Solving Tile Drainage
Problems by Using Model Data

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CONTENTS

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Our purpose in this bulletin is to report, to analyze, and to use in problem solving, extensive model data of tile drainage of land.

The data were obtained with a glassbead-glycerol model (Grover et al., 1960; Grover and Kirkham, 1961) and include: (a) values of depths and of corresponding times of fall of the surface of saturation to these depths at various distances from the drain tubes and (b) values of the drain tube discharge rates. The zero reference time for the fall of the surface of saturation and also for the discharge rate is the instant at which the surface of saturation passes through the simulated soil surface from a ponded condition.

Models were made of 109 different combinations of drain depth, drain spacing and soil stratification. For each of these 109 model conditions, the surfaces of saturation were photographed at about eight different depths through the transparent front face of the model. Photographs were read under a magnifying glass to obtain distances and times of fall. Times were obtained from a clock that was started at the zero reference time and photographed with the water tables.

A glycerol-water solution is used in the model to provide the fluid or "water" to obey Darcy's law which is applicable to ground water seeping to drain tubes in the field (Luthin, 1957). The glycerol, because of its viscosity, slows the fluid movement which would be too rapid to obey Darcy's law in the model if water alone were used. Glass beads are used instead of soil to provide a porous medium of small capillary rise, and 16-mesh-per-inch wovenwire drain tubes are used instead of clay tile drain tubes. The bottom of the model simulates an impermeable subsoil layer, called a barrier.

The model data are reported in the "Results" section. Formulas are derived and detailed examples for using the model data for solving field drainage problems are presented in the "Discussion" section.

GEOMETRIES OF DRAINAGE CONDITIONS

Part A of fig. 1 represents a typical water table geometry as it might be in the field and gives symbols $2r$, x, y, z, Z, $a/2$, d, h and $h-d$, needed (in part) to describe the geometry. Part B of fig. 1 illustrates a front elevation of the model and gives model dimensions. In fig. 1, drain "tiles" are designated by small circles; these "tiles" can be opened or closed by stopcocks to give four spacings and two distances of the tile centers to the barrier. The scale of the model ordinarily is: 1 cm. in the model equals 1 ft. in the field. But other scales are useful, and just as valid, as long as relative dimensions in the field correspond to those in the model.

Symbols not indicated in Part A of fig. 1 are:

L, length of drain tubes ("tiles");

K, hydraulic conductivity of the porous medium

K_1, K_2, hydraulic conductivities, respectively, of an upper stratum and of a lower stratum of a two-layered porous medium, when one is used;

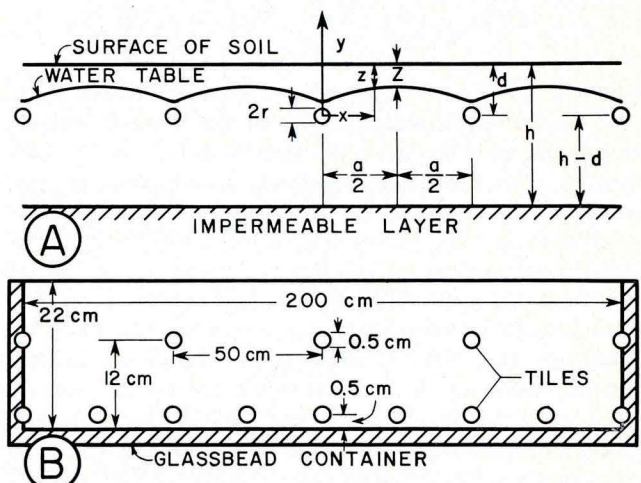f, drainable porosity (drainable fraction) of the upper layer [the drainable porosity of

FIG. 1. Schematic diagrams of drainage conditions: (A) field geometry, (B) model geometry (not to scale).

the lower layer doesn't enter into calculations]; and

t, time for the water table to fall the distance Z (and z) of fig. 1. (Some other symbols will be defined when introduced.)

Three sizes of glass beads, ½, 2 and 5 mm., were used as porous media for the model. Beads of 2 mm. diameter were used for homogeneous, nonstratified soil; and the pair sizes, 2 mm. and ½ mm., and 2 mm. and 5 mm., were used for stratified soil. Only soils of one or two layers were considered, not counting an impermeable soil layer represented by the impermeable tank bottom of the model. Since temperature was carefully controlled (to less than 1°C) and since the glycerol density also was controlled, K_1 and K_2 were constant. The values were:

$K = K_1 = 1.23$ cm./min. (when 2 mm. beads, only, were in the model)

$K_1/K_2 = 20$ (when ½ mm. beads were in the sublayer and 2 mm. beads, in the surface layer)

$K_1/K_2 = 0.4$ (when 5 mm. beads were in the sublayer and 2 mm. beads, in the surface layer)

The drainable porosity f of the surface layer of porous medium was always 0.4 cm.^{3} of air space per cm.^{3} of bulk medium. Therefore, we have, for the model,

$$f = 0.4 \text{ cm.}^3/\text{cm.}^3$$

The drainable porosities of the sublayers were not observed. The sublayer porosities are not needed because the water table fall was observed (and analyzed) only when it existed in the surface layer of beads.

The width of the porous medium was the same as the length of the drain tubes,

$$L = 1.9 \text{ cm.}$$

Drain Tubes

The drain tubes need particular comment. The radius r of the drains of the model was always

$$r = 0.25 \text{ cm.}$$

The drains flowed full, with negligible loss of head over their length compared with the loss of head in the beads, and the drains outletted into a trough containing glycerol-water solution standing at the level of the drain axes. With the drains outletting at the level of their axes, one would expect air to back up into the drains and cause a surface of seepage (difficult to deal with in models) to develop at the upper part of the drain tube-bead interface. Air did not back up, however, and surfaces of seepage, therefore, did not develop because (a) at the higher discharge rates, the drains ran full in all events and (b) at the lower discharge rates, a capillary fringe effect (discussed in the next section) prevented air backflow. The outflow reference level for the hydraulic head was taken at the level of the axes of the drain tubes.

The radius, r = 0.25 cm., in the model corresponds to a single field condition of 6-inch diameter drain tubes, when the model scale is 1 cm. of model to 1 foot in the field. If, in the model, we had used drain tubes half or twice as large, then, the rates of fall of the water table would not have differed more than about 18.3 percent from those reported here.[4] Thus, our model results should apply (without making a drain size correction) with an accuracy of about 20 percent to field drain tubes of 3 to 12 inch diameter when the model scale is 1 cm. of model to 1 ft. in the field.

If field drains run partially full, as they ordinarily do, the water tables will be lower in the field than is indicated by the model results. Therefore, application of the model results to field drainage design ordinarily will be on the "safe" side. That is, the field water table, after drain tile installation should not be as high as the design height. The model drain tubes were formed of wire screen to correspond to a field condition of drain tile being surrounded by highly permeable material, such as coarse gravel or large stable soil aggregates. If, in the field, gravel or coarse soil aggregates do not surround the tile, the water tables in the field will be higher than those indicated by the model data (see Kirkham, 1950; or Luthin, 1957, pp. 302-303).

Drainage Cases

Table 1 provides an index and gives further details of the drainage cases studied. The first column in the table gives the source figures of photographs as numbered in Grover (1959). These photographs are the raw data. The second column gives general geometry designations, A, B, . . ., for reference. The symbols d, a, y, h and r are as in fig. 1. In table 1, distances from the drain axes to the barrier, or to the interface of different layers of beads, are given as 12, 6, 3, 0.5 and 0.25 cm. The values 12, 6 and 3 cm., although so found in the model, are not so recorded in Grover. By error, Grover gives 11, 5.5 and 2.75 cm. (No computations were made in the work cited with these incorrectly recorded values.) In table 1, the words "isotropic system" in the subheadings may imply that "anisotropic systems" are included in the study when they are not. For application of the data to anisotropic conditions, see Maasland (1957) and references cited there.

Capillary Fringe

The glass beads and glycerol in the model were used to minimize the height of a capillary fringe. By capillary fringe, we mean the fluid-saturated medium above the "water table" (that is, glycerol-water table). The upper surface of the capillary fringe is called the surface of saturation.

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[4]The influence of drain size on water table fall will be brought out in the "Discussion" section, *Problem 6;* the value of 18.3 percent will be found as 1 minus 0.817 of eq. 80.

Table 1. Index of drainage cases photographed; compare fig. 1.

Grover fig. No.[a]	Case	Drain depth d (cm.)	Drain spacing a (cm.)	(cm.)	(cm.)	(cm.)
HOMOGENEOUS ISOTROPIC SYSTEM, K = CONST.						
(a) Barrier at $y = -(h-d) = 12$ cm. below drain centers						
24	A	2	50	100	200	400
25	A	4	50	100	200	400
26	A	6	50	100	200	400
27	A	8	50	100	200	400
(b) Barrier at $y = -2r = 0.5$ cm. below drain centers						
28	B	2	50	100	...	...
29	B	4	50	100	200	400
30	B	6	50	100	200	400
31	B	8	50	100	200	400
TWO-LAYER ISOTROPIC SYSTEM, $K_1/K_2 = 0.4$ (UPPER LAYER ONLY 4/10 AS CONDUCTIVE AS THE LOWER) BARRIER AT $y = -(h-d) = 12$ cm. BELOW DRAIN CENTERS						
(a) Interface of soil layers at $y = -\frac{1}{2}(h-d) = 6$ cm. below drain centers						
32	E	2	50	100	200	400
33	E	4	50	100	200	400
34	E	6	50	100	200	400
35	E	8	50	100	200	400
(b) Interface of soil layers at $y = -\frac{1}{4}(h-d) = 3$ cm. below drain centers						
36	F	2	50	100	200	400
37	F	4	50	100	200	400
38	F	6	50	100	200	400
39	F	8	50	100	200	400
(c) Interface of soil layers at $y = -r = 0.25$ cm. below drain centers						
40	G	2	50	100	200	400
41	G	4	50	100	200	400
42	G	6	50	100	200	400
43	G	8	50	100	200	400
TWO-LAYER ISOTROPIC SYSTEM, $K_1/K_2 = 20$ (UPPER LAYER 20 TIMES MORE CONDUCTIVE THAN THE LOWER) BARRIER AT $y = -(h-d) = 12$ cm. BELOW DRAIN CENTERS						
(a) Interface of soil layers at $y = -\frac{1}{2}(h-d) = 6$ cm. below drain centers						
44	C	2	50	100	200	400
45	C	4	50	100	200	400
46	C	6	50	100	200	400
47	C	8	50	100	200	400
(b) Interface of soil layers at $y = -\frac{1}{4}(h-d) = 3$ cm. below drain centers						
28c	D	2	50			
48	D	4	50	100	200	400
49	D	6	50	100	200	400
50	D	8	50	100	200	400
SYSTEM WITH DRAINS AT GREAT (19 cm.) DEPTH BARRIER AT $y = -2r = 0.50$ cm. BELOW DRAIN CENTERS						
(a) Homogeneous system, K = Const.						
51	H	19		100	200	...
(b) Two-layer system with $K_1/K_2 = 0.4$, and with the interface at $y = 11$ cm. above the drain centers, that is, 8 cm. below the soil surface.						
51	I	19			200	

[a] Numbers under "Grover fig. No." are numbers of source photographs in Grover (1959).

Although fairly large beads (2 mm. diameter) were used to minimize the height of the capillary fringe, the beads were silicone treated as a further preventative of capillary rise. Nevertheless, a pseudo capillary fringe could not be avoided. Experiments (Grover and Kirkham, 1961) showed that a suction head of 0.75 cm. of glycerol was required to pull the glycerol through the beads. (But the glycerol will not rise 0.75 cm. in the beads by capillarity—hence the term, "pseudo.") That is, with the drain tubes outletting, as they did, at the level of their centers, the surface of the glycerol in the beads would never get lower than 0.75 cm. above the drain tube centers. This 0.75 cm. level could correspond to a capillary fringe height of about 0.75 ft. in the field, when 1 cm. in the model is 1 ft. in the field. The height, 0.75 ft., is realistic, at least for "structureless" (slightly aggregated) field soil—because, by the well-known capillary rise form-

ula, the diameter of a glass tube in which water will rise to a height of 0.75 ft. is $2[(2 \times 73)/(0.75 \times 30 \times 980)] = 132$ microns, the value 132 microns (0.132 mm.) being about the same size as cavities in fine sand. In aggregated field soils where the aggregates are larger than for fine sand, the capillary fringe height will be smaller than 0.75 ft.

The symbol c is used to represent capillary fringe height. This height is the same as the cm. of capillary suction at the surface of saturation. Thus, for the model, we have

$$c = \text{height of capillary fringe} = 0.75 \text{ cm.}$$

When the height of a capillary fringe in the field does not correspond to the fixed capillary fringe height of the model, a correction must be made. This correction is described in a later section.[5]

RESULTS

The sets of photographs of the (modeled) water tables, as they vary with time, are the raw data for the results; fig. 2 is a sample set. Other photographs are not reproduced here because (aside from the space requirement) the screening necessary to make a half-tone engraving for printed reproduction confounds images of screen mesh points with images of beads so that one could not locate the correct depth of the surface of saturation. In fig. 2 there are four photographs, and each photograph contains eight strips called subphotographs (cut and remounted from original photographs) of the different depths of water tables. In each of the subphotographs of fig. 2, a grid may be seen superposed on the front of the model. The grid mesh is 2 cm. vertical by 5 cm. horizontal. The depth of fall of the water table at various distances from the drain tubes was determined with the aid of this grid. For these determinations, larger photographs than those shown in fig. 2 were used. They were such that 200 cm. of the actual model distance was 8.9 cm. distance on the photograph. On such photographs the depths of fall could be and were read, with the aid of a magnifying glass, to within about 1 mm. of actual distance of glycerol fall.

Depth to Surface of Saturation
Versus Time Relations

Because it was not practical to reproduce the $(109 \times 8 = 872)$ photographs and since, in any event, numerical values are needed—not just photographs—we (two readers working independently) have read from the photographs, like those of fig. 2, numerical values of z, the depth of fluid fall, for the various times t. An example of such readings is presented as table 2. Notice in this table that the values of z of the two readers do not agree to within 1 mm. This is because

[5] See *Problems 4* and *6*.

25
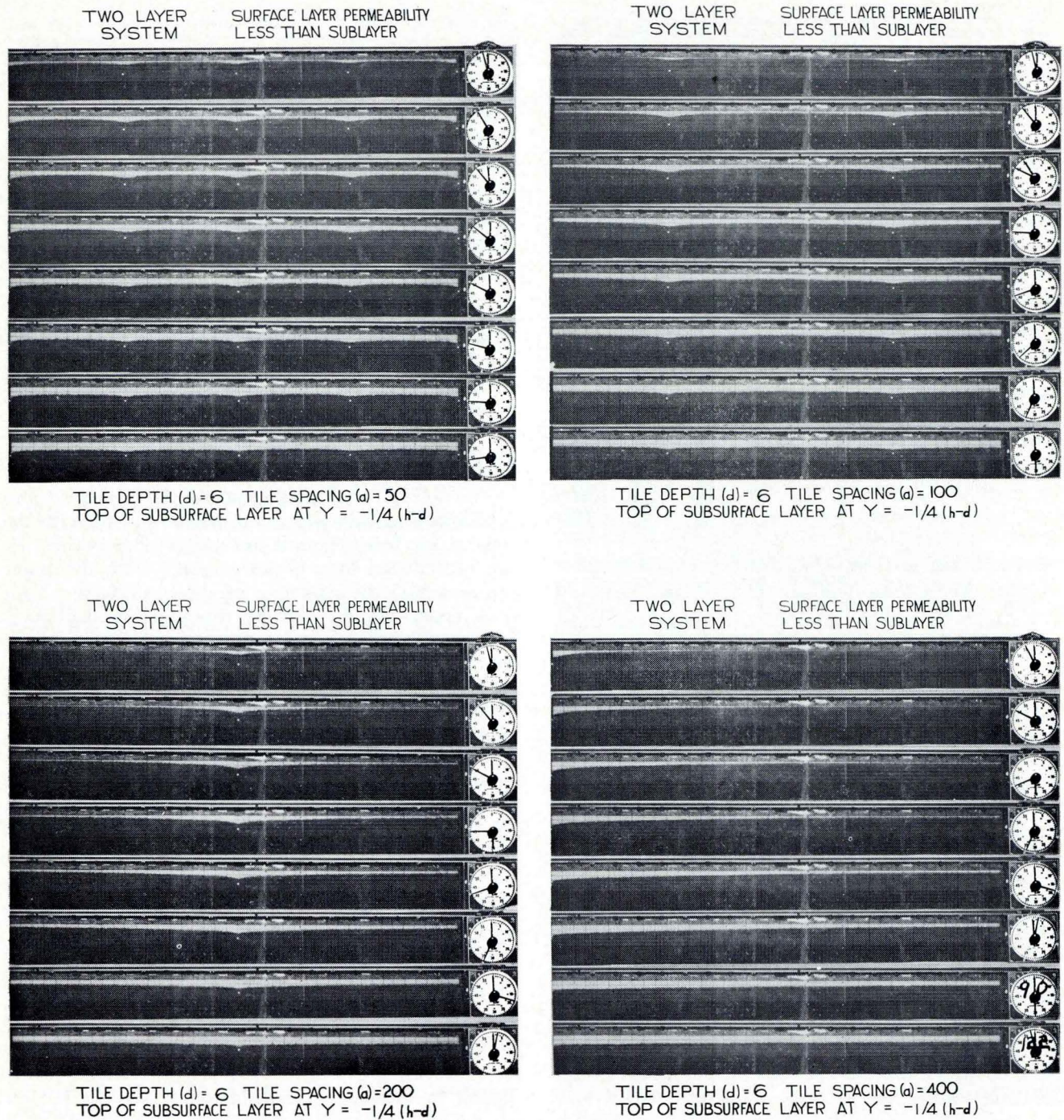
FIG. 2. Sets of falling water tables for the four spacings of drain tubes of Case F (d = 6 cm.) of table 1. Note that the clock runs counterclockwise. The long vertical striations are due to joining plates on the model front.

different drain tubes were used by each reader for the reference position of x (fig. 1). Use of different drain tubes as reference positions for measuring x prevents bias in the (averaged) results.

Table A-1, of Appendix A, with 109 subtables designated by the numbers 24a, 24b, 24c, 24d, 25a, etc., gives the values of depth of fall z as a function of

x and t, as obtained from average values, such as those shown in table 2 for the 109[6] conditions of table 1. The values of x are: x=0, x=0.1a, x=0.2a and x= 0.5a. The letters a, b, c and d, attached to the numbers 24, 25, 26, etc., in the subtable headings, refer, respectively, to the spacings 50, 100, 200 and 400 cm.

[6]Case A, 16; case B, 14; C, 16; D, 12; E, 16; F, 16; G, 16; H, 2; and I, 1--total 109.

Table 2. Sample table of values of z for x/a = 0.1 and 0.9; for Case A of table 1 when d = 2 cm. and a = 50 cm. Theoretically, because of symmetry, the values of z for x/a = 0.1 and x/a = 0.9 should be the same. For meaning of x, a, d and z, see fig. 1.

Clock reading	Lapsed time	First Reader Horiz. dist. from drain		Second Reader Horiz. dist.° from drain		Average
		x/a=0.1 (x=5 cm.)	x/a=0.9 (x=45 cm.)	x/a=0.1 (x=5 cm.)	x/a=0.9 (x=45 cm.)	
	(min.)	(cm.)	(cm.)	(cm.)	(cm.)	(cm.)
60	0	0	0	0	0	0
58	2	0.3	0.5	0.4	0.4	0.4
56	4	0.6	0.5	0.6	1.0	0.7
54	6	0.9	0 7	0.7	0.9	0.8
50	10	0.9	0.7	1.0	1.1	0.9
48	12	0.9	0.8	1.0	1.0	0.9
46	14	0.9	0.8	0.9	1.0	0.9
44	16	0.9	0.85	1.0	1.1	1.0
42	18	0.9	0.95	1.1	1.1	1.0

° A different drain tube was used as reference (for x = 0) by the second reader (see text).

in table 1. Thus, subtable 24a is for 50 cm. spacing. In table A-1, subtable 24a, the numbers, 0.4, 0.7, 0.8, etc., in the third column (for x=0.1a) are the same numbers, 0, 0.4, 0.7, 0.8, etc., occurring in the right hand column of table 2. The zero value of z at t=0 is to be understood in subtable 24a and in all the other subtables in table A-1.

Observe two other points about table A-1. When x=0.5a, we have, in accordance with fig. 1, the relation z=Z, Z being the maximum water table height; so table A-1 includes values of Z versus t as well as of z versus t.

Time for the Surface of Saturation to Fall Unit Depth

Figure 3 is a chart of values of Z (z at x=0.5a) versus t prepared, respectively, from the first and last columns of data of each of subtables 30a, 30b, 30c and 30d of table A-1 of Appendix A. The figure is for case B, d=6 cm. of table 1, and is presented to show some typical results as well as to show how one may, from such curves as are shown, obtain the values of the time for the mid-drain surface of saturation to fall a unit depth, Z=1 cm., (or to fall other depths) from the just-ponded or just-unponded condition. The times for Z=1 cm. for fig. 3 are seen to be: 7.8, 31.5, 108 and 454 min., respectively. These four times and other times for Z=1 cm. are given in Appendix B, table B-1,
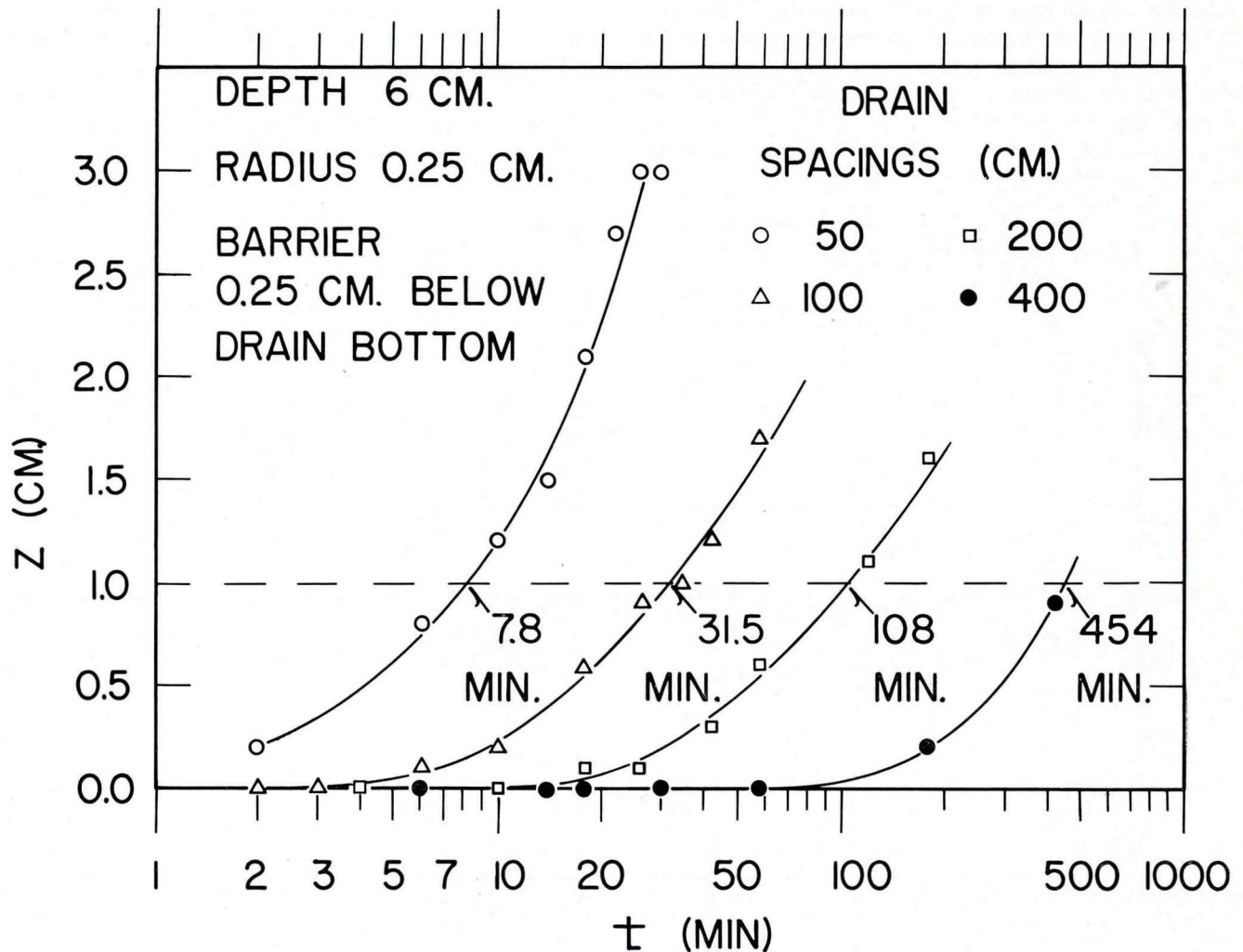
FIG. 3. Semilogarithmic curves of distance Z of water table fall midway between drains versus time t, for the four drain spacings of Case B (d = 6 cm.) of table 1. The intersections of the horizontal dashed line with the curves yield values of t for a unit fall, Z = 1 cm., of the surface of saturation midway between drains.

columns 8, 9, 10 and 11. Table B-1 also includes the times for Z=0.5 (columns 4, 5, 6, 7), 1.5 (columns 12, 13, 14, 15), and 2 cm. (columns 16, 17, 18, 19), which were obtained in the same way as were the times for Z=1 cm.

In table B-1, the time values for Z=1 cm., corresponding ordinarily to the field value Z=1 ft., are of particular interest because (compare Luthin, 1957, p. 387) drainage systems are generally considered satisfactory if the water table falls 1 ft. below the soil surface in a reasonable time (24 to 48 hours). But we emphasize that the times for the field cases corresponding to table B-1 cannot be read from table B-1 directly. We shall see in later sections how to calculate the field times from the model data. In table B-1 the numbers are not accurate to more than 2 or 3 significant figures.

Since the values of the times for Z=1 cm. in table B-1 are of particular interest, we have prepared graphs (fig. 4) of the drain spacing, a, versus the expression $(tK/fZ)^{1/2}$ for Z=1 cm. The expression $(tK/fZ)^{1/2}$ is used for several reasons. It is dimensionless; in it, t/Z is an average reciprocal velocity of fall; the exponent $\frac{1}{2}$ is used since Kirkham and Gaskell (1951) found theoretically, at least in a certain range of values, that a was proportional to $t^{1/2}$ for Z=constant. The expression $(tK/fZ)^{1/2}$ is easy to obtain for the abscissas of fig. 4 for which Z=1 cm. Since we have K=1.23 cm./

min., f=0.4 and Z=1 cm., we obtain the expression by multiplying the time periods in table B-1 by 1.23/0.4=3.075 and taking the square root of the product. The significance of the grouped symbols tK/fZ will be brought out further in the "Discussion" (see especially eq. 35).

Fig. 4 is for Z=1 cm. Curves like fig. 4 for Z=0.5, 1.5 and 2 cm. are given for comparison with fig. 4 in Appendix C, figs. C-1, C-2 and C-3. When fig. 4 and fig. C-1 (drawn to the same scale) were superposed, the curves for Z=0.5 cm. and Z=1 cm. would superpose very nearly, which was also true in some cases for Z=1 and Z=1.5, but not for Z=1 and Z=2 cm. The reason for the superposition, when it occurred, was that Z varied directly with t, making Z/t constant. This direct variation of Z with t usually occurred only when the values of Z and of Z/(d-Z) were both simultaneously small. Values of t for the "water table" to fall from the depth Z=0.5 to depth Z=1.0 cm. may be obtained by subtracting the time for Z=0.5 cm. from the time for Z=1 cm.; similarly for other depths, as for Z=0.5 cm. to Z=1.5 cm. One sees in the curves for Z=0.5 cm. in fig. C-1 that the influence of the 0.75 cm. "capillary" fringe is more marked for the curves for d=2 cm. than for the curves for d=4, 6 and 8 cm. The more marked effect of "capillarity" is seen especially at the large values of t where the curves for d=2 cm. are abnormally steep compared with the steep-
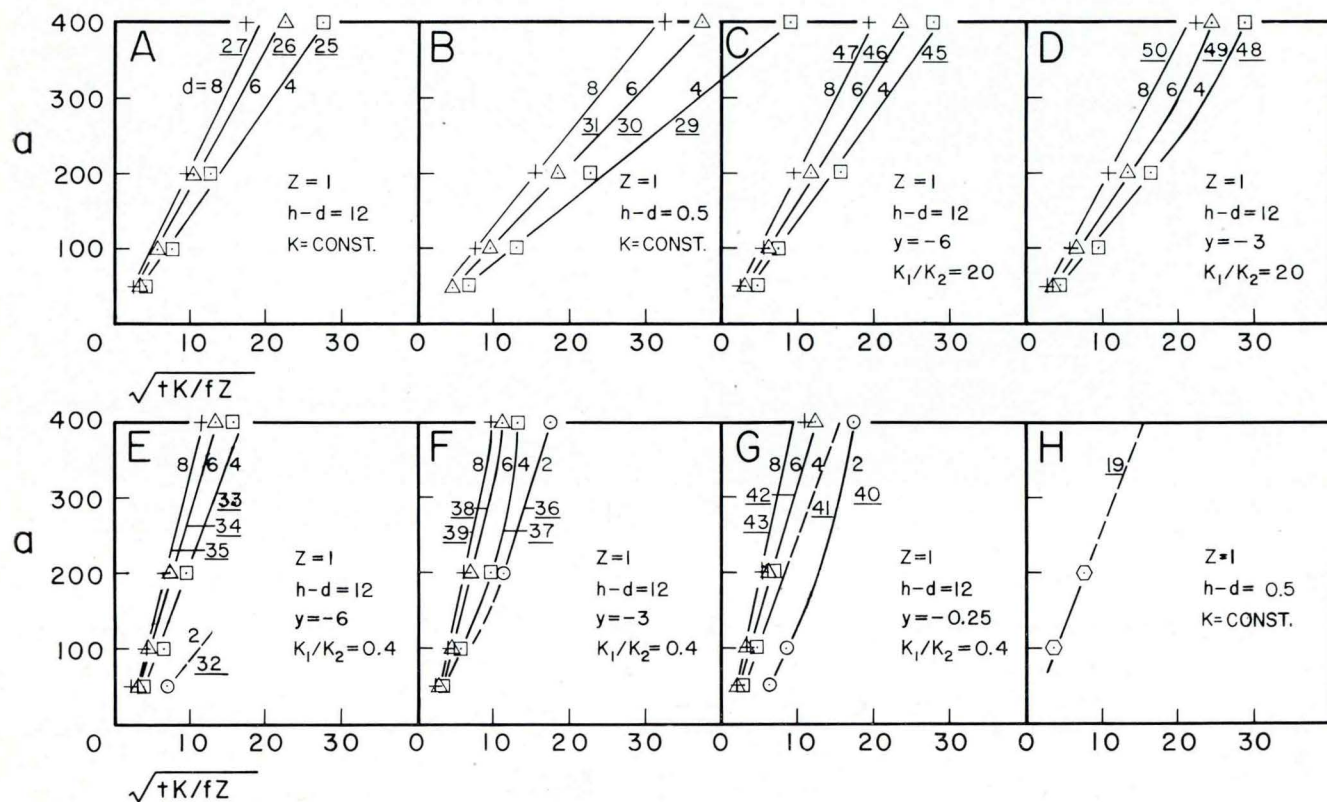
FIG. 4. Plots of model drain distance a versus $(tK/fZ)^{1/2}$ for Z = 1 cm., for cases A, B, . . ., H of table 1. The numbers 8, 6, 4 and 2 are depths to tiles in cm. The underlined numbers are those of table 1, column 1. The dimensions of a, Z, h–d and y are cm.; K = K₁ = 1.23 cm./min.; f = 0.4; fringe height = 0.75 cm.; t, in min.; tK/fZ is dimensionless.

28

ness of the curves for d=4, 6 and 8 cm. See especially cases A, F and G of fig. C-1.

In fig. 4, the curves do not, theoretically, go through the origin (where a=0, t=0) even though they seem to in some cases. To prove that the curves do not go through the origin, consider the following reasoning and calculations, where we shall take a general value of Z rather than restricting ourselves to Z=1 cm. as in fig. 4. We first observe that the situation, a = 0, that is, zero drain spacing, corresponds physically to an infinite plane sink under the soil at the tile depth. Therefore, with the tiles outletting at atmospheric pressure into this infinite plane sink, we see that the Darcy velocity v, of v=Ki, will, because the hydraulic gradient i is now unity, just be K, the hydraulic conductivity. The velocity of water table fall will be K/f, and the time for the water table to fall the Z cm. will be t=Z/(K/f). But we want tK/fZ. Therefore, we multiply both sides of the last equation by K/fZ and find, after simplification, the result, tK/fZ=1. Therefore, we have the interesting result:

$$(tK/fZ)^{1/2}=1, \text{ when } a=0;$$

which is the same as

$(tK/fZ)^{1/2}=1$, when $a/d=0$, for all values of Z. (1)

In words, the intercept on the $(tK/fZ)^{1/2}$ axis of all the curves of fig. 4 should theoretically be 1—if capillary effects are negligible. When they are not negligible the values of $(tK/fZ)^{1/2}$ will be greater than 1 (Swartzendruber and Kirkham, 1956).

Approximate Equations for Water Table Depths

To see if data such as those in fig. 4 could be further compressed, logarithmic plots were made of a/d versus tK/fZ, one of which (fig. 5) is shown for Case B (d=2 cm. excluded) of fig. 4.

The data points in fig. 5 do not fall on a single curve but on three separate curves, one curve for each value of d. Theoretically, the data points cannot be expected to fall on a single curve, because as a/d varies, the depth below the tiles of the barrier, h—d, remains constant, as is seen in fig. 1, rather than varying proportionately with a and d. Thus, the "population" of the drain cases, statistically speaking, is not homogeneous. Nevertheless, for a given value of tK/fZ in fig. 5, a/d does not change much from curve to
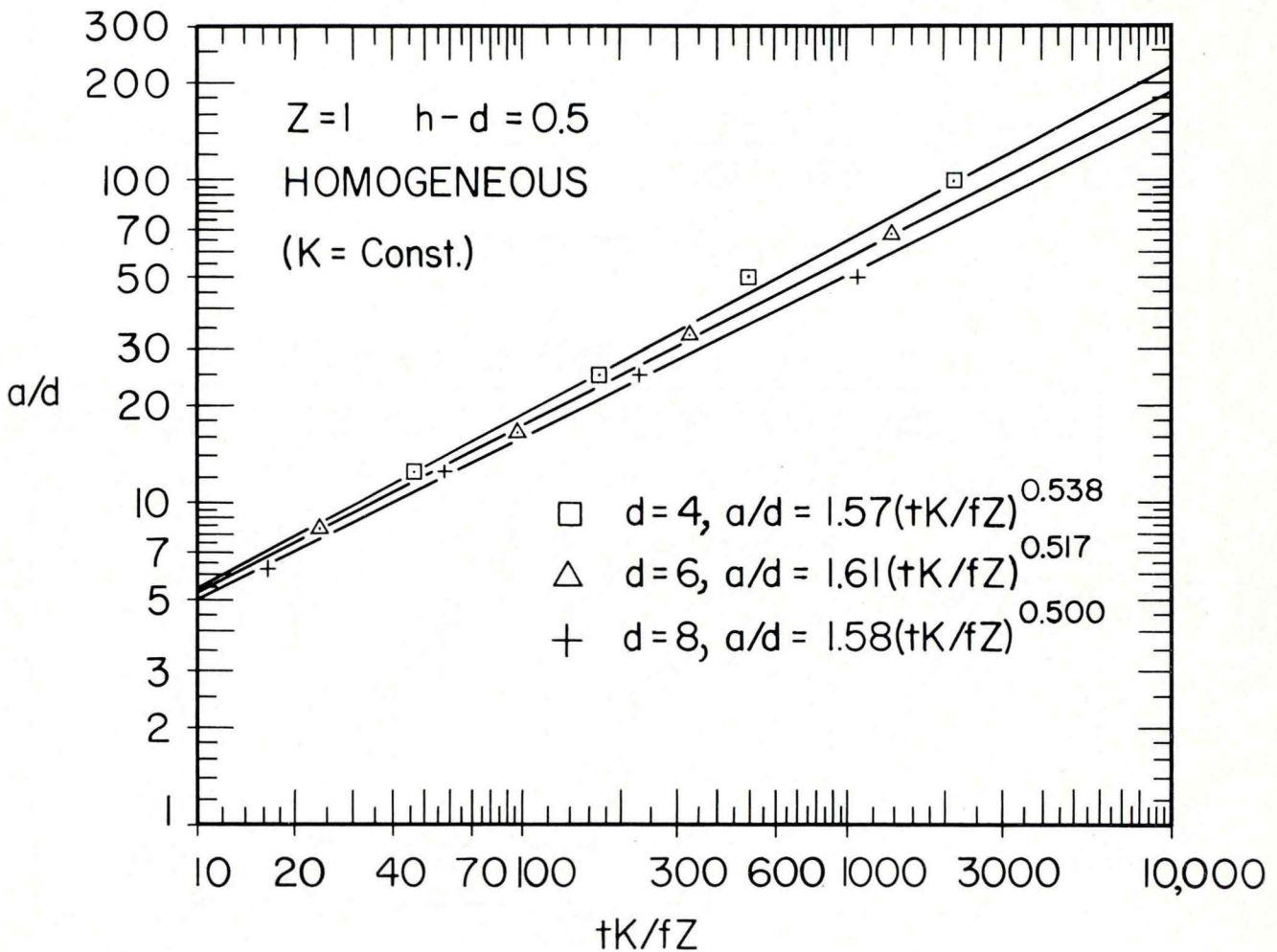
FIG. 5. Logarithmic plots of a/d versus tK/fZ for three drain depths, d = 4, 6 and 8 cm., Z = 1 cm. and h − d = 0.5 cm. (Case B of table 1).

curve. So the middle curve may be taken as an approximation for each of the relations for Case B. The equation of this middle curve is

$$a/d = 1.61 \ (tK/fZ)^{0.517}, \qquad (2)$$

which is the same as

$$\log_{10}(tK/fZ) = -0.400 + 1.933 \log_{10} a/d. \qquad (3)$$

Equations like eq. 3 have been prepared, using logarithmic regression, for all seven cases, A, B, C, ..., of table 1. These equations are presented as eqs. 4-10. In them, eq. 5 corresponds to, but is not the same as, eq. 3. When eqs. 3 and 5 are each solved for a/d, in the range of interest, they each yield approximately the same result. The equations do not give identical results because eq. 5 is based on three times more data than is eq. 3.

The equations for cases A, B, . . ., G, are (for Z=1 cm.) as follows:

$$\text{A:} \ \log_{10}(tK/fZ) = -0.487 + 1.606 \log_{10}(a/d) \qquad (4)$$
$$\text{B:} \ \log_{10}(tK/fZ) = -0.555 + 1.986 \log_{10}(a/d) \qquad (5)$$
$$\text{C:} \ \log_{10}(tK/fZ) = -0.547 + 1.696 \log_{10}(a/d) \qquad (6)$$
$$\text{D:} \ \log_{10}(tK/fZ) = -0.563 + 1.759 \log_{10}(a/d) \qquad (7)$$
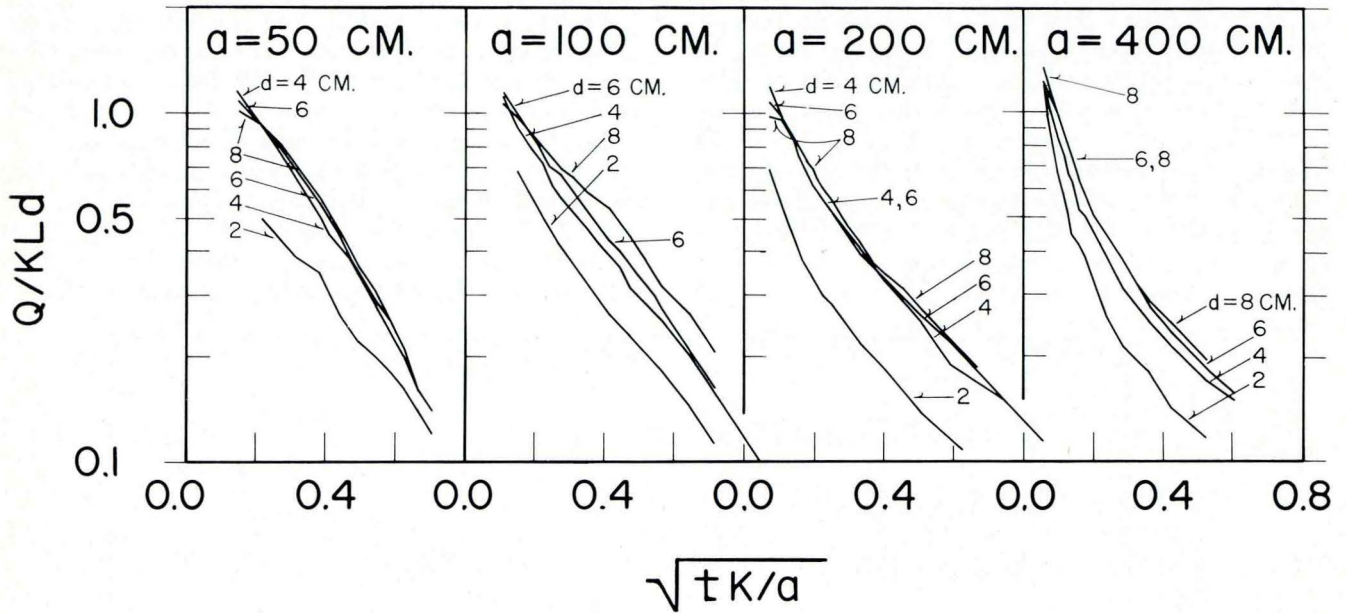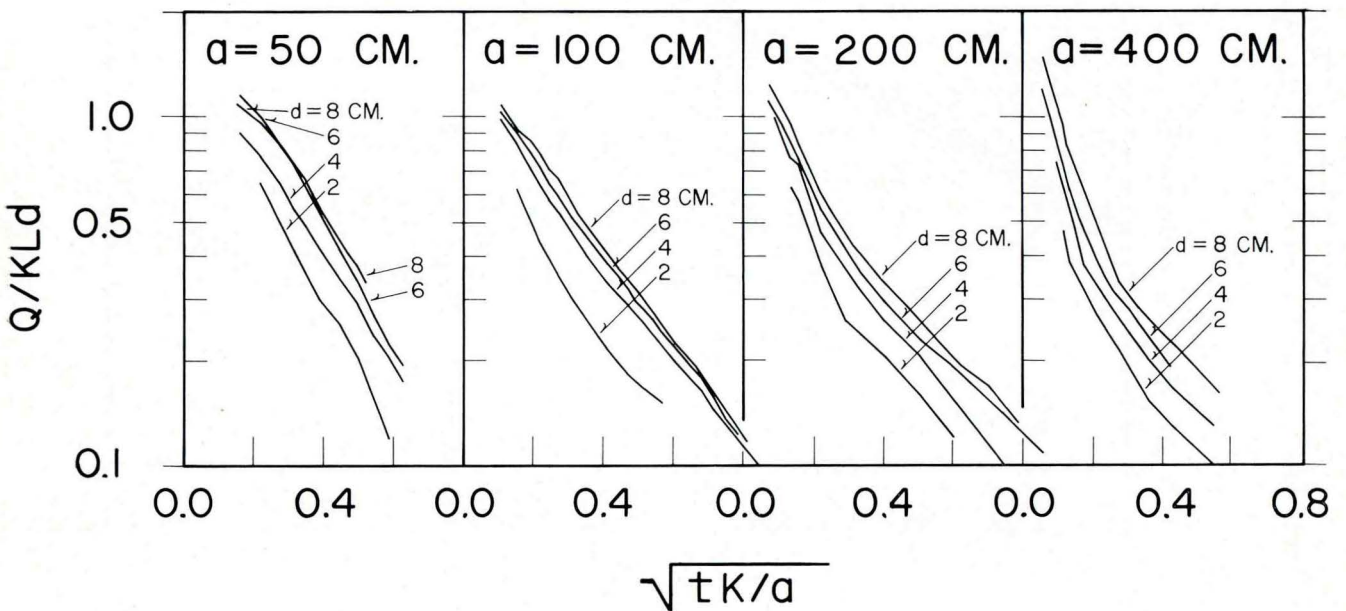
FIG. 6. Q/KLd versus $(tK/a)^{1/2}$ for cases A and C of table 1; Q is the drain discharge rate in cm.^{3}/min.; the other symbols are as before.

30

The correlation coefficients r, for the logarithmic regression equations, eqs. 4-10, have been computed. The value of r in each case was 0.99 except for case G, where r was 0.97. But these high correlation coefficients do not mean theoretically, as we saw in fig. 5, that there is just one curve for all of the individual depths, and an r value of 0.99 does not mean a 99-percent accurate result.

Drain Discharge Rate Versus Time Relations

Let Q be the discharge rate (cm.³/min.) of one of the drain tubes, and let t, as before, be the lapsed time after opening the drain tubes, at the instant the ponded fluid disappears, for one of the geometries. Then the results for the discharge rate are as in figs. 6 and 7. Here, instead of plotting Q versus t, we have
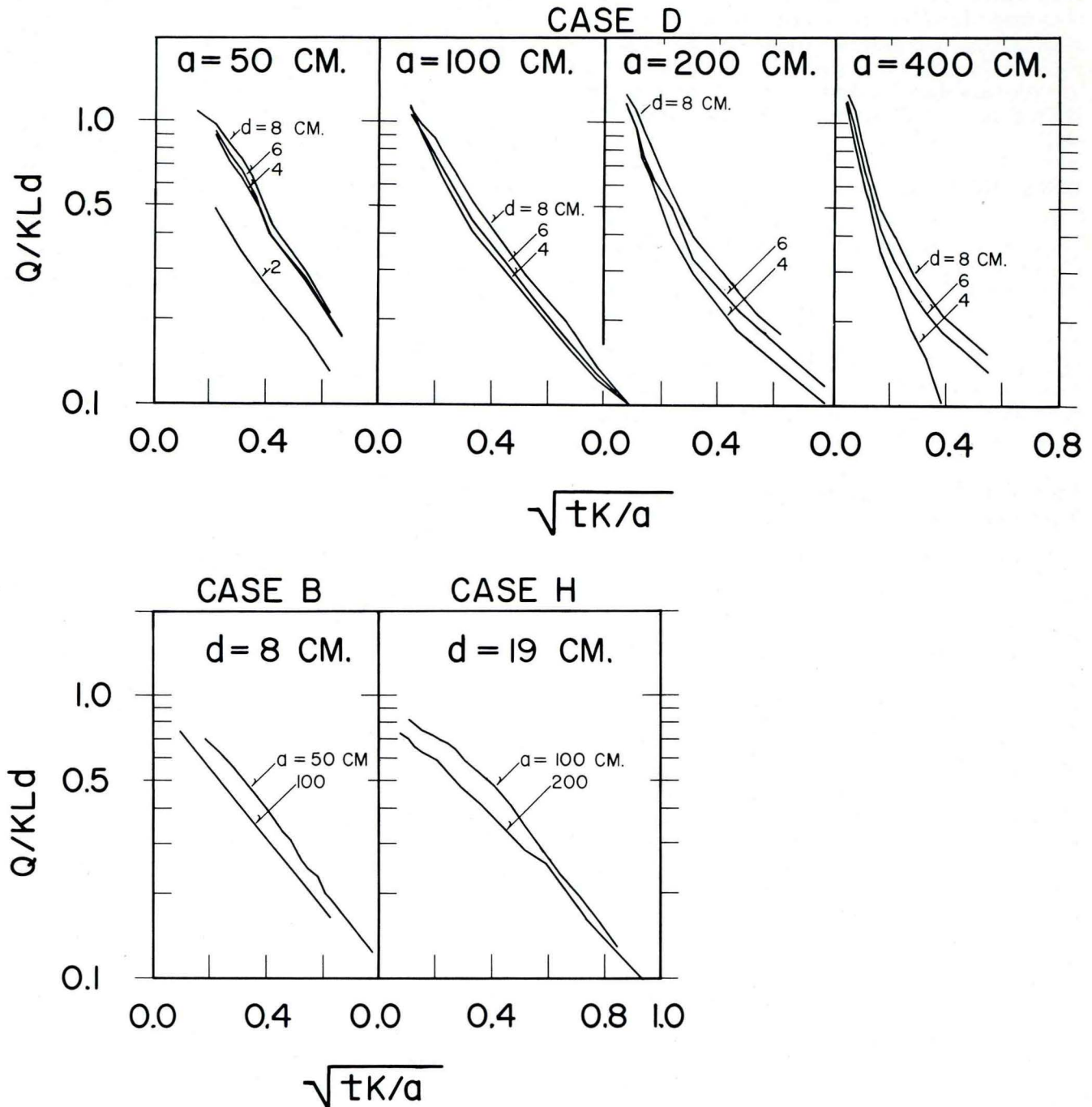
FIG. 7. Same as fig. 6 except for cases B, D and H of table 1. Discharge data were not taken for cases E, F and G of table 1.

plotted Q/KLd versus $(tK/a)^{1/2}$. The exponent ½ tends to make the data linear. A better independent variable would be $(tK/af)^{1/2}$. The values of Q were obtained during the experiments by collecting the discharge $\triangle q$ of the drains at small time intervals $\triangle t$ and by calculating the ratio, $\triangle q/\triangle t$ $(=Q)$. The experimental points are not shown on figs. 6 and 7. There were, in all cases, at least 10 points per curve approximately equally distributed over the curves. The curves were made by connecting successive points with straight lines. A duplicate run (not shown here) was made for Case D, a=100 cm., d=8 cm. The duplicate curves agreed to within less than 0.05 unit in Q/KLd except in the part of the curve where $(tK/a)^{1/2}$ was less than 0.2. In this part of the curve, the discrepancy in duplicate-run values was less than 0.10 unit.

DISCUSSION

In this discussion, we shall derive fundamental equations connecting model and field results and shall use these relations to solve, as examples, some field problems. Also, we shall compare briefly some of our model data with data of other workers. We assume that we may deal with the pseudo capillary fringe in the model as if it were a true capillary fringe. We therefore drop the term "pseudo." Ligon et al. (1962) have made a study of the glycerol fringe in glass beads.

Equations Connecting Model and Field Drawdown Data

In fig. 8, consider one of the model geometries, and its associated streamline pattern, at the instant the surface of saturation has fallen midway between drain tubes from its just-unponded condition to a depth Z. In the streamline pattern, fix attention on the streamline ABCD, midway between a pair of drains, and on an adjacent streamline EFG. Now imagine, at the instant the surface of saturation has reached Z, that we have placed infinitesimally thin, fictitious sheets of rigid, impervious material perpendicular to the plane of the paper and coincident with the two streamlines ABCD and EFG. Imagine also that we have placed other such sheets coincident with all other streamlines, such as NH and MI starting at about depth z. Imagine further, that all these sheets are constrained to remain fixed while the water table falls midway between drain tubes from Z to a slightly greater depth, $Z+\triangle Z$, during a time $\triangle T$. *Here $\triangle Z$ is small but arbitrary*, and we use T for time instead of t because we wish to reserve t for times referred to an initial, just-ponded condition rather than to an initial time when the water table is at a subsurface position JMNEAP shown in the figure.

Now let $\triangle q$ be the volume of water that discharges, between the stream surfaces ABCD and EFG, into the
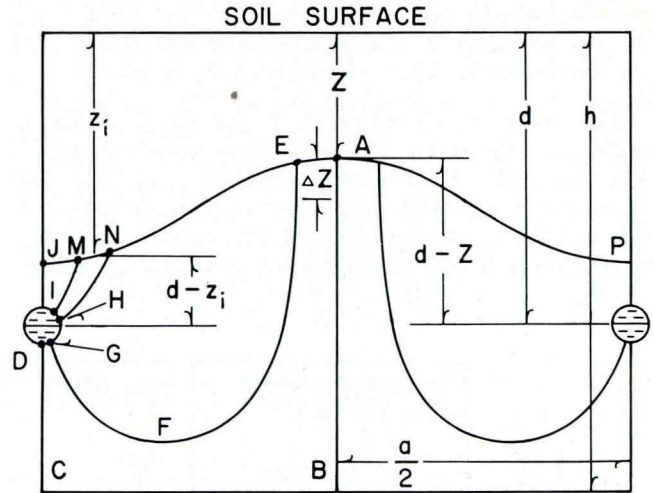
FIG. 8. Streamlines ABCD and EFG for an instantaneous position of the water table.

drain tube, for the full length of the tube L (not per unit length) and during $\triangle T$. Next, remember that, in the model, the hydraulic head level where the drain tube discharges is a distance d below the surface of the beads so that the head difference existing across AD of ABCD and across EG of EFG will, for $\triangle T$, be, on the average and taking the capillary fringe height (pull) c into account, $[(d-Z-c)-\triangle Z/2]$. Hence obtain, by inspection, the relation,

$$\frac{\triangle q}{\triangle T}=K[(d-Z-c)-\triangle Z/2]G. \qquad (11)$$

Here c, the capillary fringe height, is just equal to the centimeters of glycerol-water suction existing at the surface of saturation; G is a geometrical constant; and the product KG may be called, analogous with electrical nomenclature, the hydraulic conductance of the medium ABCDEFG. That is, we have

$$KG=\text{hydraulic conductance.} \qquad (12)$$

In eqs. 11 and 12 remember, for the corresponding electricity flow problem, that $\triangle q/\triangle T$ would correspond to I, the electric current, that $[(d-Z-c)-\triangle Z/2]$ would correspond to the average voltage difference across the medium and that KG would correspond to 1/R where R is the electrical resistance. The writing down of eq. 11 "by inspection" implies that, as for electric current flow, a linear flow law (here Darcy's law) governs the soil water flow.

Now, in fig. 8, abbreviate the distance AE by

$$AE=w$$

and see, then, that we may express $\triangle q$ by

$$\triangle q=fLw \triangle Z, \qquad (13)$$

where f is the drainable porosity.

Now return to eq. 11 and take $\triangle Z$ small enough to be negligible compared with d−Z−c. Then solve for $\triangle q$ to find

$$\triangle q=K(d-Z-c)G \triangle T. \qquad (14)$$

Next, equate the right sides of eqs. 13 and 14 and find, when the time is $\triangle T$, the result

$$fLw \triangle Z = K(d-Z-c)G \triangle T, \quad (15)$$

which is true for any Z, if $\triangle Z$ and $\triangle T$, but not $\triangle Z/\triangle T$, approach zero.

To emphasize that eq. 15 has been derived to apply to the model, place a subscript m on each symbol to obtain from eq. 15 the expression,

$$f_m L_m w_m \triangle Z_m = K_m(d_m - Z_m - c_m)G_m \triangle T_m. \quad (16)$$

For a field situation, not one necessarily geometrically similar to the model situation, instead of eq. 16, we could have found the expression

$$f_f L_f w_f \triangle Z_f = K_f(d_f - Z_f - c_f)G_f \triangle T_f. \quad (17)$$

We have been discussing conditions about the flow region between streamlines ABCD and EFG of fig. 8. If we had singled out the streamlines NH and MI in fig. 8 rather than ABCD and EFG, we should have obtained equations exactly like the last two, except that z_m and z_f would appear instead of Z_m and Z_f. Now consider again the flow region between the streamlines ABCD and EFG.

In eqs. 16 and 17, no special relation between pairs of quantities as d_m and d_f needs to exist. But suppose that the shape of the field geometry, for which we have written down eq. 17, now is similar to the shape of the model geometry so that each dimension in the field is n times that of the model (that is, the scale factor is n); and suppose further that the capillary suction distances c_f and c_m are related by the same similarity factor. (When c_f and c_m are not so related, one can make a correction, as is done later in *Problem 4*, provided that the surface of saturation is near the just-unponded condition.) Then, by definition of "similarity," we may write an expression which defines n and includes z_f and z_m and other quantities. The expression is

$$\frac{c_f}{c_m} = \frac{2r_f}{2r_u} = \frac{L_f}{L_m} = \frac{w_f}{w_m} = \frac{a_f}{a_m} = \frac{d_f}{d_m} =$$

$$\frac{h_f}{h_m} = \frac{h_f - d_f}{h_m - d_m} = \frac{z_f}{z_m} = \frac{Z_f}{Z_m} = n, \quad (18)$$

where n is a constant.

Relation Connecting G_f and G_m

We have yet to find a relation connecting G_f and G_m of eqs. 16 and 17. To find it, first imagine, referring to fig. 8 (which we are taking to pertain to the model), that the space between the streamlines ABCD and EFG, in addition to having the many stream surfaces, also has a large number of equipotential surfaces with these sets of surfaces dividing the space into volume elements. Next, let a typical volume element have (a) a length $\triangle S$ in the direction of the fluid flow, (b) a

breadth $\triangle b$ and (c) a length perpendicular to the plane of the paper L. Now imagine that each dimension of the model, as a_m and d_m, and each flow region length of the model, as $d_m - Z_m$ and $d_m - z_m$, is magnified by the factor n of eqs. 18 to bring the model up to field size. Then, the volume element which had a base area L $\triangle b$ and a length $\triangle S$ will now have a base area $n^2 L \triangle b$ and a length $n \triangle S$. Remember that G is a geometrical constant and that the values of G_m and G_f, therefore, will not depend on hydraulic conductivities. Therefore, we take, for the moment, $K_f = K_m = K$ which could be accomplished physically by using glycerol and (the same size of) glass beads in both the original model and in the magnified model. Therefore, the hydraulic conductance of the original volume element will be, by our definition, eq. 12, and by Darcy's law, KL $\triangle b/\triangle S$. And the hydraulic conductance of the magnified element will be $Kn^2L \triangle b/(n \triangle S)$; the ratio of the latter conductance to the former will be simply n. But since this factor n will also apply to every volume element constituting the space between the streamlines ABCD and EFG, we see that the whole space between ABCD and EFG will have its conductance increased by a factor n. That is, we find

$$\frac{KG_f}{KG_m} = n,$$

or,

$$\frac{G_f}{G_m} = n, \quad (19)$$

which is the needed relation connecting G_f and G_m — like the relations in eq. 18.

Condition Imposed on the $\triangle Z$'s

We next impose a condition which relates $\triangle Z_f$ and $\triangle Z_m$. We can do this because up to now, although we have imposed the conditions given by eqs. 18 (and the condition that the $\triangle Z$'s be small), we still have kept, if tacitly, $\triangle Z_m$ and $\triangle Z_f$ arbitrary. The condition imposed is that we must choose $\triangle Z_f$ and $\triangle Z_m$ such that the relation,

$$\frac{\triangle Z_f}{\triangle Z_m} = n, \quad (20)$$

is satisfied. Eq. 20 does *not* imply that $\triangle T_f$ will be equal to $n\triangle T_m$. But we can quickly obtain a needed relation connecting $\triangle T_f$ and $\triangle T_m$. We find from eqs. 18, 19 and 20 the values: $c_f = nc_m$, $L_f = nL_m$, $w_f = nw_m$, $\triangle Z_f = n \triangle Z_m$, $G_f = nG_m$. These are put into eq. 17 to find

$$f_f nL_m nw_m n \triangle Z_m = K_f n(d_m - Z_m - c_m)nG_m \triangle T_f;$$

which, when divided by eq. 16 and solved for $\triangle T_f$, yields

$$\triangle T_f = n \frac{f_f}{f_m} \frac{K_m}{K_f} \triangle T_m, \quad (21)$$

which is the needed relation.

Eq. 21 may be abbreviated to

$$\triangle T_f = A \triangle T_m, \qquad (22)$$

where A is given by

$$A = n(f_f/f_m)(K_m/K_f). \qquad (23)$$

The constant A in eq. 23 is a geometrical-physical constant, through the factors f, n, K_m and K_f (K_m and K_f are now not equal, as in the paragraph below eqs. 18). The constant A does not depend on the time because the porosities and conductivities have been taken to be constants with respect to both space and time.

Completion of Drawdown Derivations

With eqs. 21, 22 and 23, we can continue our derivation which must relate total times of fall, not just incremental times, to the total distances of fall in model and field. First, we notice that eq. 21 is valid for any pair of water table situations that are similar for model and field. In particular, eq. 21 is valid if the water table initially is an infinitesimal distance below the soil surface when the term c_f and the term c_m involved in the derivation enter. In this event, the increment $\triangle Z_f$ of eq. 20 will refer to a small drop in the water table starting from the just-unponded condition (midway between drains); we designate this first increment by the notation $\triangle Z_{f1}$. Corresponding to this $\triangle Z_{f1}$, we will have from eq. 22

$$\triangle t_{f1} = A \triangle t_{m1}, \qquad (24)$$

where T and t now both have the same significance.

Next let us compute the time increment $\triangle t_{f2}$ for a second increment of time. Imagine, at the instant the water has fallen the distance $\triangle Z_{f1}$, that the fictitious impermeable sheets coincident with ABCD and EFG are instantaneously removed so that the pressures in the flow mediums, model and field, will adjust themselves essentially instantaneously, see Muskat (1946), to a new steady-state condition that has a corresponding new set of streamlines. Now, instantaneously insert the sheets again along the new set of streamlines and then let the water table fall a distance $\triangle Z_{f2}$. Corresponding to $\triangle Z_{f2}$, we will have a time $\triangle t_{f2}$ given by

$$\triangle t_{f2} = A \triangle t_{m2}, \qquad (25)$$

where A, as is seen in eq. 23, remains the same constant as in eq. 24. Repeat the procedures of eqs. 24 and 25 for a number of increments $\triangle Z_{f3}, \triangle Z_{f4}, \ldots \triangle Z_{fL}$, where $\triangle Z_{fL}$ is the last increment, to find

$$\begin{aligned} \triangle t_{f1} &= A \triangle t_{m1} \\ \triangle t_{f2} &= A \triangle t_{m2} \\ &\ldots\ldots\ldots\ldots \\ \triangle t_{fL} &= A \triangle t_{mL}. \end{aligned} \qquad (26)$$

Now add all the $\triangle t_f$ and $\triangle Z_f$ to find, respectively,

$$\triangle t_{f1} + \triangle t_{f2} + \ldots + \triangle t_{fL} = t_f, \qquad (27)$$

$$\triangle Z_{f1} + \triangle Z_{f2} + \ldots + \triangle Z_{fL} = Z_f, \qquad (28)$$

where, in eq. 27, t_f is the total time lapsed in the field case for the water table to fall the distance Z_f of eq. 28.

Now put eqs. 26 in eq. 27 to find

$$A[\triangle t_{m1} + \triangle t_{m2} + \ldots + \triangle t_{mL}] = t_f \qquad (29)$$

But, if t_m is defined as the time for the surface of saturation in the model to fall from the just-unponded condition to the position Z_f of eq. 28, we have

$$\triangle t_{m1} + \triangle t_{m2} + \ldots + \triangle t_{mL} = t_m; \qquad (30)$$

and eqs. 29 and 30 yield

$$A t_m = t_f. \qquad (31)$$

From eqs. 23 and 31 we now find

$$t_f = n(f_f/f_m)(K_m/K_f)t_m. \qquad (32)$$

But, by expression 18, we have

$$n = Z_f/Z_m. \qquad (33)$$

Therefore, from eqs. 32 and 33 we find, finally,

$$t_f = \frac{Z_f}{Z_m} \frac{f_f}{f_m} \frac{K_m}{K_f} t_m, \qquad (34)$$

which is a final important result connecting model time t_m and field time t_f.

Three comments about eq. 34 are pertinent. First, rearranging eq. 34 we can write

$$\frac{t_f K_f}{f_f Z_f} = \frac{t_m K_m}{f_m Z_m} \text{ (subject to eqs. 18)}, \qquad (35)$$

which is dimensionless on each side and has on each side the same value for field and model. The equation shows why a number of results may profitably be expressed, as they have been, in terms of the expression tK/fZ, or as a function of it.

Second, in eqs. 34 and 35, it is basic to remember that Z_m and Z_f must be connected by the relations of eqs. 18. Third, if, in eqs. 24 through 35, we had started measuring times from the instant the surface of saturation was at any position JMNEAP in fig. 8 and had added up increments $\triangle Z$ referred to this same position JMNEAP, we would have found, instead of eq. 35, the result

$$\frac{T_f K_f}{f_f Z'_f} = \frac{T_m K_m}{f_m Z'_m} \text{ (subject to eqs. 18)}. \qquad (36)$$

where the primes on the Z's are to indicate that we now have distances that are referred to the mid-water-table height at T=0 (not t=0). Eq. 36 would be true for any shape of the surface of saturation at time T=0 — even shapes including mounds. Mounds might be due to ponded depressions in the soil surface through which water would seep after the soil surface was, in general, free of ponded water. Eq. 36 is valid only if eqs. 18 are valid.

Equations Connecting Model and Field Discharge Data

Eq. 35 connects distances Z_m, Z_f and times t_m and t_f of the fall of the surface of saturation for model and field. We can get a similar relation connecting discharge by proceeding somewhat as in the last section.

In fig. 8, imagine two streamlines such as NH and MI starting at the surface of saturation JMNEAP, respectively at $x = x_i$ and at $x = x_i + \triangle x_i$. Take the average depth of the starting points M and N of the two streamlines to be z_i. Assume that these two streamlines and other features about fig. 8 apply to a model situation. Imagine, as we did for the streamlines ABCD and EFG, that fictitious impervious sheets are coincident with the presently singled out streamlines; and let the volume of water *per unit time* passing between them be $\triangle Q_i$. Then, as for eq. 11, we may write (since we now assume, as we did before, that the tube is emptying a distance d below the soil surface) the expression,

$$\triangle Q_i = K(d - z_i - c)G_i. \qquad (37)$$

The total discharge per unit time is obtained by summing over the index i, in eq. 37, to cover all pairs of streamlines. Let there be $j - 1$ streamlines between $x = 0$ and $x = a$, the 0-th line being at $x = 0$ and the j-th at $x = a$; then, the total volume per unit time discharging into half a drain tube at $x = 0$ and into half a drain tube at $x = a$ is (with $i = 0, 1, 2, ..., j$),

$$Q = \sum_{i=1}^{j} \triangle Q_i = \sum_{i=1}^{j} K(d - z_i - c_i)G_i, \qquad (38)$$

where the subscript i on c may be dropped, if c, as assumed here, is constant. To emphasize that expression 38 applies to the model, write it in the form,

$$Q_m = \sum_{i=1}^{j} \triangle Q_{im} = \sum_{i=1}^{j} K_m(d_m - z_{im} - c_{im})G_{im}. \qquad (39)$$

For a field situation, the last expression would become

$$Q_f = \sum_{i=1}^{j} \triangle Q_{if} = \sum_{i=1}^{j} K_f(d_f - z_{if} - c_{if})G_{if}. \qquad (40)$$

Now we suppose that the dimensions and capillary fringe heights of the model and field are similar so that, from eqs. 18 and 19, we have

$$\frac{d_f}{d_m} = \frac{c_{if}}{c_{im}} = \frac{z_{if}}{z_{im}} = \frac{G_{if}}{G_{im}} = n. \qquad (41)$$

Next, using values of eqs. 41 in the last member of eqs. 40 we find

$$Q_f = \sum_{i=1}^{j} K_f(nd_m - nz_{im} - nc_{im})nG_{im}. \qquad (42)$$

In eq. 42, since n and K_f are constants, we can write

$$Q_f = K_f n^2 \sum_{i=1}^{j} (d_m - z_{im} - c_{im})G_{im}. \qquad (43)$$

Divide eq. 43 by the first and last members of eqs. 39, remembering in eqs. 39 that K_m is a constant. Find, after simplification, the basic result,

$$\frac{Q_f}{Q_m} = \frac{K_f}{K_m} n^2, \qquad (44)$$

where n is given by any and all expressions of eqs. 18.

Notice two points about eq. 44: (1) The drainable porosities of field and model, whatever they may be, do not mathematically enter, explicitly, in eq. 44 (but do enter physically and implicitly through the expression, $c_f = nc_m$, and through the values K_f and K_m). (2) Equation 44 says, when applied to two field situations (we could use subscripts f_1 and f_2 to denote them instead of f and m as in eq. 44), that the discharge rates in similar drainage systems—i.e., systems which satisfy eqs. 18—vary as the square of the ratio of corresponding dimensions.

Only a few more equations are necessary to complete the derivations.

In eq. 44 we find it convenient to take, from eqs. 18,

$$n^2 = \frac{L_f d_f}{L_m d_m};$$

so that eq. 44 becomes

$$\frac{Q_f}{Q_m} = \frac{K_f}{K_m} \frac{L_f d_f}{L_m d_m}.$$

Rearranging we find

$$\frac{Q_f}{K_f L_f d_f} = \frac{Q_m}{K_m L_m d_m} \quad \text{(subject to eqs. 18),} \qquad (45)$$

an expression in which the left-hand side and right-hand side are dimensionless.

The left-hand side of eq. 44 applies to a certain model time and, the right-hand side, to a certain field time. These times may be related because, if t_m is the known model time pertinent to the rate Q_m, then the corresponding field time t_f is given by eq. 32 in which n is given by one of eqs. 18. For example, if we take $n = a_f/a_m$, we see from eq. 32 that t_f, at the instant of validity of the left-hand side of eq. 45, is

$$t_f = \frac{a_f}{a_m} \frac{f_f}{f_m} \frac{K_m}{K_f} t_m, \qquad (46)$$

an expression which involves both the model and field porosities.

Eq. 46 may be written

$$\frac{t_f K_f}{a_f f_f} = \frac{t_m K_m}{a_m f_m}, \qquad (47)$$

each side of which is dimensionless. Eqs. 45 and 47 indicate that, in presenting model data, one may profitably plot $Q_m/K_m L_m d_m$ versus $t_m K_m/a_m f_m$, or functions of these quantities. Actually, because we did not know eq. 47 when the model data were collected, we plotted (figs. 6 and 7) the expression Q/KLd versus $(tK/a)^{1/2}$.

35

The latter factor should have had an f in the denominator.

It is common in model work to derive equations by using "dimensional analysis." We did not use dimensional analysis in the conventional sense (see Ligon, 1961) to derive our key eqs. 35, 36 and 45. In essence, we developed our key equations from "the equation of continuity" (see eq. 13) and Darcy's law (see eq. 14, and text following eqs. 18).

Our results are for a sharp surface of saturation and for a constant drainable porosity. In the field, the surface of saturation is not sharp, and some conductivity occurs above it. This conductivity is known to be small (Swartzendruber and Kirkham, 1956). For consideration of the variability in the drainable porosity, see Taylor (1960). For measuring K_f for eqs. 35, 36 and 45, see Kirkham (1946, 1955).

Solving Field Problems by Using Model Data

Time of drawdown problems

PROBLEM 1. Suppose we wish to know the time it will take for the water table midway between each pair of a series of equally spaced drains to drop 1 ft. from the just unponded condition when the drains are 100 ft. apart, 4 ft. deep, have diameter 0.5 ft., there is a barrier 12 ft. below the drain tube centers and when the soil properties are: capillary fringe height, 0.75 ft. (capillary fringe height is analytically also a geometrical property); drainable porosity, 5.4 percent; and hydraulic conductivity, 0.81 ft. per day.

Solution. Given data are: $Z_f = 1$ ft., $a_f = 100$ ft., $d_f = 4$ ft., $2r_f = 0.5$ ft., $h_f - d_f = 12$ ft. (so $h_f = 16$ ft.), $c_f = 0.75$ ft., $f_f = 0.054$ and $K_f = 0.81$ ft./day ($K_f/f_f = 15$). We want t_f.

To solve, we first see that Z_f, a_f, etc., and the other geometrical quantities obviously have been chosen to correspond to Case A of fig. 4, provided the scale is 1 ft. in the field for 1 cm. in the model. So we take this scale. In particular we note that the capillary fringe height $c_f = 0.75$ ft. has been chosen to correspond to the 0.75 cm. model value. The problem does not involve the length of the field drains; one must assume that the field drains are long compared with their spacing, the situation for which the model was designed.

So we use Case A of fig. 4, and, to deal with its "x axis," we have from our field data and eq. 35 the expression,

$$\frac{t_f(0.81 \text{ ft./day})}{(0.054)(1 \text{ ft.})} = \frac{t_m K_m}{f_m Z_m}. \quad (48)$$

To get the right-hand side of eq. 48, draw a horizontal line through the "y axis" point, $a = 100$ cm. on fig. 4, Case A, and, where this line intersects the curve $d = 4$ cm., read off on the "x axis," the value

$$(t_m K_m / f_m Z_m)^{1/2} = 7.0. \quad (49)$$

From eqs. 48 and 49, now find

$$\frac{t_f(0.81 \text{ ft./day})}{(0.054)(1 \text{ ft.})} = 7.0^2, \quad (50)$$

which, when solved for t_f, yields

$$t_f = 3.27 \text{ days. } (Answer \ Problem \ 1) \quad (51)$$

PROBLEM 2. The given data are as in Problem 1 except that we take K_f as 8.1 ft./day rather than 0.81 ft./day. We want t_f.

Solution. Since the geometry is that of Problem 1, we use the right-hand side of eq. 50, modifying only the left-hand side to find, in keeping with eq. 35, the result,

$$\frac{t_f(8.1 \text{ ft./day})}{(0.054)(1 \text{ ft.})} = 7.0^2, \quad (52)$$

which, when solved for t_f, yields
$$t_f = 0.327 \text{ day. } (Answer \ Problem \ 2)$$

PROBLEM 3. The given data are as in Problem 1 except that $f_f = 0.027$. We want t_f.

Solution. Since the geometry is as in Problem 1, we use the right-hand side of eq. 50, modifying only the left-hand side to find, in keeping with eq. 35, the result,

$$\frac{t_f(0.81 \text{ ft./day})}{(0.027)(1 \text{ ft.})} = 7.0^2,$$

which, when solved for t_f, yields (half the value of eq. 51)

$$t_f = 1.63 \text{ days. } (Answer \ Problem \ 3)$$

PROBLEM 4. The given data are as in Problem 1, except that $c_f = 0.375$ ft. rather than 0.75 ft. We want t_f.

Solution. Our model was not designed to give different capillary fringe heights. Therefore, we cannot give an exact model solution. Two approximate solutions are possible. The first is to say that, with the lessened capillary pull across the top of the flow medium, the time for the fall of the 1 ft. will be less than, but approximately equal to, the 3.27 days of Problem 1. The second approximate solution, but a more accurate one, is obtained as follows: First return to eqs. 11 and 13 to find from them

$$fLw_a \triangle Z = K(d - Z - c - \triangle Z/2)G_a \triangle T, \ (53)$$

where the subscript a has been added to w and to G to show that average values of w and G are required for exactness of eqs. 11 and 13 if finite space and time intervals $\triangle Z$ and $\triangle t$, as here, are used. Notice that the "$\triangle Z/2$" takes care of the average value of Z in eq. 53.

Next apply eq. 53 to the just-unponded condition of Problem 1 to find (changing only T to t in eq. 53) the result,

$$fLw_a \triangle Z = K(d - Z - c - \triangle Z/2)G_a \triangle t. \ (54)$$

Next apply eq. 54 to the data of Problem 1 to find

$$(0.054)Lw_a(1\text{ ft.}) =$$
$$(0.81\text{ ft./day})(4\text{ ft.} - 1\text{ ft.} - 0.75\text{ ft.} -$$
$$1\text{ ft.}/2)G_a \triangle t. \qquad (55)$$

Next apply eq. 54 to the data of Problem 4, to find

$$(0.054)Lw_a'(1\text{ ft.}) =$$
$$(0.81\text{ ft./day})(4\text{ ft.} - 1\text{ ft.} - 0.375\text{ ft.} -$$
$$1\text{ ft.}/2)G_a' \triangle t', \qquad (56)$$

where the primes show that w_a', G_a' and $\triangle t'$ are not, as is evident physically, the same as w_a, G_a and $\triangle t$ of eq. 55.

Next notice, by physical consideration of flownet changes which occur in the interval $\triangle t$ of eq. 55, as compared with the corresponding changes that occur for the interval $\triangle t'$ of eq. 56, that w_a and G_a will be approximately equal to w_a' and G_a', respectively, if the first parenthetical expression to the left of G_a in eq. 55 is approximately equal to the corresponding parenthetical expression in eq. 56. Now, since these parenthetical expressions are approximately equal, divide eq. 55 by eq. 56, taking $G_a = G_a'$ and $w_a = w_a'$. Then find, after simplification and solving for $\triangle t'$, the result,

$$\triangle t' = \frac{(4\text{ ft.} - 1\text{ ft.} - 0.75\text{ ft.} - 1\text{ ft.}/2)}{(4\text{ ft.} - 1\text{ ft.} - 0.375\text{ ft.} - 1\text{ ft.}/2)} \triangle t; \quad (57)$$

That is, find

$$\triangle t' = [(1.75\text{ ft.})/(2.125\text{ ft.})]\triangle t = 0.823 \triangle t. \quad (58)$$

But the $\triangle t$ of eqs. 58 is just equal to $t_f = 3.27$ days of Problem 1 (eq. 51); and the $\triangle t'$ of eq. 57 is just equal to t_f of our present Problem 4. So find, finally, from eqs. 57 and the noted values

$$t_f = 0.823 \times 3.27\text{ days} = 2.69\text{ days.}$$
(Answer Problem 4)

Comment 1: Comparing the answer of Problem 1 with the answer of Problem 4, we see that the change in the capillary fringe height from 0.75 ft. (9 in.) to 0.375 ft. (4.5 in.) caused the time needed for the surface of saturation to fall 1 ft. to change from 3.27 days to 2.69 days.

Comment 2: We have given considerable detail in solving Problem 4 since the problem shows that capillary fringe effects on drawdown times (a) may be important and (b) may need to be taken into account for a variety of fringe heights, even though our model data are based on a single capillary fringe height, 0.75 cm.

Comment on problems 1-4: If, in problems 1-4, we had $d_f = 5$ ft. instead of 4 ft., we should then have worked from an interpolated line, $d = 5$ cm., on fig. 4, Case A, instead of from the line $d = 4$ cm. For other depths, the procedure would be similar.

PROBLEM 5. Suppose all field data are as for Problem 1 except that the barrier is 0.50 ft. below the center of the drain tube; i.e., $h_f - d_f = 0.5$ ft. What is t_f then?

Solution. We proceed as for Problem 1 except that we use Case B of fig. 4. From it we find (for a $= 100$ cm. and d $= 4$ cm.) the value,

$$(t_m K_m/f_m Z_m)^{\frac{1}{2}} = 13,$$

so that eq. 48 becomes

$$\frac{t_f(0.81\text{ ft./day})}{(0.054)(1\text{ ft.})} = 13^2,$$

from which we find

$$t_f = (169/15)\text{ days} = 11.27\text{ days.} \qquad (59)$$
(Answer Problem 5)

Comment: The result of eq. 59 shows, when compared with that of eq. 51, that "raising" the barrier from 12 ft. below the drain centers to 0.5 ft. below their centers increases the drawdown time for the 1 ft. from 3.27 days to 11.27 days.

PROBLEM 6. Suppose the field data are as in Problem 1 except that we have $h_f - d_f = 6$ ft. (and except, since $d_f = 4$ ft., we have $h_f = 10$ ft.). What will t_f then be?

Solution. We do not have model data for $h_m - d_m = 6$ cm., when we take, as we have taken so far, 1 cm. of model to correspond to 1 ft. in the field. But 1 cm. of model does not have to be 1 ft. in the field. We can use instead (see eqs. 18) the relations

$$\frac{h_f - d_f}{h_m - d_m} = \frac{6\text{ ft.}}{12\text{ cm.}} = \frac{1\text{ ft.}}{2\text{ cm.}} = n. \qquad (60)$$

Then, the geometrical field values, namely, $Z_f = 1$ ft., $a_f = 100$ ft., $d_f = 4$ ft., $2r_f = 0.5$ ft., $h_f - d_f = 6$ ft. (with deduced value $h_f = 10$ ft.) and $c_f = 0.75$ ft., needed on the model would be:

$$\left.\begin{array}{l} Z_m = 2\text{ cm.}, a_m = 200\text{ cm.}, d_m = 8\text{ cm.},\\ 2r_m = 1\text{ cm.}, h_m - d_m = 12\text{ cm.},\\ h_m = 20\text{ cm.}, c_m = 0.375\text{ cm.} \end{array}\right\} \quad (61)$$

The closest model case we have, corresponding to eqs. 61, is that of part A of fig. C-3 (when d $= 8$ cm., and a $= 200$ cm.) for which we have

$$\left.\begin{array}{l} Z_m = 2\text{ cm.}, a_m = 200\text{ cm.}, d_m = 8\text{ cm.},\\ 2r_m = 0.5\text{ cm.}, h_m - d_m = 12\text{ cm.},\\ h_m = 20\text{ cm.}, c_m = 0.75\text{ cm.} \end{array}\right\} \quad (62)$$

So, keeping in mind the discrepancies in $2r_m$ and c_m in eqs. 61 and 62, let us use fig. C-3, part A, and its associated conditions, eqs. 62; and then see if the result we obtain can be modified to fit eqs. 61.

Drawing a horizontal line through the "y axis" of fig. C-3, part A, at a $= 200$ cm. and reading, on the "x axis," the value corresponding to the intersection of the horizontal line with the curve d $= 8$ cm., we obtain

$$(t_m K_m/f_m Z_m)^{\frac{1}{2}} = 8.5; \qquad (63)$$

that is, we have

$$t_m K_m/f_m Z_m = 72.25;$$

or, using subscripts 2 to denote conditions of eqs. 62, we have

$$t_2K_2/f_2Z_2 = 72.25, \quad (2r_2 = 0.5 \text{ cm.,} \quad (64)$$
$$c_2 = 0.75 \text{ cm.}).$$

Now we ask: How can eq. 64 be modified for the conditions of eqs. 61?

To get the answer, we call upon a result of Kirkham (1949, p. 376, eq. 16) which shows that the rate of intake Q' of water per unit area into the soil, at any distance x from the drain and so at $x = a/2$ as for our present case, in a just-ponded condition, is directly proportional to a quantity Q (not the Q of our eq. 38). This Q is the outflow rate per unit length of drain tube, and is given by Kirkham (1949, p. 374, eq. 11) as

$$Q = 2\pi K(t + d - r)/D, \quad (65)$$

in which (with $m = 1,2,...,\infty$) D is defined by

$$D = \ln \frac{\tan[\pi(2d - r)/4h]}{\tan(\pi r/4h)}$$

$$+ \sum_{m=1}^{\infty} \ln \frac{\cosh \dfrac{m\pi a}{2h} + \cos \dfrac{\pi r}{2h}}{\cosh \dfrac{m\pi a}{2h} - \cos \dfrac{\pi r}{2h}}$$

$$\frac{\cosh \dfrac{m\pi a}{2h} - \cos \dfrac{\pi(2d - r)}{2h}}{\cosh \dfrac{m\pi a}{2h} + \cos \dfrac{\pi(2d - r)}{2h}}. \quad (66)$$

In eqs. 65 and 66, we remark that Kirkham's notation is the same as ours except that he uses t for thickness of ponded surface water, whereas we use t for time; he also uses Q for discharge volume per unit time per unit length of drain tube, whereas we use Q/L. Kirkham's t must, for our case, be replaced by —c, since we are concerned with water that has just become *unponded*. Also, Kirkham's r in the expression for head difference, "t + d — r," must be replaced by zero, since Kirkham's drains discharged at the level $y = r$, and ours discharge at $y = 0$.

So eq. 65 in our notation should read (Q now being as in eq. 38, etc.)

$$Q/L = 2\pi K(d - c)/D, \quad (67)$$

where

$$D = \text{right-hand side of eq. 66.} \quad (68)$$

In eq. 67 notice that $2\pi K/D$ is the hydraulic conductance of the flow medium and that d—c is the driving head.

For eq. 67 to apply, we have seen that there could exist no ponded surface fluid, so that Q/L now becomes proportional to the amount of fluid passing unit area per unit time through the soil, as the surface of saturation falls. But this amount of fluid, for small Z and t is equal to (Z/t)f. So, in view of our statement preceding eq. 65 we can say, for geometries as indicated by fig. 8 (but with Z much smaller than in fig. 8) that (Z/t)f is, if L is constant, given by

$$(Z/t)f = B \, 2\pi K(d - c)/D,$$

where B is a constant of proportionality. Or, we can say, for two models, 1 and 2, in which L is equal and Z is small, that we have

$$\frac{t_1K_1/f_1Z_1}{t_2K_2/f_2Z_2} = \frac{D_1(d_2 - c_2)}{D_2(d_1 - c_1)}. \quad (69)$$

Now, consistent with our use of subscripts in eq. 64, we may write

$$\text{subscripts 1 apply to eqs. 61} \quad (70)$$

$$\text{subscripts 2 apply to eqs. 62} \quad (71)$$

From statements 70 and 71 and eqs. 64 and 69, we now have

$$\frac{t_1K_1/f_1Z_1}{72.25} = \frac{D_1(8 - 0.75)}{D_2(8 - 0.375)}, \quad (72)$$

in which D_1 and D_2 need to be determined.

For D_1 we find, in view of eq. 66 and statement 70

$$D_1 = \ln \frac{\tan[\pi(16 - 0.5)/80]}{\tan(\pi \, 0.5/80)} + \text{sums,} \quad (73)$$

where the "sums" involve $a_1 = 200$ cm., and are here negligible.

So performing the operations in eq. 73 and ignoring the sums we find

$$D_1 = 3.56. \quad (74)$$

Likewise, we find from eq. 66 and statement 71

$$D_2 = \ln \frac{\tan[\pi(16 - 0.25)/80]}{\tan(\pi \, 0.25/80)} = 4.28 \quad (75)$$

Putting the right-hand side of eqs. 74 and 75 in eq. 72 we find

$$\frac{t_1K_1}{f_1Z_1} = 72.25 \, \frac{3.56}{4.28} \, \frac{7.25}{7.625} =$$

$$(72.25)(0.832)(0.951) = 57.1 \quad (76)$$

which is the sought-for result for the conditions of eq. 61. Eq. 76 shows, in keeping with our physical intuition, that: (a) for the larger drain tube, the drawdown time is reduced (here by the factor, 3.56/4.28 = 0.832, see eq. 74 and 75); and (b) for the smaller capillary pull, the drawdown time is also reduced (here by the factor, 7.25/7.625 = 0.951, see eq. 72). Our result (b), that the smaller the capillary fringe the faster is the drawdown, has been observed in an electric analogue model by Childs (1947).

The value of $t_1 K_1 / f_1 Z_1$ given by eq. 76, is not as accurate a value as we can obtain because our correction factors 3.56/4.28 and 7.25/7.625 in eq. 76 were based on the conditions for $Z = 0$ (just unponded fluid); whereas our time t_1 of eq. 76 depends on the continuously changing conditions of the flow medium as it changes from the condition when $Z = 0$ to the condition when $Z = 2$ cm. To get better correction factors, we refer to table A-1, subtable 27c, which applies to fig. C-3. In subtable 27c, in the fifth and second columns and for $t = 58$ min., we see that the surface of saturation falls from $Z = 0$ to $Z = 2.3$ cm. — say 2 cm. — midway between drains; while over the drain $(x = 0)$, we see that the surface of saturation drops from $z = 0$ to $z = 6.1$ cm. — say 6.0 cm. Thus, this subtable shows that a space and time average value of d (for the equivalent rectangular flow medium of Kirkham) to use in our correction factors D_1 and D_2 in the right-hand side of eq. 72 should be equal to $[8 + (6 + 2)/2]/2 = 6$ cm. rather than the 8 cm. we used.

So the right-hand side of eq. 73 would be replaced by (neglecting the sums)

$$\ln \frac{\tan \pi (12 - 0.5)/80}{\tan \pi \, 0.5/80} = 3.21 \qquad (77)$$

and the "ln" expression in eq. 75 would be replaced by

$$\ln \frac{\tan \pi (12 - 0.25)/80}{\tan \pi \, 0.25/80} = 3.93. \qquad (78)$$

It would seem now offhand that the ratio $(d_2 - c_2)/(d_1 - c_1)$ for our improved correction factor, right-hand side of eq. 69, should be $(6 - 0.75)/(6 - 0.375)$. But this is not true because $(d_2 - c_2)/(d_1 - c_1)$ should apply to the central flow line of our interest, ABCD of fig. 8—not to an average (with space and time) flow line starting somewhere between A and J of fig. 8. So we consider the average head across the flow line ABCD which, for our case of 2 cm. fall, is $(8 + 6)/2 = 7$ cm., if we ignore capillary pull for the moment. Taking the capillary pull into account, our head correction ratio becomes

$$(7 - 0.75)/(7 - 0.375) = 6.25/6.675; \qquad (79)$$

and instead of eq. 76 we now have, using, in eq. 72, the right-hand sides of eqs. 77, 78 and 79, the result

$$\frac{t_1 K_1}{f_1 Z_1} = 72.25 \, \frac{3.21}{3.93} \, \frac{6.25}{6.675} =$$

$$(72.25)(0.817)(0.937) = 55.3. \qquad (80)$$

We may now quickly obtain the answer to Problem 6. Since eq. 80 applies to a model with the conditions of eq. 61, we may write, in keeping with our notation in problems 1-5, the equations,

$$\frac{t_m K_m}{f_m Z_m} = 55.3 = \frac{t_f K_f}{f_f Z_f}, \qquad (81)$$

where the right-hand side applies to the field conditions given above eqs. 61 and where K_f and f_f are as given in Problem 1. So we have

$$\frac{t_f(0.81 \text{ ft./day})}{(0.054)(1 \text{ ft.})} = 55.3.$$

That is, we have $t_f = (55.3)(0.15)$, or

$$t_f = 8.30 \text{ days. } (Answer \text{ } Problem \text{ } 6) \qquad (82)$$

Barrier effect problem (Problem 7)

PROBLEM 7. Noting that the answers t_f, to problems 1, 5 and 6 are all for $Z_f = 1$ ft., $a_f = 100$ ft., $d_f = 4$ ft., $2r_f = 0.5$ ft., $c_f = 0.75$ ft., $f_f = 0.054$, and $K_f = 0.81$ ft./day, but for $h_f - d_f$ are, respectively, 12 ft., 0.5 ft. and 6.0 ft., determine the "barrier effect," that is, obtain a plot of t_f versus $h_f - d_f$.

Solution. We first prepare the following schedule:

Prob. No.	Eq. No.	$h_f - d_f$	t_f
5	59	0.5 ft.	11.27 days
6	82	6.0 ft.	8.30 days
1	51	12.0 ft.	3.27 days

and then make the curve of fig. 9 which is the *Answer, Problem 7*.
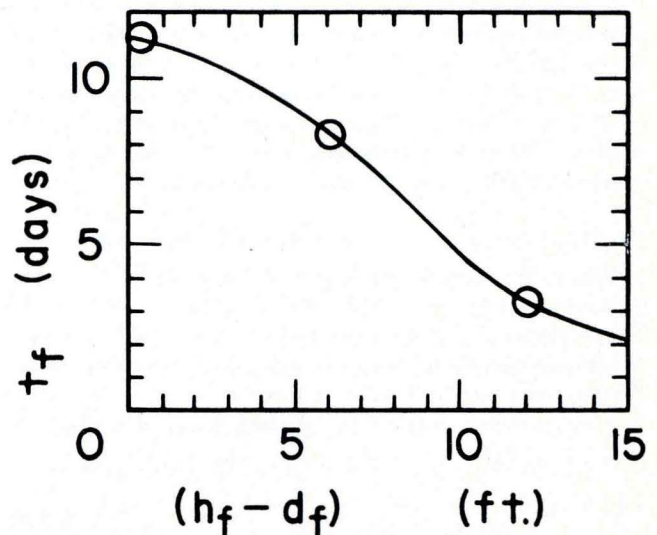
FIG. 9. Time t_f in the field for the water table to fall 1 ft. from the just-ponded condition for various depths $(h_f - d_f)$ of the barrier layer below the tile centers when $a_f = 100$ ft., $d_f = 4$ ft., $2r_f = 0.5$ ft., $f_f = 0.054$ and $K_f = 0.81$ ft./day. Theoretically, t_f should become constant as $h_f - d_f$ becomes large.

Drain tube discharge problem (Problem 8)

PROBLEM 8. Suppose we wish to know the cubic ft. per sec. of water discharging from each drain tube of Problem 1 per 100 ft. of drain tube length at the instant when Z_f is 1 ft. Our problem thus is: Find Q_f when $Z_f = 1$ ft. and $L_f = 100$ ft.

Solution. We observe that fig. 6, Case A, $a = 100$ cm., applies; and that, to obtain values for its "x axis,"

we should use eq. 47. Therefore, we put our data from Problem 1 into eq. 47 to find

$$\frac{(3.27 \text{ days})(0.81 \text{ ft./day})}{(100 \text{ ft.})(0.054)} = 0.490 = \frac{t_m K_m}{a_m f_m}$$

which gives, since for our model we have $f_m = 0.4$, the result,

$$t_m K_m / a_m = (0.4)(0.490) = 0.1960,$$

or

$$(t_m K_m / a_m)^{1/2} = 0.443.$$

We now spot this "x axis" value 0.443 on fig. 6, Case A, for the spacing $a = 100$ cm., and read off for $d = d_m = 4$ cm., the "y axis" value

$$Q/KLd = Q_m / K_m L_m d_m = 0.38. \qquad (83)$$

Next, looking at eqs. 45 and 83, we write down

$$Q_f / K_f L_f d_f = 0.38,$$

which yields, with our given field data,

$$Q_f = (0.38)(0.81 \text{ ft./day})(100 \text{ ft.})(4 \text{ ft.})$$
$$= 123 \text{ ft.}^3/\text{day}. \ (Answer \ Problem \ 8.)$$

This is a small rate, but it is the value of Q as found a fairly long time, 3.27 days, after surface water disappeared, and it is a value for the low conductivity $K_f = 0.81$ ft./day. The largest value of Q would be for $t = 0$; but we cannot extrapolate this largest value from figs. 6 and 7. The maximum value of Q can be obtained by using the ponded water drainage formulas given in Kirkham's article in Luthin (1957).

Problem Solution by an Alternate Method

The problem examples have all been for Case A or B of table 1. Principles, however, have been brought out for solving field problems for the other cases. We conclude our problem examples by solving Problem 1 by an alternate method using eq. 4.

First put the data of Problem 1 in eq. 4 to find

$$\log_{10}(tK/fZ) = -0.487 + 1.606 \log_{10}(100/4);$$

which yields

$$(tK/fZ)^{1/2} = 7.52. \qquad (84)$$

Now replace 7.0 in eqs. 49 and 50 by the 7.52 of eq. 84 to find upon solving the resulting eq. 50 for the time, now denoted by $t_f{}'$ rather than t_f, the result

$$t_f{}' = 3.77 \text{ days.}$$

Here $t_f{}'$ is larger than $t_f = 3.27$ days of eq. 51, because in eq. 4 all the model values for a/d for Case A, table 1, were combined into a single equation. The latter value, 3.77 days, is more straightforward to calculate but is not as accurate as the former value, 3.27 days, because the 3.77 value resulted from combining data not accurately belonging to the same statistical population, seen before eq. 2.

Test of Model Data With Glover's Theory; Other Theories and Data

To keep this report within bounds, we test our data against only one theory, namely, Glover's "first equation," as reported by van Schilfgaarde, et al. (1956, esp. eq. 17). The Glover equation for the drain spacing a is

$$a = \left[\frac{k(h - d + 0.5d)t}{f \ln(4/\pi)d/(d - Z)} \right]^{1/2}, \qquad (85)$$

which may be written in the form,

$$a = At^{1/2}, \qquad (86)$$

where A is the coefficient of $t^{1/2}$ in the right-hand side of eq. 85. The test consists of comparing slopes A of experimentally obtained graphs (fig. 10), values, 40, 32 and 26, with the corresponding slopes, 36.2, 32.8 and 28.4 (fig. 10, the inset table), computed from eq. 85. The agreement is considered good in view of Glover's limitations which are: Equation 85 should have $h - d = d << a$ (we have, for fig. 10, $h - d = 12$ cm. and $a = 50$ to 400 cm.) and should have $d << h - d$ (we have for fig. 10, $d = 4, 6$ and 8 cm. and have always $h - d = 12$ cm.); and should have no capillary fringe (we have 0.75 cm. fringe) and, finally, should have the time period t sufficiently large that the second and subsequent terms can be dropped from an infinite series which occurs in the derivation of eq. 86 (the times we used made the second term of the series about 12 percent of the first). Notice that eq. 86 is not of the form $a/d = At^{1/2}$, a form which would be correct only if the coefficient of $\log_{10}(a/d)$ in eq. 4 were $1/0.500 = 2.000$, instead of 1.606, as there. But eq. 5, Case B (but not the other cases) gives the form $a/d = At^{1/2}$, very nearly, since the coefficient 1.986 in eq. 5 is very nearly 2.000.

The verification of eq. 86 indicates that (under the conditions of fig. 10) drain spacing increases as the square root of time. Thus, if a crop, such as potatoes (see Luthin 1957, p. 540), needs 1 ft. of drawdown in 36 hrs., and alfalfa needs 1 ft. of drawdown in 72 hrs., then the drain spacing for the alfalfa could be $(72/36)^{1/2} = 1.41$ times as great (not twice as great) as that for potatoes. Little is known, however, about drawdown times needed by different crops.

Equation 86 may be called a "drawdown law." The law appears to apply well to most of the cases of fig. 4 and of figs. C-1, C-2 and C-3. But the fact that a square root of time law appears valid for most of the cases does not mean that the coefficient A of eq. 86 can be computed from eq. 85 in all cases.

The "drawdown law" may be interpreted a different way. Sometimes a field may be tiled at large spacing, with the thought that if drainage is not adequate, tiles may be placed midway beween the original ones. The law indicates that such halving of the spacing would
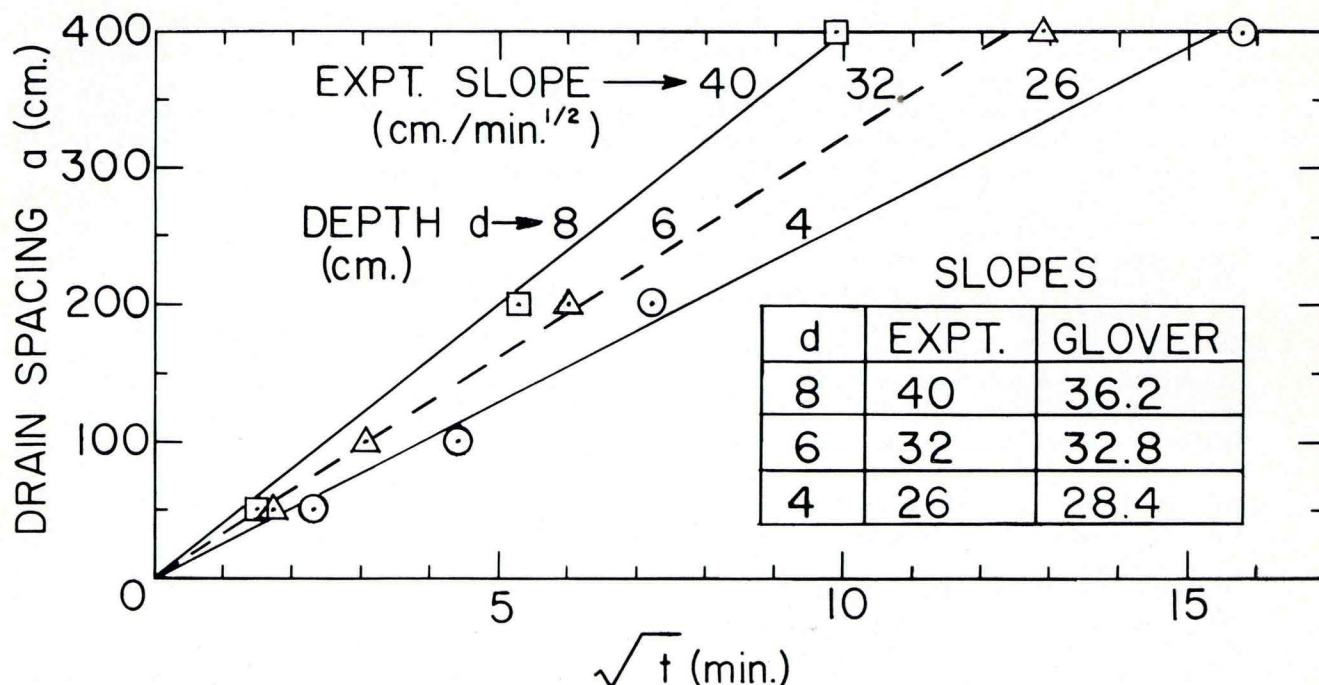
FIG. 10. Comparison of experimental slopes of curves of model data with slopes computed by Glover's theory (see van Schilfgaarde et al., 1956). The data points are for the three tile depths, d = 4, 6 and 8 cm., of case A of table B-1 where Z = 1 cm. The depth of the tile barrier below the tile centers is 12 cm.

cut the time period for a certain drawdown to one-fourth the time for this same drawdown under the conditions of the original tiling.

Some other theories and other data, which in future work may be compared and analyzed with those of this report, may be mentioned as follows: Childs (1947); Kirkham and Gaskell (1951); Wesseling (1956); Visser (see Luthin, 1957, pp. 96-98); Isherwood (1959); Breitenöder and Zanker (1960) [who used a "Hele-Shaw model," one in which a viscous fluid flows, under gravity, between plane sheets a small distance apart—a description of a recently made Hele-Shaw model is given by van Wijk (1960)]; Brut-

saert, Taylor and Luthin (1961); Brooks (1961); and Visser (1962). One finds that the data in these references are generally for narrower spacings of drains than we used and, thus, are not usually applicable in practice, except where close spacings (less than 50 ft. —our data go to 400 ft.) drains are used. The most extensive data cited are those of Breitenöder and Zanker (1960) who consider drain spacings up to 15 meters. The data in the cited literature are mainly for the water table height midway between drains and seldom cover conditions over the whole water table arch (as does our table A-1); layered soils are not considered.

SUMMARY

Over 800 "water tables," in a glassbead-glycerol model, have been photographed. Data from them have been tabulated and, in part, graphed to show how water tables fall with time for a large number (109) of different geometries in simulated tile-drained soil. The "soil" is either homogeneous or stratified. The tabulated basic data are depths to water table versus time for positions (a) midway between pairs of drain tile, (b) above the tiles and (c) at two intermediate points, so that the whole water table arch is defined. The tile spacings are 50, 100, 200 and 400 cm.; depths below the tile axes to an impermeable barrier are 12 cm. and 0.5 cm. One cm. of model may be conveniently taken as 1 foot in the field, but other scales are

shown to be suitable. For about half of the 109 geometries, tile discharge rates were measured as they varied with time of water table fall; these discharge rates are presented graphically. The water table and discharge data are for spacings up to 400 ft.; previous model data usually have been for spacings less than 50 ft.

Convenient, dimensionless equations are derived which relate the model data to field conditions. Eight examples of field problems are worked out in numerical detail by use of the derived equations. The tables, graphs and derived equations aid in understanding and in solving practical drainage problems; furthermore the tables and graphs furnish a source of

experimental data for checking theories of falling water tables yet to be discovered. One "defect" which could not be avoided in the data is the approximately 0.75 cm. high "capillary" fringe (apparently resulting more from viscosity than capillarity) of the model. This 0.75 cm. fringe corresponds to about 0.75 ft. of capillary fringe in the field for one modeling scale noted, and about 0.375 ft. for another. Corrections for other capillary fringe heights may be made. The existence of a capillary fringe increases drawdown time. A square-root-of-time "drawdown law" is indicated by the data for many tile drainage situations.

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Table A-1. Value of z (cm.) versus x (cf. part A of fig. 1) for x = 0, 0.1a, 0.2a and 0.5a, for the various clock times t (min.) of the photographed falling surfaces of saturation for the conditions listed in table 1.

24a; d=2cm., a=50cm.

t (min.)	0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	0.8	0.4	0.3	0.0
4	1.1	0.7	0.3	0.1
6	1.2	0.8	0.4	0.2
10	1.3	0.9	0.7	0.4
12	1.4	0.9	0.7	0.5
14	1.4	0.9	0.7	0.5
16	1.4	1.0	0.7	0.6
18	1.4	1.0	0.8	0.6

24b; d=2cm., a=100cm.

t	0	0.1a	0.2a	0.5a
2	0.3	0.1	0.0	0.0
4	0.5	0.3	0.1	0.0
6	0.6	0.4	0.1	0.0
10	0.8	0.4	0.2	0.1
14	0.8	0.5	0.3	0.2
18	0.8	0.6	0.4	0.3
26	0.8	0.7	0.6	0.3
42	0.9	0.9	0.9	0.7

24c; d=2cm., a=200cm.

t	0	0.1a	0.2a	0.5a
2	0.6	0.2	0.0	0 0
6	0.8	0.2	0.1	0.0
10	0.9	0.3	0.2	0.1
18	0.9	0.5	0.3	0.2
42	0.9	0.6	0.5	0.3
58	0.9	0.8	0.5	0.5
90	0.9	1.0	0.8	0.6
120	0.9	1.1	0.9	0.8

24d; d=2cm., a=400cm.

t	0	0.1a	0.2a	0.5a
4	0.7	0.0	0.0	0.0
10	0.7	0.0	0.0	0.0
18	0.8	0.1	0.0	0.0
26	0.9	0.2	0.0	0.0
42	0.9	0.2	0.1	0.1
58	1.0	0.3	0.1	0.1
91	1.0	0.5	0.2	0.1
120	1.0	0.6	0.3	0.2

25a; d=4, a=50

t	0	0.1a	0.2a	0.5a
2	1.9	1.1	0.5	0.2
4	2.0	1.6	1.2	0.7
6	2.1	1.9	1.5	1.1
8	2.4	2.0	1.8	1.5
10	2.4	2.0	1.9	1.7
12	2.4	2.1	2.0	1.8
14	2.5	2.3	2.1	2.0
16	2.5	2.4	2.2	2.0

25b; d=4, a=100

t	0	0.1a	0.2a	0.5a
2	1.6	0.4	0.1	0.0
4	2.0	0.9	0.4	0.0
6	2.2	1.3	0.7	0.1
10	2.2	1.6	1.0	0.3
14	2.3	1.7	1.4	0.6
18	2.4	1.9	1.5	0.9
42	2.7	2.3	2.0	1.9
58	2.8	2.5	2.2	2.0

25c; d=4, a=200

t	0	0.1a	0.2a	0.5a
2	1.9	0.2	0	0.0
6	2.4	0.6	0.1	0.0
12	2.6	1.3	0.3	0.0
18	2.8	1.5	0.7	0.2
43	2.8	2.0	1.5	0.8
58	2.8	2.1	1.8	1.1
90	2.9	2.3	2.0	1.6
120	2.9	2.4	2.2	1.9

25d; d=4, a=400

t	0	0.1a	0.2a	0.5a
4	2.3	0.1	0.0	0.0
10	2.8	0.3	0.0	0.0
18	2.9	0.7	0.1	0.0
26	3.0	1.1	0.2	0.1
42	3.1	1.4	0.4	0.1
58	3.3	1.6	0.7	0.2
120	3.4	2.0	1.3	0.5
720	3.4	2.5	2.2	2.2

26a; d=6, a=50

t	0	0.1a	0.2a	0.5a
2	2.2	1.8	1.2	0.6
4	3.1	2.2	1.9	1.3
6	3.7	3.0	2.1	1.9
8	3.8	3.2	2.7	2.0
10	4.0	3.4	3.0	2.6
12	4.0	3.7	3.2	3.0
14	4.0	3.8	3.7	3.3
16	4.1	4.0	3.9	3.3

26b; d=6, a=100

t	0	0.1a	0.2a	0.5a
2	2.1	1.0	0.3	0 0
4	2.9	1.8	0.9	0.2
6	3.4	2.0	1.4	0.4
10	3.9	2.3	1.9	1.1
14	4.0	2.9	2.1	1.6
18	4.1	3.2	2.5	2.0
26	4.2	3.9	3.0	2.4
34	4.3	3.9	3.7	3.0

26c; d=6, a=200

t	0	0.1a	0.2a	0.5a
6	3.2	1.2	0.3	0.0
10	3.7	1.8	0.6	0.1
18	4.0	2.2	1.4	0.2
26	4.0	2.5	1.9	0.5
42	4.2	3.3	2.1	1.5
58	4.2	3.4	2.7	1.8
90	4.3	3.8	3.4	2.7

26d; d=6, a=400

t	0	0.1a	0.2a	0.5a
4	3.7	0.1	0.0	0.0
10	4.0	0.8	0.0	0.0
18	4.6	1.5	0.1	0 0
42	4.6	2.1	0.9	0.0
58	4.9	2.5	1.4	0.1
90	4.8	2.9	2.0	0.3
240	5.0	3.7	3.0	1.8

27a; d=8, a=50

t	0	0.1a	0.2a	0.5a
2	2.2	2.0	1.5	0.9
4	3.7	2.9	2.3	1.8
6	4.5	3.9	3.1	2.3
8	4.9	4.2	3.7	3.1
10	5.3	4.8	4.1	3.8
12	5.8	5.0	4.4	4.0

27b; d=8, a=100

t	0	0.1a	0.2a	0.5a
18	5.2	4.1	3.5	2.5
22	5.8	4.4	3.9	3.0
26	5.8	4.7	4.0	3.4
34	6.0	5.1	4.7	4.0

27c; d=8, a=200

t	0	0.1a	0.2a	0.5a
2	2.2	0.6	0.1	0.0
6	4.2	1.8	0.5	0 0
10	4.9	2.3	1.2	0.1
18	5.7	3.2	1.9	0.4
26	5.8	3.7	2.3	0.9
42	6.0	4.2	3.1	1.8
58	6.1	4.6	3.8	2.3
90	6.1	5.2	4.4	3.6

27d; d=8, a=400

t	0	0.1a	0.2a	0.5a
2	2.7	0.0	0.0	0.0
6	5.3	0.6	0 0	0.0
10	5.8	1.1	0.0	0.0
18	6.1	2.0	0.3	0 0
26	6.4	2.3	0.8	0.1
42	6.4	3.1	1.5	0.2
58	6.5	3.6	2.4	0.3
100	6.5	4.1	3.2	1.0
180	6.5	4.3	3.8	2.1
300	6.5	5.2	4.4	3.4

28a; d=2, a=50

t	0	0.1a	0.2a	0.5a
2	0.8	0.1	0.0	0.0
6	1.0	0.5	0.2	0.1
10	1.0	0.8	0.4	0.1
14	1.2	0.8	0.5	0.2
18	1.3	0 8	0 6	0.3
22	1.3	0.9	0.6	0.3
26	1.4	1.0	0.8	0.4
30	1.4	1.0	0.8	0.5

28b; d=2, a=100

t	0	0.1a	0.2a	0.5a
2	0.7	0.0	0 0	0.0
6	0.9	0.1	0 0	0.0
10	1.0	0.2	0.0	0.0
18	1.0	0.6	0.1	0.0
26	1.0	0.6	0.2	0 0
42	1.0	0.8	0.4	0.2
58	1.0	0.9	0.6	0.2

29a; d=4, a=50

t	0	0.1a	0.2a	0.5a
2	1.6	0.7	0.2	0.0
4	2.1	1.1	0.7	0.2
6	2.3	1.4	0.9	0.3
10	2.5	1.9	1.1	0.8
14	2.7	2.0	1.4	1.0
18	2.9	2.4	1.6	1.1
26	3.0	2.5	2.0	1.4
34	3.0	2.7	2.2	1.7

29b; d=4, a=100

t	0	0.1a	0.2a	0.5a
2	1.5	0.2	0.0	0.0
6	2.8	0.9	0.1	0.0
10	2.9	1.1	0.4	0.1
18	3.0	1.6	0.8	0.2
42	3.2	2.3	1.3	0.8
58	3.2	2.3	1.6	1.1
90	3.2	2.5	2.1	1.4
120	3.2	2.7	2.3	1.8

29c; d=4, a=200

t	0	0.1a	0.2a	0.5a
4	1.7	0.1	0.0	0.0
10	2.2	0.5	0 0	0.0
26	2.5	1.0	0.3	0.1
42	2.7	1.1	0.7	0.2
58	2.8	1.2	0.9	0.3
120	2.8	2.1	1.1	1.0
180	2.8	2.2	1.4	1.1
280	2.9	2.3	1.6	1.2

29d; d=4, a=400

t	0	0.1a	0.2a	0.5a
26	3.6	0.2	0.0	0.0
60	3.6	0.9	0.1	0.1
120	3.6	1.1	0.6	0.1
180	3.9	1.2	0.9	0.2
240	3.9	1.3	1.0	0.3
780	3.9	1.9	1.3	1.1

30a; d=6, a=50

t	0	0.1a	0.2a	0.5a
2	2.3	1.3	0.7	0.2
6	3.5	2.3	1.8	0.8
10	3.9	2.9	2.3	1.2
14	4.3	3.1	2.7	1.5
18	4.6	3.2	2.9	2.1
22	4.7	3.3	3.0	2.7
26	4.7	3.6	3.1	3.0
30	4.9	3.7	3.2	3.0

30b; d=6, a=100

t	0	0.1a	0.2a	0.5a
3	2.4	0.8	0.3	0.0
6	3.6	1.4	0.8	0.1
10	4.2	2.0	1.0	0.2
18	4.4	2.7	1.5	0.6
26	4.7	2.7	2.0	0.9
34	4.8	2.9	2.3	1.0
42	4.9	3.0	2.5	1.2
58	4.9	3.3	2.8	1.7

30c; d=6, a=200

t	0	0.1a	0.2a	0.5a
4	2.6	0.3	0 1	0.0
10	3.6	0.9	0.2	0.0
18	4.0	1.4	0.4	0 1
26	4.1	1.8	0 8	0.1
42	4.4	2.1	1.1	0 3
58	4.7	2.6	1.3	0.6
120	4.7	3.0	2.2	1.1
180	4.8	3.1	2.5	1.6

30d; d=6, a=400

t	0	0.1a	0.2a	0.5a
2	3.0	0.0	0.0	0.0
6	4.6	0.1	0.0	0.0
14	5.0	0.3	0.0	0.0
18	5 0	0.5	0 0	0.0
30	5.0	0.8	0.1	0.0
58	5 0	1.2	0.3	0 0
180	5.1	2.2	1.0	0.2
420	5.1	2.9	1.8	0.9

31a; d=8, a=50

t	0	0.1a	0.2a	0.5a
4	4.2	2.6	1.7	0.7
6	4.8	3.0	2.2	1.0
10	5.6	4.0	2.9	1.9
15	5.9	4.5	3.7	3.2
18	6.1	4.8	3.9	3.3
26	6.8	5.2	4.7	4.4

31b; d=8, a=100

t	0	0.1a	0.2a	0.5a
6	4.7	2.0	1.0	0.2
10	5.2	2.8	1.3	0.5
15	5.7	3.1	1.9	0 9
19	6.0	3.4	2.3	1.0
26	6.3	3.9	2.8	1.3
42	6.7	4.6	3.3	2.5
58	6.7	4.9	3.9	3.9
90	6.8	5.2	4.9	3.9

31c; d=8, a=200

t	0	0.1a	0.2a	0.5a
3	3.0	0.4	0.1	0.0
14	5.3	1.8	0.6	0.1
26	5.9	2.8	1.1	0.2
42	6.2	3.0	1.8	0.6
58	6.5	3.4	2.0	0.8
90	6.6	3.7	2.7	1.1
180	6.7	4.6	3.2	2.8
240	6.8	5.0	4.0	3.4

31d; d=8, a=400

t	0	0.1a	0.2a	0.5a
2	3.3	0.0	0.0	0.0
6	6 2	0.1	0 0	0.0
14	7.0	0.5	0.0	0.0
30	7 0	1.2	0.2	0.0
58	7.0	2.1	0.8	0.1
90	7.1	2.3	1.0	0.1
300	7.2	3.6	2.9	0.9
600	7.2	4.6	3.4	2.3

32a; d=2, a=50

t	0	0.1a	0.2a	0.5a
1	0.4	0.0	0.0	0.0
2	0.8	0.1	0.1	0.0
4	1.2	0.3	0.2	0.1
6	1.3	0.5	0.5	0 3
8	1.4	0.7	0.6	0.5
10	1.5	0.8	0.9	0.7
12	1.5	0.9	1.0	0.7
14	1.6	1.1	1.1	0.9

32b; d=2, a=100

t	0	0.1a	0.2a	0.5a
2	0.5	0 0	0.0	0.0
4	0.8	0.1	0.1	0.0
6	0.8	0.2	0.1	0.1
8	0.9	0.3	0.2	0.2
12	1.0	0.5	0.3	0.2
16	1.1	0.5	0.3	0.3
20	1.1	0.7	0.4	0.5
30	1.1	0.9	0.7	0.7

32c; d=2, a=200

t	0	0.1a	0.2a	0.5a
2	0.6	0 0	0 0	0.0
4	0.7	0.0	0.0	0 0
6	0.7	0.1	0.0	0.1
8	0.7	0.2	0.1	0.1
10	0.8	0.2	0.1	0.1
20	0.9	0.4	0.3	0.1
30	1.0	0.5	0.4	0.3
59	1.1	0.7	0.6	0.6

32d; d=2, a=400

t	0	0.1a	0.2a	0.5a
1	0.4	0.2	0.0	0.0
4	0.6	0.2	0.0	0.1
5	0.9	0.2	0.1	0.1

Table A-1. Continued.

33a; d=4, a=50 — 33b; d=4, a=100 — 33c; d=4, a=200 — 33d; d=4, a=400

t (min)	0 (cm)	0.1a (cm)	0.2a (cm)	0.5a (cm)	t (min)	0 (cm)	0.1a (cm)	0.2a (cm)	0.5a (cm)	t (min)	0 (cm)	0.1a (cm)	0.2a (cm)	0.5a (cm)	t (min)	0 (cm)	0.1a (cm)	0.2a (cm)	0.5a (cm)
1	0.7	0.4	0.0	0.1	1	0.6	0.0	0.0	0.0	2	1.5	0.3	0.0	0.0	2	2.0	0.0	0.0	0.0
2	1.6	0.8	0.3	0.2	2	1.3	0.1	0.0	0.0	4	2.0	0.4	0.1	0.0	6	2.2	0.1	0.0	0.0
4	2.2	1.4	0.9	0.7	4	2.0	0.5	0.2	0.1	6	2.2	0.6	0.2	0.1	10	2.5	0.2	0.0	0.0
6	2.5	1.9	1.3	1.3	6	2.2	0.7	0.3	0.2	10	2.3	1.2	0.4	0.2					
8	2.5	2.0	1.8	1.7	10	2.3	1.4	0.8	0.5	19	2.7	1.7	1.0	0.5	18	3.1	0.5	0.1	0.0
10	2.7	2.2	2.0	1.9	14	2.5	1.8	1.3	1.0	30	2.8	1.9	1.5	1.0					
12	2.8	2.4	2.1	2.0	20	2.6	2.0	1.8	1.7	46	3.0	2.0	1.9	1.6	86	3.2	0.8	0.2	0.0
14	2.8	2.4	2.3	2.2	30	2.7	2.2	2.0	2.0	62	3.1	2.3	2.0	1.9	106	3.4	1.9	1.9	1.2

34a; d=6, a=50 — 34b; d=6, a=100 — 34c; d=6, a=200 — 34d; d=6, a=400

t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a
1	1.2	0.8	0.4	0.2	1	1.3	0.3	0.2	0.2	2	2.0	0.2	0.1	0.0	1	1.8	0.1	0.0	0.0
2	2.1	1.4	0.8	0.6	2	2.0	0.8	0.3	0.3	6	3.4	0.9	0.3	0.1	4	3.5	0.2	0 1	0.0
4	3.2	2.3	1.9	1.5	4	2.8	1.7	1.0	0.6	10	3.9	1.6	0.8	0.3	10	4.1	0.9	0.2	0.1
6	3.9	2.6	2.4	2.0	6	3.4	1.9	1.5	0.9	18	4.1	2.2	1.7	1.1	18	4.3	1.8	0.6	0.1
10	4.2	3.6	3.3	2.9	10	3.7	2.3	2.0	1.5	26	4.3	2.6	2.0	1.5	26	4.4	1.9	1.0	0.2
14	4.5	4.0	3.9	3.8	14	4.0	2.7	2.3	2.0	34	4.4	3.0	2.2	1.9	34	4.9	2.1	1.5	0.3
18	4.7	4.1	4.0	4.0	18	4.2	3.2	2.6	2.3	42	4.5	3.4	2.5	2.2	42	4.9	2.3	1.7	0.5
26	4.8	4.5	4.4	4.3	26	4.3	3.7	3.3	3.2	58	4.6	3.8	3.2	2.8	58	4.9	2.8	2.0	1.0

35a; d=8, a=50 — 35b; d=8, a=100 — 35c; d=8, a=200 — 35d; d=8, a=400

t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a
3	3.4	2.5	2.0	1.5	1	1.1	0.6	0.1	0.1	2	2.2	0.6	0.1	0.0	4	3.5	0.2	0.1	0.0
4	4.0	3.1	2.4	2.0	4	3.6	1.9	1.4	0.6						10	5.1	1.2	0.3	0.1
6	5.0	4.1	3.3	2.5	6	4.2	3.0	1.9	1.0	6	4.2	1.9	0.7	0.1	22	6.0	2.3	1.1	0.3
10	6.1	5.1	4.5	4.1	10	5.3	3.7	2.7	2.0	10	5.2	2.4	1.3	0.4	26	6.0	2.5	1.4	0.4
14	6.3	5.8	5.4	5.1	14	5.9	4.4	3.6	2.7	18	6.0	3.5	2.2	1.3	42	6.6	3.6	2.1	1.0
18	6.7	6.1	5.9	5.8	18	6.0	4.8	4.0	3.4	26	6.2	4.1	3.1	2.0	58	6.6	4.0	2.7	1.5
					22	6.2	5.0	4.5	4.0	42	6.6	4.7	4.1	2.9	74	6.7	4.2	3.2	1.9
					26	6.3	5.3	5.0	4.2	58	6.7	5.3	4.7	4.0	90	6.7	4.6	3.8	2.2

36a; d=2, a=50 — 36b; d=2, a=100 — 36c; d=2, a=200 — 36d; d=2, a=400

t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a
2					2	0.2	0.1	0.1	0.1	2	0.0	0.0	0.0	0.0	4	1.0	0.1	0.0	0.0
4	0.4	0.2	0.2	0.1	4	0.4	0.2	0.2	0.1	6	0.5	0.2	0.1	0.0	10	1.4	0.1	0.1	0.1
6	0.6	0.4	0.4	0.1	6	0.5	0.2	0.3	0.2	10	0.7	0.3	0.2	0.1	18	1.6	0.2	0.2	0.2
8	0.6	0.5	0.4	0 3	10	0.6	0.4	0.4	0.3	14	0.8	0.4	0.4	0.2	26	1.5	0.3	0.4	0.4
10	0.6	0.6	0.5	0.5	14	0.7	0.4	0.4	0.3	18	0.9	0.6	0.4	0.3	42	1.6	0.4	0.5	0.4
12	0.7	0.7	0.6	0.6	18	0.7	0.4	0.4	0.4	26	0.9	0.8	0.5	0.4	58	1.6	0.5	0.6	0.6
18	0.7	0.7	0.7	0.7	22	0.8	0.6	0.5	0.5	43	1.0	1.2	0.8	0.9	90	1.6	0.8	0.9	0.9
					26	0.8	0.6	0.5	0.5						122	1.6	0.9	1.1	1.2

37a; d=4, a=50 — 37b; d=4, a=100 — 37c; d=4, a=200 — 37d; d=4, a=400

t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a
2	1.5	0.8	0.4	0.2	2	1.5	0.2	0.1	0.0	3	1.8	0.1	0.1	0.0	4	2.3	0.1	0.0	0.0
6	2.2	1.9	1.5	1.3	4	2.0	0.7	0.4	0.2	4	2.0	0.2	0.1	0.1	10	2.7	0.6	0.2	0.1
8	2.3	2.1	1.9	1.8	6	2.1	1.1	0.7	0.4	10	2.3	1.0	0.5	0.1	19	2.7	1.0	0.6	0.1
10	2.4	2.1	2.0	2.0	10	2.2	1.5	1.2	0.9	14	2.3	1.3	0.8	0.3	26	2.7	1.4	0.8	0.2
12	2.6	2.2	2.1	2.0	14	2.2	1.9	1.6	1.3	18	2.4	1.6	1.1	0.3	42	2.8	1.7	1.2	0.6
14	2.6	2.2	2.1	2.1	18	2.3	2.0	1.9	1.6	26	2.4	1.8	1.4	0.8	58	2.8	1.9	1.4	0 8
16	2.7	2.4	2.3	2.3	22	2.4	2.0	2.0	1.9	42	2.5	2.1	2.0	1.5	90	2.9	2.0	1.9	1.7
					26	2.4	2.1	2.0	2.0	58	2.8	2.2	2.0	1.9	122	3.0	2.2	2.0	2.0

38a; d=6, a=50 — 38b; d=6, a=100 — 38c; d=6, a=200 — 38d; d=6, a=400

t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a
2	1.5	1.2	0.8	0.5	2	1.2	0.4	0.2	0.2	2	1.8	0.4	0.2	0.0	4	3.0	0.4	0.2	0.1
4½	2.6	2.1	1.8	1.5	6	2.6	1.7	1.2	0.9	6	2.8	1.2	0.5	0.2	10	4.1	1.2	0.5	0.1
6	3.0	2.4	2.0	1.9	10	3.2	2.0	1.9	1.5	10	3.4	1.8	1.0	0 4	18½	4.4	1.9	1.0	0.3
8	3.4	2.7	2.4	2.1	14	3.6	2.4	2.2	1.8	14½	3.9	2.1	1.4	0.8	26	4.8	2.0	1.4	0.4
10	3.8	3.1	2.8	2.6	18	3.9	2.9	2.6	2.3	18	3.9	2.2	1.7	1.1	42	5.1	2.6	2.0	1.0
12	4.0	3.4	3.1	2.9	22	4.0	3.2	2.9	2.6	26	4.0	2.5	2.1	1.6	58	5.1	3.1	2.2	1.9
14	4.1	3.6	3.2	3.1	26	4.2	3.5	3.4	3.0	42	4.1	3.3	2.6	2.2	90	5.1	3.5	2.9	2.2
16	4.2	3.9	3.6	3.5	30	4.2	3.7	3.7	3.3	58	4.3	3.4	3.3	2.7	122	4.8	3.9	3.4	2.9

39a; d=8, a=50 — 39b; d=8, a=100 — 39c; d=8, a=200 — 39d; d=8, a=400

t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a
2	2.2	1.8	1.3	1.1	2	1.8	0.9	0.4	0.2	2	2.1	0.5	0.2	0.1	4	3.2	0.5	0.1	0.1
4	3.5	2.9	2.2	2.1	4	2.7	1.8	1.0	0.6	6	3.6	1.6	0.9	0.3	10	4.9	1.5	0.6	0.3
6	4.5	3.9	3.2	2.8	6	3.4	2.1	1.8	1.1	10	4.3	2.3	1.5	0.7	18	5.4	2.0	1.0	0.4
8	5.0	4.4	4.0	3.7	10	4.3	3.0	2.4	2.0	18	5.0	3.0	2.1	1.8	26	6.0	2.4	1.8	0.8
10	5.5	4.7	4.2	3.7	14	4.9	3.8	3.3	2.8	26	5.4	3.8	2.8	2.0	42	6.1	3.0	2.3	1.4
12	5.6	5.3	4.8	4.5	18	5.3	4.1	3.9	3.4	34	5.9	4.1	3.5	2.7	58	6.3	3.6	2.6	1.9
14	6.0	5.6	5.0	4.9	22	5.7	4.4	4.2	4.0	42	5.9	4.4	3.9	3.3	74	6.7	4.0	3.3	2.2
16	6.1	5.8	5.2	5.2	26	6.0	4.8	4.6	4.4						90	6.8	4.3	3.8	2.8

40a; d=2, a=50 — 40b; d=2, a=100 — 40c; d=2, a=200 — 40d; d=2, a=400

t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a
2	0.7	0.4	0.2	0.2	2	0.7	0.2	0.2	0.2	2	1.1	0.2	0.1	0.0	2	0.8	0.2	0.1	0.0
4	0.9	0.6	0.5	0.4	4	0.9	0.6	0.4	0.2	4	1.3	0.4	0.2	0.1	6	1.4	0.3	0.1	0.1
6	1.1	0.8	0.8	0.6	6	1.1	0.6	0.5	0.3	6	1.5	0.4	0.2	0.1	10	1.5	0.3	0.1	0.1
8	1.1	0.8	0.9	0.8	8	1.3	0.8	0.6	0.4	10	1.5	0.6	0.3	0.2	14	1.6	0.4	0.2	0.2
10	1.2	0.9	0.9	0.9	10	1.3	0.9	0.7	0.5	14	1.5	0.6	0.4	0.4	18	1.6	0.5	0.2	0 2
12	1.2	1.0	1.0	0.9	14	1.4	0.9	0.9	0.7	18	1.5	0.8	0.5	0.4	26½	1.6	0.7	0.3	0.2
14	1.2	1.4	1.1	1.0	18	1.4	1.0	0.9	0.9	26	1.5	0.9	0.7	0.5	42	1.7	0.7	0.7	0.5
16	1.5	1.4	1.5	1.4	22	1.5	1.1	1.0	1.0	42	1.5	1.1	1.9	0.8	58	1.7	0.8	0.9	0.8

41a; d=4 a=50 — 41b; d=4, a=100 — 41c; d=4, a=200 — 41d; d=4, a=400

t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a
2	1.6	1.2	0.8	0.7	2	1.4	0.5	0.2	0.2	2	1.8	0.2	0.1	0.0	2	1.9	0.1	0.0	0.0
4	2.1	1.8	1.5	1.4	4	2.0	1.2	0.7	0.4	4	2.2	0.8	0.2	0.1	6	2.4	0.3	0.1	0.0
5	2.2	2.0	1.8	1.6	6	2.1	1.5	1.1	0.7	6	2.8	1.0	0.6	0.1	10	2.7	1.1	0.2	0.0
6	2.3	2.1	2.0	1.9	8	2.1	1.7	1.4	1.1	10	3.1	1.6	1.1	0.4	18	3.0	1.3	0.4	0.1
8	2.4	2.2	2.1	2.0	10	2.2	1.7	1.7	1.4	14	3.3	1.9	1.6	0.8	26	3.1	1.4	1.0	0.2
10	2.5	2.4	2.2	2.2	14	2.3	2.0	1.9	1.8	18	3.4	2.1	1.8	1.2					
12	2.7	2.5	2.4	2.4	18	2.4	2.2	2.1	2.0	26	3.4	2.4	2.0	1.7	42	3.1	1.9	1.5	0.5
14	2.8	2.6	2.6	2.5	22	2.4	2.3	2.2	2.1	42	3.4	2.8	2.4	2.2	59	3.1	2.0	1.6	0.7

42a; d=6 a=50 — 42b; d=6, a=100 — 42c; d=6, a=200 — 42d; d=6, a=400

t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a	t	0	0.1a	0.2a	0.5a
1	1.2	0.9	0.6	0.5	2	1.8	0.9	0.5	0.3	2	2.0	0.5	0.2	0.1	2	2.1	0.3	0.0	0.0
2	2.1	1.8	1.4	1.0	6	3.3	2.1	1.8	1.6	4	2.9	1.1	0.5	0.1	10	4.0	1.4	0.6	0.1
4	3.4	2.5	2.1	2.0	10	3.9	2.8	2.4	2.0	6	3.4	1.7	0.9	0.2	18	4.2	1.9	0.9	0.2
6	3.9	3.3	3.0	2.6	14	4.1	3.3	3.0	2.6	10	3.9	2.1	1.6	0.7	26	4.6	2.2	1.2	0.2
8	4.2	3.9	3.7	3.4	18	4.2	3.7	3.5	3.3	14	4.0	2.5	2.0	1.2	40	4.9	2.8	2.1	0.7
10	4.3	4.0	3.9	3.8	22	4.3	3.9	3.8	3.7	18	4.1	2.8	2.1	1.5	58	4.9	3.2	2.5	1.4
12	4.5	4.2	4.0	4.0	26	4.4	4.1	4.1	4.0	26	4.2	3.3	2.7	2.0	90	5.1	3.8	2.9	2.0
14	4.6	4.3	4.2	4.1	30	4.5	4.3	4.2	4.0	42	4.4	4.1	3.7	3.1	120	5.2	4.3	4.2	4.0

Table A-1. Continued.

43a; d=8, a=50

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	1.4	1.1	0.8	0.5
2	2.3	1.9	1.7	1.5
3	3.3	2.7	2.2	2.0
4	4.1	3.5	3.0	2.6
6	5.0	4.3	4.0	3.8
8	5.9	4.9	4.5	4.3
10	5.9	5.3	5.0	4.9
12	6.2	5.8	5.5	5.2

43b; d=8, a=100

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	2.0	1.3	1.0	0.5
4	3.1	2.1	1.7	1.0
6	4.0	3.0	2.1	2.0
8	4.3	3.1	2.7	2.1
10	4.8	3.8	3.2	2.9
14	5.6	4.3	4.0	3.6
18	5.8	4.7	4.4	4.1
22	6.0	5.1	5.0	4.8

43c; d=8, a=200

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	2.2	0.8	0.4	0.2
4	3.3	1.6	0.9	0.3
6	3.9	2.0	1.2	0.6
10	4.6	2.6	1.6	1.1
14	5.3	3.1	2.2	1.6
18	5.6	3.7	2.7	1.9
26	5.8	4.1	3.4	2.7
42	6.2	5.1	4.5	4.0

43d; d=8, a=400

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2¼	2.3	0.4	0.1	0.0
6	4.3	1.2	0.3	0.1
10	5.0	1.9	0.7	0.1
18	5.9	2.7	1.4	0.3
26	6.0	3.3	1.9	0.5
42	6.2	4.2	2.6	1.1
58	6.4	4.3	3.4	2.0
90	6.8	5.0	4.0	2.9

44a; d=2, a=50

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	0.7	0.1	0.1	0.0
6	0.8	0.4	0.3	0.1
8	0.9	0.5	0.3	0.2
10	0.9	0.5	0.3	0.2
12	1.0	0.6	0.4	0.3
14	1.0	0.7	0.4	0.3
18	1.1	0.7	0.5	0.4
20	1.0	0.7	0.5	0.4

44b; d=2, a=100

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	0.5	0.1	0.0	0.0
6	0.7	0.3	0.1	0.1
14	0.8	0.4	0.2	0.2
18	0.8	0.5	0.3	0.2
26	0.8	0.5	0.4	0.2
34	0.8	0.6	0.5	0.3
42	0.8	0.6	0.6	0.4

44c; d=2, a=200

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
6	0.8	0.2	0.1	0.0
14	1.0	0.4	0.1	0.1
26	1.1	0.5	0.3	0.1
42	1.2	0.8	0.3	0.4
58	1.3	0.9	0.4	0.3
90	1.3	1.0	0.4	0.6
120	1.4	1.2	0.6	0.7
180	1.4	1.1	0.8	0.9

44d; d=2, a=400

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
4	0.7	0.0	0.0	0.0
10	0.8	0.0	0.0	0.0
26	1.2	0.1	0.0	0.0
42	1.5	0.1	0.0	0.0
58	1.5	0.1	0.1	0.0
130	1.5	0.2	0.2	0.1
240	1.5	0.3	0.3	0.1
360	1.5	0.3	0.4	0.3

45a; d=4, a=50

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	1.4	0.8	0.4	0.1
6	2.1	1.6	1.2	0.8
8	2.3	1.8	1.4	1.0
10	2.3	1.9	1.6	1.4
12	2.4	2.1	1.7	1.5
14	2.4	2.1	1.7	1.7
16	2.5	2.1	1.8	1.9

45b; d=4, a=100

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	1.5	0.3	0.1	0.1
6	2.1	1.1	0.4	0.3
10	2.1	1.5	0.7	0.5
18	2.2	1.8	1.5	1.1
26	2.5	2.0	1.7	1.4
34	2.5	2.1	1.9	1.6
42	2.6	2.1	2.0	2.0

45c; d=4, a=200

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
4	2.1	0.2	0.0	0.0
10	2.5	0.7	0.2	0.1
18	2.7	1.0	0.4	0.1
26	2.7	1.4	0.6	0.3
42	2.7	1.6	0.9	0.5
58	2.8	2.2	1.7	0.8
60	2.8	2.2	1.9	1.8
120	3.0	2.4	2.2	2.0

45d; d=4, a=400

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
6	3.0	0.1	0.0	0.0
12	3.6	0.2	0.1	0.0
18	3.7	0.7	0.2	0.0
26	3.7	0.8	0.2	0.0
42	3.7	0.9	0.3	0.1
58	3.7	1.4	0.4	0.2
90	3.7	1.6	0.9	0.2
396	3.7	1.8	1.6	1.1

46a; d=6, a=50

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	2.5	1.8	1.0	0.3
4	3.9	2.3	2.0	1.3
6	4.1	2.8	2.2	1.8
8	4.3	3.2	2.8	2.1
10	4.3	3.5	2.9	2.4
12	4.3	3.7	3.3	2.8
14	4.4	3.9	3.6	3.2
16	4.5	3.9	3.8	3.4

46b; d=6, a=100

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	1.9	0.9	0.2	0.1
4	2.9	1.8	0.7	0.2
6	3.3	1.8	1.0	0.2
10	3.8	2.3	1.7	0.5
14	4.1	2.7	2.0	1.0
18	4.1	3.0	2.2	1.4
26	4.2	3.5	2.8	2.2
34	4.3	3.6	3.2	2.6
58	4.5	4.0	3.8	3.5

46c; d=6, a=200

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	2.0	0.2	0.0	0.0
6	3.6	0.5	0.2	0.0
10	3.9	1.6	0.4	0.1
18	4.0	2.1	1.1	0.2
26	4.0	2.6	1.6	0.5
42	4.3	2.9	1.9	0.9
58	4.4	3.1	2.1	1.8
90	4.4	3.6	2.8	2.1

46d; d=6, a=400

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
4	3.8	0.2	0.0	0.0
10	4.2	0.5	0.0	0.0
26	4.5	1.9	0.2	0.0
58	4.5	2.0	1.0	0.1
120	4.5	2.7	1.9	0.3
180	4.5	3.0	2.0	1.2
420	4.6	3.6	3.1	2.5

47a; d=8, a=50

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	3.1	2.1	1.6	0.9
3	4.3	2.8	2.0	1.4
4	4.9	3.4	2.4	1.9
6	5.5	4.0	3.4	2.4
7	5.7	4.2	3.5	2.8
8	5.8	4.5	3.9	3.2
9	6.0	4.8	4.1	3.6
10	6.0	5.0	4.2	3.8
11	6.0	5.0	4.3	3.7
12	6.0	5.3	4.6	4.0

47b; d=8, a=100

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	2.9	1.5	0.5	0.1
4	4.1	2.0	1.1	0.2
6	4.7	2.7	1.7	0.4
9	5.2	3.4	2.2	1.0
16	6.0	4.0	3.3	2.0
25	6.1	4.8	3.9	3.2
30	6.2	5.0	4.2	3.9
36	6.2	5.4	4.6	4.0
55	6.5	5.9	5.5	5.3

47c; d=8, a=200

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
4	4.1	1.4	0.4	0.1
18	5.9	3.4	1.9	0.4
26	6.1	3.8	2.1	0.9
42	6.3	4.2	2.8	1.9
49	6.3	4.3	3.3	2.0
64	6.4	4.7	3.8	2.6
81	6.5	5.0	4.0	3.1
100	6.5	5.4	4.5	3.8

47d; d=8, a=400

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
4	6.0	0.3	0.0	0.0
16	6.6	1.8	0.2	0.0
25	6.6	2.1	0.6	0.1
36	6.6	2.7	1.0	0.2
64	6.6	3.4	1.9	0.3
100	7.0	3.9	2.1	0.6
144	7.0	4.1	2.9	1.6
196	7.0	4.6	3.4	2.5

48a; d=4, a=50

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	1.9	0.8	0.3	0.0
4	3.3	1.9	1.2	0.5
6	3.4	2.0	1.5	1.0
8	3.4	2.0	1.8	1.3
10	3.4	2.1	1.9	1.4
12	3.5	2.2	2.0	1.7
16	3.5	2.4	2.2	2.0
20	3.5	2.5	2.3	2.0

48b; d=4, a=100

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	1.2	0.4	0.0	0.0
4	2.4	1.2	0.2	0.0
9	2.5	1.5	0.5	0.1
16	2.8	2.1	1.1	0.3
25	2.9	2.3	1.8	0.9
36	2.9	2.3	2.0	1.2
49	3.0	2.5	2.1	1.6
64	3.0	2.6	2.3	1.8

48c; d=4, a=200

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	1.0	0.0	0.0	0.0
4	2.2	0.3	0.0	0.0
9	2.9	0.7	0.1	0.0
16	2.9	1.3	0.3	0.1
25	2.9	1.5	0.7	0.2
36	2.9	1.8	0.9	0.2
49	3.0	1.9	1.3	0.3
64	3.0	2.0	1.5	0.6
100	3.0	2.1	1.8	1.2
144	3.0	2.2	2.0	1.6

48d; d=4, a=400

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	2.1	0.0	0.0	0.0
4	2.7	0.1	0.0	0.0
9	3.0	0.4	0.0	0.0
16	3.1	0.7	0.1	0.0
25	3.2	0.9	0.2	0.1
36	3.1	1.8	0.4	0.1
49	3.2	1.9	0.8	0.1
64	3.2	2.0	1.1	0.2
121	3.2	2.0	1.2	0.3
257	3.2	2.0	1.5	0.9
480	3.2	2.0	1.9	1.4

49a; d=6, a=50

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	1.7	1.4	0.9	0.3
2	2.2	1.8	1.0	0.3
4	3.4	2.3	1.8	0.9
6	3.7	2.9	2.1	1.5
9	4.0	3.3	2.6	2.0
12	4.1	3.6	2.8	2.4
16	4.3	3.7	3.4	3.0
20	4.4	4.1	3.8	3.2

49b; d=6, a=100

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	1.7	0.3	0.1	0.0
4	3.1	1.6	0.7	0.1
9	3.8	2.2	1.5	0.4
16	4.1	2.9	2.0	1.1
25	4.2	3.5	2.5	2.0
36	4.4	3.8	3.3	2.5
49	4.6	4.0	3.7	3.3
64	4.7	4.2	4.0	3.4

49c; d=6, a=200

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	1.9	0.1	0.0	0.0
4	3.5	0.7	0.1	0.0
9	4.3	1.5	0.3	0.1
16	4.4	2.0	0.8	0.1
36	4.7	2.5	1.7	0.6
64	4.9	3.2	2.1	1.1
100	4.9	3.6	2.7	2.0
144	5.0	3.9	3.3	2.5

49d; d=6, a=400

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	2.0	0.0	0.0	0.0
4	4.0	0.1	0.0	0.0
9	4.8	0.5	0.0	0.0
26	4.9	1.3	0.1	0.0
49	5.0	1.8	0.6	0.1
121	5.0	2.5	1.7	0.3
196	5.1	2.9	1.8	1.0

50a; d=8, a=50

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	2.9	2.1	1.4	0.8
4	4.3	3.2	2.4	1.4
6	5.3	4.0	3.2	2.1
8	5.7	4.4	3.7	2.9
10	6.0	4.8	4.1	3.6
12	6.3	5.2	4.6	4.0
14	6.3	5.3	4.7	4.0
16	6.3	5.6	5.0	4.5
18	6.3	5.8	5.2	4.8

50b; d=8, a=100

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	1.8	0.9	0.2	0.0
4	4.0	2.2	1.2	0.1
9	5.4	3.5	2.2	0.6
16	5.9	4.1	3.2	1.8
26	6.1	4.7	4.0	2.6
36	6.4	5.3	4.5	3.7
49	6.5	5.8	5.2	4.3
64	6.5	5.9	5.5	4.9

50c; d=8, a=200

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	1.6	0.2	0.0	0.0
4	4.4	1.2	0.1	0.0
9	5.5	2.0	0.1	0.0
16	5.9	2.9	0.7	0.1
26	6.2	3.7	1.4	0.1
36	6.3	3.9	2.0	0.5
49	6.3	4.1	2.4	1.0

50d; d=8, a=400

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	2.0	0.0	0.0	0.0
4	5.4	0.1	0.0	0.0
9	6.0	0.6	0.0	0.0
16	6.3	1.6	0.1	0.0
36	7.1	2.3	0.8	0.1
64	7.2	3.0	1.6	0.1
100	7.3	3.9	1.9	0.4
225	7.4	4.3	3.4	1.9
441	7.3	5.2	4.4	3.8

51a; d=19, a=100

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
4	4.9	3.8	2.2	0.8
6	6.4	4.9	3.3	1.6
10	9.0	6.6	4.8	2.7
14	10.5	8.4	6.0	3.9
18	12.0	9.3	7.0	4.9
26	14.1	10.9	8.8	6.8
42	16.0	13.1	11.3	9.5
58	16.8	14.1	12.6	11.4

51b; d=19, a=200

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
2	2.2	0.9	0.1	0.0
6	5.7	2.6	0.8	0.1
10	7.2	3.5	1.7	0.1
14	9.5	4.7	2.0	0.3
18	10.8	5.7	2.8	0.8
26	12.3	7.7	4.0	1.6
42	14.2	9.0	5.4	2.9
58	15.2	9.7	7.0	4.0
90	16.0	11.1	8.6	6.3
120	16.7	12.1	10.0	7.9
180	17.0	13.6	11.8	10.0

51c; K₁/K₂=0.4, d=19, a=200

t (min.)	x, 0 (cm.)	0.1a (cm.)	0.2a (cm.)	0.5a (cm.)
1	2.7	1.7	0.9	0.3
2	4.9	2.7	1.6	0.5
3	6.2	3.5	2.0	0.9
4	7.3	4.1	2.2	1.1
6	9.1	6.3	3.1	1.6
10	11.0	7.7	4.8	2.4
14	12.0	9.2	6.2	3.8
18	13.2	10.1	7.2	5.0
26	14.4	11.7	9.7	7.3

*The subtable Nos. 24a, 24b, 24c, 24d, 25a, 25b, . . ., etc., correspond to "Grover fig. Nos." 24, 25, etc. of table 1, where the a, b, c and d added on 24, 25 . . . are for drain spacings, 50, 100, 200 and 400 cm.

APPENDIX B

Table B-1. Time in minutes for the surface of saturation to fall a distance Z (see fig. 1) when Z = 0.5, 1.0, 1.5 and 2.0 cm., for various depths d and spacings a and geometry as otherwise described in table 1.

fig. No.[a]	Case	Depth d (cm.)	Z = 0.5 cm. Spacing a (cm.) Minutes				Z = 1.0 cm. Spacing a (cm.) Minutes				Z = 1.5 cm. Spacing a (cm.) Minutes				Z = 2.0 cm. Spacing a (cm.) Minutes			
			50	100	200	400	50	100	200	400	50	100	200	400	50	100	200	400
24	A	2	13.0	28.9	60.0													
25	A	4	3.2	12.7	30.8	120.0	5.4	19.5	52.0	250.0	8.0	28.7	81.8	420.0	15.0	45.0	133.0	632.0
26	A	6	1.8	6.1	25.9	107.0	3.0	9.3	36.0	167.0	4.8	13.3	48.9	232.0	7.1	18.0	65.0	300.0
27	A	8	...	...	18.5	64.0	2.2	...	28.2	97.0	3.3	10.2	39.0	130.0	4.5	13.8	50.2	177.0
28	B	2	30.0															
29	B	4	7.8	29.0	70.0	350.0	15.4	56.0	170.0	770.0	26.6	93.0	...	...	...	...	...	...
30	B	6	4.3	15.8	54.3	290.0	7.8	31.5	108.0	454.0	11.8	54.0	187.0	...	16.2	...	...	...
31	B	8	3.0	10.0	42.1	190.0	5.4	19.0	75.0	350.0	7.8	27.4	106.0	498.0	10.3	35.0	128.0	620.0
32	E	2	8.6	22.5	48.8		16.0	...	...	...	...	...	...	...	...	...	...	...
33	E	4	3.4	10.0	19.0	58.7	5.0	14.0	30.0	82.0	7.0	17.8	42.6	...	11.7	30.0	72.0	...
34	E	6	1.8	3.4	10.4	43.0	3.2	6.5	17.6	58.0	4.6	10.0	26.0	...	6.2	14.0	36.0	...
35	E	8	1.3	3.6	10.9	28.5	2.0	6.0	15.6	42.0	3.0	8.2	20.8	57.9	4.4	10.5	26.7	78.0
36	F	2	9.8	25.0	28.6	50.0	...	...	43.0	99.0	...	...	43.0	...	...	...	...	...
37	F	4	2.7	6.8	20.5	40.3	4.4	10.9	30.0	58.0	6.5	16.5	43.0	77.3	11.0	26.0	63.0	122.0
38	F	6	2.0	3.8	10.8	26.3	3.2	6.8	16.8	42.0	4.8	10.6	24.2	60.2	6.8	15.1	35.2	81.9
39	F	8	1.0	3.6	7.8	18.7	1.8	5.6	12.5	30.5	2.7	7.8	17.7	43.3	3.8	10.0	23.9	59.5
40	G	2	5.0	10.0	21.8	41.9	13.5	22.0	...	...	...	...	...	...	...	...	...	...
41	G	4	...	4.5	10.8	41.7	2.7	7.4	16.0	...	4.3	10.1	22.0	...	7.2	17.1	...	...
42	G	6	1.0	2.5	8.4	33.5	1.8	4.2	13.0	50.0	2.8	6.6	18.3	65.4	4.2	9.7	24.5	78.3
43	G	8	1.0	2.0	5.1	25.9	1.5	4.0	8.9	39.5	2.4	5.4	13.8	49.0	2.9	6.8	19.4	58.0
44	C	2	...	...	79.0		...	...	...	...	...	...	...	...	...	...	...	...
45	C	4	4.0	9.2	43.1	129.0	7.3	17.7	80.0	250.0	12.1	28.2	130.0	...	18.5	50.0	194.0	...
46	C	6	2.3	10.1	26.2	117.0	3.4	14.7	45.0	171.0	5.0	17.7	61.0	226.0	7.3	24.0	77.0	256.0
47	C	8	1.4	6.8	18.9	81.8	2.2	10.3	28.3	119.0	3.4	13.6	38.0	150.0	4.6	16.8	50.0	178.0
xx	—	—																
48	D	4	3.6	18.7	58.0	178.0	6.2	29.0	86.0	270.0	9.8	44.0	130.0	...	18.0	88.0	...	...
49	D	6	2.4	9.9	35.7	130.0	4.2	15.4	59.0	195.0	6.2	19.3	83.0	...	9.0	24.5	103.0	...
50	D	8	1.2	8.0	25.1	115.0	2.7	11.7	37.0	160.0	4.2	15.3	51.0	197.0	5.7	19.1	...	...
51	H	19	...	2.8	14.5	...	...	4.7	20.0	...	...	6.3	25.0	...	...	7.9	30.0	...
51	I	19	...	...	2.0	...	...	...	3.4	...	...	...	5.2	...	...	...	7.4	...

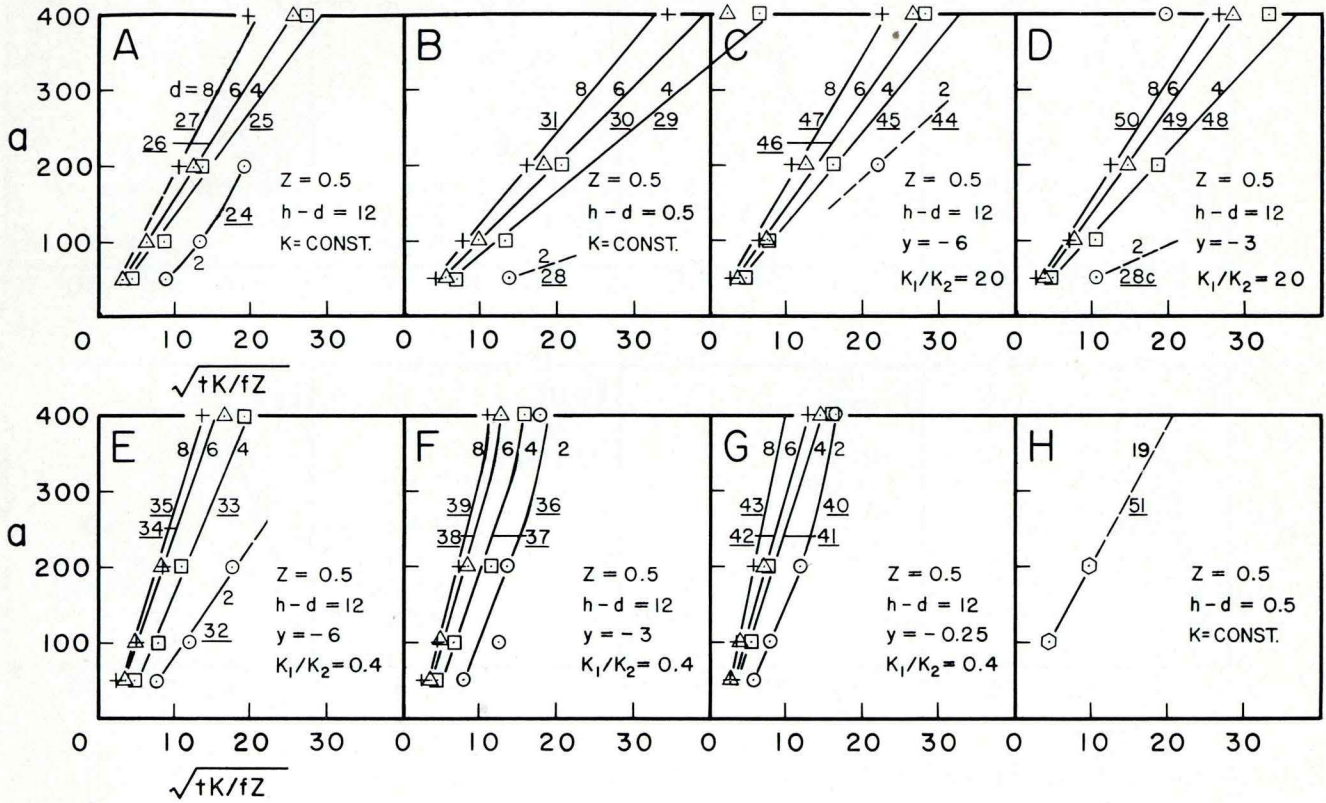[a] The heading "fig. No." refers to the source photograph number in the Ph.D. thesis of Grover (1959).
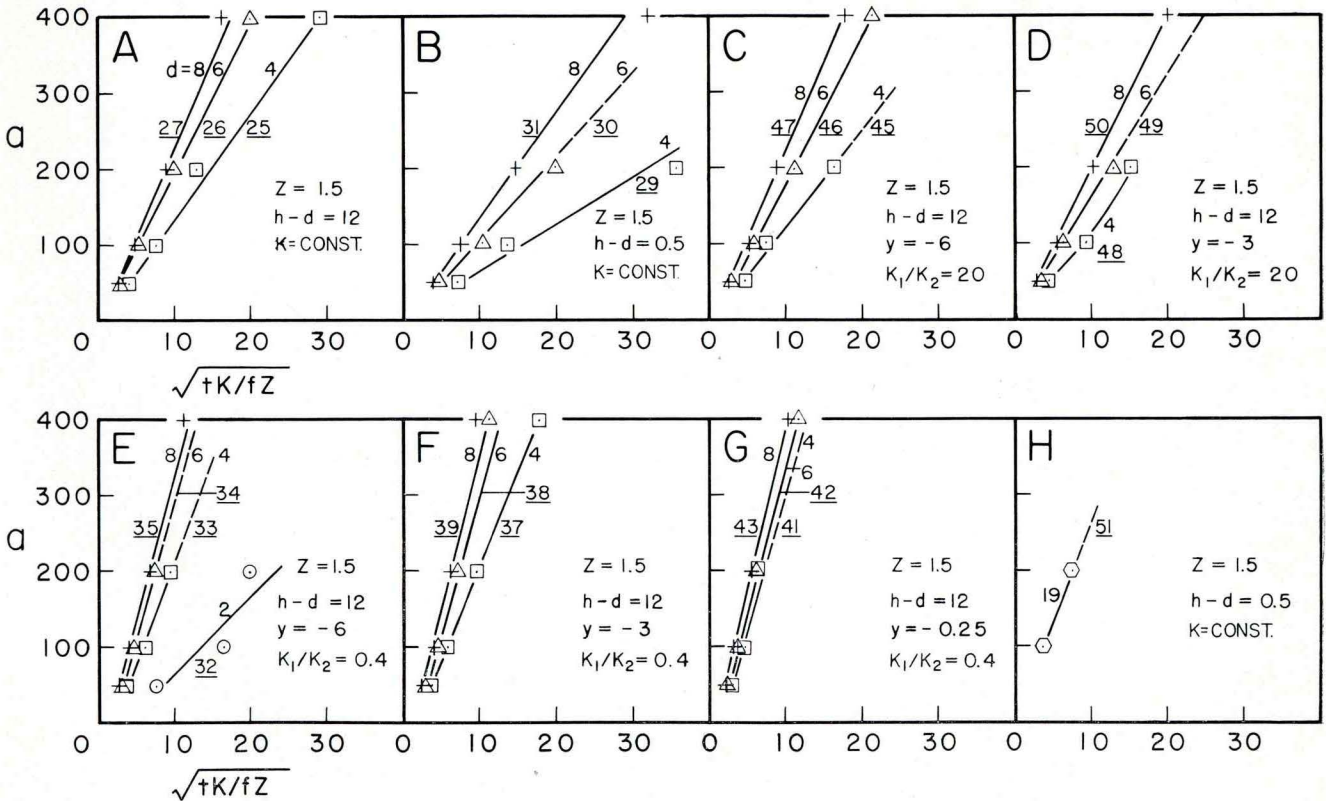
FIG. C-1.   Same as fig. 4 except $Z = 0.5$ cm.

FIG. C-2.   Same as fig. 4 except $Z = 1.5$ cm.

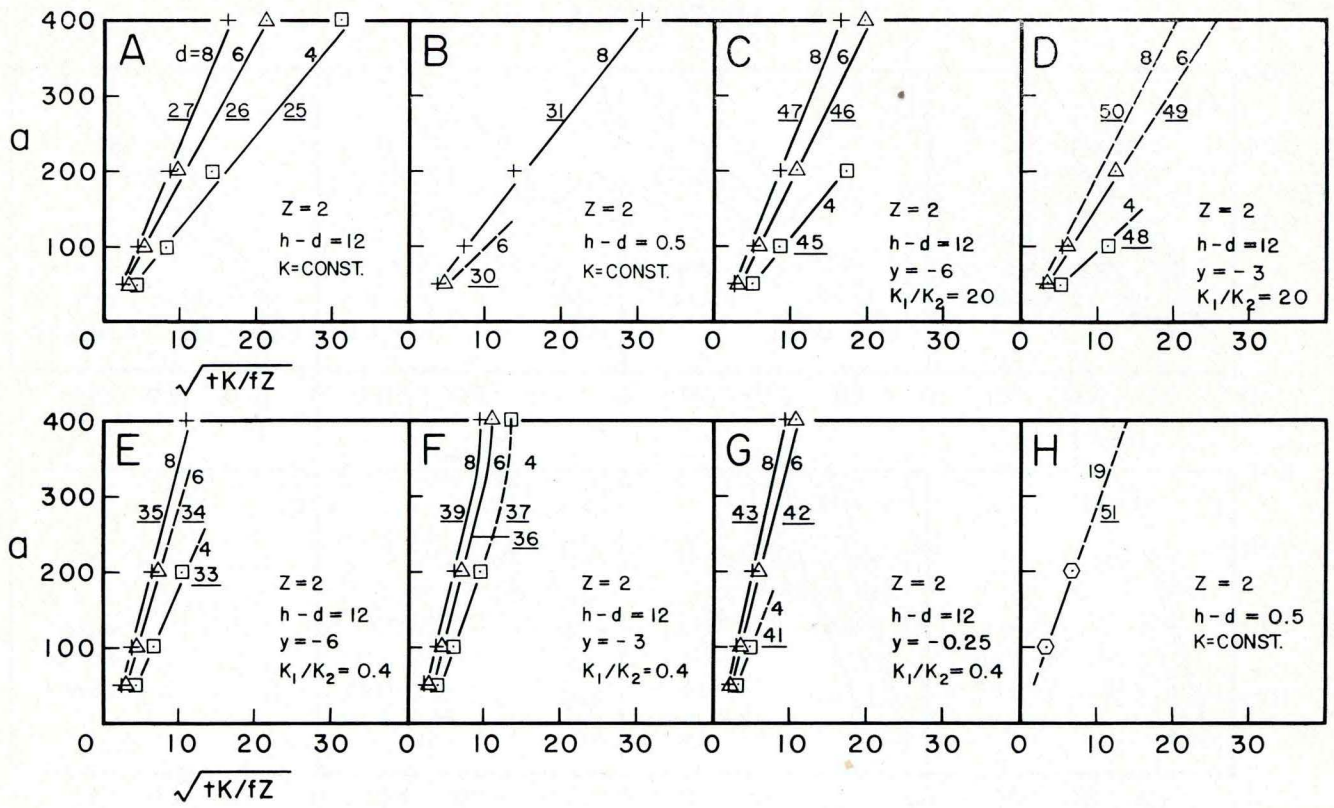FIG. C-3. Same as fig. 4 except Z = 2.0 cm.

48