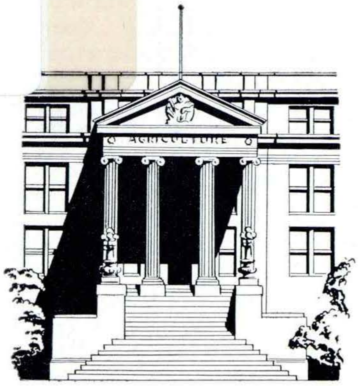

Programming Procedures For Farm and Home Planning Under Variable Price, Yield And Capital Quantities

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SUMMARY

This study was initiated to develop and improve linear programming models which might have greater application to the planning of individual farm businesses. The extension services of most states have inaugurated widespread farm and home planning projects. The increased business development and commercialization of farming causes increasing importance to fall on this type of planning. With the computational facilities available to both county extension personnel and farmers, the magnitude of variables and quantities which can be considered in planning are not great. The development of linear programming planning techniques and the availability of high capacity computers stands to allow planning for individual farmers by this method. It is possible for farm families to keep adequate farm and home records and supply certain other information, allowing several plans to be developed by high speed computers at a reasonable cost. This step can already be accomplished for simple static programs. This study has been conducted, however, to develop and apply procedures which allow analysis of stability of plans and farm and household interdependence in plans. The methods are developed to an extent that they might later be taken over in extension applications with programming services provided at a cost to individual farmers.

Other specific objectives of this study are: (1) to determine optimum resource use for an individual farm within the planning framework of the farm family; (2) to determine the range of yields over which particular cropping programs are optimum; (3) to develop price maps showing the range of price ratios over which particular livestock programs are optimum; and (4) to develop and apply dynamic linear programming procedures which consider the interdependence of the farm business and household in planning and which provide sequences of optimum plans over time as capital is accumulated.

Data used in this study were directly transferred from records kept on a case farm selected for study or based on these records and the values and goals of the farm operator and his family. Data from feed yield and labor records were in the form required for linear programming. Total fixed or overhead expense items for the farm were imputed to each enterprise by a method of successive approximations. Prices for commercial feed, fertilizer and other cost items were based on current prices; farm produce prices were expectations made by the farm operator, after checking with outlook information.

For the years 1951-56 inclusive, the cropping program on the case farm had included an average of 98 acres of corn, 66 acres of oats, 5 acres of soybeans and 42 acres of rotated meadow. (A total of 211 tillable acres was available during this 6-year period.) Average livestock production had included 45 hog litters, 61 short-fed heifers and a poultry laying flock. Based on actual prices received in 1956, preliminary programming solutions indicated that a 20-percent increase in profits could have been realized if resources had been allocated differently. This initial analysis was consistent with the farm operator's beliefs and anticipations, after he had examined initial calculations and programs. Differences between the actual farm plan in 1956 and an optimum plan differed most with respect to livestock production. The optimum computed plan specified 80 hog litters, 16 short-fed heifers and no poultry.

Additional programming solutions were then made which considered more alternatives than the farmer had previously included. Long-fed steers were added as an alternative beef-feeding enterprise. In addition, six crop rotations were defined with three fertilization levels for each. Various situations were examined to determine the profit effects of borrowing more capital, hiring labor and renting pasture. Also, a model was applied to allow a sequence of farm plans expanded over an 8-year period, from capital accumulation.

In comparison with the actual farm plan in 1956, programming solutions consistently indicated that the following changes and conditions should prevail if profits were to be maximized: (a) hog production should be increased to 80 litters and (b) a CCOMM rotation provides the least-cost source of forage when cattle are fed but a CCSb rotation provides the most profitable source of cash crops. Programming solutions for varying crop yields showed that these rotations were optimum over a yield range from 14 percent below to 50 percent above the basic yield level considered.

All but one of the farm situations programmed restricted livestock production to homegrown feeds. With this restriction and no borrowed capital, the profit-maximizing plan included 58 acres of $CCOMM_3$, 214 acres of $CCSb_3$ and 70 hog litters. With addition of borrowed capital, the plan was profitably expanded to 80 hog litters and 87 short-fed heifers. Land use was correspondingly changed to 143 acres of $CCOMM_3$ and 129 acres of $CCSb_3$.

The supply of homegrown feeds could be augmented profitably by renting 70 acres of improved pasture. This alternative source of forage permitted increased supplies of both forage and grain. Some acres previously in meadow could be shifted to corn production. The resulting optimum plan for this situation was 80 hog litters, 106 head of short-fed heifers, 92 acres of $CCOMM_3$ and 180 acres of $CCSb_3$. Correspondingly, $14,731 borrowed capital was required.

Price maps were developed with corn at $0.80, $1.00 and $1.20 to illustrate optimum livestock organization when selling prices for beef and hogs were varied. With corn at $0.80 per bushel, no livestock should be produced in the price area where beef is less than $20.13 per hundredweight and the hog price is less than $9.50 per hundredweight. Optimum livestock production for beef selling prices less than $21.74 per hundredweight and hog prices greater than $9.50 per hundredweight is 80 hog litters. Or, the "price stability area" for a plan with 80 litters of hogs and no feeder cattle is 0 to $21.74 for beef and $9.50 and above for hogs. Price boundary lines for other livestock combinations are similarly illustrated by the price maps presented in the text.

The dynamic linear programming model assumed an initial supply of operating capital of $4,813. This

amount of tenant capital was sufficient for crop production and 45 hog litters. Capital supplies and corresponding farm plans for subsequent years depended on additions to capital forthcoming from farm profits and the amount of household expenditures required in the preceding year. Annual plans changed gradually between years, but the optimum plan of any one year was dependent on the optimum plan of other years, with maximum discounted returns for the 8 years serving as the criterion.

The dynamic programming solution provided several important guides for farm and home planning activities. Optimum farm plans for successive years were dependent on the amount of family living expenses required in particular years. Projected living expenditures for the farm family affected the manner in which capital and other resources were allocated among the various crop and livestock enterprises and practices.

This study indicated several unique problems inherent in application of programming for individual farm families in their actual decisions of business and household investment plans. Plans, based on programming techniques, can be developed only if the family has kept sufficient farm and home records. These records must be kept in detail, since difficulties otherwise arise in imputing total annual costs to individual enterprises. Fuel, oil and grease, for example, may be purchased in bulk lots, with only total costs entered in the farm records. Specific enterprise costs such as veterinary fees often are recorded without indication of the enterprise involved. If programming applications are to have widespread use for planning of actual farms, record forms need to be developed accordingly. These forms should provide a means of recording business transactions unique to each enterprise on the farm. General cost items of machinery repair, fuel, etc. could be imputed at regular intervals throughout the year. An auxiliary aid would be an educational process to explain the techniques of programming. This step would provide the farmer with an understanding of the "whats and whys" of required information and simplify or lessen the "footwork" involved in programming of individual farms.

Much consultation with the farm operator is required to develop and interpret input-output data for each enterprise. Consultations with the farm family also are necessary to interpret family living expenditures and to project expenditures into the future if dynamic programming procedures are to be used.

Programming Procedures for Farm and Home Planning Under Variable Price, Yield and Capital Quantities[1]

BY LAUREL D. LOFTSGARD, EARL O. HEADY AND H. B. HOWELL

Farming has become increasingly commercialized and complex. The restriction on magnitude of profits has become less one of acquaintance with farm practices and more one of properly fitting all practices and production alternatives into an integrated plan, consistent with the scarce supplies of several resources and with market prices. This development has been recognized by state and federal extension services. All states now have farm and home planning or development programs. The major objective of these programs is to help farm families in their planning activities.

Farm planning, however, if it gives recognition to all appropriate farming practices and investment alternatives and the relative scarcity of different resources, is a complicated process. County extension personnel and farmers seldom have the computing facilities or skill to consider fully all alternatives which are relevant in determining optimum farm and family plans. Probably no individual or organization has them. Recent developments in mathematical planning techniques, however, promise to make computational facilities for these purposes available at a reasonable cost. It is entirely possible that individual farmers can have several plans computed with linear programming techniques and select the one which corresponds best with their preferences, attitudes toward risk, family responsibilities, etc. It is possible that these opportunities can be provided by extension services or private firms. But before the opportunity is fully developed, it is desirable that appropriate record forms be developed for and kept by farmers. It is also desirable that programming models be developed beyond those which simply provide a static plan as a "picture of the future." There is need to develop procedures which indicate the range of prices and yields over which a particular plan is optimum, "tie together" the farm and the household in actual computations leading to quantitative plans and develop the proper sequence of plans to be followed over time as capital accumulation takes place.

As a step toward further adapting linear programming techniques to actual use in farm planning, these problems are examined in this study. The study was made especially to adapt and to develop programming methods which might have widespread use in actual farm planning activities of the extension service.

OBJECTIVES

The general objective of this study is to develop and apply dynamic linear programming methods to individual farm situations. It attempts to combine both the farm business and the farm household into a single programming format. It is hoped that the models and procedures developed might have subsequent widespread application in farm and home planning programs such as those conducted by the extension service. Also, the study is partly an attempt to determine the feasibility of providing individual farmers with linear programming plans, when the plans are developed directly from record data of the particular farm.

The study uses data unique to a particular farm. Specific objectives are:

1. To analyze input-output data from records of an individual farm by programming optimum enterprise combinations and resource use, based on resource restraints of the actual farm.

2. To determine optimum resource use or allocation for this farm by considering only those alternative resource uses consistent with the goals and planning framework of the farm family. Family characteristics that relate to consumption needs, discount rates of future returns and managerial abilities are included in the programming model.

3. To specify shifts in optimum livestock production associated with different selling prices for hogs and beef. These prices are varied over given ranges, while corn price is held constant at three different levels.

4. To establish a land-use system, in conjunction with optimum livestock production, that indicates the range over which yields for particular crop rotations are optimum.

5. To compute a dynamic farm plan for successive years, including household expenditures as an activity

[1] Project 1328 of the Iowa Agricultural and Home Economics Experiment Station.

in the programming model. Annual expansion in these plans depends on new capital investment forthcoming from farm returns of the previous year. (Living expenses, taxes, insurance and new investments are subtracted from annual farm income. Remaining funds are available for added investment in the farm business during the next year.)

ANALYTICAL PROCEDURE

The analytical technique used to develop recommendations for optimum farm and home plans is linear programming. Within its assumptions of physical relationship in production and consumption, this method of analysis insures optimum resource allocation among the enterprise alternatives and within the limitations specified by the family's planning framework. Implicit assumptions, logic and procedure for applications of linear programming are explained and illustrated in several sources and are not detailed here.

The ordinary simplex method of linear programming does not use multivalued input-output coefficients in the programming problem. Modified simplex solutions have been developed and are used in this study. Also, solutions are computed with varying prices to specify shifts in optimum resource use as selling prices for hogs and beef are varied while corn price is held constant at three different levels. This analysis requires a variable-price solution for each level of corn price.

Under the simplex modification allowing variable input-output coefficients, the magnitude of variation in production coefficients associated with each pattern of optimum resource use is determined. In particular, the method is used to establish a stable cropping program. That is, the optimum cropping program first is determined for the expected or average yields shown later in table 4. Then, crop yields are varied in the range from 50 percent below to 50 percent above these yield levels to determine whether the optimum cropping program (determined from average yields) changes as individual crop yields change. A stable cropping program thus is defined as one that remains optimum over specified ranges of yield variability.

In the linear programming process, all livestock and cropping alternatives are considered simultaneously to give the over-all optimum plan. The resulting cropping system often is dependent on the kind of livestock production included in the final plan. For example, a plan with 100 head of feeder cattle and 20 hog litters requires more forage in the crop rotation than a plan with 30 head of feeder cattle and 80 hog litters. Both plans may specify the same rotations but different proportions of each. Hence, selecting a stable cropping program involves choice of rotations and, when two or more rotations are chosen, the number of acres in each rotation.

The procedure for selecting a stable cropping program in this study involves two phases. First, the modified simplex method of linear programming with variable crop yield coefficients was used to establish the choice of rotations. Second, the number of acres in each rotation was determined by analyzing other programming results with the farm operator. For example, optimum livestock production and field layouts provided guides for specifying number of acres in each crop.

A third modification of the simplex solution used in this study concerns variable capital restrictions. This method shows all patterns of optimum resource use as capital is varied from zero level to an unlimiting amount. A variable capital solution was used to establish yearly plans on an accumulating capital supply (i.e., when borrowing extra capital at the outset is not considered). The variable capital solution gives different plans for one point in time, however, and must be supplemented with budgeting techniques when a series of yearly plans is desired. Linear programming models that involve dynamics give optimum plans directly for different points in time.

Additional Considerations in Programming

Problems of capital rationing are of particular concern in farm planning and linear programming. A farm plan ordinarily can be expanded or intensified, with an increase in indicated profits, if more capital is made available. Farm firms do not employ borrowed capital as freely as business firms, however, and fund restrictions must be considered in planning. It is, of course, possible to assume that added capital can be borrowed at market rates of interest. It is relatively easy to establish a commercial interest charge on borrowed capital. The farmer's subjective discount rates, the rates at which he would be willing to borrow, may not coincide with the actual cost of borrowing money, however. Before capital borrowing is considered in the planning framework, the farmer must quantify the rate of return necessary to employ borrowed capital. If this rate, set by the farmer, is lower than actual borrowing charges, the farm plan is limited by external capital rationing. Returns to borrowed capital must be greater than commercial credit rates if funds are limited by internal capital rationing, an expression of the farmer's aversion to risk. For the case farm used in this study, the operator's subjective discount rates on expected capital return were higher than commercial interest rates. Hence, the use of borrowed capital was subject to internal rationing. Additional capital would be borrowed only if the returns were greater than or equal to the rate set by the farmer.

These conditions, specifying the terms under which capital might be borrowed, can be expressed in an algebraic programming format. We suppose profit maximization subject to resource supplies and other production restraints $b_1, b_2 \cdots b_m$ where $x_1, x_2 \cdots x_n$ are alternative production processes. The amount of each resource required for one unit of each production process is expressed by the requirements coefficient, a_{ij} ($i = 1 \cdots m$, $j = 1 \cdots n$). The coefficients $c_1, c_2 \cdots c_n$ are the unit returns forthcoming from each production process. In terms of these notations, the linear programming equations are expressed as:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\leqslant b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\leqslant b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\leqslant b_m \end{aligned}$$

where $x_j \geqq 0$ and the profit function

$$Z = c_1x_1 + c_2x_2 \cdots c_nx_n$$

is maximized.

In a linear programming model of the type discussed here, capital borrowing is expressed as an x_j process. Assuming $1 units and a subjective discount rate of $0.11 per $1, the corresponding c_j for the capital borrowing process is equal to —$0.11, indicating that each $1 borrowed for production must return at least $0.11. Capital will be borrowed only if it returns more than 11 percent. This particular discount rate may apply only to the first $5,000 increment of borrowed capital, however. That is, borrowed capital in excess of $5,000 may require higher returns if the farm operator is to continue borrowing. This restraint is expressed in the model by including a b_i equal to 5,000. Assuming $i = 1$ for this restraint and $j = 2$ where x_j is the capital borrowing process, the equation expressing this condition becomes:

$$0 \cdot x_1 + 1 \cdot x_2 + 0 \cdot x_3 + \cdots + 0 \cdot x_j \leqslant 5{,}000.$$

Or, all a_{ij}'s for x_j's $(j \neq 2)$ are equal to zero, and the maximum amount of x_2 (capital borrowing) cannot exceed 5,000.

Similarly, other capital restraints can be included in the programming model. If, for example, the second $5,000 increment of borrowed capital is subject to discount rates of 16 percent, a new row or equation is added to restrain borrowing. In this case, let x_3 be another capital borrowing process where the return must be at least $0.16 per $1. The new c_j is $c_3 =$ —$0.16. With $b_1 = b_2$ representing the new restraint, the corresponding equation is:

$$0 \cdot x_1 + 0 \cdot x_2 + 1 \cdot x_3 + \cdots + 0 \cdot x_j \leqslant 5{,}000.$$

All a_{ij}'s for x_j's $(j \neq 3)$ are zero, and the maximum amount of x_3 cannot exceed 5,000.

These procedures are used in developing the plans which follow. The subjective discount rates and the corresponding borrowing restrictions are those specified by the operator of the case farm for which the plans are derived.

CASE FARM SITUATION

Farm Size and Lease

The case farm selected for the programming analysis which follows is located in central Iowa on Nicollet and Clarion soils. The farm consists of 320 acres of which 272 acres are cultivated. Fence lines, farmstead, permanent pasture, roads and a railroad account for the remaining 48 acres. Of the total farm acres, 240 acres are adjoining, and 80 acres are located 1 mile from the farmstead. The entire farm lies on relatively level land; maximum slope does not exceed 3 percent.

The farm is currently operated under a father-son agreement. Although no formal lease contract has been made, the operation is actually a 50-50 livestock-share lease whereby the son furnishes all machinery and labor and half the operating capital; the father furnishes the land and other half of the operating capital. Farm profits are divided on a 50-50 basis. The many contacts made during the course of this study were with the son, or farm operator. The father, or land owner, is equally active in managerial decisions, however. Assumptions used in programming were agreed on by both parties. On the whole, there is general agreement between father and son as to managerial decisions, capital rationing, adapting to change and long-run farm goals. For example, the agreements or nature of the lease were not conducive for the tenant to engage in exploitive farming that would be economically profitable in the short run.

Labor Supply

During 1956 and 1957, the operator kept a daily labor record for all farm activities. Daily labor requirements for each crop and livestock enterprise were recorded in twelfths of an hour. Since some labor is used on jobs not identifiable with any particular enterprise, a category for service labor was entered in the daily records. Jobs classified under service labor included such items as grading the driveway, spraying weed patches, snow removal, repair on machine shop and rock removal.

Labor records gave two sources of information for this study—the labor requirements per unit of production and the labor supply available for production. Since these records were kept for 2 years, actual labor data used for programming are based on 2-year averages.

Table 1 summarizes the case farm labor supply for each month and for monthly groups.[2] The total labor supply for each month is the sum of all labor requirements as given in the records. Thus, if the labor requirements in July are 100 hours for hogs, 10 hours for beef, 25 hours for poultry, 30 hours for corn, 40 hours for soybeans, 80 hours for oats, 37 hours for haying and 3 hours for service labor, the labor supply for this month is assumed to be the sum of these increments, or 325 hours. This assumption is based on the operator's reaction to working more or less hours than he actually did. The operator stated that he was operating with maximum labor loads each month and could not devote more hours to farming. Hence,

[2] Labor supplies are grouped in units of 2 and 3 months each to establish seasonal labor supplies and permit use of seasonal labor requirements. This method allows more flexibility than a monthly analysis.

TABLE 1. CASE FARM LABOR SUPPLY IN MAN-HOURS PER MONTH AND PER MONTHLY GROUP.[a]

Month	Total labor supply	Total service labor requirements	Total labor available for specific enterprises: Per month	Total labor available for specific enterprises: Per monthly group
Dec.	272	35	237	
Jan.	260	20	240	676
Feb.	221	22	199	
March	234	25	209	
April	267	59	208	417
May	324	..	324	
June	331	..	331	655
July	325	3	322	
Aug.	301	33	268	590
Sept.	248	97	151	
Oct.	275	55	220	585
Nov.	263	49	214	
Total	3,321	398	2,923	2,923

[a] Based on 1956-57 averages from labor records kept by the farm operator.

labor restrictions were quantified from the labor records.

The second column in table 1 lists the total supply of family labor available in each month; monthly service labor requirements are indicated in the third column. Total hours of labor actually available for specific production in various time periods are obtained by subtracting service labor hours from original supplies. The resulting figures are shown by months in the fourth column and by monthly groups in the fifth, or last, column. Hours of labor available in monthly groups comprise the labor restrictions used in programming farm plans. Likewise, labor requirements for each enterprise were formulated by monthly groups.

In addition to the labor supply in table 1, labor may be hired at $1 per hour. This condition was established by the farm operator as "typical" in his farm operation. Hence, labor hiring was included in the programming situations; extra labor would not be hired unless it returned at least $1 per hour.

Capital Situation

Amount of operating capital available for production purposes was determined by a method similar to the one used for labor. That is, total capital requirements for the current farm plan represented the available capital supply for programming situations. This amount of operating capital was $13,173. These funds provided the basic capital restriction, but farm plans could be expanded by borrowing capital and, in the long run, by capital accumulation.

The capital requirements per unit of production for each enterprise are based on annual cash outlay. Value of homegrown feeds, for example, is not included in the capital requirements since this cost is merely a transfer within the firm. Cost of homegrown feeds is subtracted, however, when computing a final return figure.

CAPITAL RATIONING

Subjective discount rates for using borrowed capital on the case farm are considerably higher than commercial loan rates. The farm operator expressed his risk aversion for borrowing capital by establishing increasing discount rates for added increments of borrowed funds. Theoretically, the magnitude of discount rates depends on the degree of uncertainty or imperfect knowledge of future returns. Custom and social standing, however, also may influence personal attitudes for borrowing money. If the farm family attaches adequate satisfaction to present income and has a strong aversion to borrowing capital, its subjective discount rates may be substantially higher than rates actually expressing its risk anticipations. Regardless of motivation, subjective discount rates that exceed commercial loan rates create a situation of internal capital rationing. That is, the required returns from the amount of borrowed capital are restricted by internal conditions in the firm.

The total rates established for borrowing extra capital on the case farm as quantified by the farm operator are as follows:

1st $5,000 must yield at least 11 percent return,
2nd $5,000 must yield at least 15 percent return and
3rd $5,000 must yield at least 26 percent return.

In other words, the operator indicated that he would borrow added funds only if he felt "subjectively assured" that he could obtain returns of this level. Because of risk aversion he would be unwilling to borrow at 6 percent if prospects were only for an 8-percent return.

CAPITAL ACCUMULATION

In addition to borrowing extra capital at the outset, the farm plan could be expanded from year to year by investing profits from the preceding year. Returns available for investment are the funds remaining after subtracting fixed charges of depreciation, annual living expenses, taxes, insurance and other necessary annual expenditures. For example, an additional $500 could be invested in next year's farm plan if returns from this year's plan were $6,500 and fixed expense items were $6,000. Investing capital, generated within the farm and exceeding necessary annual expenses, each year eventually may provide a farm plan with resource limitations other than capital.

Prices

A major emphasis in this study is on the derivation of optimum plans under different price levels for hogs and beef. Constant prices, at current levels, are used for factor inputs such as seed, fertilizer and commercial feed. Other prices used here are predictions made by the farm operator. Since this study is designed to use programming techniques within an individual farmer's planning framework, it is essential that prices, as well as other coefficients, be those determined by the farm operator from the actual farm. The prices used in this study are listed in table 2.

TABLE 2. PRODUCT PRICES USED IN COMPUTING OPTIMUM FARM PLANS.

Item	Unit	Buying price (dollars)	Selling price (dollars)
Seed and fertilizer			
Corn	bushel	10.00	....
Oats	bushel	0.77	....
Soybeans	bushel	2.60	....
Legume and grass mixture	acre	5.62	....
Sorghum	cwt.	24.00	....
Nitrogen (N)	pound	0.14	....
Phosphate (P_2O_5)	pound	0.10	....
Potash (K)	pound	0.05	....
Feed and grain			
Corn[a]	bushel	1.10	1.00
Oats	bushel	0.52	0.52
Soybeans	bushel		2.00
Sorghum	cwt.	1.50	1.50
Hay (baled)	ton	16.00	16.00
Hog supplement	cwt.	4.50	....
Cattle supplement	cwt.	5.00	....
Commercial poultry feed	cwt.	4.93	....
Livestock and livestock products			
Good-to-choice steer calves[a]	cwt.	25.00	....
Medium-to-good heifer calves[a]	cwt.	22.00	
Choice fat cattle[a]	cwt.	....	27.00
Good heifers (800 lbs.)[a]	cwt.	....	26.00
Pork[a]	cwt.		14.30
Sexed chicks	each	0.55	
Eggs	dozen	....	0.34
Non-laying pullets	pound	....	0.21
Cull hens	pound	....	0.11

[a] Selling prices for corn, cattle and hogs were varied in the variable price plans shown later in figs. 3, 4 and 5.

Plans programmed for variable prices eliminate rigidity in price expectations or assumptions. While a fixed buying price is assumed for feeder cattle, selling prices are varied over a relevant range to indicate shifts in optimum plans as the ratio of beef and other prices vary. The same procedure is used for hog prices.

Land Use

Soil types on the case farm included 20 percent Nicollet and 80 percent Clarion. Detailed cropping data for the farm were available for years 1951-57, inclusive. The farm operator maintained a yearly field map showing fertilizer treatments and crop yields for each field. These data are assembled in table 3. Total acres of each crop, average fertilizer treatments and average yields are given for 1951 through 1956.

Principal rotations used in the past were corn-corn-oats-meadow (CCOM), corn-oats-meadow (COM), corn-soybeans-corn-oats-meadow (CSbCOM) and corn-oats (CO). Average acres and average per-acre fertilizer rates for each rotation during the 6-year period were determined from the field maps. This information, together with results from soil tests from the past several years, provided a framework for predicting future yields.

Historic cropping data and information from soil tests allowed agronomy specialists to establish relevant cropping alternatives. In addition to possible new crop rotations, the farm operator was interested in expected response from varying levels of fertilization. Hence, three levels of fertilization (low, medium and high) were used for each rotation. Two rotations in addition to those previously used, corn-corn-oats-meadow-meadow (CCOMM) and corn-corn-soybeans (CCSb), were included as alternatives for new plans. The crop yields for the three fertilizer treatments and six rotations are shown in table 4.

Other basic input-output data for the cropping alternatives are shown in table 5. Seed and fertilizer costs are merely the quantity of each times the price. The item listed as "other production costs" includes fuel, oil, repairs, electricity charges, telephone bills, cost of trips to town and other factors that share in the total cost of producing an acre of crops. Since total annual costs only were included in the farm records, it was necessary to impute these to each enterprise. The resulting totals for each enterprise were checked against total costs shown in the farm

TABLE 3. CROPPING DATA AND FERTILIZER TREATMENTS ON THE CASE FARM FOR YEARS 1951 THROUGH 1956.

	Corn					Oats				
		Nutrients[a] (pounds)			Yield[a]		Nutrients[a] (pounds)			Yield[a]
Year	Acres	N	P_2O_5	K_2O	(bushels)	Acres	N	P_2O_5	K_2O	(bushels)
1951	98	4.7	18.9	15.9	57	68	6.5	58.9	31.8	37
1952	78	23.3	13.3	13.3	98	78	5.9	60.0		34
1953	107	33.5	53.7	85.6	57	56	17.5	101.4	26.4	35
1954	105	72.1	61.8	53.9	45	86	19.9	60.3	...	48
1955	103	58.5	36.0	36.0	50	39	...	...	...	74
1956	95	40.8	96.3	94.4	54	69	...	...	...	30
Average	98	38.8	46.7	50.0	60	66	8.3	46.8	9.7	43
	Soybeans					Meadow				
										(tons)
1951	..	..		..	..	45	...	24.9	...	1.7
1952	8	..	60.0	..	47	47	...	...	...	4.0
1953	..	..	..	..	..	48	...	...	...	2.5
1954	..	..	..	..	..	20	...	...	...	3.0
1955	20				20	49	...	...	...	3.4
1956	5	14.5	14.5	14.5	19	42	...	...	...	1.3
Average	11	4.8	24.8	4.8	29	42	...	4.2	..	2.7

[a] Pounds of nutrients and yields are shown on a per-acre basis.

TABLE 4. ESTIMATED CROP YIELDS FROM VARIOUS FERTILIZER TREATMENTS FOR ALTERNATIVE ROTATIONS ON THE CASE FARM.

	Low fertilizer[a]				Medium fertilizer[a]				High fertilizer[a]			
Rotation	N	P_2O_5	K_2O	Yield[b]	N	P_2O_5	K_2O	Yield[b]	N	P_2O_5	K_2O	Yield[b]
Corn	5	20	20	56	5	50	20	63	35	70	20	69
Soybeans	0	0	0	23	0	0	0	26	0	0	0	28
Corn	5	20	20	54	45	60	20	62	75	60	20	65
Oats	0	30	0	43	0	40	0	49	0	40	0	53
Meadow	0	0	0	2.5	0	0	0	2.9	0	0	0	3.1
Corn	5	20	20	56	5	50	20	63	35	60	20	69
Corn	5	20	20	53	45	40	20	60	75	60	20	66
Oats	0	30	0	43	0	30	0	49	0	40	0	53
Meadow	0	0	0	2.5	0	0	0	2.9	0	0	0	3.1
Corn	5	20	20	50	35	20	20	57	80	60	20	67
Oats	0	0	0	41	0	30	0	45	0	30	0	47
Corn	5	20	20	56	5	50	20	63	35	70	20	69
Oats	0	30	0	43	0	30	0	49	0	30	0	53
Meadow	0	0	0	2.5	0	0	0	2.9	0	0	0	3.1
Corn	5	20	20	56	5	50	20	63	35	80	20	69
Corn	5	20	20	55	45	40	20	62	75	50	20	67
Oats	0	40	0	43	0	50	0	49	0	30	0	53
Meadow	0	0	0	2.5	0	0	0	2.9	0	0	0	3.1
Meadow	0	0	0	2.5	0	0	0	3.0	0	0	0	3.3
Corn	25	20	20	50	45	40	20	62	65	50	20	67
Corn	45	40	20	46	75	40	20	58	95	50	20	62
Soybeans	0	0	0	22	0	0	0	23	0	0	0	25

[a] Fertilizer treatments shown in pounds per acre of available nutrients.
[b] Yields are shown in bushels per acre for grain and tons per acre for meadow.

TABLE 5. BASIC INPUT-OUTPUT DATA FOR ALTERNATIVE CROP ROTATIONS ON THE CASE FARM.

Item	Unit	Per acre of rotation unit[a]																	
		$CSbCOM_1$	$CSbCOM_2$	$CSbCOM_3$	$CCOM_1$	$CCOM_2$	$CCOM_3$	CO_1	CO_2	CO_3	COM_1	COM_2	COM_3	$CCOMM_1$	$CCOMM_2$	$CCOMM_3$	$CCSb_1$	$CCSb_2$	$CCSb_3$
Inputs																			
Seed	\$	2.73	2.73	2.73	2.76	2.76	2.76	1.88	1.88	1.88	3.13	3.13	3.13	2.21	2.21	2.21	1.97	1.97	1.97
Fertilizer	\$	2.08	4.80	6.88	2.60	5.25	8.35	1.85	5.45	10.60	2.23	3.23	5.30	2.28	4.60	6.48	5.93	8.93	11.93
Other production costs	\$	6.73	7.00	7.14	6.46	7.01	7.21	6.96	7.07	7.22	6.57	6.95	7.15	6.50	6.99	7.28	7.74	7.98	8.07
Total variable costs	\$	11.54	14.53	16.75	11.82	15.02	18.32	10.69	14.40	19.70	11.93	13.31	15.58	10.99	13.80	15.97	15.64	18.88	21.97
Outputs																			
Corn equivalent	bu.	26.20	30.00	32.00	32.50	37.00	40.00	35.00	39.80	45.30	25.80	29.20	31.80	26.40	29.90	32.50	32.00	40.00	43.00
Hay	ton	0.50	0.58	0.62	0.62	0.72	0.77	...	...	...	0.83	0.97	1.03	1.00	1.18	1.28	....	....	....
Gross returns[b]	\$	43.67	49.78	53.43	42.84	48.72	53.04	35.66	40.20	45.72	39.45	44.96	48.72	42.67	48.98	53.54	46.66	55.34	59.66
Returns before fixed costs	\$	32.13	35.25	36.68	31.02	33.70	34.72	24.97	25.80	26.02	27.52	31.65	33.14	31.68	35.18	37.57	31.02	36.46	37.69
Labor Inputs																			
Dec.-Jan.-Feb.	hour	0.04	0.04	0.04	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.07	0.07	0.07
March-April	hour	0.30	0.30	0.30	0.37	0.37	0.37	0.55	0.55	0.55	0.37	0.37	0.37	0.30	0.30	0.30	0.12	0.12	0.12
May-June	hour	1.80	1.80	1.80	1.51	1.51	1.51	1.20	1.20	1.20	1.25	1.25	1.25	1.48	1.48	1.48	1.82	1.82	1.82
July-Aug.	hour	0.72	0.72	0.72	0.96	0.96	0.96	0.79	0.79	0.79	1.53	1.53	1.53	1.17	1.17	1.17	0.16	0.16	0.16
Sept.-Oct.-Nov.	hour	0.65	0.65	0.65	0.61	0.61	0.61	0.48	0.48	0.48	0.52	0.52	0.52	0.61	0.61	0.61	1.16	1.16	1.16

[a] Subscripts on each rotation indicate level of fertilizer treatment.
[b] Based on \$1 per bu. for corn and \$16 per ton for hay.

record books. This method resulted in the figures actually used and listed in table 5.

The subscripts shown with each rotation title in table 5 refer to level of fertilization. For example, the corn-oats rotation combined with a low level of fertilization is CO_1; a combination of the same rotation and medium fertilizer application is CO_2; the high fertilization rate on the corn-oats rotation is signified by CO_3.

Livestock Enterprises

Basic input-output data for livestock enterprises are presented in table 6. Two systems of hog production, short-fed heifers, long-fed steers and a poultry laying flock comprise the livestock alternatives considered for programming farm plans. Although other kinds of livestock production—such as dairying, beef cows and feeding yearling cattle—are possible on the case farm, the alternatives considered relevant by the farm operator are those in table 6.

Various reasons were cited by the farm operator for not considering other kinds of livestock. For example, the feeding of yearling cattle involves more risk than does the feeding of calves. For beef cows, the turnover in capital is too slow, or the production period is too long, to appeal to the farm operator. He rejected dairying because of an aversion toward milking cows.

Following is a brief explanation of each livestock enterprise considered in the programming situations.

Hogs$_1$. The subscript 1 signifies that availability of housing and equipment on the case farm limits this hog enterprise to 45 litters. This enterprise is a two-litter system whereby each sow farrows twice annually; one litter is farrowed within the period from June to August, and the second litter is farrowed within the period from December to February. Relatively long farrowing periods are used to distribute the labor load and minimize requirements for farrowing equipment. When each farrowing period is 3 months long, farrowing stalls and equipment can be used twice. Hence, if the optimum farm plan calls for 40 hog litters, 10 litters are farrowed the first days in June, 10 litters are farrowed during the latter part of July, June-farrowed sows farrow their second 10 litters in December, and July-farrowed sows farrow their second 10 litters in late January. Replacement breeding stock is supplied from June-July farrowings.

In table 6, no entries appear under Hogs$_1$ for the first two input items. Since hogs are currently produced on the case farm, basic stock, or breeding gilts, are saved from previous litters; thus, no cash outlay is required. Similarly, there is sufficient equipment to produce 45 litters. Depreciation and repairs on equipment are included in production costs. The total figure for "variable production costs other than feed" was determined for hogs and other livestock enterprises by the same method as outlined for deriving "other production costs" for crops. Data on feed inputs and meat output are directly transferred from the case farm records.

Hogs$_2$. This hog system is identical to the one just

TABLE 6. BASIC INPUT-OUTPUT DATA FOR LIVESTOCK ENTERPRISES ON THE CASE FARM.

Item	Unit	Hogs1 (per litter)	Hogs2 (per litter)	Short-fed heifers[a] (per head)	Long-fed steers[b] (per head)	Poultry (per 100 hens)
Inputs						
Basic stock	$	....	....	99.00	112.50	55.00
Equipment	$	....	28.57	....	....	....
Variable production costs other than feed	$	43.71	43.71	9.91	18.40	43.00
Corn equivalent	bu.	96.24	96.24	28.70	53.30	81.60
Hay equivalent	ton	1.06	1.06	1.31	2.44	....
Commercial feed	$	52.73	52.73	8.23	14.00	89.57
Total variable expense	$	209.64	209.64	166.80	237.24	269.17
Total capital required	$	96.44	125.01	117.14	144.90	187.57
Outputs						
Meat produced	cwt.	20.05	20.05	3.50	6.50	5.99
Eggs	doz.	....	....	....	....	804.00
Total gross return	$	286.72	286.72	208.00	297.00	297.10
Return before fixed costs	$	77.08	77.08	41.20	59.76	27.93

[a] Costs and returns for short-fed cattle are based on $22 per cwt. purchase price and $26 per cwt. selling price.
[b] Costs and returns for long-fed cattle are based on $25 per cwt. purchase price and $27 per cwt. selling price.

described except for housing and equipment restrictions. The current supply of housing and equipment limits hog production to 45 litters in the previous hog system. An added investment of $28.57 per litter, however, allows production facilities for 35 more litters, resulting in a total potential hog enterprise of 80 litters. Since production of the last 35 litters requires more capital than the first 45 litters, two separate hog activities are defined for the programming model.

Short-fed heifers. In the past, no uniform beef buying and selling program was followed on the case farm. Animals were purchased at local sales barns and fed according to their weight and quality. The buying period often extended over 6 months. Short-fed cattle were purchased in lots of two or three head, with purchases made each week or every other week. Marketing continues by various intervals in the following 6-month period. This pattern of buying and selling is assumed for short-fed heifers described here.

Medium-to-good heifers are purchased from local sources during the period from latter July to early December. Average purchase weight is 450 pounds. The feeding ration is primarily roughage, with some concentrate during the first 4 months. About 1 month before marketing time, considerably more grain is added to the ration. These animals are kept on the farm from 150 to 180 days. They are marketed as good-to-choice heifers at an average weight of 800 pounds, or a net gain of 350 pounds. Basic input-output data for this enterprise are shown in table 6. Feed inputs are determined from requirements per hundredweight of gain as evidenced in past feed records.

Long-fed steers. This enterprise involves animals of higher quality than the system just described. Good-to-choice steer calves of 450 pounds are purchased in October, wintered in corn fields and on other forage fed in drylot. Winter-fed roughages and summer pasture are supplemented with some concentrate feeds. Intensive grain feeding begins about 2 months before the steers are marketed in October or November as choice fat cattle weighing about 1,100 pounds. Input-output data, based on the same methods described for short-fed heifers, are shown in table 6.

Poultry. Sexed chicks are purchased annually to replace the old laying flock. Total produce per 100 hens includes 504 dozen eggs and 599 pounds of meat. Mortality rates are 6 percent for chicks and 5.4 percent for hens. Feed requirements and other data are presented in table 6.

Other livestock activities. The basic livestock enterprises considered in this study have just been described. Short-fed heifers and long-fed steers, however, comprise two activities each in the programming model. The two activities for short-fed heifers are identical except for labor requirements; a parallel condition exists for long-fed steers. The difference in labor requirements stems from two alternative sources of summer pasture. One source of pasture is rotated meadow on the case farm; the other source is rented pasture located about 8 miles from the farmstead. Because of extra time required for commuting to and from the rented pasture, labor requirements for cattle kept there are necessarily higher than for those pastured at home. Thus, a planning situation that includes the alternative of renting pasture must also include the exact livestock activities for utilizing this pasture.

Resource Restrictions

Resource restrictions used in this study are presented in the following equations where A_{ij} is the amount of the i-th resource required to produce one unit of the j-th enterprise and X_j is the number of units of the j-th enterprise produced. When the relationship is indicated by $\leqslant$, the supply of the resource may be greater than the amount actually used in production. In the case of labor restrictions, no labor is hired until the family labor supply is exhausted. Hence, if extra labor is hired, the equality part of the relationship applies, since labor is hired only if it is used. The same logic holds for the first equation where extra capital can be borrowed.

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{\$13,173 capital plus borrowed capital} \quad (1)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{272 acres cropland} \qquad (2)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{676 hours of December-January February labor plus hired labor} \qquad (3)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{417 hours of March-April labor plus hired labor} \qquad (4)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{635 hours of May-June labor plus hired labor} \qquad (5)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{590 hours of July-August labor plus hired labor} \qquad (6)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{585 hours of September-October-November labor plus hired labor} \qquad (7)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{45 litters of hogs}_1 \text{ (housing and equipment limitation)} \qquad (8)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{35 litters of hogs}_2 \text{ (housing and equipment limitation)} \qquad (9)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{500 hens (housing and equipment limitation)} \qquad (10)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{total feed-grain supply} = 0 \text{ at the outset} \qquad (11)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{total hay supply rented pasture} = 15 \text{ tons at the outset} \qquad (12)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{\$5,000 capital borrowed at 11 percent interest rate.} \qquad (13)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{\$5,000 capital borrowed at 15 percent interest rate} \qquad (14)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{\$5,000 capital borrowed at 26 percent interest rate} \qquad (15)$$

$$\sum_{j=1}^{n} A_{ij}X_j \leqslant \text{70 acres rented pasture} \qquad (16)$$

Since different farm situations were considered for programming, the preceding equations represent all restrictions used in the various programs. For example, when hired labor was not considered, equations 3 through 7 were confined to original labor supplies since hired labor equaled zero. Likewise, borrowed capital was equal to zero in equation 1 when capital borrowing was left out and equations 13 through 15 were omitted. Equation 16 was omitted when pasture renting was not considered in the programming situation.

ANALYSIS OF RESULTS

Optimum farm plans under various programming situations are presented in this section. In all situations, except one that includes family living expenditures, inputs and outputs are shown for the farm rather than for the tenant or landlord. Since the farm is operated under a 50-50 share-lease, input-output figures for either tenant or landlord are half the amounts shown for the farm. Each party has different items in their fixed costs, however. The landlord's major fixed cost is real estate taxes. Machinery depreciation constitutes most of the tenant's fixed cost. Items such as personal property taxes and depreciation on livestock equipment enter into fixed costs for both landlord and tenant.

Preliminary Plans

The initial step in programming plans for the case farm is presented in this section. The actual plan for 1956 is compared with a programmed plan based on the same resource, price and yield situations as experienced in 1956. This procedure is used to determine the extent to which the operator was following an optimum plan. Only a single year, without variable price programming, is used for this benchmark comparison. The programming considers only the enterprices and resource restrictions used by the operator in 1956. Added alternatives in enterprises and resource supplies, and their effect on profit, will be considered in the next section.

The actual and optimum farm plans for 1956 are illustrated in table 7; corresponding profits are given below each plan. Total cultivated land in 1956 is 211 acres. Later plans are based on 272 acres because an 80-acre tract was added to the case farm in 1957.

TABLE 7. ACTUAL PLAN AND OPTIMUM PLAN FOR THE CASE FARM IN 1956.

Actual plan		Optimum plan	
Enterprise	Quantity	Enterprise	Quantity
$CCOM_2$	80 acres	$CCOM_2$	162 acres
COM_2	52 acres	CO_2	49 acres
CO_2	54 acres		
$CSbCOM_2$	25 acres		
Short-fed heifers	61 head	Short-fed heifers	16 head
Hogs	45 litters	Hogs	80 litters
Poultry	436 hens		
Return = $11,605[a]		Return = $13,926[a]	

[a] Return before fixed charges of $2,391 are subtracted. These changes include depreciation on machinery and buildings, taxes on personal property and real estate, and insurance on buildings.

Other than a provision for increased hog production, no new production alternatives were considered for programming this situation. The answers to the following questions were sought: Did the actual farm plan represent an optimum resource allocation? If the actual plan and resource use were not optimum, what combination of enterprises would be required in an optimum plan? What amount of profit increase might be realized from an optimum plan? Answers to these questions are suggested in table 7. A major difference between the plans in table 7 is the number of rotations. The actual plan has four different rotations, whereas two rotations provide the cropping program in the optimum plan. Differences in total acres of each crop are slight, however. The actual plan calls for 95 acres of corn, 69 acres of oats, 42 acres of meadow and 5 acres of soybeans; the optimum plan contains 105 acres of corn, 66 acres of oats, 40 acres of meadow and no soybeans. These differences stem largely from the following relationships: (1) The cheapest source of roughage is from the CCOM rotation. (2) Corn returns higher revenue than other crops. Hence, rotations that include the highest proportion of corn (i.e., 50 percent in this case) and meet forage requirements are most profitable.

Livestock production in the two plans differs considerably. Poultry is not included in the optimum plan. Returns to labor and capital from poultry production are lower than for hogs and beef. A major shift from cattle to hogs is indicated. Hogs are produced at the 80-litter maximum, remaining or residual resources being used for cattle production. Extra hog facilities are profitably provided by transfer of capital investment from cattle to hogs. (Profits could be increased even more if no cattle were raised and hog production was correspondingly increased. The case farm operator established a managerial limit of 80 hog litters, however.)

The optimum plan has $2,321 more return than the 1956 actual plan, even though resource supplies for the two plans are identical. Therefore, increased profits from the optimum plan must be attributed to reallocating resources within the firm.[3]

Prices used for programming are those actually received in 1956. Thus, the optimum plan is specified for a "perfect knowledge situation." The actual plan was made within the realm of price expectations and uncertainty. Even so, the case farm operator indicated that his price expectations and prices actually received were "about the same." Because of this condition and the increased profits indicated from the optimum plan, preliminary results stimulated the farm operator's interest in the analysis of other production alternatives and resource situations.

Optimum Plans With Added Capital, Pasture Renting and Added Enterprises

The programming situations considered in this section include the production alternatives represented in tables 5 and 6 plus certain others which appeared to have profit potential. Resource supplies are those indicated in equations 1 through 16. Product prices, based on the farm operator's projections, are those listed in table 2. New plans are now programmed to determine the effects of capital borrowing and pasture renting on income, the farm business being programmed apart from the farm household as in the previous section.

Only one new livestock enterprise, long-fed steers, is considered. Several new cropping alternatives are allowed. In place of the four rotations, each with one level of fertilization as in the previous section, this section includes selection from among six rotations, each with three levels of fertilization.

TABLE 8. OPTIMUM PLANS FOR FARM SITUATIONS THAT INCLUDE NEW PRODUCTION ALTERNATIVES AND CAPITAL BORROWING.

Optimum plan without capital borrowing		Optimum plan with capital borrowing	
Enterprise	Quantity	Enterprise	Quantity
$CCOMM_3$	58 acres	$CCOMM_3$	143 acres
$CCSb_3$	214 acres	$CCSb_3$	129 acres
Hogs	70 litters	Hogs	80 litters
		Short-fed heifers	87 head
Total labor hired	56 hours	Total labor hired	455 hours
		Capital borrowed	$11,306
Return = $15,584[a]		Return = $18,852[b]	

[a] Return before fixed charges of $2,391 are subtracted.
[b] This return figure is based on 6 percent interest charged for borrowed capital. Fixed charges have not been subtracted.

The major difference in resource structure between the situations considered in this section and the one used for preliminary plans is the number of tillable acres. Plans in this section are designed for 272 tillable acres. Other resource supplies, except for capital borrowing restrictions, are unchanged. Table 8 shows optimum farm plans for the original amount of capital (i.e., $13,173, or the amount used in preliminary planning of the previous section) and the original amount plus capital borrowing.

The consideration of new cropping alternatives has a definite influence on the make-up of the optimum farm plan. As compared with the actual farm plan in 1956, the optimum plans in table 8 not only include different rotations but also a shift in applications of fertilizer nutrients. Both rotations in the optimum plans are fertilized at the third or highest rate considered. The actual plan in table 7 shows the second or medium fertilizer rate on all crops.[4] Thus, programming solutions indicate initially higher capital returns from crops than from livestock production. Livestock ranks second in level of capital return. This condition applies to the case farm because of the relatively high fertilizer response.

None of the crop rotations used in the actual farm plan is included in the optimum plans in table 8. The most intensive rotation previously considered by the farm operator is CO; also, no rotation with 2 years of meadow was previously included in the cropping program. The following conditions explain the resulting cropping system in table 8.

The CCOMM rotation provides the only source of forage in the optimum plans. The exact number of acres in this rotation depends on forage requirements of livestock. For the plan without borrowed capital, the livestock program is limited to 70 hog litters. Corresponding forage requirements are satisfied by 58 acres of $CCOMM_3$. As the farm plan is expanded by borrowing capital, hog production is increased to 80 litters, and remaining resources are invested in 87 head of short-fed heifers. The forage needs for this livestock program require 143 acres of $CCOMM_3$. Other rotation alternatives including meadow are not

[3] Original resource supplies are determined from total resource requirements in the actual plan. Hence, the actual plan has no unused resources. Although resource supplies are the same for the optimum plan, not all resources are used. The most limiting resource, capital, is used up in both plans but the optimum plan requires slightly less labor in all time periods considered.

[4] Although the actual fertilizer treatment used in 1956 does not correspond exactly with the medium rate indicated in table 4, it most nearly approximates the medium rate in terms of total pounds of nutrients per acre of rotation.

selected in the programming solution because of the oats-meadow ratio. In the CCOMM rotation, 2 years of meadow are forthcoming from only 1 year of oats. Or, one-fifth of an acre in oats results in two-fifths of an acre in meadow—a 1:2 ratio. All other crop rotations considered for the case farm have a higher ratio of oats to meadow. That is 1 acre of oats is required in the rotation to produce 1 acre of meadow —a 1:1 ratio. This analysis implies a low return from the oat crop as compared with other grains. Consequently, highest returns result from choosing a rotation for forage needs that has the least oats for the most meadow.

The other rotation in the optimum plans of table 8 is $CCSb_3$. This rotation is the most intensive cropping system considered for the case farm. Comparison of the optimum plans in tables 7 and 8 reveals the substitution effects from considering new cropping alternatives. That is, just as CCOMM substitutes for CCOM when forage is required, CCSb substitutes for CO as the most profitable land use system on remaining acres after forage needs are met. Actually, highest return per dollar invested results from all acres in $CCSb_3$. As additional funds are invested in the farm business, beyond the amount required for crops, however, total returns from all scarce resources are maximized by including livestock production in the farm plan and shifting enough acres out of CCSb into CCOMM to provide sufficient forage for livestock. The choice of CCSb over CO is a function of returns from individual crops. The latter rotation includes one-half corn, whereas CCSb includes two-thirds corn. The relatively higher returns from corn, as compared with other grains, dictate the most profitable rotation. A straight corn rotation might give even higher returns than CCSb. The rotation including soybeans, however, does provide some degree of diversification and acts as an uncertainty precaution for income stability. The advantage of one system over the other within, for example, a 10-year period, depends on price expectations and risk preferences of the farm manager. Straight corn as a cropping alternative was not considered for the case farm.

Livestock production for the optimum plans in tables 7 and 8 follows a consistent pattern. Resources remaining have been met after crop requirements are allocated first to hog production. When hog litters reach the maximum number (i.e., 80 litters) specified by the farm operator, additional resources are allocated to short-fed heifers. The optimum plan in table 7 and the optimum plan without borrowed capital in table 8 have the same supply of capital. But additional acres in the latter plan require extra capital for crops, and, consequently, remaining capital is adequate for 70 hog litters only. Also, the sequence for investing added funds (i.e., crops, hogs and cattle) is dependent on price relationships. This topic is treated in detail in a later section on variable prices.

Return figures for plans in table 8 are total returns to the farm. Tenant or landlord returns are half of the figures shown. By borrowing $11,306 capital, the farm plan is expanded to include 80 hog litters and 87 head of short-fed heifers. This plan increases income by $3,268, from $15,584 to $18,852. The average return per dollar of borrowed capital is approximately 29 percent.

At this phase of programming farm plans, the farm operator established subjective discount rates for borrowed capital. Prior to computing the plan with borrowed capital in table 8, the farm operator indicated that he would borrow capital only if expected returns were sufficiently high relative to commercial loan rates. Results of the programming solution with borrowed capital, however, served to stimulate the farm operator's interest in borrowing funds. Consequently, subjective discount rates for added increments of borrowed capital were established. As mentioned previously, his subjective discount rates require 11 percent return for the first $5,000 to be borrowed, 15 percent for the second $5,000 and 26 percent for the third $5,000 increment to be borrowed. By including these rates in the programming model, it was found that the optimum plan with borrowed capital remained unchanged.

Hours of hired labor required for plans in table 8 are 56 hours for the plan limited by the farm supply of capital and 455 hours for the plan with borrowed capital. The 56 hours of labor for the first plan are hired during the period from September through November. The plan with borrowed capital requires 455 hours of hired labor distributed throughout the year except during July and August. Costs for hiring extra labor are included in total capital requirements and accordingly deducted from total return figures shown in table 8.

PROGRAMMING ADDITIONAL SITUATIONS

Livestock production in the preceding plans is limited to homegrown feeds. No provisions are made in the programming model to obtain either hay or corn from outside sources. Although extra corn may be purchased from commercial firms or other farmers, sources for hay purchases are not very certain. In some years hay supplies are plentiful, and corresponding market prices are relatively low. On the other hand, during years of hay shortages, market prices are quite high, and in many cases, no hay is for sale.

The case farm operator could rent 70 acres of improved pasture as a source of forage for cattle. Renting this pasture permits corn production on acres previously seeded to grass. The result is a larger supply of feed for increased livestock production. The question is whether or not it is profitable to rent the improved pasture and increase livestock production.

Rental charge for the 70 acres of improved pasture is $500, or approximately $7.14 per acre. The farm operator estimated that 1 acre of pasture is adequate forage to produce one short-fed heifer, or that the 70 acres of rented pasture can provide total forage requirements for about 70 head of cattle. Some acres would be used for pasture and remaining acres harvested as baled hay. Programming results for the rented pasture situation are presented in table 9.

Comparison of results in table 9 with the plan using borrowed capital in table 8 indicates the effects on production when rented pasture provides a source of forage. Profits are increased by renting the pasture.

TABLE 9. OPTIMUM PLAN UNDER PASTURE RENTING.

Enterprise	Quantity
$CCOMM_3$	92 acres
$CCSb_3$	180 acres
Hogs	80 litters
Short-fed heifers	106 head
Rented pasture	70 acres
Total labor hired	642 hours
Capital borrowed	$14,731
Return = $20,009[a]	

[a] This return figure accounts for a 6-percent interest charge on borrowed capital. Fixed charges of $2,391 have not been subtracted.

If pasture renting was an unprofitable source of roughage, the optimum plan in table 9 would be identical to the optimum plan with borrowed capital in table 8. Since pasture renting is included with the results in table 9, the next step is to inspect the corresponding consequences on crop and livestock production.

Pasture renting provides a profitable route for increasing cattle numbers. The number of short-fed heifers is increased from 87 to 106 head by renting 70 acres of pasture. Hog production remains constant at 80 litters; this maximum limit is previously attained in the comparable plan in table 8. As additional forage becomes available, however, the cropping program is changed by shifting acres from CCOMM to CCSb. In essence, this shift substitutes meadow acres for corn acres; the final result is a greater overall feed supply. Extra hay is obtained from rented pasture, and extra corn comes from the increased corn acreage. With 92 acres of $CCOMM_3$, 180 acres of $CCSb_3$ and 70 acres of rented pasture, the livestock system that completely utilizes all feed includes 106 short-fed heifers and 80 hog litters.

Capital requirements for the optimum plan in table 9 include $14,731 of borrowed funds. Labor requirements are increased to 642 hours of hired labor. These added resource inputs are associated with an increase in return of $1,157 when a 6-percent interest rate is actually charged for borrowed capital in computing income. (The higher "discount prices" for capital borrowing were included in programming computations, so that an activity would not be included unless it met the farmer's subjective discount rate.) Total return for the plan in table 9 is $20,009. The magnitude of subjective discount rates does not alter the optimum plan, because predicted returns to capital are sufficiently higher than the discount rates.

Determining a Stable Cropping Program

The most profitable rotation when livestock is not included in the farm plan is CCSb; if livestock is included, corresponding forage needs are obtained by shifting sufficient acres from CCSb to CCOMM. This pattern of land use is based on average expected crop yields as presented in table 4. But if crop yields are below or above these averages, are the same rotations most profitable? These questions can be answered by programming of stable plans and rotations under varying yields. Optimum programs are computed as yields are varied over a certain range. In this study, yields are varied from 50 percent below the average yields used to 50 percent above. Varying degrees of complementarity between meadow and grain may exist. For the analysis which follows, however, yield relationships between different crops of a rotation are assumed to be constant. Only the high rate of fertilization is considered in these computations.

Given the price and yield expectations in tables 2 and 4, respectively, the optimum cropping program when yields are 50 percent below average is CCOMM on all acres. This rotation is optimum for yields ranging from 50 to 14 percent below average. At the latter level of yields (i.e., 14 percent below average), CCSb enters the cropping program. Within the yield range from 14 percent below average to 50 percent above average the optimum cropping program includes both CCOMM and CCSb. Thus, the cropping program included in the optimum farm plans already discussed is completely stable for the upper half of the relevant yield range. Its yield stability terminates, however, when yields are more than 14 percent below average. Or, the total span of stability is 64 percent of the range in yields studied, the average or yield level of the previous section serving as the standard of comparison. Based on these results, what cropping program should the farm operator choose?

CCOMM provides the most profitable (lowest cost) source of forage. Other rotations including meadow, such as CCOM and CSbCOM, never enter the program for the yield range investigated. Disposal of all homegrown forage on the case farm is through livestock. Hay production for sale is not profitable, and forage should be produced only to the extent necessary for livestock. Hence, having selected the specific rotations, the next step is to establish the number of acres in each rotation. The combination of CCOMM and CCSb to maximize profits must be determined in conjunction with the optimum livestock production. On the basis of computations made for yield stability, other programming results previously illustrated and consultation with the farm operator, the long-run cropping program chosen for the case farm is 157 acres of $CCSb_3$ and 115 acres of $CCOMM_3$. This pattern of land use provides 150 acres of corn, 46 acres of meadow, 23 acres of oats and 53 acres of soybeans annually. The resulting supplies of hay and corn equivalent constitute two of the resource restrictions used in the following section when optimum livestock plans are programmed for various ranges of price.

Optimum Livestock Plans for Varying Prices

Programming results discussed and illustrated in previous sections are based on the price levels in table 2. While prices might change, the same plans would be optimum if constant price relationships were maintained among products. That is, if all prices increase (decrease) by the same percentage, the make-up of the optimum plan would remain unchanged. If a constant price ratio is not maintained as prices change, however, the optimum plan will change. Accordingly, linear programming has been employed in this section to compute plans when the ratio of hogs and finished cattle varies and corn price is at three different levels. This approach allows us to establish the range of price ratios over which a particular plan is stable in the sense that it is the most profitable plan for the range indicated, although the level of income will vary. The price ranges over which particular plans are most profitable are illustrated by means of price maps.

The modified simplex solution for linear programming with variable prices is cumbersome when more than two prices are varied simultaneously. Therefore, the variable price plans presented in following sections are for price changes in beef and hogs only. Aside from the three different corn prices used, all other price data used in programming are those in table 2. Hence, costs for commercial feed, feeder cattle and other items are identical to those used in computing previous plans.

PROGRAMMING SOLUTIONS FOR VARYING BEEF AND HOG PRICES WITH CORN AT $0.80 PER BUSHEL

With corn priced at $0.80 per bushel, fig. 1 shows the ranges of prices over which particular plans are stable. Beef selling prices range from 0 to $32.00 per hundredweight, and hog selling prices range from 0 to $25.00 per hundredweight. The composition or make-up of these plans is presented in table 10.

Plan 1 in fig. 1 does not include livestock production. This plan is optimum over the selling price range of 0 to $20.13 per hundredweight for beef and 0 to $9.50 per hundredweight for hogs. In other words, for beef and hog prices lower than $20.13 and $9.50, respectively, only a cash-grain system of farming is profitable. This range of prices, with a single plan being most profitable for it, is indicated by the area or rectangle denoted as plan 1 in fig. 1. The cropping program for plan 1, as defined in table 10, is 157 acres $CCSb_3$ and 115 acres $CCOMM_3$. This system of land use is the same as the long-run cropping program outlined in an earlier section.

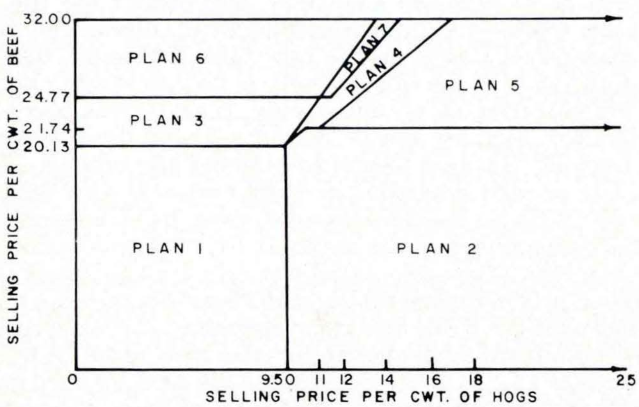

Fig. 1. Price map for optimum farm plans with varying selling prices for hogs and beef and with corn price at $0.80 per bushel.

For hog prices above $9.50 per hundredweight and beef prices below $21.74, the optimum plan includes livestock concentrated on hog production. The plan for these price ranges is that indicated as plan 2 in fig. 1 and defined in table 10 to include 80 hog litters and the cropping program indicated. Again, 80 litters are the maximum that the farm operator would produce. The price range for this optimum plan is represented by the entire area designated as plan 2 in fig. 2. When hog prices are equal to or below the minimum or break-even level of $9.50, the minimum or break-even beef price is $20.13 per hundredweight. (Minimum or break-even price defines the level at which any hogs or beef cattle become profitable.) But when hog prices increase above $9.50 per hundredweight, beef prices must be equal to or above $21.74 per hundredweight before a plan including feeder cattle becomes profitable.

Just as the area to the right of plan 1 defines the range of prices which are profitable for hog production when beef is at $21.74, the area above plan 1 (i.e., above the break-even price for beef and below the break-even price for hogs) defines the price range over which beef is profitable. The corresponding optimum program is that indicated as plan 3 and ranges over the price levels shown.

As indicated in table 10, this plan calls for livestock production which includes only 112 short-fed heifers. Optimum resource use, when beef selling price ranges from $20.13 to $24.77 per hundredweight and hog price is $9.50 or lower, does not include long-fed steers or hogs. The level of beef production in plan 3

TABLE 10. DESCRIPTION OF FARM PLANS COMPUTED WITH VARYING SELLING PRICES FOR BEEF AND HOGS.

Plan number	Enterprises in the farm plan	Limiting resouces	Capital borrowed	Labor hired
Plan 1	157 acres $CCSb_3$ 115 acres $CCOMM_3$	Land	....	....
Plan 2	157 acres $CCSb_3$ 115 acres $CCOMM_3$ 80 hog litters	Land Hog housing	$ 910	82 hrs. in Sept., Oct., Nov.
Plan 3	157 acres $CCSb_3$ 115 acres $CCOMM_3$ 112 short-fed heifers	Land Forage	$5,233	....
Plan 4	157 acres $CCSb_3$ 115 acres $CCOMM_3$ 66 short-fed heifers 57 hog litters	Land Forage	$5,728	31 hrs. in Sept., Oct., Nov. 6 hours in May, June
Plan 5	157 acres $CCSb_3$ 115 acres $CCOMM_3$ 48 short-fed heifers 80 hog litters	Land Hog housing Forage	$6,719	137 hrs. in Sept. Oct., Nov. 87 hrs. in Dec., Jan., Feb.
Plan 6	157 acres $CCSb_3$ 115 acres $CCOMM_3$ 30 short-fed heifers 44 long-fed steers	Land Forage	$2,003	....
Plan 7	157 acres $CCSb_3$ 115 acres $CCOMM_3$ 30 short-fed heifers 25 long-fed steers 45 hog litters	Land Forage	$3,690	88 hrs. in Sept., Oct., Nov. 12 hrs. in May, June

is limited by forage. Hired labor is not required for plan 3.

As beef selling price exceeds $24.77 per hundredweight, the optimum livestock combination becomes 30 short-fed heifers and 44 long-fed steers (plan 6). The price range of this plan is indicated by the relevant area in fig. 3. Plan 6 is optimum for beef prices ranging from $24.77 to $32.00 per hundredweight; correspondingly, maximum allowable hog prices for this plan range from $11.00 to $13.70 per hundredweight; as denoted by the slanting line on the right of the plan 3 area. As in plan 3, forage limits the number of cattle, and labor is not hired. The change between plan 3 and plan 6 denotes the most profitable use of forage as the beef prices change. Also, as denoted by the changes between plan 3 and plan 6, higher profits result, as beef price increases, when heavier cattle are fed at a lower margin per hundredweight. Short-fed heifers are purchased for $22.00 per hundredweight and marketed at 800 pounds for plan 3; long-fed steers are purchased for $25.00 per hundredweight and marketed at 1,100 pounds for plan 6. Since the area for each plan in fig. 1 refers to the same selling price for both types of cattle, price margins are necessarily higher for short-fed heifers than for long-fed steers.

Price ranges that include production of both hogs and cattle are illustrated in fig. 1 by the areas for plans 4, 5 and 7. Description of these plans is provided in table 10. In addition to 80 hog litters produced for the price ranges of plan 2, plan 5 includes 48 short-fed heifers. Forage is not a limiting factor in plan 2, and 82 hours of hired labor are required. Hence, with hog numbers maintained at the 80 litters of plan 2, feeder cattle in plan 5 are possible only if additional labor is hired. The lower boundary or price line of $21.74 per hundredweight in plan 5 indicates the minimum beef price at which hired labor can be used profitably for beef production. Plan 5 is stable over a beef price range from $21.74 to $32.00. The corresponding minimum hog price for this plan ranges from $11.00 to $17.20 per hundredweight, as indicated by the slanting price line or boundary to the left of the area for plan 5. Maximum limits for hog price are not defined for plans 2 and 5; these plans remain optimum for all hog selling prices greater than the minimum ones indicated by the border lines already explained.

Given the beef price range for plan 5, we can now examine the effects on resource use when hog price is lower than the minimum (i.e., $11.00 to $17.20 per hundredweight) required to produce 80 litters. Such a price level for hogs results in plan 4 with 57 hog litters and 66 short-fed heifers.

When hog price is less than that defined by the border line between plans 4 and 5, profits are maximized by substituting short-fed heifers for hogs. The optimum magnitude of substitution is indicated by the numbers of each enterprise in plan 4. The limiting resource dictating this shift between plans 4 and 5 is forage.

The specific price boundaries for plan 4 are illustrated in fig. 1. The area of price ranges for which the plan is stable is considerably less than for plans already discussed. With corn priced at $0.80 per bushel, however, the livestock prices unique to plan 4 are more realistic than the extreme price combinations (e.g., $8.00 for hogs with beef at $30.00, or $10.00 for beef with hogs at $20.00) included in plans 1, 2, 3, 5 and 6.

One price area not yet mentioned is represented by plan 7 in fig. 1. Livestock enterprises for this optimum plan include 45 hog litters, 30 short-fed heifers and 25 long-fed steers. At the minimum beef price for plan 7, hog prices range from $11.00 to $11.50; the corresponding range in hog prices for the maximum beef price is $13.70 to $14.90. The area bounded by these prices represents the only range of prices resulting in a plan which includes all three livestock enterprises. Re-examination of plans 4 and 6, which lie on either side of plan 7, however, suggests the combination of enterprises expected in plan 7. One reason that plan 7 results is that resources are profitably shifted out of hogs and short-fed heifers into production of long-fed steers when hog prices fall below the minimum hog price line for plan 4 and these prices represent the only range of prices required for plan 6. Another reason for the make-up of plan 7 is that hog production is substituted for long-fed steers when hog prices exceed the maximum hog price line for plan 6.

Some particular hog and beef price combinations in fig. 1 are denoted by more than one optimum plan. For example, the intersection point for $9.50 hogs and $20.13 beef touches four different plans. Hence, *ceteris paribus*, optimum resource use for this very unique price combination is indifferent among plans 1, 2, 3 and 4. Likewise, for a price combination of exactly $24.77 for beef and $11.00 for hogs, plans 3, 4, 6 and 7 are equally profitable.

The livestock combination requiring largest amounts of resources is plan 5 in table 10. Total capital requirements for this plan are $13,173 (the original supply of capital) plus $6,719 of borrowed capital. (As for previous programming solutions with constant prices, captial returns must offset the subjective discount rates mentioned earlier before added increments of borrowed capital are used.) A total of 224 hours of hired labor also is needed for plan 5, an amount higher than for any other plan in table 10.

PROGRAMMING SOLUTIONS FOR VARYING BEEF AND HOG PRICES WITH CORN AT $1.00 PER BUSHEL

Optimum plans varying selling prices for livestock and a corn price of $0.80 per bushel were determined for the previous section. Similarly, plans are illustrated in fig. 2 for corn priced at $1.00 per bushel. As for fig. 1, the plans in fig. 2 are described in table 10.

The price area for plan 1 in fig. 2, as compared with fig. 1, illustrates higher minimum or break-even prices for beef and hogs when corn is priced at $1.00 per bushel. Minimum selling prices before feed can be profitably processed through livestock now are $10.48 per hundredweight for hogs and $20.94 per hundredweight for beef. For selling prices less than these, total farm profits are maximized by a plan including only cash crops.

Inspection of other plans in fig. 2 indicates the increased prices associated with different livestock combinations when corn is $1.00 per bushel. Too, some

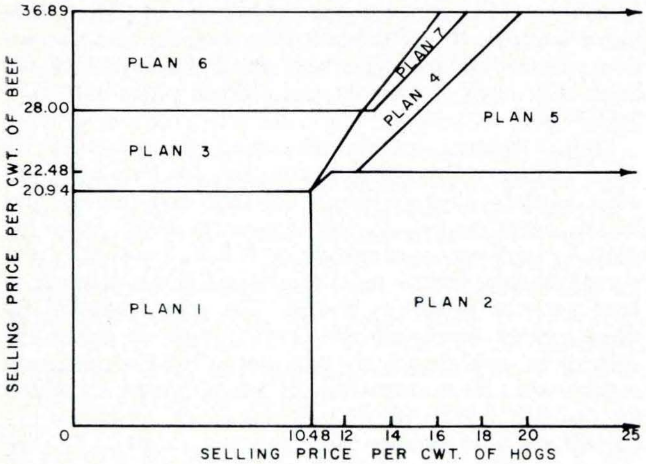

Fig. 2. Price map for optimum farm plans with varying selling prices for hogs and beef and with corn price at $1.00 per bushel.

plans have a larger area of price ratios over which a given plan is stable (i.e., is the most profitable one), resulting from higher priced corn, as compared with fig. 2.

PROGRAMMING SOLUTIONS FOR VARYING BEEF AND HOG PRICES WITH CORN AT $1.20 PER BUSHEL

In this section, the effects of varying beef and hog selling prices are examined when corn price is at $1.20 per bushel. The corresponding price map is illustrated in fig. 3. As in figs. 1 and 2 with corn prices of $0.80 and $1.00 per bushel, respectively, the same livestock plans appear in fig. 3 where corn price is $1.20 per bushel. The general effect of increasing corn price from $0.80 to $1.20 per bushel is higher minimum or break-even prices for feeding hogs and cattle. Also, higher prices define both the minimum and maximum price boundary lines for each plan. In fig. 3, the upper beef price limit on plans 4, 5, 6 and 7 is $41.13 per hundredweight. Comparable to the change in maximum beef selling price between figs. 1 and 2,

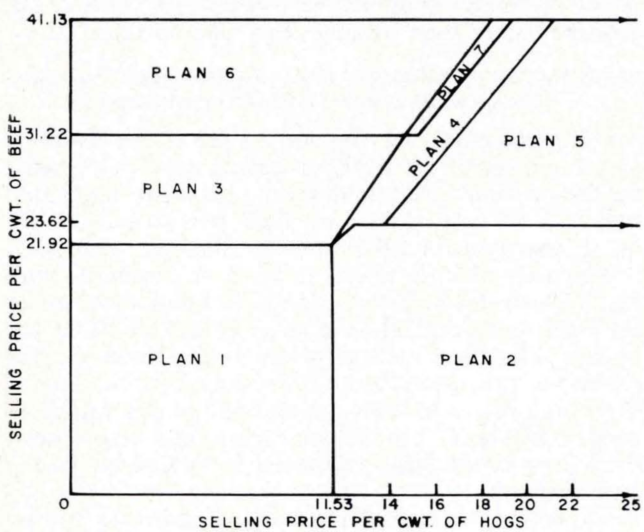

Fig. 3. Price map for optimum farm plans with varying selling prices for hogs and beef and with corn price at $1.20 per bushel.

the relevant price range for beef is again extended in fig. 3 to show the complete price areas for the plans concerned (i.e., plans 4, 5, 6 and 7).

Dynamic Programming to Indicate Optimum Plans for Successive Years

The programming solutions in preceding sections develop optimum programs for a single year. We simply suppose this year to be a "cross section picture" of a longer planning period. When capital becomes limiting, borrowed capital is automatically used if returns are sufficiently high to cover market interest rates, plus an amount necessary to offset the farmer's subjective discount rate. If, however, the farm operator decides not to borrow capital, regardless of anticipated returns, he may expand production only gradually by investing "surplus" returns at the end of each production period. In this case, optimum farm plans must be computed for several years to determine optimum investments for each increment of added capital. Under a situation of this type, one which is typical of Iowa farmers, changes in plans gradually emerge over time. The optimum plan of one year is not independent of the optimum plan of another year, though indeed these plans may differ greatly. Whether forage in a rotation should be produced in an early year, for example, depends on the amount of income which might be generated from corn in the same year and the consequent capital provided for livestock in a subsequent year. Or whether or not funds can be used profitably for fertilizer this year may depend on the level of income and consumption requirements for the family. Whether or not investment in practices which will produce large income in a future year is desirable depends on the rate at which income is discounted, and thus on the present value of this future income. Hence, a procedure is needed which considers a time span of several years and "ties together" the plans of different years. Also, the procedure should "tie together" farm planning and home planning in their competition for capital and use of income. Dynamic linear programming is such a procedure and has been used to accomplish these ends in the plans to follow.

THE DYNAMIC PROGRAMMING MODEL

In this section we outlined the dynamic programming model developed for, and used in, this study. It is an expansion of the ordinary simplex model. The concept for a dynamic model is to identify each coefficient with a particular time period. Thus, complete identification of any coefficient in the programming matrix refers to row, column and year. For example, a_{ij}, the input-output coefficient in the ordinary static model, refers to the amount of i-th resource or restraint used per unit of the j-th activity in a single year or time period. In the dynamic model, this notation is supplemented by a superscript k, which denotes a particular year. Each coefficient is now identified as a^k_{ij}, the amount of the i-th resource used per unit of the j-th activity in the k-th year.

Following this notation, each alternative production process for any one year ordinarily is expressed as x_{ij}.

To identify the activity x_j with a particular year, in our dynamic model a superscript k is added to give $x^k_j (j = 1 \cdots n,\ k = 1 \cdots t)$. Likewise, resource supplies or restraints are indicated by $s_i (i = 1 \cdots m)$, which becomes s^k_i when reference is made to the i-th resource supply in the k-th year. Unit returns to activities are denoted as c^k_j, to indicate the return for the j-th activity in the k-th year. In terms of these notations, the first dynamic programming equation is expressed as:

$$\begin{aligned} s^1_1 \geqq\ & a^1_{11}x^1_1 + a^1_{12}x^1_2 + \cdots + a^1_{1j}x^1_j + \cdots + \\ & a^1_{1n}x^1_n + a^2_{11}x^2_1 + a^2_{12}x^2_2 + \cdots + a^2_{1j}x^2_j \\ & + \cdots + a^2_{1n}x^2_n + a^k_{11}x^k_1 + a^k_{12}x^k_2 + \cdots \\ & + a^k_{1j}x^k_j + \cdots + a^k_{1n}x^k_n + \cdots + a^t_{11}x^t_1 \\ & + a^t_{12}x^t_2 + \cdots + a^t_{1j}x^t_j + \cdots + a^t_{1n}x^t_n. \end{aligned} \tag{17}$$

This equation is complete for the first resource supply in the first year (s^1_1). When $k \neq 1$, however, all $a^k_{ij} (k \neq 1)$ are equal to zero, except those representing inter-year capital flows, because activities for year 2 and beyond will not use resource supplies from year $1 (s^1_i)$. Therefore, the relevant terms (those without zero coefficients) of the preceding equation become:

$$\begin{aligned} s^1_1 \geqq\ & a^1_{11}x^1_1 + a^1_{12}x^1_2 + \cdots + x^1_{1j}x^1_j + \cdots \\ & + a^1_{1n}x^1_n. \end{aligned} \tag{18}$$

Exception to equation 18 occurs for the $s^k_i (k = 2, 3, \cdots, t)$ that represent transfer of net income from one year to operating capital of the next year for years 2 through t. The supply of operating capital is increased each year by the difference between (a) the net income of the previous year and (b) fixed costs and household withdrawals of the previous year. This process becomes automatic in the programming operation by including two conditions which represent withdrawal of funds for fixed costs and household expenditures and transfer of capital between years. An element representing the magnitude of annual fixed costs and household expenditures is entered in the resource vector (Po column) to permit a withdrawal activity to enter the plan at this exact level. This activity is "forced" into the plan at this level for each year by assigning an artificially large c_j value (+m value) to it. "False profit" so accumulated in the plan is subtracted from the final program.

The capital transfer between years might be accomplished by several methods. Here we accomplish this as follows:

Any activity produced in the k-th year has a positive coefficient in the capital equation for year k but has a negative coefficient in the capital equation for year k + 1. In simplex calculation, for example, one unit (acre) of corn may require \$20 of operating capital in year 1 and yield a net return (C^1_j value) of \$35 in year 1. A unit of this activity produced in year 1 will add \$20 + \$35 to the supply of operating capital in year 2, if the farmer is operating from his own funds and need not repay a loan with the \$20. Since \$55 is added to the capital supply of the next year, —55 becomes the coefficient in the column for the corn activity and the row for capital supply in year 2.[5]

[5] It should be noted here that only operating capital is transferred between years. If some portion of the capital requirement includes investment, only the part used for operating costs is transferable.

Algebraically, the total supply of operating capital so accumulated in year k is:

$$s^k_1 = \sum_{j=1}^{n-1} (c^{k-1}_j + a^{k-1}_{1j}) x^{k-1}_j - a^k_{1n}x^k_n \tag{19}$$

where capital is the first resource supply (s_1) and x_n is the fixed cost-family living activity.

In terms of the programming model, the set of equations for year 1 can now be expressed as:

$$\begin{aligned} s^1_1 \geqq\ & a^1_{11}x^1_1 + a^1_{12}x^1_2 + \cdots + \\ & a^1_{1j}x^1_j + \cdots + a^1_{1n}x^1_n \\ s^1_2 \geqq\ & a^1_{21}x^1_1 + a^1_{22}x^1_2 + \cdots + \\ & a^1_{2j}x^1_j + \cdots + a^1_{2n}x^1_n \\ & \cdots\cdots\cdots\cdots \\ s^1_i \geqq\ & a^1_{i1}x^1_1 + a^1_{i2}x^1_2 + \cdots + \\ & a^1_{ij}x^1_j + \cdots + a^1_{in}x^1_n \\ & \cdots\cdots\cdots\cdots \\ s^1_m \geqq\ & a^1_{m1}x^1_1 + a^1_{m2}x^1_2 + \cdots + \\ & a^1_{mj}x^1_j + \cdots + a^1_{mn}x^1_n \end{aligned} \tag{20}$$

where s^1_1 refers to the capital supply in year 1, $s^1_2 \cdots s^1_i \cdots s^1_{m-1}$ represent other resource restrictions, s^1_m is the fixed cost, including household consumption, for year 1 and all a^1_{mj} are zero for $j \neq n$.

Remaining equations for years 2 through t are somewhat enlarged in the s^k_1 row because of the capital transfer process. For example, in year $k (k \neq 1)$ these equations become:

$$\begin{aligned} s^k_1 \geqq\ & - a^{k-1}_{11}x^{k-1}_1 - a^{k-1}_{12}x^{k-1}_2 - \cdots - a^{k-1}_{1j}x^{k-1}_j \\ & - \cdots - a^{k-1}_{1n}x^{k-1}_n + a^k_{11}x^k_1 + a^k_{12}x^k_2 \\ & + \cdots + a^k_{1j}x^k_j + \cdots + a^k_{1n}x^k_n \\ s^k_2 \geqq\ & a^k_{21}x^k_1 + a^k_{22}x^k_2 + \cdots + \\ & a^k_{2j}x^k_j + \cdots + a^k_{2n}x^k_n \\ & \cdots\cdots\cdots\cdots \\ s^k_i \geqq\ & a^k_{i1}x^k_1 + a^k_{i2}x^k_2 + \cdots + \\ & a^k_{ij}x^k_j + \cdots + a^k_{in}x^k_n \\ & \cdots\cdots\cdots\cdots \\ s^k_m \geqq\ & a^k_{m1}x^k_1 + a^k_{m2}x^k_2 + \cdots + \\ & a^k_{mj}x^k_j + \cdots + a^k_{mn}x^k_n \end{aligned} \tag{21}$$

where s^k_1 refers to the supply of capital, $s^k_2 \cdots s^k_i \cdots s^k_{m-1}$ represent other types of resources which do not have inter-year transfers and s^k_m is the total fixed cost, including household consumption, and all a^{k-1}_{ij} (in the equation for s^k_1) entries, indicating additions to capital from the previous year's production, are negative. Again, all a^k_{mj} for $j \neq n$ are zero.

As in the ordinary static model, $x^k_j \geqq 0$, and the profit function

$$\begin{aligned} Z =\ & c^1_1x^1_1 + c^1_2x^1_2 + \cdots + c^1_jx^1_j + \cdots \\ & + c^1_nx^1_n + c^2_1x^2_1 + c^2_2x^2_2 + \cdots + \\ & c^2_jx^2_j + \cdots + c^2_nx^2_n + c^k_1x^k_1 + c^k_2x^k_2 \\ & + \cdots c^k_jx^k_j + \cdots + c^k_nx^k_n + \cdots \\ & + c^t_1x^t_1 + c^t_2x^t_2 + \cdots + c^t_jx^t_j + \cdots + c^t_nx^t_n \end{aligned} \tag{22}$$

is maximized. In this, Z is the maximum present value of future incomes, under the constraint that certain fixed costs and family living expenses of each are "just exactly met." Hence, each element c^k_j is a discounted quantity defined as $c^k_j = c^{\cdot k}_j \div (1 + r)^k$ where $c^{\cdot k}_j$ is the nondiscounted net revenue of the j-th activity in the k-th year. Although it is not true of this study, the objective in many studies might be maximization of capital values, with activities arranged to express values of resources at the end of the relevant period, depending on the optimum program selected.

PLANS BY DYNAMIC PROGRAMMING

Optimum farm plans for 8 consecutive years, computed by a dynamic programming model, are presented in table 11. The amount of new funds available for investment in any one year is determined by subtracting fixed charges and family living requirements from the programmed return figure of the previous year. For example, $1,000 of additional capital is available for next year if this year's return is $6,000 and fixed charges plus living costs are only $5,000. If fixed expenses and living costs are $5,500, only $500 in new funds is available for investment in the coming year. Capital and return figures are for the tenant only; plans in preceding sections illustrated these figures as an aggregate for the farm. Since the case farm is operated on a 50-50 livestock-share lease, operating capital and returns are divided equally between the tenant and landlord. Fixed and living costs are not identical for both parties. Hence, fixed and living cost deductions for plans in table 11 are unique to the tenant.

LIVING EXPENSES

Since a dynamic program of the type developed depends on the consumption needs or desires of the family, as well as on the productivity of investment alternatives, projected expenditure patterns are necessary. The farm operator and his family were asked to project their living costs for 8 years. This particular time period was arbitrarily chosen by consultation with the farm operator and his wife. Projected living expenses for the farm family are presented in table 12. Final estimates for living costs were developed from an itemized form of detailed expenses which was completed by the farm family. A home economist counselled with the family during the time the budget form was filled out. This counselling aided the farm family in appraising required expenditures for future years (e.g., expenses encountered when children begin school, future costs of replacing household equipment, etc.).

PRICE AND CAPITAL BASIS OF PLANS

Prices used for the plans in table 11 are those presented in table 2. Hence, variation between years is not considered. Too, crops assumed for each year are those from the stable cropping program established in a previous section (i.e., 115 acres of CCO-MM_3 and 157 acres of $CCSb_3$). Hence, increase in the

TABLE 11. ANNUAL EXPANSION OF A DYNAMIC FARM PLAN AS CUMULATING RETURNS ARE INVESTED.[a]

Year	Cumulative operating capital[b]	Farm plan[c]	Return	Fixed and living charges[d]	Return minus fixed and living charges (col. 5 minus col. 4)	Cumulative surplus capital
1	$4,813	Crops 45 hog litters	$6,822	$6,136	$ 686	....
2	5,499	Crops 56 hog litters	7,246	5,820	1,426	....
3	6,925	Crops 80 hog litters	8,088	7,023	1,065	....
4	7,990	Crops 80 hog litters 17 long-fed steers	8,433	7,948	485	....
5	8,475	Crops 80 hog litters 23 long-fed steers	8,540	7,293	1,247	....
6	9,579	Crops 80 hog litters 48 short-fed heifers	8,892	6,429	2,463	$ 143
7	9,579	Crops 80 hog litters 48 short-fed heifers	8,892	6,479	2,413	2,606
8	9,579	Crops 80 hog litters 48 short-fed heifers	8,892	6,479	2,413	5,019

[a] All capital and return figures are for tenant only, but the farm plan indicates total production for the farm.
[b] Operating capital does not include investment in machinery.
[c] The long-run cropping plan is established as 115 acres of $CCOMM_3$ and 157 acres of $CCSb_3$.
[d] Includes living expenses, fixed machinery depreciation, taxes, insurance, etc.

TABLE 12. PROJECTED LIVING COSTS FOR THE CASE FARM FAMILY.

Item	Living costs in dollars for years							
	1958	1959	1960	1961	1962	1963	1964	1965
1. Food purchased	1,170	1,260	1,330	1,340	1,350	1,400	1,450	1,450
2. Clothing and personals	271	400	425	425	425	450	450	450
3. Household operation	529	550	550	550	550	550	550	550
4. Repairs and minor furnishings	125	150	150	150	150	150	150	150
5. Health	729	729	829	829	829	850	850	850
6. Recreation	30	100	50	60	75	80	80	80
7. Education	32	50	50	250	60	70	70	70
8. Giving (church, charity, etc.)	185	240	195	200	200	210	210	210
9. Auto-family use	125	150	150	150	160	175	175	175
10. Income and social security taxes	458	458	458	458	458	458	458	458
Total	3,600	4,087	4,187	4,412	4,257	4,393	4,443	4,443

annual capital supply is reflected mainly in changes in livestock organization.

The initial supply of operating capital assumed for year 1 is $4,813. This amount of funds is sufficient for the tenant's share of crop production and 45 hog litters. The tenant return from this plan is $6,822. The tenant has $6,136 in fixed and living costs, leaving only $686 in additional capital to invest in the farm plan of the second year. Consequently, available capital in the second year is $5,499 ($4,813 + $686 = $5,499). The added capital is most profitably invested by increasing the number of hog litters. The resulting plan includes crops plus 56 litters. Enough added capital is available in the third year to increase hog production to the maximum of 80 litters. As also determined for other static plans developed in this study, the first three plans in table 11 indicate that profits are maximized if added capital, beyond that needed for crops, is allocated to hog production before cattle feeding. This result is, of course, based on the price levels of table 2, and, as shown in the section considering variable prices, sufficiently high prices can make cattle feeding more profitable than hogs.

The operating capital supply for year 4 is $7,990. This amount of capital is greater than available in the third year when maximum hog litters were produced. At this point, extra funds are most profitably utilized by investment in long-fed steers. Long-fed steers come into the plan before short-fed heifers because funds are more limiting than other resources, and long-fed steers give highest returns on capital. When feed supplies become limiting, however, short-fed heifers give higher total returns to the over-all mix or combination of scarce resources. (Farm plans in tables 7, 8 and 9 include short-fed heifers but no long-fed steers because plans were not restricted by capital.) Plans 4, 5 and 6 in table 11 illustrate the optimum sequence for investing increasing amounts of capital as they become available from year to year. Long-fed steers can most profitably command use of capital when it is sufficiently scarce.

Because of variations in annual fixed and living costs among years, the capital added in one year is sometimes less than the amount added in the previous year. Major changes in annual fixed charges result from new investments such as machinery and buildings. Some capital is accumulated in each year, however, until maximum capital requirements are attained in year 6. Available capital then is greater than the amount needed for full utilization of other resources. (The amount of capital available but unused is entered in the column for "cumulative surplus capital.")

Changes in livestock production between the plans for years 5 and 6 are related to forage limitations. For a capital supply of approximately $8,548, forage is not yet limiting, and 25 long-fed steers are included in the optimum plan (not shown in table 11). But beyond this capital level, forage becomes more limiting than capital. Consequently, further increases in the amount of operating capital must be invested in short-fed heifers if profits are to be maximized, because they give a higher return to forage than long-fed steers.

Optimum plans for the last 3 years of table 11 are identical. This stability in plans is due to the fact that capital is no longer limiting, and the most profitable plan consistent with other fixed resources is attained already in year 6. The optimum plan for years 6, 7 and 8 includes crops, 80 hog litters and 48 short-fed heifers. Further expansion of livestock production is prevented by the restriction that only homegrown feed is used. Surplus or unused capital at the end of 8 years is $5,019. This capital might be invested in off-farm opportunities, farm ownership or other alternatives.

Again, capital and return figures of table 11 represent only the tenant's share. Hence, while the various plans illustrate total production for the case farm, total capital and return figures for the farm are double those indicated in table 11.[6]

Potential Use of Dynamic Programming Solutions

The dynamic or multi-year farm plans just discussed provide a long-range farm and home budget for the farm family. Family goals and values are interrelated with organization of the farm business. While the technique of dynamic programming has been applied to a restricted situation, it has promise in providing more realistic farm and home plans for farm families. Some of the types of considerations which are uncovered in this approach follow.

An examination of projected family living costs in table 12 indicates the relationship of planned family expenditures to the business plan. Living costs for 1959 are $487 higher than for 1958. Although food and clothing account for nearly half of this total increase, recreation increases by $70 and giving (item #8) by $55. Thus, $125 ($70 + $55) which could be invested in the farm business is allocated to these two consumption goals. Of the $70 increase in family expenditures, $40 is for purchase of a new bicycle. Examining the returns per dollars invested in the farm plan for 1959, it is determined that the potential increase in 1959 farm income, if the $125 were invested in the farm business, is $40. Consequently, a sacrifice of $125 for recreation and giving at the outset of 1959 would yield funds for purchase of a bicycle in 1960. The investment would continue to produce $40 in subsequent years, although there is no capital limitation after year 6. The family concerned indicated that it would prefer purchase of the bicycle, however, with some profit foregone in the next year or two.

This example illustrates one of many alternative decisions between firm and household exposed for consideration by dynamic planning. Any specific cost or investment item can be similarly evaluated by the procedure.

While greater livestock production would be possible in early years of the sequence, the value placed on, or the necessity of funds for, family living items caused plans to emerge in early years which differ

[6] It should be remembered that capital requirements for hog production exceeding 45 litters are higher for the initial year of production than for succeeding years. That is, extra investment in buildings, equipment and brood sows for the last 35 litters is required only at the outset of their production. Thereafter, per-litter capital requirements are identical for all 80 litters. This condition accounts for the difference between total capital requirements (i.e., $19,158 = $9,579 x 2) for the plan in the last 3 years of table 11 and capital requirements shown for the identical plan in previous tables.

somewhat from those of later years. Also, the dynamic programming approach causes the best plan in any one year to be dependent upon the best plan in another year. Actually, the procedure defines yearly plans which are consistent with the best over-all plan for the complete span of 8 years. The criterion for the optimum over-all or 8-year plan, and thus for the best plan of any one year, is the maximization of discounted net returns over the complete span of years, subject to the restraint of meeting family living costs in each individual year. Returns of each year are discounted, at market rates of interest, back to the point of time representing initiation of the yearly plans.

In the model used, a single consumption activity or requirement was used for each year. Greater independence among investments in farm and household activities would have been attained had additional consumption activities been defined for each year, with each having a "price tag" attached to it. For example, if the family would prefer to consume one part of income from the previous year, which would become an increment of investment for the coming year, if it would not earn more than 20 percent in the business, a "price" of this level could be attached to the consumption activity. The fund available then would be reinvested in the business if it returned more than this, but consumed if it returned less. Other consumption activities could be defined with smaller "prices," each with a restraint of the relevant magnitude. The portion of income used for consumption and business investment then would be more clearly interdependent. The farm family would have to evaluate carefully the level of preference or "price" attached to particular consumption or family living items, however.

The potential for broad application of dynamic programming to individual farms depends, of course, on the availability of relevant data. Both farm business records and home accounts are essential. Too, the family concerned must project their living expenses and major household costs for future time periods. The assembly of these data should not be difficult where adequate home and farm accounts are available for the past. Perhaps the more demanding activity on the part of the family is the establishment of goals and a relative ranking of consumption outlays over time. It must be able to indicate whether a particular activity or consumption expenditure should be given priority in the next year, or at a latter time relative to the earnings possible from reinvestment of income in the farm business.