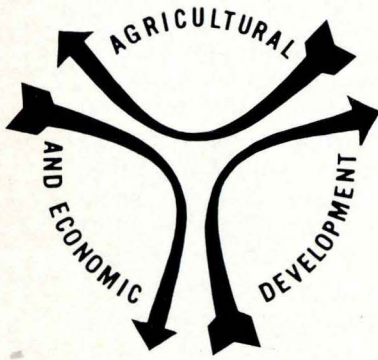


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Regional Changes in Grain Production

An Application of Spatial Linear Programming

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SUMMARY

The study reported here is a continuation of an investigation of national and regional adjustment problems in wheat and feed-grain production. Earlier analyses are reported in USDA Technical Bulletin 1241(6). Much of the background material pertinent to this study is included in that publication. Empirical results, the implications and certain procedures are stressed in this report.

The analysis attempts to answer both practical and methodological questions. Among these are: Is the grain surplus problem likely to persist? How might optimum fertilizer use increase total production potential? How might this fertilizer use, together with certain improved practices, change the structure of regional comparative advantage? What regional patterns of wheat and feed-grain production are likely or possible with equilibrium in the wheat and feed-grain sector of agriculture? What regions are likely to gain or lose in comparative advantage with respect to these commodities? Finally, are certain programming models appropriate for analysis of interregional competition?

The analysis uses three general linear programming models: These are designated *ex post*, *ex ante* and production-distribution models. All three models are based on 104 relatively homogeneous regions or geographic areas in the United States. These regions historically have produced more than 90 percent of national output.

The *ex post* model is related to the 1954 production period. It represents a base period or starting point and was designed to answer these specific questions: With balanced production and needs, but with certain constraints and structural relationships in the economy, what would have been the "best" regional pattern for the production of wheat, corn, oats, barley and grain sorghum in 1954? What was the degree of slack in the wheat and feed-grain economy in that year? And, how long could the production techniques in use at that time meet the needs of a growing economy?

Three productive activities or production processes were considered in the *ex post* model. They were food wheat, feed wheat and a feed-grain rotation. Production within each region was constrained by the acreage planted to wheat and feed grains in 1953, a year without acreage controls and with an acreage base indicative of potential planting in the absence of controls.

The formal objective of the analysis was to maximize the total net returns to farmers as a group when output and requirements are balanced at particular but increasing levels. The initial level was an estimate of normal disappearance for a perfectly competitive equilibrium situation under

zero demand elasticities or invariant demands and certain other assumptions.

The *ex ante* model is not related to any particular period in the future. This model attempts to show the consequences of the use of fertilizers by farmers at most profitable rates and, in some regions, the consequences of a change of production methods by a shift from horse-drawn to tractor equipment. These are changes that may never be completely achieved in agriculture. The industry appears, however, to be moving rapidly in these directions. Hence, we attempted to specify the final consequence of such changes as one indication of the future structure of the industry. Furthermore, fertilizer represents a factor with one of the greatest agricultural output-increasing potentials available.

The 104 basic regions are the same for the *ex ante* model as for the *ex post* model; so, too, are the productive activities and the formal programming objective.

The production-distribution model is, from an interregional competition viewpoint, more general. It is used primarily to indicate how well historical price differentials reflect distribution costs for wheat and feed grains. Hence, it suggests whether or not the *ex post* and *ex ante* models can be used as "short-cuts" for more general production and distribution models.

In addition to the 104 basic production regions used in the other two models, the production-distribution model employs 10 so-called consumption regions. These consumption regions provide the final distribution "points" for the output of the 104 production regions. Production within and shipments into these consumption regions are constrained by estimates of internal wheat and feed-grain needs. These estimated needs are based on normal per-capita consumption and the population in 1954, plus net exports, and are simple disaggregates of estimated national requirements in that year. The programming objective of this model is that of satisfying the requirements of the 10 consumption regions at a minimum total cost of production and shipping. Again, this solution is analogous to perfectly competitive equilibrium under certain assumptions.

Results of the *ex post* model show that the estimated needs of 1954, which approximate the actual disappearance of wheat and feed grains in that year, could have been supplied while leaving about 29 million acres (or 14 percent) of the 210 million base acreage unused. The unused acreage is one measure—albeit an imperfect one—of the excess capacity of the grain sector of agriculture in 1954.

When the output requirements are increased to

approximately the 1957 disappearance of wheat and feed grain, only 9.6 million acres are left unused by the model solution. Limitations in computer facilities did not permit solutions above this level of output. Estimates show, however, given the acreage base and technical coefficients used in the *ex post* model (i. e., 1954), that the potential production was less than the actual disappearance of 1958.

Given the production practices and factor prices of 1954, regions in the Southeast and high-risk areas of the Southwest show up as marginal grain producers. The intermediate or next lower marginal areas appear to be regions in eastern parts of the Mountain States, Michigan and the Appalachian area.

The optimum regional pattern of wheat and feed-grain production was markedly different in the results of the *ex ante* model as compared with the *ex post* model. On the basis of the production coefficient used, at production requirements of 1954, regions in Mississippi, Alabama and Tennessee were included in the optimum production plan as feed-grain producers. In an economic sense, and under the advanced fertilization techniques used for the *ex ante* model, these regions outcompete or have a comparative advantage over traditional feed-grain areas in Missouri, Indiana, Minnesota and Nebraska.

The results of the *ex ante* model also show that estimated demands for wheat and feed grains in 1985 could be met by using just 171 million acres, or 39 million fewer acres than were planted in 1953. Capital inputs would be increased 81 percent, but labor inputs would be only 4 percent higher than in 1954 (assuming no other labor-saving or substitution techniques over the next few decades). The technical coefficients used in this model suppose the existence of farming practices, farm organizations and, possibly, credit facilities that may not be achieved soon. They do, however, represent changes evident in agriculture today and may prove to be extremely conservative by 1985 in consideration of other changes that may likely occur in agricultural input-output coefficients.

The production-distribution model, which uses the same technical coefficients as the *ex post* model, except for the shipping charges, produces a somewhat different optimum regional pattern of production from the *ex post* model. The production-distribution model is designed as a yardstick for the other analysis. It can serve as a yardstick because it explicitly considers transportation as an economic variable. The main difference in the results of this model compared with those of the *ex ante* model is that more wheat is specified east of the Mississippi. This wheat substitutes for that specified for the Northern Plains, Montana and Colorado by the *ex post* model. The results of the production-distribution model appear

a bit unrealistic in this sense. The degree of aggregation in the consumption regions and the omission of wheat qualities may explain this.

Comparison of the results of the *ex post* and *ex ante* models leads to the conclusions: (a) If production techniques had remained at the 1954 level, average per-capita consumption rates of the recent past could not have been maintained without a rise in the real cost of food. (b) Fertilizer is a factor with great output-increasing potential—this factor, nearly alone, could more than provide the additional food requirements needed by 1985. (c) If fertilizer were used at nearly optimum rates, it appears that the South could improve its current relative competitive position.

Interpretation of the results of this analysis must be conditioned by certain limitations. These limitations are of two classes: data and comprehensiveness. It is difficult to obtain complete data on crop enterprises for most regions in the country. Production coefficients necessarily were based on fragmentary data or, in some cases, were estimated by other data. Estimates of optimum fertilizer use, for example, were based on estimated input-output relationships which were the products of limited experiments and of judgment.

Computer size and funds limited the "economic completeness" of the analysis. Ideally, we would have numerous activities representing the different levels of productive efficiency within each region. The result, then might not show entire regions "going out" of production as shown by the models used. Instead, they might show, perhaps, that only 80 percent of the farms in a particular region were in an unfavorable competitive position. Ideally, too, we would consider in the analysis all the relevant alternatives, including nonfarm work alternatives, available to farmers. In this way, the results would not be conditioned by historic prices of certain factors but would be the result of "real opportunity cost."

If research funds and data were not limited, we would have many demand regions and transportation activities that would move products from one area of the nation to another similar to what we actually observe. For each of these demand regions, we would have demand functions representing the long- as well as the short-run. We would then proceed to derive, step-by-step, general equilibrium solutions. Thus we would obtain more realistic results to serve as guides for adjustment problems in United States agriculture. Even though we were not able to complete such an ideal analysis, the results presented show the usefulness of "less complete" regional programming models. No claim is made that the results provide final answers. They do, however, give insights into current adjustment problems of agriculture and suggest changes likely to occur in the future.

Regional Changes in Grain Production¹

An Application of Spatial Linear Programming

by Alvin C. Egbert, Earl O. Heady and Ray F. Brokken

This study is the second in a series dealing with the apparent overcapacity of agriculture and the relative competitive advantages of different producing regions of the United States. Like the previous study, analysis is restricted to wheat and feed grains. Initial research was reported in United States Department of Agriculture Technical Bulletin No. 1241 (6). Interpretation and summary of this initial study, especially to provide background information for the research reported in this bulletin, is given in a following section.

Our problem was to determine how production to meet national demands for grains could be best distributed among regions to maximize net returns to farmers in aggregate or to minimize the cost of food requirements to consumers.

The analysis reported here, as well as initial research, was based on a linear programming model that reflects important spatial interrelations of United States agriculture. The spatial characteristic of the study was achieved by using different geographic regions as the basic production units of agriculture.

Studies of the nature presented here have become possible because of major developments in quantitative concepts and computing facilities. The quantitative concept used—linear programming—permits the incorporation of many relationships and variables into a set of equations and allows simultaneous determination of production patterns for many regions. The large-scale computations required are possible only because of modern developments in computer technology. But a stage has been reached, perhaps, at which quantitative concepts and computing facilities are superior to the available data. However, improvement in data inventory and more efficient solutions to problems can be best achieved perhaps only as models are formulated and solutions generated. This report and the results presented in it represent a second step in formulating such models and the generation of solutions to problems

that cut across major geographic and commodity sectors of a complex industry—agriculture.

The results of the study have practical value to the degree that they indicate the pattern of agricultural production that would be most profitable to farmers under assumed economic changes. The results also have methodological value in the sense that they require and promote data accumulation and conceptual developments which will lead to further and more detailed or efficient analyses of the interrelationships between regions and commodities of American agriculture.

Analysis is needed to provide improved knowledge of interregional adjustment potentials and needs of the nation's agriculture. Analysis also is needed for policy and educational programs to attain needed adjustments and to reduce their costs on particular regions and population groups. Rapid change has been taking place in American agriculture and in important elements in its structure. These changes have not, however, been taking place at equal rates over the many areas and commodities which make up the industry. Some regions are gaining in their relative advantage in producing particular commodities, while some are losing ground as change takes place at differential rates. At the same time, however, national policies have provided price supports and production controls that have tended to maintain historical patterns of regional production.

There is a need, which provides the basis for this study, for information about the relative advantage of different regions as producers of specific crops and for information about the possible adjustments ahead for various regional and commodity sectors of agriculture. Knowledge is needed also to indicate to what extent production patterns, which are encouraged by national policies, deviate from those that would occur with a more unconstrained general equilibrium of agriculture. This same knowledge can be useful in specifying the changing advantage of regions as producers of particular commodities and the possible adjustments in store for these regions in future decades.

In the early part of this research project, we

¹Project 1405 of the Iowa Agricultural and Home Economics Experiment Station.

have concentrated on the wheat and feed-grain sector because this sector is, both in cropland area and in value, one of the largest in American agriculture. Too, it currently is faced with adjustment problems of large magnitudes.

The analysis considers wheat and feed grains (corn, oats, barley and sorghums) as an integral complex. In other words, we analyze the production of these crops simultaneously. This is done especially because wheat is both a food grain and a feed grain but also because these crops compete for the same farm resources. We recognize that wheat and feed grains are part of a greater production complex which includes livestock, oilseeds and other crops. This greater complex is the subject of research now under way. Because of the magnitude and complexity of the analysis and availability of data, the research must be developed a step at a time.

The specific objectives of this study are:

- (a) To show the possible consequences of a static or unchanging wheat and feed-grain sector and a growing population on regional land-use patterns and acreage requirements.
- (b) To show the possible effects of raising the level of fertilizer and machinery inputs in wheat and feed-grain production on regional land-use patterns and on acreage requirements, given various population levels.
- (c) To compare the results of the two extreme cases given in objectives (a) and (b) to better describe and characterize the nature of the adjustment problem in the wheat and feed-grain sector.
- (d) To test the appropriateness of an abbreviated linear programming model for interregional competition research by comparing its results with that obtained from a more general model.

Background

The current adjustment problem in agriculture is not a unique present-day phenomenon: It existed even in the 1920's. Farm legislation of the period and articles in the professional journals attest to this fact. Because this early trouble period was followed closely by the depression, the basic problem of agriculture was obscured by the national problem of inadequate aggregate demand. Even so, toward the end of the 1930's, stocks began accumulating under federal programs. But then World War II came along and provided outlets for accumulated stocks.

After the war, while the output of European agriculture was recovering from shocks of the war, the demand for United States production rose rapidly; hence, farm prices increased. Higher prices, together with other wartime incentives, stimulated growth in domestic agricultural capacity. This larger capacity was achieved primarily by the adoption of new technology and improved practices together with increased acreages of grain crops. Markets devoured the output of this expanded

farm plant until the end of the 1940's. In the meantime, Europe went through its reconstruction period. As Europe recovered and got its own agricultural plant into production, foreign demand for the products of United States agriculture tapered off. But the domestic agricultural plant continued to expand. Then, as agricultural stocks began to pile up again, the Korean War broke out, and the pressure of output on demand was temporarily reduced.

The decline in demand following the end of the Korean War and continued rapid expansion in agricultural output brought lower farm prices through the mid-1950's. Lower prices brought about governmental assistance to overcome the dilemma of increased efficiency and reduced income. This assistance took such forms as price supports, acreage restrictions and the Soil Bank Program. Even with reduced acreages of wheat, corn and cotton, stocks continued to rise. This experience pointed up the need for detailed analysis of the agricultural sector with the objectives of (a) measuring the present output-consumption gap and (b) indicating the probable persistence of this gap in the future. It was then that the Farm Economics Division of the USDA and the Iowa Agricultural and Home Economics Experiment Station initiated cooperative research to study these problems.

Early in the planning stage, it was decided that interregional competition or spatial equilibrium theory was the relevant framework for this research. The reasons were several. Without some criterion or yardstick the term "production-consumption imbalance" is meaningless.

Given national economic efficiency as the yardstick by which we measure the production-consumption gap (be it positive or negative), any measurement analysis must take into account these factors: (a) regional differences in productivity, (b) differential changes in regional productivity arising from new technology and (c) economic and non-economic institutions that have a bearing on agriculture.

Interregional competition or spatial equilibrium theory can encompass all of these variables; and, from a practical viewpoint, there are several tools available for analyses in this general framework. These include budgeting systems, statistical supply and demand curves, and linear programming. All of these methods have been used in various degrees of elaborateness in the past (9, 10, 11, 13, 14). In our previous analyses and in the ones reported here, we have relied mainly on the technique of linear programming. As will be shown later, the linear programming technique (with its numerous possible variations, and hence flexibility) can approximately mirror the multitude of economic forces operating in a competitive economy—interregionally and intraregionally.

Previous Research

To provide a background for this report, previous related research is briefly summarized in this section.² The basic models used in the previous study are summarized here because they serve as the foundation upon which models for the current study are fashioned.

All previous analyses were developed around 104 regions (fig. 1). These regions include the major grain-producing areas in the United States and account for about 90 percent of total United States production. The primary criteria used to delineate these regions were homogeneity in production methods and yields, and degree of regional farm mechanization. Basic data for each of these regions were formulated on the basis of 1954 prices, costs and methods of production. The crops included in the analyses were wheat, corn, oats, barley and grain sorghums.

Optimum regional patterns of production were determined for five different linear programming models of the United States grain economy. Table 1 summarizes the special characteristics of each of these five models. Each model was formulated for methodological as well as analytical reasons. For example, Model B was designed to answer the

question: "To what extent does the opportunity cost of land used for grain production have a significant influence on the regional pattern of production in the United States?" Model D was designed to answer the question: "If regions specialized in production of grain crops for which they have either an absolute or comparative advantage, what would be the regional production pattern?" These and other questions are important for measuring the supply and demand balance in agriculture and the type and extent of adjustments required as the nation experiences further technical and price changes. Means were not available to include in a single analysis all of the questions implied in the characteristics defining each of the models listed in table 1. Each model was formulated to add to our knowledge of the nature of the present imbalance in the grain economy or of the possible impact of certain changes that might occur in the future.

The results of these analyses provide some significant insight into: (a) The magnitude of the imbalance in agriculture, (b) the location of regions that are marginal in grain production and (c) the degree of absolute and comparative advantage existing and developing between regions.

In this report, we continue to explore the grain economy mainly by comparative statics or comparisons of particular partial equilibrium situations.

²United States Department of Agriculture Technical Bulletin No. 1241 (6) and supplement (7) give the details of previous related research.

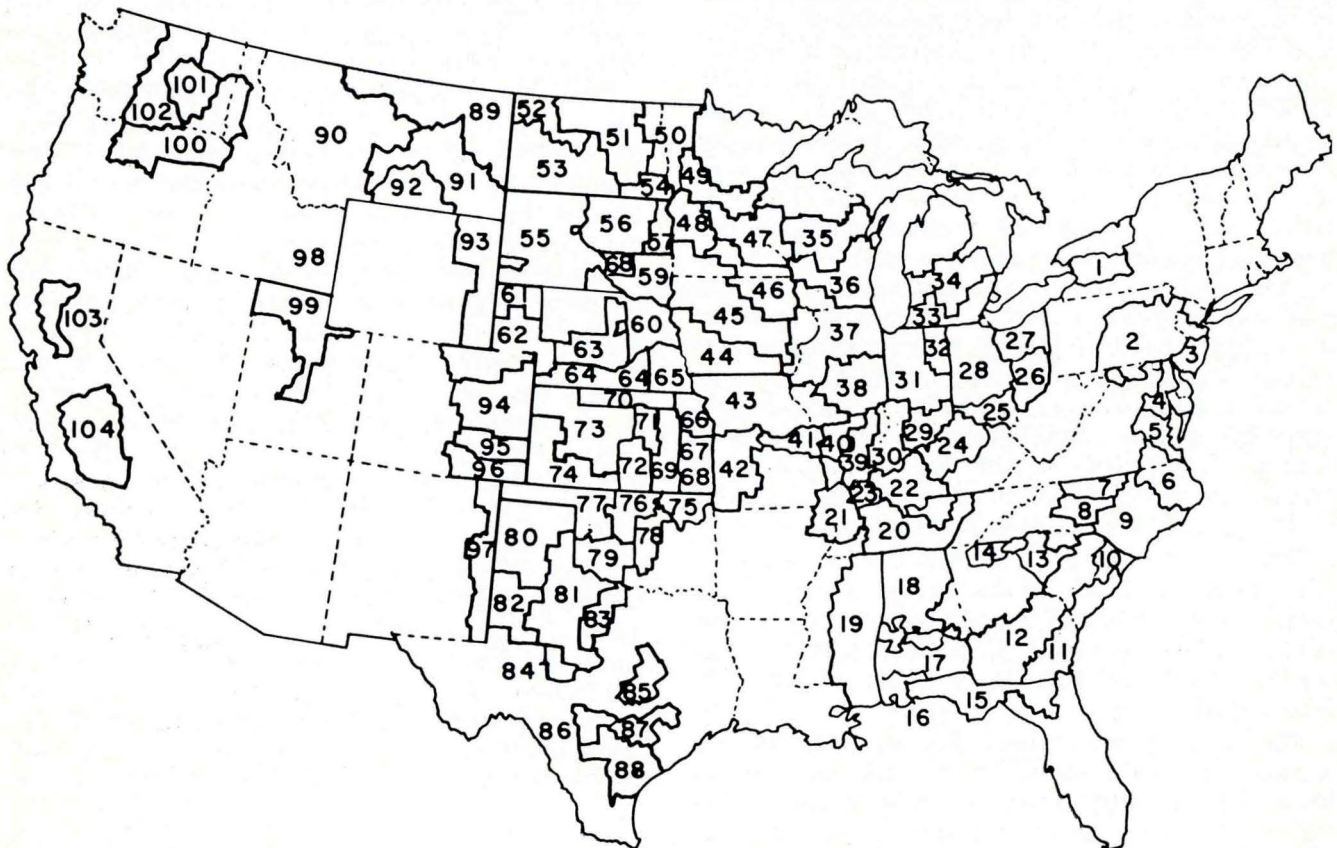


Fig. 1. Production regions.

Table 1. Summary of special characteristics of five models used in previous analysis of the wheat-feed grain economy.

Model	Regional activities	Specific cost making up total activity cost	Regional land constraints	Analysis objective
A	Food wheat Feed wheat Feed-grain rotation ^a	Labor Machinery Chemicals Seed Miscellaneous	One for all activities	Minimum total cost
B	Food wheat Feed wheat Feed-grain rotation	Land Labor Machinery Chemicals Seed Miscellaneous	One for all activities	Minimum total cost
C	Food wheat Feed wheat Feed-grain rotation	Labor Machinery Chemicals Seed Miscellaneous	One for wheat, Minimum total one for feed-grain rotation	cost
D	Food wheat Feed wheat Corn Oats Barley Sorghum	Labor Machinery Chemicals Seed Miscellaneous	One for all activities	Minimum total cost
E	Food wheat Feed wheat Feed-grain rotation	Labor Machinery Chemicals Seed Miscellaneous	One for all activities	Maximum total profit

^aThis rotation included corn, oats, barley and sorghum. Their relative weights were assumed to be the same as in the base period for acreages, 1953.

First, we analyze the grain economy with no changes in production efficiency but with projected population growth. This is called the *ex post* model. Then we look at this economy under increased production efficiency generated by optimum fertilizer use and fully mechanized production methods and the same projections in population growth. This is called the *ex ante* model. Finally, we use the production data of the *ex post* model, plus transportation data, to answer the methodological question: How well have we been able to simulate a general spatial equilibrium system? This is called the production-distribution model. Computer and resource restrictions prevented us from achieving a degree of detail and economic refinement that is conceptually possible in such analyses. National programming models become large and cumbersome for computations as details and refinements are added. Hence, certain variables of some significance must be omitted or analyzed in a "partial equilibrium" fashion if quantitative knowledge of the complex interrelationships of the various sectors of agriculture is to grow.

The basic assumptions, limitations of the analytical models and data limitations of previous analyses also are relevant to the study reported here. All are fully described in U. S. Department of Agriculture Technical Bulletin No. 1241 (6, pages 6, 9-10 and 35).

The first section which follows presents the procedural steps involved in the analyses and defines the particular economic models used. The section following presents optimum regional production patterns for wheat and feed grains; i. e.,

the solution to *ex post* and *ex ante* models at several levels and combinations of national requirements for wheat and feed grain. The latter section also shows the optimum regional pattern for the production and distribution model. Finally, comparisons of solutions are made, and conclusions are drawn concerning the implications of the results and the relative appropriateness of the two major methods of analysis used.

METHOD OF ANALYSIS

General Procedure

Three general analyses were made. These analyses are based on the *ex post*, *ex ante* and production-distribution models. Each of these models is unique in its specific formulation, but all are related in the general objective and empirical approach.

Data for all models were assembled on the basis of the 104 regions shown in fig. 1. These 104 regions were the basic building blocks or activity units of all analyses. Production costs and yields for three productive activities—food wheat, feed wheat and a feed-grain rotation—were estimated for each region. In addition, regional production constraints were estimated as equal to the maximum sum of acreages planted to the five grains—wheat, corn, oats, barley and sorghum—in the last decade. This period included years in which supply control programs were not in effect and years in which acreages of crops in particular regions approached historic records.

Long-run average prices were estimated for each region for computing the corresponding activity net returns needed for the *ex post* and *ex ante* analyses. Net returns were calculated as the difference between the estimated production costs and the value of the output per acre.

Shipping costs for the production-distribution model were based on a schedule of tariffs furnished by the Agricultural Stabilization and Conservation Service, USDA. These transportation costs, plus production costs, were used to construct activities to represent shipments of grain from the centers of the 104 production regions shown in fig. 1 to centers of the 10 consumption regions shown in fig. 2. The consumption centers represent the mean transportation cost locations of the regions.

Domestic wheat and feed-grain requirements for the base period of 1954 were estimated as the product of "normalized"³ per-capita consumption of each grain and United States population of 1954. All feed-grain quantities except those used for food were converted to corn-equivalent feed units. National requirements, so estimated, were allocated to the 10 consumption regions on the basis of

³This term is defined in supplement to USDA Technical Bulletin 1241 (7, pp. 53-55).



Fig. 2. Consumption regions and shipping destination.

relative population for the production-distribution model. Actual net exports were taken as the best estimate of export needs. These exports were allocated to the 10 consumption regions on the basis of the actual port from which they were shipped. For example, the net overseas shipment of wheat, corn, oats and barley from the ports of Boston, New York, Baltimore and other cities in the Northeast area made up part of the requirements or demand for wheat and feed grains in the Northeast.

Linear programming was used to analyze these data under the assumptions of the various models explained later. The objective of each analysis was a regional production pattern under certain economic efficiency criteria. For reasons outlined subsequently, economic efficiency was not defined in the same way for each model.

In the *ex post* model, net returns from wheat and feed grains were maximized for farmers as a

group for a range of outputs given (a) production techniques, (b) price relations of 1954 and (c) various demand combinations.

In the *ex ante* model, net returns from wheat and feed grains were maximized for farmers as a group and for a range of outputs given (a) improved production practices, (b) price relations of 1954 and (c) various demand combinations.

In the production-distribution model, total production and distribution costs for wheat and feed grains were minimized for the industry as a whole given (a) crop production techniques, (b) distribution costs, (c) factor prices of 1954 and (d) wheat and feed-grain requirements of 1954.

The special characteristics of these models are summarized in table 2.

Economic Assumptions and Implications

The methods of analysis just described involve certain economic assumptions and implications. All models have certain similarities in this respect. These assumptions and implications will be enumerated first. Then their differences will be described.

Common assumptions

The regional "firm" is the basic producing or allocative unit. This assumption means only that autonomous firms within any one region respond uniformly to economic stimuli.

Total grain acreage within any region cannot exceed the total acreage of all five grains planted in 1953.

Given the total grain acreage available, regional firms may select any one or combination of three productive activities—food wheat, feed wheat and feed-grain rotation.

Table 2. Summary of special characteristics of three models.

Model	Regional activities	Specific cost making up total activity cost	Production constraints	Demand constraints	Analysis objective
Ex post	Food wheat Feed wheat Feed-grain rotation n=312 ^a	Labor Machinery Chemicals Seed Miscellaneous	One regional land constraint for all regional activities, or 104	Variable within certain limits of national wheat and feed-grain demands ^b	Maximum total net return for each demand combination
Ex ante	Food wheat Feed wheat Feed-grain rotation n=312	Labor Machinery Chemicals Seed Miscellaneous	One regional land constraint for all regional activities, or 104	Variable within certain limits of national wheat and feed-grain demands	Maximum total net return for each demand combination
Production-distribution	Food wheat Feed wheat Feed-grain rotation Each activity is replicated for every consumption region n = 312	Freight Labor Machinery Chemicals Seed Miscellaneous	One regional land constraint for all regional activities, or 104	Fixed wheat and feed-grain demand constraints for each consumption region	Minimum total production and distribution costs

^a The letter n refers to the total number of real activities in each specific model.

^b The term variable is used here to mean that demand for wheat might range from 1 million to 2 million units, for example; on the other hand, fixed means that demand is exactly 1 million units.

Each region specializes in the most profitable of the three crop alternatives or most profitable combination in light of its production costs and realized market prices.

Ex post model assumptions

All regions or basic allocative units use production methods similar to those practiced in 1954.

Regional price differentials adequately reflect distribution costs between regions and account for quality differences. This simply means that all markets are tied together by actual or potential transfers of products and that the value of certain product qualities is reflected by price differentials.

Requirements of wheat and feed grains are a function of the domestic population level and actual net exports.

Ex ante model assumptions

Structurally, the *ex ante* model is exactly the same as the *ex post* model. These models were purposely structured alike so that certain comparisons could be made among the results obtained from each. The economic assumptions unique to the *ex ante* model are: (a) All firms apply fertilizer at optimum or maximum profit rates. (b) All firms use only tractor power and mechanized methods to produce grain. (c) Regional price differentials adequately reflect distribution costs between regions and product-quality differences. (d) Requirements of wheat and feed grains are a function of domestic population level and actual net exports.

Production-distribution model

Economic assumptions of the production-distribution model are: (a) All regional firms use production methods similar to those used to produce grain in 1954. (b) Distribution costs are dominated by transportation costs. (c) Regions produce the most profitable alternative among the three considered activities—food wheat, feed wheat and feed-grain rotation—given market prices. (d) Grain is shipped to the market that yields the highest net return.

These three models represent static analysis in that we do not specify the time involved in adjustments or trace developments leading to the equilibrium conditions and specifications. We are interested mainly in the consequences of the various conditions or changes implied by the assumptions. These consequences are important for: (a) characterizing the grain surplus problem; that is, the magnitude of the problem, its probable persistence, regional changes expected under various economic conditions and other possible structural changes in the industry; and (b) ascertaining the analytical superiority of one model over the other.

All models represent perfectly competitive economic structures. Thus, solutions to these models

represent situations toward which an unrestricted atomistic industry may be moving. How closely the solutions approximate long-run tendencies depends on how well the models reflect or incorporate important interregional, spatial and other economic differences existing within the grain economy.

Conditions for Regional Firm or Unit

The quantitative results presented later are based on the assumption that a regional producing unit can represent a collection of farm firms in spatial programming models. The conditions necessary for the assumptions to be valid are outlined as follows. Only one region is used to illustrate these conditions which generally will be the same for n regions because of the independence in decision-making units.

Let there be

- n farms ($i = 1, 2, 3, \dots, n$),
- m products ($j = 1, 2, 3, \dots, m$),
- p factors ($k = 1, 2, 3, \dots, p$),

then let

- Y_{ij} = output of the j th product by i th farm,
- X_{ijk} = k th factor used to produce the j th product on the i th farm,
- $Y_{ij} = f_{ij}(X_{ij1}, X_{ij2}, X_{ij3}, \dots, X_{ijp})$, (1)

be the production function for the j th product on the i th farm. Assume that constant returns to scale exist, at least within the relevant range; i.e.,

$$KY_{ij} = f_{ij}(KX_{ij1}, KX_{ij2}, KX_{ij3}, \dots, KX_{ijp}). \quad (2)$$

We can then express Y_{ij} as a function of one factor explicitly, say land, and some combination of all other factors (perhaps even a least-cost combination, but this is not necessary) implicitly, as in equation 3.

$$Y_{ij} = a_{ij}X_{ij1} \quad (3)$$

$$X_{ij1} = G_{ijk}(X_{ijk}) \text{ for } k, \quad (4)$$

a_{ij} in equation 3 is equal to the total derivative of Y_{ij} with respect to X_{ijk} 's. Then the marginal cost or supply curve for any farm, i , and product, j , is given by equation 5

$$\frac{P_{ij}}{a_{ij}} = MC_{ij} \quad \text{or} \quad MC_{ij} = k_{ij}, \quad (5)$$

given the side condition $\sum_j \frac{Y_{ij}}{Z_{ij}} \leq A_i$, in which MC_{ij} represents the marginal cost of Y_{ij} , Z_{ij} is the yield per acre, A_i is the number of acres on

the *i*th farm and P_{ij} is the price of the bundle of resources as given by function 6⁴

$$P_{ij} = \frac{P_{x_{ij1}} + P_{x_{ij2}} G_{ij2} (X_{ij2}) + P_{x_{ij3}} (G_{ij3})}{(X_{ij3}) + \dots + P_{x_{ijp}} G_{ijp} (X_{ijp})} \quad (6)$$

If these conditions are fulfilled, then

$$\frac{P_{1j}}{a_{1j}} = \frac{P_{2j}}{a_{2j}} = \frac{P_{3j}}{a_{3j}} = \dots = \frac{P_{nj}}{a_{nj}} \quad (7)$$

which means that $k_{1j} = k_{2j} = \dots = k_{nj}$. Hence, within a region the product supply curves are the same for all farms, even though they may have different resource organizations and constraints. Therefore, the regional supply functions are given by

$$\begin{aligned} P_{g1} &= K_1 \\ P_{g2} &= K_2 \\ &\vdots \\ P_{gm} &= K_m, \end{aligned} \quad (8)$$

and the regional side condition is

$$\sum_i \sum_j \frac{Y_{ij}}{Z_{ij}} \leq \sum_i A_i.$$

If the foregoing is the case, representing all farms in a region as an aggregate regional unit or firm in linear programming analysis is realistic. In reality, this probably will not be strictly the case. A rough approximation of these conditions, however, would produce reasonably satisfactory results. This conceptual framework was used to guide delineation of the 104 production regions.

Matrix Structure of the Ex Post and Ex Ante Models

The structure of the *ex post* and *ex ante* models can be summarized as follows:

Let

X_{ij} = the output of *j*th product in the *i*th region

r_{ij} = the net return per bushel of the *j*th product in the *i*th region or $(p_{ij} - c_{ij})$, where p_{ij} is the price per bushel and c_{ij} is the cost.

The objective then is,

$$\max f(r) = \sum_i \sum_j X_{ij} r_{ij}. \quad (9)$$

Objective 9 is maximized subject to these constraints

$$X_{ij} \geq 0, \quad (10)$$

$$\sum_j X_{ij} a_{ij} \leq A_i, \quad (11)$$

⁴The index or subscripts have been aligned to avoid certain printing difficulties. Thus, the $ij1$, $ij2$ and $ij3$ on the x subscript for P are actually subscripts of x and not of P .

$$\sum_i \sum_{j=2}^3 X_{ij} = D_1, \quad (12)$$

$$\sum_i X_{i1} = D_2, \quad (13)$$

in which a_{ij} is the fraction of an acre of land in region *i* required to produce a bushel of the *j*th product, A_i is the acres of land available to produce grain in the *i*th region. D_2 is a variable representing the output of food wheat, and D_1 is a variable representing the output of feed grain.

Data Used in the Ex Post Model

Most data on yields and cost were based on production practices existing in 1954. Because methods of estimating particular data are described in detail elsewhere (6 and 7), only a summary description will be given here.

Yields or outputs

The yields used are those expected in 1954 under average weather and typical production practices in use at that time. Table 3 presents these yields. The a_{ij} of inequality 11 are interpreted as follows: (a) The a_{i1} is the reciprocal of the food-wheat yield in region *i* or the fraction of an acre required to produce 1 bushel of food wheat. (b) The a_{i2} is the fraction of an acre required to produce, in region *i*, 1 bushel of feed units from wheat,

$$a_{i2} = \frac{1}{Y_{i1}} K_1, \quad (14)$$

where Y_{i1} is the yield of wheat in the *i*th region and $K_1 = 1.121$, the feed-unit conversion factor for wheat.

The a_{i3} is the fraction of an acre required to produce 1 bushel of feed units from feed grain in region *i* and is computed as in equation 15,

$$a_{i3} = \frac{1}{b_{i2} Y_{i2} K_2 + b_{i3} Y_{i3} K_3 + b_{i4} Y_{i4} K_4 + b_{i5} Y_{i5} K_5} \quad (15)$$

in which $K_2 = 1.000$, $K_3 = 0.495$, $K_4 = 0.789$ and $K_5 = 0.981$. The b 's are the proportions of total grain acreage planted to each crop, and the K 's are the corn-equivalent conversion factors.

Acreage constraints

The acreage constraints (A_i in equation 11) are the acreage sums of wheat, corn, oats, barley and sorghum grain planted in each region in 1953.

⁵The second position index in equations 14 and 15 does not relate to the same variable on the left side of the equality sign as on the right. On the left side it stands for programming activities where 1 = food wheat, 2 = feed wheat, 3 = feed-grain composite. On the right side it stands for specific grains where 1 = wheat, 2 = corn, 3 = oats, 4 = barley and 5 = sorghums.

Table 3. Estimated net yields in bushels per acre for wheat and feed grains,^a by regions, ex post data.

Region	Wheat	Corn	Oats	Barley	Sorghum
1	26.9	45.6	39.0	30.0	—
2	21.3	50.0	36.4	37.3	—
3	18.1	45.9	29.2	26.2	—
4	18.4	49.8	32.2	27.2	—
5	21.2	39.6	35.2	30.7	—
6	16.2	36.6	27.4	21.9	—
7	19.3	29.4	32.0	29.9	—
8	18.3	31.2	32.2	30.6	—
9	17.7	29.2	29.9	24.1	—
10	17.8	21.3	28.2	21.6	—
11	16.5	18.6	23.1	17.0	—
12	16.6	16.2	27.2	23.6	—
13	16.5	18.9	27.2	23.6	—
14	16.1	18.6	24.0	21.0	—
15	—	15.0	21.2	—	19.0
16	22.8	20.9	21.7	—	14.8
17	20.4	15.4	22.1	—	17.1
18	19.6	21.5	29.5	—	15.0
19	15.7	19.8	20.9	—	—
20	14.9	27.6	25.3	14.9	—
21	18.0	25.6	24.4	18.6	19.2
22	17.1	36.4	27.0	19.0	—
23	16.6	32.6	27.7	19.6	—
24	15.8	36.3	28.6	24.4	—
25	17.4	50.8	27.7	24.5	—
26	23.0	51.3	37.1	29.8	—
27	26.0	50.4	40.9	32.8	—
28	24.1	56.6	39.5	29.1	—
29	19.0	44.4	30.0	24.6	—
30	19.1	39.8	28.5	26.5	—
31	24.3	55.6	38.0	25.5	—
32	27.0	56.0	39.8	26.2	—
33	26.6	43.4	37.1	28.7	—
34	27.6	43.3	37.2	32.0	—
35	20.6	44.6	37.9	33.0	—
36	27.3	58.6	53.6	38.4	—
37	25.2	59.9	41.2	30.6	—
38	27.1	57.0	36.6	26.6	—
39	18.8	36.1	23.7	25.0	—
40	19.4	35.2	25.2	25.5	—
41	21.3	36.0	23.9	25.9	17.0
42	19.7	28.2	24.5	23.3	16.1
43	22.7	42.8	27.1	27.8	22.3
44	15.5	46.1	28.1	22.4	—
45	14.7	50.1	31.2	17.8	—
46	17.6	51.4	37.1	27.5	—
47	17.0	47.6	38.2	26.9	—
48	13.6	9.5	32.6	23.6	—
49	14.6	40.3	33.2	27.8	—
50	9.2	26.4	30.0	25.0	—
51	8.0	20.2	24.6	18.7	—
52	7.0	17.8	24.4	17.6	—
53	7.5	17.8	25.5	18.5	—
54	7.9	22.1	25.8	18.9	—
55	8.1	19.0	22.4	16.7	—
56	9.0	22.2	25.5	17.0	—
57	8.6	29.7	30.0	20.1	—
58	8.5	21.6	24.2	16.4	—
59	9.6	36.7	29.5	18.8	—
60	16.2	38.9	22.8	16.8	21.4
61	12.8	24.4	23.7	19.3	—
62	10.0	26.4	24.4	21.8	15.3
63	10.6	32.2	17.8	12.7	21.5
64	11.2	25.2	19.2	14.3	21.4
65	17.5	37.0	22.7	16.3	30.5
66	17.8	31.5	17.4	17.6	25.5
67	17.9	25.5	18.7	19.2	19.8
68	17.1	22.1	20.9	18.1	17.5
69	17.4	24.0	19.9	18.5	18.4
70	10.8	22.1	13.7	11.7	19.8
71	13.3	22.2	18.8	14.3	19.9
72	13.8	21.0	19.7	13.3	18.6
73	9.4	20.4	16.0	12.8	18.6
74	7.3	16.1	15.6	10.3	17.0
75	12.0	18.4	13.5	12.7	12.5
76	13.0	16.5	17.3	11.6	14.8
77	6.6	11.2	10.1	7.3	12.9
78	10.4	19.5	15.7	9.8	15.1
79	10.3	18.0	15.4	10.1	14.7
80	6.1	27.2	16.6	12.2	27.5
81	7.5	13.7	17.9	12.1	10.0
82	5.0	14.5	15.9	13.2	15.0
83	8.3	13.7	16.0	12.4	9.1
84	4.5	11.3	14.2	9.1	12.7
85	5.8	17.7	—	—	19.0
86	4.2	14.9	16.6	9.9	16.1
87	4.5	17.6	—	—	15.9
88	4.5	17.1	—	—	23.6
89	8.0	14.6	28.0	29.6	—
90	8.9	16.4	29.4	27.0	—
91	6.5	13.0	23.9	16.4	—
92	10.6	25.4	40.6	30.2	—
93	8.7	24.2	22.8	22.5	—
94	7.0	16.3	15.8	12.7	8.8
95	5.2	42.8	17.9	14.8	16.5
96	2.5	16.7	11.9	10.1	8.6
97	1.6	10.0	19.7	10.6	10.8
98	12.9	45.2	39.4	30.6	—
99	9.9	38.1	49.4	47.2	—
100	16.9	64.5	40.0	31.0	—
101	12.6	52.5	37.6	30.7	—
102	11.6	71.7	51.5	33.1	—
103	12.5	36.1	18.2	23.2	33.5
104	9.8	25.4	17.0	27.1	36.4

^aEstimated yield less seed per acre; based on a composite acre.

This was the greatest total acreage planted to these five grains in the 1950's. Acreage constraints are given in table 4.

Production costs

Production costs included these items:

- (1) Labor.
- (2) Machinery inputs including fuel, oil, grease, repairs, taxes, insurance and depreciations.
- (3) Seed inputs were accounted for by subtracting them from yields.
- (4) Fertilizer.
- (5) Lime.
- (6) Pesticides and herbicides.
- (7) Other miscellaneous items which included costs for applying fertilizer and lime not spread by integral equipment or custom hire and the cost of water for irrigation.

Table 5 presents the sum of these costs on a per-acre basis for each of the five grains. These are weighted costs for several production practices. The c_{ij} used in computing r_{ij} in equation 9 are the ratios of costs to yield per acre for food wheat, c_{i1} , and for feed wheat, c_{i2} . The c_{i3} (the c_{ij} for feed grains) were obtained by calculating the ratio of weighted costs to weighted yields as in equation 16,

Table 4. Acreage restraints, by regions.

Region	Acreage	Region	Acreage
	(thousands)		(thousands)
1	603	53	5,016
2	2,480	54	1,075
3	445	55	2,101
4	298	56	4,155
5	208	57	1,404
6	561	58	1,013
7	325	59	3,264
8	370	60	3,874
9	1,421	61	441
10	290	62	4,282
11	261	63	1,990
12	3,100	64	2,509
13	434	65	4,718
14	107	66	1,220
15	542	67	788
16	91	68	759
17	727	69	1,119
18	1,230	70	1,792
19	1,228	71	1,371
20	969	72	2,735
21	853	73	7,664
22	1,069	74	5,114
23	261	75	420
24	329	76	2,739
25	574	77	2,911
26	411	78	504
27	1,067	79	2,025
28	4,935	80	2,881
29	757	81	1,955
30	1,902	82	1,176
31	4,760	83	369
32	996	84	98
33	1,649	85	439
34	2,317	86	108
35	994	87	326
36	2,297	88	610
37	7,754	89	6,493
38	4,841	90	3,833
39	1,133	91	611
40	1,013	92	691
41	693	93	830
42	1,535	94	4,293
43	4,795	95	609
44	4,263	96	561
45	10,879	97	544
46	4,107	98	1,719
47	2,711	99	685
48	2,561	100	4,685
49	1,304	101	2,785
50	4,827	102	544
51	7,898	103	553
52	2,790	104	1,015

Table 5. Estimated per-acre^a production costs for wheat and feed grains, land omitted, by region, ex post data.

Region	Wheat	Corn	Oats	Barley	Sorghum
1	\$29.23	\$34.76	\$28.34	\$28.64	—
2	28.08	33.08	26.40	25.14	—
3	29.86	29.29	28.28	30.61	—
4	28.14	32.36	27.58	27.81	—
5	24.36	30.59	24.31	24.31	—
6	25.25	32.12	26.87	24.92	—
7	32.35	39.01	32.31	32.38	—
8	30.17	32.57	29.04	30.16	—
9	28.35	35.48	28.22	28.56	—
10	22.79	30.85	23.62	24.08	—
11	27.24	31.93	24.53	25.43	—
12	23.37	25.49	23.39	23.49	—
13	22.79	27.56	21.90	22.30	—
14	26.46	30.08	24.57	25.91	—
15	—	26.53	25.49	—	\$27.42
16	23.36	35.09	28.85	—	29.13
17	23.73	29.61	28.66	—	28.10
18	23.42	29.17	28.21	—	26.29
19	22.87	28.65	22.92	—	—
20	25.84	28.61	24.55	24.40	—
21	22.60	24.12	19.89	19.53	—
22	24.25	30.64	23.62	23.53	—
23	26.68	28.35	25.84	26.29	—
24	28.04	30.33	25.07	27.57	—
25	25.93	33.07	27.16	21.65	—
26	29.64	34.72	24.03	24.16	—
27	30.28	34.78	26.02	26.49	—
28	25.72	32.85	21.15	21.75	—
29	26.25	30.96	22.81	20.95	—
30	20.68	20.82	18.82	17.81	—
31	23.70	26.99	20.75	19.12	—
32	20.45	26.45	18.67	17.24	—
33	28.11	29.57	23.45	26.40	—
34	30.45	30.40	29.62	28.68	—
35	31.85	30.93	24.03	22.33	—
36	21.57	30.34	21.28	22.06	—
37	20.37	23.82	18.85	19.78	—
38	18.52	18.52	14.17	15.77	—
39	20.65	20.14	15.09	16.10	—
40	18.74	22.70	14.40	15.57	22.21
41	20.06	23.34	17.12	19.85	21.60
42	20.79	23.34	16.91	18.32	26.01
43	19.86	21.78	16.76	23.29	—
44	16.59	21.58	14.08	17.20	—
45	14.74	19.43	11.40	11.94	—
46	16.90	21.67	12.63	12.45	—
47	17.67	25.08	18.69	18.39	—
48	14.71	19.51	9.65	13.93	—
49	13.40	23.22	14.66	13.59	—
50	8.52	18.70	12.57	11.77	—
51	6.57	17.83	8.53	8.70	—
52	5.84	19.41	8.50	8.75	—
53	7.23	16.26	9.16	9.31	—
54	8.25	16.39	9.92	10.15	—
55	6.16	11.53	7.75	7.64	—
56	7.23	11.62	8.00	8.05	—
57	10.23	17.50	12.88	12.83	—
58	7.01	11.53	8.71	9.62	—
59	10.12	16.45	9.44	12.20	—
60	11.74	14.40	10.24	11.14	13.31
61	7.20	14.50	11.72	10.70	16.04
62	7.05	20.12	13.50	14.16	15.53
63	10.28	18.68	16.07	14.80	20.75
64	6.44	17.06	12.20	10.82	16.17
65	12.68	17.57	11.80	10.57	14.19
66	17.56	18.01	14.20	12.83	15.71
67	18.91	21.83	14.88	12.84	19.26
68	20.20	22.47	15.97	16.70	17.68
69	16.65	19.77	12.82	14.25	18.83
70	9.21	16.23	12.54	10.54	16.41
71	11.21	18.53	12.28	10.42	16.82
72	9.49	19.28	10.62	9.19	15.57
73	5.80	11.22	8.85	7.52	10.05
74	3.88	17.20	6.45	6.03	8.54
75	15.40	19.97	16.70	15.29	17.85
76	9.41	21.89	9.67	8.79	17.16
77	6.08	13.30	7.75	6.65	9.06
78	10.93	19.36	12.03	11.20	17.23
79	7.55	16.89	8.43	7.41	10.65
80	4.90	22.62	5.58	5.39	13.78
81	5.54	11.35	7.49	6.69	8.04
82	5.13	21.25	6.56	6.06	9.10
83	7.06	12.79	8.54	7.92	8.68
84	5.19	9.52	7.77	7.09	8.55
85	7.15	14.07	—	—	13.47
86	4.77	14.47	9.24	8.06	13.48
87	7.73	16.30	—	—	13.66
88	6.30	13.11	—	—	10.54
89	5.07	32.38	9.42	9.11	—
90	6.83	35.84	18.46	14.10	—
91	6.76	34.48	13.71	12.56	—
92	8.88	44.92	24.44	20.90	—
93	8.61	23.57	15.53	16.59	—
94	5.50	12.35	9.60	9.07	10.92
95	7.63	22.71	15.21	15.94	19.98
96	3.61	14.33	10.40	9.21	12.90
97	4.04	16.46	15.59	15.21	16.28
98	10.56	3.40	26.60	20.56	—
99	10.36	50.30	31.19	31.31	—
100	10.95	51.48	17.28	16.66	—
101	6.76	57.58	13.18	14.90	—
102	8.65	73.17	27.77	23.09	—
103	10.11	40.25	13.28	14.25	32.90
104	9.21	31.36	9.33	14.17	16.11

^aBased on a composite acre.

$$c_{13} = (b_{13}C_{13} + b_{14}C_{14} + b_{15}C_{15} + b_{16}C_{16}) a_{13} \quad (16)$$

in which the notation is the same as in equation 15 and a_{13} is the reciprocal of the weighted yield as given by equation 15.

Other data

The base or minimum food-wheat and feed-grain requirements (D_2 and D_1 , respectively, in equations 12 and 13) were based on normal per-capita consumption, United States population of January 1955 and net exports of the 1954 crop year. These estimated values are 677.5 million bushels of wheat and 3,548.9 million bushels of feed grains, in corn-equivalents.

The grain prices p_{ij} , used in computing r_{ij} in equation 9, were derived from averages of 1945-54 average state prices and normal within-state price gradients. These prices are included in the Appendix. The price of corn was used to represent all feed grains, because output of other feed grains was expressed in terms of corn-equivalent feed units.

Data Used in the Ex Ante Model

Yields or outputs

The yields of the *ex ante* model are those expected when fertilizer is applied at optimum rates. (See later definition of optimum.) Fertilizer use and optimum yields were determined by fitting functions to fertilizer response data presented in USDA Handbook No. 68 (17). The response data given in this publication are for each of the major plant nutrients—N, P_2O_5 and K_2O —when it is assumed that each of the other two are used at unlimiting rates.⁶ Because the data were presented in this way, simple quadratic functions of the form $Y = a + bN + cN^2$ (in which Y is the estimated yield, N stands for nitrogen, and a, b and c are estimates of the true parameters) were fitted to the nitrogen response data in most cases. In some areas, where little response to nitrogen was evident in the data, a P_2O_5 or K_2O production function was fitted and used if it showed evident response because of the nature of the data. These fitted functions were considered as reduced form functions in which the response to the other nutrients are accounted for in the estimated parameters a, b and c.

If the explicit relationship between N and the other nutrients were known, then the optimum

rate for N would be $\frac{dY}{dN} = [P_n + P_p g(N) + P_k h(N)] P_y$. Given N, then P_2O_5 and K_2O would be given by $g(N)$ and $h(N)$ which represent phosphorus and potash, respectively, as functions of nitrogen. In the absence of these explicit func-

⁶See USDA Handbook 68, p. 3 (17).

tions, optimum rates were found by successive approximations as follows:

- (a) An optimum yield (Y') was estimated.
- (b) For this yield ΔN , ΔP_2O_5 and ΔK_2O (where Δ represents a small change in the specific nutrient) were computed.
- (c) A price was computed by the following linear combinations:

$$P'_f = \frac{\Delta N}{\Delta N} P_n + \frac{\Delta P_2O_5}{\Delta N} P_p + \frac{\Delta K_2O}{\Delta N} P_k,$$
 in which P'_f is the aggregate price of fertilizer inputs, P_n is the price of nitrogen, etc.
- (d) P'_f was set equal to $\frac{dY}{dN} P_y$ and N was solved for, \hat{Y} was then derived.
- (e) \hat{Y} was compared with Y' .
- (f) If \hat{Y} was nearly equal to Y' , then N was taken to be the optimum nitrogen application, and P_2O_5 and K_2O were found by linear interpolation from the data given in Handbook 68 (17).
- (g) If \hat{Y} was not nearly equal to Y' , then steps (a) through (d) were repeated until \hat{Y} was nearly equal to Y' .

This procedure gave the optimum yields by states. These yields were compared with economic optimum yields estimated by USDA agronomists at Beltsville, Maryland.⁷ In about 50 percent of the cases, the separate estimates were very close. When they differed by as much as 2 bushels, the new optimum yield was estimated by averaging the two independent estimates. Finally, the intercept value of the fitted fertilizer response functions was adjusted so that \hat{Y} was equal to this average. Hence, fertilization rates did not change because of yield adjustments. To estimate regional yields a further assumption was made. We assumed that the response curves of fertilizer use within states were the same and that yield differences observed for the same application of fertilizer in separate regions were due to different levels of nutrients in the soil. Therefore, optimum yields for regions differed only because of differences in the grain and fertilizer prices. There was, of course a wide range in fertilizer application rates within states. Table 6 presents the estimated optimum yield by crops and regions.

Production costs

Labor costs in the *ex ante* model were made up of the labor inputs of the fully mechanized production activities used in the *ex post* model plus

Table 6. Estimated net^a yields for wheat and feed grains (in bushels), optimum fertilizer use, by regions, ex ante data.

Region	Wheat	Corn	Oats	Barley	Sorghum
1	29.4	68.8	50.1	48.0	—
2	22.0	82.0	34.3	41.1	—
3	25.4	69.2	29.1	30.4	—
4	26.2	70.8	32.2	39.3	—
5	32.9	84.5	47.0	40.9	—
6	24.9	78.4	46.8	34.7	—
7	28.0	75.6	66.3	43.4	—
8	28.4	74.8	67.0	43.8	—
9	25.0	75.6	59.2	39.5	—
10	22.0	65.9	43.4	27.5	—
11	22.0	62.6	45.3	27.6	—
12	23.5	59.6	48.8	29.4	—
13	22.6	65.9	44.4	28.0	—
14	23.0	61.7	42.8	28.7	—
15	—	55.5	46.5	—	—
16	26.2	61.6	56.6	41.4	40.9
17	24.9	61.6	52.9	40.2	40.9
18	26.5	60.3	56.0	41.4	40.9
19	20.8	72.6	38.9	—	40.9
20	25.0	73.0	54.6	36.8	—
21	23.5	67.4	52.7	35.6	—
22	23.9	61.5	50.8	29.9	—
23	24.4	59.8	48.7	29.3	—
24	24.6	59.8	49.2	29.3	—
25	30.5	77.6	50.8	30.6	—
26	30.8	78.3	51.3	34.7	—
27	30.8	77.7	51.5	37.7	—
28	30.7	78.2	50.8	34.0	—
29	31.2	78.6	65.5	41.4	—
30	32.2	74.7	67.9	38.0	—
31	31.2	78.3	65.4	41.0	—
32	31.4	77.6	66.1	41.0	—
33	34.2	53.5	56.4	69.5	—
34	33.9	53.5	56.4	69.5	—
35	31.3	73.1	60.8	55.0	—
36	31.1	73.7	60.6	55.2	—
37	31.7	85.9	68.3	42.3	—
38	32.8	87.5	73.0	40.5	—
39	32.6	86.7	72.3	39.8	—
40	32.6	85.8	73.0	40.0	—
41	25.8	65.2	52.2	29.7	28.0
42	26.2	65.3	52.6	30.2	28.5
43	27.1	71.4	55.8	31.9	28.6
44	24.2	84.7	58.1	32.2	—
45	19.7	79.6	50.2	26.8	—
46	20.1	76.0	55.4	31.3	—
47	21.5	64.0	52.7	34.2	—
48	19.8	61.2	50.3	31.0	—
49	19.3	60.7	50.3	30.8	—
50	12.4	49.8	48.6	28.3	—
51	12.4	37.8	43.2	24.9	—
52	11.3	37.8	43.8	25.6	—
53	13.4	37.8	42.2	24.6	—
54	14.5	38.2	43.1	24.9	—
55	14.6	41.1	42.4	22.2	—
56	18.0	42.4	43.6	23.3	—
57	15.7	42.8	43.2	23.0	—
58	15.6	41.9	43.2	22.8	—
59	15.8	42.8	43.3	22.9	—
60	23.1	44.3	39.1	18.5	27.2
61	16.0	42.5	40.9	19.8	26.4
62	12.5	44.3	45.1	22.7	22.5
63	16.8	44.3	40.1	19.5	27.2
64	15.6	43.9	42.8	20.7	26.6
65	24.3	44.3	42.8	20.9	30.5
66	20.6	37.1	27.0	24.2	29.9
67	19.9	36.3	26.4	23.7	29.1
68	20.1	36.0	26.4	22.9	29.0
69	19.9	35.9	26.4	23.5	28.8
70	16.3	36.4	26.2	23.1	29.5
71	19.6	36.0	26.0	23.5	28.9
72	20.1	35.6	26.6	22.9	28.9
73	13.1	36.0	26.8	22.4	29.0
74	10.6	35.6	22.9	20.8	28.1
75	18.5	47.8	48.4	22.1	25.8
76	19.9	47.3	50.1	23.2	25.2
77	14.3	47.9	42.2	19.2	25.5
78	17.7	48.8	47.3	21.8	26.0
79	17.9	48.3	46.2	21.4	25.7
80	11.3	48.8	43.6	26.9	30.0
81	14.4	46.9	47.7	29.7	27.1
82	9.6	45.9	42.3	26.6	26.1
83	16.2	47.5	49.4	30.7	27.4
84	11.5	47.4	44.9	27.9	27.1
85	9.4	47.9	—	—	27.6
86	9.3	47.9	47.9	29.9	27.7
87	9.3	46.4	—	—	26.5
88	9.3	44.9	—	—	25.5
89	9.7	27.2	53.7	29.6	—
90	9.0	27.8	52.7	28.3	—
91	10.3	25.8	52.8	27.4	—
92	10.2	27.4	53.2	30.3	—
93	9.0	24.4	50.7	22.7	—
94	9.3	44.2	50.3	23.0	8.8
95	7.7	45.1	42.4	21.2	17.7
96	3.7	42.4	37.3	16.5	8.6
97	3.2	36.2	22.1	—	11.5
98	14.3	—	48.2	46.7	—
99	11.7	64.4	52.5	47.7	—
100	17.0	74.3	56.2	48.0	—
101	14.6	93.1	72.6	51.9	—
102	14.7	93.1	72.2	51.6	—
103	13.1	60.1	33.9	23.2	41.2
104	14.9	61.0	31.0	28.0	38.8

⁷Unpublished data of the Farm Economics Division, U. S. Department of Agriculture.

^aSeed has been subtracted from yield; yields based on composite acre which includes cultivated summer-fallow.

additional labor costs associated with the increased yields.

Machinery costs, as for labor, include those associated with mechanized methods of the *ex post* model plus costs associated with increased yields.

The seed and lime inputs were the same as those used in the *ex post* model.

Given the regional yields, the steps used to calculate fertilizer costs by crops and by regions were as follows:

- (a) With regional yields of 1954, and the particular fertilizer production function, N' (nitrogen associated with 1954 yield) was derived from the functions of the type $Y = a + bN + cN^2$.
- (b) This N' was subtracted from N (nitrogen associated with \hat{Y}) to obtain the additional nitrogen required to produce optimum yield.
- (c) Additional quantities of P_2O_5 and K_2O associated with the increment in N were obtained by interpolation from data given in USDA Handbook 68.
- (d) Total fertilizer costs for each crop were finally computed by weighting the additional quantities of N , P_2O_5 and K_2O by their respective regional prices and then summing these costs and adding the sum of 1954 estimates of fertilizer costs.

Miscellaneous costs were adjusted to include the cost of applying additional fertilizer.

Table 7 presents a summary of these costs for the individual crops by region.

Matrix Structure of the Production-Distribution Model

The matrix structure of the production-distribution model can be summarized as follows:⁸

Let

X_{ijk} = the quantity of the k th crop produced in the i th production region and shipped to the j th consumption region,

C_{ijk} = the cost of producing the k th crop in the i th production region and shipping it to the j th consumption region,

B_{ijk} = the land required to produce one unit of the k th crop in the i th production region (B_{ijk} are the same for all j)

T_i = acreage of land available for grain production in the i th region,

a_{jk} = the consumption requirement of the k th product in the j th consumption region.

The programming objective is to

Table 7. Estimated costs per acre for wheat and feed grains, land cost omitted, optimum fertilizer use, by regions.

Region	Wheat	Corn	Oats	Barley	Sorghum
1	\$40.0	\$53.4	\$33.7	\$40.5	—
2	44.7	59.3	26.3	26.6	—
3	41.4	56.4	28.3	34.7	—
4	40.4	57.2	27.6	39.3	—
5	36.7	50.2	32.1	32.6	—
6	31.8	52.3	30.6	31.9	—
7	34.7	57.5	35.1	34.9	—
8	33.0	53.9	31.9	32.5	—
9	31.2	55.1	31.5	33.0	—
10	25.6	53.1	24.7	24.0	—
11	32.5	50.8	30.7	29.7	—
12	26.2	42.8	28.3	23.2	—
13	28.6	52.7	25.6	23.6	—
14	32.1	50.2	29.5	28.7	—
15	—	50.9	32.2	—	—
16	20.5	50.8	36.6	—	\$28.1
17	22.3	45.2	35.8	—	31.6
18	24.2	43.2	35.0	—	31.1
19	22.1	38.9	23.6	—	28.1
20	29.9	48.8	34.5	35.2	—
21	25.8	45.3	28.3	29.4	—
22	31.4	47.4	33.0	32.8	—
23	36.4	47.1	34.4	35.6	—
24	39.3	48.6	36.3	34.2	—
25	37.1	44.7	39.8	28.4	—
26	34.8	49.3	30.9	29.4	—
27	32.2	50.3	29.4	30.0	—
28	29.4	41.4	25.4	26.2	—
29	35.4	54.6	35.7	31.7	—
30	33.9	46.2	34.0	25.4	—
31	28.5	44.5	31.5	28.8	—
32	23.1	43.0	29.6	26.3	—
33	35.6	40.0	37.0	51.4	—
34	34.8	41.1	43.1	52.1	—
35	31.8	57.6	35.9	38.2	—
36	27.4	40.1	25.4	35.2	—
37	27.6	46.4	26.1	24.6	—
38	21.3	45.3	28.5	24.7	—
39	41.1	57.0	36.1	26.9	—
40	38.5	59.5	35.3	25.9	—
41	24.5	48.0	25.6	19.9	29.6
42	26.1	52.3	23.3	21.1	34.1
43	26.0	47.0	24.8	24.6	34.1
44	25.6	45.8	23.2	23.2	—
45	19.8	39.4	19.1	18.7	—
46	18.1	39.1	21.5	13.2	—
47	23.1	39.9	28.7	22.5	—
48	24.2	36.6	26.1	19.3	—
49	21.1	39.6	28.5	14.3	—
50	12.4	30.9	27.0	14.0	—
51	9.8	28.4	14.8	13.5	—
52	8.9	35.6	15.0	14.7	—
53	11.2	32.5	14.8	14.1	—
54	12.6	21.8	15.8	14.8	—
55	10.9	27.1	14.4	11.5	—
56	13.7	26.1	14.1	14.4	—
57	15.4	25.5	17.4	15.1	—
58	12.2	26.0	15.0	14.0	—
59	14.6	20.3	14.2	15.2	—
60	18.9	18.5	15.7	12.4	18.3
61	10.8	25.4	17.5	10.8	23.5
62	9.7	31.1	20.1	15.5	21.1
63	16.2	27.0	23.4	19.0	25.4
64	10.9	28.1	19.9	14.9	20.7
65	19.8	28.1	18.4	13.6	14.2
66	19.9	19.3	20.0	13.4	20.4
67	20.7	27.1	18.7	17.3	28.2
68	22.8	29.6	19.6	21.4	28.1
69	18.8	25.7	17.0	19.1	28.5
70	13.5	23.6	19.7	19.8	25.6
71	16.3	25.6	16.8	18.3	25.6
72	14.5	26.8	15.0	17.4	25.3
73	8.8	19.3	14.6	15.7	19.6
74	6.5	26.9	11.1	14.6	18.6
75	17.7	28.0	26.0	22.3	29.0
76	11.8	44.2	18.9	17.4	26.4
77	8.7	39.2	16.3	15.1	19.8
78	13.5	40.6	20.7	19.9	28.3
79	10.8	38.8	17.0	15.7	20.4
80	8.1	39.7	21.8	16.6	17.8
81	9.9	38.8	17.0	15.7	20.4
82	7.9	42.7	22.8	16.6	21.3
83	12.0	34.7	27.9	21.2	25.7
84	9.4	35.0	25.9	20.7	23.3
85	9.4	23.1	—	—	23.2
86	7.9	36.4	27.7	22.6	25.8
87	10.7	33.3	—	—	25.1
88	9.3	31.1	—	—	22.3
89	7.5	43.1	16.6	8.6	—
90	6.8	35.8	25.2	14.7	—
91	11.4	35.2	21.5	20.6	—
92	8.9	38.6	28.4	21.0	—
93	8.6	23.5	23.1	16.7	—
94	8.1	26.5	18.7	15.2	10.9
95	10.3	26.3	22.3	19.9	20.6
96	5.0	27.6	17.4	13.0	12.9
97	5.1	40.4	15.3	16.2	16.7
98	12.7	—	34.0	30.2	—
99	11.9	64.4	31.5	31.3	—
100	11.2	74.3	26.4	25.6	—
101	8.2	93.1	27.3	23.3	—
102	10.5	93.0	37.1	30.1	—
103	11.8	60.1	22.5	14.2	36.4
104	14.9	60.0	17.7	14.2	18.7

⁸The size of the programming matrix could have been reduced by defining separate transporting activities to transfer wheat from the food sectors (constraints) to the feed sectors of each region. However, the matrix was constructed as described because of the method used to obtain the solution. This was made necessary by the size of the computer used.

$$\text{Min. } f(c) = \sum_i \sum_j \sum_k X_{ijk} C_{ijk} \quad (17)$$

subject to these constraints

$$\sum_j \sum_k X_{ijk} B_{ijk} \leq T_i, \quad (18)$$

$$\sum_i X_{ijk} = a_{jk}, \quad (19)$$

$$X_{ijk} \geq 0. \quad (20)$$

Data Used in the Production-Distribution Model

The production costs used in the production-distribution model were the same as those used in the *ex post* model and are given in table 5.

Freight cost or shipping charges for each programming activity were derived from tariffs furnished by the Transportation and Storage Division of the Commodity Stabilization Service.

The specific freight charges used for programming are presented in Appendix tables A-2 through A-6. The charges listed in tables A-2 through A-6 represent two tariff schedules — Commodity and Class I. Commodity tariffs are available only for routes and commodities for which the volume usually shipped warrants the setting of a special rate. If this is not the case, Class I rates prevail. The Class I rates are higher than Commodity rates in nearly all cases.

The activity cost of the production-distribution model were computed by adding the appropriate production cost and shipping charges as given in equation 21,

$$C_{ijk} = C_{ij} + t_{ijk} \quad (21)$$

where t_{ijk} is the estimated cost of shipping the k th commodity from the i th production region to the j th consumption region.

The t_{ijk} 's in equation 21 were computed as follows:

$$\text{For food wheat: } t_{ij1} = t'_{ij1} \quad (22)$$

$$\text{For feed wheat: } t_{ij2} = \frac{t_{ij1}}{K_1} \quad (23)$$

For the composite feed-grain activity:

$$t_{ij3} = \frac{Y_{12}b_{12}t'_{ij2} + Y_{13}b_{13}t'_{ij3} + Y_{14}b_{14}t'_{ij4} + Y_{15}b_{15}t'_{ij5}}{Y_{12}b_{12}K_2 + Y_{13}b_{13}K_3 + Y_{14}b_{14}K_4 + Y_{15}b_{15}K_5}$$

In which the t'_{ij2} stands for the cost per unit of shipping corn from the i th production region to the j th consumption region; t'_{ij3} represents the same cost for oats, t'_{ij4} is the same cost for barley and t'_{ij5} is the cost for sorghum. All other symbols have the same meaning as used in the *ex post* model.

The acreage constraints function 18, were the same as those in the *ex post* model.

Regional consumption requirements (a_{jk}) were

calculated by allocating national requirements of 1954 to regions on the basis of normal consumption rates, population and livestock numbers. Actual net exports were allocated to consumption regions by the port of exit.

QUANTITATIVE RESULTS

Results of the *ex post* and *ex ante* models will be presented first. Charts or maps will be used for presentation of most quantitative results. Then, comparisons will be made of regional production patterns specified by solutions to these two models to show how production location might change with (a) change in regional production techniques and (b) change in national requirements or needs. Next, certain results will be presented in tabular form to allow comparison of changing resource needs arising from changes in resource combinations and changing national requirements. Following this, the results derived by the production-distribution model will be presented. Finally, this result will be compared with the result of the *ex post* model for comparable output mix and national requirements levels.

As stated earlier, optimum regional patterns of production for the *ex post* and *ex ante* models were defined for several levels of wheat and feed-grain requirements. These locational patterns were derived by the technique of variable resource of parametric programming. By this technique, a unique program (or, in technical terms, a unique basis) is obtained for each level of a particular resource or combination of resources. In the analysis of wheat and feed grains, a program (or regional production pattern) was obtained for numerous combinations of wheat and feed-grain outputs or demand levels. Although a large number of programs (i. e., the specific grains and quantities to be produced in each region) were available for presentation within certain ranges of the requirements variables, only a few of the possible programs will be presented. These should provide a general picture of how regional adjustments in production might occur under demand expansion and technological change.⁹

In the following section, production specifications will be stated in terms of net production. As used here, net production of wheat is equal to gross national production of wheat minus (a) the seed wheat required to produce the net production, (b) wheat used for feed and (c) wheat produced in the unnumbered areas (fig. 1). In 1954, the sum of these three items was about 200 million bushels. For feed grains, the term net production refers to

⁹Note that for certain changes the production pattern appears quite stable, while for other changes the production pattern appears quite dynamic. Such phenomena are due to the economic structure as postulated in a linear programming framework. Constant input-output coefficients with particular bounds lead to great stability in some instances. In others, they lead to significant changes.

the gross national production of the four feed grains (stated in corn-equivalents) minus (a) seed required to produce the net production (b) corn consumed as silage and (c) feed grains produced in the unnumbered areas of fig. 1, plus wheat used for feed. The sum of these four items in 1954 was about 750 million bushels in corn-equivalents.¹⁰

Regional Production Pattern, Ex Post Model

The *ex post* model uses data related to the year 1954. The objective is to define a regional production pattern that would give farmers, as a group, the greatest net return while, at the same time, keeping output in balance with particular wheat and feed-grain requirements or consumption mixes at the national level.

Figure 3 represents the economic optimum pattern of regional grain production to produce 663 million bushels of food wheat and 3,561 million bushels of feed grains. This figure results from a model (E in table 1) formulated for previous analysis and is explained elsewhere (6, p. 54). It is presented here to provide a benchmark with which to compare results. Figure 3 presents a regional pattern that reasonably might have occurred in 1954 with production in balance with

¹⁰The reasons for using net production in the analysis are given in USDA Technical Bulletin No. 1241 (6).

demand, as characterized by discrete demand constraints. In interpreting this figure and others that follow, remember that the analysis objective was to determine, within the limitations of our model, a pattern of production that would yield United States farmers, as a whole, maximum net returns. Furthermore, the regional prices used in the programming model were generated by a slightly different regional configuration of grain production than that depicted in fig. 3. Even so, the pattern shown in the figure resembles our *a priori* preconceptions of a balanced grain production, given the assumption of regional producing units as aggregates.

Figure 4 shows the regional production pattern to produce 700 million bushels of food wheat and 4,000 million bushels of feed grains. In other words, the "discrete" demands or requirements have been increased over those upon which the results in fig. 3 are based. Compared with fig. 3, grain production "has moved" into "marginal" regions in Michigan, western Kentucky, southern Alabama, northern Wisconsin, east central Texas and northeast North Carolina.

Finally, fig. 5 shows the regional production pattern to produce 800 million bushels of food wheat and 4,000 million bushels of feed grains. Com-

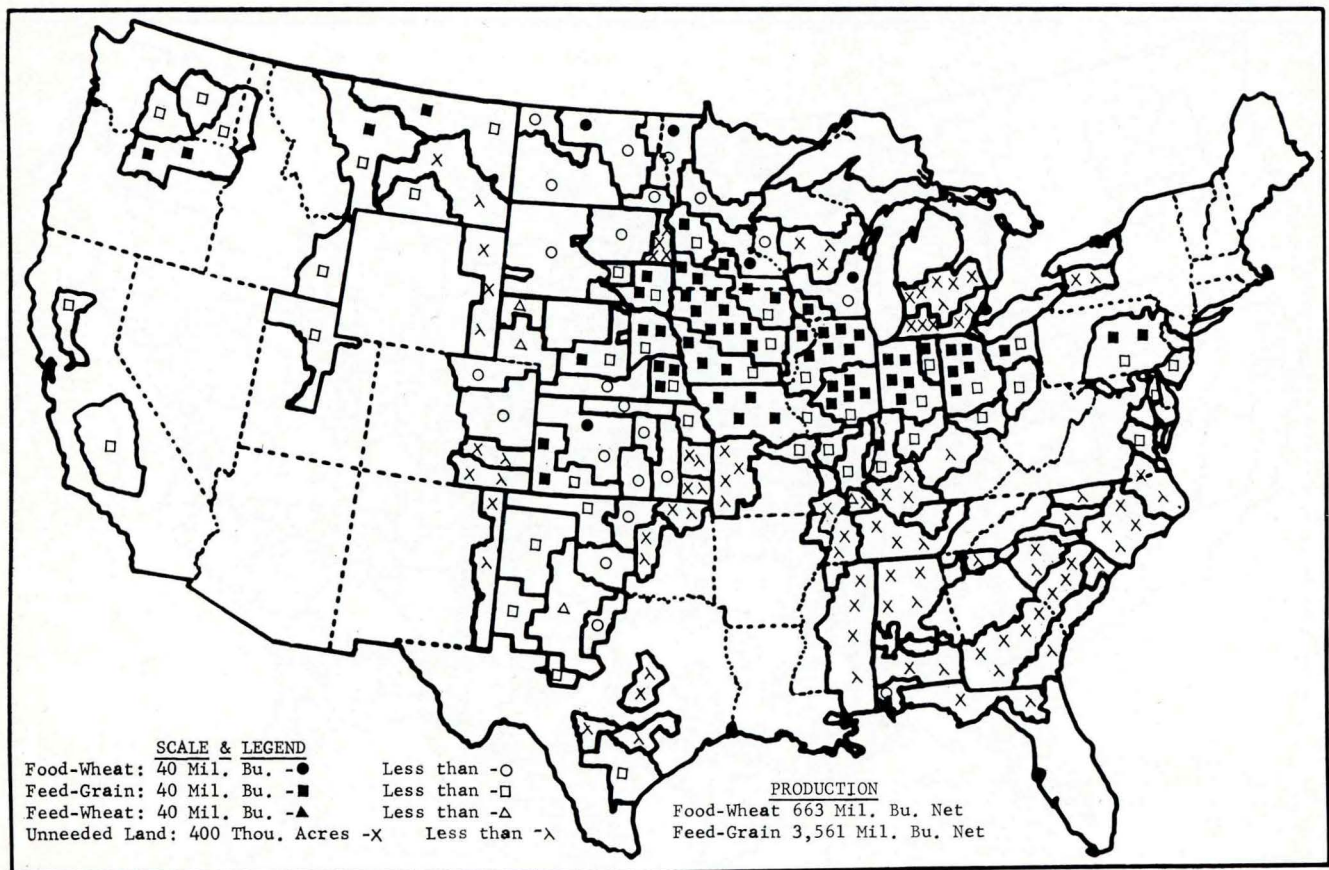


Fig. 3. Economic optimum pattern of regional grain production for specified production levels, ex post data.

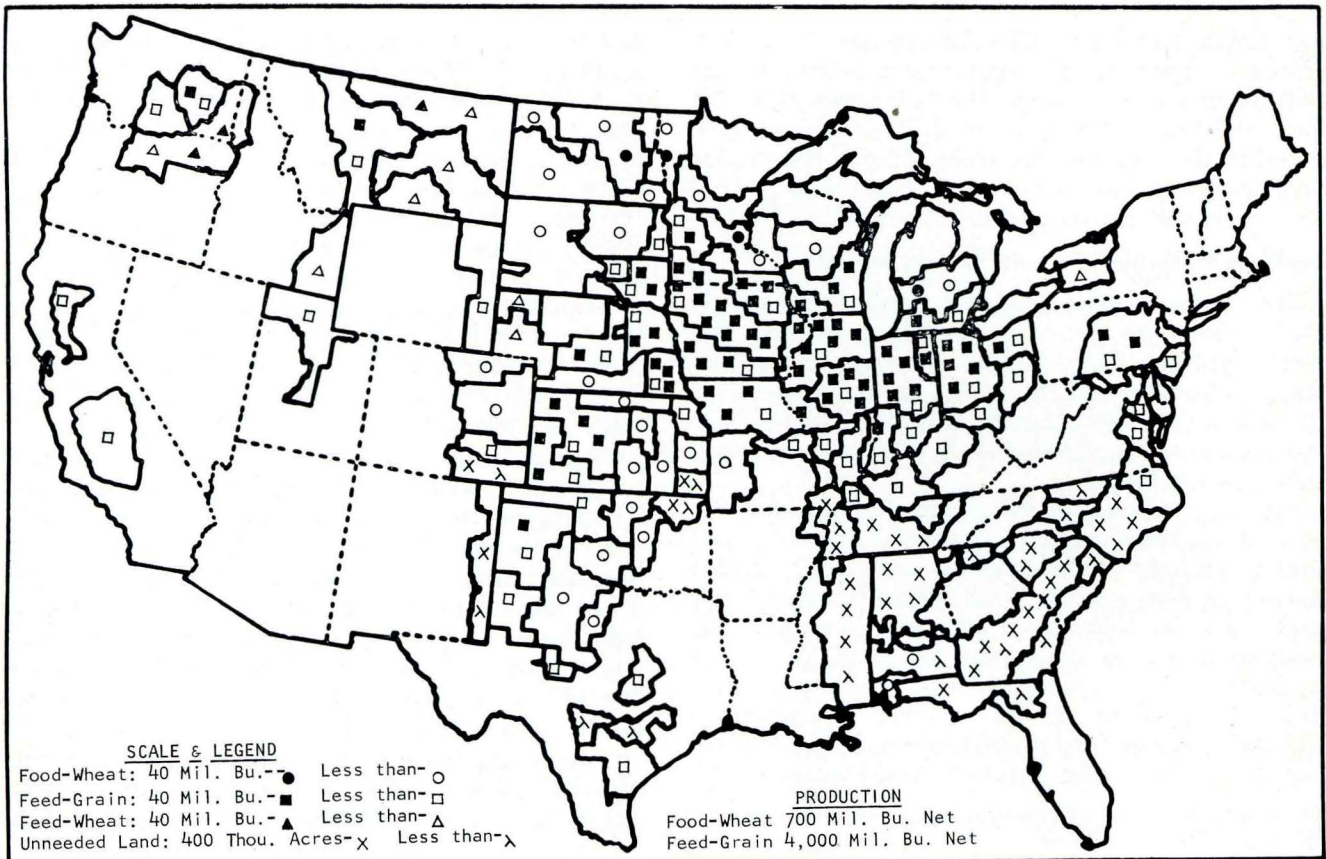


Fig. 4. Economic optimum pattern of regional grain production for specified production levels, ex post data.

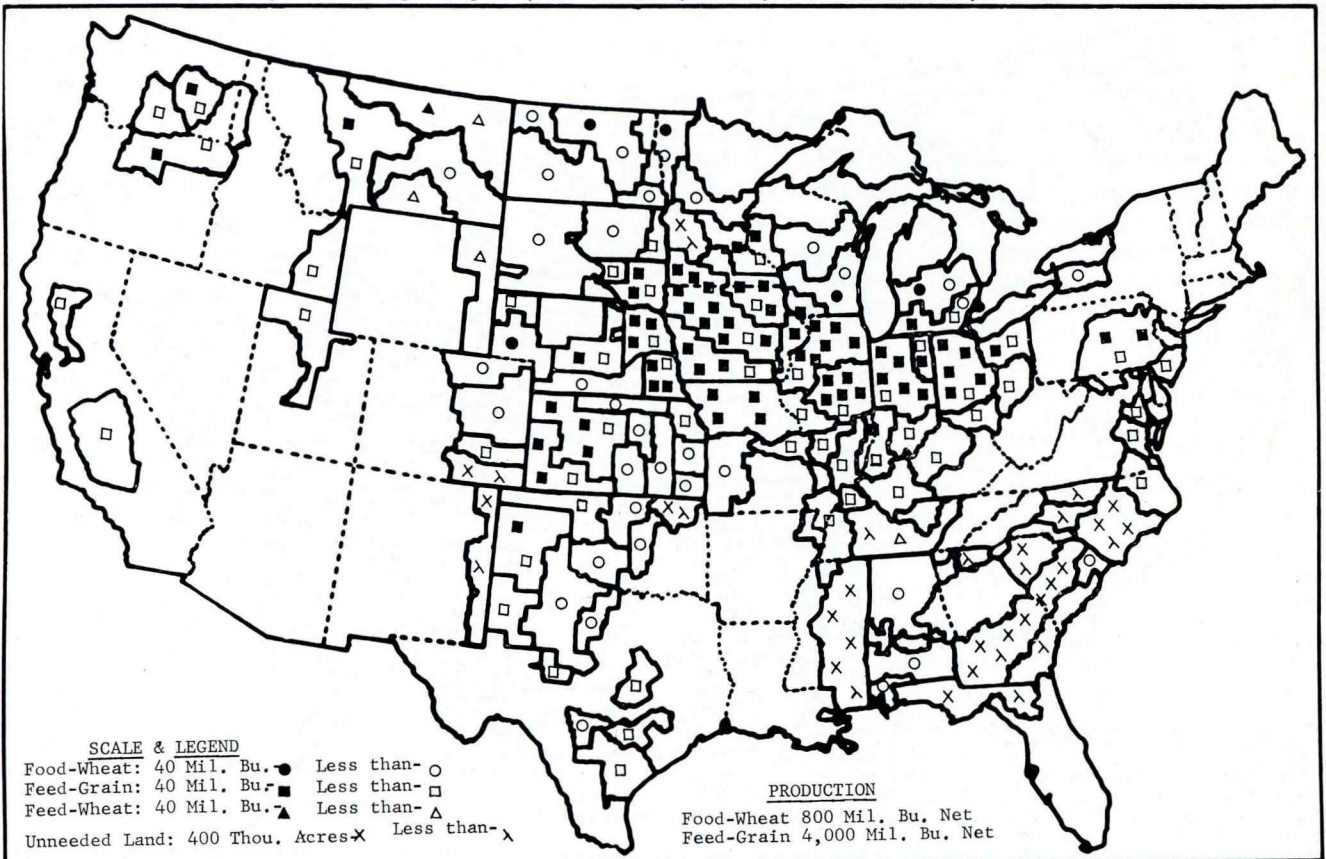


Fig. 5. Economic optimum pattern of regional grain production for specified production levels, ex post data.

pared with fig. 4, grain production now moves into regions 20 and 21, which include parts of Tennessee, Missouri and Arkansas, and into central Texas—region 87. These regions supply the wheat withdrawn from region 33 in southern Michigan and shifted to feed-grain production. The rest of the additional 100 million bushels of wheat required is supplied by region 86 (Texas), regions 17 and 18 (Alabama), region 10 (South Carolina) and region 68 (Kansas).

If demand or requirements had been raised even higher, regions in addition to those shown in fig. 5 would have been drawn into production. Limits in the computing facilities at the time prevented us from specifying additional production (i. e., raising demand or requirements levels) to the point that all available land area for crops was used and all national demand constraints filled. Of course, such a point could be reached with an infinite number of combinations of outputs or regional production patterns for wheat and feed grains, but only one combination would satisfy the criterion of profit maximization.

Regional Production Pattern, Ex Ante Model

In developing the data used in the analysis we assumed that (a) farmers applied fertilizer at maximum profit rates and (b) only mechanized produc-

tion techniques were used to produce wheat and feed grains. These changes in production technique are perhaps those that have had, and promised to have, the greatest impact on output and shifts in comparative advantage over time. Again regional production patterns are presented for several combinations of wheat and feed-grain outputs or national requirements level.

Figure 6 presents the optimum pattern of grain production, under the *ex ante* model, to meet food-wheat requirements of 678 million bushels and feed-grain requirements of 3,549 million bushels. The figures are in net terms and are approximately equal to the disappearance of 1954. If we compare fig. 6 with fig. 3—the figure showing the pattern of the *ex post* model at a comparable requirements level—we see that these two figures differ significantly. The acreages in the Corn Belt fringes have shrunk considerably. The same is true, but to a lesser degree, for the acreages of wheat in the Great Plains. Part of the contraction in the Corn Belt results from increased per-acre yields because of much higher rates of commercial fertilizer use in the more productive parts of this area. Part of the contraction is due to a substitution of grain acreage in the Delta, the eastern and western Appalachian area and the Southwest. This latter result indicates that these southern areas could in-

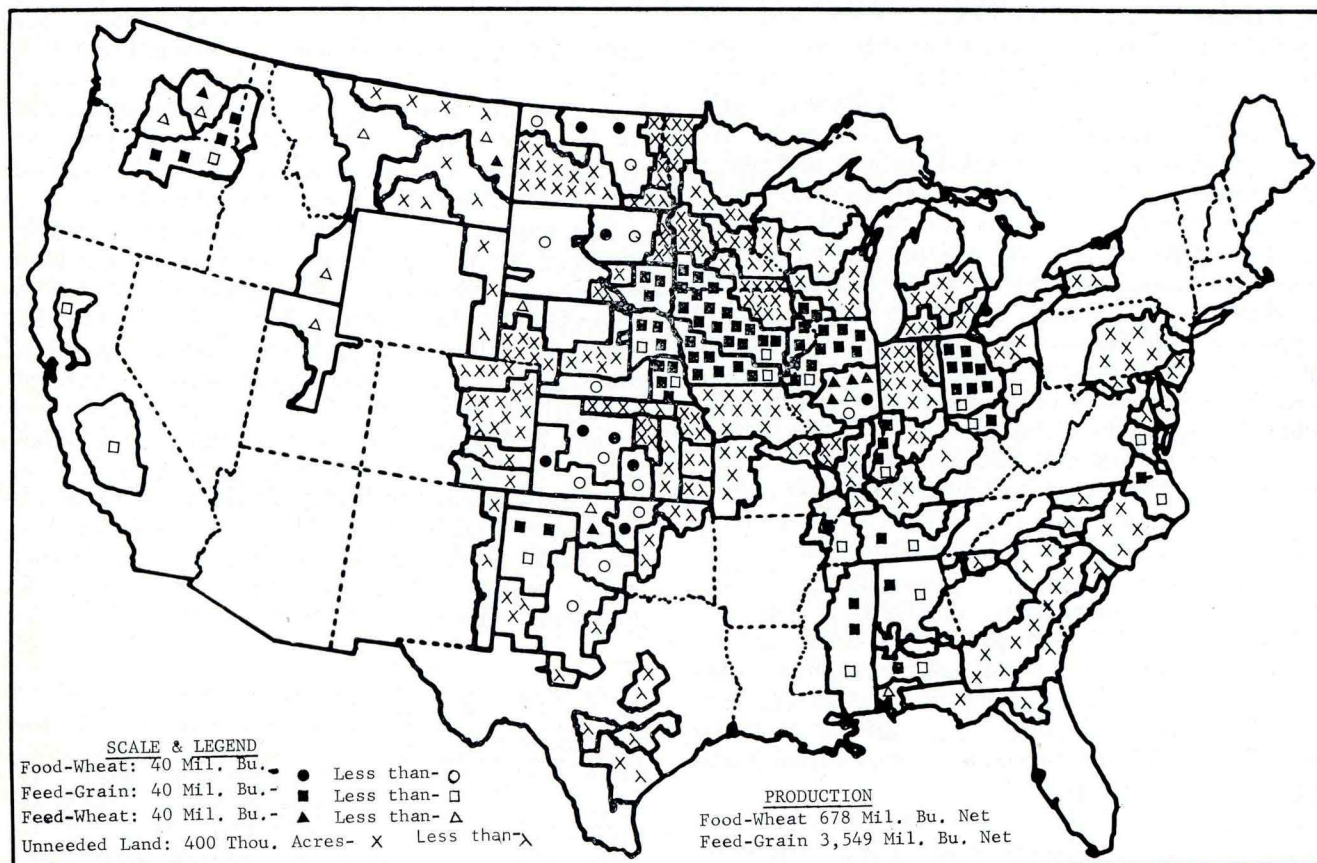


Fig. 6. Economic optimum pattern of regional grain production for specified production levels, ex ante data.

crease their competitive positions in grain production if more intensive cultivation methods were used to produce corn and other feed grains. The changes in cultural practices implied in the production coefficients underlying the yields (tables 6 and 7) of the *ex ante* model are greatly different from those of the present. Fertilization rates are assumed to be much higher than those currently being used. And this change requires that (a) farmers be aware of the output-increasing effects of fertilizer use and (b) they have the money to buy it. The latter perhaps is a significant obstacle to raising the competitive advantage of the Southeast in grain production. But the use of mechanized production methods in many areas not only means sizable investments in machinery but also may mean farm consolidation sufficient to cause the necessary machinery inputs and outputs to be economically attainable. Such changes may take time and involve considerable adjustments in the economic and social structure of an area. Nonetheless, these results suggest that improvements in the relative competitive positions of these areas are possible, if not probable.

Figure 7 shows, for the *ex ante* model, the regional production pattern when the wheat requirement is increased to 700 million bushels and feed-grain requirements are increased to 4,000 million bushels. These figures represent increases of approximately 3 and 12 percent in wheat and feed grain, respectively, as compared with the requirement levels used for fig. 6. They are met as production is extended to region 89 (Montana), region 53 (North Dakota), region 46 (Minnesota and Iowa), regions 7 and 8 (North Carolina) and region 32 (Indiana). A shift in production from wheat to all feed grains takes place in region 38 (Illinois).

Figure 8 shows the regional production pattern to meet national requirements of 700 million bushels of wheat and 5,000 million bushels of feed grains, supposing that farmers used the fertilization and mechanization techniques outlined. The additional 1,000 million bushels of feed grains are to be produced in region 43 (Missouri and Illinois), region 12 (South Carolina and Georgia), region 31 (Indiana) and regions 7 and 9 (North Carolina); and by some shifts in acreages from wheat to feed grains in region 74 (Kansas) and region 32 (Indiana).

Similarly, fig. 9 shows the regional production pattern to meet demand requirements at levels of 700 million bushels of wheat and 5,600 million bushels of feed grains with these production practices. The additional 600 million bushels of feed grains (as compared with fig. 8) are forthcoming from regions 39 and 43 (Missouri and Illinois) and region 14 (Georgia).

Figures 10 through 12 show the changes in regional production patterns when wheat production is raised from 700 to 800 million bushels and feed-

grain production is then increased stepwise from 4,000 to 5,600 million bushels. Comparing figs. 7 and 10, we see that the additional 100 million bushels of wheat are forthcoming by extending production to region 53 (North Dakota) and region 38 (Illinois). The feed grain replaced by wheat in region 38 is supplied in region 2 as feed wheat (eastern Pennsylvania). By comparing figs. 10 through 12, we can trace the regional changes and extensions of production that take place as the requirement for feed grains is increased from 4,000 to 5,600 million bushels while the national requirement for wheat is held constant at 800 million bushels. At the highest level (fig. 12), the Southeast (except for mountain areas) concentrates heavily in grain production.

The quantities of wheat and feed grains estimated to be produced by the regions shown in fig. 12 under the assumed production coefficients are approximately equal to projected national demands of 1985. These demands are 1,120 million bushels of wheat and 182.4 million tons of feed grains.¹¹ Because we are dealing with net production in this analysis, the figures presented are not identical. However, the demand figures can be reduced to comparable output figures by subtracting, from the estimated wheat disappearance or use, wheat usually used for feed, that used for seed and the wheat normally produced in the unnumbered areas in fig. 1 and by subtracting from the estimated feed-grain disappearance or use (an amount equal to 182.4 million tons after conversion to corn-equivalent bushels) the corn fed as silage, feed grains used for seed, and feed grains normally produced in the unnumbered areas of fig. 1, and then adding wheat used for feed (in corn-equivalent).

The analysis, thus, indicates that one output-increasing factor—fertilizer—would insure adequate supplies of wheat and feed grains to meet the projected population and other needs of 1985. The fertilization technique is known, and the assumed rates to be used are economically optimum in terms of fertilizer and crop prices. It was supposed, of course, that capital and farmer knowledge is available. No new biological techniques, except those technically related to higher fertilization rates, are assumed. And it appears that increased fertilizer use alone would provide additional needs beyond that year because the production program (fig. 12) specified to meet the 1985 wheat and feed-grains

¹¹These values represent liberal extrapolations based on the work of Rex Daly (4). For example, if a population of 179 million and 230 million for 1960 and 1975, respectively, are assumed, and the implied linear rate of increase in population is extrapolated to 1985 (i. e., 257.2 million) and the trend in per-capita consumption of wheat likewise is extrapolated, the indicated requirements of wheat for 1985 are 1,138 million bushels, or just 18 million bushels more than the estimate. A population of 230 million for 1975 is the upper limit of current population estimates. Daly's highest rate of increase in feed-grain consumption from 1952-53 to 1975 would amount to 53 percent over 1952-53 if this rate were extrapolated to 1985. However, the 182.4 million tons of feed grains is 159 percent of 1952-53 disappearance.

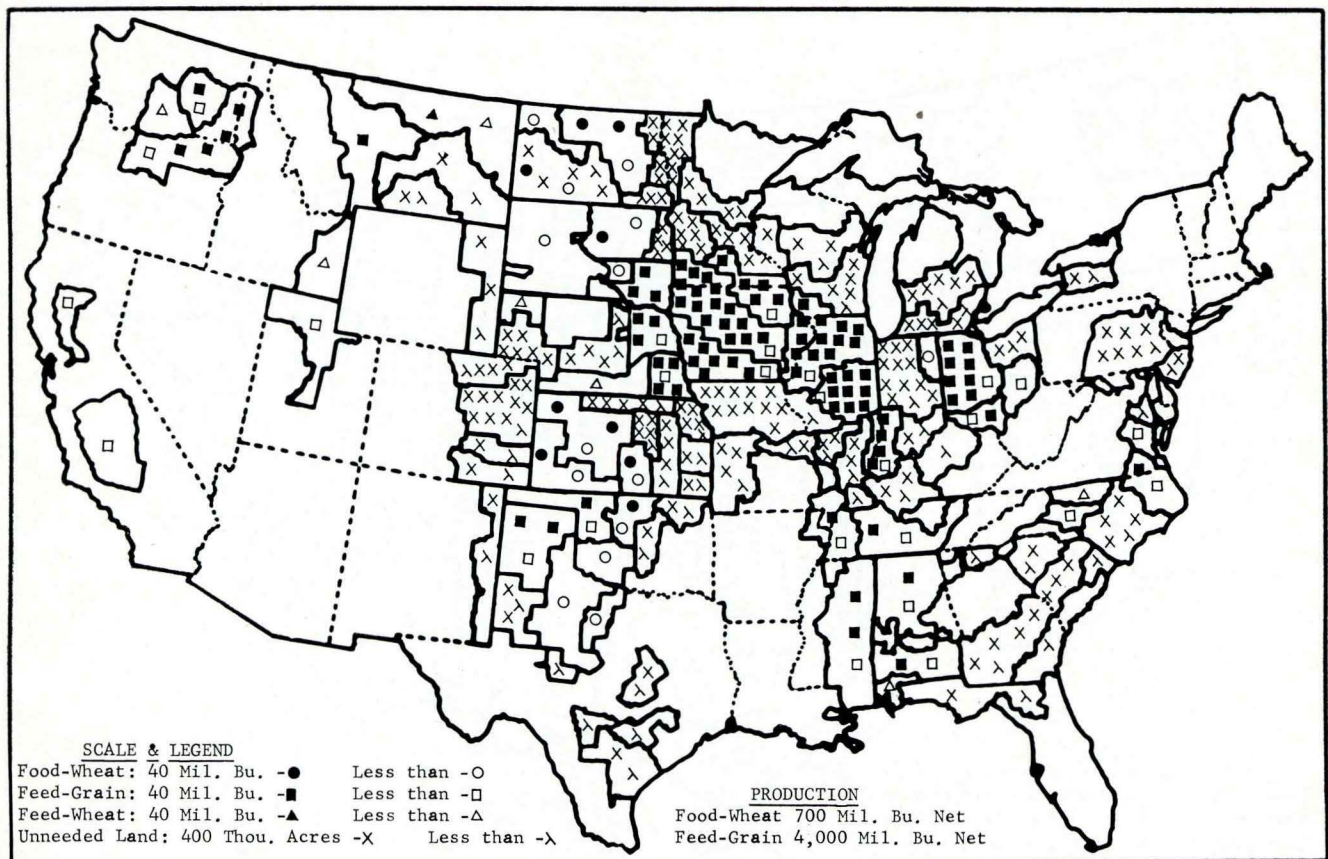


Fig. 7. Economic optimum pattern of regional grain production for specified production levels, ex ante data.

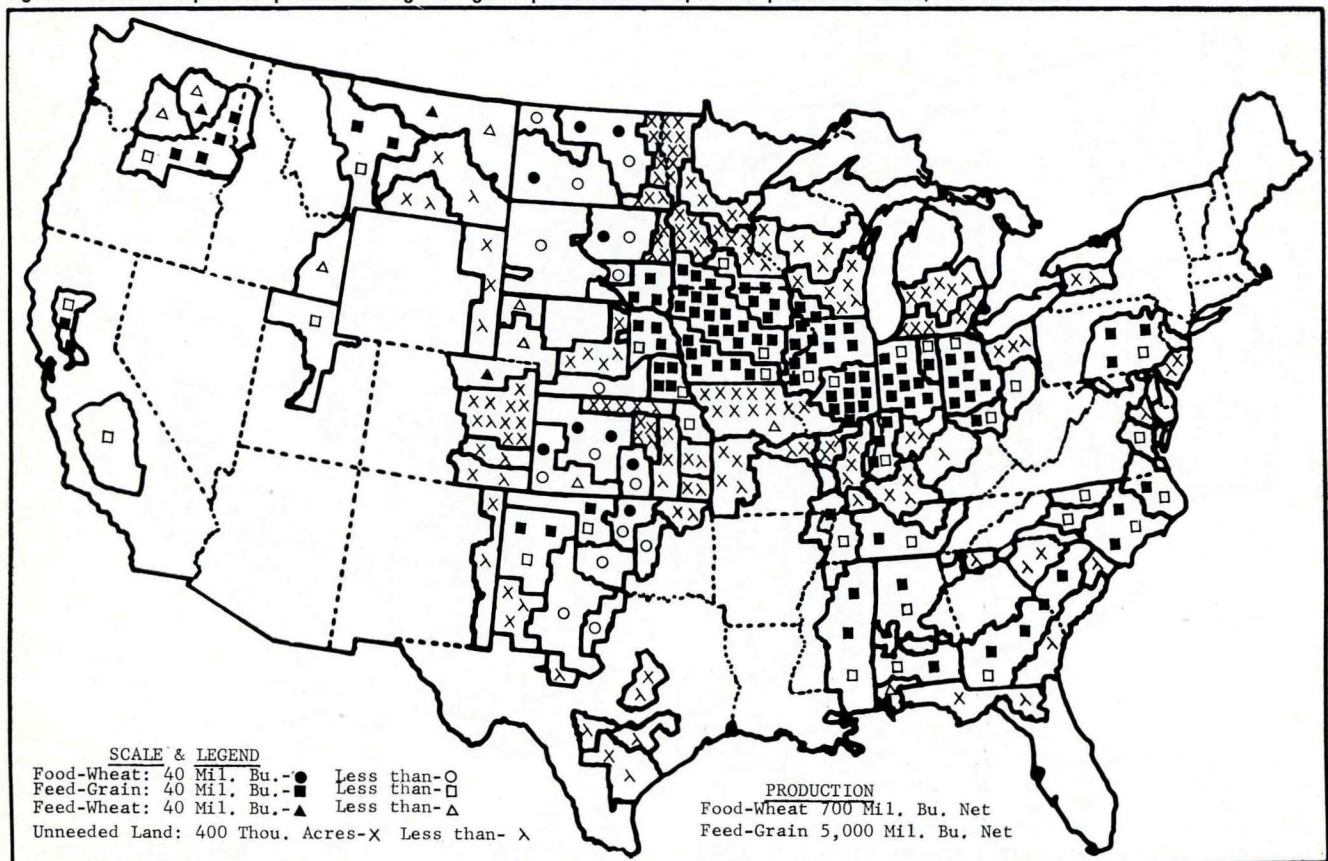


Fig. 8. Economic optimum pattern of regional grain production for specified production levels, ex ante data.

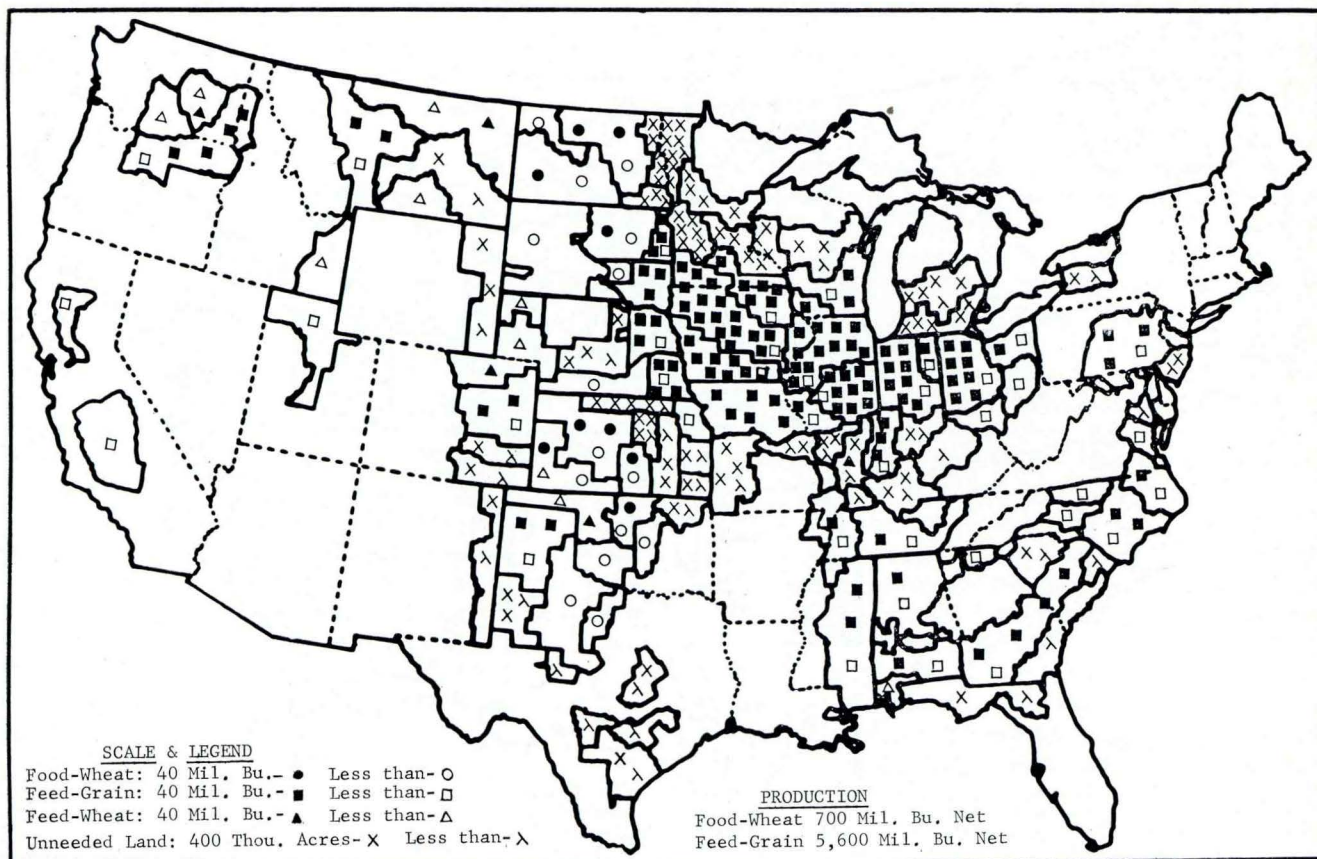


Fig. 9. Economic optimum pattern of regional grain production for specified production levels, ex post data.

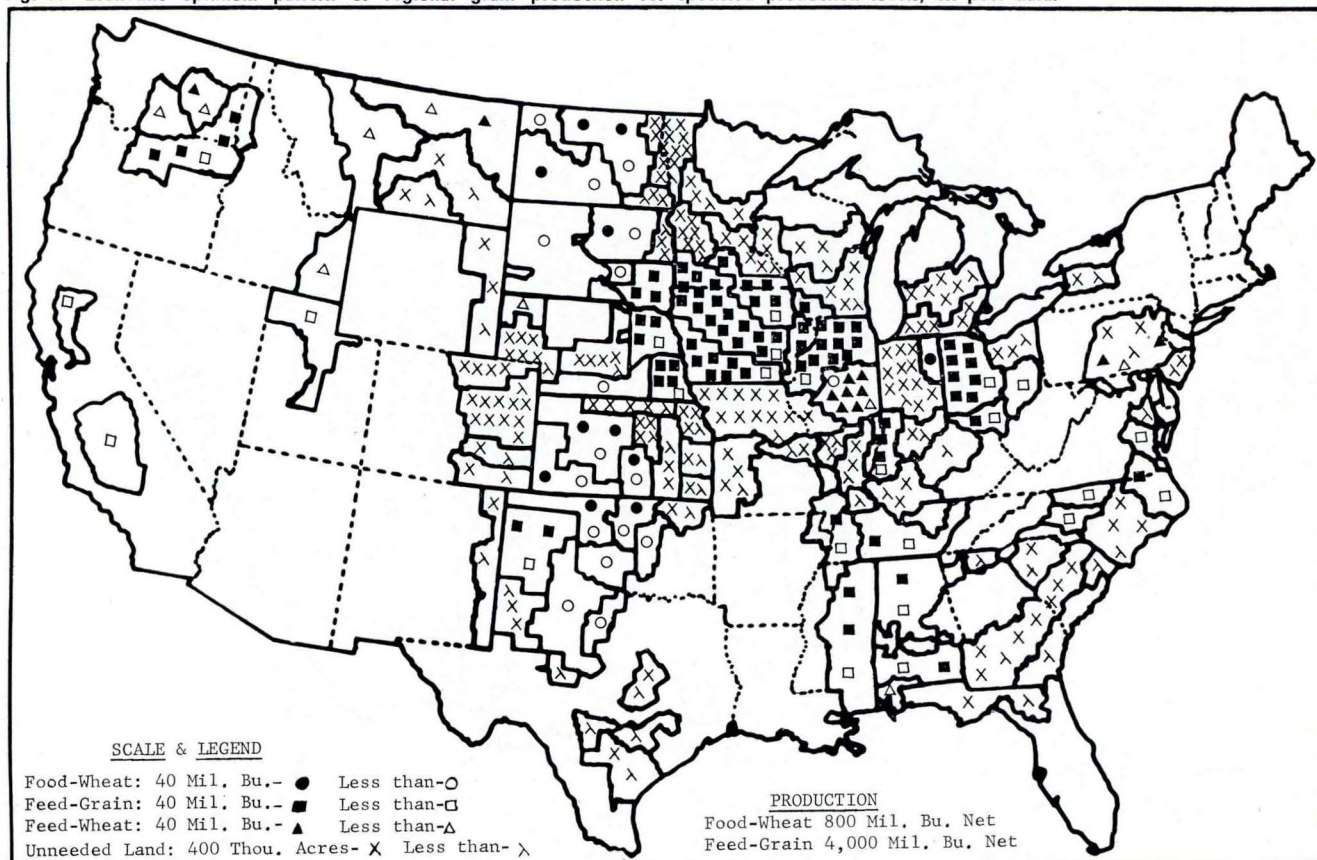


Fig. 10. Economic optimum pattern of regional grain production for specified production levels, ex ante data.

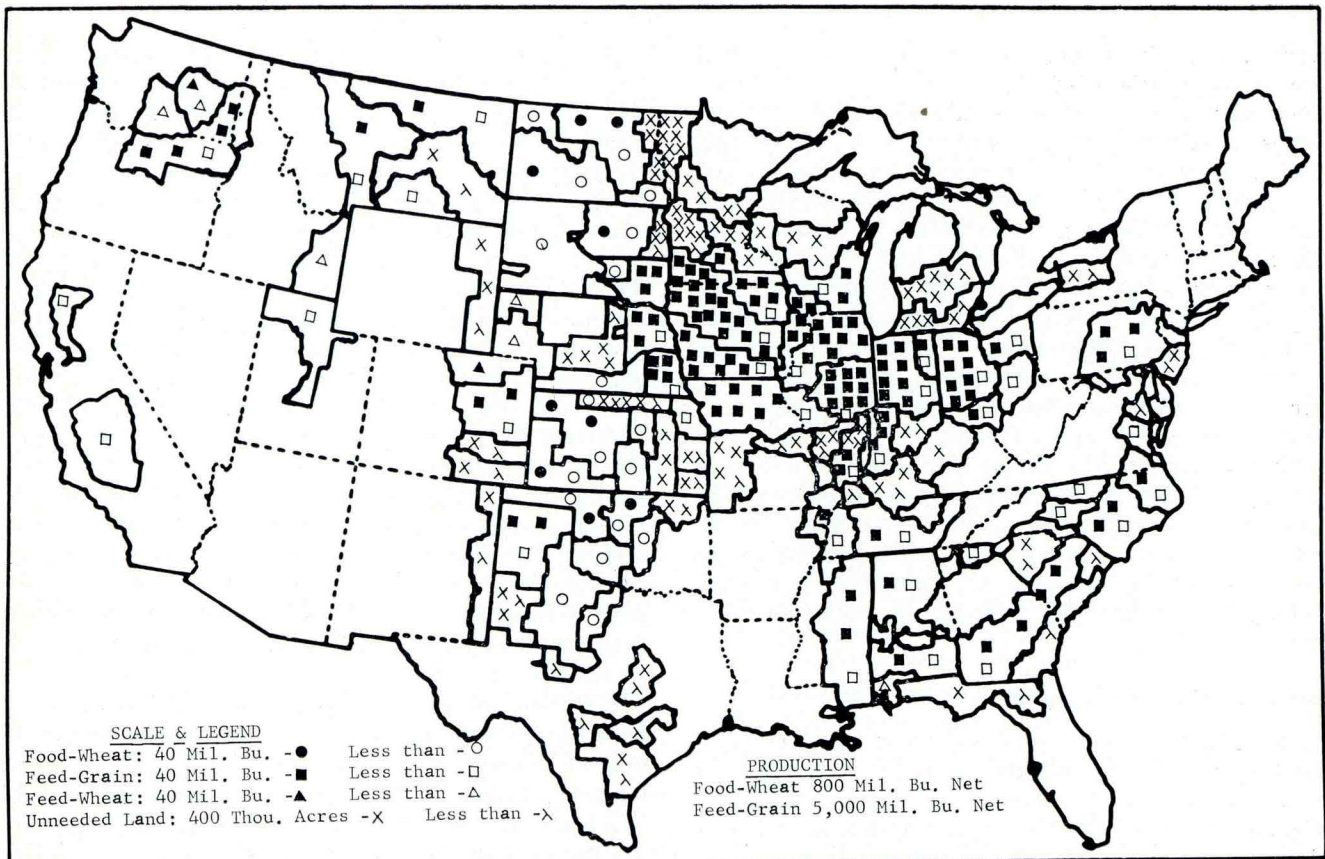


Fig. 11. Economic optimum pattern of regional grain production for specified production levels, ex ante data.

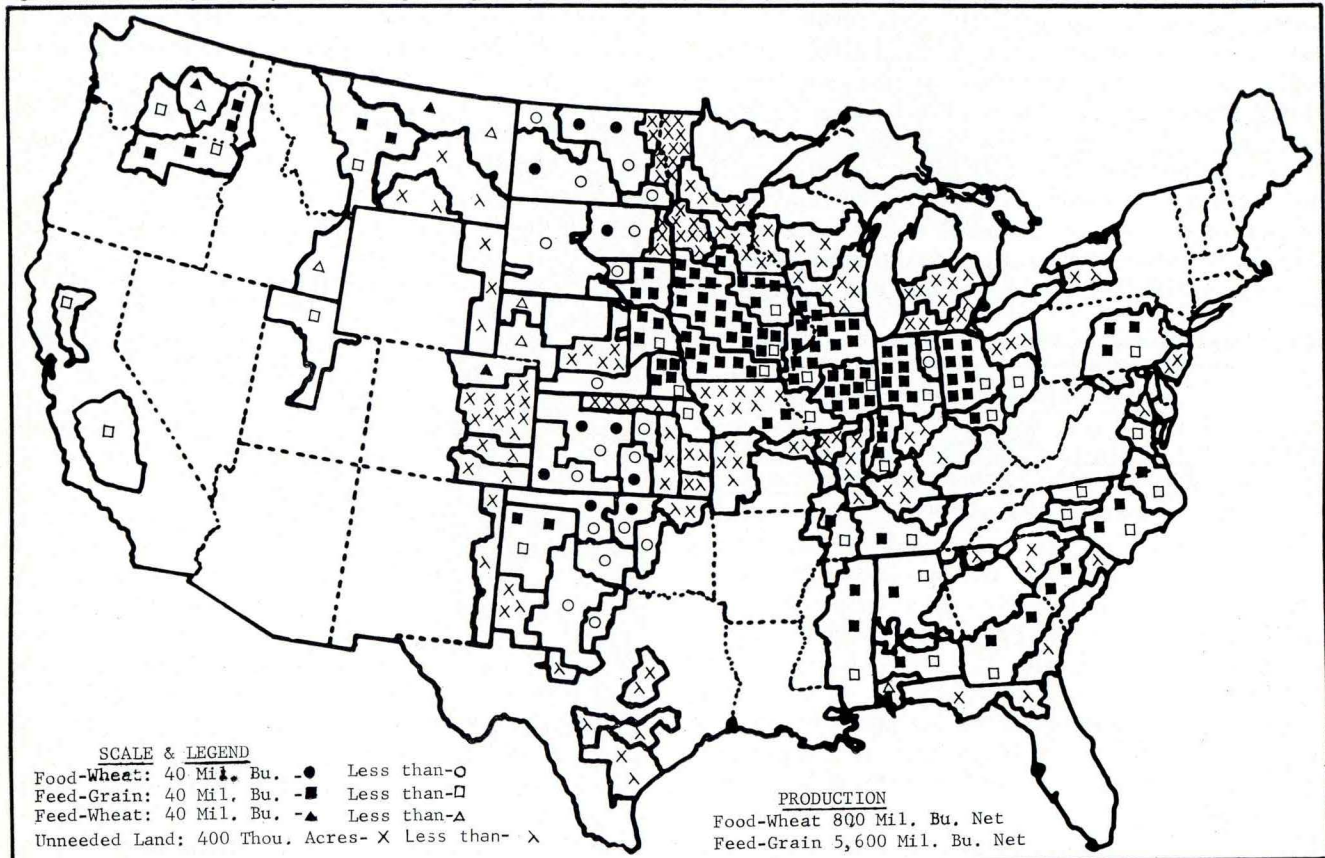


Fig. 12. Economic optimum pattern of regional grain production for specified production levels, ex ante data.

needs leaves approximately 39 million of the 1953 base acreage idle (table 8). Even if the yield of these 39 million acres were only 30 bushels of feed grains, a billion bushels more than the 5.6 billion used in table 8 (last row) could be produced. Or, on the other hand, this same acreage could produce, with wheat yields at a conservative 15-bushel average, an additional 600 million bushels of wheat.

Fertilizer is only one of several factors that could greatly increase production potential of grain during the next few decades. Other practices such as improved hybrids, insecticides, pesticides, herbicides, and new crop rotations also would have output-increasing effects. Hence, they would allow further expansion in production before 1985. Attainment of 1985 domestic food needs appears easily possible. The problem for agriculture is more nearly that of balancing supply and demand and making interregional adjustments to change rather than seeking new means for keeping up with domestic needs.

Resources Required for Specified Production Levels

The quantities and kinds of resources needed to produce future wheat and feed-grain needs are of considerable importance. They are important because they indicate the nature and magnitude of adjustments required for an important sector of agriculture. Data, which are generated on the resources needed along with the production results just presented, have been summarized for the nation as a whole. Only the aggregates for the United States are presented here because of space limitations. The figure showing regional location of production, together with other data given in the Appendix, however, will permit calculation, if desired, of the resources needed in each region.

Table 8 shows the total acres of land required to produce specified levels of wheat and feed grains

Table 8. Land resources required for specified levels of wheat and feed-grain production, ex post and ex ante models.

Combinations of food-wheat and feed-grain production ^a	Ex post model		Ex ante model	
	Acreage required	Difference of 1953 acreage and acreage required ^b	Acreage required	Difference of 1953 acreage and acreage required
(mil. bu.)	(mil. acres)	(mil. acres)	(mil. acres)	(mil. acres)
F-W 678 F-G 3,549	181.2	28.8	122.6	87.4
F-W 700 F-G 4,000	195.1	14.9	134.4	75.6
F-W 700 F-G 5,000	—	—	154.6	55.4
F-W 700 F-G 5,600	—	—	167.6	42.4
F-W 800 F-G 4,000	200.4	9.6	138.4	71.6
F-W 800 F-G 5,000	—	—	158.3	51.7
F-W 800 F-G 5,600	—	—	171.1	38.9

^aProduction levels are in net terms—see text.
^bThe 1953 acreage was 210 million acres.

—both for the *ex post* and *ex ante* models. The first columns under the headings *ex post* model and *ex ante* model indicate the acreage needed to produce the food-wheat (F-W) and feed-grain (F-G) requirements given in the first column of the table. The figure in the second column under each model indicates the acreage not needed, as compared with the acreage actually used in 1953, to meet the food-wheat and feed-grain requirements in the column at the left. For example, 28.8 million acres used in 1953 are not needed to meet demand requirements of 678 million bushels of wheat, and 3,549 million of feed grains under the *ex post* model, and 87.4 million acres are not needed to meet these requirements under the *ex ante* model. As noted before, the range in output combinations which would be specified for the *ex post* model was restricted because of (a) the maximum production that could be attained in the system and (b) computing facilities available. Consequently, the acreages required for only three combinations of output under the *ex post* model are given in table 8.

With the technology implied in the *ex post* model, it would require 181.2 million acres to produce 678 million bushels of wheat (net) and 3,549 million bushels of feed grains (net). On the other hand, these same quantities could be produced under the assumptions of the *ex ante* model with only 122.6 million acres. When the requirements are increased to 800 million bushels of wheat and 4,000 million bushels of feed grains (quantities approaching average 1956-61 annual disappearance), the acreage required under the *ex post* model is 200.4 million, while the comparable figure for the *ex ante* model is 138.4 million. Hence, as compared with 1953 base acreage, 71.6 million acres of cropland are surplus in the sense of "not being needed" to produce these requirements under the *ex ante* model. Even when the net requirements are set at 800 million bushels for wheat and at 5,600 million bushels for feed grains, the projected demand of 1985, only 171.1 acres are required to produce this mix. Compared with the 1953 base acreage, 38.9 million acres would still be in the surplus position for wheat and feed grains if the technology implied by the *ex ante* model were attained by 1985.

Only at the highest level of demand explored does the acreage requirement under the *ex ante* model approach that needed by the *ex post* model to supply 1954 requirements. Furthermore, the *ex post* model could supply, at most, about 800 million bushels of wheat and 4,500 million bushels of feed grains. This mix is approximately equal to the actual disappearance of wheat and feed grains in 1958 (18) and less than the actual disappearance in 1959. This relationship points up this fact: Our present level of per-capita consumption could not prevail very long from domestic sources without the adoption of some output-increasing technologies or improved

practices. Even though output-increasing technologies have been adopted at a faster rate than the resulting production could be absorbed in the market at reasonable prices over the last decade, elimination of all technological development in agriculture would lead to higher food prices as population expands. On the other hand, the results of the *ex ante* model indicate that technological developments already possible or in prospect could easily cause output to increase more rapidly than domestic demand in the next two decades.

Let us turn now to other resources required to produce certain levels of wheat and feed-grain output. Table 9 presents the level of labor, capital and land required under the assumptions of the *ex post* and *ex ante* models. The resources needed to produce only one combination of wheat and feed-grain outputs have been summarized for each model. This level is equal to 1954 consumption for the *ex post* model, but, for the *ex ante* model, it is equal to the projected consumption of 1985.

Table 9. Estimated resources needed to produce specified requirements, *ex post* and *ex ante* models.

Model	Requirements ^a		Resources needed		
	Food wheat	Feed grains	Labor	Capital	Land
	(mil. bu.)	(mil. bu.)	(mil. man-hours)	(mil. \$)	(mil. acres)
Ex post.....	678	3,549	630	1,817	181.2
Ex ante.....	800	5,600	654	3,297	171.1

^aDemand or requirement level is approximately equal to that of 1954 for the *ex post* model and the projected level of 1985 for the *ex ante* model.

The indicated labor required to produce the estimated 1985 wheat and feed-grain needs under the *ex ante* model is 654 million man-hours. This is only 24 million more than required to produce the 1954 output mix under the *ex post* model. It was assumed for the *ex ante* model that no labor-saving techniques or substitutions are involved except those associated with the mechanization assumption. It is likely, of course, that other labor-saving devices will be adopted and that the 1985 projected production levels can be met with less labor than that indicated in table 9. The magnitude of the surplus labor existing is perhaps better described by examining man-hour requirements for the *ex ante* model when requirements also are at the lower level of 678 million bushels of wheat and 3,549 million bushels of feed grains. At this level, the *ex ante* model specifies only 428 million man-hours, as compared with the 630 million for the *ex post* model. The figures of the *ex post* model should approximate those actually used in 1954 to produce wheat and feed grains in the programmed regions. They do not, of course, include labor for other crops and livestock.

Capital, on the other hand, increases by 81 percent from 1,817 million dollars under the specifications of the *ex post* model to 3,297 million dollars

under the specifications of the *ex ante* model for requirements approximating those of 1985. Although this increase in capital represents some additional machinery inputs, additional fertilizer inputs make up the bulk of this increase. Methods of aggregating per-acre crop cost prevented us from breaking down capital cost into its several components. This step can be done for any output combination that might be selected; however, the task would involve much time and many computations. Hence, we have presented only one illustrative comparison in table 9. This comparison does point out important changes in the levels of resource use in the future.

In conclusion, the foregoing analysis reveals several things: (a) If production techniques had remained at the 1954 level, average per-capita consumption rates of the recent past could not have been maintained without a rise in the real cost of food. (b) Fertilizer represents a factor with tremendous output-increasing potential—this factor, nearly alone, could more than provide the additional food requirements needed by 1985. (c) If fertilizer were used at nearly optimum rates, it appears that the South could improve its current relative competitive position in the grain economy.

We now turn to results from a model that attempts analysis of the wheat and feed-grain economy in a more general spatial context. The computations of the production-distribution model are more cumbersome, even in the imperfect current form of this model, than those used in the foregoing analysis. The results of the production-distribution model are used for comparison with the results that have been presented for the *ex post* and *ex ante* models.

Regional Production Pattern, Production-Distribution Model

The production-distribution model specifies not only where wheat and feed grains would be produced under economic efficiency criteria but also to which destination they would flow. It specifies the regions (given in fig. 1) where each grain is to be produced as well as the centers or regions (shown in fig. 2) to which this grain flows for consumption. Both primary production costs and distribution costs make up the objective to be minimized in the model. The national requirements of food wheat and feed grains were distributed among the 10 consumption regions in fig. 2 as shown in table 10. This distribution was made on the basis of the January 1955 population in each of the 10 regions, U. S. average per-capita consumption rates and actual net exports shipped from each of these 10 regions.

Figure 13 presents the location of the regions where production is to take place under the formulation of the production-distribution model. Be-

Table 10. Requirements of food wheat and feed grains by consumption regions.^a

Consumption region	Food wheat	Feed grains ^b
	(1,000 bu.)	(1,000 bu.)
Northeast	227,348	426,222
Appalachian	46,184	235,133
Southeast	41,369	212,370
Lake States	39,435	386,914
Corn Belt	87,626	1,384,356
Delta States	59,850	129,167
Northern Plains	13,808	359,603
Southern Plains	70,150	187,201
Mountain	5	58,646
Pacific	91,680	168,904
Total	677,455	3,548,516

^aThese requirements are net figures and are the differences between gross requirements and the estimated amounts produced in the unnumbered areas of fig. 1.

^bThese figures are in terms of corn-equivalent feed units.

fore turning to the distribution pattern of this specified production, we shall compare fig. 13 with fig. 3. Figure 3 is for the *ex post* model, which used the same production coefficients and output specifications.

First of all, we see that more regions are specified to produce two products in fig. 13 than in fig. 3. This phenomenon is simply the function of the number of demand constraints in the system and the number of activities available per region.¹²

¹²If the number of demand constraints is equal to or greater than the number of activities per region, then all activities of any one region could be specified by a production plan. While such is the case for one region, it cannot be true for all, because the number of

Figure 13 shows that the location of food-wheat production has moved eastward as compared with that shown in fig. 3. For example, in fig. 3, wheat production is specified in only three regions east of the Mississippi. These are in southern Wisconsin, east central Wisconsin and southern Alabama. On the other hand, fig. 13 shows that either all or part of 13 regions are designated for food-wheat production by the solution to the production-distribution model. Likewise, in the Pacific States, fig. 3 designates regions 100, 101, 102 and 103 for wheat production earmarked for feed. But fig. 13 shows all or part of three of these regions—100, 101 and 103—earmarked for food wheat.

The *ex post* model, upon which fig. 3 is based, implies that wheat for food would be shipped to the West Coast from regions east of the Rocky Mountains, presumably from the Northern Plains States. The production-distribution model (fig.

(footnote 12 cont'd.)

non-zero activity levels cannot be greater than the number of constraints. For example, we could not have two non-zero activities for each of 100 regions if there are but 199 constraints. One significant point here is that regional diversification is not precluded because linear input-output coefficients are used. Furthermore, we could say the same would be true for individual firms if they were large enough to experience price changes with changes in output. However, a direct awareness of price changes would not be necessary for firms to practice diversification because of adjustment to price. See the supplement to USDA Tech. Bul. 1241 (7, pp. 2-3) for an elaboration on this point.

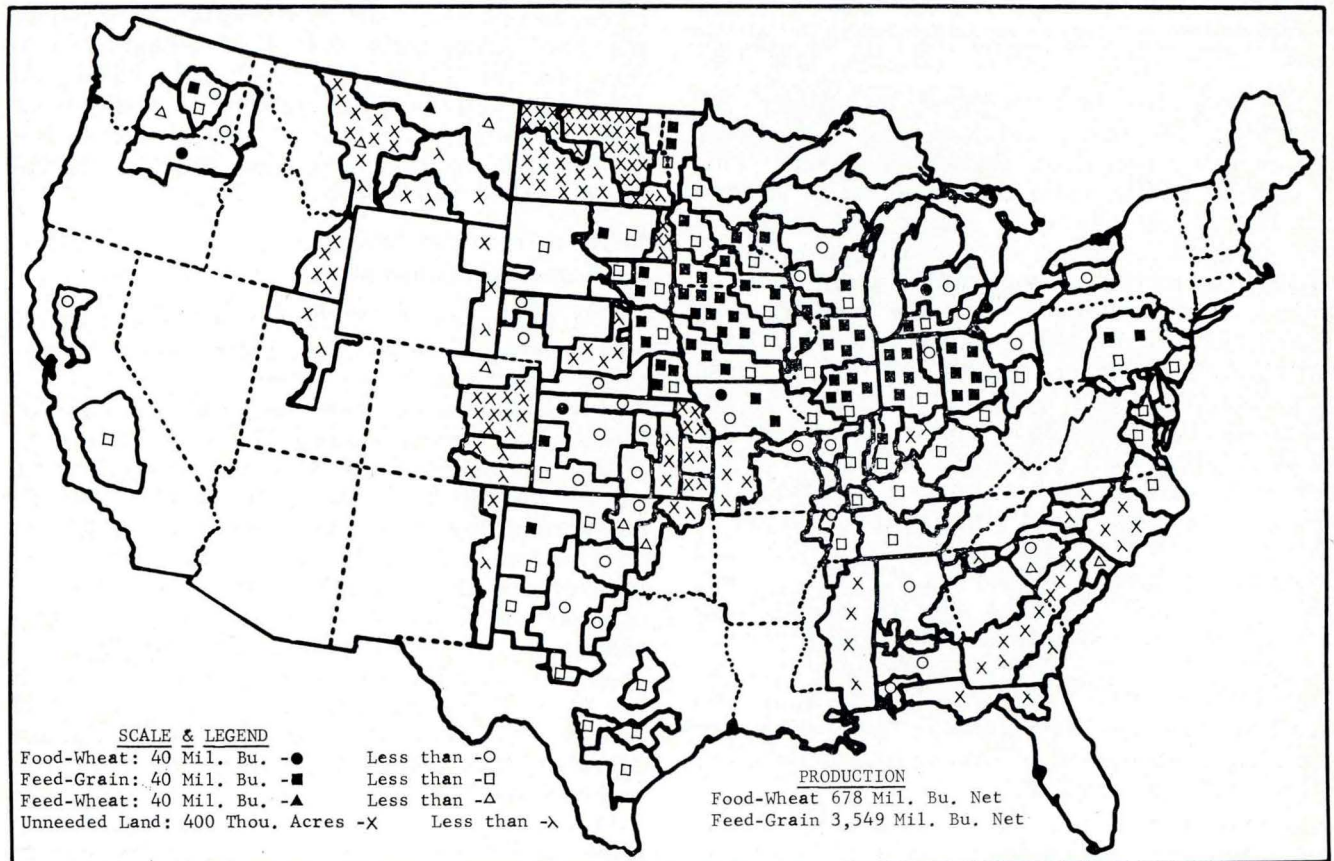


Fig. 13. Economic optimum pattern of regional grain production for specified production levels, production-distribution model.

13) indicates, on the other hand, that such a production allocation is not the most economic spatial production pattern. This model indicates that it would be cheaper to produce all of the food wheat locally and to import some feed grains instead of producing, as implied by the *ex post* model (fig. 3), all required feed locally (as wheat) plus some for outshipment.

The two models also differ concerning the production of feed grains in the Mountain States. Inasmuch as the feed wheat produced by regions 89 and 90 (Montana, fig. 13) is equal to the feed-grain needs of the Mountain States, production specified for regions 89, 90, 93 and 99 by the *ex post* model exceeds the needs of Mountain States. Hence, the *ex post* model implicitly specifies exports of feed from this area.

There are also differences between the results of the two models with respect to food-wheat production in the Northern Plains. The *ex post* model specifies food-wheat production for regions 55 and 56 (South Dakota). The production-distribution model, in contrast, specifies feed grains for these regions. Another example: The *ex post* model specifies food wheat for regions 51, 52 and 53 (North Dakota) and region 69 (Kansas), whereas the production-distribution model specifies no production of grains for these regions.

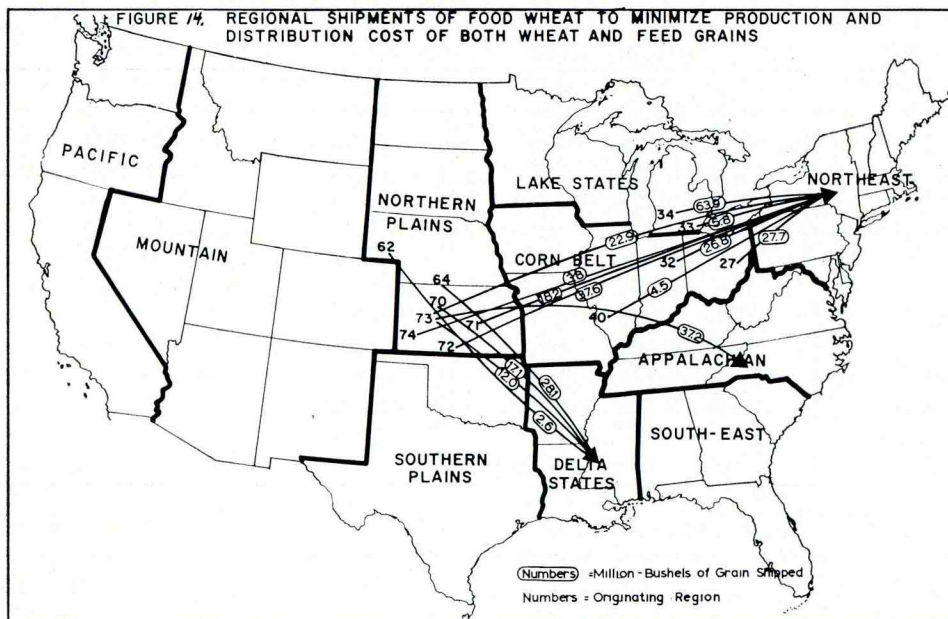
The foregoing comparisons simply mean that, when distance and shipping charges are taken into account, the Northern Plains loses its relatively prime competitive position in grain production. However, countervailing factors other than shipping charges appear to be important in establishing the actual competitive status of any one area. Some of these factors will be discussed later.

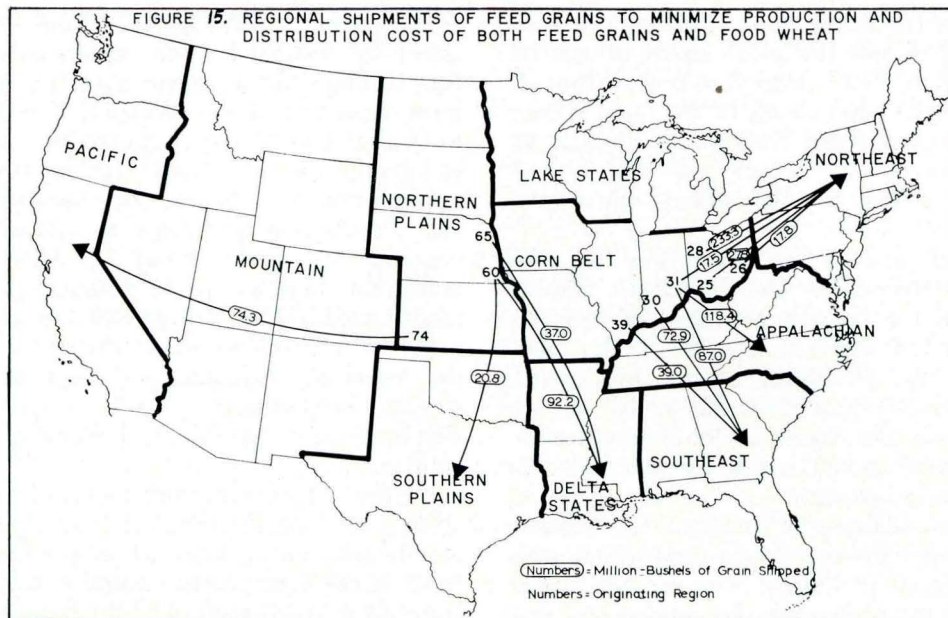
The food-wheat and feed-grain requirements of

all 10 consumption regions (table 10) are not satisfied by the production region pattern shown by fig. 13. By this we mean that the specified aggregate output of all the production regions contained in the outlines of the 10 consumption regions, group by group, do not match these consumption requirements, and, hence, surpluses and deficits exist. Production surpluses of certain consumption regions, of course, must be distributed to consumption regions with deficits. The analytical model used solves the production and distribution problems simultaneously. Figures 14 and 15 show the regional shipments of food wheat and feed grains, respectively, which balance the implied surpluses and deficits as designated by the model solution.

Figure 14 shows that most of the food-wheat shipments specified by the solution are to the Northeast. Of a total of 211 million bushels of food wheat going to the Northeast 27.7 million are from Ohio, 26.8 million from Indiana, 69.7 million from Michigan, 4.5 million from Illinois and 82.5 million from Kansas. Kansas also is designated to ship 37.2 million bushels of wheat for food to the Appalachian area and 29.1 million to the Delta States.

Again, fig. 15 shows that most feed-grain shipments are to the Northeast under the production-distribution model. Of the 296 million bushels to be shipped to this area, 278.9 million are to be supplied by Ohio and 17.5 million by Indiana. Indiana also ships 118.4 million bushels to the Appalachian area and 159.9 million to the Southeast. Illinois ships 39 million bushels to the Southeast, also. Nebraska ships 129.2 million bushels of feed grains to the Delta States, and also 21 million bushels to the Southern Plains. Finally, Kansas is desig-





nated to ship 74.3 million bushels to the Pacific States. (Other shipments within consumption regions can and are expected to take place, of course, but are not specified. For example, feed grain can flow from Iowa to Illinois or Missouri, or wheat, from Colorado to Arizona.)

Derived Regional Prices

The dual solution to the production-distribution problem provides the competitive equilibrium

prices of 10 consumption regions. It can be shown that these prices are equal to the cost of the marginal unit that fulfills the demand or consumption requirement in each region. Hence, they are competitive prices from the supply viewpoint, and they are equilibrium prices under the assumption of zero demand elasticities, or predetermined demand.

Table 11 presents imputed prices given by dual solution for each of the 10 regions along with

Table 11. Programming delivered price of wheat and corn.

Area	X ₁ ^a	X ₂ ^b	X ₃ ^c	X ₄ ^d	X ₅ ^e	X ₆ ^f	Comparison	Coefficient of determination (r ²)— With prices omitted for—			
								Of all prices	Northern Plains (wheat only)	Mountain States (corn only)	Mountain States and Southern Plains (corn only)
WHEAT											
Northeast	\$1.48	\$1.19	\$1.14	\$2.03	\$2.05	\$2.04	X ₁ and X ₂	0.71	0.74	—	—
Appalachian	1.53	1.17	1.18	2.05	2.06	2.04	X ₁ and X ₃	0.66	0.67	—	—
Southeast	1.39	1.17	1.16	2.06	2.06	2.04	X ₁ and X ₃	0.66	0.67	—	—
Lake States	1.11	1.08	1.05	2.03	1.99	1.95	X ₁ and X ₄	0.34	0.70	—	—
Corn Belt	1.04	1.07	1.08	1.99	2.00	2.00	X ₁ and X ₄	0.34	0.70	—	—
Delta States	1.22	1.11	1.18	1.99	1.97	1.96	X ₁ and X ₄	0.51	0.72	—	—
Northern Plains	0.73	1.03	1.02	2.03	2.00	2.04	X ₁ and X ₅	0.51	0.72	—	—
Southern Plains	1.21	1.01	1.02	1.99	1.98	2.03	X ₁ and X ₅	0.26	0.58	—	—
Mountain States	0.77	0.99	0.95	1.95	1.87	1.89	X ₁ and X ₆	0.26	0.58	—	—
Pacific	1.28	0.99	1.08	2.02	2.09	2.10	X ₁ and X ₆	0.26	0.58	—	—
CORN											
Northeast	\$1.05	\$1.01	\$1.00	\$1.79	\$1.69	\$1.53	X ₁ and X ₂	0.47	—	0.74	0.94
Appalachian	1.16	1.02	1.00	1.66	1.66	1.50	X ₁ and X ₂	0.47	—	0.74	0.94
Southeast	1.24	1.04	1.04	1.64	1.63	1.45	X ₁ and X ₃	0.04	—	0.53	0.90
Lake States	0.88	0.86	0.86	1.48	1.51	1.35	X ₁ and X ₃	0.04	—	0.53	0.90
Corn Belt	0.67	0.82	0.81	1.52	1.52	1.41	X ₁ and X ₄	0.26	—	0.42	0.55
Delta States	0.98	0.95	0.98	1.61	1.63	1.42	X ₁ and X ₄	0.26	—	0.42	0.55
Northern Plains	0.65	0.76	0.79	1.43	1.43	1.32	X ₁ and X ₅	0.05	—	0.37	0.52
Southern Plains	1.08	0.84	0.83	1.50	1.50	1.39	X ₁ and X ₅	0.05	—	0.37	0.52
Mountain States	0.69	0.98	1.07	1.77	1.88	1.80	X ₁ and X ₆	0.01	—	0.28	0.34
Pacific	1.12	1.00	1.07	1.82	1.89	1.76	X ₁ and X ₆	0.01	—	0.28	0.34

^aDelivered prices at center of consumption region.

^bAverage of average 1936-45 prices for states in consumption region.

^c1936-45 average price for state containing delivery center for consumption region.

^dAverage of average 1936-45 prices for states in consumption region.

^e1936-45 average price for state containing delivery center for consumption region.

^f1948-57 average price for state containing delivery center for consumption region.

several formulations of aggregate historical prices. The X_1 's in this table are the dual prices. The X_2 's are the averages of the 1936-45 state average prices for all states in a consumption region. The X_3 's are the 1936-45 average prices for the state (s) in the approximate center of the consumption region. The X_4 's and X_5 's are the same in construction as X_2 's and X_3 's, but are for the period 1945-54. Finally, the X_6 's are similar to the X_3 's but are for the period 1948-57. Coefficients of determination between the dual prices and each of the historic price series are given at the right-hand side of the table.

Several items of interest are evident in the coefficients of determination:

(a) The degree of association between the dual prices and the historic prices is less for the more recent time period (1945-54, X_4) than for the earlier time period (1936-45, X_3). This difference in association may result from the price-support program of the recent period. If this is true, it could have happened in two ways. First, the price-support system could reduce the differences in relative or absolute prices between regions. This indeed seems to be the case, especially for wheat. The variance of regional wheat prices declined from 44 to 10 for those two periods (in terms of cents per bushel). On the other hand, the variance of corn price increased for the same period, but the coefficient of variation declined from 0.10 to 0.08, which indicates a decrease in the relative price variance. Second, the price-support program could have distorted the competitive regional price structure. This phenomenon would, of course, reduce the correlation between the two price series.

(b) As shown in table 11, additional coefficients of determination were derived with certain pairs of regional prices omitted. For wheat, the pair omitted was the programmed price and the historical price for the Northern Plains. This pair appeared to be the most inconsistent of the lot, and, as can be seen in the table, the r^2 is increased for all comparisons when this pair is dropped, but more so for recent time periods.

(c) For corn, first the pair of prices for the Mountain States was omitted, and then, in addition, the pair for the Southern Plains was dropped. An r^2 was computed at each step. The values for the r^2 for each of the price series comparisons are given in the last two columns of table 11 under corn. As with wheat, the value of r^2 increased for each comparison as pairs were sequentially omitted. Tests of significance of the r^2 were not made because of the ways in which the price series were derived.

To sum up, we have shown that there is a fairly high degree of association between the dual prices obtained by linear programming analysis and historical prices. There is not, however, a one-to-one

correspondence. The historical prices of more recent periods are higher, on the average, than the dual prices. This is expected, given surplus production under price supports. The dual prices would not be expected to be identical to the historic prices, even under market equilibrium, because of aggregates involved in deriving the dual prices and because regional demand was considered to be predetermined. The omission of quality from the programming analysis may explain some of the discrepancy between the dual price and the historic price for wheat in the Northern Plains. Also, price-support programs that hold wheat at food-price levels, rather than letting it gravitate nearer feed-grain prices, constitute another possible explanation of the difference in the price levels. The surplus grain situation may partially explain this discrepancy for corn in the Mountain States. But the use of wheat in the analysis as a perfect substitute for corn is another important reason for the discrepancy.

Limitations of the Analysis

In addition to assumptions inherent in linear programming that do not mirror the economic environment, other limitations are evident in the results just presented. These limitations stem largely from the inadequacy of the computing facilities. Limitations of the maximum-profit models (*ex post* and *ex ante*) and the minimum-cost model (production-distribution) will be discussed in that order.

The maximum-profit models assume that production specifications in any one region depend on regional price differentials and not on profits. This is an important point in appraising these models. If the solution program implies a set of regional price differentials other than the set of historical prices used in the analysis, the solution cannot be strictly correct. For example, suppose that a programming solution, when analyzed, shows that feed-grain production specified for California is less than actual requirements. Hence, feed grain must be shipped into the state. Feed grain will be shipped into California from some surplus area only if the price differential (a given constant in the analysis) is great enough to cover the transport costs involved. If this is not the case, some regional production plan other than the one derived would be more efficient economically. Even so, results may not differ greatly from the best plan.

Analysis of this point, the realism of the regional price differentials used in the *ex post* and *ex ante* models, was attempted with the production-distribution model. We pointed out that the discrepancies between solutions of the two types of models may result from such things as degree of aggregation or quality differences of regional products and not from using the wrong set of

price differentials for programming. Though quality is not taken into account directly in the "maximum-profit" models, some weight is given to this factor by price differentials. Specifically, regional prices below the national average, because of low-quality grain produced in a region, would be reflected in price differentials of the region.

The production-distribution model has these apparent limitations: (a) The consumption regions are much too broad. These broad regions obviously cannot generate a completely satisfactory regional price structure, and they can distort the regional production patterns given by the programming solution. (b) Quality is not considered in the model indirectly or directly. If it had been, certainly more of the hard spring and durum wheat production areas would have been included in the

solution. Also certain regions in the southeast probably would not have been shown as wheat-producing areas because of their history for garlicky wheat which is highly discounted. (c) The model assumes that consumption occurs near production, except for interregional shipments. This assumption, too, probably has had considerable effect on the programming solution. (d) The production-distribution model assumes that milling takes place enroute to the consumption centers or near production locations. Given the present location of mills, the solution depicted by fig. 13 may be unrealistic.

Data, too, place limitations on the results of all models. Because these limitations have been discussed fully elsewhere, we will not elaborate on them here.

APPENDIX

This Appendix includes two sets of data used in the programming analysis of the text. Table A-1 includes the estimated normal wheat and corn prices by regions for 1954. Table A-2 includes freight tariffs per bushel of wheat by origin and

destination—while table A-3 includes the same information for corn. Tables A-4, A-5 and A-6, respectively, provide the same data for oats, barley and grain sorghum.

Table A-1. Estimated normal wheat and corn prices per bushel by regions, 1954.

Region	Wheat price	Corn price	Region	Wheat price	Corn price	Region	Wheat price	Corn price
1	\$1.88	\$1.66	36	1.85	1.51	71	1.88	1.50
2	1.86	1.68	37	1.85	1.51	72	1.85	1.51
3	1.91	1.66	38	1.87	1.50	73	1.85	1.52
4	1.90	1.65	39	1.87	1.51	74	1.84	1.54
5	1.92	1.68	40	1.87	1.51	75	1.85	1.50
6	1.92	1.60	41	1.86	1.52	76	1.85	1.51
7	1.96	1.68	42	1.85	1.56	77	1.84	1.55
8	1.96	1.68	43	1.83	1.54	78	1.85	1.50
9	1.94	1.62	44	1.88	1.52	79	1.84	1.53
10	1.93	1.62	45	1.88	1.50	80	1.84	1.54
11	1.94	1.62	46	1.90	1.46	81	1.85	1.48
12	1.93	1.62	47	1.92	1.40	82	1.85	1.48
13	1.93	1.67	48	1.91	1.38	83	1.85	1.48
14	1.92	1.69	49	1.94	1.40	84	1.85	1.50
15	1.66	1.66	50	1.95	1.38	85	1.86	1.49
16	1.90	1.66	51	1.94	1.36	86	1.86	1.49
17	1.91	1.66	52	1.94	1.48	87	1.87	1.50
18	1.92	1.68	53	1.94	1.46	88	1.87	1.50
19	1.83	1.63	54	1.92	1.36	89	1.79	1.60
20	1.92	1.66	55	1.89	1.45	90	1.74	1.65
21	1.87	1.60	56	1.92	1.40	91	1.79	1.60
22	1.87	1.60	57	1.93	1.37	92	1.74	1.64
23	1.90	1.62	58	1.89	1.45	93	1.76	1.58
24	1.89	1.61	59	1.92	1.46	94	1.76	1.58
25	1.88	1.55	60	1.87	1.50	95	1.83	1.57
26	1.88	1.60	61	1.76	1.58	96	1.83	1.58
27	1.88	1.58	62	1.79	1.57	97	1.81	1.54
28	1.86	1.51	63	1.86	1.47	98	1.72	1.80
29	1.86	1.55	64	1.86	1.49	99	1.73	1.88
30	1.86	1.54	65	1.87	1.50	100	1.85	1.79
31	1.83	1.49	66	1.88	1.51	101	1.86	1.83
32	1.85	1.51	67	1.87	1.51	102	1.88	1.85
33	1.87	1.52	68	1.86	1.51	103	1.95	1.89
34	1.88	1.54	69	1.86	1.51	104	1.95	1.89
35	1.86	1.52	70	1.86	1.49			

Table A-2. Freight tariffs per bushel for wheat, by origin and destination.

Originating production region	Destination									
	North- east (1)	Appala- chian (2)	South- east (3)	Lake States (4)	Corn Belt (5)	Delta States (6)	Northern Plains (7)	Southern Plains (8)	Mountain (9)	Pacific (10)
1										
2										
3										
4										
5	\$0.65	0	\$0.78	\$1.26	\$0.99	\$1.07	\$1.62	\$1.69		\$2.45
6	0.68	0	0.73	1.30	1.02	1.05	1.64	1.68		2.50
7	0.74	0	0.70	1.26	0.99	0.92	1.62	1.66		2.32
8	0.79	0	0.66	1.26	1.01	0.88	1.63	1.65		2.36
9	0.78	0	0.66	1.28	1.04	0.96	1.66	1.67		2.43
10	1.26	\$0.55	0	1.32	1.07	0.87	1.68	1.65		2.42
11	1.35	0.65	0	1.35	0.98	0.82	1.62	1.56		2.33
12	1.33	0.56	0	1.35	0.96	0.80	1.60	1.54		2.30
13	1.39	0.53	0	1.30	0.95	0.80	1.58	1.52		2.28
14	1.34	0.55	0	1.26	0.94	0.77	1.66	1.50		2.23
15										
16	1.34	0.83	0	1.07	0.85	0.50	1.56	1.53		1.93
17	1.22	0.74	0	1.04	0.82	0.55	1.46	1.56		2.02
18	1.25	0.67	0	1.13	0.77	0.55	1.41	1.56		2.02
19						0				
20	1.16	0	0.76	1.01	0.64	0.69	1.40	1.25		2.00
21	1.30	0	0.79	0.97	0.48	0.54	1.12	1.12		1.86
22	1.06	0	0.80	0.93	0.55	0.69	1.22	1.24		2.00
23	1.03	0	0.81	0.93	0.47	0.58	1.07	1.19		1.87
24	0.78	0	0.86	0.85	0.45	0.60	1.17	1.29		1.96
25	0.30	0.49	0.54	0.41	0	0.45	0.62	0.56	\$0.83	1.13
26	0.29	0.47	0.56	0.41	0	0.45	0.65	0.59	0.87	1.14
27	0.29	0.47	0.58	0.41	0	0.48	0.65	0.61	0.87	1.15
28	0.30	0.47	0.55	0.34	0	0.44	0.58	0.58	0.83	1.11
29	0.38	0.50	0.53	0.37	0	0.44	0.56	0.49	0.80	1.06
30	0.42	0.54	0.51	0.38	0	0.39	0.52	0.44	0.84	1.03
31	0.38	0.50	0.57	0.32	0	0.47	0.53	0.52	0.80	1.03
32	0.33	0.46	0.57	0.35	0	0.49	0.56	0.53	0.82	1.07
33	0.34	0.59	0.65	0	0.31	0.47	0.65	0.59	0.92	1.14
34	0.34	0.59	0.66	0	0.33	0.50	0.60	0.62	0.94	1.15
35	0.48	0.64	0.69	0	0.30	0.50	0.41	0.53	0.82	0.91
36	0.45	0.60	0.68	0	0.27	0.45	0.39	0.52	0.80	0.93
37	0.46	0.55	0.67	0.26	0	0.44	0.48	0.50	0.73	0.84
38	0.44	0.53	0.62	0.32	0	0.38	0.51	0.48	0.76	0.93
39	0.44	0.53	0.59	0.35	0	0.36	0.51	0.44	0.76	0.93
40	0.44	0.53	0.60	0.32	0	0.34	0.49	0.42	0.74	0.89
41	0.44	0.49	0.59	0.44	0	0.34	0.50	0.47	0.74	0.89
42	0.61	0.65	0.68	0.46	0	0.41	0.40	0.42	0.70	0.75
43	0.52	0.58	0.65	0.40	0	0.43	0.43	0.49	0.70	0.84
44	0.60	0.71	0.74	0.40	0	0.46	0.31	0.52	0.64	0.75
45	0.55	0.66	0.73	0.37	0	0.51	0.37	0.56	0.68	0.84
46	0.53	0.65	0.74	0.34	0	0.51	0.39	0.59	0.70	0.84
47	0.57	0.68	0.75	0	0.35	0.47	0.46	0.65	0.77	0.81
48	0.61	0.72	0.80	0	0.38	0.57	0.49	0.68	0.83	0.81
49	0.63	0.74	0.83	0	0.41	0.53	0.53	0.73	0.85	0.80
50	0.67	0.78	0.86	0	0.45	0.65	0.57	0.76	0.90	0.78
51	0.69	0.87	0.95	0.43	0.47	0.67	0	0.77	0.74	0.81
52	0.75	0.93	1.01	0.49	0.53	0.68	0	0.78	0.71	0.81
53	0.73	0.92	0.99	0.47	0.51	0.65	0	0.83	0.70	0.81
54	0.63	0.82	0.89	0.39	0.42	0.60	0	0.70	0.71	0.81
55	0.77	0.88	0.95	0.55	0.49	0.63	0	0.73	0.58	0.81
56	0.66	0.79	0.87	0.41	0.38	0.55	0	0.65	0.66	0.81
57	0.64	0.75	0.86	1.37	0.36	0.53	0	0.63	0.68	0.81
58	0.66	0.77	0.85	0.48	0.38	0.52	0	0.62	0.62	0.81
59	0.62	0.73	0.80	0.40	0.34	0.47	0	0.57	0.66	0.81
60	0.62	0.70	0.77	0.39	0.31	0.41	0	0.54	0.67	0.75
61	0.77	0.84	0.92	0.53	0.45	0.56	0	0.68	0.57	0.75
62	0.83	0.86	0.91	0.51	0.46	0.50	0	0.62	0.44	0.75
63	0.67	0.74	0.82	0.44	0.35	0.46	0	0.59	0.58	0.75
64	0.69	0.77	0.86	0.45	0.37	0.42	0	0.55	0.57	0.75
65	0.58	0.67	0.77	0.37	0.28	0.35	0	0.48	0.63	0.75
66	0.59	0.67	0.74	0.42	0.30	0.33	0	0.49	0.63	0.75
67	0.61	0.67	0.74	0.45	0.30	0.33	0	0.49	0.66	0.75
68	0.62	0.68	0.76	0.48	0.31	0.31	0	0.44	0.70	0.75
69	0.62	0.68	0.76	0.47	0.31	0.37	0	0.49	0.63	0.75
70	0.66	0.71	0.79	0.44	0.34	0.37	0	0.51	0.61	0.75
71	0.60	0.72	0.80	0.50	0.35	0.40	0	0.53	0.63	0.75
72	0.59	0.72	0.80	0.50	0.35	0.38	0	0.47	0.63	0.75
73	0.69	0.74	0.82	0.54	0.37	0.43	0	0.55	0.58	0.75
74	0.66	0.77	0.84	0.56	0.40	0.45	0	0.58	0.60	0.75
75	0.73	0.62	0.67	0.60	0.40	0.38	0.54	0	0.82	0.75
76	0.78	0.70	0.72	0.63	0.46	0.40	0.59	0	0.77	0.75
77	0.85	0.78	0.77	0.67	0.49	0.47	0.65	0	0.71	0.75
78	0.85	0.68	0.69	0.66	0.46	0.40	0.65	0	0.77	0.75
79	0.89	0.77	0.74	0.67	0.52	0.40	0.69	0	0.75	0.75
80	0.86	0.80	0.83	0.74	0.58	0.50	0.66	0	0.59	0.75
81	0.92	0.78	0.79	0.67	0.57	0.44	0.67	0	0.68	0.75
82	0.93	0.83	0.83	0.73	0.63	0.50	0.73	0	0.63	0.75
83	0.86	0.74	0.75	0.77	0.54	0.44	0.68	0	0.68	0.75
84	0.95	0.81	0.76	0.76	0.59	0.50	0.77	0	0.69	0.75
85	0.84	0.72	0.74	0.74	0.58	0.40	0.67	0	0.73	0.75
86	0.87	0.79	0.80	0.77	0.60	0.48	0.80	0	0.80	0.75
87	0.85	0.75	0.76	0.78	0.58	0.40	0.80	0	0.80	0.75
88	0.85	0.82	0.80	0.76	0.61	0.45	0.81	0	0.80	0.75
89	0.88	0.96	1.10	0.55	0.67	0.88	0.50	0.87	0	0.81
90	0.93	1.01	1.16	0.59	0.71	0.89	0.52	0.86	0	0.75
91	0.86	0.94	1.08	0.52	0.64	0.75	0.44	0.81	0	0.81
92	0.93	0.98	1.13	0.59	0.71	0.76	0.45	0.80	0	0.75
93	0.81	0.89	0.96	0.54	0.50	0.65	0.33	0.72	0	0.94
94	0.78	0.83	0.91	0.55	0.46	0.62	0.30	0.58	0	0.75
95	0.78	0.83	0.91	0.58	0.46	0.61	0.30	0.58	0	0.75
96	0.78	0.79	0.91	0.61	0.49	0.61	0.30	0.58	0	0.75
97	0.92	1.04	0.80	0.68	0.60	0.42	0.72	0.34	0	0.75
98	0.99	1.01	1.17	0.75	0.67	0.82	0.51	0.82	0	0.56
99	0.96	1.17	1.09	0.73	0.64	0.84	0.48	0.64	0	0.55
100	1.12	1.17	1.17	0.75	0.80	0.89	0.64	0.95	0.33	0
101	1.12	1.17	1.17	0.75	0.80	0.89	0.64	0.95	0.33	0
102	1.12	1.17	1.17	0.75	0.80	0.89	0.64	0.95	0.33	0
103	1.12	1.17	1.17	0.85	0.80	0.84	0.64	0.64	0.33	0
104	1.12	1.17	1.17	0.85	0.80	0.84	0.64	0.64	0.33	0

Table A-3. Freight tariffs per bushel for corn, by origin and destination.

Originating production region	Destination									
	North- east (1)	Appala- chian (2)	South- east (3)	Lake States (4)	Corn Belt (5)	Delta States (6)	Northern Plains (7)	Southern Plains (8)	Mountain (9)	Pacific (10)
1-24										
25	\$0.28	\$0.45	\$0.50	\$0.38	0	\$0.42	\$0.58	\$0.52	\$0.80	\$0.96
26	0.27	0.44	0.52	0.38	0	0.42	0.60	0.55	0.81	0.98
27	0.27	0.44	0.54	0.38	0	0.44	0.60	0.56	0.81	0.98
28	0.28	0.44	0.51	0.34	0	0.41	0.54	0.54	0.78	0.94
29	0.35	0.46	0.49	0.35	0	0.41	0.52	0.46	0.75	0.90
30	0.40	0.50	0.48	0.38	0	0.37	0.48	0.41	0.78	0.87
31	0.35	0.46	0.54	0.30	0	0.44	0.50	0.48	0.74	0.89
32	0.31	0.43	0.53	0.32	0	0.46	0.53	0.50	0.76	0.91
33	0.31	0.55	0.61	0	\$0.29	0.44	0.61	0.55	0.85	0.96
34	0.31	0.55	0.62	0	0.30	0.46	0.56	0.58	0.88	0.96
35	0.45	0.60	0.64	0	0.28	0.47	0.38	0.50	0.76	0.75
36	0.42	0.56	0.63	0	0.26	0.42	0.36	0.48	0.75	0.77
37	0.43	0.52	0.63	0.25	0	0.41	0.45	0.47	0.68	0.69
38	0.41	0.50	0.57	0.30	0	0.36	0.48	0.45	0.71	0.77
39	0.41	0.50	0.55	0.33	0	0.33	0.47	0.41	0.71	0.77
40	0.41	0.50	0.56	0.30	0	0.32	0.46	0.39	0.69	0.73
41	0.41	0.46	0.55	0.31	0	0.32	0.47	0.44	0.69	0.73
42	0.57	0.61	0.63	0.43	0	0.38	0.37	0.39	0.66	0.60
43	0.49	0.54	0.61	0.38	0	0.40	0.40	0.46	0.66	0.69
44	0.56	0.66	0.69	0.38	0	0.43	0.29	0.48	0.60	0.60
45	0.52	0.61	0.68	0.34	0	0.47	0.34	0.52	0.63	0.69
46	0.50	0.60	0.69	0.32	0	0.47	0.36	0.55	0.65	0.69
47	0.53	0.64	0.70	0	0.32	0.44	0.43	0.61	0.72	0.70
48	0.57	0.67	0.74	0	0.36	0.53	0.45	0.63	0.78	0.67
49	0.58	0.69	0.76	0	0.38	0.50	0.50	0.68	0.79	0.66
50	0.62	0.73	0.80	0	0.42	0.61	0.53	0.71	0.84	0.63
51	0.64	0.82	0.88	0.40	0.44	0.62	0	0.72	0.69	0.66
52	0.70	0.87	0.94	0.45	0.50	0.64	0	0.73	0.66	0.67
53	0.68	0.85	0.92	0.44	0.48	0.60	0	0.78	0.66	0.76
54	0.58	0.76	0.83	0.37	0.39	0.56	0	0.66	0.66	0.67
55	0.72	0.82	0.89	0.51	0.46	0.58	0	0.68	0.54	0.67
56	0.62	0.73	0.82	0.39	0.36	0.51	0	0.60	0.61	0.67
57	0.60	0.70	0.80	0.35	0.34	0.49	0	0.58	0.64	0.67
58	0.61	0.72	0.79	0.45	0.36	0.48	0	0.58	0.58	0.67
59	0.57	0.68	0.75	0.37	0.32	0.44	0	0.54	0.61	0.67
60	0.58	0.65	0.72	0.37	0.29	0.39	0	0.50	0.63	0.60
61	0.72	0.78	0.85	0.49	0.42	0.52	0	0.64	0.54	0.60
62	0.77	0.80	0.85	0.48	0.43	0.46	0	0.57	0.40	0.60
63	0.62	0.69	0.76	0.41	0.33	0.43	0	0.55	0.45	0.60
64	0.64	0.71	0.80	0.42	0.35	0.40	0	0.51	0.54	0.60
65	0.54	0.63	0.72	0.35	0.26	0.33	0	0.45	0.58	0.60
66	0.55	0.62	0.69	0.38	0.28	0.31	0	0.46	0.58	0.48
67	0.57	0.62	0.69	0.42	0.28	0.31	0	0.46	0.62	0.48
68	0.58	0.64	0.71	0.45	0.29	0.29	0	0.41	0.65	0.48
69	0.58	0.64	0.71	0.44	0.29	0.34	0	0.46	0.59	0.48
70	0.61	0.66	0.73	0.41	0.32	0.34	0	0.48	0.58	0.48
71	0.56	0.67	0.74	0.46	0.33	0.38	0	0.49	0.58	0.48
72	0.55	0.67	0.74	0.45	0.33	0.35	0	0.44	0.58	0.48
73	0.64	0.69	0.76	0.50	0.35	0.40	0	0.51	0.50	0.48
74	0.61	0.72	0.79	0.52	0.37	0.42	0	0.54	0.53	0.48
75	0.68	0.58	0.62	0.56	0.37	0.36	0.50	0	0.76	0.60
76	0.72	0.65	0.67	0.59	0.43	0.37	0.55	0	0.71	0.60
77	0.79	0.72	0.72	0.63	0.46	0.44	0.61	0	0.66	0.60
78	0.79	0.64	0.64	0.62	0.43	0.37	0.61	0	0.72	0.60
79	0.83	0.72	0.69	0.65	0.49	0.37	0.64	0	0.70	0.60
80	0.80	0.74	0.78	0.63	0.54	0.46	0.61	0	0.55	0.60
81	0.86	0.73	0.74	0.69	0.53	0.41	0.63	0	0.64	0.60
82	0.86	0.77	0.77	0.68	0.59	0.46	0.68	0	0.59	0.60
83	0.80	0.69	0.70	0.72	0.51	0.41	0.64	0	0.64	0.60
84	0.88	0.75	0.71	0.71	0.55	0.46	0.72	0	0.65	0.60
85	0.79	0.68	0.69	0.69	0.54	0.38	0.65	0	0.68	0.60
86	0.81	8.74	0.75	0.72	0.56	0.45	0.75	0	0.74	0.60
87	0.79	0.70	0.71	0.73	0.54	0.38	0.75	0	0.74	0.60
88	0.79	0.77	0.74	0.71	0.57	0.42	0.76	0	0.75	0.60
89	0.82	0.89	1.03	0.51	0.62	0.82	0.47	0.81	0	0.67
90	0.87	0.95	1.08	0.55	0.66	0.83	0.48	0.81	0	0.67
91	0.80	0.88	1.01	0.48	0.60	0.70	0.41	0.76	0	0.67
92	0.87	0.91	1.05	0.55	0.66	0.71	0.42	0.75	0	0.70
93	0.76	0.83	0.90	0.51	0.46	0.61	0.31	0.67	0	0.78
94	0.73	0.78	0.85	0.51	0.43	0.58	0.28	0.54	0	0.45
95	0.73	0.78	0.85	0.54	0.43	0.57	0.28	0.54	0	0.59
96	0.73	0.78	0.85	0.57	0.46	0.57	0.28	0.54	0	0.59
97	0.86	0.73	0.75	0.64	0.56	0.39	0.67	0.22	0	0.45
98	0.92	0.97	1.09	0.70	0.62	0.77	0.47	0.76	0	0.52
99	0.89	0.94	1.01	0.68	0.60	0.78	0.44	0.60	0	0.52
100	1.05	1.09	1.09	0.70	0.75	0.83	0.60	0.89	0.31	0
101	1.05	1.09	1.09	0.70	0.75	0.83	0.60	0.89	0.31	0
102	1.05	1.09	1.09	0.70	0.75	0.83	0.60	0.89	0.31	0
103	1.05	1.09	1.09	0.80	0.75	0.78	0.60	0.60	0.31	0
104	1.05	1.09	1.09	0.80	0.75	0.78	0.60	0.60	0.31	0

Table A-4. Freight tariff for oats per bushel, by origin and destination.

Originating production region	Destination									
	North-east (1)	Appalachian (2)	South-east (3)	Lake States (4)	Corn Belt (5)	Delta States (6)	Northern Plains (7)	Southern Plains (8)	Mountain (9)	Pacific (10)
1-24										
25	\$0.16	\$0.26	\$0.29	\$0.22	0	\$0.24	\$0.33	\$0.30	\$0.45	\$0.60
26	0.15	0.25	0.30	0.22	0	0.24	0.35	0.31	0.46	0.61
27	0.15	0.25	0.31	0.22	0	0.25	0.35	0.32	0.46	0.61
28	0.16	0.25	0.29	0.20	0	0.24	0.31	0.31	0.44	0.59
29	0.20	0.26	0.28	0.20	0	0.23	0.30	0.26	0.43	0.56
30	0.23	0.29	0.27	0.20	0	0.21	0.28	0.24	0.45	0.55
31	0.20	0.26	0.31	0.17	0	0.25	0.28	0.28	0.43	0.56
32	0.18	0.24	0.30	0.19	0	0.26	0.30	0.28	0.44	0.57
33	0.18	0.31	0.35	0	\$0.16	0.25	0.35	0.32	0.49	0.61
34	0.18	0.31	0.35	0	0.17	0.26	0.32	0.33	0.50	0.61
35	0.26	0.34	0.37	0	0.16	0.27	0.32	0.28	0.44	0.48
36	0.24	0.32	0.36	0	0.15	0.24	0.21	0.28	0.43	0.50
37	0.24	0.29	0.36	0.14	0	0.24	0.26	0.27	0.39	0.45
38	0.24	0.29	0.33	0.17	0	0.20	0.27	0.26	0.41	0.50
39	0.24	0.28	0.31	0.19	0	0.19	0.27	0.23	0.40	0.50
40	0.24	0.28	0.32	0.17	0	0.18	0.26	0.22	0.39	0.47
41	0.24	0.26	0.31	0.24	0	0.18	0.27	0.25	0.39	0.47
42	0.32	0.35	0.36	0.25	0	0.22	0.21	0.22	0.37	0.40
43	0.28	0.31	0.35	0.21	0	0.23	0.23	0.26	0.37	0.45
44	0.32	0.38	0.40	0.21	0	0.25	0.17	0.28	0.34	0.40
45	0.29	0.35	0.39	0.20	0	0.27	0.20	0.30	0.36	0.45
46	0.28	0.35	0.40	0.18	0	0.27	0.21	0.32	0.36	0.45
47	0.30	0.36	0.40	0	0.19	0.25	0.24	0.35	0.41	0.43
48	0.32	0.38	0.42	0	0.20	0.30	0.26	0.36	0.44	0.43
49	0.33	0.40	0.43	0	0.22	0.28	0.28	0.39	0.45	0.43
50	0.36	0.42	0.46	0	0.24	0.35	0.30	0.40	0.48	0.42
51	0.37	0.47	0.50	0.23	0.25	0.36	0	0.41	0.39	0.43
52	0.40	0.50	0.54	0.26	0.28	0.36	0	0.42	0.38	0.43
53	0.39	0.49	0.53	0.25	0.27	0.35	0	0.44	0.38	0.43
54	0.34	0.44	0.48	0.21	0.22	0.32	0	0.38	0.38	0.43
55	0.41	0.47	0.51	0.29	0.26	0.33	0	0.38	0.31	0.43
56	0.35	0.42	0.47	0.22	0.20	0.29	0	0.34	0.35	0.43
57	0.34	0.40	0.46	0.20	0.19	0.28	0	0.33	0.36	0.43
58	0.35	0.41	0.45	0.26	0.20	0.28	0	0.33	0.33	0.43
59	0.33	0.40	0.43	0.21	0.18	0.25	0	0.31	0.35	0.43
60	0.33	0.37	0.41	0.21	0.16	0.22	0	0.29	0.36	0.40
61	0.41	0.45	0.49	0.28	0.24	0.30	0	0.36	0.31	0.40
62	0.44	0.46	0.48	0.27	0.25	0.26	0	0.33	0.24	0.40
63	0.36	0.40	0.43	0.23	0.19	0.24	0	0.31	0.31	0.40
64	0.37	0.41	0.46	0.24	0.20	0.23	0	0.29	0.31	0.40
65	0.31	0.36	0.41	0.20	0.15	0.19	0	0.25	0.34	0.40
66	0.31	0.36	0.40	0.22	0.16	0.18	0	0.26	0.34	0.40
67	0.33	0.36	0.40	0.24	0.16	0.18	0	0.26	0.35	0.40
68	0.33	0.36	0.40	0.26	0.17	0.16	0	0.24	0.37	0.40
69	0.33	0.36	0.40	0.25	0.17	0.20	0	0.26	0.34	0.40
70	0.35	0.38	0.42	0.23	0.18	0.20	0	0.28	0.33	0.40
71	0.32	0.38	0.42	0.26	0.19	0.22	0	0.28	0.33	0.40
72	0.31	0.38	0.42	0.27	0.19	0.20	0	0.25	0.34	0.40
73	0.37	0.40	0.43	0.29	0.20	0.23	0	0.29	0.31	0.40
74	0.35	0.41	0.45	0.30	0.21	0.24	0	0.31	0.32	0.40
75	0.40	0.33	0.36	0.32	0.21	0.20	0.29	0	0.44	0.40
76	0.41	0.37	0.38	0.34	0.25	0.21	0.31	0	0.41	0.40
77	0.45	0.41	0.41	0.36	0.26	0.25	0.35	0	0.38	0.40
78	0.45	0.36	0.37	0.35	0.25	0.21	0.35	0	0.41	0.40
79	0.47	0.41	0.39	0.37	0.28	0.21	0.37	0	0.40	0.40
80	0.46	0.43	0.44	0.36	0.31	0.27	0.35	0	0.32	0.40
81	0.49	0.42	0.42	0.39	0.30	0.24	0.36	0	0.36	0.40
82	0.49	0.44	0.44	0.39	0.34	0.27	0.39	0	0.34	0.40
83	0.46	0.40	0.40	0.41	0.29	0.23	0.36	0	0.36	0.40
84	0.51	0.43	0.41	0.40	0.31	0.27	0.41	0	0.37	0.40
85	0.45	0.39	0.39	0.40	0.31	0.22	0.38	0	0.39	0.40
86	0.46	0.42	0.43	0.41	0.32	0.26	0.43	0	0.43	0.40
87	0.45	0.40	0.40	0.42	0.31	0.22	0.43	0	0.43	0.40
88	0.45	0.44	0.43	0.40	0.32	0.24	0.43	0	0.43	0.40
89	0.47	0.51	0.59	0.29	0.36	0.47	0.27	0.46	0	0.43
90	0.50	0.54	0.62	0.31	0.38	0.48	0.28	0.46	0	0.40
91	0.46	0.50	0.58	0.28	0.34	0.40	0.24	0.43	0	0.43
92	0.50	0.52	0.60	0.32	0.38	0.41	0.24	0.43	0	0.40
93	0.43	0.47	0.51	0.29	0.26	0.35	0.18	0.38	0	0.50
94	0.42	0.44	0.48	0.29	0.25	0.33	0.16	0.31	0	0.40
95	0.42	0.44	0.48	0.31	0.25	0.33	0.16	0.31	0	0.40
96	0.42	0.44	0.48	0.32	0.26	0.33	0.16	0.31	0	0.40
97	0.49	0.42	0.43	0.36	0.32	0.22	0.38	0.13	0	0.40
98	0.53	0.55	0.62	0.40	0.36	0.44	0.27	0.44	0	0.30
99	0.51	0.54	0.58	0.39	0.34	0.45	0.25	0.34	0	0.29
100	0.60	0.62	0.62	0.40	0.38	0.48	0.34	0.51	0.18	0
101	0.60	0.62	0.62	0.40	0.38	0.48	0.34	0.51	0.18	0
102	0.60	0.62	0.62	0.40	0.38	0.48	0.34	0.51	0.18	0
103	0.60	0.62	0.62	0.45	0.43	0.45	0.34	0.34	0.18	0
104	0.60	0.62	0.62	0.45	0.43	0.45	0.34	0.34	0.18	0

Table A-5. Freight tariff for barley per bushel, by origin and destination.

Originating production region	Destination									
	North-east (1)	Appalachian (2)	South-east (3)	Lake States (4)	Corn Belt (5)	Delta States (6)	Northern Plains (7)	Southern Plains (8)	Mountain (9)	Pacific (10)
1-24										
25	\$0.24	\$0.40	\$0.43	\$0.33	0	\$0.36	\$0.49	\$0.45	\$0.68	\$0.90
26	0.23	0.37	0.45	0.33	0	0.36	0.52	0.47	0.70	0.81
27	0.23	0.37	0.46	0.33	0	0.38	0.52	0.48	0.70	0.92
28	0.24	0.37	0.44	0.30	0	0.35	0.46	0.46	0.67	0.89
29	0.30	0.40	0.42	0.30	0	0.35	0.45	0.39	0.64	0.85
30	0.34	0.43	0.41	0.31	0	0.31	0.41	0.35	0.67	0.83
31	0.30	0.40	0.46	0.26	0	0.37	0.43	0.41	0.64	0.84
32	0.27	0.36	0.46	0.28	0	0.39	0.45	0.43	0.66	0.86
33	0.27	0.47	0.52	0	\$0.25	0.38	0.52	0.47	0.73	0.91
34	0.27	0.47	0.53	0	0.26	0.40	0.48	0.49	0.76	0.92
35	0.39	0.51	0.55	0	0.24	0.40	0.33	0.42	0.66	0.72
36	0.36	0.48	0.54	0	0.22	0.36	0.31	0.41	0.64	0.74
37	0.37	0.44	0.54	0.21	0	0.35	0.39	0.40	0.58	0.67
38	0.35	0.43	0.49	0.26	0	0.31	0.41	0.40	0.61	0.74
39	0.35	0.43	0.47	0.28	0	0.29	0.40	0.35	0.61	0.74
40	0.35	0.43	0.48	0.26	0	0.28	0.39	0.34	0.59	0.71
41	0.41	0.46	0.55	0.41	0	0.32	0.47	0.44	0.69	0.73
42	0.49	0.52	0.54	0.37	0	0.33	0.32	0.34	0.56	0.60
43	0.42	0.46	0.52	0.32	0	0.34	0.34	0.39	0.56	0.67
44	0.48	0.56	0.59	0.32	0	0.37	0.25	0.41	0.51	0.60
45	0.44	0.53	0.59	0.29	0	0.41	0.30	0.45	0.54	0.67
46	0.43	0.52	0.60	0.27	0	0.41	0.31	0.47	0.56	0.67
47	0.46	0.55	0.60	0	0.28	0.38	0.36	0.52	0.61	0.65
48	0.48	0.58	0.64	0	0.31	0.46	0.40	0.54	0.68	0.65
49	0.50	0.59	0.65	0	0.32	0.43	0.43	0.58	0.68	0.64
50	0.54	0.63	0.69	0	0.36	0.52	0.45	0.61	0.72	0.62
51	0.55	0.70	0.76	0.34	0.38	0.54	0	0.54	0.59	0.65
52	0.60	0.75	0.81	0.38	0.42	0.54	0	0.55	0.57	0.65
53	0.59	0.73	0.79	0.37	0.41	0.52	0	0.58	0.56	0.65
54	0.51	0.66	0.72	0.31	0.33	0.48	0	0.58	0.57	0.65
55	0.62	0.70	0.76	0.44	0.39	0.50	0	0.58	0.47	0.65
56	0.53	0.63	0.70	0.33	0.31	0.44	0	0.52	0.53	0.65
57	0.60	0.68	0.68	0.30	0.29	0.42	0	0.50	0.55	0.65
58	0.53	0.62	0.68	0.39	0.31	0.42	0	0.49	0.50	0.65
59	0.49	0.58	0.64	0.32	0.27	0.38	0	0.46	0.52	0.65
60	0.50	0.56	0.62	0.31	0.24	0.33	0	0.43	0.54	0.60
61	0.62	0.67	0.73	0.32	0.36	0.45	0	0.55	0.46	0.60
62	0.66	0.68	0.73	0.41	0.37	0.39	0	0.49	0.36	0.60
63	0.54	0.60	0.65	0.35	0.28	0.37	0	0.47	0.47	0.60
64	0.55	0.61	0.69	0.36	0.30	0.34	0	0.44	0.46	0.60
65	0.45	0.54	0.62	0.30	0.22	0.28	0	0.38	0.51	0.60
66	0.47	0.53	0.59	0.34	0.24	0.27	0	0.39	0.51	0.60
67	0.49	0.53	0.59	0.36	0.24	0.27	0	0.39	0.53	0.60
68	0.50	0.54	0.61	0.38	0.25	0.25	0	0.36	0.56	0.60
69	0.50	0.54	0.61	0.37	0.25	0.25	0	0.39	0.51	0.60
70	0.53	0.57	0.63	0.35	0.27	0.30	0	0.41	0.49	0.60
71	0.48	0.58	0.64	0.40	0.28	0.32	0	0.42	0.50	0.60
72	0.47	0.58	0.64	0.40	0.28	0.30	0	0.38	0.51	0.60
73	0.55	0.59	0.65	0.43	0.30	0.34	0	0.44	0.47	0.60
74	0.53	0.61	0.67	0.44	0.32	0.36	0	0.46	0.48	0.60
75	0.58	0.50	0.53	0.48	0.32	0.30	0.43	0	0.66	0.60
76	0.62	0.56	0.57	0.50	0.37	0.32	0.47	0	0.61	0.60
77	0.68	0.62	0.61	0.54	0.39	0.38	0.52	0	0.57	0.60
78	0.68	0.55	0.55	0.53	0.37	0.32	0.52	0	0.62	0.60
79	0.71	0.61	0.59	0.56	0.42	0.32	0.55	0	0.60	0.60
80	0.69	0.64	0.67	0.54	0.46	0.40	0.53	0	0.48	0.60
81	0.73	0.63	0.63	0.59	0.46	0.36	0.54	0	0.54	0.60
82	0.74	0.66	0.66	0.58	0.51	0.40	0.59	0	0.51	0.60
83	0.70	0.60	0.60	0.61	0.43	0.35	0.55	0	0.54	0.60
84	0.76	0.65	0.61	0.60	0.47	0.40	0.62	0	0.55	0.60
85	0.67	0.58	0.59	0.60	0.46	0.32	0.56	0	0.58	0.60
86	0.70	0.63	0.64	0.62	0.48	0.39	0.64	0	0.64	0.60
87	0.68	0.60	0.61	0.62	0.47	0.32	0.64	0	0.64	0.60
88	0.68	0.66	0.64	0.60	0.49	0.36	0.65	0	0.64	0.60
89	0.70	0.77	0.88	0.44	0.54	0.70	0.40	0.70	0	0.65
90	0.75	0.81	0.93	0.47	0.57	0.71	0.41	0.69	0	0.60
91	0.69	0.75	0.87	0.42	0.51	0.60	0.35	0.65	0	0.65
92	0.75	0.78	0.90	0.47	0.57	0.61	0.36	0.64	0	0.60
93	0.65	0.71	0.77	0.43	0.40	0.52	0.27	0.58	0	0.75
94	0.62	0.66	0.72	0.44	0.37	0.50	0.24	0.46	0	0.60
95	0.62	0.66	0.72	0.46	0.37	0.40	0.24	0.46	0	0.60
96	0.62	0.66	0.72	0.49	0.39	0.49	0.24	0.46	0	0.60
97	0.74	0.63	0.64	0.55	0.48	0.34	0.58	0.19	0	0.60
98	0.79	0.83	0.93	0.60	0.54	0.66	0.41	0.55	0	0.45
99	0.77	0.81	0.87	0.58	0.51	0.55	0.38	0.51	0	0.44
100	0.90	0.93	0.93	0.60	0.64	0.71	0.51	0.55	0.26	0
101	0.90	0.93	0.93	0.60	0.64	0.71	0.51	0.55	0.26	0
102	0.90	0.93	0.93	0.60	0.64	0.71	0.51	0.55	0.26	0
103	0.90	0.93	0.93	0.68	0.64	0.67	0.51	0.51	0.26	0
104	0.90	0.93	0.93	0.68	0.64	0.67	0.51	0.51	0.26	0

Table A-6. Freight tariff for grain sorghums per bushel, by origin and destination.

Originating production region	Destination									
	North-east (1)	Appala-chian (2)	South-east (3)	Lake States (4)	Corn Belt (5)	Delta States (6)	* Northern Plains (7)	Southern Plains (8)	Mountain (9)	Pacific (10)
1-40										
41	\$0.41	\$0.46	\$0.55	\$0.41	0	\$0.32	\$0.47	\$0.44	\$0.69	\$0.73
42	0.57	0.61	0.63	0.43	0	0.38	0.37	0.39	0.66	0.60
43	0.49	0.54	0.61	0.38	0	0.40	0.40	0.46	0.66	0.69
44-61										
62	0.77	0.80	0.85	0.48	\$0.43	0.46	0	0.57	0.40	0.60
63	0.62	0.69	0.76	0.41	0.33	0.43	0	0.55	0.44	0.60
64	0.64	0.71	0.80	0.42	0.35	0.40	0	0.51	0.54	0.60
65	0.54	0.63	0.72	0.35	0.26	0.33	0	0.44	0.58	0.60
66	0.55	0.62	0.69	0.39	0.28	0.31	0	0.46	0.58	0.48
67	0.57	0.62	0.69	0.42	0.28	0.31	0	0.46	0.62	0.48
68	0.58	0.64	0.71	0.45	0.29	0.29	0	0.41	0.65	0.48
69	0.58	0.64	0.71	0.44	0.29	0.34	0	0.46	0.59	0.48
70	0.61	0.66	0.73	0.41	0.32	0.34	0	0.48	0.58	0.48
71	0.56	0.67	0.74	0.46	0.33	0.38	0	0.49	0.58	0.48
72	0.55	0.67	0.74	0.47	0.33	0.35	0	0.44	0.58	0.48
73	0.64	0.69	0.76	0.50	0.35	0.40	0	0.51	0.50	0.48
74	0.61	0.72	0.79	0.52	0.37	0.42	0	0.54	0.53	0.45
75	0.68	0.58	0.62	0.56	0.37	0.36	0.50	0	0.76	0.60
76	0.72	0.65	0.67	0.59	0.43	0.37	0.55	0	0.71	0.60
77	0.79	0.72	0.72	0.63	0.46	0.44	0.61	0	0.66	0.58
78	0.79	0.64	0.64	0.62	0.43	0.37	0.61	0	0.72	0.60
79	0.83	0.72	0.69	0.65	0.49	0.37	0.64	0	0.70	0.58
80	0.80	0.74	0.78	0.63	0.54	0.46	0.61	0	0.55	0.57
81	0.86	0.73	0.74	0.70	0.53	0.41	0.63	0	0.64	0.58
82	0.86	0.77	0.77	0.68	0.59	0.46	0.68	0	0.59	0.57
83	0.80	0.69	0.70	0.72	0.51	0.41	0.64	0	0.64	0.60
84	0.88	0.75	0.71	0.71	0.55	0.46	0.72	0	0.65	0.57
85	0.79	0.68	0.70	0.69	0.54	0.38	0.65	0	0.68	0.60
86	0.81	0.74	0.75	0.72	0.56	0.45	0.75	0	0.74	0.46
87	0.79	0.70	0.71	0.73	0.54	0.38	0.75	0	0.74	0.60
88	0.79	0.77	0.74	0.71	0.57	0.42	0.76	0	0.75	0.60
89-94										
95	0.73	0.78	0.85	0.54	0.43	0.57	0.28	0.54	0	0.45
96	0.73	0.78	0.85	0.57	0.46	0.57	0.28	0.54	0	0.45
97	0.86	0.73	0.75	0.64	0.56	0.39	0.67	0.22	0	0.45
98-102										
103	1.05	1.09	1.09	0.80	0.75	0.78	0.60	0.60	0.31	0
104	1.05	1.09	1.09	0.80	0.75	0.78	0.60	0.60	0.31	0

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