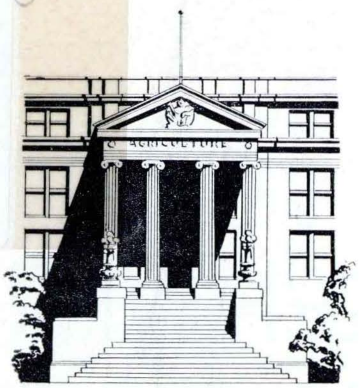

Use of a Dynamic Model In Programming Optimum Conservation Farm Plans on Ida-Monona Soils

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CONTENTS

SUMMARY

This study develops a sequence of yearly plans which provide optimum 5-year farm programs for a 160-acre farm under three alternative conservation situations. The 160-acre farm is located on the Ida-Monona soil association of western Iowa. In each of the 5-year plans, family living or household consumption is considered to be an "exogenous" activity because an "exact" capital allowance for this activity must be met each year. Family living, therefore, competes with farm production in the use of available operating capital. In each optimum plan, capital generated from crop and livestock production in any one year is used for farm production and household consumption for the following year. Dynamic linear programming is used to obtain the optimum farm plans.

Two dynamic linear programming models (the *expansion model* and the *rotation model*) were developed. Each model allows the programming of t years of activities and restrictions. The models, therefore, should extend the magnitude of farm problems which can be analyzed by dynamic programming approaches.

The three conservation situations studied are as follows: Situation I – a 160-acre farm on which crops *are not assumed* to be fertilized and cropland *is not assumed* to be terraced and contoured. Situation II—the same as Situation I, except crops *are assumed* to be fertilized. Situation III—a 160-acre farm on which crops *are assumed* to be fertilized and cropland *is assumed* to be terraced and contoured.

The various soil types on the 160-acre farm are combined to form two soil productivity classes: Land A, a "low" productivity class, and Land B, a "high" productivity class. In each situation, average management, adequate machinery and hay and grain storage facilities are assumed. Only one price level is assumed. Also, in each situation, $9,900 of capital for operations and family living is available in the first year. In each of the other years (years 2, 3, 4 and 5), however, the amount of capital available is a function of the returns obtained from farm production the preceding year. All future returns are discounted back to present value. No capital is assumed to be borrowed.

The crop enterprises considered in each situation include all possible combinations and rotations of corn, oats and hay for a 5-year period within the following limits: (1) not more than 3 years of continuous corn or hay, (2) only 1 year of oats and (3) no hay following corn. Noncrop enterprises included in each situation are: a two-litter hog system, deferred-fed calves, grain buying and household consumption (or family living). The technique of dynamic programming caused each of the 5 years in each situation to be interrelated. Hence, the crop and livestock plan in any one year depends upon crop and livestock production in previous and future years. This is so because the activities included in the plan for any one year are those activities which will maximize profits for the 5-year period after allowances have been made for expenditures for family living.

In all three situations, Land A was used mostly for hay production and Land B for corn production over the 5-year period. Hay was grown on Land B only to supplement hay production on Land A or to meet the cropping limitations assumed. In the early years of each plan, forage was grown on Land A to build up productivity. Corn was grown on Land A in the latter years of each plan after the productivity had been increased through forage production. In Situation I, because fertilizer and terracing and contouring were not included, crop yields were relatively low, and a larger proportion of Land A and Land B was required for forage production to meet the livestock forage feed requirements than in the other two situations. Also, because yields were low, more corn was purchased for feed and less was sold for cash in Situation I than in situations II and III. Thus, by using fertilizer or fertilizer and terracing and contouring, it was possible to decrease forage acreage in situations II and III, while maintaining soil productivity.

Under the pricing system used, hogs were more profitable than cattle. Accordingly, the maximum number of hogs allowed by available hog building space or capital was produced each year. (Capital restricted hog production in year 1 of Situation III.) Thus, deferred-fed calves were included only after crop and hog production. In year 1 of Situation I, because family living, fertilizer and terracing and contouring were not included, more capital was available for livestock production than in year 1 of situations II and III. As a result, 33 head of cattle were included in year 1 of Situation I, whereas in year 1 of situations II and III the larger capital requirement of crops, plus the capital requirement for family living, caused cattle numbers to be reduced to 4 head and 0 head, respectively. In the latter years of each situation, capital was a nonlimitational resource, and the number of cattle included in each plan increased.

Net returns for the 5-year period were highest when fertilizer and terracing and contouring were included. Returns were lowest when neither fertilizer nor terracing and contouring were included. The inclusion of crop fertilization increased total farm returns much more than did the inclusion of terracing and contouring of cropland. Return per $1 on investment was greatest, however, for terracing practices and the farm reorganization attached to them. House-

hold consumption did not restrict the adoption of terracing and contouring in year 1 of Situation III. Household consumption did restrict hog and cattle production in year 1 of Situation III and cattle production in years 1, 2 and 3 of Situation II.

The above results point up several important considerations for future conservation planning: (1) The same crop and livestock plan should not be recommended each year if profit maximization over time is the relevant goal. (2) In long run conservation plans, the years should be interrelated so that changes in resource structure (particularly in the accumulation of capital and the change in soil fertility) may be incorporated into the conservation plan. The recommended plan thus should provide step-by-step yearly plans which do give consideration to the level of capital and soil fertility. (3) Household consumption, as well as farm production, should be considered in making conservation recommendations. (4) Livestock should be "fitted into" the plan to utilize forages produced, and (5) because different farms vary in size and amount of resources available, different conservation plans should be recommended for different farms.

Use of a Dynamic Model in Programming Optimum Conservation Farm Plans on Ida-Monona Soils[1]

BY WESLEY G. SMITH[2] AND EARL O. HEADY

Establishing soil conservation plans on farms requires time. Several years must elapse before new seedings and mechanical erosion control practices can be established and have their full effect on yields and income. A farm which has been heavily cropped must devote a sequence of years to adjusting from the present system to a conservation system. For the first year, land which has been in row crops must be planted to oats, with a grass seeding. In the second year, additional hay may be produced, and livestock herds will need to be increased to utilize it. Additional equipment and buildings may be required. Terraces, dams and similar structures may be applied in the first and latter years. The main yield-increasing effects of new rotational systems and mechanical practices will not be realized short of the 4-6 years required to complete a crop sequence. Accordingly, the opportunities in income and the capital requirements for any additional livestock, made possible from different cropping practices, will be spread over a similar number of years. Typically, perhaps, income declines in the years immediately following initiation of a conservation plan, even though the system may eventually increase annual income.

These adjustment problems are especially complex in the Ida-Monona soil area of western Iowa. On the average, farms are small relative to the income needed for family living, debt reduction and capital accumulation. To meet annual financial commitments, many farm families may not be able to withstand a decline in income for 4 or 5 years, if this is required for establishing a conservation plan. Little is known, however, about the effects of family living requirements and capital limitations on the best farm plan over time. This study has been made to explore the interactions of system of farming, capital level and family living requirements over time.

Several economic studies of soil conservation practices have been made in the Ida-Monona soil area of western Iowa.[3] These studies, however, have not specified the transition adjustments over time which a farmer must make in adopting a final conservation plan. What is needed is a series of intermediate, or transition plans, as well as the final conservation plan. In this study, a series of yearly plans covering a 5-year period is developed. The plan for each year is the best possible plan in terms of the 5-year optimum, considering the capital available and the need for funds for family living. Because the plans are only for a 5-year period, they do not represent final conservation plans. Rather, they are intermediate or transition plans indicating how the necessary adjustments toward a final conservation plan can be made over a 5-year period. Dynamic linear programming techniques have been used to obtain the optimum 5-year plans.

The plans are optimum only in the sense that they allow profit to be maximized while not exceeding the capital and labor available and while providing annual income withdrawals to meet family living expenses.

Under actual farm conditions, the length of time required for a farmer to attain the final conservation plan will be a function of the resources available. Hence, the time required will vary with the productivity of the land, the farmer's capital and equity position, his managerial ability and the supply of labor. Additionally, the type of conservation program and the time involved in adjusting to it will depend upon the relative marginal return of capital invested in conservation practices, as compared with the return on the same capital invested in nonconservation practices.

THE AREA AND PROBLEM

The Ida-Monona soil association consists of strongly rolling hills and loessial soils which were originally fertile and which are productive when managed efficiently. On many farms, however, soil productivity has been progressively diminished by erosion. Loss of

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[1] Project 1085, Iowa Agricultural and Home Economics Experiment Station. This project was financed by a grant from the Tennessee Valley Authority.

[2] Formerly research assistant at Iowa State University and now assistant professor at the University of Delaware.

[3] See: Ross V. Baumann, Earl O. Heady and Andrew R. Aandahl. Costs and returns for soil-conserving systems of farming on Ida-Monona soils in Iowa. Iowa Agr. Exp. Sta. Res. Bul. 429. 1955; A. Gordon Ball, Earl O. Heady and Ross V. Baumann. Economic evaluation of use of soil conservation and improvement practices in western Iowa. U. S. Dept. Agr. Tech. Bul. 1162. 1957; Gerald W. Dean, Earl O. Heady, S.M.A. Husain and E. R. Duncan. Economic optima in soil conservation farming and fertilizer use for farms in the Ida-Monona soil area of western Iowa. Iowa Agr. and Home Econ. Exp. Sta. Res. Bul. 455. 1958; S.M.A. Husain. Optimum resource allocation for erosion control farming on Ida-Monona soils. Unpublished Ph.D. thesis. Iowa State University Library, Ames. 1957.

fertile topsoil through sheet erosion has been particularly great in some areas. Gully erosion is serious because of the topography and vertical structure of the Ida-Monona soils. Some gullies, now over 100 feet deep, cut back several hundred feet each year. Consequently, roads, bridges, fences and farm buildings must be relocated frequently. More serious than the deep gullies are the small gullies and depressions which develop in cultivated fields. It is estimated that the annual loss of soil in the area averages about 20 tons per acre.[4] On some farms, it is as high as 60 tons, an amount equivalent to nearly ½ inch of topsoil.

Farm practices common in the area intensify soil losses by erosion. Corn is the main crop grown. While many farms are operated on a cash-grain basis, the farms generally have livestock enterprises organized around the corn and oats produced. When rotations do exist, they commonly include 2 years of corn, 1 year of small grain and 1 year of hay. Even then, row crops frequently are planted up and down hills, on slopes exceeding 15 percent.

Various soil conservation practices, such as contouring and terracing, contour strip-cropping, sodded waterways, improved rotations and permanent seeding of steep land are needed on most farms in the area. Such practices would help conserve soil resources, reduce damage from floods and maintain or augment the low farm incomes through time.

OBJECTIVES

The general objective of this study is to determine optimum farm plans over a period of years—with *optimum* again referring to profit maximization — within the restraints of available capital and family living requirements. In this sense, we wish to determine which crop and livestock enterprises and conservation practices are optimum over a series of years for farm families that have varying amounts of capital and must provide annually for living expenses. Does a family which has little capital have to follow an exploitive cash and row crop system to provide funds for living from its limited capital? If so, what is the appropriate pattern of crop and livestock enterprises over time when capital accumulation can, or cannot, take place? How are optimum time plans altered as the farm family acquires more capital? These are the types of questions which the analysis is designed to answer. More specifically, the objectives of the study are:

1. To determine optimum 5-year plans which result in alternative levels of conservation for typical 160-acre, owner-operator farms on Ida-Monona soils.

2. To determine, for each of the 5 years, the best possible crop and livestock plan at different levels of conservation, after first taking into consideration funds required for household consumption.

3. To determine the effect of (a) requirements for household consumption (or family living) on the optimum rate of adoption of conservation practices and (b) present and future incomes, as household and farm business compete for funds at different capital levels.

4. To interpret the results of the dynamic programs in terms of: (1) recommendations on conservation to farmers with different amounts of capital and (2) conservation and nonconservation investment opportunities as compared with cropping and livestock opportunities, when funds are limited.

5. To develop applications of dynamic linear programming methods which may be useful in further analysis of conservation and time problems in farming.

The last specific objective is largely methodological. Nevertheless, it is considered one of the more important objectives of the study. While it has been discussed in abstract mathematical form in recent years, large-scale application of dynamic linear programming has not been made in economic problems, particularly in those of farming. As a result of the experience gained in this study, methods have been developed for the IBM-650 which permit fairly simple computations of relatively large-scale dynamic programming problems at a reasonable cost. Several months of experimenting with programming routine, model construction and coding procedures were required to attain this end.

TECHNIQUE OF ANALYSIS

Two dynamic linear programming models are used in determining 5-year plans for alternative conservation situations on 160-acre farms in the Ida-Monona area. The mechanical conservation alternatives studied include: (1) no crop fertilization *or* terracing and contouring of cropland, (2) crop fertilization *but no* terracing and contouring of cropland and (3) crop fertilization *and* terracing and contouring of cropland. Numerous rotations could be used in attaining any of these alternatives.

In each conservation alternative studied, the cost of family living (household consumption) is considered to compete with the farm business. A consumption activity is necessary in view of the fact that not all capital forthcoming as income from a year's farm production will be available for further production—some must be used for family living. Furthermore, family living is considered to take precedence over farm production in the allocation of available capital.

Method of Dynamic Linear Programming

The technique of dynamic linear programming permits the programming of activities and restrictions for t years (where t is a finite number). Outputs of any one year in the program become inputs for the following year. Thus, activities in each of the t years are interrelated. In the optimum t-year plan obtained, the plan for each year represents the best or most profitable plan in terms of the t-year optimum.

The first model used for dynamic linear program-

[4] John C. Frey. Some obstacles to soil erosion control in western Iowa. Iowa Agr. Exp. Sta. Res. Bul. 391. 1952.

ming solutions, *the expansion model,* treats individual crops and noncrop enterprises as activities. That is, crops are not introduced into the programming model as rotations, but are considered as single crops. Hence, individual crops can be placed in any time sequence which is required for the optimum plan.

The second model, *the rotation model,* treats crop rotation and single (or individual) noncrop enterprises as activities. Both models involve time and employ similar computational procedures and algebraic structure. The models differ only in the form of the input-output matrix. Since details of the rotation and expansion models are outlined elsewhere,[5] only a brief outline of the algebraic procedure for dynamic programming is presented here. The equations shown apply to both the expansion and rotation models.

Denote the year of the program (i.e., the year in which the activity or restriction occurs) by the subscript k, where k=1,2, . . ., t; the number of the row (or restriction) by i, where i=1, 2, . . ., m; and the number of the column (or activity) by j, where j=1, 2, . . ., n. Let element a_{ijk} represent the unit requirement or the output of the *jth* activity for the *ith* resource in the *kth* year; x_{jk} the level of the *jth* activity in the *kth* year; b_{ik} the level of the *ith* resource in the *kth* year and c_{jk} the net revenue of the *jth* activity in the *kth* year. The dynamic linear programming model can be expressed as in the relationships of equations

$$\begin{aligned}
&a_{111}x_{11} + \dots + a_{1j1}x_{j1} + a_{1j'2}x_{j'2} + \dots + a_{1nt}x_{nt} \leqq b_{11}\\
&a_{211}x_{11} + \dots + a_{2j1}x_{j1} + a_{2j'2}x_{j'2} + \dots + a_{2nt}x_{nt} \leqq b_{21}\\
&\quad\vdots\\
&a_{i11}x_{11} + \dots + a_{ij1}x_{j1} + a_{ij'2}x_{j'2} + \dots + a_{int}x_{nt} \leqq b_{i1}\\
&a_{i12}x_{12} + \dots + a_{ij2}x_{j2} + a_{ij'2}x_{j'2} + \dots a_{int}x_{nt} \leqq b_{i2}\\
&\quad\vdots\\
&a_{i1k}x_{1k} + \dots + a_{ijk}x_{jk} + a_{ij'k'}x_{j'k'} + \dots + a_{int}x_{nt} \leqq b_{ik}\\
&\quad\vdots\\
&a_{m1t}x_{1t} + \dots + a_{mjt}x_{jt} + a_{mj't}x_{j't} + \dots + a_{mnt}x_{nt} \leqq b_{mt}
\end{aligned} \tag{1}$$

where k = 1, 2, . . ., t; i = 1, 2, . . ., m; j = 1, 2, . . ., n; and where $j \neq j'$ and $k \neq k'$. The objective is make

$$f(X) = c_{11}x_{11} + c_{21}x_{21} + \dots + c_{jk}x_{jk} + \dots + c_{nt}x_{nt} \tag{2}$$

a maximum, subject to the non-negative condition

$$x_{jk} \geqq 0. \tag{3}$$

To facilitate solution of the system and to maximize the objective, which is profit in this case, we introduce m "slack" or "disposal" variables, and the inequalities of the relationships in equations 1 are replaced with the equalities in equations 4. The variables x_{jk} ($j = n + 1, n + 2, \dots, n + m$) is a "slack" variable because it accounts for the excess of the right-hand side of equations 1 over the left-hand side. We now have r activities where n + m = r, and j now has the range j = 1, 2, . . ., r. The input-output coefficients, corresponding to the "slack" variables are in the form of

$$a_{ijk} = 1 \quad (i = 1, 2, \dots, m, \text{ and } j = n + 1, n + 2, \dots, n + m)$$

where i = j − n, and

$$a_{ijk} = 0$$

where $i \neq j - n$

which is an identity matrix. The "slack" vectors thus change the inequalities in equations 1 into equalities in equations 4 at a cost of introducing m additional non-negative unknowns. The "new" set of equations is:

$$\begin{aligned}
&a_{111}x_{11} + a_{121}x_{21} + \dots + a_{1j1}x_{j1} + \dots + a_{1rt}x_{rt} = b_{11}\\
&a_{211}x_{11} + a_{221}x_{21} + \dots + a_{2j1}x_{j1} + \dots + a_{2rt}x_{rt} = b_{21}\\
&\quad\vdots\\
&a_{i11}x_{i1} + a_{i21}x_{i1} + \dots + a_{ij1}x_{j1} + \dots + a_{irt}x_{rt} = b_{i1}\\
&a_{i12}x_{i2} + a_{i22}x_{i2} + \dots + a_{ij2}x_{j2} + \dots + a_{irt}x_{rt} = b_{i2}\\
&\quad\vdots\\
&a_{i1k}x_{1k} + a_{i2k}x_{2k} + \dots + a_{ijk}x_{jk} + \dots + a_{irt}x_{rt} = b_{ik}\\
&\quad\vdots\\
&a_{m1t}x_{1t} + a_{m2t}x_{2t} + \dots + a_{mjt}x_{jt} + \dots + a_{mrt}x_{rt} = b_{mt}
\end{aligned} \tag{4}$$

where $x_{jk} \geqq 0$, and we maximize

$$f(X) = \Sigma\, c_{jk}x_{jk}$$

where c_{jk} is the discounted value of $\bar{c}_{jk}$, the net price of the *jth* activity in the *kth* year, computed as

$$c_{jk} = \bar{c}_{jk}\,(1 + r)^{-k}$$

where r is the market rate of interest. In computing the $\bar{c}_{jk}$, no interest rate has been subtracted where capital might be borrowed. Most farmers do use borrowed capital, but the amount is variable between farms. Hence, the net income figures presented later would need to be adjusted downward to account for the amount of capital borrowed by an individual farmer.

Equations 4 and the condition $x_{jk} \geqq 0$ guarantee that no activity will be carried on at a negative level. The discount equation considers that a farmer's capital could be loaned at market rates, as well as used for farming.

[5] For details of the rotation model and a fuller explanation of the expansion model see: Wesley G. Smith. Dynamic linear programming of conservation alternatives, including household consumption. Unpublished Ph.D. thesis. Iowa State University Library, Ames. 1958. pp. 7-31.

In the expansion model, many of the a_{ijk} coefficients in equations 1 and 4 are zero. In fact, the a_{ijk} coefficients for all real activities, x_{jk} where $j \leqq n$, will normally be zero in any year (k) which does not correspond to the year being considered. For example, if the activities and restrictions in year 3 are being considered (i.e., $k = 3$), then the a_{ijk} coefficients in years where $k \neq 3$ normally will be zero. The a_{ijk} values are zero because activities and restrictions in years 1, 2, 4, 5, . . ., t are separated from the activities and restrictions in year 3. If the activities in year 3 include interyear intermediate products (outputs of 1 year which are inputs of the following year), however, the corresponding a_{ijk} coefficients of year 3 activities that are opposite the resource restrictions of year 4 will not equal zero because they are outputs. These outputs (a_{ijk} coefficients) thus will necessarily have a negative sign. Hence, all a_{ijk} coefficients of year 3's activities opposite years 1, 2, 4, 5, . . ., t will be zero, except those a_{ijk} coefficients which represent transfers of outputs from year 3 to inputs for year 4. Furthermore, all coefficients representing outputs, whether within any one year or between 2 years, have a negative sign, because they add to available resource supplies. Similarly, all inputs, a_{ijk} coefficients, within a year representing resource requirements have a positive sign, because they subtract from available resource supplies. Other coefficients are zero.

More specifically, interyear transfers in capital and feed supplies are accomplished as follows: Let P_j be a column vector of resource requirements as indicated in equation 5. All elements in the *kth* year are positive, except those representing transfer of feed from field crops to feed supplies. In year $k + 1$, all elements are zero, except for the capital restriction where the corresponding element $a_{ijk} + 1$ is negative and of the magnitude 1. In all $a_{ijk} + 2$ coefficients, however, the elements of the $k + 2$ year are zero. Thus, if capital is represented by the second restriction (a_{2jk}), then we have $a_{2jk} > 0$, $a_{2jk+}1 = 1$ and $a_{2jk+}2 = 0$.

$$P'_j = (a_{1jk}\ a_{2jk} \ldots a_{mjk}\ a_{1jk+1}\ a_{2jk+1} \ldots a_{mjk+1}\ a_{1jk+2}\ a_{2jk+2} \ldots a_{mjk+2}) \qquad (5)$$

FARM PROGRAMMING SITUATIONS

The application of the dynamic linear programming model is made for the 160-acre, owner-operator farms with different alternatives in conservation.[6] The farm situation represents the Ida-Monona soil association in western Iowa. This study is a continuation of previous studies on the same area.[7] For this reason, some of the background on the farm situations and the area analyzed has not been included here.

Optimum 5-year plans have been computed for the following situations:

Situation I: 160-acre farm *without* crop fertilization and *without* the land being terraced and contoured.

Situation II: 160-acre farm *with* crop fertilization but *without* the land being terraced and contoured.

Situation III: 160-acre farm *with* crop ferilization and *with* the land terraced and contoured.

In each situation, average management is assumed for crop and livestock production. Fertilizer in situations II and III is considered to be applied to corn, oats and second-year hay at a single rate.[8] Only one-price level is used in programming all situations.

The programming solutions for these situations were computed with an IBM-650 Magnetic Drum Processing Machine. A modified simplex method developed by Herman O. Hartley and Dale D. Grosvenor of the Department of Statistics, Iowa State University of Science and Technology, was used.

FARM RESOURCE SITUATIONS

Because farm resources are not present in unlimited supplies, it is necessary to define the restrictions which limit the plan in each farm situation analyzed. The resource restrictions which limit the optimum time plans are presented below.

Land

Land is one of the most important resource restrictions in western Iowa. An average 160-acre farm includes 143 acres of cropland. The remaining 17 acres are in farmstead, roads, fences and wasteland. Because of the magnitude of the programming problem, it was necessary to hold land restrictions to two categories of soils. The various soil types were classified into two groups—Land A and Land B. Table 1 shows the classification of cropland by soil type and slope of land. Table 2 shows the composition of Land A and Land B. Because of restrictions on the size of the matrix which could be handled by the IBM-650, it was not possible to use more soil groups in the programming.

The farm situation considered in this study includes 48.6 acres of Land A and 94.4 acres of Land B. Land A consists of 65 percent Ida and 35 percent Monona soils. Land B is made up of 5 percent Ida, 58 percent Monona and 37 percent Napier soils. Because of the

[6] Optimum 5-year farm plans are presented in the appendix for a 280-acre farm on the Ida-Monona soil area.

[7] Husain, *op. cit.*, p. 2; and Dean, Heady, Husain and Duncan, *op. cit.*, p. 2.

[8] Information on the single rate of fertilization—the rate necessary to obtain the estimated crop yields used in this study—was obtained in 1957 from F. F. Riecken, W. D. Shrader, J. T. Pesek, F. W. Schaller, J. J. Hanway and R. C. Prill of the Department of Agronomy, Iowa State University of Science and Technology, Ames, Iowa, and from D. F. Slusher of the Soil Conservation Service, Ames. Both the estimated crop yields and the rate of fertilization were determined by these members of the Department of Agronomy, Iowa State.

TABLE 1. CLASSIFICATION OF CROPLAND BY SOIL TYPE AND SLOPE OF LAND.

Percent slope interval	Soil Type				
	Ida	Castana	Monona	Napier	Total
	Acres				
0-6	2.8	0.0	28.0	33.8	64.6
7-14	6.6	0 0	26.1	0.0	32.7
15-20	27.4	0.0	11.3	0.0	38.7
Above 20	1.2	0.0	5.8	0.0	7.0
Total	38.0	0.0	71.2	33.8	143.0

TABLE 2. CLASSIFICATION OF CROPLAND BY SOIL PRODUCTIVITY CLASS AND CONVENIENCE OF FIELD OPERATIONS.

Percent slope and land class	Ida	Soil Type Castana	Monona	Napier	Total
	(Percent)				
LAND A					
0-6	10.41	0.00	0.00	0.00	10.41
7-14	0.00	0.00	11.33	0.00	11.33
15-20	53.49	0.00	23.74	0.00	77.23
Above 20	1.03	0.00	0.00	0.00	1.03
Total	64.93	0.00	35.07	0.00	100.00
LAND B					
0-6	0.00	0.00	30.48	36.86	67.34
7-14	4.92	0.00	22.59	0.00	27.51
15-20	0.00	0.00	5.15	0.00	5.15
Above 20	0.00	0.00	0.00	0.00	0.00
Total	4.92	0.00	58.22	36.86	100.00

differences in soil types and slopes and, therefore, productivity levels, corn yields are lower on Ida soils and the steeper Monona soils than on Napier soils. In classifying the three soil types into two productivity levels, most of the Ida and Monona soils were grouped together to form Land A—the "low" productivity soil class. Land B consists of the majority of more level Monona and Napier soils. In grouping the soil types into soil productivity classes, it was necessary to consider field size. For example, rather than put 3 acres of group B soil in a separate field because of the location, they would be included with the adjacent group A soils. Hence, Land A does not consist entirely of "low" productivity soils, nor Land B of "high" productivity soils.

Labor

The labor supply on the 160-acre farm consists of that provided by the operator and other family members. The operator supplies 260 hours each month. The family is considered to supply 26 hours in each of the months January through April and October through December, 130 hours during each of June, July and August and 40 hours in each of May and September. The family labor supplies are in terms of an operator-equivalent basis. That is, the labor shown is assumed to be, on an hourly basis, as efficient as operator labor. The labor supply is assumed to represent the modal labor situation for 160-acre farms in western Iowa.

Total available hours of labor for each month are presented in table 3. Subgroup totals have been made because the labor supply is limiting from March through June and from July through November.

TABLE 3. LABOR SUPPLY AVAILABLE FOR CROP AND LIVESTOCK PRODUCTION BY MONTHS.

Month	Operator	Available hours Family labor	Total
January	260	26	286
February	260	26	286
March	260	26	286
April	260	26	286
May	260	40	300
June	260	130	390
March–June subtotal			1,262
July	260	130	390
August	260	130	390
September	260	40	300
October	260	26	286
November	260	26	286
July–November subtotal			1,652
December	260	26	286

Operating Capital

One of the most limiting resources of farmers in western Iowa is operating capital. *Operating capital* may be defined as that capital not invested in machinery, buildings and land. The amount of operating capital available to farmers varies greatly. Even on the same farm, the most profitable combination of crops and livestock differs with the amount of operating capital available for production.

Since the various years are interrelated in dynamic linear programming, only operating capital in year 1 can be specified. The amount of operating capital available on the 160-acre farm in year 1 is $9,900. This capital level was selected because it allows all land to be cropped in year 1 after a deduction for family living has been made.[9] Thus, in year 1, $9,900 is available for family living and crop and livestock production. In years 2, 3, 4 and 5, the amount of available operating capital depends upon the total revenue from crop and livestock production in the preceding year. Hence, in years 2, 3, 4 and 5, the amount of operating capital will vary (except by coincidence) under each situation studied. It is assumed that no capital is borrowed in any of the 5 years.

Machinery and Buildings

It is assumed that a complete line of the necessary machinery for crop and livestock production is available and does not have to be purchased out of the $9,900 of operating capital. It is also assumed that adequate building facilities for crop and livestock production are already present. The floor area of the building space for hog production is 720 square feet; for cattle production it amounts to 1,960 square feet. A maximum of 20 litters of pigs and 65 head of cattle can be produced in the available building space under each situation. Adequate facilities for grain and hay storage and for farm machinery also are assumed to be on hand.

Total annual fixed costs (taxes, insurance, building repairs and depreciation on machinery and buildings) amount to $2,397.[10] This figure must be subtracted from the net returns figure to obtain net income. Fixed costs are independent of the level of crop and livestock combinations selected.

Family Living

In farming, available operating capital is used for both farm production and household consumption (i.e., operating capital generally is allocated for family living and farm production from the same fund). Therefore, not all operating capital is available for crop and livestock production. In this study, the annual cost of family living (household consumption) is assumed to be $3,697. The deduction for family living is assumed to represent the cost of family living

[9] An annual deduction of $3,697 is made from operating capital for family living in situations II and III. In Situation I, the cost of family living is deducted from available operating capital in years 2, 3, 4 and 5.

[10] Taken from the 1955 "Iowa Farm Record Summary" for western Iowa. The 1955 total fixed cost farm size group of 140-199 acres is used as an estimate for the 160-acre farm

for a family of two adults and two children.[11] In year 1 of Situation I, $9,900 is available for crop and livestock production, since it is assumed that the cost of family living has already been deducted from available operating capital (see footnote 9). In all situations, operating capital is used for family living before it is used for crop and livestock production because family living is forced into the plan by an artificially high net revenue.

PRICES

The prices used in this study are given in table 4 and are the same as those used previously.[12] Grain prices used in this study are somewhat higher than those prevailing more recently but are at levels which existed when this study was initiated.

DESCRIPTION OF ACTIVITIES

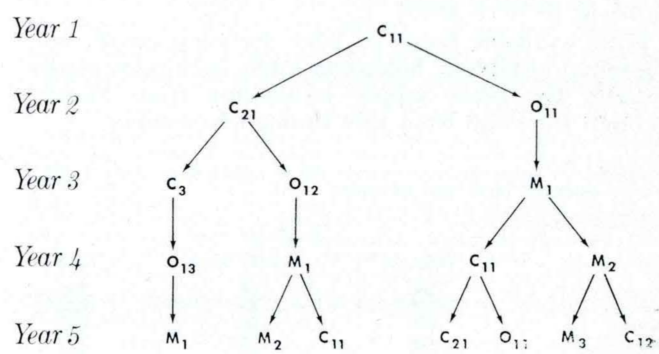
The basic programming activities which compete for scarce resources are household consumption and crop and livestock enterprises produced under different practices or methods. The crop enterprises consist of corn, oats and hay crops. The livestock enterprises include a two-litter hog system and deferred-fed calves. Household consumption has been included as a basic enterprise or activity because this activity is forced into all optimum plans. A larger number of livestock enterprises was not included because, from previous studies, deferred-fed calves and two-litter hog system were found to be the most profitable in a plan representing a single point in time, for the capital level used. Crop enterprises were handled in a special manner outlined in the next section.

Crop Enterprises and Yields

Many farmers do not follow a specific sequence of crops from year to year. Instead, in any particular year, they produce those crops that they think will maximize profits for that year, after taking weather conditions, expected prices, preceding crops, yields and feed requirements into consideration. In this study individual crop enterprises are allowed to be fitted into sequences so that, within certain limits, all possible combinations of corn, oats and hay over a 5-year period are allowed. The limits for all situations are: (1) not more than 3 consecutive years of corn or hay may be produced, (2) only first-year oats may be grown and (3) oats are used as a nurse crop for hay production in years 2, 3, 4 and 5.

The important aspect of the cropping combinations which result from this approach is that every possible combination of corn, oats and hay (within the previously defined limits) has been included, whether viewed as individual crops in each year of the 5-year plans, or as 5-year crop rotations. Thus, by using this approach, the most profitable crops are grown in each of the 5 years in terms of the 5-year optimum plan. It is assumed that by using this approach the plans obtained will approximate more closely the decision-making realm of farmers, than had only several 5-year crop rotations been considered as possible crop enterprises.

The number of possible combinations of corn, oats and hay can be explained as follows: Let C_{11} represent first-year corn after 1 year of hay; C_{12}, first-year corn after 2 years of hay; C_{13}, first-year corn after 3 years of hay; C_{21}, second-year corn after 1 year of hay; C_{22}, second-year corn after 2 years of hay; C_{23}, second-year corn after 3 years of hay and C_3 third-year corn. Also, let O_{11} represent first-year oats after 1 year of corn; O_{12}, first-year oats after 2 years of corn and O_{13}, first-year oats after 3 years of corn. Denote first-year hay by M_1 and second- and third-year hay by M_2 and M_3, respectively. Starting with first-year corn in year 1, the following 5-year cropping combinations are possible:



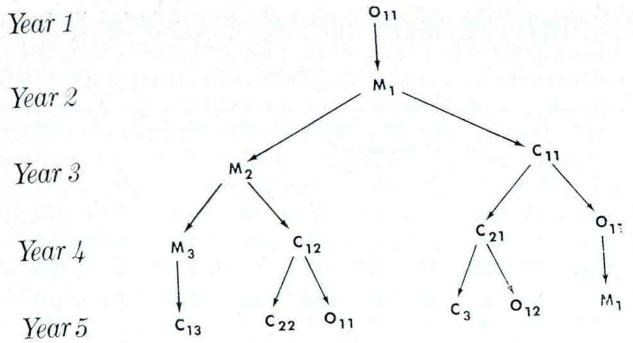
If first-year oats are grown in year 1, the following combinations are possible:

[11] Source: Cooperative Extension Service in Agriculture and Home Economics. 1955 family living expenditures of eighty-six Iowa farm families. FM-1231. Ames, Iowa. July, 1956.

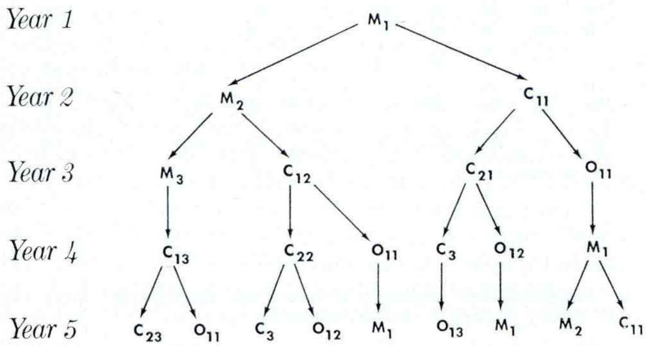
[12] Dean, Heady, Husain and Duncan, *op. cit.*, p. 2.

TABLE 4. PRICES USED IN DETERMINING OPTIMUM PLANS ON THE 160-ACRE FARM.

Item	Unit	Purchase price ($)	Selling price ($)
Corn (selling)	bu.	. . .	1.33
Corn (buying)	bu.	1.43	. . .
Oats	bu.	. . .	0.70
Hay	tons	0.00	0.00
Alfalfa seed	lb.	0.43	. . .
Bromegrass seed	lb.	0.25	. . .
Nitrogen (N)	cwt.	14.40	. . .
Phosphorus (P_2O_5)	cwt.	11.00	. . .
Cattle supplement	cwt.	4.40	. . .
Hog supplement	cwt.	4.40	. . .
Steer feeder calves	cwt.	23.68	. . .
Choice fat cattle	cwt.	. . .	26.08
March market hogs	cwt.	. . .	18.43
Sept. market hogs	cwt.	. . .	19.87
Old sows	cwt.	. . .	16.98
Terracing cost	ft.	0.04	. . .
Contouring cost	acre	0.25	. . .



If hay is produced in year 1, the following cropping combinations are possible:



In addition to the preceding cropping combinations, land may be put into permanent pasture or rented out (i.e., left in disposal) for one or more years. For example, it would be possible to have: $C_{11} - d_2 - d_3 - d_4 - d_5$; $C_{11} - C_{21} - d_3 - d_4 - d_5$ — etc., where d_k represents disposal land in year k. The total number of possible cropping combinations (including $d_1 - d_2 - d_3 - d_4 - d_5$) on either Land A or Land B over the 5-year period is 52.

By using this approach, it is assumed that each crop produces a specific soil productivity level for crop production the following year (i.e., C_{11} produces a different soil productivity level for crop production the following year than C_{21} does). Since the same cropping possibilities exist whether viewed as rotations or individual crops, either the expansion model or the rotation model of dynamic programming will produce the same 5-year plan. Nevertheless, 52 distinct rotations or crop combinations must be considered for the rotation model on either land type.

Thus, while there are only two land groups representing restrictions, there are numerous levels of fertility within each of the land groups, and each fertility level represents a different restriction. Corn produced in one year represents a specific crop in a sequence of crops and requires land of a particular fertility level in this year. At the same time, the corn produces land of a different fertility level for the next year. Hence, it subtracts from the supply of one land restriction in the given year, but adds to the supply of another land restriction in the following year. The land used by corn in the given year will be of different productivity than the land produced by it in the next year. The same holds true for each of the other crops, or for each other crop activity representing a different year in a sequence (i.e., C_{23}, C_{11}, C_3, etc.). This "interaction" between soil productivity supplies or restrictions of different years is accomplished by using positive a_{ijk} values for the requirements of the *jth* crop activity on the *ith* soil productivity restriction in the *kth*, or current, year and using a negative a_{ijk} value for a different soil productivity restriction in the $k + 1$ year. With the possibility of h different soil fertility restrictions in any one year, g different crop activities and t years, the equations of crop production possibilities take on the general form indicated in equations 6. Assume the crop activities represented as x_{11} and x_{21} draw from soil fertility restriction represented as b_{11} in year 1, but produce the soil restriction represented as b_{i2} in year 2. Similarly, the crop activities represented by x_{31} and x_{41} use land from soil restriction b_{21} in year 1, but produce soil of the fertility level represented by $b_{i+1,2}$ in year 2; they do not use land represented by soil restriction b_{11} in year 1. Thus we have

$$\begin{aligned}
&a_{111}\,x_{11} + a_{121}\,x_{21} + 0\ \ x_{31} + 0\ \ x_{41} + 0\,x_{51} + \ldots + 0\,x_{g1} = b_{11}\\
&0\ \ x_{11} + 0\ \ x_{21} + a_{231}\,x_{31} + a_{241}\,x_{41} + 0\,x_{51} + \ldots + 0\,x_{g1} = b_{21}\\
&\quad\vdots\\
&-a_{i12}\,x_{12} - a_{i22}\,x_{22} + 0\ \ x_{32} + 0\ \ x_{42} + 0\,x_{52} + \ldots + 0\,x_{g2} = b_{i2} \qquad (6)\\
&0\ \ x_{12} + 0\ \ x_{22} - a_{i+1\ 32}\,x_{32} - a_{i+1\ 42}\,x_{42} + 0\,x_{52} + \ldots + 0\,x_{g2} = b_{i+1\ 2}\\
&\quad\vdots\\
&0\ \ x_{1t} + 0\ \ x_{2t} + 0\ \ x_{3t} + 0\ \ x_{4t} + 0\,x_{5t} + \ldots + a_{hgt}\,x_{gt} = b_{ht}.
\end{aligned}$$

A similar procedure is used in relating the capital supplies of one year to those of the next year.

Crop yield estimates for corn, oats and hay by soil type and soil productivity class at alternative levels of fertilization and terracing and contouring are given in tables 5, 6 and 7. In tables 5, 6, and 7, the crops (i.e., C_{11}, C_{12}, etc.) are defined as before. The yields are in bushels per acre for corn and oats and tons per acre for hay. Yields of grain and hay are lower on Land A (the "low" productivity soil class) than on Land B.

Table 7 shows the rate of fertilizer application for corn, oats and hay and table 8, the cost per acre of fertilizer application for crop production on Land A and Land B. From tables 5 and 6, it is suggested that crop fertilization increases yield much more than does the use of terracing and contouring. Fertilizer is applied at a single recommended rate.[13] Fertilizer costs are much higher on Land A than on Land B. It is assumed that for activities including terracing and contouring, all cropland is terraced and contoured.

Farmer cost of terracing and contouring is esti-

[13] It is assumed that the yield per acre of C_{23} on Land A and Land B is the same as the yield of C_{22}.

TABLE 5. ESTIMATED CORN, OATS AND HAY YIELDS ON IDA, MONONA, NAPIER AND CASTANA SOILS AT ALTERNATIVE FERTILIZATION AND TERRACING AND CONTOURING LEVELS. CORN AND OAT YIELDS IN BUSHELS PER ACRE, MEADOW YIELD IN TONS PER ACRE.

			Crop															
			C_{11},C_{12},C_{13}	C_{11},C_{12},C_{13}	C_{21}	C_{21}	C_{22}	C_{22}	C_3	C_3	O_{11}	O_{11}	O_{12},O_{13}	O_{12},O_{13}	M_1,M_2	M_1,M_2	M_3	M_3
		Conservation	Fertilizer application*															
Soil type	Percent slope	practices	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1
Ida	7-14	None	15	45	15	42	15	42	15	42	15	30	13	30	0.5	2.5	0.5	2.3
		Terracing and contouring	20	52	20	50	20	50	20	50	18	35	16	35	0.5	2.7	0.5	2.3
Ida	15-20	None	15	40	15	38	15	38	15	38	14	27	12	27	0.5	2.2	0.5	2.0
		Terracing and contouring	20	46	20	44	20	44	20	44	16	30	14	30	0.5	2.4	0.5	2.2
Ida	Above 20	None	12	36	12	34	12	34	12	34	12	24	10	24	0.4	1.8	0.4	1.6
		Terracing and contouring	12	37	12	36	12	36	12	36	12	26	10	26	0.4	2.0	0.4	1.8
Monona	0-6	None	55	70	48	65	52	65	42	65	33	42	30	42	2.5	3.0	2.2	2.7
		Terracing and contouring	60	75	52	70	56	70	46	70	35	45	31	45	2.5	3.0	2.2	3.0
Monona	7-14	None	48	60	40	55	44	55	35	55	30	38	27	38	2.2	2.8	2.0	2.6
		Terracing and contouring	55	70	46	65	50	65	40	65	32	40	29	40	2.3	2.9	2.0	2.8
Monona	15-20	None	44	55	35	50	40	50	30	50	28	36	26	36	1.8	2.4	1.8	2.2
		Terracing and contouring	50	64	40	58	45	58	35	58	30	38	27	38	2.0	2.6	1.8	2.4
Napier	0-6	None	62	75	54	70	58	70	50	70	35	45	32	45	2.8	3.2	2.8	3.2
Castana	15-20	None	50	64	42	58	46	58	38	58	32	40	29	40	2.0	2.6	2.0	2.6
		Terracing and contouring	54	68	45	62	50	62	40	62	32	40	29	40	2.0	2.6	2.0	2.6

* For fertilizer application: F_0 = no fertilizer applied; F_1 = fertilizer applied at a single rate.

TABLE 6. CORN, OAT AND HAY YIELDS ON LAND A AND LAND B AT ALTERNATIVE FERTILIZATION AND TERRACING AND CONTOURING LEVELS. CORN AND OAT YIELDS IN BUSHELS PER ACRE, MEADOW YIELD IN TONS PER ACRE.

		Crop															
		C_{11},C_{12},C_{13}	C_{11},C_{12},C_{13}	C_{21}	C_{21}	C_{22}	C_{22}	C_3	C_3	O_{11}	O_{11}	O_{12},O_{13}	O_{12},O_{13}	M_1,M_2	M_1,M_2	M_3	M_3
Land class	Conservation	Fertilizer application*															
	practices	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1	F_0	F_1
Land A	None	25.6	46.3	22.6	43.2	24.2	43.2	20.8	43.2	19.2	30.7	17.1	30.7	1.0	2.3	1.0	2.1
	Terracing and contouring	31.1	53.6	27.7	50.3	29.3	50.3	25.8	50.3	21.3	33.5	19.0	33.5	1.0	2.5	1.0	2.3
Land B	None	53.5	67.6	46.1	62.7	50.0	62.7	41.4	62.7	31.9	41.3	29.0	41.3	2.4	3.0	2.3	2.8
	Terracing and contouring	57.1	72.2	49.2	67.3	53.0	67.3	44.3	67.3	33.2	43.0	30.0	43.0	2.4	3.0	2.3	3.0

* For fertilizer application: F_0 = no fertilizer applied; F_1 = fertilizer applied at a single rate.

TABLE 7. ESTIMATED FERTILIZER REQUIREMENTS OF N, P_2O_5 AND K FOR ESTIMATED YIELDS ON IDA, MONONA, NAPIER AND CASTANA SOIL TYPES.

Crop	C_{11},C_{12},C_{13}	C_{21}	C_{22}	C_3	O_{11}	O_{12},O_{13}	M_2
Soil type	N P_2O_5 K	N P_2O_5 K	N P_2O_5 K	N P_2O_5 K	N P_2O_5 K	N P_2O_5 K	N P_2O_5 K
Ida	30 + 80 + 0	60 + 40 + 0	50 + 40 + 0	60 + 40 + 0	15 + 40 + 0	20 + 40 + 0	0 + 40 + 0
Monona	10 + 20 + 0	45 + 20 + 0	35 + 20 + 0	45 + 20 + 0	10 + 30 + 0	15 + 30 + 0	0
Napier	10 + 0 + 0	45 + 0 + 0	35 + 0 + 0	45 + 0 + 0	10 + 0 + 0	15 + 0 + 0	0
Castana	10 + 20 + 0	45 + 20 + 0	35 + 20 + 0	45 + 20 + 0	10 + 30 + 0	15 + 30 + 0	0

TABLE 8. COST IN DOLLARS PER ACRE FOR FERTILIZER REQUIREMENTS* FOR ESTIMATED CROP YIELDS ON LAND A AND LAND B.

Land	Crop						
class	$C_{11},C_{12}C_{13}$	C_{21}	C_{22}	C_3	O_{11}	O_{12},O_{13}	M_2
Land A	$9.80	$11.51	$10.07	$11.51	$5.92	$6.64	$2.86
Land B	3.30	8.08	6.65	8.08	3.61	4.34	0.22

* Fertilizer prices are: N = 14.4 cents per pound; P_2O_5 = 11 cents per pound.

mated at \$7.92 per acre for Land A and \$6.62 for Land B.[14] These figures represent only 30 percent of the total cost. The other 70 percent is considered to be paid by the federal government as a soil conservation payment—a common practice in the area. In situations that include terracing and contouring, the costs for terracing and contouring are charged only against crop production in year 1; it is assumed that if terraces are constructed in year 1 they will be present in later years. The investment must be made, however, in the year of introduction.

CROP RESOURCE REQUIREMENTS

Crop resource requirements for labor and capital are presented in table 9. Only seed, fertilizer and terracing and contouring costs are charged against hay production. All other costs incurred in hay production are charged against the livestock enterprise which consumes the hay.

The "fixed" cost per acre for corn, oats and hay represents those cost items (i.e., fuel, seed, insecticides, fixed machinery, tractor and building costs) which are incurred independent of crop yield. These "fixed" costs vary with the number of acres grown, but not with the quantity of production per acre. "Variable" costs are those which vary directly with yield per acre and include operating costs such as hauling and elevating. For example, the yield of C_{11} on Land B, when fertilizer and terracing and contouring are not included, is 53.5 bushels per acre. Therefore, total operating capital requirement for 1 acre of C_{11} is \$17.08 (the "fixed" cost) plus \$0.08 x 53.5 (the variable cost). The March-June labor coefficient for 1 acre of C_{11} is 4.69 hours, and the July-November labor requirement is 4.79 hours.

Since the basic data for crop coefficient computations have been presented in tables 3, 4, 5, 6, 7 and 8, separate tables of input-output coefficients and net revenues for each of the cropping possibilities previously defined have not been included. Net revenues of crop activities in year 1 in each situation are simply yield per acre times price of crop minus total cost per acre for producing the crop. In years 2, 3, 4 and 5, however, net revenue of individual crops or crop rotations has been discounted, because time must be considered in dynamic programming. The discounted net revenue is the worth of the future net revenue at the present time. The rate of interest used in discounting all future net revenues is 6 percent. As an illustration of discounting net revenue, consider the following example: The net revenue from 1 acre of C_{11} on Land B in year 1 when fertilizer and terracing and contouring are not included is

$$53.5 \times \$1.33 - [\$17.08 + (\$0.08 \times 53.5)] = \$49.80$$

where the yield of corn is 53.5 bushels per acre and the price of corn \$1.33 per bushel. In year 4, however, the discounted net revenue from 1 acre of C_{11} is

$$\frac{53.5 \times \$1.33 - [\$17.08 + (\$0.08 \times 53.5)]}{(1.0 + 0.06)^4} = \$39.45.$$

That it, \$49.80 in year 4 discounted at 6 percent is

[14] Farmer costs for terracing and contouring Land A and Land B are calculated as follows: Assume Land A has a slope of more than 8 percent and Land B, a slope of less than 8 percent. One mile of terracing land with a slope of more than 8 percent equals 10 acres protected. One mile of terracing land with a slope of less than 8 percent equals 12 acres protected. Terracing costs \$0.045 per foot and contouring \$0.25 per acre. Hence, farmer cost of terracing and contouring 1 acre of Land A is $\left(\frac{5{,}280 \times \$0.045}{10} + \$0.25\right) 0.33 = \$7.92$; for 1 acre of Land B it is $\left(\frac{5{,}280 \times \$0.045}{12} + \$0.25\right) 0.33 = \$6.62$.

Source: Agricultural conservation program handbook for 1956, Iowa. U. S. Dept. Agr., Agricultural Conservation Program Service. August 1955.

TABLE 9. RESOURCE REQUIREMENTS PER ACRE FOR CORN, OATS AND HAY ON LAND A OR LAND B.

Item	Unit	Corn*	Oats*	Hay*†
Labor requirement	hours			
March	hours		0.36	. . .
April	hours	1.18	0.90	. . .
May	hours	2.20	. . .	. . .
June	hours	1.31	. . .	6.22
Total March-June labor requirement	hours	4.69	1.26	6.22
July	hours	1.07	1.88	5.30
August	hours		1.88	
September	hours	0.20	. . .	4.48
October	hours	1.48	. . .	. . .
November	hours	2.04	. .	. . .
Total July-November labor requirement	hours	4.79	3.76	9.78
Fixed cost per acre	\$	17.08	13.11	4.97‡
Variable cost per bushel	\$	0.08	0.05	0.00
Cost of terracing and contouring – Land A	\$	7.92	7.92	7.92
Cost of terracing and contouring – Land B	\$	6.62	6.62	6.62

* Add on a per-acre basis: 0.3 hour of April labor for oats and 0.1 hour of May and June labor for corn when these crops are fertilized.

† The labor and variable capital requirements of hay are charged against the livestock enterprise that uses the hay for feed.

‡ Meadow seed cost composed of 8 pounds of alfalfa seed at \$0.43 per pound and 6 pounds of bromegrass seed at \$0.255 per pound.

TABLE 10. BASIC INPUT-OUTPUT DATA AND NET RETURNS FOR DEFERRED-FED CALVES AND A TWO-LITTER HOG SYSTEM ON A UNIT BASIS.*

Item	Unit	Deferred-fed calves	Two-litter hog system
Purchase date		October	. . . . .
Market date		December	. . . . .
Initial weight	lbs.	450.00	. . . . .
Marketing weight	lbs.	1,000.00	. . . .
Death loss	percent	2.50	5.00
Pigs weaned	no.	. . . .	14.16
Pigs sold	no.	. . . . .	12.45
Market hogs	lbs.	. . . . .	2,739.44
Market sow	lbs.	. . . .	400.00
Total pork	lbs.	. . . .	3,139.44
Feed:			
Corn equivalent	bu.	52.00	190.00
Supplement	lbs.	125.00	1,523.00
Hay equivalent†	tons	2.24	0.70
March-June labor	man-hrs.	7.96	24.13
July-November labor	man-hrs.	17.77	21.42
Building space requirement	sq. ft.	30.00	71.00
Annual cash expense:			
Supplement	\$	5.50	67.01
Building use	\$	2.09	3.25
Power use	\$	2.31	20.41
Equipment use	\$	2.42	21.03
Miscellaneous cost	\$	8.97	26.06
Boar service	\$	.	4.00
Death loss	\$	2.66	. . . . .
Feeder stock	\$	106.56	
Breeding gilt	\$	. . . . .	62.13
Total annual expense	\$	130.51	203.89
Investment in equipment	\$	13.50	27.74
Total capital outlay	\$	144.01	231.63
Net return	\$	61.13	196.48

* The unit of the deferred-fed calves enterprise is one head. The unit of the two-litter hog system is one sow with two litters of pigs.

† Pasture requirements have been converted into tons of hay equivalent.

only worth $39.45 in year 1. All activities in years 2, 3, 4 and 5 are discounted in this manner.

LIVESTOCK ENTERPRISES

The annual livestock enterprises included in each situation are two-litter hog systems and deferred-fed calves. Table 10 presents the basic input-output data and net revenues of these livestock enterprises. Net revenues of each livestock enterprise are discounted in years 2, 3, 4 and 5 for all situations. The resource requirements, of course, are the same in each of the five years. The cost of forage harvesting is included in the miscellaneous cost item for all situations.

The deferred-fed calf enterprise consists of choice steer calves purchased in October at 450 pounds, wintered, grazed 60 days on pasture and then full-fed to 1,000 pounds and sold in December. In the two-litter hog system, pigs are farrowed in March and September and are sold 6 months later at 220 pounds. The average number of pigs per litter is 7.08. An average death loss after weaning of 5 percent is assumed. One gilt is saved for replacement. The total quantity of pork sold during the year is 3,139 pounds. This includes 400 pounds from the sale of one sow.

ANALYSIS OF RESULTS AND OPTIMUM PLANS

Profit-maximizing, or optimum 5-year, farm plans for the conservation situations outlined earlier are presented in this section. Initial resource supplies, except in Situation I where family living does not have to be taken from capital supply in year 1, are the same for each situation studied. Hence, variations between plans mainly are due to differences in conservation levels, because the same cropping and livestock opportunities are available in each situation. In all situations, it is possible for land to be left in disposal (i.e., put into permanent pasture or rented out) for 5, 4, 3, 2 or 1 year(s).

All optimum 5-year plans have been computed within the limits of available resource supplies. Corn may be purchased off the farm, however, to expand livestock production. In the tables that follow, the "corn surplus or deficit" column shows the bushels of corn bought or sold each year. A plus sign signifies corn sold, and a minus sign indicates corn bought. It is assumed that all hay is produced on the farm. When needed, surplus hay can be transferred from one year to the following year. Similarly, unused capital can be transferred from one year to the next, if it can be invested profitably the following year. (All of the capital flow, plus the initial capital, representing surplus of net income over living expenses can be transferred between years.)

In each plan, the annual discounted net return is given in the "net returns" column. This figure represents the farm's annual net return discounted back to the present after family living expenses have been subtracted. Returns have been computed in this manner to express the amount of capital which might be accumulated and transferred between years. In this case, no cost has been subtracted to represent interest on borrowed funds. The discounted net returns figure in the last column would need to be reduced by an amount corresponding to the interest cost of mortgages and other credit for owners with borrowed funds. In all situations studied, the return on capital used for the plans indicated is considerably above interest rates for capital.

The data in tables 11, 12 and 13 have been adjusted to compensate for rounding errors.

Plan I: Optimum 5-Year Plan for a 160-Acre Farm Without Fertilizer or Terracing and Contouring Under Situation I

Table 11 presents an optimum 5-year plan for a 160-acre farm that does not use fertilizer, terracing and contouring. Forage crops are grown to provide feed for livestock. The 5-year rotations formed by dynamic programming represent the most profitable combinations of crops on Land A and Land B for the 5-year period, supposing a planning horizon of this period and a goal of maximized discounted net profit, subject to meeting the restraint of living costs.

The year represented by a particular crop is that indicated by the following sequence. Thus, for the 48.6 acres of Land A, first-year hay is grown in year 1; second-year hay, following hay, is grown in year 2; third-year hay, following hay, is grown in year 3; first-year corn, following hay, is grown in year 4; and second-year corn, following corn, is grown in year 5. The symbols have similar meaning for the crop indicated on the various acreages or tracts of Land B. It should be remembered that crops were not forced into these sequences through prior selection of specific rotations; these sequences were generated as the optimum cropping plans within the framework of a profit-maximizing plan over a 5-year period.

Land Class	*Rotation*	*Acres*
Land A	$M_1 - M_2 - M_3 - C_{13} - C_{23}$	48.6
Land B	$M_1 - M_2 - M_3 - C_{13} - C_{23}$	5.9
	$M_1 - M_2 - C_{12} - C_{22} - C_3$	11.9
	$C_{11} - C_{21} - O_{12} - M_1 - C_{11}$	23.1
	$C_{11} - C_{21} - O_{12} - M_1 - M_2$	36.1
	$C_{11} - O_{11} - M_1 - C_{11} - C_{12}$	17.4

In the above crop rotations on Land A and Land B, the same symbols are used to define crop production over the 5-year period as were used in the section on crop enterprises. For example, the rotation M_1 - M_2 - M_3 - C_{13} - C_{23} on Land A means that first-year hay (M_1) is grown in year 1, second-year hay (M_2) is grown in year 2, third-year hay (M_3) is grown in year 3, first-year corn after 3 years of hay (C_{13}) is grown in year 4 and second-year corn after 3 years of hay (C_{23}) is grown in year 5. In each of the 5 years, 48.6 acres are grown.

Over the 5-year period, the cheapest source of forage for livestock production is obtained from hay grown on Land A. Hay yields are relatively higher than corn yields on this land class. On the more productive soil, Land B, the reverse is true, and in terms of acreage, corn is the main crop grown. Oats are relatively unprofitable on both classes of land.

TABLE 11. PLAN I: OPTIMUM 5-YEAR PLAN FOR A 160-ACRE FARM WITHOUT CROP FERTILIZATION AND WITHOUT TERRACING AND CONTOURING OF CROPLAND UNDER SITUATION I.

Year of plan	Production capital used*	Optimum combination of enterprises: Crops: Land class	Crop	Acres	Livestock: Type	Number	Other: Type	Value	Corn surplus or deficit (bu.)	Limiting resources	Discounted net returns†
1	$ 9,900	A B B	M_1 M_1 C_{11}	48.6 17.8 76.6	Deferred-fed calves two-litter hog system	33 head 20 litters	Family living	$3,697	+ 446	Land A Land B Forage feed Hog building space	$3,348
2	$12,815	A B B B	M_2 M_2 C_{21} O_{11}	48.6 17.8 59.2 17.4	Deferred-fed calves two-litter hog system	33 head 20 litters	Family living	$3,697	− 644	Land A Land B Forage feed Hog building space	$3,600
3	$13,526	A B B B B	M_3 M_3 C_{12} O_{12} M_1	48.6 5.9 11.9 59.2 17.4	Deferred-fed calves two-litter hog system	43 head 20 litters	Family living	$3,697	− 2,662	Land A Land B Forage feed Hog building space	$2,341
4	$13,414	A B B B B	C_{13} C_{13} C_{22} M_1 C_{11}	48.6 5.9 11.9 59.2 17.4	Deferred-fed calves two-litter hog system	58 head 20 litters	Family living	$3,697	− 579	Land A Land B Forage feed July-Nov. labor Hog building space	$3,425
5	$10,857	A B B B B B	C_{23} C_{23} C_3 C_{11} M_2 C_{21}	48.6 5.9 11.9 23.1 36.1 17.4	Deferred-fed calves two-litter hog system	43 head 20 litters	Family living	$3,697	0	Land A Land B Hog building space Forage feed	$4,037

* Capital available for crop and livestock production after family living expenses have been met.
† Net returns after living expenses are subtracted.

Their function is to provide a nurse crop for forage. In actual practice, many farmers with small acreages of grain would not incorporate more than one rotation. As the preceding plan indicates, however, Land B should be used mostly for grain production, while forage is primarily produced on Land A. Because of the cropping restriction—that not more than 3 years of continuous corn or hay may be grown—some corn is produced on Land A. Likewise, some hay production occurs on Land B. Moreover, some forage production is required on Land B to provide adequate feed for livestock.

YEAR 1

In year 1, no deduction is made from capital to allow for family living. Hence, $9,900 is available for crop and livestock production. Because fertilizer and terracing and contouring are not included in Situation I, capital requirements for crops are lower than in the other situations studied. Consequently, more capital is available for livestock production in the first year. Under the pricing system used, hogs give higher returns to capital than cattle. The maximum hog production allowed by building space is 20 litters, however, even though the supply of available operating capital is greater than required for this number of hogs. The next highest return from capital is in feeding cattle. The resulting plan for year 1 is 20 litters of pigs and 33 head of deferred-fed calves. To provide the necessary forage feed, all of Land A plus 17.8 acres of Land B are used to grow hay. The remaining acres of Land B are used for corn. A total of 446 bushels of corn is not needed for feed and is sold for cash. The limiting resources for this plan are Land A, Land B, forage feed and hog building space. Discounted net return, after living costs have been paid, in year 1 is $3,348. If expenses for household consumption had been subtracted from capital supply in the first year, less operating capital would have been available for cattle production. Hence, cattle numbers, and therefore net return, would have been lower than that shown.

For the plan in table 11, the amount of disposal or unused capital in year 1 is $854. Disposal capital in any one year is the amount of funds not transferred to the supply of available capital for the year following. It might seem that this surplus capital could be used for increased production in year 2. In this dynamic programming model, however, profits are maximized for a multiyear period, and crop and livestock production is interrelated over all years of the whole period. Consequently, profits are maximized for the 5-year period by investing only $12,815 in year 2; $854 is available, but is not invested. When more captial is needed in year 3, most of the addition is generated by the plan in year 2. If the optimum plan had been for years 1 and 2 only, disposal capital in year 1 would have been transferred for use in year 2, because production and corresponding returns in subsequent years would not have been considered.

YEAR 2

In year 2, the number of deferred-fed calves and hogs produced is the same as in year 1. Similarly, all of Land A and 17.8 acres of Land B are used to produce the necessary forage feed. In year 2, however, only 59.2 acres of corn are grown. The remainder of Land B is used to grow oats, which provide a nurse crop for forage the following year. Substitution of oat acres for corn acres, plus the decreased yields of second-year corn, diminishes the grain supply and necessitates purchase of 644 bushels of corn for livestock. A surplus of income of $3,600 over family living costs is generated in year 2. Hence, this amount of operating capital is transferred to year 3, in addition to the $12,815 available for production at the outset of year 2, but which is returned by the production in year 2. This result contrasts the parallel situation in year 1. Unused capital in year 1 could not be profitably invested in year 2, and surplus capital was not transferred from year 1 to year 2. The net gain in capital during year 2, however, can be profitably invested in year 3 and is transferred with the original capital fund. The limiting resources in the optimum plan for year 2 are Land A, Land B, forage feed and hog building space—the same resources that restricted production in year 1. Discounted net return after subtraction of family living costs is $3,600.

YEAR 3

In year 3, increased capital and the sequence of crops in previous years permits cattle production to be expanded to 43 head. Because hogs have a higher return to capital and labor in combination, the maximum number of litters permitted by building space again are produced. Crop production in year 3 includes 71.9 acres of hay, 59.2 acres of oats and only 11.9 acres of corn. (See sequence on various acreages as mentioned earlier.) The increased hay acreage provides the additional forage required for feeding the larger number of cattle in year 3. Because of the decreased corn acreage, however, along with an increase in cattle numbers, it is necessary to purchase 2,662 bushels of corn for feed. As before, interyear interdependence between crop and livestock enterprises specifies the best plan for any one year. Hence, profits are maximized for the 5-year period if most of the feed grain requirements in year 3 are purchased. Thereby, more hay acres are allowed in year 3, thus providing corn land of higher yield potential in years 4 and 5. The limiting resources in year 3 are Land A, Land B, forage feed and hog building space. Discounted net return, after subtracting family living costs, is $2,341. The decrease in discounted net return in year 3, as compared with year 2, is explained by the fact that there are fewer acres of corn, and discounting is over a longer period, causing the same enterprise in year 3 to have a lower discounted net return than in year 2. The major decrease in net return in year 3, however, is caused by a decreased corn acreage.

While even nondiscounted net return is smaller in year 3 than in year 2, the plan is not in a "stage of deterioration." As one year's plan, in a complex of 5 interdependent years, the plan for year 5 is one which allows a maximum of discounted return over a 5-year period, subject to the restraint that family living costs must be met in each year. To select a sequence of yearly plans which would result in a larger income in year 3 would cause the sum of discounted returns over the 5-year period to be less than under the plan selected.

YEAR 4

In year 4, $13,526 is used for farm production. As before, the amount of available operating capital depends on the level and types of enterprises in the previous year. All of Land A is now used for corn, because only 3 years of consecutive hay are allowed under the cropping restrictions explained earlier. On Land B, 59.2 acres are used for forage, and the remainder is used for corn. Higher hay yields on Land B allow cattle numbers to be expanded to 58 head, even though fewer acres are used for forage production in year 4 than in year 3. As in previous years, the 20 litters of pigs allowed by building space are produced. Also, since the majority of corn is produced on Land A where yields are lower, 579 bushels of corn must be purchased, even though corn acreage is greater in year 4 than in year 3. The limiting resources in year 4 are Land A, Land B, forage feed, hog building space and July-November labor. Labor is now restrictive because of the increased cattle numbers. Discounted net return, after subtracting family living and fixed costs, is $3,425. Discounted net return in year 4 is greater than in year 3, because cattle numbers and corn acreage are increased.

YEAR 5

In year 5, $10,857 is used in production. Since this is the last year of the time plan, only corn and hay are grown; oats are not needed as a nurse crop since no crops are indicated for a sixth year. Fewer cattle are produced in year 5 than in year 4, because the cropping sequence in previous years specifies decreased hay acreage in year 5. The maximum number of hogs allowed by building space is produced. Because of lower yields, only enough corn is produced to satisfy the livestock feed-grain requirements, even though corn acreage is larger in year 5 than in year 4. Limiting resources in year 5 are Land A, Land B, hog building space and forage feed. In year 5, discounted net return, after subtracting living costs, is $4,037.

It should be noted that land use in year 4 would allow the same number of acres of forage in year 5 as in year 4, and therefore the same cattle numbers. The optimum plan in year 5, however, specifies only 36.1 acres of hay, with the remaining acreage to be allocated to corn. In other words, discounted net return is maximized over the 5-year period by decreasing cattle numbers and forage acreage and increasing corn acreage in year 5. Similarly, no other pattern of grain acreage and livestock production over the 5-year period would result in discounted net returns as high as those represented in this optimum plan.

Over the 5-year period, net discounted returns, after living costs have been met, total $16,751. Forage feed is the principal limiting resource in cattle feeding. Limited hay production results not only because acreage is limited but also because fertilizer is not included for crops. Soil fertility, aside from that for starting seedings, must be generated by the crops grown and the livestock manure returned to the fields.

Land is the main limiting resource in crop and livestock production in each year. Capital is not limitational in any of the 5 years under Situation I for two reasons: (1) capital requirements of crops are low since no fertilizer, terracing and contouring are used for cropping activities and (2) production of cattle is limited because of the low forage yields, which in turn result from rotations which do not include fertilization.

Plan II: Optimum 5-Year Plan for a 160-Acre Farm With Fertilizer But Without Terracing and Contouring Under Situation II

The main difference between Situation I and Situation II is that the latter situation includes use of commercial fertilizer on crops. A second difference lies in the fact that family living is an activity requiring capital in all 5 years in Situation II. In Situation I, it was not subtracted from the initial capital supply of $9,046 under the supposition that consumption withdrawals could come from income generated during the year. In Situation II, however, it is supposed that consumption requirements in year 1 must come from capital available at the beginning of the year, rather than from income generated during the year. Actually, the two situations might be viewed as representing different initial amounts of capital. Situation II could be considered to have $3,697 less initial capital, if consumption requirements were specified to come from current income. Otherwise, resource supplies and production possibilities are the same as in Situation I. The optimum 5-year plan for Situation II is presented in table 12.

The following crop rotations represent the most profitable combinations of crops for Situation II over the 5-year period.

Land Class	*Rotation*	*Acres*
Land A	$C_{11} - O_{11} - M_1 - M_2 - M_3$	23.3
	$M_1 - M_2 - M_3 - C_{13} - C_{23}$	10.0
	$M_1 - M_2 - C_{12} - C_{22} - C_3$	15.3
Land B	$C_{11} - C_{21} - C_3 - O_{13} - M_1$	40.2
	$C_{11} - O_{11} - M_1 - C_{11} - C_{21}$	35.8
	$C_{11} - C_{21} - O_{12} - M_1 - C_{11}$	18.4

As in Plan I, the optimum 5-year cropping plan includes using the most productive soils (Land B) mainly for corn production. The less productive soils (Land A) are used mainly for hay. Forage and, therefore, oats as a nurse crop for forage, are grown on Land B after 3 consecutive years of corn, because of the cropping limitations explained earlier.

YEAR 1

In year 1 of Plan II, only $6,203 of operating capital is available for crop and livestock production, because $3,697 is used directly for family living. Since (a) the same prices are used in Situation II as in Situation I and (b) hogs give higher capital returns than cattle, capital is first allocated to production. The maximum number of pig litters allowed by building space (20 litters) is produced. The next most profitable use of capital is first-year corn. All of Land B and 23.3 acres of Land A are used for corn. In Situation II, because yields are higher as a result of crop fertilization, funds are allocated to corn production before cattle production. Finally, the remaining capital is used in cattle production. In summary for Situation II, capital in year 1 is first allocated to family living;

TABLE 12. PLAN II: OPTIMUM 5-YEAR PLAN FOR A 160-ACRE FARM WITH CROP FERTILIZATION BUT WITHOUT TERRACING AND CONTOURING OF CROPLAND UNDER SITUATION II.

Year of plan	Production capital used*	Optimum combination of enterprises: Crops			Livestock		Other		Corn surplus or deficit (bu.)	Disposal forage (tons)	Limiting resources	Discounted net returns†
		Land class	Crop	Acres	Type	Number	Type	Value				
1	$ 6,203	A A B	C_{11} M_1 C_{11}	23.3 25.3 94.4	Deferred-fed calves two-litter hog system	4 head 20 litters	Family living	$3,697	+ 5,319	43.6	Land A Land B Capital Hog building space	$6,099
2	$11,377	A A B B	O_{11} M_2 C_{21} O_{11}	23.3 25.3 58.6 35.8	Deferred-fed calves two-litter hog system	42 head 20 litters	Family living	$3,697	+ 699	0	Land A Land B Capital Hog building space Forage feed	$4,945
3	$15,681	A A A B B B	M_1 M_3 C_{12} C_3 M_1 O_{12}	23.3 10.0 15.3 40.2 35.8 18.4	Deferred-fed calves two-litter hog system	61 head 20 litters	Family living	$3,697	− 1,469	41	Land A Land B Capital Hog building space Feed grain	$4,755
4	$13,318	A A A B B B	M_2 C_{13} C_{22} O_{13} C_{11} M_1	23.3 10.0 15.3 40.2 35.8 18.4	Deferred-fed calves two-litter hog system	55 head 20 litters	Family living	$3,697	− 329	0	Land A Land B Hog building space Forage feed July-Nov. labor	$4,867
5	$13,850	A A A B B B	M_3 C_{23} C_3 M_1 C_{21} C_{11}	23.3 10.0 15.3 40.2 35.8 18.4	Deferred-fed calves two-litter hog system	59 head 20 litters	Family living	$3,697	− 430	49.8	Land A Land B Hog building space July-Nov. labor	$4,322

* Capital available for crop and livestock production after family living expenses have been met. For this plan, it is supposed that family living in year 1 must come from the original $9,900. Hence, only $6,203 is available for farm production.

† Net returns after living expenses have been met. For this plan, it is assumed that family living in year 1 must come from the original capital of $9,900. Only $6,203 is available in year 1. Living expense, however, is not subtracted from discounted net return in year 1 for this plan. Living expense has been subtracted from net return in other years to indicate the income surplus which might be transferred to the following years.

secondly, to hog production; thirdly, to corn production; and finally the remaining capital is used for cattle production. As a result, only 4 head of deferred-fed calves are included in the plan for year 1. As in Plan I, crop and livestock production in any one year is interrelated with crop and livestock production in all other years. In year 1, 117.7 acres of corn and 25.3 acres of hay are grown. A different plan emerges than under Situation I because of differences in capital availability and in fertilization practices for crops. More forage feed is produced than is required by the limited livestock enterprise. The surplus, 43.6 tons, is transferred to, and utilized during the next year. Also, because of a limited livestock enterprise and a large corn acreage, a surplus of 5,319 bushels of corn is not needed for feed and is sold. The limiting resources in year 1 are Land A, Land B, capital and hog building space. Capital limits cattle production, and building space limits hog production. Net return, without subtracting family living expense, amounts to $6,099.[15]

YEAR 2

In year 2, capital is transferred from year 1 and is available in year 2 for family living and increased farm production. Because of the increase in capital, plus the forage carryover from year 1, cattle numbers can be expanded to 42 head in year 2. Since hogs are still more profitable than cattle, the maximum number of hogs allowed by building space is produced. Correspondingly, crop production includes 58.6 acres of corn, 59.1 acres of oats and 25.3 acres of hay. Since the interyear dependence between crops and livestock specifies the optimum plan for any one year, oat acres are substituted for corn acres in year 2. This substitution permits more forage feed and, therefore, more livestock to be produced in year 3. Even though corn acreage is decreased in year 2, however, corn yields from fertilization and Land B productivity provide more grain feed than is required for expanded livestock production. The surplus corn is sold for cash. Capital in year 2 is allocated in this order: family living, hog production, corn, hay, cattle production and oat production. The limiting resources in year 2 are Land A, Land B, capital, hog building space and forage feed. Capital and forage feed limit cattle production, while building space limits hog production. Discounted net return, after family living expenses are subtracted, is $4,945. The decrease in discounted net return in year 2 as compared with year 1 is caused by a smaller corn acreage and by the fact that income is discounted over a longer period. Capital transferred from year 2 to year 3 is $15,681 and includes the capital available at the outset of year 2, plus the surplus of income (over costs and expenses) generated in year 2.

YEAR 3

In year 3, the increase in operating capital and the sequence of crops in previous years permits further expansion in the cattle enterprise. The year 3 plan calls for 20 litters of pigs and 61 deferred-fed calves. The corresponding crop plan includes 69 acres of forage, 55 acres of corn and 16 acres of oats. Much more hay is produced than is needed for the livestock enterprise. The surplus hay (41 tons) is transferred to, and is utilized in year 4, however, thus permitting a smaller hay acreage in that year. Again, this transfer illustrates the interrelationship between crops and livestock within each year and between years. The increased livestock enterprise and decreased grain acreage in year 3 results in the purchase of 1,469 bushels of corn off the farm. In other words, profits are maximized for the 5-year period by purchasing feed grain off the farm in year 3, thereby allowing more hay acres which in turn provide potential corn acres in years 4 and 5. The limiting resources in the plan for year 3 are Land A, Land B, hog building space, capital and feed grain. Hog production is limited by building space and cattle production by capital and by feed grain, which limits cattle feeding since capital is limitational and does not allow grain purchase. Discounted net return, after subtracting family living and fixed costs amounts to $4,755. Even with increased livestock production in year 3, discounted net return is lower than in year 2 because of a smaller corn acreage and the length of the discounting period.

YEAR 4

In year 4, capital transferred from production and as a surplus of net income over family living expenses, is a nonlimitational resource. Hence, feed grain is also nonlimitational, since it may be purchased. Grain crop acreage is expanded, and forage acreage is decreased. The cropping plan includes 61 acres of corn, 42 acres of forage and 40 acres of oats. As corn acreage increases, cattle production declines, because of a smaller hay acreage. Only 55 head of deferred-fed calves are included in the plans for year 4. The increased grain acreage also causes July-November labor to become a limiting resource. Since hogs and grain acreage have a higher return on July-November labor than cattle, the number of calves fed is reduced. Even with the increased grain acreage, however, the purchase of 329 bushels of corn for feed is necessary. Sufficient forage for livestock feed is attained from the smaller hay acreage in year 4, because it is supplemented by hay carried over from year 3. Other limiting resources are Land A, Land B and hog building space. Discounted net return, after subtracting family living expense, is $4,867. The greater acreage of first-year corn permits discounted net return in year 4 to be as high as that of year 3, even though cattle numbers are decreased and the discounting coefficient is larger because of the longer time period.

YEAR 5

In year 5, capital is again a nonlimitational resource. A total of $13,850 of operating capital is used and is available as a transfer from the initial capital supply plus the income surplus in the preceding year. The

[15] Living expense is not subtracted from income in year 1 since we suppose it to be drawn from initial caiptal of $9,900. For other years, however, the net discounted return shown supposes that living expenses have been deducted from the amount shown.

only crops grown in year 5 are corn and hay. Oats are not produced because this is the final year of the plan and, therefore, a nurse crop for further forage production is not required. More forage (50 tons) is produced in year 5 than is required for feed. This surplus forage production results from the cropping restrictions discussed previously. The surplus forage might be either sold or used as a green manure crop.

The cropping sequence of the optimum 5-year plan calls for 65.3 acres of hay and 79.5 acres of corn in year 5. As in the other years, 20 litters of pigs are produced. The increased forage acreage (and hence, decreased crop labor requirements) permits 4 more cattle to be produced in year 5 than in year 4. Specifically, the plan calls for 20 litters of pigs and 59 deferred-fed calves. Because of the large forage acreage, it is necessary to purchase 430 bushels of corn for feed. The limiting resources are Land A, Land B, hog building space and July-November labor. Hog production is limited by hog building space and cattle production by July-November labor. Discounted net return, after subtracting family living expense, is $4,322. Surplus capital, available but not required by the plan, in year 5 amounts to $4,437.

In Plan II, discounted net return, in excess of family living expense, amounts to a total of $21,291 over the 5-year period, an increase of $4,540 over Plan I. (The sum of the last column in table 11 is compared with the sum of the same column in table 12, except that $3,697 has been subtracted from the $6,099 in the first year. This adjustment is made to account for the fact that consumption in the first year for table 12 is assumed to come from the initial $9,900 capital supply and income for year 1 was not adjusted for living, as were the figures for the other years.) Hog building space is the resource which limits hog production in each of the 5 years for both situations I and II. Hogs are more profitable, for limited labor, capital and feed, than cattle. Hence, the maximum number of litters allowed by building space is produced each year. Capital is thus allocated to hog production before it is allocated to cattle production. Land A and Land B are limiting resources each year; both land A and B are fully utilized in each of the 5-year periods. The resources which primarily restrict cattle production are capital in the first 3 years and labor in the last 2 years of both plans. While capital is in short supply at the outset, the surplus of income over consumption and expenses allows funds to accumulate for the last 2 years. Directly, forage limits cattle numbers in the first 2 years, but indirectly it is capital, since this resource also limits hay production. Because of crop fertilization, and therefore higher grain and hay yields in Situation II, more livestock is included in the optimum plan than in the case of Situation I. Also, more corn is sold (or less is purchased in some years) in Plan II than in Plan I, even though livestock numbers are greater in Plan II. In both plans, Land A is used mainly for hay production and Land B for corn. In both optimum 5-year plans the farm firm and farm household are considered as an interrelated economic unit. Crop and livestock production in each of the 5 years is not independent of living needs by the household.

A somewhat different "time pattern" for crops also emerges under the two situations. Since capital is less and living expenses must be met at the outset, more corn is grown in years 1, 2 and 3 under Situation II. Greater amounts of corn early in the period also are more profitable, however, in terms of the criterion of maximized discounted net returns, because of the fertilization practices allowed under Situation II. Under Situation I, discounted net returns are greatest over the 5 years if more land is planted to hay at the outset to provide a fertility build-up for the greater corn acreage grown near the end of the period. The acreage patterns under the two situations are summarized thus:

	Situation I			Situation II		
	Corn	Oats	Hay	Corn	Oats	Hay
Year 1	76.6	0.0	66.4	115.7	0.0	25.3
Year 2	59.2	0.0	66.4	58.6	59.1	25.3
Year 3	11.9	59.2	71.9	55.5	18.4	69.1
Year 4	83.8	0.0	59.2	61.1	40.2	41.7
Year 5	83.8	0.0	36.1	79.5	0.0	73.5

Total production capital used over the 5 years, exclusive of machinery and related investment, is $60,512 for Situation I without fertilization practices and $60,429 for Situation II with fertilization practices. Hence, approximately the same amount of capital allows a greater aggregate output and income under Situation II where fertility can be purchased, as compared with Situation I where more forage must be used to provide fertility for grain crops.

Plan III: Optimum 5-Year Plan for a 160-Acre Farm With Fertilizer and With Terracing and Contouring Under Situation III

Table 13 presents the optimum 5-year plan for Situation III. In this situation, crops are fertilized at a single recommended rate, and all cropland is terraced and contoured. This situation differs from Situation II only in the addition of terracing and contouring. As a result of these practices, crop yields are higher than in the other two situations. Because crop yields are higher, fewer acres of Land A or Land B are needed for forage production and the corresponding nurse crop of oats. The maximum amount of hay grown in a single year (3) is 52.4 acres. The optimum 5-year cropping program for Plan III is shown thus:

Land Class	*Rotation*	*Acres*
Land A	$M_1 - M_2 - C_{12} - C_{22} - C_3$	31.0
	$C_{11} - O_{11} - M_1 - M_2 - M_3$	17.8
Land B	$C_{11} - C_{21} - O_{12} - M_1 - C_{11}$	26.0
	$C_{11} - O_{11} - M_1 - C_{11} - C_{21}$	34.6
	$C_{11} - C_{21} - C_3 - O_{13} - M_1$	33.8

As in plans I and II, rotations grown on Land A include relatively more forage crops than do rotations on Land B. Hay production on either land class is included in the cropping plan only to produce forage for feed or to meet the cropping restrictions. Oats are grown only to establish meadow seedings.

YEAR 1

In year 1, the same initial amount of capital ($9,900) is available as in Situation II. Of this, $3,697 again is

TABLE 13. PLAN III: OPTIMUM 5-YEAR PLAN FOR A 160-ACRE FARM WITH CROP FERTILIZATION AND TERRACING AND CONTOURING OF CROPLAND UNDER SITUATION III.

Year of plan	Production capital used*	Optimum combination of enterprises: Crops: Land class	Crops: Crop	Crops: Acres	Livestock: Type	Livestock: Number	Other: Type	Other: Value	Corn surplus or deficit (bu.)	Disposal forage (tons)	Limiting resources	Discounted net returns†
1	$ 6,203	A A B	M_1 C_{11} C_{11}	31.0 17.8 94.4	Two-litter hog system	18 litters	Family living	$3,697	+ 5,972	68.6	Land A Land B Capital	$5,207
2	$10,293	A A B B	M_2 O_{11} C_{21} O_{11}	31.0 17.8 59.8 34.6	Deferred-fed calves two-litter hog system	35 head 20 litters	Family living	$3,697	+ 1,305	0	Land A Land B Hog building space Forage feed	$4,968
3	$13,929	A A B B B	C_{12} M_1 O_{12} M_1 C_3	31.0 17.8 26.0 34.6 33.8	Deferred-fed calves two-litter hog system	57 head 20 litters	Family living	$3,697	− 445	11.6	Land A Land B Hog building space July-Nov. labor	$5,226
4	$13,252	A A B B B	C_{22} M_2 M_1 C_{11} O_{13}	31.0 17.8 26.0 34.6 33.8	Deferred-fed calves two-litter hog system	56 head 20 litters	Family living	$3,697	− 98	0	Land A Land B Hog building space Forage feed July-Nov. labor	$5,410
5	$14,297	A A B B B	C_3 M_3 C_{11} C_{21} M_1	31.0 17.8 26.0 34.6 33.8	Deferred-fed calves two-litter hog system	56 head 20 litters	Family living	$3,697	+ 881	7.5	Land A Land B Hog building space July-Nov. labor	$5,861

* Capital available for crop and livestock production after family living expenses have been met. For this plan, it is supposed that family living in year 1 must come from the original $9,900. Hence, only $6,203 is available for farm production.

† Net returns after living expenses have been met. For this plan, it is assumed that family living in year 1 must come from the original capital of $9,900. Only $6,203 is available in year 1. Living expense, however, is not subtracted from discounted net return in year 1 for this plan. Living expense has been subtracted from net return in other years to indicate the income surplus which might be transferred to the following years.

used for household consumption in year 1. Hence, only $6,203 in operating capital is available for farm production in this year. Terracing and contouring costs for the 5-year period are included in the capital requirements of crops in this year. Therefore, crop capital requirements are higher for this situation than for the other two situations in year 1. If land is to be used for crop production during the 5-year period, and under this situation, it must be both cropped and terraced and contoured in year 1.

As in the previous plans, hogs are more profitable than deferred-fed calves. Because of the additional cost of terracing and contouring, however, the limited capital specifies that no cattle and only 18 litters of pigs can be produced in year 1 of Plan III. In plans I and II, 20 litters of pigs plus some cattle were included in the plans for year 1. After withdrawals for family living, capital is so restrictive that it is first allocated to terracing and contouring and crop production, with the remaining funds used for hog production in Plan III.

Crop production, corresponding to the optimum 5-year plan for Situation III includes 112 acres of corn and 31 acres of hay. All of Land B and some of Land A are used for corn production. A surplus of 68.6 tons of hay and 5,972 bushels of corn are produced but are not needed for the limited livestock enterprise. The surplus hay becomes available mostly in the last half of year 1 and is utilized mainly in year 2. The surplus corn is sold for cash in year 1. At first glance, it might appear unprofitable to produce so much surplus hay in the first year (i.e., more land could have been used for corn production and less for hay production). Crop and livestock production is interrelated within any one year and between years, however, and this interrelationship affects the optimum plan for each year of the 5-year period. Thus, surplus hay production in year 1 is necessary to allow increased livestock production in subsequent years, as part of a 5-year sequence for maximizing discounted net income. Any other plan would result in a lower discounted net return for the 5-year period. Net return in year 1, excluding family living, amounts to $5,207. Crop and livestock production in year 1, along with the capital invested in year 1 production but recovered within the same year, provides $14,116 of operating capital for farm production and household consumption in year 2.

YEAR 2

In year 2, $10,293 is available and used for crop and livestock production. Because more capital is available in year 2, the livestock enterprise can be expanded. Capital, beyond that needed for crop production, is first allocated to the more profitable hog production, with the remaining funds used for cattle. Hog production is increased from 18 litters in year 1 to 20 litters in year 2, and cattle production is increased from zero to 35 head. The corresponding cropping system includes 60 acres of corn, 52 acres of oats and 31 acres of hay. Oats are substituted for corn in year 2 to allow increased forage acreage and livestock production in years 3, 4 and 5. Even with the decreased corn average and the increased livestock numbers, however, 1,305 bushels of corn are not required for feed in year 2 and are sold for cash. Crop and livestock production are limited by Land A, Land B, hog building space and forage feed. Hogs are only limited by building space. It should be noted that capital is nonlimitational in year 2; surplus capital is available for expanded production. The other restrictions, however, cause this capital to go unused. Had more forage feed been available, surplus capital would have been used for expanded cattle production and would have become limitational. Discounted net return in year 2, after subtracting family living expense, is $4,968.

YEAR 3

In year 3, $13,929 is used for family living and crop and livestock production. Hay acreage is increased by 21 acres over year 2. Because more forage and capital are available, cattle production is increased by 22 head. Hog production is limited to the maximum number allowed by building space. Crops produced in year 3 include 65 acres of corn, 52 acres of hay and 26 acres of oats. More hay is produced than is used for livestock in year 3, but the surplus is utilized in year 4. The larger hay acreage and increased cattle numbers in year 3 necessitate purchase of 445 bushels of corn to meet feed requirements. The expanded livestock enterprise also causes July-November labor to be restrictive in cattle production. Thus, cattle numbers are restricted by July-November labor rather than by capital, feed grain, forage feed or building space. Land A and Land B limit further crop production, and hog building space, further hog production. Discounted net return, after subtracting family living expense, is $5,226.

YEAR 4

In year 4, slightly less operating capital is used. Because of the large forage production in year 3, and since some of this is carried into the next year, forage acreage in year 4 can be decreased without causing a decrease in livestock numbers. In year 4, 65.6 acres of corn, 43.8 acres of forage and 33.8 acres of oats are produced. Corresponding livestock production includes 20 litters of pigs and 56 head of deferred-fed calves. Because of the cropping restrictions previously discussed, it is necessary to substitute oat acres for corn acres in year 4. The oats provide a nurse crop for hay production in year 5. Because oats are substituted for corn in year 4, not enough feed grain is produced to meet the livestock feed requirements. Therefore, 98 bushels of corn are purchased for feed. In addition, only sufficient forage feed is available, from production in the 2 overlapping feed years, to produce the 20 litters of pigs and 56 head of deferred-fed calves. Limiting resources in year 4 are Land A, Land B, hog building space, forage feed and July-November labor. Further crop production is limited by Land A and Land B, hog production by building space and cattle production by forage feed and July-November labor. Capital again is not a limitational resource, and there is a surplus of $5,630 for use in paying fixed costs at the beginning of the year, for paying off indebtedness, for expanded consumption or for outside investment. Discounted net return in year 4, after subtracting family living expenses, is $5,410.

YEAR 5

In year 5, $17,994 is available for family living and farm production. Of this amount, $3,697 is required for household consumption. Exactly the same number of deferred-fed calves and litters of pigs are included in the plan for year 5 as in the plan for year 4. Therefore, the two plans differ only in crop enterprises. In year 5, only corn and hay are grown because oats are not required as a nurse crop. Since crop production in any one year is partly interdependent with crop and livestock production in previous years, more forage (7.5 tons) is produced than is required by the livestock within the calendar year. The limiting resources in year 5 are Land A, Land B, hog building space and July-November labor. Hog space restricts hog production, while labor restricts cattle numbers. The land restriction, of course, limits further crop production. Discounted net return, after subtracting family living expense, is $5,861. Discounted net return is higher for year 5 than for year 4, even though income is discounted for a longer time, because of increased corn acreage. Surplus capital in year 5 amounts to $5,192 and could be used for the purpose previously mentioned.

Over the 5-year period, discounted net returns, after subtracting family living expense, total $22,975 in Plan III, an increase of $6,224 over Plan I, and $1,684 over Plan II. Yields are higher in this plan than in plans I and II because of the inclusion of crop fertilization and terracing and contouring. As a result of the higher yields in Plan III, fewer acres of forage are required per unit of livestock produced. In year 1, because of the additional cost of contouring and terracing and because cattle are less profitable than hogs, only crops and hogs are produced. In years 2, 3, 4 and 5, increased capital allows cattle production to be included in the optimum 5-year plan. In Plan III, as in plans I and II, the farm firm and household are treated as a single economic unit. The plan thus specifies the best combination of crops and livestock over a 5-year period after an allowance has been made for family living.

While the addition to discounted net income is slight under Situation III, this increment is possible even with less capital than in Situation II. Total operating capital used over the 5 years is $60,429 under Situation II and $57,974 under Situation III. Some of the capital is used for terracing under Situation III. This practice allows somewhat higher yields and thus contributes to income but restricts livestock investment through the limitations in labor supply. The capital added for terracing is less than the reduction made for livestock. On the other hand, the income added through terracing is less than the income reduction brought about by a slight reduction in livestock enterprises, when Situation III is compared with Situation II.

In comparing plans for the three situations, it is obvious that a much larger increase in income for the farm as a whole—given the restrictions on labor and other resources—is attributed to the fertilization practices than to the mechanical conservation practices. Addition of fertilizer, under Situation II as compared with Situation I, allows a shift of farm organization in more profitable directions, and less forage need be produced as a means of providing fertility in corn land and for cattle feeding. In years where capital limits production, it allows more corn for cash sale. (Corn is a more profitable cash crop than hay. While the mechanical erosion control practices added in Situation III cause income to increase somewhat, the income increment for the farm as a whole, within the resource restraints, is not nearly as great as when fertilization is added under Situation II.) Return per $1 of investment, however, considering shifts in organization of the entire farm, is greatest for the terracing effects under Situation III as compared with Situation II.

IMPLICATIONS OF FINDINGS FOR CONSERVATION PLANNING

The optimum plans shown in this study point up important considerations for future conservation planning. The same crop and livestock plan should not be recommended each year if profit maximization over time is the relevant goal. This is so because, in a long-run conservation plan, the years are interrelated, and changes in resource structure over time, particularly in the accumulation of capital and in soil fertility, cause a different plan to be optimum each year until an equilibrium plan is attained. Thus, production possibilities in the plans for the first year are dependent on those which exist in the second, third, fourth and fifth years, and vice versa.

Although the current study presents plans for only a 5-year period, these same interyear planning considerations would hold true for conservation plans for a 10- or 20-year period; crop and livestock activities in each of the 10 or 20 years should be interrelated. Clearly, there is a need for more research to provide conservation recommendations which outline step-by-step plans which do give consideration to the level of capital and soil fertility over a multiyear period. Also, since expenditures for household consumption on most farms draw from the same cash fund as those for farm production, family living expenditures need to be taken into consideration in time plans. This becomes especially important when making conservation recommendations to farmers who are short on capital. For example, profits may be maximized over a period of years by the production of cash crops in the early years of the conservation plan in order to meet family living requirements and build up a capital fund which will allow the introduction of conservation practices during the latter years of the plan. Clearly, conservation recommendations need to consider the farmer's capital position and the cash requirements of household consumption if the conservation plan is to be adopted. There is no point, for example, in recommending the planting of forages or terracing and contouring of cropland if the farmer has very few livestock and needs most of his capital for family living, even though these practices would increase his income in the long run.

Likewise, changes in soil productivity over time need to be considered. Profits may be maximized over a period of years by using the "most productive" land for grain production while the "poor" land is used for hay production in the early years of the plan, thus building up the productivity of the "poor" land, and at the same time, decreasing income as little as possible in the early years of adoption of the conservation plan. In the latter years, because of previous forage crops, the "poor" land will be more productive and can be used for grain production, while forage can be grown on the "most productive" land to maintain its productivity. (In the optimum plans for the 160-acre farm, tables 11, 12 and 13, the "most productive" land was used primarily for grain production in the early years, while the "least productive" land was used for hay production.) In addition, if livestock are to be included in the conservation plan, they should be "fitted into" the plan in such a way as to use the forage produced.

Finally, because size of farm and the amount of available resources vary between farms, different conservation plans should be recommended for different farms. This, of course, would be done if the goal of the long-run conservation plan was profit maximization, because it would consider each of the years to be interrelated and would treat the farm firm and farm household as an economic unit.

REFINED DYNAMIC PROGRAMMING MODELS

Dynamic programming procedures have been used in this study to define a sequence of yearly plans which provide an optimum over-all plan for a 5-year period. The model employed is relatively simple, but in this respect parallels, in terms of refinement, those which farmers might use. It has illustrated, however, that the empirical procedure can be used to provide dynamic plans for farmers. As a dynamic planning mechanism it might have use in farm plans tested and recommended by such organizations as the Soil Conservation Service, the Extension Service and the Farmers Home Administration, as well as greater application in research. Use of this study for development of the model used, the construction of programming routines for the IBM-650 and the elimination of problems of coding and other empirical steps should extend the magnitude of farm problems which can be analyzed by dynamic programming approaches.

The high cost of computations at the time this study was initiated caused several refinements in the model to be eliminated. Improvement in programming routines and coding procedures, however, now would make them possible. Some improvements which might be included in future studies are outlined here. The length of the planning period used was only 5 years. Some specialists on farm decision making suggest that crop and livestock plans do not, because of uncertainty considerations, extend beyond a time span of this length. Five years were used in this study largely because of the limitations of computor capacity. In subsequent studies, it should be possible to extend the time period to 10 or 20 years. Plans should be made extending over periods of varying lengths so that plans can be specified for farmers with planning horizons of varying length before them. Similarly, optimum plans should be made using different discount rates.

Household activities included in this study were relatively simple. Living expenses were "exogenous" to the determination of yearly plans, since they represented a restriction which had to be met "exactly" in each year. Farm plans were, in the competition for capital, different from what they would have been in the absence of the consumption activities, especially in the early years of situations I and II. The combination and magnitude of household activities in the over-all plan, however, were not determined by the nature of the farm plan. (The household activities only helped determine the nature of the farm plan.) Nevertheless, procedures can be used which allow both household and farm activities in the over-all plan to be interdependent with each other.

A simple way to accomplish this end might be to include several consumption activities in each year. One of these might represent basic living needs and would constitute a restriction to be met in each year. Of course, it need not be a constant amount each year, but might well change between years, depending on the composition of the family and the ages of members. It would still represent a restriction affecting the farm plan, but would not be affected by the farm plan. Additional consumption activities for each year, however, with prices representing family values attached to them, might be added with restrictions of relevant magnitude. Whether or not these latter consumption activities are included in the final plan would depend on their "interaction" with the farming activities. For example, if the "price" attached to one of these consumption activities was 4 percent on each dollar of funds used, scarce capital would be allocated to production when a farm enterprise returned more than this amount. But if the return from farm enterprises were less than 4 percent, funds would be allocated to the consumption activity. Other consumption activities and related restrictions could be added, and each could have a different value attached to it. While this procedure does not allow incorporation of an indifference curve into the analysis, it does provide a simple substitute for the more complicated procedures of nonlinear programming.

The magnitude of the time-programming problem analyzed in this study was limited by the capacity of the calculator available. With greater machine capacity, more years and activities could be included in the model. The time period programmed should be long enough that it begins to "suggest equilibrium" in yearly plans. While subsequent programming studies have indicated that this might be attained in 5 years under farm ownership and ample capital, the current study included only this number of years. Hence, we have no basis for knowing the degree of stability in the plan at the end of the period. For example. if more years were included, would the combination of crop and livestock enterprises be about the same as those indicated, or would they continue to shift toward another pattern?

The model employed here is one of comparative statics, in the sense that time is involved, but future quantities are assumed to be known with certainty. Use of stochastic programming models would overcome the limitations of this assumption but are too complex, in a computational sense, to allow analysis of meaningful dynamic problems with machine facilities now available.

The objective function used in this study was one of maximizing discounted net returns, within the framework outlined earlier. For some conservation problems, a relevant objective function would be maximization of capital value over the planning period. In this case, the value of capital (including appreciation in assets through such things as greater investment in livestock, increased cash balance and build-up in soil fertility, as well as income withdrawn in individual years) would be discounted back to the present. This system would give added emphasis to cropping sequences which increase the productivity of land over the planning period.

This type of objective function, however, may not be appropriate for periods of time as short as the one considered here, especially where the operator plans to continue farming at the end of the period. The 5-year time period was used not only because it gave rise to a programming model taxing available computational facilities, but also because it was thought that farmers in western Iowa generally do not plan crops and livestock beyond a 5-year period. In fact, some persons acquainted with agriculture would argue that farmers are prone to "break the planning chain" after each individual year.

To the extent that farmers do consider profit maximization over a single year ahead, this sequence of plans would be derived best by making up an independent optimum plan for each year, with the resources available in the subsequent year depending on those supplied or "left over," somewhat as "windfalls," from the previous year's plan. It would appear that a more appropriate sequence of plans would be one which does cause all years to be interdependent, at least over a limited time span. The planning procedure may be of a "moving average" nature. A plan may be made up in year t for the next 5 years, but revised for 5 more years after results from the first year are known in year t+1, with the same procedure followed in years t+2, t+3, . . ., t+n. Models are being developed to allow these changes in planning and will be applied in later studies.