Beef-Cattle Production Functions in Forage Utilization

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RESEARCH BULLETIN 517 JULY 1963 AMES, IOWA

CONTENTS

SUMMARY

Little information has existed on substitution rates between pasture forages and corn in a beef-fattening enterprise. Without this knowledge it is difficult to determine which combinations of pasture forage and corn would maximize profits. Profits in feeding depend not only on the cost of feed but also on the time of marketing. The pasture forage-corn ration that minimizes costs may not necessarily be the ration that maximizes profits, since profits are affected by the time of marketing. Both the quality and the price of beef are subject to change during the beef-fattening period. Consequently, the beef-cattle feeder is confronted with the problem of selecting (a) the least-cost pasture forage-corn ration (b) that will place the beef cattle on the market finished to a grade (c) at the time when the expected market price will maximize profits.

A beef-feeding experiment was designed to determine the feed relationships between soilage (fresh-chopped pasture forage) and corn. It was conducted at two locations over a period of 3 years – 1957, 1958 and 1959. Six different soilage-corn rations, ranging from all soilage to 2 parts soilage and 1 part corn, were fed to different lots of feeder steers at each location. The rations at each location were also fortified with a feed supplement. Stilbestrol was included in the rations at one of the locations. The results of this feeding experiment are based on the performance of 336 head of good-to-choice feeder steers.

Several alternative regression equations, including quadratic, modified Cobb-Douglas and exponential functions, were used in this study to estimate production surfaces. In each of the functions, an attempt was made to remove the effects of autocorrelation by estimating an autocorrelation coefficient and then making an autoregressive transformation.

The quadratic functions gave better results than either the modified Cobb-Douglas or the exponential functions. However, more research is needed to determine which functions are best under different situations. In some cases the modified Cobb-Douglas function gave good results. In other cases, it gave increasing returns to scale, denoting that a small proportional increase in the quantity of feed fed results in a more than proportionate increase in beef gain. These results are inconsistent with theory. The exponential functions, which merit further research, gave sigmoid isoquant contours, denoting, first, increasing marginal rates of substitution between feeds and, then, decreasing marginal rates of substitution. Again, these results are inconsistent with logic.

The quadratic production functions for rations with and without stilbestrol and the aggregate function with the stilbestrol and nonstilbestrol rations combined are:

I. With stilbestrol

$$G=0.11637150C+0.02316051F -0.0000049955C^2-0.0000007455F^2 +0.0000000374CF-1.2236046H$$

II. Without stilbestrol

$$G=0.14971812C+0.02128774F -0.0000122612C^2-0.0000007455F^2 \; 0.0000037907CF-2.2005042H$$

III. The aggregate function

$$G=0.13628727+0.02193828F -0.00000819C^2-0.00000063F^2 -0.00000253CF-1.75011550H$$

In these equations, G refers to pounds of beef gain, C refers to pounds of corn, F refers to pounds of soilage, and H refers to the deviations of the average maximum temperature of each observation interval from the mean maximum temperature for the over-all feeding period. From these production functions, the basic input-output relationships can be derived.

The marginal rates of substitution of corn for soilage have been derived for various soilage-corn rations at various levels of beef gain. The marginal rates of substitution indicate, for a given level of gain, the pounds of soilage that could be replaced if an additional pound of corn were added to the ration. For 100 pounds of beef gain, the marginal rate of substitution of corn for soilage is 6.57 for the 20:1 soilage-corn ration with stilbestrol; it is 5.17 for the 2:1 ration. For the same level of gain, the marginal rate of substitution of corn for soilage is 8.11 for the 20:1 soilage-corn ration without stilbestrol and is 7.36 for the 2:1 ration. The marginal rates of substitution of corn for soilage are diminishing. Similar results were obtained for higher levels of beef gains.

Time equations, which were derived from estimated soilage-consumption functions, express the total time (T) required to consume a given quantity of corn and soilage as a function of the soilage (F) and corn (C) fed. The time equations for the rations with and without stilbestrol are:

I. With stilbestrol

$$T = -558.36128626+0.05781948C \pm 6.7475645\,(6{,}847.57044400 -0.00000523C^2-0.82767919C -0.26107315H+0.29640324F)^{1/2}$$

II. Without stilbestrol

$$T = -1{,}176.48647060+0.00763899C \pm 11.436640\,(10{,}582.2245560 -0.00001571C^2+0.45460922C -3.07787452H+0.17108144F)^{1/2}$$

These equations were used to predict the time required to produce various levels of gain for different soilage-corn rations. The time required to produce a given level of gain, for the rations both with and without stilbestrol, decreases as the proportion of corn in the ration increases. Also, for a given feeding period the maximum level of gain is attained with the heaviest corn ration (i.e., the 2:1 soilage-corn ration).

The beef steers were graded at definite intervals during the feeding period. After the grade observa-

tions (which were in subjective grade terms) had been coded, a functional relationship that expressed grade as a function of the corn and soilage fed was estimated for both the stilbestrol and nonstilbestrol rations. The over-all equations for estimating the grade of beef steers fed various soilage-corn rations either with or without stilbestrol are:

I. With stilbestrol

$$Q = 21.67 + 0.0024655079C + 0.0000220680F - 0.0000003510C^2$$

II. Without stilbestrol

$$Q = 21.67 + 0.0016294178C + 0.0000270836F - 0.0000000330C^2$$

where Q is the predicted slaughter grade which can be interpreted in subjective grade terms. From these grade functions, the isograde equations, as well as the marginal rates of substitution equations, can be derived.

To estimate the profits from feeding different soilage-corn rations for feeding periods of different lengths with different feed-price assumptions, it was necessary to derive a price function that would estimate the price of the beef steers during the feeding period. These price functions represented the grade of the beef steers during the feeding period, as well as the general market price associated with the grade. The estimated price functions were used in the over-all stilbestrol and nonstilbestrol profit functions. The profit equations are:

I. With stilbestrol

$$\pi = (850.00 + 0.11637150C + 0.02316051F - 0.0000049955C^2 - 0.0000007455F^2 + 0.0000000374CF - 1.2236046H)(0.2500 - 0.0000040158F + 0.0000382807T + 0.0000017537T^2 - 0.0000000048FT) - P_C C - P_F F - 0.2TP_S - K$$

II. Without stilbestrol

$$\pi = (850.00 + 0.14971812C + 0.02128774F - 0.0000122612C^2 - 0.0000005775F^2 - 0.0000037907CF - 2.2005042H)(0.2500 - 0.0000004996F - 0.0002393978T + 0.0000036576T^2 - 0.0000000281FT) - P_C C - P_F F - 0.2TP_S - K$$

where π refers to the profit, P_C refers to the price of corn, P_F refers to the price of soilage, P_S refers to the price of the supplement, K is the value of the feeder steer at the beginning of the feeding period, and all of the other symbols are the same as explained for the earlier equations.

Estimated profits from feeding beef steers various soilage-corn rations for feeding periods of various lengths under various feed-price assumptions have been tabulated in the text. Usually, for this experiment, the greatest profits are obtained by feeding the heaviest corn ration. However, when the price of soilage is low relative to the price of corn, the most profitable ration includes less corn and more soilage.

The equations and the procedure also are given in the text for obtaining the optimum feeding period for any given soilage-corn ration under different feed-price conditions. Similarly, the equations and the procedure are given for obtaining the optimum soilage-corn ration with different feed-price assumptions.

Beef-Cattle Production Functions in Forage Utilization[1]

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The optimum proportion of land which should be devoted to forage or grasses and grain has long been studied by agriculturists. Similarly, the optimum proportion of forage, grain and other feed materials to be included in a livestock ration has been a continuous concern. These are not unrelated problems since, as has been shown elsewhere, the optimum proportion of forage and grain is not independent of the optimum proportion of the two crops grown in the crop rotation (and vice versa).[2] For a physical maximum of livestock production from a given land area, the marginal rate of substitution of feeds produced in the crop rotation must equal the marginal rate of substitution of the same feeds used in the livestock ration. Even where attempts are not made to maximize the livestock product from a given land area, these two sets of marginal rates of substitution are basic to decisions on profit maximizing in growing crops or feeding livestock.[3]

These rates of substitution provide fundamental knowledge for determining optimum pasturage systems. Many experiments have been conducted in recent years to evaluate returns and feasibility of pasture. Experiments, in the Corn Belt especially, have been based on beef feeding where the cattle are handled differently on pasture. Several such experiments have been conducted in Iowa. One difficulty has prevailed in these studies; namely, while the corn consumed by the animals was easily measured, the forage grazed could not be measured with similar accuracy. Without ability to measure forage consumption, the beef production function could not be estimated.

This study has been designed to allow prediction of grain surfaces and marginal substitution rates through the feeding of soilage (green-chopped forage) as the forage input. In providing forage in this manner, feed quantities are measurable. Hence, basic relationships in crop use and animal nutrition can be estimated. It is recognized, of course, that some variance still exists in the measurement of feed intake when forage is provided in this form. Too, it is known that the feed composition, as among species of plants, differs under soilage feeding in drylot and unrestricted pasturage.

Other differences also exist: Cattle in pasture trample grass and otherwise render some of it unusable. Accordingly, beef production from soilage production is not identical to that from pasture production. Gains in drylot differ from the performance resulting from wider animal movement over pasture. Finally, these results, as those of other feeding experiments, apply only to the types, grades and ages of livestock used in the study.

However, although these differences exist, this study, with cattle fed soilage in drylot, has been conducted to provide fundamental data relative to pasture production and utilization. The method at least provides measurable quantities and basic predictions in livestock nutrition. It is expected that the quantities which result might be transformed to fit other feeding systems and forage production conditions. For example, if the marginal productivities and substitution rates of forage as measured in this study can be transformed, even though imperfectly, into equivalent quantities of hay or other green forages, or into green forages of other locations, progress will have been attained. That is, the experimental results then would have some predictive value for locations and forage conditions other than those of the particular experimental sites.

The data also provide information for estimating the fundamental mathematical basis of animal nutrition; an important possibility which has not been explored. The data also are suited to other purposes in agronomy, animal science and economics. However, only the basic predictions from the experiment, which can provide the foundation for these other uses, are made and reported in this phase of the research. This study involves the prediction of beef-gain production functions and the associated relationships of gain isoquants, marginal substitution equations, isoclines, expansion paths, quality isoquants, isotime contours and associated numerical quantities. The predictions are used in specifying some least-cost and profit-maximizing rations in the feeding of soilage.

SCOPE AND OBJECTIVES OF THE STUDY

The experiment was designed to provide data for estimating a beef-cattle production function and substitution rates between two kinds of feed—corn and fresh-chopped pasture forage (soilage).

The inferences possible from this study are restricted by the experimental data. The feeder cattle used

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[1] Projects 848 and 1135 of the Iowa Agricultural and Home Economics Experiment Station.

[2] See: Earl O. Heady. Resource and revenue relationships in agricultural production control programs. Rev. Econ. and Stat. 33:226-240. 1951; and Earl O. Heady. Economics of agricultural production and resource use. Prentice-Hall, Inc., New York. 1952. pp. 167-265.

[3] Earl O. Heady and John L. Dillon. Agricultural production functions. Iowa State University Press, Ames. 1961. pp. 31-73.

were good-to-choice steers, averaging about 850 pounds. The feeding period is limited to the pasture growing season, since the rations fed in the experiment were restricted to various combinations of corn and fresh-chopped pasture forage (soilage).

The beef-feeding trials were conducted at two separate locations over a 3-year period. The experimental design at the two locations was the same except that stilbestrol was included in rations at one location and not in those at the other location. Thus, to the extent that location had an effect on gains, differences between rations with and without stilbestrol cannot be attributed simply to stilbestrol. While an attempt was made to keep the conditions comparable, management of cattle and feeding may have differed somewhat at the two locations. While the beef-feeding experiment may be treated as one including stilbestrol and one without it, comparisons between stilbestrol and non-stilbestrol rations and interpretations must be made with knowledge that there may or may not have been a location effect.

The specific objectives of this study were: (a) to determine the rates at which pasture forage and corn substitute in the beef-fattening process, (b) to determine the rate at which such feeds are transformed into beef gains for different pasture forage-corn rations, (c) to determine the time required to produce different levels of gain for different pasture forage-corn rations, (d) to determine the quality of beef cattle (i.e., the grade) produced from various pasture forage-corn rations, (e) to estimate, under different price conditions, the combination of feed, gain and grade that will maximize profits for the soilage growing season, (f) to compare various rations with and without stilbestrol and (g) to consider alternative functional forms for evaluating feed-gain relationships.

EMPIRICAL RESULTS

A beef-feeding experiment extending over 1957, 1958 and 1959 provided the data for this study. This experiment, conducted cooperatively by the Department of Animal Science, the Department of Agronomy and the Department of Economics and Sociology, Iowa State University, was designed specifically to provide data for estimating the production function and feed relationships for beef steers fed on soilage (fresh-chopped pasture forage) and corn.

Experimental Design

The beef-feeding experiment, designed to determine the feed relationships of soilage and corn for fattening beef steers, was conducted at the Western Iowa Experimental Farm at Castana and the Soil Conservation Experimental Farm at Shenandoah. The rations at Castana contained 10 mg. of stilbestrol daily, while those at Shenandoah did not. The soilage and corn at both experimental farms were mixed and full fed in fixed proportions.

Experimental Cattle

The cattle were Hereford steers purchased the preceding fall as choice feeders at Omaha. After purchase, they were divided between the farms and wintered on a ration to gain about a pound per day. A week before the beginning of the soilage-feeding experiment in the spring, half of the steers from each farm were transferred to the other farm. Then steers were allowed access to pasture for conditioning; next, the steers were individually weighed on 2 successive days. On the basis of the average of these two weights, the wintering location and the winter gains, they were placed in eight lots of seven steers each at each location. Four of the steers in each lot had been wintered at each location, and experimental treatments were assigned randomly to the eight lots of steers. In 1957 the steers were weighed individually at 28-day intervals. In 1958 and 1959 they were weighed at 21-day intervals. The steers also were weighed individually on 2 successive days at the end of the soilage-feeding experiment and at any time the cattle were sold. The average of the two weights was used as the weight for that particular time.

The Castana cattle were fed for 133 days in 1957, 144 days in 1958 and 132 days in 1959, for an average feeding period of 136 days. The Shenandoah cattle were fed for 138 days in 1957, 144 days in 1958 and 132 days in 1959, for an average feeding period of 138 days. The steers were appraised at definite intervals during the experiment. Whenever the average grade of any one lot was appraised as low-choice, they were sold. Similar lots of both farms were sold at the same time. For each year, 56 steers were required for the experiment at each farm, with a total of 122 head per year. The results, covering a 3-year period, were based on the performance of 336 steers.

Experimental Treatments

Six different treatments at both locations and two replicated treatments, one at each location, were included each year. With two of the six treatments duplicated each year, all treatments were included in the experiment the same number of times after three years. Table 1 shows the number and kind of experimental treatments, including those replicated each year, for the 3-year period. These same treatments, aside from the stilbestrol mentioned elsewhere, were used at each location. The first replication was at Castana.

Feed Supply

The six feed combinations, or rations, fed ranged from all soilage to 2 parts soilage and 1 part corn. The forage, fed as soilage, was an alfalfa-bromegrass mixture—predominantly alfalfa, with bromegrass making its main contribution during the first clipping. It

Table 1. The experimental treatments or rations for the 3-year period—1957, 1958 and 1959.

Lot number	Ration[a] 1957	1958	1959
1	All soilage	All soilage	All soilage
2	20:1	20:1	20:1
3	10:1	10:1	10:1
4	5:1	5:1	5:1
5	3:1	3:1	3:1
6	2:1	2:1	2:1
7	All soilage	20:1	10:1
8	5:1	3:1	2:1

[a] The ratio of soilage to corn by weight.

Table 2. Composition of the supplement fed at Castana and Shenandoah.

Ingredient	Castana: All soilage lots (lbs.)	Castana: Lots receiving corn and soilage (lbs.)	Shenandoah: All soilage lots (lbs.)	Shenandoah: Lots receiving corn and soilage (lbs.)
Alfalfa meal	80.0		80.0	
Ground corn		80.0		80.0
Dried molasses	10.0	10.0	10.0	10.0
Dicalcium phosphate	6.0	6.0	6.0	6.0
Salt	2.5	2.5	3.5	3.5
Trace mineral premix	0.5	0.5	0.5	0.5
Stilbestrol premix	1.0	1.0	...	...
Total	100.0	100.0	100.0	100.0

was chopped once daily and was fed fresh with the proper amount of concentrate and supplement.

The concentrate was ground, shelled corn of about 14 percent moisture. The supplement at each location was fed to all lots at 1 pound per head per day throughout the experiment. Table 2 shows the composition of the supplement fed at the two farms.

Estimation of the Production Function

Based on economic and nutritional theory, several different algebraic equations were estimated for the data from each farm. The functions for the full 3 years at each farm are denoted as the over-all functions,[4] either with or without stilbestrol. Interest is mainly in the over-all functions, since they parallel the environment within which a farmer makes decisions. Unable to predict the outcome for individual years, he must make decisions on the basis of an "expected" or average outcome. The over-all equations express total gain as a function of the feed consumed and temperature.

The alfalfa meal fed in the supplement was converted to a soilage basis[5] and then combined with the soilage fed to give a total soilage (forage) input. The corn fed in the supplement was combined with other corn in the rations to give a total corn input. Table 3 shows the composition of the supplement, fed at the rate of 0.2 of a pound per head per day, after the corn and alfalfa meal were thus "deleted."

Gain and feed observations, on a per-steer basis, were added progressively to give a cumulative series of gains and of soilage and corn consumption from the beginning of the feeding experiment. The daily maximum temperatures for each feed-gain observation period were listed;[6] then the temperatures for each interval were averaged to give an average maximum temperature for each feed-gain observation interval. A temperature series was obtained by computing the difference between the average maximum temperature for each feed-gain observation interval and the mean maximum temperature for the over-all feeding period. This series was used, with the cumulative series of gain, soilage consumption and corn consumption, to estimate the production surface.

[4] "Over-all" refers to the combined feeding periods of all 3 years at any one farm.

[5] Alfalfa meal was converted to soilage by the following method:

$$\text{Lbs. of soilage} = (\text{lbs. of alfalfa meal}) \frac{(\%\ \text{alfalfa meal dry matter})}{(\%\ \text{soilage dry matter})}$$

The percent dry matter of good alfalfa meal was obtained from: Frank B. Morrison. Feeds and feeding, a handbook for the student and stockman. 21st Ed. The Morrison Publishing Co., Ithaca, N. Y. 1949. p. 1086. The percent dry matter of soilage was obtained by taking the mean percent dry matter from samples of the soilage that was fed.

Table 3. Composition of the supplement for the stilbestrol and the nonstilbestrol rations.

Ingredient	Feeder steers receiving stilbestrol (pounds)	Feeder steers not receiving stilbestrol (pounds)
Dried molasses	50.0	50.0
Dicalcium phosphate	30.0	30.0
Salt	12.5	17.5
Trace mineral premix	2.5	2.5
Stilbestrol premix	5.0	. .
Total	100.0	100.0

Autocorrelation

Coefficients estimated when the observations for the same steer are related over time give rise to problems of autocorrelation. While feed-gain observations between lots of steers are independent, successive observations on any one lot of steers are not independent. If the observations on any one ration were to be independent, the number of lots of steers would need to be as great as the number of observations, each lot being observed one time only.

If the coefficients of the production functions were estimated by least squares under the assumption that the error terms, u_t (where t is an index of time), have the following properties:

(1)
- (a) The errors, u_t, are uncorrelated with each of the independent variables in the equation
- (b) $E(u_t) = 0$, and the u_t's are normally distributed
- (c) $E(u_t^2) = \sigma^2 < \infty$
- (d) $E(u_t u_s) = 0 \quad t \neq s$

then the coefficient estimates are the best linear unbiased estimates. However, if there is autocorrelation in the errors, u_t, and they follow the autoregressive scheme:

$$u_t = \beta u_{t-1} + e_t$$

where β is the autocorrelation coefficient and e_t is a random variable with the following properties:

(2)
- (a) The errors, e_t, are uncorrelated with each of the independent variables in the equation
- (b) $E(e_t) = 0$, and the e_t's are normally distributed
- (c) $E(e_t^2) = \sigma^2 < \infty$
- (d) $E(e_t e_s) = 0 \quad t \neq s$

and if the production function is estimated under the assumptions given by equations 1 when the errors are really autocorrelated, then the estimates remain unbiased and consistent but are no longer efficient.[7]

The presence of autocorrelation in the estimating equation does not bias the regression coefficients or make them inconsistent. It does, however, affect their variances and covariances.[8] Wold and Jureen state that, if the residuals are not autocorrelated, the coefficients estimated by least squares are unbiased, and

[6] Climatological data for the Western Iowa and the Soil Conservation Experimental farms are available through United States Department of Commerce climatological reports.

[7] D. Cochrane and G. H. Orcutt. Application of least squares regression to relationships containing auto-correlated error terms. Jour. Amer. Stat. Assoc. 44: 32-61. 1949.

[8] Stefan Valavanis. Econometrics. McGraw-Hill Book Co., Inc., New York. 1959.

the usual statistical test of the coefficients is valid. If, however, the residuals are autocorrelated, the question of significance is "... subject to a considerable margin of indeterminancy."[9]

Cochrane and Orcutt show that the method of least squares, when applied in the usual manner to relationships that contain "... highly positively auto-correlated error terms results in an extremely inefficient use of data. . . ."[10] Furthermore, they point out that most of the efficiency may be regained by a transformation that will make the error terms approximately normal.

To make tests of significance and to construct confidence limits, the error terms must be random. If the error terms that were highly autocorrelated have been made random by a transformation, then it is possible to make tests of significance and to construct confidence limits in the usual manner.[11]

Basic Equations

One of the equations used to estimate the production surface was a quadratic function[12] of the type:

$$(3)\quad G_t = a_1C_t + a_2F_t + a_3C_t^2 + a_4F_t^2 + a_5CF_t + u_t$$

where G refers to pounds of beef gain, C refers to pounds of corn, F refers to pounds of soilage, the a_i's $(i=1, \ldots, 5)$ are constants to be estimated, u is a random variable and t is an index of time. The quadratic production function is estimated without a constant term under the assumption that, when corn and forage intake is zero, beef gain also will be zero.

To remove the effects of autocorrelation, the assumption was made that the random variable, u_t, was generated by the autoregressive scheme

$$(4)\quad u_t = \beta u_{t-1} + a_6H_t + e_t$$

where β is the autocorrelation coefficient, H is a temperature variable, a_6 is a constant to be estimated and e_t is a random variable with the properties given by equations 2.

The temperature variable is included in equation 4 under the assumption that temperature would increase or decrease beef gains depending upon the temperature for each observation interval.

Equation 3 can be written for t-1 as:

$$(5)\quad G_{t-1} = a_1C_{t-1} + a_2F_{t-1} + a_3C^2_{t-1} + a_4F^2_{t-1} + a_5CF_{t-1} + u_{t-1}.$$

Now equation 5 can be solved for u_{t-1} and substituted into equation 4 to give

$$(6)\quad u_t = \beta(G_{t-1} - a_1C_{t-1} - a_2F_{t-1} - a_3C^2_{t-1} - a_4F^2_{t-1} - a_5CF_{t-1}) + a_6H_t + e_t.$$

[9] Herman Wold and Lars Jureen. Demand analysis. John Wiley and Sons, Inc., New York. 1953.

[10] Cochrane and Orcutt, *op. cit.*

[11] *Ibid.*

[12] Previous work with the data indicated that the quadratic function consistently gave a better statistical fit than did the linear, Cobb-Douglas or square-root functions. The results of a modified Cobb-Douglas function and an exponential function are reported in Appendix A.

If equation 6 is now substituted into equation 3, the following equation is obtained:

$$(7)\quad (G_t - \beta G_{t-1}) = a_1(C_t - \beta C_{t-1}) + a_2(F_t - \beta F_{t-1}) + a_3(C^2_t - \beta C^2_{t-1}) + a_4(F^2_t - \beta F^2_{t-1}) + a_5(CF_t - \beta CF_{t-1}) + a_6H_t + e_t.$$

If the variables in equation 3 are replaced by the transformed variables in equation 7, then the errors, e_t, are not autocorrelated, and the least squares method of estimation will apply.[13] Such a transformation requires knowledge of the autocorrelation coefficient β. An empirical estimate of the autocorrelation coefficient was made independently of the functional form used to estimate the production surface. This estimate was obtained from the gain observations by taking the deviations from the observation period means of the replicated lots and then regressing the deviations for observation period t on the deviations for observation period t-1. This gives a maximum likelihood estimate of β. This same procedure was used for all 3 years at both locations to obtain an average autocorrelation coefficient. The autocorrelation coefficient, β, estimated by this procedure was 0.8954, with a standard error of 0.0709. This coefficient is significant at the 0.01 probability level.[14]

Using this estimate of the autocorrelation coefficient, the variables in equation 3 were transformed as indicated in equation 7. The transformed variables were used to obtain least-square estimates of the coefficients in the production function.

The production functions so estimated using the quadratic function are:[15]

I. The over-all stilbestrol function

$$(8)\quad G = 0.11637150C + 0.2316051F - 0.0000049955C^2 - 0.0000007455F^2 + 0.0000000374CF - 1.2236046H$$

II. The over-all nonstilbestrol function

$$(9)\quad G = 0.14971812C + 0.02128774F - 0.0000122612C^2 - 0.0000005775F^2 - 0.0000037907CF - 2.2005042H.$$

The coefficient of determination, standard errors and the "t" values for the over-all stilbestrol and nonstilbestrol production functions are presented in tables 4 and 5. The coefficient of determination is quite high for both the stilbestrol and the nonstilbestrol functions indicating that the quadratic function explains a major portion of the variance in beef gains. All of the variables in the nonstilbestrol function are significant at a probability level of 0.05 or less. How-

[13] G. Tintner. Econometrics. John Wiley and Sons, Inc., New York. 1952.

[14] The "t" value of the estimated coefficient is 12.6283 with 143 degrees of freedom.

[15] In addition to the two over-all production functions, an "aggregate" production function also has been computed. This aggregate function is obtained by fitting the quadratic function collectively to both the stilbestrol and nonstilbestrol data. The aggregate production function, along with the isoquant schedules and the marginal rates of substitution, is presented in Appendix B.

Table 4. Coefficient of determination, standard errors and "t" values for the over-all stilbestrol production function (equation 8).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.9784	C	0.016265	7.155	$p<0.001$
	F	0.002312	10.017	$p<0.001$
	C^2	0.000006	0.855	$0.20<p<0.40$
	F^2	0.000001	4.202	$p<0.001$
	CF	0.000002	0.019	$p<0.50$
	H	0.307973	3.973	$p<0.005$

Table 5. Coefficient of determination, standard errors and "t" values for the over-all nonstilbestrol production function (equation 9).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.9718	C	0.015948	9.388	$p<0.001$
	F	0.002163	9.843	$p<0.001$
	C^2	0.000006	2.148	$0.01<p<0.05$
	F^2	0.0000001	4.197	$p<0.001$
	CF	0.000002	2.288	$0.01<p<0.05$
	H	0.306167	7.187	$p<0.001$

ever, two variables in the stilbestrol function are acceptable only at a probability exceeding 0.4. Nevertheless, these variables have been retained in the production function since they appear to be consistent with logic.

The coefficient on the temperature variable (H) for both the stilbestrol and nonstilbestrol functions is significant at least at the 0.005 level of probability. The negative sign on the temperature coefficient indicates that gains decrease as the temperature rises.

Other Functions

Particular forms of exponential and Cobb-Douglas functions also were fitted to the data. The estimates of both functions are reported in Appendix A. They were rejected because of the peculiarities of the function so derived. The exponential functions gave rise to sigmoid gain isoquants, and the Cobb-Douglas function gave increasing marginal productivity of feeds. Square-root functions, not reported in the Appendix, were much less efficient for estimation than the quadratic equations used.

Experiments for the two locations also were pooled to obtain the "aggregate" relationships reported in Appendix B. While the stilbestrol and nonstilbestrol functions did not differ significantly at the 0.01 level of probability, they were significant at the 0.05 level. Hence, the two sets of functions are kept separate in the text discussion which follows.

Gain Isoquants

The beef-gain isoquant equations for rations with and without stilbestrol, derived from the two over-all production function equations (8 and 9, respectively) are:

I. With stilbestrol

$$(10)\quad F = 15{,}533.54124 + 0.0250838C \pm (-670{,}690.811)\,[(0.02316051 + 0.0000000374C)^2 + 0.00000298\,(0.11637150C - 0.0000049955C^2 - 1.2236046H - G)]^{1/2}$$

II. Without stilbestrol

$$(11)\quad F = 18{,}430.94372 - 3.28199134C \pm (-868{,}800.865)\,[(0.02128774 - 0.0000037907C)^2 + 0.00000231\,(0.14971812C - 0.0000122612C^2 - 2.2005042H - G)]^{1/2}.$$

The isoquant equations express soilage (F) as a function of corn (C), the level of gain (G) and temperature (H).[16] With beef gains held constant at a given level, the isoquant equations specify all possible combinations of soilage and corn that will produce this given level of gain.

Substitution Rates

Equations for determining the marginal rates of substitution between soilage and corn for the rations with and without stilbestrol can be derived from isoquant equations 10 and 11, respectively. The equations for predicting the marginal rates of substitution of corn for soilage are:

I. With stilbestrol

$$(12)\quad \frac{\delta F}{\delta C} = \frac{0.11637150 + 0.0000000374F - 0.000009991C}{0.02316051 + 0.0000000374C - 0.000001491F}$$

II. Without stilbestrol

$$(13)\quad \frac{\delta F}{\delta C} = \frac{0.14971812 - 0.0000037907F - 0.0000245224C}{0.02128774 - 0.0000037907C - 0.000001155F}$$

Beef-gain isoquant schedules and the marginal rates of substitution associated with them have been derived for 100, 200, 300, 350 and 400 pounds of beef gain and are presented in tables 6 and 7. Corresponding gain isoquants are presented in figs. 1 and 2. Since the study did not include soilage-corn ratios beyond 2:1, the isoquant schedules beyond a 2:1 ration are extrapolations.

The rate at which corn will substitute for soilage in any one beef-fattening ration for a given level of gain is indicated by the slope at a particular point on the isoquant. The rate of substitution indicates, for any one level of gain, the amount of soilage that may be replaced by a 1-pound increase in corn. Since the isoquants in figs. 1 and 2 are curved and convex to the origin, the marginal rates of substitution of corn for soilage for all levels of gain are at a diminishing rate. Substitution rates for any one level of gain are large for rations with a small proportion of corn and diminish as the proportion of corn in the ration increases. For example, in table 6, 11,127 pounds of soilage and 300 pounds of corn can be fed in a ration to produce

[16] While the temperature variable is included in the isoquant equations, as it will be in all other equations, the temperature will be fixed at the over-all mean for most of the analysis which follows, unless otherwise stated. The over-all mean temperature for the stilbestrol feeding period was 79.36 degrees F., while the over-all mean temperature for the nonstilbestrol feeding period was 83.69 degrees F.

Table 6. Isoquant schedules, derived from the over-all stilbestrol quadratic function, showing possible feed combinations[a] and marginal rates of substitution of corn for soilage at five gain levels, for 850-pound good-to-choice feeder steers (temperature held constant at the over-all mean).

	100 lbs. gain			200 lbs. gain			300 lbs. gain			350 lbs. gain			400 lbs. gain		
Lbs. corn	Lbs. soilage	Ration[b]	$\frac{\partial F}{\partial C}$[c]	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$
0	5,182	[d]	7.55												
100	4,456	44.56	6.99												
300	3,147	10.49	6.14	11,127	37.09	17.29									
500	1,984	3.97	5.51	8,490	16.98	10.62									
700	933	1.33[e]	5.01	6,632	9.47	8.24									
900				5,126	5.70	6.92									
1,100				3,836	3.49	6.04	13,732[e]	12.48	38.81						
1,300				2,696	2.07	5.39	9,954[e]	7.66	12.40						
1,500							7,882	5.25	8.87						
1,700							6,291	3.70	7.19	11,201[e]	6.59	15.30			
1,900							4,962	2.61	6.16	8,822[e]	4.64	9.69			
2,100							3,806	1.81[f]	5.44	7,118	3.39	7.58	13,431[e]	6.40	29.83
2,300										5,733	2.49	6.37	10,103[e]	4.39	11.46
2,500										4,545	1.82[f]	5.56	8,176[e]	3.27	8.29
2,700													6,687[e]	2.48	6.74
2,900													5,441	1.88[f]	5.78

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 pound per day. The estimated number of feeding days for each of the feed combinations in this table are shown in table 30.
[b] Ration is the ratio of soilage to corn.
[c] The marginal rate of substitution of corn for soilage.
[d] The all-soilage ration.
[e] The estimated feeding period exceeds the 136-day average feeding period in the experiment.
[f] All feed combinations at this point exceed the 2:1 ration and, hence, are outside the limits of this experiment.

Table 7. Isoquant schedules, derived from the over-all nonstilbestrol quadratic function, showing possible feed combinations[a] and marginal rates of substitution of corn for soilage at four gain levels, for 850-pound good-to-choice feeder steers (temperature held constant at the over-all mean).

	100 lbs. gain			200 lbs. gain			300 lbs. gain			350 lbs. gain		
Lbs. corn	Lbs. soilage	Ration[b]	$\frac{\partial F}{\partial C}$[c]	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$
0	5,526	[d]	8.64	17,773[e]		142.53						
100	4,677	46.77	8.35	15,441[e]	154.41	28.88						
300	3,056	10.19	7.87	11,623	38.74	14.62						
500	1,524	3.05	7.47	9,051	18.10	11.54						
700				6,912	9.87	9.99						
900				5,020	5.58	8.99						
1,100				3,296	3.00	8.28						
1,300				1,696	1.30[f]	7.74						
1,400												
1,500							11,488[e]	7.66	29.75			
1,700							7,847	4.62	13.54			
1,900							5,474	2.88	10.61			
2,100							3,510	1.67[f]	9.16			
2,200												
2,300												
2,400										8,266[e]	3.44	22.53
2,500										6,492	2.60	14.80
2,600										5,161	1.98[f]	12.14

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 of a pound per day. The estimated number of feeding days for each of the feed combinations in this table are shown in table 31.
[b] Ration is the ratio of soilage to corn.
[c] The marginal rate of substitution of corn for soilage.
[d] The all-soilage ration.
[e] The estimated feeding period exceeds the 138-day average feeding period in the experiment. See table 31.
[f] All feed combinations at this point exceed the 2:1 ration and, hence, are outside the limits of the experiment.

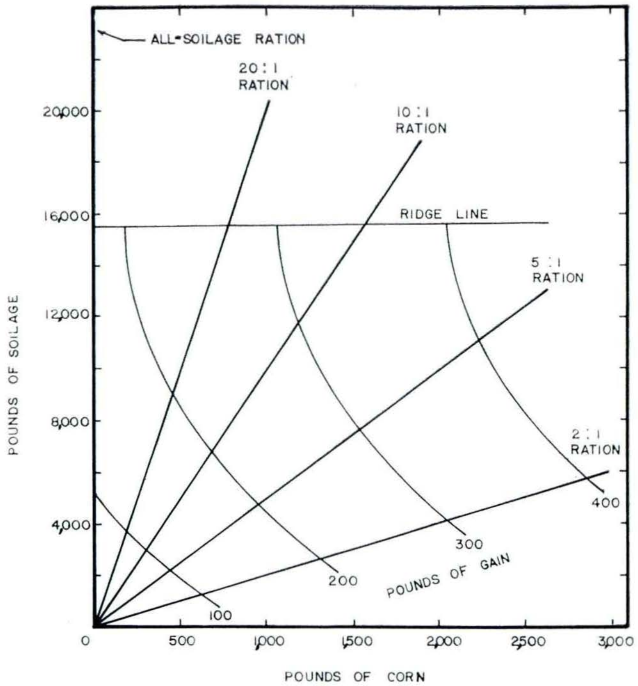

Fig. 1. Gain isoquants and selected ration lines for the over-all stilbestrol function (temperature held constant at the over-all mean).

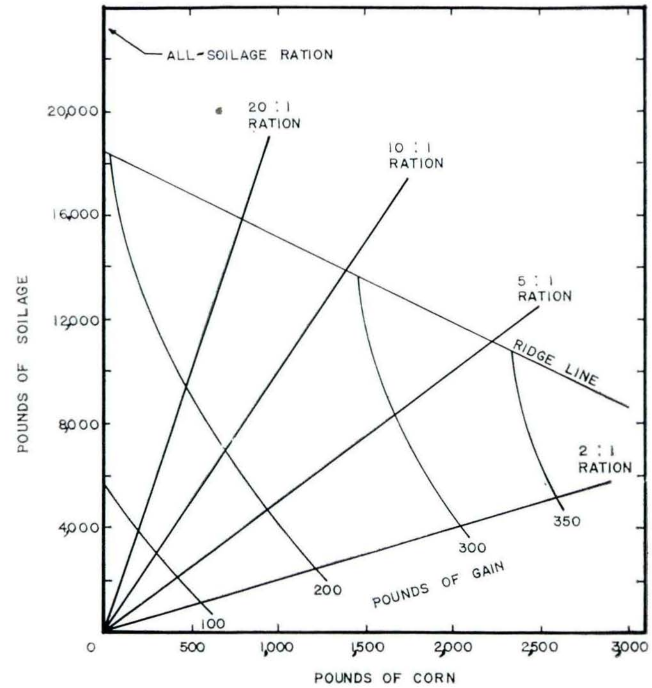

Fig. 2. Gain isoquants and selected ration lines for the over-all nonstilbestrol function (temperature held constant at the over-all mean).

200 pounds of gain, and the marginal rate of substitution of corn for soilage is 17.29. That is, 1 additional pound of corn replaced 17.29 pounds of soilage. Alternatively, 3,836 pounds of soilage and 1,100 pounds of corn can be fed to produce 200 pounds of gain, and the estimated rate of substitution of corn for soilage is now only 6.04.

Tables 6 and 7 do not show the marginal rate of substitution of corn for soilage clearly along any one ration line for different levels of gain. The rate of substitution along a ration line is of interest since it indicates the relative productivity of the ration as cattle take on heavier weights. The prediction equations for estimating the quantities of corn and soilage in this fixed ratio that are required to produce various levels of gain may be derived from the over-all production function and from ration equation 14.

$$\frac{F}{C} = \alpha. \tag{14}$$

The ration equation defines α as the ratio of soilage to corn. If equation 14 is rewritten as

$$F = \alpha C, \tag{15}$$

then, by substituting αC into the production function for F, it is possible to derive for various soilage-corn rations the quantities of corn that are required to produce various levels of gain. Once the corn requirements have been determined for any given ration, the soilage requirements are readily determined from equation 15. However, for the all-soilage ration, the isoquant equation can be used directly to determine the quantities of soilage required for various levels of gain.

The derived equations for predicting the quantities of corn that are required to produce various levels of gain for various stilbestrol and nonstilbestrol soilage-corn rations are:

I. With stilbestrol

$$\begin{aligned} C = &-(0.11637150 + 0.02316051\alpha) \\ &(-0.000001491\,\alpha^2 + 0.0000000748\,\alpha \\ &-0.000009991)^{-1} \pm (-0.000001491\,\alpha^2 \\ &+ 0.0000000748\,\alpha - 0.00000991)^{-1} \\ &[(0.11637150 + 0.02316051\,\alpha)^2 \\ &- (-0.000002982\,\alpha^2 + 0.0000001496\,\alpha \\ &- 0.000019982)\,(-1.2236046H - G)]^{1/2} \end{aligned} \tag{16}$$

II. Without stilbestrol

$$\begin{aligned} C = &-(0.14971812 + 0.02128774\,\alpha) \\ &(-0.000001155\,\alpha^2 - 0.0000075814\,\alpha \\ &-0.0000245224)^{-1} \pm (-0.000001155\,\alpha^2 \\ &-0.0000075814\,\alpha - 0.0000245224)^{-1} \\ &[(0.14971812 + 0.02128774\,\alpha)^2 \\ &-(0.00000231\,\alpha^2 - 0.0000151628\,\alpha \\ &-0.0000490448)\,(-2.2005042H - G)]^{1/2} \end{aligned} \tag{17}$$

The marginal rates of substitution are estimated for the stilbestrol rations by using equation 12, and equation 13 is used for the nonstilbestrol rations.

The predicted quantities of corn and soilage for selected rations at various levels of gain (i.e., 100, 200, 300, 350 and 400 pounds) and the associated marginal rates of substitution of corn for soilage are presented in table 8 for the stilbestrol rations and in table 9 for the nonstilbestrol rations. The ration lines corresponding to the data in tables 8 and 9 have been plotted in figs. 3 and 4.

The data in tables 8 and 9 indicate that, as a feeder steer takes on more weight and is fed any given ration,

Table 8. Corn and soilage quantities[a] and the marginal rates of substitution along the 100, 200, 300, 350 and 400 pound beef-gain isoquants for selected stilbestrol rations (temperature is held constant at the over-all mean).

Ration (ratio of soilage to corn)	100 lbs. gain			200 lbs. gain			300 lbs. gain			350 lbs. gain			400 lbs. gain		
	Lbs.[b] soilage	Lbs.[c] corn	$\frac{\partial F}{\partial C}$[d]	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$
All soilage	5,182	0	7.55												
20:1	3,834	192	6.57	9,028	451	11.55									
15:1	3,545	236	6.38	8,087	539	10.01									
10:1	3,091	309	6.11	6,799	680	8.42	11,777[e]	1,178	18.61						
8:1	2,825	353	5.96	6,117	765	7.74	10,228[e]	1,278	13.07	12,891[e]	1,611	25.19			
5:1	2,256	451	5.65	4,766	953	6.65	7,639	1,528	8.57	9,275	1,855	10.44	11,105[e]	2,221	14.15
3:1	1,672	557	5.36	3,480	1,160	5.82	5,463	1,821	6.52	6,539	2,180	7.03	7,685[e]	2,562	7.72
2:1	1,268	634	5.17	2,626	1,313	5.36	4,095	2,048	5.61	4,881	2,441	5.77	5,709	2,854	5.97

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 of a pound per day. The estimated number of feeding days for each of the feed combinations in this table are shown later.
[b] The all-soilage value was derived from equation 10, all other values were derived using equation 15.
[c] Derived from equation 16.
[d] The marginal rate of substitution of corn for soilage.
[e] The estimated feeding period exceeds the 136-day average feeding period in the experiment.

Table 9. Corn and soilage quantities[a] and marginal rates of substitution along the 100, 200, 300 and 350 pound beef-gain isoquants for selected nonstilbestrol rations (temperature is held constant at the over-all mean).

Ration (ratio of soilage to corn)	100 lbs. gain			200 lbs. gain			300 lbs. gain			350 lbs. gain		
	Lbs.[b] soilage	Lbs.[c] corn	$\frac{\partial F}{\partial C}$[d]	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$
All soilage	5,526	0	8.64									
20:1	3,896	195	8.11	9,401	470	11.86						
15:1	3,556	237	8.01	8,386	559	10.99						
10:1	3,031	303	7.86	6,956	696	10.01	12,445[e]	1,474	50.06			
8:1	2,732	341	7.78	6,190	774	9.57	11,898[e]	1,487	35.72			
5:1	2,111	422	7.62	4,687	937	8.84	8,328	1,666	14.43	9,646[e]	2,354	45.25
3:1	1,507	502	7.46	3,299	1,100	8.28	5,650	1,883	10.77	7,341[e]	2,447	17.52
2:1	1,111	556	7.36	2,417	1,208	7.97	4,078	2,039	9.52	5,195	2,597	12.19

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 of a pound per day. The estimated number of feeding days for each of the feed combinations in this table are shown later.
[b] The all-soilage value was derived from equation 11; all other values were derived from equation 15.
[c] Derived from equation 17.
[d] The marginal rate of substitution of corn for soilage.
[e] The estimated feeding period exceeds the 138-day average feeding period in the experiment.

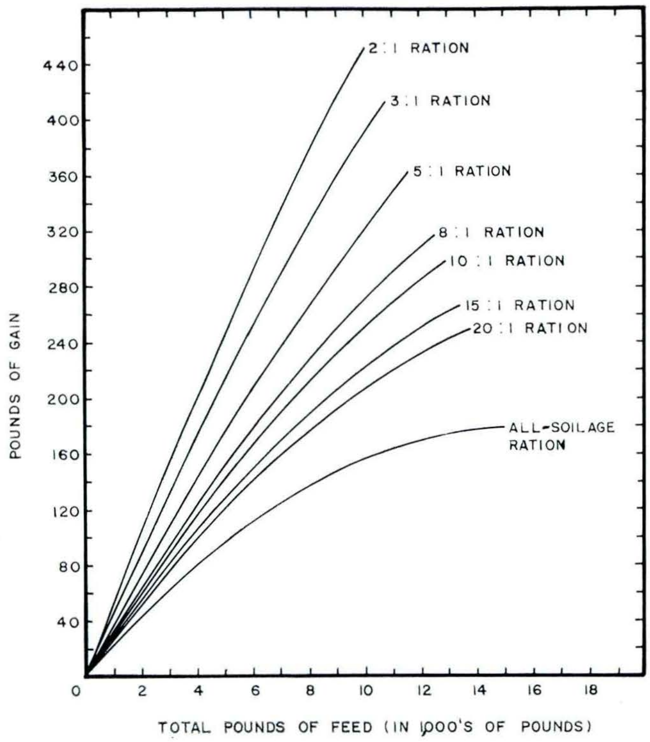

Fig. 3: Feed-gain transformation curves for selected rations derived from the over-all stilbestrol production function (temperature held constant at the over-all mean).

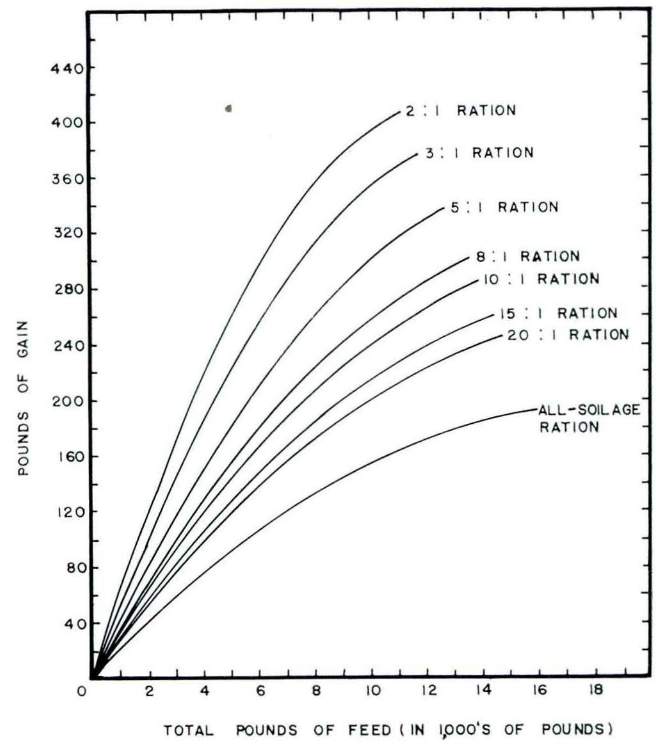

Fig. 4. Feed-gain transformation curves for selected rations derived from the over-all nonstilbestrol production function (temperature held constant at the over-all mean).

the rate of substitution of corn for soilage increases. For example, in table 8, with the 15:1 ration, a feeder steer fed 3,545 pounds of soilage and 236 pounds of corn is predicted to gain 100 pounds, and the marginal rate of substitution of corn for soilage will be 6.38. If the steer is fed the same ration until he consumes 8,087 pounds of soilage and 539 pounds of corn, it is estimated that he will have gained 200 pounds, and the predicted marginal rate of substitution of corn for soilage will be 10.01. The increase in the rate of substitution of corn for soilage, along any one ration line, indicates that corn becomes more important in the fattening ration relative to soilage as the feeder steer increases in weight.

Ration Lines

The production surface may be further examined by investigating the input-output relationships when the two feeds—corn and soilage—are fed in fixed proportions. Since, for any given ration line, the two feeds are held in fixed proportions, it is possible to derive feed-gain transformation equations from the production functions. The feed-gain transformation equations are derived by defining a new variable, γ, as the total pounds of feed of a given ration. Then, for each fixed ration, each feed input variable is redefined in terms of γ and substituted into the production function equation to give the feed-gain transformation equation or a total-gain equation for that particular fixed ration. Thus, the total-gain equation for each ration predicts the total amount of gain from various quantities of feed of a fixed ration. The marginal, or additional, gain equations may be derived from the total-gain equations by taking the first derivative of gain (G) with respect to total feed (γ).[17] The marginal-gain equation is used to estimate the additional gain from the last pound of feed fed of a given ration.

Total- and marginal-gain equations, for eight selected rations, are derived from over-all stilbestrol production function 8 and are shown in table 10. Similar equations derived from over-all nonstilbestrol production function 9 are shown in table 11.

The predicted total-gain values for various levels of feed input for the eight selected rations, are shown in table 12 and plotted in fig. 3 for beef steers fed stilbestrol. The estimated marginal gain values corresponding to the total-gain values are presented in table 13. Similarly, the predicted total-gain values for steers that were not fed stilbestrol are shown in table 14 and plotted in fig. 4, and the associated marginal gains are shown in table 15. The predicted values in both tables 12 and 13 show that, from the same total pounds of feed, total gain and marginal gain monotonically increase as the proportion of corn in the ration increases. In table 12, 7,000 pounds of feed of an all-soilage ration is predicted to produce 125.6 pounds of gain; whereas, if the ration is 20:1, the 7,000 pounds of feed will produce 159.6 pounds of beef gains. Other columns in table 12 are interpreted in the same manner. Table 13 shows that with 7,000 pounds of all soilage ration, the marginal gain is 0.0127; with 7,000 pounds of the 20:1 ration, the marginal gain is 0.0180.

[17] For the method used, see: Earl O. Heady and John L. Dillon. Agricultural production functions. Iowa State University Press, Ames, Iowa. 1961.

Table 10. Total and marginal gain equations, derived from the over-all stilbestrol quadratic function, for selected rations for 850-pound good-to-choice feeder steers.[a]

Ration[b]	Prediction equations for: Total gain	Marginal gain
Ration A All soilage	$G_A = 0.023160514\ \gamma_A - 0.000000745\ \gamma^2_A - 1.223605\ H$	$\frac{\partial G_A}{\partial \gamma_A} = 0.023160514 - 0.000001508\ \gamma_A$
Ration B 20:1	$G_B = 0.02759913\ \gamma_B - 0.000000686\ \gamma^2_B - 1.223605\ H$	$\frac{\partial G_B}{\partial \gamma_B} = 0.02759913 - 0.000001372\ \gamma_B$
Ration C 15:1	$G_C = 0.02898620\ \gamma_C - 0.000000672\ \gamma^2_C - 1.223605\ H$	$\frac{\partial G_C}{\partial \gamma_C} = 0.02898620 - 0.000001344\ \gamma_C$
Ration D 10:1	$G_D = 0.0316342399\ \gamma_D - 0.000000554\ \gamma^2_D - 1.223605\ H$	$\frac{\partial G_D}{\partial \gamma_D} = 0.0316342399 - 0.000001308\ \gamma_D$
Ration E 8:1	$G_E = 0.0335172901\ \gamma_E - 0.000000647\ \gamma^2_E - 1.223605\ H$	$\frac{\partial G_E}{\partial \gamma_E} = 0.0335172901 - 0.000001294\ \gamma_E$
Ration F 5:1	$G_F = 0.0386956787\ \gamma_F - 0.000000651\ \gamma^2_F - 1.223605\ H$	$\frac{\partial G_F}{\partial \gamma_F} = 0.0386956787 - 0.000001302\ \gamma_F$
Ration G 3:1	$G_G = 0.0484632605\ \gamma_G - 0.000000724\ \gamma^2_G - 1.223605\ H$	$\frac{\partial G_G}{\partial \gamma_G} = 0.0464632605 - 0.000001448\ \gamma_G$
Ration H 2:1	$G_H = 0.0542308423\ \gamma_H - 0.000000878\ \gamma^2_H - 1.223605\ H$	$\frac{\partial G_H}{\partial \gamma_H} = 0.0542308423 - 0.000001756\ \gamma_H$

[a] In each equation, γ denotes total pounds of feed of the particular ration indicated by the small capital letter following γ.
[b] Ration is the ratio of soilage to corn.

Table 11. Total and marginal gain equations, derived from the over-all nonstilbestrol quadratic function, for selected rations for 850-pound good-to-choice feeder steers.[a]

Ration[b]	Prediction equations for: Total gain	Marginal gain
Ration A All soilage	$G_A = 0.021287744\ \gamma_A - 0.000000578\ \gamma^2_A - 2.2005042\ H$	$\frac{\partial G_A}{\partial \gamma_A} = 0.021287744 - 0.000001156\ \gamma_A$
Ration B 20:1	$G_B = 0.0274034765\ \gamma_B - 0.000000724\ \gamma^2_B - 2.2005042\ H$	$\frac{\partial G_B}{\partial \gamma_B} = 0.0274034765 - 0.000001448\ \gamma_B$
Ration C 15:1	$G_C = 0.0293146425\ \gamma_C - 0.000000777\ \gamma^2_C - 2.2005042\ H$	$\frac{\partial G_C}{\partial \gamma_C} = 0.0293146425 - 0.000001556\ \gamma_C$
Ration D 10:1	$G_D = 0.0329632326\ \gamma_D - 0.000000892\ \gamma^2_D - 2.2005042\ H$	$\frac{\partial G_D}{\partial \gamma_D} = 0.0329632326 - 0.000001784\ \gamma_D$
Ration E 8:1	$G_E = 0.0355577856\ \gamma_E - 0.000000982\ \gamma^2_E - 2.2005042\ H$	$\frac{\partial G_E}{\partial \gamma_E} = 0.0355577856 - 0.000001964\ \gamma_E$
Ration F 5:1	$G_F = 0.0426928071\ \gamma_F - 0.000001268\ \gamma^2_F - 2.2005042\ H$	$\frac{\partial G_F}{\partial \gamma_F} = 0.0426928071 - 0.000002536\ \gamma_F$
Ration G 3:1	$G_G = 0.0533953380\ \gamma_G - 0.000001802\ \gamma^2_G - 2.2005042\ H$	$\frac{\partial G_G}{\partial \gamma_G} = 0.0533953380 - 0.000003604\ \gamma_G$
Ration H 2:1	$G_H = 0.0640978689\ \gamma_H - 0.000002461\ \gamma^2_H - 2.2005042\ H$	$\frac{\partial G_H}{\partial \gamma_H} = 0.0640978689 - 0.000004922\ \gamma_H$

[b] In each equation, γ denotes total pounds of feed of the particular ration indicated by the small capital letter following γ.
[b] Ration is the ratio of soilage to corn.

Table 12. Estimated total gain from various total feed quantities[a] (γ) selected stilbestrol rations fed to 850-pound good-to-choice feeder steers.[b]

Pounds of feed fed	Total gain[c] in pounds for selected rations:[d] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	11.4	13.6	14.3	15.7	16.6	19.2	23.1	26.9
1,000	22.4	26.9	28.3	31.0	32.9	38.0	45.7	53.4(e)
2,000	43.3	52.5	55.3	60.7	64.4	74.8	90.0	104.9
3,000	62.8	76.6	80.9	89.0	94.7	110.2	132.9	154.8(f)
4,000	80.7	99.4	105.2	116.1	123.7	144.4	174.3	202.9
5,000	97.2	120.9	128.1	141.8	151.4	177.2	214.2	249.2(g)
6,000	112.1	140.9	149.7	166.3	177.8	208.7	252.7	293.8
7,000	125.6	159.6	170.0	189.4	202.9	239.0	289.7	336.6(h)
8,000	137.6	176.9	188.8	211.2	226.7	267.9	325.3	377.7(i)
9,000	148.1	192.8	206.4	231.7	249.3	295.5	359.5	417.0
10,000	157.1	207.4	222.6	250.9	270.5	321.8	392.2	454.5
11,000	164.6	220.6	237.5	268.8	290.4	346.9	423.4	
12,000	170.6	232.4	251.0	285.4	309.0	370.6		
13,000	175.1	242.9	263.2	300.7				
14,000	178.1							
15,000	179.7							

[a] In addition to the feed fed of selected rations, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 of a pound per day. The estimated number of feeding days for each of the feed quantities is shown in table 30.
[b] Temperature is held constant at the over-all mean.
[c] All values are derived from the equations in table 10.
[d] The ration is the ratio of soilage to corn, and the letters in parentheses in the last column refer to the feeding periods as follows (see table 29): e=30 days, f=60 days, g=90 days, h=120 days and i=140 days, for all quantities above the horizontal line.

Table 13. Estimated marginal gain from various total feed quantities (γ) of selected soilage-corn ration fed to 850-pound good-to-choice feeder steers (with stilbestrol).

Pounds of feed fed	Marginal gain[a] in pounds for selected rations:[b] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	0.0224	0.0269	0.0283	0.0310	0.0329	0.0380	0.0457	0.0534
1,000	0.0217	0.0262	0.0276	0.0303	0.0322	0.0374	0.0450	0.0525
2,000	0.0202	0.0249	0.0263	0.0290	0.0309	0.0361	0.0436	0.0507
3,000	0.0187	0.0235	0.0250	0.0277	0.0296	0.0348	0.0421	0.0490
4,000	0.0171	0.0221	0.0236	0.0264	0.0283	0.0335	0.0407	0.0472
5,000	0.0157	0.0207	0.0223	0.0251	0.0270	0.0322	0.0392	0.0455
6,000	0.0142	0.0194	0.0209	0.0238	0.0258	0.0309	0.0378	0.0437
7,000	0.0127	0.0180	0.0196	0.0225	0.0245	0.0296	0.0363	0.0419
8,000	0.0112	0.0166	0.0182	0.0212	0.0232	0.0283	0.0349	0.0402
9,000	0.0097	0.0153	0.0169	0.0199	0.0219	0.0270	0.0334	0.0384
10,000	0.0083	0.0139	0.0155	0.0185	0.0206	0.0257	0.0320	0.0367
11,000	0.0068	0.0125	0.0142	0.0172	0.0193	0.0244	0.0305	
12,000	0.0053	0.0111	0.0128	0.0159	0.0180	0.0230		
13,000	0.0038	0.0098	0.0115	0.0146				
14,000	0.0023							
15,000	0.0008							

[a] All values have been derived from the equations in table 10.
[b] The ration is the ratio of soilage to corn.

Table 14. Estimated total gain from various total feed quantities[a] (γ) of selected soilage-corn rations fed to 850-pound good-to-choice feeder steers (without stilbestrol).[b]

Pounds of feed fed	Total gain[c] in pounds for selected rations:[d] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	10.5	13.5	14.5	16.3	17.5	21.0	26.2	31.4
1,000	20.7	26.7	28.5	32.1	34.6	41.4	51.6	61.6(e)
2,000	40.3	51.9	55.5	62.4	67.2	80.3	99.6	118.4
3,000	58.7	75.7	80.9	90.9	97.8	116.7	144.0	170.1(f)
4,000	75.9	98.0	104.8	117.6	126.5	150.5	184.8	217.0
5,000	92.0	118.9	127.1	142.5	153.2	181.8	221.9	259.0(g)
6,000	106.9	138.4	147.9	165.7	178.0	210.5	255.5	296.0
7,000	120.7	156.4	167.1	187.0	200.8	236.7	285.5	328.1(h)
8,000	133.3	172.9	184.7	206.6	221.6	260.4	311.8	355.3(i)
9,000	144.8	188.0	200.8	224.4	240.5	281.5	334.6	377.5
10,000	155.1	201.7	215.4	240.4	257.4	300.1	353.8	394.8
11,000	164.3	213.9	228.4	254.7	272.3	316.2	369.3	
12,000	172.3	224.6	239.8	267.1	285.3	329.7		
13,000	179.1	234.0	249.7	277.8	296.3			
14,000	184.8	241.8	258.0	286.7	305.3			
15,000	189.4	248.2	264.8					
16,000	192.8							
17,000	195.0							

[a] In addition to the feed fed, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 of a pound per day. The estimated number of feeding days for each of the feed quantities is shown in table 31.
[b] Temperature is held constant at the over-all mean.
[c] All values are derived from the equations in table 11.
[d] The ration is the ratio of soilage to corn. The letters in parentheses above each horizontal line refer to the feeding periods: e=30 days, f=60 days, g=90 days, h=120 days and i=over 120 days and up to 140 days for quantities below the line, traced out by the horizontal lines.

Table 15. Estimated marginal gain from various total feed quantities (γ) of selected soilage-corn ration fed to 850-pound good-to-choice feeder steers (without stilbestrol).

Pounds of feed fed	Marginal gain[a] in pounds for selected rations:[b] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	0.0207	0.0267	0.0288	0.0321	0.0346	0.0414	0.0516	0.0616
1,000	0.0201	0.0260	0.0278	0.0312	0.0336	0.0402	0.0498	0.0592
2,000	0.0190	0.0245	0.0262	0.0294	0.0316	0.0376	0.0462	0.0543
3,000	0.0178	0.0231	0.0246	0.0276	0.0297	0.0351	0.0426	0.0493
4,000	0.0167	0.0216	0.0231	0.0258	0.0277	0.0325	0.0390	0.0444
5,000	0.0155	0.0202	0.0215	0.0240	0.0257	0.0300	0.0354	0.0395
6,000	0.0144	0.0187	0.0200	0.0223	0.0238	0.0275	0.0318	0.0346
7,000	0.0132	0.0173	0.0184	0.0205	0.0218	0.0249	0.0282	0.0296
8,000	0.0120	0.0158	0.0169	0.0187	0.0198	0.0224	0.0246	0.0247
9,000	0.0109	0.0144	0.0153	0.0169	0.0179	0.0199	0.0210	0.0198
10,000	0.0097	0.0129	0.0138	0.0151	0.0159	0.0173	0.0174	0.0149
11,000	0.0086	0.0115	0.0122	0.0133	0.0140	0.0148	0.0138	
12,000	0.0074	0.0100	0.0107	0.0116	0.0120	0.0122		
13,000	0.0063	0.0086	0.0091	0.0097	0.0100			
14,000	0.0051	0.0071	0.0075	0.0080	0.0081			
15,000	0.0040	0.0057	0.0060					
16,000	0.0028							
17,000	0.0017							

[a] All values have been derived from the equations in table 11.
[b] The ration is the ratio of soilage to corn.

The marginal gain indicates the amount of gain added to total gain from the last pound of feed fed. For any given ration, the marginal gains, as shown in tables 13 and 15, are monotonically decreasing, indicating diminishing returns to feed. For the 10:1 ration, in table 13, the 2,000th pound of feed adds 0.0290 pound of gain to total gain while the 10,000th pound of feed adds only 0.0185 pound of gain. For any one level of feed fed, the marginal productivity of feed increases as the proportion of corn in the ration increases.

Least-Cost Rations

The least-cost ration or combination of corn and soilage for producing a given level of gain is specified whenever the marginal rate of substitution between the feeds is equal to their inverse price ratio. That is, the least-cost ration for a given level of gain is determined when

(18) $\frac{\partial F}{\partial C} = \frac{P_C}{P_F}$, where $\frac{\partial F}{\partial C}$ is the marginal rate of substitution of corn for soilage, P_C is the price of corn and P_F is the price of soilage.

The beef steers in this experiment were fed a fixed ration throughout the course of the experiment. Therefore, the production function, which expresses total gains as a function of the corn and soilage consumed, is determined for rations in which a fixed proportion of corn and soilage has been fed for the entire feeding period. However, the isoquants derived from the production function show all the possible combinations of corn and soilage that will produce a given level of gain under a fixed-ration feeding system. Hence, ration effects and their costs can be predicted for feed ratios other than those represented in the treatments.[18] Corn prices ranging from 75 cents per bushel through $1.75 per bushel and soilage prices ranging from $1 per ton through $8 per ton were used in estimating least-cost rations in terms of beef gains.

Predicted least-cost rations for various gain levels are presented in table 16 for the stilbestrol rations. (Least-cost rations without stilbestrol are shown only in graphs.) The least-cost ration, when stilbestrol is fed in the ration, can be determined in the following manner: If the price of corn is $1.12 per bushel and the price of soilage is $4 per ton, the corn-soilage price ratio is 10.0. For this corn-soilage price ratio, 100 pounds of gain can be produced at least cost by feeding 5,182 pounds of the all-soilage ration. The time required for the beef steer to gain 100 pounds is estimated to be 59.5 days.[19] Using the same price ratio, 200 pounds of gain can be produced by feeding 539 pounds of corn and 8,084 pounds of soilage, which is a soilage-corn ration of 15:1. The estimated time re-

[18] The isocline equations for the rations with and without stilbestrol are (where K is the corn-soilage price ratio):

I. With stilbestrol

$$F = \frac{0.02316051K - 0.11637150 + (0.0000000374K + 0.0000099910)C}{0.0000000374 + 0.000001491K}$$

II. Without stilbestrol

$$F = \frac{0.02128774K - 0.14971812 + (-0.0000037907K + 0.0000245224)C}{-0.0000037907 + 0.000001155K}$$

[19] The time equation, equation 28, presented in a later section provides the basis for the time estimates.

Table 16. Least-cost rations for selected levels of gain and various corn-soilage price ratios for 850-pound good-to-choice feeder steers (with stilbestrol).[a]

Corn-soilage price ratio	100 lbs. gain: Feed required[b]: Lbs. corn	Lbs. soilage	Ration[c]	Number of days[d]	200 lbs. gain: Feed required: Lbs. corn	Lbs soilage	Ration	Number of days	300 lbs. gain: Feed required: Lbs. corn	Lbs soilage	Ration	Number of days	400 lbs. gain: Feed required: Lbs. corn	Lbs soilage	Ration	Number of days
5.0	(634)	(1,268)	(2.0)[e]	(30.5)	(1,313)	(2,626)	(2.0)[e]	(62.8)	(2,048)	(4,095)	(2.0)[e]	(97.5)	(2,854)	(5,709)	(2.0)[e]	(135.2)
6.0	340	2,902	8.5	42.8	1,110	3,777	3.4	72.0	1,940	4,721	2.4	102.9	2,847	5,753	2.0	135.6
7.0	99	4,465	45.2	54.2	885	5,234	5.9	83.4	1,732	6,064	3.5	114.2	2,659	6,971	2.6	146.7[f]
8.0	..	5,182	all soilage	59.5	730	6,388	8.7	92.4	1,590	7,127	4.5	123.0	2,530	7,935	3.1	155.4[f]
9.0	..	5,182	all soilage	59.5	620	7,319	11.8	99.6	1,488	7,985	5.4	130.1				
10.0	..	5,182	all soilage	59.5	539	8,084	15.0	105.7	1,414	8,690	6.1	136.0				
11:0	..	5,182	all soilage	59.5	479	8,722	18.2	110.7	1,358	9,279	6.8	140.9[f]				
12.0	..	5,182	all soilage	59.5	431	9,262	21.5	115.0	1,315	9,777	7.4	145.1[f]				
13.0	..	5,182	all soilage	59.5	394	9,725	24.7	118.7	1,280	10,203	8.0	148.8[f]				
14.0	..	5,182	all soilage	59.5	365	10,125	27.8	122.0	1,253	10,571	8.4	151.9[f]				
15.0	..	5,182	all soilage	59.5	341	10,474	30.7	124.8	1,231	10,893	8.9	154.7[f]				
16.0	..	5,182	all soilage	59.5	321	10,782	33.6	127.4								
17.0	..	5,182	all soilage	59.5	304	11,054	36.3	129.6								
18.0	..	5,182	all soilage	59.5	290	11,298	38.9	131.7								
19.0	..	5,182	all soilage	59.5	278	11,516	41.4	133.5								
20.0	..	5,182	all soilage	59.5	268	11,714	43.6	135.2								
21.0	..	5,182	all-soilage	59.5	260	11,893	45.8	136.7[f]								
22.0	..	5,182	all soilage	59.5	252	12,056	47.8	138.1[f]								
23.0	..	5,182	all soilage	59.5	245	12,205	49.7	139.4[f]								
24.0	..	5,182	all soilage	59.5	240	12,342	51.5	140.5[f]								
25.0	..	5,182	all soilage	59.5	234	12,468	53.2	141.6[f]								

[a] Temperature is held constant at the over-all mean.
[b] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 of a pound per day.
[c] Ration is the ratio of soilage to corn.
[d] The time equation, equation 28, presented in a later section provides the basis for these estimates.
[e] The predicted feed requirements for the least-cost ration resulted in a soilage-corn ration of less than 2:1 which is outside the limits of the experiment. Therefore, the highest corn ration (i.e., the 2:1 soilage-corn ration) has been substituted as the least-cost ration.
[f] The estimated feeding period exceeds the 136-day average feeding period in the experiment.

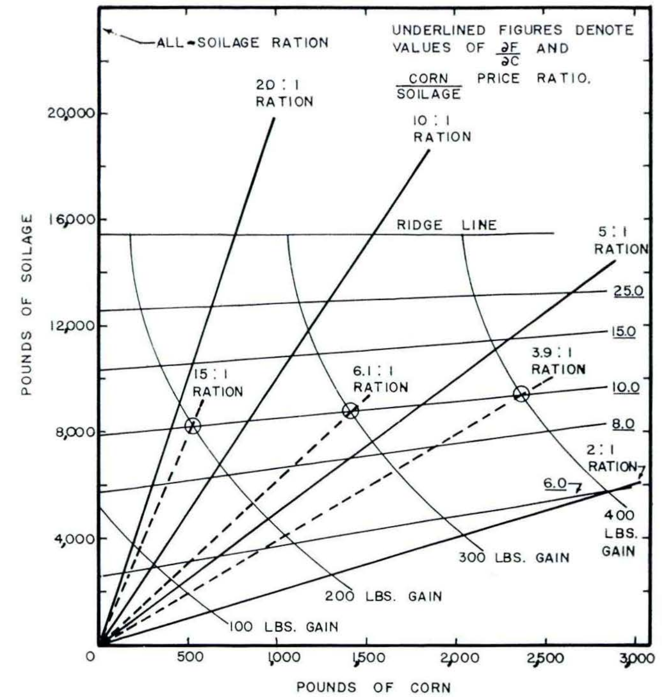

Fig. 5. Gain isoquants and isoclines showing the path of least-cost rations for the stilbestrol rations (temperature held constant at the over-all mean).

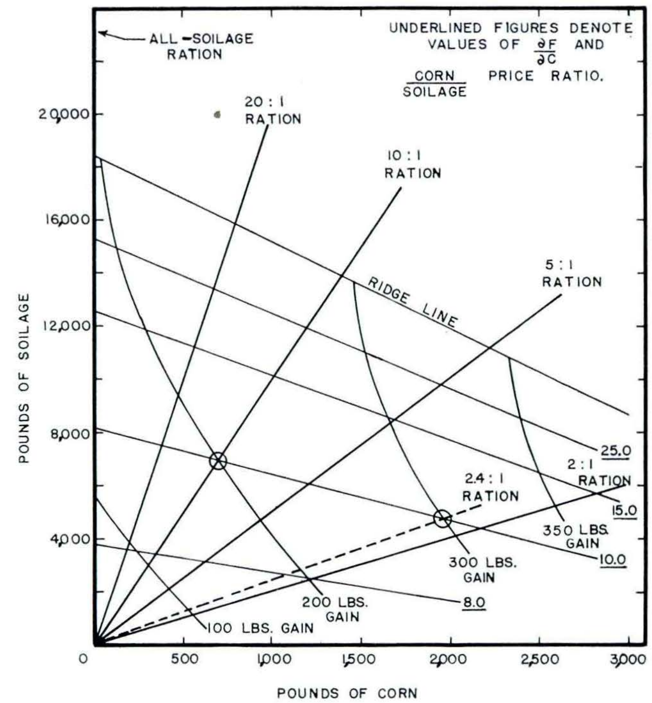

Fig. 6. Gain isoquants and isoclines showing the path of least-cost rations for the nonstilbestrol rations (temperature held constant at the over-all mean).

quired for the beef steer to attain this gain is 105.7 days. With a corn price of $1.12 and soilage at $8 per ton, the corn-soilage price ratio is 5.0. The predicted least-cost ration for this feed-price ratio (table 16) is less than the 2:1 ration for all levels of gain. Since high corn-concentrate rations outside the limits of the experiment may be physiologically unfeasible, the highest corn ration fed in the experiment has been substituted for the estimated ration (i.e., the 2:1 ration has been used as the least-cost ration in all such cases).

The isocline, a line connecting all points on successive isoquants with equal substitution ratios, specifies (at the intersection point with each isoquant) the combination of corn and soilage that will produce the gain level at least cost. Isoclines showing the path of least-cost stilbestrol rations are plotted in fig. 5 for a few of the corn-soilage price ratios presented in table 16. The least-cost stilbestrol rations for 200, 300 and 400 pounds of gain with a corn-soilage price ratio of 10.0 are shown in fig. 5. The least-cost ration is the 15:1 soilage-corn ration for 200 pounds of gain and the 6.1:1 soilage-corn ration for 300 pounds of gain. Isoclines showing the path of least cost for some rations without stilbestrol are plotted in fig. 6.

The corn-soilage price ratio map in fig. 7 provides a simplified method of estimating the least-cost ration for various corn and soilage prices. The price ratio map is so designed that it indicates an optimum least-cost ration for a range of corn-soilage price ratios, rather than "the" optimum least-cost ration for all possible corn-soilage price ratios. The diagonal lines on the price ratio map, in fig. 7, may be called iso-price ratio lines since they depict the various combinations of corn and soilage prices that have the same

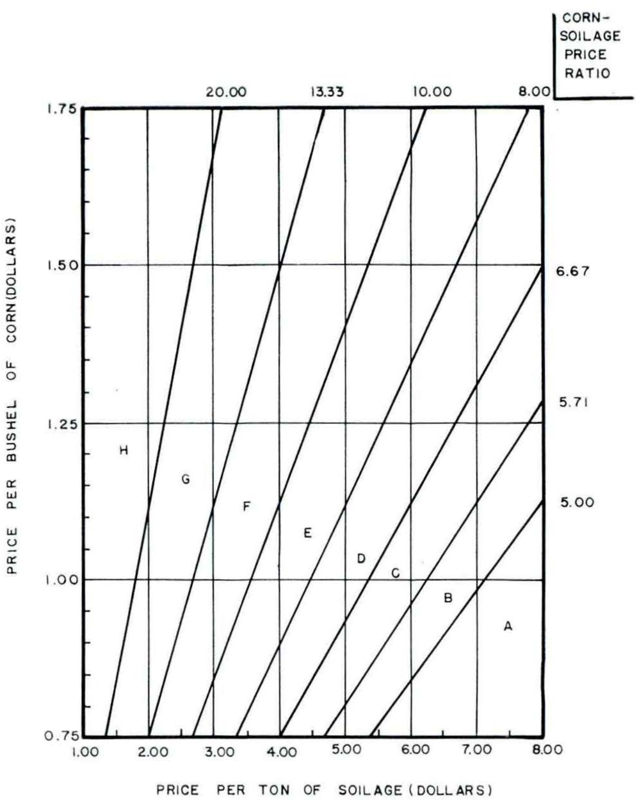

Fig. 7. Corn-soilage price ratio map for stilbestrol rations.

corn-soilage price ratio. They divide the corn-soilage price map into price ratio areas which are indicated as A, B, . . ., H. All price ratios that lie within any one of these areas specify the same least-cost ration. Given the price of corn and soilage, the level of depicted gain and the inclusion of stilbestrol, the least-cost ration is specified in the following manner: Assume that the price of corn is $1.20 per bushel and the price of soilage is $6 per ton. The coordinates of these two prices lie in price ratio area "D." The least-cost ration for 300 pounds of gain when stilbestrol is fed is found in table 17. The least-cost ration will be a soilage-corn ration of 3.8:1, which requires 563 pounds of corn and 2,125 pounds of soilage per 100 pounds of gain. This ration requires a feeding period of 117 days[20] with an average daily gain of 2.57 pounds. The least-cost ration for 100 pounds of gain when stilbestrol is fed in the ration, assuming the same feed price ratio, is found to be a soilage-corn ration of 100.7:1, including 48 pounds of corn and 4,827 pounds of soilage. The feeding period will be 57 days with an average daily gain of 1.76 pounds.[21]

[20] The time equation, equation 28, presented in a later section provides the basis for the time estimate.

Table 17. Least-cost rations for 850-pound good-to-choice feeder steers in terms of feed per 100 pounds of gain (with stilbestrol).[a]

Lbs. of gain	Price ratio area	Lbs. corn[b]	Lbs. soilage[b]	Ration[c]	Average daily gain[d]	Number of days[e]
100	A	(634)	(1,268)	(2.0)[f]	(3.28)	(30.5)
	B	569	1,612	2.8	3.02	33.1
	C	298	3,162	10.6	2.24	44.7
	D	48	4,827	100.7	1.76	56.9
	E	..	5,182	All soilage	1.68	59.5
	F	..	5,182	All soilage	1.68	59.5
	G	..	5,182	All soilage	1.68	59.5
	H	..	5,182	All soilage	1.68	59.5
200	A	(656)	(1,313)	(2.0)[f]	(3.18)	(62.8)
	B	(656)	(1,313)	(2.0)[f]	(3.18)	(62.8)
	C	536	2,010	3.8	2.70	74.0
	D	418	2,786	6.7	2.32	86.1
	E	316	3,612	11.5	2.02	98.9
	F	228	4,482	19.6	1.78	112.6
	G	160	5,391	33.7	1.57	127.4
	H	114	6,330	55.8	1.40	143.3[g]
300	A	(683)	(1,365)	(2.0)[f]	(3.08)	(97.5)
	B	(683)	(1,365)	(2.0)[f]	(3.08)	(97.5)
	C	634	1,648	2.6	2.86	104.8
	D	563	2,125	3.8	2.57	116.7
	E	499	2,633	5.3	2.32	129.4
	F	446	3,168	7.1	2.10	142.8[g]
	G	404	3,726	9.2	1.91	157.2[g]
	H	..	..	..	...	..[h]
350	A	(697)	(1,395)	(2.0)[f]	(3.02)	(115.9)
	B	(697)	(1,395)	(2.0)[f]	(3.02)	(115.9)
	C	671	1,554	2.3	2.90	120.8
	D	612	1,944	3.2	2.64	132.7
	E	560	2,359	4.2	2.41	145.3[g]
	F	517	2,797	5.4	2.21	158.5[g]
	G	..	..	..	..	..[h]
400	A	(714)	(1,427)	(2.0)[f]	(2.96)	(135.2)
	B	(714)	(1,427)	(2.0)[f]	(2.96)	(135.2)
	C	704	1,489	2.1	2.91	137.4[g]
	D	655	1,813	2.8	2.68	149.2[g]
	E	..	...	..	...	..[h]
	F	..	...	..	...	..[h]
	G	..	...	..	...	..[h]
	H				..	[h]

[a] Temperature is held constant at the over-all mean.

[b] In addition to the corn and soilage, there would also be fed a certain amount of supplement shown in table 3 at the rate of 0.2 of a pound per day.

[c] Ration is the ratio of soilage to corn.

[d] Average daily gain is determined by dividing total gain by number of days.

[e] The time equation, equation 28, presented in a later section, provides the basis for these estimates.

[f] The highest concentrate ration that was fed during the experiment has been substituted whenever the predicted feed requirements resulted in a soilage-corn ratio of less than 2:1.

[g] The estimated feeding period exceeds the 136-day feeding period in the experiment.

[h] Requires a feeding period in excess of 160 days.

Estimation of the Time Function

The proportion of corn in the soilage-corn rations affects the rate of gain as well as the cost of gain. The ration that produces the fastest gains need not coincide with the least-cost ration. To estimate the effects of different feed rations on the rate of gain, a function expressing the quantity of soilage consumed as a function of the quantity of corn fed and time was computed from the basic experimental data. The function, $F=f(C, T)$, where F denotes pounds of soilage, C denotes pounds of corn and T denotes time in days, can be used directly to derive isotime curves. The total time required to consume given quantities of corn and soilage may be obtained by solving the function, $F=f(C, T)$, for time. The function then expresses time as a function of corn and soilage; i.e., $T=t(C, F)$. The function was used to predict soilage consumption:

$$(19)\quad F_t = a_1C_t + a_2T_t + a_3C^2_t + a_4T^2_t + a_5CT_t + u_t$$

where F refers to pounds of soilage consumed, C refers to pounds of corn consumed, T refers to time in days, the a_i's $(i=1, \ldots, 5)$ are constants to be estimated, u is a random variable and t is an index of time. The function is estimated without a constant term under the assumption that zero corn and time inputs give zero consumption. A further assumption was made; namely, that the random variable u_t was generated by the autoregressive scheme:

$$(20)\qquad u_t = \beta u_{t-1} + a_6H_t + e_t$$

where β is the autocorrelation coefficient, H is a temperature variable, a is a constant to be estimated and e_t is a random variable with the properties given by equations 2.

The temperature variable has been included in equation 20 under the assumption that the temperature during any one observation interval may increase or decrease the quantity of forage consumed during that time. Equation 19 can be rewritten for u_{t-1} to give:

$$(21)\quad F_{t-1} = a_1C_{t-1} + a_2T_{t-1} + a_3C^2_{t-1} + a_4T^2_{t-1} + a_5CT_{t-1} + u_{t-1}$$

Equation 21 can now be solved for u_{t-1} and substituted into equation 20 to give:

$$(22)\quad u_t = \beta(F_{t-1} - a_1C_{t-1} - a_2T_{t-1} - a_3C^2_{t-1} - a_4T^2_{t-1} - a_5CT_{t-1}) + a_6H_t + e_t$$

By substituting equation 22 into equation 19 and collecting terms, the following equation is obtained:

$$(23)\quad (F_t - \beta F_{t-1}) = a_1(C_t - \beta C_{t-1}) + a_2(T_t - \beta T_{t-1}) + a_3(C^2_t - \beta C^2_{t-1}) + a_4(T^2_t - \beta T^2_{t-1}) + a_5(CT_t - \beta CT_{t-1}) + a_6H_t + e_t$$

[21] The time equation, equation 28, presented in a later section provides the basis for the time estimate. The feeds—corn and soilage—reported in table 17 are in pounds of feed required per 100 pounds of gain. This method of stating feed requirements is consistent with the general practice followed in the animal sciences.

Thus, if the variables in equation 19 are replaced by the transformed variables in equation 23, then the errors, e_t, are not autocorrelated, and the least-squares method of estimation will apply.

The autocorrelation coefficient used in the soilage-consumption transformation equation was the same as the one used in transforming data for the production function for two reasons: first when the autocorrelation coefficient for the soilage-consumption function was estimated in the same manner as for the production function, a biased estimate of β was obtained. The experimental data show that there was a tendency to feed the same quantities of corn and soilage to all lots that were fed the same ration. Some of the lots that were fed the same ration were actually fed the same quantities of corn and soilage for the entire feeding period. However, other lots that were fed the same rations may have been fed the same quantities of soilage and corn for a portion of the feeding period, or at least until it was evident that one of the two lots would actually eat more soilage and corn than the other lot. Only then would there be a definite difference in the quantities of feed fed to each lot, and the differences were always in the same direction. Thus, the autocorrelation coefficient tends to be biased upward because of the tendency to feed the same quantities of corn and soilage to all lots that were fed the same ration. Second, since the data had already been transformed for the production function, it was decided to use these transformed data in estimating the soilage-consumption functions.

The over-all soilage-consumption functions estimated, using the transformed data in the quadratic functions, are:

I. The over-all stilbestrol function

$$(24)\quad F = -1.992155C + 82.750048T + 0.00026539C^2 + 0.07410081T^2 - 0.00856894CT + 0.88080396H$$

II. The over-all nonstilbestrol function

$$(25)\quad F = -3.4605222C + 102.8699400T + 0.00009447C^2 + 0.04277036T^2 - 0.00066794CT + 17.99069800H$$

The coefficient of determination, standard errors and "t" values for the over-all stilbestrol and nonstilbestrol soilage-consumption functions are presented in tables 18 and 19. The coefficient of determination is quite high for both the stilbestrol and nonstilbestrol functions. This high coefficient of determination indicates that a major portion of the variance in soilage consumption has been explained by the quadratic function. Most of the variables that were used in the regression are acceptable at a very high level of significance. Even though certain variables are acceptable only at lower probability levels, they have been retained in the regression since the basis for including the variables in the regression appeared to be consistent with logic.

The temperature coefficient for both the stilbestrol and nonstilbestrol soilage-consumption functions must be interpreted in light of the experimental feeding period which was started the second week in May

Table 18. Coefficient of determination, standard errors and "t" values for the over-all stilbestrol soilage-consumption function (equation 24).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.9954	C	0.3041	6.552	$p<0.001$
	T	0.3046	27.161	$p<0.001$
	C^2	0.0001	1.977	$0.05<p<0.10$
	T^2	0.0221	3.343	$0.001<p<0.005$
	CT	0.0034	2.556	$0.01<p<0.025$
	H	4.4157	0.199	$p>0.50$

Table 19. Coefficient of determination, standard errors and "t" values for the over-all nonstilbestrol soilage-consumption function (equation 25).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.9947	C	0.2867	12.070	$p<0.001$
	T	2.9635	34.712	$p<0.001$
	C^2	0.0001	0.706	$p>0.40$
	T^2	0.0210	2.036	$0.05<p<0.025$
	CT	0.0033	0.204	$p>0.50$
	H	4.0030	4.494	$p<0.001$

and continued to the latter part of September. When feeder cattle are first put on a new ration, some time is required for them to adjust to the new ration. After adjustment, they tend to consume larger quantities of the given ration than of the old one. Furthermore, as cattle put on more weight they also eat more of the given ration. Therefore, the coefficient for the temperature variable is expected to have a positive sign, since temperature is positively correlated with the time conditions for consumption. While the time coefficient for the stilbestrol function is not significant at usual probability levels, it has been retained in the consumption function to be consistent with the nonstilbestrol function.

The soilage-consumption equations, or the isotime equations, (equations 24 and 25) express the quantity of soilage (F) that will be consumed as a function of corn (C) and time (T). If time is held constant, the soilage-consumption equations will specify all possible combinations of soilage and corn that will be consumed within this given time period. Since the feeder steers have been on full feed, the isotime function will predict the "stomach capacity" of the feeder steers for any given feeding period.

The slope of the isotime curve, or the "stomach capacity" curve, indicates the substitution rate between feeds when time is held constant. It indicates the amount by which one feed must be decreased to increase the consumption of the other feed by one unit if time is constant. The equations, derived from the soilage-consumption functions, for predicting the rate at which corn substitutes for soilage in consumption for any given feeding period are:

I. With stilbestrol

$$(26)\quad \frac{\partial F}{\partial C} = -1.992155 + 0.14820162C - 0.00856894T$$

II. Without stilbestrol

$$(27)\quad \frac{\partial F}{\partial C} = -3.4605222 + 0.00018094C - 0.00066794T$$

The isotime schedules and the associated rates of substitution have been derived for 30, 60, 90, 120, 130 and 140 feeding days and are presented in table 20 for the over-all stilbestrol function. The isotime schedules have been plotted in fig. 8 (the dashed lines) for the over-all stilbestrol function and, in fig. 9, for

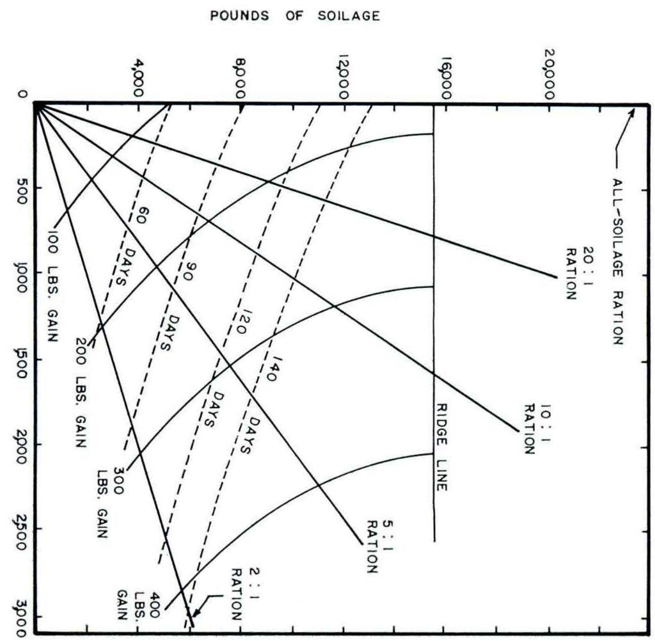

Fig. 8. Isotime curves, gain isoquants and selected ration lines for the over-all stilbestrol function (temperature held constant at the over-all mean).

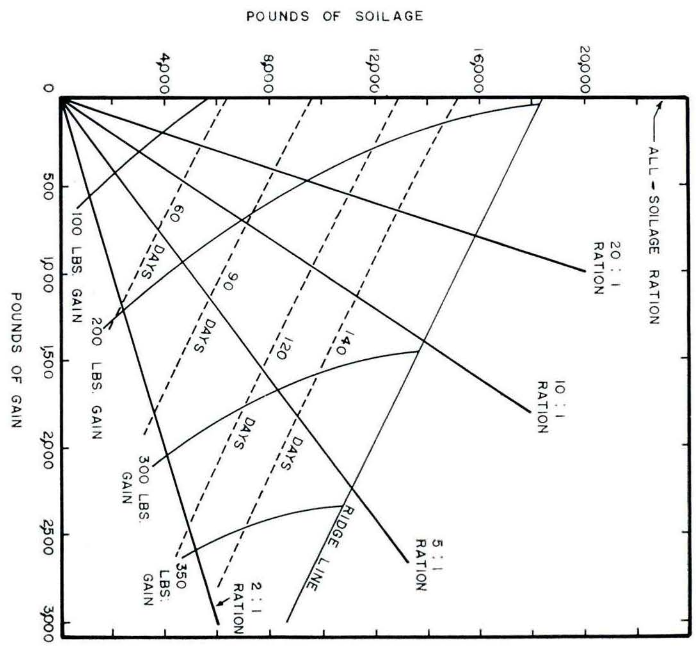

Fig. 9. Isotime curves, gain isoquants and selected ration lines for the over-all nonstilbestrol function (temperature held constant at the over-all mean).

Table 20. Isotime schedules showing quantities of various feed combinations[a] that could possibly be fed the marginal rate of substitution of corn for soilage in consumption for six different feeding intervals (with stilbestrol).[b]

	30 days			60 days			90 days			120 days			130 days			140 days		
Lbs. corn	Lbs. soilage	Ration[c]	∂F/∂C[d]	Lbs. soilage	Ration	∂F/∂C	Lbs. soilage	Ration	∂F/∂C	Lbs. soilage	Ration	∂F/∂C	Lbs. soilage	Ration	∂F/∂C	Lbs. soilage	Ration	∂F/∂C
0	2,549	[e]	2.25	5,232	[e]	2.51	8,048	[e]	2.76	10,997	[e]	3.02	12,010	[e]	3.11	13,037	[e]	3.19
100	2,327	23.3	2.20	4,984	49.8	2.45	7,774	77.7	2.71	10,697	10.70	2.97	11,701	117.0	3.05	12,721	127.2	3.14
300	1,898	6.3	2.09	4,504	15.0	2.35	7,243	24.1	2.60	10,115	33.7	2.86	11,102	37.0	2.95	12,104	40.3	3.03
500	1,491	3.0	1.98	4,045	8.1	2.24	6,732	13.5	2.50	9,553	19.1	2.76	10,523	21.0	2.84	11,508	23.0	2.93
700	1,105	1.6[f]	1.88	3,607	5.2	2.13	6,243	8.9	2.39	9,013	12.9	2.65	9,966	14.2	2.73	10,933	15.6	2.82
900				3,191	3.5	2.02	5,776	6.4	2.29	8,494	9.4	2.54	9,429	10.5	2.63	10,380	11.5	2.71
1,100				2,796	2.5	1.92	5,329	4.8	2.18	7,996	7.3	2.44	8,914	8.1	2.52	9,848	9.0	2.61
1,300				2,422	1.9[f]	1.82	4,904	3.8	2.07	7,519	5.8	2.33	8,420	6.5	2.42	9,336	7.2	2.50
1,500							4,500	3.0	1.97	7,064	4.7	2.22	7,948	5.3	2.31	8,847	5.9	2.40
1,700							4,117	2.4	1.86	6,629	3.9	2.12	7,496	4.4	2.20	8,378	4.9	2.29
1,900							3,755	2.0[f]	1.75	6,216	3.3	2.01	7,066	3.7	2.10	7,931	4.2	2.18
2,100										5,825	2.8	1.91	6,657	3.2	1.99	7,505	3.6	2.08
2,300										5,454	2.4	1.80	6,270	2.7	1.89	7,100	3.1	1.97
2,500										5,105	2.0[f]	1.69	5,903	2.4	1.78	6,717	2.7	1.86
2,700													5,558	2.1[f]	1.67	6,354	2.4	1.76
2,900																6,013	2.1	1.65

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 of a pound per day.
[b] Temperature is held constant at the over-all mean.
[c] Ration is the ratio of soilage to corn.
[d] Indicates the marginal rate of substitution of corn for soilage in consumption.
[e] The all-soilage ration.
[f] All feed combinations at this point exceed the 2:1 ration and, hence, are outside the limits of the experiment.

the over-all nonstilbestrol function. The slope of the isotime curves at any given point indicates the rate at which corn substitutes for soilage in consumption. The curvature of the isotime curves changes little over their length, suggesting that the substitution rates between the two feeds in consumption are "highly" constant. The isotime curves in both figs. 8 and 9 are slightly convex to the origin. However, if the range on the experimental rations had been extended past the 2:1 ration, the isotime curves, or "stomach capacity" curves, might have become slightly concave to the origin, at least for the heavy corn rations.

By superimposing the gain isoquants on the isotime curves, it is possible to get some idea of the portion of the production surface that is relevant for various feeding periods. Figs. 8 and 9 show the predicted gain isoquants (the solid lines) superimposed over the predicted isotime curves (the dashed lines) for the rations with and without stilbestrol, respectively. In both figs. 8 and 9, the all-soilage ration line, the 140-day isotime curve and the 2:1 ration line depict the boundary lines for the 140-day feeding period.

Time Relationships

Equations that express the total time required to consume various quantities of soilage and corn may be derived from the over-all soilage-consumption functions. Thus, the over-all time equations for the rations with and without stilbestrol, as derived from the two over-all soilage-consumption equations (24 and 25), are as follows:

I. With stilbestrol

$$(28)\quad T = -558.36128626 + 0.05781948C \pm 6.7475645\,(6{,}847.57044400 - 0.00000523C^2 - 0.82767919C - 0.26107315H + 0.29640324F)^{1/2}$$

II. Without stilbestrol

$$(29)\quad T = -1{,}176.48647060 + 0.00763899C \pm 11.436640\,(10{,}582.22455560 - 0.00001571C^2 + 0.45460922C - 3.07787452H + 0.17108144F)^{1/2}$$

Time equations 28 and 29 express the total time (T) required to consume a given quantity of corn and soilage as a function of the soilage (F) and corn (C) fed. Thus, it is possible to predict the time required to produce various levels of gain, when different soilage-corn rations are fed, by substituting into the time equations the predicted feed requirements for the various levels of gain. Table 21 shows, for a selected number of stilbestrol soilage-corn rations, the time required to produce various levels of gain.[22] In all cases, the time required to produce a given level of gain decreases as the proportion of corn in the ration increases. Too, the predicted values indicate that, for a given feeding period, the maximum level of gain is attained with the heaviest corn ration.

The average daily rate of gain for various levels of

[22] The predicted feed requirements for selected rations with and without stilbestrol for various levels of gain are shown in tables 8 and 9.

Table 21. Predicted total time required to produce various levels of gain for eight selected stilbestrol soilage - corn rations (temperature is held constant at the over-all mean).[a]

Selected rations[b]	100 lbs. gain			200 lbs. gain			300 lbs. gain			350 lbs. gain			400 lbs. gain		
	Lbs. corn	Lbs. soilage	Time (days)	Lbs. corn	Lbs. soilage	Time (days)	Lbs. corn	Lbs. soilage	Time (days)	Lbs. corn	Lbs. soilage	Time (days)	Lbs. corn	Lbs. soilage	Time (days)
All soilage		5,182	59.5												
20:1	192	3,834	49.6	451	9,028	113.1									
15:1	236	3,545	47.5	539	8,087	105.7									
10:1	309	3,091	44.2	680	6,799	95.6	1,178	11,777	162.4[c]						
8:1	353	2,825	42.2	765	6,117	90.3	1,278	10,228	149.0[c]						
5:1	451	2,256	38.0	953	4,766	79.8	1,528	7,639	127.2	1,855	9,275	154.0[c]	2,221	11,105	183.9[c]
3:1	557	1,672	33.6	1,160	3,480	69.7	1,821	5,463	109.1	2,180	6,539	130.5	2,562	4,685	153.1[c]
2:1	634	1,268	30.5	1,313	2,626	62.8	2,048	4,095	97.5	2,441	4,881	115.9	2,854	5,709	135.2

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 of a pound per day.
[b] Ration is the ratio of soilage to corn.
[c] The estimated feeding period exceeds the 136-day average feeding period in the experiment.

gain are presented in table 22 for a selected number of stilbestrol rations. The average daily rate of gain is found by dividing total gain by the number of days required to attain this gain. For any given level of gain, the average daily gain increases as the proportion of corn in the ration increases. For any given ration, the average daily rate of gain diminishes as the beef animal takes on heavier weights. The estimates in table 23 for the rations without stilbestrol follow a pattern similar to those for the stilbestrol rations, except for the magnitude of the average daily rates of gain.

Table 22. Average daily gains for various levels of gain when 850-pound good-to-choice feeder steers are fed selected stilbestrol soilage-corn rations (temperature is held constant at the over-all mean).

Total pounds of gain	Average daily rate of gain for selected rations:[a] (in lbs.)							
	All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
50	1.818	2.101	2.183	2.336	2.439	2.688	3.030	3.333
100	1.681	2.016	2.105	2.262	2.370	2.632	2.976	3.279
150	1.472	1.908	2.011	2.183	2.297	2.573	2.924	3.233
200		1.768	1.892	2.092	2.215	2.506	2.869	3.185
250		1.556[b]	1.739	1.983	2.122	2.434	2.812	3.133
300			1.568[b]	1.847[b]	2.013[b]	2.358	2.750	3.077
350						2.273[b]	2.684	3.020
400							2.613[b]	2.959

[a] Ration is the ratio of soilage to corn.
[b] Indicates a feeding period of more than 136 days.

Table 23. Average daily gains for various levels of gain when 850-pound good-to-choice feeder steers are fed selected nonstilbestrol soilage-corn rations (temperature is held constant at the average mean).

Total pounds of gain	Average daily rate of gain for selected rations:[a] (in lbs.)							
	All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
50	2.058	2.415	2.513	2.674	2.778	3.030	3.311	3.546
100	1.901	2.288	2.392	2.564	2.674	2.924	3.226	3.448
150	1.685	2.140	2.249	2.435	2.551	2.814	3.119	3.356
200		1.944	2.073	2.278	2.404	2.688	3.003	3.241
250		1.598[b]	1.806	2.073	2.218	2.530	2.867	3.113
300				1.881[b]	1.933[b]	2.327	2.698	2.956
350						2.192[b]	2.465[b]	2.754
400								2.426[b]

[a] Ration is the ratio of soilage to corn.
[b] Indicates a feeding period of more than 138 days.

Time Equations for Selected Rations

The over-all time equations may be reduced to individual time equations for selected rations in the same manner as the over-all production functions were reduced to individual gain equations. These individual time equations for selected rations are shown in tables 24 and 25 for the rations with and without stilbestrol, respectively.

The estimated number of days required to feed various quantities of the selected rations are shown in table 26 for the stilbestrol rations. The predicted values show that time to consume a given quantity (pounds) of feed increases with the proportion of corn in the ration. For example, 5,000 pounds of the all-soilage ration (table 26) can be consumed in 57.5 days, but 61.2 days are required to consume 5,000 pounds of a 20:1 ration; 62.4 days, for a 15:1 ration; and 64.5 days, for a 10:1 ration.

Fresh-chopped pasture forage has a very high moisture content and is highly palatable. Hence, a steer consumes and digests, in a given time period, larger quantities of rations that contain successively

Table 24. Time equations, derived from the over-all stilbestrol time function, to predict the time required for 850-pound good-to-choice feeder steers to consume various quantities of selected rations.

Ration[a]	Prediction equations for time in days[b]
Ration A All soilage	$T_A=-558.3613+6.7476\gamma_A\pm6.7476\ (6{,}847.5704+0.2964\gamma_A-0.2611H)^{1/2}$
Ration B 20:1	$T_B=-558.3613+0.0028\gamma_B\pm6.7476\ (6{,}847.5704-0.00000001\gamma^2_B+0.2429\gamma_B-0.2611H)^{1/2}$
Ration C 15:1	$T_C=-558.3613+0.0036\gamma_C\pm6.748\ (6{,}847.5704-0.00000002\gamma^2_C+0.2261\gamma_C-0.2611H)^{1/2}$
Ration D 10:1	$T_D=-558.3613+0.0053\gamma_D\pm6.7476\ (6{,}847.5704-0.00000004\gamma^2_D+0.1942\gamma_D-0.2611H)^{1/2}$
Ration E 8:1	$T_E=-558.3613+0.0064\gamma_E\pm6.7476\ (6{,}847.5704-0.00000006\gamma^2_E+0.1715\gamma_E-0.2611H)^{1/2}$
Ration F 5:1	$T_F=-558.3613+0.0096\gamma_F\pm6.7476\ (6{,}847.5704-0.00000014\gamma^2_F+0.1090\gamma_F-0.2611H)^{1/2}$
Ration G 3:1	$T_G=-558.3613+0.0144\gamma_G\pm6.7476\ (6{,}847.5704-0.00000033\gamma^2_G+0.0154\gamma_G-0.2611H)^{1/2}$
Ration H 2:1	$T_H=-558.3613+0.0193\gamma_H\pm6.7476\ (6{,}847.5704-0.00000058\gamma^2_H-0.0783\gamma_H-0.2611H)^{1/2}$

[a] Ration is the ratio of soilage to corn.
[b] In each equation, γ denotes total pounds of feed of the particular ration indicated by the small capital letter following γ.

Table 25. Time equations, derived from the over-all nonstilbestrol time function, to predict the time required for 850- pound good-to-choice feeder steers to consume various quantities of selected rations.

Ration[a]	Prediction equations for time in days[b]
Ration A All soilage	$T_A=-1{,}176.4865\pm11.4366\ (10{,}582.2246+0.1711\gamma_A-3.0779H)^{1/2}$
Ration B 20:1	$T_B=-1{,}176.4865+0.0004\gamma_B\pm11.4366\ (10{,}582.2246-0.00000004\gamma^2_B+0.1846\gamma_B-3.0779H)^{1/2}$
Ration C 15:1	$T_C=-1{,}176.4865+0.0005\gamma_C\pm11.4366\ (10{,}582.2246-0.00000006\gamma^2_C+0.1888\gamma_C-3.0779H)^{1/2}$
Ration D 10:1	$T_D=-1{,}176.4865+0.0007\gamma_D\pm11.4366\ (10{,}582.2246-0.00000013\gamma^2_D+0.1969\gamma_D-3.0779H)^{1/2}$
Ration E 8:1	$T_E=-1{,}176.4865+0.0008\gamma_E\pm11.4366\ (10{,}582.2246-0.00000019\gamma^2_E+0.2026\gamma_E-3.0779H)^{1/2}$
Ration F 5:1	$T_F=-1{,}176.4865+0.0013\gamma_F\pm11.4366\ (10{,}582.2246-0.00000044\gamma^2_F+0.2183\gamma_F-3.0779H)^{1/2}$
Ration G 3:1	$T_G=-1{,}176.4865+0.0019\gamma_G\pm11.4366\ (10{,}582.2246-0.00000098\gamma^2_G+0.2420\gamma_G-3.0779H)^{1/2}$
Ration H 2:1	$T_H=-1{,}176.4865+0.0025\gamma_H\pm11.4366\ (10{,}582.2246-0.00000175\gamma^2_H+0.2656\gamma_H-3.0779H)^{1/2}$

[a] Ration is the ratio of soilage to corn.
[b] In each equation, γ denotes total pounds of feed of the particular ration indicated by the small capital letter following γ.

Table 26. Estimated total time required for 850-pound good-to-choice feeder steers to consume various amounts of selected soilage-corn rations (with stilbestrol).[a]

Pounds of feed fed	Total days[b] required to feed various quantities of selected rations:[c]							
	All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	6.0	6.3	6.4	6.6	6.7	7.0	7.5	8.0
1,000	12.0	12.6	12.8	13.1	13.4	14.1	15.1	16.0[d]
2,000	23.7	25.0	25.4	26.1	26.7	28.1	30.1	32.0
3,000	35.1	37.2	37.8	39.0	39.8	42.0	45.1	47.9[e]
4,000	46.4	49.3	50.2	51.8	53.0	56.0	60.1	63.8
5,000	57.5	61.2	62.4	64.5	66.0	69.8	75.1	79.6
6,000	68.3	73.1	74.5	77.1	78.9	83.7	90.0	95.3[f]
7,000	79.0	84.7	86.4	89.6	91.8	97.5	104.9	110.9
8,000	89.5	96.3	98.3	102.1	104.6	111.2	119.8	126.4[g]
9,000	99.8	107.7	110.1	114.4	117.4	125.0	134.6	141.9[h]
10,000	110.0	119.0	121.7	126.7	130.0	138.6[h]	149.5	157.3
11,000	120.0	130.2	133.2	138.8[h]	142.6	152.3	164.3	
12,000	129.9	141.3[h]	144.7	150.9	155.2	165.9		
13,000	139.6[h]	152.3	156.0	163.0				
14,000	149.2							
15,000	158.7							

[a] Temperature is held constant at the over-all mean.
[b] All values are derived from the equations in table 24.
[c] Ration is the ratio of soilage to corn. The horizontal lines refer to these gains in pounds: d=100, e=200, f=300 and g=400. The letter h indicates a period of more than 136 days for numbers below the last line.

greater proportions of soilage. Of course, the total digestible nutrient intake is less in soilage rations than in rations containing successively greater proportions of corn. This relationship is implied when tables 12 and 26 are compared. In tables 12 and 14, time lines have been drawn across the various ration columns to indicate feeding periods of equal length. Thus, tables 12 and 26 for the stilbestrol rations show gains to be greater, for a given time, for the heavier corn rations. Furthermore, this greater gain is attained with less total pounds of feed, as compared with rations with more soilage.

The average daily gains from feeding various quantities of selected rations are shown in table 27 for the rations with stilbestrol and in table 28 for the rations without stilbestrol. The average daily gains increase for any given level of feed consumption as the proportion of corn in the ration increases. Similarly, for any given ration, the average daily gains decrease as the quantity of feed fed increases.

By using the over-all soilage-consumption equations and ration equation 15, it is possible to derive equations to predict for various soilage-corn rations the quantities of corn and soilage that will be consumed in various time periods. By substituting ration equation 15 into soilage-consumption equation 24, the following can be derived for predicting corn values for various stilbestrol soilage-corn rations:

I. With stilbestrol

$$(30)\quad C=3{,}753.25935+1{,}884.01974\,\alpha + 16.14405T \pm 1{,}884.01974 [(-1.992155-\alpha-0.00856894T)^2 - 0.87844142T - 0.00007866T^2 - 0.000935503H]^{1/2}$$

After the corn values have been determined for any given soilage-corn ration, the predicted soilage values corresponding to each predicted corn value are derived from ration equation 15. Once the various combinations of corn and soilage have been determined for the different rations, these combinations can be substituted into the over-all production function (equation 8) to predict the levels of gain. Table 29 includes feed and gain predictions for specified time periods.

The corn quantities for the soilage-corn rations without stilbestrol, can be obtained by using the following equation, which was derived in a manner similar to that for the stilbestrol rations:

II. Without stilbestrol

$$(31)\quad C=5{,}640.651186+1{,}630.0\,\alpha+1.08874220T \pm 1{,}630.0[(-3.4605222-\alpha -0.00066794T)^2-0.03887249T - 0.00001616T^2-0.00679832H]^{1/2}$$

Tables 30 and 31 are presented to show the estimated feeding periods for the various possible feed combinations presented in tables 6 and 7. These estimated feeding periods in tables 30 and 31 have been used in tables 6 and 7, respectively, as a basis for designating the relevant marginal rates of substitution.

Table 27. Average daily gains from feeding various quantities of selected soilage-corn rations to 850-pound good-to-choice feeder steers (with stilbestrol).[a]

Pounds of feed fed	Average daily gains in pounds for selected rations:[b] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	1.90	2.16	2.23	2.38	2.48	2.74	3.08	3.36
1,000	1.87	2.13	2.21	2.37	2.46	2.70	3.03	3.34
2,000	1.83	2.10	2.18	2.33	2.41	2.66	2.99	3.28
3,000	1.79	2.06	2.14	2.28	2.38	2.62	2.95	3.23
4,000	1.74	2.02	2.10	2.24	2.33	2.58	2.90	3.18
5,000	1.69	1.98	2.05	2.20	2.29	2.54	2.85	3.13
6,000	1.64	1.93	2.01	2.16	2.25	2.49	2.81	3.08
7,000	1.59	1.88	1.97	2.11	2.21	2.45	2.76	3.04
8,000	1.54	1.84	1.92	2.07	2.17	2.41	2.72	2.99
9,000	1.48	1.79	1.87	2.03	2.12	2.36	2.67[c]	2.94[c]
10,000	1.43	1.74	1.83	1.98	2.08[c]	2.32[c]	2.62	2.89
11,000	1.37	1.69	1.78[c]	1.94[c]	2.04	2.28	2.58	
12,000	1.31	1.64[c]	1.73	1.89	1.99	2.23		
13,000	1.25[c]	1.59	1.69	1.84				
14,000	1.19							
15,000	1.13							

[a] Temperature is held constant at the over-all mean.
[b] Ration is the ratio of soilage to corn.
[c] Indicates a feeding period of more than 136 days.

Table 28. Average daily gains from feeding various quantities of selected soilage-corn rations to 850-pound good-to-choice feeder steers (without stilbestrol).[a]

Pounds of feed fed	Average daily gains in pounds for selected rations:[b] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	2.14	2.50	2.59	2.76	2.82	3.09	3.36	3.57
1,000	2.13	2.47	2.54	2.72	2.81	3.04	3.31	3.52
1,500	2.11	2.44	2.53	2.68	2.78	3.00	3.26	3.47
2,000	2.09	2.41	2.50	2.64	2.74	2.96	3.22	3.42
2,500	2.06	2.38	2.47	2.61	2.71	2.93	3.18	3.36
3,000	2.04	2.35	2.44	2.58	2.67	2.90	3.14	3.31
3,500	2.01	2.33	2.41	2.55	2.64	2.86	3.09	3.25
4.000	1.98	2.30	2.38	2.52	2.61	2.81	3.04	3.20
4,500	1.96	2.27	2.35	2.49	2.58	2.77	2.99	3.14
5,000	1.93	2.23	2.32	2.45	2.54	2.74	2.95	3.09
5,500	1.90	2.21	2.29	2.42	2.51	2.70	2.90	3.03
6,000	1.88	2.18	2.26	2.39	2.47	2.66	2.85	2.97
6,500	1.85	2.15	2.22	2.35	2.44	2.62	2.80	2.91
7,000	1.82	2.12	2.20	2.32	2.40	2.58	2.75	2.85
7,500	1.80	2.09	2.16	2.29	2.36	2.53	2.70	2.79
8,000	1.77	2.06	2.13	2.26	2.33	2.49	2.65	2.73
8,500	1.74	2.03	2.10	2.22	2.29	2.45	2.60	2.67
9,000	1.71	2.00	2.07	2.19	2.26	2.41	2.55	2.60[c]
9,500	1.68	1.97	2.04	2.15	2.22	2.37	2.49[c]	2.54
10,000	1.66	1.94	2.01	2.12	2.19	2.33	2.44	2.47
10,500	1.63	1.90	1.97	2.08	2.15	2.28	2.39	2.41
11,000	1.60	1.87	1.94	2.05	2.11	2.24[c]	2.33	
11,500	1.57	1.84	1.91	2.01	2.07	2.20	2.28	
12,000	1.55	1.81	1.87	1.98	2.04[c]	2.15		
12.500	1.52	1.78	1.84	1.94[c]	2.00	2.11		
13,000	1.49	1.75	1.81	1.91	1.96			
13,500	1.46	1.72[c]	1.78[c]	1.87	1.92			
14,000	1.43	1.68	1.74	1.84				
14,500	1.40	1.65	1.71					
15,000	1.37	1.62	1.68					
15,500	1.34[c]	1.59						
16,000	1.32							
16,500	1.29							
17,000	1.26							

[a] Temperature is held constant at the over-all mean.
[b] Ration is the ratio of soilage to corn.
[c] Indicates a feeding period of more than 138 days.

GRADE FUNCTIONS

This section develops estimates of beef grades for steers fed different soilage-corn rations. A functional relationship has been estimated for beef grades in relation to feed inputs, making it possible to construct isograde contours and to derive the marginal rates of substitution of corn for soilage in producing a given grade of beef.

The procedure adopted was to estimate the functional relationship: Grade=g (pounds of corn fed, pound of soilage fed). To estimate this functional relationship, however, it was necessary to code the beef grades which were measured in the usual subjective terms as high standard, average good, low choice, etc.

Table 29. Estimated quantities of corn and soilage[a] that would be fed and the predicted beef gains for eight selected stilbestrol rations for six different feeding intervals (temperature is held constant at the over-all mean).

	30 days			60 days			90 days			120 days			130 days			140 days		
Ration[b]	Lbs. corn[c]	Lbs. soilage[d]	Lbs. gain[e]	Lbs. corn	Lbs. soilage	Lbs. gain	Lbs. corn	Lbs. soilage	Lbs. gain	Lbs. corn	Lbs. soilage	Lbs. gain	Lbs. corn	Lbs. soilage	Lbs. gain	Lbs. corn	Lbs. soilage	Lbs. gain
All soilage	. . .	2,549	40.0	. . .	5,232	100.8	. . .	8,048	138.1	. . .	10,997	164.5	. . .	12,010	170.6	. . .	13,037	175.2
20:1	116	2,322	63.2	236	4,716	119.8	359	7,180	169.1	486	9,713	210.1	529	10,572	221.9	572	11,439	232.6
15:1	150	2,248	65.6	304	4,555	125.0	461	6,919	177.3	623	9,339	222.0	677	10,158	235.1	732	10,983	247.3
10:1	212	2,115	70.1	427	4,270	134.2	646	6,463	191.8	869	8,692	242.7	944	9,443	258.0	1,020	10,198	272.5
8:1	253	2,026	73.0	510	4,082	140.3	771	6,166	201.4	1,035	8,276	256.0	1,123	8,986	272.7	1,212	9,698	288.7
5:1	361	1,803	80.7	723	3,617	155.7	1,089	5,444	225.0	1,456	7,281	288.4	1,579	7,896	308.2	1,703	8,513	327.3
3:1	504	1,513	90.8	1,010	3,031	175.9	1,518	4,554	256.4	2,028	6,083	320.2	2,198	6,594	352.5	2,369	7,106	375.2
2:1	632	1,264	99.7	1,269	2,538	193.8	1,912	3,824	282.2	2,561	5,122	364.8	2,779	5,558	391.1	2,997	5,995	416.7

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 of a pound per day.
[b] Ration is the ratio of soilage to corn.
[c] Derived from equation 30.
[d] The all-soilage value was derived from equation 24, all other values were derived from equation 16.
[e] Derived from equation 8.

Table 30. Predicted feeding time for various possible feeding combinations[a] for various levels of gain—with stilbestrol (temperature held constant at the over-all mean).

	100 lbs. gain		200 lbs. gain		300 lbs. gain		350 lbs. gain		400 lbs. gain	
Lbs. corn	Lbs. soilage[b]	No. of days[c]	Lbs. soilage	No. of days	Lbs. soilage	No. of days	Lbs. soilage	No. of days	Lbs. soilage	No. of days
0	5,182	59.5								
100	4,456	54.2								
200	3,780	49.2	13,506	150.7[d]						
300	3,147	44.6	11,127	130.3						
400	2,550	40.2	9,652	118.2						
500	1,984	35.9	8,490	108.9						
600	1,447	31.8	7,503	101.1						
700	933[e]	27.9	6,632	94.3						
800			5,846	88.2						
900			5,126	82.6						
1,000			4,459	77.4						
1,100			3,836	72.5	13,732	180.0[d]				
1,200			3,250	67.9	11,382	158.9[d]				
1,300			2,696	63.4	9,954	146.6[d]				
1,400			2,170[e]	59.1	8,832	137.2[d]				
1,500					7,882	129.3				
1,600					7,045	122.3	13,192	189.0[d]		
1,700					6,291	116.1	11,201	170.9[d]		
1,800					5,601	110.3	9,879	159.3[d]		
1,900					4,692	104.9	8,822	150.1[d]		
2,000					4,366	99.8	7,918	142.3[d]		
2,100					3,806[e]	95.0	7,118	135.5	13,431	205.6[d]
2,200							6,396	129.2	11,412	186.7[d]
2,300							5,733	123.4	10,103	174.8[d]
2,400							5,119	118.0	9,063	165.4[d]
2,500							4,545	112.9	8,176	157.5[d]
2,600									7,393	150.5[d]
2.700									6,687	1.441[d]
2,800									6,040	138.2[d]
2,900									5,441[e]	132.7

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3, fed at the rate of 0.2 of a pound per day.
[b] Derived from equation 24.
[c] Derived from equation 28.
[d] The estimated feeding period exceeds the 136-day average feeding period in the experiment.
[e] The feed combination at this point exceeds the 2:1 ration and, hence, lies outside the limits of the experiment.

Table 31. Predicted feeding time for various possible feeding combinations[a] for various levels of gain—without stilbestrol (temperature held constant at the over-all mean).

Lbs. corn	100 lbs. gain: Lbs. soilage[b]	100 lbs. gain: No. of days[c]	200 lbs. gain: Lbs. soilage	200 lbs. gain: No. of days	300 lbs. gain: Lbs. soilage	300 lbs. gain: No. of days	350 lbs. gain: Lbs. soilage	350 lbs. gain: No. of days
0	5,526	52.6	17,773	161.9[d]				
100	4,677	47.9	15,441	144.8[d]				
200	3,854	43.4	13,236	128.6				
300	3,056	39.2	11,623	117.5				
400	2,280	35.1	10,262	108.5				
500	1,524	31.1	9,051	100.7				
600	786[e]	27.3	7,943	93.8				
700			6,912	87.5				
800			5,941	81.7				
900			5,020	76.4				
1,000			4,140	71.3				
1,100			3,296	66.6				
1,200			2,482	62.1				
1,300			1,696[e]	57.8				
1,400								
1,500					11,488	151.9[d]		
1,600					9,350	136.1		
1,700					7,847	125.6		
1,800					6,589	117.3		
1,900					5,474	110.1		
2,000					4,456	103.8		
2,100					3,510[e]	98.1		
2,200								
2,300								
2,400							8,266	148.9[d]
2,500							6,492	136.0
2,600							5,161[e]	126.9

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3, fed at the rate of 0.2 of a pound per day.
[b] Derived from equation 25.
[c] Derived from equation 29.
[d] The estimated feeding period exceeds the 138-day average feeding period of the experiment.
[e] The feed combination at this point exceeds the 2:1 ration and, hence, lies outside the limits of the experiment.

The beef-steer grades were coded by using a 10-year average yearly market price of the slaughter steer grades. Specifically, the yearly average prices for the various grades of slaughter steers at the Chicago market for the 10-year period, 1951-60, were listed. Then the 10-year average price for each grade was determined and used as the *average* grade. (If the 10-year average yearly price for good steers at the Chicago market was $20, this price was considered to be the value for *average* good steers.) The *high* and *low* "good" grade values were then determined by making a linear interpolation between the *average* "good" grade values for the different "good" grades. The grade index that was used to code the subjective grade terms is shown in table 32. The computed range on each of the various beef grades is shown in table 33.

After the observed subjective grade terms had been coded with numerical values, the grade value of the steers at the beginning of the feeding experiment was subtracted from each of the observed grade values for each lot of steers. This procedure gave a grade series in terms of the change in beef grade since the beginning of the feeding period. A quadratic function was used to express the functional relationship between the change in beef grade (Q') and the consumption of various quantities of the two feeds—corn (C) and soilage (F). The over-all[23] equations for estimating the change in grade of beef steers since the beginning of the feeding period for the rations with and without stilbestrol are:

I. With stilbestrol

$$(32)\quad Q' = 0.0024655079C + 0.0000220680F - 0.0000003510C^2$$

II. Without stilbestrol

$$(33)\quad Q' = 0.0016294178C + 0.0000270836F - 0.0000000330C^2$$

[23] Since only two grade observations were made in 1957, the estimated grade functions are based only on the combined feeding periods of 1958 and 1959. Hence, "over-all" refers to the combined feeding periods of 1958 and 1959 at any one location.

Table 32. An index for coding market grades of slaughter steers.[a]

Slaughter steer grades		Numerical code
Prime:	High	29.53
	Average	28.87
	Low	28.21
Choice:	High	27.55
	Average	26.87
	Low	26.13
Good:	High	25.38
	Average	24.64
	Low	23.70
Standard:	High	22.75
	Average	21.81
	Low	20.93
Utility:	High	20.04
	Average	19.16
	Low	18.28

[a] The numerical coding value for the *average* grade of each particular slaughter grade is the 10-year, 1951-60 average yearly price for that grade of slaughter steers at Chicago. (See footnote 25.)

Table 33. The range on the index values for market grades of slaughter steers.[a]

Slaughter steer grades	Range of coded values for subjective slaughter steer grades		
Prime:	29.20 ≤	high prime	
	28.54 ≤	average prime	< 29.20
	27.87 ≤	low prime	< 28.54
Choice:	27.21 ≤	high choice	< 27.87
	26.50 ≤	average choice	< 27.21
	25.75 ≤	low choice	< 26.50
Good:	25.01 ≤	high good	< 25.75
	24.17 ≤	average good	< 25.01
	23.22 ≤	low good	< 24.17
Standard:	22.28 ≤	high standard	< 23.22
	21.37 ≤	average standard	< 22.28
	20.48 ≤	low standard	< 21.37
Utility:	19.60 ≤	high utility	< 20.48
	18.72 ≤	average utility	< 19.60
		low utility	< 18.72

[a] The range of coded values for each subjective slaughter steer grade was obtained by making a linear interpolation between each of the grade values in table 32.

In these equations, C is the total intake of corn in pounds, measured from the beginning of the feeding period to each particular observation period when an observation was made on grade. The feeder steers were first graded[24] at the beginning of the feeding period. The next two grade observations were made at 6-week intervals, after which the beef steers were graded every 21 days until the end of the feeding experiment. F is total pounds of soilage intake, measured in the same manner as was corn. Q' is the change in grade of the beef steer since the beginning of the feeding experiment.

The coefficient of determination, standard errors and "t" values for the over-all stilbestrol and nonstilbestrol grade functions are presented in tables 34 and 35. The coefficient of determination is 0.906 for the stilbestrol function and 0.842 for the nonstilbestrol function. Certain of the variables in both the stilbestrol and nonstilbestrol functions are acceptable only at a very low level of probability. Nevertheless, these variables have been retained in the function since they appear to be consistent with logic.

If a constant term is added to the change in grade functions (i.e., equations 32 and 33) and if this constant term represents the grade of the beef steers at the beginning of the feeding period, the equations with the constant term added can be used to predict the slaughter grade of good-to-choice feeder steers after being fed various quantities of corn and soilage. The predicted grade values can then be interpreted in subjective grade terms with the aid of table 33.

The over-all average grade value of the beef steers at the beginning of the feeding experiment was 21.67. When this value of 21.67 is used as the constant term in equations 32 and 33, the over-all grade functions (Q) for the rations with and without stilbestrol can be written as:

I. With stilbestrol

(34) $$Q = 21.67 + 0.0024655079C + 0.00002206680F - 0.0000003510C^2$$

II. Without stilbestrol

(35) $$Q = 21.67 + 0.0016294178C + 0.0000270836F - 0.0000000330C^2$$

where Q is the predicted slaughter grade which can be interpreted in subjective grade terms with the use of table 33.

Isograde Contours

The beef isograde equations can be derived for the rations with and without stilbestrol from over-all grade equations 34 and 35, respectively. The beef isograde equations are:

[24] In 1958, the feeder steers were graded at the beginning of the feeding experiment on both a feeder and slaughter steer basis. However, in 1959 the feeder steers were graded at the beginning of the feeding experiment on only a feeder basis. To construct a grade surface it is necessary that the beef grades all be on the same basis. Therefore, the first grade observations in 1959 were converted from a feeder basis to a slaughter basis. The 1958 data where the feeder steers were graded at the beginning of the feeding experiment on both a feeder and slaughter steer basis was used as a basis for converting the first grade observations in 1959 from a feeder to a slaughter basis. Thereafter, only the grade observations that were on a slaughter basis were used to determine the beef-grade surface.

Table 34. Coefficient of determination, standard errors and "t" values for the over-all stilbestrol grade function (equation 32).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.9066	C	0.0002031230	12.138	$p<0.001$
	F	0.0000128474	1.718	$0.05<p<0.10$
	C^2	0.0000000928	3.782	$p<0.001$

Table 35. Coefficient of determination, standard errors and "t" values for the over-all nonstilbestrol grade function (equation 33).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.8420	C	0.0002638487	6.176	$p<0.001$
	F	0.0000152410	1.777	$0.05<p<0.10$
	C^2	0.0000000330	0.266	$p<0.50$

I. With stilbestrol

(36) $$F = \frac{Q - 21.67 - 0.0024655079C + 0.0000003510C^2}{0.0000220680}$$

II. Without stilbestrol

(37) $$F = \frac{Q - 21.67 - 0.0016294178C + 0.0000000330C^2}{0.0000270836}$$

The isograde equations can be used to determine the isograde contours that specify the various quantities of corn and soilage required to attain a given grade of beef. The slope of the isograde contours is the substitution rate between the two feeds in the production of a given grade of beef. The equations for predicting the marginal rate of substitution of corn for soilage in the production of a given grade of beef can be obtained from the isograde equations by taking the partial derivative of soilage with respect to corn. The equations for predicting the marginal rates of substitution of corn for soilage in the production of a given grade of beef are:

I. With stilbestrol

(38) $$\frac{\partial F}{\partial C} = \frac{0.000000702C - 0.002466}{0.000022}$$

II. Without stilbestrol

(39) $$\frac{\partial F}{\partial C} = \frac{0.00000066C - 0.001629}{0.000027}$$

Beef isograde schedules, and the marginal rates of substitution associated with them, have been derived for the following beef grades: high standard, low good, average good, high good and low choice. The beef isograde schedules and associated marginal rates of substitution are presented in table 36 for the over-all stilbestrol function. The isograde schedules (dashed lines) have been plotted in fig. 10 for the over-all stilbestrol function and in fig. 11 for the over-all nonstilbestrol function. The slope of the isograde curves (dashed lines) at any given point indicates the rate at which corn substitutes for soilage in the production of a given grade of beef. The curvature of the isograde curves, as indicated in both figs. 10 and 11, changes but little, suggesting that the substitution rates between the two feeds in the production of a given grade of beef is nearly constant. The isograde curves in both figs. 10 and 11 are, although not readily apparent, slightly convex of the origin, indicating that the marginal rates of substitution of corn for soilage

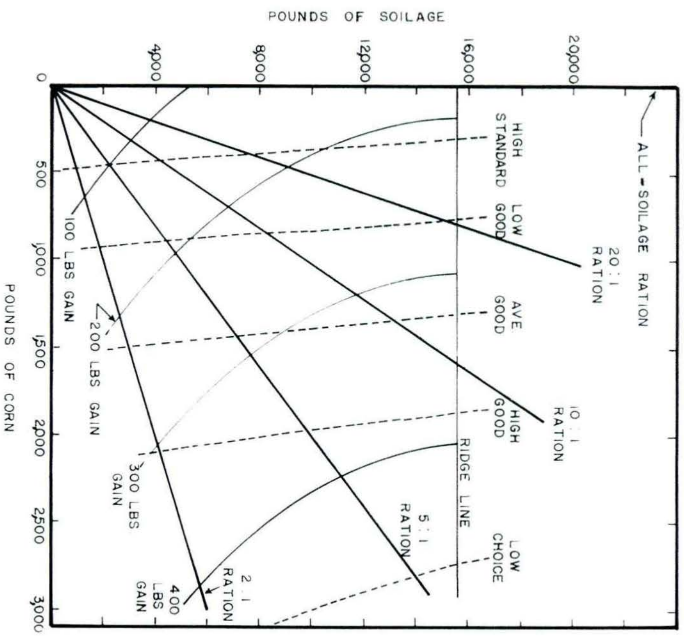

Fig. 10. Gain isoquants and isograde contours for the stilbestrol rations (temperature held constant at the over-all mean).

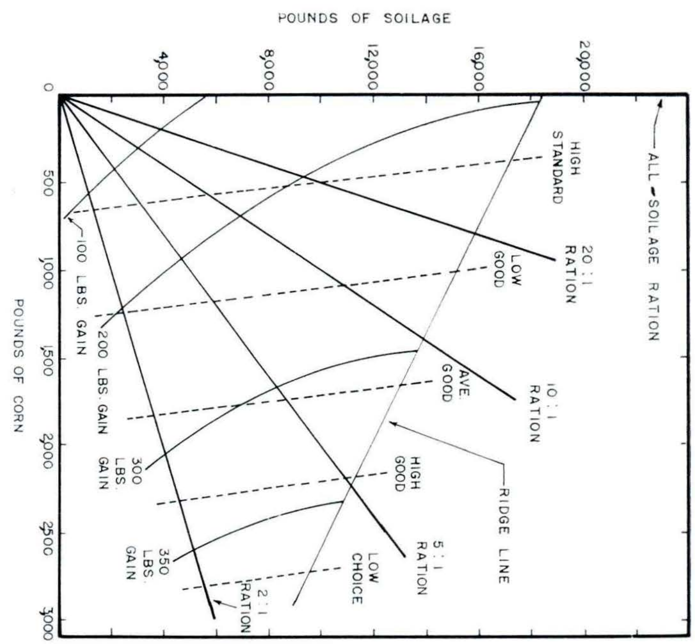

Fig. 11. Gain isoquants and isograde contours for the non-stilbestrol rations (temperature held constant at the over-all mean).

Table 36. Isograde schedules, derived from the over-all stilbestrol grade function (equation 34), showing possible feed combinations[a] and marginal rates of substitution of corn for soilage at five slaughter steer grade levels, for 850-pound good-to-choice feeder steers (temperature is held constant at the over-all mean).

	High standard[b]			Low good[b]			Average good[b]			High good[b]			Low choice[b]		
Lbs. corn	Lbs. soilage	Ration[c]	$\frac{\partial F}{\partial C}$[d]	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$
0															
100															
200	27,231[e]	136.16	105.36												
300	16,854[e]	56.18	102.18												
400	6,795	14.49	100.00												
500															
600				30,680[e]	51.13	92.63									
700				21,575[e]	30.82	89.46									
800				12,789	15.99	86.28									
900				4,320	4.80	83.10									
1,000															
1,100															
1,200							23,418[e]	19.52	73.55						
1,300							16,222[e]	12.48	70.37						
1,400							9,344	6.67	67.19						
1,500							2,784	1.86[f]	64.01						
1,600															
1,700															
1,800										18,535[e]	10.30	54.47			
1,900										13,257[e]	6.98	51.29			
2,000										8,287	4.14	48.11			
2,100										3,636	1.73[f]	44.93			
2,200															
2,300															
2,400															
2,500															
2,600													19,135[e]	7.36	29.02
2,700													16,392[e]	6.07	25.84
2,800													13,967[e]	4.99	22.66
2,900													11,860[e]	4.09	19.47
3,000													10,071[e]	3.36	16.30

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 of a pound per day.
[b] The numerical value of the subjective slaughter grades used in deriving the isograde schedules is the *average* value of each particular grade as shown in table 32.
[c] Ration is the ratio of soilage to corn.
[d] Indicates the marginal rate of substitution of corn for soilage.
[e] Estimated time required to consume this combination of corn and soilage exceeds the 136-day average feeding period.
[f] All feed combinations at this point exceed the 2:1 ration, and, hence, are outside the limits of the experiment.

for any given grade of beef are at a very slightly diminishing rate.

By superimposing the gain isoquants (the solid curves with negative slopes) over the isograde curves, it is possible to see certain relationships between the the levels of beef gains and beef grades. Figs. 10 and 11 show the predicted gain isoquants (solid curves) superimposed over the predicted isograde curves (dashed lines) for the rations with and without stilbestrol, respectively. In fig. 10, the average good isograde contour is represented by a coded numerical grade value of 24.64, shown in table 32. However, in subjective grade terms the average good grade, as well as all other grades, can be considered to extend over a range of numerical values. The average good grade in coded numerical values, as shown in table 33, extends from 24.17 to 25.01. Furthermore, the entire grade surface can be broken down into grade "areas" as indicated in table 33. The average good grade range in fig. 10, for example, would extend both above and below the average good isograde contour indicated. Therefore, each of the isograde curves can be thought of as a "wide band" extending over the grade surface denoting the various subjective beef grade "areas" such as high standard, low good, average good, etc.

PROFIT MAXIMIZATION

Estimates of the last section were merely for the grade of slaughter steers fed different soilage-corn rations. No attempt was made to derive the value of steers at the end of a given feeding period. We now attempt to estimate (a) the expected profits from feeding various soilage-corn rations for various feeding periods with different soilage-corn price conditions, (b) the optimum feeding period for different soilage-corn rations with different soilage-corn price conditions and (c) the optimum soilage-corn ration to maximize profits under different soilage-corn price conditions.

The price at which beef cattle sell upon the end of a feeding period depends, *ceteris paribus,* upon their grade (see fig. 12). Thus, one of the objects of fattening beef cattle is to improve their grade (quality). While the price of beef cattle will vary among (and even within broad) grades, the price of the different grades will also vary over any given feeding period because of seasonal price changes. Therefore, the value or the price for which the beef cattle will sell at any given time depends upon the grade of the cattle and the price for that particular grade.

To estimate the price for which slaughter steers will sell at any given time, the functional relationship that expresses the price of slaughter steers as a function of the quantity of soilage consumed and time was computed. To estimate this functional relationship, however, it was necessary to have a price series to represent the grade of beef steers during the feeding period as well as the general market price associated with the grade. Since the beef steers were graded at definite intervals throughout the beef-feeding experiment, the subjective grade terms can be replaced with the market price for that grade at the

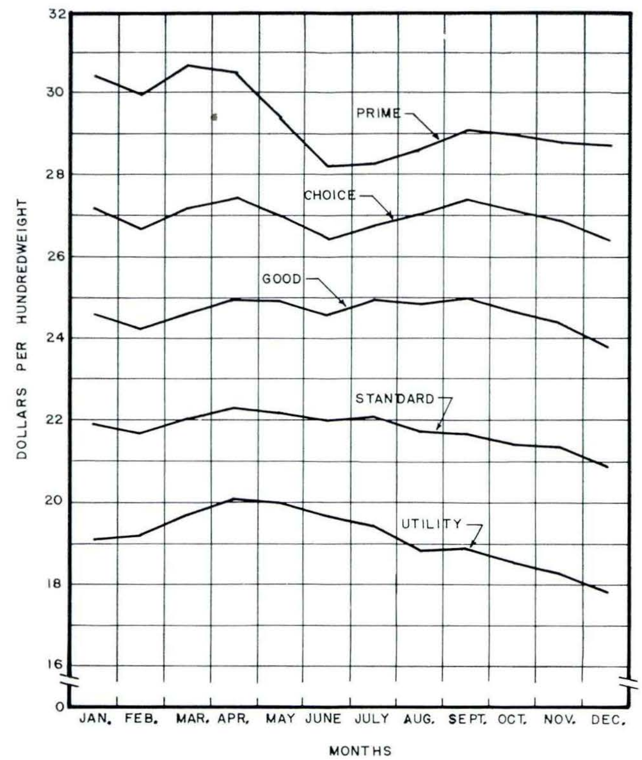

Fig. 12. Seasonal change in slaughter steer prices at Chicago, 1951-60 average.

time the steers were appraised. This procedure gives the subjective grade terms a numerical value for analysis and also furnishes a price series to represent the value or price of the steer at various stages of the feeding period.

The price of steers for this analysis is based on weekly Chicago prices.[25] For each week throughout the beef-feeding experiment, a 10-year (1951-60) weekly average price was computed for each of the various grades of slaughter and feeder steers. The 10-year average weekly price of each grade was considered to be the *average* price of that particular grade.[26]

After the 10-year average weekly price had been computed for each of the beef grades, the subjective grade observations were then replaced by the corresponding average weekly price. In some instances, the beef steers were graded on both a feeder and slaughter basis, while in most cases, the steers were graded on either a feeder or a slaughter basis. In the former case, the basis that resulted in the highest price was the one used in the analysis. The procedure assumes that a beef steer is sold on the grade basis that brings the greatest return.

To estimate the change in the price of beef steers from the beginning of the feeding period, a quad-

[25] U. S. Department of Agriculture, Agricultural Marketing Service. Livestock, meat and wool market news. Vols. 19-29. 1951-61.

[26] For example, if for the second week in August the 10-year average weekly price for choice slaughter steers at Chicago was $25.00 per hundred pounds, this price was considered to be the price for *average* choice slaughter steers. The prices for the high and low grades of each particular grade were then determined by making a linear interpolation between the average grade values.

ratic equation was used to determine the functional relationship: The change in the price of beef steers $(P') = p'$ (pounds of soilage, time in days). This relationship was estimated for both the rations with stilbestrol and those without. The "over-all"[27] change in price equations for the rations is:

I. With stilbestrol

$$(40)\ P' = -0.0000040158F + 0.0000382807T + 0.0000017537T^2 - 0.0000000048FT$$

II. Without stilbestrol

$$(41)\ P' = -0.0000004996F - 0.0002393978T + 0.0000036576T^2 - 0.0000000281FT.$$

In these equations, F refers to pounds of soilage, T refers to time in days and P′ refers to the change in the price of beef steers measured in cents per pound. All of the variables are measured from the beginning of the feeding period to each particular period when an observation was made on grade.

The price series (P′) used in this analysis was obtained by subtracting the price of the steers at the beginning of the feeding period from all price values in the series. Thus, the first price observation value would be zero. Consequently, price equations 40 and 41 have been estimated without a constant term.

The coefficient of determination, standard errors and "t" values for the stilbestrol and nonstilbestrol price functions are presented in tables 37 and 38.

If a constant term is added to equations 40 and 41 and if this constant term is the value of the steers at the beginning of the feeding period, then the price functions (i.e., those functions with the constant term added) can be used to predict the price at which the beef steers will sell. The average price of the feeder steers at the beginning of the feeding period was 25 cents per pound. When 25 cents is used as the constant term, the price function (P) for the rations with and without stilbestrol can be written as:

[27] "Over-all" refers to the combined feeding period of 1959 at any one location.

Table 37. Coefficient of determination, standard errors and "t" values for the over-all stilbestrol price function (equation 40).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.8218[a]	F	0.00000051	7.806	$p<0.001$
	T	0.00005077	0.754	$0.40<p<0.50$
	T^2	0.00000040	4.407	$p<0.001$
	FT	0.000000004	1.228	$0.20<p<0.40$

[a] The coefficient of determination is based on the "raw" sum of squares.

Table 38. Coefficient of determination, standard errors and "t" values for over-all nonstilbestrol price function (equation 41).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.7484[a]	F	0.00000254	0.197	$p>0.50$
	T	0.00000190	1.263	$0.20<p<0.40$
	T^2	0.00000002	2.180	$0.025<p<0.05$
	FT	0.00000002	1.250	$0.20<p<0.40$

[a] The coefficient of determination is based on the "raw" sum of squares.

I. With stilbestrol

$$(42)\ P=0.2500-0.0000040158F+0.0000382807T+0.0000017537T^2-0.0000000048FT$$

II. Without stilbestrol

$$(43)\ P=0.2500-0.0000004996F-0.0002393978T+0.0000036576T^2-0.0000000281FT$$

Similarly, if a constant term is added to the production functions in equations 8 and 9 and if this constant term represents the average weight of the steers at the beginning of the feeding period, then the production functions with this constant term can be used to predict the total weight (W) of the beef steers. The equations for estimating the total weight (W) of the beef steers for the rations with and without stilbestrol can be written as:

I. With stilbestrol

$$(44)\ W=850.00+0.11637150C+0.02316051F-0.0000049955C^2-0.0000007455F^2+0.000000374CF-1.2236046H$$

II. Without stilbestrol

$$(45)\ W=850.0+0.14971812C+0.02128774F-0.0000122612C^2-0.0000005775F^2-0.0000037907CF-2.2005042H$$

Profit Function

Profit is defined as the difference between total revenue and the total expenditure for all inputs. The profit function as it is related to beef-cattle feeding can be represented as:

$$(46)\ \pi=WP-P_CC-P_FF-0.2TP_S-K$$

where π refers to the profit, W refers to the total weight of the steer, P refers to the selling price, P_C refers to the price of corn, C refers to the pounds of corn fed, P_F refers to the price of soilage, F refers to the pounds of soilage fed, T refers to time in days, P_S refers to the price of the supplement, and K is the value of the feeder steer at the beginning of the feeding period. The equation includes only feed costs (other costs would need to be subtracted to compute net profit).

Thus, the over-all profit functions for the rations are:

I. With stilbestrol

$$(47)\ \pi=(850.00+0.11637150C+0.02316051F-0.0000049955C^2-0.0000007455F^2+0.0000000374CF-1.2236046H)(0.2500-0.0000040158F+0.0000382807T+0.0000017537T^2-0.0000000048FT)-P_CC-P_FF-0.2TP_S-K$$

II. Without stilbestrol

$$(48)\ \pi=(850.00+0.14971812C+0.02128774F-0.0000122612C^2-0.0000005775F^2-0.0000037907CF-2.2005042H)(0.2500-0.0000004996F-0.0002393978T+0.0000036576T^2-0.0000000281FT)-P_CC-P_FF-0.2TP_S-K$$

The profit equations can be used to estimate profits from feeding any given soilage-corn ration — from the all-soilage ration to the 2:1 soilage-corn ration — for any given feeding period within the pasture growing season. For example, the estimated profits from feeding the 10:1 soilage-corn stilbestrol ration for 140 days can be determined if the cost of the feeder steer and the prices of the feed inputs are known. The quantity of corn that will be fed in the 10:1 soilage-corn stilbestrol ration can be determined from equation 40. The soilage value corresponding to this corn value is then readily determined from ration equation 15. Therefore, given the cost of the feeder steer and the prices of the feed inputs, the expected profits can be predicted.

The profit equation can also be used to estimate profits from feeding some given total quantity of feed of a specific ration. The time required to consume this given total quantity of feed of a given ration can be determined from the time equations in table 24 or table 25. A time equation can be derived for rations other than those listed by following the same procedure used in deriving the equations in tables 24 or 25. Again, if the cost of the feeder steer and the prices of the feed inputs are known, the expected profits can be determined.

The expected profits from feeding various stilbestrol rations for 140, 130, 120 and 90 days with various feed price assumptions are presented in tables 39, 40, 41 and 42, respectively. In table 39, a feeder steer fed the 20:1 ration for 140 days is predicted to consume 11,439 pounds of soilage, 572 pounds of corn and 28 pounds of supplement. At the end of the 140-day feeding period, the steer is predicted to weigh 1,083 pounds, to grade high standard, to sell for a price of \$23.62 per hundredweight and to be worth \$255.67. The steer at the beginning of the feeding period has been valued at \$25 per hundredweight for a total value of \$212.50. If the price of soilage is \$3 per ton and the price of corn is \$1 per bushel, the total feed costs for feeding a steer 140 days will be \$28.35, which includes the cost of the supplement valued at \$3.50 per hundredweight. The profit above feed costs from feeding the 20:1 ration for 140 days is \$14.82. All of the other rations and feed-price combinations are interpreted in a similar manner. The expected profits from feeding various nonstilbestrol rations for 140 days under various feed-price assumptions are presented in table 43.

With most of the feed-price combinations, the greatest profits are obtained when the heaviest corn ration is fed. However, when the price of soilage is low relative to the price of corn, the most profitable ration has less corn and more soilage.

While tables 39 through 43 show the expected profits from feeding various soilage-corn rations for various periods of time with various feed-price combinations, they do not clearly show the optimum feeding period for any given ration and feed-price combination.

The profit functions shown in equations 47 and 48 can be written in general terms as:

$$(49)\quad \pi=(a_1+a_2C+a_3F+a_4C^2+a_5F^2+a_6CF+a_7H)\,(b_1+b_2F+b_3T+b_4T^2+b_5FT)-P_CC-P_FF-0.2TP_S-K$$

where the a_i's ($i=1, \ldots, 7$) refer to the constants in the total weight equations and b_i's ($i=1, \ldots, 5$) refer to the constants in the price equations. Since a relationship between feed inputs and time exists as specified by the soilage-consumption functions, profits must be maximized subject to the conditions specified by the soilage-consumption functions. With this restriction the profit function, equation 49, can be written as:

$$(50)\quad \pi=(a_1+a_2C+a_3F+a_4C^2+a_5F^2+a_6CF+a_7H)\,(b_1+b_2F+b_3T+b_4T^2+b_5FT)-P_CC-P_FF-0.2TP_S-K-\lambda\,(F-c_1C-c_2T-c_3C^2-c_4T^2-c_5CT-c_5H)$$

where λ is an undetermined Lagrange multiplier and the c_i's ($i=1, \ldots, 5$) are the constants in the soilage-consumption functions.

If the ration equation is defined as:

$$(51)\quad \frac{C}{F}=W$$

then

$$(52)\quad C=WF.$$

Now, by substituting WF for C in the profit function, equation 50, it will be possible to determine the optimum feeding period for any given ration and the quantities of corn and soilage that will be fed during this optimum feeding period. Thus, the profit function, equation 50, can be written as:

$$(53)\quad \pi=[(a_4W^2+a_5+a_6W)F^2+(a_2W+a_3)F+a_1+a_7H]\;[b_1+b_2F+b_3T+b_4T^2+b_5FT]-(P_CW+P_F)F-0.2TP_S-K-\lambda\;[(1-c_1W)F-c_2T-c_3W^2F^2-c_4T^2-c_5WFT-c_6H].$$

Maximization of the profit function subject to the conditions of the soilage-consumption function results in the following set of necessary conditions:

$$(54)\quad \frac{\partial\pi}{\partial F}=[(a_4W^2+a_5+a_6W)F^2+(a_2W+a_3)F+a_1+a_7H]\;[b_2+b_5T]+[b_1+b_2F+b_3T+b_4T^2+b_5FT]\;[2(a_4W^2+a_5+a_6W)F+(a_2W+a_3)]-(P_CW+P_F)-[(1-c_1W)\lambda-2c_3W^2\lambda F-c_5W\lambda T]=0$$

$$(55)\quad \frac{\partial\pi}{\partial T}=[(a_4W^2+a_5+a_6W)F^2+(a_2W+a_3)F+a_1+a_7H]\;[b_3+2b_4T+b_5F]-0.2P_S-[-\lambda c_2-2c_4\lambda T^2-c_5WF\lambda]=0.$$

There are now three equations (the soilage-consumption function and equations 54 and 55) and three unknowns (F, T and λ), and the solution of these equations will determine the optimum feeding time and the quantity of soilage (F) that will be fed given the ration (W) and the feed-price combination. If corn is included in the ration, then the quantity of corn that will be fed can be determined from ration equation 52. Once the optimum feeding period and

Table 39. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs and net revenue for good-to-choice feeder steers, weighing 850 pounds at the outset, for eight selected soilage-corn stilbestrol rations fed for 140 days (equation 47)[a]

		Ration:															
Items under rations[b]		All soilage		20:1		15:1		10:1		8:1		5:1		3:1		2:1	
Soilage		13,037 lbs.		11,439 lbs.		10,983 lbs.		10,198 lbs.		9,698 lbs.		8,513 lbs.		7,106 lbs.		5,995 lbs.	
Corn		0 lbs.		572 lbs.		732 lbs.		1,020 lbs.		1,212 lbs.		1,703 lbs.		2,369 lbs.		2,997 lbs.	
Supplement[c]		28 lbs.		28 lbs.		28 lbs.		28 lbs.		28 lbs.		28 lbs.		28 lbs.		28 lbs.	
Cost of feeder steer[d]		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50	
Final weight		1,025 lbs.		1,083 lbs.		1,097 lbs.		1,123 lbs.		1,130 lbs.		1,177 lbs.		1,225 lbs.		1,267 lbs.	
Grade[e]		Av. standard		High standard		Low good		Low good		Av. good		High good		High good		Low choice	
Selling price		$ 22.87		$ 23.62		$ 23.83		$ 24.20		$ 24.43		$ 24.99		$ 25.65		$ 26.17	
Total revenue		$234.46		$255.67		$261.49		$271.63		$278.20		$294.18		$314.21		$331.44	
Price of soilage	Price of corn	Total feed cost[f]	Net revenue	Total feed cost[f]	Net revenue	Total feed cost[f]	Net revenue	Total feed cost[f]	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue
($/ton)	($/bu.)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)
1.00	..	7.50	14.46														
	0.75			14.36	28.81	16.28	32.71	19.74	39.40	22.06	43.64	28.04	53.65	36.26	65.46	44.12	74.82
	1.00			16.91	26.26	19.55	29.45	24.29	34.84	27.48	38.23	35.64	46.05	46.83	54.88	57.50	61.44
	1.25			19.47	23.71	22.81	26.18	28.84	30.29	32.89	32.82	43.24	38.44	57.41	44.31	70.88	48.05
	1.50			22.02	21.15	26.08	22.91	33.40	25.74	38.30	27.40	50.84	30.84	67.98	33.73	84.26	34.68
	1.75			24.57	18.60	29.35	19.64	37.95	21.19	43.71	21.99	58.44	23.24	78.55	23.16	97.64	21.30
3.00	..	20.54	1.43														
	0.75			25.80	17.37	27.26	21.73	29.94	29.20	31.76	33.94	36.55	45.13	43.36	58.35	50.12	68.82
	1.00			28.35	14.82	30.53	18.46	34.49	24.65	37.17	28.53	44.15	37.53	53.94	47.78	63.50	55.44
	1.25			30.90	12.27	33.80	15.19	39.04	20.09	42.59	23.12	51.75	29.93	64.51	37.20	76.88	42.06
	1.50			33.46	9.71	37.07	11.92	43.59	15.54	48.00	17.71	59.35	22.33	75.09	26.63	90.26	28.68
	1.75			36.01	7.16	40.33	8.66	48.15	10.99	53.41	12.29	66.95	14.73	85.66	16.05	103.64	15.30
5.00	..	33.57	–11.61														
	0.75			37.24	5.93	38.24	10.75	40.13	19.00	41.46	24.24	45.06	36.62	50.47	51.24	56.11	62.83
	1.00			37.79	3.30	41.51	7.48	44.69	14.45	46.87	18.83	52.66	27.02	61.04	40.61	67.47	49.45
	1.25			42.34	0.83	44.78	4.21	49.24	9.89	52.28	13.42	60.26	21.42	71.62	30.09	82.87	36.07
	1.50			44.90	–1.72	48.05	0.94	53.79	5.34	57.70	8.01	67.86	13.82	82.19	19.52	96.25	22.69
	1.75			47.45	–4.28	51.32	–2.33	58.35	0.79	63.11	2.60	75.46	6.22	92.77	8.95	109.63	9.31
7.00	..																
	0.75	46.61	–24.65	48.68	–5.50	49.23	–0.23	50.33	8.80	51.16	14.55	53.58	28.11	57.57	44.14	62.10	56.84
	1.00			51.23	–8.06	52.49	–3.50	54.89	4.25	56.57	9.13	61.18	20.51	68.15	33.57	75.49	43.45
	1.25			53.78	–10.61	55.76	–6.77	59.44	–0.30	61.98	3.72	68.78	12.91	78.72	22.99	88.87	30.07
	1.50			56.34	–13.16	59.03	–10.04	63.99	–4.86	67.39	–1.69	76.38	5.31	89.30	12.41	102.25	16.69
	1.75			58.89	–15.72	62.30	–13.31	68.54	–9.41	72.81	–7.10	83.98	–2.29	99.87	1.84	115.63	3.31

[a] Temperature is held constant at the over-all mean.
[b] The soilage and corn quantities are derived in the manner of those in table 29.
[c] The supplement in table 3 is fed at the rate of 0.2 of a pound per day.
[d] The feeder steer is valued at $25.00/cwt.
[e] Derived from equation 34.
[f] The total feed cost includes the cost of corn and soilage plus 28 pounds of supplement valued at $3.50/cwt.

Table 40. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs and net revenue for good-to-choice feeder steers, weighing 850 pounds at the outset, for eight selected soilage-corn stilbestrol rations fed for 130 days (equation 47).[a]

Items under rations[b]		Ration: All soilage		20:1		15:1		10:1		8:1		5:1		3:1		2:1	
Soilage		12,010 lbs.		10,572 lbs.		10,158 lbs.		9,443 lbs.		8,986 lbs.		7,896 lbs.		6,594 lbs.		5,558 lbs.	
Corn		0 lbs.		529 lbs.		577 lbs.		944 lbs.		1,123 lbs.		1,579 lbs.		2,198 lbs.		2,779 lbs.	
Supplement[c]		26 lbs.		26 lbs.		26 lbs.		26 lbs.		26 lbs.		26 lbs.		26 lbs.		26 lbs.	
Cost of feeder steer[d]		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50	
Final weight		1,021 lbs.		1,072 lbs.		1,085 lbs.		1,108 lbs.		1,123 lbs.		1,158 lbs.		1,203 lbs.		1,241 lbs.	
Grade[e]		Av. standard		High standard		Low good		Low good		Av. good		Av. good		High good		Low choice	
Selling price		$ 22.90		$ 23.56		$ 23.75		$ 24.08		$ 24.30		$ 24.80		$ 25.41		$ 25.89	
Total revenue		$233.68		$252.55		$257.75		$266.86		$272.78		$287.25		$305.50		$321.26	
Price of soilage	Price of corn	Total feed cost[f]	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue
($/ton)	($/bu.)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)
1.00	. .	6.92	14.26														
	0.75			13.28	26.77	15.06	30.19	18.28	36.08	20.45	39.83	26.01	48.74	33.65	59.35	40.91	67.85
	1.00			15.64	24.41	18.08	27.17	22.49	31.87	25.46	34.82	33.06	41.69	43.46	49.54	53.31	55.45
	1.25			18.00	22.05	21.11	24.14	26.71	27.65	30.47	29.81	40.11	34.64	53.27	39.73	65.72	43.04
	1.50			20.36	19.69	24.13	21.12	30.93	23.43	35.49	24.79	47.16	27.59	63.09	29.91	78.12	30.64
	1.75			22.71	17.34	27.15	18.10	35.14	19.22	40.50	19.78	54.21	20.54	72.90	20.10	90.53	18.23
3.00	. .	18.93	2.25														
	0.75			23.85	16.20	25.22	20.03	27.72	26.64	29.43	30.85	33.91	40.84	40.24	52.76	46.46	62.30
	1.00			26.21	13.84	28.24	17.01	31.94	22.43	34.44	25.84	40.96	33.79	50.05	42.95	58.87	49.89
	1.25			28.57	11.48	31.26	13.99	36.15	18.21	39.46	20.82	48.01	26.74	59.87	33.13	71.27	37.49
	1.50			30.93	9.12	34.29	10.96	40.37	13.99	44.47	15.81	55.06	19.69	69.68	23.32	83.68	25.08
	1.75			33.29	6.76	37.31	7.94	44.58	9.78	49.49	10.79	62.11	12.64	79.49	13.51	96.09	12.67
5.00	. .	30.94	—9.76														
	0.75			34.42	5.63	35.37	9.87	37.17	17.19	38.42	21.86	41.80	32.95	46.84	46.16	52.02	56.74
	1.00			36.78	3.27	38.40	6.85	41.38	12.98	43.43	16.85	48.85	25.90	56.65	36.35	64.43	44.33
	1.25			39.14	0.91	41.42	3.83	45.60	8.76	48.45	11.83	55.90	18.85	66.46	26.54	76.83	31.93
	1.50			41.50	—1.45	44.44	0.81	49.81	4.55	53.46	6.82	62.95	11.80	76.27	16.73	89.24	19.52
	1.75			43.86	—3.81	47.47	—2.22	54.03	0.33	58.48	1.80	70.00	4.75	86.09	6.91	101.64	7.12
7.00	. .	42.95	—21.77														
	0.75			44.99	—4.94	45.53	—0.28	46.61	7.75	47.40	12.88	49.70	25.05	53.43	39.57	57.58	51.18
	1.00			47.35	—7.30	48.56	—3.31	50.82	3.54	52.42	7.86	56.75	18.00	63.24	29.76	69.98	38.78
	1.25			49.71	—9.66	51.58	—6.33	55.04	—0.68	57.43	2.85	63.80	10.95	73.06	19.94	82.39	26.37
	1.50			52.07	—12.02	54.60	—9.35	59.26	—4.90	62.45	—2.17	70.85	3.90	82.87	10.13	94.80	13.96
	1.75			54.43	—14.38	57.63	—12.38	63.47	—9.99	67.46	—7.18	77.90	—3.15	92.68	0.32	107.20	1.56

[a] Temperature is held constant at the over-all mean.
[b] The soilage and corn quantities are derived in the manner of those in table 29.
[c] The supplement in table 3 is fed at the rate of 0.2 of a pound per day.
[d] The feeder steer is valued at $25.00/cwt.
[e] Derived from equation 34.
[f] The total feed cost includes the cost of corn and soilage plus 26 pounds of supplement valued at $3.50/cwt.

Table 41. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs and net revenue for good-to-choice feeder steers weighing 850 pounds at the outset, for eight selected soilage-corn stilbestrol rations fed for 120 days (equation 47)[a].

Items under rations[b]	Ration: All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
Soilage	10,997 lbs.	9,713 lbs.	9,339 lbs.	8,692 lbs.	8,276 lbs.	7,281 lbs.	6,083 lbs.	5,122 lbs.
Corn	0 lbs.	486 lbs.	623 lbs.	869 lbs.	1,035 lbs.	1,456 lbs.	2,028 lbs.	2,561 lbs.
Supplement[c]	24 lbs.	24 lbs.	24 lbs.	24 lbs.	24 lbs.	24 lbs.	24 lbs.	24 lbs.
Cost of feeder steer[d]	$212.50	$212.50	$212.50	$212.50	$212.50	$212.50	$212.50	$212.50
Final weight	1,015 lbs.	1,060 lbs.	1,072 lbs.	1,093 lbs.	1,106 lbs.	1,138 lbs.	1,179 lbs.	1,215 lbs.
Grade[e]	Av. standard	High standard	Low good	Low good	Low good	Av. good	High good	Low choice
Selling price	$ 22.94	$ 23.53	$ 23.70	$ 24.00	$ 24.19	$ 24.64	$ 25.19	$ 25.64
Total revenue	$232.74	$249.45	$254.08	$262.21	$267.52	$280.55	$297.09	$311.43

Price of soilage ($/ton)	Price of corn ($/bu.)	All soilage: Total feed cost[f] ($)	All soilage: Net revenue ($)	20:1: Total feed cost ($)	20:1: Net revenue ($)	15:1: Total feed cost ($)	15:1: Net revenue ($)	10:1: Total feed cost ($)	10:1: Net revenue ($)	8:1: Total feed cost ($)	8:1: Net revenue ($)	5:1: Total feed cost ($)	5:1: Net revenue ($)	3:1: Total feed cost ($)	3:1: Net revenue ($)	2:1: Total feed cost ($)	2:1: Net revenue ($)
1.00	...	6.34	13.90														
	0.75			12.20	24.74	13.85	27.73	16.83	32.88	18.83	36.19	23.98	44.07	31.04	53.55	37.70	61.22
	1.00			14.37	22.58	16.63	24.95	20.71	29.00	23.45	31.57	30.49	37.57	40.09	44.50	49.14	49.79
	1.25			16.54	20.41	19.41	22.17	24.59	25.12	28.07	26.95	36.99	31.07	49.15	35.45	60.57	38.36
	1.50			18.70	18.24	22.19	19.39	28.47	21.24	32.69	22.33	43.49	24.57	58.20	26.39	72.00	26.92
	1.75			20.87	16.07	24.97	16.61	32.35	17.36	37.31	17.71	49.99	18.06	67.25	17.34	83.44	15.49
3.00	...	17.34	2.91														
	0.75			21.91	15.03	23.19	18.39	25.52	24.19	27.11	27.91	31.27	36.79	37.12	47.47	42.82	56.10
	1.00			24.08	12.86	25.97	15.61	29.40	20.31	31.73	23.29	37.77	30.29	46.18	38.42	54.26	44.67
	1.25			26.25	10.70	28.75	12.83	33.28	16.43	36.35	18.67	44.27	23.79	55.23	29.36	65.69	33.23
	1.50			28.42	8.53	31.53	10.05	37.16	12.55	40.97	14.05	50.77	17.28	64.28	20.31	77.12	21.80
	1.75			30.59	6.36	34.31	7.27	41.04	8.67	45.58	9.44	57.27	10.78	73.33	11.26	88.56	10.37
5.00	...	28.33	−8.09														
	0.75			31.63	5.32	32.53	9.05	34.21	15.50	35.39	19.63	38.55	29.51	43.21	41.39	47.95	50.98
	1.00			33.79	3.15	36.31	6.27	38.09	11.62	40.00	15.02	45.05	23.01	52.26	32.33	59.38	39.55
	1.25			35.96	0.98	38.09	3.49	41.97	7.74	44.62	10.40	51.55	16.50	61.31	23.28	70.81	28.11
	1.50			38.13	−1.18	40.87	0.71	45.85	3.86	49.24	5.78	58.05	10.00	70.37	14.23	82.25	16.68
	1.75			40.30	−3.35	43.65	−2.07	49.73	−0.02	53.86	1.16	64.55	3.50	79.42	5.17	93.68	5.25
7.00	...	39.33	−19.09														
	0.75			41.34	−4.39	41.87	−0.29	42.90	6.81	43.66	11.36	45.83	22.22	49.29	35.30	53.07	45.86
	1.00			43.51	−6.56	44.65	−3.07	46.78	2.93	48.28	6.74	52.33	15.72	58.34	26.25	64.50	34.42
	1.25			45.67	−8.73	47.43	−5.85	50.66	−0.95	52.90	2.12	58.83	9.22	67.40	17.20	75.94	22.99
	1.50			47.84	−10.90	50.21	−8.63	54.55	−4.83	57.52	−2.50	65.33	2.72	76.45	8.14	87.37	11.56
	1.75			50.01	−13.07	52.98	−11.41	58.43	−8.71	62.14	−7.12	71.83	−3.78	85.50	−0.91	98.80	0.12

[a] Temperature is held constant at the over-all mean.
[b] The soilage and corn quantities are derived in the manner of those in table 29.
[c] The supplement in table 3 is fed at the rate of 0.2 of a pound per day.
[d] The feeder steer is valued at $25.00/cwt.
[e] Derived from equation 34.
[f] The total feed cost includes the cost of corn and soilage plus 24 pounds of supplement valued at $3.50/cwt.

Table 42. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs and net revenue for good-to-choice feeder steers, weighing 850 pounds at the outset, for eight selected soilage-corn stilbestrol rations fed for 90 days (equation 47).[a]

		Ration:															
Items under rations[b]		All soilage		20:1		15:1		10:1		8:1		5:1		3:1		2:1	
Soilage		8,048 lbs.		7,180 lbs.		6,919 lbs.		6,463 lbs.		6,166 lbs.		5,444 lbs.		4,554 lbs.		3,824 lbs.	
Corn		0 lbs.		359 lbs.		461 lbs.		646 lbs.		771 lbs.		1,089 lbs.		1,518 lbs.		1,912 lbs.	
Supplement[c]		18 lbs.		18 lbs.		18 lbs.		18 lbs.		18 lbs.		18 lbs.		18 lbs.		18 lbs.	
Cost of feeder steer[d]		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50	
Final weight		988 lbs.		1,019 lbs.		1,027 lbs.		1,042 lbs.		1,051 lbs.		1,075 lbs.		1,105 lbs.		1,132 lbs.	
Grade[e]		Av. standard		High standard		High standard		Low good		Low good		Low good		Av. good		High good	
Selling price		$ 23.19		$ 23.57		$ 23.69		$ 23.89		$ 24.02		$ 24.35		$ 24.74		$ 25.07	
Total revenue		$229.13		$240.24		$243.37		$248.92		$252.59		$261.71		$273.49		$283.79	
Price of soilage	Price of corn	Total feed cost[f]	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue
($/ton)	($/bu.)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)
1.00	..	4.65	11.97														
	0.75			9.03	18.71	10.27	20.60	12.52	23.90	14.03	26.05	17.93	31.28	23.24	37.76	28.15	43.14
	1.00			10.63	17.11	12.33	18.54	15.40	21 02	17.48	22.61	22.79	26.42	30.02	30.98	36.69	34.60
	1.25			12.23	15.51	14.39	16.48	18.29	18.13	20.92	19.17	27.65	21.56	36.79	24.20	45.22	26.07
	1.50			13.84	13.91	16.45	14.42	21.17	15.25	24.36	15.73	32.51	16.70	43.57	17.42	53.76	17.53
	1.75			15.44	12.30	18.51	12.36	24.06	12.36	27.80	12.29	37.38	11.84	50.35	10.65	62.30	8.99
3.00	..	12.70	3.92														
	0.75			16.21	11.53	17.19	13.68	18.98	17.44	20.20	19.89	23.38	25.84	27.79	33.20	31.97	39.31
	1.00			17.81	9.93	19.25	11.62	21.86	14.56	23.64	16.45	28.24	20.98	34.57	26.42	40.51	30.78
	1.25			19.41	8.33	21.31	9.56	22.75	11.67	27.08	13.01	33.10	16.12	41.35	19.65	49.05	22.24
	1.50			21.02	6.73	23.37	7.50	27.63	8.79	30.52	9.57	37.96	11.26	48.12	12.87	57.58	13.71
	1.75			22.62	5.12	25.42	5.44	30.52	5.90	33.96	6.13	42.82	6.39	54.90	6.09	66.12	5.17
5.00	..	20.75	—4.12														
	0.75			23.39	4.35	24.11	6.76	25.44	10.98	26.37	13.72	28.82	20.39	32.35	28.65	35.80	35.49
	1.00			24.99	2.75	26.17	4.70	28.33	8.09	29.81	10.28	33.68	15.53	39.12	21.87	44.34	26.95
	1.25			26.59	1.15	28.23	2.64	31.21	5.21	33.25	6.84	38.54	10.67	45.90	15.09	52.87	18.42
	1.50			28.20	—0.45	30.28	0.58	34.10	2.32	36.69	3.40	43.40	5.81	52.68	8.31	61.41	9.88
	1.75			29.80	—2.06	32.34	—1.48	36.98	—0.56	40.13	—0.04	48.26	0.95	59.46	1.54	69.94	1.35
7:00		28.80	—12.17														
	0.75			30.57	—2.83	31.03	—0.16	31.90	4.52	32.53	7.56	34.26	14.95	36.90	24.09	39.62	31.67
	1.00			32.17	—4.43	33.09	—2.22	34.79	1.63	35.97	4.12	39.12	10.09	43.68	17.32	48.16	23.13
	1.25			33.77	—6.03	35.15	—4.28	37.67	—1.25	39.41	0.68	43.99	5.23	50.46	10.54	56.70	14.59
	1.50			35.38	—7.63	37.20	—6.34	40.56	—4.14	42.85	—2.77	48.85	0.37	57.23	3.76	65.23	6.06
	1.75			36.98	—9.24	39.26	—8.40	43.45	—7.02	46.29	—6.21	53.71	—4.49	64.01	—3.02	73.77	—2.48

[a] Temperature is held constant at the over-all mean.
[b] The soilage and corn quantities are derived in the manner of those in table 29.
[c] The supplement in table 3 is fed at the rate of 0.2 of a pound per day.
[d] The feeder steer is valued at $25.00/cwt.
[e] Derived from equation 34.
[f] The total feed cost includes the cost of corn and soilage plus 18 pounds of supplement valued at $3.50/cwt.

Table 43. Predicted total feed consumption, total weight, grade, selling price, total revenue, total feed costs and net revenue for good-to-choice feeder steers, weighing 850 pounds at the outset, for eight selected soilage-corn nonstilbestrol rations fed for 140 days.[a]

		Ration:															
Items under rations[b]		All soilage		20:1		15:1		10:1		8:1		5:1		3:1		2:1	
Soilage		15,240 lbs.		13,094 lbs.		12,487 lbs.		11,438 lbs.		10,768 lbs.		9,176 lbs.		7,296 lbs.		5,827 lbs.	
Corn		0 lbs.		655 lbs.		832 lbs.		1,144 lbs.		1,346 lbs.		1,835 lbs.		2,432 lbs.		2,914 lbs.	
Supplement[c]		28 lbs.		28 lbs.		28 lbs.		28 lbs.		28 lbs.		28 lbs.		28 lbs.		28 lbs.	
Cost of feeder steer[d]		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50		$212.50	
Final weight		1,040 lbs.		1,090 lbs.		1,103 lbs.		1,124 lbs.		1,137 lbs.		1,166 lbs.		1,199 lbs.		1,222 lbs.	
Grade[e]		Av. standard		High standard		Low good		Low good		Low good		Av. good		High good		Low choice	
Selling price		$ 22.06		$ 23.01		$ 23.28		$ 23.75		$ 24.04		$ 24.75		$ 25.58		$ 26.23	
Total revenue		$229.50		$250.84		$256.68		$266.80		$273.29		$288.66		$306.71		$320.64	
Price of soilage	Price of corn	Total feed cost[f]	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue	Total feed cost	Net revenue
($/ton)	($/bu.)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)	($)
1.00	..	8.32	8.68														
	0.75			16.02	22.32	18.09	26.09	21.74	32.57	24.11	36.68	29.87	46.29	36.92	57.29	42.63	65.50
	1.00			18.94	19.40	21.81	22.37	26.85	27.45	30.12	30.67	38.06	38.10	47.77	46.44	55.64	52.50
	1.25			21.86	16.48	25.53	18.65	31.95	22.35	36.13	24.66	46.25	29.91	58.63	35.58	68.65	39.49
	1.50			24.78	13.56	29.24	14.94	37.06	17.24	42.14	18.65	54.44	21.72	69.49	24.72	81.66	26.48
	1.75			27.71	10.63	32.96	11.22	42.16	12.14	48.15	12.64	62.64	13.52	80.34	13.87	94.66	13.48
3.00	..	23.56	−6.56														
	0.75			29.11	9.23	30.58	13.60	33.18	21.12	34.88	25.91	39.04	37.12	44.21	50.00	48.46	59.68
	1.00			32.03	6.31	34.30	9.88	38.28	16.02	40.89	19.90	47.24	28.92	55.07	39.14	61.47	46.67
	1.25			34.95	3.39	39.01	6.17	43.39	10.91	46.90	13.89	55.43	20.73	65.93	28.28	74.48	33.66
	1.50			37.88	0.46	41.73	2.45	48.50	5.80	52.90	7.89	63.62	12.54	76.78	17.43	87.48	20.66
	1.75			40.80	−2.46	45.45	−1.27	53.60	0.70	58.91	1.88	71.81	4.35	87.64	6.57	100.49	7.65
5.00	..	38.80	−21.80														
	0.75			42.20	−3.86	43.07	1.11	44.62	9.68	45.65	15.14	48.22	27.94	51.51	42.70	54.29	53.85
	1.00			45.13	−6.79	46.78	−2.60	49.72	4.58	51.65	9.14	56.41	19.75	62.37	31.84	67.30	40.84
	1.25			48.05	−9.71	50.50	−6.32	54.83	−0.53	57.66	3.13	64.60	11.56	73.22	20.99	80.30	27.84
	1.50			50.97	−12.63	54.22	−10.04	59.93	−5.63	63.67	−2.88	72.80	3.36	84.08	10.13	93.31	14.83
	1.75			53.89	−15.55	57.93	−13.75	65.04	−10.74	69.68	−8.89	80.99	−4.83	94.94	−0.73	106.32	8.82
7.00	..	54.04	−37.04														
	0.75			55.30	−16.96	55.55	−11.37	56.05	−1.75	56.41	4.38	57.39	18.77	58.81	35.40	60.12	48.02
	1.00			58.22	−19.88	59.27	−15.09	61.16	−6.86	62.42	−1.63	65.59	10.57	69.66	24.55	73.12	35.02
	1.25			61.14	−22.80	62.99	−18.81	66.27	−11.97	68.43	−7.64	73.78	2.38	80.52	13.69	86.13	22.01
	1.50			64.06	−25.72	66.70	−22.52	71.37	−17.07	74.44	−13.65	81.97	−5.81	91.38	2.83	99.14	9.00
	1.75			66.99	−28.65	70.42	−26.24	76.48	−22.18	80.45	−19.66	90.17	−14.01	102.23	−8.02	112.14	−4.00

[a] Temperature is held constant at the over-all mean.
[b] The soilage and corn quantities are derived in the manner of those in table 29.
[c] The supplement in table 3 is fed at the rate of 0.2 of a pound per day.
[d] The feeder steer is valued at $25.00/cwt.
[e] Derived from equation 34.
[f] The total feed cost includes the cost of corn and soilage plus 28 pounds of supplement valued at $2.50/cwt.

the quantities of soilage and corn have been determined, it is possible to determine also what the profits will be for this optimum feeding period by substituting the values of soilage (F), corn (C) and time (T) into the profit equation (either equation 47 or equation 48 depending on whether or not stilbestrol has been fed in the ration).

For any given soilage-corn ration and feed-price combination, the optimum feeding period is limited by the pasture growing season, approximately 140 days. Therefore, for any given soilage-corn ration and feed-price combination, the optimum feeding period cannot exceed the pasture growing season.

The optimum feeding period for the 20:1 soilage-corn stilbestrol ration with soilage valued at $6.00 per ton and corn valued at $1.00 per bushel is a feeding period of 28 days. During this feeding period of 28 days, 2,139 pounds of soilage, 107 pounds of corn and 5.6 pounds of supplement would be fed. The profit at the end of the 28-day feeding period is predicted to be 28 cents – the maximum amount of profit that may be expected from feeding the 20:1 soilage-corn stilbestrol ration with soilage valued at $6.00 per ton and corn valued at $1.00 per bushel.

The optimum feeding period for the 20:1 soilage-corn stilbestrol ration with different feed-price assumptions could be solved in a similar manner. Moreover, the same procedure could be applied to all possible soilage-corn rations either with or without stilbestrol.

While the above procedure can be used to determine the optimum feeding period for any given soilage-corn ration and feed-price combination, it does not specify the optimum soilage-corn ration. To determine the optimum soilage-corn ration, one additional necessary condition must be added to the necessary conditions already mentioned (i.e., equations 54 and 55). This additional necessary condition is:

$$(56)\quad \frac{\partial \pi}{\partial W} = [b_1 + b_2F + b_3T + b_4T^2 + b_5FT] \\ [2a_4WF^2 + a_6F^2 + a_2F] - P_CF - \\ [-c_1\lambda F - 2c_3W\lambda F - c_5\lambda FT] = O.$$

There are now four equations (the soilage-consumption function and equations 54, 55 and 56) and four unknowns (F, T, λ and W). The solution of these equations will determine the optimum ration, the optimum feeding time and the quantity of soilage (F) that will be fed. Once the quantity of soilage and the ration (W) have been determined, the quantity of corn that will be fed is readily determined from ration equation 52.

For any given feed-price combination, the optimum ration is limited by the 2:1 soilage-corn ration. Rations of less than 2 parts soilage to 1 part corn are outside the limits of this study. Therefore, the optimum ration cannot be less than 2 parts soilage to 1 part corn, and the optimum feeding period cannot exceed the pasture growing season, which is approximately 140 days.

As one example, the optimum soilage-corn stilbestrol ration and the optimum feeding period with soilage valued at $6 per ton and corn valued at $1 per bushel is the 2:1 soilage-corn ration fed for the entire pasture season, or 140 days. The profit is predicted to be $46.45. Similarly, optimum rations can be predicted for other price relationships. The optimum soilage-corn ration and the optimum feeding period for the soilage-corn rations without stilbestrol under various feed-price assumptions would be determined in the same way as for stilbestrol rations.

APPENDIX A

The Exponential and Modified Cobb-Douglas Production Functions

In addition to the single-equation quadratic model discussed in the text, two other models were investigated in an attempt to estimate the beef-cattle production function. The first model is an exponential model involving a system of equations, and the second is a modified Cobb-Douglas function. The exponential model had the special form of a recursive system. The recursive system of equations included the production function, the ration relation, the gain relation and the consumption function.[28]

The model includes two endogenous variables (G and F) and four exogenous variables (T, R, R^2 and H) where R is the ration or ratio of corn to soilage. The reasoning behind these relations is that both the beef gains (G) and the soilage consumption (F) are experimentally determined, whereas time (T) and the ration (R) and ration squared (R^2) are predetermined variables, while temperature (H) is truly an exogenous variable. To consider autocorrelation, as with the quadratic function, the random variables were assumed to be generated by an autoregressive scheme. An empirical estimate of the autocorrelation coefficient was made in a manner similar to the procedure discussed in the text. The autocorrelation coefficient estimated was 0.57596153 with a standard error of 0.07509728. This coefficient was highly significant at the 0.001 level of probability.[29]

When the original data were transformed to logarithms, the variances between the time periods (i.e., the observation periods) were no longer homogeneous. Since the variance for the first time period (i.e., the first observation period) was approximately four times the variance of the other time periods, the first observations were weighted by dividing all the variables for the first observation period by two. This procedure tended to restore the homogeneity of the variance between time periods.

The estimated equations. The estimated gain functions for the over-all stilbestrol and nonstilbestrol rations are:

I. With stilbestrol

$$(57)\quad \log G = 0.89782288 + 0.72323783 \log T + 1.47167010R - 1.91775510R^2 - 0.00236429H$$

II. Without stilbestrol

$$(58)\quad \log G = 1.06433880 + 0.64511669 \log T + 1.2632370R - 1.69572540R^2 - 0.00188145H$$

The estimated soilage-consumption functions for the over-all stilbestrol and nonstilbestrol rations are:

I. With stilbestrol

$$(59)\quad \log F = -0.11743472 + 1.04132470 \log T - 1.03246990R + 0.54436501R^2 - 0.00002339H$$

II. Without stilbestrol

$$(60)\quad \log F = -0.20054633 + 1.08377530 \log T - 1.24860120R + 0.75196945R^2 + 0.00073851H$$

The production functions for the over-all stilbestrol and nonstilbestrol rations are:

I. With stilbestrol

$$(61)\quad G = 0.389326\, F^{0.6945363}\, e^{5.0398013R - 5.2863577R^2 - 0.0054066H}$$

II. Without stilbestrol

$$(62)\quad G = 0.623860\, F^{0.6440521}\, e^{4.8521485R - 5.1475925R^2 - 0.0054288H}$$

The coefficient of determination, standard errors and "t" values for the over-all gain and soilage-consumption functions, respectively, are presented in tables A-1 and A-2 for the stilbestrol rations and in tables A-3 and A-4 for the rations without stilbestrol.

Table A-1. Coefficient of determination, standard errors and "t" values for the over-all stilbestrol gain function (equation 57).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.9788	(constant)	0.06226914	14.418	$p<0.001$
	log T	0.02165814	33.393	$p<0.001$
	R	0.16934600	8.690	$p<0.001$
	R^2	0.42343700	4.529	$p<0.001$
	H	0.00062402	3.789	$p<0.001$

Table A-2. Coefficient of determination, standard errors and "t" values for the over-all stilbestrol soilage-consumption function (equation 59).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.9989	(constant)	0.01670231	7.031	$p<0.001$
	log T	0.00580931	179.251	$p<0.001$
	R	0.04542330	22.730	$p<0.001$
	R^2	0.11357800	4.793	$p<0.001$
	H	0.00016733	0.140	$p<0.50$

Table A-3. Coefficient of determination, standard errors and "t" values for the over-all nonstilbestrol gain function (equation 58).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.9675	(constant)	0.06564631	16.213	$p<0.001$
	log T	0.02309004	27.939	$p<0.001$
	R	0.18298530	6.904	$p<0.001$
	R^2	0.45061000	3.763	$p<0.001$
	H	0.00061368	3.066	$0.001<p<0.005$

Table A-4. Coefficient of determination, standard errors and "t" values for the over-all nonstilbestrol soilage-consumption function (equation 60).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.9980	(constant)	0.02320912	8.641	$p<0.001$
	log T	0.00816344	132.760	$p<0.001$
	R	0.06469410	19.300	$p<0.001$
	R^2	0.15931100	4.720	$p<0.001$
	H	0.00021700	3.403	$p<0.001$

[28] The procedure and logic were developed by Dr. Wayne A. Fuller of the Statistical Laboratory, Iowa State University.

[29] The "t" value for the estimated coefficient was 7.6695 with 143 degrees of freedom.

The variances, which are only approximate, have been computed.

Even though the coefficients of determination for this model of the beef-cattle production function were quite high, the model was rejected on the basis of logic. The beef-gain isoquants were sigmoid curves, denoting first increasing marginal rates of substitution between feeds and then decreasing marginal rates of substitution. However, the model merits further research.

The Modified Cobb-Douglas Function

To use the Cobb-Douglas equation to estimate the beef-cattle production function, it is desirable to modify the function to overcome its symmetrical shortcomings. Various rations, from all-soilage to various combinations of corn and forage, were fed in the experiment. The corn input, thus, is zero for the all-corn ration. If beef gains are to be estimated with the classical Cobb-Douglas function, where beef gains= g (corn, forage), gains are zero when the cattle are fed the all-forage ration. However, the function can be modified by replacing the feed-input variable corn (C) with $(C+\alpha)$.[30] This procedure also allows derivation of isoclines which do not pass through the origin (thus, lifting the restraint of the same optimum ration at all weight levels for the same soilage-corn price ratio).

[30] All of the variables (G, F, C, and H) are measured in the same manner as with the exponential function.

To consider autocorrelation, the assumption was made that the random variable, u_t, was generated by an autoregressive scheme. The autocorrelation coefficient used to transform the data was the same as the one used to transform the data in the exponential function discussed in the previous section. Similarly, the first observations were weighted in the same manner as in the exponential function, and for the same reasons.

The estimated equations. The estimated production functions for the over-all rations with and without stilbestrol are:

I. With stilbestrol

$$(63)\quad G=0.06413187F^{0.455354483}(C+400)^{0.59896829}e^{-0.00185945H}$$

II. Without stilbestrol

$$(64)\quad G=0.091115840F^{0.38061675}(C+600)^{0.62111655}e^{-0.00155832H}$$

The computed coefficient of determination for the over-all stilbestrol production function is 0.9759; the coefficient of determination for the over-all nonstilbestrol production function is 0.9631. The approximate variances of the estimated regression coefficients and the constant α may be computed. However, the standard errors and "t" values have not been computed. Even though the coefficients of determination were quite high for this model of the production function, it was rejected because it gave increasing returns to scale.

APPENDIX B

The Aggregate Production Function[31]

The aggregate production function presented in this section is based on the same statistical assumptions as the over-all stilbestrol and the over-all nonstilbestrol functions presented in an earlier section. It is derived from pooling the observations at the two experimental locations. All the variables used in the aggregate function are defined and measured in the same manner as with the over-all functions.

The estimated aggregate production function is:

$$(65)\quad G=0.13628727C+0.02193828F-0.00000819C^2-0.00000063F^2-0.00000253CF-1.75011550H$$

The coefficient of determination, standard errors and "t" values for the aggregate production function are presented in table B-1.

The beef-gain isoquant equation, as derived from the aggregate production function, is as follows:

$$(66)\quad F=17{,}411.33333-2.00793651C\pm(-793{,}650.793)\left[(0.02193828-0.00000253C)^2-0.00000252(0.13628727C-0.00000819C^2-1.75011550H-G)\right]^{1/2}$$

The equation for predicting the marginal rates of substitution of corn for soilage is:

$$(67)\quad \frac{\partial F}{\partial C}=\frac{0.13628727-0.00001638C-0.00000253F}{0.02193828-0.00000126F-0.00000253C}$$

[31] The aggregate production function has been tested against the over-all stilbestrol production function and the over-all nonstilbestrol production function to determine if there is a difference between the two over-all production functions. The following F test was used:

$$F=\frac{\dfrac{SSE_{ag\ ftn}-SSE_{os\ ftn}-SSE_{on\ ftn}}{df_{ag\ ftn}-df_{os\ ftn}-df_{on\ ftn}}}{\dfrac{SSE_{os\ ftn}+SSE_{on\ ftn}}{df_{os\ ftn}+df_{on\ ftn}}}$$

where: SSE=sum of squares for error
df=degrees of freedom
ag ftn=aggregate production function
os ftn=over-all stilbestrol production function
on ftn=over-all nonstilbestrol production function

The computed value of F with 6 and 276 degrees of freedom is:

$$F^{6}_{276}=2.3448$$

The table values for F with 6 and 276 degrees of freedom are approximately:

F^{6}_{276} equals approximately 2.13 (the 0.05 probability level)

F^{6}_{276} equals approximately 2.87 (the 0.01 probability level)

Therefore, at the 0.05 probability level there is reason to believe that there is a difference between the over-all stilbestrol and the over-all nonstilbestrol production functions. However, at the 0.01 probability level this disparity between the two functions is no longer significant.

Table B-1. Coefficient of determination, standard errors and "t" values for the aggregate production function (equation 65).

R^2	Independent variable	Standard error of regression coefficient	"t" value	Level of significance
0.9725	C	0.01144547	11.908	$p<0.001$
	F	0.00157833	13.900	$p<0.001$
	C^2	0.00000404	2.027	$0.025<p<0.05$
	F^2	0.00000011	5.727	$p<0.001$
	CF	0.00000128	1.977	$0.05<p<0.10$
	H	0.22004461	7.954	$p<0.001$

The beef-gain isoquant schedules and the marginal rates of substitution associated with them have been derived for 100, 200, 300, 350 and 400 pounds of beef gain and are presented in table B-2.

The prediction equation for estimating the quantities of corn and soilage that are required to produce various levels of gain for different soilage-corn rations is derived from the aggregate production function and the ration equation $\frac{F}{C}=\alpha$. The derived equation for predicting the quantities of corn that are required to produce various levels of gain for various soilage-corn rations is:

$$(68)\quad C=-(0.13628727+0.02193828\alpha)(-0.00001638-0.00000506\alpha-0.00000126\alpha^2)^{-1}\pm(-0.00001638-0.00000506\alpha-0.00000126\alpha^2)^{-1}[(0.13628727+0.02193828\alpha)^2-(-0.00003276-0.00001012\alpha-0.00000252\alpha^2)(-1.75011550H-G)]^{1/2}$$

Once the corn values for any given ration have been determined, the corresponding soilage values are readily determined with the ration equation $F=\alpha C$.

The predicted quantities of corn and soilage, for selected rations at various levels of gain (i.e., 100, 200, 300, 350 and 400 pounds) and the associated marginal rates of substitution of corn for soilage are presented in table B-3.

Ration lines

Total and marginal gain equations, for eight selected rations, are derived from the aggregate production function and are shown in table B-4. The estimated marginal gain values corresponding to the total gain values are presented in table B-5.

Table B-2. Isoquant schedules, derived from the aggregate quadratic function, showing posisble feed combinations[a] and marginal rates of substitution of corn for soilage at five gain levels, for 850-pound good-to-choice feeder steers (temperature is held constant at the over-all mean).

Lbs. corn	100 lbs. gain: Lbs. soilage	Ration[b]	$\frac{\partial F}{\partial C}$[c]	200 lbs. gain: Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	300 lbs. gain: Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$	400 lbs. gain: Lbs. soilage	Ration	$\frac{\partial F}{\partial C}$
0	5,394	[d]	8.10									
100	4,602	46.02	7.74	16,715	169.15	148.01						
300	3,114	10.38	7.16	11,442	38.14	15.15						
500	1,731	3.46	6.69	8,880	17.76	11.14						
700				6,852	9.79	9.32						
900				5,107	5.67	8.21						
1,100				3,545	3.22	7.44						
1,300				2,118	1.63	6.86	13,339	10.26	44.11			
1,500							9,262	6.17	13.64			
1,700							6,931	4.08	10.21			
1,900							5,066	2.67	8.59			
2,100							3,455	1.65	7.59			
2,600												
2,700												
2,800										8,985	3.21	19.16
2,900										7,418	2.56	13.32
3,000										6,217	2.07	10.96

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement in table 3. This supplement would be fed at the rate of 0.2 of a pound per day.
[b] Ration is the ratio of soilage to corn.
[c] The marginal rate of substitution of corn for soilage.
[d] The all-soilage ration.

Table B-3. Corn and soilage quantities[a] and the marginal rate of substitution along the 100, 200, 300 and 400 pound beef-gain isoquants for selected rations (temperature is held constant at the over-all mean).

Ration (ratio of soilage to corn)	100 lbs. gain: Lbs.[b] soilage	Lbs.[c] corn	$\frac{\partial F}{\partial C}$[d]	200 lbs. gain: Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	300 lbs. gain: Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$	400 lbs. gain: Lbs. soilage	Lbs. corn	$\frac{\partial F}{\partial C}$
All soilage	5,394		8.10									
20:1	3,886	194	7.45	9,286	464	11.61						
15:1	3,565	238	7.32	8,302	553	10.54						
10:1	3,067	307	7.14	6,924	692	9.37	13,067	1,307	37.74			
8:1	2,779	347	7.04	6,186	773	8.86	11,083	1,385	19.14			
5:1	2,173	435	6.83	4,731	946	8.01	8,004	1,601	11.51			
3:1	1,572	524	6.64	3,371	1,123	7.36	5,538	1,846	8.94	8,484	2,828	16.73
2:1	1,170	585	6.52	2,492	1,246	7.00	4,047	2,024	7.93	6,034	3,017	10.68

[a] For each of the feed combinations, there would also be fed a certain amount of the supplement shown in table 3. This supplement would be fed at the rate of 0.2 of a pound per day.
[b] The all-soilage value was derived from equation 66, all other values were derived from the ration equation $F=\alpha C$.
[c] Derived from equation 68.
[d] The marginal rate of substitution of corn for soilage.

Table B-4. Total and marginal gain equations, derived from aggregate production function, for selected rations for 850-pound good-to-choice feeder steers.

Ration[a]	Prediction equations for: Total gain[b]	Marginal gain[b]
Ration A All soilage	$G_A = 0.02193828\gamma_A - 0.00000063\gamma^2_A - 1.75011550H$	$\frac{\partial G_A}{\partial \gamma_A} = 0.02193828 - 0.00000126\gamma_A$
Ration B 20:1	$G_B = 0.02738347\gamma_B - 0.00000070\gamma^2_B - 1.75011550H$	$\frac{\partial G_B}{\partial \gamma_B} = 0.02738347 - 0.00000140\gamma_B$
Ration C 15:1	$G_C = 0.02908509\gamma_C - 0.00000073\gamma^2_C - 1.75011550H$	$\frac{\partial G_C}{\partial \gamma_C} = 0.02908509 - 0.00000146\gamma_C$
Ration D 10:1	$G_D = 0.03233364\gamma_D - 0.00000080\gamma^2_D - 1.75011550H$	$\frac{\partial G_D}{\partial \gamma_D} = 0.03233364 - 0.00000160\gamma_D$
Ration E 8:1	$G_E = 0.03464372\gamma_E - 0.00000085\gamma^2_E - 1.75011550H$	$\frac{\partial G_E}{\partial \gamma_E} = 0.03464372 - 0.00000170\gamma_E$
Ration F 5:1	$G_F = 0.04099645\gamma_F - 0.00000102\gamma^2_F - 1.75011550H$	$\frac{\partial G_F}{\partial \gamma_F} = 0.04099645 - 0.00000204\gamma_F$
Ration G 3:1	$G_G = 0.05052553\gamma_G - 0.00000134\gamma^2_G - 1.75011550H$	$\frac{\partial G_G}{\partial \gamma_G} = 0.05052553 - 0.00000268\gamma_G$
Ration H 2:1	$G_H = 0.06005461\gamma_H - 0.00000175\gamma^2_H - 1.75011550H$	$\frac{\partial G_H}{\partial \gamma_H} = 0.06005461 - 0.00000350\gamma_H$

[a] Ration is the ratio of soilage to corn.
[a] In each equation, γ denotes total pounds of feed of the particular ration indicated by the small capital letter following γ.

Table B-5. Estimated marginal gain from various total feed quantities of selected soilage-corn rations fed to 850-pound good-to-choice feeder steers.

Pounds of feed fed	Marginal gain[a] in pounds for selected rations:[b] All soilage	20:1	15:1	10:1	8:1	5:1	3:1	2:1
500	0.0213	0.0267	0 0284	0.0315	0.0338	0.0400	0.0492	0.0583
1,000	0.0207	0.0260	0.0276	0.0307	0 0329	0.0390	0.0478	0.0566
2,000	0.0194	0.0246	0.0261	0.0291	0.0312	0.0369	0.0452	0.0530
3,000	0.0182	0.0232	0.0247	0.0275	0.0296	0.0349	0.0425	0.0495
4,000	0.0169	0.0217	0.0232	0 0260	0.0279	0.0329	0.0398	0.0460
5,000	0.0156	0.0203	0.0217	0.0244	0.0262	0.0308	0.0371	0.0425
6,000	0.0144	0.0189	0.0203	0.0228	0.0245	0.0288	0.0344	0.0390
7,000	0.0131	0.0175	0.0188	0.0212	0.0228	0.0268	0.0318	0.0355
8,000	0.0119	0.0161	0.0173	0.0196	0.0211	0.0247	0.0291	0.0320
9,000	0.0106	0.0147	0.0159	0.0180	0.0194	0.0227	0.0264	0.0285
10,000	0.0093	0.0133	0.0144	0 0164	0.0177	0.0207	0 0237	0.0250
11,000	0.0081	0.0119	0.0129	0.0148	0.0160	0.0186	0.0210	0.0215
12,000	0 0068	0.0105	0.0115	0.0132	0.0143	0.0166	0.0184	0.0180
13,000	0.0056	0.0091	0.0100	0.0116	0.0126	0.0146	0.0157	0.0145
14,000	0.0043	0.0077	0.0085	0.0100	0.0109	0.0125		
15,000	0.0030	0.0062	0.0071	0.0084				
16,000	0 0018	0.0048						
17,000	0.0005							

[a] All values are derived from the equations in table B-4.
[b] The ration is the ratio of soilage to corn.