 90 degrees. Conversely, when a negative interaction occurs, the ridge lines form an angle of greater than 90 degrees.

Since the area between the ridge lines is considered the "rational" area in fertilizer use, it will also be referred to here as the "rational" area for the relationship between yield and percent P and percent K in the upper soybean leaves. However, much of the figure is an extrapolation because it occurs outside the upper limits of the observations for percent P and percent K as indicated by the straight dashed lines (lower left corner of fig. 4).

This concept of relationship of yield to percent P and percent K appears adaptable to the economic approach to yield response functions of fertilization described by Heady et al. (9), Brown et al. (3) and Pesek et al. (12). In general, they expressed nutrient combinations in terms of their substitution or replacement rates, since similar yields could be obtained with different nutrient combinations. However, it is unlikely that the nutrients actually substitute for each other in the numerous chemical and biological processes within the plant.

In this study, concentrations of P and K in the upper leaves at the 26- to 30-bushel yield levels were within the "rational" area and well within the limits of the observed values for percent P and percent K. As the observed values ranged from 0.23 to 0.55 for percent P and, from 0.69 to 1.77, for percent K, any estimated yield isoquants resulting from calculated percent P and percent K outside these observed ranges are less reliable estimates and are shown by dashed yield isoquant lines. It is evident that the range of data for an adequate representation of the yield-chemical composition relationships when using this plant part in growth stage 7 was not wide enough in this case.

The relationships between percent P and percent K, in the upper petioles sampled in growth stage 7, at different estimated yield levels are shown in fig. 4. The previous general discussion about the yield-chemical composition relationships in the upper leaves is also applicable to these relationships in the upper petioles.

Percentages of P and K, especially of P, in the upper petioles at the specified yield levels varied somewhat from those percentages in the upper leaves for comparable yield levels. Practically all estimated values for percent P and percent K within the "rational" area for the upper petioles fell within the range of observed values (fig. 4). The range of observed percent P and percent K values for the upper petioles was such that the maximum yield

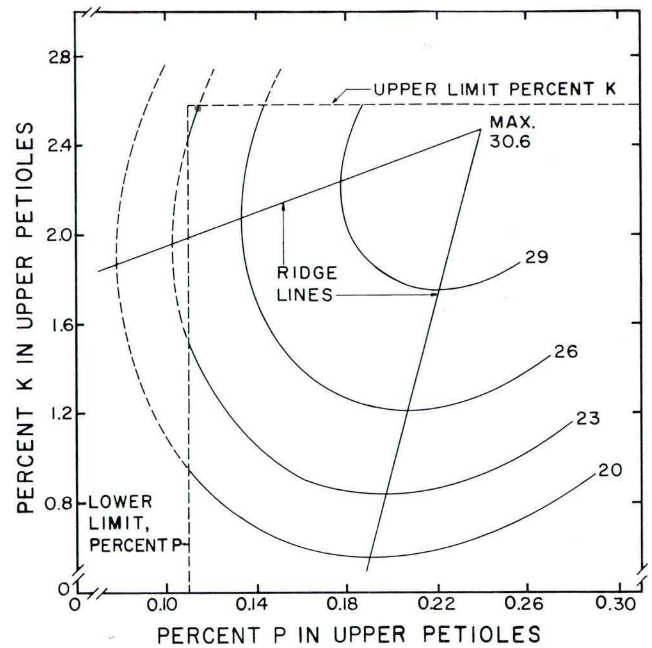

Fig. 4. Yield isoquants calculated from the quadratic equation, relating yield to percent P and percent K in the upper petioles in growth stage 7 from the combined data of experiments 1, 2 and 4 at specified estimated yield levels. (Numbers at ends of isoquants are bushels of grain per acre.)

estimate occurred within the range of levels of percent P and percent K actually observed.

SIGNIFICANCE OF THE RESULTS

The graphs of the equations for yield as a function of the composition of upper leaves or upper petioles in figs. 3 and 4, respectively, are similar in that they both illustrate positive interaction between percent P and percent K. These graphs also exhibit diminishing returns to increased percent P and percent K and indicate that given yield levels below the maximum may be attained with a jointly varying range of composition of petioles or leaves. Coefficients of determination of these two equations also are almost identical (table 12). The maximum predicted yields, however, are different, the lower being predicted on the basis of petiole composition. Perhaps this means that relatively high yields are more likely to be predicted by leaf composition, because this organ seems to have a greater flexibility in accumulating and holding nutrients for subsequent development of the grain.

The upper leaves would probably be the most practical, and perhaps even the most logical, part to use for future studies of this nature because: (a) Plant leaves play a major role in nutrient storage, while petioles function largely as conducting tissues, and the petiole nutrient content may be more sensitive to temporary environmental changes than is the nutrient content of the leaves. (b) Upper leaves are more convenient to sample in the field than are the other plant parts. Although it is less convenient to sample upper petioles than upper leaves in the field, the R^2 values, maximum yields and the associated critical points of the regression equations with respect to percent P and percent K for the upper petioles (table 12) indicate that the upper petioles also may be a suitable plant part to use in this type of study.

Of the square-root and quadratic forms of the regression equations used in this work, the quadratic equations may be preferred, since they are relatively easy to use, and there was little difference found between the precision of the two forms for estimating yields in this investigation.

It appears that a much wider range of soybean yields and nutrient percentages within the various plant parts is needed before more reliable regression equations can be calculated for use in estimating maximum soybean yields. A greater number of points on the soybean yield response surface would have been beneficial in this investigation.

Although future studies may show that different nutrients stored in other plant parts at some specific growth stage may provide a better relationship between yields and the nutrient composition of the plant, considerable precision was attained by using the content of the two nutrients in this study. It was possible to account for 73 percent of the variation in soybean yields by the multiple regression equations containing only percent P and percent K as independent variates. Including other variables such as variety, lodging, climatic factors, etc., also should increase the precision in estimating yields.

LITERATURE CITED

1. Anderson, R. L. and T. A. Bancroft. Statistical theory in research. McGraw-Hill Book Pub. Co., Inc., New York. 1952.

2. Bennett, W. F., G. Stanford and L. C. Dumenil. Nitrogen, phosphorus and potassium content of the corn leaf and grain as related to nitrogen fertilization and yield. Soil Sci. Soc. Amer. Proc. 17:252-258. 1953.

3. Brown, W. G., E. O. Heady, J. T. Pesek and J. A. Stritzel. Production functions, isoquants, isoclines and economic optima in corn fertilization for experiments with two and three nutrients. Iowa Agr. Exp. Sta. Res. Bul. 441. 1956.

4. Dumenil, Lloyd. Nitrogen and phosphorus composition of corn leaves and corn yields in relation to critical levels and nutrient balance. Soil Sci. Soc. Amer. Proc. 25:295-298. 1961.

5. Ezekiel, M. Method of correlation analysis. 2nd ed. John Wiley and Sons, Inc., New York. 1941.

6. Hammond, L. C., C. A. Black and A. G. Norman. Nutrient uptake by soybeans on two Iowa soils. Iowa Agr. Exp. Sta. Res. Bul. 384. 1951.

7. Hanway, J. J. Corn growth and composition in relation to soil fertility: II. Uptake of N, P and K and their distribution in different plant parts during the growing season. Agron. Jour. 54:217-222. 1962.

8. Hanway, J. J. and H. Heidel. Soil analysis methods as used in the Iowa State College Soil Testing Laboratory. Agron. 57 (Rev.) Iowa Agr. Exp. Sta., Ames, Iowa. 1952. (mimeo).

9. Heady, E. O., J. T. Pesek and W. G. Brown. Crop response surfaces and economic optima in fertilizer use. Iowa Agr. Exp. Sta. Res. Bul. 424. 1955.

10. Kalton, R. R., C. R. Weber and J. C. Eldredge. The effect of injury simulating hail damage to soybeans. Iowa Agr. Exp. Sta. Res. Bul. 359. 1949.

11. Nelson, W. L., L. Burkhart and W. E. Colwell. Fruit development, seed quality, chemical composition and yield of soybeans as affected by potassium and magnesium. Soil Sci. Soc. Amer. Proc. 10:224-229. 1945.

12. Pesek, J. T., E. O. Heady, J. P. Doll and R. P. Nicholson. Production surfaces and economic optima for corn yields with respect to stand and nitrogen levels. Iowa Agr. and Home Econ. Exp. Sta. Res. Bul. 472. 1959.

13. Sayre, J. D. Mineral accumulation in corn. Plant Physiol. 23:267-281. 1948.

14. Snedecor, G. W. Statistical methods. 5th ed. Iowa State University Press, Ames. 1956.

15. Tremblay, F. T. and K. E. Baur. Plant analysis—a method of determining the phosphorus requirements of peas. Agron. Jour. 44:614-618. 1952.

16. Tyner, E. H. The relation of corn yields to leaf nitrogen, phosphorus and potassium content. Soil Sci. Soc. Amer. Proc. 11:317-323. 1946.

17. Tyner, E. H. and J. R. Webb. The relation of corn yields to nutrient balance as revealed by leaf analysis. Jour. Amer. Soc. Agron. 38:173-185. 1946.

18. Webb, J. R. and J. T. Pesek. An evaluation of phosphorus fertilizers varying in water solubility: I. Hill applications for corn. Soil Sci. Soc. Amer. Proc. 22:533-538. 1958.

APPENDIX

Table A-1. Soil test results for all experiments.

Exp.	Soil depth	Soil test results pH	P^a	K^b Moist	Dry
1	0 to 6	6.6	3.0	—	58
	6 to 12	6.8	0.5	20	—
	12 to 18	7.2	0.5	14	—
	18 to 24	7.4	0.5	14	—
2	0 to 6	5.8	5.3	63	142
	6 to 12	6.3	1.4	40	—
	12 to 18	6.6	1.2	34	—
	18 to 24	7.0	0.5	30	—
3	0 to 6	5.3	5.5	62	108
	6 to 12	5.3	1.0	39	—
	12 to 18	5.6	0.8	21	—
	18 to 24	6.0	0.5	19	—
4	0 to 6	6.4	5.5	125	224
	6 to 12	6.0	0.5	50	—
	12 to 18	6.1	0.5	38	—
	18 to 24	6.6	0.5	34	—

[a]Soil test results are given in pounds per acre as determined by Iowa State University Soil Testing Laboratory.
[b]Soil tests were run on field-moist and air-dry samples.

Table A-2. Lodging score for soybean experiments on per-plot basis at various rates of P and K fertilizers.

Fertilizer treatment[a] P	K	Exp. 1 I[b]	II[b]	Exp. 2 I	II	Exp. 3 I	II	Exp. 4 I	II
0	0	1.0[c]	1.8	1.4	2.0	1.0	1.0	1.1	1.5
0	53	4.0	4.7	1.5	1.5	1.1	1.1	1.5	1.0
1.8	3.3	1.1	3.2	2.0	1.5	1.1	1.0	2.8	1.2
1.8	163	4.8	4.5	1.5	1.4	1.2	1.2	1.5	2.0
7.0	13	1.8	3.2	1.5	1.8	1.1	1.5	1.5	1.5
7.0	53	3.5	4.9	2.2	2.0	1.5	1.6	1.8	2.0
7.0	119	4.8	4.8	1.8	1.6	1.1	1.2	1.5	1.2
16	30	4.6	1.8	1.8	1.6	1.2	1.5	2.2	1.5
16	83	4.8	4.5	1.5	2.4	1.8	1.5	2.8	2.2
28	0	1.0	1.0	1.5	1.5	1.3	1.4	3.0	2.5
28	13	1.2	4.6	1.8	1.5	1.4	2.2	2.0	2.0
28	53	4.5	4.7	2.5	2.0	1.2	1.1	2.0	2.0
28	119	5.0	5.0	2.3	1.6	2.3	1.8	2.2	1.5
28	212	5.0	4.8	1.2	2.3	1.5	1.5	2.0	1.5
44	30	4.9	2.0	2.0	2.2	2.0	1.8	2.0	2.8
44	83	5.0	4.6	2.8	1.8	1.3	1.2	2.4	2.2
63	13	1.2	3.6	1.8	2.6	1.8	1.3	2.2	2.5
63	53	4.9	4.9	2.2	1.4	1.5	1.5	2.0	3.2
63	119	4.5	4.7	1.8	2.3	2.0	1.5	2.0	1.8
85	3.3	1.4	1.1	2.3	2.3	2.5	1.4	2.5	2.0
85	163	4.5	4.8	2.0	1.8	1.5	1.4	2.2	3.4
112	53	4.6	4.9	1.8	2.6	1.3	2.5	2.5	2.0

[a]Rates of P and K are in pounds per acre.
[c]Based on scale of 1 to 5 where, 1—all plants are erect and 5—almost all plants are down. Score was taken just prior to harvest.
[b]Replications.

Table A-3. Multiple regression statistics b_0, b_i, $s(b_i)$ and R^2 values for the square-root form of estimated yield $(\hat{Y})$ on the X variates for four plant parts sampled during two growth stages from four combined experiments.

Growth stage	Plant part[b]	b_0[c]	b_i[a] and $s(b_i)$ for the variates $p^{1/2}$	p^d	$k^{1/2}$	k^d	$p^{1/2}k^{1/2}$	R^2
5	LL	40.38	—50.55	—80.70	—23.51	—21.87**	156.51**	0.44**
			59.18	60.40	17.11	5.91	27.28	
	UL	—126.80	321.37**	—321.14**	76.64**	—41.91**	60.72**	0.41**
			89.23	73.74	21.09	6.71	18.97	
	LP	44.67	—75.78	—21.01	—16.15	—4.11	81.95**	0.34**
			75.42	78.31	9.95	2.44	20.35	
	UP	65.73	—99.21	—94.30	—20.15	—11.06**	115.95**	0.35**
			100.24	81.88	13.74	2.68	20.96	
7	LL	—82.25	80.82	—134.66	—171.23**	—80.69**	16.47	0.36**
			81.58	75.28	33.50	11.26	40.63	
	UL	—132.60	125.50*	—200.20**	228.53**	—105.16**	39.18	0.57**
			54.37	42.48	31.94	10.36	29.71	
	LP	—1.48	48.73	—122.77*	29.48**	—17.67**	40.12*	0.39**
			50.64	55.86	10.27	2.97	19.20	
	UP	5.20	9.83	—145.18*	31.36	—25.58**	82.49**	0.43**
			72.76	67.86	15.99	4.11	23.17	

[a]b_i and $s(b_i)$ values are the upper and lower figures, respectively.
[b]Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles and UP is upper petioles.
[c]Regression constant.
[d]Values used in calculations for the lower and upper leaves were coded by subtracting 0.10 percent P and 0.30 percent K from the observed values in growth stage 5 and by subtracting 0.15 percent P from the observed values for lower and upper leaves and 0.04 percent P from the lower and upper petioles in growth stage 7.

Table A-4. Multiple regression statistics b_0, b_i, $s(b_i)$ and R^2 values for the quadratic form of estimated yield $(\hat{Y})$ on the X variates for four plant parts sampled during two growth stages from four combined experiments.

Growth stage	Plant part[b]	b_0[c]	b_i[a] and $s(b_i)$ for the variates p^d	p^2	k^d	k^2	pk	R^2
5	LL	0.87	116.56**	—310.91**	28.09**	—16.05**	16.15	0.32**
			39.24	103.71	4.79	2.91	12.51	
	UL	18.10	34.12	—199.60**	—2.19	—6.14**	70.24**	0.45**
			54.04	67.30	5.37	1.09	10.49	
	LP	18.50	18.07	—157.54	4.16	—1.43**	20.71*	0.30**
			47.06	114.60	2.20	0.54	9.70	
	UP	14.99	62.03	—244.51*	0.03	—0.93**	24.27**	0.30**
			63.58	101.40	2.39	0.28	6.01	
7	LL	—13.58	118.08	—353.89*	51.44**	—19.42**	—18.31	0.31**
			75.38	176.50	10.02	2.95	31.01	
	UL	—17.38	85.39*	—285.40**	51.10**	—19.26**	19.45	0.58**
			33.82	55.46	7.06	1.87	13.51	
	LP	16.81	27.20	—222.55	7.53**	—3.35**	36.60*	0.33**
			42.03	113.59	2.89	0.66	15.39	
	UP	10.86	100.10*	—422.53**	8.17*	—3.43**	23.63*	0.43**
			49.34	102.72	3.37	0.63	11.42	

[a]b_i and $s(b_i)$ values are the upper and lower figures, respectively.
[b]Soybean plant parts are designated as follows: LL is lower leaves, UL is upper leaves, LP is lower petioles and UP is upper petioles.
[c]Regression constant.
[d]Values used in calculations for the lower and upper leaves were coded by subtracting 0.10 percent P and 0.30 percent K from the observed values in growth stage 5 and by subtracting 0.15 percent P from the observed values for lower and upper leaves and 0.04 percent P from the lower and upper petioles in growth stage 7.