Production Functions and Methods of Specifying Optimum Fertilizer Use Under Various Uncertainty Conditions for Hay

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CONTENTS

SUMMARY

Farmers who use fertilizer on a crop are concerned with at least two uncertainty problems relating to the amount of fertilizer that maximizes profits. Both of these problems arise, in part, from weather, a common underlying cause. First, crop yield is uncertain. Thus, a quantity of fertilizer applied, with the decision made before planting, may not be optimum for the yield actually obtained. Second, at the time of planting, future crop use or price may also be indefinite. The value of hay, for example, may differ according to the method of utilization. Hence, a particular amount of fertilizer applied in the spring may not provide the optimum value product at the end of the season.

This study is concerned with the estimation of production functions and the analysis of uncertainty problems as they relate to fertilization of alfalfa with P_2O_5 and K_2O. The basic data are taken from an experiment in which three hay cuttings were obtained during the 1952 growing season on Weller silt loam in Van Buren County, Iowa.

For estimation of production surfaces and derivation of economic optima, a quadratic function provided the best fit for the data. The equations for the first, second and third cuttings were used to predict production surfaces for fertilization of hay. The positively sloped portions of surfaces presented in the text express the range of fertilization levels and the mixtures that are relevant to decision-making under the particular environmental conditions of the study. Isoquants, denoting marginal rates of substitution between nutrients, also were computed. In general, the substitution rates do not decline rapidly for different proportions of nutrients at a given yield level. Too, the isoquants for the three cuttings were similar in slope relative to a given proportion of nutrients. The production function equations provided the information used to predict (a) profit-maximizing levels and ratios of P_2O_5 to K_2O and (b) the levels of fertilization that maximize the return on fertilizer investment.

The fertilizer levels that maximize profits per acre under various prices are higher than those that maximize the return on the fertilization investment. This relationship holds under all conditions where fertilization is profitable. At prices of \$20 per ton for hay and 10 and 5 cents per pound, respectively, for P_2O_5 and K_2O, the profit-maximizing level is 64 pounds of P_2O_5 and 79 pounds of K_2O. It takes 55 pounds of K_2O per acre to maximize profit on the fertilization investment when a uniform application of 20 pounds of P_2O_5 per acre is applied and when application costs are \$1.30 per acre. For the prices and applications of P_2O_5 studied, the rates of K_2O that maximize return on the fertilization investment range only 11 pounds—from 52 to 63 pounds per acre.

The initial analysis included estimation of optimum quantities of fertilizer under different price, capital and resource-use situations. For this analysis, the number of cuttings per year was assumed to be known with certainty. The optimum quantities and optimum combination of the two nutrients were computed. However, since many farmers use fertilizer grades available in the market (e.g., 0-20-20), computations were made for the three most common ones to determine the effect of their application on net returns, as compared with the optimum mix. Decreases in profits resulting from the use of these three grades, or the optimum mix, varied from nil to 32 percent in the cases examined.

Next, a situation was analyzed where it was assumed that uncertainty exists with respect to the number of cuttings harvested each year. Climatic data were available to indicate the probability of drouth conditions. On the basis of these data, it was concluded that, over a 5-year planning period, three cuttings could be expected in 4 years and two cuttings, in the remaining year. Ex ante, the decision-maker must anticipate the number of cuttings and apply fertilizer accordingly. His expectations may or may not be correct. Within this setting, it was shown that losses could be minimized if the decision-maker assumed that three cuttings could be expected every year.

The effect of on-farm utilization of alfalfa in forms other than hay on optimum rates of fertilizer was considered. This effect was assumed to be that of price alone rather than that of "side-benefits." A procedure was outlined whereby a value could be imputed to alfalfa hay depending on its method of utilization. Ex ante knowledge of the form of the production function was assumed for this purpose. Fertilizer grades that minimize the cost of producing a required output were then computed.

The most complex situation analyzed was that in which both the number of cuttings and the price (as a result of utilization) were assumed to be uncertain at the time of fertilizer application. This problem was treated as one of decision-making under absolute uncertainty (a game against nature). Three decision-making models (Wald, Hurwicz and Laplace) were then applied to provide criteria for the course of action that the farmer should follow. Each criterion indicated the same act or decision with respect to fertilization as being optimal.

For the data analyzed under uncertainty conditions, the effect on profits resulting from errors in grade of fertilizer used was greater, within limits, than the effect of the rate of use of a particular grade. This stresses the need for ex ante information, such as soil tests, to predict "ideal" fertilizer ratios or combinations.

The data indicate that differences in net profits arising from the use of various fertilizer grades and different levels of application are not necessarily large. Hence, at times, the analysis has been given an unwarranted definiteness, particularly when concerning the

use of one grade rather than another. The objection may be considered even more justifiable if the environment concerned is judged to be at all variable or uncertain.

There are some dangers in using production function data for predictive purposes. Recommendations may be made on the basis of one experiment carried out under particular environmental conditions during a single year. Unless the circumstances are similar in future years, the suggested rates of fertilization may not maximize profit. If, however, data are built up for various soil types under changeable environmental conditions, greater accuracy can be achieved in advice to farmers.

Production Functions and Methods of Specifying Optimum Fertilizer Use Under Various Uncertainty Conditions for Hay[1]

By Earl O. Heady, John T. Pesek
and W. Owen McCarthy

One problem confronting a farmer, acting in his capacity as a decision-maker, concerns planning for a future that is uncertain with respect to yields, prices or both. This study examines the significance of decisions centering around fertilizer use, assuming that it has been decided to use **some** fertilizer.

Once a positive decision has been made to use fertilizer, a whole new series of choices must be faced. Ideally, a choice of specific fertilizer elements should be made on the basis of soil-test data and known crop requirements and response; in practice, however, often complete information is not available to the individual farmer. This study seeks to demonstrate some alternative types of choices and recommendations which can be applied to fertilization problems under uncertainty. The data are not expected to have general use. The applications made use of a particular set of data to illustrate choices under alternative decision criteria and assumed production and price outcomes.

OBJECTIVES

The main objective of this study was to develop methods for estimating or specifying optimum fertilizer quantities under different settings with respect to price, capital, resource availability and uncertainty of number of cuttings and method of utilization under conditions found in a selected fertilizer topdressing experiment with alfalfa. To achieve this objective, it was necessary to estimate production or response surfaces for alfalfa fertilized at a particular site. These response surfaces are then used to predict isoquants, isoclines and related quantities that are basic to decisions on optimum fertilizer use.

Initially, the problem of decision-making is limited to determining the rate of fertilization of alfalfa that would maximize profits when one, two or three cuttings can be expected with certainty. A study is made of the optimum quantities and ratios of nutrients under assumed price situations, and comparisons are made of the sacrifice in profit resulting from the use of other than optimal combinations. Next, it is assumed that the farmer is uncertain as to whether he will get one, two or three cuts from his alfalfa, and the decision-making problem is explored within this setting.

The optimum application of fertilizer depends on expected yields, the method of utilization or the market value of the crop and the cost of fertilizer. Since a farmer may sell alfalfa as hay or use it for feed, the optimum rate of fertilization will depend on the market price of hay or imputed value for the livestock that use it. Therefore, a decision-making situation is examined where it is assumed that the method of utilization and, hence, the price, is uncertain. Optimum strategies for fertilizer use are developed for those situations in which both yield and ultimate disposal are uncertain.

Decision theory or criteria are used to specify optimum choices under the several situations just outlined. The situations do not include all of the uncertainties, especially those of price or of weather, confronting the farmer when he makes decisions on quantities and mixtures of fertilizer nutrients. However, the situations included are thought to be more realistic than the situations usually assumed for recommendations to farmers.

SOURCE OF BASIC DATA

The alfalfa fertilization experiment from which the basic data were obtained was on Weller silt loam in Van Buren County in 1952. Three levels each of phosphorus and potassium fertilizer were topdressed on an established alfalfa stand early in the spring. The design was a

Table 1. Yields of alfalfa on Weller silt loam in 1952 (tons oven-dry material per acre).

Rate of fertilization (lbs./acre)		1st cut		2nd cut		3rd cut	
		Replicate		Replicate		Replicate	
P_2O_5	K_2O	I	II	I	II	I	II
0	0	0.73	0.84	0.69	0.95	0.45	0.55
60	0	0.94	1.41	0.85	0.95	0.60	0.50
120	0	1.16	1.38	0.83	0.92	0.62	0.57
0	60	1.05	1.21	0.85	0.85	0.55	0.57
60	60	1.32	1.49	0.92	0.92	0.65	0.67
120	60	1.27	1.56	0.97	1.16	0.62	0.72
0	120	1.05	1.32	0.90	0.92	0.57	0.62
60	120	1.49	1.27	1.00	1.00	0.69	0.57
120	120	1.68	1.38	1.07	1.00	0.67	0.65

[1]Projects 1189 and 1293 of the Iowa Agricultural and Home Economics Experiment Station, in cooperation with the Tennessee Valley Authority.

3 x 3 factorial, replicated twice, and three cuttings were made during the growing season. Table 1 presents the treatments and yields of dry matter from the experiment.

Regression Analysis

As a foundation for the analysis which follows, a regression equation was fitted to the data from each cutting. It was decided, after preliminary examination of

Table 2. Regression coefficients, standard errors and t-values and their probabilities for equations 1 through 5 expressing tons of alfalfa dry matter per acre as a function of pounds of P_2O_5 (P) and K_2O (K) applied per acre in Van Buren County, Iowa, in 1952.

	Coefficients of[a]					
	P	K	P^2	K^2	PK	Intercept
Equation 1[b]						
Partial regression coefficients	0.007042	0.006181	—0.000028	—0.000027	—0.000010	0.8222
Standard errors	0.002865	0.002723	0.000020	0.000019	0.000015	
t-values	2.59	2.27	1.36	1.29	0.67	
Probability levels of t[c]	0.02	0.05	0.20	0.20	0.50	
Equation 2						
Partial regression coefficients	0.001278	0.001403	—0.000004	—0.000006	0.000005	0.8119
Standard errors	0.002028	0.002033	0.000015	0.000015	0.000011	
t-values	0.63	0.69	0.26	0.39	0.45	
Probability levels of t	0.50	0.50	0.50	0.50	0.50	
Equation 3						
Partial regression coefficients	0.001431	0.002181	—0.000005	—0.000012	—0.000002	0.4903
Standard errors	0.001044	0.001048	0.000008	0.000008	0.000005	
t-values	1.37	2.08	0.58	1.46	0.37	
Probability levels of t	0.20	0.05	0.50	0.20	0.50	
Equation 4						
Partial regression coefficients	0.008320	0.007584	—0.000032	—0.000033	—0.000005	1.6342
Standard errors	0.002432	0.002430	0.000018	0.000019	0.000013	
t-values	3.42	3.12	1.73	1.77	0.37	
Probability levels of t	0.01	0.01	0.10	0.10	0.50	
Equations 5						
Partial regression coefficients	0.009751	0.009765	—0.000037	—0.000045	—0.000007	2.1245
Standard errors	0.003038	0.003033	0.000023	0.000023	0.000016	
t-values	3.21	3.22	1.59	1.92	0.43	
Probability levels of t	0.01	0.01	0.15	0.05	0.50	

[a]Variables P and K refer to pounds per acre of P_2O_5 and K_2O, respectively.

[b]Equations 1, 2 and 3 express the yields of dry matter of the first, second and third cuttings, respectively, while equation 4 expresses the yield of the first two cuttings and equation 5 expresses the total yield of dry matter.

[c]Probability of drawing a t-value as large or larger given the null hypothesis.

Table 3. Analyses of variance for alfalfa cuttings on Weller silt loam in 1952.

	Source of variation	Degrees of freedom	Sum of squares		Mean squares	F
Cutting 1	Total	17	1.0904			
Equation 1	Replicates	1	0.0760			
	Treatments	8	0.7747			
	Due to regression	5		0.7434	0.1487	4.96*
$R^2=0.959^{**}$	Lack of fit	3		0.0313		
	Error	8	0.2396		0.0300	
Cutting 2	Total	17	0.1777			
Equation 2	Replicates	1	0.0193			
	Treatments	8	0.1141			
	Due to regression	5		0.0985	0.0197	3.57+
$R^2=0.863^{**}$	Lack of fit	3		0.0156		
	Error	8	0.0442		0.0055	
Cutting 3	Total	17	0.0795			
Equation 3	Replicates	1	0.0000			
	Treatments	8	0.0542			
	Due to regression	5		0.0498	0.0100	3.15+
$R^2=0.918^{**}$	Lack of fit	3		0.0045		
	Error	8	0.0253		0.0032	
Cutting 1+2	Total	17	1.4346			
Equation 4	Replicates	1	0.1721			
	Treatments	8	1.3785			
	Due to regression	5		1.3534	0.2707	5.04*
$R^2=0.982$	Lack of fit	3		0.0251		
	Error	8	0.3840		0.0480	
Cuttings 1+2+3	Total	17	2.6018			
Equation 5	Replicates	1	0.1721			
	Treatments	8	1.9530			
	Due to regression	5		1.9118	0.3824	6.42**
$R^2=0.979^{**}$	Lack of fit	3		0.0412		
	Error	8	0.4767		0.0596	

\+ $P > 0.05$
* $P \leq 0.05$
** $P \leq 0.01$

several functions, to use a second-degree polynomial function with an interaction term. Yields for the first and second and for the first, second and third cuttings were added together, and functions also were fitted to these totals. The five resulting equations are presented, in the order mentioned, in table 2. Note that equation 4 can be obtained by adding equations 1 and 2. Equation 5 can be obtained by adding equations 1, 2 and 3. The **t**-values and the standard errors of each coefficient for the equations are also included in table 2.

Table 3 presents the analyses of variance for the yield data corresponding to each of the five regression equations. The over-all significance of the regressions was tested by means of the **F**-ratio. The **F**-values are all significant at less than the 5-percent level, except for the second and third cuts taken alone where the **F**-values fall just slightly above the 5-percent level of significance.

All regression coefficients have been retained in the predicting equations because they play a logical role in fertilizer response. All partial regression coefficients have the expected signs, and all except the interaction term coefficients are greater than their respective standard errors for equations 1, 4 and 5 which are used in subsequent analyses. Omission of the crossproduct terms would not appreciably affect the estimates.

Nature of the Production Surfaces

Equations 1, 4 and 5 were used to derive expected yields of alfalfa for various P_2O_5 and K_2O levels within the experimental range. These are shown in table 4, and, together with additional estimates, have been used to construct the production surfaces of fig. 1. The relevant range of fertilization for the experiment was from 0 to 120 pounds per acre for both nutrients, but, for illustrative purposes, extrapolation has been made beyond this range.

The comparative heights of each surface in fig. 1 are a reflection of the accumulated yield after each cutting. The differences in heights represent the addition

Table 4. Expected yields of alfalfa (tons oven-dry material per acre) for various P_2O_5 and K_2O levels (lbs. per acre).

		K_2O			
	P_2O_5	0	40	80	120
1st cut	0	0.82	1.03	1.14	1.18
	40	1.06	1.27	1.35	1.36
	80	1.21	1.38	1.46	1.46
	120	1.26	1.42	1.49	1.47
1st + 2nd cut	0	1.63	1.88	2.03	2.07
	40	1.92	2.16	2.30	2.33
	80	2.10	2.33	2.46	2.48
	120	2.17	2.40	2.52	2.53
1st + 2nd + 3rd cut	0	2.12	2.44	2.62	2.65
	40	2.66	2.76	2.93	2.94
	80	2.67	2.96	3.12	3.12
	120	2.76	3.05	3.19	3.18

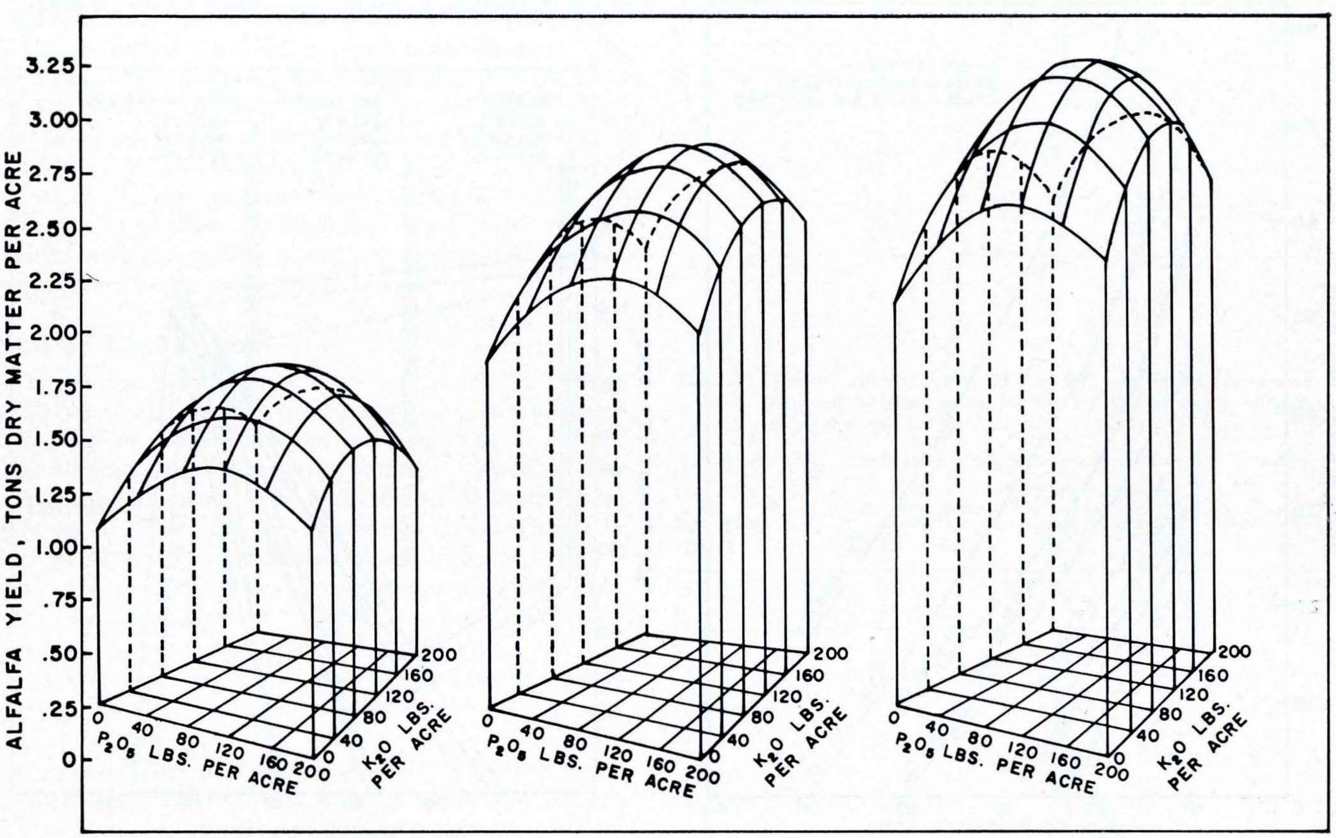

Fig. 1. Dry matter production surfaces for the first, first and second, and first, second and third cuttings of alfalfa, respectively, from left to right.

to total yield from the extra cutting. The second cutting was heavier than the third (as is reflected in equations 2 and 3). Hence, the increase in height between the first and second surfaces is greater than the increase between the second and third.

Nature of the Yield Isoquants

Isoquants[2] were derived from production functions 1, 4 and 5 after they were adjusted for the moisture content of hay.[3] These adjusted functions were used to graph a set of three isoquants for each equation, and these isoquants are shown in fig. 2. The isoquants predict the various combinations of P_2O_5 and K_2O that were required to produce a particular alfalfa hay yield. Some of these combinations are shown in table 5 along with the marginal rates of substitution of P_2O_5 for K_2O. Thus, for one cutting, 5 pounds of K_2O and 54 pounds of P_2O_5, or 30 pounds of K_2O and 14 pounds of P_2O_5, both gave a yield of 1.3 tons of hay, and the marginal rates of substitution are 0.744 and 1.346 pounds P_2O_5 per pound K_2O, respectively.

[2]For a complete procedure for calculating isoquants see: Earl O. Heady, John T. Pesek and William G. Brown. Crop response surfaces and economic optima in fertilizer use. Iowa Agr. Exp. Sta. Res. Bul. 424. 1955.

[3]Commercially, hay is usually considered to contain 12 percent moisture; prices are quoted for a product with this characteristic, and deviations are adjusted for the actual price paid. The following analyses are less cumbersome with this change reflected in the equations.

Table 5. Fertilizer combinations and corresponding marginal rates of substitution for various hay yields.

1st cut 1.3 tons per acre			1st+2nd cut 2.5 tons per acre			1st+2nd+3rd cut 3.0 tons per acre		
		MRS[a]			MRS[a]			MRS[a]
Lb. K_2O	Lb. P_2O_5	P_2O_5 for K_2O	Lb. K_2O	Lb. P_2O_5	P_2O_5 for K_2O	Lb. K_2O	Lb. P_2O_5	P_2O_5 for K_2O
5	54	0.744	5	75	0.507	5	69	0.528
10	45	0.856	10	59	0.677	10	59	0.634
20	28	1.095	20	37	0.962	20	40	0.868
30	14	1.346	30	15	1.303	30	23	1.134
40	0	1.656	35	6	1.479	40	9	1.437

[a]Pounds of K_2O replaced by 1 pound of P_2O_5 for particular yield levels.

The isoquants are curved and indicate diminishing marginal rates of substitution. The change in slope from left to right is gradual, indicating that the nutrients are close substitutes, within the range of the experiment, for attaining a given yield increase.

Nature of the Yield Isoclines

Yield isoclines[4] were derived from the basic production functions for the first, first plus second, and first plus second plus third cuttings corresponding to the respective production functions 1, 4 and 5. An isocline family has been drawn in fig. 3 for each of the three equations or production functions. Relative slopes of each set of isoclines are quite similar for the price ratios used here. This means that expansion paths of produc-

[4]See: Earl O. Heady, et. al., loc. cit. for a complete discussion of isoclines.

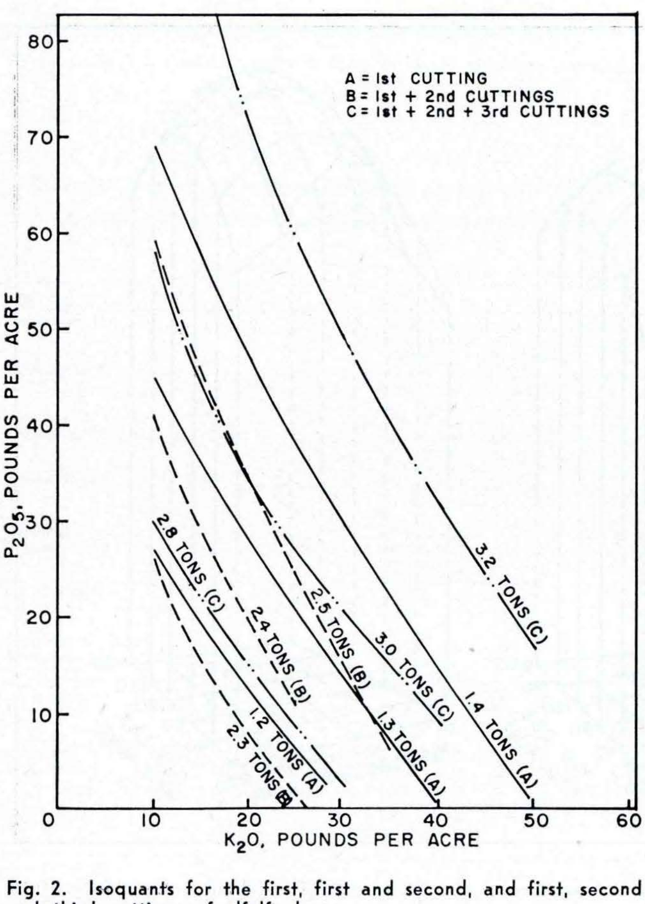

Fig. 2. Isoquants for the first, first and second, and first, second and third cuttings of alfalfa hay.

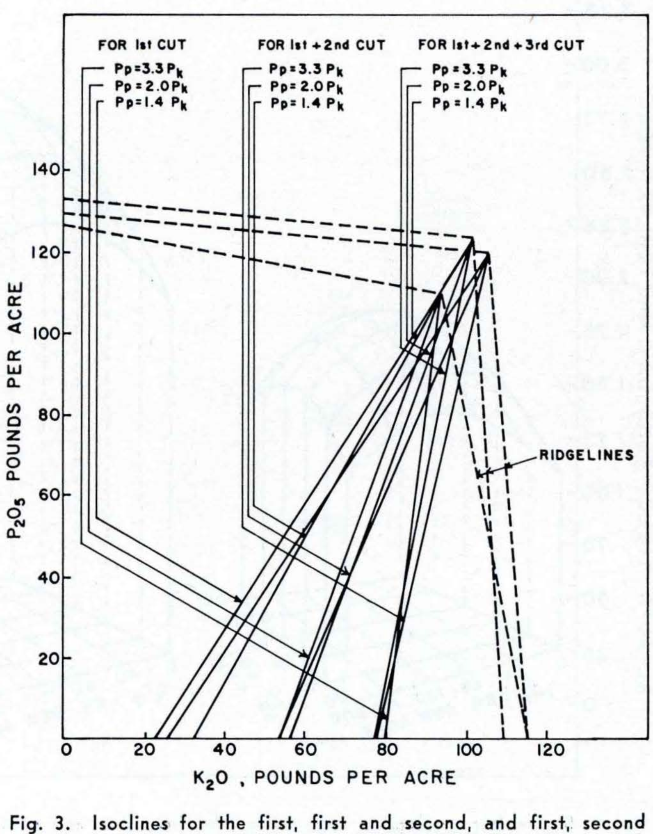

Fig. 3. Isoclines for the first, first and second, and first, second and third cuttings of alfalfa hay. (Dashed lines are ridgelines.)

tion are about the same whether one, two or three cuttings are expected; i.e., least-cost mixes of P_2O_5 and K_2O do not change much with number of cuttings. The maximum is predicted to come with less of both nutrients for a single cutting than for two or three cuttings, but the location of isocline convergence differs only slightly for two and three cuttings. In this case, maximum yield for three cuttings is predicted with slightly more P_2O_5 but slightly less K_2O than for two cuttings.

DERIVATION OF ECONOMIC OPTIMA

Optimum fertilizer quantities are derived in this section under the assumptions of certainty with respect to yields, utilization, number of cuttings and price. Prices used are the monthly averages in the Iowa market. The justification for this approach is that hay has a value no greater than the current market price for a farmer who wishes to sell. On the other hand, the farmer cannot impute a value higher than the market price to his own hay. If his cost of production were higher than this, other things equal, he should buy hay. Appendix A shows the range in monthly prices received by Iowa farmers for alfalfa hay for the period 1944-58.

Three possibilities relating to various optima are examined in this section. The first and simplest possibility involves deriving profit-maximizing quantities of fertilizer under the assumption that the farmer has unlimited capital available for its purchase. The second possibility involves deriving the quantity of fertilizer that maximizes return per dollar invested in fertilization. The third possibility involves determining the relative profitability among common fertilizer grades marketed in Iowa and as compared with profit-maximizing (optimum) blends. In computing these several fertilization quantities, the only cost considered is that of the fertilizer and its application. If fixed harvesting costs were included, indicated profits would be lowered, but fertilizer quantities would generally be the same.

Unlimited Capital Situation

Profit-maximizing quantities of fertilizer, where capital is unlimited and prices and yields are assumed to be known with certainty, were derived from the original production function equations 1, 4 and 5. The partial derivatives of yield with respect to both P and K for each function were equated to the nutrient/hay price ratio, and the profit-maximizing rates of fertilization were determined by simultaneously solving each pair of equations for P and K. The results of these calculations are presented in tables 6, 7 and 8.

As the price of hay increases, the data in the tables indicate that it is profitable to apply more fertilizer; but, as fertilizer prices increase, with the price of hay remaining constant, net profits are maximized by restricting fertilizer use. The profit-maximizing quantity of fertilizer also increases as the number of cuttings expected increases.

As hay prices rise, the nutrient proportions that

Table 6. Profit-maximizing rates of P and K fertilization for various hay and fertilizer prices for the first cutting of alfalfa hay.

Hay price ($/ton)	Fertilizer price (cents/lb.)		Profit-maximizing quantities of fertilizer (lbs./A.)		Hay yield (tons/A.)
	P_2O_5	K_2O	P_2O_5	K_2O	
15	8	3	27	77	1.44
15	10	5	9	58	1.28
15	12	7	0	27	1.09
20	8	3	48	82	1.55
20	10	5	35	67	1.46
20	12	7	21	53	1.34
25	8	3	61	84	1.60
25	10	5	50	73	1.54
25	12	7	39	62	1.47
30	8	3	69	86	1.63
30	10	5	60	77	1.59
30	12	7	51	67	1.53

Table 7. Profit-maximizing rates of P and K fertilization for various hay and fertilizer prices for the total of the first two cuttings of alfalfa hay.

Hay price ($/ton)	Fertilizer price (cents/lb.)		Profit-maximizing quantities of fertilizer (lbs./A.)		Hay yield (tons/A.)
	P_2O_5	K_2O	P_2O_5	K_2O	
10	8	3	12	73	2.35
10	10	5	0	47	2.15
10	12	7	0	20	1.99
15	8	3	48	84	2.62
15	10	5	31	67	2.48
15	12	7	14	51	2.29
20	8	3	66	89	2.72
20	10	5	54	77	2.63
20	12	7	41	64	2.53
25	8	3	77	92	2.76
25	10	5	67	82	2.71
25	12	7	57	72	2.64
30	8	3	84	94	2.78
30	10	5	76	86	2.75
30	12	7	67	78	2.70

Table 8. Profit-maximizing rates of P and K fertilization for various hay and fertilizer prices for the total of all three cuttings of alfalfa hay.

Hay price ($/ton)	Fertilizer price (cents/lb.)		Profit-maximizing quantities of fertilizer (lbs./A.)		Hay yield (tons/A.)
	P_2O_5	K_2O	P_2O_5	K_2O	
10	8	3	28	77	3.18
10	10	5	5	59	2.90
10	12	7	0	39	2.73
15	8	3	60	85	3.41
15	10	5	45	72	3.29
15	12	7	30	60	3.13
20	8	3	76	88	3.49
20	10	5	64	79	3.42
20	12	7	53	70	3.33
25	8	3	85	91	3.53
25	10	5	76	83	3.49
25	12	7	67	76	3.43
30	8	3	92	92	3.55
30	10	5	84	86	3.52
30	12	7	77	80	3.48

maximize profits change considerably. In table 8, assume that P_2O_5 costs 8 cents per pound and that K_2O costs 3 cents per pound. When hay is selling at $10 per ton, the proportion of P_2O_5 to K_2O that maximizes profits is approximately 1:3. But if the hay price rises to $30, the ratio changes to 1:1. As nutrient prices change relative to each other, however, the profit-maximizing proportions of fertilizer nutrients change in such a way that the relative quantity of the nutrient becoming relatively more expensive will be decreased.

Comparison of tables 6, 7 and 8 emphasizes that correct anticipation of the number of cuts (and thus the ex ante decision to fertilize accordingly) can have significant consequences on costs of fertilization and on profit. For example, the farmer may expect two cuttings and apply the corresponding amount of fertilizer but realize only one cutting. Assuming prices of $15 for hay, 8 cents for P_2O_5 and 3 cents for K_2O, table 7 shows that, on the basis of expectations of two cuttings, 48 pounds of P_2O_5 and 84 pounds of K_2O should be applied. However, if only one cutting is realized, with the yield levels indicated by equation 1, only 27 pounds of P_2O_5 and 77 pounds of K_2O should be applied. The farmer will have applied an excess of 21 pounds of P_2O_5 and 7 pounds of K_2O. Without accounting for residual value, the cost of the excess fertilizer is $1.89 per acre. If prices per pound for P_2O_5 and K_2O are now assumed to rise to 12 and 7 cents, respectively, the excess fertilizer has a value of $3.36 per acre. Or, suppose that the farmer fertilizes for three cuts (table 8) but gets only two (table 7). With hay at $15 per ton, P_2O_5 at 12 cents and K_2O at 7 cents per pound, the excess fertilizer has a value of $2.55 per acre.

Limited Capital Situation

The previous section indicated profit-maximizing quantities of fertilizer where a farmer is not limited on capital for purchase of fertilizer. But most farmers must allocate limited capital among competing investment alternatives. Under these conditions, profit for the farm as a whole is maximized if investments in fertilizer, feed, livestock and other alternatives are pushed to levels so that marginal value returns on investment are equal among them. This criterion, rather than the one discussed in the preceding section then is relevant. However, to make fertilizer recommendations on the basis of equal marginal returns on investment would require knowledge of (1) the amount of capital available and (2) the return from various increments of it invested in different alternatives. In the absence of this knowledge, a substitute criterion is one of nutrient quantities that maximize return on the investment in fertilization.[5] These quantities have been derived for specified prices in this section.

The amount of fertilizer that maximizes returns per dollar invested in fertilization may be derived as follows where we have a production function of the form:

$$Y = a + bF + cF^2. \quad (6)$$

Y is yield, and F is fertilizer applied. Then $\triangle Y$, yield increase from fertilization, is

$$\triangle Y = bF + cF^2. \quad (7)$$

If e is price per unit of product, a value function for the response can be constructed from the production function as follows

$$V = ebF + ecF^2. \quad (8)$$

The following cost function, C, may also be constructed

$$C = f + gF \quad (9)$$

where f is the fixed cost associated with application of fertilizer per unit of area, and g is the price per pound of F. The return per dollar invested in fertilization may be expressed as

$$I = \frac{ebF + ecF^2}{f + gF}. \quad (10)$$

The return on the money invested is maximized by setting the first derivative of I with respect to F equal to zero and by solving for F at relevant values of e, f and g.

The alfalfa experimental results include two nutrients, P_2O_5 and K_2O. If one nutrient is held constant in the basic production functions 1, 4 and 5 (converted to a constant in hay yields), a value function may be derived for the remaining nutrient. Thus returns are maximized for the nutrient allowed to vary—given the other nutrient fixed at specified rates.[6] If fixed application and fertilizer costs and a hay price are then assumed as in table 9, the amounts of fertilizer maximizing returns per dollar invested can be derived. The different figures for each price situation illustrate the amount of potassium fertilizer needed to maximize returns on the ferti-

[5]See also: John Pesek and Earl O. Heady. Derivation and application of a method for determining minimum recommended rates of fertilization. Proc. Soil Sci. Soc. Amer. 22: 419-423. 1958.

[6]This procedure is an approximation of the procedure implied in equation 10 wherein the cost of one nutrient is fixed value at any one rate and, like the cost of application, it appears in the term f in equations 9 and 10 when calculations are made.

Table 9. Quantities of K_2O maximizing return per dollar invested in fertilization of three cuttings of alfalfa hay at different fertilizer prices given; $20 per ton for hay, fixed rates of P_2O_5 applied and different costs of application.

Fixed inputs: Cost of application per acre	Fixed inputs: Lbs. P_2O_5 applied per acre	Fertilizer prices (cents/lb.) P_2O_5	Fertilizer prices (cents/lb.) K_2O	Maximizing rate of K_2O (lbs./A.)	Hay yield (tons/A.)
$0.80	0	8	3	54	2.82
1.30	0	10	5	54	2.82
1.80	0	12	7	54	2.82
0.80	10	8	3	58	2.95
1.30	10	10	5	54	2.92
1.80	10	12	7	52	2.91
0.80	20	8	3	60	3.05
1.30	20	10	5	55	3.02
1.80	20	12	7	52	3.01
0.80	40	8	3	63	3.22
1.30	40	10	5	56	3.19
1.80	40	12	7	52	3.17

lization investment (including the cost of the fertilizer and the fixed costs per acre of applying it) when different fixed amounts of P_2O_5 are used.

Table 9 indicates, for a hay price of $20, the amounts of fertilizer to be used if the return per dollar invested in K_2O fertilizer and its application is maximum at given fixed costs and P_2O_5 rates. The fixed costs of application, including depreciation, interest, housing, repairs, fuel and labor, are based on records kept at Iowa State University.[7] The average fixed cost per acre is taken as $1.30, but high and low cost levels have also been assumed for illustrative purposes. These correspond with high and low fertilizer prices. As the amount of P_2O_5 applied per acre grows heavier, the amount of K_2O required does not increase in proportion.

A main conclusion to be drawn from table 9 is that, if three cuttings are expected, the rates of K_2O needed to maximize returns on fertilizations have a small range. Compared with the relevant portion of table 8, fertilization rates are lower when based on the criterion of maximum return on fertilizer investment, but the difference is modest. The reasons for this are, first, that the response to the fixed P_2O_5 application is adequate to pay for the fixed cost of application and cost of P_2O_5, and, second, that fertilization of alfalfa at $20 per ton for hay in this case is not a highly profitable practice. The less profit there is to be gained by fertilization, the more nearly alike will be the rates of fertilization based on the two criteria.

Relative Profitability of Market Grades

Many farmers use pre-mixed fertilizer grades commonly found in the market. Reports of tonnage of fertilizer sold in Iowa prepared by the Iowa Department of Agriculture in recent years show that commonly used ratios of P-K fertilizers are 0:1:1, 0:2:1 and 0:1:3. The most popular grades in these ratios are 0-20-20, 0-20-10 and 0-12-36. In this section, profits from these grades, in 50-pound increments, are compared with each other by computing expected alfalfa yields from the basic production functions 1, 4 and 5.

Tables 10 and 11 indicate the net returns to fertilizer when the alfalfa is sold as hay. The market prices for the fertilizer grades were taken to be $60, $50 and $60 per ton for the three grades, respectively. Table 10 represents data for the first cut. With a price of hay at $15 per ton, the greatest net return to fertilizer above fixed costs is obtained from 150 pounds per acre of 0-12-36. The net return is $1.61 per acre. By comparison, 150 pounds of 0-20-20 give a net return of $1.19 per acre, while 150 pounds of 0-20-10 give a net return of $0.77. The profit-maximizing quantity of fertilizer again increases with the price of hay. For a hay price of $25 per ton, maximum net returns are obtained with 300 pounds of 0-20-20. The net return is $6.75 per acre. When 250 pounds per acre of 0-12-36 are used (the most profitable level for this grade), net return is $6.48 compared with the $5.72 for the most profitable level of 350 pounds per acre of 0-20-10. Profit differences, when measured in absolute terms, are not great among the three grades analyzed, but the percentage differences are substantial.

[7]Midwest Farm Handbook, 4th ed. Iowa State University Press, Ames, Iowa. 1958.

Table 10. Net returns to fertilizer for the first cutting of alfalfa at various fertilizer grades and quantities and hay prices.[a]

Fertilizer grade	Fertilizer applied (lbs./A.)	Cost ($/A.)	Hay yield (tons/A.)	Net returns to fertilizer when hay price per ton is $15	$20	$25
0-20-20	0	0.00	0.92	0.00	0.00	0.00
	50	1.50	1.07	0.69	1.42	2.15
	100	3.00	1.19	1.01	2.34	3.68
	150	4.50	1.30	1.19	3.08	4.98
	200	6.00	1.40	1.16	3.54	5.93
	250	7.50	1.48	0.90	4.00	6.50
	300	9.00	1.55	0.45	3.60	6.75
	350	10.50	1.61	—	3.18	6.60
0-20-10	0	0.00	0.92	0.00	0.00	0.00
	50	1.25	1.03	0.39	0.93	1.48
	100	2.50	1.13	0.64	1.69	2.74
	150	3.75	1.22	0.77	2.27	3.77
	200	5.00	1.30	0.76	2.67	4.59
	250	6.25	1.38	0.61	2.90	5.19
	300	7.50	1.44	0.34	2.95	5.57
	350	8.75	1.50	—	2.82	5.72
	400	10.00	1.55	—	2.52	5.66
0-12-36	0	0.00	0.92	0.00	0.00	0.00
	50	1.50	1.08	0.90	1.70	2.50
	100	3.00	1.22	1.44	2.92	4.40
	150	4.50	1.33	1.61	3.64	5.68
	200	6.00	1.42	1.39	3.90	6.38
	250	7.50	1.48	0.89	3.68	6.48
	300	9.00	1.52	—	2.98	5.98

[a]Table derived values for the last three columns do not agree precisely with tabulated values because of rounding errors in the **Hay Yield** column.

Table 11. Net returns to fertilizer for three cuttings of alfalfa at various fertilizer grades and quantities and hay prices on a per-acre basis.[a]

Fertilizer grade	Fertilizer applied: Amount (lbs./A.)	Cost ($/A.)	Hay yield (tons/A.)	Net returns to fertilizer when hay price per ton is $15	$20	$25
0-20-20	0	0.00	2.38	0.00	0.00	0.00
	50	1.50	2.59	1.64	2.68	3.73
	100	3.00	2.78	2.97	4.96	6.95
	150	4.50	2.95	4.01	6.84	9.68
	200	6.00	3.10	4.74	8.32	13.90
	250	7.50	3.23	5.19	9.42	13.65
	300	9.00	3.33	5.33	10.10	14.88
	350	10.50	3.42	5.18	10.40	15.63
	400	12.00	3.49	4.73	10.30	15.88
	450	13.50	3.55	3.99	9.82	15.65
0-20-10	0	0.00	2.38	0.00	0.00	0.00
	50	1.25	2.54	1.12	1.91	2.70
	100	2.50	2.68	2.07	3.60	5.12
	150	3.75	2.82	2.85	5.05	7.25
	200	5.00	2.94	3.45	6.27	9.09
	250	6.25	3.06	3.89	7.26	10.64
	300	7.50	3.16	4.15	8.03	11.91
	350	8.75	3.24	4.23	8.56	12.89
	400	10.00	3.32	4.15	8.86	13.58
	450	11.25	3.39	3.89	8.94	13.98
	500	12.50	3.44	3.46	8.78	14.10
	550	13.75	3.49	2.86	8.39	13.93
0-12-36	0	0.00	2.38	0.00	0.00	0.00
	50	1.50	2.62	2.16	3.38	4.60
	100	3.00	2.83	3.77	6.02	8.28
	150	4.50	3.00	4.82	7.92	11.03
	200	6.00	3.13	5.30	9.06	12.83
	250	7.50	3.23	5.24	9.48	13.73
	300	9.00	3.29	4.61	9.14	13.68

[a]Table derived values for the last three columns do not agree precisely with tabulated values because of rounding errors in the *Hay Yield* column.

Table 11 indicates net returns per acre when three cuts of hay are realized. For a hay price of $25 per ton, the highest net return of $15.88 is realized with 400 pounds of 0-20-20. If 500 pounds of 0-20-10 are used, net return per acre is $1.78 less. The decrease in profit resulting from the use of 300 pounds 0-12-36 per acre, as compared with 400 pounds 0-20-20, is $2.20 per acre.

These differences from using alternative fertilizer grades are not great. However, consider again the data in table 11. If hay is priced conservatively at $15 per ton, the difference in value product resulting from using 300 pounds of 0-20-20, rather than 350 pounds of 0-20-10, is $1.10 per acre. When the hay price rises to $20 per acre, the amount is $1.54. With hay priced at $25 per ton, the figure is $1.99 per acre.

We now examine the question: Are the profit differences great when common mixed grades are used in place of the optimum mixture? This comparison is on the basis of the data that underly production function 5 for three cuttings. Using the predicted isoclines, we first compute the optimum quantity of 0-20-20, 0-20-10 and 0-12-36 grades that should be used under specified price situations. Next, the amount of profit forthcoming from each of these quantities as compared with using no fertilizer, is computed. Finally, these quantities are compared with optimum quantities of K_20 and P_20_5 from table 8 in which it is assumed that nutrients can be combined in the proportion indicated as best by the isocline equations. Results of these calculations are presented in table 12.

Use of either the optimum rate or 0-20-20 fertilizer grade results in approximately the same net value product, the difference between gross value of hay produced and the fertilizer cost. The greatest difference in net profit occurs when hay is selling at $15 per ton. In this situation, application of 45 pounds of P_20_5 and 72 pounds of K_20 returns $0.24 per acre more than use of 60 pounds each of P_20_5 and K_20 in the 0-20-20 grade. If 0-20-10 or 0-12-36 grade is used, rather than optimum amounts of K_20 and P_20_5 specified by the isoclines, net profits are reduced by as much as $1.62 or $1.02 per acre, respectively. It appears, therefore, that indiscriminate use of fertilizer grades may result in a considerable reduction in profits, as compared with those possible from using optimum mixes of nutrients. On the other hand, optimum quantities may sometimes be approximated closely enough by using a pre-mixed grade. In this case, the difference in net profit may be less important than the inconvenience of purchasing two kinds of fertilizer materials and mixing them or applying them separately.

Table 12. Effect of using optimum rates of pre-mixed fertilizer grades on profit as compared with optimum combinations of P and K fertilizers at different hay prices.

Fertilizer grade	Hay price ($/ton)	Fertilizer price ($/ton)	Applied rate of P_2O_5 (lbs./A.)	Applied rate of K_2O (lbs./A.)	Net value product ($/A.)	Net profit over no fertilization ($/A.)
Optimum	15	a	45	72	41.25	5.57
Optimum	20	a	64	79	58.09	10.50
Optimum	25	a	76	83	75.38	15.91
0-20-20	15	60	60	60	41.01	5.33
0-20-20	20	60	73	73	58.00	10.42
0-20-20	25	60	80	80	75.35	15.88
0-20-10	15	50	76	38	39.89	4.21
0-20-10	20	50	96	48	56.46	8.88
0-20-10	25	50	108	54	73.47	14.00
0-12-36	15	60	26	78	41.02	5.34
0-12-36	20	60	30	90	57.06	9.48
0-12-36	25	60	33	99	73.29	13.82

[a]Fertilizer nutrients in optimum grade are priced the same as for pre-mixed grades, or 10 and 5 cents per pound for P_2O_5 and K_2O respectively.

NUMBER OF CUTTINGS UNCERTAIN

Analyses in previous sections assumed that number of cuttings and price were known with certainty and that optimum fertilizer quantities were computed accordingly. In this section, it is assumed also that the number of cuttings to be realized is uncertain at the time of fertilization. On this basis, the use or applicability of game theory models in decisions and recommendations for fertilizer can be examined. This analysis assumes that the farmer might be uncertain about the number of cuttings to be obtained but is certain about the form of the production function when a particular cutting is obtained.

The assumption of prices known with certainty is retained; although somewhat unrealistic, retention of this assumption allows us to simplify the analysis. However, this assumption is not as unrealistic as it may first seem for hay fertilized in the spring and is perhaps less unrealistic than assumptions about the production function. Table 13, derived from the alfalfa hay prices in Appendix A, expresses the June, July and August prices as percentages of the prices prevailing the previous April. Prices during the summer tend to be lower than the price the preceding April. In no case was the price in June, July or August less than 80 percent of the April price, and, in about one-third of the years, the monthly price was 90 percent or more of the April price. August showed the greatest interyear fluctuation.

Table 13. Alfalfa hay prices for the months of June, July and August, 1944-58, expressed as a percentage of the April price.[a]

Year	April	June	July	August
1944	100	86	84	86
1945	100	89	85	81
1946	100	96	95	100
1947	100	98	87	92
1948	100	81	104	106
1949	100	86	80	83
1950	100	91	81	88
1951	100	93	81	83
1952	100	86	90	105
1953	100	88	93	94
1954	100	88	85	89
1955	100	88	83	82
1956	100	112	115	116
1957	100	82	82	82
1958	100	87	85	83

[a]See Appendix A.

Variations in Income Resulting From Differences in the Number of Cuttings

Decisions with respect to the number of cuttings to be harvested during the year and the optimum fertilization level may err in either of two directions, ignoring indirect and residual effects of fertilizer. The number of cuttings expected may be greater than the number realized, resulting in more fertilizer being applied than necessary to maximize profits. On the other hand, the number of cuts harvested may be greater than the number planned; then the amount of fertilizer applied will be short of that necessary to maximize profits. With regard to the alfalfa data, six outcomes thus are possible.

Too much fertilizer may be applied:

(a) Two cuttings expected—one obtained.
(b) Three cuttings expected—one obtained.
(c) Three cuttings expected—two obtained.

Alternatively, too little fertilizer may be used:

(d) One cutting expected—two obtained.
(e) One cutting expected—three obtained.
(f) Two cuttings expected—three obtained.

The deviations from expected profits can be computed for each of these situations. In case a, for example: The alfalfa yields for different fertilizer mixtures and rates are derived from production functions 1 and 4. A range of hay prices ($15, $20 and $25 per ton) is assumed with the price of fertilizer known. Net returns to fertilizer then can be calculated for different rates of fertilizer application, and the profit-maximizing rate can readily be determined.

In example (a) two cuttings are expected, but only one is obtained. The amount of fertilizer maximizing returns for production function 4 is thus applied to returns conforming to production function 1. Using the prices assumed, differences in net cash returns can be worked out. These differences can be regarded as gains or losses in profit. The justification for this is that ex ante expectations are assumed to be the relevant ones in the mind of the decision-maker. If a yield other than the one anticipated is realized, profits are either increased or reduced. If cases b through f are treated similarly, variations in net returns can be tabulated in a similar fashion.

Tables 14 and 15 indicate the extent by which net returns are reduced when the number of cuttings is overestimated. The optimum mixtures included in each table are the combinations of P_2O_5 and K_2O that maximize returns under the price conditions assumed. Earlier, it was shown that these amounts lead to somewhat greater net returns than any of the commonly used pre-mixed grades. The amount of fertilizer applied is the quantity that maximizes profits if the number of cuttings is correct. When the number of cuttings is overestimated, returns are reduced. The total value product is less—mostly because of fewer cuttings—and also, though less important, the expenditure on fertilizer is greater than warranted by the ex post optimum.[8]

Table 14. Reduction in anticipated net returns when fertilizing in anticipation of two cuttings, only one cutting obtained.

Fertilizer grade	Lbs. per acre nutrients or fertilizer applied: P_2O_5	K_2O	Decline in expected net return ($/A.) when hay price per ton is: $15	$20	$25
Optimum	31	67	1.85	—	—
	54	77	—	3.26	—
	67	82	—	—	4.57
0-20-20	200		1.63	2.18	2.72
	250[a]		1.99	2.66	3.32
	300[b]		2.32	3.10	3.87
	350[c]		2.64	3.52	4.40
0-20-10	100		0.64	0.85	1.06
	150[a]		0.93	1.25	1.57
	200		1.22	1.64	2.05
	250		1.50	2.00	2.50
	300[b]		1.76	2.35	2.93
	350[c]		2.01	2.69	3.36
0-12-36	150		1.40	1.88	2.34
	200[a]		1.75	2.34	2.92
	250[b, c]		2.05	2.74	3.42

[a]Quantity maximizing net returns for two cuttings when hay price is $15 per ton.

[b]Quantity maximizing net returns for two cuttings when hay price is $20 per ton.

[c]Quantity maximizing net returns for two cuttings when hay price is $25 per ton.

Table 15. Reduction in anticipated net returns when fertilizing in anticipation of three cuttings, only one cutting obtained.

Fertilizer grade	Lbs. per acre nutrients or fertilizer applied: P_2O_5	K_2O	Decline in expected net return ($/A.) when hay price per ton is: $15	$20	$25
Optimum	45	72	4.68	—	—
	64	79	—	7.24	—
	76	83	—	—	9.67
0-20-20	250		4.29	5.72	7.15
	300[a]		4.88	6.50	8.13
	350[b]		5.42	7.22	9.03
	400[c]		5.87	7.82	9.78
0-20-10	300		3.81	5.08	6.34
	350[a]		4.29	5.74	7.17
	400		4.75	6.34	7.92
	450[b]		5.16	6.89	8.60
	500[c]		5.53	7.38	9.23
0-12-36	150		3.21	4.28	5.35
	200[a]		3.87	5.16	6.45
	250[b, c]		4.35	5.80	7.25

[a]Quantity maximizing net returns for three cuttings when hay price is $15 per ton.

[b]Quantity maximizing net returns for three cuttings when hay price is $20 per ton.

[c]Quantity maximizing net returns for three cuttings when hay price is $25 per ton.

Net returns are reduced most when three cuttings are expected but only one is obtained (table 15). If we assume the price of alfalfa hay to be $20, then, on an ex ante basis, 350 pounds of 0-20-20 should be applied. Since the number of cuttings obtained is only

[8]If expectations of the number of cuttings are correct, the quantities of fertilizer given in the tables maximize profits. If expectations are incorrect, the amount of fertilizer applied no longer maximizes profits ex post.

one, rather than three, the realized net returns per acre will be $7.22 less than was anticipated. If the hay price is $25, a reduction of $9.78 per acre occurs. Under these circumstances, minimization of loss might be considered an alternative to profit maximization where decisions must be made under uncertainty of number of cuttings.

If there is uncertainty as to whether one or three cuttings are likely, application of 200 pounds of 0-12-36 reduces anticipated net value product least (by $3.87 per acre when the price of hay is $15 per ton). On the other hand, expectations may be correct, and three cuttings may be obtained. Then, 200 pounds of 0-12-36 fertilizer does not give greatest profit from fertilizer use when compared with other combinations (tables 8 and 11). The expected net value product is reduced least when the crop is fertilized in anticipation of two cuttings but when only one is obtained (table 14). If 67 pounds of P_2O_5 and 82 pounds of K_2O are applied per acre, expected net returns are reduced by only $4.57 (when hay is selling for $25 per ton). When three cuttings are expected, 76 pounds of P_2O_5 and 83 pounds of K_2O are applied, and, if only one cutting is obtained, the reduction in anticipated net returns is $9.67 per acre (table 15).

Most of the losses discussed in this section are due to the failure of realizing a cutting rather than to overfertilization. Using 0-20-20 and a hay price of $20 per ton, the profit-maximizing rate of 350 pounds per acre results in a net profit of $10.40 per acre from fertilization, table 11. If only one cutting is obtained, the net profit (table 10) is maximized with 250 pounds of fertilizer and is $4.00. The net profit from one cutting fertilized with 350 pounds is $3.18. Although the net return was reduced $7.22 per acre, only $0.82 of this was due to overfertilization for the single cutting. With hay still at $20 per ton but fertilizing with 0-12-36, 250 pounds per acre maximizes profit at $9.48 per acre. This much fertilizer returns $3.68 per acre in one cutting while the optimum rate for one cutting is 200 pounds, returning $3.90 profit. Here, only $0.22 of the $5.80 per acre decrease in profit is due to overfertilization.

Tables 16 and 17 relate to the situations in which expectations are too conservative. The number of cuttings obtained are greater than anticipated. Quantities of fertilizer that were (subjectively) presumed sufficient to maximize profits are less than required. The largest addition to anticipated net returns occurs when one cutting is expected but three are harvested (results not shown in table). Here, if hay is selling at $20 per ton, unanticipated returns amount to $5.72 per acre if 250 pounds of 0-20-20 are used. Or, if the hay price is $25 per ton, an addition of $8.13 per acre to anticipated profits is possible if 300 pounds of 0-20-20 are used.

If one cutting is expected but two are obtained (table 16), the increase in expected value product is smallest. Nevertheless, even when hay is only $15 per ton, the addition to expected net returns is between $0.93 and $1.40 per acre. The difference depends on the grade of fertilizer applied. As the hay price rises, the addition to anticipated net returns becomes greater. When hay is selling at $25 per ton, the increase in anticipated returns is at least 50 percent greater than when the price of hay is $15 per ton.

Table 16. Addition to anticipated net returns when fertilizing in anticipation of one cutting but two cuttings obtained.

Fertilizer grade	Lbs. per acre nutrients or fertilizer applied: P_2O_5		K_2O	Addition to expected net returns ($/A.) when hay price per ton is: $15	$20	$25
Optimum	9		58	1.24	—	—
	35		67	—	2.60	—
	50		73	—	—	3.87
0-20-20		100		0.86	1.16	1.44
		150[a]		1.25	1.68	2.09
		200		1.63	2.18	2.72
		250[b]		1.99	2.66	3.32
		300[c]		2.32	3.10	3.87
0-20-10		100		0.64	0.85	1.06
		150[a]		0.93	1.25	1.57
		200		1.22	1.64	2.05
		250		1.50	2.00	2.50
		300[b]		1.76	2.35	2.93
		350[c]		2.01	2.69	3.36
0-12-36		100		0.99	1.32	1.65
		150[a]		1.40	1.88	2.34
		200[b]		1.75	2.34	2.92
		250[c]		2.05	2.74	3.42

[a]Quantity maximizing net returns for one cutting when hay price is $15 per ton.

[b]Quantity maximizing net returns for one cutting when hay price is $20 per ton.

[c]Quantity maximizing net returns for one cutting when hay price is $25 per ton.

Table 17. Addition to expected net returns when fertilizing in anticipation of two cuttings but three cuttings obtained.

Fertilizer grade	Lbs. per acre nutrients or fertilizer applied: P_2O_5		K_2O	Addition to expected net returns ($/A.) when hay price per ton is: $15	$20	$25
Optimum	31		67	2.20	—	—
	54		77	—	3.50	—
	67		82	—	—	4.68
0-20-20		200		1.95	2.60	3.25
		250[a]		2.30	3.06	3.83
		300[b]		2.56	3.40	4.26
		350[c]		2.78	3.70	4.63
0-20-10		200		1.47	1.96	2.45
		250[a]		1.78	2.36	2.95
		300		2.05	2.73	3.41
		350		2.28	3.05	3.81
		400[b]		2.50	3.33	4.17
		450[c]		2.69	3.59	4.48
0-12-36		150		1.81	2.40	3.01
		200[a]		2.12	2.82	3.53
		250[b], [c]		2.30	3.06	3.83

[a]Quantity maximizing net returns for two cuttings when hay price is $15 per ton.

[b]Quantity maximizing net returns for two cuttings when hay price is $20 per ton.

[c]Quantity maximizing net returns for two cuttings when hay price is $25 per ton.

Reduction of Uncertainty Resulting From Knowledge of the Probability Distribution of the Number of Cuts

Myers[9] estimated the probabilities of runs of consecutive dry days at Corydon in south-central Iowa. He took a "dry" day as having less than 0.2 inch of rainfall. He then estimated the probabilty of the middle day in a 5-day period being part of a series of successive dry days; these series of dry days were taken to be 5, 10, 15, 20, 25, 30 or more days.

Most farmers in southern Iowa take the first cut of alfalfa by about June 10; the second cut, by July 15; and, the third cut, not later than Sept. 1. It is assumed that a 4- to 5-week dry period starting toward the end of June would result in only one cutting being taken. A 3- to 4-week dry period starting around the middle of July would preclude a third cut. Table 18, adapted from Myers' data, indicates that, in 1 year out of 20, only one cutting can be expected. In 2 years out of 10, a third cutting is unlikely.

The period of time the operator expects to be on his farm also is relevant in his decisions. We need to consider some span of years in decision-making for supposing the frequency distribution of "correct" and "incorrect" choices. The 1954 Census of Agriculture[10] shows that the average length of time the Iowa farm occupier (tenant or owner) has been on his present farm is 13 years. The planning period for fertilizer-use decisions is probably considerably less than this, especially for tenant operators. Accordingly, a 5-year horizon is assumed for the analysis which follows. An added proviso is that, in each of 4 years, three cuttings are obtained. In the remaining year, only two are harvested. The probability of getting only one cutting in the 5-year period is ignored.

For discussion here, it is also assumed that loss of the third cutting does not alter the yield function for the first two cuttings and that loss of the last two cuttings does not influence the yield function for the first. This simplifying assumption may not always be true in practice, since loss of the second cutting does not necessarily mean the loss of a cutting taken at the usual time for the third. Considering this possibility of this cutting, however, would introduce a degree of complexity with which we are not prepared to deal at this time.

Under these circumstances, two possible courses of action are considered:

(a) The alfalfa is fertilized in expectation of three cuttings every year.

(b) In 1 out of the 5 years, fertilizer is applied at the rate that maximizes returns if two cuttings are obtained. Ex post, this decision is correct or incorrect. If the latter is true, it is further assumed that, in 1 year, only two cuttings are obtained when three are expected.

At the end of a 5-year period, the net returns situation based on ex ante expectations conforms to one of the possibilities outlined in table 19.[11] Profit-maximizing quantities of fertilizer for the various situations were obtained from tables 6, 7, 8, 10 and 11. Hay yields and net returns were then computed. Hay prices used were \$15, \$20 and \$25 per ton and, fertilizer prices, \$0.10 per pound for P_2O_5 and \$0.05 per pound for K_2O.

Situation A is the one in which anticipations prove correct over the whole 5-year period. Common grades may be used in contrast to optimum proportions derived from production functions. Use of the former may result in a reduction of income of up to \$9.93 per acre over the whole period (when the price of hay is \$25 and 0-12-36 is used rather than the optimum grade). Of the pre-mixed grades, 0-20-20 gives the greatest net returns if hay prices are high. If hay is

[9]Richard E. Myers. Estimation of consecutive dry days at Ames and Corydon, Iowa. Unpublished M. S. thesis. Iowa State University Library, Ames, Iowa. 1959.

[10]U. S. Bureau of the Census. United States Census of Agriculture, 1954. Vol. 1, part 9. U. S. Govt. Print. Off., Washington, D. C. 1956.

[11]Because situation A is the only one in which expectations are wholly correct, net returns here should be highest. Table 19 does not confirm this belief. The reason is that fertilizer is applied in 50-pound increments. For maximum net returns using a particular mixture, 229 pounds per acre may be necessary when one cutting is obtained. If two cuttings are realized, 253 pounds may be needed to maximize profits. For three cuttings, 269 pounds may maximize returns. But the tables are drawn up so that it is possible that the profit-maximizing amount appears as 250 pounds in each case.

Table 18. Probability of a length of run of dry days greater than the number of days indicated.[a]

Period	Number of dry days 0	5	10	15	20	25	30	35
May 10-16	0.82	0.69	0.38	0.19	0.09			
	0.80	0.61	0.35	0.17	0.08			
	0.80	0.58	0.32	0.15	0.08			
	0.79	0.56	0.31	0.13	0.05			
	0.80	0.59	0.35	0.15	0.08			
June 14-20	0.82	0.65	0.40	0.20	0.10			
	0.84	0.68	0.43	0.26	0.15	0.07	0.05	
	0.84	0.68	0.44	0.29	0.19	0.10	0.06	
	0.85	0.68	0.46	0.31	0.21	0.13	0.08	0 05
	0.86	0.69	0.50	0.34	0.22	0.14	0.10	0.06
July 19-25	0.88	0.78	0.57	0.39	0.25	0.14	0.09	0.05
	0.88	0.79	0.55	0.39	0.24	0.14	0.09	0.05
	0.84	0.70	0.50	0.34	0.21	0.14	0.08	0.05
	0.84	0.68	0.48	0.31	0.20	0.14	0.08	0 05
	0.85	0.70	0.52	0.35	0.23	0.14	0.10	0.06

[a]See Myers, op. cit.

Table 19. Net returns from fertilizing over a 5-year period assuming various methods of fertilization.

Situation	Grade	Net returns per acre from fertilization when hay price per ton is: \$15	\$20	\$25
A. For 4 years expects 3 cuts, gets 3; for 1 year expects 2 cuts, gets 2	Optimum	25.50	48.95	74.75
	0-20-20	24.21	48.30	74.52
	0-20-10	19.03	41.29	65.90
	0-12-36	24.38	44.34	64.82
B. For 4 years expects 3 cuts, gets 3; for 1 year expects 3 cuts, gets 2	Optimum	25.38	48.87	74.67
	0-20-20	24.09	48.30	74.52
	0-20-10	18.87	41.11	65.75
	0-12-36	24.38	44.34	64.82
C. For 3 years expects 3 cuts, gets 3; for 1 year expects 2 cuts, gets 3; for 1 year expects 3 cuts, gets 2	Optimum	25.23	48.75	74.60
	0-20-20	23.95	47.60	74.27
	0-20-10	18.53	41.03	65.63
	0-12-36	24.38	43.60	64.82

selling for $25 per ton, use of 0-20-20 rather than 0-12-36 results in extra returns of $8.62 over the period.

The decision may be made to disregard the probability of getting only two cuts in 1 of the 5 years. The assumption may be that three cuttings can be expected every year. Situation B gives the net returns when this course is followed. Compared with situation A, returns are reduced slightly. The largest observed difference between these two situations is 18 cents or less on a per-acre basis. Moreover, this reduction is spread over a 5-year period.

In situation C, expectations prove correct in 3 years out of the 5. The net value product is not reduced by a large amount when compared with situation A. The most unfavorable case is when 0-12-36 is used throughout the period and when hay is priced at $20 per ton. Net profits fall by $0.74 per acre compared with the 0-12-36 case in situation A. This amounts to a reduction of 15 cents per year. The use of different mixtures leads to substantial differences in returns. However, the main source of variation in such returns seems to originate with the use of a particular mixture rather than with the possible discrepancies between expected and realized cuttings.

We assumed profits to be influenced by two factors: (a) the choice of a particular fertilizer grade and (b) the amount of fertilizer applied when expectations of the number of cuttings prove incorrect. On the basis of this study, it appears that profits are especially dependent upon the choice of a particular fertilizer grade.

The basic assumption of this section was that only once, in a 5-year planning period, three cuttings of alfalfa could not be harvested. The probability of getting only one cut in any one year has been rejected altogether. Uncertainty still remains as to the actual year in which two cuttings are obtained. For the data used in this study, it has been shown that decreases in net income because of incorrect fertilizer-use decisions can be minimized by assuming that three cuttings will always be obtained. This conclusion holds for all situations examined. The reduction in net income by acting as though three cuttings will always be obtained amounts to about 4 cents per acre per year when measured against correct anticipation of situation A in table 22. This loss is small. However, differences in net returns arising from use of different fertilizer grades are such that meaningful recommendations can still be made concerning the grade to use.

THE UTILIZATION PROBLEM

Previous analysis has assumed that the alfalfa crop is harvested as hay. However, this is not always true. Here we examine two cases when alfalfa is utilized in a form other than hay: (a) standing alfalfa harvested by field chopper and fed to dairy cows and (b) alfalfa used as a summer pasture for pigs.

To derive economic optima, a value must be assigned to the crop. For alfalfa used as hay, this was taken to be the local market price. However, alfalfa is not usually sold green-chopped or as pasture, and there is no established price in the latter two instances. The crop may be used for dairy cow or hog enterprises, thus replacing other feedstuffs. Hence, it assumes a value equal to the cost of the feeds for which it substitutes.

When green-chopped alfalfa is used for dairy cows, its value is derived from the substitution of alfalfa for concentrate and grain mixtures. The value of the green-chopped alfalfa can be estimated in terms of the value of the part of the ration it replaces. Consequently the imputed value of green alfalfa varies directly with the price of concentrates and grain mixtures. This value is also directly dependent upon the amount of concentrates and grains that green alfalfa can replace without affecting the nutritive value of the ration. It was calculated that, when considerable latitude is allowed for either one of these factors, the imputed value of green alfalfa will not exceed the range of $15 to $25 on a per-ton, hay-equivalent basis.

In case of hogs using summer pasture, the value of alfalfa is derived mainly from its replacement of the protein supplement in the ration otherwise fed. By a procedure similar to the one used for dairy cows, it was found that alfalfa pasture might be worth anywhere from about $17 to $25 per ton hay-equivalent when used for summer pasture.

All of these prices associated with alternative uses of alfalfa fall within the range of prices previously considered in estimating economic optima. If a reasonably accurate estimate can be obtained concerning the imputed value of alfalfa, the foregoing framework suffices to determine the corresponding optimum fertilizer mix and level of application.

UTILIZATION AND NUMBER OF CUTTINGS UNCERTAIN

In the successive sections, the analyses have become increasingly complex. This section deals with the problem of levels of fertilization when both the number of cuttings expected and the utilization or price of the crop are unknown at the time fertilizer is applied. Uncertainty as to the former arises especially from fluctuating weather conditions. The assumption that there is no ex ante knowledge regarding the use of the alfalfa is also justifiable, since hay and other farm product prices may change in response to weather or a changing economic environment.

The postulate underlying this section is that the farmer who grows alfalfa as an intermediate product regards either its replacement value or its market price as the relevant price in decision-making. The replacement value is the price he is willing to pay (himself, in effect) for use of the crop in a further stage of the production process. The price depends on the market for hay, and the imputed value of hay depends on the market prices of other feeds and livestock products. The fertilization problem thus becomes one of decision-making under price uncertainty.

Application of Game Theory to Decision-Making Under Uncertainty

A series of prices is assumed to be known, corresponding to the various uses of hay. But, as utilization is uncertain, there is no ex ante knowledge of which price or value will be realized. If the alfalfa is kept, or sold as hay, its price is $15, $20 or $25 per ton, depending on the state of the market. Price is no longer assumed certain. When the crop is fed green-chopped to dairy cows, the price per ton of hay-equivalent is $16.20, $21.40 or $26.60 and, as pasture for hogs, the price is $17.45, 21.44 or $25.43 per ton. The relevant price out of each set depends on whether prices for grain and protein concentrate are low, average or high. It is assumed that, when there is a low price for hay, prices for feed also are low.

The problem of level of fertilization now becomes one of decision-making under absolute uncertainty, sometimes known as "a game against nature."[12] In games against nature, a matrix is given, and one player must choose a strategy represented by a row, the column representing the strategy chosen by "nature"— a fictitious player having no known objective and no known strategy. As far as this study is concerned, the farmer must choose from among a set of strategies $a_1, a_2 \ldots a_m$, but the relative desirability of each act depends upon "nature's strategy" (either $s_1, s_2 \ldots s_n$). To each pair (a_i, s_j) consisting of a farmer strategy and a nature strategy, there is a consequence or outcome. For the alfalfa fertilization situation, the game matrix is presented in table 20.

Table 20. Game matrix for alfalfa fertilization problem.

Farmer strategies (utilization method)	"Nature strategies" (price outcome)		
	s_1	s_2	s_3
a_1 Sells or keeps as hay	$15.00	$20.00	$25.00
a_2 Feeds green-chopped to dairy cows	$16.20	$21.40	$26.60
a_3 Feeds as pasture to pigs	$17.45	$21.44	$25.43

In table 20 the method of utilization corresponds to the farmer's strategies; prices refer to nature's strategies. Each value corresponding to a row (a_i) and a column (s_j) indicates the outcome if the farmer selects one strategy and the price outcome is that indicated. The problem now is in a game theory context. There are a number of possible "nature strategies," as well as several strategies available to the farmer. He does not know which "state of nature" will hold true, but he still has the problem of deciding which course to select. The decision concerning the strategy to select can be based on certain criteria. These criteria have been discussed in an attempt to resolve the decision problem under uncertainty. The criteria select the farmer strategy that is optimal according to the particular criterion used.

THE MINIMAX CRITERION

This criterion has been suggested by Wald.[13] Each farmer strategy is appraised by looking at the "worst" "nature strategy" coresponding to it, and the optimum choice is the one with the "best worst payoff." The 'best worst payoff" supposes that nature will select the strategy which is "worst" from the standpoint of the farmer and that the farmer will select the course which is then best to him. To apply this criterion, each farmer strategy is assigned its security level as an index. The security level is the least amount receivable under any "nature strategy." For table 20, the index for strategy a_1 is $15.00; for strategy a_2, $16.20; and, for strategy a_3; $17.45; all under s_1 in this problem. The farmer strategy with a miximum security index is a_3. Therefore, according to the maximum criterion, the farmer should fertilize the alfalfa in expectation of feeding it to hogs. The criterion is conservative: Relative to each farmer strategy, it concentrates on the "nature strategy" having the worst consequence.

THE PESSIMISM-OPTIMISM INDEX CRITERION

This criterion, first formulated by Hurwicz,[14] is less conservative. A judgment is formed, based on a weighted combination of the best and worst "nature strategies." The best and worst "nature strategies" are weighted according to a pessimism-optimism index. Compilation of this index supposedly requires a judgment by the farmer, depending on whether he is pessimistic or optimistic. The procedure can be explained as follows: For strategy a_i, let m_i be the minimum and M_i the maximum of the $s_{i1}, s_{i2} \ldots s_{in}$ where s_{ij} is the cell element or value in table 20. A fixed number α between 0 and 1 called the pessimism-optimism index is chosen. With each a_i is associated the index $\alpha m_i + (1-\alpha) M_i$. Of farmer strategies, the one with the higher index is chosen. If farmers are considered conservative, α might be taken as being between 0.5 and 0.8. In table 21, α indexes for values of α ranging from 0.3 through 0.8 are included. For the α values 0.5 through 0.8, the index shows that strategy a_3 is optimum. At $\alpha = 0.5$, the choice between a_3 and a_2 is very close, but, at $\alpha \leqq 0.4$, farmer strategy a_2, is optimum.

PRINCIPLE OF INSUFFICIENT REASON CRITERION

This principle was first systematized by Jacob Bernoulli (1654-1745).[15] It states that, if there is no evi-

[12]Absolute uncertainty means only that a series of prices is known, but the probability attaching to each price is unknown. For example, see: Duncan R. Luce and Howard Raiffa. Games and decisions. John Wiley and Sons, Inc., New York, N. Y. 1957.

[13]A. Wald. Statistical decision functions. John Wiley and Sons, Inc., New York, N. Y. 1950. pp 231-249.

[14]L. Hurwicz. Optimality criteria for decision making under ignorance. Cowles Commission discussion paper, Statistics, No. 370. 1951. (Mimeo.)

[15]Luce and Raiffa, *op. cit.*

Table 21. Pessimism-optimism index criterion, αm_i and $(1-\alpha)M_i$ values.

Pessimism-optimism index, α	Farmer strategies	αm_i	$(1-\alpha)M_i$	$\alpha m_i+(1-\alpha)M_i$
0.3	a_1	4.50	17.50	22.00
	a_2	4.86	18.62	23.48
	a_3	5.23	17.80	23.03
0.4	a_1	6.00	15.00	21.00
	a_2	6.48	15.96	22.44
	a_3	6.98	15.26	22.24
0.5	a_1	7.50	12.50	20.00
	a_2	8.10	13.30	21.40
	a_3	8.72	12.71	21.43
0.6	a_1	9.00	10.00	19.00
	a_2	9.72	10.64	20.36
	a_3	10.47	10.17	20.64
0.7	a_1	10.50	7.50	18.00
	a_2	11.34	7.98	19.32
	a_3	12.21	7.63	19.84
0.8	a_1	12.00	5.00	17.00
	a_2	12.96	5.32	18.28
	a_3	13.96	5.09	19.05

dence showing that one event from an exhaustive set of mutually exclusive events is more likely to occur than another, then the events should be judged equally probable. As far as game theory is concerned, this principle is usually associated with the name Laplace.[16]

If there is complete ignorance for the fertilizer problem in table 20 with respect to which "nature strategy" among $s_1, s_2 \ldots s_n$ is relevant, behavior should be based on the assumption that they are all equally likely. The situation then becomes one of risk, with a uniform probability distribution over all of the "nature strategies." To decide which course to follow, each farmer strategy is assigned an index as follows:

$$\frac{s_{i1}+s_{i2}+\ .\ .\ .\ +s_{in}}{n}.$$

The farmer strategy with the largest index is chosen.

[16]John Milnor. Games against nature. In, R. M. Thrall, C. H. Coombs and R. L. Davis, eds. Decision processes. John Wiley and Sons, Inc. New York, N. Y. 1957.

For table 20, the index is \$20.00 for strategy a_1, \$21.40 for strategy a_2 and \$21.44 for strategy a_3.

Now, although this application of game theory has indicated which farmer strategies are considered optimum, the expected price remains uncertain. Thus, there still is doubt as to the optimum quantity of fertilizer to use. The changes in value product resulting from applying other than the profit-maximizing quantity of fertilizer are examined in the next section.

Consequences of Incorrect Decision-Making

While the decision to apply fertilizer in expectation that the crop will be used as pasture for hogs may prove correct, the value of the pasture is still absolutely uncertain before the decision is made. Thus, there is always the chance that the amount of fertilizer applied may not maximize net value product. Variations in net value product when alfalfa replacement value or hay prices change are shown in table 22. The figures represent the increase in value product (less fertilizer cost) from applying fertilizer, as compared with using no fertilizer.

If strategy a_3 is selected and 0-20-20 is applied, 350 pounds of fertilizer maximize net returns when the alfalfa[17] is valued at \$17.45 or \$21.44 per ton. If the replacement value of the alfalfa is \$25.43 per ton, then 400 pounds of 0-20-20 are optimum. In the latter case, use of only 350 pounds decreases the value product by only 28 cents per acre. If 400 pounds are applied when only 350 pounds maximize profits, the decline in net value product is also 28 cents per acre for the \$17.45 price and, zero, for the \$21.44 price.

Use of 0-20-20 fertilizer at a given rate gives a greater net value product than either 0-20-10 or 0-12-36, but the latter two mixes may be used for one reason or another. In the case of 0-12-36, 250 pounds per acre always gives maximum net returns for that grade whatever the alfalfa replacement value. For 0-20-10, use of 400 pounds maximizes net value product at the low price, but the quantity needed increases to 450

[17]More precisely, the equivalent weight of alfalfa expressed as hay.

Table 22. Increase in net value product of alfalfa from fertilization for various farmer strategies (a_i) and hay prices.

Fertilizer grade	(lbs./A)	a_1 Hay price per ton:			a_2 Use green-chopped Replacement value per ton:			a_3 Used as pasture Replacement value per ton:		
		\$15	\$20	\$25	\$16.20	\$21.40	\$26.60	\$17.45	\$21.44	\$25.43
0-20-20	250	\$5.19	\$ 9.42	\$13.65	\$6.20	\$10.60	\$15.00	\$7.27	\$10.63	\$14.01
	300	5.33	10.10	14.88	6.47	11.44	16.40	7.67	11.47	15.28
	350	5.18	10.40	15.63	6.43	11.86	17.30	7.74	11.90	16.07
	400	4.73	10.30	15.88	6.06	11.86	17.66	7.46	11.90	16.35
	450	3.99	9.82	15.63	5.39	11.45	17.52	6.85	11.49	16.15
	500	2.94	8.92	14.90	4.44	10.68	16.92	5.94	10.73	15.52
0-20-10	300	4.15	8.03	11.91	5.08	9.11	13.15	6.05	9.15	12.24
	350	4.23	8.56	12.89	5.27	9.77	14.27	6.35	9.81	13.26
	400	4.15	8.86	13.58	5.28	10.18	15.09	6.46	10.22	13.98
	450	3.89	8.94	13.98	5.10	10.35	15.60	6.36	10.39	14.42
	500	3.46	8.78	14.10	4.74	10.27	15.80	6.07	10.31	14.56
	550	2.86	8.39	13.93	4.18	9.94	15.70	5.57	9.99	14.40
0-12-36	150	4.82	7.92	11.03	5.56	8.79	12.02	6.34	8.81	11.79
	200	5.30	9.06	12.83	6.20	10.11	14.03	7.14	10.14	13.15
	250	5.24	9.48	13.73	6.25	10.69	15.08	7.32	10.70	14.09
	300	4.61	9.14	13.68	5.69	10.41	15.13	6.83	10.44	14.06
	350	3.44	8.08	12.73	4.55	9.38	14.21	5.71	9.41	13.12

pounds and 500 pounds, respectively, for the median and high prices. The decrease in returns from using 400 pounds per acre, rather than the profit-maximizing quantity of 500 pounds, at the highest replacement value is insignificant, only 58 cents per acre.

If farmer strategy a_3 is selected, application of the optimal amount of 0-20-20 fertilizer gives a net value product of \$7.74, \$11.90 or \$16.35 per acre, depending on the replacement value or price of the alfalfa. On the other hand, if 0-20-10 fertilizer is used, maximum net returns are \$6.46, \$10.39 or \$14.56 per acre. Thus, use of 0-20-10 rather than 0-20-20 may result in a reduction of net returns of \$1.28, \$1.51 or \$1.79 per acre. On the basis of these results for the particular experimental data, it is concluded that variations in net value product resulting from using different fertilizer grades are greater than changes in net returns attributable to incorrect decision with respect to amount of a single fertilizer used.

Variations in net returns are not large when deviations from the optimum quantities of a particular fertilizer grade applied are not greater than 50 pounds per acre in this case. However, the changes in value product by using profit-maximizing quantities of one fertilizer grade rather than another have yet to be examined. These variations may be greater.

Assume that, for some reason, the 0-20-10 grade is used rather than 0-20-20 fertilizer and that value-product maximizing quantities of fertilizer for strategy a_3 are applied in each case. The reduction in net value product per acre from using 0-20-10 fertilizer rather than 0-20-20 is as follows for the three price situations explained earlier:

If strategy a_1 is selected—\$0.58, \$0.88 or \$0.80

If strategy a_2 is selected—\$0.78, \$1.10 or \$1.12

The actual reduction will depend on the alfalfa replacement price. The three columns of figures correspond to the low, medium or high prices for hay or hay-equivalent in the relevant part of table 22.[18]

Alternatively, if 0-12-36 fertilizer is used rather than 0-20-20, the reduction (addition in one case) in net value product per acre, assuming the same prices, is as follows:

If strategy a_1 is selected—(–0.06), \$0.92 or \$1.90

If strategy a_2 is selected—\$0.18, \$1.17 or \$2.22

It is apparent that these differences are large relative to the variations in profit arising from the use of a nonoptimum quantity of a single fertilizer. They may still be too small to make great differences in farmer decisions. Whether the conclusion has general application would again depend on the data arising from production functions derived under other soil and climatic and crop conditions.

[18]These are \$15, \$20 and \$25 under strategy a_1 and \$17.45, \$21.44 and \$25.43 for strategy a_3.

Appendix A

Midmonth prices received by Iowa farmers for alfalfa hay at local markets.

Year[a]	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1944	21.00	20.50	20.20	20.00	20.30	17.30	16.90	17.30	17.00	17.50	18.40	20.00
1945	21.50	22.00	21.20	21.10	19.50	18.70	18.00	17.00	16.70	16.80	17.00	17.00
1946	18.00	18.00	18.10	17.30	16.60	16.60	16.50	17.30	17.30	18.30	18.80	21.10
1947	20.30	19.00	19.60	19.00	19.00	18.30	16.50	17.50	20.00	20.00	22.00	26.00
1948	26.00	23.50	23.70	24.00	24.00	23.60	25.00	25.50	27.00	27.50	26.30	27.00
1949	28.40	28.00	27.30	26.00	25.00	21.00	20.50	21.50	20.00	21.50	22.20	22.50
1950	22.30	22.20	21.50	21.50	21.50	18.50	17.50	19.00	17.50	18.00	18.10	20.00
1951	20.00	21.00	19.50	20.10	19.40	18.30	16.20	16.70	16.80	18.40	18.30	20.70
1952	20.40	19.90	19.90	18.80	18.70	17.40	16.90	19.70	21.20	21.60	22.20	22.10
1953	23.40	23.00	21.50	20.20	20.50	17.30	18.70	19.00	20.30	21.90	22.50	24.00
1954	24.00	21.70	21.70	21.60	10.00	19.00	18.30	19.30	20.30	19.80	20.40	21.00
1955	21.00	21.00	20.50	19.00	18.00	16.70	15.70	15.60	17.50	17.00	18.00	18.20
1956	19.00	18.00	17.80	17.80	21.00	20.00	20.40	20.70	20.00	17.70	20.50	22.10
1957	21.20	20.30	20.20	19.20	18.60	15.80	15.70	15.70	14.80	15.70	15.30	16.60
1958	16.60	15.20	15.00	14.90	13.90	13.00	12.60	12.30	12.80	13.10		

[a]Source: Iowa Crop and Livestock Reporting Service, Des Moines, Iowa.