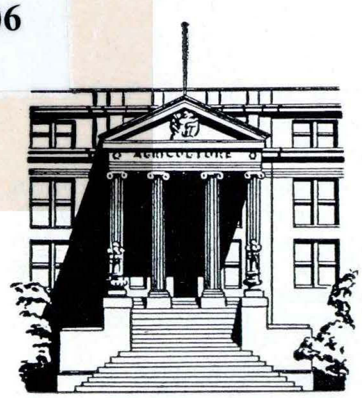

Derivation of Hydrographs for Small Watersheds From Measurable Physical Characteristics

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SYNOPSIS

This study was concerned with the development of a method for synthesizing the unit hydrograph for small watersheds from topographic characteristics. The topographic and hydrologic characteristics from 42 watersheds located in Illinois, Iowa, Missouri, Nebraska, Ohio and Wisconsin were investigated. These watersheds varied in size from 0.23 to 33.00 square miles.

Topographic Characteristics

Five watershed characteristics were measured on each basin: drainage-area size, A; length of the main stream, L; length to the center of area, L_{ca}; slope of the main stream, S_c; and mean land slope, S_L. An initial attempt to relate these factors with hydrograph properties by employing the principles of dimensional analysis proved unsuccessful. As a result, a preliminary analysis of the topographic data was undertaken to ascertain the reason for this failure.

The study indicated that the three length factors—A, L and L_{ca}—for the watersheds were highly correlated and could not be used as independent terms. In addition, the results showed that the watersheds adopted a relatively consistent shape, intermediary between ovoid and pear-shaped. For all practical purposes, in the watersheds studied, $L_{ca}=0.50L$.

Tests indicated that the variables, L and S_c, are related if consideration is given to the effect of regional influence.

An empirical relation was found between the mean land slope of a watershed, S_L, and the mean slope of a representative sample of first-order streams taken from the same watershed, s_1.

Hydrologic Characteristics

The rainfall and runoff characteristics from a number of unit storms occurring over each watershed were analyzed. The results showed that the period of rise, P_R, could be used in place of lag time, t_L, as a time parameter to relate the salient features of rainfall and runoff. For practical work, $P_R=t_L$.

For each watershed, a representative distribution graph was developed and modified to a dimensionless form based on the use of P_R, as the time parameter. Recent hydrologic investigations have shown that the unit hydrograph can be described by a two-parameter equation which is identical in form to the equation describing the two-parameter statistical gamma distribution. This distribution was fitted to each dimensionless graph, and estimators of its parameters, q and γ', were obtained by machine calculation. It was found that, in most cases, the two-parameter gamma distribution could be employed to describe the dimensionless graph; however, additional work is required in evaluating the goodness of fit in terms of hydrologic acceptance.

A set of relationships was derived to enable evaluation of the three variables, P_R, q and γ'. With these values known, the dimensionless graph, distribution graph or unit hydrograph of a given area can be described. A successful linkage between hydrograph properties and watershed characteristics was obtained by relating the storage factor P_R/γ', with the watershed factor $L/\sqrt{S_c}$.

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Derivation of Hydrographs for Small Watersheds From Measurable Physical Characteristics[1]

by Don M. Gray[2]

Wisler and Brater (59, p. 1) define hydrology as "the science that deals with the processes governing the depletion and replenishment of the waters of the land areas of the earth." Thus, hydrology is concerned with the transportation of the water through the air, over the ground surface and through the strata of the earth.

The manner in which water passes to a stream channel governs the terminology of the flow. The accepted components of stream-flow are interflow, ground water, channel precipitation and surface runoff. Of primary importance in this study is surface runoff, or water which passes to a stream channel by traveling over the soil surface. Its origin may be water arising from melting snow or ice, or rainfall which falls at rates in excess of the soil infiltration capacity.

The majority of work completed concerning the phenomena of surface runoff has been in two distinct area groups: (a) those hydrologic studies applied to large basins varying in size from 100 to several thousand square miles and (b) those studies applied to small areas of a hundredth-acre to a few acres. Work on the larger areas has been initiated largely by the United States Army Corps of Engineers and the Bureau of Reclamation for construction of large hydraulic structures. In contrast, agricultural research has investigated erosion, water yield and rates of surface runoff from small plots having varied physical and cultural treatments. The number of hydrologic investigations on watersheds of intermediate size is relatively small.

For design purposes engineers require a knowledge of the time-rate distribution of surface-runoff volumes. This distribution is depicted graphically by the hydrograph as a continuous plot of the instantaneous discharge rate with time. The design of small hydraulic structures—such as road culverts and chutes, water-conveyance channels, detention structures, weirs, spillways, drop inlets and others, as recommended for use for conveyance, control or conservation of surface runoff by the Bureau of Public Roads and the Soil Conservation Service—depends largely on the discharge-time relationships resulting from intense rains occurring on basins of an area of only a few square miles.

In many areas of the country for which rainfall records are available, there is a lack of stream-gaging stations in operation. For these ungaged areas, the surface-runoff hydrograph for a given storm may be approximated by two techniques:

1. Use of a recorded hydrograph from a like storm obtained from a physically similar area; or
2. Use of a synthetic hydrograph.

The success of the first method is limited by the degree of similarity between the significant runoff-producing characteristics of the watersheds involved. If they are not closely alike, an erroneous approximation of the true hydrograph may result. The latter method is limited by the reliability of the synthetic technique applied, which in many cases will have been developed from empirical data collected from large areas located in a different region.

Further hydrologic investigations on watersheds of intermediate size can be easily justified in view of the high annual expenditures of state and federal funds for the control and conservation of surface runoff and the relative inadequacy of the data on which the designs of the facilities are based. The application of economic principles at the watershed level requires that damages and benefits arising from structural and conservation programs be associated with individual subunits or subbasins within a large area. For example, the relative proportion of offsite damages attrib-

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utable to a given area because of flooding downstream should be prorated according to the contribution of this area to the flooding process. Such an estimate can be made properly only after the runoff characteristics of the area are known.

This bulletin describes a procedure whereby the unit hydrograph of surface runoff for small watershed areas may be synthesized. It presents the methodology and necessary relationships to perform this approximation once the pertinent physical characteristics of the watershed are determined.

REVIEW OF LITERATURE

Surface Runoff Phenomena

Depending upon the rate at which rain falls, the water may either infiltrate into the soil or accumulate and flow from the area as surface runoff. If the rainfall intensity, neglecting interception and evaporation losses, is less than the infiltration capacity, all of the water will enter the soil profile. If the rainfall intensity is in excess of the soil-infiltration capacity, a sequence of events occurs which ultimately produces surface runoff.

Excess water produced by a high-intensity rain must first satisfy soil and vegetal storage, detention and interception requirements. When the surface depressions are filled, the surface water then begins to move down the slopes in thin films and tiny streams. At this stage, the overland flow is influenced greatly by surface tension and friction forces. Horton (21) shows that, as precipitation continues, the depth of surface detention increases and is distributed according to the distance from the outlet (see fig. 1). With the increase in depth or volume of supply, there is a corresponding increase in the rate of discharge. Therefore, the rate of outflow is a function of the depth of water detained over the area.

The paths of the small streams are tortuous in nature, and every small obstruction causes a delay until sufficient head is built up to overcome this resistance (23). Upon its release, the stream is suddenly speeded on its way again. Each time that there is a merging of two or more streams, the water is accelerated further in its downhill path. It is the culmination of all of these small contributions which produces the ultimate hydrograph of surface runoff. After the excess rain ends, the water remaining on the area as surface detention disappears progressively from the watershed as a result of the combined action of surface runoff and infiltration.

The Hydrograph

A hydrograph of a stream is the graphical representation of the instantaneous rate of discharge with time. It includes the integrated contributions from ground-water, interflow, surface-run-

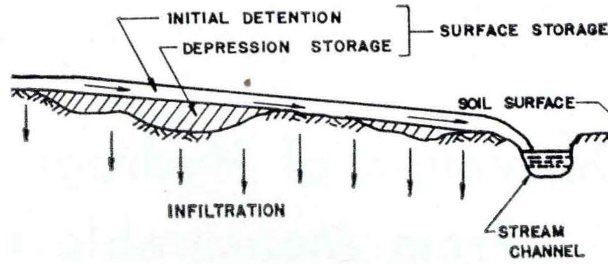

Fig. 1. Surface runoff phenomena.

off and channel-precipitation sources. For any stream, the nature of the hydrograph produced by a single, short-duration, excessive storm occurring over the drainage area follows a general pattern. This pattern shows a period of rise or a period of increasing discharge which culminates in a peak or crest followed by a recession of flow which may or may not recede to zero depending on the amount of ground-water flow. A typical hydrograph divided into its three principal parts is shown in fig. 2. For small watershed areas, the total contribution to the runoff hydrograph by ground-water flow, channel precipitation and interflow usually is small in comparison with the amount received from surface runoff. For this reason, the ensuing discussions will be directed toward hydrographs resulting mainly from surface runoff with small amounts of channel precipitation.

RISING LIMB OR CONCENTRATION CURVE

The rising limb extends from the time of beginning of surface runoff to the first inflection point on the hydrograph and represents the increase in discharge produced by an increase in storage or detention on the watershed. Its geometry is characterized by the distribution of the time-area histogram of the basin and the duration, intensity and uniformity of the rain. The initial portion is concave upward as a result of two factors: (a) the greater concentration of area between adjacent isochromes within the middle and upper

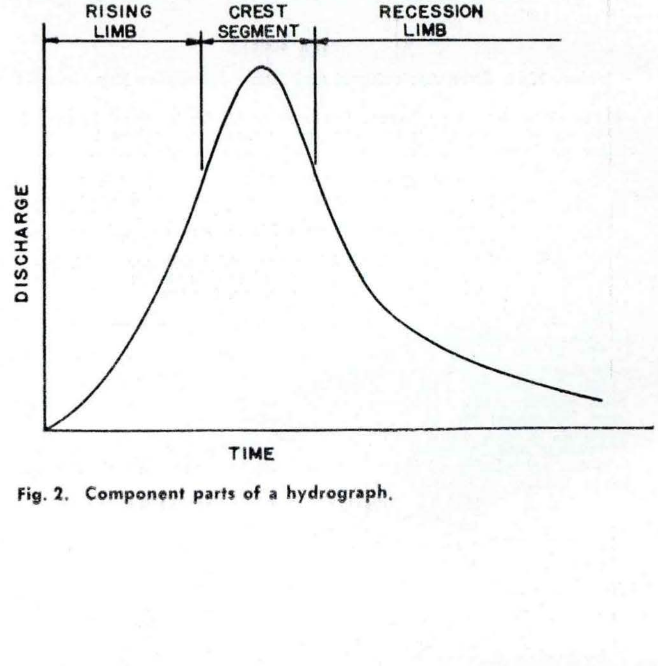

Fig. 2. Component parts of a hydrograph.

reaches of the basin and (b) the greater opportunity for infiltration, evaporation, surface detention and interception during the initial periods of the storm (32, p. 390).

CREST SEGMENT

The crest segment includes that part of the hydrograph from the inflection point on the rising limb to a corresponding point on the recession limb. The peak of the hydrograph or the maximum instantaneous discharge rate occurs within this time interval. The peak represents the arrival of flow from that portion of the basin receiving the highest concentration of area-inches of runoff. Ramser (41, p. 799) states:

> The maximum rate of runoff from any watershed area for a given intensity rainfall occurs when all parts of the area are contributing to flow. That part of the watershed nearest the outlet must still be contributing to the flow when the water from the most remote point on the watershed reaches the outlet.

That is, the duration of rain must be equal to or exceed the time of concentration.

RECESSION LIMB

The recession limb includes the remaining part of the hydrograph. It represents the withdrawal of water from storage after all of the excess rainfall has ceased. Consequently, it may be considered as the natural decrease in the rate of discharge resulting from the draining-off process. The shape of the curve is independent of time variations in rainfall or infiltration and is dependent essentially upon the physical features of the channel alone. Horner and Flynt (19) and Barnes (4) have listed mathematical expressions describing the recession limb. The general equation is of the form

$$q_t = q_o k^{dt} \qquad (1)$$

where q_t = instantaneous discharge at time, t,
q_o = instantaneous discharge at time, t_o,
k = recession constant, and
dt = elapsed time interval, $t - t_o$.

This equation produces a straight line when plotted on semilogarithmic paper. The value of the recession constant, k, is generally not constant throughout all discharge rates. Frequently, the recession curve is broken into a series of line segments to obtain several values of k, with each value applicable within a given range of flows.

Topographic Factors and the Hydrograph

The surface-runoff hydrograph for a watershed represents the integrated effect of all the basin physical characteristics and their modifying influence on the translation and storage of a rainfall-excess volume. The factors involved are numerous; some have a major bearing on the phenomena, whereas others are of negligible consequence. Sherman (44) suggests that the dominant factors are:

1. Drainage-area size and shape;
2. Distribution of the watercourses;
3. Slope of the valley sides or general land slope;
4. Slope of the main stream; and
5. Pondage resulting from surface or channel obstructions forming natural detention reservoirs.

DRAINAGE-AREA SIZE AND SHAPE

The major effect of increasing drainage-area size on the geometry of the surface-runoff hydrograph is lengthening of the time-base of the hydrograph (59, p. 40). It follows therefrom that, for a given rainfall excess, the peak ordinate, when expressed in units of cubic feet per second (cfs) per square mile, will likewise decrease with area.

Drainage-area shape is instrumental in governing the rate at which water is supplied to the main stream as it proceeds to the outlet (59, p. 42). It is, therefore, a significant feature which influences the period of rise. For example, a semicircular basin in which the flow converges from all points to the outlet will define a hydrograph with a shorter time to peak than one produced on a long narrow basin of equal area. Langbein and others (30, p. 133) summarize the effect as follows:

> A drainage basin whose drainage tributaries are compactly organized so that water from all parts of the basin has a comparatively short distance to travel will discharge its runoff more quickly and reach greater flood crests than one in which the larger part of the basin is remote from the outlet.

Although drainage areas can adopt a multiplicity of shapes they generally are ovoid or pear-shaped. Dooge (12, p. 57) found that, unless the shape of a watershed deviated appreciably from generally ovoid, the geometry of the hydrograph remained relatively constant.

DISTRIBUTION OF WATER COURSES

The pattern and arrangement of the natural stream channels determine the efficiency of the drainage system. Other factors being constant, the time required for water to flow a given distance is directly proportional to length. Since a well-defined system reduces the distance water must move in overland flow, the corresponding reduction in time involved is reflected by an outflow hydrograph having a short, pronounced concentration of runoff.

SLOPE OF MAIN STREAM

After reaching the main drainageway, the time necessary for a flood wave to pass the outlet is related to the length of traverse and the slope of the waterway. The velocity of flow of water, v, in an open channel may be expressed in the general form

$$v = AR^m S_c^n \quad (2)$$

where A = constant whose magnitude depends on the roughness of the channel,
R = hydraulic radius,
S_c = channel slope, and
m and n = exponents.

It follows from equation 2 that the time, t, required for a particle of water to move a given distance, l, is inversely related to some power of the slope value. According to Manning, the values of the exponents are respectively, $m = 2/3$ and $n = 1/2$. Dooge (12, p. 95) shows that in loose boundary hydraulics, however, roughness and slope are not independent, and that the velocity relationship depends on the size of the bed material. He indicates that, for a channel in equilibrium, the travel time varies inversely with the cube root of the channel slope.

The influence of channel slope is reflected in the time elements of the hydrograph. Since the recession limb represents the withdrawal of water from channel storage, the effect of channel slope should be influential in that portion of the hydrograph. Correspondingly, with increased channel slope, the slope of the recession limb increases, and the base time of the hydrograph decreases.

SLOPE OF VALLEY SIDES OR GENERAL LAND SLOPE

The general land slope has a complex relationship to the surface runoff phenomena because of its influence on infiltration, soil moisture content and vegetative growth. The influence of land slope on hydrograph shape is manifested in the time of concentration of the runoff volumes to defined stream channels. On large watershed areas, the time involved in overland flow is small in comparison with the time of flow in the stream channel. Conversely, on smaller areas, the overland flow regime exerts a dominating effect on the time relationships and the peak of the hydrograph (11).

The velocity of overland flow is not readily computed because of the variations in types of flow that may exist along the paths of transit. Overland flow over smooth slopes may range from purely laminar to purely turbulent. Horton (22, 23) describes an additional type of flow, subdivided flow, in which flow is subdivided by grass or vegetal matter so as to produce a condition where the velocity is practically uniform over the depth of flow. Under this flow condition, resistance is very great.

Theoretical and empirical considerations of the overland flow regime were expressed by Butler (8, p. 316) in the following relationship:

$$q = ay^b S_L^c \quad (3)$$

where q = rate of outflow per unit width,
y = average depth of surface storage,
S_L = land slope, and
a, b and c = coefficient and exponents which vary with Reynold's number, raindrop impact and roughness.

Equation 3 indicates that the effect of land slope is similar to that of channel slope. With increasing land slope, the time elements of the hydrograph decrease.

PONDAGE OR STORAGE

Since storage must first be filled, then emptied, its delaying and modifying effect on the excess precipitation volumes is instrumental in determining hydrograph shape. Much of the variation caused by differences in subintensity patterns and areal distribution of a rain and by differences in the time of travel of runoff volumes from individual subbasins to the outlet is evened out.

Storage effects exist in both overland and channel flow. Sherman (45) summarizes the effect on the unit graph of differences in storage caused by differences in topography as follows:

> Topography with steep slopes and few pondage pockets gives a unit graph with a high sharp peak and short time period. A flat country with large pondage pockets gives a graph with a flat rounded peak and a long time period.

During its passage through a watercourse, a flood wave may be considered to undergo a simple translation (uniformly progressive flow) and reservoir or pondage action (29, p. 562). The extent of modification of the flood wave can be ascertained by employing flood routing procedures if the flow characteristics and the geometrical properties of the stream channel are known. In general, storage causes a decrease in the peak discharge and a lengthening of the time base of the hydrograph.

The foregoing discussion considers only the generalized influences of topographic factors on hydrograph shape. It is impossible, within the bounds of this study, to cover the influence of each individual factor in detail. The effect of each factor may be obscured by the effect of another. The final hydrograph will depend on the cumulative effect of all of the factors as they act alone or in combination with others.

Unit Hydrograph

In 1932, L. K. Sherman (45) advanced the theory of the unit hydrograph or unit graph, now recognized as one of the most important contributions to hydrology related to the surface runoff phenomena. A unit hydrograph is a discharge hydrograph resulting from "1 inch" of direct runoff generated uniformly over the tributary area at a uniform rate during a specified period of time.

The theory is based in principle on the criteria (26, p. 137):

1. For a given watershed, runoff-producing storms of equal duration will produce surface runoff hydrographs with equivalent time bases, regardless of the intensity of the rain;
2. For a given watershed, the magnitude of the ordinates representing the instantaneous discharge from an

area will be proportional to the volumes of surface runoff produced by storms of equal duration; and

3. For a given watershed, the time distribution of runoff from a given storm period is independent of precipitation from antecedent or subsequent storm periods.

The first criterion cannot be exactly correct because the effect of channel storage will vary with stage. Since the recession curves approach zero asymptotically, however, a practical compromise is possible without excessive error (32, p. 445). In addition, the effective gradient and the resistance to flow change with the magnitude of the flood wave.

Sherman (43) confirmed the hypothesis regarding the proportionality of ordinates provided that the selected time unit is less than the minimum concentration period. This was accomplished by reducing the quantitative phenomena of rainfall, loss, pondage and runoff to a problem of hydraulics that could be solved by well-known and accepted hydraulic formulas.

With respect to the third criterion, antecedent precipitation is important to the runoff phenomena primarily because of its effect on the soil infiltration capacity and the resultant total volume of runoff occurring from a given storm.

The unit-graph theory has been generally accepted by most hydrologists. Its use as a hydrologic tool is perhaps best summarized by Mitchell (34, p. 14):

> There has been developed no rigorous theory by which the unit-hydrograph relations may be proven. However, the results which have been obtained by a judicious application of the relationship have been so predominantly satisfactory that there can be no doubt that it is indeed, a tool of considerable value for resolving to some extent the complex relations of rainfall and runoff and for advancing the science of hydrology.

UNIT-STORM AND UNIT-HYDROGRAPH DURATION

Theoretically, an infinite number of unit hydrographs are possible for a given basin because of the effects of rainfall duration and distribution. It is necessary for practical considerations, however, to know the tolerance or range of unit-storm periods within which a given unit graph is applicable. This information is required for the synthesis of a hydrograph for a storm of long duration and the development of a representative unit graph for an area.

Several investigators have expressed different opinions, based on experience, regarding the critical rainfall duration for a given basin. Wisler and Brater (59, p. 247) employ a unit storm defined as "a storm of such duration that the period of surface runoff is not appreciably less for any storm of shorter duration." The authors found that an appropriate duration of the unit storm varies with characteristics of the basin. For small watersheds (areas less than 10 square miles), unit hydrographs result from short, isolated storms whose durations are less than the period of rise. For large watersheds, however, the unit-storm duration may be less than the period of rise, possibly no more than half as long (59, p. 248). Wisler and Brater recommend that, in applying the distribution graph[3] to a given storm sequence on small watersheds:

> The volume of rainfall excess may be converted to runoff by means of a single application of the distribution graph, if its duration is no longer than the period of rise. The graph resulting from a longer rain must be derived by successive applications of the distribution graph to unit durations of rainfall excess.

For the larger areas they conclude:

> The distribution graph is not a sufficiently precise tool to be sensitive to differences in duration of rainfall excess that are small compared with the period of rise . . . . It will require further research before enough experimental evidence is available to establish the nature of the variation for small changes in duration.

The more common principle is to associate the unit graph with the storm from which it was produced. For example, for a given area there may be a 2-hour unit graph or a 6-hour unit graph, depending on whether the unit-storm duration was either 2 hours or 6 hours, respectively, and provided that the time of concentration of the basin had not been exceeded. Unit graphs for various storm durations can be developed from one of known duration using the S-curve technique as outlined by Linsley, *et al.* (32, p. 451 ff.).

The selection of a proper time period for unit hydrographs is important. Sherman (46, p. 524) suggests the following criteria to be used in its selection:

> For areas over 1000 square miles use 12-hour units in preference to 24 hours. For areas between 100 and 1000 square miles use units of 6, 8 or 12 hours. For areas of 20 square miles use 2 hours. For smaller areas use a time unit of about one-third or one-fourth of the approximate concentration time of the basin.

Mitchell (34, p. 30) recommends that the storm duration or unit-hydrograph duration which is most convenient for use on any basin is about 20 percent of the time between the occurrence of a short storm of high intensity and the occurrence of peak discharge. He relates (34, p. 35):

> The effect upon the unit hydrograph becomes significant only when there is substantial variation between the unit-hydrograph duration and the storm duration . . . . It is usually permissible to allow the storm duration to vary between 50 per cent and 200 per cent of the unit-hydrograph duration before any correction factor for this effect will become necessary.

Linsley, *et al.* (33, p. 195) cite that in practical applications, experience has shown that the time unit employed should approximate one-fourth of the basin lag time (time from the center of mass of effective precipitation to the peak of the unit graph). They suggest that the effect of small differences in storm duration is not large and that a tolerance of ± 25 percent from the adopted unit-hydrograph duration is acceptable.

Yet another criterion is adopted by the United States Army Corps of Engineers (56, p. 8). They found that, for drainage areas of less than 100 square miles, values of the unit-storm duration equal to about half the basin lag time appear to be satisfactory.

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[3]Since the distribution graph is simply a modified form of the unit hydrograph, all principles governing the selection, development, synthesis and application of one graph also apply in the case of the other.

MATHEMATICAL INTERPRETATION OF THE UNIT HYDROGRAPH

Among the most recent contributions to the field of hydrology has been the development of theoretical expressions which define the geometry of the unit graph. Two such mathematical expressions have been proposed, one by Edson (13) and the other by Nash (38). Since these results occupy an important role in the current study, the complete derivation given by each author is listed in Appendix B. Although the resultant equations 30h and 31f, Appendix B, were founded on different underlying assumptions, both may be reduced to the common form

$$Q_t = \frac{V(\alpha)^{\beta}}{\Gamma(\beta)} e^{-\alpha t} t^{\beta - 1} \qquad (4)$$

where Q_t = instantaneous ordinate of the unit graph at time, t,
V = volume,
α = parameter having the dimensions of the reciprocal of time,
β = dimensionless parameter,
e = base of the natural logarithms, and
$\Gamma(\beta)$ = gamma function of β.

The result is especially applicable to the formulation of a synthetic procedure. Foremost, for this purpose, it offers the investigator a useful tool whereby a solution can be obtained in logical sequence from reason to result. Edson explains that the general failure encountered in correlating physical characteristics of the basin and the hydrograph properties, peak discharge and period of rise, may be attributed to the fact that the functional relationships between this latter set of factors and the parameters α and β are sufficiently complex to restrict a satisfactory tie-in.

Use of the two-parameter equation enables description of the complete unit graph once the relationships between the physical characteristics and the parameters α and β have been established. Thus, the necessity for single-point correlations, as used almost exclusively in the past, can be eliminated. In addition, the mathematical form is particularly adaptable for use in high-speed computers. The application of the continuous curve is advantageous to practically all hydrologic problems.

Distribution Graph

As an outgrowth of the unit-hydrograph principle, Bernard (5) conceived the concept of the distribution graph. A distribution graph is a unit hydrograph of surface runoff modified to show the proportional relation of its ordinates, expressed as percentages of the total surface-runoff volume. In accordance with the unit-hydrograph principle, if the base time of the unit hydrograph is divided into any given number of equal time increments, the percentage of the total volume of flow that occurs during a given time interval will be approximately the same, regardless of the magnitude of total runoff.

Since the area under each distribution graph is equal to 100 percent, differences in the runoff characteristics between watersheds are reflected in the respective shapes of their distribution graphs. The distribution graph is used in preference to the unit graph when hydrograph characteristics from areas of different size are compared.

Synthetic Unit Hydrographs

Numerous procedures have been derived whereby the unit hydrograph for an ungaged area can be constructed. Each procedure, however, differs somewhat from another—either in the relationships established or in the methodology employed. The ensuing discussions are confined to brief summaries of the more pertinent synthetic techniques published in the literature.

SNYDER

Snyder (48), in 1938, was the first hydrologist to establish a set of formulas relating the physical geometry of the basin and properties of the resulting hydrograph. In a study of watersheds located mainly in the Appalachian Highlands, which varied in size from 10 to 10,000 square miles, he found that three points of the unit hydrograph could be defined by the following expressions:[4]

$$t_L = C_t \, (L L_{ca})^{0.3} \qquad (5a)$$

where t_L is the basin lag (time difference in hours between the centroid of rainfall and the hydrograph peak), L is the length of the main stream in miles from the outlet to divide, and L_{ca} is the distance in miles from the outlet to a point on the stream nearest the center of area of the watershed. For the watersheds studied, the coefficient, C_t, varied from 1.8 to 2.2.

$$Q_p = (640 \, C_p \, A)/t_L \qquad (5b)$$

where Q_p is the peak discharge of the unit hydrograph in cfs, and A is the drainage area in square miles. The coefficient, C_p, ranged in magnitude from 0.56 to 0.69.

$$T_B = 3 + 3 \, (t_L/24) \qquad (5c)$$

where T_B is the length of the base of the unit hydrograph in days.

Equations 5a, 5b and 5c define points of a unit hydrograph produced by an excess rain of duration, $t_r = t_L/5.5$. For storms of different rainfall durations, t_R, an adjusted form of lag, t_{LR}, determined by the equation

$$t_{LR} = t_L + (t_R - t_r)/4 \qquad (5d)$$

must be substituted in equations 5b and 5c.

Once the three quantities, t_L, Q_p and T_B, are known, the unit hydrograph can be sketched. It is constructed so that the area under the curve

[4]To be consistent, the symbols have been changed from those appearing in the original articles to conform to the designations used throughout the bulletin.

represents a 1-inch volume of direct runoff accruing from the watershed. As an aid to this sketching process, the Corps of Engineers (56) has developed a relation between the peak discharge and the width of the unit hydrograph at values of 50 percent and 75 percent of the peak ordinate.

A study similar to that of Snyder's was conducted by Taylor and Schwarz (52) on 20 watersheds which varied in size from 20 to 1,600 square miles, and were located in the Atlantic States. In this study, the relationships given for lag and peak discharge included a weighted slope term.

COMMONS

In 1942, Commons (11) suggested that a dimensionless hydrograph, the so-called basic hydrograph, would give an acceptable approximation of the flood hydrograph on any basin. This hydrograph was developed from flood hydrographs in Texas. It is divided so that the base time is expressed as 100 units, the peak discharge as 60 units and the area as a constant 1,196.5 units.

The absolute values for a hydrograph are established once the volume of runoff and peak discharge are known. The volume in second-foot-days is divided by 1,196.5 to establish the value of each square unit. Dividing the peak flow by 60 gives the value of one unit of flow in cfs. The magnitude of one time unit is then computed by dividing the value of the square unit by that of the flow unit. Finally, the hydrograph is synthesized by converting listed coordinates of the basic graph to absolute time and discharge readings according to the calculated conversions.

SOIL CONSERVATION SERVICE (SCS)

The method of hydrograph synthesis used by the SCS employs an average dimensionless hydrograph developed from an analysis of a large number of natural unit hydrographs for watersheds varying widely in size and geographical location (55, pp. 3.16-4ff). This dimensionless hydrograph has its ordinate values expressed as the dimensionless ratio, Q_t/Q_p, and its abscissa values as the dimensionless ratio, t/P_R. Q_t is the discharge at any time, t, and P_R is the period of rise. For a given watershed, once the values of Q_p and P_R are defined, the unit hydrograph can be constructed. The following expressions are given for this purpose:

$$Q_p = (484\ AV)/P_R. \tag{6a}$$

where V is the volume of runoff in inches, which for a unit hydrograph is unity. With A expressed in square miles, V in inches and P_R in hours, the units of Q_p are cubic feet per second. P_R is determined from the expression

$$P_R = t_R/2 + t_L. \tag{6b}$$

The lag, t_L, can be estimated in two ways, either by the expression

$$t_L = \frac{\Sigma A_x V_x T_x}{\Sigma A_x V_x} \tag{6c}$$

where A_x and V_x are, respectively, the area and depth of runoff of subarea x, and T_x is the time required for water to travel from the centroid of the subarea to the basin outlet, or by the expression

$$t_L = 0.6T_c \tag{6d}$$

where T_c is the time of concentration. T_c can be obtained from expressions given in the SCS handbook (55) or from data reported by Kirpich (27).

HICKOK, KEPPEL AND RAFFERTY

The approach to hydrograph synthesis given by Hickok, *et al.* (17) is very similar to that employed by the SCS. However, the Hickok, *et al.* investigations were confined entirely to small watershed areas. The runoff characteristics of 14 watersheds which vary in size from 11 to 790 acres, located in semiarid regions, were investigated, and an average dimensionless graph (Q_t/Q_p versus t/t_L') was developed. In this study, lag t_L' was taken as the time difference between the centroid of a limited block of intense rainfall and the resultant peak discharge. The authors presented two different methods of determining lag.

For reasonably homogenous semiarid rangelands up to about 1,000 acres in area,

$$t_L' = K_1 (A^{0.3}/S_L\sqrt{DD})^{0.61} \tag{7a}$$

where S_L is the average land slope of the watershed and DD is the drainage density. With A in acres, S_L in percent, DD in feet per acre, and K_1 equal to 106, lag is given in minutes.

For watersheds with widely different physiographic characteristics,

$$t_L' = K_2\left(\frac{\sqrt{L_{csa}} + W_{sa}}{S_{La}\sqrt{DD}}\right)^{0.65} \tag{7b}$$

where L_{csa} is the length from the outlet of the watershed to the center of gravity of the source area in feet, W_{sa} is the average width of the source area in feet and S_{La} is the average land slope of the source area. The source area was considered to be the half of the watershed with the highest average land slope. The coefficient, K_2, is taken equal to 23.

The authors suggested that Q_p could be obtained from the relation

$$\frac{Q_p}{V} = \frac{K_3}{t_L'} \tag{7c}$$

which gives Q_p in cfs when V is expressed in acre feet, t_L' in minutes, and K_3 taken equal to 545.

CLARK

In 1943, Clark (10) suggested that the unit hydrograph for an area could be derived by routing its time-area concentration curve through an appropriate amount of reservoir storage. In the routing procedure, an instantaneous unit hydrograph (hydrograph resulting from an instantaneous rainfall of 1-inch depth and duration equal to zero time) is formed. The unit hydrograph for any rainfall duration, t_R, can be obtained from the instantaneous graph by averaging the ordinates of the instantaneous graph t_R-units of time apart and then plotting the average discharge at the end of the interval.

Clark used the Muskingham method of flood routing. The basic equations employed in this method are:

$$I - O = dS/dt \quad (8a)$$
$$S = KQ \quad (8b)$$
$$Q = xI + (1-x)O \quad (8c)$$

where I = inflow rate,
O = outflow rate,
S = storage,
t = time,
K = storage constant,
Q = weighted discharge, and
x = dimensionless weight factor.

In flood routing through a reservoir, storage is directly related to outflow; thus, the factor, x, is equal to zero. Equations 8a, 8b and 8c can be combined to the simplified form

$$I - O = KdO/dt. \quad (8d)$$

To apply this procedure to a given watershed, estimates of the storage constant, K, and lag through the basin must be obtained. Clark suggested that K can be approximated by the relation

$$K = cL/\sqrt{S_c} \quad (8e)$$

where S_c is the mean channel slope. For L expressed in miles, c varies from about 0.8 to 2.2.

Linsley (31) in a discussion of Clark's paper conceived that the comparative magnitude of flood flows and storage in the tributaries would affect the relationship. He recommended the inclusion of the square root of area term in equation 8e as a measure of these factors. The equation formed is

$$K = \frac{bL\sqrt{A}}{\sqrt{S_c}} \quad (8f)$$

where b is a coefficient.

Linsley, *et al.* (33, p. 241) suggest that the value of t_L computed from a recognized formula can be used as an approximation of basin lag.

For further information on the use of routing techniques for hydrograph synthesis, refer to the works of Horton (24), Dooge (12) and Nash (38).

INVESTIGATIONS, RESULTS AND DISCUSSION

The basic format of this bulletin has been designed to combine the individual sections of Investigations, Results and Discussion for each phase of the problem.

The material is presented in a sequence similar to that in which the work was completed. The initial phase entailed the procurement, organization and basic analyses of the topographic and hydrologic data. The important features of this part of the study included: the derivation of geometric properties of the watersheds, the listing of significant storm characteristics, the plotting of hydrographs and the development of a representative distribution graph for each basin, and a discussion of the salient relation between rainfall and runoff characteristics.

These results provided the basis upon which the synthetic technique was formulated. The theoretical work by Edson and Nash shows that the geometry of a unit hydrograph can be described by a two-parameter equation (see equation 4). The necessity for point correlations is thus eliminated provided that the two constants can be evaluated and their relation with physical properties of the watersheds established.

These parameters were approximated by the best-fit estimators of q and γ', of the two-parameter gamma distribution, obtained by fitting this distribution to a dimensionless form of a representative distribution graph of each watershed. In the dimensionless form, the time relationships of the distribution graph were based on the period of rise, P_R.

Once the three variables, P_R, q and γ', for any watershed are known, its dimensionless graph, distribution graph and unit hydrograph can be constructed. The final step in the development of the synthetic method involved the determination of prediction equations from which values of the three parameters could be estimated from topographic characteristics.

Basic Data

A complete listing of the basic topographic and hydrologic data employed in the study is given in tables C-1, C-2, C-3 and C-4, Appendix C. These records were obtained from the listed collection agencies either by onsite visits to the location or through personal communication. A complete file of these data is maintained at the Department of Agricultural Engineering, Iowa State University of Science and Technology, Ames, Iowa.

Topographic Characteristics

The unit graph or distribution graph represents the integrated effect of all of the sensibly constant basin factors and their modifying influence on the translation and storage of runoff from unit storm. It follows, therefrom, that pertinent characteristics of these graphs should be related to significant features of the basin. Five physical characteristics of each watershed were measured in an attempt to determine these relations. They included: drainage-area size, A; length of the main stream, L; length to center of area, L_{ca}; slope of the main stream, S_c; and mean land slope,

S_L. A complete definition of each term as applied to this bulletin is given in the glossary of terms (Appendix A).

The initial approach used to establish relationships between hydrograph geometry and basin properties was to employ the principles of dimensional analysis (37) to reduce the number of variables involved. Some work regarding the application of these principles to the field of geomorphology has been reported by Strahler (50). Accordingly, in applying these principles, the variables employed must be selected with great care such that a dependent variable can be functionally related to a system of independent variables and to no others.

The use of dimensional analysis to obtain the desired relations proved relatively unsuccessful, however. A possible reason for this failure was the lack of independence of the variables used. As a consequence, a study was initiated to determine whether the various topographic factors were related.

LENGTH OF THE MAIN DRAINAGEWAY, L, AND DRAINAGE-AREA SIZE, A

Superficially, an investigator might presume that the variables L and A would be poorly related because of the diversity in shapes expected between watersheds. In an effort to test this assumption, the values of L and A were plotted on logarithmic paper as shown in fig. 3. These data were supplemented with similar results reported by Taylor and Schwarz (52) to increase range of the resultant plot. The regression line fitted to the points is defined by the equation

$$L = 1.40A^{0.568}. \tag{9}$$

An "F" test (40, p. 49), applied to the result indicated that for the experimental data, the regression line significantly defines the relation between L and A. This result provides evidence that the two factors are not independent and, therefore, prohibits their use as independent terms in dimensional analysis techniques.

The length of the main stream corresponding to a given watershed area can be obtained with reasonable accuracy from equation 9. The standard error of estimate for the regression was determined to be 24.8 percent (58).

LENGTH TO CENTER OF AREA, L_{ca}, AND LENGTH OF THE MAIN DRAINAGEWAY, L

Equation 9 suggests that the watersheds studied do not deviate appreciably in geometric form. If this characteristic persists, it follows that L_{ca}, the shape parameter, and L should be closely related. These data are presented graphically in fig. 4. As in the previous case, data reported by Taylor and Schwarz (52) were included.

A regression analysis applied to these values showed that the relationship between L_{ca} and L was significantly defined by the equation

$$L_{ca} = 0.54\,L^{0.96}. \tag{10a}$$

The standard error of estimate from regression was determined to be 14.8 percent.

Equation 10a suggests two important implications. First, the interdependence of the two parameters, L_{ca} and L, restricts their usage as independent terms in dimensional analysis. Second, the use of the product term, LL_{ca}, as used in many synthetic procedures, has little advantage over the use of either L or L_{ca} alone.

For practical purposes, equation 10a may be reduced to the form

$$L_{ca} = 0.50L. \tag{10b}$$

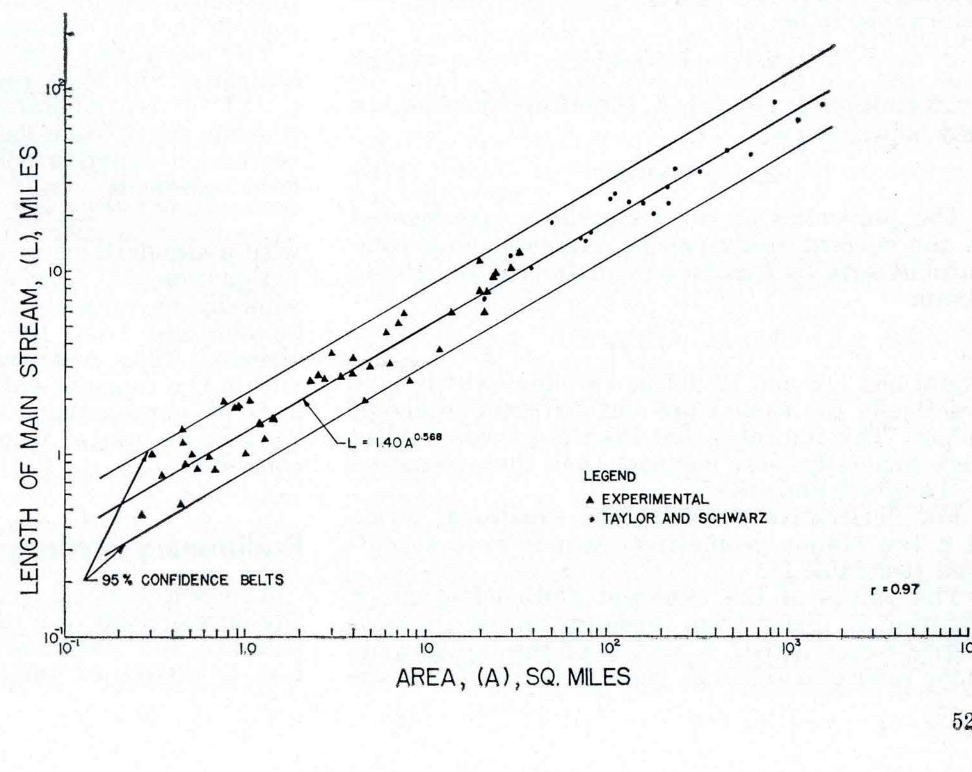

Fig. 3. Relation of length of main stream, L, and watershed area, A.

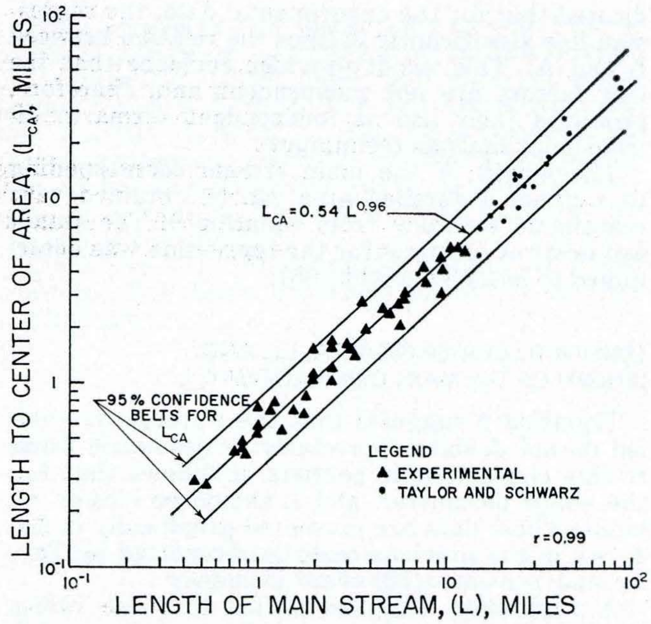

Fig. 4. Relation of length to center of area, L_{ca}, and length of main stream, L.

DISCUSSION

The general compactness or shape characteristics of the watersheds listed were compared with those of 340 drainage basins from the northeastern United States reported by Langbein and others (30). They evaluated the area-distance property for each of the watersheds by the factor, Σal, or the product of each partial area, a, by the channel distance from the midpoint of the main stream serving it downstream to the gaging station, l. The regression of the factor, Σal, with drainage area, A, for the 340 drainage basins was determined to be

$$\Sigma al = 0.90\ A^{1.56}. \qquad (11a)$$

By definition, $L_{ca} = \Sigma al/A$, therefore equation 11a may be written as

$$L_{ca} = 0.90\ A^{0.56}. \qquad (11b)$$

The properties of the watersheds investigated in the current study can be expressed in a comparable form by combining equations 9 and 10a to obtain

$$L_{ca} = 0.73\ A^{0.55}. \qquad (12)$$

Equations 11b and 12 define two lines which have practically the same slope but different intercept values. This indicates that the watersheds studied were generally more compact than those reported by Langbein and others.

For illustrative purposes, the equational forms of a few simple geometrical shapes were considered (see table 1).

The values of the exponent and coefficient of equation 12 differ from those for any of the geometric forms listed. A review of the topographic maps indicated that the general shape of the watersheds was intermediate between ovoid and pear-shaped. This observation does not contradict the proportion given by equation 10b which also defines a square, rectangle or circle because of the influence of the sinuosity of the streams.

Table 1. Relationship between basin characteristics, L_{ca} and A, for three simple geometric forms (30, p. 135).

Geometric shape	Equational form between Lca and A
Glory hole or funnel	$L_{ca} = 0.375\ A^{0.50}$
Equilateral triangle with outlet at one of the vertices	$L_{ca} = 0.94\ A^{0.50}$
Square with outlet at one of the corners	$L_{ca} = 0.76\ A^{0.50}$

MEAN LAND SLOPE, S_L, AND SLOPE OF THE FIRST-ORDER STREAMS, s_1

In the study, the mean land slope, S_L, was taken as a quantitative measure of the general land slope of a watershed. Several methods are available whereby S_L can be determined for a given area. Two common methods are the intersection-line method and the grid-intersection method (20). Regardless of the method employed, the labor involved in computation is extensive, and, in addition, the task requires topographic maps.

In an effort to minimize labor and to overcome difficulties arising from limited topographic information in the determination of S_L, an attempt was made to relate the variable with a more readily measurable basin characteristic. It was hypothesized that the slopes of the first-order streams (21, p. 281) were related to their respective values of S_L. For a given area, the slopes of first-order streams can be determined either from topographic maps or by field investigations with the aid of a barometric altimeter. When topographic maps are used, the delineation of the first-order streams should be accomplished by the contour method discussed by Morisawa (36).

The mean land slopes from 16 watersheds were compared with their respective mean-slope values, s_1, of a representative sample of first-order streams taken from each basin (see fig. 5). The regression equation computed by the method of least squares is

$$S_L = 0.86 s_1^{0.67} \qquad (13)$$

with a standard error of estimate of 28.6 percent.

Equation 13 furnishes a simple relationship whereby an estimate of the mean land slope can be obtained from the slopes of the first-order streams. The empirical results are valid only within the range of data included. Because of the ease of measurement of s_1, however, additional work is warranted to establish the relation more concretely.

Preliminary Hydrograph Analysis

The selection of hydrologic data suitable for the development of a distribution graph tests the patience and judgment of the investigator. The task is simplified when both rainfall and runoff

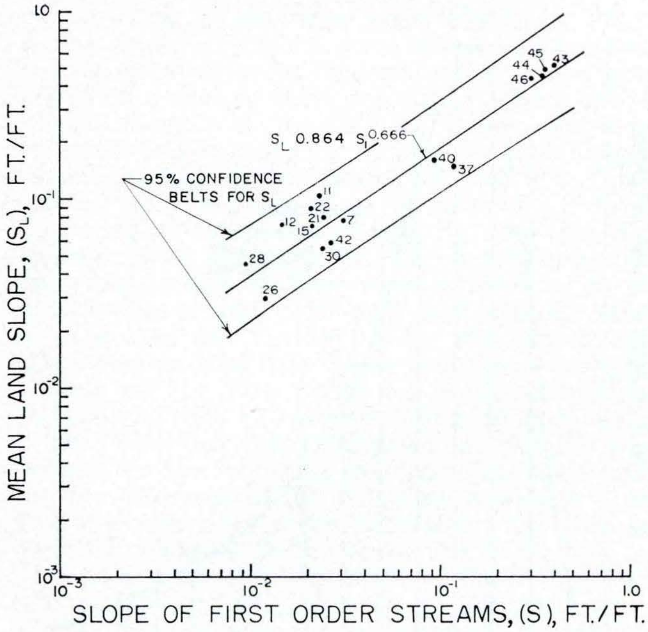

Fig. 5. Relation of mean land slope, S_L, with the average slope of a representative sample of first-order streams, s_1, from the same watershed.

records are available. Frequently, however, the difficulty encountered is that of selecting suitable records without the aid of adequate precipitation data. The influence of rainfall duration and distribution on the shape of the hydrograph and on the salient relationships between rainfall and runoff may cause considerable variation between distribution graphs and between the lag times of a given basin. To reduce the possibility of large errors in the results, consistent procedures were used in selecting the data.

Barnes (3), Bernard (5) and Brater (7, p. 1,196) have suggested various criteria requisite for the selection of hydrologic data suitable for distribution-graph and/or unit-hydrograph development. These were summarized to formulate the basis of the following list of standards used in this study:

1. The rain must have fallen within the selected time interval or time unit and must not have extended beyond the period of rise of the hydrograph.
2. The storm must have been well-distributed over the watershed, all stations showing an appreciable amount.
3. The storm period must have occupied a place of comparative isolation in the record.
4. The runoff following a storm must have been uninterrupted by the effects of low temperatures and unaccompanied by melting snow or ice.
5. The stage graphs or hydrographs must have a sharp, defined, rising limb culminating to a single peak and followed by an uninterrupted recession.
6. All stage graphs or hydrographs for the same watershed must show approximately the same period of rise.

The degree of adherence to these criteria in selecting data from a particular watershed was dictated by the accessibility and availability of these data. In some cases, because of an insufficient number of hydrographs, it was necessary to select those which were affected by small rains occurring either before or after the principal burst. In these cases, the "parasite" graphs were separated from the main graph by accepted hydrologic procedures (32, p. 447, 449).

A list of the storms selected for each watershed and pertinent information related to their characteristics appears in table C-4, Appendix C.

DEVELOPMENT OF AN EMPIRICAL DISTRIBUTION GRAPH

The hydrographs and stage graphs selected were reduced to distribution graphs in a manner outlined in Appendix D.

A representative distribution graph for a given basin may be developed using one of several methods recommended by hydrologists. Linsley, *et al.* (33, p. 198) advise that the correct unit graph may be obtained by plotting the separate unit graphs with a common time of beginning of excess precipitation and sketching a mean graph which conforms to the individual graphs as closely as possible and passes through the average peak discharge and period of rise for the group.

Brater (7, p. 1,201) developed a composite distribution graph for each of the Coweeta watersheds by the following procedure: All of the distribution graphs for each stream were first superimposed as nearly as possible on each other. The composite graph for the area was then developed either by selecting one of the individual graphs as representing an average or by drawing the average graph through the cluster and listing the percentages at selected time intervals.

Another technique utilized by Mitchell (34, p. 34) recommends that the separate graphs be superimposed to a position of best fit and then the ordinates averaged to obtain the average distribution graph. In determining the position of best fit, the timing of the various elements are given weight in the following order of decreasing importance:

1. Maximum ordinate,
2. Time of occurrence of precipitation excess,
3. Ascending limb of the hydrograph and
4. Descending limb of the hydrograph.

The major difference between the methods arises in positioning the separate graphs to the position of best fit. Care must be given to this aspect, otherwise an incorrect representative graph may result. If, for instance, positioning is disregarded and the concurrent ordinates simply averaged, the resultant graph will have a lower peak and broader time base.

In this study, the method of resolving a representative graph for an area was controlled by the scatter of the original data. The times-of-occurrence and magnitudes of the peak discharges were considered the most significant factors. When the individual graphs plotted with a common time of beginning of surface runoff showed small time variations at the peak discharge, the average graph was obtained by the method described by Linsley, *et al.* (33). If, on the other hand, the composite plot indicated large differences in timing of the peaks so as to restrict the graphic determination of an average peak, the graphs were positioned to a location of best fit in accordance with Mitchell. The average period of rise and peak discharge then were obtained, and an average distribution graph was constructed by trial plottings.

It is relatively easy to check the final graph selected since the sum of the ordinates of a distribution graph must equal 100 percent. It is necessary in final results to complete adjustments of the initial trial graphs to satisfy this criterion.

The representative distribution graph of an area developed in this manner was designated as the empirical graph of the watershed. The terminology "empirical" was adopted to infer that the graph was developed from empirical data and to avoid the possibility of misinterpretation conveyed by the words mean or average.

The empirical graphs for the 42 watersheds are presented in figs. D-5 through D-15, Appendix D.

RELATIONSHIP BETWEEN EMPIRICAL GRAPH AND STATISTICAL GAMMA DISTRIBUTION

The mathematical expressions proposed by Edson and Nash (see Appendix B) to describe the unit graph may be replaced by the generalized form given by equation 4. Since the characteristic shape of a unit graph is retained by the distribution graph, this equation is also applicable in describing the latter. Only appropriate changes to the dimensions of the constants must be considered.

The shape of the unit graph or distribution graph appears to follow the form of a skew statistical frequency curve. This property is easily perceived when the distribution graph of a watershed is plotted as a discrete frequency histogram (see fig. 6). The analogy is further supported by presenting the ordinate values as a percent flow based on a given time increment.

One of the most common and most flexible of the frequency curves, which has been used numerous times in the analysis of hydrologic data, is Pearson's Type III curve. The equation for this distribution is given by Elderton (14) in the form

$$f(x) = y = (N/a)\frac{(p)^{p+1}}{e^{p}\Gamma(p+1)} e^{-cx}(1 + x/a)^{ca} \qquad (14a)$$

where the origin is at a, the mode. The origin can be transferred to zero by making the appropriate substitutions $x - a = x$ and $p = ca$, into equation 14a to obtain

$$f(x) = y = \frac{N}{\Gamma(p+1)}(c)^{p+1} e^{-cx} x^{p}. \qquad (14b)$$

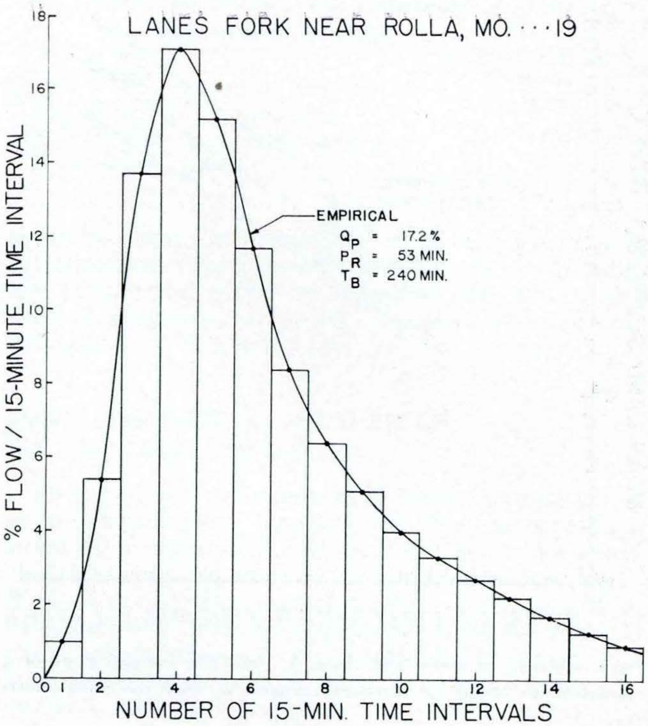

Fig. 6. Empirical graph for watershed 19 plotted as a frequency histogram.

It follows, if $q = p-1$ and $\gamma = c$, equation 14b further reduces to

$$f(x) = y = \frac{N(\gamma)^{q}}{\Gamma(q)} e^{-\gamma x}\, x^{q-1} \qquad (14c)$$

where $f(x) = y$ = any "y" value,
x = any "x" value,
N = total frequency or number of observations of x,
γ and q = scale and shape parameters, respectively, estimated from observed x values,
$\Gamma(q)$ = gamma function of q and
e = base of the natural logarithms.

Equation 14c defines a particular type of Pearson's Type III curve which is commonly referred to as the two-parameter or incomplete gamma distribution.

It is easily recognizable that equations 4 and 14c are identical when the following equalities exist:

$$f(x) = y = Q_t$$

$$N = V$$

$$x = t$$

$$\gamma \text{ and } q = \alpha, \beta.$$

On the basis of this evidence, it was assumed that the empirical graphs could be defined using the two-parameter gamma distribution as the model with values of q and γ estimated from the experimental data by statistical procedures.

DEVELOPMENT OF DIMENSIONLESS GRAPHS

The empirical graphs were reduced to a standardized form to avoid inconsistencies in the time increments used in their description. Each graph was adjusted with its ordinate values expressed in percent flow based on a time increment equal to one-quarter of the period of rise (% flow/ $0.25P_R$) and the abscissa as the ratio of any time, t, divided by the period of rise, P_R, (see figs. E-2 through E-12, Appendix E). The empirical graphs described in this manner were referred to as dimensionless graphs. Although each ordinate value is expressed as % flow/$0.25P_R$, the connotation simply infers that it is the percentage of the total volume of the flow based on a time-increment duration of 0.25 P_R; percent being dimensionless.

The time-increment duration of $0.25P_R$ was chosen for the following reasons: (a) The period of rise was ascertained to be an important time characteristic of a given watershed; (b) the use of $0.25P_R$ enables definition of the rising limb at four points; and (c) the shape of the hydrograph was retained by using this size of increment.

The dimensionless graph represents a modified form of the unit hydrograph in which the basic shape has been retained. Its geometry can be described by modifying the constants of equation 4. The general equation for the dimensionless graph can thus be expressed as:

$$Q_{t/P_R} = \frac{V'(\alpha')^{\beta}}{\Gamma(\beta)} e^{-\alpha' t/P_R} \; t/P_R{}^{\beta-1} \quad (15a)$$

where Q_{t/P_R} is the % flow/0.25 P_R at any value of t/P_R, V' is the volume in percent and α' is a dimensionless parameter.

Fitting the Two-Parameter Gamma Distribution to the Dimensionless Graph

The evaluation of the parameters, α' and β, of equation 15a from empirically-derived data by the usual curve-fitting procedures of the method of least squares or the method of moments is a cumbersome and laborious task. Nash (38, 39) has given a procedure for evaluating the parameters, k and n, of equation 31f (see Appendix B) from storm data by the method of moments. The application of this technique was prohibited, however, because of the limited rainfall data available.

The equality of the equational forms for the two-parameter gamma distribution and the unit hydrograph has been established. It can be assumed that each dimensionless graph represents a sample of t/P_R values taken from the gamma population (defined by the parameters q and γ'); in which γ', a dimensionless quantity, replaces the scale parameter, γ, of equation 14c. Thom (53) found that efficient estimates of the parameters of the two-parameter gamma distribution could be obtained by the method of maximum likelihood. This method was used exclusively in the application of the distribution for the evaluation of the drouth hazards in Iowa as reported by Barger and Thom (2).

The latter study is cited further because of an additional contribution made: The programming of the two-parameter gamma distribution to the IBM-650 computer (16). Consequently, the maximum likelihood estimates of q and γ' could be obtained from the dimensionless graphs by machine calculations. The use of this program resulted in a material reduction in time, labor and cost in the current study.

The procedures involved in organizing and processing the dimensionless graphs to obtain the parameters by machine calculation are given in Appendix E. Each of the dimensionless graphs of the 42 watersheds included in the study was treated the same. In addition, it is shown in Appendix E that the equational form of the two-parameter gamma distribution describing the dimensionless graph (see equation 14c) can be written

$$Q_{t/P_R} = \frac{25.0(\gamma')^{q}}{\Gamma(q)} e^{-\gamma' t/P_R} \; t/P_R{}^{q-1}. \quad (16)$$

The work involved in solving equation 16 is reduced considerably by the use of appropriate mathematical tables (1, 18).

GOODNESS OF FIT OF FITTED DISTRIBUTIONS

Figures E-2 through E-12, Appendix E, show the best-fit gamma distributions plotted with their respective dimensionless graphs. It is evident from the figures, that the relative degree to which the fitted curve approximates the actual graph varies considerably. This is well-illustrated by comparing the curves for watershed 9, fig. E-4, and those for watershed 25, fig. E-8, Appendix E.

An attempt was made to minimize the effect of these differences to the precision of fit in further correlation studies involving the parameters, q and γ', by choosing the values of the parameters from curves which exhibited good fit. Great difficulty was encountered, however, in selecting a suitable index of goodness of fit. The problem was manifested when considering both statistical and practical aspects.

The chi-square test may be used to obtain a statistical measure of the goodness of fit (40, p. 65). Chi-square values, χ^2, are obtained by the formula

$$\chi^2 = \sum_{i=1}^{k} (O_i - E_i)^2/E_i \quad (17)$$

where O_i = observed percent flow,
E_i = theoretical or expected percent flow, and
k = number of classes or increments, t/P_R = 0.25.

The probability level, P, of obtaining the calculated χ^2 value is obtained by comparing its magnitude with tabulated values at k−3 degrees of freedom (40, p. 445).

This test was completed for the two curves of each watershed. The probability levels of the calculated χ^2 values ranged from a minimum, P =

0.25, for watershed 13 (fig. E-5, Appendix E) to a maximum, P = >0.9995, for watersheds 4, 23 and 25 (figs. E-2, E-7 and E-8, Appendix E).

On the basis of this evidence, the hypothesis that the actual curve is of the same population as the fitted curve cannot be rejected. By the same reasoning, the goodness of fit cannot be considered highly significant except in special cases, such as for the latter-mentioned watersheds. For these watersheds, the evidence is conclusive that the fit is good, and the dimensionless graph can be represented by the two-parameter gamma distribution.

This discussion does not conclude the argument that the goodness of fit is adequate in all cases from a practical aspect. Hydrologists are concerned primarily with the agreement of the graphs within the portion bounded by the crest segment. Large discrepancies within this segment invalidate the usefulness of the fitted curves for design purposes, especially for full-flow type structures. An example of wide variation is shown between the curves for watershed 11, fig. E-4, Appendix E.

The major problem is one of quantitatively defining hydrologic acceptance. Any measure used must take into consideration the accuracy of the measuring instruments, the scatter and deviations of the original data from the empirical graphs, and the effect of the lack of agreement of the curves on calculated hydrographs for design storms of long duration. At the time of this phase of work, additional consideration had to be given to the uncertainty of the expected relationships involving the parameters, q and γ', and basin characteristics. As a result, rather than attempting to determine an elaborate test for evaluating the precision of fit within the crest segment, an arbitrary "point" criterion was established. Hereafter, a satisfactory fit connotes that the fitted curve agreed within $\pm$ 20 percent of the dimensionless graph at the peak ordinate. The parameters, q and γ', from the fitted curves which adhered to this criterion were used in further investigations.

Additional basic studies are needed concerning the application of statistics as a measure of the variation of hydrologic data. For the particular problem indicated, a significant contribution could be made in developing a method to test the agreement of the curves within the crest segment. The association of the adopted measure and practical considerations will be resolved in the application of the synthetic method to actual storm data.

MODIFIED DIMENSIONLESS GRAPHS

Experience indicated that poor agreement between the fitted and dimensionless graphs generally occurs either:

1. When the dimensionless graph is of apparent different geometric shape than the gamma distribution (see watershed 12, fig. E-4, Appendix E), or
2. When the dimensionless graph exhibits a prolonged recession (see watershed 11, fig. E-4, Appendix E).

Obtaining a dimensionless graph which exhibits a different shape than the gamma distribution is not unlikely considering the numerous factors affecting its geometry. For such cases, the agreement between the two curves would be poor since the comparison is essentially between empirically derived data from one population and a theoretical model describing another population. Closer approximations would result by fitting these data to a more appropriate model, or possibly to two different models—one describing the rising limb, the other the recession limb.

The prolonged, extended, recession limb of a dimensionless graph for a given area is probably the result of one of two causes: (a) Either the area in question has very large storage characteristics or (b) an appreciable contribution of flow has occurred as interflow (35). For these data, the fitted and the experimental curves deviate appreciably within the crest segment. Since the method of maximum likelihood provides the "best-fit" line over the entire curve, greater significance is given to the recession limb than to the crest segment. Thus, greater error is induced to the "best-fit" line near the center. Nash (38) encountered similar difficulties by using the method of moments as the fitting procedure.

The magnitude of this difference possibly may be reduced by (a) increasing the number of points describing the dimensionless graph, (b) applying different statistical fitting methods, or (c) force-fitting. Of the three alternatives, the third offers the greatest potential with minimum labor. The versatility of the gamma distribution, as demonstrated in fig. 10, suggests that by sacrificing accuracy within the relatively unimportant hydrologic portion of the curve as the recession limb, values of q and γ' could be chosen to obtain a closer approximation of the dimensionless graph near the center. The use of this technique of force-fitting was considered permissible because there is evidence that the dimensionless graph is of a gamma population.

FORCE-FITTING

An arbitrary procedure was established to exemplify the results that could be obtained with simple manipulation of the original data. Alternate values of t/P_R at increments of $t/P_R = 0.125$ were removed from the recession limb, and the respective ordinate values were summed. This total was then prorated over the crest segment in accordance with the ratio: % flow/$0.25P_R$ for the given ordinate value divided by the sum of the ordinates (% flow/$0.25P_R$) at increments of $t/P_R = 0.125$, within the crest segment. The respective additions were made to the dimensionless graph to form the pseudo-dimensionless graph (see fig. 7). The modified or "best-fit," two-parameter gamma distribution for the pseudo-dimensionless graph then was obtained by procedures outlined in Appendix E.

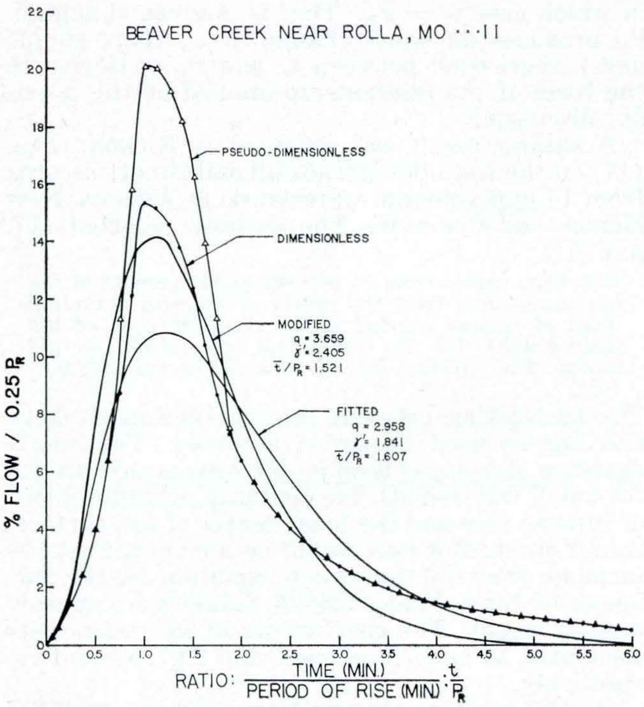

Fig. 7. Influence of modification of the input data on the fit of the two-parameter gamma distribution to the dimensionless graph for watershed 11.

Fig. 7 shows the dimensionless, fitted, pseudo-dimensionless and modified curves for watershed 11. As would be expected, the modified curve shows closer agreement with the dimensionless graph at the peak ordinate and greater deviation on the recession limb. Similar results were obtained for four other watersheds: watershed 15, fig. E-5; watershed 17, fig. E-6; watershed 24, fig. E-7; and watershed 33, fig. E-10 (see Appendix E). In all cases, the agreement between the curves has been improved within the crest segment, although greater variation is noted in other portions of the curves. This observation is particularly evident on watershed 17. For this watershed, the rising limb appears to adopt different geometry than that described by the fitted distribution.

The results suggest that by minor adaptation of the input data, a two-parameter gamma distribution can be forced to fit dimensionless graphs with extended recession characteristics to give more practical results. Additional developmental work is required in the methodology of fitting to alleviate the successive trial procedures. Here, as before, the problem of evaluating the fit in terms of "hydrologic acceptance" remains.

Selection of the Time Parameter

Before synthetic techniques can be employed in synthesizing a hydrograph for a given area, it is necessary to have available a time parameter relating the salient features of rainfall and runoff for the area in question. Several forms of lag have been proposed for this purpose (3).

Two of the most widely used forms are those proposed by Horner and Flint (19) and by Snyder (48). Horner and Flint define lag as the time difference between the center of mass of precipitation excess and the center of mass of the resulting hydrograph. The authors found that lag for a given area was nearly constant and, therefore, independent of precipitation and topographic effects.

Snyder in 1938 introduced lag to define the time difference between the center of mass of a surface-runoff-producing rain and the occurrence of peak discharge. In using this definition it was necessary to specify the storm type; otherwise, because of the unsymmetrical nature of the hydrograph, the magnitude of lag for a given area will vary with storm duration.

The constant property of lag is consistent with unit-graph theory. In addition, it is of major importance in synthetic studies since differences in lag values can be related to differences in physical conditions of the watersheds such as size, shape, slope and storage.

To avoid possible confusion in the remainder of this bulletin, the term lag, t_L, as used hereafter refers to the definition as proposed by Snyder. An attempt was made to determine the lag for each basin studied from an analysis of the available rainfall records. The results of this analysis are presented in table C-4, Appendix C.

COMPUTING LAG FROM AVAILABLE RAINFALL RECORDS

It is evident from table C-4, Appendix C, that the individual lag values within certain areas exhibit considerable scatter. These variations may be explained in part by the incomplete restriction of storm type. Moreover, the lack of agreement of the time properties reported on the rainfall and runoff charts was a major source of variation. In some cases this disagreement completely prohibited the calculation of lag.

These inconsistencies in time properties can be attributed to several factors, including: (a) inadequate raingage placement and coverage, (b) direction of storm movement, (c) distribution of rainfall, (d) inaccuracies arising from malfunctioning of the recording instruments and difficulties encountered in prorating time errors over long periods, (e) errors induced in the recording of data and (f) restrictions imposed by time-scale limitations on the original data.

This last factor, time-scale limitations, is especialy significant on data collected by the United States Geological Survey (USGS). On stage graphs obtained from this source, 1 hour of time is represented by 0.10-inch or 0.20-inch increments. The time of occurrence of peak discharge could only be approximated with reasonable accuracy to the nearest 15-minute period on the former scale or to the nearest 7.5-minute period on the latter scale. This limitation is particularly critical in lag computations for small watershed areas.

Because of the difficulties encountered, lag was determined only for those storms in which there was reasonable agreement in the time properties of the precipitation and runoff data. In table C-4, Appendix C, the inconsistency between the recorded rainfall depths and peak discharges also can be discerned, particularly on the larger watersheds. This incongruity is not unexpected considering the interaction of inadequate raingage placement and storm characteristics. For such cases, the lag values were determined assuming that the time and shape of the recorded mass curve depicted the rainfall characteristics over the entire area.

RELATION BETWEEN LAG AND PERIOD OF RISE

In spite of the simplifications introduced, it was impossible to obtain lag for all of the watersheds studied. To avoid these deletions from other investigations, an additional study was undertaken in an attempt to find a more suitable time parameter which could be measured for each basin.

As shown by the Soil Conservation Service (55), lag, t_L, and time of concentration of a watershed, T_c, are related in the form $t_L = 0.60T_c$. For watersheds of the size used in the study, it is reasonable to assume that P_R approximates T_c. Thus, on this assumption it follows that t_L and P_R would also be related. These variables from 94 selected storms are shown plotted on a logarithmic scale in fig. 8. The regression line, fitted to these data by the method of least squares, is defined by the equation

$$t_L = 0.996 P_R^{1.005} . \quad (18)$$

For all practical cases, the values of the constant and exponent of equation 18 can be taken as unity,

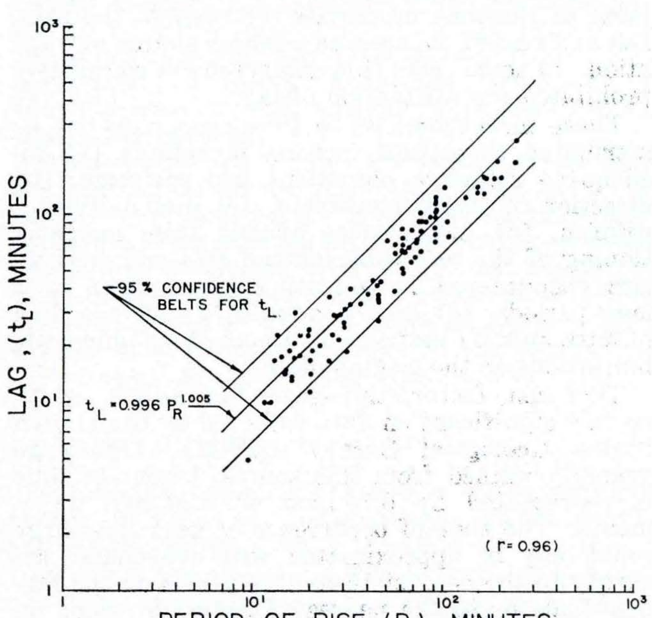

Fig. 8. Relation of lag, t_L, and period of rise, P_R, for 94 selected storms.

in which case $t_L = P_R$. That is, a given change in P_R produces an equal change in t_L. This simple linear regression between t_L and P_R conforms to the form of the relationship implied by the previous discussion.

A similar result was obtained by Hickok, *et al.* (17) in their studies of rainfall and runoff records from 14 experimental watersheds in Arizona, New Mexico and Colorado. The authors reported (17, p. 615):

> Rise time varied from 74 percent to 145 percent of the lag time (time from the center of mass of a limited block of intense rainfall to the resulting peak of the hydrograph) for the individual watersheds in the study. The average for all watersheds was 102 percent.

The association between the lag time used above and lag as used herein is assumed. For short-duration storms, as used in developing unit graphs for small watersheds, the center of a limited block of intense rain and the mass center of the surface-runoff-producing rain would be approximately coincident. For the regression, equation 18, the variances of the t_L values and P_R values were approximately equal. The coefficients of variation were calculated to be 27.1 percent and 25.7 percent respectively.

On the basis of this evidence, it was concluded that the period of rise, P_R, could be used as an effective time parameter to relate the salient features of rainfall and runoff on a given watershed. The result is generally applicable only for uniformly distributed, short-duration, high-intensity storms occurring over small watershed areas.

Relation Between Parameters, q and γ'

The parameters, q and γ', of equation 16 describing the dimensionless graph are linearly related. This relationship can be developed considering that at the peak, $dQ_{t/P_R}/d(t/P_R) = 0$, $t/P_R = 1$ and Q_{t/P_R} is a maximum. By setting the first differential of equation 16 equal to zero and substituting $t/P_R = 1$ into the result, it follows that

$$q = 1 + \gamma'. \quad (19)$$

Equation 19 states that for the dimensionless graph the variables plot as a straight line with an intercept value and slope equal to unity.

As shown in fig. 9, the values of the parameters from the experimental data deviate somewhat from the theoretical result. The least squares line fitted to these data is defined by the regression

$$q = 1.445 + 0.873\gamma'. \quad (20)$$

Figure 9 shows that, according to the regression, the values of q have been overestimated at the smaller values of γ' and underestimated at the larger γ' values. The influence of this property on the geometry of the dimensionless graph can be seen in fig. 10. With increasing values of the peak, Q_p, the ratio, t/P_R, is less than unity; whereas, at small values of Q_p, the values for t/P_R are greater than unity.

The failure of experimental results to follow

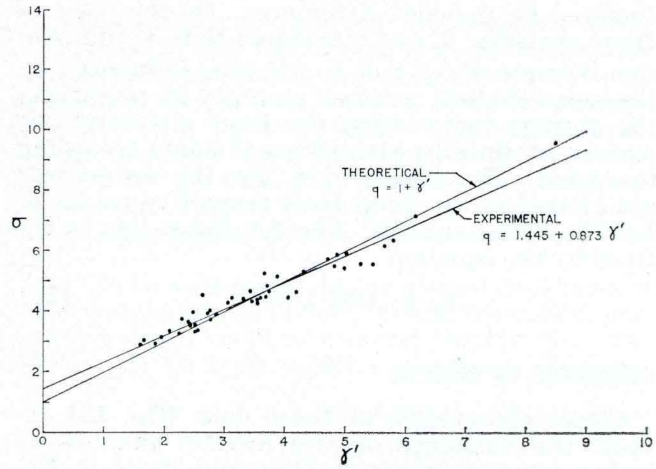

Fig. 9. Theoretical and experimental relationships of parameters q and γ' for dimensionless graphs.

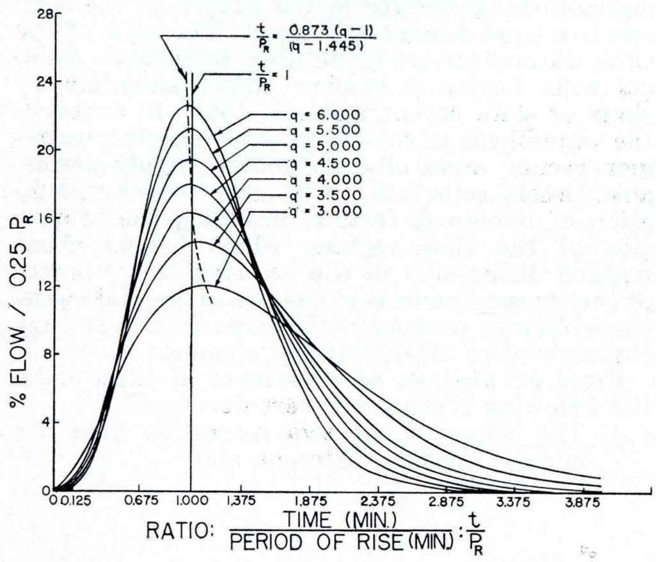

Fig. 10. Variation in the geometry of the dimensionless graph resulting from selection of parameters q and γ', in accordance with the regression equation, $q = 1.445 + 0.873\ \gamma'$.

equation 19 is a measure of the inability of the fitting procedure to achieve proper positioning of the peak ordinates of the fitted graphs. It was not believed, however, that this discrepancy was of sufficient magnitude to restrict the validity of the fitted curves. This error becomes less important if consideration is given to the subjectiveness in developing the empirical graph of a given watershed and to the magnitudes of the deviations among the periods of rise of the distribution graphs of a watershed.

Estimation of the Storage Factor, P_R/γ', From Basin Characteristics

The reliability of a workable synthetic procedure depends on the success with which the empirical hydrologic results can be related to measurable physical characteristics. Edson (13) has shown that rainfall duration influences the magnitude of the parameters, m and y, of equation 30h. He suggests that all hydrographs under consideration should be reduced to a common rainfall duration before an evaluation of the parameters is attempted.

In this study the unit-storm concept proposed by Wisler and Brater (59, p. 38) was accepted, and the representative unit hydrograph for each watershed was described by the two-parameter gamma distribution defined by the parameters q and γ (see equation 14c). According to this principle, the parameters, q and γ, for the unit hydrograph of a given basin are relatively independent of storm duration. It would appear, therefore, that differences in the magnitude of these parameters for the unit hydrographs from different watersheds could be attributed mainly to differences in the physical characteristics of the watersheds.

The effect of unit-storm duration is eliminated if consideration is given only to the parameter, γ, which replaces the exponents y and k of equations 30h and 31f. As discussed by Edson (13) and Nash (38), the exponents y and k reflect the storage properties of a given watershed. Thus, it would be expected that their magnitude would not be influenced by rainfall-duration effects and that y and k would be relatively constant for all unit graphs of a given basin.

The values of γ for the unit hydrograph or distribution graph may be derived from the values for the dimensionless graph, γ', in the following manner: Since, for the dimensionless graph,

$$q/\gamma' = \bar{t}/P_R \tag{21a}$$

where $\bar{t}$ is the mean time, by substituting, $\gamma = \gamma'/P_R$, into equation 21a, it follows that

$$q/\gamma = \bar{t} \tag{21b}$$

in which γ is dimensionally equal to the reciprocal of time.

The required correlation is expedited by considering the relationship between γ and k. For the instantaneous unit graph

$$k = 1/\gamma = P_R/\gamma' \tag{22}$$

in which γ and γ' are the parameters of the two-parameter gamma distribution for the unit hydrograph and for the dimensionless graph, respectively. The relationship is correct dimensionally.

Equation 22 suggests that the ratio, P_R/γ', measures the storage characteristics of a basin and thus was termed the storage factor. In addition, the equation shows that the ratio should be dependent on the same basin characteristics that influence the storage constant, k.

The prediction of the storage constant, k, from measurable physical characteristics has been attained only with limited success. Clark (10) and Linsley (31) have suggested relationships for this purpose. These are given by equations 8e and 8f.

RELATION OF THE STORAGE FACTOR P_R/γ', AND THE WATERSHED PARAMETER $L/\sqrt{S_c}$, FOR 33 SELECTED WATERSHEDS

It was assumed that the storage factor, P_R/γ', like the storage constant, k, is a measure of the lag or travel time of water through a given reach. Thus, for purely hydraulic reasons, its magnitude would vary directly with the length of the main stream, L, and inversely with some power of the channel slope, S_c. Watershed area, A, was not included in the relation for two reasons: first, because the watersheds used in this study were small, the storage in the tributary streams was assumed to be negligible compared with that in the main stream; and second, the high degree of association between L and A (see fig. 3) prohibits the development of a significantly better relation when using both factors over the relation which would result from the use of either L or A individually.

The experimental results showing the storage factor, P_R/γ', plotted with the respective values of $L/\sqrt{S_c}$ for 33 selected watersheds are given in fig. 11. The "least-squares" line for these data is defined by the equation

$$P_R/\gamma' = 9.77\ (L/\sqrt{S_c})^{0.475} \qquad (23)$$

in which L, the length of the main stream, is expressed in miles and S_c, the average slope of the channel, is in percent. The standard error of estimate for the P_R/γ' values was calculated to be 34.0 percent.

RELATION BETWEEN AVERAGE CHANNEL SLOPE, S_c, AND LENGTH OF MAIN STREAM, L

Linsley (31) suggests that the relation between the storage constant, k, and basin factors is influenced by regional differences. On the basis of these remarks, it was considered that, if the relation between P_R/γ' and $L/\sqrt{S_c}$ was influenced by the same factors, greater accuracy in predicting the storage factor from the basin characteristic may be attained by stratifying the data according to region. The values of S_c for the watersheds are plotted in fig. 12 at their respective values of L. The "least-squares" line for these data is defined by the equation

$$S_c = 1.62\ L^{-0.663}. \qquad (24)$$

INFLUENCE OF REGION

Closer observation of these data (fig. 12) reveals the existence of two distinct families of points for watersheds in Ohio and those in Nebraska-Western Iowa (see fig. 13). These two areas represent regions of widely divergent geologic and climatic conditions. Probably the most distinguishing feature is the nature of their respective predominant soil types. The soils of the Ohio watersheds are moderately permeable, residual soils having a shallow solum underlain by shale or slate parent material (54). In contrast, the watersheds of the Nebraska-Western Iowa region occupy areas of deep, coarse, highly permeable, loessial soils (42). It is, however, the culmination of numerous factors, including the properties of the flow regime, which brings about marked differences in the erosional development of the stream channels in the two areas. Likewise, these factors produce differences in the storage characteristics of the stream channels.

From an analysis of covariance of these data, the following conclusions were developed:

1. The slopes of the two regression lines are not significantly different, and

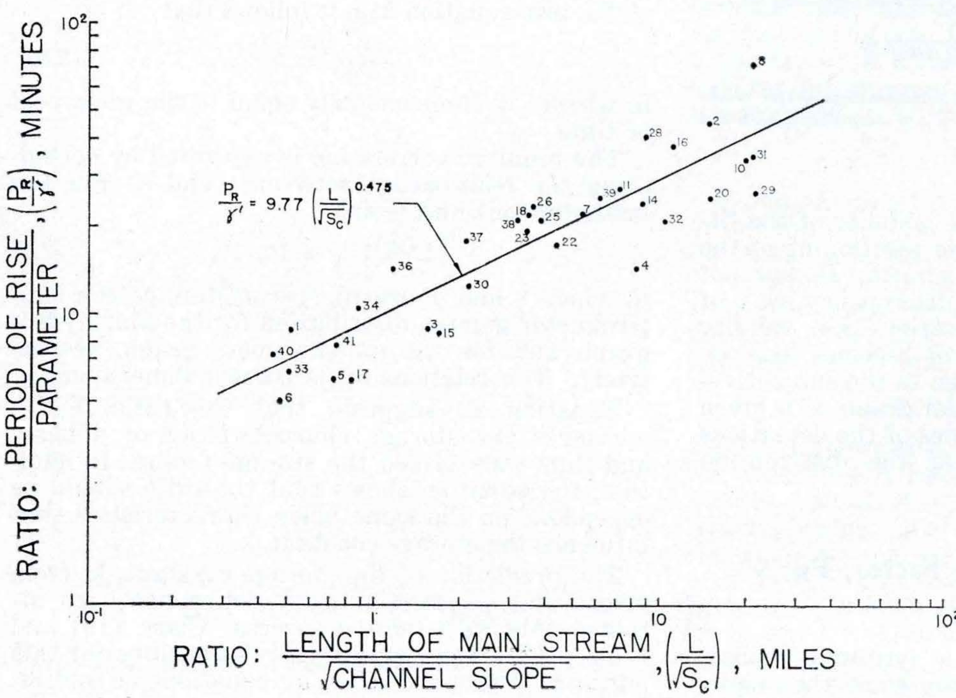

Fig. 11. Relation of the storage factor, P_R/γ', and watershed factor, $L/\sqrt{S_c}$, for 33 selected watersheds.

2. The difference between the adjusted mean values of the two groups is greater than can be accounted for by sampling variation.

In essence, the analysis makes it possible to represent the data by two parallel regression lines passing through the mean logarithmic values of S_c and L for the Ohio and Nebraska-Western Iowa watersheds. The above result gives evidence that the relationship between S_c and L varies with region.

As can be seen in fig. 12 the plotted data for the other watersheds in Illinois, central Iowa, Missouri and Wisconsin adopt no general pattern but vary appreciably in their relative positions. In some cases they approach the regression line for the Ohio watersheds; in others, they approach that of the Nebraska-Western Iowa region, or appear to occur in their own individual class. Since the characteristics of these basins were not available, the development of a complete family of curves was not attempted.

SELECTED GROUPING OF WATERSHEDS FOR THE PREDICTION OF THE STORAGE FACTOR, P_R/γ'

Considering the evidence that the relationship between S_c and L varies from region to region, it follows that in predicting P_R/γ' from the ratio,

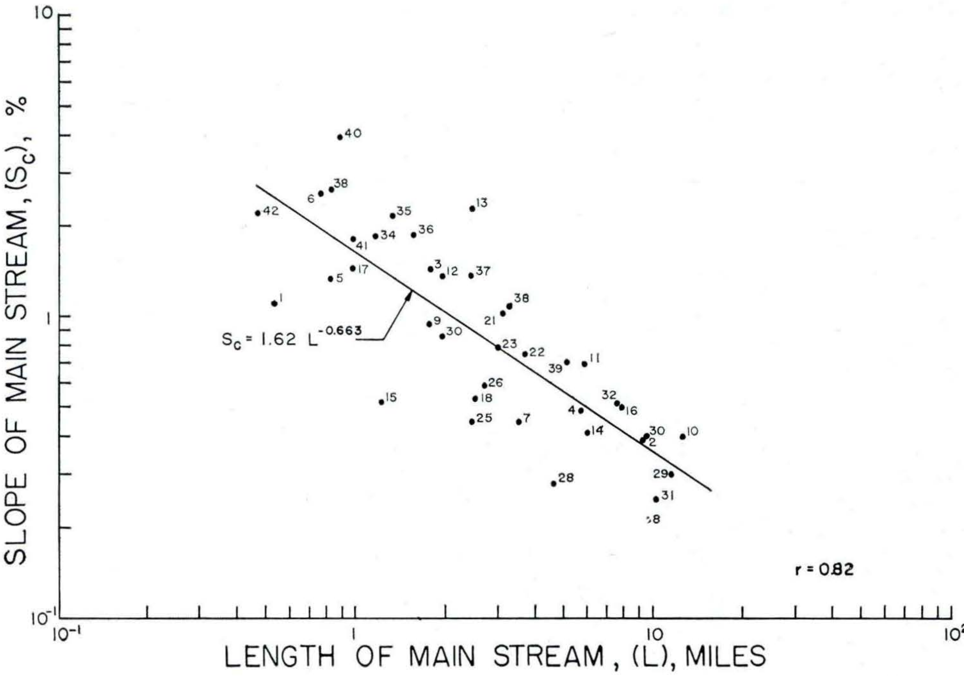

Fig. 12. Relation of slope of the main stream, S_c, and length of the main stream, L.

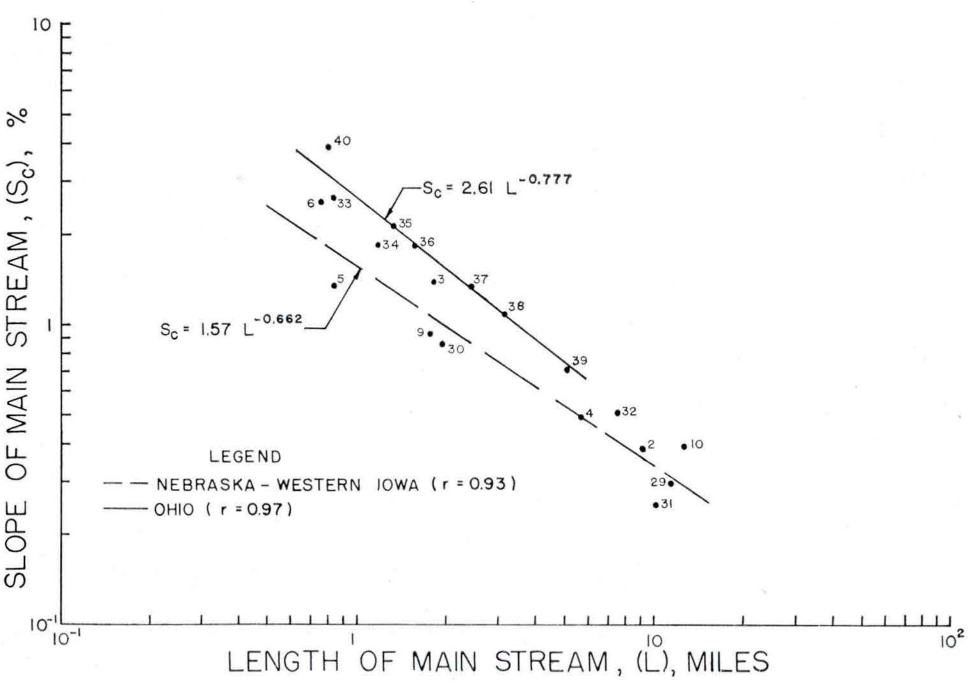

Fig. 13. Relation of slope of the main stream, S_c, and length of the main stream, L, for watersheds in the two regions: Nebraska-Western Iowa and Ohio.

$L/\sqrt{S_c}$, those watersheds from areas in which S_c and L vary in the same proportion should be combined. Otherwise, the results obtained would be inconsistent. In this study the following grouping appeared to be the most appropriate:

1. Nebraska-Western Iowa,
2. Central Iowa-Missouri-Illinois-Wisconsin, and
3. Ohio.

Figures 14a, 14b and 14c show the storage factor, P_R/γ', plotted with the ratio $L/\sqrt{S_c}$ for these three groups. The regression equations calculated by the method of least squares were, respectively:

Nebraska-Western Iowa

$$P_R/\gamma' = 7.40(L/\sqrt{S_c})^{0.498} \quad (25a)$$

Central Iowa-Missouri-Illinois-Wisconsin

$$P_R/\gamma' = 9.27(L/\sqrt{S_c})^{0.562} \quad (25b)$$

Ohio

$$P_R/\gamma' = 11.4(L/\sqrt{S_c})^{0.531} \quad (25c)$$

The coefficients of variation were, respectively, 28.0 percent, 30.7 percent and 29.1 percent.

An analysis of covariance of these data yielded the following results:

1. The slopes of the regression lines do not differ significantly.
2. The adjusted mean values of P_R/γ' for the Ohio watersheds and the Nebraska-Western Iowa watersheds are significantly different from the adjusted mean values of either of the other two groups.

The analysis statistically confirms that storage factors computed from a given value of $L/\sqrt{S_c}$ differ significantly because of regional influence, provided that the regions exhibit distinct differences in their characteristics. The fact that the watersheds in Central Iowa-Missouri-Illinois-Wisconsin adopt storage properties common to both the Ohio and Nebraska-Western Iowa groups is indicative by the nonsignificance between the adjusted mean value and slope of the regression for this group, and the same properties for the others. All of the data can be expressed by two parallel lines passing through the respective mean logarithmic values of P_R/γ' and $L/\sqrt{S_c}$ for the Ohio and Nebraska-Western Iowa groups. Because of the difficulty in associating a given basin in central Iowa, Missouri, Illinois or Wisconsin with either of the two regions, all of the watersheds from these areas were retained as a separate group, as shown in fig. 14b. The 95-percent confidence belts have been added to figs. 14a, 14b and 14c to facilitate the use of equations 25a, 25b and 25c as prediction equations.

DISCUSSION

It is evident from figs. 14a and 14c that, for a given value of $L/\sqrt{S_c}$ the storage factor, P_R/γ' is higher for the Ohio watersheds than for the Nebraska-Western Iowa watersheds. This difference can be associated with differences in the geometry of the stream channels in the two regions. In Ohio, low flows are confined to shallow, V-

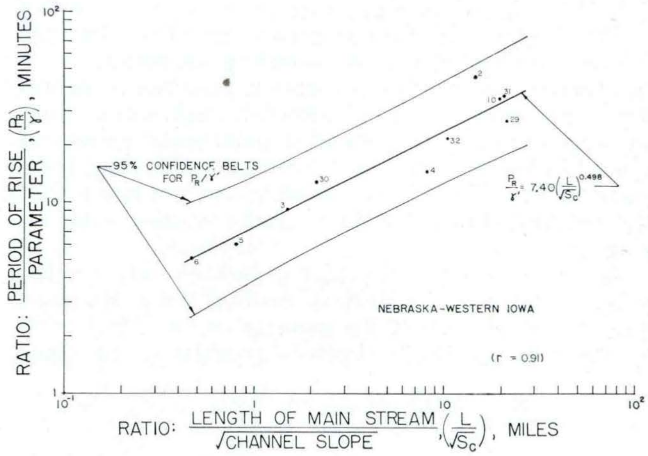

Fig. 14a. Relation of storage factor, P_R/γ', and watershed parameter, $L/\sqrt{S_c}$, for watersheds in the Nebraska-Western Iowa region.

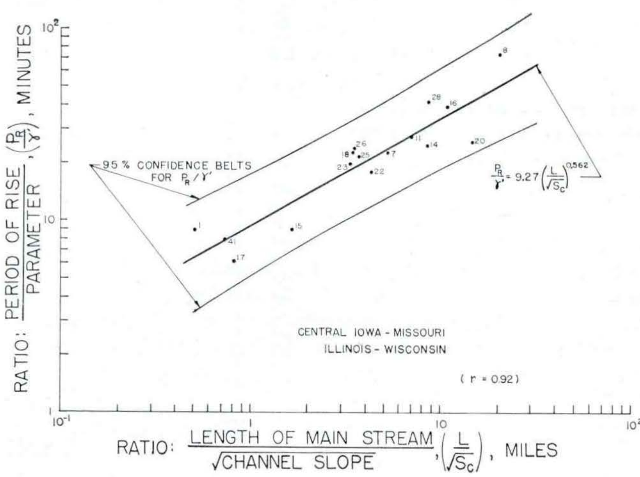

Fig. 14b. Relation of storage factor, P_R/γ', and watershed parameter, $L/\sqrt{S_c}$ for watersheds in the Central Iowa-Missouri-Illinois-Wisconsin region.

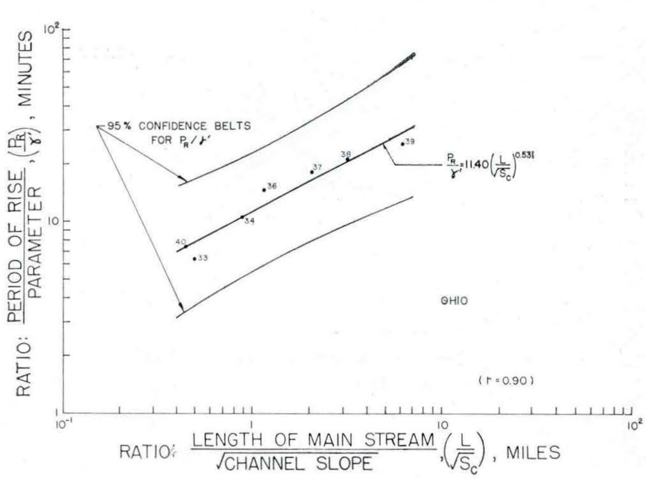

Fig. 14c. Relation of storage factor, P_R/γ', and watershed parameter, $L/\sqrt{S_c}$, for watersheds in Ohio.

shaped channels which top to a narrow, rounded valley bottom. Even in the case of small flood waves, characteristic of those originating from unit storms, overbank storage would be appreciable. In contrast, stream channels in the loessial area are in the form of deeply entrenched, U-shaped gullies. For watersheds within this region, most flood flows resulting from unit storms would be confined within the channel.

The use of equation 25b applied to watersheds in Illinois or Wisconsin may be questioned, because only one watershed from each state was included in the analysis. The P_R/γ' values obtained by this equation fall intermediate between those for the Ohio and Nebraska-Western Iowa regions. This positioning corresponds roughly to that which would be expected were the general geologic, physiographic and climatic conditions of the three regions compared.

In summary, it can be stated that the storage factor, P_R/γ', can be predicted with reasonable accuracy from the watershed parameter, $L/\sqrt{S_c}$, when consideration is given to the effect of regional influence. A possible method of accounting for this influence is to stratify the data into groups in which S_c and L have the same relation. Additional study is required to exploit this possibility more fully. In applying the results, it is recommended that the empirical relation be selected from that group whose geologic, physiographic and climatic conditions are most nearly representative of those of the watershed in question.

Relation of Period of Rise, P_R, and the Parameter γ'

The results presented in the preceding sections give a relationship between dimensionless-graph properties and a relationship between these properties and basin characteristics. These may be expressed in equational form as

$$q = \phi(\gamma') \text{ and}$$
$$P_R/\gamma' = \phi'(L/\sqrt{S_c})$$

where ϕ and ϕ' designate the function. The equations contain three unknowns, P_R, q and γ'; hence, an additional expression is required to allow simultaneous solution. Two possibilities of meeting this requirement are: (a) relating the variables either individually or in combination with some watershed characteristics other than L or S_c, or (b) relating q or γ' with P_R. Use of the first alternative is questionable, however, because of the interrelationships among watershed characteristics and the bias that would be introduced to the relation by using dependent terms.

Equations 19 and 21a can be combined to form

$$(1 + \gamma')/\gamma' = \bar{t}/P_R \qquad (26)$$

in which $\bar{t}/P_R$ is the mean value of t/P_R of the dimensionless graph. Obviously, if $\bar{t}/P_R$ were a constant, γ' would likewise be constant, and a common shape could be employed to describe all of the dimensionless graphs. As shown previously, this is not the case. In addition, however, equation 26 suggests that if $\bar{t}/P_R$ can be expressed as a function of P_R alone, then P_R and γ' would be related. Figure 15 shows these latter values plotted on a rectangular coordinate base.

It was found that the variation in γ' could be significantly explained in linear regression with P_R. The regression equation fitted to these data by the method of least squares is

$$\gamma' = 2.676 + 0.0139 P_R. \qquad (27a)$$

For this regression, the standard deviation from regression was calculated as 1.253.

Since equation 27a is to be used in conjunction with equations 25a, 25b and 25c, it is more convenient for computational purposes to express the result in the form:

$$P_R/\gamma' = \frac{1}{2.676 + \frac{0.0139}{P_R}}. \qquad (27b)$$

Equation 27b is plotted in fig. 16, which can be used to solve for P_R when P_R/γ' is known.

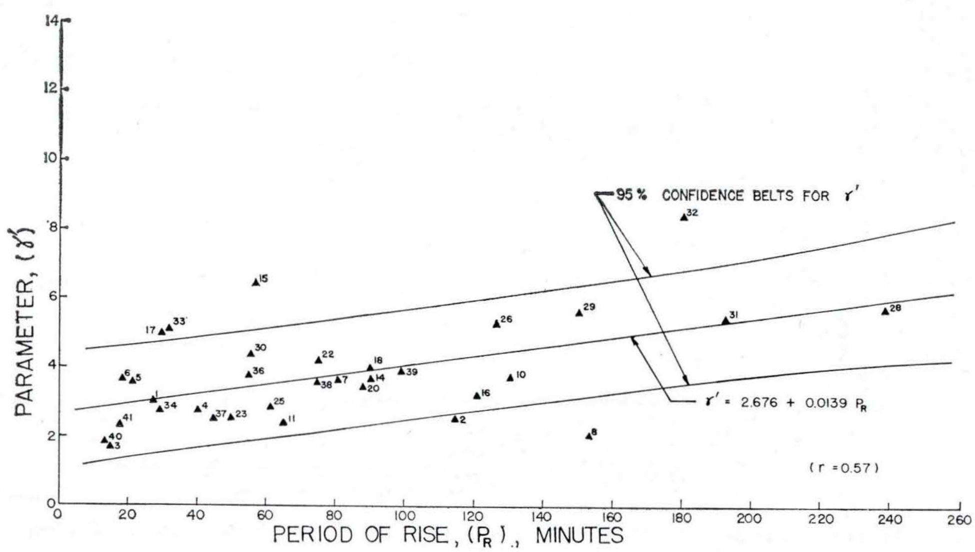

Fig. 15. Relation of parameter, γ', and period of rise, P_R, for 33 selected watersheds.

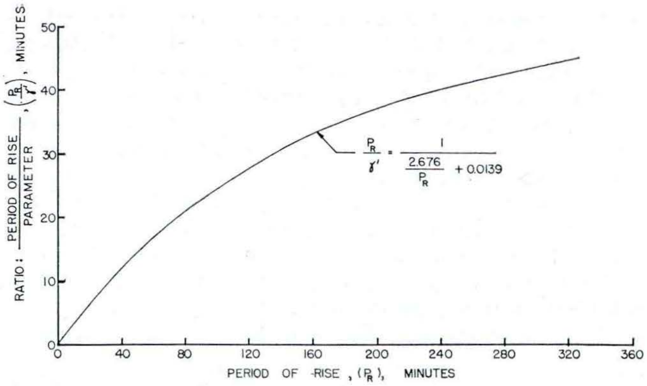

Fig. 16. Relation of storage factor, P_R/γ', and period of rise, P_R.

DISCUSSION

In using equation 27b, it can be readily shown that, as P_R approaches infinity, P_R/γ' approaches a maximum value of 71.9 minutes. Hence, the application of the results is necessarily restricted to watersheds having P_R/γ' values less than this maximum. This limit, however, includes all of the data used in this study.

With the values of P_R/γ' and $L/\sqrt{S_c}$ plotted on rectangular coordinates (fig. 17), it is evident that there is a tendency for the P_R/γ' values to show wider deviations from the regression lines at the larger values of $L/\sqrt{S_c}$. A possible reason for this property is inherent in remarks made by Wisler and Brater (59, p. 305).

> The term "large watersheds," applies to basins having an area greater than 10 sq. miles. However, the distinguishing feature of large watersheds is not that their area is greater than some arbitrary limit but rather that they are of such size that, within the basin, there are likely to be major differences in rainfall duration and intensity and in soil permeability. On large watersheds, major floods are frequently the result of high rates of runoff from only a portion of the basin. Consequently, it is necessary to determine unit hydrographs for several different rainfall-distribution patterns.

Equations 9 and 24 can be solved for an area, A, equal to 10 square miles to obtain a value of $L/\sqrt{S_c}$ equal to approximately 7 miles. Figure 17 shows that within this range the experimental data agree very well with the fitted regression lines.

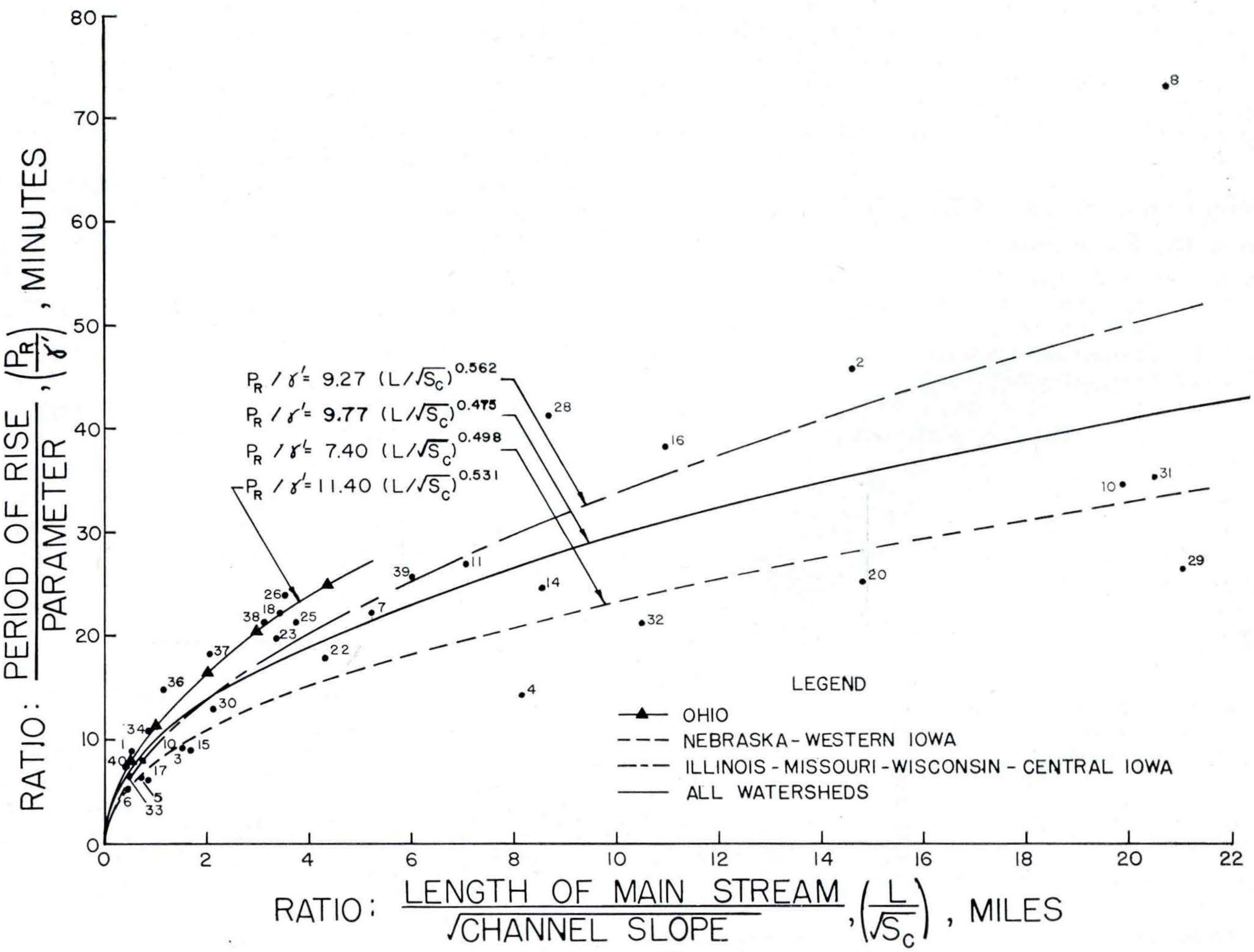

Fig. 17. Relation of storage factor, P_R/γ', and watershed parameter, $L/\sqrt{S_c}$, plotted on a rectangular coordinate scale.

The remarks by Wisler and Brater to the effect that no sharp, distinct arbitrary limit can be established to define the boundary between large and small watersheds may be reiterated. The coincidence of the results of this study and these remarks indicates that the unit-hydrograph techniques discussed here can be applied with reasonable success on areas less than 10 square miles in size. On larger areas, additional factors must be considered.

It is recommended that the results reported in figs. 14a, 14b, 14c and 16 be limited to watersheds having characteristics which fall within the limits of the experimental data. Where, because of necessity, it becomes necessary to extrapolate these results, the investigator must be cognizant of the increased chance of obtaining larger errors. By combining the results of this study it can be shown that the limit of equation 27b restricts the application of the results to watersheds less than the approximate sizes given in table 2.

Table 2. Approximate maximum watershed sizes for which the prediction equations are applicable.

Region	Watershed area (square miles)
Nebraska-Western Iowa	362
Central Iowa-Missouri-Illinois-Wisconsin	94
Ohio	82

Application of Results

An illustrative example showing the synthesis of a unit hydrograph for an area from basin characteristics and using the relationships given in this bulletin is given in Appendix F.

SUMMARY

Topographic and hydrologic characteristics from 42 selected watersheds varying in size from 0.23 to 33.00 square miles and located in the states of Illinois, Iowa, Missouri, Nebraska, Ohio and Wisconsin were studied. The general conclusions listed are valid only for topographic and hydrologic conditions comparable to those used in the study.

Topographic Characteristics

Five watershed properties were obtained for each watershed where the data permitted: drainage-area size, A; length of the main stream, L; length to center of area, L_{ca}; slope of the main stream, S_c; and mean land slope, S_L. From analyses of these data, the following general conclusions were formed:

1. The factors L, L_{ca} and A are highly correlated, and thus their use as independent terms in dimensional analysis techniques is prohibited.

2. For practical purposes, the value of L_{ca} may be taken to be equal to one-half the value of L.

3. The general shape of small watersheds is intermediate between ovoid and pear-shaped.

4. For watersheds in a given region, the factors S_c and L show a distinct relationship.

5. The mean land slope, S_L, of a given basin can be estimated with reasonable accuracy from the mean slope of a representative sample of first-order streams, s_1, taken from the same basin.

Hydrologic Characteristics

The rainfall and runoff characteristics from a number of selected unit storms occurring over each watershed were studied. For each basin, a representative distribution graph—the so-called empirical graph—was derived and modified to a dimensionless form based on the period of rise, P_R, as the time parameter.

The two-parameter, gamma distribution described by the parameters, q and γ', was fitted to each dimensionless graph, and the maximum likelihood estimators of the parameters were obtained. Relationships were established so that the parameters P_R, q and γ' could be evaluated from the topographic characteristics L and S_c of a given basin. With P_R, q and γ' known, the dimensionless graph, distribution graph and unit hydrograph for the basin can be described.

The following conclusions were derived from this study:

1. The period of rise can be used to replace lag time as a time parameter.

2. For practical purposes, the period of rise may be taken to be equal to the lag time.

3. In general, the two-parameter gamma distribution can be used to describe the dimensionless graph, distribution graph, or unit hydrograph.

4. Additional work is required on the methodology of fitting the two-parameter gamma distribution to the unit hydrograph and in the evaluation of the goodness of fit in terms of hydrologic acceptance.

5. The storage factor, P_R/γ', can be predicted with reasonable success from the watershed factor, $L/\sqrt{S_c}$, provided consideration is given to regional influence.

6. The parameter, γ', of the two-parameter gamma distribution describing the dimensionless graph can be estimated from the period of rise.

7. For a given watershed, the dimensionless graph, distribution graph and unit hydrograph can be derived from the watershed characteristic, $L/\sqrt{S_c}$.

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APPENDIX A: GLOSSARY OF TERMS AND SYMBOLS

Centroid of precipitation—mass center of the unit-storm rainfall histrogram about which the sum of the product moments of rainfall volume times time are equal to zero.

cfs—cubic feet per second.

Channel storage—the volume of water confined within a stream channel.

Dimensionless graph — a special dimensionless form of the unit hydrograph showing the ordinate values expressed as a percentage of the total flow volume based on a time increment equal to one-quarter the period of rise (% flow/$0.25P_R$) and its abscissa expressed as the ratio of any time, t, divided by the period of rise.

Distribution graph—a unit hydrograph of surface runoff modified to show the proportional relation of its ordinates expressed as percentages of the total surface runoff volume occurring in selected time intervals.

Drainage-area size, A—plane area of the watershed in square miles which is enclosed within the topographic divide above the gaging station.

Empirical graph—the representative distribution graph of a watershed.

Excess precipitation — that portion of rainfall which is in excess of soil infiltration and other losses, and which appears as surface runoff at the gaging station.

First-order streams—the smallest, unbranched, fingertip tributary streams of a drainage net.

Lag time, t_L—time difference in minutes between the centroid of precipitation and the peak discharge rate of the hydrograph.

Length of main stream, L—distance in miles along the main stream from gaging station to the outermost point defined on the topographic map (fig. A-1).

Length to center of area, L_{ca}—distance in miles along the main stream from the gaging station to the point nearest the mass center of the area (fig. A-1).

Main stream—stream of the highest order which passes through the gaging station. To delineate the main stream at bifurcation the following rules established by Horton (21, p. 281) were used: (a) Starting below the junction, the main stream was projected upstream from the bifurcation in the same direction. The stream joining the main stream at the greatest angle was taken as the lower order. (b) If both streams were at about the same angle to the main stream at the junction, the shorter was taken as the lower order (fig. A-1).

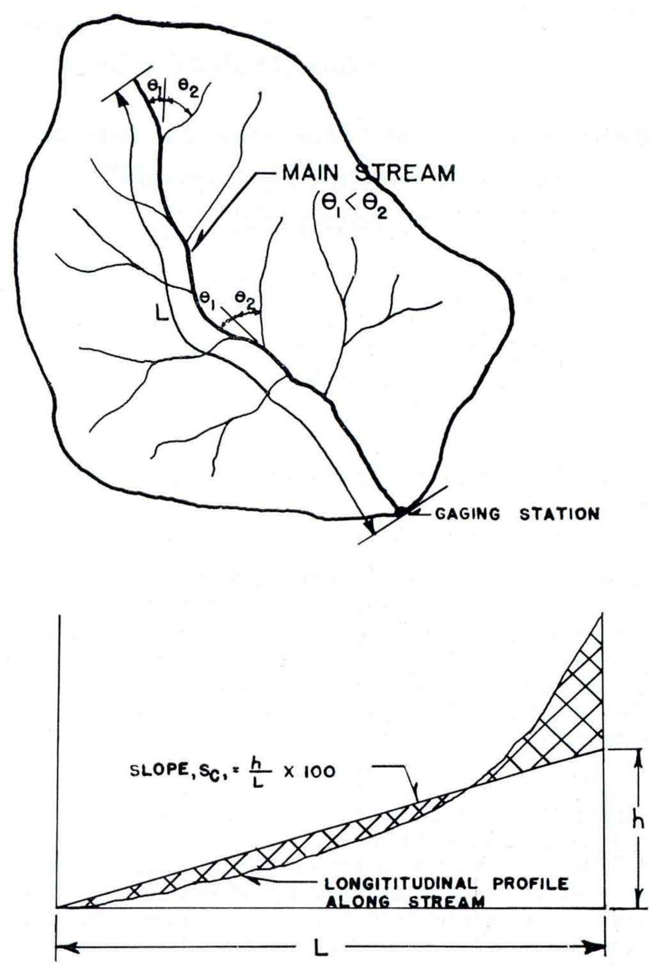

Fig. A-1. Watershed characteristics.

Mean land slope, S_L—mean land slope in percent determined by the grid-intersection method.

Mean slope of first-order streams, s_1—slope in percent obtained by averaging the slopes from a representative number of first-order streams in a given watershed.

% flow/$0.25P_R$—ordinate scale of the dimensionless graph representing the percentage of total volume of surface runoff occurring in a time interval equal to one-quarter the period of rise.

Period of rise—time lapse in minutes from the beginning of surface runoff to the occurrence of the peak discharge rate.

q, γ—shape and scale parameters, respectively, of the two-parameter gamma distribution which describes the distribution graph or unit hydrograph of a given watershed.

q, γ'—dimensionless parameters of the two-parameter gamma distribution which describes the dimensionless graph of a given watershed.

Q_t—ordinate of the unit hydrograph in cfs.

Q_t/P_R—ordinate of the dimensionless graph in % flow/$0.25P_R$.

Slope of the main stream, S_c—slope in percent of a line drawn along the longitudinal section of the main channel in such a manner that the area between the line and a horizontal line drawn through the channel outlet elevation is equal to the area between the channel grade line and the same horizontal line (fig. A-1).

r—correlation coefficient.

Unit hydrograph—a discharge hydrograph resulting from 1-inch of surface runoff generated uniformly over the watershed area at a uniform rate during a specified period of time.

Unit storm—that storm which produces a unit hydrograph. The duration of a unit storm is such that the period of surface runoff is not appreciably less for any storm of shorter duration.

APPENDIX B: EQUATIONAL FORMS OF THE UNIT HYDROGRAPH

Basic Elements of "Mathematical Interpretation of the Unit Hydrograph" by Edson (13)

If isochrones could be drawn to represent the time required for each local element of effective rainfall to reach the mouth of a watershed, the culmination of area, A, with time, t, would result in an approximate parabola

$$A \alpha t^x,\ x > 1$$

so that the runoff discharge rate, Q, might become

$$Q \alpha t^x,\ x > 1. \qquad (30a)$$

However, the time of travel required for each component is so affected by other components that the hypothetical isochrones are invalidated. It is generally considered that the consequent delay in delivery is the result of valley storage. The discharge from storage is known to decrease exponentially with time

$$Q \alpha e^{-yt} \qquad (30b)$$

where y is the recession constant whose magnitude is greater than zero.

Thus, the reservoir action of the valley storage is seen to have a dampening effect on the flow implied by proportion 30a. Accordingly, proportion 30a must continue in effect indefinitely. On the other hand, since valley storage must exist for even a small amount of discharge, proportion 30b is seen to be in effect from the very inception of runoff. The combined effect becomes

$$Q \alpha t^x\, e^{-yt}. \qquad (30c)$$

The fact that the recession limb of a unit hydrograph becomes approximately linear when plotted on semilogarithmic paper simply means that proportion 30b is dominant sometime after the peak discharge. At no time prior to the peak discharge, however, is proportion 30b dominated by proportion 30a, so that proportion 30c cannot be developed by the usual curve-fitting methods.

The total discharge volume, V, is obtained from

$$V = \int_0^\infty Q\,dt \qquad (30d)$$

but

$$Q = Bt^x\, e^{-yt} \qquad (30e)$$

where B is a proportionality constant. Substituting equation 30e into equation 30d

$$V = \int_0^\infty Bt^x\, e^{-yt}\, dt\,. \qquad (30f)$$

To facilitate the integration of equation 30f, let

$x = m - 1$,
$z = yt$, and
$dz = y\,dt$.

By substitution, equation 30f becomes

$$V = \int_0^\infty B(z/y)^{m-1} e^{-z} dz/y$$

$$= By^{-m} \int_0^\infty z^{m-1} e^{-z} dz.$$

The quantity, $\int_0^\infty z^{m-1} e^{-z} dz$, is recognized as the gamma function of m, $\Gamma(m)$. Therefore

$$V = By^{-m} \Gamma(m)$$

$$\text{and } B = \frac{V}{y^{-m} \Gamma(m)}. \qquad (30g)$$

By substituting equation 30g into equation 30e and making the appropriate substitutions

$$Q = \frac{V y^m}{\Gamma(m)} e^{-yt} t^{m-1}. \qquad (30h)$$

Basic Elements of "The Form of the Instantaneous Hydrograph" by Nash (39)

It is assumed that any watershed may be replaced by a series of n reservoirs each having the storage characteristics

$$S = kQ \qquad (31a)$$

where S = storage volume.
k = proportionality constant having dimensions of time, and
Q = discharge rate.

When an instantaneous inflow of volume, V, takes place to the first reservoir, its level is raised by an amount sufficient to accommodate the increased storage. The discharge rises instantaneously from zero to V/k and diminishes with time according to the equation

$$Q_1 = \frac{V}{k} e^{-t/k} \qquad (31b)$$

where t is the time and e is the base of the natural logarithms. Q_1 becomes the inflow, I, to the second reservoir. Therefore, the discharge from the second reservoir, Q_2, is

$$Q_2 = \frac{1}{k} e^{-t/k} \int_0^t I e^{t/k} dt$$

$$Q_2 = \frac{1}{k} e^{-t/k} \int_0^t \frac{V}{k} dt$$

$$Q_2 = \frac{V}{k^2} e^{-t/k} t. \qquad (31c)$$

With successive routings through n reservoirs, the discharge rate, Q_n, becomes

$$Q_n = \frac{V}{k^n (n-1)!} e^{-t/k} t^{n-1}. \qquad (31d)$$

$$\text{But, } (n-1)! = \Gamma(n) \qquad (31e)$$

where Γ is the gamma function. Substituting the equality 31e into equation 31d, the relation can be written

$$Q_n = \frac{V}{\Gamma(n)} k^{-n} e^{-t/k} t^{n-1}. \qquad (31f)$$

APPENDIX C: TOPOGRAPHIC AND HYDROLOGIC DATA

In an effort to alleviate overcrowding of tables and illustrations by using watershed names, each watershed was given a number designation (see table C-1). The number designation was employed exclusively to define the watersheds and to associate topographic and hydrologic properties with a given watershed throughout the bulletin.

Table C-1. Watershed name and corresponding number designation.

State	Number	Watershed
Illinois	1	W-IV, Edwardsville
Iowa	2	Davids Creek near Hamlin
	3	Hayworth Main Outlet near Climbing Hill
	4	Indian Creek at Council Bluffs
	5	Muckey Creek near Mapleton
	6	Nepper Main Outlet near Mapleton
	7	Ralston Creek near Iowa City
	8	Rapid Creek near Iowa City
	9	Renneker Main Outlet near Anthon
	10	Waubonsie Creek near Bartlett
Missouri	11	Beaver Creek near Rolla
	12	Behmke Branch near Rolla
	13	Big Creek near Yukon
	14	Bourbeuse Creek near St. James
	15	Coyle Branch at Houston
	16	East Fork Fishing River at Excelsior Springs
	17	Green Acre Branch near Rolla
	18	Jenkins Branch at Gower
	19	Lanes Fork near Rolla
	20	Lanes Fork near Vichy
	21	Little Beaver Creek near Rolla
	22	Lost Creek at Elsberry
	23	Mill Creek at Oregon
	24	Oak Grove Branch near Brighton
	25	Shiloh Branch near Marshall
	26	Stahl Creek near Miller
	27	Stark's Creek at Preston
	28	White Cloud Creek near Maryville
Nebraska	29	Dry Creek near Curtis
	30	W-3, Hastings
	31	New York Creek near Herman
	32	Tekamah Creek at Tekamah
Ohio	33	W-5, Coshocton
	34	W-11, Coshocton
	35	W-91, Coshocton
	36	W-92, Coshocton
	37	W-94, Coshocton
	38	W-95, Coshocton
	39	W-97, Coshocton
	40	W-196, Coshocton
Wisconsin	41	W-I, Fennimore
	42	W-IV, Fennimore
North Carolina	43	W-7, Coweeta
	44	W-8, Coweeta
	45	W-9, Coweeta
	46	W-10, Coweeta

Table C-2. Collection agencies for raw topographic and hydrologic data.

Letter designation	Agency and location
ARS	United States Department of Agriculture, Agricultural Research Service, Soil and Water Conservation Divisions; Beltsville, Maryland; Hastings, Nebraska and Coshocton, Ohio
FS	United States Department of Agriculture, Forest Service, Coweeta Hydrology Laboratory, Dillard, Georgia
ISU	Iowa State University of Science and Technology, Department of Agricultural Engineering, Ames, Iowa
SUI	State University of Iowa, Department of Mechanics and Hydraulics, Iowa City, Iowa
USGS	United States Department of Interior, Geological Survey, Topographic and Water Resources Divisions; States of Iowa, Missouri, Nebraska and Ohio
USWB	United States Department of Commerce, Weather Bureau, National Weather Records Center, Asheville, North Carolina

Table C-3. Summary of topographic characteristics.

Watershed number[a]	Collection agencies[b]	A (sq. miles)	L (miles)	Lca (miles)	S_c (%)	S_L (%)
1	ARS	0.45	0.54	0.28	1.10	5.68[c]
2	ISU, USGS	26.01	9.14	4.95	0.39	4.15[c]
3	ISU	0.91	1.80	0.85	1.41	8.05[c]
4	USGS	7.56	5.69	2.08	0.49	8.45[c]
5	ISU	0.69	0.83	0.45	1.34	12.30[c]
6	ISU	0.28	0.75	0.43	2.56	——
7	SUI	3.00	3.50	2.80	0.45	7.76[d]
8	ISU, USGS	24.57	9.50	4.15	0.21	4.10[c]
9	ISU	0.89	1.78	0.68	0.94	5.26[c]
10	ISU, USGS	32.64	12.50	5.30	0.40	2.90[c]
11	USGS	13.70	5.95	2.90	0.70	10.30[d]
12	USGS	1.03	1.95	1.15	1.37	7.05[d]
13	USGS	8.36	2.45	1.65	2.28	——
14	USGS	21.30	6.00	3.02	0.41	——
15	USGS	1.30	1.21	0.80	0.52	7.18[d]
16	USGS	20.00	7.80	3.60	0.50	6.31[c]
17	USGS	0.62	0.98	0.60	1.45	——
18	USGS	2.72	2.50	1.20	0.53	5.06[c]
19	USGS	0.23	——	——	——	——
20	USGS	24.10	9.40	3.10	0.40	——
21	USGS	6.27	3.10	1.60	1.02	7.86[d]
22	USGS	12.20	3.70	1.98	0.74	8.69[d]
23	USGS	4.90	3.00	1.60	0.79	6.56[c]
24	USGS	1.00	1.00	0.75	——	——
25	USGS	2.87	2.45	1.60	0.45	3.72[c]
26	USGS	3.86	2.70	1.40	0.59	2.93[d]
27	USGS	4.72	1.98	1.10	——	——
28	USGS	6.06	4.60	2.60	0.28	4.46[d]
29	USGS	20.00	11.59	5.49	0.30	4.74[c]
30	ARS	0.75	1.96	1.54	0.86	5.44[d]
31	USGS	30.00	10.25	5.45	0.25	3.88[c]
32	USGS	21.53	7.50	4.25	0.52	5.01[c]
33	USGS, ARS	0.55	0.82	0.48	2.64	18.90[c]
34	USGS, ARS	0.46	1.17	0.74	1.83	24.60[c]
35	USGS, ARS	0.46	1.31	0.57	2.13	25.60[c]
36	USGS, ARS	1.44	1.56	0.72	1.84	25.40[c]
37	USGS, ARS	2.37	2.41	1.02	1.37	14.80[d]
38	USGS, ARS	4.02	3.25	1.45	1.09	22.60[c]
39	USGS, ARS	7.15	5.11	2.44	0.72	20.40[c]
40	ARS	0.47	0.88	0.42	3.94	15.60[d]
41	ARS	0.52	0.99	0.55	1.80	8.10[c]
42	ARS	0.27	0.46	0.29	2.20	5.77[d]
43	FS	——	——	——	——	51.20[d]
44	FS	——	——	——	——	45.50[d]
45	FS	——	——	——	——	46.10[d]
46	FS	——	——	——	——	43.30[d]

[a]Refer to table C-1 for code designation.
[b]Refer to table C-2 for interpretation.
[c]Mean land slope computed from regression equation 13.
[d]Slope determination by grid-intersection method (21).

Table C-4. **Summary of storm characteristics and hydrograph properties.**

Raingage			Storm characteristics			Hydrograph properties			
				Rainfall					
Watershed number[a]	Collection agency[b]	Station	Storm date	Excess period (min.)	Total depth (in.)	Collection agency	Period of rise (min.)	Peak discharge (cfs)	Lag time (min.)
1	ARS	Weighted average R-1, R-2, R-3, R-4, R-5, R-6, R-7	July 8-9, 1942	56	2.11	ARS	28	423	22
			Aug. 14-15, 1946	53	1.98		39	667	39
			Aug. 15-16, 1946	40	1.00		25	260	17
2	USWB	Coon Rapids	Aug. 15, 1952	......	-- ----	USGS	120	840	......
			June 4-5, 1953	......	-- ----		120	362	......
			June 6-7, 1956	100	1.57		105	533	93
3	ISU	Weighted average H-1, H-2	June 15, 1950	20	1.57	ISU	15	860	14
			June 23, 1951	30	1.01		14	820	10
			June 25, 1951	15	0.98		16	980	14
4		....	July 8-9, 1955	......	-- ----	USGS	51	540	......
		....	July 13, 1956	......	-- ----		50	712	......
		....	June 15-16, 1957	......	-- ----		40	2,050	......
5	ISU	Weighted average M-1, M-2	June 19, 1951	20	0.60	ISU	23	420	20
			1. Aug. 17, 1951	15	0.85		20	600	17
			2. Aug. 17, 1951	15	0.71		26	592	30
			June 24, 1953	25	1.05		20	557	14
6	ISU	Weighted average N-1, N-2, N-3	June 17-18, 1951	25	2.02	ISU	20	700	25
			1. June 24, 1953	25	1.36		17	426	23
			2. June 24, 1953	20	0.86		14	290	18
7	SUI[c]	....	June 27, 1941	55	2.34	SUI	63	1,345	64
		....	June 30, 1941	45	1.01		90	817	73
		....	July 30, 1950	25	0.84		67	241	58
		....	May 24, 1953	......	-- ----		80	290	......
8	USWB	Morse 1N	July 12, 1943	20	0.23	USGS	153	279	......
			June 1, 1945	20	0.47		121	377	......
			July 31-Aug. 1, 1950	25	0.76		153	261	......
			July 31, 1956	......	-- ----		243	1,025	......
9	ISU	Weighted average R-1, R-2	April 30, 1951	10	0.52	ISU	16	493	13
			June 23, 1951	25	0.84		18	765	17
			July 2, 1951	20	1.18		20	1,450	12
10		....	Aug. 23, 1954	......	-- ----	USGS	135	3,500	......
			July 15-16, 1956	......	-- ----		90	4,200	......
			July 1, 1957	......	-- ----		165	2,460	......
			June 7, 1957	......	-- ----		70	2,448	......
11	USWB	Rolla 7S	April 23, 1950	......	-- ----	USGS	35	742	......
			Aug. 9-10, 1951	60	1.54		30	1,080	32
			Aug. 15-16, 1951	15	0.70		75	640	......
			July 7, 1955	50	0.95		60	1,047	......
12	USWB	Rolla 4SE	June 9, 1950	40	1.41	USGS	45	1,190	44
			June 9, 1954	65	2.17		45	845	36
13	USWB	Tyrone 2N	Sept. 12, 1949	30	1.35	USGS	55	351	......
			May 31, 1957	45	0.35		60	940	......
			July 14, 1957	50	1.70		60	490	75
14	USWB	St. James 3NW	June 20-21, 1948	60	1.20	USGS	105	4,050	77
			July 12, 1948	45	1.08		90	3,270	80
			June 26, 1949	45	1.00		90	1,090	107
			May 25, 1957	30	0.82		90	3,400	123
15	USWB	Houston 1SE	June 9-10, 1950	......	-- ----	USGS	47	265	......
			April 6, 1951	90	2.16		43	648	45
			June 29, 1951	57	1.80		67	315	84
			June 30, 1951	50	1.45		55	996	40
16	USWB	....	June 21, 1951	......	-- ----	USGS	99	1,030	......
			Aug. 8, 1951	......	-- ----		135	5,550	......
			June 24, 1955	......	-- ----		123	1,450	......
			May 1, 1954	......	-- ----		126	833	......
17	USWB	Rolla 5SE	April 23, 1953	13	0.94	USGS	30	577	26
			June 9, 1954	30	1.89		31	821	19
			May 12, 1955	35	1.15		15	337	15
18	USWB	Gower 2N	July 16-17, 1950	......	-- ----	USGS	90	385	......
			June 2, 1954	75	1.49		90	657	122
			June 24, 1955	45	1.10		90	463	107
19	USWB[d]	....	April 23-24, 1953	......	-- ----	USGS	58	120	......
			June 10, 1954	......	-- ----		60	120	......
			May 25, 1957	25	0.63		45	48	35
20	USWB	Vichy 2SE	Aug. 15, 1950	60	1.54	USGS	60	1,790	85
			July 23, 1955	60	1.66		70	1,530	......
			May 22, 1957	35	2.30		90	6,230	98
21	USWB	Rolla 3W	July 22, 1951	......	-- ----	USGS	60	864	......
			April 23, 1953	15	1.31		75	2,050	65
			July 6-7, 1955	15	0.75		50	950	74
			Aug. 7, 1955	15	1.45		72	564	60
22		....	Oct. 11, 1954	......	-- ----	USGS	80	1,325	......
			May 28, 1955	......	-- ----		66	400	......
			Aug. 7, 1955	......	-- ----		79	1,600	......

Table C-4. (Continued)

Watershed number[a]	Raingage: Collection agency[b]	Raingage: Station	Storm characteristics: Storm date	Rainfall: Excess period (min.)	Rainfall: Total depth (in.)	Hydrograph properties: Collection agency	Period of rise (min.)	Peak discharge (cfs)	Lag time (min.)
23	USWB	Oregon 1NE	Aug. 14, 1951	45	0.60	USGS	60	548	50
			Aug. 15, 1951	25	0.45		60	680	52
			Aug. 21, 1954	45	1.25		45	580	28
24	USWB	Brighton	May 22, 1957	25	1.85	USGS	71	845	73
25	USWB	Marshall	May 27-28, 1955	30	1.40	USGS	70	658	71
			June 2, 1955	......	-- ----		45	885	......
			June 29, 1957	......	-- ----		68	503	......
26	USWB	Miller 2SE	June 7, 1956	70	2.25	USGS	90	747	118
			June 24-25, 1956	40	1.34		130	432	126
			May 22, 1957	30	1.00		165	556	165
27	USWB	Preston	April 21, 1957	70	1.15	USGS	150	832	141
			May 9, 1957	25	0.70		75	160	81
			May 22, 1957	55	0.60		65	635	69
28	USWB	Maryville 7NW	June 24, 1949	25	0.55	USGS	164	158	150
			May 25, 1951	75	0.60		238	171	193
			June 19, 1951	45	1.10		238	396	253
			June 20-21, 1951	......	-- ----		361	443	......
			June 21-22, 1951	......	-- ----		419	410	......
29	USWB	Curtis 4N	May 30-31, 1951	30	0.86	USGS	165	2,375	129
		Curtis 4N	June 8, 1951	40	1.62		150	4,430	136
		Stockville 6SSW	June 21-22, 1951	45	1.51		70	3,956	60
30	ARS	B-32R	June 18, 1947	38	1.00	ARS	55	143	64
			May 5-6, 1949	29	0.88		60	307	65
			June 8, 1949	......	-- ----		63	288	......
			July 10, 1951	50	1.84		41	845	42
31	USWB	Spiker 4NW	May 31, 1951	50	1.82	USGS	190	2,980	198
			Aug. 14, 1951	30	0.60		200	1,046	160
			Aug. 20, 1951	30	0.60		185	3,151	160
32	USWB	Rosalie	May 27-28, 1954	100	0.32	USGS	180	1,676	......
			May 31-June 1, 1954	55	0.42		90	1,135	......
			May 12-13, 1956	25	0.79		210	1,294	......
33	ARS	91	June 4, 1941	53	1.00	ARS	32	293	34
34	ARS	27	Sept. 23, 1945	32	1.21	ARS	26	310	23
			July 21, 1956	45	1.61		30	134	27
			June 12, 1957	23	1.60		25	88	37
			July 14, 1958	33	1.13		32	110	41
35	ARS	91	June 4, 1941	52	1.00	ARS	28	214	29
			Sept. 23, 1945	30	1.20		26	130	29
			June 28, 1946	22	0.86		17	235	30
			July 14, 1958	27	1.08		36	86	34
36	ARS	27	Sept. 23, 1945	30	1.75	ARS	45	212	38
			June 16, 1946	16	0.57		55	192	69
			July 11, 1946	......	-- ----		70	404	......
			June 12, 1957	......	-- ----		40	262	......
37	ARS	27	June 18, 1940	26	0.98	ARS	45	248	46
38	ARS	27	June 4, 1941	35	1.02	ARS	74	880	93
			June 21, 1946	20	0.75		75	753	102
			June 12, 1957	53	2.00		82	896	99
39	ARS	27	Aug. 4, 1938	64	1.34	ARS	76	410	94
			July 11, 1946	84	2.52		100	974	142
			July 21, 1946	60	1.18		100	840	113
			June 12, 1957	......	-- ----		80	1,270	94
40	ARS	108	July 8, 1939	28	0.78	ARS	13	177	17
			Aug. 15, 1941	......	-- ----		14	140	......
			June 6, 1947	50	1.26		20	126	23
			Aug. 16, 1947	26	1.11		13	179	23
			July 21, 1949	28	1.14		14	116	23
41	ARS	Weighted average R-2, R-9	Aug. 12, 1943	23	2.07	ARS	18	306	18
			June 28, 1945	16	0.93		16	340	19
			July 15-16, 1950	11	1.07		20	350	19
42	ARS	R-2	Aug. 12, 1943	23	1.89	ARS	10	212	5
			June 28, 1945	18	0.96		12	229	10
			July 15-16, 1950	10	0.93		16	183	21

[a]Refer to table C-1 for code designation.
[b]Refer to table C-2 for interpretation.
[c]Raingage station unknown, available from SUI.
[d]Raingage station unknown, rainfall chart for storm on May 25, 1957, obtained from USGS, Rolla, Missouri.

APPENDIX D: DISTRIBUTION GRAPHS AND EMPIRICAL GRAPHS

Development of an Empirical Graph for a Given Watershed

An outline of the procedures used to develop the empirical graph of a watershed is given as follows: The data collected from watershed 19 is used for illustrative purposes.

1. DEVELOPMENT OF DISCHARGE HYDROGRAPHS.

   A. By using the appropriate rating tables for the station, the stage graphs from selected storms were reduced to discharge hydrographs and plotted as shown in figs. D-1, D-2 and D-3. In this procedure, all major fluctuations on the stage graphs were noted so that a faithful reproduction of the orginal was obtained.

2. SEPARATION OF BASE FLOW.

Since the distribution-graph or unit-graph principle is applicable only to surface runoff, it was necessary to separate the base flow component from each discharge hydrograph. Several techniques are available to accomplish this separation; however, the selection of one in preference to another is subject to personal opinion. With reference to the methodology employed, Wisler and Brater (59, p. 30) state, "The exact location of the end of surface runoff usually cannot be determined, but this is not of great importance as long as one always follows a consistent procedure."

For the watersheds used in this study, the contribution of base flow during the flood period was assumed to be practically negligible. It was considered impractical, therefore, to adopt a complex, time-consuming technique for base-flow separation. A simple, arbitrary procedure was developed to accomplish this purpose.

   A. A straight line was drawn tangent to the recession curve where the curve showed a relatively constant depletion rate over a long period of time.

   B. The initial point of rise on the recession limb was connected with the point at which the tangent line departed from the recession curve by a straight line (see figs. D-1, D-2 and D-3).

The area above this line was taken to represent surface runoff; the area below, base flow. In an attempt to obtain congruency in the time-bases of the hydrographs, the period of surface runoff was temporarily defined as the time from the initial point of rise to the point at which the surface runoff rate decreased to 5 percent of the peak discharge rate, $0.05Q_p$.

Where a parasite storm complexed the recession limb, as in the hydrograph for April 23, 1953, the normal recession limb was plotted according to a composite recession developed from the other hydrographs of record (see fig. D-4).

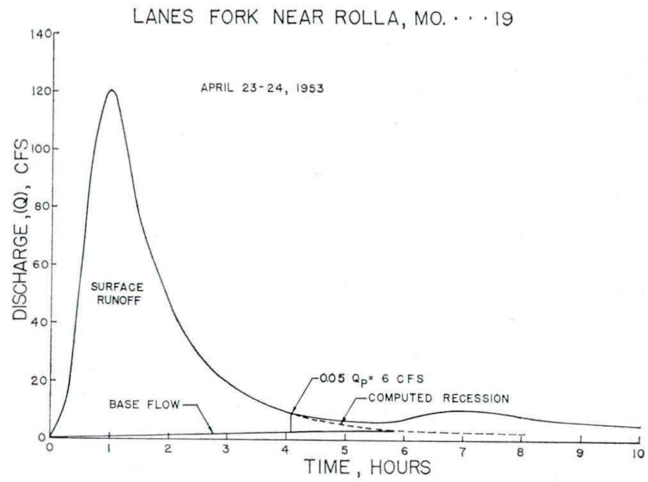

Fig. D-1. Discharge hydrograph for storm of April 23-24, 1953, on watershed 19.

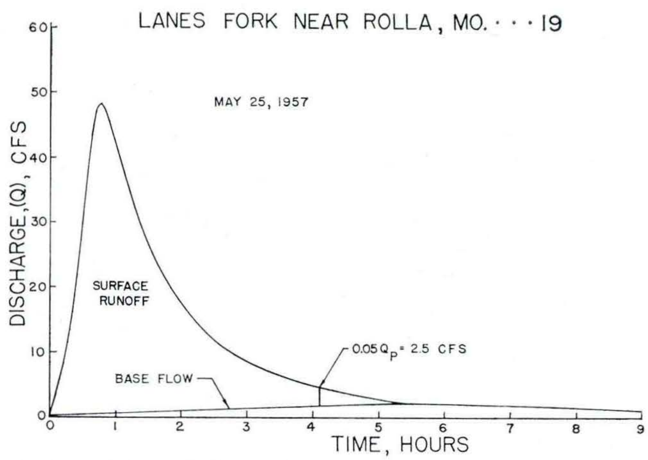

Fig. D-3. Discharge hydrograph for storm of May 25, 1957, on watershed 19.

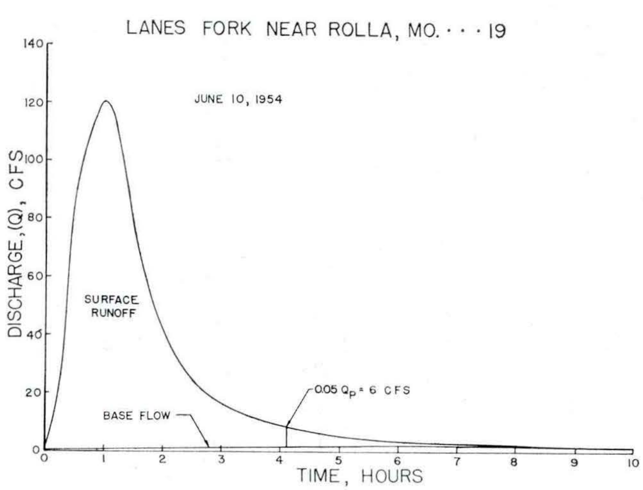

Fig. D-2. Discharge hydrograph for storm of June 10, 1954, on watershed 19.

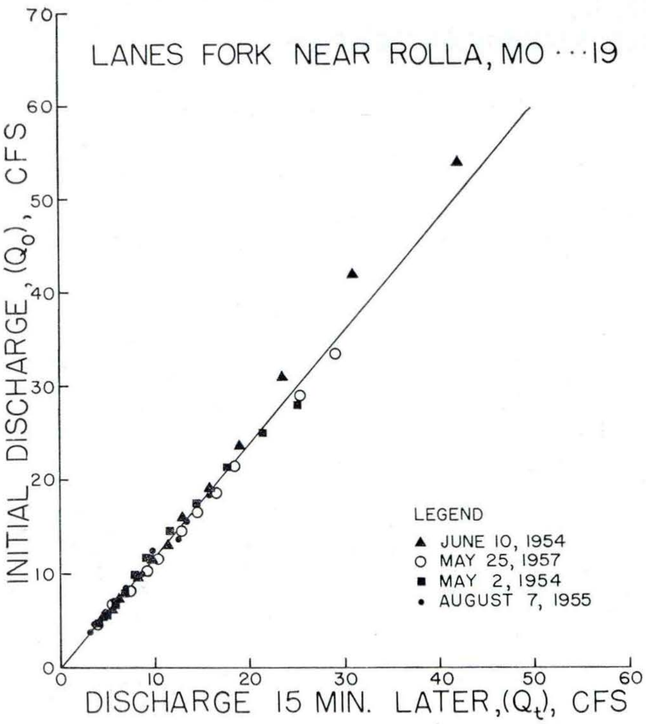

Fig. D-4. Recession curve for watershed 19.

3. DEVELOPMENT OF THE DISTRIBUTION GRAPHS.

A. The time-bases of the surface-runoff hydrographs were divided into at least 10 and preferably 14-15, equal-time increments of Δt-minutes' duration. To avoid irregular time increments, Δt was chosen to the nearest 5 minutes and the last increment taken to include the discharge $0.05Q_p$. The same time unit was used for all hydrographs of a given watershed.

B. The ordinate values of the surface-runoff hydrographs were tabulated at the respective times from the beginning of surface runoff,

$$\frac{\Delta t}{2}, \frac{3\Delta t}{2}, \frac{5\Delta t}{2} \ldots\ldots\ldots \frac{(2n-1)\,\Delta t}{2}$$

where n is the number Δt-increments. For each hydrograph, the peak discharge was always recorded.

C. The distribution graph was developed from each hydrograph by the relationship

$$\%\ \text{flow}/\Delta t\text{-increment} = \frac{\Sigma\text{cfs for a given } \Delta t\text{-period}}{\Sigma\text{cfs for n } \Delta t\text{-periods}} \times 100\,.$$

See tables D-1, D-2 and D-3.

Table D-1. Distribution graph for watershed 19 from storm of April 23-24, 1953.

Accumulated time (min.)	Number of 15-minute periods	Corrected discharge (cfs)	Flow per 15-minute time interval (percent)
0.0	0	0	0.0
7.5	1	4	0.6
22.5	2	27	3.9
37.5	3	82	11.8
52.5	4	113	16.2
57.5		119[a]	17.1
67.5	5	112	16.0
82.5	6	85	12.2
97.5	7	65	9.3
112.5	8	51	7.3
127.5	9	38	5.5
142.5	10	30	4.3
157.5	11	24	3.4
172.5	12	19	2.7
187.5	13	15	2.2
202.5	14	12	1.7
217.5	15	9	1.3
232.5	16	7	1.0
247.5	17	4	0.6
Total		697	100.0

[a]Peak discharge rate; not included in total.

Table D-2. Distribution graph for watershed 19 from storm of June 10, 1954.

Accumulated time (min.)	Number of 15-minute periods	Corrected discharge (cfs)	Flow per 15-minute time interval (percent)
0.0	0	0	0.0
7.5	1	8	1.1
22.5	2	50	6.9
37.5	3	98	13.6
52.5	4	115	15.9
60.0		119[a]	16.5
67.5	5	114	15.8
82.5	6	90	12.3
97.5	7	62	8.6
112.5	8	46	6.4
127.5	9	34	4.7
142.5	10	25	3.5
157.5	11	20	2.8
172.5	12	16	2.2
187.5	13	13	1.8
202.5	14	11	1.5
217.5	15	9	1.2
232.5	16	7	1.0
247.5	17	5	0.7
Total		723	100.0

[a]Peak discharge rate; not included in total.

Table D-3. Distribution graph for watershed 19 from storm of May 25, 1957.

Accumulated time (min.)	Number of 15-minute periods	Corrected discharge (cfs)	Flow per 15-minute time interval (percent)
0.0	0	0	0.0
7.5	1	4	1.4
22.5	2	18	6.6
37.5	3	41	15.0
45.0		48[a]	17.5
52.5	4	44	16.0
67.5	5	36	13.1
82.5	6	28	10.2
97.5	7	22	8.1
112.5	8	18	6.6
127.5	9	15	5.5
142.5	10	12	4.2
157.5	11	9	3.5
172.5	12	7	2.6
187.5	13	6	2.2
202.5	14	5	1.8
217.5	15	4	1.4
232.5	16	3	1.1
247.5	17	2	0.7
Total		274	100.0

[a]Peak discharge rate; not included in total.

4. DEVELOPMENT OF EMPIRICAL GRAPH

The empirical graph for each watershed was developed by procedures described previously in the text. The graph for watershed 19 is given in fig. D-9.

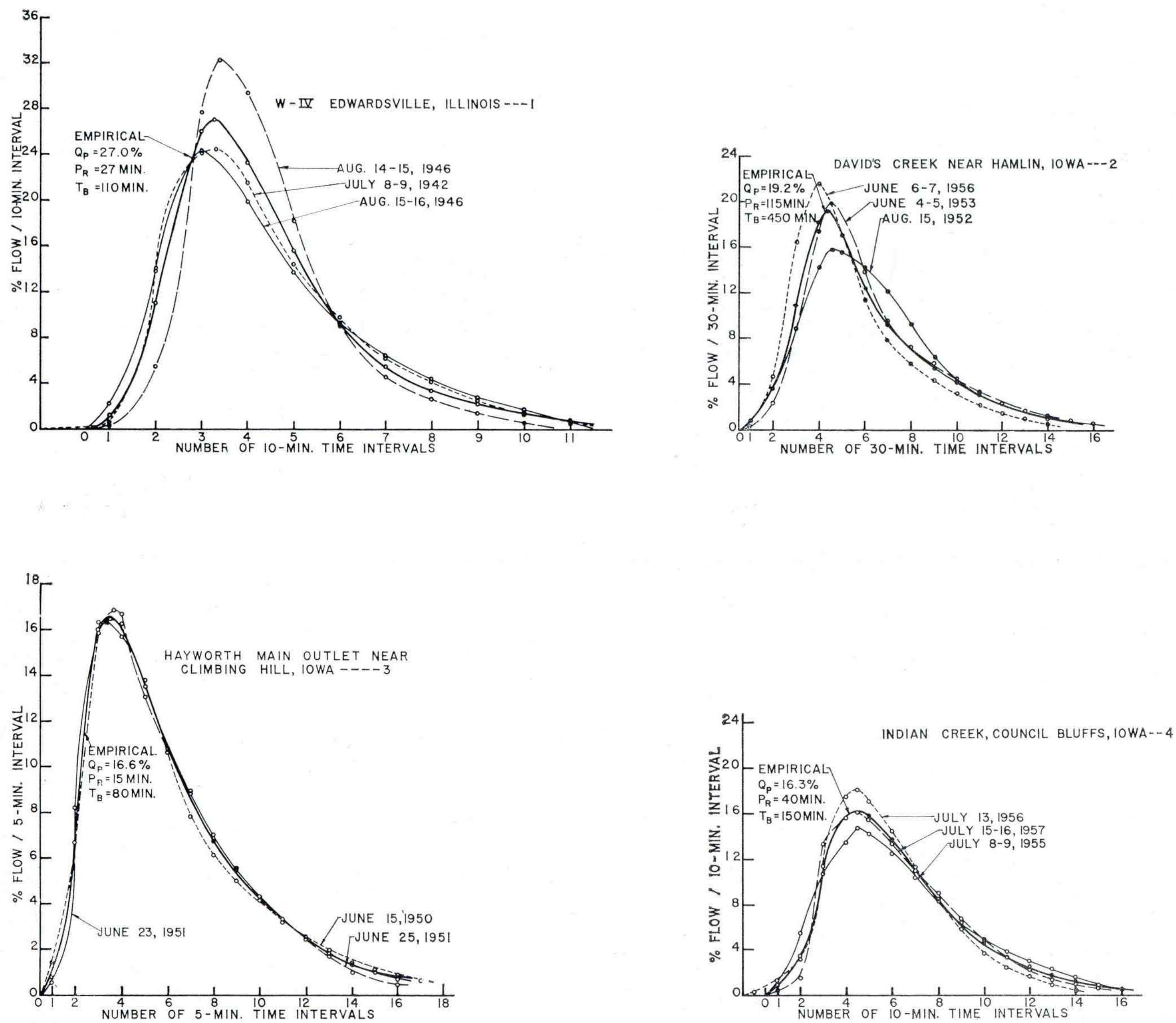

Fig. D-5. Distribution graphs for selected storms and empirical graphs for watersheds 1, 2, 3 and 4.

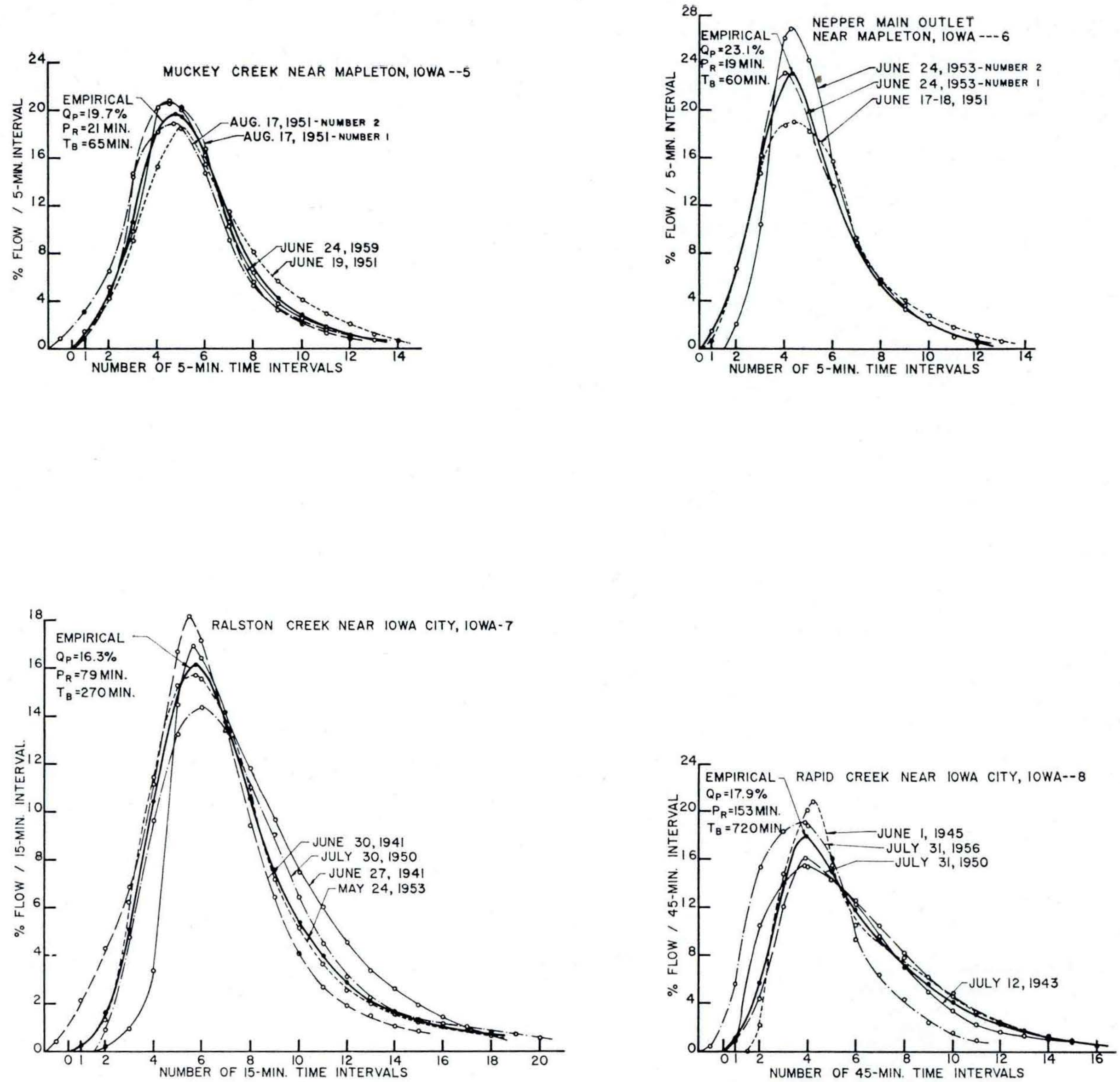

Fig. D-6. Distribution graphs for selected storms and empirical graphs for watersheds 5, 6, 7 and 8.

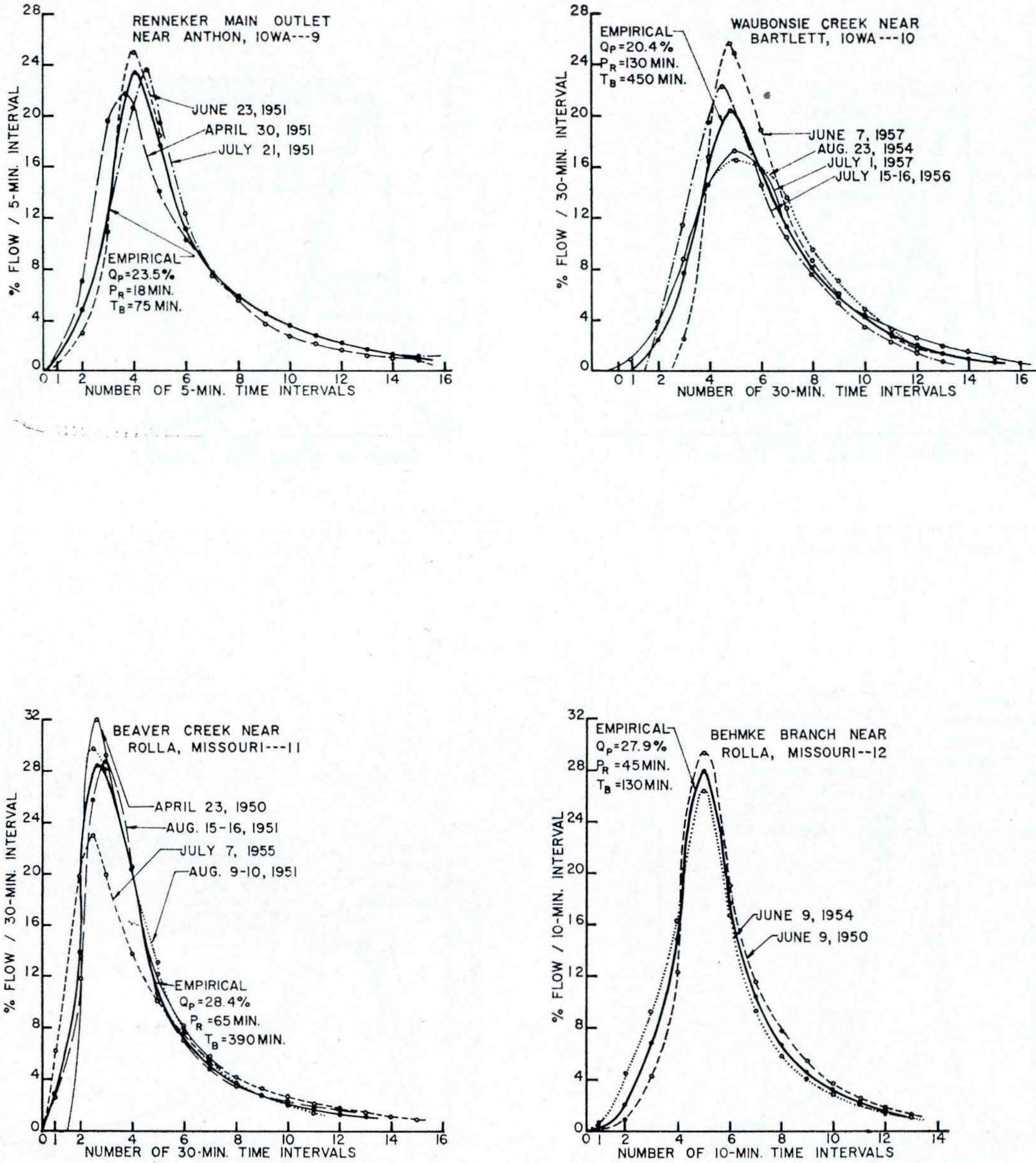

Fig. D-7. Distribution graphs for selected storms and empirical graphs for watersheds 9, 10, 11 and 12.

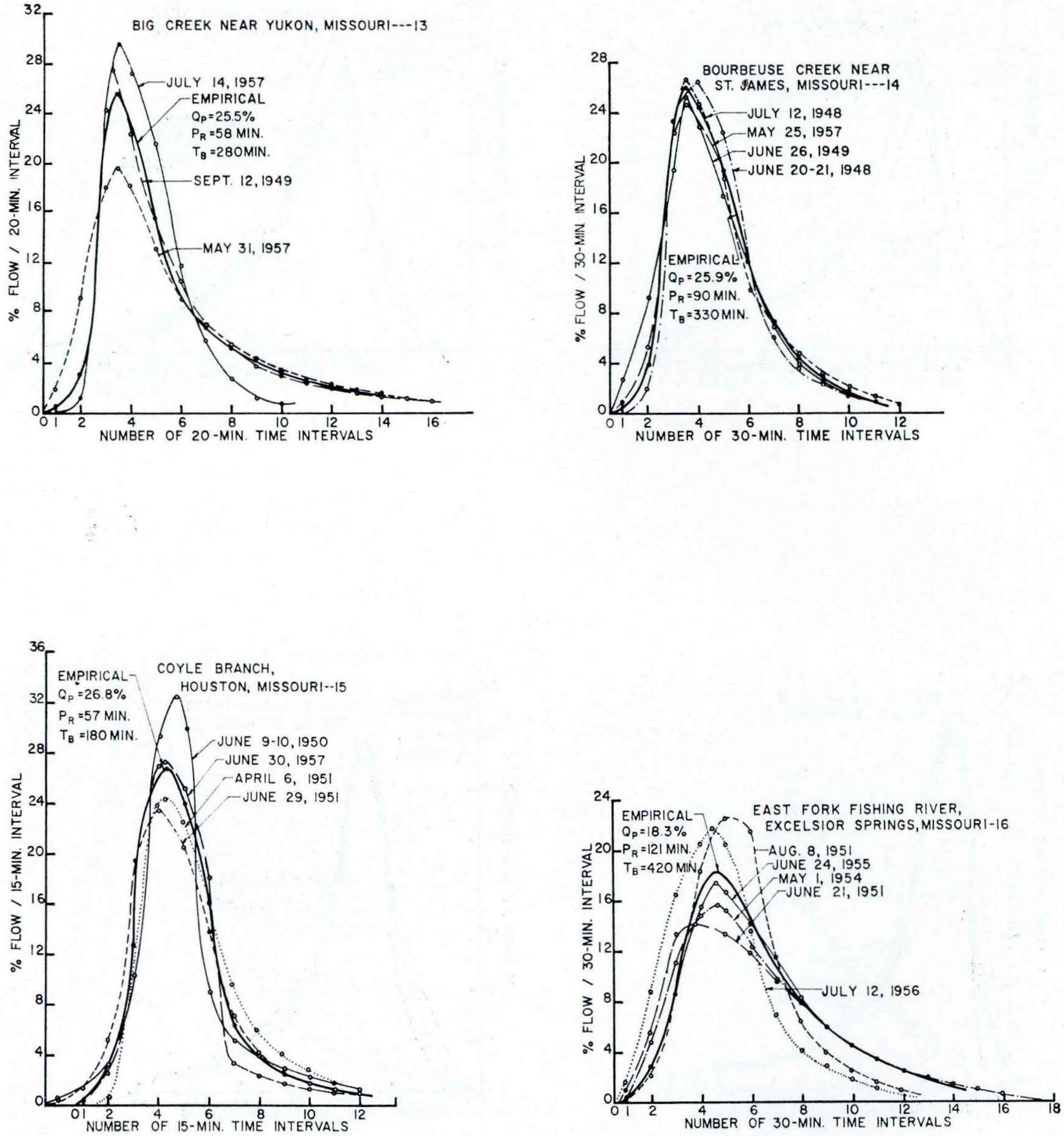

Fig. D-8. Distribution graphs for selected storms and empirical graphs for watersheds 13, 14, 15 and 16.

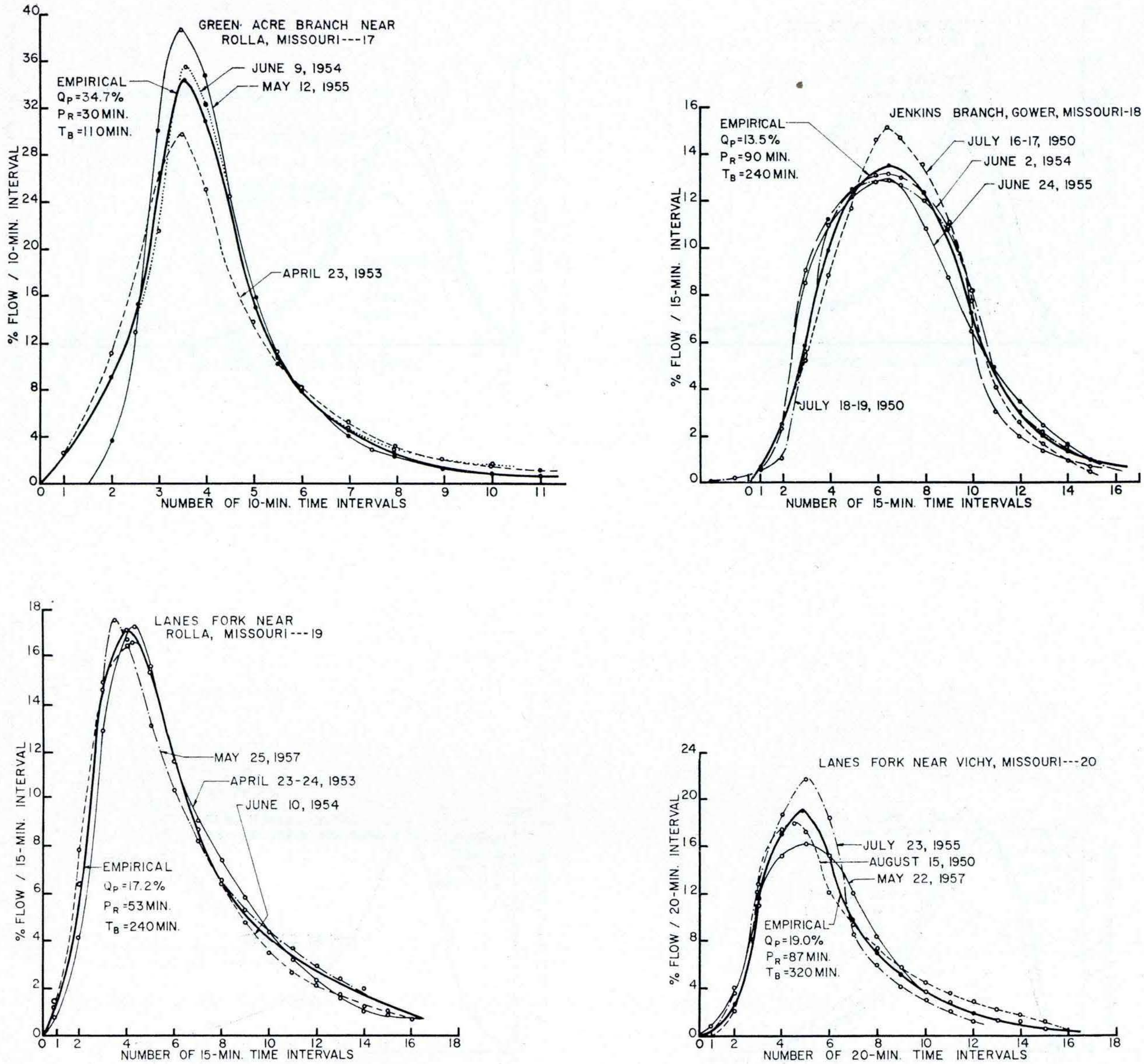

Fig. D-9. Distribution graphs for selected storms and empirical graphs for watersheds 17, 18, 19 and 20.

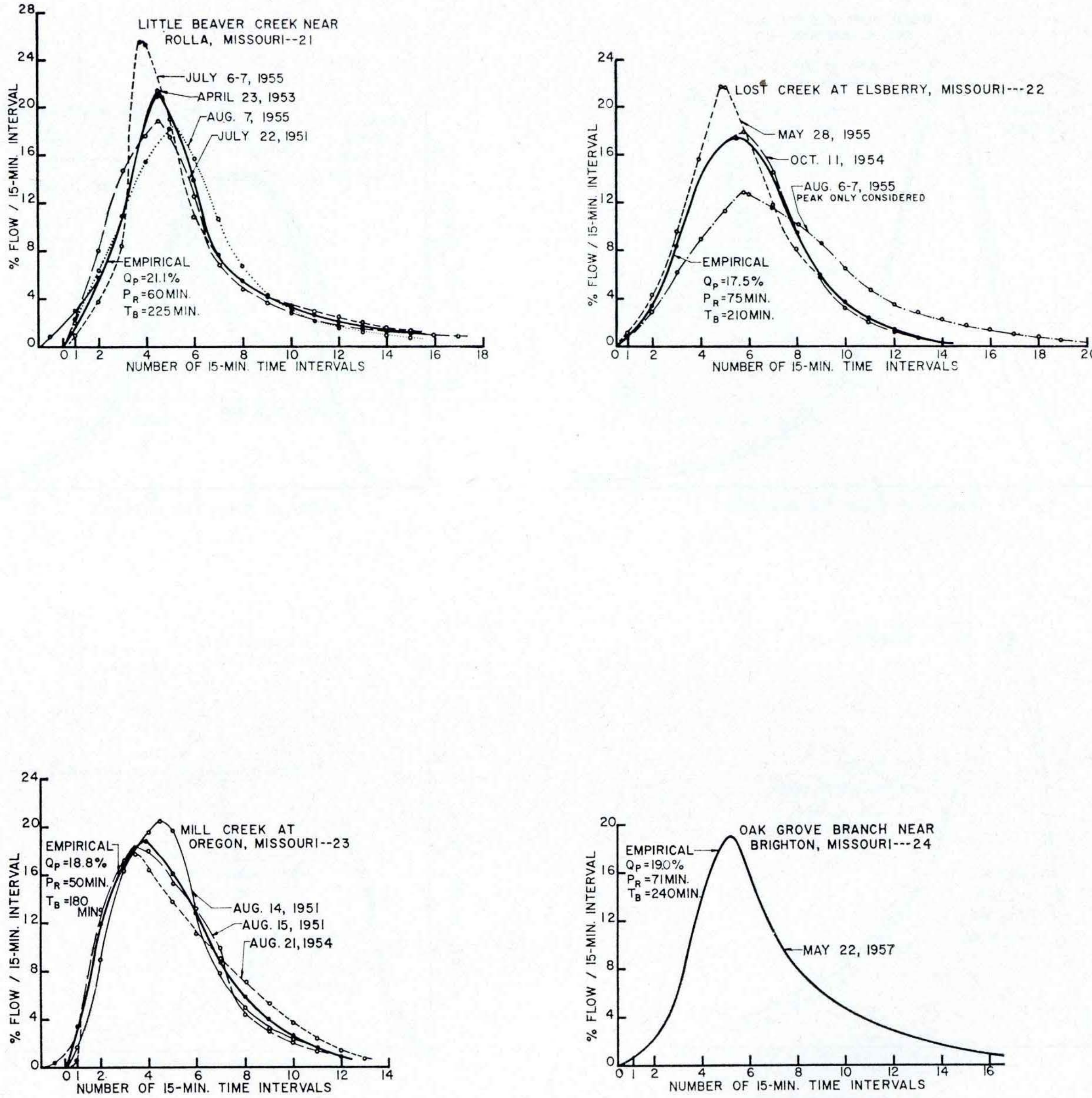

Fig. D-10. Distribution graphs for selected storms and empirical graphs for watersheds 21, 22, 23 and 24.

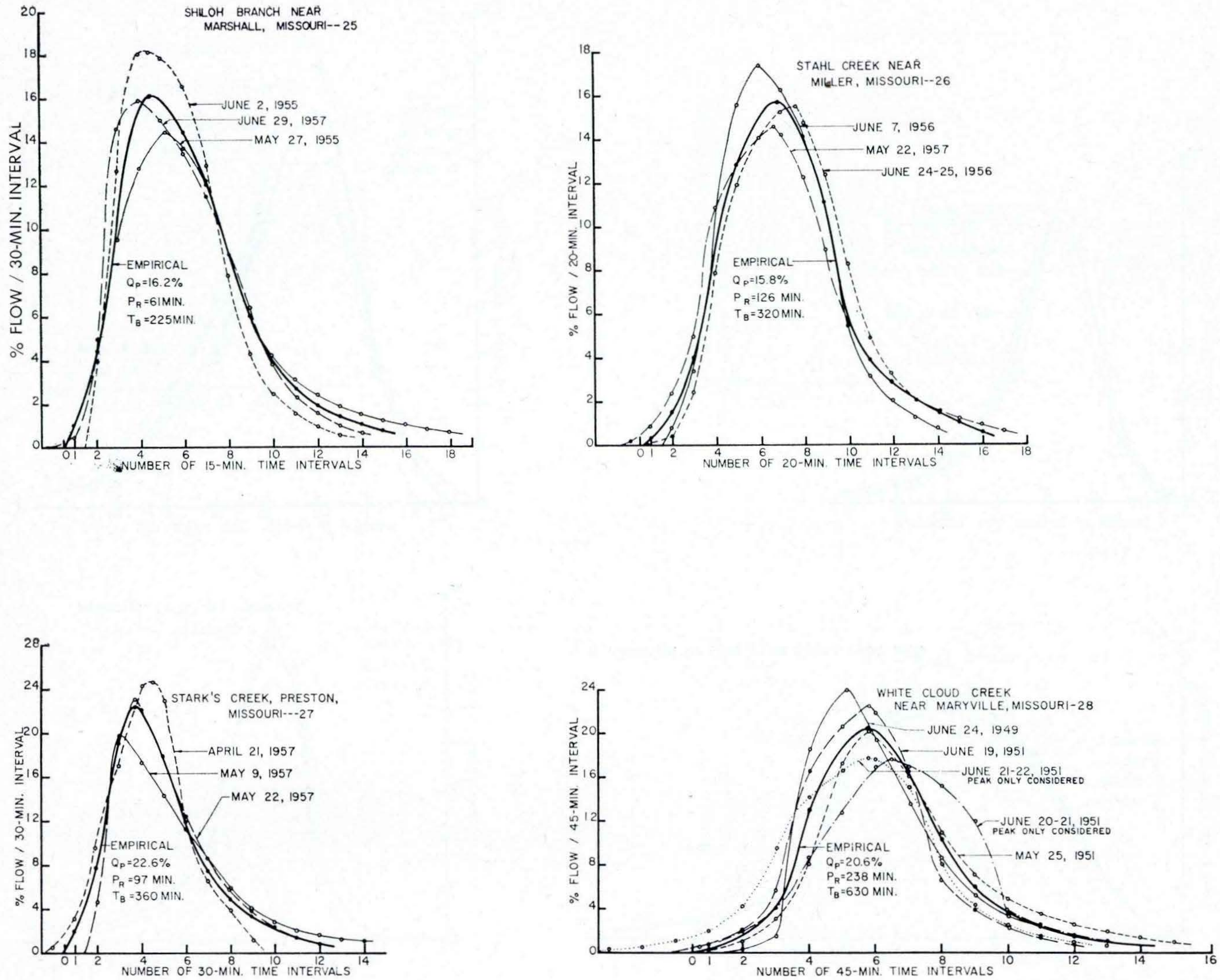

Fig. D-11. Distribution graphs for selected storms and empirical graphs for watersheds 25, 26, 27 and 28.

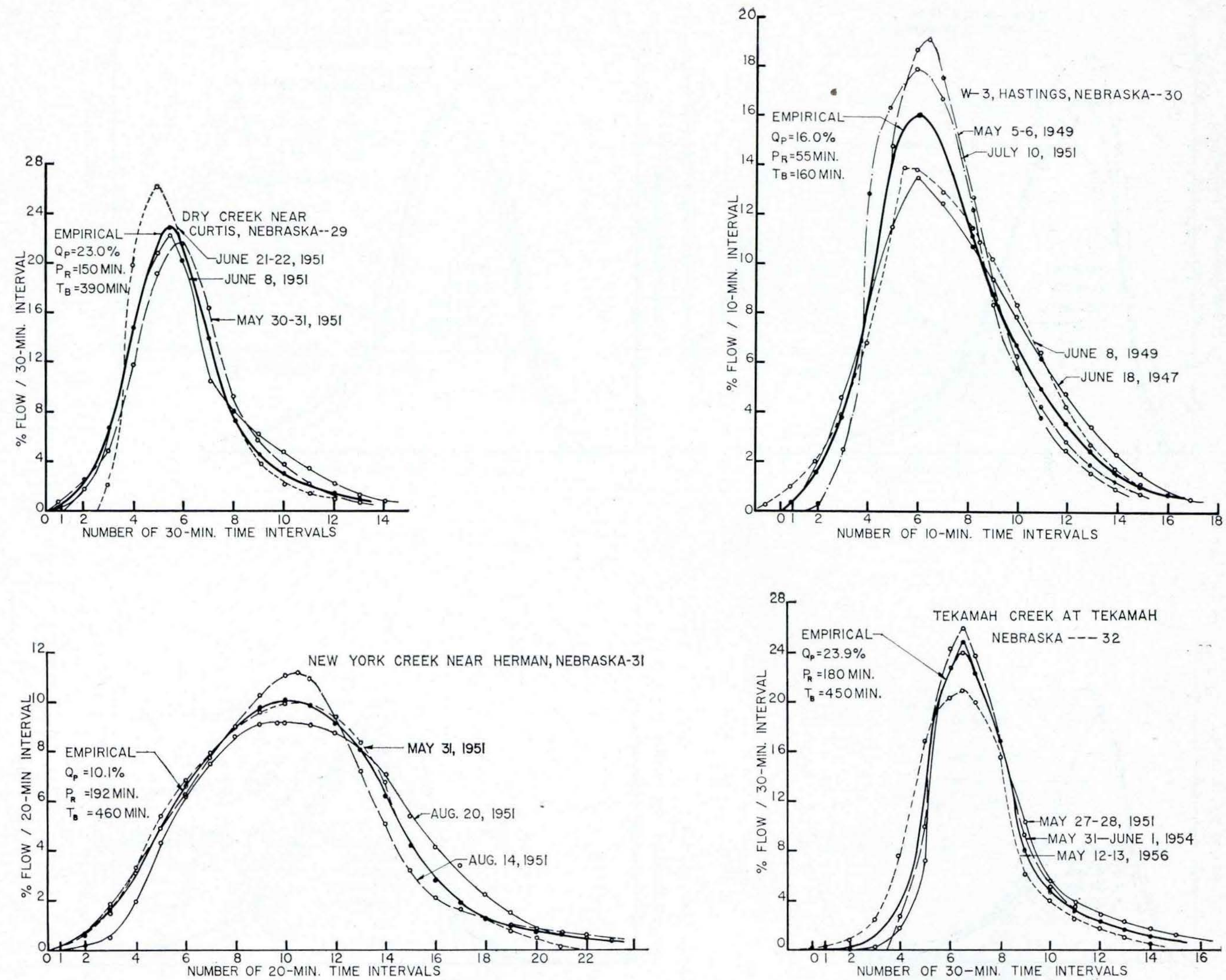

Fig. D-12. Distribution graphs for selected storms and empirical graphs for watersheds 29, 30, 31 and 32.

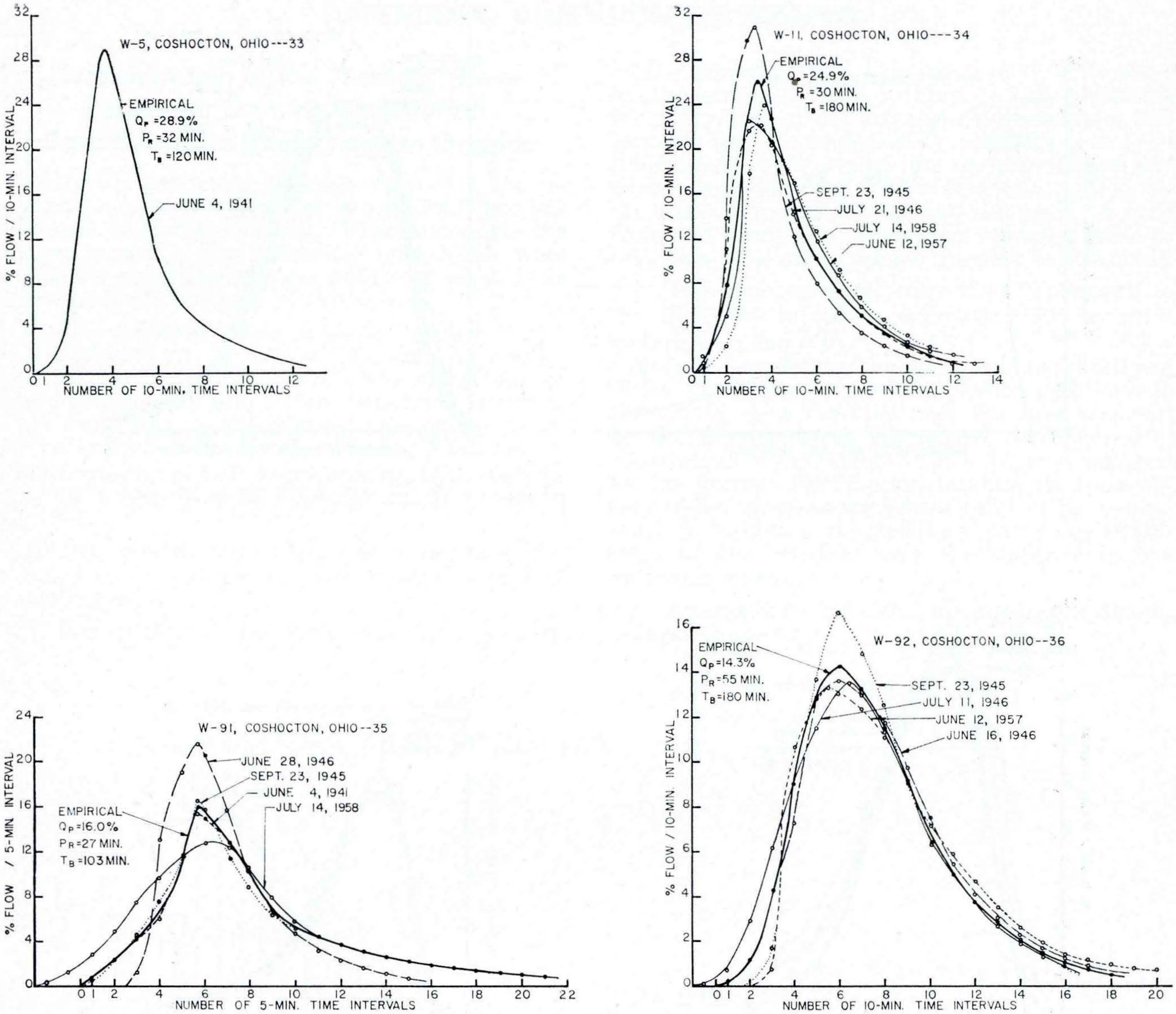

Fig. D-13. Distribution graphs for selected storms and empirical graphs for watersheds 33, 34, 35 and 36.

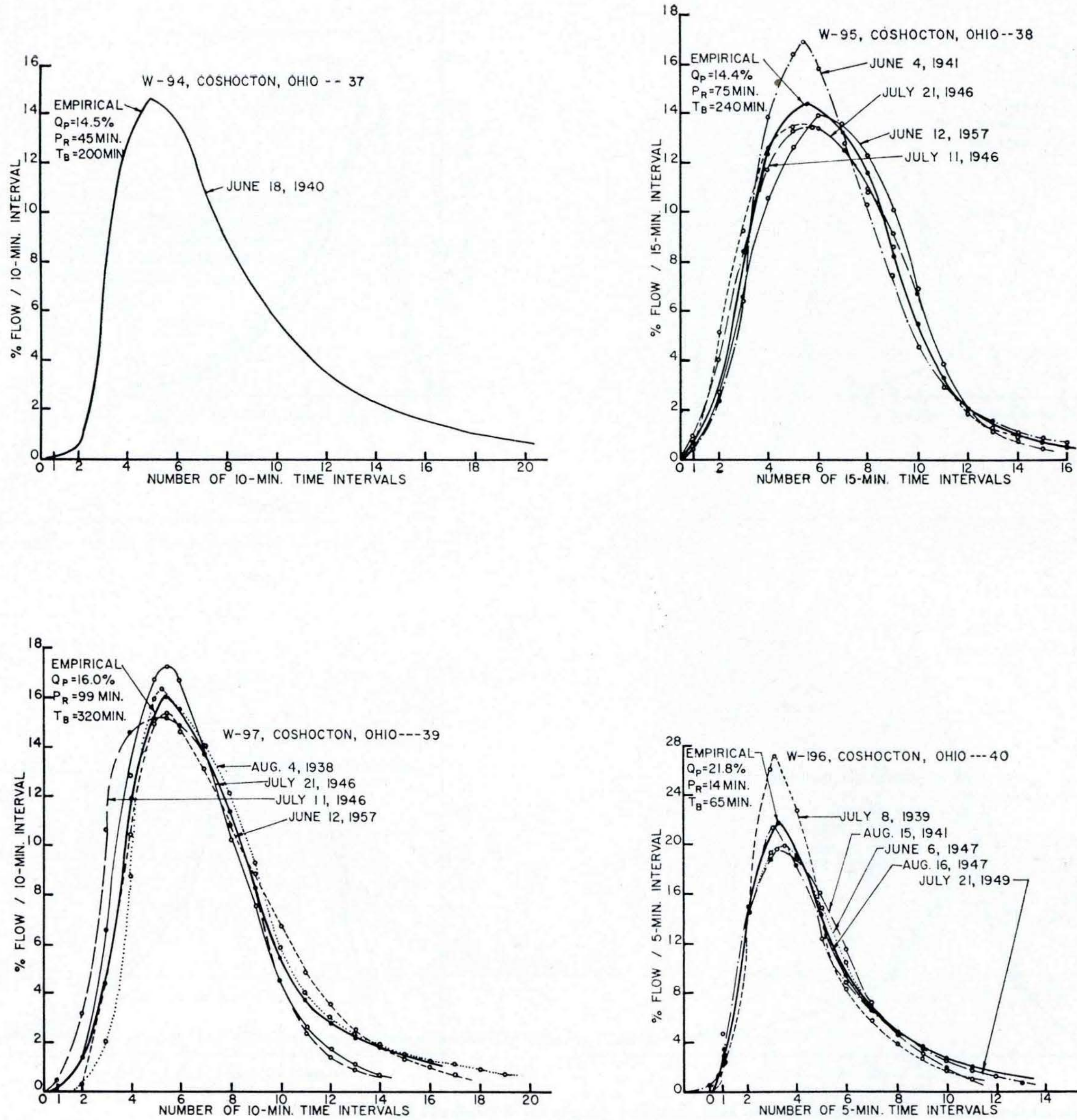

Fig. D-14. Distribution graphs for selected storms and empirical graphs for watersheds 37, 38, 39 and 40.

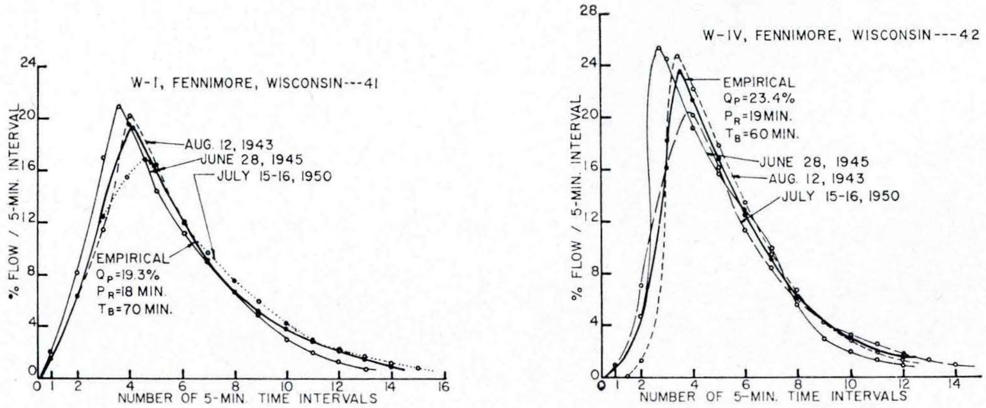

Fig. D-15. Distribution graphs for selected storms and empirical graphs for watersheds 41 and 42.

APPENDIX E: DIMENSIONLESS GRAPHS

Determination of the "Best-Fit" Two-Parameter Gamma Distribution Describing the Dimensionless Graphs

With the particular program employed, the input capacity of the IBM 650 was restricted to 999 numbers of 10 digits or less. To accommodate the entire capacity, the following procedures were applied to the dimensionless graphs of each of the 42 watersheds included in this study:

1. The ordinate values of Q_1, Q_2, Q_3, Q_4, ... Q_n, expressed in % flow/0.25P_R, for the respective increments, $t/P_R = 0.125$, along the base of the dimensionless graph were listed and summed. This sum must be 200 percent because the number of abscissa values chosen has been doubled. A dimensionless graph expressed in this manner may be represented as a histogram, as shown in fig. E-1.

2. The ordinate values given in Step 1 were increased by a multiple of five, to give a sum of 1,000 percent.

3. Each value of the ratio, t/P_R, was punched on the predetermined number of IBM cards given by the ordinate value in Step 2. The correction for odd values of the ordinates, for example, 14.5 percent, was accomplished by placing 14 cards of the respective t/P_R-value into the distribution and placing an additional card of the value, t/P_R, for the next odd ordinate nearest the peak. A card from the group for the largest recorded value of t/P_R was removed to reduce the deck to 999 cards.

4. The punched cards were then introduced to the IBM 650 to obtain estimators of the parameters, γ', q and $\bar{t}/P_R$.

Before completing the analysis, an additional factor must be considered. A basic hypothesis in fitting the data required that the area enclosed by the dimensionless graph and the theoretical distribution be equivalent. Since the area enclosed by the gamma distribution is unity, it is necessary to include the appropriate value of N in equation 14c to obtain the desired result. The evaluation of the constant was accomplished in the following manner:

1. Approximate area, A_D, bounded by a dimensionless graph.

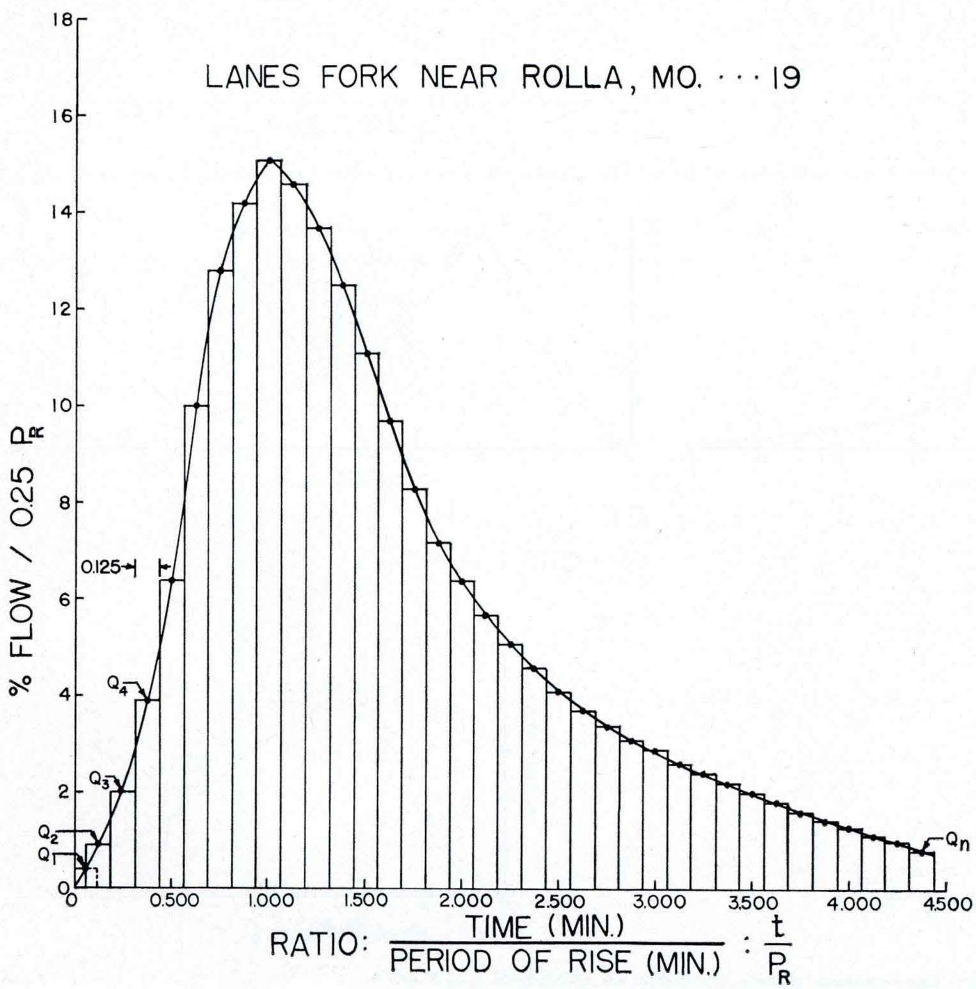

Fig. E-1. Dimensionless graph of watershed 19 as histogram.

$$A_D = (Q_1/2)\ (0.083) + (Q_2)\ (0.125) + (Q_3)\ (0.125) + (Q_4)\ (0.125) \ldots \ldots + (Q_n)\ (0.125)$$

$$A_D = (Q_1/2)\ (0.083) + 0.125 \sum_{n=2}^{n} Q_n. \quad (32a)$$

For practical work, only small error will be introduced if it is assumed,

$$(Q_1/2)\ (0.083) \cong 0.125\ Q_1$$

therefore, equation 32a reduces to

$$A_D \cong 0.125 \sum_{n=1}^{n} Q_n. \quad (32b)$$

But,

$$\sum_{n=1}^{n} Q_n = 200 \text{ percent.} \quad (32c)$$

Substituting equation 32c into equation 32b, it follows that

$$A_D \cong 25.0 \text{ percent.}$$

2. Area bounded by the two-parameter gamma distribution of the dimensionless graph, A_G (see equation 14c)

$$A_G = N \int_{t/P_R=0}^{\infty} \frac{(\gamma')^q}{\Gamma(q)}\, e^{\gamma' t/P_R}\ t/P_R^{\,q-1}\ d(t/P_R)$$

but

$$\int_{t/P_R=0}^{\infty} \frac{(\gamma')^q}{\Gamma(q)}\, e^{\gamma' t/P_R}\ t/P_R^{\,q-1}\ d(t/P_R) = 1\,.$$

It follows that for A_D to be equal to A_G, the constant, N, of the two-parameter gamma distribution must have a numerical value of 25.0 percent.

On the plotted figures (see figs. E-2 - E-12), the theoretical curves have been given a finite maximum value of t/P_R. Obviously, this is not theoretically correct, because the distribution is defined by the integral from $t/P_R = 0$ to $t/P_R = \infty$. The volume of flow occurring beyond these maximum values is usually very small, however, and in part has been compensated for by the increased value of the constant.

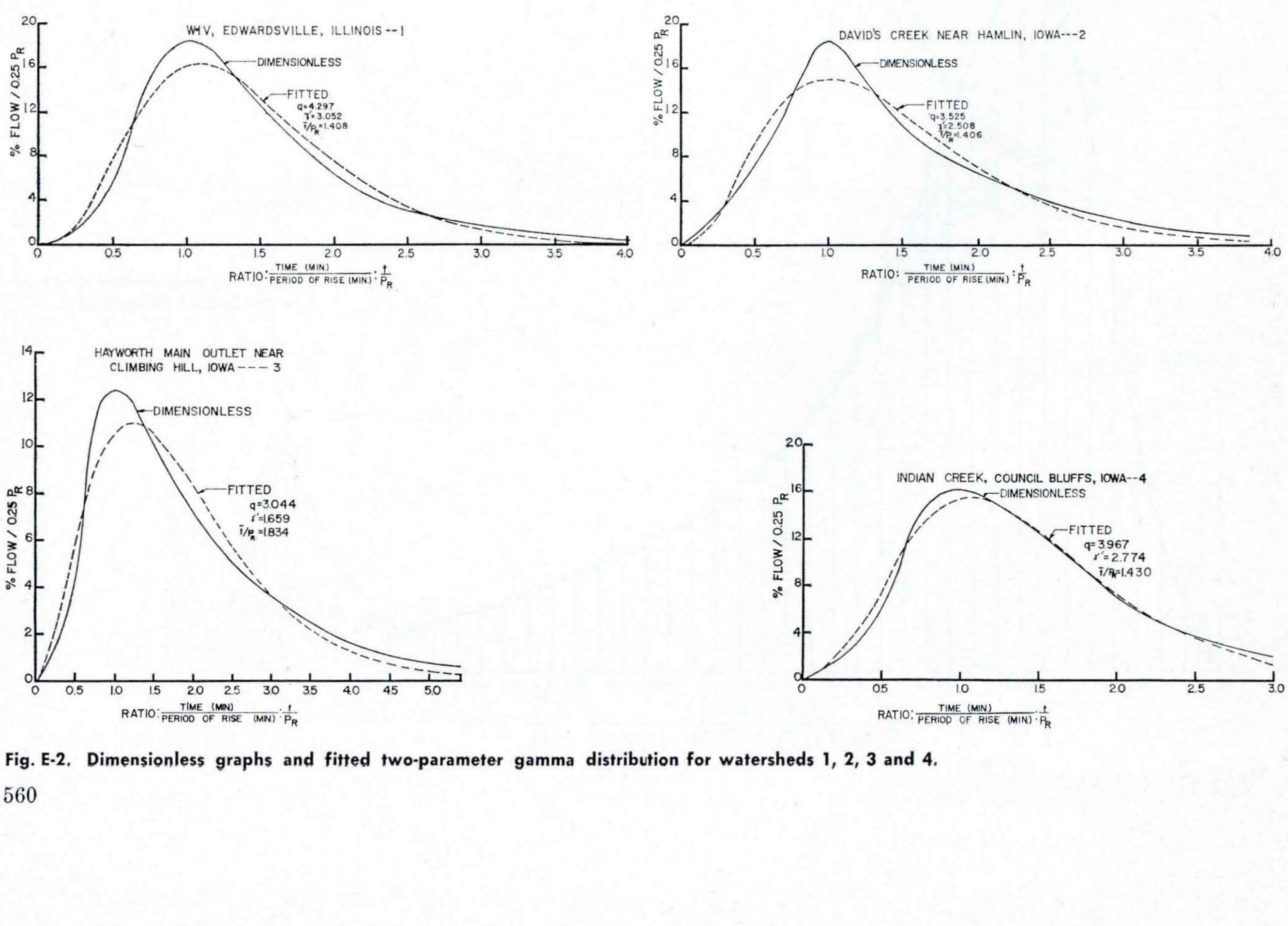

Fig. E-2. Dimensionless graphs and fitted two-parameter gamma distribution for watersheds 1, 2, 3 and 4.

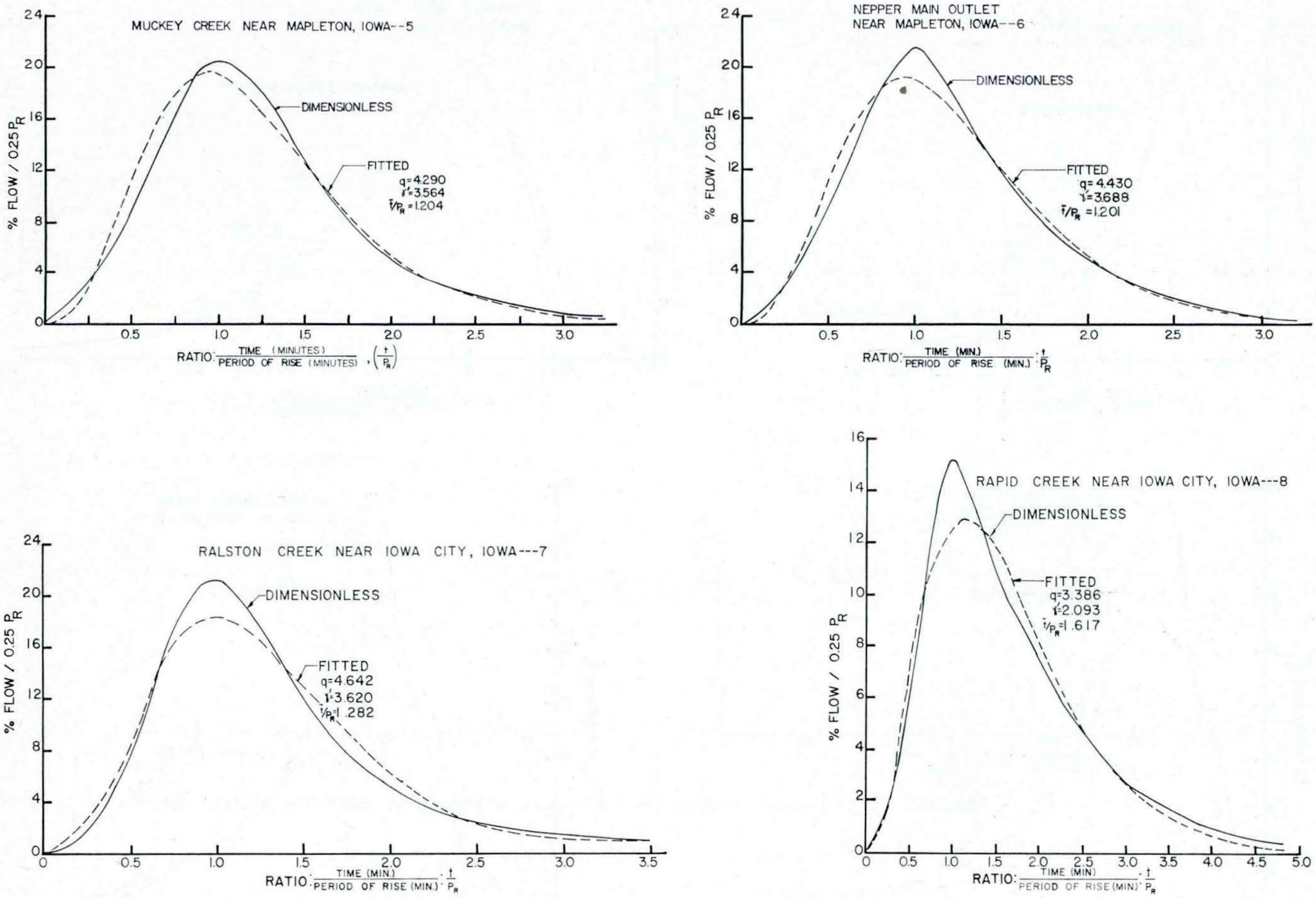

Fig. E-3. Dimensionless graphs and fitted two-parameter gamma distribution for watersheds 5, 6, 7 and 8.

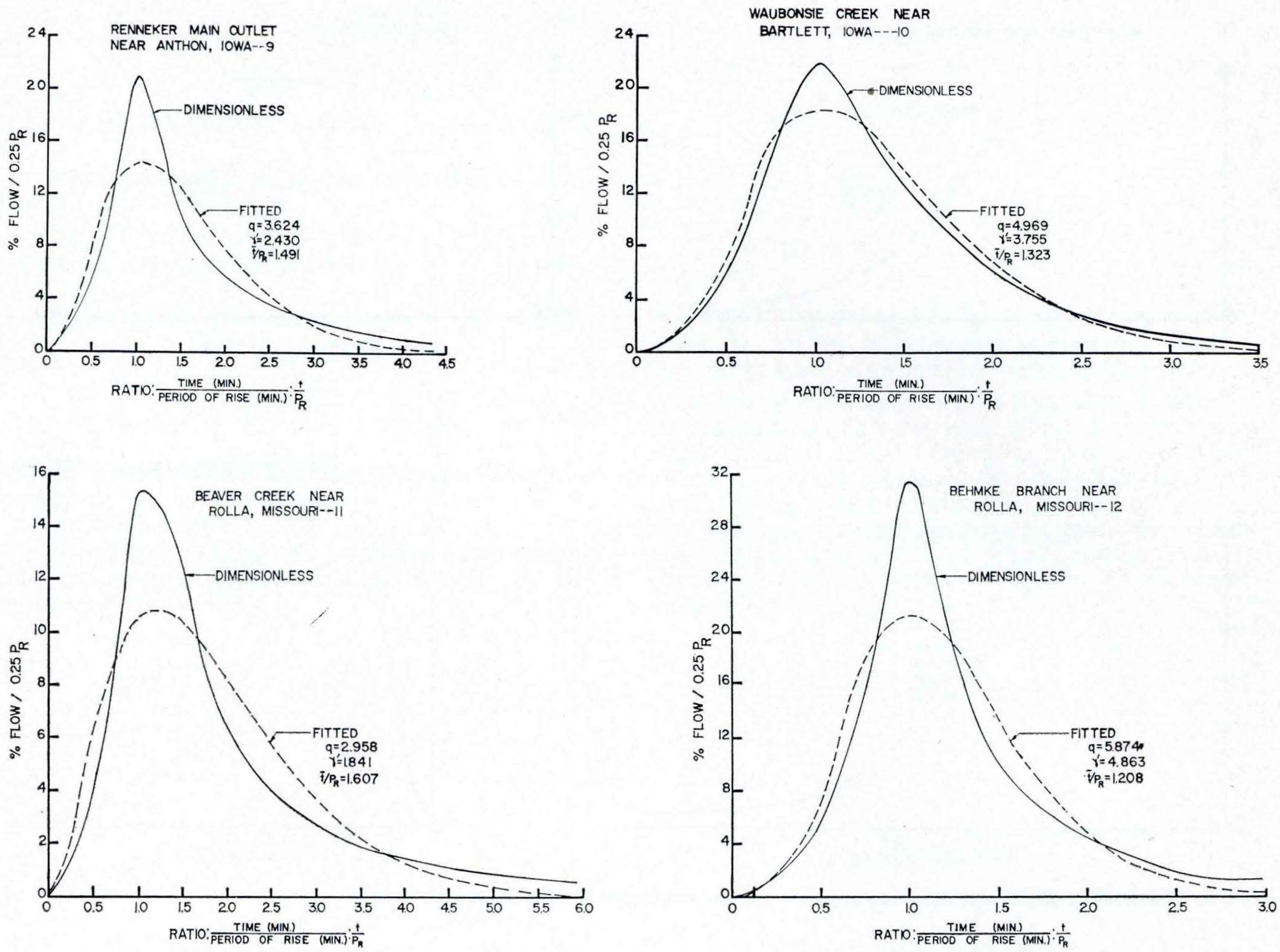

Fig. E-4. Dimensionless graphs and fitted two-parameter gamma distribution for watersheds 9, 10, 11 and 12.

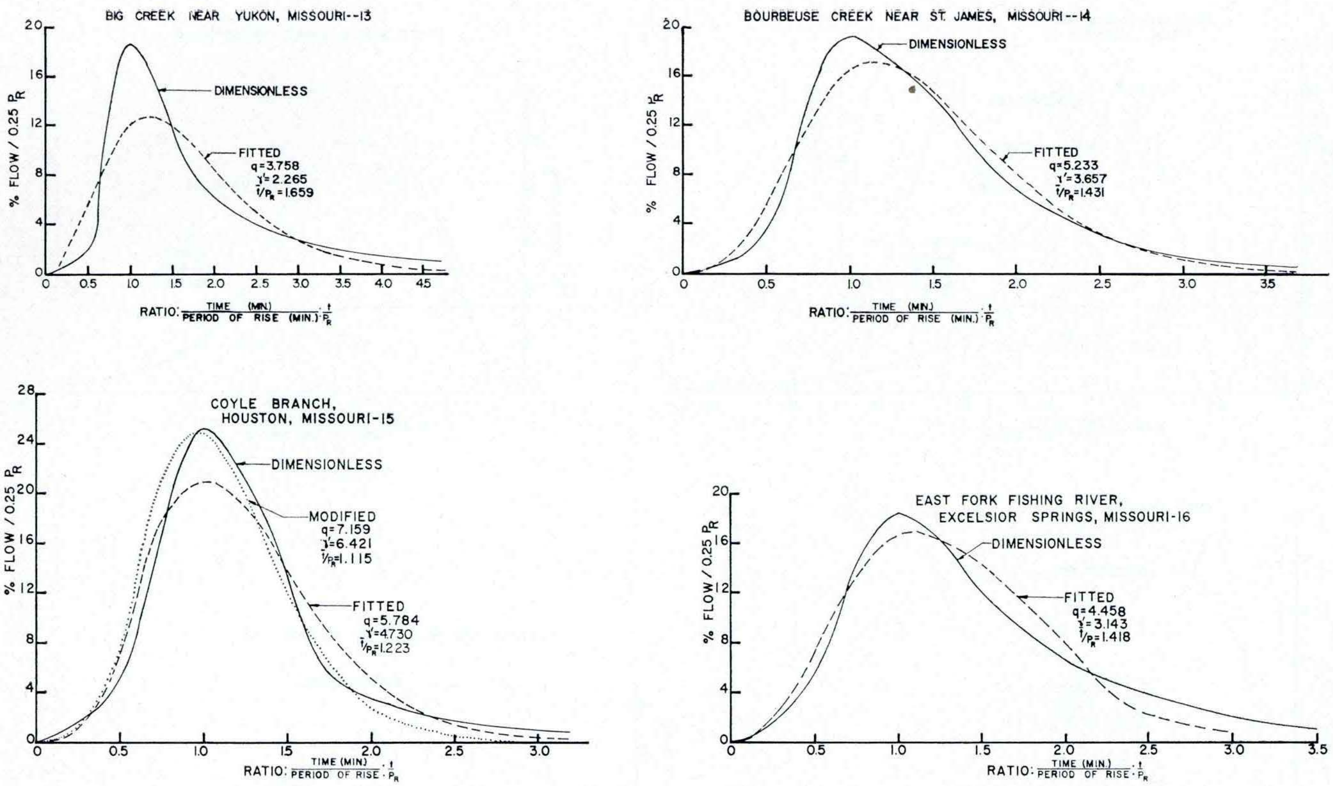

Fig. E-5. Dimensionless graphs and fitted two-parameter gamma distribution for watersheds 13, 14, 15 and 16.

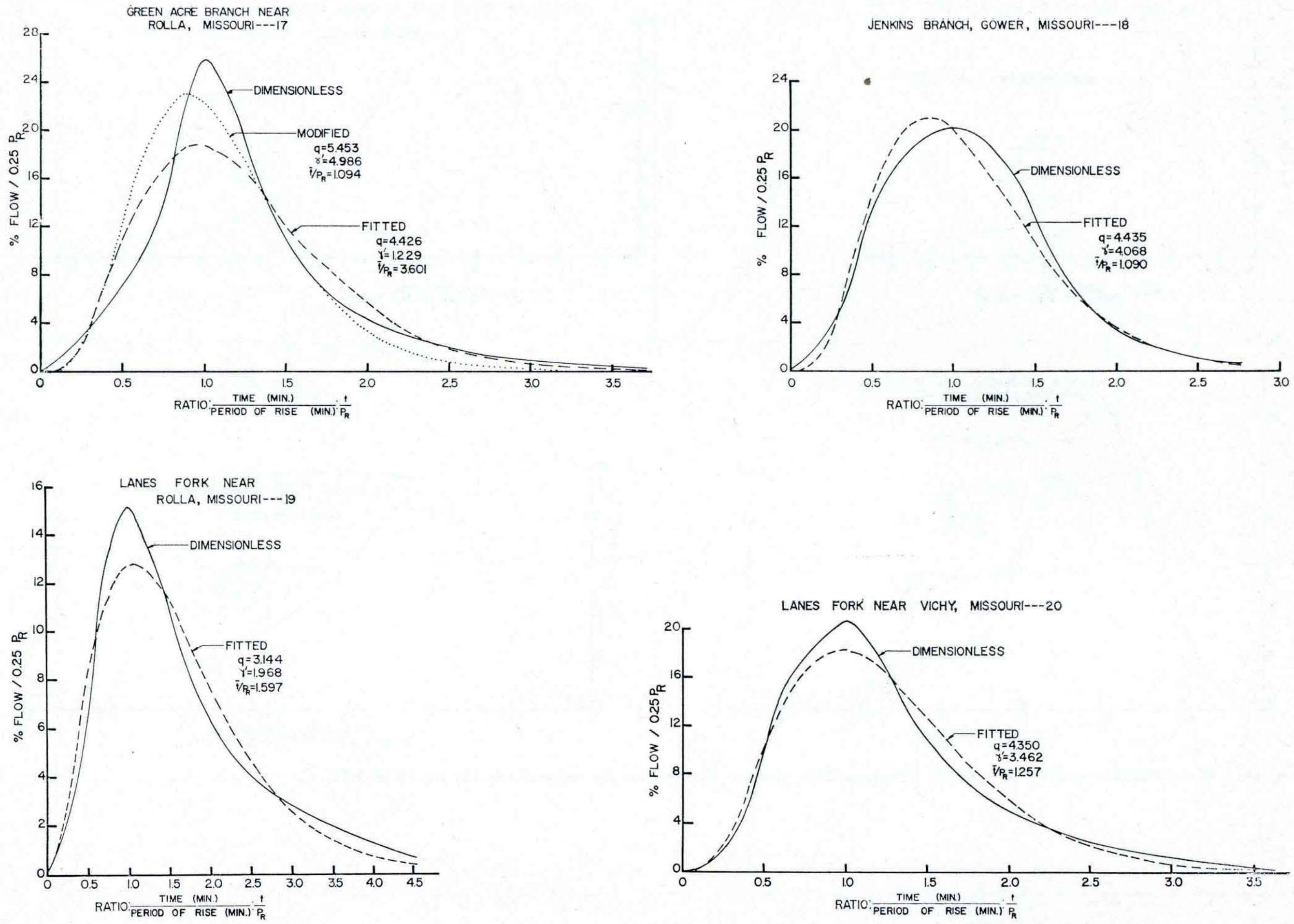

Fig. E-6. Dimensionless graphs and fitted two-parameter gamma distribution for watersheds 17, 18, 19 and 20.

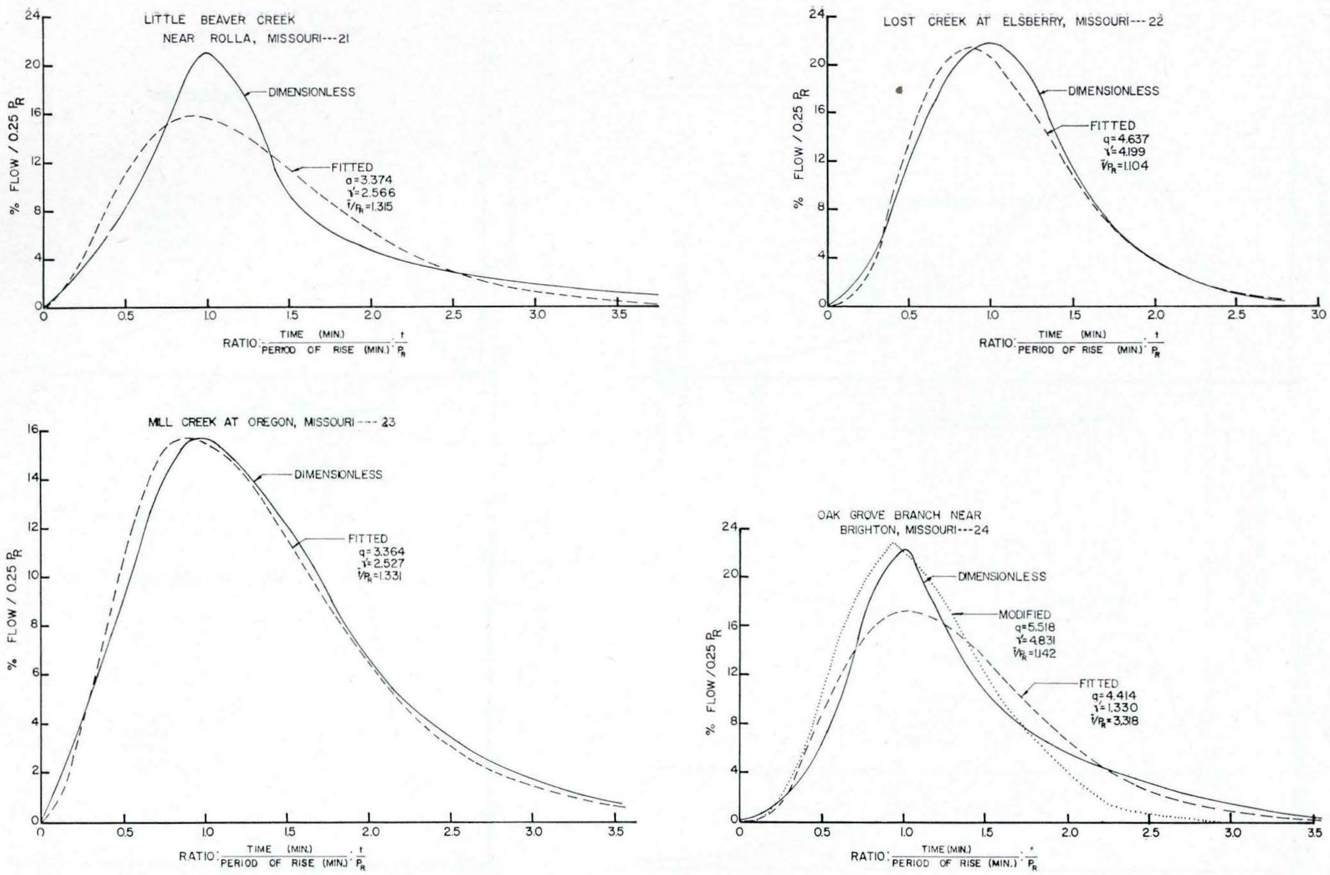

Fig. E-7. Dimensionless graphs and fitted two-parameter gamma distribution for watersheds 21, 22, 23 and 24.

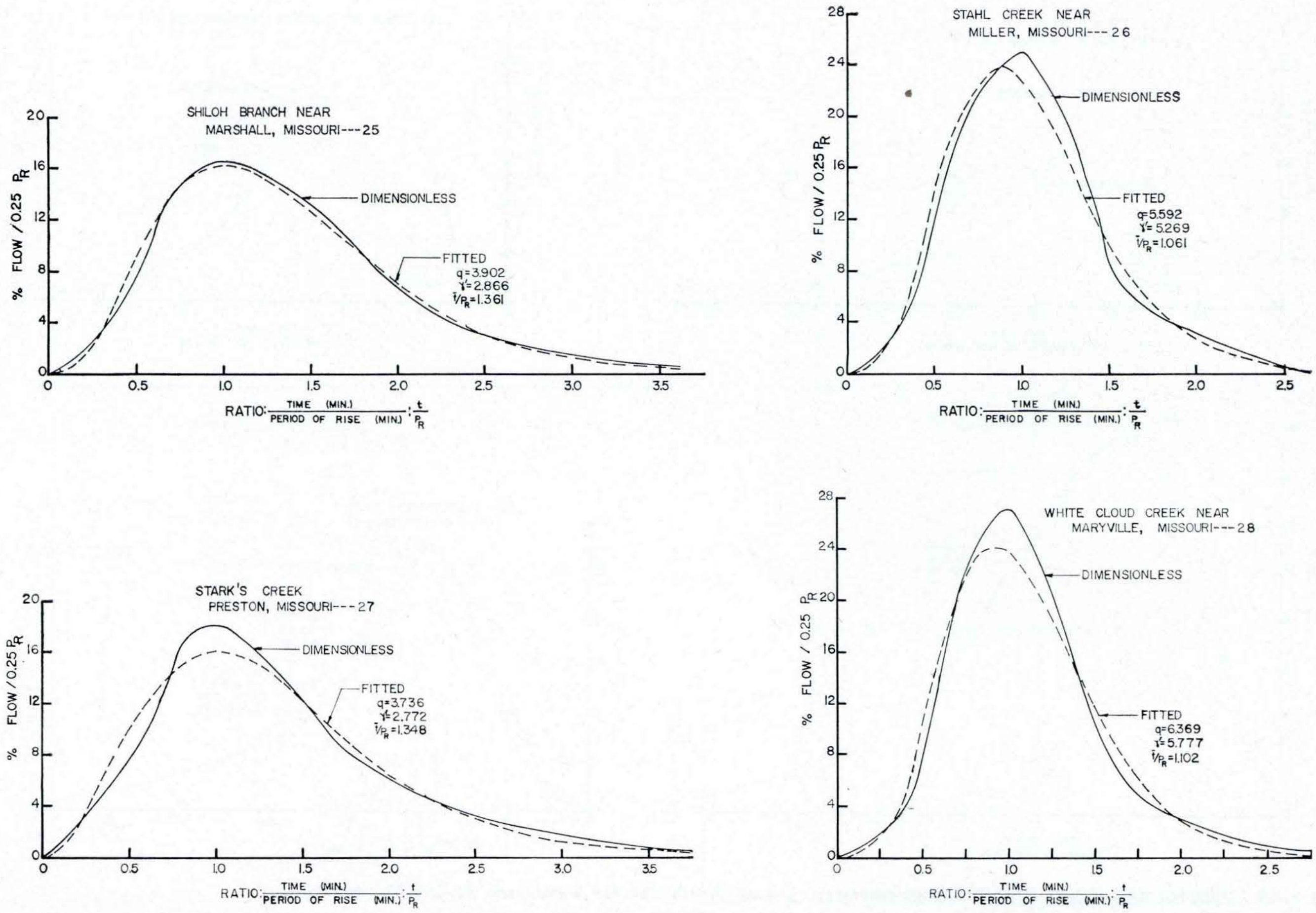

Fig. E-8. Dimensionless graphs and fitted two-parameter gamma distribution for watersheds 25, 26, 27 and 28.

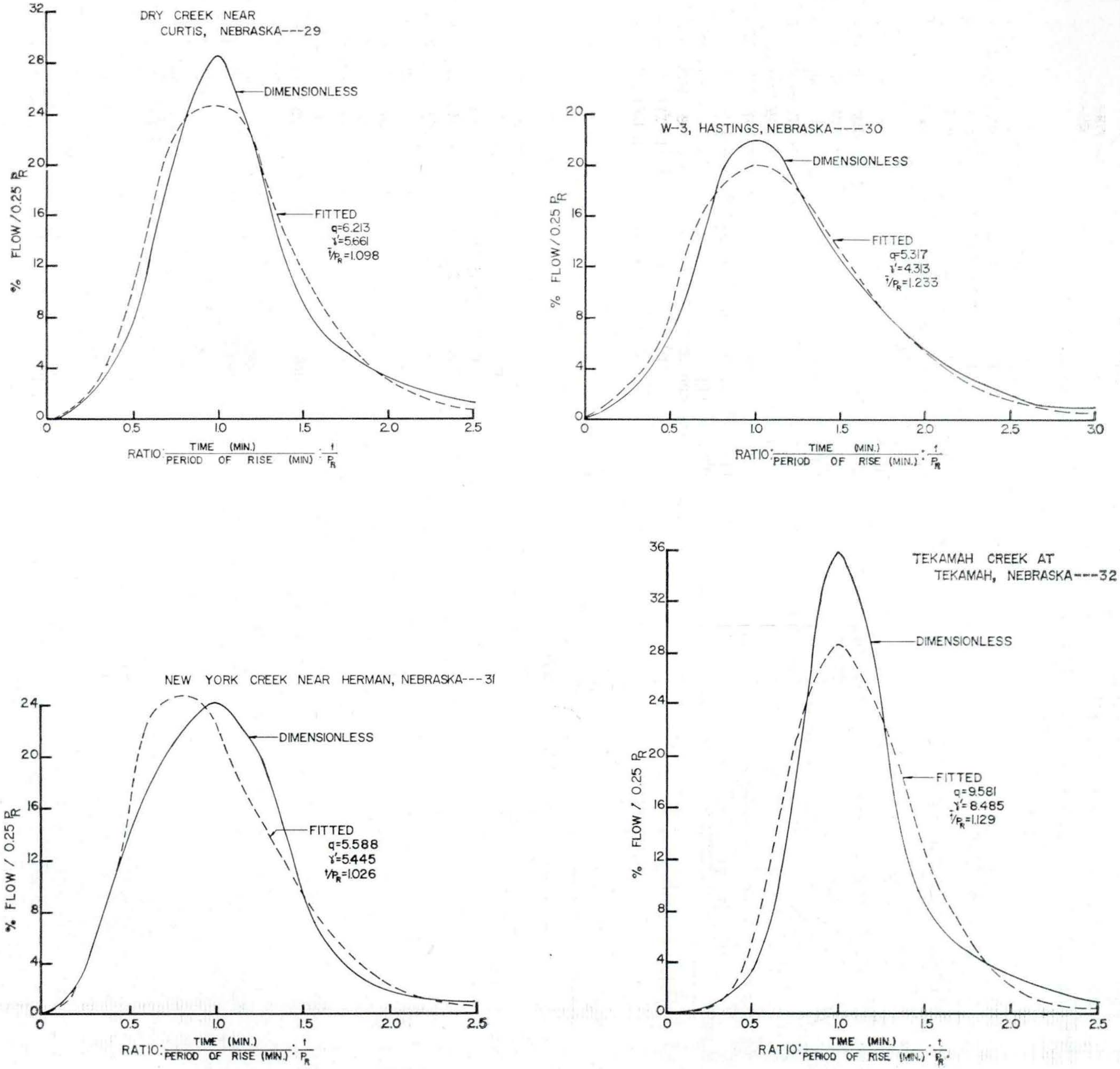

Fig. E-9. Dimensionless graphs and fitted two-parameter gamma distribution for watersheds 29, 30, 31 and 32.

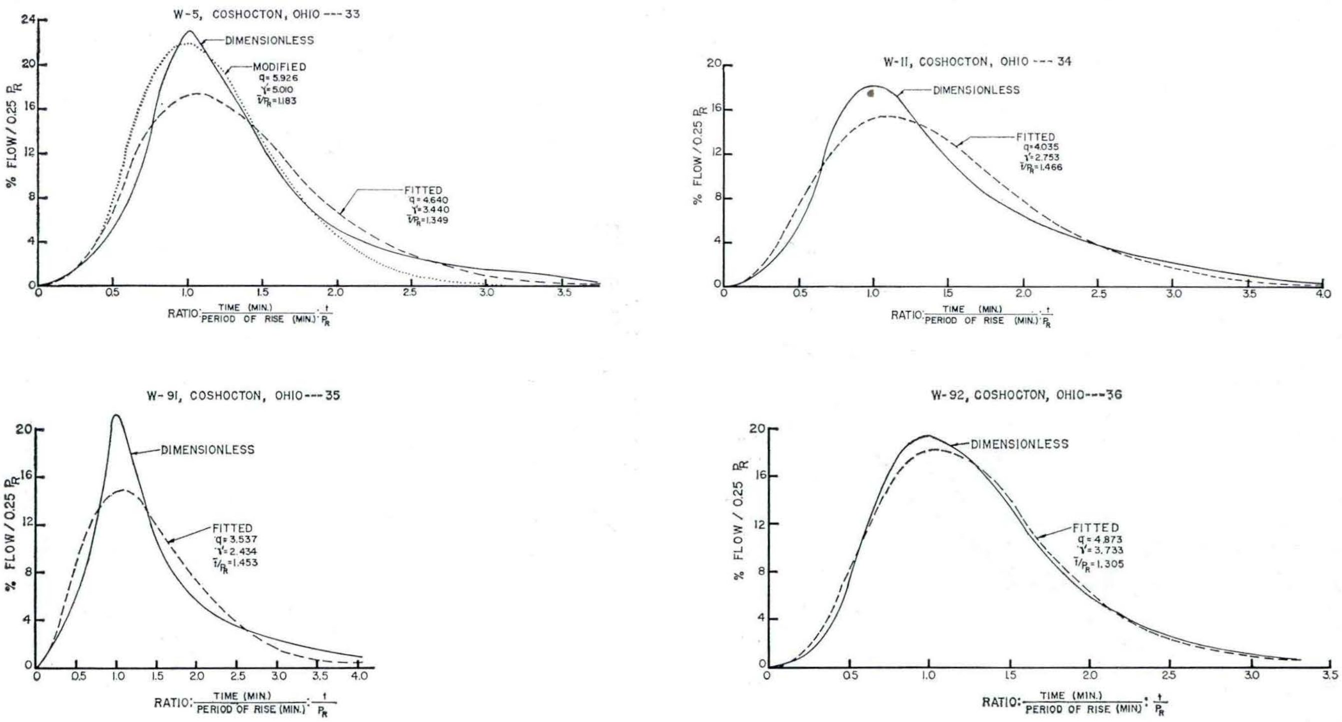

Fig. E-10. Dimensionless graphs and fitted two-parameter gamma distribution for watersheds 33, 34, 35 and 36.

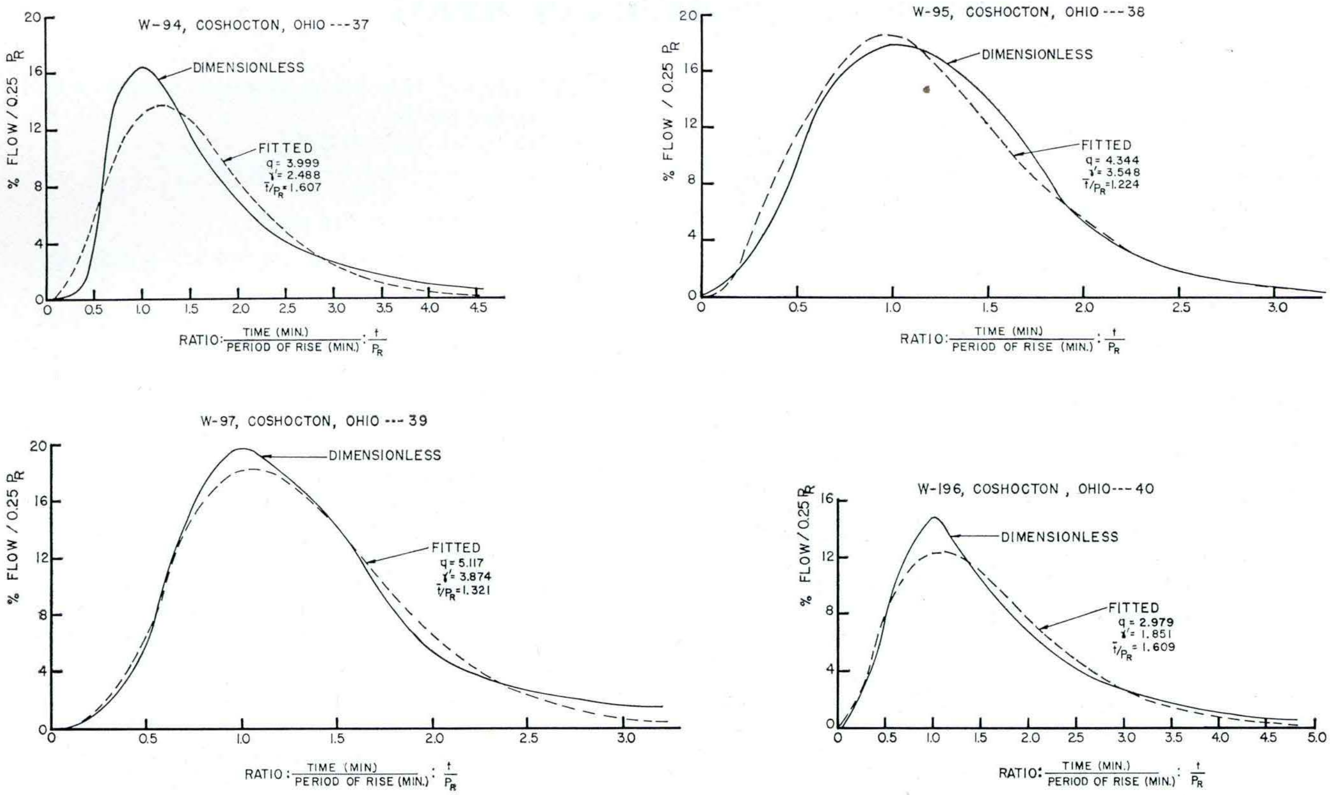

Fig. E-11. Dimensionless graphs and fitted two-parameter gamma distribution for watersheds 37, 38, 39 and 40.

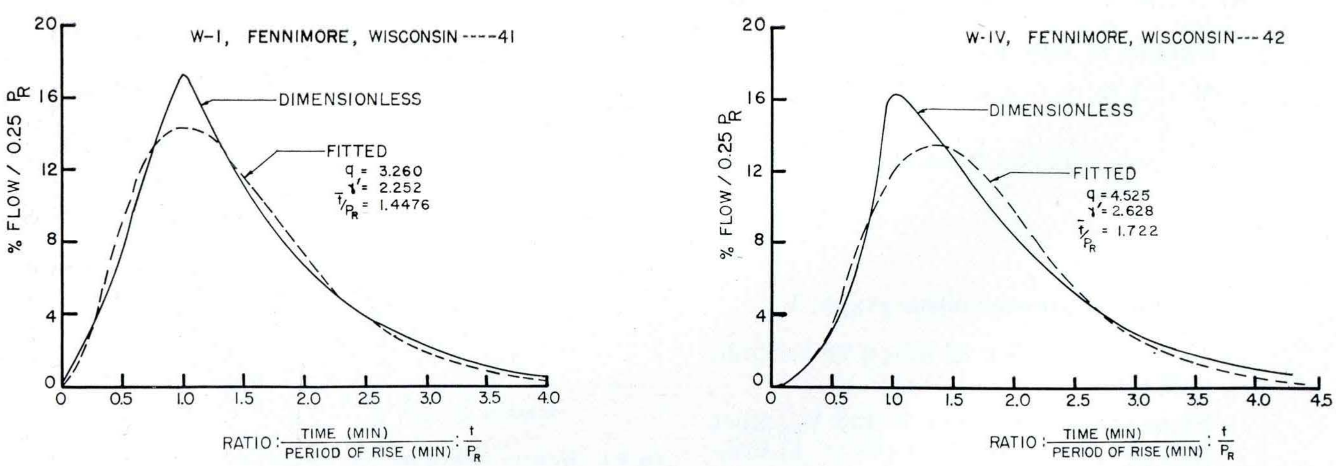

Fig. E-12. Dimensionless graphs and fitted two-parameter gamma distribution for watersheds 41 and 42.

APPENDIX F: APPLICATION OF RESULTS

PROBLEM

Define the unit hydrograph for a watershed, 5 square miles in area, which falls within a region of comparable geologic, physiographic, and climatic conditions as those of western Iowa. The following information was obtained from an available topographic map: $L = 3.80$ miles and $S_c = 0.57$ percent.

PROCEDURES

Step 1. Determine parameters: P_R, γ' and q.

A. With $L/\sqrt{S_c} = 5.03$ miles, enter figure 5 and select; $P_R/\gamma' = 16.6$ minutes.

B. With $P_R/\gamma' = 16.6$ minutes, enter figure 9 and obtain; $P_R = 58$ minutes. Therefore, $\gamma' = 58/16.6 = 3.494$.

Step 2. Compute the ordinates of the dimensionless graph.

A. Using equation 4, compute the % flow/$0.25P_R$ at the respective values of $t/P_R = 0.125, 0.375, 0.625$ . . . . . and every succeeding increment of $t/P_R = 0.250$, until the sum of the ordinates approximates 100 percent (see table F-1). Also calculate the peak percentage. At the peak,

$$Q_{(1)} = \frac{25.0\ (3.494)^{4.494}}{\Gamma(4.494)} e^{-3.494(1)}$$

$$(1)^{4.494} = 18.2 \text{ percent.}$$

Step 3. Develop the unit hydrograph

A. Compute the necessary conversion factor

Volume of unit hydrograph, V

$$V = 1 \text{ in. x } 5 \text{ mile}^2 \text{ x } 640 \text{ acre/mile}^2 \text{ x } \frac{1}{12 \text{ in/ft}} \text{ x } 43{,}560 \text{ ft}^2\text{/acre} = 11{,}616{,}000 \text{ ft}^3.$$

Volume of dimensionless graph, V_D

V_D = cfs x 0.25 x 58 min x 60 sec/min = 870 cfs — sec

Since the two volumes, V and V_D, must be equal, it follows that cfs = 11,616,000/870 = 13,352 cfs.

B. Convert the dimensionless graph ordinates to cfs.

$$Q_t = \frac{\% \text{ flow}/0.25P_R}{100} \text{ x cfs}$$

Therefore, at the peak,

$$Q_p = 18.2/100 \text{ x } 13{,}352 = 2{,}430 \text{ cfs.}$$

C. Convert the time base of the dimensionless graph to absolute time units.
At the peak, $t/P_R = 1$; therefore, t = 58 minutes.

Step 4. Plot the unit hydrograph (see fig. F-1).

According to the data in fig. 8, the time of beginning of surface runoff should be placed coincident with the centroid of precipitation. For convenience of calculation, the unit hydrograph should be associated with unit-storm periods of 0.25 P_R duration.

Table F-1. Coordinates of the synthesized unit hydrograph.

t/P_R	Accumulated time (min.)	% flow[a] ($0.25P_R$)	Cumulative % flow ($0.25P_R$)	Unit graphs (cfs)
0.000	0.0	0.0	0.0	0
0.125	7.3	0.3	0.3	40
0.375	21.8	5.2	5.5	694
0.625	36.3	13.0	18.5	1,736
0.875	50.8	17.6	36.1	2,350
1.000	58.0	......	......	2,430[b]
1.125	65.3	17.7	53.8	2,363
1.375	79.8	14.9	68.7	1,989
1.625	94.3	11.2	79.9	1,495
1.875	108.0	7.7	87.6	1,028
2.125	123.3	5.0	92.6	668
2.375	137.8	3.1	95.7	414
2.625	152.3	1.9	97.6	254
2.875	166.8	1.1	98.7	147
3.125	181.3	0.6	99.3	80
3.625	210.3	0.2	99.8	27
3.875	224.8	0.1	99.9	14
4.125	239.3	0.1	100.0	13
		Total	100.0	13,352

[a]Rounded to nearest 0.10 percent.
[b]Peak discharge rate; not included in total.

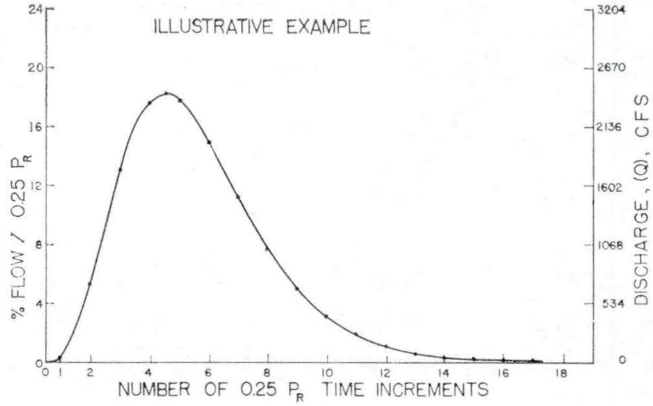

Fig. F-1. Synthetic unit graph for 5-square-mile watershed used in illustrative problem.