Distributed Lag Inventory Analyses

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CONTENTS

SUMMARY

The objective of this study was to measure determinants of short-term inventory behavior for selected commodities: beef, pork, butter, cheese, department store stocks, manufacturers' nondurable inventories and manufacturers' durable inventories. The last two were studied using monthly data; the others, with quarterly data.

Dynamic considerations must enter into any adequate explanation of inventories. Distributed lag models were used in this study because they are one reasonable way of treating such dynamic phenomenon as expectations, frictions and lags. Such models are useful for study of inventory behavior. Nevertheless, there are some problems in using them. They commonly lead to equations to be estimated which are nonlinear in the parameters. Reduced equations containing exactly the same variables but different nonlinear combinations of parameters may be obtained from different models containing different behavioral assumptions. Whenever linear estimation is used, as in this study, we must be cautious about placing specific behavioral interpretations on the resulting coefficients.

Important determinants of end-of-quarter beef inventories are lagged inventories and current changes in farm production of beef and pork. Beef inventories are more responsive to changes in farm marketings in the fourth quarter than in other quarters. Pork inventories are affected by lagged inventories, changes in farm pork production and changes in farrowings. Pork inventories are less responsive to changes in farm marketings during the second quarter than during other quarters. Both beef and pork inventories undergo autonomous seasonal variation. This is seasonal variation which is not explained by economic variables (such as prices and sales) but is measured by seasonal shift variables included in the equations. Beef and pork inventories are not affected by sales level, changes in sales or price changes. Pork inventory behavior underwent a change in early 1952. Pork inventories were affected by price ceilings which were in effect during the Korean War.

Significant determinants of quarterly butter and cheese inventories during 1929-41 were lagged inventories, current and lagged changes in farm milk production, lagged butter and cheese wholesale price changes and current sales. Current changes in sales affected cheese inventories but not butter inventories. Current price changes were not significant. Butter and cheese stocks underwent no autonomous seasonal variation. Both were less responsive to expected price changes during the second quarter of the year, when milk production normally reached a seasonal peak.

Quarterly department store stocks are affected by lagged stocks, current level of sales and change in level of sales. End-of-quarter inventories are most responsive to current sales conditions during the third quarter and least responsive during the first and fourth quarters. These stocks do undergo autonomous seasonal variation.

Monthly manufacturers' nondurable inventories: (1) are affected by lagged inventories, level of sales and changes in the level of sales and volume of unfilled orders; (2) undergo autonomous seasonal variation and (3) do not appear to be affected by changes in input prices or by the volume of new orders for nondurables.

Monthly manufacturers' durable inventories: (1) are affected by lagged inventories, level of sales and changes in the level of sales, changes in volume of unfilled orders or volume of new orders and changes in input prices and (2) do undergo autonomous seasonal variation.

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Inventories have attracted a great deal of study among economists. It is generally agreed that inventories play an important role in causing or accentuating cyclical fluctuations in the economy (1, pp. 6,7). The understanding and prediction of inventory behavior is, therefore, useful in planning public fiscal and monetary policy. The ability to predict inventory behavior is helpful to private businessmen in planning for future periods. Predictions of inventory investment can be used to determine business demand for raw materials, supplies and semifinished items. A knowledge of inventory behavior is also useful in predicting employment, consumer income and consumer demand. Agricultural marketing firms can use inventory predictions in determining prices to pay or prices to charge. Knowledge of inventory behavior can be used in determining the short-term outlook for agricultural prices and income.

The objectives of this study are to find and measure the effects of significant determinants of inventory investment for various products. Stocks of butter, cheese, beef and pork and department store inventories are analyzed using quarter-year data. Manufacturers' durable goods and nondurable goods inventories are studied using monthly data.

DISTRIBUTED LAG MODELS

For a number of reasons, all having to do with economic dynamics, distributed lag models seem well suited to the study of short-term inventory behavior.

Here a distributed lag model is taken to mean a model, designed to depict behavior of economic agents, in which the equation to be estimated contains one or more lagged values of the dependent variable among the independent variables. Other independent variables may appear in either their current or lagged values.

In the analysis of short-term inventory behavior, distributed lags may arise from various sources. Inventories are a bridge between the present and the future. They are held at one date to be disposed of at a later date. Hence, expectations of future conditions must play a role in determining present levels of inventories. Since inventories are ultimately intended to be used or sold, expectations as to future sales levels or future prices may be relevant.

A reasonable model might be

$$(1.1) \quad i_t = a\, s^*_{t+1}$$

where s^*_{t+1} = value of sales expected next period; i.e., the expectation formed this period as to the value of s_{t+1}. The parameter a is then the actual inventory-expected sales ratio.

Since s^*_{t+1} is not an observable variable, it is necessary to make some assumption about how s^*_{t+1} is determined. Nerlove has suggested one model for generating price expectations (12, p. 53). Applying his model to the generation of sales expectations,

$$(1.2a) \quad s^*_{t+1} - s^*_t = \beta(s_t - s^*_t)$$

If $0<\beta<2$, equation 1.2a is equivalent to stating expected sales as a weighted average of current and past sales,

$$(1.2b) \quad s^*_{t+1} = \beta \sum_{i=0}^{n} (1-\beta)^i s_{t-i}$$

If $0<\beta<1$, equation 1.2a says: Next period's expectation will be determined by adding to this period's expectation, some fraction of the amount by which actual current sales exceed expected current sales. If $1<\beta<2$, the amount added to s^*_t will be greater than the excess of s_t over s^*_t. (This corresponds to an assumption of cyclical sales expectations.) Equation 1.2a cannot be estimated since expected values are assumed to be nonobservable.

Equations 1.1 and 1.2a can be solved to obtain the reduced or estimation equation,

[1] Project 1355 of the Iowa Agricultural and Home Economics Experiment Station. This research was partially supported by a grant from the National Science Foundation.

$$(1.3) \quad i_t = a\beta s_t + (1-\beta)\, i_{t-1}$$

This contains a distributed lag because of the nature of the assumption concerning the generation of s^*_{t+1}. This is the same as Goodwin's flexible accelerator (7); also see Lovell (10). This model suggests the accelerator to represent an expectational mechanism. This seems to be a logical interpretation of the inventory accelerator. If sales rise during period t, why desire to hold a larger inventory at the end of period t (i.e., at the end of the sales increase) unless sales are expected to remain high or to continue rising?

The speculative motive may also be operating in determining desired levels of inventories. In this event, anticipated profits are maximized when marginal storage costs equal anticipated price rise. To generate expected prices, one might assume a model of the type represented by equation 1.2a. For agricultural commodities, expected levels of farm production may be relevant to the determination of expected prices.

The simple procedure is to assume that actual inventories at any point in time, i_t, are equal to the level of inventories businessmen desire to be holding at that time; i.e., to i'_t, the equilibrium level of inventories. In some recent studies, it has been explicitly assumed that observed inventories are not equal to desired inventories (4, pp. 795-800; 10). Nerlove has argued that, in general, it is unwarranted to assume observed values to be equal to equilibrium values (11, pp. 5-7, 15-16; 12, p. 24). If one assumes that they are not equal, it also is necessary to make some assumption about the relation between observed and equilibrium values.

If one would not feel safe in assuming $i'_t = i_t$, equation 1.1 might be replaced by

$$(1.4) \quad i'_t = a\, s^*_{t+1}$$

Here a is the desired inventory-expected sales ratio. For generating s^*_{t+1}, Duesenberry et al. have proposed (4, p. 796)

$$(1.5) \quad s^*_{t+1} = s_t$$

Various people have proposed the relation 1.6 between actual and desired levels of a variable

$$(1.6) \quad \Delta i_t = \beta(i'_t - i_{t-1})$$

The model consisting of equations 1.4, 1.5 and 1.6 reduces to

$$(1.7) \quad i_t = a\beta s_t + \beta i_{t-1}.$$

This contains a distributed lag because of the nature of the assumed relation between i_t and i'_t.

Among the reasons for assuming $i'_t \neq i_t$ and equation 1.6 are these: (a) The level of s^*_{t+1} is not known or expected with certainty; the businessman may have much more confidence in his prediction of direction of change of expected sales or of desired inventory than in his prediction of amount of change. He may, therefore, decide to adjust actual inventories by some fraction of the desired change in inventories, as represented by equation 1.6 if $0<\beta<1$. (b) Institutional factors are relevant. It may be necessary to adjust actual inventories by less than the desired amount because of high costs encountered in large or rapid changes in the level of production. Lags between the time at which decisions are reached and the time at which they can be carried out may also result in partial adjustments. (c) Logistical considerations enter into the picture. It may be impossible to obtain additional raw materials or supplies quickly enough to permit increasing actual inventories to the desired level at the desired time. Similarly it may be impossible or undesirable to sell off excess inventories as rapidly as they become excessive.

Equations 1.3 and 1.7 contain exactly the same variables, but the parameters composing their coefficients have quite different behavioral interpretations. This illustrates, in simple form, two problems encountered in the use of distributed lag models: (a) the reduced equations may be nonlinear in the parameters (e.g., the coefficient of s_t in equation 1.3 is the product of a and β); (b) different models lead to equations containing exactly the same variables, but the coefficients are composed of different parameter combinations.

The nonlinearity is not a serious problem here, since the coefficient of i_{t-1} may be divided into the coefficient of s_t to obtain an estimate of a. And a confidence interval for a can be obtained without great difficulty. But consider this model (used in the study of butter and cheese inventories):

$$(1.8) \quad i'_t = as^*_{t+1} + b\Delta p^*_{t+1} = as^*_{t+1} + b(p^*_{t+1} - p_t)$$

$$(1.9) \quad s^*_{t+1} - s^*_t = \beta_0(s_t - s^*_t)$$

$$(1.10) \quad \Delta p^*_{t+1} = \beta_1 \Delta p_t + \beta_2 \Delta F^*_{t+1}$$

$$(1.11) \quad \Delta F^*_{t+1} = \beta_3 \Delta F_t$$

$$(1.12) \quad i_t - i_{t-1} = c\,(i'_t - i_{t-1})$$

F_t represents current farm milk production, and p_t represents wholesale butter or cheese price. These equations can be reduced to

$$(1.13) \quad \begin{aligned} i_t = {} & a\beta_0 cs_t + b\beta_1 c\Delta p_t - b\beta_1 \\ & c(1-\beta_0)\Delta p_{t-1} + b\beta_2\beta_3 c\Delta F_t \\ & - b(1-\beta_0)\beta_2\beta_3 c\Delta F_{t-1} \\ & + (2-\beta_0-c)i_{t-1} \\ & - (1-\beta_0)(1-c)i_{t-2} \end{aligned}$$

The seven coefficients in equation 1.13 might be estimated by least squares. But it will not be possible to obtain unique estimates of the seven parameters in equations 1.8 to 1.12. One estimate of β_0 can be obtained from the ratio of the coefficients of Δp_{t-1} and Δp_t; another, from ΔF_{t-1} and ΔF_t. There is no need for them to be equal. Using each estimate of β_0, the coefficients of i_{t-1} and i_{t-2} can be used to obtain two estimates of c, for a total of four estimates. Estimates of a can then be obtained from the coefficient of s_t. On the other hand, it is not possible to compute estimates of b, β_1, β_2 and β_3. In another model used for the study of butter and cheese inventories, there are nine parameters, but the reduced equation has only seven coefficients.

Even if unique estimates of the parameters in the model can be obtained from the estimated coefficients, it may be virtually impossible to compute measures of reliability.

All this suggests the desirability of using some form of nonlinear estimation on equations such as 1.13, to permit computing unique estimates of each parameter and measures of reliability (6,8).

In this study, all reduced equations were estimated by least squares; t ratios were computed for each coefficient, and F ratios frequently were computed to test the significance of the contribution of added variables to the coefficients of determination. Usually, results will be interpreted as though there were a simple and unique one-to-one correspondence between estimated coefficient and initial hypothesis: that is, as though the coefficients were linear.

Such a procedure can be justified on various grounds. One is economy; nonlinear least squares estimates of equation 1.13 will cost from 5 to 15 or 20 or more times as much as linear estimates. Nonlinear estimates are justifiable, therefore, only if they have some superiority over linear estimates. It is difficult to see where this superiority lies.

Application of nonlinear estimation to equation 1.13, for example, would be an effort to test hypotheses that a, b, β_0, β_1, β_2, β_3 and c equal zero. That is, it would be an effort to test various hypotheses concerning adjustment of actual inventory levels to desired inventory levels, price and sales expectations and determination of the level of desired inventories. Application of linear least squares to equation 1.13 can be interpreted as an effort to test hypotheses concerning the effect of various variables on actual inventories.

Conceivably, the linear estimates of the coefficients of ΔF_t and ΔF_{t-1} could be nonsignificant, and their addition could result in a negligible increase in the value of R^2, while nonlinear estimates of b, β_2, β_3, c and $(1-\beta_0)$ were significant. Even so, if all five were less than unity in absolute value, the nonlinear estimation might lead to the conclusion that the products were zero. One might then conclude that ΔF^*_{t+1} does enter into the butter and cheese inventory decision process, but in such a way that it has no significant effect on final inventories. It does not seem useful to be able to identify those variables which enter into the decision process but have negligible effect on the final outcome.

It seems more reasonable to argue that such a situation will not exist. Businessmen would soon observe that this was so and identify the variables. A desire (and necessity) for economy of effort in decision-making and implementation would lead to the deletion of these variables from the decision process.

If this is not true and if variables which have no measureable effect on the final outcome do enter into decision processes, it raises a question as to the usefulness of studies of decision-making or of decision processes.

A decision to apply nonlinear estimation raises another question. Different models containing different assumptions about behavioral patterns and decision processes will sometimes lead to reduced equations containing exactly the same variables. Equations 1.3 and 1.7 are one example. These even contain the same parameters, a and β, but a and β have different meanings in the two models. If nonlinear estimation leads to the conclusions $a \neq 0$ and $\beta \neq 0$, which model does one accept—1.1 and 1.2a or 1.4, 1.5 and 1.6?

Sometimes the reduced equation from two models will contain exactly the same variables, but their coefficients will consist of different combinations of parameters having different interpretations. Consider the model consisting of

$$(1.8b) \quad i'_t = a\, s^*_{t+1} + b\Delta p^*_{t+1} + c\Delta F^*_{t+1}$$

$$(1.9b) \quad s^*_{t+1} = s_t$$

$$(1.10b) \quad \Delta p^*_{t+1} = \beta_0 \Delta p_t + \beta_1 \Delta p_{t-1} + \beta_2 \Delta F^*_{t+1}$$

$$(1.11b) \quad \Delta F^*_{t+1} = \alpha_0 \Delta F_t + \alpha_1 \Delta F_{t-1}$$

$$(1.12b) \quad \Delta i_t = c_0 (i'_t - i_{t-1}) + c_1 (i_{t-1} - i_{t-2}).$$

The reduced equation for this model is

$$\begin{aligned}(1.13a) \quad i_t = {} & a c_0 s_t + b\beta_0 c_0 \Delta p_t + b\beta_1 c_0 \Delta p_{t-1} \\ & + (b\beta_2 + c)\, \alpha_0 c_0 \Delta F_t \\ & + (b\beta_2 + c)\, \alpha_1 c_0 \Delta F_{t-1} \\ & + (1 + c_1 - c_0) i_{t-1} - c_1 i_{t-2}.\end{aligned}$$

On a priori grounds, this model is just as reasonable as equations 1.8 through 1.12. Then if one is to estimate equation 1.13 by nonlinear least squares, he ought also to estimate equation 1.13a by nonlinear least squares. And suppose his t tests indicate equal proportions of significant coefficients in the two models and the two estimates yield equal values of R^2. If they do not, then the choice as to the more relevant model may be clear. The clear-cut decision reached, however, will have cost 10 to 30 or 40 times as much as it would have cost to obtain linear estimates.

The preceding discussion raises two questions about studies of decision-making or decision processes. These studies assume that the nature of the process followed in reaching a decision determines the decision reached and the action taken. So far as I know, this basic assumption has never been subjected to empirical test. The preceding discussion indicates the desirability of testing this assumption and differentiating those conditions under which it holds true from those under which it does not. There are possible situations in which some elements of the decision process have no effect on the final outcome. There are also situations in which the same result is attained from different decision processes.

The whole discussion constitutes a warning note against placing great confidence in the interpretation of coefficients in distributed lag models. Distributed lag concepts are a fruitful source of hypotheses concerning dynamic behavior; the trouble is that they are too fruitful.

CHOICE OF DEPENDENT VARIABLE

In equations such as 1.7 or 1.13, where i_{t-1} is one of the independent variables, it makes little difference whether i_t or Δi_t is used as the dependent variable. Specifically: (a) The estimated coefficients of i_{t-1} will differ by unity, (b) all other coefficients will be the same, (c) all standard errors will be the same and (d) the R^2 obtained from the use of Δi_t will usually be smaller, but the value of F will be the same in both cases.

Suppose the model to be estimated is, in matrix notation

$$(1.14) \quad y_t = (X \; y_{t-1}) \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} + \varepsilon = Z\beta + \varepsilon$$

where y_t and y_{t-1} are T x 1 column vectors of observations, X is a T x m matrix, β_0 is an m x 1 column vector of coefficients, β_1 is a scaler and ε is a T x 1 column vector of random disturbances. Writing $Z'Z$ as a partitioned matrix and using a method for calculating the inverse of a partitioned matrix (27), the least squares estimates are

$$(1.15) \quad \begin{pmatrix} b_0 \\ b_1 \end{pmatrix} = \begin{pmatrix} (X'X)^{-1} X'y_t - C(\mu'\mu)^{-1} \mu' y_t \\ (\mu'\mu)^{-1} \mu' y_t \end{pmatrix}$$

where

$$(1.16) \quad C = (X'X)^{-1} X' y_{t-1}$$

and

$$(1.17) \quad \mu = y_{t-1} - XC$$

Now suppose we estimate

$$(1.18) \quad \Delta y_t = (X \; y_{t-1}) \begin{pmatrix} \beta_{\Delta 0} \\ \beta_{\Delta 1} \end{pmatrix} + \varepsilon_\Delta = Z\beta_\Delta + \varepsilon_\Delta$$

Substituting Δy_t into equation 1.15 in place of y_t, it can be seen that

$$(1.19) \quad \begin{pmatrix} b_{\Delta 0} \\ b_{\Delta 1} \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 - 1 \end{pmatrix}$$

The coefficients of y_{t-1} differ by unity; all other coefficients are equal.

To show that all standard errors and the F ratios will be equal, it is necessary to show the equality of $e'e$ and $e'_\Delta e_\Delta$.

$$(1.20) \quad e = y_t - Zb = y_t - Xb_0 - y_{t-1}b_1$$

$$(1.21) \quad e_\Delta = y_t - y_{t-1} - Xb_0 - y_{t-1}(b_1 - 1) = e$$

Since $e = e_\Delta$, $e'e = e'_\Delta e_\Delta$ and the variances and covariances of the coefficients are equal.

The F ratio for testing the significance of regression is

$$(1.22) \quad F = \frac{\begin{pmatrix} b_0 - \beta_0 \\ b_1 - \beta_1 \end{pmatrix}' Z'Z \begin{pmatrix} b_0 - \beta_0 \\ b_1 - \beta_1 \end{pmatrix}}{e'e\,(T - m - 1)}$$

$F_\Delta = F$ since $b_1 - \beta_1 = b_1 - 1 - (\beta_1 - 1)$ and $e'e = e'_\Delta e_\Delta$.

DATA FORM

A number of studies of inventories, and of other aspects of short-term economic behavior, have used seasonally adjusted data. In this study unadjusted data were used in a model allowing for seasonal shifts in the intercepts,

$$(1.23) \quad y_t = \beta_0 + \sum_i \beta_i x_{it} + \sum_i c_i D_{it} + \mu_t.$$

The values assigned to the D_{it} are presented in tables 1 and 2.

It is also possible that seasonal rotations occur in the inventory equations. To check this possibility, residuals from fitted equations were plotted against the various independent variables. When monthly or quarterly differences appeared

Table 1. Quarterly seasonal variables.

Quarter	D_1	D_2	D_3
1	−1	0	1
2	−1	0	−1
3	1	−1	0
4	1	1	0

(Variable: D_1, D_2, D_3)

Table 2. Monthly seasonal variables.[a]

$D_1 = \sin iR$	$D_7 = \sin 4iR$
$D_2 = \cos iR$	$D_8 = \cos 4iR$
$D_3 = \sin 2iR$	$D_9 = \sin 5iR$
$D_4 = \cos 2iR$	$D_{10} = \cos 5iR$
$D_5 = \sin 3iR$	$D_{11} = \cos 6iR$
$D_6 = \cos 3iR$	

[a] i ranges from 1 in January to 12 in December each year. $R = 30^\circ$. Thus, each year, D_3 varies from sin 60° in January to sin 360° in June to sin 720° in December. sin $6iR = 0$ every month.

in the slopes, additional variables were added to the regressions.

Equation 1.23 assumes that the slopes remain constant from season to season but that intercepts may change. In contrast, the use of seasonally adjusted data implies the assumption that both intercepts and slopes vary seasonally, and that they vary in a certain way. Write the model as

$$(1.24) \quad y_t = \beta_{0t} + \sum_i \beta_{it} x_{it} + \mu_t.$$

Use s_t to denote the seasonal index for y_t and r_{it} to denote the index for x_{it}.
Divide equation 1.24 by s_t

$$(1.25) \quad y_t/s_t = \beta_{0t}/s_t + \sum_i (\beta_{it}/s_t) x_{it} + \mu_t/s_t$$

The equation actually estimated with seasonally adjusted data is

$$(1.26) \quad y_t/s_t = \beta_{0t}/s_t + \sum_i (\beta_{it}\, r_{it}/s_t)\, x_{it}/r_{it} + \mu_t/s_t$$

The use of seasonally adjusted data implies the assumptions

$$(1.27) \quad \beta_{0s} = \beta_{0t}/s_t$$

$$(1.28) \quad \beta_{is} = \beta_{it}\, r_{it}/s_t$$

$$(1.29) \quad \mu_{st} = \mu_t/s_t.$$

β_{0s} and β_{is} are the parameters estimated by use of seasonally adjusted data.

According to equation 1.27, the intercepts vary seasonally in a certain way: $\beta_{0t} = \beta_{0s} s_t$. According to assumption 1.28, the slopes also vary seasonally in a certain way: $\beta_{it} = \beta_{is}\, s_t/r_{it}$. Although intercept and slopes may vary seasonally, there is no reason they should happen to vary in these particular ways. If s_t or r_{it} varies cyclically or secularly (i.e., changing seasonals), the use of seasonally adjusted data then implies cyclical or secular variations of certain types in the parameters.

It is well known that the process of taking a moving average can introduce auto- and serial-correlation into random time series (14, pp. 203-205). Dividing a random series by a moving average can introduce autocorrelation into the derived series. If the original disturbances are temporally random, the disturbances in the seasonally adjusted equation may be autocorrelated. This will only bias the t and F tests if no lagged values of y_t appear among the x_{it}. If lagged values of y_t do appear among the x_{it}, biased coefficients will result. If the original disturbances are autocorrelated, there is no reason that dividing by s_t should remove the autocorrelation.

ESTIMATION PROCEDURE

At least three separate statistical considerations enter into the choice of the estimation procedure in this study: (1) the previously mentioned nonlinearity in the parameters, (2) the possibility of autocorrelated disturbances in the equations (which introduces additional nonlinearities) and (3) possible simultaneous determination of inventories with other endogenous variables. A general estimation procedure would have to be a highly nonlinear simultaneous equations procedure. Such a procedure could possibly be developed by synthesizing two-stage-least-squares with procedures discussed by Hartley (8) and Fuller and Martin (6).

The reasons for ignoring the first complication have been discussed. As for the second, it is likely that the hypothesis of autocorrelated errors will usually be rejected in equations which make generous use of lagged dependent variables, as do the ones used here (11, 12). This hypothesis can be tested by use of the Durbin-Watson d statistic. It was not used in this study since it is not appropriate when lagged values of the dependent variable appear as independent variables. In a few cases in which it was tried, the maximum and minimum values of d were 2.12 and 1.92.

In the models used here, the variables relevant to the third problem are current values of sales, prices, new or unfilled orders and farm marketings and current first differences of these variables. It is not an oversimplification of reality to argue that quarterly farm marketings can be treated as predetermined. It seems justifiable to treat monthly and perhaps quarterly price changes as predetermined; i.e., to argue that they represent current dynamic response to previous changes or to current changes in predetermined variables.

It is questionable whether sales, new orders and unfilled orders can reasonably be treated as predetermined or exogenous. This question merits further investigation.

In the studies reported here, the values of R^2 are sufficiently large that bias arising from simultaneous determination is generally small.

QUARTERLY BEEF AND PORK INVENTORIES

Variables

i_t = Cold storage holdings of all frozen and cured meat, end-of-quarter t, in millions of pounds (18, 20).

i'_t = Equilibrium (i.e., desired) level of end-of-period stocks. Throughout this report, this symbol will have the same meaning.

Q_t = Commercial meat production during quarter t, millions of pounds (18, 20).

Q^*_t = Expectation during period t—1 of the value of Q_t.

s_t = sales during quarter t, millions of pounds (derived as $Q_t + i_{t-1} - i_t$).

s^*_t = Predicted value during period t—1 of the value s_t. Throughout this report x^*_{t+j} will mean the expectation formed during period t+j—1 as to the value of x_{t+j}.

p_{Pt} = Average wholesale value at Chicago of 100 pounds of major pork cuts, dollars (19, 21).

p_{Bt} = Average wholesale value of 100 pounds U.S. Choice grade beef carcass, dollars (16, 19).

ΔF_t = 0 in second and third quarters of each year.

= previous spring pig crop minus previous fall pig crop, tens of thousands of pigs saved, in fourth quarter of year and first quarter of next year (18, 20).

Most of the other variables are derivatives of these—either lagged values or first differences.

Beef and pork are indicated by the subscripts B and P.

$\Delta Q'_{Bt}$ = ΔQ_{Bt} during first, second and third quarters
= $-3\Delta Q_{Bt}$ in fourth quarter

$\Delta Q'_{Pt}$ = $-2\Delta Q_{Pt}$ during second quarter
= ΔQ_{Pt} in first, third and fourth quarters

D_{Pt} = 0 each quarter 1949-III through 1952-I,
= 1 each quarter after 1952-I

G_t = 1 each quarter 1951-II through 1953-I
= 0 all other quarters

$\Delta Q'_{Bt}$ and $\Delta Q'_{Pt}$ were added to the regressions after inspection of residuals. D_{Pt} was included at the suggestion of Wilbur Maki, who had previously noted in his work a rather sharp break in the pattern of pork inventories in late 1951 or early 1952. He attributes this to the adoption of a new method of curing certain pork products which shortens the time required for curing. The end of the first quarter of 1952 was selected as the break point after inspection of residuals from equations not including D_{Pt}.

From May 1951 to February 1953, ceilings were in effect on beef prices. Wholesale ceiling prices were in effect on pork from October 1951 to February 1953. G_t was included to allow for the possible effect of these ceilings on cold storage holdings of meat.

Models

Two principal models were used to study beef and pork inventories. In each case, the pork model will be presented and its differences from the beef model will be discussed.

Model A

For discussion of this model, see Fuller and Ladd (5).

(2.1) $i'_{Pt} = a\Delta p^*_{Pt+1} = a(p^*_{Pt+1} - p_{Pt})$

(2.2) $\Delta p^*_{Pt+1} = b\Delta Q^*_{Pt+1} + e\Delta Q^*_{Bt+1} = b(Q^*_{Pt+1} - Q_{Pt}) + e(Q^*_{Bt+1} - Q_{Bt})$

(2.3) $\Delta Q^*_{Pt+1} = \beta_0(\Delta Q^*_{Pt} - \Delta Q_{Pt}) + \beta_1\Delta F_t$

(2.4) $\Delta Q^*_{Bt+1} = \beta_0(\Delta Q^*_{Bt} - \Delta Q_{Bt})$

(2.5) $\Delta i_{Pt} = c(i'_{Pt} - i_{Pt-1})$

These five equations can be reduced to the one equation to be estimated

(2.6) $\Delta i_{Pt} = (\beta_0 - c)i_{Pt-1} + \beta_0(c-1)i_{Pt-2} - abc\beta_0\Delta Q_{Pt} - aec\beta_0\Delta Q_{Bt} + abc\beta_1\Delta F_t$

The quantity change variables (ΔQ^*_t, ΔQ_t, etc.) are interpreted as referring to farm marketings. Livestock sold by farmers is processed (i.e., commercially produced) within a few days after being sold. Over a period of a quarter, commercial production can be taken as equal to farm marketings, ΔQ^*_{t+1}, therefore, represents expected change in farm supplies.

Because of the large seasonal variation in meat production, large quantities of meat are stored in anticipation of seasonal price rises. Tolley and Harrell found that packers were fairly successful in predicting changes in supply but not in predicting changes in demand and that packers used the U.S. Department of Agriculture pig crop reports in making decisions on size of stocks (15). Equation 2.1 states desired level of stocks as a function of the expected price change. Expected price change, in turn, is hypothesized to be a function of expected changes in both beef and pork production. The expected production of each meat is stated as a weighted average of past levels of production. The equation for expected pork production also includes the change in farrowings during the fourth and first quarters, when pork inventories typically increase. The estimated equations contain the three quarterly variables defined earlier. Including these in equation 2.6 is equivalent to including variables which reflect normal seasonal variation in farm production in the equations for expected production. Because of uncertainty as to the magnitude of price and production changes and possible institutional limitations on speed of adjustment, actual inventory change is assumed to be a positive fraction of desired change.

Model B

(2.9) $i'_{Pt} = a\,\Delta p^*_{Pt+1} + bs^*_{Pt+1}$

$$(2.10)\quad s^*_{Pt+1} = e_0 s_{Pt-1} + e_1 \Delta s_{Pt}$$

$$(2.11)\quad \Delta p^*_{Pt+1} = \alpha_0 \Delta Q^*_{Pt+1} + \alpha_1 \Delta Q^*_{Bt+1} + \beta_0 \Delta p_{Pt} + \beta_1 \Delta p_{Pt-1}$$

$$(2.12)\quad \Delta Q^*_{Pt+1} = \varepsilon_0 \Delta Q_{Pt} + \varepsilon_1 \Delta Q_{Pt-1} + \varepsilon_2 \Delta F_t$$

$$(2.13)\quad \Delta Q^*_{Bt+1} = \varepsilon_3 \Delta Q_{Bt} + \varepsilon_4 \Delta Q_{Bt-1}$$

$$(2.14)\quad \Delta i_{Pt} = c_0 (i'_{Pt} - i_{Pt-1}) + c_1 \Delta i_{Pt-1} + c_2 (s_{Pt} - s^*_{Pt})$$

Some time, no doubt, elapses between the beginning of a quarter and the time when decisions are made and communicated to all relevant agents as to the desired level of end-of-quarter inventories. In the interim, it is possible that inventory continues to be accumulated at the same rate as last quarter ($c_1 = 1$), or at some fraction of that rate. Alternatively, action in the interim may be guided by $i'_{t-1} - i_{t-2}$; i.e., by desired change in end-of-quarter stocks last quarter. This second alternative is not likely to be useful here since the reduced equation contains 15 independent variables, including the seasonal variables. Using i'_{Pt-1} would add four more variables: Δp_{Pt-2}, ΔQ_{Bt-2}, ΔQ_{Pt-2} and ΔF_{t-1}. Considering multicollinearity problems alone, it seems unlikely that the inclusion of these variables would add anything to the explanatory or predictive value of the equation. (As it is, singular matrices were encountered with several of the beef inventory equations.)

The inclusion of $s_{Pt} - s^*_{Pt}$ arises from the hypothesis: (a) Planned change in inventory is a fraction of the desired change and (b) actual change differs from planned change by a fraction of the excess of actual sales over expected sales (4, p. 796).

These equations can be reduced to the following equation to be estimated.

$$\begin{aligned}(2.15)\quad \Delta i_{Pt} = {} & ac_0\beta_0 \Delta p_{Pt} + ac_0\beta_1 \Delta p_{Pt-1} \\ & + ac_0\alpha_0\varepsilon_0 \Delta Q_{Pt} + ac_0\alpha_1\varepsilon_3 \Delta Q_{Bt} \\ & + ac_0\alpha_0\varepsilon_1 \Delta Q_{Pt-1} + ac_0\alpha_1\varepsilon_4 \Delta Q_{Bt-1} \\ & + ac_0\alpha_0\varepsilon_2 \Delta F_t \\ & + (bc_0e_0 + c_2 - c_2e_0) s_{Pt-1} \\ & + (bc_0e_1 + c_2) \Delta s_{Pt} \\ & + c_2(e_0 - e_1) \Delta s_{Pt-1} + (c_1 - c_0) i_{Pt-1} \\ & - c_1 i_{Pt-2}\end{aligned}$$

Before estimating Model B, ΔQ_{Pt-1} was dropped from the beef equation, and ΔQ_{Bt-1} was dropped from the pork equation. Model B was also fitted under the assumption $b = c_2 = 0$. This will be referred to as Model B.1; the more general model, as B.2.

Results

Results are presented in tables 3 and 4. In each table, the first three equations represent Models A, B.1 and B.2, respectively. In these and all later tables, coefficients for each equa-

Table 3. Selected statistical results from quarterly beef stocks regressions, 1949-III to 1960-IV.[a]

	i_{Bt-1}	i_{Bt-2}	ΔQ_{Bt}	ΔQ_{Pt}	ΔQ_{Bt-1}	$\Delta Q'_{Bt}$	s_{Bt-1}	Δs_{Bt-1}	Δp_{Bt}	Δp_{Bt-1}	D_{1t}	D_{2t}	D_{3t}	1	R^2
1	−0.021	−0.269	0.136	0.055		−0.041					10	18	−3	42	0.9211
	0.102	0.102**	0.022***	0.016***		0.012***					6	8**	6		
2	0.107	−0.284	0.155	0.052	−0.009	−0.027			0.013	0.021	11	23	−2	24	0.9304
	0.146	0.121**	0.029***	0.017***	0.032	0.014*			0.014	0.013	7	9**	7		
3	0.082	−0.275	0.156	0.050		−0.028	0.001	0.005	0.013	0.024	9	23	−2	23	0.9304
	0.138	0.119**	0.030***	0.017***		0.014*	0.005	0.036	0.014	0.014*	7	9**	7		
4		−0.286	0.136	0.056		−0.041					10	17	−4	41	0.9210
		0.064***	0.022***	0.016***		0.012***					6	7***	4		

[a] Dependent variable = Δi_{Bt}.

Table 4. Selected results from quarterly pork stock regressions, 1949-III to 1960-IV.[a]

	i_{Pt-1}	i_{Pt-2}	ΔQ_{Bt}	ΔQ_{Pt}	ΔQ_{Pt-1}	s_{Pt-1}	Δs_{Pt}	Δs_{Pt-1}	Δp_{Pt}	Δp_{Pt-1}	$\Delta Q'_{Pt}$	ΔF_t	D_{1t}	D_{2t}	D_{3t}	D_{Pt}	G_t	1	R^2
1	0.124	−0.317	−0.045	0.277							0.056	0.093	−66	−49	−7			−31	0.9660
	0.094	0.082***	0.053	0.042***							0.024**	0.028***	15***	26*	29				
2	0.064	−0.292	−0.011	0.313	0.077				0.041	0.024	0.084	0.061	−64	−46	5	−33		25	0.9755
	0.147	0.151*	0.052	0.061***	0.070				0.026	0.022	0.023***	0.028**	14***	25*	33	14**			
3	0.077	−0.294	0.091	0.227	0.110	0.009	0.101	−0.035	0.036	0.019	0.061	0.053	−77	−45	−5	−38		13	0.9818
	0.148	0.147*	0.078	0.092**	0.072	0.024	0.068	0.044	0.025	0.020	0.023**	0.027*	14***	24*	33	14**			
4		−0.382		0.233							0.088	0.078	−60	−43	14	−44	41	67	0.9767
		0.055***		0.029***							0.021***	0.020***	12***	18**	22	11***	15**		

[a] Dependent variable = Δi_{Pt}.

tion are presented on the top line, and standard errors are on the line underneath. An * indicates significance at the 10-percent level; ** indicates significance at the 5-percent level; *** indicates significance at the 1-percent level.

Dichotomous variables

Addition of the three variables D_1, D_2 and D_3 increased the value of R^2 by significant amounts for both beef and pork. The coefficient of D_3 was never significant.

D_{Pt} was always significant and negative, indicating a downward shift in the pork inventory equation occurring in 1952. G_t was always significant and positive in the pork inventory equations and positive but nonsignificant in the beef inventory equations. Ceiling prices evidently caused packers to hold substantial amounts of pork that normally would have been sold. The existence of ceiling prices evidently had little effect on beef inventory holdings.

Production variables

Beef inventories are affected by both beef and pork production changes. Pork inventories, on the other hand, are affected only by pork production changes. Beef stocks absorb about 12 percent of the increase in beef production during the first, second and third quarters, which are periods of stock depletion. They absorb 25 percent during the fourth quarter, which is a period of accumulation.

Around 12 percent of the increase in farm pork production is absorbed by stocks during the second quarter, which is generally followed by increasing prices, and about 25 percent is absorbed during the other quarters.

ΔQ_{Bt-1} and ΔQ_{Pt-1} had no significant effect upon stocks. The same is also true of $Q_{Bt} - Q_{Bt-4}$ and $Q_{Pt} - Q_{Pt-4}$, which were used in a few equations.

Prices

Δp_{Pt} and Δp_{Pt-1} have no effect on pork inventories. Δp_{Bt} and Δp_{Bt-1} have little or no effect on beef inventories. The coefficient of Δp_{Bt} was never significant. The coefficient of Δp_{Bt-1} was commonly significant at the 10-percent level (averaging 0.022 in value), and its addition raised the value of R^2 by an amount significant at about the 12-percent level. The addition of both beef price variables simultaneously, however, did not significantly affect the value of R^2. These findings are consistent with Tolley and Harrell's conclusion that packers are able to predict changes in supply fairly accurately but unable to predict changes in demand accurately. They use predictions of supply (i.e., marketings) but not of price, which reflects marketings and demand (15).

Sales

Problems of multicollinearity were encountered when the three beef sales variables were used; the matrix of sums of squares and cross products of independent variables was singular. When s_{Bt-1} and Δs_{Bt-1} were used, this difficulty was not encountered. These two variables, however, were nonsignificant.

In adding s_{Pt-1}, Δs_{Pt} and Δs_{Pt-1} to pork equation 2, table 4, to obtain equation 3, the resulting increase in R^2 was significant at the 5-percent level, although none of the coefficients was significant. In three other comparisons by the F test, however, nonsignificant increases in R^2 resulted from their addition. The coefficient of Δs_{Pt} was infrequently significant at the 10-percent level and never at a higher level, and the other two coefficients were always nonsignificant.

The conclusion that these sales variables have no effect on inventories is further confirmation of Tolley and Harrell's finding that packers are unsuccessful in predicting demand (15). If they were successful, sales variables would be expected to be significant determinants of meat inventories (provided the model accurately reflects packers' sales prediction mechanism).

Lagged inventories

In simpler models which contained only i_{t-1}, the two ΔQ_t and $\Delta Q'_t$, the coefficient of i_{Bt-1} was significant at the 1-percent level whereas the coefficient of i_{Pt-1} was nonsignificant. The addition of i_{Pt-2} and i_{Bt-2} to the pork and beef equations, respectively, resulted in significant increases in the value of R^2 and reduced the coefficient of i_{Bt-1} to nonsignificance. In none of the equations fitted, would the t test reject the hypotheses that the partial regression coefficients of i_{Bt} on i_{Bt-1} and i_{Pt} on i_{Pt-1} are unity. (The coefficients of Δi_t on i_{t-1} do not differ significantly from zero.)

Δi_{Bt-1} and Δi_{Pt-1} were used in a few cases; they were always nonsignificant.

Model comparisons

A vs. B.1 (Δp_t, Δp_{t-1}, ΔQ_{t-1}). The addition of these three variables resulted in nonsignificant increases in the values of R^2. The only coefficient ever significant was the coefficient of Δp_{Bt-1} and it was never significant at a higher level than 10 percent.

B.1 vs. B.2 (s_{t-1}, Δs_t, Δs_{t-1}). As indicated previously, the evidence is clear that beef sales expectations play no effective role in determining beef inventories. The evidence is less clear on pork, but the preponderance of evidence indicates that pork sales variables have no effect on pork inventories.

In addition, the signs of all coefficients in Model A, except for the nonsignificant coefficient of ΔQ_{Bt} in the pork inventory equations, conform with expectations. The evidence, then, is that Model A is a more accurate representation of the meat inventory process than are Models B.1 or B.2. For pork inventories, a simpler model than A is appropriate—the model obtained by setting $e = 0$.

Dynamic properties

The results indicate that beef and pork inventory behavior may be depicted by second-order-difference equations.

$$(2.18) \quad i_t = b_1 i_{t-1} + b_2 i_{t-2} + \sum_j a_j x_{jt} + \mu_t$$

The solution to such a difference equation has two parts. One is the general solution to $i_t = b_1 i_{t-1} + b_2 i_{t-2}$; the other is a particular solution to the entire equation (2, pp. 169-215). Set $\sum_j a_j x_{jt} + \mu_t = L_t$ and fix L_t at the value of L_0.

Write:

$$(2.19) \quad \begin{aligned} x_1 &= [b_1 + (b_1^2 + 4b_2)^{1/2}]/2 \\ x_2 &= [b_1 - (b_1^2 + 4b_2)^{1/2}]/2 \end{aligned}$$

If x_1 and x_2 are real numbers, a solution is

$$(2.20) \quad i_t = \frac{(i_0 - L'_0)x_2 - i_1 + L'_0}{x_2 - x_1} x_1^t + \frac{i_1 - L'_0 - (i_0 - L'_0)\, x_1}{x_2 - x_1} x_2^t + L'_0$$

where $L'_0 = L_0/(1-b_1 - b_2)$ and i_0 and i_1 are initial conditions.

Assume inventories have been constant and in equilibrium for some time and that one variable, say, x_{j0} in L_0 changes in value. After this disturbance, the time path followed by actual inventories will be determined by the values of x_1 and x_2. If both are positive and less than unity, inventories will follow a monotonic time path in their asymptotic approach to L'_0, the new equilibrium value of the inventories. That is,

$$(2.21) \quad \lim_{t \to \infty} \frac{\partial i_t}{\partial x_{j0}} = \frac{a_j}{1-b_1-b_2}$$

Beef equations 2 and 3 are of this type.
If $(b_1^2 + 4b_2) < 0$, set

$$(2.22) \quad \begin{aligned} c &= b_1/2 \\ d &= [(-1)\ (b_1^2 + 4b_2)]^{1/2}/2 \\ D &= (c^2 + d^2)^{1/2} \\ \sin R &= c/D \\ \cos R &= d/D \end{aligned}$$

Then the solution is

$$(2.23) \quad i_t = D^t[(i_0 - L'_0)\cos(tR) + \frac{i_1 - (1-c)L'_0 - ci_0}{d} \sin(tR)] + L'_0$$

When $0 < D < 1$ (as in beef equations 1 and 4 and in the pork equations), inventories follow a damped cyclical path in moving from one equilibrium level to another. The length of the cycle is $360°/R$.
Again

$$(2.24) \quad \lim_{t \to \infty} \frac{\partial i_t}{\partial x_{j0}} = \frac{a_j}{1-b_1-b_2}$$

Table 5 presents data on dynamic properties. The evidence demonstrates that pork inventories follow a damped cycle in moving from one equilibrium position to another. The evidence also strongly indicates a cycle of 9 to 10 quarters. The 10-year cycle indicated by equation 1 must be rejected because equation 1 is suspect. This is because it excludes two relevant variables: D_{Pt} and G_t. Evidence from other equations not presented shows that the reason for the difference between equations 2 and 4 is the addition of G_t. Dropping the five variables i_{Pt-1}, ΔQ_{Bt}, ΔQ_{Pt-1}, Δp_{Pt} and Δp_{Pt-1} has negligible influence on the coefficient of i_{Pt-2} and on the values of the roots of the difference equation. (These five variables also have a negligible effect on the value of R^2. Dropping them from equations 2 and 3 reduces the values of R^2 by only 0.0033 and 0.0005, respectively.) The addition of G_t raises the absolute value of the coefficient of i_{Pt-2} by 30 to 40 percent and affects the values of the roots of the

Table 5. Results describing dynamic properties of beef and pork inventory equations.

Commodity and equation number	X_1	X_2	D	R (degrees)	$1-b_1-b_2$	$360°/R$ (quarters)
Beef 1	0.49 + 0.17i	0.49 − 0.17i	0.52	19	0.29	19
2	0.704	0.403			0.18	
3	0.674	0.408			0.19	
4	0.50 + 0.19i	0.50 − 0.19i	0.53	20		18
Pork 1	0.56 + 0.03i	0.56 − 0.03i	0.56	3	0.19	120
2	0.53 + 0.09i	0.53 − 0.09i	0.54	10	0.23	36
3	0.54 + 0.07i	0.54 − 0.07i	0.54	7	0.22	51
4	0.50 + 0.36i	0.50 − 0.36i	0.62	36	0.38	10

difference equation. Because of the significance of G_t, the results from equation 4 should give a more accurate description of dynamic properties than the results from the other equations.

In the case of beef inventory equations, the estimated dynamic properties depend on whether or not the regression contains Δp_{Bt-1}. Equations 1 and 4, which do not contain it, indicate a cyclical adjustment path with a period of about 5 years. The other equations, which do contain Δp_{Bt-1}, suggest a monotonic approach to a new equilibrium. Since the evidence in support of the hypothesis that Δp_{Bt-1} affects beef inventories is weak, it may be concluded that beef inventories follow a cyclical adjustment path.

QUARTERLY BUTTER AND CHEESE INVENTORIES

Variables

i_t = End-of-quarter total cold storage holdings of creamery butter or American cheese, thousands of pounds (17).

s_t = Total quarterly sales of creamery butter or American cheese, thousands of pounds. Computed as quarterly production—Δi_t. Production data from U. S. Department of Agriculture (17).

p_t = Average wholesale price, in cents per pound, of 92-score butter at Chicago or average wholesale price of cheese, in cents per pound for fresh single daisies at Chicago (17).

p'_t = p_t divided by wholesale price index for all commodities other than farm products and foods, 1947-49:1.00 (22).

F_t = Quarterly milk production on farms, millions of pounds (17).

$\Delta p'_{b1,\,2t-1} = \Delta p'_{bt-1}$, first quarter
$= -\Delta p'_{bt-1}$, second quarter
$= 0$, third and fourth quarters

$\Delta p^2_{c2t-1} = \Delta p^2_{ct-1}$, second quarter
$= 0$, all other quarters

Where necessary for clarity, subscripts b and c are used to denote butter and cheese.

Models

Two basic models are employed. The fundamental concepts in each are similar to those in Beef and Pork Model B. The expectation generating mechanisms for prices are different.

Model A

$$(3.1)\quad i'_t = a\, s^*_{t+1} + b\Delta p^*_{t+1}$$

$$(3.2)\quad s^*_{t+1} - s^*_t = \beta_0(s_t - s^*_t)$$

$$(3.3)\quad p^*_{t+1} - p^*_t = \beta_1(p_t - p^*_t) + \beta_2\Delta F^*_{t+1}$$

$$(3.4)\quad \Delta F^*_{t+1} = \beta_3\Delta F_t$$

$$(3.5)\quad \Delta i_t = c\,(i'_t - i_{t-1})$$

Analytically, this is similar to a model proposed by Nerlove for studying consumer demand when equilibrium demand depends upon one expected price and expected income (11, pp. 27-29). The method of reduction is also presented by Nerlove. The reduced equation is

$$\begin{aligned}(3.6)\quad i_t = {} & a\beta_0\beta_1 c s_t + a\beta_0 c(1-\beta_1)\Delta s_t \\ & + bc(\beta_1 - 1)\Delta p_t + bc(1-\beta_0)(1-\beta_1)\Delta p_{t-1} \\ & + b\beta_2\beta_3 c\Delta F_t - b\beta_2\ (1-\beta_0)\beta_3 c\Delta F_{t-1} \\ & + (3-\beta_0-\beta_1-c)i_{t-1} \\ & - [(1-c)(2-\beta_0-\beta_1) + (1-\beta_0)(1-\beta_1)]i_{t-2} \\ & + (1-\beta_0)(1-\beta_1)(1-c)\ i_{t-3}\end{aligned}$$

Model B

$$(3.1)\quad i'_t = a\, s^*_{t+1} + b\Delta p^*_{t+1}$$

$$(3.2)\quad s^*_{t+1} - s^*_t = \beta_0(s_t - s_t^*)$$

$$(3.3a)\quad \Delta p^*_{t+1} = \beta_1\Delta p_t + \beta_2\Delta F^*_{t+1}$$

$$(3.4)\quad \Delta F^*_{t+1} = \beta_3\Delta F_t$$

$$(3.5)\quad i_t - i_{t-1} = c(i'_t - i_{t-1})$$

The sole difference between models A and B lies in the price expectation generation mechanism. Equations 3.3 and 3.4 are equivalent to

$$(3.6)\quad p^*_{t+1} = \beta_1\sum_i (1-\beta_1)^i p_{t-i} + \beta_2\beta_3 \sum_i (1-\beta_1)^i\ \Delta F_{t-i}$$

Equations 3.3a and 3.4 are equivalent to

$$(3.7)\quad p^*_{t+1} = p_t + \beta_1\Delta p_t + \beta_2\beta_3\Delta F_t$$

The five equations of Model B reduce to

$$\begin{aligned}(3.8)\quad i_t = {} & a\beta_0 c s_t + b\beta_1\, c\Delta p_t \\ & - b\beta_1\, c(1-\beta_0)\ \Delta p_{t-1} + b\beta_2\beta_3\, c\Delta F_t \\ & - b(1-\beta_0)\beta_2\beta_3\, c\Delta F_{t-1} \\ & + (2-\beta_0 - c)i_{t-1} \\ & - (1-\beta_0)\ (1-c)i_{t-2}\end{aligned}$$

Results

Selected statistical results for butter and cheese inventory regression equations are presented in tables 6 and 7. Equations 1 and 2 represent models A and B. Because of the importance of federal price-support purchases of these products in several post-World War II years, the analysis was restricted to 1929-41 data.

Seasonal variables

For both butter and cheese, the three seasonal variables are substitutes for ΔF_{t-1}. The addition of the three variables to butter inventory equations did not significantly increase the value of R^2. The value of F was less than 1. The main

Table 6. Selected statistical results for regression analysis of quarterly butter stocks, 1929-3rd quarter to 1941-4th quarter (dependent variable $= i_{bt}$).

	1	i_{bt-1}	i_{bt-2}	i_{bt-3}	ΔF_t	ΔF_{t-1}	$\Delta p'_{bt}$	$\Delta p'_{bt-1}$	$\Delta p'_{b1\ 2t-1}$	s_{bt}	Δs_{bt}	D_{1t}	D_{2t}	D_{3t}	R^2
1	78,398	1.1415 0.1554***	−0.0542 0.2192	−0.3513 0.1367**	13.8015 2.8480***	7.4261 3.5915**	−56 509	1,772 482***		−0.1348 0.1133	−0.0499 0.1039	3,137 11,239	−21,852 14,314	−14,621 12,748	0.9526
2	92,028	1.3159 0.1420***	−0.4616 0.1452***		13.1747 3.0010***	3.3688 3.0710	192 501	1,405 475***		−0.1866 0.0925*		6,894 11,760	-23,266 14,664	− 7,579 12,403	0.9441
3	86,778	1.0880 0.0765***		−0.3500 0.0657***	14.3500 1.1594***	11.8758 .5089***		1,955 386***		−0.1553 0.0732**					0.9495
4	86,573	1.1747 0.0679***		−0.3500 0.0557***	14.8524 .9887***	12.4604 .4521***		2,344 339***	1,796 417***	−0.1770 0.0621***					0.9647

Table 7. Selected statistical results for regression analyses of quarterly cheese stocks, 1929-III to 1941-IV (dependent variable $= i_{ct}$).

		i_{ct-1}	i_{ct-2}	i_{ct-3}	ΔF_t	ΔF_{t-1}	Δp^2_{ct}	Δp^2_{ct-1}	Δp^2_{c2t-1}	s_{ct}	Δs_{ct}	D_{1t}	D_{2t}	D_{3t}	R^2
1	−2,258	1.4346 0.1855***	−0.6359 0.2538**	−0.0360 0.1749	6.6458 1.7540***	1.3103 1.7858	19 27	61.5503 25.8239**		0.1909 0.1315	−0.6413 0.1390***	6,120 5,334	−4,147 7,472	−2,888 5,818	0.9554
2	−2,132	1.1890 0.2004***	−0.2401 0.2072		2.3430 1.8162	3.4776 1.9548*	−10 31	85.2075 29.2921***		0.0257 0.1478		1,050 6,380	− 167 9,070	−7,126 7,015	0.9298
3	−3,681	1.4959 0.1492***	−0.7888 0.1273***		6.0539 0.8671***	2.1356 0.5472***		68.9927 21.7305***		0.2393 0.1126**	−0.6614 0.1250***				0.9500
4	−1,363	1.4403 0.1407***	−0.6751 0.1261***		5.6167 0.8253***	2.6039 0.5395***		104.1176 24.1194***	−110.4743 41.0759**	0.1743 0.1078	−0.6129 0.1180***				0.9575

effect of including these variables was to reduce the coefficient of ΔF_{t-1} and increase its standard error. The addition of D_{1t}, D_{2t} and D_{3t} to cheese inventory equations consistently increased the value of R^2 significantly when ΔF_{t-1} was excluded and had a nonsignificant effect when ΔF_{t-1} was included. D_{2t} and D_{3t} were almost invariably significant by the t test in equations excluding ΔF_{t-1}, but they were never significant in equations including ΔF_{t-1}.

Table 8 shows some relevant coefficients of determination among the seasonal variables and other variables. The high R^2 pertaining to ΔF_{t-1} shows why this substitution takes place. The high R^2 pertaining to ΔF_t suggests that these seasonal variables could substitute for ΔF_t if it were deleted. No equations were fitted using seasonally adjusted data. The size of some of the R^2s suggests that quite different conclusions might have been reached if such data were used.

Table 8. Values of r^2 and R^2 between seasonal variables and butter and cheese variables.

Variable j	r^2jD D_1	D_2	D_3	$R^2j.123$
ΔF_t	0.72	0.02	0.22	0.97
ΔF_{t-1}	0.19	0.61	0.17	0.97
i_{ct}	0.24	0.03	0.07	0.34
i_{ct-1}	0.17	0.13	0.12	0.42
i_{ct-2}	0.25	0.12	0.06	0.43
Δp^2_{ct-1}	0.01	0.15	0.08	0.24
s_{ct}	0.03	0.03	0.12	0.18
Δs_{ct}	0.63	0.00	0.12	0.74
i_{bt}	0.18	0.26	0.26	0.70
i_{bt-1}	0.56	0.09	0.08	0.72
i_{bt-2}	0.16	0.31	0.26	0.73
i_{bt-3}	0.57	0.09	0.08	0.73
Δp_{bt-1}	0.01	0.15	0.13	0.29
s_{bt}	0.00	0.00	0.37	0.37

The higher R^2 for i_{bt} than for i_{ct} in table 8 can be explained by the different functions served by butter and cheese inventories. Butter demand exhibits less seasonal variation than does farm milk production or butter production. The main reason for holding butter inventories is to carry butter from periods of peak production to periods of low production. Most of the reasons for holding inventories of cheese are the same as the reasons for holding butter, but there is an additional reason for the existence of cheese inventories. Holding cheese in inventory to age it is an inherent part of the process of production.

Prices

Plotting residuals from original equations resulted in the addition of Δp^2_{c2t-1} to the cheese inventory equations and $\Delta p'_{b1,2t-1}$ to the butter inventory equations. Their coefficients were significant, and their use resulted in significant increases in the coefficients of determination.

Several forms of the price variables were tried for both products; Δp_t, $\Delta p'_t$, Δp^2_t and $\Delta p'^2_t$ and lagged values. Δp_t^2 is the first difference of p_t^2. The price variable selected had little effect on the size and level of significance of other coefficients. Nor did it have an appreciable effect on the values of R^2. The price variables in tables 6 and 7 were selected for presentation because they consistently yielded larger values of R^2.

The coefficient of current price change was never significant; the coefficient of lagged price change was almost invariably significant. The average results from several equations indicate a coefficient of Δp^2_{ct-1} of 110 for the first, third and fourth quarters and, 8 for the second quarter. Average results indicate these values of the coefficient of $\Delta p'_{bt-1}$: first quarter, 4,034; second quarter, 364; third and fourth quarters, 2,199.

The peak volume of milk production occurs in the second quarter. The typical pattern during this sample period was for milk production to rise during the first and second quarters (the second quarter increase being much greater than the first quarter increase) and to fall during the third and fourth quarters. Thus, substantial volumes of butter and cheese moved into storage during the second quarter to be sold during the last half of the year when farm milk production declined. Evidently, given ΔF_t and ΔF_{t-1}, it took a much larger increase in expected price to obtain a given speculative increase in storage during the second quarter than during other quarters.

Sales

Δs_{bt} had no discernable effect on end-of-quarter butter inventories. Its coefficient was never significant. The other sales variables do have an effect on inventories. The coefficient of Δs_{ct} was consistently significantly negative; its average value was -0.62. The value of s_{bt} does affect i_{bt}; the coefficient being significantly negative. Its average value in the equations estimated was -0.19. The coefficient of s_{ct} was usually significant at the 5- or 1-percent level in equations excluding $\Delta p^2_{c2,t-1}$; its average coefficient was 0.22. The addition of $\Delta p^2_{c2,t-1}$ reduced its coefficient slightly to 0.17 and reduced its level of significance to about 12 percent. The reason for the difference in signs between the coefficients of s_{bt} and s_{ct} will be discussed under the section, Model Comparisons.

Farm production

ΔF_t and ΔF_{t-1} are significantly positively related to i_{bt} and i_{ct}. In the butter inventory equations estimated, the average values of their coefficients were 15 and 12; in the cheese inventory equations, 5.6 and 2.9.

Lagged inventories

The coefficients of i_{bt-1} and i_{ct-1} were always significant. The coefficient of i_{bt-2} was significant when i_{bt-3} was excluded but nonsignificant when i_{bt-3} was included. A number of F tests found no significant increase in the value of R^2 from the addition of i_{bt-2} when i_{bt-3} was included. When i_{bt-2} was included, the addition of i_{bt-3} resulted in an increase in the value of R^2 significant at the 2- to 3-percent level.

The variable i_{ct-3} has a nonsignificant effect on i_{ct} (its coefficient was smaller than the standard error); i_{ct-2} does, however, have a significant effect.

Average values of significant coefficients from the equations fitted were as follows: i_{bt-1}, 1.15; i_{ct-1}, 1.41; i_{bt-2}, -0.47; i_{ct-2}, -0.62; i_{bt-3}, -0.33.

Trend

Time was included in a number of both butter and cheese inventory equations. It was nonsignificant.

Model comparisons

The estimation equation for Model A contains the same independent variables as Model B, plus i_{t-3} and Δs_t. In a number of comparisons with different price variables and with and without a time trend, the addition of these two variables to the cheese inventory equations always raised R^2 by significant amounts. Their addition also increased the absolute value of the coefficients of i_{t-2}, s_t and ΔF_t and caused them to become significant. Evidently Δs_{ct} was responsible for this since the addition or deletion of i_{ct-3} by itself had a negligible effect on the values of R^2 or of the other coefficients.

The effect of adding i_{bt-3} and Δs_{bt} to butter inventory equations was to reduce i_{bt-2} to nonsignificance, to reduce the level of significance of s_{bt}, and to significantly increase the value of R^2. Although the additional variables of Model A make a significant contribution to R^2, the signicant coefficients—those of i_{bt-3} and Δs_{ct}—do not have the expected sign.

The significant coefficients of s_{bt} and ΔF_t also have signs opposite from the expected sign.

A reasonable set of assumptions about the parameters in models A and B is listed in the first column of table 9. The expected and observed signs of the coefficients of the reduced equations are also presented.

The explanation for the inconsistency between the expected and observed sign of the coefficient of s_{bt} probably lies in the nature of the assumed process for determining expected sales, equation 3.2. It is unreasonable to assume β_0 to be negative,

Table 9. Assumed signs of parameters, expected and observed signs of coefficients, butter and cheese models A and B.

Expected signs of parameters	Variable	Expected sign of coefficient: Model A	Expected sign of coefficient: Model B	Observed sign of coefficient: Butter	Observed sign of coefficient: Cheese
Model A	s_t	+	+	−	+
$a>0$	Δs_t	+	0	−[a]	−
$b>0$	Δp_t	−	+	+, −[a]	+[a]
$1>\beta_0>0$	Δp_{t-1}	+	−	+	+
$1>\beta_1>0$	ΔF_t	−	−	+	+
$\beta_2<0$	ΔF_{t-1}	+	+	+	+
$\beta_3>0$	i_{t-1}	+	+	+	+
$1>c>0$	i_{t-2}	−	−	−[b]	−
Model B	i_{t-3}	+	0	−	−[a]
Same as A except $\beta_1>0$					

[a] Nonsignificant.
[b] Nonsignificant with i_{bt-3}.

since this amounts to saying that s^*_{t+1} is negative. This is easily seen if equation 3.2 is written in its alternative form.

$$(3.9) \quad s^*_{t+1} = \beta_0 \sum_{i=0}^{n} (1-\beta_0)^i s_{t-i}$$

Evidently some alternative procedure is used to determine expected butter sales. After allowing for seasonal effects, the partial correlation between s_{bt} and s_{bt-1} is negative.

$$(3.10) \quad s_{bt} = \underset{(0.44)^{**}}{-0.94} \, s_{bt-1} + \underset{(3,630)^{***}}{15,641} \, D_{1t} - \underset{(4,863)^{***}}{17,337} \, D_{2t} - \underset{(6,673)}{6,447} \, D_{3t} + \mu_t$$

Possibly businessmen used a process of this type in predicting butter sales. It is probable that they would at least have and use knowledge of the existence of this negative partial correlation in making their sales predictions. This is one justification for including s_{bt} to account for sales expectations, even though its coefficient is negative.

The inconsistency between the expected and observed signs of the coefficients of ΔF_t indicates the desirability of another modification of the model. The level of production of butter and cheese is closely tied to the level of farm production of milk. As farm production rises butter and cheese production tend to rise. The increased production cannot be sold off immediately, and, hence, inventories tend to rise. As farm production falls off, it takes some time, even if it were desired, for individual butter and cheese plants to obtain additional supplies from farmers previously supplying plants producing other products. Hence, equation 3.5 should be revised to

$$(3.5a) \quad \Delta i_t = c_0(i'_t - i_{t-1}) + c_1 \Delta F_t + c_2 \Delta F_{t-1}; \quad c_1>0,\ c_2>0$$

A much larger proportion of total farm milk production went into butter than into American cheese during 1929-41: 34 percent vs. 4 percent (17). Equation 3.5a would then lead to expectations of larger coefficients of ΔF_t and ΔF_{t-1} in the butter than in the cheese inventory equations. This was the case.

The explanation of the nonsignificance of the coefficients of Δp_t may lie in the existence of a time lag in the process of obtaining and using information about current prices. The signs of the coefficients of Δp_{t-1} are consistent with the expectations of Model A. Model A, however, is unsatisfactory for reasons presented previously.

The reason i_{bt-3} was significant and i_{ct-3} was not may lie in differences in the structure of the two industries. Large firms played a more important role in the marketing of cheese than of butter. Large firms are apt to be more sensitive to changes in market conditions. They would be expected generally to have more and better information on indicated future sales and price movements and correspondingly greater confidence in their predictions. The greater sensitivity and the greater confidence would tend to result in a value of c which was close to unity. As c approaches unity, the coefficient of i_{t-3} becomes smaller.

A model for cheese inventories that is consistent with the observed results consists of equation 3.1 and

$$(3.11) \quad s^*_{t+1} = s_t$$

$$(3.12) \quad \Delta p^*_{t+1} = \beta \Delta p_{t-1}$$

$$(3.13) \quad \Delta i_t = c_0(i'_t - i_{t-1}) + c_1 \Delta i_{t-1} + c_2 \Delta F_t + c_3 \Delta F_{t-1} + c_4 \Delta s_t$$

All parameters are positive except c_4.

The reduced equation is

$$(3.14) \quad i_t = (1-c_0+c_1) i_{t-1} - c_1 i_{t-2} + a c_0 s_t + b\beta\, c_0 \Delta p_{t-1} + c_2 \Delta F_t + c_3 \Delta F_{t-1} + c_4 \Delta s_t$$

From equation 4, table 7, we obtain the following estimates of the parameters:

(3.15) est $a = 0.74$
est $(b\beta) = 443$ in first, third and fourth quarters
$= -27$ in second quarter
est $c_0 = 0.24$
est $c_1 = 0.68$
est $c_2 = 5.62$
est $c_3 = 2.60$
est $c_4 = -0.61$

It was mentioned that there may be a substantial time lapse between the beginning of a quarter and the time at which inventory decisions are made and acted upon and that, in the interim, inventory change will be a function of inventory change last quarter. The magnitude of est c_1 and the fact that est $c_1 >$ est c_0 are consistent with this hypothesis.

A model of butter inventories consistent with observed results consists of equations 3.1, 3.12 and

$$(3.16) \quad s^*_{t+1} = es_t$$

$$(3.17) \quad \Delta i_t = c_0(i'_t - i_{t-1}) + c_1\Delta i_{t-1} + c_2\Delta i_{t-2} + c_3\Delta F_t + c_4\Delta F_{t-1}$$

The reduced equation is

$$(3.18) \quad i_t = (1-c_0 + c_1)i_{t-1} + (c_2 - c_1)i_{t-2} - c_2 i_{t-3} + c_3\Delta F_t + c_4\Delta F_{t-1} + aec_0 s_t + b\beta\, c_0\Delta p_{t-1}$$

This model is consistent with the results in table 6 only if we assume $c_2 = c_1$. From equation 1 we obtain the following estimates: est $c_0 = 0.26$, est $c_1 = 0.40$, est $c_2 = 0.35$. Assuming $c_1 = c_2$, equation 4 yields the following estimates:

$$(3.19) \quad \begin{aligned} \text{est } (ae) &= -1.01 \\ \text{est } (b\beta) &= 23{,}600 \text{ first quarter} \\ &= 3{,}122 \text{ second quarter} \\ &= 13{,}361 \text{ third and fourth quarters} \\ \text{est } c_0 &= 0.18 \\ \text{est } c_1 &= \text{est } c_2 = 0.35 \\ \text{est } c_3 &= 14.85 \\ \text{est } c_4 &= 12.46 \end{aligned}$$

From equation 3.10, the partial regression coefficient of s_{bt} on s_{bt-1} is -0.94. Assuming $e = -0.94$, est $a = 1.07$.

Dynamic properties

These are summarized in table 10. A second degree difference equation appears to be adequate for the description of quarterly cheese inventories. The nature of the solution of second degree difference equations was discussed in connection with beef and pork inventories.

The results from cheese equation 2 are open to question since this equation excludes the relevant variable Δs_{ct}. Equation 3 and other equations solved but not presented here, indicate a damped oscillatory movement of i_{ct} in moving from one equilibrium position to another. The cycle is about 3 years in length.

A third degree difference equation appears to be necessary for the description of the butter inventory process. Write the general third order difference equation as

$$(3.20) \quad i_t = b_1 i_{t-1} + b_2 i_{t-2} + b_3 i_{t-3} + L_0$$

The solution is

$$(3.21) \quad i_t = D^t[e \cos(tR) + f \sin(tR)] + a\,x_3^{\,t} + \frac{L_0}{1-b_1-b_2-b_3};$$

e, f and a are functions of the initial conditions, the roots and $L_0/(1-b_1-b_2-b_3)$, and x_3 is the third root of the solution. In all of the third degree difference equations for butter, x_3 was about -0.45 and $0<D<1$. Hence,

$$(3.22) \quad \lim_{t\to\infty} \frac{\partial i_t}{\partial x_{j0}} = \frac{a_j}{1-b_1-b_2-b_3}$$

In moving from one equilibrium position to another, butter inventories follow a damped oscillatory time path with a cycle of 4 to 5 years in length.

QUARTERLY DEPARTMENT STORE STOCKS

Variables

$I_t = (P_{HFt} + 2P_{At})/3$. This is the same deflator Robinson used (13).

P_{HFt} = Bureau of Labor Statistics consumer price index for house furnishings, last month of quarter t, 1947-49: 1.00 (23).

P_{At} = Bureau of Labor Statistics consumer price index for apparel, last month of quarter t, 1947-49:1.00 (23).

i_t = Index of department store stocks, end of quarter t (3) deflated by I_t.

s_t = Sum of indexes of department store sales for the 3 months of quarter t (3) deflated by the sum of I_t and corresponding price indexes for the first 2 months of the quarter.

$s_{2t}^2 = s_t^2$, in second quarter
= 0, other quarters

$\Delta s_{3t} = \Delta s_t$, in third quarter
= 0, other quarters

$\Delta s_{4t} = \Delta s_t$, in the fourth quarter
= 0, other quarters

E_t = 0, in the second, third and fourth quarters
= 0, first quarter if Easter falls in March
= the date of Easter if Easter comes in April, cf, table 11.

Table 10. Results describing dynamic properties of butter and cheese inventory equations.

Commodity and equation number	X_1	X_2	X_3	D	R (degrees)	$\frac{360^0}{R}$ (quarters)	$1-b_1-b_2-b_3$
Butter 1	0.80 + 0.37i	0.80 − 0.37i	−0.45	0.88	25	14	0.26
2	0.66 + 0.17i	0.66 − 0.17i		0.68	14	25	0.15
3	0.78 + 0.36i	0.78 − 0.36i	−0.47	0.60	25	14	0.26
4	0.82 + 0.29i	0.82 − 0.29i	−0.46	0.87	20	18	0.18
Cheese 2	0.93	0.26					0.05
3	0.75 + 0.48i	0.75 − 0.48i		0.89	33	11	0.29

Table 11. First quarter values of E_t.

Year	E_t first quarter
1948	0
1949	17
1950	9
1951	0
1952	13
1953	5
1954	18
1955	10
1956	1
1957	21
1958	6
1959	0
1960	17

Models

Two models were used in the analysis of department store stocks. Each has a number of variants. These furnish examples of a problem presented earlier: Reduced equations, containing exactly the same variables but different combinations of parameters, can be obtained from distributed lag models.

Model A.1

(4.1) $i_t = \Sigma a_i s_{t-i}, i = 0, 1, 2, \ldots, n$

(4.2) $a_i = \lambda a_{i-1}, i \geqslant 1$

These two equations reduce to

(4.3) $i_t = a_0 s_t + \lambda i_{t-1}$.

Equation 4.1 is the sum form of the equation used by Robinson (13) in first difference form. He estimated values of the a_i without making any assumption such as equation 4.2. Equation 4.2 is the Koyck assumption of a geometrically distributed lag (9, p. 20).

Model A.2

(4.4) $i_t = a\, s^*_{t+1}$

(4.5) $s^*_{t+1} - s^*_t = c\, (s_t - s^*_t)$

These equations reduce to

(4.6) $i_t = a\, cs_t + (1-c) i_{t-1}$.

The variables in equation 4.6 are the same as the variables in equation 4.3 but the parameters are different. In Model A.2 the parameter a represents a stock-expected sales ratio. The parameter c is an adjustment coefficient assumed not equal to one because of uncertainty, technological and logistical frictions. Equation 4.3 derives from the assumed operation of the accelerator principle in a special way. It need not carry any expectational implications, but it can be interpreted as representing an expectation generating mechanism. Equation 4.1 can be derived from equations 4.1a and 4.7.

(4.1a) $i_t = \alpha s^*_{t+1}$

(4.7) $s^*_{t+1} = \dfrac{1}{\alpha} \Sigma\, a_i s_{t-i}, i = 0, 1, 2, \ldots, n.$

Applying equation 4.2 to 4.7 yields equation 4.3. Model A.2 also furnishes a basis for interpreting the accelerator principle as a sales expectation phenomenon.

Model B.1

(4.8) $i'_t = a\, s^*_{t+1}$

(4.9) $s^*_{t+1} - s_t = \beta (s_t - s_{t-1})$

(4.10) $\Delta i_t = c (i'_t - i_{t-1})$

Model B.1 differs from A.2 in two respects: It contains a different sales expectation generation mechanism, and it allows for a difference between actual and desired inventories. These equations can be reduced to

(4.11) $i_t = a\, cs_t + a\beta\, c\Delta s_t + (1-c)\, i_{t-1}$.

Model B.2

(4.8) $i'_t = a\, s^*_{t+1}$

(4.12) $s^*_{t+1} = s_t$

(4.13) $\Delta i_{Pt} = c\, (i'_t - i_{t-1})$

(4.14) $\Delta i_t = \Delta i_{Pt} + b\Delta s_t$

This is Duesenberry's basic model (4, pp. 795-796). Equation 4.13 states that planned inventory change is a fraction of desired inventory change. Equation 4.14 states that actual inventory change differs from planned because of unexpected variations in sales. These equations reduce to

(4.15) $i_t = a\, cs_t + b\Delta s_t + (1-c)\, i_{t-1}$

The interpretation to be placed on these coefficients is quite different from the interpretation to be placed upon the coefficients in equation 4.10, although both equations contain exactly the same variables.

A third model also was estimated. Its reduced equation included the variables in equation 4.15 plus i_{t-2}, i_{t-3} and current and lagged changes in an index of wholesale prices of house furnishings and apparel. The addition of these four variables made no significant contribution to the value of R^2, and their coefficients were nonsignificant.

Results

Results are presented in table 12. Equation 1 represents Model A; equations 2 and 3 represent Model B. In fitting these equations, the Korean War period was excluded. Robinson also excluded this period (13).

Trend

Time was significant in equations derived from the various forms of Model A and nonsignificant in equations derived from the various forms of Model B.

Table 12. Selected statistical results for end-of-quarter department store stocks regressions, unadjusted data, 1948-I to 1950-II, 1952-I to 1960-II, $(y = i_t)$.

	b_0	s_t	Δs_t	s^2_{2t}	Δs_{3t}	Δs_{4t}	E_t	i_{t-1}	D_{1t}	D_{2t}	D_{3t}	t	R^2
1	16.2045	1.3100						−0.5108	−3.2830	−11.2742	4.2043	0.1972	0.9926
		0.1391***						0.1430***	0.2892***	0.5421***	.8543***	0.0692***	
2	12.0833	1.6817	−0.2919	0.000318	0.6913	−0.1487	0.1116	−0.8433	−0.5911	−8.9793	2.2685	0.0989	0.9955
		0.1905***	0.1197**	0.000136**	0.3017**	0.2348	0.0649*	0.1856***	0.7559	1.2833***	1.8064	0.0689	
3	5.3073	1.9945	−0.5157	0.000414	0.8760		0.1265	−1.0912		−8.4355			0.9949
		0.0867***	0.0512***	0.000044***	0.2395***		0.0600**	0.0876***		0.4150***			

Seasonal shift variables

D_{1t}, D_{3t} and time were competitive with Δs_t. The addition of Δs_t reduced all three to nonsignificance. The addition of these three variables had a nonsignificant effect on the value of R^2 when Δs_t was included. The coefficient of D_{2t} was always significant and negative.

As will be discussed later, one effect of adding D_{1t} and D_{3t} was to dampen the oscillations in i_{t-1}. This is reasonable since these terms alternately raise and lower i_t, as shown in table 13.

Table 13. Values of the seasonal contribution, $\sum a_i D_{it}$ to department store inventories.

Quarter	Equation number 1	2
I	7.48	2.86
II	−0.92	−1.66
III	7.99	8.39
IV	−14.55	−9.57

Seasonal rotation variables

Graphic study of residuals lead to the addition of $s_{2t}{}^2$, Δs_{3t} and Δs_{4t}. E_t was included for obvious reasons. Department store sales rise shortly before Easter time. When Easter comes in April, end-of-March inventories will reflect the build-up in inventories in anticipation of the sales rise. The volume of inventories would be expected to be greater the greater number of days remaining until Easter. When Easter falls in March, these inventories acquired in anticipation of Easter sales would be liquidated before the end of March. As expected, the sign of the coefficient of E_t was positive and was usually significant at the 5-percent level. The addition of these four variables had a significant effect on the value of R^2. The coefficient of Δs_{4t}, however, was never significant, and dropping it had a negligible effect on the value of R^2 and on other coefficients.

Sales

The variable s_t had a significant effect in every equation. Its average coefficient was 1.45 in equations including D_1, D_3 and t and was 2.00 in equations excluding them.

The coefficient of Δs_t averaged −0.50 in equations excluding D_1, D_3 and t and averaged −0.30 in equations which included them and the rotation variables. The coefficient of Δs_t was nonsignificant in equations which excluded the rotation variables and E_t.

Equation 3 indicates that a 1-unit change in Δs_t produces a change in the same direction of 0.36 unit in i_t in the third quarter and a change in the opposite direction of about 0.50 unit in the first, second and fourth quarters. Model B.2 implies a negative coefficient on Δs_t; Model B.1, a positive coefficient. In different quarters, the results are consistent with different hypotheses.

One possible explanation is: Model B.2 is relevant for the first, second and fourth quarters. In these quarters, s^*_{t+1} is equal to s_t with allowance for normal seasonal variations. For the third quarter, Model B.1 is relevant. An increase in sales during the third quarter sets up optimistic expectations concerning sales during the Christmas shopping season, and inventories are raised accordingly. A decrease in sales has the reverse effect.

There seems to be no similar explanation for the significance of s_{2t}^2. Even though significant, the coefficient has a small impact on inventories. Letting $t = 0, 1, 2, 3, 4$ represent the last quarter of the previous year and the four quarters of the current year, the sales terms in equation 3 can be written as

$$(4.16)\quad \begin{aligned} i_1 &= 1.47s_1 + 0.52s_0 \\ i_2 &= 1.47s_2 + 0.52s_1 + 0.00041s_2^2 \\ i_3 &= 2.35s_3 + 0.36s_2 \\ i_4 &= 1.47s_4 + 0.52s_3 \end{aligned}$$

Setting s_2 at its mean value for the period, $\partial i_t/\partial s_t = 1.47$ for the first and fourth quarters, 1.57 for the second quarter and 2.35 for the third quarter. Equation 2 yields 1.39, 1.47 and 2.08, respectively.

In his study of department store inventories, Robinson (13) used an equation of the form

$$(4.15)\quad \Delta i_t = \sum_i b_i \Delta s_{t-i} + \sum_j c_j \left|\Delta s_{t-j}\right|$$

Δi_t and Δs_t were computed as first differences of deflated seasonally adjusted inventory or sales divided by moving averages of deflated seasonally adjusted stocks or sales. It was suggested earlier that the seasonal adjustment process may alter the properties of the data, possibly including its auto- and serial-correlation properties. Dividing by a moving average of seasonally adjusted data and then taking first differences would further affect the auto- and serial-correlation properties. Robinson used a shorter sample period than the one used in this study. Hence, different results would be expected from the two studies.

Robinson computes a "total acceleration coefficient" which has the same meaning as the limiting value of $\partial i_t/\partial s_0$ as t approaches infinity. His estimate is 1.86. Values of less than unity were obtained in this study.

Robinson also found that:

> "Department store officials were rather good at forecasting changes in sales. This follows from the fact that the partial correlation between sales change for period n and inventory change for the same period was generally positive. Negative partial correlations might have been expected, since an unforeseen increase in sales tends to cause a temporary drop in inventories." (13, p. 356)

The results presented here lead to a different conclusion. The partial regression of i_t or of Δi_t on Δs_t was significantly negative in three quarters and positive in the third quarter of the year. Following Robinson's reasoning, one would conclude that department store officials were rather poor at forecasting changes in sales during three quarters and rather good in the other.

A high level of sales forecasting skill is consistent with a negative, positive or zero partial correlation between Δi_t and Δs_t. Assume that Δs_t was perfectly foreseen in period $t-1$. Then the partial correlation might be positive if a positive value of Δs_t were taken to indicate a positive value for Δs^*_{t+1}. A negative partial correlation would exist if a positive value of Δs_t were taken to indicate a negative Δs^*_{t+1}. A zero correlation could exist if Δs^*_{t+1} were assumed to be independent of Δs_t or if department stores made inventory plans for only one quarter ahead.

Through the use of absolute value of sales changes, $|\Delta s_{t-j}|$, Robinson concluded that the amount of adjustment to a change in sales depends on whether the sales change is positive or negative (13). In the present study, two additional sales variables were included in a number of equations. One variable was $s_t^+ = s_t$ if $s_t \geqslant s_{t-1}$, otherwise, $s_t^+ = 0$; the other was $s_t^- = s_t$ if $s_t \leqslant s_{t-1}$, otherwise, $s_t^- = 0$. The two coefficients were not significantly different; they were equal to three decimal places.

Lagged inventories

Evidently, i_{t-2} and i_{t-3} have a nonsignificant effect. The coefficient of i_{t-1} varied from -0.51 in Model A to -0.84 in Model B equations that included D_1, D_3 and t and to -1.08 in Model B equations that excluded these three variables.

Model comparisons

Analyses including s_{2t}^2, Δs_{3t} and E_t lead to the conclusion that Model B is more appropriate than Model A. Δs_t was nonsignificant in equations excluding these variables.

Dynamic properties

These are summarized in table 14. A first-order difference equation is adequate for the study of quarterly department store stocks. Let the difference equation be written as

$$(4.16)\quad i_t = b_1 i_{t-1} + \Sigma a_j x_{jt} + \mu_t = b_1 i_{t-1} + L_0.$$

The solution is

$$(4.17)\quad i_t = b_1^t i_0 + \frac{L_0}{1-b_1}(1 - b_1^t)$$

Assume that inventories are initially in equilibrium and then undergo a single disturbance. The

Table 14. Dynamic properties of selected department store inventory equations.

Equation No.	$\lim\limits_{t \to \infty} \frac{\partial i_t}{\partial i_0}$	$\lim\limits_{t \to \infty} \frac{\partial i_t}{\partial s_0}$	$\lim\limits_{t \to \infty} \frac{\partial i_t}{\partial u_0}$	Adjustment path
1	0	0.87	0.66	Damped cycle
2	0	0.76	0.55	Damped cycle
3	$\pm\infty$	$\pm\infty$	$\pm\infty$	Explosive cycle

nature of the time path followed by actual inventories in moving from the old to the new equilibrium is determined by the value of b_1. If $0 > b_1 > -1$, as in equations 1 and 2, inventories follow a damped cyclical path. If $b_1 < -1$, as in equation 3, actual inventories undergo explosive oscillations and diverge from, rather than converge to, the new equilibrium. These explosive oscillations occur only in equations which do not contain D_{1t} and D_{3t}. The damping effect exercised by these variables is reasonable in view of the evidence in table 13.

In equation 3, we cannot reject the hypothesis that b_1 is greater than but close to -1, say -0.99. Assuming this, this equation leads to the same conclusion as do the other equations:

$$\lim_{t \to \infty} \frac{\partial i_t}{\partial s_0} < 1 \quad \text{and} \lim_{t \to \infty} \frac{\partial i_t}{\partial \mu_0} < 1.$$

MONTHLY MANUFACTURERS' NONDURABLES INVENTORIES

Variables

P_{TNt} = Wholesale price index of all nondurable goods, month t, 1947-49:1.00 (23, 24).

i_t = Total nondurable goods industries manufacturer's inventories, millions of dollars, end of month t, deflated by P_{TNt} (25, 26).

P_{NMt} = Wholesale price index, nondurable manufactures, 1947-49:1.00 (23, 24).

s_t = Total nondurable goods industries monthly sales, millions of dollars (25, 26) deflated by P_{NMt}.

O_{ut} = Total nondurable goods industries manufacturers' unfilled orders, end of month t, millions of dollars (25, 26), deflated by P_{NMt}.

P_t = Wholesale price index of nondurable raw or slightly processed goods, month t, 1947-49:1.00 (23, 24).

$s_{5,8t} = s_t$ in May and August
$= 0$ in all other months

$\Delta s_{7,11t} = -\Delta s_t$ in July and November
$= 0$ in all other months

$\Delta O_{u2,3t-1} = -\Delta O_{ut-1}$ in February
$= \Delta O_{ut-1}$ in March
$= 0$ in all other months

Models

In their explanation of the inventory investment component of gross national product, Duesenberry, et al. (4, p. 789) used this equation

$$(5.1) \quad \Delta i_t = b_0 + b_1 s_t + b_2 \Delta s_t + b_3 O_{ut-1} + b_4 \Delta O_{ut-1} + b_5 i_{t-1} + b_6 \Delta i_{t-1}$$

Model A

One model, but not the only one, from which this equation can be derived is the following:

$$(5.2) \quad i'_t = a\, s^*_{t+1}$$

$$(5.3) \quad s^*_{t+1} = s_t$$

$$(5.4) \quad \Delta i_{Pt} = \alpha(i'_t - i_{t-1}) + bO_{ut-1} + d\Delta O_{ut-1}$$

$$(5.5) \quad \Delta i_t = c\,(\Delta i_{Pt} - \Delta i_{t-1}) + e\Delta s_t$$

The reduced equation of this model is

$$(5.6) \quad \Delta i_t = a\alpha c s_t + e\Delta s_t + bcO_{ut-1} + dc\Delta O_{ut-1} - \alpha c i_{t-1} - c\Delta i_{t-1}$$

To the extent that unfilled orders represent a demonstrated demand, businessmen may find it desirable to hold larger inventories when unfilled orders are large to hedge against possible shortages, delays and materials price increases. As unfilled orders rise and steps are taken to work them off, inventories of raw materials and goods in process will rise. O_{ut-1} and ΔO_{ut-1} are therefore included as determinants of planned inventory change (10, pp. 297-298).

Model B

This is slightly more general than Model A. It allows for the possibility that the speculative motive may play a role in determining desired inventories of raw and semifinished materials.

$$(5.7) \quad i'_t = a\, s^*_{t+1} + b\Delta P^*_{t+1}$$

$$(5.8) \quad \Delta P^*_{t+1} = \varepsilon \Delta P_t$$

$$(5.9) \quad s^*_{t+1} = s_{t-1}$$

$$(5.10) \quad \Delta i_{Pt} = \alpha(i'_t - i_{t-1}) + \beta_1 O_{ut-1} + \beta_2 \Delta O_{ut-1}$$

Actual change in monthly inventories may depend on Δi_{Pt-1} as well as on Δi_{Pt}, and perhaps also on Δi_{t-2}.

$$(5.11) \quad \Delta i_t = c_0 \Delta i_{Pt} + c_1 \Delta i_{Pt-1} + c_2 \Delta s_t + c_3 \Delta i_{t-2}$$

These equations reduce to

$$(5.12) \quad \Delta i_t = -\alpha(c_0 + c_1) i_{t-1} + \alpha c_1 \Delta i_{t-1} + c_3 \Delta i_{t-2} + \beta_1 (c_0 + c_1)\, O_{ut-1} + (\beta_2 c_0 - \beta_1 c_1)\, \Delta O_{ut-1} + \beta_2 c_1 \Delta O_{ut-2} + a\,\alpha(c_0 + c_1) s_{t-1} + c_2 \Delta s_t - a\,\alpha c_1 \Delta s_{t-1} + b\,\alpha\,\varepsilon\, c_0 \Delta P_t + b\,\alpha\,\varepsilon\, c_1 \Delta P_{t-1}$$

Table 15. Selected statistical results from analyses of monthly manufacturers' nondurable inventories, April 1948 to December 1960, $y = \Delta i_t$.

	i_{t-1}	Δi_{t-1}	Δi_{t-2}	s_{t-1}	Δs_t	Δs_{t-1}	0_{ut-1}	$\Delta 0_{ut-1}$	$\Delta 0_{ut-2}$	ΔP_t	ΔP_{t-1}	$s_{5,8t}$	$\Delta s_{7,11t}$	$\Delta 0_{u2,3t-1}$	1	R^2
1	−0.0242 0.0188	−0.0060 0.0856		0.0388[a] 0.0251	−0.0004 0.0256		0.1328 0.0267***	−0.1689 0.0919*							−363.85	0.4246
2	−0.0545 0.0165***	0.0404 0.0807	−0.0069 0.0769	0.0777 0.0218***	−0.0491 0.0265*	−0.1276 0.0255***	0.1166 0.0276***	−0.0903 0.0927	−0.0835 0.0955	−3.0100 4.0737	−4.3001 4.1322	−0.0010 0.0037	−0.0070 0.0525	0.5735 0.1836***	−203.46	0.4948
3	−0.0520 0.0169***	0.0429 0.0778	−0.0525 0.0766	0.0756 0.0223***	−0.0461 0.0235*	−0.1310 0.0230***	0.1145 0.0283***	−0.1765 0.0857**	−0.0483 0.0889						−222.00	0.4454
4	−0.0529 0.0159***			0.0764 0.0211***	−0.0488 0.0259*	−0.1313 0.0244***	0.1174 0.0228***	−0.1199 0.0865	−0.1095 0.0900			−0.0017 0.0035	0.0071 0.0500	0.5911 0.1811***	−217.76	0.4855
5	−0.0427 0.0185**			0.0627 0.0247**	−0.0206 0.0317	−0.0986 0.0329***	0.1195 0.0230***	−0.1589 0.0807*				−0.0016 0.0035	0.0040 0.0499	0.5139 0.1727***	−252.54	0.4917
							0_{ut}	$\Delta 0_{ut}$	$\Delta 0_{ut-1}$							
6	−0.0582 0.0170***	0.0484 0.0768	−0.0587 0.0744	0.0821 0.0222***	−0.0497 0.0229**	−0.1373 0.0227***	0.0959 0.0282***	−0.2732 0.0856***	−0.1146 0.0871						−127.67	0.4595

[a] Coefficient of s_t.

Model C

This was obtained by setting $b = 0$ in Model B. The reduced equation is identical to equation 5.12 except that the two price change terms are absent.

Results

Results obtained from monthly seasonally unadjusted data are presented in tables 15 and 16. The first three equations represent models A, B and C, respectively.

The values of R^2 are low. There is a good deal of random variability in change in nondurable inventories. Equation 4 was fitted using i_t as the dependent variable. The value of R^2 was 0.9954, more than twice the value in equation 4.

Table 16. Coefficients of seasonal variables.

	sin iR	cos iR	sin 2iR	cos 2iR	sin 3iR	cos 3iR
1	−5.58 2.107***	6.57 2.02***	−3.35 2.37	104.02 21.92***	− 2.95 18.85	−16.84 18.09
2	−6.469 2.104***	3.212 2.305				
3	−6.358 2.010***	3.514 2.112*				
4	−6.474 2.019***	3.111 2.092				
5	−6.398 2.115***	4.219 2.207*	−1.878 2.203	43.035 26.794		
6	−5.874 1.998***	3.318 2.086				

Seasonal variables

Sin iR and cos iR were the only important seasonal variables. Sin 3iR, cos 3iR, sin 4iR and cos 4iR were nonsignificant in all equations. Sin 2iR and cos 2iR are competitive with Δs_t. Its exclusion from an equation raised the coefficients of sin 2iR and cos 2iR from nonsignificance to significance at the 5-percent level. Excluding sin 2iR and cos 2iR increased the absolute size of the coefficient of Δs_t by about 200 percent from nonsignificance to significance at the 5- or 10-percent level. Excluding them also increased the absolute size of the coefficient of Δs_{t-1} by half.

Seasonal rotation variables

Plotted residuals indicated the possibility of some intermonth variations in the slopes of s_t, Δs_t and $\Delta 0_{ut-1}$. The addition of $s_{5,8t}$, $\Delta s_{7,11t}$ and $\Delta 0_{u2,3t-1}$ significantly increased the value of R^2, although only the coefficient of $\Delta 0_{u2,3t-1}$ was significant.

Prices

Model B is not superior to Model C. ΔP_t and Δp_{t-1} always had nonsignificant coefficients and resulted in nonsignificant increases in the value of R^2, indicating that expectations as to raw materials prices play a negligible role in determining the level of nondurable inventories. This is not in agreement with Lovell's findings (10). Using deseasonalized deflated quarterly data, he found evidence that speculation on input product prices does influence inventories. He obtained a highly significant coefficient of -0.62 for the variable $\Delta P_{t+1}/P_t$.

Sales

Some experimenting was done to compare results obtained using s_t or s_{t-1}. They yielded the same values of R^2; the use of s_t generally resulted in a higher level of significance for the coefficient of Δs_t. Decisions as to the level of inventories desired at the end of the current month likely must be made before information is available on current rate of sales. Therefore, s_{t-1} was used in most of the regressions. The coefficient of s_{t-1} was significant at the 5- or 1-percent level in nearly every equation. The average value of its coefficient was 0.074.

Although equation 6 happens to be the only one in table 16 in which the coefficient of Δs_t was significant at the 5-percent level, its coefficient was frequently significant at the 5-percent level. In equations 2, 3 and 4, the coefficient is significant at the 6- to 7-percent level. The average value of the coefficient of Δs_t was -0.048 when s_{t-1} was used. When s_t was used, the average value fell to -0.080.

Lovell's analysis of quarterly data yielded a coefficient of Δs_t of -0.17; this corresponds to -0.057 in monthly data. He found s_t to be nonsignificant, whereas the present analysis indicates it to be significant (10, p. 302).

The average coefficient of Δs_{t-1} in the equations estimated was -0.13 when sin iR and cos iR were the only seasonal variables used. Other times its average value was somewhat smaller, though still significant at the 5- or 1-percent level.

Unfilled orders

Some experimenting was carried out to test the effect of various lags in these variables. The use of 0_{ut}, $\Delta 0_{ut}$ and $\Delta 0_{ut-1}$ always yielded a slightly larger value of R^2 than did the use of 0_{ut-1}, $\Delta 0_{ut-1}$ and $\Delta 0_{ut-2}$. Equations 3 and 6 give a typical comparison.

Evidently $\Delta 0_{ut-2}$ is not relevant to the determination of current inventories. Its coefficient was never significant, frequently being smaller than the standard error.

When using 0_{ut-1} and $\Delta 0_{ut-1}$ (with or without $\Delta 0_{ut-2}$), the level of significance of $\Delta 0_{ut-1}$ was commonly reduced by the addition of the seasonal rotation variables. Otherwise its average value was -0.18. Equation 5 indicates the coefficient of $\Delta 0_{ut-1}$ to be -0.67 in February, 0.35 in March and -0.16 in all other months.

Although the results leave some question as to the effect of $\Delta 0_{ut-1}$, they clearly show that 0_{ut-1} is important. Its coefficient was invariably significant at the 1-percent level, and its use resulted in a highly significant increase in the value of R^2. The average value of the coefficient was 0.12. When 0_{ut-1} was replaced by 0_{ut}, the coefficient of 0_{ut} was 0.10. When converted to the same time period this agrees with Lovell's estimate (10, p. 302).

The results also clearly demonstrate that $\Delta 0_{ut-1}$ is irrelevant when $\Delta 0_{ut}$ is included. The coefficient $\Delta 0_{ut-1}$ was never significant in these circumstances, whereas the coefficient of $\Delta 0_{ut}$ invariably was significant. The coefficients of $\Delta 0_{ut}$ averaged -0.27.

Several equations were estimated using new orders in place of unfilled orders. They yielded much inferior results. The values of R^2 were much smaller; the residual sums of squares were 11 percent larger. The new-order variables were nonsignifiant by t tests.

These results indicate that a high, but constant ($\Delta 0_{ut-1} = 0$ or $\Delta 0_{ut}=0$), level of unfilled orders leads to an increase in inventories, presumably of raw materials and goods in process. An equal volume of unfilled orders brought about by recent growth ($\Delta 0_{ut-1}>0$ or $\Delta 0_{ut}>0$) leads to a smaller increase in the level of inventories. This seems quite reasonable considering the possibile existence of time lags in adjustment.

Lovell obtained a coefficient of 0.33 for 0_{ut} using deseasonalized deflated data (10, p. 302).

Lagged inventories

The average coefficient of i_{t-1} was -0.05, highly significant. Rewriting Lovell's results to have Δi_t as dependent, his estimate does not differ significantly from zero (10, p. 302).

Δi_{t-1} and Δi_{t-2} make no contribution to our understanding of the determinants of i_t. Their coefficients were nonsignificant and their addition resulted in negligible increases in the value of R^2. In several equations, i_{t-2} was used with i_{t-1}. Neither coefficient was significant. The simple correlation between these two variables is 0.996.

Model comparisons

It was previously pointed out that Model B is not superior to C. Model C is superior to A, since Δs_{t-1} was highly significant and the addition of Δs_{t-1}, $\Delta 0_{ut-2}$ and Δi_{t-2} resulted in a significant increase (at the 5-percent level) in the value of R^2.

Model A is unsatisfactory, because Δs_t was nonsignificant whereas it was commonly significant in models B and C. The coefficient of i_{t-1} was also nonsignificant in Model A equations but significant in equations based on models B and C. All three models contain nonsignificant lagged changes in inventories.

An approriate model appears to be one derived from Model C by setting c_1 and c_3 equal to zero, assuming $\beta_2<0$ and specifying a different mechanism for determining s^*_{t+1}.

$$(5.13) \quad i'_t = a\, s^*_{t+1}$$
$$(5.14) \quad s^*_{t+1} = s_{t-1} + \beta_0 \Delta s_t$$
$$(5.15) \quad \Delta i_{Pt} = \alpha(i'_t - i_{t-1}) + \beta_1 0_{ut-1} + \beta_2 \Delta 0_{ut-1}$$
$$(5.16) \quad \Delta i_t = c_0 \Delta i_{Pt} + c_1(s_t - s^*_t)$$

The reduced equation is

$$(5.17) \quad \Delta i_t = a\alpha c_0\, s_{t-1} + (a\alpha\beta_0 c_0 + c_1)\Delta s_t - c_1(\beta_0 - 1)\Delta s_{t-1} - \alpha c_0\, i_{t-1} + \beta_1 c_0\, 0_{ut-1} + \beta_2 c_0\, \Delta 0_{ut-1}$$

Assuming $\beta_2<0$, the estimated coefficients then have the expected signs provided $|c_1| > a\alpha\, \beta_0\, c_0$ and $0<\beta_0<1$.

An equally useful alternative is obtained if 0_{ut-1} and $\Delta 0_{ut-1}$ in (5.15) are replaced by 0_{ut} and $\Delta 0_{ut}$.

Dynamic properties

The evidence is clear that a first degree difference equation is adequate for the study of monthly inventories of nondurable goods. Write it as

$$(5.13) \quad i_t = b_1 i_{t-1} + \sum_j a_j x_{jt} + u_t$$

The estimated value of b_1 is 0.95. This is close to Lovell's value of 0.93 (10, p. 302). Since this is less than unity, this is a stable system. If inventories have been in equilibrium and are disturbed, they will follow a monotonic convergent time path to the new equilibrium in the absence of further disturbances. The proportion of the equilibrium change that will have actually occurred at the end of t periods after the initial disturbance is $1 - 0.95^t$. It takes nearly 45 months for nine-tenths of the complete adjustment to take place.

$\partial i_t/\partial x_{jt} = a_j$ represents the instantaneous or contemporaneous rate of change in i_t. $a_j/(1-0.95) = 20\, a_j$ represents the limiting value of $\partial i_t/\partial x_{j0}$ as t approaches infinity.

MONTHLY MANUFACTURERS' DURABLES INVENTORIES

Variables

P_{TDt} = Wholesale price index, total durable goods, month t, 1947-49:1.00 (23,24).

P_{DMt} = Wholesale price index, durable manufactured goods, month t, 1947-49:1.00 (23,24).

i_t = Total durable goods industries manufacturers' inventories, millions of dollars, end of month t (25,26) divided by P_{TDt}.

s_t = Manufacturers' durable goods sales, month t, millions of dollars (25,26) divided by P_{DMt}.

0_{ut} = durable goods industries manufacturers' unfilled orders, end of month t, millions of dollars (25,26) divided by P_{DMt}.

0_{nt} = Durable goods industries manufacturers' new orders, month t, millions of dollars (25,26) divided by P_{DMt}.

P_t = Wholesale price index, durable raw or slightly processed goods, month t, 1947-49:1.00 (23,24).

$0_{u*,t-1} = 0^2_{ut-1} \times 10^{-6}$ in September
$= 0^2_{ut-1} \times 10^{-6}$ in December
$= 0$ in all other months.

Models

The same three models were used here as in the study of nondurables inventories.

Results

Results are presented in tables 17 and 18. The values of R^2 are 25 to 50 percent larger than the values for corresponding models in tables 15 and 16. Equation 2 was estimated with i_t as dependent variable; the value of R^2 was 0.9980.

Seasonal variables

Cos iR and sin 3iR were the only two important seasonal variables. Their coefficients were generally significant at the 1-percent level. The level of significance of the coefficient of cos 4iR depended upon which unfilled order variables were used. Its coefficient was generally nonsignificant when 0_{ut-1}, $\Delta 0_{ut-1}$ and $\Delta 0_{ut-2}$ were used and significant at the 5-percent level when 0_{ut-1}, $\Delta 0_{ut}$ and $\Delta 0_{ut-1}$ were used.

Prices

Model B is superior to Model C. The coefficients of ΔP_t and ΔP_{t-1} were significant, and their addition resulted in a highly significant increase in the value of R^2. The average value of the coefficients of ΔP_t was -1.52. The average value of the coefficients of ΔP_{t-1} was 0.82. In contrast, ΔP_{t-1} and ΔP_{t-2} were used in a few equations but were never significant.

In his study, using seasonally adjusted deflated data, Lovell found a negative but nonsignificant relation between i_t and proportional input price change from quarter t to quarter t + 1 (10, p. 302).

Sales

The coefficient of s_{t-1} was almost invariably significant at the 1-percent level. The coefficients averaged 0.075 in value. The coefficient of Δs_t was generally significant at the 1-percent level. Its coefficient averaged 0.059 when using unfilled orders and averaged nearly twice this in equations which included new rather than unfilled orders.

The coefficients of Δs_{t-1} were nonsignificant. Using quarterly data, Lovell obtained coefficients of 0.1256 for s_t and –0.1043 for Δs_t. These corres-

Table 17. Selected statistical results from analyses of monthly manufacturers' durable inventories, unadjusted data, April 1948 to December 1960, $y = \Delta i_t$.

	i_{t-1}	Δi_{t-1}	Δi_{t-2}	s_{t-1}	Δs_t	Δs_{t-1}	0_{ut-1}	$\Delta 0_{ut-1}$	$\Delta 0_{ut-2}$	ΔP_t	ΔP_{t-1}	0_{u*t-1}	1	R^2
1	−0.0369 0.0086***	0.4732 0.0662***		0.0797 0.0190***	0.0612 0.0217***		0.0031 0.0014**	0.0183 0.0133					−118	0.6746
2	−0.0302 0.0087***	0.3506 0.0734***	0.1642 0.0684**	0.0668 0.0195***	0.0622 0.0201***	0.0048 0.0216	0.0020 0.0012*	−0.0181 0.0171	0.0557 0.0182***	−1.565 0.304***	0.825 0.327**	0.0526 0.0191***	− 91	0.7515
3	−0.0342 0.0091***	0.3730 0.0801***	0.1143 0.0799	0.0776 0.0204***	0.0469 0.0232**	−0.0210 0.0235	0.0023 0.0014*	−0.0207 0.0186	0.0529 0.0198***				−122	0.6951
									$\Delta 0_{ut}$					
4	−0.0394 0.0085***	0.3923 0.0749***	0.1660 0.0697**	0.0805 0.0194***	0.0736 0.0205***	−0.0025 0.0219	0.0024 0.0012*	0.0430 0.0173**	−0.0368 0.0172**	−1.531 0.312***	0.848 0.332**	0.0372 0.0187*	− 57	0.7430
								$\Delta 0_{nt}$	$\Delta 0_{nt-1}$					
5	−0.0370 0.0071***	0.4099 0.0726***	0.2160 0.0638***	0.0848 0.0177***	0.1141 0.0226***			−0.0561 0.0162***	−0.0321 0.0129**	−1.285 0.309***	0.889 0.321***		− 55	0.7427

Table 18. Coefficients of seasonal variables.

	sin iR	cos iR	sin 3iR	sin 4iR	cos 4iR
1	35 20*	93 18***	52 18***	−32 18*	35 19*
2		85 17***	59 16***		26 17
3	37 21*	105 18***	54 18***	− 9 20	30 19
4		85 18***	58 17***		43 18**
5		96 17***	45 16***		47 18***

pond to 0.043 and −0.035 in monthly data (10, p. 302).

Unfilled orders

Plotting residuals indicated the advisability of adding $0_{u*,t-1}$. Its coefficient was significant and it resulted in highly significant increases in the value of R^2.

In spite of low intercorrelations among the various variables used there seems to be substantial interaction among them. The coefficient of $\Delta 0_{ut-2}$ was significant at the 5- or 1-percent level in every equation, and its addition resulted in a significant increase in the value of R^2. Its coefficient averaged 0.049 in value. The addition of $\Delta 0_{ut-2}$ reduced the level of significance of 0_{ut-1}. Its addition also changed the coefficient of $\Delta 0_{ut-1}$ from significantly positive (averaging 0.045) to negative and nonsignificant.

The addition of $\Delta 0_{ut-1}$ also reduced the level of significance of the coefficient of 0_{ut-1}. The coefficients of 0_{ut-1} happen to be significant at the 10-percent level in the model B and C equations presented here. It was rarely significant at a higher level of probability and was commonly nonsignificant at even the 10-percent level. Evidently variations in 0_{ut-1} have a negligible impact on durables inventories.

$\Delta 0_{ut}$ was used in some equations in place of $\Delta 0_{ut-2}$. Its coefficient was significant at the 5-percent level, averaging −0.042 in value.

The preponderance of evidence indicates that 0_{ut-1} has no effect on inventories. The use either of $\Delta 0_{ut-2}$ or of $\Delta 0_{ut}$ and $\Delta 0_{ut-1}$ is appropriate.

New orders

A few analyses were run using 0_{nt-1}, $\Delta 0_{nt-1}$ and $\Delta 0_{nt-2}$ or $\Delta 0_{nt}$ in place of, or in addition to, unfilled-orders variables. The use of new orders generally yielded a larger value of R^2, the difference in residual sums of squares running from 1 to 6 percent. 0_{nt-1} was nonsignificant. The coefficient of $\Delta 0_{nt}$ was highly significant, averaging −0.064. The coefficient of $\Delta 0_{nt-2}$ was also highly significant, averaging −0.049.

Lagged inventories

The coefficients of i_{t-1} and Δi_{t-1} were almost invariably significant at the 1-percent level. The coefficients of i_{t-1} averaged −0.034 in value. This is half the size of the coefficient obtained by Lovell: -0.07 (10, p. 302). The coefficients of Δi_{t-1} had an average value of 0.39. The coefficients of Δi_{t-2} were significant, averaging 0.19 in value.

Model comparisons

Model B is the most appropriate of the three models used. It is better than Model C, since the price variables were significant and their addition resulted in a significant increase in the value of R^2. F tests also showed Model B to be superior to Model A. Of the five variables added to Model A to obtain Model B, only one—Δs_{t-1}—was nonsignificant by t tests. The coefficient of ΔP_t was negative, whereas it was expected to be positive. The other three coefficients had the expected signs.

In Model B, Δs_{t-1}, 0_{ut-1} and $\Delta 0_{ut-1}$ were generally nonsignificant when $\Delta 0_{ut-2}$ was included. When $\Delta 0_{ut}$ was included, Δs_{t-1}, and 0_{ut-1} were nonsignificant. Thus it appears that Model B is more complex than is necessary.

The results suggest any one of three models as being valid ones. One is

$$(6.1)\quad i'_t = a\, s^*_{t+1} + b\Delta P^*_{t+1}$$

$$(6.2)\quad \Delta P^*_{t+1} = \varepsilon_0 \Delta P_t + \varepsilon_1 \Delta P_{t-1}$$

$$(6.3)\quad s^*_{t+1} = s_{t-1} + \beta \Delta s_t$$

$$(6.4)\quad \Delta i_{Pt} = \alpha_0 (i'_t - i_{t-1}) + \alpha_1 \Delta 0_{ut-2}$$

$$(6.5)\quad \Delta i_t = \Delta i_{Pt} + c_0 \Delta i_{t-1} + c_1 \Delta i_{t-2}$$

The reduced equation is

$$(6.7)\quad \Delta i_t = a\, \alpha_0 s_{t-1} + a\, \alpha_0 \beta \Delta s_t + b\, \alpha_0 \varepsilon_0 \Delta P_t + b\, \alpha_0 \varepsilon_1 \Delta P_{t-1} + \alpha_1 \Delta 0_{ut-2} - \alpha_0 i_{t-1} + c_0 \Delta i_{t-1} + c_1 \Delta i_{t-2}$$

The second is obtained if $\alpha_1 \Delta 0_{ut-2}$ in equation 6.4 is replaced by $\alpha_1 \Delta 0_{ut} + \alpha_2 \Delta 0_{ut-1}$. The third is obtained if $\alpha_1 \Delta 0_{ut-2}$ is deleted from equation 6.4 and $c_2 \Delta 0_{nt} + c_3 \Delta 0_{nt-1}$ are added to equation 6.5. A few regressions which were computed indicate an appropriate model would contain $\alpha_1 \Delta 0_{ut} + \alpha_2 \Delta 0_{ut-1}$ in equation 6.4 and $c_2 \Delta 0_{nt} + c_3 \Delta 0_{nt-1}$ in equation 6.5. The negative signs on the coefficients of $\Delta 0_{nt}$ and $\Delta 0_{nt-1}$ indicate that these variables represent unexpected changes in demands. These have an inverse effect on inventories in the same way that unexpected changes in sales do.

The negative coefficient of ΔP_t may be interpreted in either of two ways. Perhaps $\varepsilon_0 < 0$ and $\varepsilon_1 > 0$; i.e., monthly price fluctuations are expected. The second, and more reasonable, explanation is this: Both ε_0 and ε_1 are positive. But ΔP_t enters into the model as a surrogate variable for tightness of supply conditions as well as an indicator of expected future price changes. A current increase in price is taken to mean further increase in prices. It also indicates tight market conditions which delay the execution of the desired increases in inventories.

There is no obvious reason why price changes should influence durables inventories but have no impact on nondurables inventories. Nor is there an obvious explanation of why Δs_{t-1} affects nondurables inventories but has no effect on durables inventories.

The other differences between the results for durables and nondurables are probably explicable on the grounds that technical and logistical frictions are more prevalent in the production and marketing of durables. This would explain why Δi_{t-1} and Δi_{t-2} are significant determinants of durables inventories but not of nondurables inventories. It might also explain why the relevant unfilled-orders variables have a longer lag for durables than for nondurables.

Dynamic properties

The third order difference equation appropriate for durables inventories can be written as

$$(6.8)\quad i_t = b_1 i_{t-1} + b_2 i_{t-2} + b_3 i_{t-3} + \sum_j a_j x_{jt} + u_t$$

The general solution to $i_t = b_1 i_{t-1} + b_2 i_{t-2} + b_3 i_{t-3}$ is $ax_1^t + bx_2^t + cx_3^t$ where x_1, x_2, and x_3 are the roots of $x^3 - b_1 x^2 - b_2 x - b_3 = 0$. The solution to equation 6.8 is

$$(6.9)\quad i_t = ax_1^t + bx_2^t + cx_3^t + L'_0$$

where $L'_0 = (\Sigma a_j x_{jt} + u_t) / (1 - b_1 - b_2 - b_3)$. The constants a, b, and c are functions of initial conditions and of the roots.

Table 19 contains the roots x_1, x_2 and x_3 for the equations in table 16. Suppose durables inventories are in equilibrium and undergo a single perturbation. The exact time path followed by actual inventories in adjusting to the new situation cannot be determined without knowing the values of a, b and c. Since the root of largest absolute value is negative, inventories will eventually undergo explosive oscillations. They will do so immediately if b exceeds a and c in absolute value. The second order difference equation derived from Model A also indicates that inventories will ultimately oscillate explosively.

Table 19. Roots of difference equations.

Equation	X_1	X_2	X_3
1	0.276	−1.713	
2	0.371	−1.369	−0.323
3	0.348	−1.462	−0.225
4	0.381	−1.430	−0.304
5	0.407	−1.401	−0.379

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