New Procedures in Estimating Feed Substitution Rates and in Determining Economic Efficiency in Pork Production

II. Replacement Rates of Corn and Soybean Oilmeal in Fortified Rations for Growing-Fattening Swine on Pasture

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II. Replacement Rates of Corn and Soybean Oilmeal in Fortified Rations for Growing-Fattening Swine on Pasture[1]

BY EARL O. HEADY, DAMON V. CATRON, DEAN E. MCKEE, GORDON C. ASHTON AND VAUGHN C. SPEER

A previous bulletin reported results from an experiment designed to predict substitution rates and economic optima in corn/soybean oilmeal rations for growing and fattening hogs in drylot.[2] Principles and analytical models were included which illustrate that the least-cost ration depends both on (1) the marginal rate of substitution between feeds and (2) the ratio of feed prices. These basic concepts will not be repeated in this bulletin.

Since more hogs are farrowed in spring than in fall, the research reported in this study was conducted for growing and fattening hogs raised on pasture. Like the drylot study, the objectives of the pasture experiment were to estimate: (1) the production function, (2) the substitution rate between corn and soybean oilmeal at different points on the production surface, (3) the least-cost ration for different soybean oilmeal/corn price ratios, (4) the relationship between the rate of hog gains and the input of corn and soybean oilmeal and (5) the proportion of the years in which a least-cost feeding system results in greater profits than a least-time feeding system. Substitution between major classes of feed such as corn and soybean oilmeal is possible mainly where the rations are fortified with appropriate quantities of trace minerals (as well as antibiotics in the case of drylot feeding). These fortifying elements have been included in the rations of this study.

DESCRIPTION OF THE EXPERIMENTS

Two experiments were conducted cooperatively by the Department of Animal Husbandry and the Department of Economics and Sociology to obtain data for estimating feed relationships for hogs fed on pasture. The first experiment, A. H. 597, was conducted during the summer of 1953. The second experiment, A. H. 597A, was conducted during the summer of 1954. Both experiments were conducted on an alfalfa pasture. The data from the two experiments were combined for the purposes of this study.

Both experiments were randomized complete block designs and included 12 treatment combinations with three replications each. Treatment combinations consisted of six rations, an antibiotic treatment and an antibiotic check. The rations were: 8 percent, 10 percent, 12 percent, 14 percent, 16 percent and 18 percent protein. The antibiotic treatment consisted of crystalline chlorotetracyline (aureomycin) fed at the rate of 5 mg. per pound of ration. The rations were composed of ground yellow corn and solvent-extracted soybean oilmeal fortified with dicalcium phosphate, calcium carbonate, salt, trace minerals and vitamins (table 1). The experimental unit was an individual hog, and each hog received the same ration throughout the entire experiment.

The hogs were fed individually in portable field units. Each field unit consisted of three pens equipped with individual self-feeders and waterers. The units were aligned side by side on pasture. They were moved each Monday, Wednesday and Friday during the experiment. The original order of the units on the field was maintained at all times.

The pasture sward for experiment 597 was composed of a mixture of alfalfa and bromegrass. Mower clipping was used to maintain a maximum herbage height of about 8 inches; the pasture area was clipped several times during the trial to prevent excessive growth.

[1] Project 1135, Iowa Agricultural and Home Economics Experiment Station.

[2] Heady, Earl O., Woodworth, Roger, Catron, Damon V. and Ashton, Gordon C. New procedures in estimating feed substitution rates and determining economic efficiency in pork producton. Iowa Agr. Exp. Sta. Res. Bul. 409.

TABLE 1. COMPOSITION OF RATIONS FED IN EXPERIMENTS A. H. 597 AND A. H. 597A (POUNDS OF EACH INGREDIENT INCLUDED IN 100 POUNDS OF FEED).

Ingredients	Percent protein: A. H. 597						Percent protein: A. H. 597A					
	18	16	14	12	10	8	18	16	14	12	10	8
Ground yellow corn*	71.60	78.75	83.70	88.85	91.95	97.00	73.05	78.50	83.95	89.35	94.75	97.25
Solvent soybean oilmeal	25.90	18.70	13.70	8.40	5.30	0.15	24.50	19.00	13.50	8.00	2.50	....
Dical. phosphate	0.95	1.00	1.05	1.30	1.30	1.40	0.70	1.30	1.40	1.60	1.70	1.70
Calcium carbonate	1.00	1.00	1.00	0.90	0.90	0.90	1.20	0.65	0.60	0.50	0.50	0.50
Salt	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Trace minerals	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Total pounds	100	100	100	100	100	100	100	100	100	100	100	100

*The protein content of the corn fed in both experiments was 8.2 percent.

Moisture was sufficient in both seasons so that the herbage remained of good quality over the experimental period.

Treatment combinations and the pigs were randomly assigned to pens within a block. Of the three replications, one included females, and the other two included males. The hogs were weighed every second week while they were on the experiment and were removed from the experiment as each hog reached 200 pounds.

The breeding of the hogs used in experiment A. H. 597 was Duroc x Poland China x Landrace x Duroc and Poland China x Landrace x Duroc. A Poland China x Landrace x Duroc cross was used in experiment A. H. 597A. Thirty-six hogs were required for each experiment, a total of 72 hogs for both experiments.

ESTIMATION OF THE PRODUCTION FUNCTION

Two steps have been followed in estimating the production function. First, three alternative types of functions have been fitted to all observations of the two experiments.[3] These functions are denoted as *over-all functions*. Second, each of the three types of functions have been fitted to the observations on each of the six rations separately. The latter functions are called *individual ration functions*. Interest is mainly in the over-all functions; they express the relationship between hog gains and the input of any one of many combinations of the feeds. Individual ration functions express the relationship between hog gains and feed input when feeds are held in fixed proportions. The input-output curves for different rations varied in fixed proportions can be readily obtained from the over-all function. Therefore, comparisons of the feed-gain relationship estimated by the over-all function with that estimated from the individual ration function provides a simple means of checking the reliability of the over-all function.

The production functions express total gain beyond weaning as a function of total feed consumption beyond weaning. Experimental observations were taken on the consumption of feed and the amount of gain over 2-week intervals. The interval observations were progressively totaled over the entire feeding period to obtain a series of cumulative summations of gain, corn consumption and soybean oilmeal consumption beyond weaning for each hog. The over-all production functions were then fitted to the 72 series of observations. Each individual ration function is fitted to 12 such series of observations.

AUTOCORRELATION

Fitting of the functions for the cumulative series introduces a problem of autocorrelation. The different observations for each hog are not independent since (1) the second observation taken on a hog is the sum of the feed consumption and gain over the first and second 2-week intervals; (2) the third observation is the sum of the feed consumption and gains in each of three 2-week periods, etc. Although the series of observations taken on a hog is itself autocorrelated, it is independent of the series of observations taken on other hogs. Since the over-all production function is fitted to all observations in each series, the autocorrelation coefficient for the entire collection of data is likely to have a value greater than zero.

The presence of autocorrelation in the observations does not present problems in predicting the relationship between the dependent and the independent variables, but it does introduce problems in making tests of significance. The effect of autocorrelation is to reduce the number of effective observations to which the function is fitted. In other words, the number of degrees of freedom used for tests of significance of uncorrelated series is fewer than when autocorrelation is present.

Procedures are available for approximating the effective number of degrees of freedom in autocorrelated series.[4] However, the necessity of calculating the autocorrelation coefficient and approximating the effective number of observations may be avoided by basing the tests of significance on a minimum number of effective observations to which the series would be reduced by autocorrelation.

Since the observations taken on different animals are independent, the minimum number of effective observations may be regarded as equal to the number of hogs from which observations were taken. The minimum number of effective observations is 72 for the over-all function and 12 for the individual ration functions. If the tests are significant on the basis of the minimum number of effective observations, the null hypothesis may be rejected. If the tests are not significant, the null hypothesis cannot be accepted without further testing. In the latter case, the test must be conducted on the basis of the actual number of observations used, disregarding autocorrelation for the moment. If the test still is not significant at an acceptable probability level with the greater number of degrees of freedom, the null hypothesis may then be accepted.[5]

CONSIDERATIONS IN THE CHOICE OF THE FUNCTION

Choice of the appropriate form of the relation between feed inputs and hog gains should be related to the nutritional logic underlying the problem. The protein requirement of hogs relative to their requirement for carbohydrate declines from weaning to market weight. Young pigs in a stage of rapid growth require

[3]Analyses of variance are presented in table 2 for both experiments and for two weight intervals. They indicate no significant antibiotic effects for daily gain or feed consumption under conditions given. For feed consumption per pound of gain, antibiotic effects were significant only for the initial-to-200-pound weight interval in experiment 597. Similar results appear in table 3 for pooled data of the two experiments. Antibiotic effects upon feed per 100 pounds gain are significant at the 5-percent probability level for only the linear term for the initial-to-the-200-pound weight interval. In general, the analyses of variance do not support the hypothesis that the antibiotic treatment and the check lots constitute separate populations. Consequently, data from treatment and check lots are pooled for estimation of the production function and each protein level includes observations from 12 hogs.

[4]Tintner, Gerhard. Econometrics. John Wiley and Sons, Inc., New York. 1952. pp. 240-252.

[5]The effective number of observations need to be approximated only if the tests are not significant on the basis of the minimum effective number of observations but are significant on the basis of the actual number of observations taken. For example, if the calculated "t" for a regression coefficient in the over-all function were 2.616, the regression coefficient would be significant at the 0.01 level of probability on the basis of 500 degrees of freedom. On the basis of the minimum number of effective observations for the over-all function, 72, the regression coefficient would not be significant. If the autocorrelation reduces the number of effective observations to less than 125, the null hypothesis would be accepted at the 0.01 level of probability. If the effective number of observations is 126 or greater, the null hypothesis would be accepted.

TABLE 2. INDIVIDUAL SUMMARIES OF MEAN SQUARES FOR EXPERIMENTS 597 AND 597A.

Source of variation	Experiment 597 D.F.	Experiment 597 Mean squares Daily gain	Daily feed	Feed/lb. gain	Experiment 597A D.F.	Experiment 597A Mean squares Daily gain	Daily feed	Feed/lb. gain
				Initial weight to 75 pounds				
Replicate	2	0.0016	0.3101	0.1056	2	0.0603	0.4848	0.1896
Protein level	5	0.8522	2.8067	6.5172	5	0.7934	1.3989	1.8664
Linear component	1	3.4110	10.2024	20.1655	1	3.5733	5.1637	8.4689
Quadratic component	1	0.7005	2.7961	8.9280	1	0.2476	1.3714	0.6209
Remainder	3	0.0498	0.3451	1.1642	3	0.0488	0.1531	0.0808
Antibiotic	1	0.0001	0.0093	0.5160	1	0.0004	0.0051	0.0004
Protein level x antibiotic	5	0.0320	0.1237	0.6345	5	0.0422	0.4086	0.0241
Experimental error	22	0.0304	0.2794	0.5516	22	0.0212	0.1288	0.0524
Total	35	....	....	....	35	....	....	....
				Initial weight to 200 pounds				
Replicate	2	0.0188	0.2122	0.0548	2	0.1526	1.9210	0.0054
Protein level*	4	0.1121	0.3863	0.1453	5	0.4445	2.8237	0.2494
Linear component	1	0.1893	0.7958	0.1288	1	1.5616	10.1308	0.6380
Quadratic component	1	0.2294	0.3060	0.4488	1	0.6167	3.7891	0.3236
Remainder	2	0.0149	0.2218	0.0018	3	0.0148	0.0663	0.0951
Antibiotic	1	0.0104	0.0832	0.1307	1	0.0148	0.6861	0.0910
Protein level x antibiotic	4	0.0124	0.0764	0.0119	5	0.0342	0.3692	0.0277
Experimental error	17	0.0135	0.1729	0.0248	22	0.0152	0.1632	0.0314
Total†	28	....	....	....	35	....	....	....

*There were no data for the 8-percent protein rations after the pigs attained 75 lbs. weight in Experiment 597.
†Values for one pig estimated in Experiment 597.

TABLE 3. COMBINED SUMMARIES OF MEAN SQUARES FOR EXPERIMENTS 597 AND 597A.

Source of variation	D.F.	Mean squares Daily gain	Daily feed	Feed/lb. gain	D.F.	Mean squares Daily gain	Daily feed	Feed/lb. gain
		Initial weight to 75 pounds				Initial weight to 200 pounds		
Replicate	5	0.0961	0.3860	0.7273	5	0.0743	1.0672	0.0765
Protein level*	5	1.5776	3.8398	7.1813	4	0.2837	1.1735	0.2508
Linear component	1	6.9834	14.9413	27.3855	1	0.5240	2.8060	0.2585
Quadratic component	1	0.8905	4.0419	7.1289	1	0.5612	1.8042	0.6254
Remainder	3	0.0047	0.0719	0.4641	2	0.0248	0.0419	0.0596
Antibiotic	1	0.0055	0.0003	0.2726	1	0.0029	0.1949	0.1344
Protein level x antibiotic	5	0.0402	0.2498	0.3848	4	0.0262	0.2152	0.0034
Experimental error	55	0.0298	0.2225	0.3802	44	0.0143	0.1640	0.0304
Total†	71	....	....	....	58	....	....	....

*There were no data for the 8-percent protein rations after the pigs attained 75 pounds weight.
†Values for one pig estimated for the initial-to-200-pound period.

relatively large amounts of protein for tissue building. As pigs mature and approach heavier weights, the nutrient requirement for growth declines, and more of the nutrients are required for the production of finish. Therefore, as the finishing process becomes more and more prominent relative to the growth process, the requirement for carbohydrate feeds becomes greater relative to the requirement for protein feeds. The shift in the nutrient requirements of the hog from weaning to maturity implies a decline in the rate at which soybean oilmeal substitutes for corn.

The foregoing considerations provide a basis for specifying general characteristics of the production surface and the type of mathematical equation needed in predictions: The production function should allow changing elasticities of production as hog weight increases. The elasticity of production for corn should increase with increasing hog weight, while the elasticity for soybean oilmeal should decrease. Corn should be allowed to become a limiting factor of production.

OVER-ALL FUNCTIONS

Three types of equations examined as alternatives for expressing the over-all relationship between hog gains and the input of corn and soybean oilmeal are: (1) a quadratic, (2) a modified form of the quadratic—quadratic root function—and (3) a power function. The quadratic equation is considered because it allows changing elasticities of production. The modified quadratic, with the squared terms replaced with root terms, results in a slow decline in marginal productivity of feed as feed inputs reach higher levels. The power or Cobb-Douglas function expresses both feeds as limitational, but assumes constant elasticities of production and linear isoclines through the origin. In the functions which follow, C refers to pounds of corn, P refers to pounds of soybean oilmeal and Y refers to pounds of gain, all measured beyond weaning.

(1) Quadratic: $$Y = -1.7536 + 0.2988C + 0.9828P - 0.00003012C^2 - 0.003880P^2 - 0.0001684CP$$

(2) Square root: $$Y = -17.4939 + 0.2472C + 0.03568P + 1.4249\sqrt{C} + 6.6133\sqrt{P} - 0.08138\sqrt{C}\sqrt{P}$$

(3) Cobb-Douglas: $$Y = 0.5493C^{0.8426}P^{0.1604}$$

The quadratic function explains 98.3 percent of the variance in hog gains (table 4), the square root function explains 98.1 percent, while the power function explains only 94.2 percent of gain variance. Using the minimum number of degrees of freedom, the linear and squared terms of equation 1 are significant at the 0.01 and 0.05 probability levels. The cross-product term is acceptable at a probability level between 0.10 and 0.15. The linear term for P in equation 2 is significant only at a probability level greater than 0.30. Both terms for the Cobb-

TABLE 4. CORRELATION COEFFICIENTS, STANDARD ERRORS AND "t" VALUES FOR OVER-ALL FUNCTIONS. STANDARD ERRORS AND "t" VALUES IN ORDER GIVEN IN EQUATIONS.

Equation	n	R^2	Standard errors					Value of t				
			s_{b1}	s_{b2}	s_{b3}	s_{b4}	s_{b5}	t_{b1}	t_{b2}	t_{b3}	t_{b4}	t_{b5}
1	521	0.983	0.0091	0.0348	0.00001	0.00026	0.00011	32.89*	28.25*	3.00*	15.06*	1.55‡
2	521	0.981	0.0146	0.0353	0.3760	0.5107	0.0339	16.89*	1.01§	3.79*	12.95*	2.40†
3	521	0.942	0.0213	0.0072	....	....	....	68.78*	22.41*	....	....	....

* $p < 0.01$
† $0.05 > p > 0.01$
‡ $0.15 > p > 0.10$
§ $p > 0.30$

TABLE 5. CORRELATION COEFFICIENTS, STANDARD ERRORS OF REGRESSION COEFFICIENTS AND "t" VALUES FOR INDIVIDUAL RATION FUNCTIONS.

Equation	R^2	Standard errors		t values	
		s_{b1}	s_{b2}	t_{b1}	t_{b2}
		$Y = a + b_1C + b_2C^2$			
8-percent ration	0.965	0.0288	0.000048	10.06	0.16
10-percent ration	0.983	0.0187	0.000031	17.94	1.30
12-percent ration	0.992	0.0136	0.000025	31.95	5.60
14-percent ration	0.991	0.0160	0.000032	31.27	7.43
16-percent ration	0.976	0.0997	0.000210	5.72	1.87
18-percent ration	0.983	0.0232	0.000052	25.28	8.08
		$Y = a + b_1C + b_2\sqrt{C}$			
8-percent ration	0.965	0.0412	1.2257	7.77	0.64
10-percent ration	0.982	0.0268	0.8159	10.35	1.26
12-percent ration	0.992	0.0199	0.5691	13.04	5.06
14-percent ration	0.991	0.0242	0.6614	9.20	6.78
16-percent ration	0.975	0.0406	1.1010	3.13	6.54
18-percent ration	0.987	0.0298	0.7772	3.24	10.47
		$Y = a\,C^{b_1}$			
8-percent ration	0.932	0.0424	....	27.24	....
10-percent ration	0.978	0.0165	....	62.22	....
12-percent ration	0.947	0.0256	....	41.63	....
14-percent ration	0.987	0.0122	....	79.58	....
16-percent ration	0.939	0.0252	....	37.73	....
18-percent ration	0.971	0.0154	....	56.19	....

Douglas function, equation 3, are significant at an 0.01 probability level.

The sum of the elasticities for corn and soybean oilmeal in the power function is equal to 1.003, indicating slightly increasing returns to proportional increases in the input of the two feeds. This relationship appears unlikely in pork production, and, for the pasture data, the function appears to overestimate gains for higher levels of feed inputs. (This characteristic holds true only for the particular observations of this study and is not a characteristic of the same function fitted to other data.) The quadratic and the quadratic root functions express decreasing returns to proportional increases in both feeds.

COMPARISONS BETWEEN OVER-ALL FUNCTIONS

Relationships among predictions from the three functions are shown in fig. 1 for a 12-percent protein ration. Similar estimates are obtained from all three functions up to a feed input of about 250 pounds. Beyond 250 pounds of feed, the curve estimated by the Cobb-Douglas function rises above the curves estimated by the other functions. The quadratic and quadratic root functions give very similar results throughout the entire range of the curves. Below feed inputs of 350 pounds, the curve for the quadratic root function lies below the curve for the quadratic root function. Beyond feed inputs of 350 pounds, the positions of the two curves are reversed.

The relationships between the three functions at other protein levels are similar. At lower protein levels, the curves from the two quadratic-type functions are more nearly linear and correspond more closely to the estimates obtained from the Cobb-Douglas function. At higher protein levels, the curves for the quadratic type functions have greater curvature and fall away from the Cobb-Douglas curve more rapidly. The quadratic functions produce curves that are most consistent with the scatter diagrams at all protein levels.

INDIVIDUAL RATION FUNCTIONS (TABLE 5)

Since the over-all functions have been fitted to all observations on the production surface, they might result in "abnormal" predictions for individual rations. Single-variable functions express the result of a single ration without encountering some of the "joint relationships" inherent in the over-all functions, and are, therefore, compared with the over-all functions. These com-

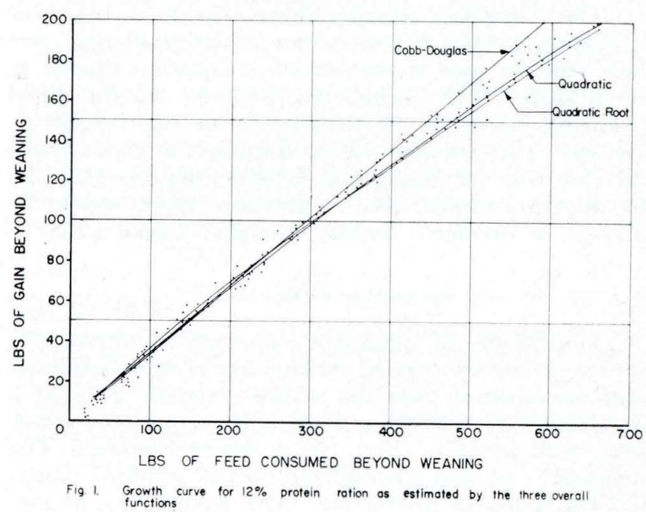

Fig. 1. Growth curve for 12% protein ration as estimated by the three overall functions

parisons show that "spurious" predictions do not arise from the over-all functions. In these graphic comparisons, corn alone is the independent variable. In any one ration, the ratio of corn to soybean oilmeal is fixed; an increase in corn consumption must be accompanied by a constant proportion of soybean oilmeal. There is no necessity for measurements to include both feeds in the individual ration function.

The individual ration functions, paralleling the three over-all functions, with gain as the dependent variable and corn as the independent variable, are as follows:

Quadratic functions:

(4) 8-percent protein ration,
$$Y = -5.102 + 0.290C + 0.000009C^2$$

(5) 10-percent protein ration,
$$Y = -1.200 + 0.335C - 0.00004C^2$$

(6) 12-percent protein ration,
$$Y = -3.062 + 0.433C - 0.0001C^2$$

(7) 14-percent protein ration,
$$Y = -1.982 + 0.500C - 0.0002C^2$$

(8) 16-percent protein ration,
$$Y = -1.664 + 0.570C - 0.0004C^2$$

(9) 18-percent protein ration,
$$Y = 1.260 + 0.586C - 0.0004C^2$$

Square root functions:

(10) 8-percent protein ration,
$$Y = -0.189 + 0.320C - 0.784\sqrt{C}$$

(11) 10-percent protein ration,
$$Y = -5.854 + 0.278C + 1.028\sqrt{C}$$

(12) 12-percent protein ration,
$$Y = -14.831 + 0.260C + 2.880\sqrt{C}$$

(13) 14-percent protein ration,
$$Y = -20.064 + 0.223C + 4.484\sqrt{C}$$

(14) 16-percent protein ration,
$$Y = -31.031 + 0.127C + 7.200\sqrt{C}$$

(15) 18-percent protein ration,
$$Y = -32.685 + 0.096C + 8.137\sqrt{C}$$

Cobb-Douglas functions:

(16) 8-percent protein ration,
$$Y = 0.111C^{1.156}$$

(17) 10-percent protein ration.
$$Y = 0.272C^{1.026}$$

(18) 12-percent protein ration,
$$Y = 0.258C^{1.067}$$

(19) 14-percent protein ration,
$$Y = 0.505C^{0.967}$$

(20) 16-percent protein ration,
$$Y = 0.598C^{0.948}$$

(21) 18-percent protein ration,
$$Y = 1.000C^{0.865}$$

For all three equations, estimates for the 8-percent ration show an increasing marginal productivity of feed. The quadratic and square root functions show a decreasing marginal productivity for rations with 10 percent or more of protein. The Cobb-Douglas function shows increasing marginal productivity through the 12-percent protein ration.[6] Increasing marginal feed productivity for low protein rations may be an effect of pasture.

[6]Only equation 17 of the single ration power functions has an elasticity which does not differ significantly from one ($P>0.05$).

Young pigs consume very little forage, but, as they mature, they consume increasingly greater amounts. Hence, with the low palatability of a low-protein ration, small pigs may obtain insufficient amounts of protein from forage. As they grow, however, forage intake and, hence, gain per pound of concentrates may increase sharply, even for low-protein rations. Forage then becomes a substitute source of protein for hogs obtaining a small proportion of soybean oilmeal in the concentrate ration. However, this substitution is possible mainly as the hog grows. The tendency to substitute forage protein for concentrate protein is less with rations high in protein because of their greater palatability and nutritional "completeness." This phenomena would not have been expressed if feed value of forages could have been measured and used in predictions.

COMPARISON OF OVER-ALL AND SINGLE-VARIABLE ESTIMATES

After examination of the various statistics for the three over-all functions, the quadratic equation (1) was selected as the best estimator for the production surface. The Cobb-Douglas over-all function (3) was eliminated because of the smaller proportion of the gain variance explained and the greater algebraic restrictions imposed by its logarithmic form. Square root over-all function 2 provides estimates highly similar to the quadratic function. However, since it explains a slightly lower portion of variance in gains and has a relatively greater standard error for the P terms, it was rejected in favor of the quadratic function.[7] Hence, the text comparisons which follow compare estimates of single-line, input-output curves derived from over-all and single-variable equations for the latter functions.

"Growth curves" for six rations estimated by the single-variable and the over-all quadratic functions are shown in figs. 2 through 7. Similar curves are obtained from the estimates of the two types of quadratic functions. At the 8-percent, 10-percent and 12-percent protein levels, the curve estimated from the over-all quadratic function is almost identical to the curve estimated from the function fitted to each ration separately. The curves for the 14-percent, 16-percent and 18-percent protein levels are similar up to feed inputs of about 500 pounds. Beyond this, the curve estimated by the over-all function has slightly greater slope than the curve estimated by the individual ration function for 14-percent and 16-percent protein levels; the reverse is true for the 18-percent protein level.

INTERVAL ESTIMATES FROM COBB-DOUGLAS (TABLE 6)

Farmers normally change rations only two or three times over the growing-fattening period. Since the isoclines for the Cobb-Douglas function are linear, passing through the origin, they provide estimates of such "average rations." Hence, if the marginal rate of substitution for any isoquant is equated to the price ratio

[7]While the regression coefficient for the cross-product term was significant at a probability level greater than 0.10 but less than 0.15, it has been retained in over-all quadratic function 1 since it adds some precision to estimates.

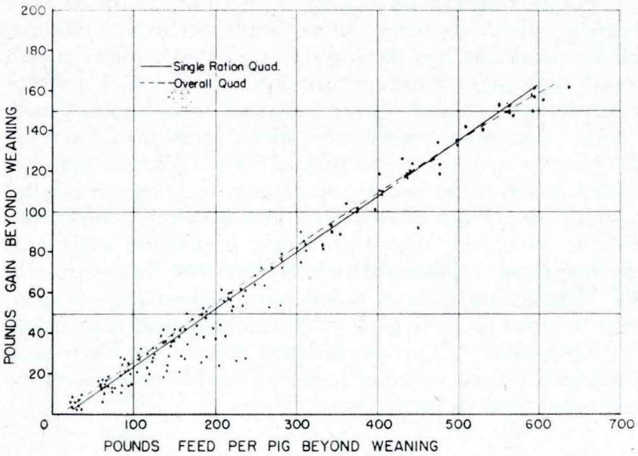

Fig. 2. Growth curves for the 8% protein ration estimated from individual ration and the several quadratic functions

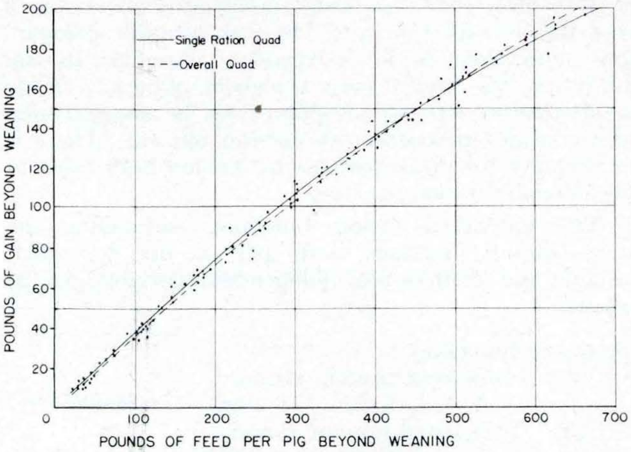

Fig. 5. Growth curves for the 14% protein ration estimated from the individual ration and overall quadratic functions

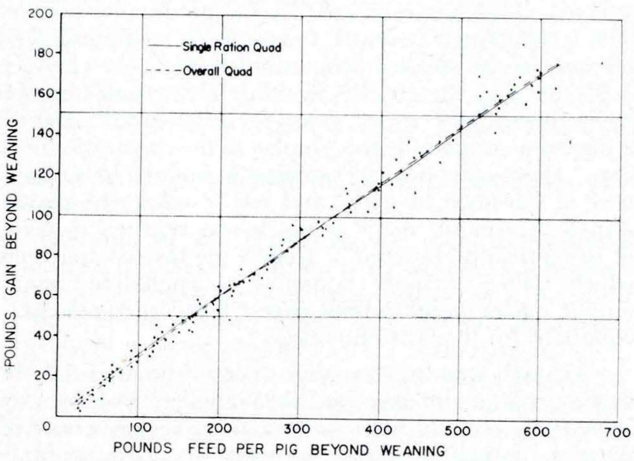

Fig. 3. Growth curves for the 10% protein ration estimated from the individual ration and overall quadratic functions

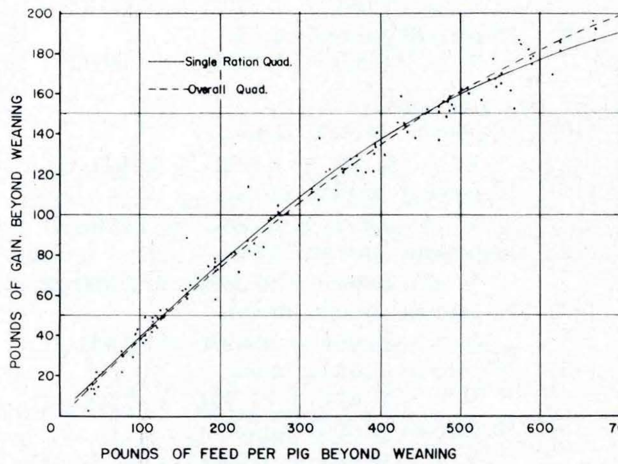

Fig. 6. Growth curves for the 16% protein ration estimated from the individual ration and overall quadratic functions

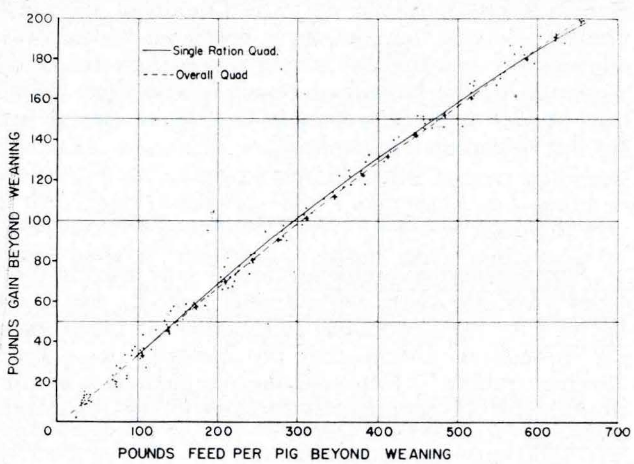

Fig. 4. Growth curves for the 12% protein ration estimated from the individual ration and overall quadratic functions

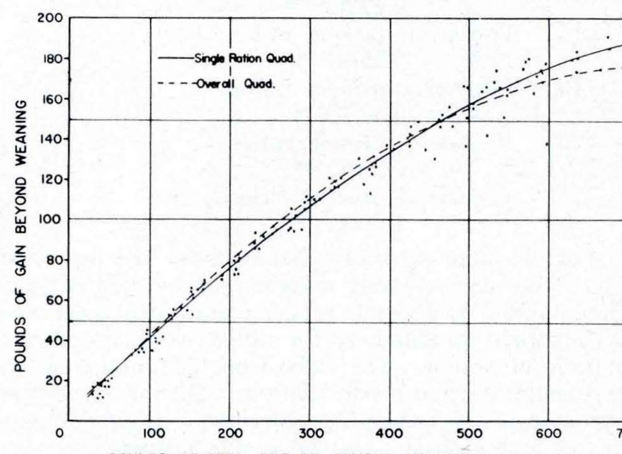

Fig. 7. Growth curves for the 18% protein ration estimated from the individual ration and overall quadratic functions

TABLE 6. CORRELATION COEFFICIENTS, STANDARD ERRORS AND "t" VALUES FOR INTERVAL COBB-DOUGLAS FUNCTIONS.

Equation	R^2	Standard errors		t values	
		s_{b1}	s_{b2}	t_{b1}	t_{b2}
Weaning to 75 lb. liveweight	0.824	0.0359	0.0196	25.28	13.77‡
75 lb. to 150 lb. liveweight	0.887	0.0236	0.0125	34.19	11.28‡
150 lb. to 200 lb. liveweight	0.734	0.0553	0.0211	17.85	1.28*

*Significant at 20-percent level.
‡Significant at 1-percent level.

of the two feeds, it indicates the ration which, on the average, is the least-cost one for the particular gain interval. A quadratic function also provides linear isoclines. However, since they do not pass through the origin, a single "average ration for a weight interval" cannot be specified for marginal rates of substitution predicted from individual gain isoquants. Instead, each gain isoquant within a gain interval would specify a different ration.

If the ration specified for one gain isoquant is used for all other unit gains, the ration so selected need not be the least-cost feed combination for the entire gain interval. For this reason, a Cobb-Douglas function provides useful estimates for the purposes at hand, if it can be accepted statistically. Since it appeared less satisfactory than other functions for the over-all surface, an attempt was made to predict three interval functions; and hence, to eliminate "overestimates" of gain at high feed inputs. Another reason for this attempt was to provide "average rations" for three gain intervals and to conform with the normal practice of changing rations two or three times during the growing-fattening period.

In fitting these interval Cobb-Douglas functions, the observations have been divided into the following liveweight groups; (1) weaning to 75 pounds, (2) 75 pounds to 150 pounds and (3) 150 pounds to 200 pounds. A separate function was fitted to each interval over the observations from all rations. The estimated relations are:

(22) Weaning to 75 pounds:
$$Y = 0.3350C^{0.9087}\ P^{0.2704}$$

(23) 75 pounds to 150 pounds:
$$Y = 0.6543C^{0.8072}\ P^{0.1408}$$

(24) 150 pounds to 200 pounds:
$$Y = 0.3127C^{0.9875}\ P^{0.0270}$$

The elasticity of production for soybean oilmeal declines, from low weights to higher weights, as expected. However, the elasticity of production for corn falls and then rises, instead of consistently rising from low weights to high weights as expected. The sum of the elasticities of production for the two feeds are 1.1791 for the first interval, 0.9480 for the second and 1.0145 for the third. This relationship—increasing feed productivity followed by decreasing feed productivity and then increasing feed productivity—is inconsistent with known biological conditions.

While the interval Cobb-Douglas approach gave satisfactory results in estimating "average rations" to be fed over a gain interval in the earlier drylot study,[8] it does not appear to be appropriate for the pasture data. The quadratic over-all function again appears to be the best choice among the various alternative functions examined, although modifications must be made in its use for determining rations to be used as "averages over gain intervals."

PRODUCTION SURFACE ESTIMATES

The pork production surface for corn and soybean oilmeal, based on equation 1 is shown in fig. 8. Consumption of corn and soybean oilmeal is measured by the vertical distance of the surface. The gains in hog weight, between weaning and market weight, follow a path over the face of the surfaces. The location of the path upon the surface is determined by the ration fed.

A ration consists of a fixed combination of corn and soybean oilmeal and represents a vertical slice of the surface through the origin. The ration is represented by a straight line drawn in the horizontal or feed plane of the surface passing through the origin of the graph. The growth curve for a particular ration is the vertical distance between the ration line and the face of the surface. The growth curve and the ration line for the 18-percent protein ration are shown in fig. 8.[9] The 18-percent ration growth curve traces the path of the hog gains over the surface throughout the production period. Each point along the ration line in the feed plane measures the total consumption of the two feeds from weaning. Each point on the growth curve measures the total gain in weight associated with the feed quantity. The slope of the ration growth curve represents the marginal productivity of feed for the particular quantity and combination of feeds represented.

[8]Heady, E. O., Woodworth, R., et al., op. cit.

[9]All portions of the surface outside the limits of 8- and 18-percent rations are beyond the limits of observations in this study. Hence, the appropriate surface is actually a "wedge" bounded by the 8- and 18-percent ration lines.

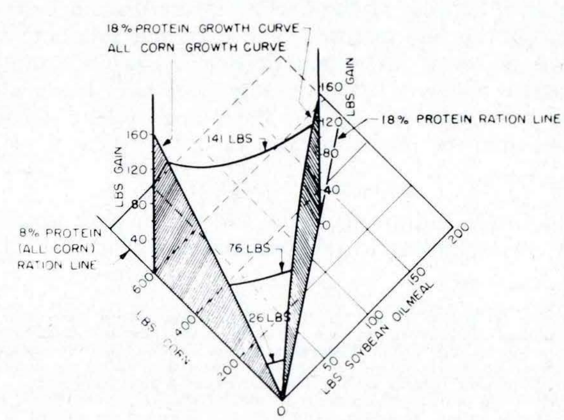

Fig. 8. Production surface for hogs on pasture, based on equation 1. (Contours on surface are gain isoquants.)

TABLE 7. TOTAL GAIN BEYOND WEANING AND MARGINAL PRODUCTIVITIES OF A POUND OF RATION FOR VARIOUS PROTEIN RATIONS.

Pounds of feed consumed beyond weaning	Total gain beyond weaning						Marginal productivity of feed*					
	Percent protein in the ration						Percent protein in the ration					
	8	10	12	14	16	18	8	10	12	14	16	18
50	12.54	13.67	15.76	17.30	18.93	20.64	0.296	0.334	0.399	0.461	0.529	0.611
100	26.70	28.90	32.91	35.72	38.64	41.55	0.293	0.330	0.390	0.447	0.503	0.568
150	40.72	43.96	49.70	53.54	57.36	60.96	0.290	0.326	0.382	0.433	0.477	0.526
200	54.60	58.84	66.13	70.74	75.10	78.88	0.287	0.322	0.374	0.418	0.452	0.484
250	68.34	73.54	82.20	87.32	91.87	95.31	0.284	0.318	0.365	0.405	0.426	0.442
300	81.95	88.06	97.90	103.30	107.65	110.24	0.282	0.314	0.357	0.390	0.400	0.400
350	95.41	102.39	113.25	118.65	122.45	123.68	0.279	0.311	0.349	0.376	0.375	0.358
400	108.74	116.55	128.24	133.38	136.27	135.63	0.276	0.309	0.340	0.362	0.349	0.316
450	121.92	130.52	142.86	147.50	149.11	146.09	0.273	0.303	0.332	0.348	0.323	0.274
500	134.97	144.32	157.13	161.01	160.97	155.06	0.270	0.299	0.324	0.333	0.298	0.232
550	147.88	157.93	171.03	173.90	171.85	162.53	0.267	0.295	0.316	0.319	0.272	0.190
600	160.65	171.36	184.57	186.18	181.75	168.51	0.264	0.291	0.307	0.305	0.246	0.148
650	173.29	184.61	197.76	197.84	190.67	173.00	0.261	0.287	0.299	0.291	0.221	0.105
700	185.78	197.68	210.58	208.88	198.61	176.00	0.258	0.283	0.291	0.276	0.195	0.063

*Added gain resulting from an added pound of ration. All figures predicted as derivatives of equation 1.

MARGINAL PRODUCTIVITY OF FEEDS IN FIXED PROPORTIONS

Predicted total gains beyond weaning and the marginal productivity of feed at several levels of total feed consumption are shown in table 7 for six rations. Marginal productivities are calculated on the basis of a proportional increase in the consumption of both corn and soybean oilmeal. In other words, the marginal productivities show the increase in hog weight from a 1-pound increase in the quantity of ration consumed.

The change in nutrient requirements as the hog approaches maturity is partly reflected in the total gains for each ration (table 7). Fifty pounds of an 8-percent protein ration produce 12.54 pounds of gain; 50 pounds of an 18-percent protein ration produce 20.64 pounds of gain. The higher protein ration supplies more of the protein necessary for tissue building and growth at low weights. However, 700 pounds of the 8-percent protein ration produce 185.78 pounds of gain, while the same amount of 18-percent protein ration produces only 176 pounds of gain. The high-protein ration does not supply a sufficient amount of carbohydrate for production of fat at later stages of growth.[10] These differences are brought out very clearly by the marginal productivity figures. Up to a total feed intake of 250 pounds, marginal feed productivity is highest with an 18-percent ration. At the 300-pound feed level, an additional pound of the 16-percent ration has the same marginal productivity as an 18-percent ration. The marginal productivity of the 18-percent protein ration declines from 0.611 at the 50-pound feed level to 0.063 at the 700-pound feed level because of the decline in the protein requirements as the hog matures. The marginal productivities of rations lower in protein decline less rapidly and do not reach as low a level, although they have lower marginal productivities at the beginning of the feeding period than the 18-percent ration.

ISO-PRODUCT CONTOURS

Figure 9, a drawing of the contour or pork isoquant map corresponding to the production surface shown in fig. 8 is estimated from quadratic function 1. Contours or pork isoquants have been derived for 26, 76 and 141 pounds of gain beyond weaning. The equation for the iso-product contours, equation 25, is obtained by solving the production function for corn (C) in terms of soybean oilmeal (P) and gain (Y). Setting Y equal to the desired amount of gain and assuming a series of values for soybean oilmeal, the associated quantities of corn to produce the given amount of gain can then be calculated.

$$(25)\quad C = 4960.36 - 2.7961P \pm (-16{,}600.2656)\,[-0.00000044P^2 + 0.00001774P + 0.08907733 - 0.00012048\,Y]^{1/2}$$

The quantity of corn and soybean oilmeal required to produce 26, 76 and 141 pounds of gain beyond weaning also has been determined from the single-variable equations 4 through 9 for each of the six rations. These feed quantities have been plotted in fig. 9. (In every instance the quantities estimated from the individual ration functions fall very close to the contours estimated from the over-all equation.) The close agreement between the estimates from the individual ration functions and the over-all function is further proof that the over-all equation provides reliable predictions of the relationship expressed within the experimental data.

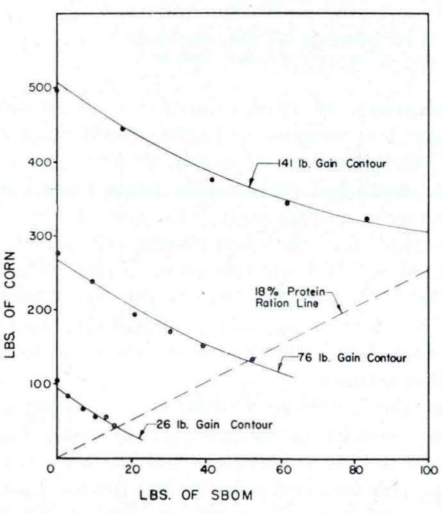

Fig. 9. Iso - product contours for pasture fed swine

[10]The first 50 pounds of a 12-percent ration produce more gain than the same amount of an 8-percent ration, but less than would be produced with the first 50 pounds of an 18-percent ration. Seven hundred pounds of the 12-percent protein ration produced more gain than either the 8-percent or the 18-percent protein ration. The 12-percent protein ration meets the relatively high protein requirements at early stages of growth better than the 8-percent ration. In terms of total gain, it does not do as well as the 18-percent ration. However, at heavier weights, the relatively higher carbohydrate requirement of the mature hog is more adequately met by the 12-percent ration than the 18-percent ration.

The experiments conducted did not include rations beyond the 18-percent protein level. Therefore, the portion of the iso-product contours lying below the 18-percent protein ration line is an extrapolation beyond the range of the data. The present study provides no information on the shape of the contours in that section of the production surface. However, with increasing levels of protein, the contours should flatten out and eventually approach a zero slope.

RATES OF SUBSTITUTION

The rate at which soybean oilmeal substitutes for corn in the hog ration at a given level of output is indicated by the slope at a particular point on the iso-product contour. The substitution rate indicates the amount of corn replaced by adding one more pound of soybean oilmeal to the ration, with gain constant at a particular magnitude. The iso-product contours in fig. 9 are curved, and consequently the rate of substitution of soybean oilmeal for corn declines as the ration includes a greater percentage of protein.

Marginal rates of substitution of soybean oilmeal for corn can be derived from equation 26 which is based on equation 25:

$$(26)\quad \frac{dC}{dP} = \frac{-0.982769 + 0.007760P + 0.000168C}{-0.298812 + 0.000060C + 0.000168P}$$

Table 8 includes prediction of the pork isoquants and the marginal rates of substitution associated with them. With a corn/SBOM ratio of 0.20, a 14-percent protein ration, 57.5 pounds of corn and 11.5 pounds of soybean oilmeal are required to produce 26 pounds of gain. The rate of substitution on the 26-pound gain contour with a 14-percent protein ration is 3.01 (i.e., a pound of soybean oilmeal replaces 3.01 pounds of corn at the particular point on the 26-pound contour). For the same ration, 169.7 pounds of corn and 33.94 pounds of soybean oilmeal are required to produce a 76-pound gain. However, the quantity of corn replaced by a 1-pound increase in soybean oilmeal drops to 2.44 pounds for this level of gain. For a gain of 141 pounds, 338.9 pounds of corn and 67.78 pounds of soybean oilmeal are required, and the rate of substitution drops to 1.50. Hence, the substitution rate and relative feed value of soybean oilmeal declines as the hog increases in weight. While this point has been illustrated for a 14-percent ration only, it also holds true for rations containing other percentages of protein.

CHANGES IN SUBSTITUTION RATES FOR A GIVEN GAIN

Not only do the substitution rates for protein decline as the hog attains greater weight, but also they decline as the proportion of protein increases for growth to a given weight. For example, substitution rates vary from 3.25 to 2.88 over the range for which the 26-pound gain contour has been predicted. In a ration containing 0.02 pounds of soybean oilmeal per pound of corn, 1 pound of soybean oilmeal replaces 3.25 pounds of corn. A pound of soybean oilmeal replaces only 2.88 pounds of corn at the same gain level in a ration with 0.40 pound of soybean oilmeal per pound of corn. One pound of the former replaces only 2.99 pounds of the latter when the ratio of soybean oilmeal is 0.22.

For the 76-pound gain contour, the marginal rate of substitution varies from 3.19 with a soybean oilmeal/corn ratio of 0.02 to 1.98 with a soybean oilmeal/corn ratio of 0.40. The rate of substitution declines much more rapidly along the 76-pound gain contour than along the 26-pound gain contour because of the greater hog weight. The range in magnitude of substitution rates is even greater along the 141-pound gain contour. A pound of soybean oilmeal replaces 3.09 pounds of corn with a soybean oilmeal/corn ratio of 0.20, but only 0.01 pound of corn with a ratio of 0.40.

Special aspects in the interpretation of the substitution rates presented in table 8 should be mentioned. The feed quantities are the predicted total amounts of corn

TABLE 8. CORN AND SOYBEAN OILMEAL QUANTITIES AND SUBSTITUTION RATES ALONG THE 26-, 76- AND 141-POUND GAIN ISOQUANTS (DERIVED FROM EQUATIONS 25 AND 26).

Proportion of SBOM in ration*	Percent protein†	26 pounds of gain: Feed required Corn (lbs)	SBOM (lbs)	Marginal rate of substitution‡ dC/dP	76 pounds of gain: Feed required Corn (lbs)	SBOM (lbs)	Marginal rate of substitution‡ dC/dP	141 pounds of gain: Feed required Corn (lbs)	SBOM (lbs)	Marginal rate of substitution‡ dC/dP
0.02	8.7	88.0	1.76	3.25	251.1	5.02	3.19	472.8	9.46	3.09
0.04	9.4	82.9	3.32	3.22	237.1	9.48	3.07	447.6	17.90	2.86
0.06	10.0	78.5	4.71	3.18	225.0	13.50	2.97	426.5	25.59	2.65
0.08	10.6	74.5	5.96	3.15	214.3	17.15	2.87	408.4	32.67	2.46
0.10	11.2	71.0	7.10	3.12	204.9	20.49	2.79	392.6	39.28	2.28
0.12	11.8	67.8	8.13	3.10	196.4	23.57	2.71	379.1	45.49	2.11
0.14	12.4	64.8	9.08	3.07	188.7	26.42	2.64	367.1	51.40	1.95
0.16	12.9	62.2	9.95	3.05	181.8	29.09	2.57	356.6	57.05	1.79
0.18	13.5	59.7	10.75	3.03	175.5	31.59	2.50	347.2	62.50	1.64
0.20	14.0	57.5	11.50	3.01	169.7	33.94	2.44	338.9	67.78	1.50
0.22	14.5	55.4	12.19	2.99	164.3	36.15	2.38	331.6	72.94	1.35
0.24	14.9	53.5	12.83	2.98	159.4	38.25	2.33	325.0	78.01	1.21
0.26	15.4	51.7	13.44	2.96	154.8	40.24	2.28	319.3	83.02	1.07
0.28	15.8	50.0	14.00	2.95	150.5	42.14	2.23	314.3	88.01	0.93
0.30	16.3	48.4	14.53	2.94	146.5	43.95	2.18	310.0	93.01	0.79
0.32	16.7	47.0	15.04	2.92	142.8	45.69	2.14	306.4	98.05	0.65
0.34	17.1	45.6	15.51	2.91	139.3	47.35	2.10	303.5	103.18	0.50
0.36	17.5	44.3	15.95	2.90	135.9	48.94	2.05	301.3	108.45	0.35
0.38	17.9	43.1	16.37	2.89	132.8	50.47	2.01	299.8	113.93	0.18
0.40	18.2	41.9	16.77	2.88	129.9	51.95	1.98	299.2	119.68	0.01

*The figures show the pounds of soybean oilmeal for each pound of corn. Hence, the figure 0.20 refers to 2 pounds of soybean oilmeal for each 1 pound of corn.

†Based upon a protein content of 45 percent for soybean oilmeal and 8.2 percent for corn.

‡The negative signs have been omitted from the substitution rates. The substitution ratios are derivatives from equation 26. The feed combinations for specified gains have been derived from equation 25.

and soybean oilmeal consumed beyond weaning to the level of gain represented by respective isoquants. The substitution rates are an expression of the rate at which soybean oilmeal replaces corn at exactly (or very near) the gain level specified. The substitution rates are not averages for all gain levels from weaning up to the specified gain. In other words, substitution rates along a 10-pound gain isoquant would differ from those shown in table 8 for a 26-pound gain isoquant.

The production function, equation 1, predicts the gain resulting from various total amounts of feed consumed beyond weaning. Hence, each contour, such as in fig. 9 and table 8, is derived with reference to the origin or weaning weight. The predictions suppose that the quantities of corn and soybean oilmeal specified by the coordinates of a point on a contour are fed in that proportion from weaning to the level of gain represented by the contour.

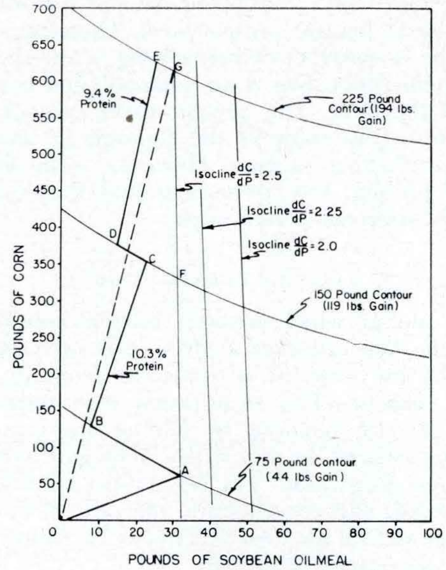

Fig. 10. Expansion path of the least cost ration with a price ratio of 2.5, 2.25, and 2.0.

LEAST-COST RATIONS

The least-cost ration can be defined by equation 27 where $\frac{\partial C}{\partial P}$ refers to the marginal rate of substitution of corn for soybean oilmeal and P_p/P_c is the price ratio of soybean oilmeal and corn. If the substitution ratio is greater than the price ratio, more soybean oilmeal should be used, since the value of the corn replaced is greater than the value of the soybean oilmeal added. If the substitution ratio is less than the price ratio, the ration is not lowest in cost and corn should be substituted for soybean oilmeal.

$$(27) \quad \frac{\partial C}{\partial P} = \frac{P_p}{P_c}$$

Least cost rations are determined by equating the derivative ($\frac{\partial C}{\partial P}$, equation 26) to the feed price ratio. However, least-cost rations determined by use of substitution rates estimated from the over-all production function must be carefully interpreted. The hogs in these experiments were each fed a constant ration throughout the entire course of the experiments. The production function therefore, expresses, under a constant ration system of feeding, the relationship of total weight gain to total consumption of corn and soybean oilmeal from weaning weight. The iso-product contours (fig. 10) derived from the over-all production function, therefore, show the possible combinations of corn and soybean oilmeal to produce various levels of gain under a single ration technique of feeding. The ration which is determined by equating the substitution rate on these contours to the feed price ratio gives the total quantity of corn and soybean oilmeal which will produce the amount of gain *represented by the respective contour,* and not over-all gain contours, with the lowest outlay for the feed under a system of feeding a single ration throughout the feeding period. For example, point G on the 225-pound contour in fig. 10 is the locus, on that contour, where soybean oilmeal substitutes for corn at the rate of 2.5. Therefore, the coordinates of point G are the quantities of corn and soybean oilmeal which will produce 194 pounds of gain beyond weaning under a constant ration system of feeding. With a soybean oilmeal price of 2.5 times the price of corn, corn and soybean oilmeal would be fed from weaning to the 225-pound weight level, in the proportions represented by the line OG.

A single ration fed throughout the entire feeding period is one possible system to follow; but it is obvious from the relationships shown in fig. 10 that this system does not result in the lowest possible feed cost. Feed costs can be further reduced by adjusting the proportion of corn and soybean oilmeal fed at intermediate points throughout the feeding period. The line AFG in fig. 10 is an isocline joining all points on successive contours having a slope of 2.5 (i.e., a substitution rate of 2.5). Comparing line AFG to line OG it is obvious that below 194 pounds, least-cost gains are not attained if the same ration is fed throughout the entire feeding period. One hundred and nineteen pounds of gain can be produced at lower cost by feeding corn and soybean oilmeal in the proportions represented by the feed quantity at point F. This ration has a higher proportion of soybean oilmeal than the ration represented by the line OG. Similarly, the ration for producing 44 pounds of gain has a higher proportion of soybean oilmeal than the ration for producing 119 pounds of gain under a single ration feeding system. The rate of substitution between corn and soybean oilmeal is changing continuously as the hog gains in weight. Consequently, if feed costs are to be minimized, a different ration should be fed for each successive pound of gain produced.

PARTICULAR INTERPRETATION OF ISOCLINES

The isoclines (say $\frac{\partial C}{\partial P} = 2.5$) in fig. 10 might appear to be interpreted as indicating that the least-cost ration to reach 75 pounds includes the amount of protein shown at A, with some of this taken away from the pig as he attained 119 and then 225 pounds. However, this is not the case. Point A shows only the feed combination, fed as a single ration, which would be used to attain minimum cost if the pig were to be taken only to 75 pounds; point F shows the least-cost feed combination if the pig were to be fed to exactly 150 pounds; point G shows the feed combination foı lowest cost if the hog were fed a single ration and taken to exactly 225 pounds. These isoclines do not directly show how the corn and soybean oilmeal proportion are to be adjusted throughout the period from weaning to market weight in order to minimize feed cost per each successive pound of gain produced. The isoclines refer only to a constant ration feeding system, to the gain level indicated. They are, however, indicative of the necessity for feeding a lower protein ration for each successive pound of gain produced.

The quadratic function provides isoclines which are linear but do not pass through the origin.[11] Hence, they indicate, for each successive gain contour, the ration which would produce the particular gain at lowest cost if only this ration were fed to the weight level. However, since the isoclines intersect other contours at points representing different rations, this one ration would not represent the least-cost method of producing smaller gains on the same hog. The least-cost ration for each gain level is indicated by the point at which the isocline corresponding to a particular price ratio intersects the relevant isoquant. It is for this reason that an isocline such as $\frac{\partial C}{\partial P} = 2.5$ in fig. 10 represents a smaller proportion of protein for hogs of heavier weights.

Generally isoclines have a positive slope. In fig. 10 they appear to be vertical, or to have a slight negative slope. This phenomenon arises mainly because of the nature of the experiment and measurements. The quantities measured in the feed plane are corn and soybean oilmeal. The third feed, forage, is not measured. If the protein in forage were added to that in soybean oilmeal, the isoclines would have a positive slope. Small pigs eat very little forage because their digestive organs cannot handle it. However, as hogs progress in weight, they can and do consume much more forage relative to concentrates. Thus for a 225-pound hog, the feed equivalent of soybean oilmeal in forage would, if added to the soybean oilmeal measured in the study, fall at a point to the right of G in fig. 10.

In a study designed to relate the gain surface physically with feed input, forage should be measured and introduced into the production function. The current study did not, however, have this objective. It was designed to allow specification of least-cost rations under conditions representing the environment in which most farmers make their decisions. Most farmers turn their pigs on pasture as a disease control precaution, as well as to obtain some feed advantage. Yet hog pasture usually includes an abundance of forage, and no attempt is made to fully utilize it in matching costs of forage against concentrate feeds. The farmer is concerned, given an ample supply of forage and the quantity that is consumed when different concentrate rations are fed, with *balancing* corn and protein supplement feeds in a manner to minimize concentrate costs. In a subsequent study, it is anticipated that an experimental design will be included to allow physical measurement of production relationships for all three feeds.

INTERVAL RATIONS

A fixed ratio of corn and soybean oilmeal fed over an interval of gains does not result in the lowest possible feed costs for the entire gain interval because substitution rates change continuously as the hog increases in weight. Therefore, if feed costs are to be minimized the proportions in which the corn and soybean oilmeal are fed should be changed for each unit of gain produced (i.e., should follow an isocline). In practice, it is impossible to make such extremely small changes in the ration. To adjust the ration for gains even as small as a pound, the hogs would have to be fed individually and the rations changed daily. Farmers are concerned with the least-cost ration for rather wide intervals of gain. From a practical standpoint, they may consider changing the ration only two or three times in the course of the entire feeding period from weaning to market weight.

Hence, in providing practical figures for farmer recommendations, the production surface has been divided into three weight intervals. The three weight intervals are: Weaning to 75 pounds liveweight, 75 to 150 pounds liveweight and 150 to 225 pounds liveweight (i.e., the total weight contours shown in fig. 10). For a given ratio of the feed prices, a constant ration is selected for each interval. The ration selected should produce the gain at a lower feed cost than any other constant ration fed over the same interval.[12]

The least-cost ration for each interval is given in table 9 for a series of soybean oilmeal/corn price ratios. These rations were approximated from the over-all quadratic production function in this manner: A series of ration lines was projected through the surface from weaning to a liveweight of 225 pounds. The total feed requirements beyond weaning then were computed for producing a 75-, 150- and 225-pound hog along each of the ration lines. In other words, the various quantities of corn and soybean oilmeal which can be used for producing 44, 119 and 194 pounds of gain beyond weaning were determined. The difference for a particular ration, in respect to protein percentage, between the total feed requirements at the beginning and the end of each weight interval was used as the feed requirements for the particular interval. The least-cost ration for the interval is then determined by summing the value of corn and soybean oilmeal for the numerous rations. While the procedure is an approximation (in contrast to the equation of the derivative of the isoquant equation with the price ratio), it gives estimates of the least-cost ration,

[11]The equation of isoclines is as follows where —K is a stated substitution rate or price ratio:

$$\frac{\partial C}{\partial P} = \frac{-0.9828 + 0.007760P + 0.0001684C}{-0.2988 + 0.0006024C + 0.0001684P} = -K$$

The equation of isoclines then is

$$C = \frac{0.2988K + 0.007760P - 0.0001684PK - 0.9828}{0.0006024K - 0.0001684}$$

[12]Greater refinement in the least-cost feeding system can be achieved by dividing the surface into a greater number of weight intervals. The ration then can be altered more frequently over the total production period.

TABLE 9. LEAST-COST RATIONS ON ALFALFA PASTURE FOR VARIOUS SOYBEAN OILMEAL/CORN PRICE RATIOS.

Soybean oilmeal-corn price ratio	Weaning to 75 pounds				75 to 150 pounds				150 to 225 pounds			
	Feed required		Percent protein	Days to feed over interval	Feed required		Percent protein	Days to feed over interval	Feed required		Percent protein	Days to feed over interval
	Corn	SBOM			Corn	SBOM			Corn	SBOM		
1.1					174	42	14.9	43	232	23	11.2	32
1.2					175	41	14.7	43	235	22	11.0	32
1.3					177	39	14.5	43	236	20	10.8	32
1.4					178	38	14.3	43	238	19	10.6	32
1.5					181	37	14.1	43	239	18	10.5	32
1.6					182	36	13.9	43	241	16	10.2	33
1.7					186	34	13.4	43	243	15	10.1	33
1.8					190	30	12.9	43	245	14	9.9	33
1.9					196	27	12.4	43	248	13	9.8	33
2.0					204	23	11.7	43	249	12	9.6	34
2.1					209	20	11.0	44	249	12	9.6	34
2.2					213	19	10.9	44	249	12	9.6	34
2.3					216	17	10.6	44	251	11	9.5	34
2.4					218	16	10.5	44	251	11	9.5	34
2.5					221	15	10.3	45	254	10	9.4	34
2.6					223	14	10.1	46	254	10	9.4	34
2.7	85	24	15.8	33	230	12	9.7	47	256	9	9.2	34
2.8	93	20	14.5	36	234	10	9.5	47	256	9	9.2	34
2.9	104	17	12.9	40	236	9	9.3	48	256	9	9.2	34
3.0	118	12	11.2	46	242	8	9.1	49	259	8	9.1	34
3.1	130	8	10.0	52	242	8	9.1	49	259	8	9.1	34

accurate within a few tenths of a percent in protein and sufficiently accurate for practical uses.

INDICATION OF RATIONS

In table 9, least-cost rations can be determined as follows: With a price of 4.05 cents per pound for soybean oilmeal and 1.5 cents per pound for corn, the price ratio is 2.7. At this price ratio, the least-cost ration over the weaning to the 75-pound interval is 15.8 percent protein. The ration will include 85 pounds of corn and 24 pounds of soybean oilmeal for this amount of gain, plus the 2.5 pounds of the minerals indicated in table 1 for each 100 pounds of feed. For gains in the 75-150 pound interval for a price of $1.12 per bushel for corn and $4 per hundredweight for soybean oilmeal, a price ratio of 2 on a per-pound basis, the least-cost ration includes 204 pounds of corn and 23 pounds of soybean oilmeal, plus 2.5 pounds of minerals per 100 pounds of feed. With 11.7 percent protein, growth over this gain interval requires 43 days.

The procedure described above for finding the least-cost ration can be contrasted to other concepts of feeding in fig. 10. With a price ratio of 2.5, the rations for each of the three weight intervals would be represented by line OABCDE. From weaning to 75 pounds liveweight, corn and soybean oilmeal would be fed in the proportion represented by the line OA. At the 75-pound contour a shift is made along the contour to point B. From 75 to 150 pounds, corn and soybean oilmeal would be fed in the proportion represented by line BC. The ration is shifted again at the 150-pound contour to the proportions represented by line DE for the interval from 150 to 225 pounds liveweight. The line OABCDE might be called the "practical expansion path" of rations for a price ratio of 2.5. It represents a compromise between feeding of a different ration for each pound of gain (line AG) and feeding the same ration over the entire feeding period (line OG). The first alternative is impractical while the second does not minimize feed costs.

It can be seen by comparing the line OG with line OABCDE that the constant ration over the entire production period results in underfeeding of soybean oilmeal throughout the first and second weight intervals and overfeeding of soybean oilmeal throughout the third weight interval. Cost of feed is not minimized in any of the three weight intervals by feeding the constant ration. The same condition may hold true for OABCDE within each weight interval. Soybean oilmeal may be underfed through the first portion of the weight interval and overfed through the last portion. Consequently, feed costs can be further reduced by lessening the magnitude of the weight interval, but still feeding a constant ration over each of the smaller intervals. Again, however, it is believed that determination of "average least-cost" rations for the three intervals is sufficient for practical purposes.

LEAST-COST GRAPH

Figure 11, based on the rations in table 9, provides a graph for calculation of least-cost rations under different prices for corn and soybean oilmeal. The series of iso-price ratio lines in fig. 11 show all combinations of corn and soybean oilmeal prices giving the same price ratio. Rather than to consider an infinite series of price ratio lines with minute changes in the proportions of corn and soybean oilmeal in the ration, only eight lines have been drawn. This procedure amounts to assuming that the gain isoquant is made up of a series of linear segments (rather than of continuous points on a smooth curve).[13]

The least-cost ration for a given set of prices for corn and soybean oilmeal is found by reading up the corn axis of fig. 11 to the given price of corn, and then reading across in a horizontal direction until a point is reached directly above the given price of soybean oilmeal. The area of the graph in which the point lies determines which of the rations in table 10 is the least-cost ration to be fed for the gain intervals of 75-150 pounds and 150-225 pounds for growing-fattening hogs on alfalfa pasture. For example, if the price of corn is $1.40 per bushel and the price of soybean oilmeal is $6.50 per cwt., these prices form the coordinates of point X on the graph. Point X falls in area G. The least-cost ration for hogs between 75 and 150 pounds liveweight

[13]The substitution rate of soybean oilmeal for corn is constant along any one segment of a linear-segment iso-product curve. The ratio of feed prices may then vary between the numerical value of the slopes of two adjacent segments of the iso-product contour without affecting the least-cost combination of the feeds.

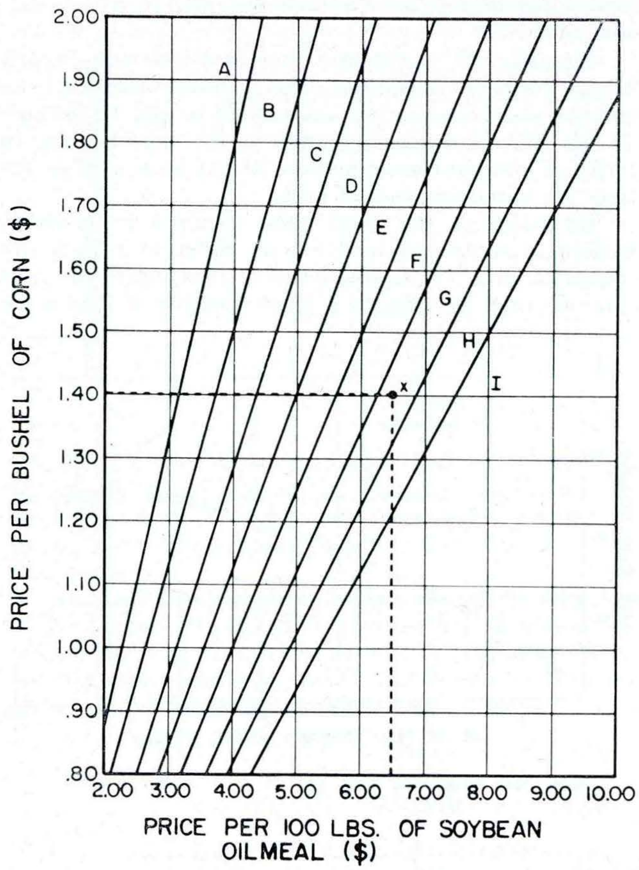

Fig. 11. Least-cost ration graph

is given opposite line G in section I of table 10. For hogs between 150 and 225 pounds liveweight, the least-cost ration is given opposite line G in section II. The rations in table 10 are in terms of feed requirements per hundred pounds of gain; rather than in terms of the quantity of feed required for a single hog to produce the amount of gain for each weight interval. This measure is used since it conforms with customs in animal science for quoting feed requirements.

RATE OF GAIN

The proportion of protein in the ration affects the time of marketing as well as the rate and the cost of gains. Savings in feed costs must be balanced against gains or losses from marketing hogs at different times of the year.

To allow prediction of the effect of rations on rate of gain, two types of functions are examined as alternatives for expressing the relationship between the inputs of corn (C) and soybean oilmeal (P) and the number of days (T) required to consume various quantities of the two feeds. The two functions are the quadratic (28) and the square root (29). Each function has been fitted over the observations from all six rations in both experiments.

$$(28) \quad T = 4.2477 + 0.4414C - 0.3673P - 0.0003C^2 + 0.0047P^2 - 0.0010CP$$

$$(29) \quad T = -23.0421 - 0.0064C + 0.5304P + 7.9090\sqrt{C} - 4.0347\sqrt{P} - 0.2120\sqrt{C}\sqrt{P}$$

The statistics for the two functions are presented in table 11. Equation 29, the quadratic root, explains only a slightly greater proportion of the variation in the de-

TABLE 10. LEAST-COST RATIONS FOR PASTURE FED HOGS IN TERMS OF FEED PER 100 POUNDS OF GAIN.

	Corn	SBOM	Percent protein	Average daily gain	Number of days
I. 75 to 150 pounds					
A	232	56	14.9	1.74	43
B	237	51	14.3	1.75	43
C	243	48	13.9	1.76	43
D	261	36	12.4	1.75	43
E	279	27	11.0	1.71	44
F	291	21	10.5	1.68	44
G	297	19	10.1	1.64	46
H	315	12	9.3	1.54	48
I	323	11	9.1	1.52	49
II. 150 to 225 pounds					
A	309	31	11.2	2.33	32
B	317	25	10.6	2.33	32
C	321	21	10.2	2.31	33
D	331	17	9.8	2.29	33
E	332	16	9.6	2.27	34
F	335	15	9.5	2.25	34
G	339	13	9.4	2.23	34
H	341	12	9.2	2.20	34
I	345	11	9.0	2.18	34

TABLE 11. CORRELATION COEFFICIENTS, STANDARD ERRORS AND "t" VALUES FOR TIME FUNCTIONS IN ORDER OF REGRESSION COEFFICIENTS IN EQUATIONS 28 AND 29.

Equation	R^2	R	Standard errors					t values*				
			s_{b1}	s_{b2}	s_{b3}	s_{b4}	s_{b5}	t_{b1}	t_{b2}	t_{b3}	t_{b4}	t_{b5}
(28)	0.912	0.955	0.0141	0.0539	0.000027	0.00040	0.00018	31.35	6.81	12.04	11.83	5.60
(29)	0.918	0.958	0.0021	0.0503	0.5358	0.7283	0.0483	3.05	10.55	14.76	5.54	4.39

*All t values exceed a probability level of 0.05 or less.

pendent variable, (T), than does equation 28. Equation 28 gives a relation showing time as a maximum at a level of feed consumption within the range of experimental observations. For total time to reach a maximum would mean that the hog would have to "die off" and cease feed intake, an unrealistic situation. It appears more logical that the slope of the total time function should fall off rapidly at low total feed input and approach linearity as total feed consumption reaches a high level and the hog approaches maturity. A mature hog has nearly a constant daily feed intake. Conceivably, the hog could live several years, with total time continuing to increase with age and continued feed consumption.

Equation 29, the square root function, more nearly allows the latter conditions. The relations obtained from the two types of equations are plotted in figs. 12 through 17 for the six rations included in the experiments. In terms of comparisons, equation 29 has been used as the basis for estimating rate of gain.

Estimates of the total time required to consume various quantities of feed for six different rations are shown in table 12. At low levels of feed input, the least time required to consume a given quantity of feed is ob-

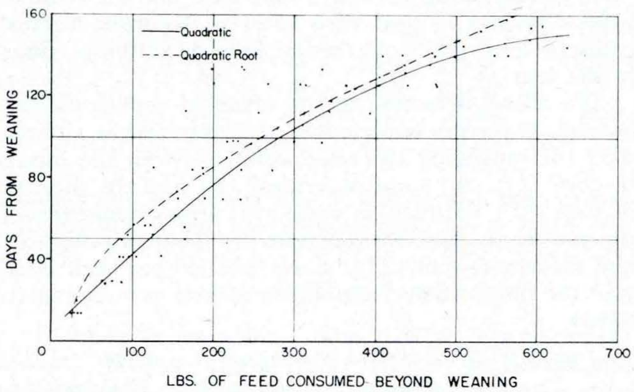

Fig. 12. Time relation for 8% protein ration

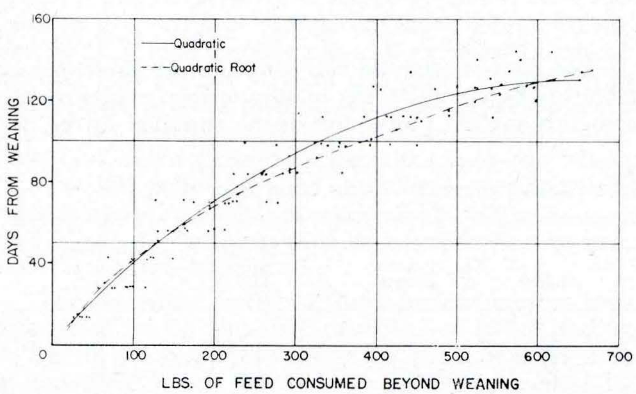

Fig. 13. Time relation for 10% protein ration

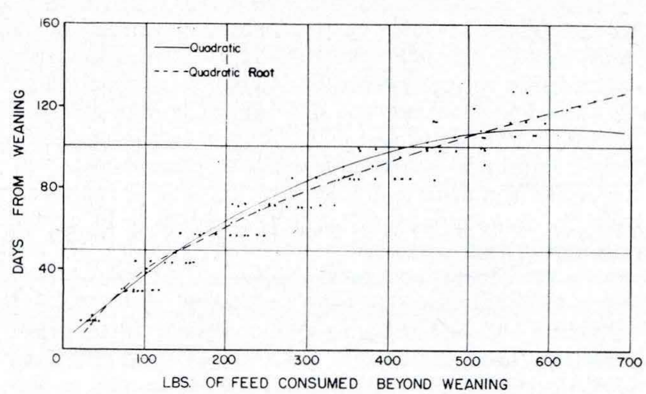

Fig. 14. Time relation for 12% protein ration

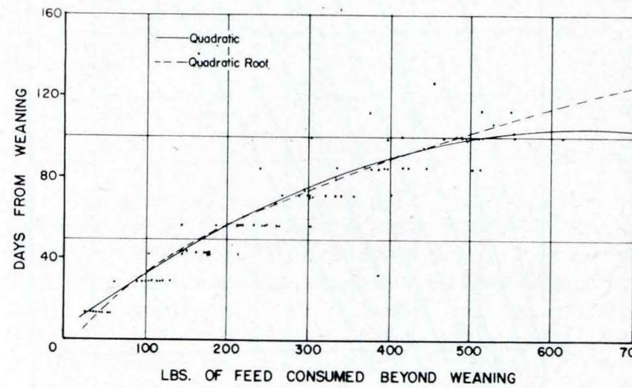

Fig. 15. Time relation for 14% protein ration

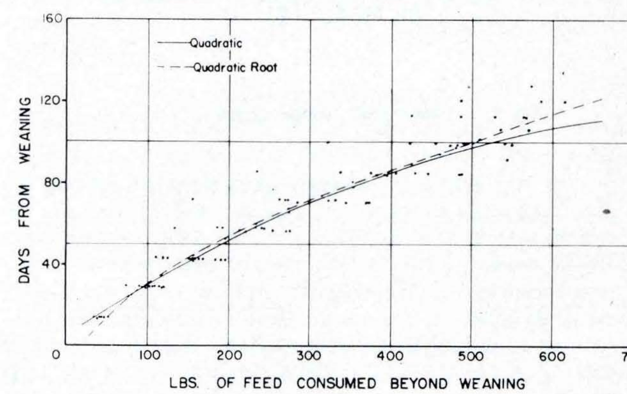

Fig. 16. Time relation for 16% protein ration

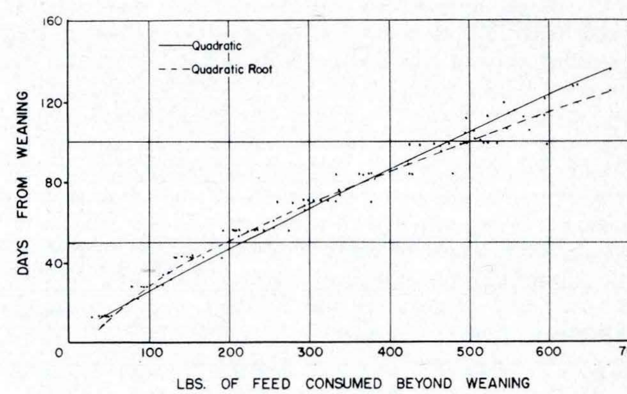

Fig. 17. Time relation for 18% protein ration.

TABLE 12. TOTAL TIME REQUIRED TO CONSUME A GIVEN AMOUNT OF FEED IN RATIONS OF VARIOUS PROTEIN LEVELS.

Lbs. of feed	Total days to consume specified feed quantity for protein levels of: 8%	10%	12%	14%	16%	18%
50	31.5	23.8	19.5	16.5	14.0	11.7
100	53.9	42.4	36.6	32.8	29.9	27.2
150	71.0	56.5	49.6	45.3	42.3	39.7
200	85.4	68.1	60.4	55.9	52.9	50.5
250	98.0	78.2	69.8	65.2	62.3	60.2
300	109.4	87.2	78.3	73.6	70.9	69.2
350	119.9	95.4	86.0	81.3	78.9	77.7
400	129.6	102.9	93.1	88.5	86.4	85.7
450	138.7	109.9	99.7	95.2	93.5	93.3
500	147.3	116.4	106.0	101.6	100.3	100.6
550	155.4	122.6	111.9	107.7	106.7	107.7
600	163.2	128.4	117.4	113.4	113.0	114.6
650	170.7	133.9	122.8	119.0	119.0	121.2
700	177.8	139.2	127.9	124.3	124.8	127.7
750	184.7	144.2	132.8	129.5	130.5	134.0
800	191.4	149.1	137.5	134.5	135.9	140.2
850	197.8	153.7	142.0	139.3	141.3	146.3
900	204.0	158.2	146.4	144.0	146.5	152.2

tained with the 18-percent protein ration. As the pigs become older and heavier, the advantage of the extremely high protein ration becomes increasingly smaller and finally gives slower gains than a ration with somewhat less protein.

An equation of daily rates of gain can be expressed as in equation 30 where D is gain per day, T is total time to consume a given amount of feed, Y is gain forthcoming from the same feed and C and P refer to corn and soybean oilmeal consumption per pig.

$$(30)\quad D = \frac{Y - 1.75 + 0.2988C + 0.9828P - 0.00003C^2 - 0.0039P^2 - 0.00017CP}{T - 23.0421 - 0.00064C + 0.5304P + 7.909\sqrt{C} - 4.035\sqrt{P} - 0.212\sqrt{C}\sqrt{P}}$$

The average daily rate of gain, between 50-pound feed increments, of the 18-percent protein ration rises up to a total feed input of 250 pounds (table 13). Between inputs of 200 and 250 pounds, a maximum average daily rate of gain of 1.685 pounds is reached. Beyond the 250-pound level of feed consumption, each additional 50-pound increment of the ration results in lower daily gains.

The 16-percent protein ration produces gains at a slower rate than the 18-percent protein ration up to an input of 200 pounds of feed. The same comparison holds true between the 14-percent and the 16-percent protein rations. As the hog consumes more feed and increases in weight, the rate of gain falls off with the higher protein ration. The rate of gain between 50-pound feed increments rises with the 16-percent protein ration only up to a total feed consumption of 350 pounds. Rates of gain are higher with the 14-percent ration than with the 16-percent ration beginning at 250 pounds of feed. At 350 pounds of feed, the 12-percent protein ration produces gains at a faster rate than the 14-percent ration. While the rates of gain for the 8- and 10-percent rations continue to increase up to 700 pounds of feed, they never become greater than for the 12-percent ration.

It is evident from the data in table 13 that the protein content of the ration must be decreased over the production period if the rates of gain are to be kept at a maximum. The minimum time ration to feed from weaning to 75 pounds lightweight consists of 63.42 pounds of corn and 31.71 pounds of soybean oilmeal. The protein content of the ration is 19.9 percent, and the time to feed out over the interval is 24 days. The interval from 75 pounds to 150 pounds can be covered in a minimum of 43 days with a ration of 190.16 pounds of corn and 30.43 pounds of soybean oilmeal. The protein level of the ration is 12.9 percent. A 10.6-percent ration results in gains from 150 pounds to 225 pounds in the minimum of 32 days and requires 237.51 pounds of corn and 19.0 pounds of soybean oilmeal.

LEAST-COST VERSUS LEAST-TIME RATIONS

The least-cost ration always results in feed costs equal to or lower than a least-time feeding system. However, the least-cost ration is not necessarily the most profitable management practice. The two systems of feeding result in the same profit only when the soybean oilmeal/corn price ratio is such that the least-cost and the least-time rations are identical. The least-cost and least-time rations for pasture-fed hogs are identical when the

TABLE 13. AVERAGE DAILY GAIN BETWEEN FEED INTERVALS FOR VARIOUS PROTEIN LEVELS.

Lbs. of feed	Daily gain computed as an average between 50-lb. feed increments with protein percentages of: 8	10	12	14	16	18
100	0.632	0.817	1.000	1.128	1.241	1.343
150	0.819	1.074	1.294	1.422	1.511	1.563
200	0.965	1.280	1.520	1.630	1.676	1.661
250	1.088	1.457	1.704	1.784	1.777	1.685
300	1.194	1.613	1.860	1.901	1.833	1.658
350	1.288	1.754	1.992	1.988	1.852	1.591
400	1.373	1.882	2.106	2.051	1.843	1.493
450	1.449	2.000	2.205	2.094	1.809	1.369
500	1.519	2.110	2.289	2.118	1.754	1.222
550	1.583	2.211	2.362	2.126	1.680	1.056
600	1.642	2.305	2.423	2.119	1.590	0.873
650	1.696	2.394	2.474	2.099	1.484	0.675
700	1.746	2.476	2.516	2.067	1.365	0.463

soybean oilmeal/corn price ratio is 2.5 or less for hogs in the first interval; 1.8 for hogs in the second interval; and 1.4 for hogs being fed through the third interval. For price ratios higher than these, the least-cost ration produces gains at a slower rate than the least-time ration. Also, the least-time ration then has a higher feed cost than the least-cost ration.

If the hog producer is faced with the prospect of a declining market price for hogs, hogs may be sold at a lower price under the least-cost rather than under the least-time feeding system (because of the greater length of time ordinarily required to produce the gain with least-cost rations). For hogs fed on pasture, differences in market price may be great enough to offset the feed economies obtained by feeding the least-cost rations. With a rising market price for hogs, the least-cost feeding system ordinarily will give the greater return over feed cost: Within limits, the value of the hog is increased the longer it is held off the market; also gains are produced at a lower cost.

Rations for least-cost and least-time rations are much more similar on pasture than drylot. This condition holds true on pasture because of the availability of protein in the forage. If the price ratio favors the use of a small percentage of protein in the concentrate mix, the hog can supplement the protein intake by consuming more forage. Accordingly, the rate of gain for hogs on pasture is not decreased much when the concentrate ration is adjusted to include less protein. In drylot, however, a shift in ration to meet a higher protein/corn price ratio cannot be offset by a greater intake of protein from forage.

HISTORIC OUTCOMES

The effect of the seasonal fluctuation in hog prices on returns over feed cost has been examined over the 20-year period from 1935 through 1954 for spring hogs farrowed at four different dates. These figures indicate the number of years in which either the least-time or least-cost rations would have been most profitable. The market price at which the hogs would have been sold under each system has been determined by taking into account the time required to produce a 225-pound hog with least-cost and least-time rations. The feed prices used were the average annual price of soybean oilmeal in each of the years and the price of corn in the month at which the hogs reach weaning weight, 75 pounds liveweight and 150 pounds liveweight. The price of corn was assumed constant for the duration of each weight interval. It was further assumed that 6 weeks would be required to raise pigs from farrowing to weaning weight. The farrowing dates considered were: Feb. 1, March 1, April 1 and May 1.

In the 20 years, with hogs farrowed on Feb. 1, the least-cost rations would have resulted in the greater return over feed costs in 15 of the years (table 14). The least-time ration would have given the greater returns over feed cost in only 5 years. With hogs farrowed on March 1, the least-cost ration would have given the greatest return over feed cost in 19 years; the least-time ration would have given greater returns in only 1 of the 20 years. For hogs farrowed on April 1, the least-cost rations would have been more profitable in 13 years, while the least-time ration would have been more profitable in 7 of the 20 years. For a May 1 farrowing date, the least-cost ration was more profitable in 9 years, while the least-time ration was more profitable in 11 of the 20 years.

Rate of gain is of lesser importance with hogs farrowed early in the season. Hogs farrowed in February, March and April can be produced more profitably on the least-cost ration a greater proportion of the time than on the least-time ration. Rate of gain is of much greater importance for hogs farrowed late in the season, because of sharp seasonal price declines in October. The lower market price often more than offsets the feed economies obtained by feeding the least-cost ration.

The average feed costs for the 20-year period in producing a 225-pound market hog on pasture differ only slightly for least-cost and least-time rations. The small difference probably results from the hogs being on pasture. Protein from legumes replaces some of that which would otherwise be obtained at a cost from soybean oilmeal in drylot. The rates at which soybean oilmeal substitutes for corn in the hog ration under a pasture feeding system are such that the least-cost ration deviates only slightly from the rations which maximize the rate of gain. The modal gain for the least-cost ration

TABLE 14. COMPARISON OF RETURNS PER HOG OVER FEED COST AFTER WEANING FOR LEAST-COST AND LEAST-TIME RATIONS IN THE PERIOD, 1935 TO 1954.

Farrowing date	No. of times least-cost gave greater returns over feed cost	No. of times least-time gave greater returns over feed cost	Average return over feed cost 1935-54		Average feed cost 1935-54	
			L.C.	L.T.	L.C.	L.T.
February 1	15	5	$24.81	$23.94	$12.02	$12.08
March 1	19	1	26.26	26.19	12.34	12.41
April 1	13	7	25.13	25.21	12.39	12.44
May 1	9	11	22.14	22.17	12.42	12.65

TABLE 15. FREQUENCY DISTRIBUTION OF THE DIFFERENCES IN RETURNS ($) BETWEEN THE LEAST-COST AND LEAST-TIME RATIONS.

Farrowing date	Amount by which returns over feed costs for a least-cost ration exceed those of a least-time ration											
	Greater than 2.51	2.01 to 2.50	1.51 to 2.00	1.01 to 1.50	0.51 to 1.00	0 to 0.50	0 to -0.50	-0.51 to -1.00	-1.01 to -1.50	-1.51 to -2.00	-2.01 to -2.50	Less than -2.51
February 1	2	0	1	6	3	3	4	0	0	0	1	0
March 1	0	0	0	0	1	18	1	0	0	0	0	0
April 1	2	0	0	0	2	9	2	1	2	0	0	2
May 1	1	0	0	2	0	6	4	4	2	0	1	0

is less than 50 cents over the 20 years (table 15). The largest difference was for the Feb. 1 farrowing date. In 2 of the 20 years the returns for the least-cost ration exceed the returns for the least-time ration by more than $2.50. In 10 of the years, the returns with the least-cost ration were within $1 of the returns with the least-time ration. The returns from the least-cost and least-time rations differed by $1 or less for all 20 of the years under a March 1 farrowing date. On the average, over a period of years, little gain is forthcoming from feeding the least-cost, as compared with the least-time ration on pasture. However, in a few individual years the economic advantages of the least-cost ration with a pasture feeding system is quite large.

GRINDING AND MIXING VERSUS FREE CHOICE

Feeding either least-time or least-cost rations requires grinding and mixing of concentrate feeds. These extra steps add to the cost of the rations. Hence, an additional study is needed for comparison between (a) the costs of rations fed free choice and (b) the costs of rations plus the costs of grinding and mixing for least-time and least-cost rations. These comparisons are not possible from the data of this study.

SUMMARY

Two experiments were conducted with corn and soybean oilmeal rations fed to growing-fattening hogs on alfalfa pasture. These experiments were designed to determine feed substitution rates and optimum rations. Each experiment included six different rations, ranging in protein content from 8 to 18 percent.

Several algebraic forms of production functions were examined as alternatives in expressing the relationship between hog gains and corn and soybean oilmeal inputs under pasture feeding. Functions were fitted to all observations to provide estimates of the over-all gain surface. Then the production period was divided into weight intervals, and a function was fitted to each interval. Of the alternative forms of functions considered, a quadratic over-all equation gave greatest accuracy in estimates and was used for analysis of the corn/soybean oilmeal substitution rates for hogs produced on pasture.

The production function, isoquant and marginal substitution equations used for predictions are indicated below as equations a, b and c, respectively. In these equations, Y refers to gain, C refers to corn

$$\text{(a)}\quad Y = -1.75 + 0.299C + 0.983P - 0.00003C^2 - 0.00388P^2 - 0.00017CP$$

$$\text{(b)}\quad C = 4960.36 - 2.796P \pm (-16{,}600.3)[-0.0000004P + 0.000018P + 0.0891 - 0.00012Y]^{1/2}$$

$$\text{(c)}\quad \frac{\partial C}{\partial P} = -\frac{0.983 - 0.0078P - 0.00017C}{0.299 - 0.00006C - 0.00017C}$$

consumed and P refers to soybean oilmeal. All variables are measured in pounds per pig after weaning.

From these equations have been predicted: (1) marginal feed productivities, (2) gain isoquants for different weight levels, (3) marginal rates of substitution and (4) feed isoclines. Table A shows possible feed combinations in producing a gain of 76 pounds. The first two columns show alternative combinations, in pounds of soybean oilmeal and corn, for producing the specified gains. The third column includes derivatives from equation c and indicates the rate at which soybean oilmeal substitutes for corn.

With a SBOM/corn price ratio of 2.5, the least-cost ration on pasture includes 13.5 percent protein. With this ration, the price ratio is equal to the marginal rate of substitution, a necessary condition for minimizing feed costs. Similar data have been worked out for other gain levels and are presented in tabular and graphic form in the text.

The rate at which soybean oilmeal substitutes for corn in the hog ration declines as the hog increases in weight. To minimize feed costs for any set of corn and soybean oilmeal prices, the proportion of soybean oilmeal must be reduced as hog weight increases. For example, at a 141-pound gain level and with the 2.5 price ratio, the ration which minimizes feed costs includes only 10.6 percent protein on pasture.

Maintenance of the maximum rates of gain also requires that the ration be altered as the production period progresses. A high-protein ration must be fed in the early part of the feeding period, with protein content gradually diminished, if the rate of gain is to be at a maximum over the entire growing and fattening period.

One choice which must be made by the hog producer is whether to feed the least-cost or the least-time ration. Over the past 20 years, a least-cost ration would have given greatest profit for hogs farrowed on Feb. 1, March 1 and April 1. The least-time ration would have given greater returns in most years for hogs farrowed on May 1.

The differences in returns between the two types of rations, however, were usually small for three reasons: First, altering the protein level of the ration changes the rates of gain in the 75- to 150-pound and 150- to 225-pound weight intervals only by small amounts for pasture-fed hogs because they can use protein from forage to substitute for that in grain or supplement.

TABLE A. ALTERNATIVE FEED COMBINATIONS IN PRODUCING 76 POUNDS OF GAIN.

Pounds corn	Pounds SBOM	Marginal rate of substitution, $\frac{\partial C}{\partial P}$	Percent protein in ration
225	14	2.97	10.0
214	17	2.87	10.6
205	20	2.79	11.2
196	24	2.71	11.8
189	26	2.64	12.4
182	29	2.57	12.9
176	32	2.50	13.5
170	34	2.44	14.0
164	36	2.38	14.5
159	38	2.33	14.9
155	40	2.28	15.4
150	42	2.23	15.8
146	44	2.18	16.3
143	45	2.14	16.7
139	46	2.10	17.1

Second, the predominant soybean oilmeal/corn price ratio was 2.5 or less over the 20-year period. Consequently, in most years, the ration which resulted in the least-time also resulted in least-cost gains in the weaning-to-75-pound weight interval. Third, since the hogs were on pasture, part of their protein requirements could be obtained from pasture, even for a ration containing a small proportion of soybean oilmeal. Thus, least-cost and least-time rations are much more similar for hogs on pasture than for hogs produced in drylot.

APPENDIX

DETERMINATION OF INTERVAL LEAST-COST RATIONS BY MEANS OF INTERVAL FUNCTION

An alternative procedure for determining the least-cost ration over a segment of the production period is that of using interval production functions. The number of times that the ration is to be reconsidered in the course of the entire feeding period can be arbitrarily decided upon beforehand. This procedure has been followed in fitting the interval Cobb-Douglas functions presented earlier. The production period has been divided into the following liveweight intervals: weaning to 75 pounds, 75 to 150 pounds, and 150 to 200 pounds. A separate function has been fitted to the observations for each interval. The object is to find a constant ration for each weight interval which will produce the gain from the beginning to the end of the interval at the lowest possible feed cost given the price of corn and soybean oilmeal.

From each interval production function, an iso-product contour can be derived to represent the end of the weight interval. These contours, estimated from the interval Cobb-Douglas functions, are shown in fig. 1-A. The lower contour shows the combinations of corn and soybean oilmeal which produce 44 pounds of gain beyond weaning (i.e., will feed a hog from a weaning weight of 31 pounds to a weight of 75 pounds). The second contour indicates the combination of corn and soybean oilmeal required to produce 75 pounds of gain beyond a liveweight of 75 pounds. The third contour shows the feed quantities to produce 75 pounds of gain beyond a liveweight of 150 pounds. The experimental observations extended up to a liveweight of 200 pounds. Hence, the third contour involves some extrapolation beyond the range of the data. Equating the marginal rate of substitution along each contour to the inverse price ratio of the feeds gives the combination of feeds which will produce the gains in each weight interval at minimum cost. The feed quantities are determined on the assumption that a constant ration will be fed over each weight interval.

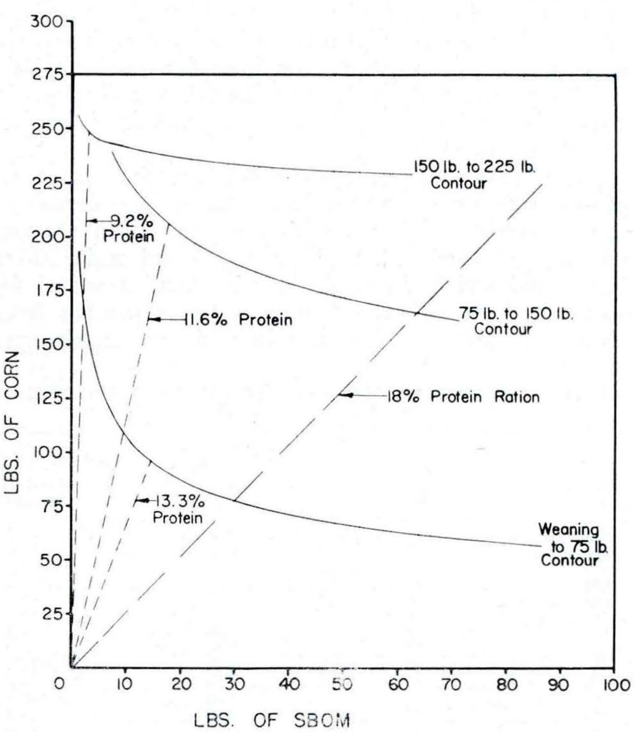

Fig. 1-A. Iso-product contours estimated from the interval Cobb-Douglas function.

If the soybean oilmeal/corn price ratio is equal to 2, the least-cost ration will be one of 13.3 percent protein for the weaning to 75-pound interval; 11.6 percent protein for the 75- to 150-pound interval; and 9.2 percent protein for the 150- to 225-pound interval. The feed quantities are shown by the ration lines in fig. 1-A and are measured from the origin to the contour in all three cases. Figure 1-A is a drawing of three iso-product contours each taken from a separate surface. By using interval production functions, a separate production surface is estimated for each weight interval.

The three interval contours are assembled in fig. 1-A in the form of an over-all production surface extending from weaning to the 225-pound weight. The expansion path of the least-cost ration with a soybean oilmeal/-corn price ratio of 2.0 also is shown. The iso-product contour at the end of the first interval is drawn, and the dashed line shows the ration fed throughout that interval. The point at which the least-cost ration line of the first weight interval intersects the 75-pound contour becomes the origin for the graph of the 75-pound to 150-pound contour. The second segment of the dashed line shows the least-cost ration fed throughout the second weight interval. The point of origin for the graph of the 150- to 225-pound contour lies on the point where the ration line for the second weight interval intersects the 150-pound contour. The three segments of the dashed line illustrate the manner in which the feeds are fed throughout the entire feeding period.

A shift in the price ratios of the feeds will alter the expansion path for the least-cost ration. If the price of corn falls relative to the price of soybean oilmeal, it will be economical to feed rations containing a higher proportion of corn than previously. The ration lines in fig. 1-A would now be shifted to the left. When the contours are assembled in the manner of fig. 1-A, the points of origin for the second and third contours would be shifted to the left along the respective contours. The contours would then be shifted to the right and upward from their present position. The new expansion path for the least-cost ration would be more steeply inclined towards the corn axis.