An Application of Linear Programming to the Study of Supply Responses in Dairying

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CONTENTS

SUMMARY

This study is an application of continuous capital and variable price programming to an analysis of the farm supply of milk and cream. Sioux County, Iowa, was selected as the region to be studied because of its varied farming programs and the importance of dairy production in the area. The 160-acre farm selected for consideration is typical of the soil type, farm size, livestock and cropping programs, farm machinery and building facilities for the locale.

The basic enterprises considered are five dairy activities, five cattle feeding enterprises, spring and fall hog farrowing systems, a supplementary poultry enterprise and five crop rotation systems with four levels of fertilization. Cream production, grade B milk in cans and in bulk, and grade A milk in cans and in bulk are the dairy activities assumed feasible for producers in the area. (In this bulletin manufacturing grade milk is referred to as such or as grade B milk.)

The initial plans in this study are restricted to the resources available to the typical farmer. These resource restrictions include 135 acres of rotated land, 17 acres of permanent pasture, 390 hours of operator's labor for each of June, July and August and 260 hours for each of the remaining months, housing for 200 poultry, 15 litters of spring and fall pigs and 30 dairy cows. Some of these restrictions are relaxed in later phases of the analysis.

All prices used in the study except the dairy prices are projected estimates for Iowa, 1960. The dairy prices used in the fixed price programs are average market prices for the products in the Sioux County marketing area.

One objective of the study was to evaluate linear programming as a tool for the analysis of the supply of agricultural products. The experience of this present study shows that linear programming is a useful tool for the investigation of supply.

Answers to four general questions were sought in the analyses of the five dairy activities. These questions are: (1) How do variations in the operating capital level affect the number of dairy cows included in the optimum plans? (2) What changes in dairy cow numbers occur as the prices of the dairy enterprises are varied? (3) How do simultaneous variations in hog and milk prices affect the optimum plans? (4) What changes occur in the optimum plans as certain changes are made in the price of corn and in the availability of labor and hog housing?

As the level of operating capital rises, the size of the dairy herd kept for cream increases and then decreases. The level of operating capital has no influence on the number of dairy cows kept for grade B milk production in cans beyond the $10,640 level, where the maximum herd size is reached. Fall and spring labor restrictions prevent further expansion of these two dairy enterprises. The other three dairy enterprises, however, continue to expand as the capital level increases.

At high capital levels, bulk milk production permits a larger dairy operation than does can milk production. The assumed saving in labor for the bulk tank operation permits this expansion over the can operation. At low capital levels, can production involves a larger dairy enterprise. The capital intensive bulk operation exhausts the small capital supply at a smaller herd size than does can production. At each capital level, the net income with a grade A operation exceeds the net income with a grade B or cream operation. Above the $11,000 capital level, bulk production of grade A milk or grade B milk is more profitable than can production of the same grade. The differences in net income among the various types of operation are positively related to the level of operating capital.

The size of the bulk milk premiums required to maintain a given volume of production depends on the level of operating capital, with much larger premiums required at low capital levels. The required price differential for grade A milk over grade B milk is the same at all capital levels studied.

The linear programming analysis results in stepped supply functions. Smoothing these curves indicates that the supply of milk or cream is generally highly elastic at low dairy prices and quite inelastic at high prices. In a few cases elasticity constantly decreases as price rises; in most cases, however, it falls, rises and then falls again. In some cases, elasticity fluctuates sharply within a relatively small price range.

At a given price, price elasticity usually varies as the level of operating capital varies.

The price level at which dairying goes out of the optimum plans is lower at higher capital levels. Apparently, once a producer is set up for milk production, it takes lower prices to squeeze him out of dairy production as his capital level increases.

Spring labor is the most restrictive resource for dairy production. At high levels of investment, fall labor and, less frequently, hay and corn supplies become limitational.

Grade A milk production in bulk is the only dairy enterprise that can compete successfully for the limited resources when unlimited hog housing is available and average prices are used. Grade A in bulk enters the optimum plans only at the high capital levels. When variable hog and dairy prices are considered with unlimited hog housing facilities, cream production is not included in the optimum plans for any realistic cream price. The negative relationship between the price of hogs and milk output is clearly exhibited when hog price variations are considered with grade A milk production. Milk supply is quite elastic with respect to hog prices.

The results of the analyses with a labor-hiring activity suggest the following generalizations: (1) The availability of hired labor has little effect on the optimum farm organization except at the higher capital levels. (2) At these higher capital levels, the dairy enterprise is increased; the cattle feeding enterprise is decreased, if included in the plan with no hired labor; and the hog enterprises remain about the same.

Twenty-cent increases and decreases in the price of corn from $1.30 per bushel were considered for

grade A milk production in cans with unlimited hog housing facilities. The corn price of $1.50 results in a decrease in hog production and an increase in the dairy herd over the $1.30 corn-price situation. The plans with the corn price of $1.10 are almost identical with the plans for a price of $1.30. Factors other than price of corn—hay and labor supply—combine to prevent further increases in the already large investments in hogs.

This study shows that the inflexibility of agricultural production as product prices fluctuate is not inconsistent with profit maximization. The prices of some commodities may vary widely without changing the optimum enterprise combination. Small production changes are sometimes associated with different optimum plans as prices change. In many situations the income lost by adopting a suboptimum plan is almost negligible. This suggests that dairy output may not be responsive to price changes even in those situations in which the smoothed supply curves based on optimum farm plans are elastic. They also support the position that the current period's dairy output is an important variable in predicting the next period's output.

This bulletin presents partial results of the linear programming analyses. More complete detail can be found in a Technical Appendix available from the senior author. This appendix contains: (a) basic data used in constructing the input-output coefficients, (b) the complete optimum plans for each situation studied and (c) tables showing income losses suffered by operating at various suboptimum plans.

An Application of Linear Programming to the Study of Supply Responses in Dairying[1]

BY GEORGE W. LADD AND EDDIE V. EASLEY

SCOPE OF STUDY

Inadequate knowledge about the supply response of agricultural products has hampered the formulation of a sound national agricultural price and marketing program. Without a more adequate understanding of supply, agricultural economists cannot fully evaluate the effects of various agricultural programs on the volume of production and consumption, on the incomes of producers and on the welfare of society.

Previous work in dairy supply response is not adequate to meet the current production and marketing problems in dairying. Many of these studies were made in specialized dairy areas and consequently do not apply to those areas where farmers have many possible alternative enterprises (10, 14, 25.) Many studies made before World War II cannot be applied to present-day conditions because of technological advances (2, 10, 14, 20, 26). Most of these studies have been restricted in scope—estimating output per cow or total output from a few variables selected from a long list of pertinent variables.

Recent developments in the application of activity analysis to agricultural research suggest that this technique has promising possibilities for the study of supply response. It permits analyzing the effects of several factors which have hitherto received scant attention. In addition, an activity analysis study of supply develops much information that can be of use in farm management.

The objectives of this study are to investigate the applicability of linear programming to the study of agricultural supply and to obtain quantitative information on the effects of various factors on the farm production of dairy products.

FARM SITUATION STUDIED

Sioux County, Iowa, was selected as the area to be studied because of its varied farming programs. This diversified pattern of farming extends into southwestern Minnesota and southeastern South Dakota. It is believed that this area approximates the types of farming characteristic of large areas of the Plains States. Since the estimation of the profitability of different dairy enterprises relative to other farm enterprises is the main task of this study, Sioux County's selection is further justified, as a substantial quantity of both cream and whole milk is produced there.

The Galva-Primghar-Sac, Marcus-Primghar-Sac and Moody soil types are characteristic of this area. Soil analysts recommend high nitrogen-phosphate combinations for successful fertilization on most soils in this area, while potash is seldom needed (27). According to the 1954 agricultural census (28), the average farm size for the county was approximately 166 acres. Cattle and hog production, cream and whole milk production, and poultry raising were the enterprises with the highest value of products sold, in the order outlined. Corn, oats, hay and soybeans dominate the crop enterprises.

Plans in this study are restricted by the resources available to the typical farmer in the Sioux County area. The farm selected for consideration is typical of the soil type, farm size, livestock and cropping program, farm machinery and building facilities in the area. It is assumed that the farmer has an established farming business and has certain fixed resources at his disposal. These resources include land, machinery and buildings. Since the average farm size is 166 acres, 160-acre farms were selected for analysis. The farm studied is typical of 160-acre farms in this area, having 135 acres devoted to crops and rotated pasture, 17 acres to permanent pasture and 8 acres to farmsteads, roads and fences.

The service buildings on the farm consist of adequate housing for 200 hens, grain and hay storage facilities, a dairy barn which can house up to 30 dairy cows, and adequate space for 15 litters of spring and 15 litters of fall pigs. The machinery and equipment for these quantities of livestock represent part of the existing stock of capital, along with the buildings, land and livestock found on the typical 160-acre farm. It is assumed that the capital in livestock and supplies can be converted to forms allowing reorganization and reinvestment, but that capital in buildings, land and machinery will be retained in these forms even for new farm plans. The grain and hay storage facilities are adequate to handle the production from the cropland. The dairy barn is such that the necessary facilities can be provided

[1]Project 1318, Iowa Agricultural and Home Economics Experiment Station. The authors gratefully acknowledge the help received from Wilfred Candler, John Pesek, Ross Baumann, Sheldon Williams and Paul L. Kelley, who contributed data, ideas, encouragement and constructive criticism.

This report is one of a series of coordinated studies of dairy marketing problems in the Northern Great Plains. These studies have been made by various states cooperating in the North Central Regional Committee on Dairy Marketing Research (NCM-12) and financed partly by regional research funds. The Plains States subcommittee, consisting of representatives on the regional committee from North Dakota, South Dakota, Nebraska, Kansas and Iowa, has had primary responsibility for this research.

when a shift is made from selling cream to selling whole milk or when additional space is needed for an expanded hog enterprise.

The farm studied is owner-operated, and the labor available is that of one operator, plus additional family labor during the months of June, July and August. Total hours assumed available are 390 hours for each of the three summer months and 260 hours for each of the remaining months. This available labor is utilized for all competitive livestock and cropping programs. In addition there are 200 hours of annual labor available for a supplementary poultry enterprise.

The annual cash outlay is used for the purchase of fertilizer, seed, protein supplement for livestock, taxes, fuel and oil, power use, annual veterinary expenses, building and equipment repair, depreciation, and other variable expenses associated with the farm operations. Operating capital is the operator's cash, bank deposits and other liquid assets which are used to pay these expenses.

Prices Used

All prices, except the milk prices, used for the fixed price programs are projected estimates for Iowa, 1960, made by the United States Department of Agriculture. The base milk prices are 1956 average prices for the local Sioux County markets. In various phases of the analysis the dairy, hog and corn prices are varied. The fixed base prices are given in Orazem (24).

Alternative Enterprises

The enterprises to be considered are five dairy enterprises, five cattle feeder plans, spring and fall hog farrowing systems, a poultry enterprise and five crop rotation plans with four different fertilizer levels. All these enterprises compete freely for the available resources, except poultry, which competes for capital and feed only.

Besides the livestock and crop enterprises, which are described in the following paragraphs, milk selling and hog selling activities are included in the analysis. These selling activities are of special interest in this study of supply response. For convenient reference, table 1 lists all of the activities.

TABLE 1. LIST OF ACTIVITIES OR ENTERPRISES INCLUDED IN THE STUDY.

Enterprise number	Activity or enterprise
P_1	Cream production
P_2	Grade B milk production in cans
P_3	Grade B milk production in bulk
P_4	Grade A milk production in cans
P_5	Grade A milk production in bulk
P_6	Short-fed yearlings
P_7	Drylot-fed yearlings
P_8	Drylot-fed steer calves
P_9	Beef cow-calf enterprise
P_{10}	Spring-farrowed hogs
P_{11}	Fall-farrowed hogs
P_{12}	Poultry
P_{13}	CO_0
P_{14}	CO_1
P_{15}	CO_2
P_{16}	CO_3
P_{17}	CCO_0
P_{18}	CCO_1
P_{19}	CCO_2
P_{20}	CCO_3
P_{21}	$CSbCOM_0$
P_{22}	$CSbCOM_1$
P_{23}	$CSbCOM_2$
P_{24}	$CSbCOM_3$
P_{25}	$CCOMM_0$
P_{26}	$CCOMM_1$
P_{27}	$CCOMM_2$
P_{28}	$CCOMM_3$
P_{29}	Dairy selling activity
P_{30}	Hog selling activity
P_{31}	Labor hiring activity

DAIRY ENTERPRISES

Five dairy enterprises are considered feasible for farmers in Sioux County. Farmers in this area may produce either cream, manufacturing milk (sometimes referred to here as grade B milk) in cans or in bulk, or grade A milk in cans or in bulk. The average productive life of the cows in each enterprise is 4 to 5 years. The annual replacement stock for each cow is one-third of a cow, one-third of a 1-year-old heifer and one-fourth of a 2-year-old heifer. Net returns include the value of the dairy stock sold.

Cream production. This enterprise includes cows of medium producing capacity on fair permanent pasture with average management. Each cow is fed 43 bushels of corn and corn equivalent, 5.5 tons of hay and hay equivalent, and 160 pounds of soybean meal. The average production of each cow is 275 pounds of butterfat, 5,000 pounds of skimmilk, and the meat sold as beef. Feed costs and net returns are calculated on a per cow basis. Net returns for the enterprise reflect the market value of the butterfat and the meat plus the feed value of the skimmilk.

Grade B milk production in cans. The annual production capacity of each cow is 8,000 pounds of 3.5-percent grade B milk. The same feeding practices are followed for grade B milk production as for cream production.

Grade B milk production in bulk. This enterprise consists of average management and medium producing cows whose milk is handled with a bulk-tank setup. Each cow produces 8,100 pounds of 3.5-percent milk annually, an assumed savings of about 1 percent over the can operation.[2] The equipment necessary for this enterprise is the same as for the can enterprise except that a bulk tank replaces cans, rack and a milk cooler, and a pipeline installation is added.

Grade A milk production in cans. Above-average management is assumed for this enterprise, where greater care is given to sanitation and feeding practices of the herd. Each cow is fed 47 bushels of corn and corn equivalent, 5.7 tons of hay and hay equivalent and 280 pounds of soybean meal. The annual production per cow is 9,000 pounds.

Grade A milk production in bulk. In this enterprise, all of the necessary equipment for marketing

[2]Some studies have suggested that the savings for a bulk operation are not over 1 percent of the total production from a can operation (6, 23); hence the 100-pound increase between the can and bulk operation.

milk in cans is replaced with a bulk tank and a pipeline installation. Above-average cows produce 9,100 pounds of milk annually. The same feeding practices as outlined for the grade A milk enterprise are followed here.

BEEF CATTLE ENTERPRISES WITH AVERAGE MANAGEMENT

The beef cattle enterprises considered feasible for Sioux County are short-fed yearlings, medium yearlings fed in drylot, choice steer calves fed on drylot and a beef cow-calf enterprise.

Short-fed yearlings. This enterprise consists of medium yearlings bought in November and sold in May and another group bought in May and sold the following November, at an average weight gain of 370 pounds. The yearlings, purchased in November at an average weight of 670 pounds, are put on a moderately high grain ration as soon as possible and are marketed the following May. The second lot of yearlings are purchased in May, put on pasture and a moderate grain ration, and sold the following November. Market weight averages 1,040 pounds per head.

Medium yearlings, fed in drylot. The yearlings, purchased in November at approximately 610 pounds, are sold the following September at 1,070 pounds after being wintered primarily on roughage and put on full feed in early summer. The ration includes 55 bushels of grain and 200 pounds of protein.

Choice steer calves, fed on drylot. The purchase weight of the steer calves is 430 pounds. They are bought in October and sold the following August at 980 pounds. The same feeding practice is followed as in the case of drylot-fed yearlings except that a greater amount of grain is necessary.

Beef cow-calf enterprise. The beef cow is used mainly to produce 1,000-pound calves for sale. A 90-percent calf crop is assumed with replacement of the cow every 8 years; 150 pounds of cull cow and 750 pounds of calf are sold per cow each year. The ration includes 46 bushels of corn, 6.82 tons of hay and 178 pounds of supplement.

HOG ENTERPRISES WITH AVERAGE MANAGEMENT

Spring and fall farrowing of hogs are considered in this study. All data are calculated on a per litter basis. For both systems it is assumed that 7.8 pigs are weaned, 5 percent are lost and 6.5 pigs are sold at a market weight of 220 pounds. One gilt is kept from each litter for farrowing in the following year.

Spring hogs. In this system, pigs are farrowed in April, fed out in pasture, and marketed in October. Each litter consumes 110 bushels of corn, 520 pounds of protein supplement and 0.7 ton of hay and hay equivalent. Pork sold per litter, including a 300-pound sow, averages approximately 1,730 pounds.

Fall hogs. In this system, pigs are farrowed in October, fed out in drylot and marketed in April at 220 pounds. The amount of pork sold per litter is the same as in the spring farrowing. A decline in net return from the spring crop is due to the feeding of more corn and protein supplement.

POULTRY ENTERPRISE

This is a supplementary farm laying flock and is replaced with new stock each year. It utilizes only the homemaker's labor; therefore, it does not compete with the other enterprises for labor. It does compete, however, for capital and feed resources. The average annual production per bird is 15 dozen eggs and 4.87 pounds of meat. An average of 1.73 sexed chicks per hen must be purchased each year for potential laying. The mortality rates for hens and chicks are estimated at 15 and 10 percent, respectively.

CROP ENTERPRISES[3]

The following rotations are considered feasible for farmers in Sioux County: corn-oats (CO), corn-corn-oats (CCO), corn-soybeans-corn-oats-meadow (CSbCOM), corn-corn-oats-meadow (CCOM), and corn-corn-oats-meadow-meadow. (CCOMM). Four levels of fertilization are considered in this study for each rotation: (a) no fertilizer, denoted by the subscript zero, (b) 25 pounds of available nitrogen, denoted by the subscript 1, (c) 50 pounds of available nitrogen, denoted by the subscript 2, and (d) 80 pounds of available nitrogen, denoted by the subscript 3. For example, a $CCOM_2$ is a corn-corn-oats-meadow rotation, with 50 pounds of available nitrogen. Hence, there are 20 crop alternatives, five rotations with four different fertility levels.

ANALYTICAL PROCEDURE

This study used the continuous capital and variable price modifications of the simplex method of linear programming (7, 8). The logic and technique of linear programming have been adequately dealt with in the literature (3, 4, 9, 12, 13, 17). Several applications of the method have been made to farm adjustment and planning problems (5, 16, 18, 22, 24).

For the details of the two computational procedures used here see Candler (7, 8). Some computational economy in the simultaneous use of these two procedures can be achieved in the following way. Set up the matrix for variable pricing but apply the continuous capital procedure first to obtain the capital optimum programs. Variable pricing may then be applied at any of these capital optimums. For any other capital level of interest, choose the capital optimum just before the level of interest. By adding the difference between the optimum and the desired levels of capital to the capital supply of the optimum program, a new plan is brought in at the desired capital level. For example, if a capital optimum is \$8,530 and a desired capital level is \$9,000, simply add \$470 to the capital supply in the optimum plan and use the continuous capital criteria to determine which activity to introduce into the plan to obtain the \$9,000 optimum.

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[3]The feasible rotations, fertility estimates and production responses were obtained from Dr. John Pesek, Department of Agronomy, Iowa State College.

TABLE 2. SITUATIONS ANALYZED.

Situation number	Situation	Figure
1	Continuous capital for P_1, P_6 - P_{28}	1
2	Continuous capital for P_2, P_6 - P_{28}	1
3	Continuous capital for P_3, P_6 - P_{28}	1
4	Continuous capital for P_4, P_6 - P_{28}	2
5	Continuous capital for P_5, P_6 - P_{28}	2
6	Variable pricing for P_1 with P_6 - P_{29} for \$6,000, \$9,000 and \$15,000 capital levels	3
7	Variable pricing for P_2, with P_6 - P_{29} for \$6,000, \$9,000 and \$12,000 capital levels	4
8	Variable pricing for P_3, with P_6 - P_{29} for \$9,000, \$12,000 and \$15,000 capital levels	5
9	Variable pricing for P_4 with P_6 - P_{29} for \$6,000, \$9,000 and \$12,000 capital levels	6
10	Variable pricing for P_5 with P_6 - P_{29} for \$9,000, \$12,000 and \$15,000 capital levels	7
11	Continuous capital, relaxed hog restriction with P_1, P_6 - P_{28}	
12	Continuous capital, relaxed hog restriction with P_4, P_6 - P_{28}	
13	Continuous capital, relaxed hog restriction for P_5, P_6 - P_{28}	2
14	Variable pricing for P_1, P_{10} and P_{11}, relaxed hog restriction with P_6 - P_{30} for \$9,000 capital level	8
15	Variable pricing for P_4, P_{10} and P_{11}, relaxed hog restriction with P_6 - P_{30} for \$9,000 capital level	9
16	Variable pricing for P_5, P_{10} and P_{11}, relaxed hog restriction with P_6 - P_{30} for \$9,000 capital level	10
17	Continuous capital, restricted hog production with P_1, P_6 - P_{31}	11
18	Continuous capital, relaxed hog restriction with P_4, P_6 - P_{31}	11

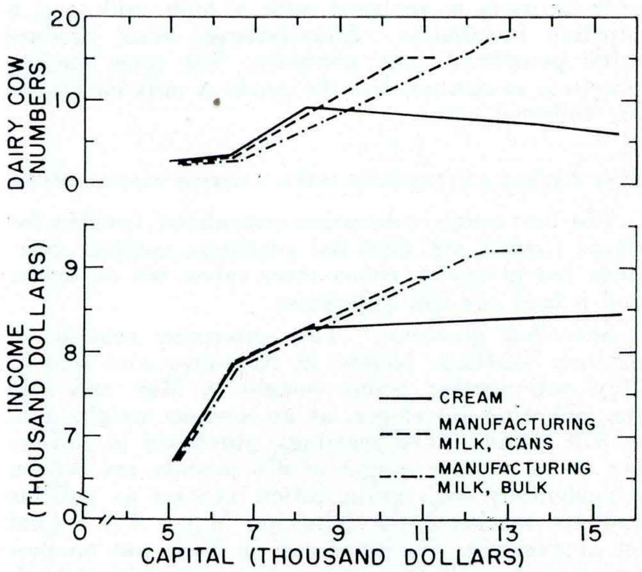

Fig. 1. Capital-income and herd size curves for cream and manufacturing milk.

In this study the continuous capital procedure was carried to the point at which operating capital became nonlimitational; i.e., to the point at which further increases in the level of operating capital caused no further change in the composition of output. The variable price procedure was arbitrarily stopped when unrealistically high dairy prices were reached.

A number of factors were analyzed for their effect on dairy supply. They are operating capital, price of dairy products, hog prices, corn prices, hog housing restrictions and the hiring of labor. The situations analyzed are listed briefly in table 2.

RESULTS

Results of the linear programming analyses are presented in figs. 1 to 11. Since interest in this study focuses on dairy supply, only data on dairy supply and income are generally presented. The income figures presented have not been adjusted for fixed costs. They are Z_j - C_j values obtained in the matrix; Z_j is the price received per unit of product, and C_j is the average variable cost of production. Deduction of machinery costs, real estate costs, personal property taxes, insurance and miscellaneous fixed costs will adjust these figures to net returns.

Restricted Hog Housing

CONTINUOUS CAPITAL

These results are presented in figs. 1 and 2. A dairy herd for cream production reaches its maximum size at the lower capital levels, then yields to a cattle feeding enterprise as the level of capital increases. The capital level has no influence on dairy cow numbers kept for grade B milk production in cans beyond the \$10,640 level. As soon as these two dairy enterprises reach their maximum sizes, the spring labor supply is exhausted. The relative profitability of other enterprises prevents dairying from drawing labor away from them for further expansion of the dairy herd as the capital level increases. The other three dairy enterprises studied continue to expand even after the spring labor supply is exhausted, by drawing resources away from the nondairy enterprises. These three dairy enterprises have a higher net return per unit of output than do cream and grade B in cans. Grade A in bulk, which has the

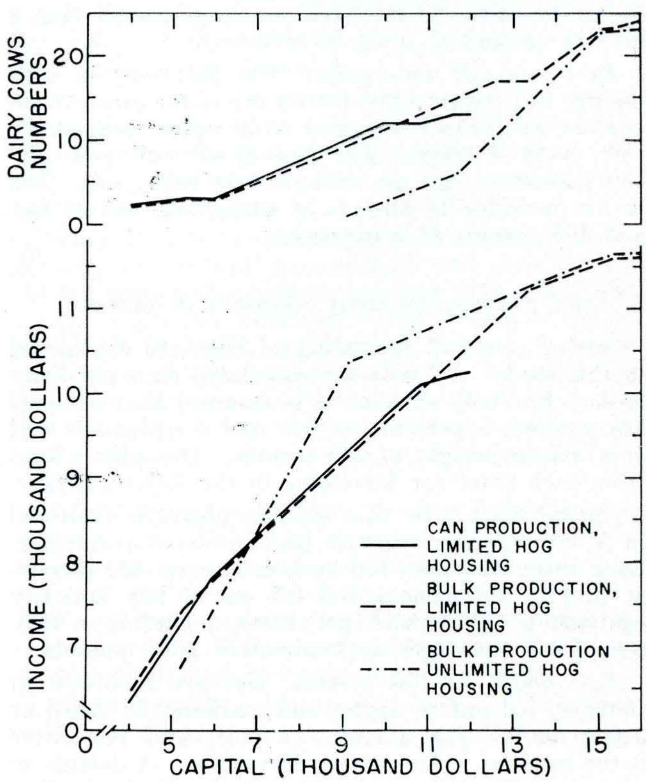

Fig. 2. Capital-income and herd size curves for grade A milk in cans and bulk.

highest net return per unit, is the only one which expands substantially after spring labor becomes limitational.

With sufficient capital, bulk production permits a larger herd size than can production. Spring labor becomes limitational at a lower capital level and with a smaller herd size for the can operations. The maximum herd sizes for the bulk operations are reached at higher capital levels than for the can operations.

At each capital level, grade A bulk production yields a larger net return than does grade B bulk production. At capital levels below $11,000, the average return to capital is highest for grade A in cans; above this level, grade A in bulk has the highest average return. At capital levels above $11,000, grade B in bulk has a higher average return to capital than grade B in cans.

The differences in income among the various situations are positively related to the level of operating capital. At the $6,000 level, the difference in income between cream production and either of the grade A techniques is $400, from which must also be deducted any cost of transfer; grade B production yields less net income than cream production. At the higher capital levels the difference between the income-capital curves is wider. At the $14,000 capital level, for instance, the income difference between grade A in bulk and cream is $2,900. This income difference is $1,800 when grade A in cans is considered; $800 when grade B in bulk and $500 when grade B in cans is considered.

VARIABLE DAIRY PRICES

The results from the variable pricing modification of the simplex method are presented in figs. 3

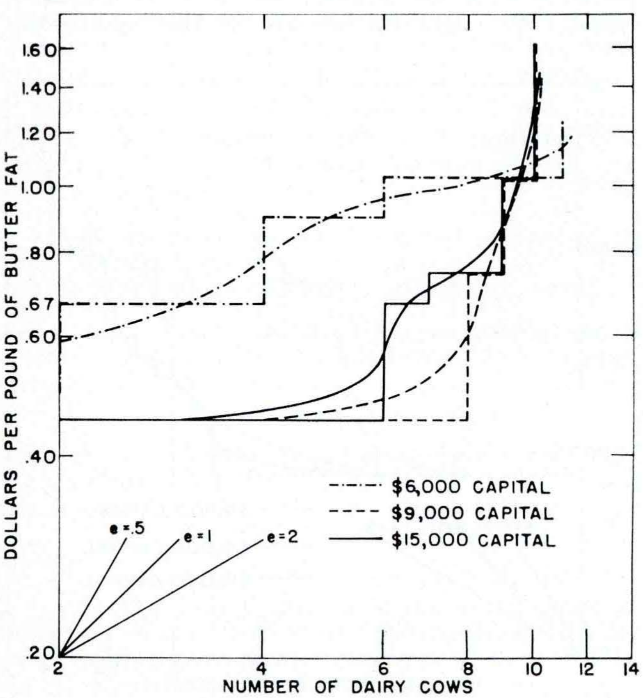

Fig. 3. Stepped and smooth supply functions for cream.

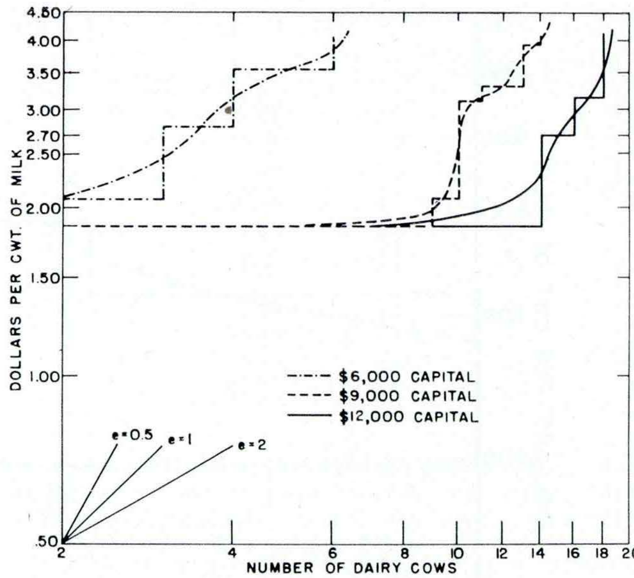

Fig. 4. Stepped and smooth supply functions for grade B milk in cans.

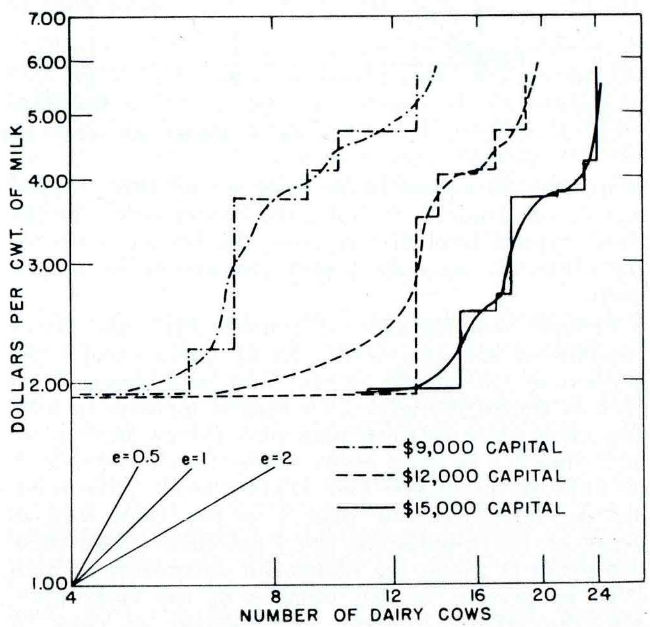

Fig. 5. Stepped and smooth supply functions for grade B milk in bulk.

through 7.[4] For all dairy enterprises except grade B in bulk, there is an inverse relationship between the capital level and the price at which dairying goes out of the optimum plan. The relationship is clearest for grade A production in cans. For the others, the price at which dairying goes out is lower for intermediate and high capital levels than for low capital levels. At the intermediate and high capital levels, when dairying does enter the optimum plan, it enters with larger herd sizes.

For every dairy enterprise, the price at which dairy goes out of the optimum plan is the same for at least two capital levels. In the case of grade

[4]These are logarithmic charts to facilitate visual comparison of elasticities. As an aid in making these comparisons, three constant elasticity curves are drawn in the lower left-hand corner of each graph.

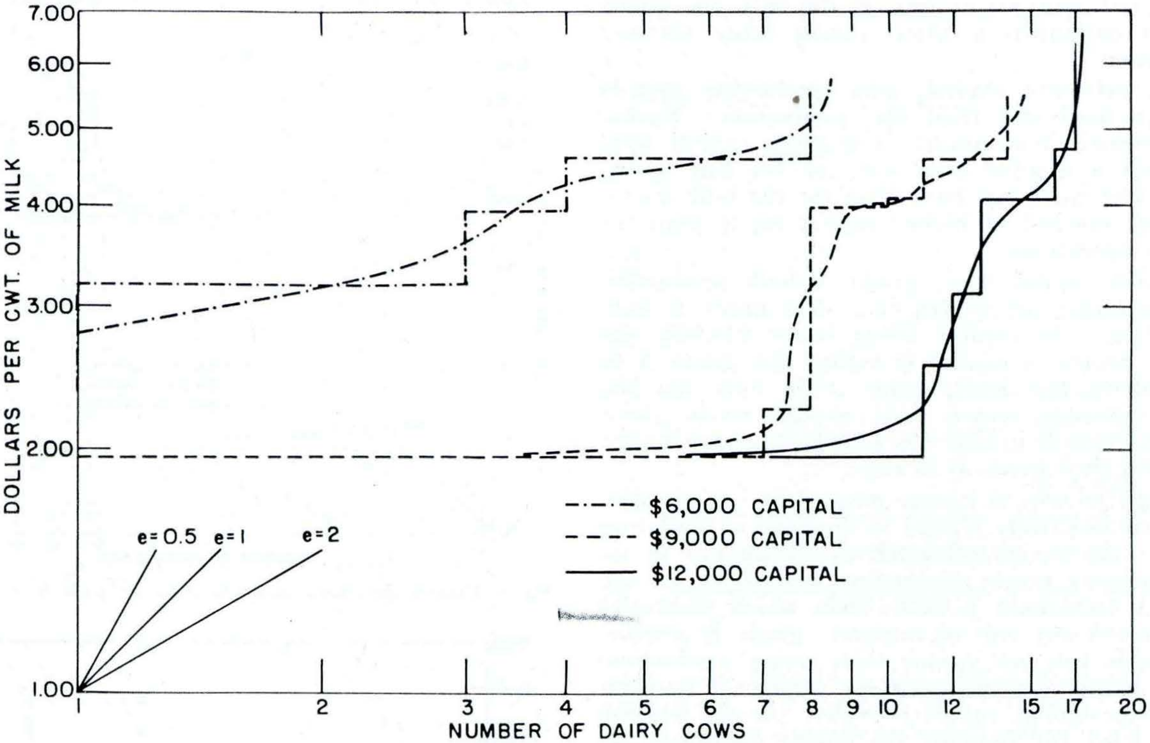

Fig. 6. Stepped and smooth supply curves for grade A milk in cans.

B in bulk, this price is the same for all three capital levels; for grade A in bulk, the border price for the third capital level differs from the border price for the other two by only 1 cent; for cream, by only 4 cents.

Fifteen combinations of capital level and dairy enterprises are analyzed.[5] In all cases except two (cream at the $6,000 capital level and grade A in cans at the $6,000 level), the largest increase in herd size takes place between plan zero, the no-dairy plan, and plan 1. In four plans (these two and grade A in bulk at the $9,000 and $15,000 levels), the number of dairy cows in plan 1 is less than half as great as the number in the final plan. For these two grade A plans, the number of dairy cows in plan 1 is 46 percent of the number in the final plan. For the other 11 plans, the number of cows in plan 1 equals or exceeds 50 percent of the number in the final plan. These results indicate that, at low dairy prices, both the single farm dairy supply curve and the total supply curve for an area are highly price responsive.

At the lower capital levels, capital is the limitational resource which restricts the expansion of the dairy enterprise. At the higher capital levels, especially in combination with the higher prices, spring and fall labor are the limiting resources.

The concept of supply elasticity is not useful in connection with these curves since the point elasticity is zero (on the vertical segments of the supply curve), infinite (on the horizontal segments) or indeterminate (at the corners). The arc elasticity varies from zero to nearly infinity. It seems worthwhile to attempt to obtain some measure of supply elasticity for these curves for purposes of comparison even though the measures are arbitrary.

We might suppose that each dairy enterprise-capital level combination studied here is a representative firm for a group of fairly homogeneous firms. We might further assume that aggregating

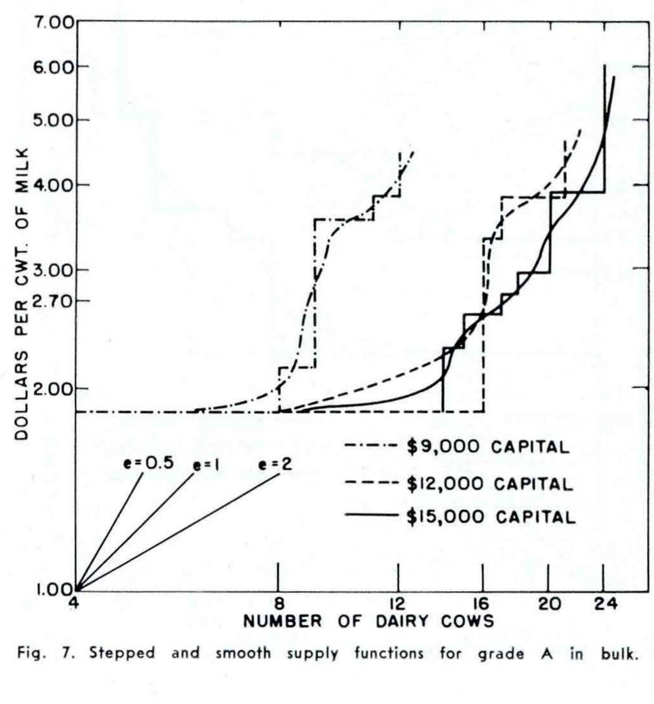

Fig. 7. Stepped and smooth supply functions for grade A in bulk.

[5]An analysis was run for grade B in bulk at the $6,000 capital level. The plans obtained had only four or five cows at the highest prices. It is reasonable to assume, however, that the expense of installing a bulk tank would make it unprofitable to operate with such small herd sizes.

the supply curves for these firms and dividing by the number of firms would give a smooth curve whose general characteristics are those of a smooth curve drawn through the stepped supply curves determined by linear programming. Having stated these two assumptions, it is necessary to specify the way in which the characteristics of the stepped supply curve determine the characteristics of the smooth curve. One reasonable procedure is to suppose that the average curve so obtained is the smooth curve drawn through the midpoints of the vertical segments and the midpoints of the horizontal segments (Method I). Alternatively, one could fit least squares regressions to these same points. Some experimenting indicated that the curves obtained by this latter method are substantially the same as those obtained by connecting the midpoints of the horizontal segments with smooth curves (Method II).

Because of the arbitrary nature of any criteria that might be used, the expense of calculating least squares curves was avoided by using Method I and free-hand Method II. The smoothed curves shown in figs. 3 through 7 are those fitted graphically by Method I. From a visual analysis it appears that these curves give a reasonably good fit, with the possible exception of the curves for the intermediate and high capital levels for grade A in bulk and the intermediate capital level for grade A in cans. The elasticity for each of these three curves is changing rapidly at the base price.

Table 3 shows the elasticity for each curve at the base price, determined graphically from the Method I curves and the Method II curves. If the previously described process of obtaining a smooth curve by averaging a total supply curve is valid, the Method I curves seem to be superior to the Method II curves on grounds of logic and consistency. In many instances there are substantial differences between the elasticities given by the two methods. It is hoped that continuing work in this field can lead to clearer criteria for drawing the smoothed supply curves.

The smoothed curves in figs. 4 to 7 give some idea as to the impact of bulk milk handling in a market. Comparing optimum volumes of can and bulk production:

1. At the $9,000 capital level, can production of grade B milk exceeds bulk production at each price, but the price elasticities are approximately equal.

2. At this same capital level, can production of grade A milk exceeds bulk production and the elasticities are again approximately equal at each price except in the neighborhood of the base price.

3. At the $12,000 capital level, bulk supply and its elasticity successively exceed, fall short of, and exceed can supply and can supply elasticity as prices rise. The differences, however, are fairly small.

4. At this capital level, can supply of grade A milk falls somewhat short of bulk supply at all relevant prices; can supply elasticity is equal to or slightly higher than bulk supply elasticity except around the base price.

TABLE 3. ESTIMATED SUPPLY ELASTICITIES AT BASE PRICES.

		Elasticity	
Base price	Capital level	Method I	Method II
$0.67	$ 6,000	2.1	1.6
	9,000	0.3	0.7
	15,000	1.0	2.0
2.70	6,000	1.0	1.7
	9,000	0.4	0.4
	12,000	0.5	0.8
2.70	9,000	0.2	0.2
	12,000	0.2	1.1
	15,000	1.1	1.2
4.05	6,000	1.3	3.3
	9,000	1.0	2.6
	12,000	1.3	1.3
4.05	9,000	0.8	1.9
	12,000	1.3	1.3
	15,000	1.2	0.5

5. At and above the $15,000 capital level, bulk supply of grade B milk exceeds can supply, and bulk supply elasticity exceeds can supply elasticity.

6. At and above $15,000, bulk supply of grade A milk substantially exceeds can supply at each price, but bulk and can supply elasticities are about equal at each price.

The relationship between the optimum volumes of bulk milk and can milk production varies with the level of operating capital because of the saving in labor for the bulk operation with a pipeline installation over the can operation and because of the larger operating capital requirement per cow for bulk production.

Unlimited Hog Housing

The previous analyses assumed available hog housing facilities could handle no more than 15 litters of spring pigs or 15 litters of fall pigs. However, 15 litters is an average for Sioux County 160-acre farms (28), not necessarily a maximum. In many of the previous plans, hog housing was a limitational resource for spring pigs. In the continuous capital analyses, it was limitational in the programs for cream and grade B in cans at every capital level. In the grade A in bulk programs hog housing was limitational at every capital level but the highest. In the variable pricing programs it was generally limitational at all dairy prices for the lowest levels of operating capital and at all but the higher dairy prices for the other two levels of operating capital.

Some analyses were made in which unlimited hog housing was available. A farmer might obtain the effect of unlimited housing if he used the loose housing system for dairy cattle (11). This system offers a highly flexible arrangement that can be easily converted to housing for other types of livestock if not used by the dairy herd.

CONTINUOUS CAPITAL

The optimum plans for grade A bulk milk production are presented in fig. 2. With unlimited hog housing facilities and average hog and dairy prices, cream and grade A can production are eliminated from the optimum farm organization, and net incomes are substantially increased. The limited hay (which includes the pasture equivalent of hay), corn and labor supplies are consumed by large investments in hog production. Even the volume of grade A bulk production is substantially smaller than formerly except at the highest capital levels.

VARIABLE PRICES FOR HOGS AND DAIRY

Optimum plans with variable hog and dairy prices are presented in figs. 8, 9 and 10. At a hog price of $16, no cream is sold until butterfat reaches a price of $1.45. The production of grade A milk enters the optimum plan when the ratio of dairy to hog prices is about 30 percent. Grade A production is highly sensitive to variations in hog prices. Cream production is highly responsive to hog price changes when this price reaches low levels. In fact, dairy production seems to be more responsive to hog price changes than to dairy price changes. Relatively small hog price changes are sufficient to change dairy herd size from zero to 12 or so.

CONTINUOUS CAPITAL WITH A LABOR-HIRING ACTIVITY

Spring and fall labor became limitational in so many of the programs that it was decided to explore the effects of adding a labor-hiring activity to the matrix. The wage rate used is $1.04 per hour, which was the average monthly wage rate without room and board for spring labor in Iowa in 1956. The results are presented in fig. 11.

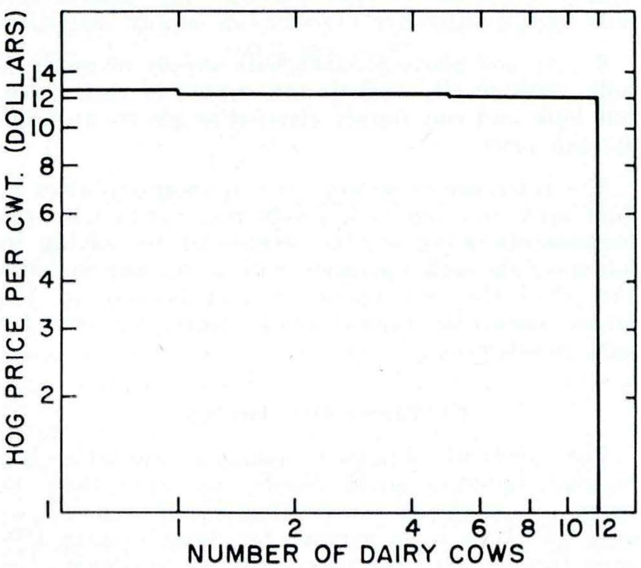

Fig. 8. Stepped supply function for cream as a function of hog price when cream is 67 cents per pound of fat.

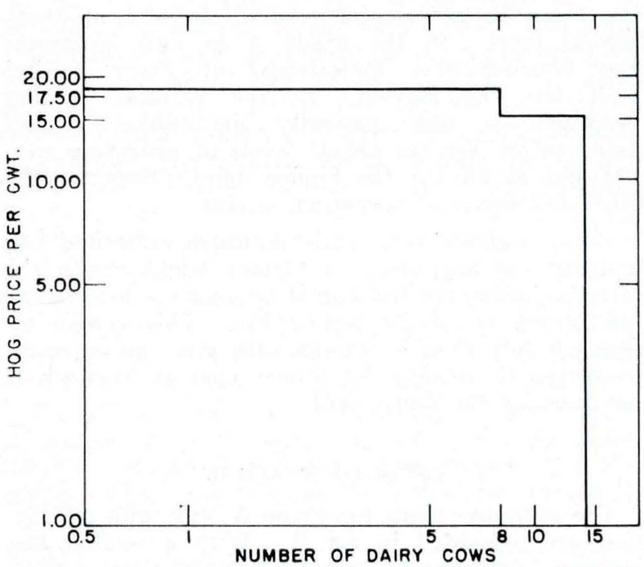

Fig. 9. Stepped supply function for grade A milk in cans as a function of hog price when milk is $4.70 per cwt.

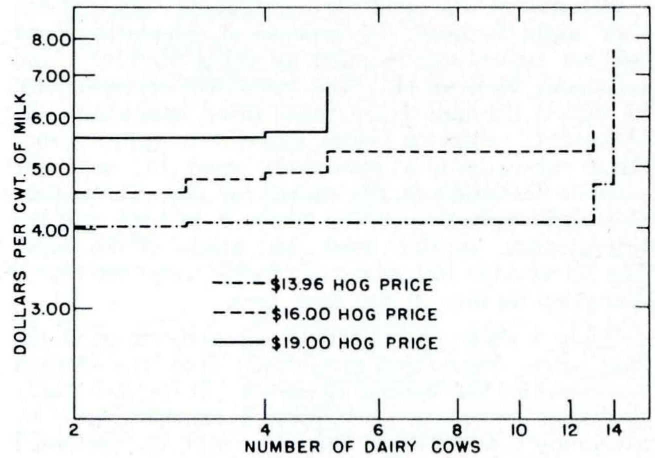

Fig. 10. Stepped supply functions for grade A milk production in bulk at various hog prices.

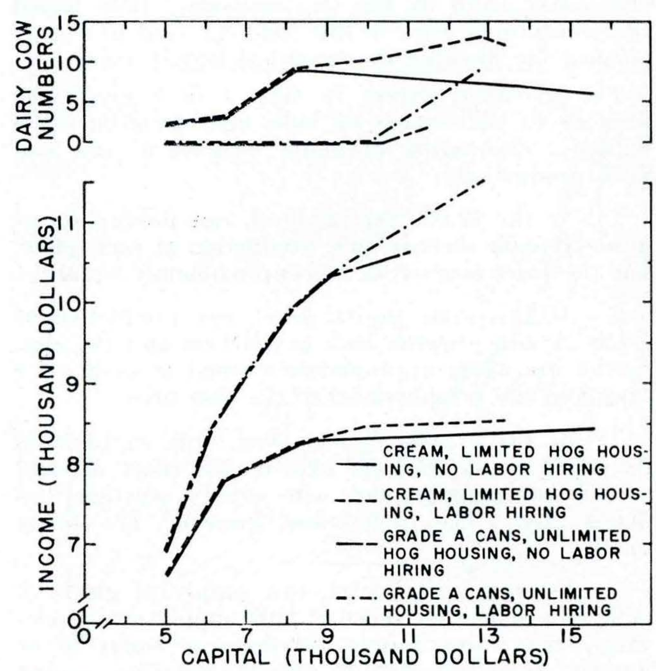

Fig. 11. Capital-income and herd size curves with and without labor-hiring activity.

RESTRICTED HOG HOUSING, CREAM PRODUCTION

In the absence of a labor-hiring activity, the spring labor supply is exhausted at a capital level of $8,193. Labor hiring alters the optimum plans only at higher capital levels, where cream production, which is a high labor-consuming enterprise, expands sharply.

UNLIMITED HOG HOUSING, GRADE A MILK IN CANS

In the absence of a labor-hiring activity, the spring labor supply is exhausted at a capital level of $8,229 and the fall labor supply is exhausted at a level of $11,218. The addition of a labor-hiring activity has little effect on the optimum plans until the $8,500 capital level. Beyond this level it increases the number of hog litters slightly in a few plans; it increases the number of dairy cows in the final capital optimum by 7.

Any conclusions reached from only two analyses are highly tentative. These two analyses do have two points in common, however. The availability of hired labor has little effect on the optimum farm organization except at the higher capital levels. At these levels the dairy enterprise is increased.

Continuous Capital With Unrestricted Hog Production and Changes in the Corn Price

Twenty-cent increases and decreases in the price of corn from the base price were considered in an analysis of grade A milk production in cans. The increase of 20 cents had a negligible effect on the optimum plans at the lower capital levels. The greatest effect was at the highest capital level of $11,789, where dairy herd size reached 17 and hog production was eliminated. The 20-cent decrease in the price of corn had a negligible effect at every capital level. Factors other than price—hay and corn requirements and spring and fall labor restrictions—combine to prevent increases in the already large investments in hog production.

Income Losses From Adopting Suboptimum Plans

The farm plans presented previously are those plans which farmers operating under the conditions imposed in this study would follow if they desired to maximize profits and had complete and certain knowledge. Farmers' actual reactions to price changes involve at least two elements: (1) The price elasticity of supply assuming profit maximization in the presence of perfect knowledge and certainty (optimum plan elasticity). (2) The income lost by not maximizing profits because of the existence of other motives, imperfect knowledge and uncertainty. A new plan adopted in response to a price change may require greater or smaller labor inputs, once established. In most cases changing the farm organization will require added work during the transition period; certainly it will require added managerial effort. It seems worthwhile to consider how much income a farmer will lose if he remains with a plan which is optimum with one set of prices, even though prices change sufficiently to cause a different plan to become optimum.

These income losses were computed for all plans presented in figs. 3 through 7. The main features of the income loss data were the same in each case. To avoid excessive detail, only one table of income losses is presented here—table 4.[6] Obviously, the optimum plan for a given price situation must give a higher income than a suboptimum plan, so the income difference is given as the loss in income from remaining with a given plan under various price conditions. Let n be the number of dairy price ranges for a given capital level with a given dairy enterprise. There are then n different optimum plans. Each plan is optimum for one price range and suboptimum for n—1 price ranges. For each capital level the income loss section of a table contains n rows numbered from 0 to n—1 and n columns numbered likewise. Two numbers appear in the i-th row and the j-th column; call the first a_{ij} and the second b_{ij}. These two numbers represent, respectively, the minimum and maximum income losses if plan i is optimum and plan j is adopted for $i > j$, and the maximum and minimum losses, respectively, for $i < j$. For example, considering the $9,000 capital level, if plan 3 is optimum and plan 2 is adopted, the income loss ranges from $0(=a_{32})$ at a price of $3.75 to $13 $(=b_{32})$ at a price of $4.10. Net income ranges from $8,962 at the lower price to $9,194 (=$9,207—$13) at the higher price. (Note that at the border price of $3.75 both plans are optimum.) If plan 2 is optimum and plan 3 is adopted, the income loss ranges from $0(=b_{23})$ at a price of $3.75 to $262 $(=a_{23})$ at a price of $2.24; net income varies from $7,707 (=$7,969—$262) to $8,962.

One of the striking points is the number of relatively small income losses, even with relatively large changes in dairy price and in optimum herd size.

If optimum plan elasticity is high and income loss is also high, farmers will probably be quite responsive to price changes. If both are small, they will probably be quite unresponsive. If one is high and the other low, it is reasonable to expect that they will again be unresponsive to price changes. If we consider price changes of the magnitude usually experienced historically and look at the smoothed curves in figs. 3 through 7 in conjunction with the income loss data, it is evident that the second situation is the most common one.

Almost all cases in which both are high occur in the lower price ranges when plan zero is adopted while some other plan is optimum. The optimum plan elasticity is generally high in the lower price ranges and b_{10} and b_{20} are often quite large. On the other hand, if plan zero is optimum and some other plan is adopted, we commonly have the third situation—high optimum plan elasticity and low income loss. The income losses b_{0j} are often quite small even with large herd sizes in plan j. Out of 15 cases, $a_{20} > b_{02}$ in 12 and $a_{20} < b_{02}$ in 3. Together these suggest that a price increase from price level

[6] The method of computing income losses from suboptimum plans is explained in the Appendix.

TABLE 4. COMPARISON OF INCOME DIFFERENCES BETWEEN THE OPTIMUM PLAN AND SUBOPTIMUM PLAN AS PRICES VARY FOR GRADE A IN BULK AT SELECTED CAPITAL LEVELS.

Capital level	Optimum plan	Price range ($)		Dairy cows (head)	Optimum income ranges ($)		Range of income loss from suboptimum plan ($)* 0		1		2		3		4		5		6		7		8	
$ 9,000	0	0.00	1.94	0	0	7,797	....	....		0		27		339		598		1,403						
	1	1.94	2.24	6	7,797	7,969	0	172	....	....	27	0	339	262	598	491	1,403	1,215						
	2	2.24	3.75	7	7,969	8,962	172	1,165	0	128	....	....	262	0	491	80	1,215	395						
	3	3.75	4.10	9	8,962	9,207	1,165	1,410	128	173	0	13	....	....	80	0	395	220						
	4	4.10	4.70	10	9,207	9,707	1,410	1,910	173	329	13	114	0	1	....	....	220	0						
	5	4.70	5.65	13	9,707	10,593	1,910	2,796	329	671	114	370	1	98	0	3	....	....						
$12,000	0	0.00	1.93	0	0	7,930	....	....		0		106		443		565		1,346						
	1	1.93	3.49	13	7,930	9,835	0	1,905	....	....	106	0	443	107	565	140	1,346	499						
	2	3.49	4.06	14	9,835	10,547	1,905	2,617	0	14	....	....	107	0	140	2	499	206						
	3	4.06	4.10	16	10,547	10,605	2,617	2,675	14	23	0	6	....	....	2	0	206	193						
	4	4.10	4.70	17	10,605	11,470	2,675	3,540	23	153	6	98	0	3	....	....	193	0						
	5	4.70	5.65	19	11,470	12,890	3,540	4,962	153	412	98	296	3	60	0	4	....	....						
$15,000	0	0.00	1.93	0	0	8,056	....	....		0		81		191		602		978		1,107		1,157		1,170
	1	1.93	2.54	15	8,056	8,895	0	839	....	....	81	0	191	21	602	294	978	545	1,107	643	1,157	679	1,170	692
	2	2.54	2.69	17	8,895	9,122	839	1,066	0	16	....	....	21	0	294	239	545	459	643	549	679	583	692	595
	3	2.69	3.73	18	9,122	10,839	1,066	1,717	16	266	0	148	....	....	239	0	459	7	549	44	583	55	595	67
	4	3.73	3.80	21	10,839	10,957	1,717	118	266	230	148	176	0	19	....	....	7	0	44	34	55	43	67	56
	5	3.80	4.10	23	10,957	11,564	118	607	230	484	176	330	19	129	0	43	....	....	34	0	43	2	56	15
	6	4.10	4.21	23	11,564	11,799	607	235	484	564	330	399	129	182	43	71	0	6	....	....	2	0	15	13
	7	4.21	5.05	24	11,799	13,600	235	1,801	564	1,180	399	933	182	594	71	292	6	55	0	6	....	....	13	0
	8	5.05	5.80	24	13,600	15,236	1,801	1,636	1,180	1,759	933	1,438	594	989	292	518	55	126	6	40	0	17	....	....

*Income figure rounded to nearest dollar; income change less than $1 is denoted by a zero.

P_0 to P_1 or P_2 will be quite effective in inducing farmers to begin dairying, while a price decrease from P_1 or P_2 to P_0 will be much less effective in inducing them to quit dairying.

This contrasts with the situation at higher prices. A comparison of plans i and i + 1 ($i \neq 0$) in each situation shows that the maximum income loss from following plan i + 1 when i is optimum is generally greater than the maximum loss from following i when i + 1 is optimum. Out of a total of 59 such comparisons, $a_{i,\ i+1} > b_{i+1,\ i}$ in 45 cases; $a_{i,\ i+1} < b_{i+1,\ i}$ in 13 and $a_{i,\ i+1} = b_{i+1,\ i} = 0$ in 1 case. Six of the cases where $a_{i,\ i+1}$ is smaller occur with grade A in bulk. Similar results generally hold for $a_{i,\ i+2}$ and $b_{\ i+2,\ i}$. These results suggest the hypothesis that a price increase from level P_i to level P_{i+1} will be less effective in increasing dairy production than a price decrease from P_{i+1} to P_i will be in decreasing dairy production, after the farmer is set up for dairying.

This hypothesis is also suggested by the fact that $b_{i+1,\ i} < a_{i-1,\ i}$. This means that the income loss from having fewer than the optimum number of dairy cows is less than the loss from having more than the optimum number. In some price ranges, elasticity declines as price increases. Within these price ranges, it is likely that a price increase from P_i to P_{i+1} will have less effect on total dairy production than a price decrease from P_i to P_{i-1}. In other price ranges, elasticity increases with rising price. Within these ranges, the fact that $b_{i+1,\ i} < a_{i-1,\ i}$ may be offset by the increasing elasticity between P_{i-1} and P_{i+1} so that movement from P_i to P_{i+1} will have as much effect as a movement from P_i to P_{i-1}.

IMPLICATIONS OF THE RESULTS

The foregoing sections have presented the results obtained in the analyses. This section will discuss some of the over-all significance and general implications of the results. Based as they are on a limited number of analyses of a specific farming situation, the generalizations in this section are necessarily tentative and subject to revision in the light of later findings. It is hoped that more study and empirical work will lead to a broader application of linear programming to analysis of industry and regional supply relationships. Such an application may be particularly useful in formulating national dairy policies.

Implications for Dairy Marketing

The optimum farm plans show many cases in which product prices may vary widely without changing the opimum enterprise combination. Small production changes are sometimes associated with different optimum plans as prices change. Changes in the corn price and the availability of hired labor affect optimum dairy production only at higher capital levels. These results are a possible explanation of the long-noted inflexibility of farmers' production patterns. The analyses presented above suggest another reason for the inflexibility of production patterns: the small income loss suffered by the farmer if he continues his previous enterprise combination even though price changes have made that combination suboptimum and another combination optimum. In some cases, farmers may feel that the income lost by not adjusting their operation is so small it is not worthwhile to make the change in farm organization. In other cases, the income gained by shifting from a suboptimum plan to an optimum plan (which equals the income lost by not shifting) may be imperceptible to the operator because of its small size.

Let us suppose that the income gained by shifting to the optimum plan must exceed 2 percent of the net income earned by remaining with the suboptimum plan before the farmer will change his operations. By using the data on income losses, the effects on the supply curve can be determined. As an illustration, consider the $12,000 capital level in table 4; the results are shown in fig. 12, where the solid line is the supply curve from fig. 7 and the dotted lines represent price changes necessary to cause the farmer to change his farm plans under the previous assumption. For example, if the farmer has adopted plan 2, which is optimum for the price range of $3.49 to $4.06, and has 14 dairy cows, the price can rise to $5.41, where plan 5 with 19 cows is optimum, before the income lost by not changing his farm organization amounts to 2 percent of the suboptimum income he earns with plan 2 at this price of $5.41. Likewise, the price can fall to $1.93 before the income loss amounts to 2 percent of his income earned at that price from plan 2.[7]

[7]Actually, the price can fall below $1.93. Not having computed income losses at dairy prices of zero, it is not possible to find the exact price below $1.93 at which the 2-percent level is reached.

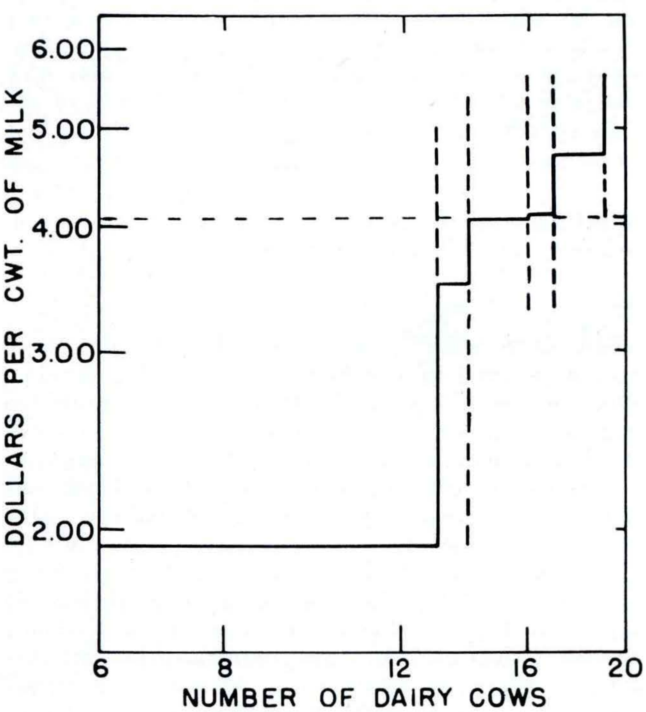

Fig. 12. Suboptimum supply curves for grade A in bulk at the $12,000 capital level.

Suppose the current price is $4.08 and suppose that the operator was in an equilibrium position last year. His current herd size is indicated by that one of the vertical dotted lines passing through $4.08 which indicates his optimum herd size last year. Thus current dairy herd size could vary between 13 and 19 cows—19 percent above or below optimum output—depending upon his preceding operations.

Figure 12 can also be used to study variation in current herd size at other prices. The same computations could be performed for other capital levels and other enterprises. This one example, however, suffices to illustrate the point. Similar results would be found from other computations. These results show that the current year's level of production is an important factor in the determination of future levels of production.

The results suggest that the length of time farmers believe a certain price situation will last and the strength of their belief may be just as important as the level of prices in determining the supply of dairy products. If a price change makes a farmer's present farm organization suboptimum, his decision on changing his farm plan is related to the expected duration of the new prices and the strength of that expectation. He is more apt to adjust his operations to avoid an annual income loss if he strongly believes the new price structure to be fairly permanent than if he believes it to be temporary or believes that prices are just as likely to return soon to their former levels as to remain at present levels. It would be expected that the element of certainty of duration would play a more important role in his decision if the annual income loss were small than if it were large. The small size of many of the suboptimum income losses, therefore, suggests that a program aimed at increasing or decreasing dairy production should place as much emphasis on the certainty of the new relative price levels as on the magnitude of those levels.

The amount of operating capital influences the optimum volume of dairy production and the type of dairy enterprise. This analysis suggests that a significant part of the variation among farms and among areas in volume of dairy production can be explained by differences in the amount of operating capital farmers possess. The impact of bulk milk handling on milk supply in a market will also depend on the level of operating capital. At the $9,000 capital level, a shift from can to bulk production of either manufacturing milk or grade A milk must be accompanied by substantial premium payments if the farmer's optimum herd size and net income are to be maintained. If base prices prevail for can milk—$2.70 and $4.05, respectively—the bulk milk bonus must be in the neighborhood of 50 cents for grade A milk and 60 to 70 cents for manufacturing milk. These premiums will maintain optimum herd size at their previous levels and will maintain or increase the net income of producers. At a capital level of $12,000, the optimum herd size will remain the same even if the bulk producer receives a 10- to 30-cent lower price than the can producer, if prices for can milk are at their base level. (At somewhat higher prices for grade B milk in cans, a premium will be required to maintain optimum production levels.) Net income will be nearly maintained at previous levels even after these price reductions. At and above the $15,000 capital level, the optimum herd size for grade A milk production would be maintained even if the bulk producer received $1.20 less for his milk; the optimum herd size for manufacturing milk production would remain the same in the face of a 30-cent price decline.

Although the capital level affects the size of the bulk milk premium required to maintain production, the capital level has little effect on the necessary price differential of grade A over grade B milk. At each capital level, with can or bulk handling, if the grade B price is $2.70, the grade A price must be in the neighborhood of $3.80 to $4.00 to maintain grade A production at the same level as grade B production.

Feed prices are commonly included as an important variable in the prediction of supply. Present results suggest that variations in feed prices may affect only those farmers with large volumes of operating capital. Even for them, the effect may be small and in the opposite direction from what one would expect. The effect of the 20-cent corn price increase was to leave dairy production unaffected or to increase it. In a diversified farming area, where farmers grow much of their feed, the volume of feed production may be just as important as the prices paid for purchased feed. In this study, the available supplies of hay and hay equivalent and corn and corn equivalent are related to the size of the dairy enterprise.

The importance of labor in dairy production is emphasized by the number of times family labor is a limiting resource in the optimum plans and by the effect of adding labor hiring as an activity. The results show that dairy output is a function of the available family labor supply as well as of the cost of hired labor. But they also show that the availability of hired labor affects only the dairy production of farmers possessing large amounts of operating capital.

The optimum level of dairy production seems to be more responsive to hog price changes than to dairy price changes, suggesting that in a hog producing area, the level of hog prices may be more important than the level of dairy prices in determining dairy production. Past levels of hog production are also relevant because of the relationship between current dairy production and availability of hog housing. As the amount of hog housing rises, optimum levels of dairy production fall sharply.

Implications for Dairy Supply Analysis

Everything in the preceding section is, of course, pertinent to the analysis of supply. Certain other aspects need to be brought out in their relationship to supply analysis. Certain hypotheses have been advanced by agricultural economists as explanations of the inflexibility of farmers' production patterns. Among them are the suggestions that (a) farmers are not primarily motivated by profit maximization and (b) certain technical relationships within the farm firm prevent the farmer from readily adjusting to price changes (16, page 675.) Within the framework of the linear programming model, the analysis reveals that inflexibility of dairy output is consistent with profit maximization and with (b). The causes of inflexibility mentioned previously emphasize the importance of distinguishing between immediate and delayed, or short-run and long-run, responses in supply analysis and lend empirical support to the use of lagged production as an independent variable.

Dynamic elements may also be important for other reasons. Within the linear programming framework, feed production is determined simultaneously with dairy production and other livestock production. In actual operations, their values may be determined by a sequence of separate but interrelated decisions rather than by several simultaneous decisions. In this case dynamic elements are introduced into the relationship between dairy and other livestock production and between dairy and feed production. Dynamic factors are also introduced by the relationship between dairy production and the amount of available dairy and hog housing and equipment, the latter having been determined by decisions made in the past.

Another conclusion to be drawn from this study is that it is not surprising that our quantitative knowledge about dairy supply is in an unsatisfactory state in spite of the number of competent people who have studied it. Even this limited study shows the multiplicity of forces affecting dairy production in a diversified farming area. It also shows that many of them do not affect it in any easily quantified manner. For example, the variable price and continuous capital analyses indicate that the relationship between dairy price and dairy production cannot be adequately represented by an equation that is linear or linear in the logarithms. The price elasticity varies with the dairy price level and also with the level of operating capital. Thus, instead of using an equation such as

$$Q = aP^{b}K^{c}$$

to estimate supply response it might be more appropriate to use an equation such as

$$Q = aP^{(bP+cK)}K^{d}$$

where Q is quantity of output, P is output price and K is level of operating capital. Hog production furnishes another example. With unlimited hog housing, hog production expands sharply until the hay or corn supply is exhausted. The large investments in the hog enterprise either block out the dairy enterprise completely or reduce dairy production, depending on the relative prices for hogs and dairy products. It follows then that the price of hogs is an important factor in determining the supply of dairy products in the competitive livestock area of the grain belt. The variable price analyses suggest that the relationship between hog prices and dairy production may be quite difficult to find empirically. The relationship is quite close within a small range of hog prices; above and below this small range there is no relationship. The previous analysis of suboptimum income losses also indicated the possibility that farmers respond differently to a price increase than to a price decrease. Such asymmetry would make it difficult to measure the relationship between dairy price and dairy output.

LIMITATIONS

This study shares with other empirical studies the limitations arising from the nature of the four basic assumptions of linear programming: (1) constant returns, (2) additive processes, (3) finite number of processes and (4) fixed supplies of certain factors (12, 13, 21). The question of the reality of (1) is especially pertinent. Interwoven with the task of obtaining adequate input-output coefficients is the task of handling those enterprises whose input-output relationships are not linear. With the usual assumption of linear input-output relationships, the problem is simply to maximize these linear functions subject to the resource restrictions.

However, there are many situations where these relationships may be nonlinear. This is particularly evident when the relationship between capital or labor, for example, and dairy output is considered.

If the relationship is one of decreasing returns to the inputs as shown in fig. 13 the problem can be handled in a linear programming model by approximating the relationship with a series of linear segments. Each linear segment becomes a separate activity in the matrix. The function of $y = f(x)$ is approximated by the function $y' = f(x)$. The function $y' = f(x)$ can be embodied in the model, as segment a (as shown in fig. 13) has a higher return than segment b and likewise b has a higher return than segment c.

Nonlinearities that correspond to increasing returns to the inputs as shown in fig. 14 present greater complications. Each successive segment of the function $y'' = f(x)$ has a greater slope than the one preceding. This relationship is not consistent with

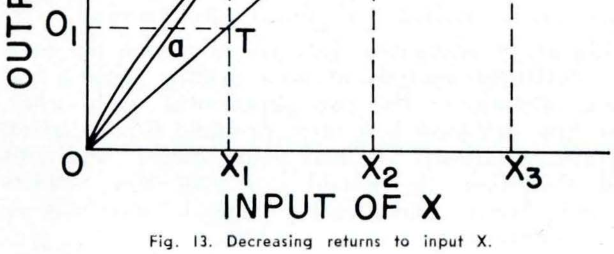

Fig. 13. Decreasing returns to input X.

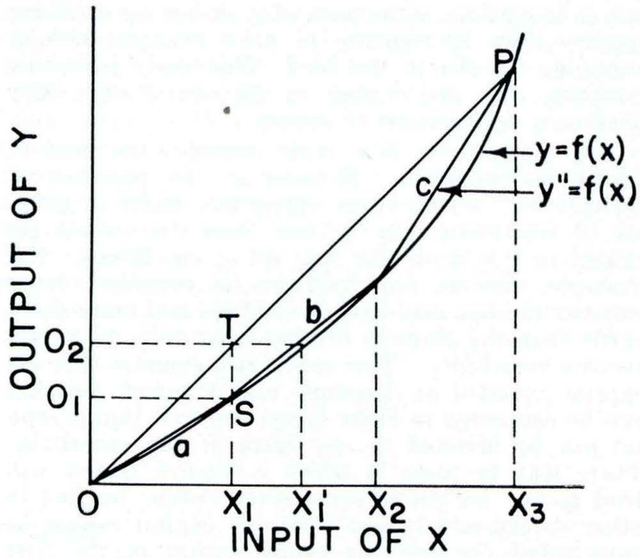

Fig. 14. Increasing returns to input X.

the maximizing procedure of linear programming; therefore, the function $y'' = f(x)$ cannot be readily incorporated into the usual simplex model. If the segments are considered as separate activities in the matrix, segment c would obviously enter the solution first, prohibiting the entrance of either a or b. With the entrance of c, the model would then proceed as if constant returns existed.

An attempt might be made to approximate $y = f(x)$ by drawing the straight line, OP, from the origin. Activity y would then enter the final plan at, say, point T, at level O_2 with an input of X_1. An output of O_2 requires an input of x_1', however; an input of X_1 is sufficient to produce only O_1.

To circumvent the problem created by the increasing returns situation, we limited the dairy activity to only one segment of the curve by setting up each enterprise for the upper limit of a given herd size range. For example, segment a in fig. 14 may represent dairy production from 0 to 15 cows. The coefficients, then, are computed for a herd size of 15 and the equipment restrictions are specified so as to prevent the herd size from going above 15. It is true, then, that the bias in the results varies inversely with the size of the herd for each technique. This procedure has the obvious shortcoming of not considering the entire range of production possibilities, but it does permit the consideration of possible increasing returns situations in the linear programming model.

Dairy production is expressed solely in terms of the number of dairy cows, a constant ratio existing between total output and the number of dairy cows. Dairy production can also be increased or decreased by changing the levels of feeding as prices fluctuate. A number of dairy processes reflecting various levels of feeding could be included in a linear programming model, if the necessary coefficients were known. The present procedure is justified if the milk production function is sufficiently close to being linear that dairy price changes cause little change in optimum feeding levels. Another factor, which is probably not as important, is the possibility of buying different quality cows in response to price changes without changing the size of the herd. This study considers, however, only the change in the size of the dairy herd as a determinant of supply.

The analysis in this study assumes the goal of profit maximization. Because of the presence of uncertainty, actual farm operations under a given set of conditions may deviate from the optima obtained in this study for that set of conditions. For example, farmers with facilities for housing a large number of hogs may have fewer hogs and more dairy cattle than the plans in this study in order to reduce income variability. This study also assumes that the capital invested in livestock and livestock supplies can be converted to other forms and that liquid capital can be invested in any form in any enterprise. There may be cases in which a finance agency will lend money for use in some enterprise(s) but not in other enterprise(s), and borrowed capital cannot be substituted for operator-owned capital in the first enterprise(s) to free operator-owned capital for use in the second enterprises(s). Then it would be necessary to have separate capital input items for the two groups of enterprises.

Most marketing and policy questions require some concept of an aggregate supply relationship for a region or an area. This would require the aggregation of the supply functions for all the individual firms in the area. Besides knowing the supply functions for a 160-acre farm under various resource restrictions, we would also have to know these relationships for, say, 100-, 200-, and 320-acre farms, and the numbers of farms in each of these group sizes. This study considers only the single firm analysis, but additional inquiry will be concerned with an extension of the analysis to derive an aggregate function.

This bulletin, being concerned only with the supply aspects of linear programming, contains no discussion of the farm management implications of the results. One problem that arises in applying linear programming results to farm management is the composition effect: What is profitable for one or a few farmers to do may not be profitable for all farmers to do. If linear programming can be extended to the derivation of aggregate supply curves, this problem may be solved. Aggregate supply, before and after a large number of farmers made a particular change in their operations, could be compared with aggregate demand to determine the effect of the change on farm prices. Then it could be determined whether farmers as a group would be better or worse off if they made the change.

This analysis implies an infinite marginal rate of substitution of income for leisure up to a point (given by the labor restriction) and a zero marginal rate of substitution beyond that point. The labor restrictions used here represent the best available estimates of the hours of work typical farmers actually put into their farm operations. It is quite possible that the marginal rate of substitution of income for leisure of many farmers is somewhere between these two extremes and decreases with increasing income. In this event, a larger increase in income will be required to induce an operator to expand his operations by a certain amount when his income is high than when it is low.

Some of the optimum plans are impractical because of the small size of operations involved. Some call for 126 acres in one rotation and only 9 in another rotation; some call for two beef cows and others for two litters of pigs. In the few cases in which optimum plans do contain small-scale operations, the matrix was not recalculated, blocking out these operations. The reason is that the small operations involved such a small proportion of total resources, the remainder of the plan would be expected to change very little.

USEFULNESS OF LINEAR PROGRAMMING FOR SUPPLY ANALYSIS

One objective of the study was to investigate the usefulness of linear programming as a tool for the analysis of the supply of farm products. In research in economics many problems arise which do not arise in the physical sciences because the physical scientists can perform controlled experiments in the laboratory. Linear programming can serve as the economist's laboratory in the analysis of supply. In the real world, many forces operate to determine the volume of milk and cream produced. For a number of reasons, it is difficult or impossible to analyze the effects of many of these forces.

Economists often use statistical methods in place of controlled laboratory experiments. Because of the multiplicity of forces affecting dairy production, the expense of collecting and analyzing data, and the limited number of observations available from time series data, a great deal of aggregation is required to carry out the statistical analyses. For example, an index of other livestock prices is used. It is entirely possible that the response of dairy production to poultry price change differs substantially from the response to a hog price change, which in turn differs from the response to a beef price change.

Variations in many of the forces affecting dairy production are correlated with each other, giving rise to the problem of multicollinearity in statistical analyses. For some of the variables affecting dairy output, for example, level of operating capital and amount of hog housing available, data are not available and the expense of collecting adequate data would be great.

The linear programming method avoids some of these problems. The separate effects of many variables can be analyzed under rigidly specified conditions; their effects can be analyzed singly or in various combinations. Interrelationships among various forces can be closely scrutinized. Many things which would be studied if laboratory experimentation were possible can be studied with linear programming.

Linear programming does not eliminate the need for statistical analysis of data dealing with actual farm operations; the two supplement each other. The first can show how farmers would behave, given certain specifically defined assumptions, conditions and objectives; the second can show how farmers actually behave under rather general (and more or less accurately measured) conditions.

Linear programming will not reach its greatest effectiveness in the area of supply until it can be used to derive aggregate supply curves. When that stage is reached, data may have to be collected from farmers on various variables such as hog housing, operating capital and available family labor. At this stage, one of the advantages of the linear programming method—avoidance of the cost of collecting and analyzing volumes of data—seems to disappear. It is not eliminated, however. The linear programming analyses will have already shown which variables are relevant and which are not. It is almost certain that the acquisition of this same knowledge through the statistical procedure would have resulted in the collection of data on, and the analysis of, variables which are not significant. Thus there will still be economies in data collection.

Of course, the data used in the linear programming analyses do not flow without cost or limit from the mind of an omniscient investigator; they must be collected. Some of the data, however, are already available from agronomists, animal husbandry specialists, engineers, etc. Some are available from farm records and census reports. Other data need not be collected at all; they can be left free for analysis, as was done in this study with dairy and hog prices, operating capital level and hog housing restrictions.

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APPENDIX. METHOD OF COMPUTING INCOME LOSSES FROM SUBOPTIMUM PLANS.

I. Income loss from a suboptimum plan without dairying (plan zero).

1. Since the changes in dairy prices will not affect the income of this suboptimum plan, simply subtract the income for plan zero from the income for each optimum plan.
2. This difference in income will represent the loss that results from adopting plan zero if some other plan is optimum.

II. Income loss from a suboptimum plan with dairying.

1. Multiply the difference in the dairy prices for the suboptimum and the optimum plan by the dairy production of the suboptimum plan: $\triangle P_D$ X dairy production = additional income. $\triangle P_D$ will be positive if the number of the optimum plan exceeds the number of the suboptimum plan and negative if the number of the optimum plan is less than the number of the suboptimum plan. Any prices appropriate to the two plans may be used; in this study, border prices were used.
2. Add the additional income to the income of the suboptimum plan: Income of suboptimum plan + additional income = new income.
3. Loss in income = income of the optimum plan minus the new income.

Applying this procedure to the optimum plan data in table 4 will not yield exactly the income losses shown in this table. Some of the optimum plans contained a fractional number of dairy cows, whereas the dairy cow numbers in table 4 (and in figs. 3 to 7) have been rounded to the nearest whole number. The income losses were computed from the unrounded data.