# Short-Run Corn Supply and Fertilizer Demand Functions Based on Production Functions Derived From Experimental Data; a Static Analysis

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## SUMMARY


Physical conditions of production are the foundation of product supply and factor demand in agriculture. This study is the first of its type to relate technology, as expressed in production functions estimated from experimental data, to the market phenomena of price determination—demand and supply. The main objective is to examine the nature of corn supply and fertilizer demand functions for a within-season period. That is, the functions specify the yield conponent of supply elasticity, or the supply elasticity assuming corn acreage is fixed and fertilizer is the variable resource.

Because this report is an initial effort and because available empirical data are few, the major emphasis throughout the report is on methodology.

The approach is normative since the functions indicate what the supply and demand **would be** based on production functions derived from fertilizer experiments if farmers maximized profits under conditions where capital, institutional and behavioral restraints are unimportant. Such normative concepts are referred to simply as "static supply" and "static demand."

Because farmers operate in a dynamic world in which prices and input-output relationships are not known with certainty and because the physical conditions on farms do not entirely parallel experimental conditions, the static supply and demand elasticities estimated in this study do not entirely parallel such quantities as they might be expressed in the market. Analysis of these differences suggests that the elasticity estimates in this study represent the upper boundary of the actual short-run supply and demand elasticities. As such, the estimates indicate the maximum short-run production response which farmers might be expected to make to changes in price.

Three algebraic forms of the production function, the quadratic, square root and logarithmic, were examined to determine the advantages and restraints which each possesses for projecting physical relationship in nature into estimates of supply and demand curves and elasticities. The algebraic form of the production function was found to have a highly significant effect on the estimated supply and demand functions. Of the three algebraic forms examined, the quadratic and square root forms appeared most appropriate for the type of analysis reported in this publication. Examples of the three algebraic forms expressing static supply from a 1953 experiment on Ida silt loam in Iowa are:

(a) Quadratic: $$Y = 100.7 - \frac{2.67}{P_y^2}$$

(b) Square root:

$$Y = 37.1 + \frac{22.98P_y + 27.93P_y^2}{0.07 + 0.33P_y + 0.40P_y^2}$$

(c) Logarithmic: $$Y = 1.85P_y^{0.40}$$

where Y is the supply quantity of corn per acre and $P_y$ is the price of corn. $P_2O_5$ is fixed at 80 pounds per acre, and the price of nitrogen, the variable resource, is 13 cents per pound.

Ten production functions fitted to experimental data obtained in Iowa, Kansas, Michigan, North Carolina and Tennessee provide the basis for inferences about static supply and demand curves and elasticities. Because the sample of physical production functions is small, no attempt is made to aggregate functions and to infer quantitative results for United States agriculture. Instead, the procedure in the empirical section is to examine the degree of consistency of the estimated quantities with certain hypotheses suggested by economic and agronomic theory. The results of the analysis are consistent with the hypothesis that short-run corn supply is highly inelastic. For all soil and weather conditions, and for all prices considered in the empirical section, static supply elasticity is low. Without exception, static supply is inelastic ($E_s < 1$) for corn prices over 40 cents per bushel. The supply elasticity ranges from zero to less than 0.3 for corn prices above \$1 and from zero to less than 0.2 for corn prices above \$1.20 per bushel. Supply tends to be most elastic in situations where the soil is low in fertility but is otherwise satisfactory for corn production; i.e., adequate rainfall, good soil structure, etc. The analysis supports the hypothesis that considerable variation in supply elasticity exists among soil types and years within a given area such as Iowa.

The study shows that static supply elasticity increases as the price of corn falls. Because of limited data, supply elasticities estimated from historic results of actual response by farmers to price changes generally consider the elasticity to be single-valued. Thus normative models of the type used in this study, which provide information on supply outside the range of historic data, are a useful supplement to descriptive supply analysis.

Static factor demand tends to be more elastic than static product supply. The price elasticity of the short-run demand for nitrogen, for example,

lies between 0.2 and 1.7 (with the exception of Wisner loam) when the price of nitrogen is 13 cents per pound. The demand for $K_2O$ is more elastic than the demand for $P_2O_5$, which, in turn, is more elastic than the demand for nitrogen. As might be expected, the level of static demand for nitrogen is higher than for the other two nutrients. The soils which are low in the particular nutrient but which are otherwise suitable for corn production, tend to display the highest and least elastic static demands.

The static demand in marginal corn production areas tends to be lower and more elastic than static demand in the Corn Belt. Although the small sample size precludes making strong inferences, the results emphasize the need for price-quantity data as well as elasticities. That is, because of the high level of demand for fertilizer and the large areas suited for corn production in the Corn Belt, the greatest change in pounds of fertilizer applied to corn resulting from price changes would occur in this area. Nevertheless, since marginal areas initially produce less corn, the greatest percentage change in fertilizer purchases may be in these areas.

# Short-Run Corn Supply and Fertilizer Demand Functions Based on Production Functions Derived From Experimental Data; a Static Analysis[1]

by Luther G. Tweeten and Earl O. Heady

Need exists to relate technology in farming to the phenomena of product supply and factor demand. Technology is expressed on a purely physical basis in production functions which relate output to input. A number of such functions have been estimated in recent years from controlled experimental data. These functions are readily adaptable to estimation of economic phenomena. Previously, they have been used to estimate least-cost input combinations and profit-maximizing output levels. Yet, the economic applications have not been extended to short-run product supply and factor demand. These basic data can be used for such purposes and, thus, might serve to extend knowledge of product supply and factor demand phenomena in agriculture.

## Need for Study

Problems of large or surplus production and low returns to resources stem from the nature of product supply functions and resource demand functions in agriculture. The nature of these functions determines the level of output and the quantity of resources used in the industry. Along with the structure of commodity demand and resource supply, product supply and resource demand determine the level of prices and incomes of farmers. Although quantities expressing supply and demand relationships for products and resources, respectively, have extreme importance in lessening farm problems, existing empirical knowledge is meager. Data are needed for both short-run and long-run aspects of product supply and factor demand. The time and dollar cost involved in solving farm problems depends on the nature of these functions over various periods of time.

This study deals with supply and demand relationships for an extremely short-run period and for a single product and a restricted set of resources. More specifically, the study provides estimates of normative supply functions for corn and normative demand functions for fertilizer as these are expressed in controlled experiments. The "length of run" considered supposes land and other resources to be fixed, while fertilizer is considered to be variable. Product supply functions and factor demand functions then are derived from the physical production functions estimated under experimental conditions. The general purpose of this approach is to determine whether response in production of a particular crop and use of a particular resource might be large or small in relation to price changes. For example, if the supply and demand functions are highly elastic, we would expect a policy which results in lower crop prices to have a great effect in causing crop ouput and factor use to be restricted.[2]

[1] Project 1135, Iowa Agricultural and Home Economics Experiment Station, Center for Agricultural and Economic Adjustment, cooperating.

[2] Price elasticity of supply or demand, E, relates precentage changes in quantity, Q, and price, P.

$$E = \frac{\text{percentage change in quantity (Q)}}{\text{percentage change in price (P)}} .$$

The exact mathematical form is:

$$E = \frac{dQ}{dP} \cdot \frac{P}{Q} .$$

The price elasticity formulas which we find most useful in this study are elasticity of product supply, $E_S$, which relates product price and quantity supplied, and elasticity of factor demand, $E_d$, which relates factor price and quantity demanded. In the "long run", the acreage also would be affected by product price. The total quantity supplied at any product price would be composed of two components, acreage and yield. The "short-run" or yield component only is considered in this study. Knowledge of the yield component and its elasticity, $E_S$, is useful in explaining the elasticity of total supply, $E_T$. If we know the elasticity of acreage, $E_A$, with respect to the product price, then $E_T$ may be found, since $E_T = E_S + E_A$. The proof follows:

Total production, T, equals the yield, S, multiplied by the number of acres, A.

$$\text{(a)} \quad T = SA .$$

The elasticity of total production is

$$\text{(b)} \quad E = \frac{dT}{dP_y} \cdot \frac{P_y}{T} .$$

Assuming no interaction between yield, S, and acreage, A, the total derivative of T with respect to product price, $P_y$, is

$$\text{(c)} \quad \frac{dT}{dP_y} = \frac{\delta T}{\delta S} \cdot \frac{dS}{dP_y} + \frac{\delta T}{\delta A} \cdot \frac{dA}{dP_y}$$

or

$$\text{(d)} \quad \frac{dT}{dP_y} = \frac{dS}{dP_y} \cdot A + \frac{dA}{dP_y} \cdot S .$$

Multiplying (d) by $\frac{P_y}{T}$, we obtain

$$\text{(e)} \quad \frac{dT}{dP_y} \cdot \frac{P_y}{T} = \frac{dS}{dP_y} \cdot \frac{P_yA}{SA} + \frac{dA}{dP_y} \cdot \frac{P_yS}{SA} ,$$

and, therefore,

$$\text{(f)} \quad E_T = E_S + E_A .$$

If these functions have low elasticity, however, a considerable drop in crop prices would have only small effect in reducing output and factor use. These and similar kinds of questions can be examined from the type of analysis in this study.

### Objectives

The over-all purpose of this study is to examine the nature of corn supply and fertilizer demand functions in the short run. Specifically, the two major objectives are (1) to develop the methodology of estimating demand and supply functions from production functions and (2) to derive empirical estimates of corn supply functions, fertilizer demand functions and their associated elasticities from experimental production functions.

Because the study is the first of its type and because empirical data are limited, a major portion of the study is devoted to the first objective. The logic and assumptions of the approach are discussed in some detail as a foundation for interpreting the empirically derived coefficients. Algebraic forms of production functions possess unique properties which impose significant restraints on the estimated curves and elasticities. Accordingly, the characteristics of three algebraic forms commonly used (logarithmic, square root and quadratic) are discussed and illustrated graphically. The empirical section of the study is grounded on the methodological section and is based quantitatively on 10 production functions estimated under experimental conditions. The derived supply and demand functions apply to fixed land inputs with fertilizer as the only variable resource. These empirical functions are normative: They show what supply and demand functions would be on the basis of physical production functions derived from fertilizer experiments if farmers maximized profits under conditions where capital is not limited and uncertainty or other psychological restraints are unimportant.

Empirical supply and demand functions are derived separately for each year and location of the experiments studied. No attempt is made to aggregate the functions or to generalize the results for United States agriculture. Rather, we wish to determine whether the empirical quantities are consistent with hypotheses suggested by economic and agronomic theory. The analysis of supply is focused on evaluating the hypothesis that short-run supply is relatively inelastic. Because policy decisions relating to this and similar hypotheses are being made, the results of the empirical section, particularly when supplemented by additional data, are basic for making optimum choices from alternative courses of action.

### Approach

"Actual supply" and "actual demand" functions express the quantities which farmers do, in fact, sell and buy at various prices. Since the behavioral characteristics of farmer response cannot be measured because of conditions arising from uncertainty and changing technology, actual supply and demand functions do not exist in a broad empirical sense. Various approaches have been used to estimate the nature of the actual functions. Traditionally, producer supply and demand functions have been estimated by a "descriptive" approach, usually characterized by least-squares statistical models and time-series data for the industry in aggregate. The approach embodies estimation of parameters on the basis of the past response of farmers to changes in relevant economic variables. The term "descriptive" is used since the historical behavior of farmers is described in the model. The results are useful and meaningful to the extent that techniques are adequate and that farmers' past behavior is a reasonable indication of their future behavior.

Derivation of supply and demand functions from production functions is a normative approach. The approach explains the nature of economic phenomena on the basis of what farmers "could do" to maximize profits under given conditions of production and prices. The conditions of production may be expressed by a production function derived from controlled experimental data or from farm surveys. Neither approach appears adequate for all purposes, and limitations of each suggest that they be considered supplements and not substitutes. Production functions obtained from controlled experimental data are particularly appropriate for examining product supply and resource demand in the short run.

### Data

Only corn-fertilizer production functions estimated under nonirrigated conditions are used in this study for several reasons. First, a number of such functions have been fitted which represent various soil, moisture and other conditions influencing parameters of product supply and factor demand. These functions provide a more meaningful foundation for analysis of supply and demand than do the limited number of functions fitted for other farm products and factors.

Second, fertilizer inputs primarly determine the short-run (fixed acreage) corn-supply response within the control of farmers. Agronomic experiments indicate that it is possible to increase corn yields by as much as 50 percent or more by application of fertilizer.[3] The opportunity within a year for farmers to adjust corn output per acre depends largely on fertilizer application.

A third reason for selection of corn-fertilizer production functions is the importance of corn supply in the current feed-grain surplus and the possible effect that various price policies might have on feed input and quantity of resources used.

Although corn output is potentially responsive to fertilizer, farmers do not base production decisions on physical possibilities alone. Their action is determined by a complex of conditions including input-output and price ratios, behavioral and institutional factors. It is well to consider the logic

[3] Earl O. Heady, John T. Pesek and William G. Brown. Crop response surfaces and economic optima in fertilizer use. Iowa Agr. Exp. Sta. Res. Bul. 424. 1955. p.304.

and assumptions relating production functions to supply and demand within this complex of conditions.

## LOGIC AND ASSUMPTIONS

In this section, the logic, assumptions and steps in the analysis are made explicit. The experimental conditions under which the production functions were derived differ somewhat from actual conditions on farms. Furthermore, the conceptual framework underlying the statistical demand and supply functions in this study does not entirely parallel the actual behavioral, institutional and economic framework within which farmers operate. For these reasons, the framework for derivation of supply and demand from production functions is established in the following pages. The relation between these supply and demand functions and logically similar functions expressed on farms and in the market also is discussed.

### Static Product Supply

For purposes of this analysis, we define short-run supply of a farm product as the various quantities which farmers would produce at all possible prices (a) if they maximized profits, given the production function and prices of inputs and outputs, and (b) if all factors but fertilizer were fixed. In subsequent sections, this concept of short-run supply of a farm product is called "static supply."

To illustrate the logic of the derivation of static supply from physical production functions, we first consider the marginal cost; i.e., the addition to total cost from one more unit of output. From the production function, we can determine the number of additional inputs required to produce one more unit of output. The cost of this unit of output is found merely by multiplying the number of units of inputs by the unit price of the inputs. Input prices are constant for an individual farmer, regardless of the level of output. The marginal cost, therefore, is determined by the production function and the fixed or constant input prices.

A farmer who maximizes profits, and who has no institutional or capital restrictions on output, would produce a commodity in a quantity such that the return or price per unit just equals the cost of one more unit; i.e., where marginal cost equals marginal revenue. If output is smaller or greater than this quantity, profit will be decreased. The marginal revenue or return from an additional unit is, of course, the product price. It follows that for any given product price, the supply quantity is uniquely determined by the marginal cost. Hence, the marginal cost function derived from the production function for a given level of factor prices is essentially equivalent to the static supply function for the particular producing unit.[4] Marginal cost is a function of output; however, the "static" supply quantity is a function of product price. This difference in functional forms is easily handled in mathematical formulations. Since they are equivalent, an algebraic expression in one form can be converted into the other by a simple algebraic manipulation.

#### STATIC SUPPLY ON FARMS

Given the goal of profit maximization and knowledge of input-output and price relationships by farmers, the static supply (marginal cost) functions in this study may differ from those derived from actual farm data.[5] These functions are comparable to the extent that: (1) The "fixed" conditions such as technology, soil and weather, under which the controlled experiments are conducted, are at levels which represent farm conditions. (2) All relevant short-run variables are specified, including inputs and competing or complementary outputs. (3) The algebraic form of the production function is adequate to express the physical relationships.

A distribution of production functions exists for the various soil, technological and weather conditions found on farms throughout the country. The production functions contained in this study were estimated under experimental conditions where the variety, soil type and weather were "fixed." That is, each production function was estimated with various levels of fertilizer, but with given moisture, soil, seed variety, etc. These fixed conditions are probably more favorable for use of fertilizer than conditions found on most farms because (1) experiments are likely to take place on soils where yields are responsive to fertilizer, and (2) experimental data showing little or no yield response from fertilizer are not often published. Hence, the production functions in this study probably represent an above-average response to fertilizer (above-average marginal product of fertilizer) in terms of the total distribution of functions on farms.

Considerable emphasis is given to the price elasticity of static supply in subsequent sections. There appears to be little clear **a priori** basis for expecting supply elasticities computed from data showing above-average yield response to overestimate or underestimate static supply elasticity on farms. The elasticity is influenced by experimental conditions through a base effect and a slope effect. The base effect is due to the position of the static supply curve, given the slope. If static supply is estimated under more favorable moisture, etc., than found on farms, the supply curve is likely to lie further to the right than is the farm static supply curve. Assuming that the slopes are the same, the elasticity of the farm static supply curve is underestimated. That is, the absolute change in supply quantity (slope effect) is the same, but the percentage change in quantity computed from experi-

[4] In an exact sense, a static supply curve is equivalent only to that segment of the marginal cost curve which lies above the average variable cost of production. If average variable cost is not covered, losses are minimized by discontinuing production. In this study all production functions are essentially in stage II or III of production, hence average variable cost is always less than marginal cost. On the basis of the above assumptions, it follows that production theoretically would not be discontinued because variable costs are not covered.

[5] Strictly speaking, the concept of a static supply function for a commodity on a farm is only an approximation. The marginal cost for a commodity on a farm is a static supply curve to the extent that the assumptions of profit maximization, sufficient capital, etc., are met.

mentally derived functions is smaller because it is computed from a larger base.

The slope of the static supply curve relates to the production function through the slope of the marginal physical product.[6] If the marginal product falls sharply to the right, the slope of the supply curve is steep. If resources other than fertilizer are not as limiting, under experimental conditions, as those found on farms, the marginal products of fertilizer may not fall as sharply, and therefore, the supply curves may rise less steeply. The result of this condition is a tendency for the slope effect to overestimate the static supply elasticity on farms. We conclude that if experimental conditions are more favorable for fertilizer response than those found on farms, the result may be underestimation of static supply elasticity on farms through the base effect and overestimation through the slope effect. These effects may offset one another to some extent.

Failure to specify all relevant economic factors which are variable in the short run in the production function may cause static supply elasticity on farms to differ from supply elasticity estimated from production functions. "Relevant" economic factors are those which potentially influence production, can be controlled by farmers and have a price. In this study, static supply is estimated from production functions with only one, two and, in one instance, three variable factors, all of which are fertilizer nutrients. In general, only those fertilizer nutrients which gave no response were excluded. But other inputs, including measures to control weeds and insects, are relevant economic inputs in the short run on farms. Farmers can exhibit greater responsiveness to price changes when more inputs are variable. Hence, failure to specify inputs in the production function may cause underestimation of static supply elasticity on farms.

Production functions do not specify the effect of competing and complementing crops on corn output. The functions do not indicate how corn production would change in response to legume or soybean production through physical effects on corn yield. Also, the extent of residual response from fertilizer application is not specified. Although some fertilizer remains in the soil for longer periods, the production functions indicate only the corn-yield response in the same year the fertilizer is applied. Individual static supply curves exist for the second and subsequent years of residual response. The "total" static supply curve can be considered to be the sum of these annual curves. The first-year curve necessarily would lie to the left of the total supply curve. Because of the base effect, the first-year curve likely would be more elastic than the total static supply curve.

The estimation of static supply also depends on the adequacy of the algebraic forms used to express the physical relationships found in nature and the economic relationships in the market. Assuming that the "fixed" conditions, under which the functions were estimated, were similar to those found on farms, restraints imposed by algebraic forms of the production functions may result in unrealistic estimates of costs and the static supply functions. In subsequent sections, the adequacy of three algebraic forms is examined in some detail. For present purposes, it appears that the two algebraic forms most commonly used, the quadratic and the square root, are reasonably adequate to express data derived from controlled experiments with fertilizer as the variable input. These forms would not be adequate, however, to express more complex physical relationships where many additional inputs and competing and complementary physical outputs would be included. The algebraic forms of the supply and demand relationship assume that corn and fertilizer are independent of other outputs and inputs in the market. To some degree, corn and fertilizer prices are determined interdependently with the prices of other commodities. The difficulties of estimation and manipulation of a simultaneous system of equations preclude the use of this method for the present.

To summarize, the elasticity of static supply found from experimentally derived production functions may differ from static supply (marginal cost) found on farms for a number of reasons. Three of the most important are (a) above-average experimental conditions, (b) omission of relevant short-run inputs and (c) failure to specify the residual response. Above-average experimental conditions may tend to underestimate static supply elasticity on farms through the base effect and to overestimate elasticity through the slope effect. Omission of relevant short-run inputs results in underestimation of elasticity on farms. The failure to specify residual response may cause overestimation of farm elasticity. The conclusion is that supply elasticity estimated from controlled experimental data parallels that found on farms to the extent (1) that experimental conditions are similar to the physical conditions of production on farms and (2) that tendencies for overestimation or underestimation of elasticities offset one another.

## AGGREGATION OF STATIC SUPPLY FUNCTIONS

From a policy standpoint, we are interested principally in estimates of static supply for the whole agricultural industry, but the use of production functions enables us to estimate static supply only for single units of production. To what extent can we generalize about the industry from a single unit of production?

If the quantities of product forthcoming from

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[6] Consider a product, Y, produced with factors $X_1, X_2, \ldots X_n$. The total cost of production, TC, is $\sum_{i=1}^{n} X_i P_i$ where $P_i$ is the price of factor $X_i$. The marginal physical product of factor $X_i$ is $\frac{\delta Y}{\delta X_i}$. The marginal cost, MC, is

$$\text{(a)} \quad MC = \frac{d(TC)}{dY} = \frac{d(\Sigma X_i P_i)}{dY} = \Sigma \frac{\delta X_i}{\delta Y} P_i \ .$$

The slope of the marginal cost curve is the derivative of equation a, or the second derivative of total cost.

$$\text{(b)} \quad \frac{d(MC)}{dY} = \Sigma \frac{\delta^2 X_i}{\delta Y^2} P_i \ .$$

It is apparent that the slope of the marginal cost curve relates to the production function through the derivative of the marginal product. The same conclusion applies to the static supply since it essentially is equivalent to the marginal cost.

each production unit at all product prices were known, we could determine the industry static supply curve by summing these quantities. The elasticity of static supply for corn could be readily computed from the industry static supply. In actual practice, of course, only a representative sample of these producing units would be needed to estimate the relevant quantities for the industry.

This study contains static supply curves only for producing units and soils where experimental production functions have been fitted. They are not estimates for a random sample of producing units or soils. Hence, it is not expected that the estimates can be aggregated to provide an image of the industry static supply curve for corn. This is not the purpose of the study. The purpose is to use production functions which are available to estimate supply (and demand) functions and their elasticities based on the assumptions just outlined. Such quantities cannot be estimated for a representative sample of soils or farms because the required production functions do not exist. Thus, rather than to attempt estimation of supply functions from experiments and aggregate them for the industry, we wish only to examine the algebraic nature and properties of these functions at particular locations and for particular years of the experiments. If we find that all of these exhibit low elasticity at usually experienced prices, basic knowledge important to farmers' decisions and to policy will have been uncovered. If we find, however, that no consistency exists among locations and years, our conclusions will be in a different direction.

## MARKET SUPPLY — INTRODUCTION OF UNCERTAINTY

Thus far, we have discussed static supply; i.e., the nature of supply given the production function, prices and profit-maximizing behavior. The data on which the study is based are suited only for analyzing static supply. It is of interest, however, to consider the relation between static supply and actual short-run supply of products as expressed in the market. These concepts differ largely because of conditions associated with uncertainty. Farmers operate in a dynamic world where prices and the production function (marginal cost) are not known with accuracy. Because of uncertainty, only expected marginal costs and returns can be equated. Farmers avoid "going out on a limb" to increase production although price and production conditions may appear favorable. Response to price changes may be dampened because farmers are unaware of or indifferent to the changes, or because they consider the changes temporary. Farmers who produce corn for livestock feed on their own farms are often unconcerned with short-run changes in the market price of corn. Motives other than profit, such as the desire for a stable or a "certain" minimum level of income, also influence decisions on inputs and outputs. Farmers often stop short of profit-maximizing output because of capital rationing or government restrictions.

These conditions of uncertainty lead to a lagged response by farmers to price changes. That is, farmers do not increase the corn output to the extent indicated by the static supply functions when the price of corn increases. Rather they increase output by some proportion of the indicated amount during the first year and continue the adjustment during subsequent years. After several production periods, they may be very close to the change indicated by the static supply function. If the adjustment to a price increase is distributed over several periods, the results of this study may be of interest in explaining the yield component of supply and demand elasticity over several production periods.

In summary, because of conditions arising from uncertainty, farmers exhibit less than the optimum response necessary to maximize profit. Farmers probably are less responsive to price stimuli than predicted by the elasticity of static supply because of behavioral and institutional restraints on production. Although there is no clear **a priori** basis for concluding that **static** supply elasticities estimated from experimental or actual farm data differ appreciably, the introduction of uncertainty strongly suggests that static supply elasticities estimated in this report tend to overestimate dynamic supply elasticity as expressed in the markets. The empirical estimates in this report are expected to represent the upper boundary of the actual short-run supply response that might be experienced in the market.

## STATIC SUPPLY WITH RESPECT TO FACTOR PRICE

Static supply, with respect to a factor price, may be defined as the quantity of a **product** produced at all possible prices of a **factor.** The product price and other factor prices are assumed constant. This concept with the static assumption listed earlier is labeled "static cross-supply" for convenience.

Static cross supply and static supply can be found from the same curve in some instances. Because graphs of static supply include only product prices on the quantity axis, we often are not fully aware that the supply quantity is a function of price ratios. Hence, the static supply quantity is a function of real prices and is independent of the general price level. In the case of static supply with a single variable factor, the supply quantity is a function of the simple ratio of product-factor prices. If this price ratio is measured on the vertical axis, it is quite simple to find the supply quantity at various factor prices as well as at various product prices. The supply quantity is the same for any given price ratio whether we consider the quantity to be a function of factor price or of product price. The elasticity is also the same numerical value but opposite in sign for each curve at any given price ratio.[7] The opposite

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[7] Consider a static supply function

$$\text{(a)}\quad Y = g\left(\frac{P_y}{P_1}, \frac{P_y}{P_2}, \ldots, \frac{P_y}{P_n}\right)$$

where the supply quantity is a function of n product-factor price ratios. The price elasticity of supply is

$$\text{(b)}\quad E_s = \frac{dY}{dP_y} \cdot \frac{P_y}{Y}.$$

From (a)

$$\text{(c)}\quad \frac{dY}{dP_y} = \frac{g'_1}{P_1} + \frac{g'_2}{P_2} + \ldots + \frac{g'_n}{P_n} = \sum_{i=1}^{n} \frac{g'_i}{P_i},$$

hence,

(Footnote 7 continued on page 582)

sign reflects the reverse slope of the static cross-supply.

The supply quantity (and elasticity) at various factor prices may also be found if the product price only is given on the vertical axis. Simply convert the product prices to ratios or "fix" the product price at some level and compute the factor prices. Moving up the vertical axis, the factor price becomes smaller, and the supply quantity of the product becomes larger (assuming a positive slope on the static supply curve).

Static cross-supply is particularly useful in appraising the effect of changing factor prices on product output. The elasticity of static supply, with respect to the product price, is equal numerically but opposite in sign to the elasticity of static supply for a "bundle" of resources—the elasticity of static cross-supply. This "bundle" may be several fertilizer elements applied in a fixed ratio, or applied in a least-cost mix. It follows that, if the elasticity of static corn supply with respect to the price of corn is low, the elasticity of corn supply with respect to the prices of the variable fertilizer elements also is low.

### Static Factor Demand

Short-run factor demand may be defined as the various quantities which farmers will purchase at all possible prices of the particular factor. Prices of other factors and of the product(s) from which the factor demand is derived are assumed constant. This definition of short-run factor demand with the added assumptions of profit maximization and knowledge of input-output and price relationships by farmers is henceforth referred to as "static demand."

To understand the logic relating the production function and static demand, it is useful to consider the marginal value product (i.e., the addition to total revenue from using an additional unit of a factor). The additional product forthcoming from the use of an additional unit of a factor (marginal physical product) is found from the production function. The additional product multiplied by the product price is the marginal value product. A farmer maximizing profits in the absence of capital restrictions would use a resource in a quantity such that the marginal return (marginal value product) from the resource equals its marginal cost. In agriculture, the marginal cost is the factor price. For any given factor price, under these conditions, the demand quantity of the factor would be uniquely determined by the marginal value product. Thus, marginal value product and static demand would be equivalent under the assumptions of a representative production function complete knowledge, profit maximization and absence of capital and institutional restrictions.

The marginal value product relates to static demand in the same way that marginal cost relates to static supply. Marginal cost and marginal value product are expressions of respective costs and returns which may be derived with knowledge of the production function and prices. These concepts do not indicate what farmers will do but only describe quantities existing in nature. When the assumptions of profit maximization, rational action, etc., are made, these concepts form the basis for the behavior of farmers. Defined as static supply and static demand, these concepts form an expository link between physical relationships and price determination in the market.

### STATIC DEMAND ON FARMS

Static demand estimated from controlled experimental data may differ from static demand on farms (marginal value product) because of above-average experimental conditions, failure to include residual response and to specify other relevant inputs, and other reasons. Above-average experimental conditions may shift the static demand curve to the right and cause underestimation of static demand elasticity on farms. The favorable response from fertilizer under experimental conditions partially is a result of low carryover of nutrients from past years. But with a given soil fertility level, ignoring residual response from fertilizer applied in the current year reduces demand for nutrients and causes overestimation of actual static demand elasticity (assuming the slope remains unchanged). Failure to specify all relevant short-run inputs may result in underestimation of static demand elasticity on farms.

The net influence on demand estimates because of differences between farm and experimental conditions is not apparent from **a priori** logic. Of course, the static demand elasticities estimated in this study parallel those found on farms to the extent that (1) the experimental conditions under which the production functions were derived are similar to those found on farms and (2) the tendencies for overestimation and underestimation offset each other.

It is of interest to consider how the static demand elasticities estimated in this study—assuming that they adequately represent static demand elasticity on farms—compare with actual factor demand elasticity as might be expressed by a farmer in the market. Because of conditions broadly associated with uncertainty, such as motives other than profit, capital limitations and inadequate knowledge of prices and the production function, farmers are probably less responsive to input price changes than is indicated by static demand elasticity. It appears reasonable to conclude that static demand elasticity as found in this study (or on farms) is probably greater than the short-run factor demand elasticity as expressed in the market.

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(Footnote 7 cont'd)

$$\text{(d)}\quad E_s = \left( \sum_{i=1}^{n} \frac{g'_i}{P_i} \frac{P_y}{Y} \right) .$$

The elasticity of supply, with respect to a factor price, $P_i$, is

$$\text{(e)}\quad E_{cs_i} = \frac{\delta Y}{\delta P_i} \cdot \frac{P_i}{Y} = -g'_i \frac{P_y}{P_i^2} \cdot \frac{P_i}{Y} = -\left( \frac{g'_i}{P_i} \cdot \frac{P_y}{Y} \right)$$

therefore,

$$\text{(f)}\quad \sum_{i=1}^{n} E_{cs_i} = -\left( \Sigma \frac{g'_i}{P_i} \right) \frac{P_y}{Y} = -E_s .$$

The conclusion is that the sum of the elasticities, with respect to a change in the prices of the variable factors, is equal numerically but opposite in sign to the elasticity of supply. If only one factor is variable,

$$\text{(g)}\quad E_{cs} = -E_s .$$

## STATIC DEMAND WITH RESPECT TO PRODUCT PRICES

Static demand with respect to a product price may be defined as the various quantities of a **factor** which farmers will purchase at all possible **product** prices. The prices of other products, of the factor demanded and of related factors in the production process are considered fixed. With the added conditions that farmers maximize profits and know prices and the production process, this concept is called "static cross-demand."

Static cross-demand can be found from a static demand curve in the same manner that static cross-supply can be found from the static supply curve. The demand quantity is a function of the factor-product price ratios. If demand for a factor is derived from a single product and other factor levels are fixed, the demand quantity is a function of the simple factor-product price ratio. For any given price ratio, the demand quantity (or the elasticity) is the same whether the quantity (or the elasticity) is considered a function of the factor price or the product price. Of course, the elasticities are opposite in sign, indicating reverse slopes of the static demand and cross-demand curves.

There are two reasons for interest in static cross-demand. First, changes in product prices, more often than changes in an input price, may cause variations in the demand quantity of an input in agriculture. Prices of inputs supplied by nonfarm sectors often are more stable than are farm product prices. For example, the demand quantity of fertilizer may change more often because of changes in the price of corn than as a result of changes in the price of fertilizer.

A second reason for interest in static cross-factor demand is its role in explaining the relationship among static supply, static factor demand and technology in farming. The relationship among the price elasticity of static supply $E_s$, the elasticity of production $E_{p(i)}$ and the price elasticity of static cross-demand $E_{cd(i)}$ for the i-th resource is expressed as[8]

$$(1)\quad E_s = \sum_{i=1}^{n} E_{p(i)} E_{cd(i)}$$

In the case when one factor $X_i$ is variable, all others fixed, the elasticity of static supply is

$$(2)\quad \frac{dY}{dP_y}\cdot\frac{P_y}{Y} = \left(\frac{dY}{dX_i}\cdot\frac{X_i}{Y}\right)\left(\frac{dX_i}{dP_y}\cdot\frac{P_y}{X_i}\right), \text{ or}$$

$$E_s = E_p E_{cd}.$$

Since $E_{cd} = -E_d$ when one factor $X_i$ is variable,[9] $E_s = -E_p E_d$.

If the product-factor price ratios are such that farmers are operating at the beginning of stage II of production (average product at a maximum), $E_p = 1$ and, therefore, $E_s = -E_d$. As more $X_i$ is used, $E_p$ declines, and the ratio of $E_s$ to $E_d$ also declines. Nearing the end of stage II (total product reaching a maximum), $E_s$ can be near zero and $E_d$ highly elastic. Since most production takes place within the limits of stage II, static factor demand is expected to be more elastic than static product supply when one factor is variable. This "factor" may be, of course, a composite of several factors.

## DERIVATION AND CHARACTERISTICS OF ALGEBRAIC SUPPLY AND DEMAND FUNCTIONS

The true or natural form of a production function cannot be theoretically deduced.[10] In practice, algebraic forms are chosen for their simplicity as well as for their close approximation to the supposed true algebraic form. Estimates of static supply and demand curves are affected by the algebraic form chosen for the production function as well as by environmental conditions, prices and the number of variable resources. In some instances, the algebraic form of the production function imposes restrictions which result in unrealistic and unacceptable estimates of static supply and demand although the original data are satisfactory. The purpose of this section is: (1) to show the procedure used to derive algebraic supply and demand functions, (2) to discuss and illustrate graphically the characteristics of supply and demand curves (and elasticities) which arise from the algebraic forms of the production function and (3) to demonstrate the effects of prices, the level of the fixed resources and the number of variable resources on supply and demand curves and elasticities.

A number of algebraic forms have been fitted to yield response data. We consider only three of these, the quadratic, square root and logarithmic. The quadratic and square root forms have been used most often to depict response of corn yield to fertilizer. These two related forms are computationally convenient and meet the assumptions of the physical model quite well.

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[8] Consider a production function (a)

$$(a)\quad Y = f(X_1, X_2, \ldots, X_n)$$

where output, Y, is a function of factors $(X_1, X_2, \ldots, X_n)$. The total derivative of (a) with respect to the product price, $P_y$, is

$$(b)\quad \frac{dY}{dP_y} = \frac{\delta Y}{\delta X_1}\cdot\frac{dX_1}{dP_y} + \frac{\delta Y}{\delta X_2}\cdot\frac{dX_2}{dP_y} + \ldots + \frac{\delta Y}{\delta X_n}\cdot\frac{dX_n}{dP_y}.$$

As explained previously, the elasticity of supply is

$$(c)\quad E_s = \frac{dY}{dP_y}\cdot\frac{P_y}{Y}$$

Thus, to convert (b) to the form (c), we multiply (b) by $\frac{P_y}{Y}$ and obtain (d)

$$(d)\quad \frac{dY}{dP_y}\cdot\frac{P_y}{Y} = \left(\frac{\delta Y}{\delta X_1}\cdot\frac{X_1}{Y}\right)\left(\frac{dX_1}{dP_y}\cdot\frac{P_y}{X_1}\right) + \left(\frac{\delta Y}{\delta X_2}\cdot\frac{X_2}{Y}\right)\left(\frac{dX_2}{dP_y}\cdot\frac{P_y}{X_2}\right) + \ldots + \left(\frac{\delta Y}{\delta X_n}\cdot\frac{X_n}{Y}\right)\left(\frac{dX_n}{dP_y}\cdot\frac{P_y}{X_n}\right).$$

The elasticity of production $E_{p(i)}$ and elasticity of static cross-demand for a factor $X_i$ are

$$(e)\quad E_{p(i)} = \frac{\delta Y}{\delta X_i}\cdot\frac{X_i}{Y} \quad\text{and}\quad E_{cd(i)} = \frac{dX_i}{dP_y}\cdot\frac{P_y}{X_1}.$$

Hence, (d) may be written

$$(d)\quad E_s = \sum_{i=1}^{n} E_{p(i)}E_{cd(i)}.$$

[9] The proof that the elasticities of static demand and static cross-demand are numerically equal but opposite in sign is very similar to the proof given in footnote 6.

[10] Cf. Heady, Pesek and Brown, **op. cit.**, pp. 293-304. Also, Earl O. Heady and John L. Dillon. Agricultural production functions. Iowa State Univ. Press, Ames. 1961. These references contain a more comprehensive discussion of production functions.

The steps in the computation of supply, demand and elasticity equations are shown only for the quadratic production function, but the methods are the same for other algebraic functions. The general forms of the three functions are:

$$(3)\quad \text{Quadratic (Quad)}\quad Y = b_{00} + b_{10}X_1 + b_{20}X_2 + b_{11}X_1^2 + b_{22}X_2^2 + b_{12}X_1X_2$$

$$(4)\quad \text{Square root (SR)}\quad Y = b_{00} + b_{10}X_1 + b_{20}X_2 + b_{11}X_1^{1/2} + b_{22}X_2^{1/2} + b_{12}X_1^{1/2}X_2^{1/2}$$

$$(5)\quad \text{Logarithmic (Log)}\quad Y = b_0X_1{}^{b}X_2{}^{c}$$

where Y is product, $X_1$ and $X_2$ are factors and the b's and c are coefficients. It is useful to consider the sign of the coefficients in the usual case of diminishing returns to $X_1$ and $X_2$ and a positive interaction between $X_1$ and $X_2$. In the quadratic equation, equation 3, $b_{11}$ and $b_{22}$ are negative; the other coefficients are positive. Only $b_{10}$ and $b_{20}$ are negative in the square root equation, 4. In the usual case, all coefficients are positive in the logarithmic equation, 5. When hypothetical product supply and factor demand curves are illustrated in the following pages, the coefficients are assumed to have these signs. We refer to the absolute value of all coefficients in the discussion of the derivation and characteristics of curves and elasticities, unless otherwise specified.

When one factor is fixed, $X_2$ for example, some of the terms in equations 3, 4 and 5 become constants, and the equations can be written:

$$(6)\quad \text{Quad}\quad Y = b'_{00} + b'_{10}X_1 + b_{11}X_1^2$$

where,

$$b'_{00} = b_{00} + b_{20}X_2 + b_{22}X_2^2;\quad b'_{10} = b_{10} + b_{12}X_2$$

$$(7)\quad \text{SR}\quad Y = b'_{00} + b_{10}X_1 + b'_{11}X_1^{1/2}$$

where

$$b'_{00} = b_{00} + b_{20}X_2 + b_{22}X_2^{1/2};\quad b'_{11} = b_{11} + b_{12}X_2^{1/2}$$

$$(8)\quad \text{Log}\quad Y = b'_0X_1{}^{b}$$

where,

$$b'_0 = b_0X_2{}^{c}\ .$$

We note that functions 6, 7 and 8 are the same general forms used to express yield response when only one factor is explicitly included in the experiment.

Three functions fitted to the same data are used to illustrate graphically the characteristics of the quadratic, square root and logarithmic functions.[11] These three functions are:

$$(9)\quad \text{Quad}\quad Y = -7.51 + 0.584N + 0.664P - 0.00158N^2 - 0.00180P^2 + 0.00081NP$$

$$(10)\quad \text{SR}\quad Y = -5.682 - 0.316N - 0.417P + 6.3512N^{1/2} + 8.5155P^{1/2} + 0.3410N^{1/2}P^{1/2}$$

$$(11)\quad \text{Log}\quad Y' = 2.7649N^{0.2877}P^{0.4090}$$

where Y is total yield of corn in bushels per acre, N is pounds of nitrogen and P is pounds of $P_2O_5$. Y' refers to total yield above check plot levels. Consequently, the estimates derived from the log equation are not strictly comparable with those obtained from the other two forms.

Equations 9, 10 and 11 were obtained from a controlled experiment conducted in 1952 on Ida silt loam in Iowa. The experiment included variable application of nitrogen and $P_2O_5$, each at nine different levels. The rates ranged from zero to 320 pounds for both nutrients. The soil was highly deficient in both nutrients, hence the yield without fertilizer was low. Rainfall was ample until mid-August when a drouth began. Plant population was 18,000 stalks per acre.

Current prices are used for the "fixed" prices (i.e., corn, $1.10 per bushel; nitrogen, 13 cents per pound; and $P_2O_5$, 8 cents per pound).[12]

### Short-Run Product Supply

#### QUADRATIC

First, we derive the product supply and elasticity of supply equations for quadratic function 6 with only factor $X_1$ variable. For convenience, static product supply with a single variable factor is henceforth referred to as short-run static supply and with more than one variable factor as long-run static supply. In the conventional terminology used previously, **both** of these concepts are **short-run.** That is, some resources are fixed in both of these new classifications. The equation of total profit, $\pi$, is formed by combining equation 6 with the product price, $P_y$, and with the $X_1$ factor price, $P_1$. F is the fixed cost.

$$(12)\quad \pi = P_y(b'_{00} + b'_{10}X_1 + b_{11}X_1^2) - X_1P_1 - F.$$

To maximize profit, we take the derivative in equation 12 with respect to $X_1$ and set it equal to zero.

$$(13)\quad \frac{d\pi}{dX_1} = P_y(b'_{10} + 2b_{11}X_1) - P_1 = 0$$

or

$$b'_{10} + 2b_{11}X_1 = \frac{P_1}{P_y}$$

Equation 13 is the familiar profit-maximizing concept of the marginal product equated to the inverse price ratio. Solving for $X_1$, we have

$$(14)\quad X_1 = \left(\frac{P_1}{P_y}\right)\frac{1}{2b_{11}} - \frac{b'_{10}}{2b_{11}}\ .$$

Substituting this expression for $X_1$ into the pro-

[11] Heady, Pesek and Brown, **op. cit.**, p. 304.

[12] U. S. Dept. Agr., Agricultural Marketing Service. Agricultural prices April 1959. In addition to the prices listed above, a $K_2O$ price of 5 cents per pound is used in the final, empirical section.

duction function, 6, we obtain the short-run supply equation with $X_1$ variable and $X_2$ fixed.[13]

$$(15)\quad Y = \left(b'_{00} - \frac{b'^2_{10}}{4b_{11}}\right) + \left(\frac{P_1}{P_y}\right)^2 \frac{1}{4b_{11}} \qquad P_y \geqslant \frac{P_1}{b'_{10}}$$

The supply curve obtained from supply equation 15 becomes asymptotic to a vertical line at a quantity

$$Y = b'_{00} - \frac{b'^2_{10}}{4b_{11}}$$

as $P_y$ becomes very large. The curve intersects the price axis at

$$P_y = \frac{P_1}{\sqrt{b'^2_{10} - 4b'_{00}b_{11}}}$$

The static supply curve theoretically is only that portion of the marginal cost curve lying above the average variable cost. It is not profitable to supply any quantity if variable costs are not covered. If $b'_{00}$ is positive and diminishing returns exist to $X_1$, only stages II and III of production exist. Hence, the marginal cost curve always lies above the average variable cost curve. But the restraint that $X_1$ be greater than or equal to zero normally insures that the supply curve intersects neither the price nor quantity axis. For $X_1$ to be greater than zero, $P_y$ must be greater than $P_1/b'_{10}$. The supply curve extends to the price axis only if $-4b'_{00}b_{11} \leqslant 0$ —— an unlikely condition since $b_{11}$ usually is negative.

The static product supply curve when $b'_{00}$ and $b'_{10}$ are positive and $b_{11}$ negative is shown in fig. 1. The static supply curve is that portion of the total curve lying above $P_y = P_1/b'_{10}$. Since $b'_{00}$ will be supplied at a very low product price, the static supply curve may be considered as the vertical line at $Y = b'_{00}$ plus the "curved" portion extending upward to the right at the intersection with the vertical portion. Figure 2 depicts the family of supply curves obtained from quadratic supply equation 15 with nitrogen variable and $P_2O_5$ fixed at various levels from zero to 320 pounds. Note that the supply curves shift to the right as nitrogen is varied in the presence of more fixed $P_2O_5$. After about 240 pounds of $P_2O_5$, the curves move to the left because of a decline in

$$b'_{00} = b_{00} + b_{20}X_2 + b_{22}X_2^2,$$

which indicates a diminishing total product to $X_2$. Also note that all of the curves rise steeply when the price of corn is above 80 cents per bushel. Thus, little change in supply quantity results from a change in corn price. The family of supply curves when $P_2O_5$ is variable and nitrogen is fixed is not shown because this family illustrates the same characteristics as are shown in fig. 2.

## SQUARE ROOT

The supply equation for square root production

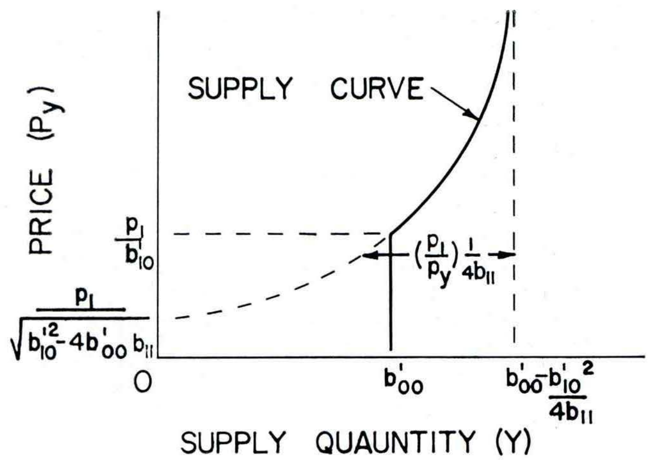


**Fig. 1. Hypothetical short-run static supply curve derived from a quadratic production function.**

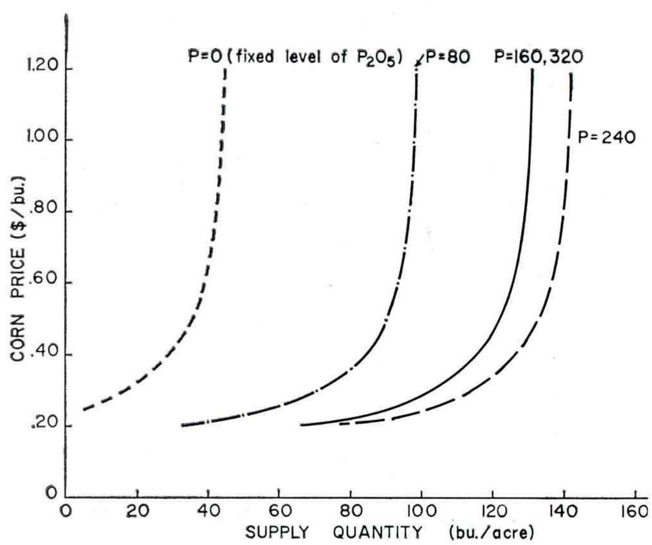


**Fig. 2. Short-run static corn supply from a quadratic production function fitted to Ida silt loam data. The price of nitrogen, the variable factor, is 13 cents per pound.**

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[13] Total variable cost, TVC, and average variable cost, AVC, when $X_1$ is variable can be found from equations 14 and 15.

$$\text{(a)}\quad TVC = X_1P_1 \qquad \text{(b)}\quad TC = X_1P_1 + F$$

where $X_1$ is the expression for $X_1$ in equation 14. Total cost, TC, is found by adding fixed costs, F, to TVC. If $X_2$ is the only fixed input, $F = X_2P_2$. Other inputs are also fixed in most instances.

$$\text{(c)}\quad AVC = \frac{TVC}{Y} \qquad \text{(d)}\quad ATC = \frac{TC}{Y}$$

where Y is the quantity supplied for a given $P_y$ computed from the supply equation, 15. It follows that TVC and AVC are functions of $P_y$ in this framework. This is for convenience; in the usual form, cost is a function of output, Y. Equation 14 is the short-run static factor demand equation. To find the above costs for other algebraic forms, simply insert the short-run demand function for $X_1$ (given later in the text) into (a), (b), (c) or (d). Formulas for costs with two variable factors are included in the appendix.

function 7 is derived in the same manner as equation 15 and is:

$$(16)\quad Y = b'_{00} + \frac{b'^{2}_{11}}{C^2}(b_{10} + C)\ ;$$

$$C = 2\left(\frac{P_1}{P_y} - b_{10}\right)$$

The supply curve becomes asymptotic to the vertical line at the quantity

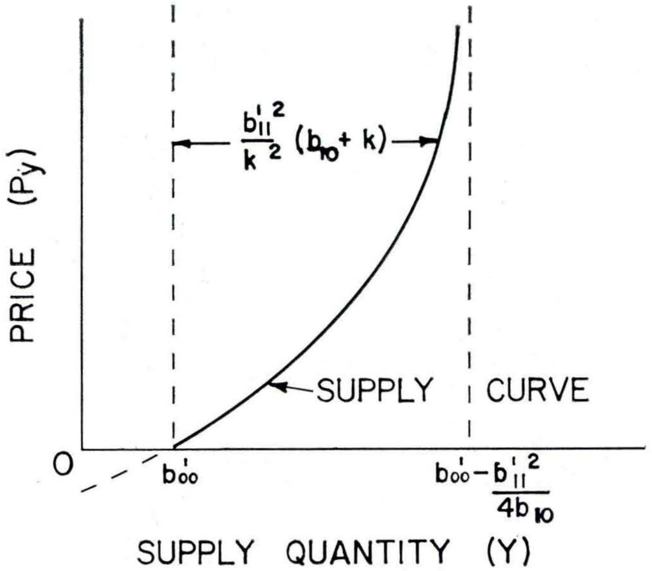



**Fig. 3. Hypothetical short-run static supply curve derived from a square root production function.**

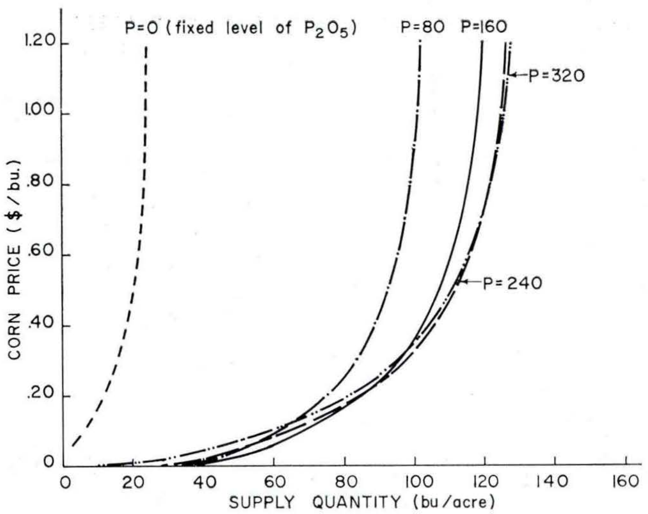



**Fig. 4. Short-run static corn supply from a square root production function fitted to Ida silt loam data. The price of nitrogen, the variable factor, is 13 cents per pound.**

$$Y = b_{00} - \frac{b'^{2}_{11}}{4b_{10}}$$

as $P_y$ becomes large. The curve intersects the quantity axis at $b'_{00}$. Figure 3 illustrates the nature of the supply curve under the usual condition that $b'_{00}$ and $b'_{11}$ are positive, while $b_{10}$ is negative. Because of the mathematical properties of the square root function, the quantity of the resource is always greater than zero. Thus, the restraint $X_1 \geqslant 0$ need not be imposed if $P_y$ is greater than zero.

The family of supply curves derived from the square root supply equation, 16, with nitrogen variable and $P_2O_5$ fixed is shown in fig. 4 These curves also rise quite sharply above a corn price of 80 cents, but not as sharply as the quadratic curves in fig. 2. The curves also shift to the left at high levels of $P_2O_5$ because of a decline in

$$b'_{00} = b_{00} + b_{20}X_2 + b_{22}X_2^{1/2}\,.$$

LOGARITHMIC

The short-run supply equation for logarithmic production function 8 is

$$(17)\quad Y = b'^{\frac{1}{1-b}}_{0}\left(b\,\frac{P_y}{P_1}\right)^{\frac{b}{1-b}}.$$

The supply curve passes through the origin; i.e., $Y = 0$ when $P_y = 0$. Assuming $b'_0$ is positive, the supply curve will slope upward at an increasing rate if $0 < b < 1/2$, at a constant rate if $b = 1/2$, and at a decreasing rate if $1/2 < b < 1$. Figure 5 shows the supply curve when $b'_0$ is positive and $0 < b < 1/2$.

Figure 6 shows the supply curves derived from the logarithmic supply equation, 17, with nitrogen variable and $P_2O_5$ fixed. Supply shifts to the right at higher levels of $P_2O_5$. To be comparable with figs. 2 and 4, the quantity supplied should be increased by the check plot levels of the original experiment.

The summary of algebraic forms is reserved until the long-run product supply and factor demand have been discussed.

### Long-Run Product Supply

Extension from one to several variable factors introduces a new concept to the supply equation. In long-run static supply, inputs are combined in proportions which allow a given output to be produced at a minimum cost. To obtain the supply equation, partial derivatives of the profit equation are taken with respect to each factor $X_1, X_2, \ldots, X_n$. The derivatives are set equal to zero and are solved simultaneously for $X_1, X_2, \ldots, X_n$. These expressions are substituted into the production function to form the supply equation. (The long-run static product supply and other equations for two variable factors are given in the appendix.)

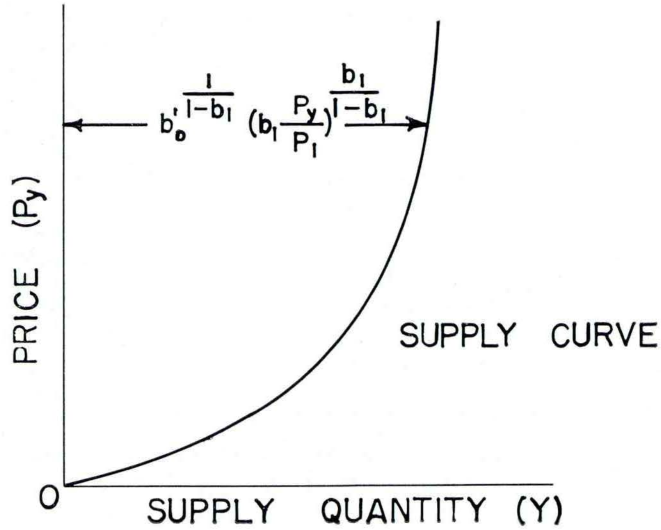



**Fig. 5. Hypothetical short-run static supply curve derived from a logarithmic production function.**

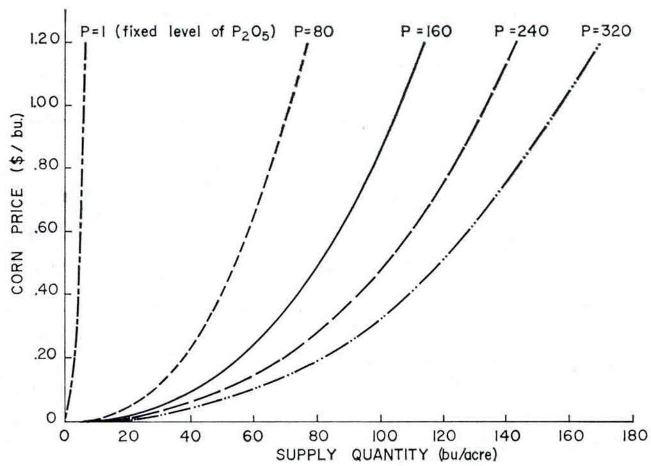



**Fig. 6. Short-run static corn supply from a logarithmic production function fitted to Ida silt loam data. The price of nitrogen, the variable factor, is 13 cents per pound.**

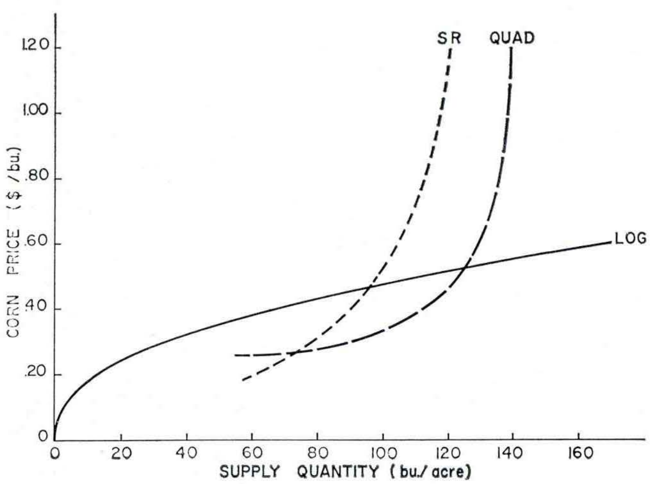



**Fig. 7. Long-run static corn supply from quadratic, square root and logarithmic production functions fitted to Ida silt loam data. The prices of nitrogen and $P_2O_5$, the variable factors, are 13 cents and 8 cents per pound, respectively.**

The general characteristics of the quadratic, square root and logarithmic long-run supply equations are broadly similar to the short-run supply equations and, therefore, are not discussed. However, the long-run static supply curves derived from equations 9, 10 and 11 with nitrogen and $P_2O_5$ variable are illustrated in fig. 7. The quadratic and square root curves are similar. The quadratic curve, however, depicts a greater supply above a 30-cent corn price and slopes more steeply above a 60-cent corn price. The logarithmic curve rises at a decreasing rate since the sum of the exponents is greater than one-half, giving a highly unrealistic estimate of supply at higher corn prices.[14]

### Price Elasticity of Product Supply

QUADRATIC

The price elasticity of short-run static supply is computed from supply equation 15 by the formula

$$(18)\quad E_s = \frac{dY}{dP_y} \cdot \frac{P_y}{Y}$$

$$(19)\quad \frac{dY}{dP_y} = \frac{d\left[\left(b'_{00} - \frac{b'^2_{10}}{4b_{11}}\right) + \left(\frac{P_1}{P_y}\right)^2 \frac{1}{4b_{11}}\right]}{dP_y} = -\left(\frac{P_1}{P_y}\right)^2 \frac{1}{2b_{11}P_y}$$

and, therefore,

$$(20)\quad E_s = -\left(\frac{P_1}{P_y}\right)^2 \frac{1}{2b_{11}P_y} \frac{P_y}{Y}$$

or,

$$= \frac{-\left(\frac{P_1}{P_y}\right)^2 \frac{1}{2b_{11}}}{\left(b'_{00} - \frac{b'^2_{10}}{4b_{11}}\right) + \left(\frac{P_1}{P_y}\right)^2 \frac{1}{4b_{11}}}.$$

The denominator in equation 20 is the supply equation. As product price becomes very large, $E_s$ approaches zero. The elasticity of supply increases as product price falls and approaches a limit

$$-\frac{b'^2_{10}}{2b'_{00}b_{11}}$$

as $P_y$ approaches $P_1/b'$. In short, the range of static supply is

[14] The product supply equation with $X_1$ and $X_2$ variable, derived from the logarithmic function, slopes upward at an increasing rate if $0 < b + c < \frac{1}{2}$, at a constant rate if $b + c = \frac{1}{2}$ and at a decreasing rate if $\frac{1}{2} < b + c < 1$. The supply elasticity in the two-variable case is $\frac{b + c}{1 - (b + c)}$. See the appendix.

$$0 < E_s < -\frac{b'^2_{10}}{2b'_{00}b_{11}}.$$

The elasticity of supply is inversely related to the values of $b'_{00}$, $b'_{10}$ and $b_{11}$. (We refer to absolute values unless otherwise specified.) That is, high values of these coefficients are associated with low values of $E_s$.

The level of the fixed factor affects elasticity through $b'_{00}$ and $b'_{10}$ since

$$b'_{00} = b_{00} + b_{20}X_2 + b_{22}X_2^2$$

and

$$b'_{10} = b_{10} + b_{12}X_2.$$

The fixed factor $X_2$ affects the elasticity of static supply through the base effect only. That is, an increase in $X_2$ increases $b'_{00}$ if $X_2$ is less than $-\frac{b_{20}}{2b_{22}}$ and also increases $b'_{10}$ if interaction is positive ($b_{12} > 0$). Increases in these coefficients, $b'_{00}$ and $b'_{10}$, shift the supply curve to the right and leave the slope unchanged. The slope of the supply curve relates to the second derivative of the production function with respect to $X_2$, or $\frac{d^2Y}{dX^2} = 2b_{22}$. The quantity is a constant, indicating that the slope of the static supply curve remains the same for a given price ratio for all levels of $X_2$. The absence of a slope effect suggests that the elasticity of supply will be highest for low fixed factor levels because of the base effect.

SQUARE ROOT

The formula for the elasticity of supply for square root equation 16 is

$$(21)\quad E_s = \frac{\left(\frac{P_1}{P_y}\right)^2 \frac{4b'^2_{11}}{C^3}}{b'_{00} + \frac{b'^2_{11}}{C^2}(b_{10} + C)}.$$

The elasticity of supply approaches zero as $P_y$ becomes large. As $P_y$ approaches zero, the elasticity approaches a constant $2b'^2_{11}/(b'_{00} + b'^2_{11})$. In general, the constant is greater than zero, and, therefore, the limits of $E_s$ are

$$0 < E_s < \frac{2b'^2_{11}}{b'_{00} + b'^2_{11}}.$$

The elasticity varies directly as $b'_{11}$ and inversely as $b'_{00}$ and $b_{10}$. In contrast to the quadratic equation, higher levels of the fixed resource increase elasticity through $b'_{11}$ if the interaction coefficient is positive ($b'_{11} = b_{11} + b_{12}X_2$). If, however, we also consider the effect on $b'_{00}$, the elasticity might be lowered by higher levels of the fixed factor ($b'_{00} = b_{20}X_2 + b_{22}X_2^{1/2}$) if $X_2 > \left(-\frac{b_{22}}{2b_{20}}\right)^2$. An increase in $X_2$ normally reduces the slope of the static supply curve and shifts the curve to the right. These two tendencies have opposing effects on the elasticity. If the slope effect is dominant, the elasticity is greatest at high fixed factor levels. When interaction is zero, the elasticity is lowered by the base effect with higher levels of $X_2$ as long as $b'_{00}$ is increasing.

LOGARITHMIC

The elasticity of supply derived from logarithmic supply equation 17 is a constant.

$$(22)\quad E_s = \frac{b}{1 - b}.$$

It depends only on the value of b and is independent of the level and number of fixed factors, price ratios, etc. The elasticity estimated by the logarithmic function can perhaps be interpreted as an "average." It probably underestimates elasticity at lower product prices and overestimates elasticity at higher product prices (fig. 8). Figure 8 depicts the elasticities of the short-run static supply curves in figs. 2, 4 and 6. Only the elasticities of supply curves for $P_2O_5$ fixed at zero and 160 pounds are illustrated. The base and slope effects exactly counterbalance in the logarithmic supply function at all levels of the fixed resource. Hence, only one graph is needed to depict the elasticity for all fixed factor levels. Figure 8 also demonstrates that the elasticity of the log function is constant over all product prices.

Figure 8 illustrates that the elasticities of supply for the quadratic and square root supply curves (figs. 2 and 4) are quite similar and decline at higher corn prices. The elasticities are uniformly higher for both algebraic forms when $P_2O_5$ is fixed at zero pounds. The base effect causes highest elasticity at low fixed factor levels for the quadratic supply function. The base effect overshadows the slope effect causing highest elasticity at the zero level of $P_2O_5$ for the square root form.

Figure 9 illustrates the elasticities of the long-run static supply curves in fig. 7. The characteristics of the curves are similar to those in fig. 8 when only nitrogen was variable. The long-run elasticities, however, are uniformly higher. The logarithmic ranks highest and the quadratic lowest in order of magnitude of the elasticities depicted at higher product prices. This characteristic was also apparent in fig. 8 and is a general pattern of the three algebraic forms.

**Short-Run Factor Demand**

QUADRATIC

We previously derived the short-run static fac-

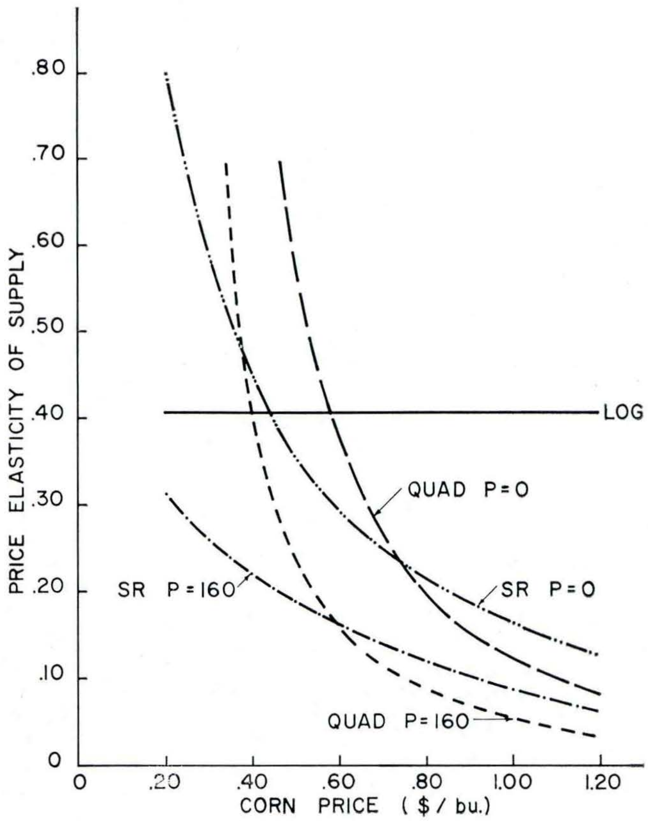



Fig. 8. Price elasticity of short-run static corn supply from quadratic, square root and logarithmic production functions fitted to Ida silt loam data. See figs. 2, 4 and 6.

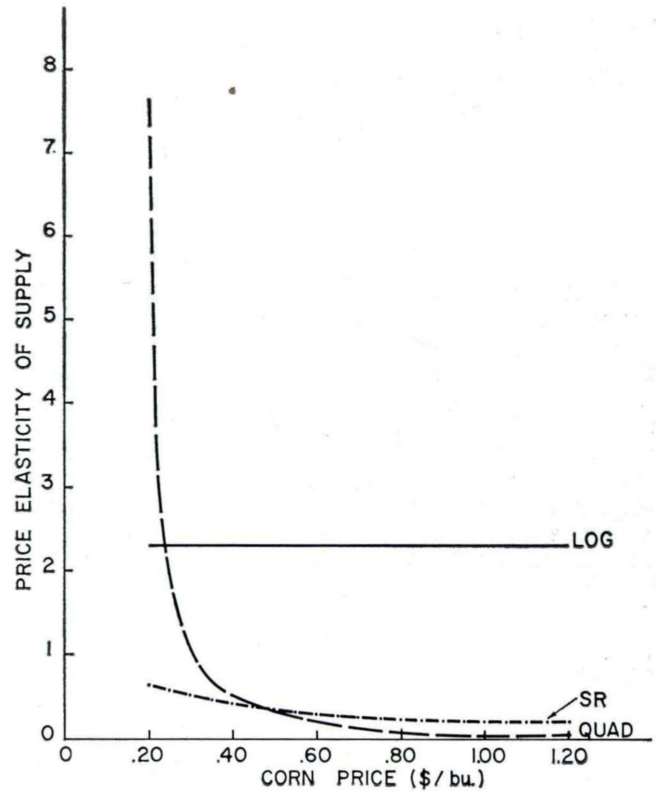



Fig. 9. Price elasticity of long-run static corn supply from quadratic, square root and logarithmic production functions fitted to Ida silt loam data. See fig. 7.

tor equation, 14, for $X_1$ by taking the derivative of the profit equation, 12, with respect to $X_1$.

$$(14)\quad X_1 = \left(\frac{P_1}{P_y}\right)\frac{1}{2b_{11}} - \frac{b'_{10}}{2b_{11}}.$$

It is apparent that $X_1$ is a linear function of $P_1$, and the static demand curve is a straight line. If $b'_{10}$ is positive, $b_{11}$ negative, the demand curve is illustrated in fig. 10. The slope $2b_{11}P_y$ of the demand curve is independent of $X_2$.

The elasticity of short-run static demand for a factor $X_1$ is found by the formula

$$(23)\quad E_d = \frac{dX_1}{dP_1}\cdot\frac{P_1}{X_1}$$

$$(24)\quad \frac{dX_1}{dP_1} = \frac{d\left[\left(\frac{P_1}{P_y}\right)\frac{1}{2b_{11}} - \frac{b'_{10}}{2b_{11}}\right]}{dP_1} = \frac{1}{P_y 2b_{11}}$$

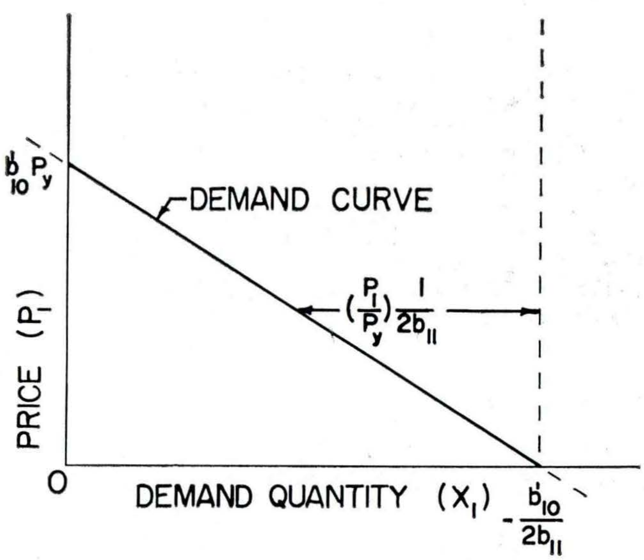



Fig. 10. Hypothetical short-run static demand curve derived from a quadratic production function.

hence,

$$(25)\quad E_d = \frac{1}{P_y 2b_{11}} \cdot \frac{P_1}{\left[\left(\frac{P_1}{P_y}\right)\frac{1}{2b_{11}} - \frac{b'_{10}}{2b_{11}}\right]} = \frac{P_1}{P_1 - b'_{10}P_y}$$

Elasticity of demand for the quadratic is independent of $b'_{00}$ and $b_{11}$. We note that $E_d$ is negative if $b'_{10}P_y$ is greater than $P_1$. Given this condition, greater values of $b'_{10}$ decrease $E_d$. Other things equal, $b'_{10}$ is inversely related to the magnitude of both $E_d$ and $E_s$. The elasticity of demand approaches infinity as the price approaches the upper end of the demand curve ($b'_{10}P_y$). Consequently, the quadratic equation is likely to overestimate elasticity at higher prices. The limits of elasticity for the quadratic equation are $0 \leqslant E_d < \infty$.

SQUARE ROOT

The equation for factor demand is

$$(26)\quad X_1 = \left[\frac{P_y b'_{11}}{2(P_1 - P_y b_{10})}\right]^2 = \left(\frac{b'_{11}}{C}\right)^2;$$

$$C = 2\left(\frac{P_1}{P_y} - b_{10}\right).$$

The demand curve is curvilinear. If $b_{10}$ is negative and $b'_{11}$ positive, the hypothetical curve is illustrated in fig. 11.

The elasticity of factor demand for the square root equation is

$$(27)\quad E_d = \frac{2P_1}{P_y b_{10} - P_1}$$

In contrast to the quadratic form, the $E_d$ for the square root form is independent of the level of the fixed factor $X_2$. The magnitude of the elasticity is inversely related to the values of $P_y$ and $b_{10}$.

As $P_1$ becomes large, the elasticity of demand approaches 2. This unusual restraint may cause the square root equation to underestimate elasticity at high factor prices. The limits of the static demand elasticity are $0 < E_d < 2$.

LOGARITHMIC

The equation for short-run factor demand derived from the logarithmic function is

$$(28)\quad X_1 = \left(\frac{P_1}{P_y b_0 b}\right)^{\frac{1}{b-1}}.$$

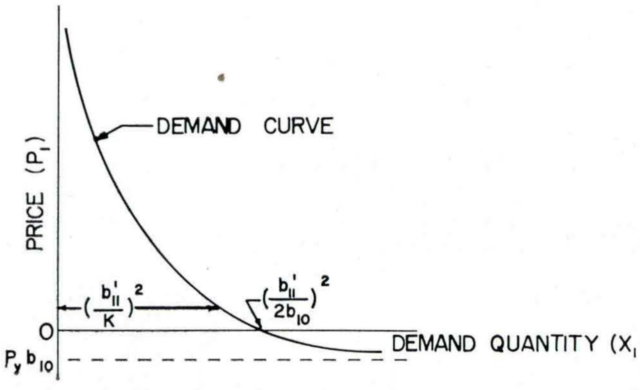



**Fig. 11. Hypothetical short-run static demand curve derived from a square root production function. In this figure, the quantity C in equation 26 is denoted by K.**

The curve which results when $0 < b < 1$ is illustrated in fig. 12.

The demand curve is asymptotic to the price and quantity axes. The unrealistic implication is that an infinite quantity is demanded as the price approaches zero, and that some quantity is demanded at any price.

The elasticity of demand for the logarithmic form is constant at all prices and at all levels of other resources.

$$(29)\quad E_d = \frac{1}{b-1}.$$

Figure 13a graphically demonstrates the characteristics of algebraic forms used to express short-run static demand for nitrogen with $P_2O_5$ fixed at zero and 160 pounds. Since the logarithmic form indicates no demand for nitrogen when $P_2O_5$ is zero, the logarithmic demand curve

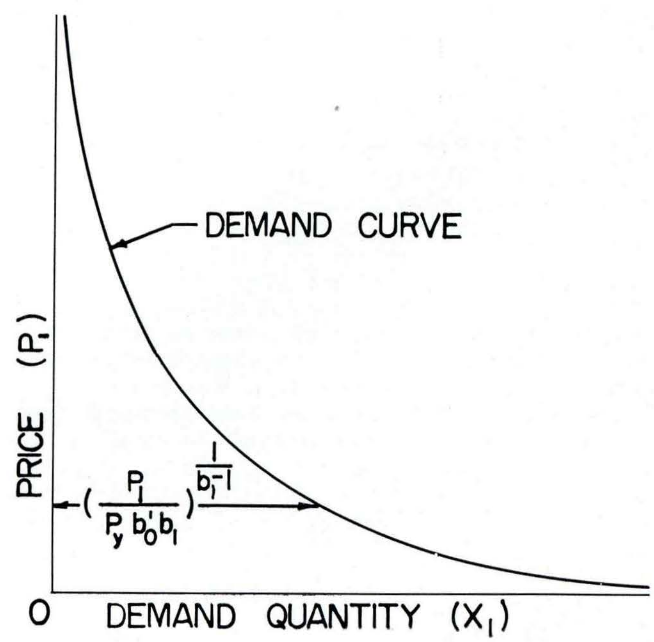



**Fig. 12. Hypothetical short-run static demand curve derived from a logarithmic production function.**

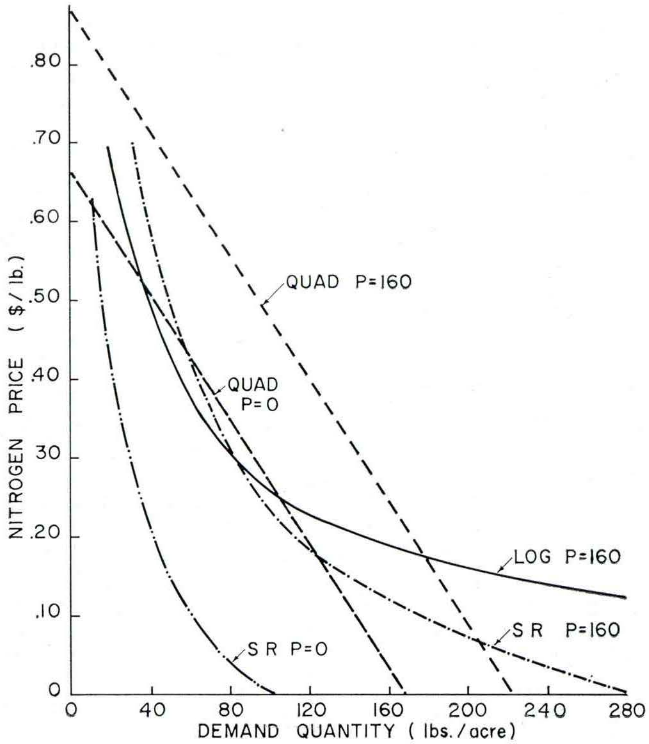



**Fig. 13a. Short-run static nitrogen demand from quadratic, square root and logarithmic production functions fitted to Ida silt loam data. $P_2O_5$ is fixed at 80 pounds; the corn price is $1.10 per bushel.**

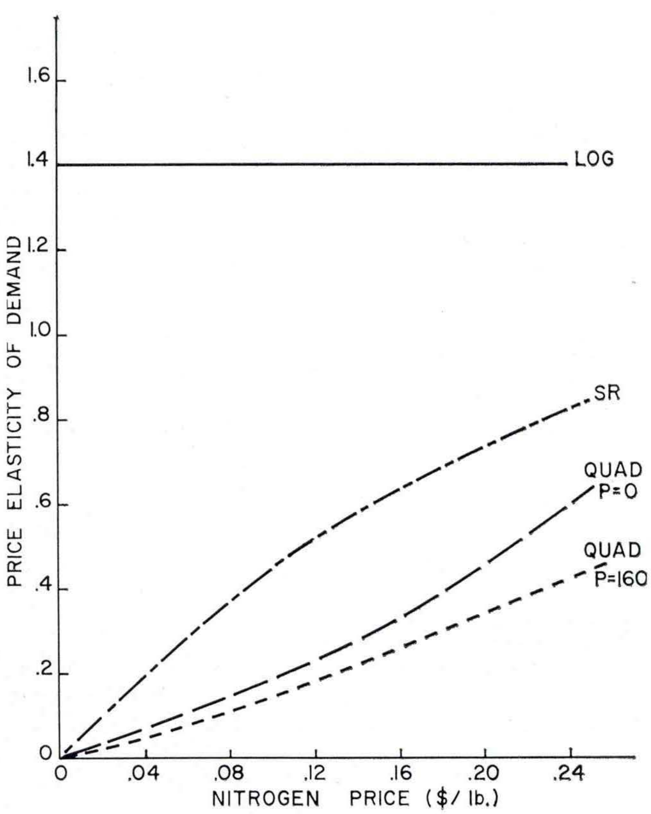



**Fig. 13b. Price elasticity of short-run static nitrogen demand illustrated in fig. 13a.**

when $P_2O_5$ is set at 160 pounds only is shown. Each algebraic form indicates the same quantity demanded as another form at some factor price. In the intermediate range of factor prices, the quadratic demand curve indicates the largest demand quantities. At higher prices and lower prices, the logarithmic and square root forms usually indicate larger demand quantities.

The elasticities of the short-run static demand curves for nitrogen in fig. 13a are shown in fig. 13b. The elasticities of the logarithmic and square root curves are independent of the level of $P_2O_5$; therefore, only one graph is presented for each. As with supply, the elasticity of factor demand depicted by the quadratic equation is highest for the zero level of $P_2O_5$. Again, the elasticity of the logarithmic form is constant for all prices of nitrogen. The elasticities of the quadratic and square root curves are somewhat similar in magnitude. Each increases with higher nitrogen prices. The elasticity of the quadratic demand curve would exceed the elasticity of square root and logarithmic demand curves if nitrogen prices were extended.

### Long-Run Factor Demand

When more than one factor is variable in the production process, the demand for any one may be called long-run static demand. The long-run static demand equations are found from the profit equation merely by taking the partial derivatives with respect to each variable, $X_1, X_2, \ldots, X_n$, and equating them to zero. The long-run static demand equations are these partial derivatives solved simultaneously for $X_1, \ldots, X_n$.

The characteristics of the long-run and the short-run static demand equations are similar and hence are not presented. (See the appendix.) However, the long-run static demand curves for nitrogen derived from production functions 9, 10 and 11 are illustrated in fig. 14a. The logarithmic production function provides an unsatisfactory estimate of short-run static supply. The square root and quadratic curves are quite similar, particularly at nitrogen prices around 70 cents and approaching zero. The quadratic indicates a greater demand between these prices. The elasticities of the two curves are more nearly similar to each other than to the logarithm elasticities below a 25-cent nitrogen price (fig. 14b). The logarithmic curve presents a different pattern and is considerably more elastic in the price range illustrated in fig. 14b.

### Cross-Product Supply and Factor Demand

The equations for static cross-supply relating the quantity of product supplied, Y, and factor price, $P_1$, are static supply equations 15, 16 and 17. Since the supply quantity is a function of the price rato, $\frac{P_y}{P_1}$, we may set $P_y$ at some fixed

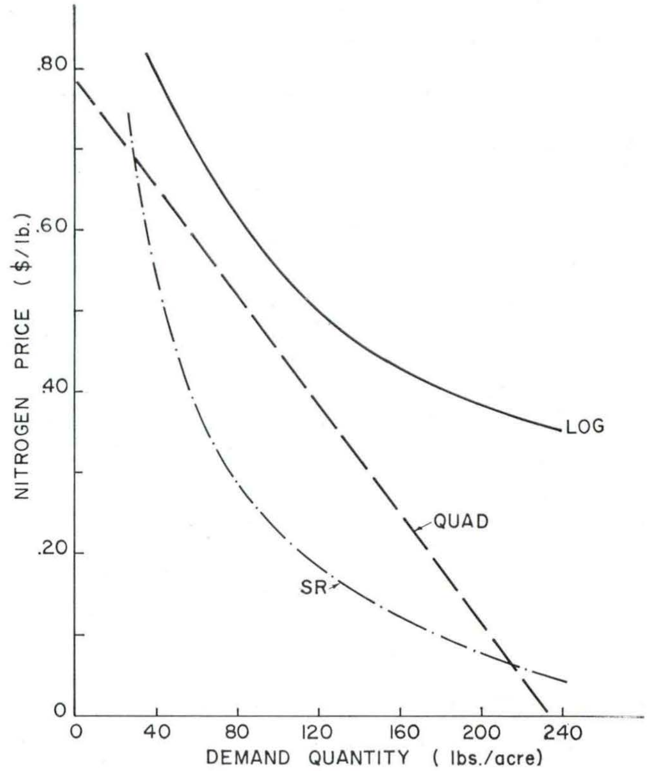



**Fig. 14a. Long-run static nitrogen demand from quadratic, square root and logarithmic production functions fitted to Ida silt loam data. The corn price is $1.10 per bushel.**

value, and the supply quantity becomes a function of $P_1$. The elasticities of cross-supply equations for Y are the elasticities of static supply equations 20, 21 and 22, but with opposite signs.

The equations for static cross-demand, the relationship between the demand quantity of a factor, $X_1$, and the product price, $P_y$, are static demand equations 14, 26 and 28. The demand quantity of a factor is a function of the price ratio, $\frac{P_1}{P_y}$. Therefore, by fixing the factor price, $P_1$, at some level, the quantity demanded, $X_1$, becomes a function of $P_y$. The elasticity of cross-demand for $X_1$ is computed from the elasticity of static demand equations 25, 27 and 29, but with the signs reversed.

The characteristics of these equations have already been discussed. The graphs of cross-demand and supply present no unique features necessary for understanding the subsequent section and, therefore, are not discussed.

### Selection of Algebraic Forms

The foregoing analysis strongly emphasizes the impact of algebraic forms on the estimates of supply and demand curves and elasticities. Each of the forms discussed possesses certain characteristics which are desirable, depending on what is being estimated and the degree of refinement desired. Nevertheless, neither the quadratic nor

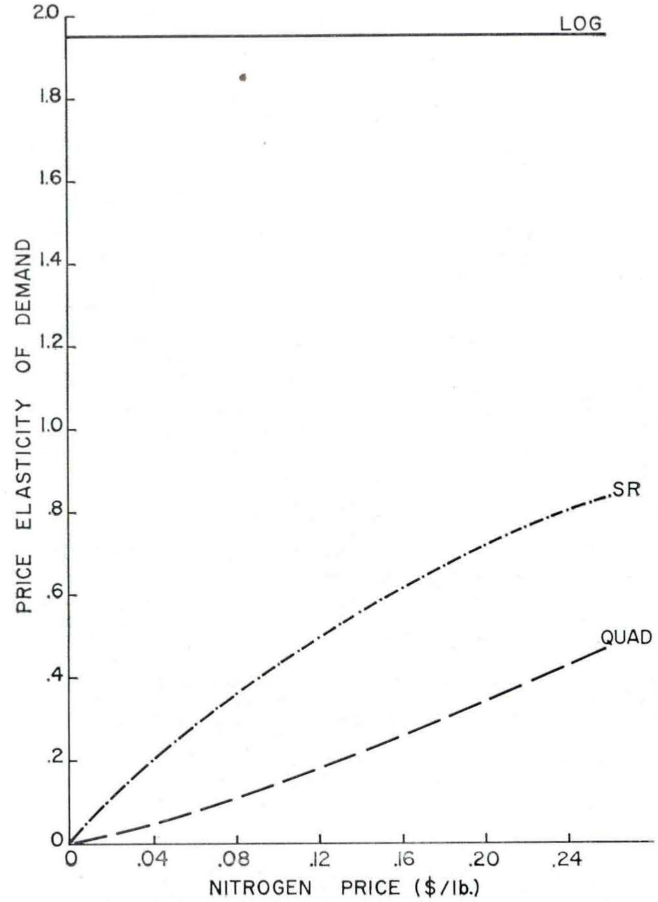



**Fig. 14b. Price elasticity of long-run static nitrogen demand illustrated in fig. 14a.**

the square root nor the logarithmic form projects all the characteristics of physical phenomena found in nature into the estimates of static supply and demand.

A logarithmic supply curve displays uniform elasticity despite the levels of the fixed factor and prices. If the sum of production elasticities is greater than one-half, as in fig. 7, the supply curve is completely unrealistic at high product prices. For example, the curve indicates that more than 500 bushels per acre are supplied for a corn price of $1 per bushel. The logarithmic form does not provide satisfactory supply and demand estimates at extreme prices and is not recommended for instances where precise estimates of supply and demand are required. Logarithmic estimates of elasticities are easily computed, however, and may provide satisfactory estimates of the **average** elasticity over the entire range of fixed factor levels and prices.

Although the quadratic and square root forms are closely related, the differences in restraints imposed by each are sometimes striking. The short-run quadratic demand is a straight line; its elasticity depends on the level of the fixed factor and approaches infinity as the factor price increases. In contrast, the square root demand is curvilinear; its elasticity is independent of the level of the fixed factor and approaches the value 2 as factor price increases. Hence, the quadratic form may overestimate, and the square root form underestimate the demand elasticity at higher factor prices. If the effect of varying fixed factor levels on demand elasticity is being determined, the quadratic form is appropriate.

The short-run quadratic supply curve generally does not intersect either the price or quantity axis, and its elasticity is independent of the fixed factor level. The square root supply curve generally intersects the price axis, and its elasticity is a function of the fixed factor level. Unlike the demand situation, the square root form is more appropriate for ascertaining the effect of fixed factor levels on supply.

These three algebraic forms, although often used, represent but a few of the possible forms. The characteristics of the logarithmic forms are somewhat similar to several other forms, such as the Spillman or Mitscherlich, which do not indicate a diminishing total product. The square root and quadratic equations are similar to many others which display a declining marginal product throughout the range of data. Although the particular characteristics of the forms discussed cannot be generalized for these related functions, the discussion does point up the need to evaluate the properties of each function in reference to the type of estimates being made.

The quadratic and square root forms appear more appropriate than the logarithmic form for analyzing static supply and demand. The logarithmic form with constant elasticity and no allowance for both an increasing and diminishing total product provides unsatisfactory estimates of supply and demand curves and elasticities. In the following section, only the quadratic and square root forms are used.

## STATIC CORN SUPPLY AND FERTILIZER DEMAND

In this section, we present static corn supply and fertilizer demand derived from production functions. Supply and demand are for a within-season period. That is, they indicate response on a per-acre basis to changes in price. Ten production functions expressing corn output as a function of fertilizer inputs are the basis for the analysis.

### Presentation of Production Functions

The production functions in this publication were fitted to controlled experimental data obtained in Iowa, Kansas, Michigan, North Carolina and Tennessee. These functions represent broad soil and weather conditions which influence yield response and also supply and demand parameters. The production functions do not represent all of the corn-fertilizer functions fitted to data. Some functions were omitted which were considered inappropriate because of an insufficient range of fertilizer application in the experimental plots. Also, functions were rejected which were fitted to logarithmic or related algebraic forms such as the Spillman or Mitscherlich. The analysis was further restricted to published functions; that is, no attempt was made to fit functions to data for use in this publication.

Perhaps some of the included functions are not significantly different from one another. If this occurs, one function might be chosen to represent the statistically similar group. The functions were not tested for significant differences because (1) the number of appropriate functions currently available is not large and (2) statistical estimates needed to test for differences are not available for some functions.

In some instances, however, it appeared appropriate to select one function from several acceptable functions fitted to the same or highly similar data. Also, it sometimes was necessary to fix the level of factors such as moisture in the production function. Where judgment was involved, we attempted to obtain the highest estimate of product supply elasticity within the bounds of the data and acceptable algebraic forms. The logic of this procedure is based on the desire to test the hypothesis that static supply elasticity is very low. If our estimates of static supply are highly inelastic, then we are more confident of a decision to not reject the hypothesis, if positive bias is anticipated in the elasticity estimates.

Certain details of the functions are important in understanding the nature of the parameters which they estimate. In the following paragraphs, the fitted functions are presented along with brief comments on the soil, weather and other pertinent data. The original sources may be consulted for further details. All functions are on a per-acre basis. Unless otherwise specified, Y is predicted bushels of corn, N is pounds of nitrogen, P is pounds of $P_2O_5$, and K is pounds of $K_2O$.

Equation 30 is a quadratic form with three independent variables.[15] The function was fitted

$$(30)\quad Y = 58.7647 + 0.2088N + 0.1388P + 0.0825K - 0.000511N^2 - 0.000859P^2 - 0.000499K^2$$

to data from a 1954 experiment on Clarion silt loam in Iowa. Heaviest application of $P_2O_5$ and $K_2O$ was 160 pounds and of nitrogen was 320 pounds. Rainfall was limited, and marginal yields diminished rapidly.

In 1953, an experiment was made on calcareous variant Webster silty clay loam in Wright County, Iowa, and equation 31 was fitted to the data.[16]

$$(31)\quad Y = 76.9263 - 0.1632N - 0.1430P + 3.6048N^{1/2} + 1.4606P^{1/2} + 0.1803N^{1/2}P^{1/2}$$

Nitrogen, $P_2O_5$ and $K_2O$ were applied at rates up to 240, 120 and 80 pounds, respectively. None of

[15] John P. Doll, Earl O. Heady and John T. Pesek, Fertilizer production functions for corn and oats; including an analysis of irrigated and residual response. Iowa Agr. and Home Econ. Exp. Sta. Res. Bul, 463. 1958. p. 367.

[16] Joseph Andrew Stritzel. Agronomic and economic evaluation of direct and residual crop responses to various fertilizer nutrients. Unpublished Ph.D. thesis, Iowa State University Library, Ames, 1958 p. 33.

the $K_2O$ terms was significant; hence these terms were omitted from the equation. Rainfall was adequate during most of the growing season.

Equation 32 was derived from a nitrogen, $P_2O_5$ and $K_2O$ experiment in 1953 on Carrington silt

$$(32)\quad Y = 99.223 - 0.04453N + 0.3162K + 0.9190N^{1/2} - 0.001813K^2$$

loam in Iowa.[17] Nitrogen was applied up to 240 pounds; $P_2O_5$ and $K_2O$ were applied up to 120 and 80 pounds, respectively. The soil was highly fertile, and a large response from fertilizer was not anticipated. $P_2O_5$ did not produce a significant response, except when interacting with $K_2O$, and was dropped from the equation.

Data for equation 33 were obtained from a 1955 experiment also on Carrington silt loam.[18] Nitro-

$$(33)\quad Y = 73.67811 + 0.06731P + 0.03000K - 0.000177P^2 - 0.000213K^2 + 0.000080PK$$

gen was included in the experiment, but none of the direct and interaction effects of nitrogen was significant above the 50-percent level; therefore, nitrogen terms were not included in the equation. The low rainfall in 1955 caused the yield response from nitrogen to be more limited than the response from other nutrients. Heaviest application of nitrogen was 240 pounds; $P_2O_5$ and $K_2O$, 160 pounds.

Equation 34 was obtained from an experiment conducted on Wisner loam soil in the "thumb"

$$(34)\quad Y = 104.1 + 0.07370N + 0.05002P - 0.0003316N^2 - 0.00005602P^2 - 0.00002546NP$$

area of Michigan in 1956.[19] The magnitude of the constant term indicates that the fertility level was probably high without any fertilizer application. The maximum application of nitrogen and $K_2O$ was 320 pounds each and 640 pounds of $P_2O_5$. The small numerical values of the coefficients of the linear and squared terms suggest very little response to fertilizer. The interaction term, though negative, does not differ significantly from zero. Only 16 percent of the variability in yield was explained by nitrogen and $P_2O_5$.

In addition to the two-nutrient equations just listed, the square root equation, 9, fitted to Ida silt loam data was selected to depict the nature of supply and demand for this particular soil and year.[20] For a discussion of the experiment, see the previous section on derivation and characteristics of algebraic supply and demand functions.

An experiment conducted on the coastal plain of North Carolina provided data for equation 35.[21]

$$(35)\quad Y = 15.4 + 0.6900N - 0.0029N^2$$

Nitrogen was applied in 20-pound increments up to 180 pounds. Weather was described as "dry."

Equation 36 was obtained from data collected at location 58 on Norfolk-like soils in North Car-

$$(36)\quad Y = 36.55 + 0.2369N - 0.00094N^2$$

olina.[22] Location 58 was one of six at which the experiment was run in 1955. The experiment included nitrogen, $P_2O_5$ and $K_2O$, but little response was exhibited to any nutrient except nitrogen. Equation 36 is a simplified, decoded form of the three-nutrient equation with $P_2O_5$ and $K_2O$ fixed at their average level—75 pounds. The heaviest application of nitrogen was 250 pounds.

Equation 37 was estimated from a 1956 experiment on Verdigris soil in eastern Kansas.[23] Nitro-

$$(37)\quad Y = 69.38 + 0.311N - 0.001379N^2$$

gen, $P_2O_5$ and $K_2O$ were applied up to 120, 80 and 40 pounds, respectively. Rainfall was adequate, and almost ideal conditions prevailed during most of the growing season. An analysis of variance indicated that nitrogen was significant at the 0.1-percent level. $P_2O_5$ and $K_2O$ were nonsignificant and were omitted from the equation.

Parks and Knetsch devised a drouth index, D, and used it to derive equation 38.[24] The highest

$$(38)\quad Y = 92.95 + 0.4834N - 0.0010N^2 - 0.5981D - 0.0028ND$$

estimate of supply elasticity resulted when the index was set at the lowest moisture level, D = 103. The experiment took place from 1954 to 1956 on Lintonia soil in Tennessee.

### Static Corn Supply

In the following pages, we examine the nature of short-run and long-run static supply. The term short run indicates that a single fertilizer nutrient is variable. The term long run indicates that more than one nutrient is variable. Both concepts are short run in the usual terminology, since inputs other than fertilizer would be variable in the conventional meaning of long-run supply.

The restraints imposed by algebraic forms of the production function particularly affect the estimates of static supply elasticity at very high or very low prices. To avoid extreme prices, the supply curves and elasticities are illustrated for

[17] William G. Brown, Earl O. Heady, John T. Pesek and Joseph A. Stritzel. Production functions, isoquants, isoclines and economic optima in corn fertilization for experiments with two and three variable nutrients. Iowa Agr. Exp. Sta. Res. Bul. 441. 1956. p. 809.

[18] Doll, Heady and Pesek, **op. cit.**, p. 390.

[19] W. B. Sundquist and L. S. Robertson, Jr. An economic analysis of some controlled fertilizer input-output experiments in Michigan. Mich. Agr. Exp. Sta. Tech. Bul. 269. 1959. p.40.

[20] Heady, Pesek and Brown, **op. cit.**, p. 304.

[21] P. R. Johnson. An economic analysis of corn fertilization in the coastal plains of North Carolina. Unpublished Ph.D. thesis. North Carolina State College Library, Raleigh. 1952.

[22] D. C. Hurst and D. C. Mason. Some statistical aspects of the TVA North Carolina Cooperative Project on determination of yield response surfaces for corn. In E. L. Baum, Earl O. Heady, J. T. Pesek and C. C. Hildreth, eds. Economic and technical analysis of fertilizer innovations and resource use. Iowa State University Press, Ames. 1959. p. 213.

[23] Frank Orazem and Floyd W. Smith. An economic approach to the use of fertilizer. Kan. Agr. Exp. Sta. Tech. Bul. 94. 1958. p.9.

[24] W. L. Parks and J. L. Knetsch. Corn yields as influenced by nitrogen level and drouth intensity. Agron. Jour. 51:363-364. 1959.

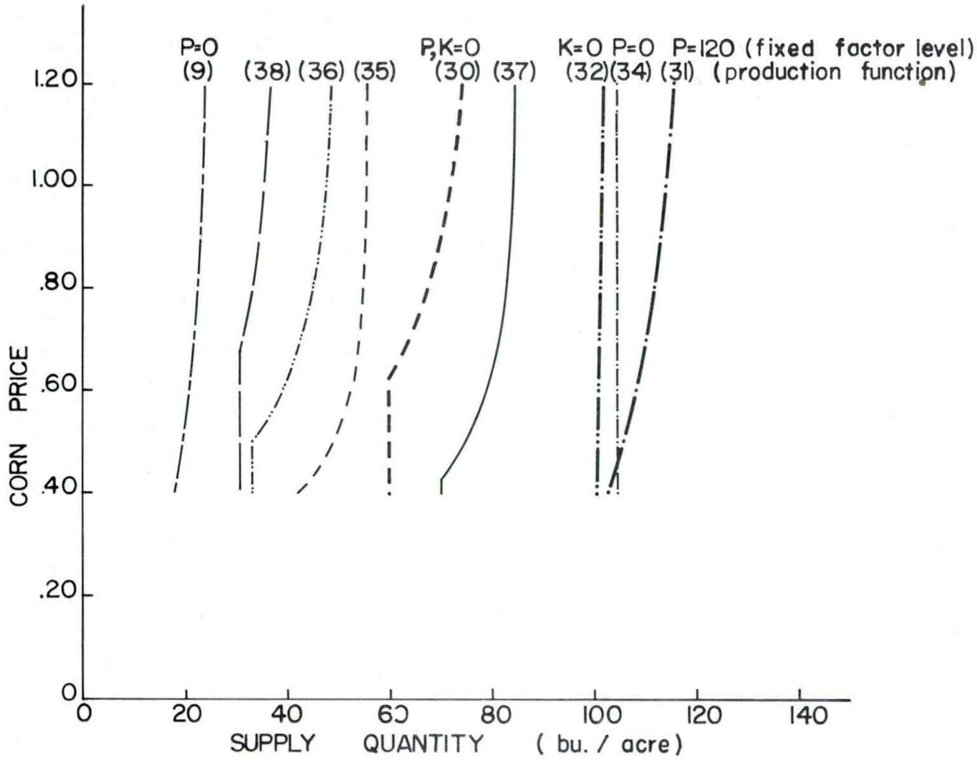



Fig. 15a. Short-run static corn supply. The price of nitrogen, the variable factor, is 13 cents per pound.

corn prices ranging from 40 cents to \$1.20 per bushel. Nitrogen, $P_2O_5$ and $K_2O$ prices are 13 cents, 8 cents and 5 cents per pound, respectively. The corn price range of 40 cents to \$1.20 appears adequate; decision makers seldom would desire information on the effects on production of changes in the corn price outside this range of prices. Supply curves and elasticities may be found for other corn and fertilizer prices by considering ratios rather than absolute prices. This procedure is demonstrated later.

## SHORT-RUN SUPPLY

It is impractical to present a complete family of short-run supply curves for all values of the fixed nutrients when two or more nutrients are included in the production function. As explained earlier, the fixed resource is set at the level that gives the highest estimate of elasticity within the range of the experimental data. A low fixed resource level resulted in the highest elasticity of supply in most instances. The low fixed factor levels did not affect the slope but shifted the quadratic supply curves to the left, increasing the elasticity. Quadratic equation 34 was an exception since the interaction coefficient $b_{12}$ was negative. In the square root equations, 9, 31 and 32, the level of the fixed factor exerts opposite influences on elasticity through the base and slope effects discussed previously. The base effect overshadowed the slope effect in equations 9 and 32 and resulted in the highest elasticity of supply at low fixed factor levels.

With nitrogen as the only variable input, the positions of the supply curves are widely dispersed, but the slopes are uniform (fig. 15a). The level of supply varies as much as 100 bushels per acre. The wide range is explained largely by (a) the soil fertility, (b) moisture conditions and (c) the level of the fixed nutrient. The $b_{00}$ value is the predicted yield level of the soil without application of fertilizer. It reflects the initial fertility level of the soil and moisture conditions. The supply curves farthest to the right, 31, 32 and 34, represent production functions with high $b_{00}$ values of 77, 99 and 104 bushels per acre, respectively. The initial yield level of the supply curve farthest to the left, 9, was almost zero. If all curves were adjusted to a common $b_{00}$ and fixed factor level, the range of supply quantities at any price would be small indeed. (Note that the number of the supply curve in fig. 15a, recorded above the curve, is also the number of the production function from which the curve was derived. The level of the fixed factor, factors other than nitrogen included in the production function, also is recorded above the supply curve.)

The steep slopes of the curves indicate that a change in price would result in little change in quantity. Supply curve 34 for Wisner loam in Michigan is a vertical straight line. No nitrogen is being used, and none would be used until corn reaches \$1.80 per bushel. The supply quantity at all indicated prices is the initial yield of 104 bushels. Curves 30, 36, 37 and 38 display vertical straight line segments. These segments indicate that nitrogen is unprofitable up to the corn price where the segments show some curvature. The supply quantity in these segments is the initial yield $b'_{00}$. The vertical segments would not extend to the quantity axis, since, at some nonzero price

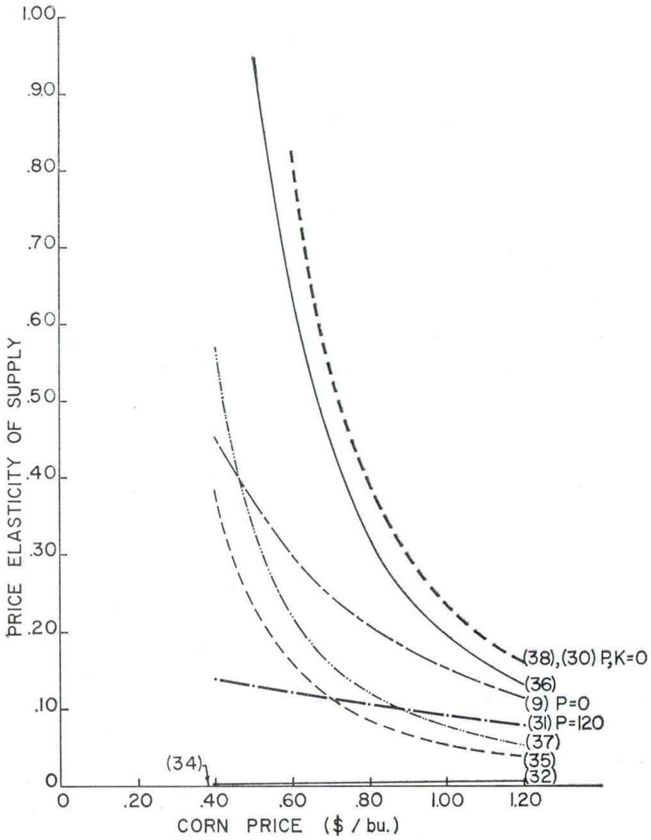



**Fig. 15b. Price elasticity of short-run static corn supply illustrated in fig. 15a.**

of corn, it would not be profitable to harvest the initial yield. The cost per bushel to harvest corn is well below the 40-cents-per-bushel minimum of fig. 15a and need not concern us.

The steep slopes of the static supply curves in fig. 15a are reflected in their low elasticities illustrated in fig. 15b. All supply curves are inelastic ($E_s < 1$) when the corn price (horizontal axis) is above 40 cents. Moving from right to left in fig. 15b, the elasticities of supply curves 30, 36 and 38 rise sharply, and it appears as if they would be greater than unity when the corn price is 40 cents. Nitrogen no longer is profitable before these curves become elastic, however. Static supply elasticity drops to zero when the corn price is below 62 cents, 50 cents and 67 cents for curves 30, 36 and 38, respectively. The elasticity of all supply curves in fig. 15a is less than 0.5 when the price of corn is above 80 cents. At a corn price of \$1.20, the elasticities range from zero (34) to 0.16 (30 and 38). We conclude that the elasticity is low for all supply curves throughout the range of prices considered in the analysis.

Figures 15a and 15b have wider application if we think in terms of price ratios rather than absolute prices. The price of nitrogen $P_n$ used to estimate the supply curves and elasticities was 13 cents per pound, but it is desirable to be able to generalize the supply quantities and the elasticities for other nitrogen prices. The corn price axes may be considered price ratio axes. For a corn price, $P_c$, of 90 cents per bushel, the ratio is

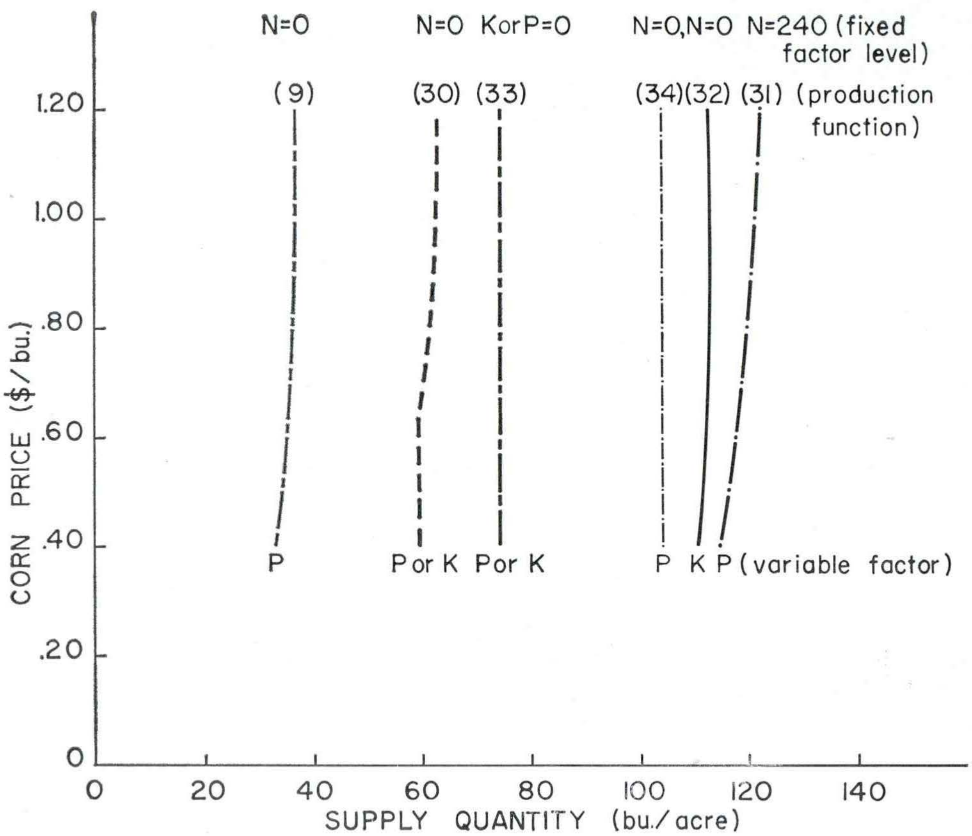



**Fig. 16a. Short-run static corn supply. The price of $P_2O_5$, the variable factor in the supply curves P, is 8 cents per pound. The price of $K_2O$, the variable in the supply curves K, is 5 cents per pound.**

$\frac{90 \text{ cents}}{13 \text{ cents}} = 7$. The supply quantity or the elasticity of supply remains the same for any absolute level of prices providing the price ratio is seven. But suppose that $P_n$ falls to 10 cents and $P_c$ remains at 90 cents. The new price ratio is nine. To find the level of supply from fig. 15a or the elasticity from fig. 15b for $P_n = 10$ cents, $P_c = 90$ cents, compute the corn price which gives a price ratio of nine when $P_n = 13$ cents; i.e., $P_c = \$1.17$. Then observe the supply quantities and elasticities from figs. 15a and 15b for $P_c = \$1.17$. This method is limited when supply is computed with two or more variable factors. It is necessary to consider the price ratios among factors as well as between factors and products. The procedure described may be used as an approximate device if interfactor price ratios remain unchanged.

Figure 16a depicts static corn supply curves with either $P_2O_5$ or $K_2O$ as the only variable factor. (The variable factor is indicated by P or K below each supply curve in fig. 16a.) The curves indicate a considerable range of supply levels. The range would be somewhat less if the border curves, 9 and 31, were estimated with nitrogen fixed at the same level. All curves except curve 34 were derived from Iowa data. Hence, there is little basis for comparisons among regions. Figure 16a demonstrates a broad range of supply by soil types and weather within Iowa. Supply curves 32 and 33 were estimated from experiments on Carrington soil in 1953 and 1955, respectively. These two curves indicate the wide range in the level of supply which can arise among years on a given soil type.

The slopes are more uniform than the positions of the supply curves. In general, they rise even more steeply than the static supply curves when only nitrogen is variable in fig. 15a. Supply curves 33 and 34 are perfectly vertical in fig. 16a. No $P_2O_5$ is used in curve 33 until the corn price reaches \$1.67 per bushel with nitrogen and $K_2O$ fixed at zero pounds, and no $K_2O$ is used until the corn price is \$1.19 per bushel. With nitrogen fixed at zero in curve 34, $P_2O_5$ is not profitable until the price of corn reaches \$1.60 per bushel. Only the initial yield level, $b_{00}$, is assumed to be supplied until these prices are reached.

The elasticities of supply curve 30 up to 60 cents and of curves 33 and 34 are zero (fig. 16b). All the static supply curves for only $P_2O_5$ or $K_2O$ variable are highly inelastic. All are below 0.20 for a corn price of 40 cents. The elasticity declines with higher prices of corn and is less than 0.05 for all supply curves when corn is \$1.20 per bushel. Although the magnitude of static supply elasticity with only $P_2O_5$ or $K_2O$ variable differs by soil type and weather, we may conclude from fig. 16b that it is uniformly low in all cases in the range of corn prices considered. This conclusion is based primarily on Iowa data. In several other experiments in other states, $P_2O_5$ and $K_2O$ were included but did not affect yield significantly. We may generalize that the static supply elasticity, with only $P_2O_5$ and $K_2O$ variable for these soil and weather conditions, also would be near zero.

Figures 16a and 16b indicate that the elasticity of supply as well as the slopes are less variable among soil types and years than is the level of supply. The level of supply, indicated by curves 32 and 33, for Carrington soil in fig. 16a differs considerably. Yet the elasticities, shown in fig. 16b, of these supply curves are very similar. Of course, the elasticities become even more uniform when corn price becomes large and the elasticities approach zero.

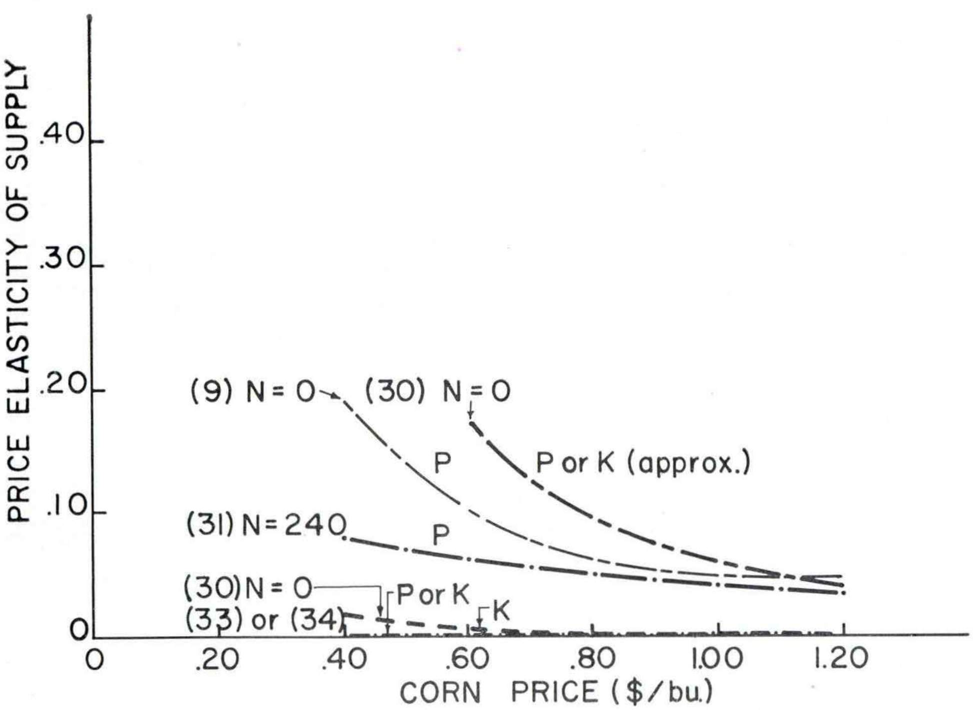



**Fig. 16b. Price elasticity of the short-run static supply illustrated in fig. 16a.**

All of the supply curves in fig. 16a were derived from production functions which include two or three fertilizer nutrients as inputs. It is unlikely that either $P_2O_5$ or $K_2O$ would be applied alone. Long-run static supply curves with $P_2O_5$ and $K_2O$ varying with other nutrients provide a more meaningful estimate of static supply.

### LONG-RUN SUPPLY

The range of supply quantities is not as broad and the curves are not as steep when more than one nutrient is variable in static supply (fig. 17a). The range of supply quantities is less than 60 bushels per acre. The long-run quantity may be less, the same or more at any price for a given curve than the short-run quantities shown in figs. 15a and 16a. But for any curve, the slope is always less as more nutrients become variable.

Three fertilizer nutrients are variable in supply curve 30 (N, P, K) ; in the remainder, only two nutrients are variable. The supply curves 30 (N, P,) for nitrogen and $P_2O_5$ variable and 30 (N, K) for nitrogen and $K_2O$ variable are similar to curve 30 (N, P, K) and, consequently, are not illustrated. Addition of the third nutrient, $P_2O_5$ or $K_2O$ in either case, caused little change in the supply curve. But adding nitrogen to supply curve 30 (P, K) shifted the curve sharply to the right. Obviously, nitrogen was the most limiting resource on the Clarion soil where curve 30 was derived.

Supply curve 30 (N, P, K) presents an interesting pattern. Nitrogen, $P_2O_5$ and $K_2O$ individually become profitable (nonzero quantity) at corn prices of 62 cents, 58 cents and 61 cents, respectively. The slope of curve 30 remains vertical until the price of corn reaches 58 cents and it becomes profitable to apply $P_2O_5$. The segment of curve 30 (N, P, K) from 58 cents to 61 cents is the same as the short-run curve 30 (P) over the same price range in fig. 16a. At 61 cents, $K_2O$ also becomes profitable, and curve 30 (N, P, K) becomes "long run" with two variable nutrients. It follows the curvature of 30 (P, K) until nitrogen becomes profitable at 62 cents. When all three nutrients become variable at 62 cents, curve 30 (N, P, K) becomes separate from other supply curves for function 30.

All the supply curves, except curve 34, in fig. 17a are from Iowa data. While it is not possible to make interregional comparisons, it is possible to isolate some of the effects on supply of moisture and of soil fertility. Curves 32 and 33 were derived on Carrington soil in 1953 and 1955, respectively. Because of more rainfall in 1953, curve 32 lies considerably to the right of curve 33. Curves 9, 31 and 32 were estimated on different soils in Iowa but under similar moisture conditions in 1953. The curves depict nearly equivalent levels of supply. The results are consistent with the hypothesis that greater divergence in the level of supply arises because of differences in moisture than arises because of differences in soil type.

The moisture and fertility level of the soil also explain the curvature of the supply curves. The greatest curvature is found in curves derived on soils low in fertilizer but otherwise favorable for

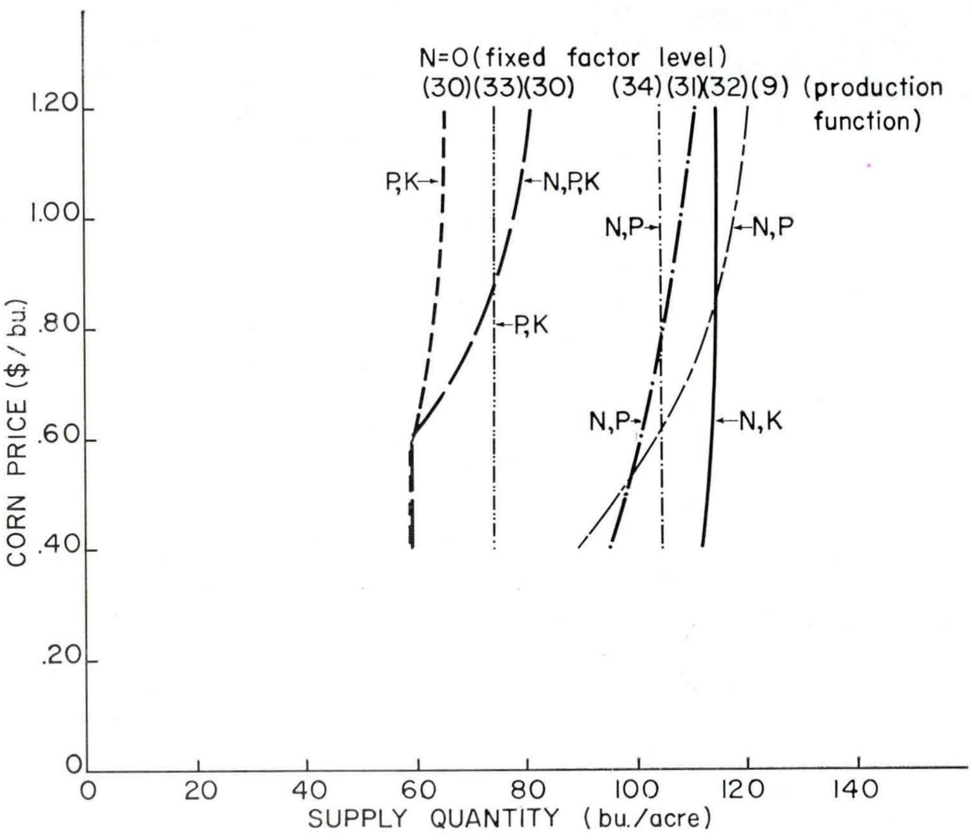



Fig. 17a. Long-run static corn supply. The prices of nitrogen, $P_2O_5$ and $K_2O$, the variable factors, are 13 cents, 8 cents and 5 cents per pound, respectively.

corn production; i.e., with adequate moisture, good soil structure, etc. Curves 9 and 31, for example, were estimated under favorable moisture conditions. Curve 30, though estimated under limited moisture, lacked fertilizer, particularly nitrogen, and hence indicated considerable curvature.

On the other hand, supply curves 33 and 34 are vertical straight lines. The corn prices at which nutrients become profitable — the slope becomes less than infinite — for supply curve 33 are \$1.23 and \$1.51 for $P_2O_5$ and $K_2O$, respectively. For supply curve 34, it is profitable to use $P_2O_5$ when the corn price reaches \$1.59 per bushel, but the price of corn must reach \$1.79 per bushel before nitrogen becomes profitable. Lack of moisture severely limited the physical response to fertilizer for production function 33 in 1955. Wisner loam is a fertile, heavy soil, and the lack of curvature in supply curve 34 is due as much to the initial fertility of the soils as to limited rainfall.

The level and slope of the supply curves in fig. 17a principally explain the elasticities illustrated in fig. 17b. The elasticity of the vertical supply curves, 33 and 34, is zero. Curves 9, 30 and 31 not only display the least slopes in fig. 17a, but also are most elastic (least inelastic). It is interesting to note that the elasticities are more uniform than the levels of supply curves 32 and 33 for Carrington soil. However, curves 9, 31 and 32 estimated under similar moisture conditions present a variety of elasticities. But the elasticities of these curves are uniform in the sense that they are low.

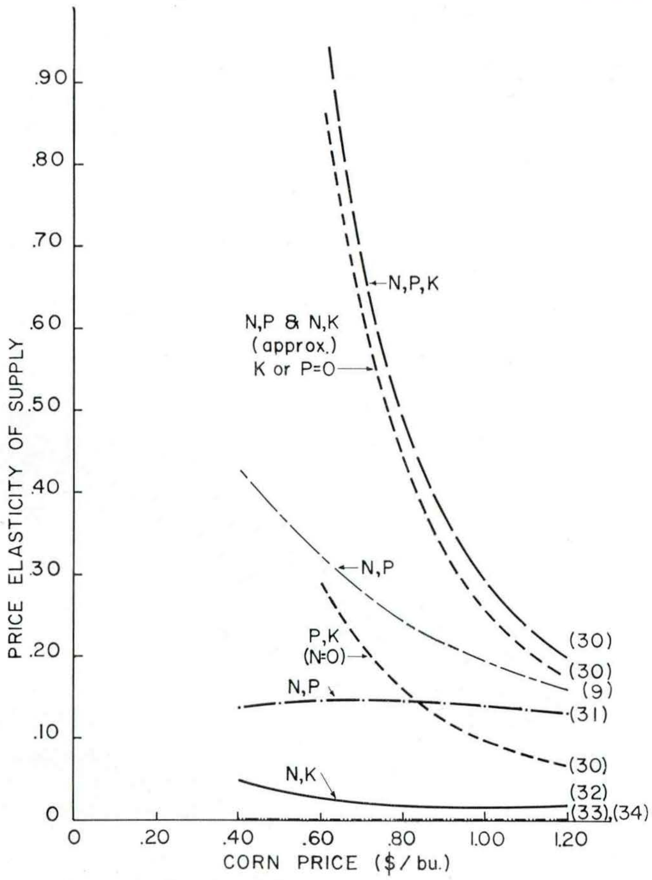



Fig. 17b. Price elasticity of long-run static corn supply illustrated in fig. 17a.

The long-run static supply curves are less inelastic than are the short-run supply curves (fig. 17b). Nevertheless, all the long-run curves are inelastic when corn is over 40 cents per bushel. The elasticity is less than 0.5 when the price of corn is greater than 80 cents and less than 0.2 when the corn price is \$1.20 or higher. If curve 30 were omitted, the elasticity of the remaining curves would lie below 0.45 for a corn price of 40 cents or more. Much of the elasticity of curve 30 is due to nitrogen. The elasticity of curve 30 with only nitrogen variable (see fig. 15b) is nearly as large as with three nutrients variable and is considerably more elastic (less inelastic) than with only $P_2O_5$ and $K_2O$ variable. The structure of Clarion soil (30) is adequate, but the soil is low in certain nutrients, particularly nitrogen.

The long-run supply elasticities of fig. 17b give a more realistic estimate of static supply than do the short-run elasticities for the same production functions shown in figs. 15b and 16b. A farmer seldom would use only a single nutrient when other nutrients give a significant yield response and also limit the response of the single nutrient.

Figure 18 is included to provide a summary of the static supply curves when all nutrients included in the production functions are allowed to vary. Since nitrogen was the only input explicitly included in most production functions fitted outside Iowa, the static supply curves in fig. 15 provide a basis for inferences about these areas. The Iowa production functions contain two or more fertilizer nutrients, hence, fig. 17 is the logical basis for inferences about the Iowa area. Figure 18 includes static supply curves from figs. 15 and 17 and allows comparisons between areas.

Figure 18a indicates that the production functions fitted to Iowa data (9, 30 through 33) generally depict a higher level of supply than do those fitted to data from other states (34 through 38). The favorable soil conditions (other than nitrogen, $P_2O_5$ and $K_2O$ content) and the weather in Iowa are possible explanations for this difference. The curves from Iowa data also may represent the intensive corn-producing areas of other Corn Belt states such as Minnesota and Illinois. The slopes of the supply curves do not indicate any general differences among areas. Of the two curves having the greatest slope, curve 33 is from Iowa data and curve 34 is from Michigan data. Of five curves having the least slope, two are from Iowa (3 and 30), one is from North Carolina (36), one is from Kansas (37), and one is from Tennessee (38).

The elasticities of the static supply curves also do not show any important differences among areas (fig. 18b). Static supply curves 30 and 33 from Iowa data rank lowest and highest in elasticity, supporting the hypothesis that greater differences may exist within an area than among areas. Despite differences within and among areas, the elasticities of all the curves are uniformly low. All supply curves are inelastic for a

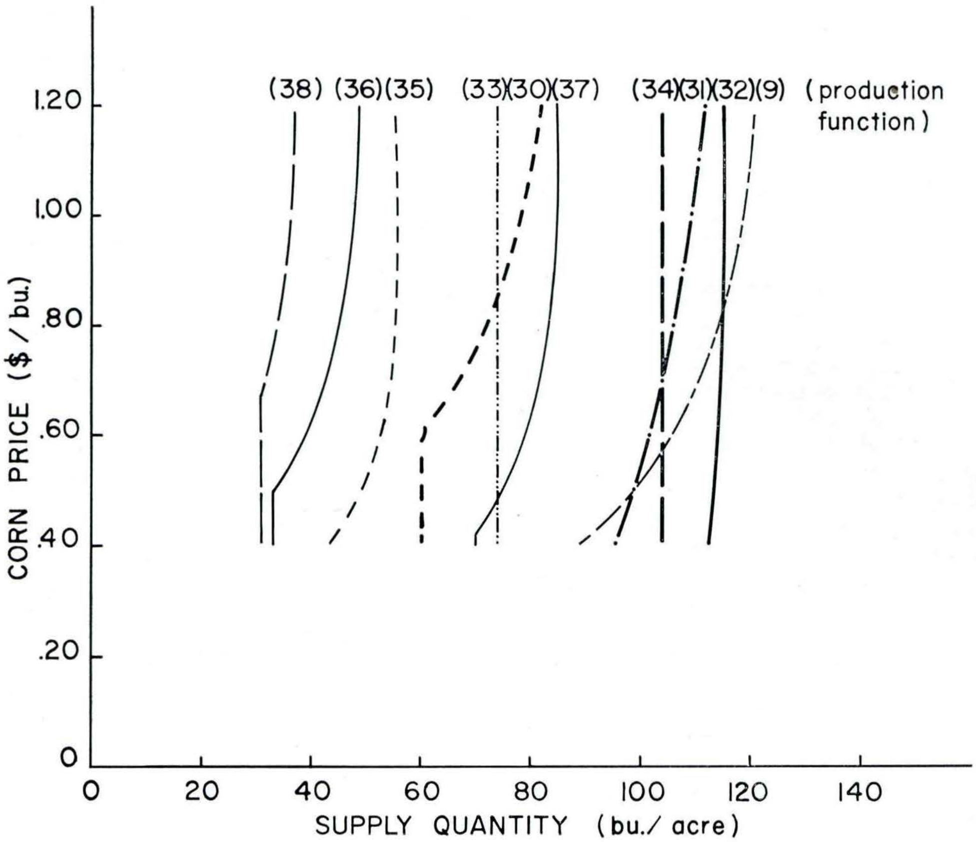
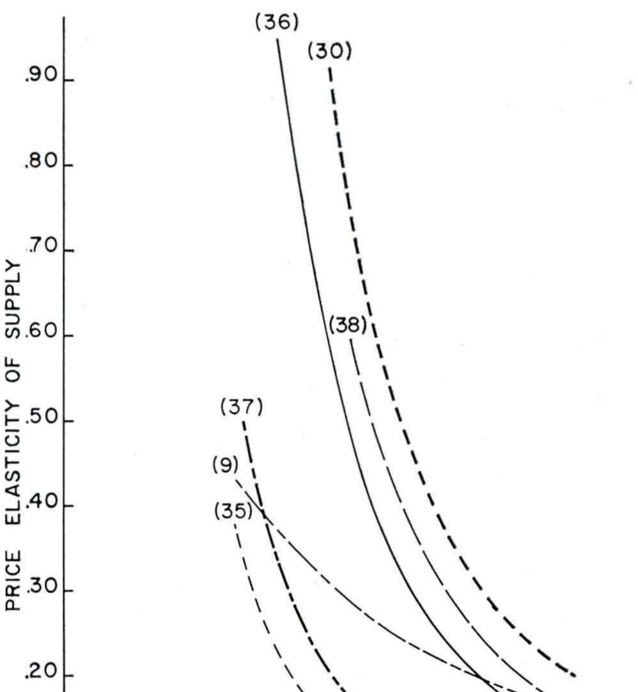



Fig. 18a. Summary of short-run and long-run static corn supply. All fertilizer nutrients included in the production function are variable. The prices of the variable factors, nitrogen, $P_2O_5$ and $K_2O$, are 13 cents, 8 cents and 5 cents per pound, respectively.




corn price over 40 cents. The elasticity falls with high corn prices. It is less than 0.3 for a corn price greater than $1 and less than 0.20 for a corn price greater than $1.20. The elasticity of supply curves 32, 33 and 34 is zero or near zero in the price range of 40 cents to $1.20.

## CONCLUSIONS AND IMPLICATIONS

Figures 15 through 18 indicate that the elasticity of static supply is low for all soil and weather conditions, prices, short- and long-run supply curves and algebraic forms considered. Without exceptions, static supply is inelastic ($E_s < 1$) for corn prices over 40 cents per bushel. The elasticity is less than 0.3 for corn prices above $1 and less than 0.2 for corn prices above $1.20. The "average" elasticity of the curves would lie well below these values since, in many instances, the elasticity is zero or near zero in the relevant price range. The results clearly support the hypothesis that static supply elasticity is low. In the earlier section on logic and assumptions, a **priori** considerations strongly implied that supply estimated from experimental data is more elastic than supply expressed in actual market behavior. That is, the estimates in this study are a sample from a physical environment representing the maximum response which might be expected from farmers to changes in price. Thus the low elasticity of

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Fig. 18b. Price elasticity of short-run and long-run static supply illustrated in fig. 18a.

static supply supports the hypothesis that market supply elasticity is low when the corn acreage is given. This hypothesis can be tested with greater precision as more empirical data become available.

The empirical results are estimates of supply elasticity essentially at the start of the growing season. The foregoing analysis indicates that there is no basis in the physical conditions of production for concluding that the supply elasticity is zero. The majority of the static supply curves display **some** positive elasticity. Thus, the short-run elasticity, although low, is probably not zero or negative. The supply elasticity may approach zero, however, at the end of the growing season. As the season progresses, opportunities diminish for increasing or decreasing the corn yield in response to price changes. The elasticity of supply declines and approaches zero at harvest time on a given corn acreage.

Differences in supply levels, slopes and elasticities among and within geographic areas agree with **a priori** considerations drawn from economic theory and soil science. These differences in static supply arise mainly from variation in soil and moisture conditions. In general, those soils which are low in fertilizer elements but are otherwise satisfactory for corn production provide the highest estimates of elasticity. The analysis supports the hypothesis that as much difference in elasticity may exist within soil types in Iowa as among general soil areas. It is important, therefore, to use caution in generalizing about the static supply elasticity for all production units within an area.

Economists sometimes estimate supply elasticities from time series data and least squares statistical methods. When the data are transformed into logarithms, the least squares coefficients are constant for all relative prices. While constant price elasticities are a useful approximation for the narrow range of prices experienced in recent years, the foregoing analysis suggests that elasticity is greater at low corn prices than at high corn prices. The results of this study indicate that an estimate of the impact of a small price change on corn production is less when the corn price is $1.50 per bushel than when the corn price is $0.80 per bushel. Thus, normative results such as found in this report, are a potential supplement to time series data by indicating the changes in supply elasticity which might occur outside the range of experienced price.

Policymakers may wish to appraise the feasibility of controlling corn production by a tax on fertilizer. In the earlier discussion on static cross-supply, we learned that elasticities of static corn supply with respect to (a) the price of corn or (b) the price of fertilizer are equal numerically but opposite in sign. Thus, previous evidence from the 10 physical production functions indicating that supply elasticity is low is also evidence that the corn production is unresponsive to changes in the price of fertilizer. On the basis of the physical conditions of production depicted in the 10 functions, a 10 percent tax on fertilizer would decrease corn production per acre somewhat less than 3 percent. Since supply elasticity increases as more fertilizer elements become variable, a tax or subsidy on several fertilizer inputs would be more effective in changing production than would the same measures on a single nutrient.

The estimates of supply derived from functions such as equations 36, 37 and 38 illustrate some of the pitfalls inherent in the use of elasticity estimates for policy purposes. These estimates for marginal areas of corn production indicate a low level but high elasticity of static supply. We may be correct in concluding that the greatest percentage increase in corn supply from higher corn prices would come from marginal areas outside the Corn Belt. It may be wrong, however, to conclude that the greatest absolute increase in production would occur in marginal areas. Because of the large number of production units and high yields per unit, the largest increase in total bushels produced likely would come from the Corn Belt. This example indicates the value of working with estimates of supply curves rather than elasticities when possible.

The analysis provides a useful basis for developing hypotheses of future trends in the static supply elasticity of corn. Two conditions potentially affecting the static supply elasticity are: (1) depletion of nutrient levels in the soil through erosion and crop attrition and (2) new technology and changing production practices such as irrigation, improved varieties, etc. Depletion of fertilizer nutrients in the soil tends to make the soil more responsive to commercial fertilizer and increases the elasticity of static supply. These supply curves on soils depleted in nutrients can be expected to lie to the left of present curves, and other things equal, will have higher elasticity.

Assuming constant fertility, new varieties tend to shift the supply curve to the right, decreasing the elasticity. Introduction of new weed and insect control measures may tend to increase supply elasticity. The tendencies for increasing the elasticity — lower fertility and new practices — probably overshadow those for reducing elasticity. This suggests the hypothesis that the short-run supply elasticity is likely to increase in the future. This hypothesis may be accepted or rejected as more data become available.

The conclusions and implications are subject to the limitations of the analysis, of course. Additional production functions are being estimated. A larger sample of functions will provide a more meaningful basis for inferences about supply for corn and other farm enterprises. Furthermore, more research is necessary to determine how normative estimates of supply and demand parameters as found in this study compare with actual farmer behavior.

### Static Factor Demand

Static demand for a factor may be either short run or long run. The term short run, as used here, means that the levels of all other factors in the production process are considered fixed. Long run means that the levels of other factors are variable. Substitution of one factor for another is possible in the long run. The direction of the substitution depends on the change in prices and the nature of the interaction among factors.

Static demand is derived with the price of corn fixed at \$1.10 per bushel. It is possible to generalize for other corn prices by considering the fertilizer-corn price ratio since the quantity demanded is a function of this ratio. The quantity demanded when the price of corn is \$1.10 per bushel and nitrogen is 11 cents per pound, for example, is the same as when corn is 80 cents per bushel and nitrogen is 8 cents per pound.

Throughout the analysis, emphasis is placed on the conditions which influence the level and elasticity of static demand. To the extent that these conditions can be identified and isolated, they will be used to predict the nature of demand in situations of interest to farm planners.

## SHORT-RUN DEMAND

A family of short-run static demand curves can be generated from a given production function for different levels of the fixed resource (fig. 19a). The data are made manageable in the following presentation by setting the fixed resource at the same levels as in the previous short-run supply analysis. We recall that the fixed resource was set at the level giving the highest estimate of supply elasticity within the bounds of the data. The section on algebraic forms indicates that this level

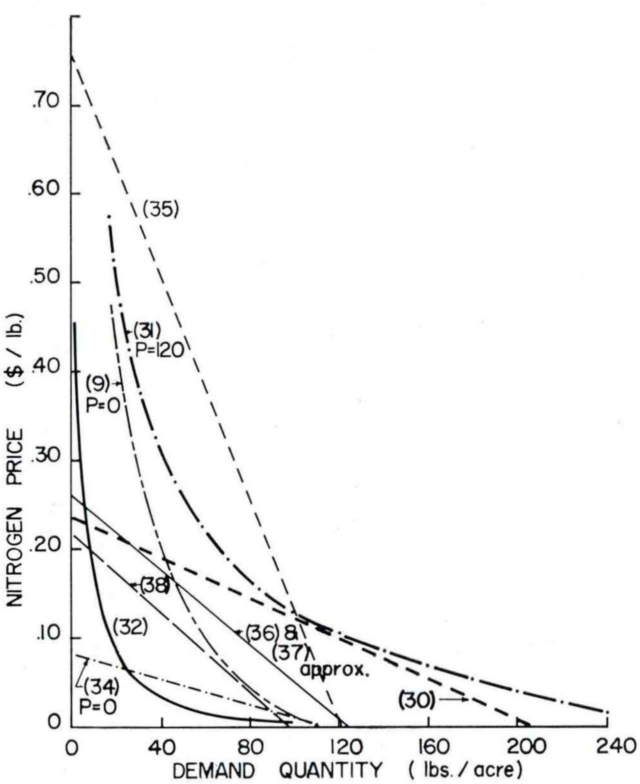


**Fig. 19a. Short-run static nitrogen demand. The corn price is \$1.10 per bushel.**

of the fixed resource also gives the highest estimate of static demand elasticity for the quadratic and square root forms.

The most striking feature of fig. 19a is the lack of uniformity in the level of static demand derived from the various production functions. If the price of nitrogen is 13 cents per pound, for example, the demand quantity ranges from zero to 100 pounds of nitrogen per acre. The possible sources of the divergent pattern of static demand are the algebraic form of the function, the moisture pattern, and the initial fertility and other properties of the soil.

The square root demand curves consistently show a higher level of demand than do the quadratic (straight line) demand curves, but only as the curves approach the price axis. Moving farther to the right from the price axis, no pattern is apparent for either algebraic form.

The computation of static demand is independent of $b'_{00}$ and, therefore, is not directly affected by the initial nutrient level of the soil. The initial fertility influences the level of demand indirectly, however. A high level of nitrogen demand reflects a large response of corn yield to additional inputs of nitrogen (marginal physical product). The marginal physical product is likely to be large if (a) the soil is not initially satiated with nitrogen and (b) other factors such as $P_2O_5$, $K_2O$ and moisture are not limiting. The level of demand indicated by each curve in fig. 19a may be explained by either of these factors.

Although rainfall was adequate in 1953, demand curve 32 depicts a low demand. The yield response to nitrogen was low for curve 32 because the initial fertility level of the Carrington soil was high ($b_{00} = 99$ bushels). The low demand for nitrogen on Wisner soil (34) is also explained by the high fertility level of the soil ($b_{00} = 104$ bushels). On such soils, a large response to fertilizer application usually is not anticipated.

Demand curve 35 was derived under dry conditions on Norfolk-like soil in North Carolina. Yet, the level of demand is high because the soil was initially low in nitrogen ($b_{00} = 15.4$ bushels) but contained adequate amounts of other nutrients. The result was a considerable response to nitrogen despite the low moisture. Curve 31, which indicates the lowest level of demand at low nitrogen prices, was derived under favorable moisture conditions and adequate amounts of $P_2O_5$ and $K_2O$ (120 pounds) on Webster soil in Iowa.

The slopes of the demand curves indicate the "intensity" of diminishing returns. If successive increments of corn production fall off rapidly with additional units of nitrogen, the demand curve for nitrogen drops sharply to the right. The slope and the level of the demand curve determine the elasticity. The magnitude of elasticity is directly related to the slope and inversely related to the level of demand or the base effect described earlier. Changes in the level of the fixed factor cause compensating changes in the position and slope of the square root form of demand. The static demand elasticity consequently is constant at all levels of

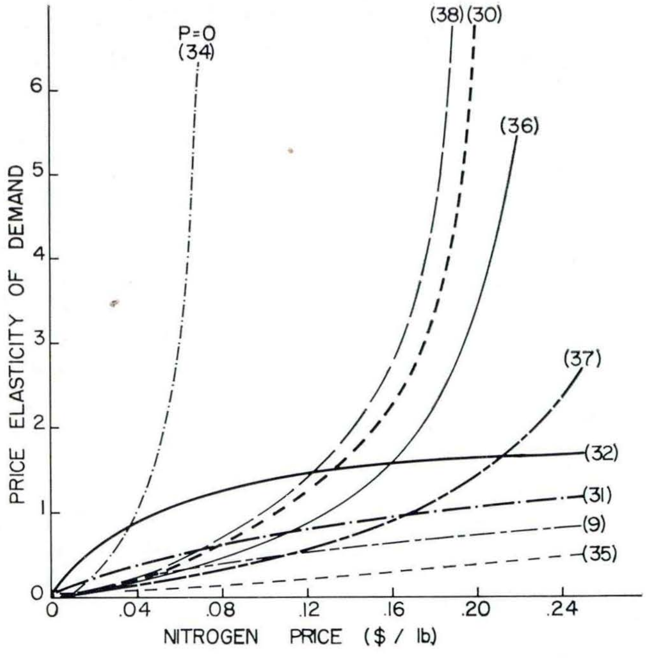


Fig. 19b. Price elasticity of static nitrogen demand illustrated in fig. 19a.

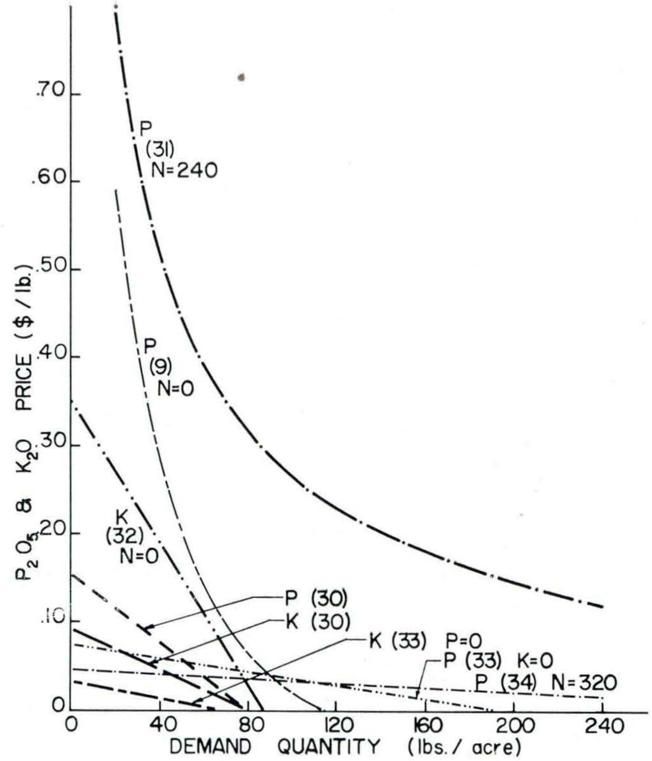


Fig. 20a. Short-run static $P_2O_5$ and $K_2O$ demand. The corn price is $1.10 per bushel.

the fixed factor. If interaction is positive, the quadratic form of the demand curve shifts to the right, and the elasticity decreases with higher fixed factor levels.

The elasticities of the static demand curves for nitrogen are quite uniform for low nitrogen prices to about 13 cents per pound. (In fig. 19b, the horizontal axis is the nitrogen price.) At approximately the current price, 13 cents, the elasticity ranges from 0.20 to 1.70 except for curve 34. Demand becomes considerably more elastic and highly divergent above 13 cents. The divergence is explained by the algebraic forms and by the experimental conditions under which the curves were estimated. The elasticity of the quadratic form approaches infinity and elasticity of the square root form approaches 2 at high factor prices. The four curves indicating the highest elasticities in fig. 19b are quadratic forms. Three of the four curves indicating the lowest elasticities are square root forms.

The low elasticity of demand curve 35 is due to the high level and steep slope of the demand curve. The level of demand is high because the soil was initially low in nitrogen; the slope is steep because low moisture restricted the yield response from large applications of nitrogen. Demand curve 34 is highly elastic when the price of nitrogen is greater than 6 cents. As the nitrogen price approaches the intersection of the demand curve with the price axis at 8 cents in fig. 19a, the elasticity approaches infinity (fig. 19b). Wisner loam, from which curve 34 was derived, is a heavy, rich soil, and the yield response to nitrogen was low. Demand curve 38 also was very elastic at most nitrogen prices. Production function 38 contains a drouth index which was set at a low moisture level to give the demand curve illustrated in fig. 19a. Had the index been set at a high moisture level, the elasticity would have been lower. We conclude that demand in these samples is most elastic under conditions where nitrogen fertilizer has little effect on yield because the soil initially contains adequate nitrogen or because the yield response is limited by lack of moisture or other factors.

Considerable variation also is apparent in the levels of short-run static demand for $P_2O_5$ and $K_2O$ illustrated in fig. 20a. (P and K on the curves indicate the demand for $P_2O_5$ and $K_2O$, respectively.) The divergent level of demand is explained by the nutrient and moisture conditions of the soil where the production functions were derived. Curves 33 for $P_2O_5$ and $K_2O$ depict two of the lowest demand levels. Both were estimated from an experiment on Carrington soil in 1955 when the yield response was severely limited by low rainfall. Demand curve 31 for $P_2O_5$ indicates the highest level of demand. It was derived from a 1953 experiment on Webster soil when rainfall was adequate. The high level of nitrogen (N = 240 pounds) also shifted demand curve 31 to the right. A high level of demand also is depicted by curve 9. It was estimated from a 1953 experiment on Ida soil in Iowa. Moisture generally was sufficient in 1953, and the soil gave a significant yield response to use of nitrogen and $P_2O_5$.

The curves depicting the highest level of demand, 9 and 31, are the least elastic (fig. 20b). The very elastic curves are those indicating the lowest level of demand, 33 and 34. The flatter slopes of curves 33 and 34 also contributed to the

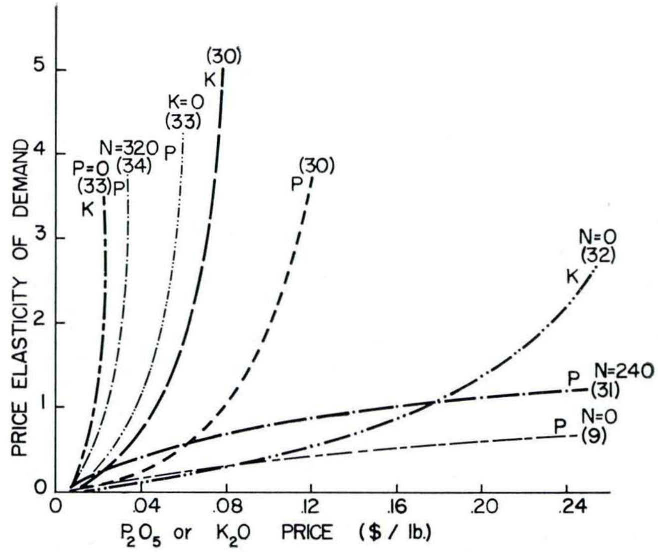


Fig. 20b. Price elasticity of short-run static $P_2O_5$ and $K_2O$ demand illustrated in fig. 20a.

high elasticity. Some of the difference is due to the restraints imposed by the square root form on the elasticities of curves 9 and 31. The difference, however, is attributed mainly to the conditions under which the functions were estimated.

The elasticities of the $P_2O_5$ and $K_2O$ demand curves are greater and more divergent than the elasticities of demand for nitrogen illustrated in fig. 19b. Much of the difference in the magnitude is due to the lower levels of demand for $P_2O_5$ and $K_2O$. For example, five demand curves for $P_2O_5$ and $K_2O$ intersect the price axis below 20 cents. But only one demand curve (34) for nitrogen intersects the price axis below 20 cents.

LONG-RUN DEMAND

Figure 21a illustrates the long-run demand for nitrogen. Factors other than nitrogen (i.e., $P_2O_5$ and $K_2O$) are not fixed as in figs. 19a and 20a but are allowed to vary as the price of nitrogen changes. Figure 21a also includes demand curves from production functions 35 to 38 which contain only one variable input. This allows comparisons of demand curves and elasticities among areas, etc., without the additional interpretation resulting from fixed factor levels.

From fig. 21a, we may observe the effects of moisture and soil type on demand. Production functions 9, 31 and 32 were estimated in 1953 in Iowa. Since the rainfall was somewhat uniform among these experiments, the level of demand differs mainly because of soil type. Demand curve 9 from Ida soil data depicts one of the highest demands, and curve 32 from Carrington data depicts one of the lowest demands. The elasticities of these curves display more uniformity, however (fig. 21b).

The effect of moisture is apparent from production functions 32 and 33 estimated in 1953 and 1955, respectively, on Carrington soil. The demand curve for nitrogen is indicated in fig. 21a

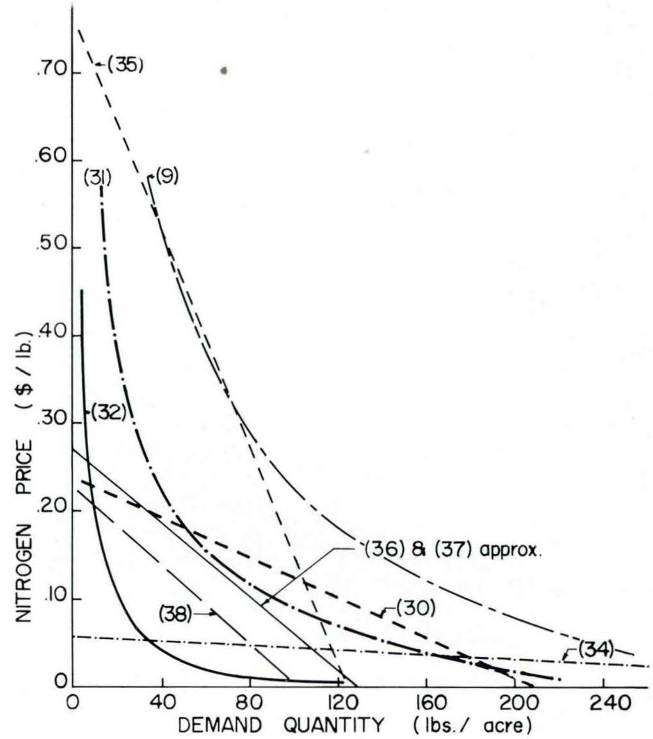


Fig. 21a. Long-run static nitrogen demand, including short-run static nitrogen demand from production functions with nitrogen the only input. The corn price is $1.10 per bushel.

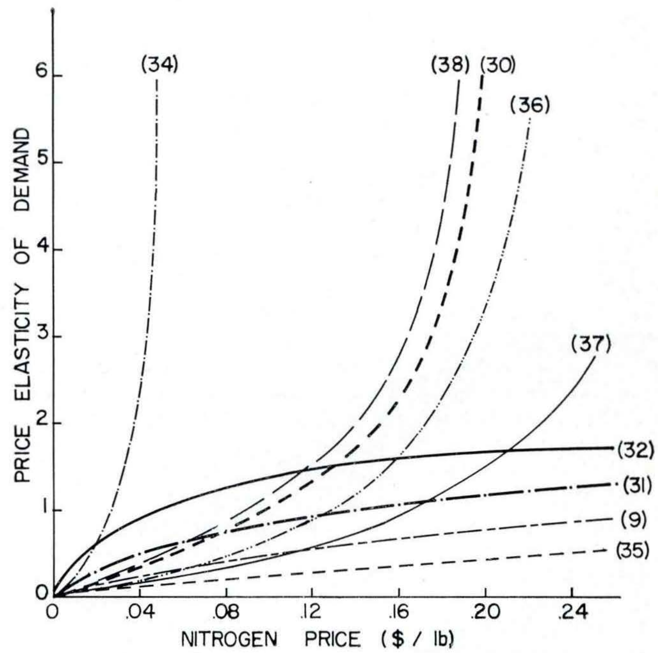


Fig. 21b. Price elasticity of long-run and short-run static nitrogen demand illustrated in fig. 21a.

for the year 1953 only. In 1955, nitrogen gave no response because of low rainfall. The demand for nitrogen in 1955 was essentially zero.

In general, the Iowa functions depict a greater demand for nitrogen than do the other functions—except function 35. Demand curve 32 from Iowa data indicates a very low demand, however. It is impossible to generalize about the level of demand of each production unit for nitrogen simply because it lies within some area.

The slope as well as the level of demand relates to the soil fertility and moisture conditions. The two quadratic forms displaying the greatest and least slopes are curves 35 and 34 in fig. 21a. Demand curve 35 was estimated on soil with sufficient nutrients other than nitrogen, but with limited moisture. The first units of nitrogen gave a large yield response, but because of insufficient moisture, the marginal product declined rapidly. The flattest demand curve (34) was estimated on heavy Wisner soil. Because the initial nitrogen level in the soil was high in relation to the available moisture, the first units of nitrogen added little to the yield. The marginal product remained almost constant as more nitrogen was applied because of the adequate amounts of other nutrients and the moisture-holding capacity of the heavy soil. These results conform with the general observation from fig. 21a that the demand curves denoting the largest quantity at a given price also decline most sharply in slope. The possible reason is: fertile soils, such as those represented by curves 30 and 34, which do not exhibit a large initial response to nitrogen fertilizer sustain some response, with application of greater amounts of nitrogen, because of the high levels of other nutrients and the mositure-holding capacity of the soil.

The elasticity of the low, flat demand curve (34) is very high (fig. 21b). Aside from curve 34, all the long-run demand curves in fig. 21a display considerable uniformity for prices ranging from very low to the level of 13 cents per pound. Figure 21b illustrates a pattern similar to the pattern of short-run elasticities in fig. 19b. The long-run demand curves are more elastic, however. An increase in the nitrogen price results in a greater decrease in the quantity in the long run since other factors may be substituted for nitrogen. If no interaction between nutrients is present, the long-run and short-run demand curves and elasticities are identical.

The demand curves derived from Iowa data appear to be less elastic than those from other areas. Much of the difference is due to the algebraic form at higher nitrogen prices. Comparisons are more realistic at the mid-range of nitrogen prices. Considering only the six demand curves with the lowest elasticity, every other one was derived from Iowa data. The differences in elasticities are perhaps better explained by soil and moisture conditions than by areas. Demand elasticity tends to be lowest for soils which are low in nitrogen and where rainfall and other fertilizer elements are plentiful.

The level of long-run demand for $P_2O_5$ and $K_2O$ illustrated in fig. 22a is somewhat lower than the long-run demand for nitrogen depicted in fig. 21a. Figure 22a also indicates that the demand for $K_2O$ is less than the demand for $P_2O_5$. In several instances, $P_2O_5$ and $K_2O$ were included in the controlled experiments from which the production functions were derived but did not give signifi-

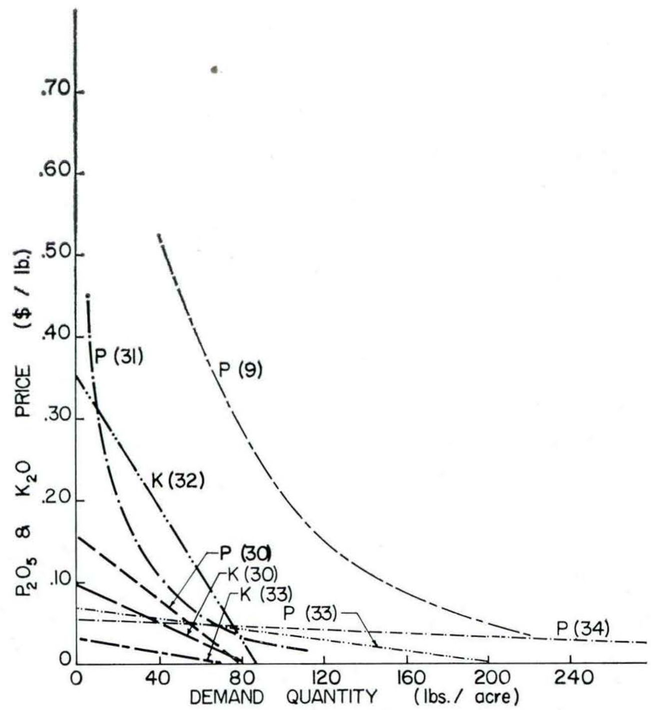


Fig. 22a. Long-run static $P_2O_5$ and $K_2O$ demand. The corn price is \$1.10 per bushel.

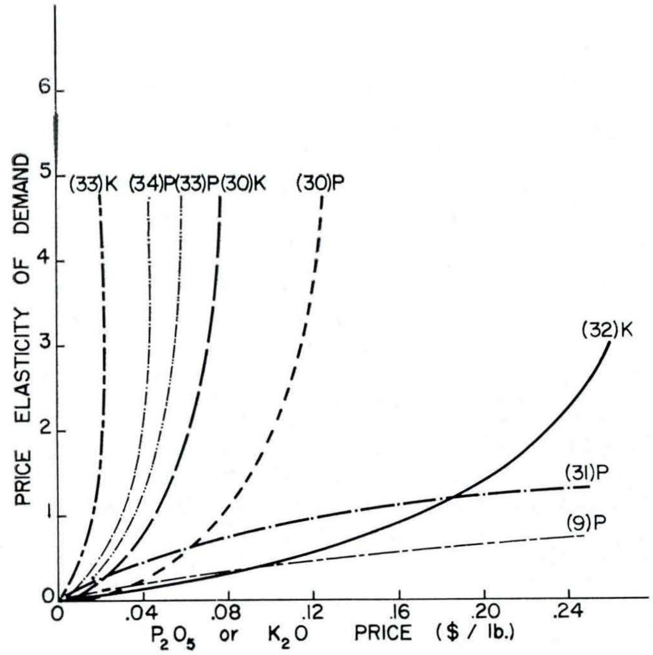


Fig. 22b. Price elasticity of long-run static $P_2O_5$ and $K_2O$ demand illustrated in fig. 22a.

cant responses. The $P_2O_5$ and $K_2O$ variables which were omitted from the functions in such instances represent a zero demand for the nutrient. Demand curve 30 for Clarion soil in Iowa illustrates the differences in demand levels for the three nutrients in a given year. That is, demand for nitrogen in fig. 21a is greater than for $P_2O_5$ in fig. 22a, which in turn is greater than that for $K_2O$.

All the demand curves except curve 34 in fig.

22a are from Iowa data. The divergent pattern in fig. 22a again suggests the wide variation in demand existing within a given area. Demand curve 32 (K), estimated in 1952, indicates a much larger demand than curve 33 (K), estimated in 1953, although both are for Carrington soil. Demand curve 32 (K) is also far less elastic than curve 33 (K) (fig. 22b). The elasticity of long-run demand for $P_2O_5$ and $K_2O$ tends to be high and divergent. The elasticity is greatest on soils giving little response to fertilizer because of an initially high nutrient level or inadequate moisture. For example, curve 34, estimated on a heavy, rich soil, gave little response to fertilizer, and the elasticity is high. Demand curve 9, estimated on a soil with plentiful moisture and low $P_2O_5$, gave a large response to fertilizer. The elasticity of curve 9 was low whether estimated with a square root or quadratic form.

## CONCLUSIONS AND IMPLICATIONS

Considerable variation exists in the level and elasticity of static demand among and within areas. These differences conform with principles from agronomic theory relating crop response from fertilizer to soil and moisture conditions.

The analysis indicates that static demand is greatest and the function is least elastic where the soil is low in the particular nutrient, but is high in moisture and other nutrients. Where moisture is limited and the soil is highly fertile, static demand tends to be low and very elastic. The implication is that, on the basis of static analysis, a tax or subsidy on fertilizer would result in the greatest percentage change in fertilizer consumption in marginal areas of fertilizer use. To the limited extent that it is possible to generalize about areas from the small sample, a change in the price of fertilizer would have the greatest proportional impact in areas such as the Great Plains. The least percentage change in fertilizer consumption would occur in the Corn Belt and Southeast where response to fertilizer is very large. Of course, the largest **absolute** change in fertilizer consumption likely would occur in areas where fertilizer is presently being used in the largest amounts. It is useful to consider the impact of fertilizer price changes by soils rather than by areas since the analysis indicates that the demand elasticity varies greatly by soil and year within areas.

In the foregoing analysis, the demand for $K_2O$ is more elastic than the demand for nitrogen. Fertilizers are often sold in fixed ratios, and it may not be meaningful to consider independently the demand for a single element. Assuming demand to be independent, however, a fertilizer manufacturer of all three elements likely would find the purchase of $K_2O$ more responsive than that of nitrogen to a lowering of both prices by the same percentage. The demand curve for nitrogen, $P_2O_5$ and $K_2O$ in some fixed ratio would likely be to the right of the demand curve for any one element. It follows that the demand for a fixed ratio of the three elements probably would be less elastic than the demand for any one element.

In the earlier section on logic and assumptions, we found that the price elasticity of static demand with respect to the price of fertilizer or with respect to the price of corn are equal but opposite in sign. Inferences about the response of fertilizer purchases to fertilizer prices also apply to corn prices. For example, a fall in the corn price would be expected to reduce fertilizer purchases proportionately more than corn production. The results of the static analysis also are consistent with the hypothesis that a change in **corn** price has the greatest percentage impact on fertilizer sales in marginal areas, but the greatest absolute impact in traditional areas of corn production.

The analysis indicates that fertilizer demand is more elastic than corn supply. Because of diminishing returns, successive inputs of fertilizer add smaller and smaller increments to corn input. Thus, fertilizer consumption must increase by a larger percentage than corn output in response to a favorable corn price. The impact on the fertilizer industry of a change in the price of corn might be relatively greater than the impact on corn production.

The analysis provides a basis for forming hypotheses of future trends in the demand for fertilizer. If the price of fertilizer falls relative to the price of corn, the largest proportional increase in fertilizer consumption in the short run is likely to occur in marginal areas of fertilizer use. The largest total increase, however, would likely be in areas where fertilizer presently is used in large amounts.

As the fertility level of the soil declines because of cropping and erosion, the demand curve for fertilizer will shift to the right and probably become less elastic. Although the demand for fertilizer will increase, the relative short-run responsiveness of fertilizer consumption to changes in the price of corn or of fertilizer probably will diminish. Introduction of irrigation and other technological improvements also will influence the demand elasticity of fertilizer. To the extent that these technological changes substitute for fertilizer, the fertilizer demand elasticity will increase. To the extent that innovations such as new crop varieties only shift the demand for fertilizer to the right, the fertilizer demand elasticity will decrease. These hypotheses of future trends in fertilizer demand may be tested and revised as additional data and methodological procedures become available.

# APPENDIX

## EQUATIONS FOR SUPPLY, COSTS AND ELASTICITIES — TWO-VARIABLE FACTORS

In the following pages, the equations for supply, costs and elasticity are given for the quadratic, square root and logarithmic forms of the production function. Only equations for two-variable factors are shown. The frequent occurrence of these functions, and the somewhat troublesome nature of the computations of supply, etc., equations suggest the convenience of having these equations readily available.

The "short-run" equations (only $X_1$ variable) for supply, demand, costs and elasticities are included in the text. These equations easily are generalized for only $X_2$ variable and for production functions containing only one independent variable. This appendix contains only "long-run" equations; i. e., both factors are variable in the production process. The supply equation, for example, allows both $X_1$ and $X_2$ to vary in least-cost proportions. The demand equation for $X_1$ allows $X_2$ to vary, and the demand equation for $X_2$ allows $X_1$ to vary.

### Quadratic Formulas

THE PRODUCTION FUNCTION

$$(39)\quad Y = b_{00} + b_{10}X_1 + b_{20}X_2 + b_{11}X_1^2 + b_{22}X_2^2 + b_{12}X_1X_2$$

THE SUPPLY EQUATION, Y, AND ELASTICITY OF SUPPLY, $E_s$

$P_1$ = price of $X_1$ $\qquad$ $P_2$ = price of $X_2$

$P_y$ = price of Y $\qquad$ $C_0 = 4b_{11}b_{22} - b_{12}^2$

$$C_1 = \frac{2b_{22}P_1 - b_{12}P_2}{C_0} \qquad C_2 = \frac{b_{20}b_{12} - 2b_{10}b_{22}}{C_0}$$

$$C_3 = \frac{2b_{11}P_2 - b_{12}P_1}{C_0} \qquad C_4 = \frac{b_{10}b_{12} - 2b_{20}b_{11}}{C_0}$$

$$C_{00} = b_{00} + b_{10}C_2 + b_{20}C_4 + b_{11}C_2^2 + b_{22}C_4^2 + b_{12}C_2C_4$$

$$C_{10} = b_{11}C_1^2 + b_{22}C_3^2 + b_{12}C_1C_3$$

$$(40)\quad Y = C_{00} + \frac{C_{10}}{P_y^2}$$

$$(41)\quad E_s = \frac{-2\dfrac{C_{10}}{P_y^2}}{C_{00} + \dfrac{C_{10}}{P_y^2}}$$ (the denominator is the supply equation)

TOTAL VARIABLE COST, TVC, AND AVERAGE VARIABLE COST, AVC

$$(42)\quad X_1 = \frac{C_1}{P_y} + C_2 \qquad (43)\quad X_2 = \frac{C_3}{P_y} + C_4$$

$$(44)\quad TVC = X_1P_1 + X_2P_2 \qquad (45)\quad AVC = \frac{TVC}{Y}$$

"Y" in the AVC equation is the supply quantity for a given $P_y$ in the supply equation. Hence, TVC and AVC are functions of $P_y$.

Although all inputs in production function 39 are variable, we do not use the terms total cost (TC) or average total cost (ATC). Production function 39 is essentially a short-run concept. Some inputs not included in the function are fixed. The cost of these fixed inputs can be added to TVC to form the TC. The ATC can be found by dividing TC by Y.

THE DEMAND EQUATIONS, $X_i$, AND ELASTICITY OF DEMAND, $E_d$

$$(46)\quad X_1 = C_2 - \frac{b_{12}P_2}{P_yC_0} + P_1\left(\frac{2b_{22}}{P_yC_0}\right)$$

$$(47)\quad X_2 = C_4 - \frac{b_{12}P_1}{P_yC_0} + P_2\left(\frac{2b_{11}}{P_yC_0}\right)$$

Equations 46 and 47 are equations 42 and 43 rewritten as functions of factor prices. The equations of demand for $X_1$ and $X_2$ fixed are given in the text. In equations 46 and 47, the alternate factor is not fixed at some level but is allowed to substitute for the other in the production process.

$$(48)\quad E_d(X_1) = \frac{P_1\left(\dfrac{2b_{22}}{P_yC_0}\right)}{C_2 - \dfrac{b_{12}P_2}{P_yC_0} + P_1\left(\dfrac{2b_{22}}{P_yC_0}\right)}$$

$$(49)\quad E_d(X_2) = \frac{P_2\left(\dfrac{2b_{11}}{P_yC_0}\right)}{C_4 - \dfrac{b_{12}P_1}{P_yC_0} + P_2\left(\dfrac{2b_{11}}{P_yC_0}\right)}$$

### Square Root Formulas

THE PRODUCTION FUNCTION

$$(50)\quad Y = b_{00} + b_{10}X + b_{20}X_2 + b_{11}X_1^{1/2} + b_{22}X_2^{1/2} + b_{12}X_1^{1/2}X_2^{1/2}$$

THE SUPPLY EQUATION, Y, AND ELASTICITY OF SUPPLY, $E_s$

$$C_0 = 2\left(\frac{P_1}{P_y} - b_{10}\right) \qquad C_1 = 2\left(\frac{P_2}{P_y} - b_{20}\right)$$

$$C_2 = C_0C_1 - b_{12}^2$$

$$C_5 = \frac{8P_1P_2}{P_y^2} - \frac{4(b_{10}P_2 + b_{20}P_1)}{P_y}$$

$$Cx_1 = \frac{b_{11}C_1 + b_{22}b_{12}}{C_2} \qquad Cx_2 = \frac{b_{22}C_0 + b_{11}b_{12}}{C_2}$$

$$(51)\quad Y = b_{00} + b_{10}Cx_1^2 + b_{20}Cx_2^2 + b_{11}Cx_1 + b_{22}Cx_2 + b_{12}Cx_1Cx_2$$

$$(52)\quad E_s = \frac{\left[\begin{array}{l} \frac{2b_{10}Cx_1}{C_2}\left[Cx_1C_5 - \frac{2b_{11}P_2}{P_y}\right] + \\ \frac{2b_{20}Cx_2}{C_2}\left[Cx_2C_5 - \frac{2b_{22}P_1}{P_y}\right] + \\ \frac{b_{11}}{C_2}\left[Cx_1C_5 - \frac{2b_{11}P_2}{P_y}\right] + \\ \frac{b_{22}}{C_2}\left[Cx_2C_5 - \frac{2b_{22}P_1}{P_y}\right] + \\ \frac{b_{12}}{C_2}\left\{Cx_1\left[Cx_2C_5 - \frac{2b_{22}P_1}{P_y}\right] + \right. \\ \left. \qquad Cx_2\left[Cx_1C_5 - \frac{2b_{11}P_2}{P_y}\right]\right\} \end{array}\right]}{b_{00} + b_{10}Cx_1^2 + b_{20}Cx_2^2 + b_{11}Cx_1 + b_{22}Cx_2 + b_{12}Cx_1Cx_2}$$

(The denominator is the supply equation.)

THE DEMAND EQUATIONS, $X_i$, AND THE ELASTICITY OF DEMAND, $E_d$

Equations 53 and 54 are the demand equations for $X_1$ and $X_2$, respectively, when the alternative factor is not fixed but is allowed to vary in the production process. The price elasticities of demand of the square root demand equations are equations 55 and 56.

$$(53)\quad X_1 = Cx_1^2 \qquad (54)\quad X_2 = Cx_2^2$$

$$(55)\quad E_d(X_1) = \frac{4P_1C_1}{P_y\,(b_{12}^2 - C_0C_1)}$$

$$(56)\quad E_d(X_2) = \frac{4P_2C_0}{P_y\,(b_{12}^2 - C_0C_1)}$$

TOTAL VARIABLE COST, TVC, AND AVERAGE VARIABLE COST, AVC

$$(57)\quad TVC = X_1P_1 + X_2P_2 \qquad (58)\quad AVC = \frac{TVC}{Y}$$

"Y" in the AVC equation is the supply quantity for a given $P_y$ in the supply equation. The total cost can be found by adding the fixed costs to total variable cost.

**Logarithmic Formulas**

THE PRODUCTION FUNCTION

$$(59)\quad Y = b_0X_1^bX_2^c$$

THE SUPPLY EQUATION, Y, AND ELASTICITY OF SUPPLY, $E_s$

$$C_0 = \frac{b}{1-(b+c)} \qquad C_1 = \frac{c}{1-(b+c)}$$

$$C_2 = \frac{1-c}{1-(b+c)} \qquad C_3 = \frac{1-b}{1-(b+c)}$$

$$C_4 = \log b_0 + \log b - \log P_1$$

$$C_5 = \log b_0 + \log c - \log P_2$$

$$\text{Log } Y = \log b_0 + C_0C_4 + C_1C_5 + (C_0 + C_1)\log P_y$$

$$(60)\quad Y = \text{antilog } (\log Y)$$

$$(61)\quad E_s = \frac{b+c}{1-(b+c)}$$

TOTAL VARIABLE COST, TVC, AND AVERAGE VARIABLE COST, AVC

$$\log X_1 = C_2C_4 + C_1C_5 + \frac{\log P_y}{1-(b+c)}$$

$$(62)\quad X_1 = \text{antilog } (\log X_1)$$

$$\log X_2 = C_0C_4 + C_3C_5 + \frac{\log P_y}{1-(b+c)}$$

$$(63)\quad X_2 = \text{antilog } (\log X_2)$$

$$(64)\quad TVC = X_1P_1 + X_2P_2 \qquad (65)\quad AVC = \frac{TVC}{Y}$$

"Y" in the AVC equation is the supply quantity for a given $P_y$ in the supply equation. To find the total cost, add fixed costs to equation 64.

THE DEMAND EQUATIONS, $X_i$, AND ELASTICITY OF DEMAND, $E_d$

Since $C_4 = f(P_1)$, equation 62 may be used as the demand equation for $X_1$ when $X_2$ is variable. Similarly, because $C_5 = f(P_2)$, equation 63 may be used as the demand equation for $X_2$. The price elasticities for $X_1$ and $X_2$ are

$$(66)\quad E_d(X_1) = \frac{c-1}{1-(b+c)} = -C_2$$

$$(67)\quad E_d(X_2) = \frac{b-1}{1-(b+c)} = -C_3\ .$$