Resource Returns and Productivity Coefficients in Selected Farming Areas of Iowa, Montana and Alabama

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RESEARCH BULLETIN 425 APRIL, 1955 AMES, IOWA

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FOREWORD

This study has been made possible through a Faculty Research Fellowship awarded to the senior author by the Social Science Research Council. It is an attempt to prepare some benchmarks in resource returns and efficiency for selected farming areas of the United States. With changes in the national economy and various stages of equilibrium and disequilibrium attained in agriculture at different times, this study provides basic data for comparisons of progress in later years. The year 1950 was selected as a benchmark year for study since, although weather variations are of some importance, it represents a point in peacetime production wherein agriculture had several previous years to adjust to a full-employment economy and to add capital and release labor in line with the favorable employment opportunities and income levels. This study is a cooperative project with the Alabama Polytechnic Institute and Montana State College. John Snare and Chester B. Baker of these two institutions are responsible for collection of data in Alabama and Montana, respectively. The statistical analysis and the overall summary have been prepared by the authors listed on this study. Professors Baker and Snare will prepare forthcoming reports and interpretations for Montana and Alabama.

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SUMMARY

1. This study analyzes resource returns in four selected farming areas of the United States where the quantity and proportions of resources used and the commodities produced are quite different. The sample areas include the Alabama Piedmont, northern Iowa, a dry-land wheat area of Montana and southern Iowa. From farm samples in each area, production function and marginal resource productivities have been derived for different classes of inputs. Other computational procedures, such as estimation of resource returns through tabular and residual procedures, have also been employed.

2. The production functions derived were as follows:

	Crops	*Livestock*
Mont.:	$Y_c = 4.85D_c^{0.503}L_c^{0.039}C_c^{0.580}$	$Y_l = 1.89L_l^{0.084}C_l^{0.937}$
N. Iowa:	$Y_c = 6.01D_c^{0.912}L_c^{0.076}C_c^{0.165}$	$Y_l = 2.29L_l^{0.077}C_l^{0.907}$
S. Iowa:	$Y_c = 5.23D_c^{0.795}L_c^{0.087}C_c^{0.393}$	$Y_l = 1.14L_l^{0.117}C_l^{0.982}$
Ala.:	$Y_c = 14.13D_c^{0.385}L_c^{0.319}C_c^{0.462}$	$Y_l = 5.46L_l^{0.233}C_l^{0.743}$

For these functions, Y refers to output in dollars, D refers to land in acres, L refers to labor in months and C refers to capital in dollars.

3. Marginal resource productivities differ greatly among the four acres. For crops, with inputs at the arithmetic mean, the marginal return per month of labor was $38.73 in Alabama, $45.98 in southern Iowa, $57.33 in Montana and $67.09 in northern Iowa. Marginal productivity of capital, per $1 input in the four areas was, respectively, $1.16, $1.26, $2.23 and $0.64 while the productivities per acre of land were $20.48, $31.61, $10.32 and $45.91, respectively. The marginal productivities of labor used on livestock were $83.18 in Alabama, $148.46 in southern Iowa, $106.86 in Montana and $130.65 in northern Iowa. Productivity of livestock capital in the four areas was, respectively, $0.92, $1.20, $1.20 and $1.05, with all inputs at the mean.

4. In terms of the estimates of this study, differentials in resource productivity are explained in the quantities and proportions of resources used. Alabama farms averaged only 23.8 acres of cropland and used 10.4 months of labor and $553 of capital services for crops. Montana farms included 975 acres and used 13.7 months of labor and $5,207 in capital services. The same figures for northern Iowa are 167 acres, 9.5 months and $2,168; for southern Iowa, they are 115 acres, 8.7 months and $1,420. An increase in capital inputs for southern Iowa to the mean of the Montana sample has the predicted effect of raising marginal productivity of labor used on crops to $204.42; a fourfold increase in capital on crops in Alabama would increase marginal labor productivity to $182.80.

5. Labor services constituted 27 percent of all inputs for farms in Montana, 20 percent in northern Iowa, 27.4 percent in southern Iowa and 51 percent in Alabama. Land represented only 7 percent of all crop inputs in Alabama, 16.2 percent in southern Iowa, 24.7 percent in Montana and 34.8 percent in northern Iowa.

Resource Returns and Productivity Coefficients in Selected Farming Areas of Iowa, Montana and Alabama[1]

BY EARL O. HEADY AND RUSSELL SHAW

OBJECTIVES AND METHOD OF STUDY

This study in production economics is one of a series dealing with economic efficiency in agriculture. It is designed to measure and compare certain aspects of efficiency in selected agricultural areas. The investigation deals only with tangible measures of economic efficiency and resource productivity; it does not relate to intangible and subjective aspects of farming such as the values which farm persons may attach to "agriculture living" *per se*. While certain of these quantities are important, they are not subject to easy measurement and likely have no great importance for the wide differences observed in the study. While the investigation is aimed particularly at some inter-regional productivity and efficiency comparisons, it gives insight into intra-regional and even intra-farm productivity conditions.

The central objective of the investigation is to measure the value productivity of resources and their services used in different farming regions and to predict, within the limitations of the data and methods, the effect of varying combinations and quantities of resources on the value of the product produced. The study is designed to be of value both to individual farm decisions and national policies.

From the standpoint of the individual, the study indicates (1) the income to be expected when different quantities and combinations of resources are used at a particular geographic location and (2) the gains or sacrifices which might attend movement of the families' resources between producing regions. From the standpoint of national programs and policy, the study indicates (1) the extent of differentials in resource productivity between farms in given agricultural areas and between agricultural areas and (2) certain causes, in as much as these are explained in the kinds and quantities of resources used, of differentials in resource productivity.

The figures of later sections provide the basis for certain (1) inter-area, (2) intra-area and inter-farm, (3) intra-area and intra-farm and (4) intra-farm and inter-product comparisons of resource productivities. They show the returns from resources and predicted contribution of specific resources to farm production when rather broad categories of resources are used in varying quantities and proportions. The objectives of this study end at this point. More detailed studies, involving budgeting procedures and highly stratified samples, are necessary to specify the exact forms that resources should take and the techniques of production which should be employed. Finally, the study has methodological aspects: It compares different inferences which might be made from productivity estimates based on alternative empirical procedures.

SOURCE OF DATA

The main statistics of this study are based on random samples of farms in four agricultural areas of the United States. Samples were drawn in 1951 for the Piedmont area of Alabama, north-central Iowa, southern Iowa and the dry-land wheat area of Montana. Samples of the two Iowa areas also are available for the year 1939. While data from many regions of the United States would have been desirable, limited funds necessitated restriction of the study to the areas mentioned. However, these give some interesting contrasts.

The Alabama Piedmont area represents a group of farms operated mainly by share-croppers where livestock is relatively unimportant and cotton is the main cash crop. It is an area of small farms where emphasis is on labor as the important resource used in production.

Northern Iowa can be identified with the highly productive central Corn Belt region where somewhat over one-half the farms are operated by owners while the rest are operated by regular tenants. The agriculture is diversified in terms of crops and livestock production although more cash income is from livestock than from crops. The amount of capital per worker is relatively high, and the farms are highly mechanized.

Southern Iowa is somewhat similar to southern areas in other Corn Belt states. Its soil is less productive and agriculture revolves largely around diversified livestock enterprises while acres per farm and the capital and income per worker are considerably less than in the central Corn Belt. The Montana dry-land area represents, in the main part, a cash-grain farming system where the amount of capital per worker is large. The capital investment for livestock is great, with emphasis on beef production, on those farms with an acreage of pasture. The main crop of the area is wheat, produced under summer-fallow methods. Farms are large in acreage with one-half the land typically in wheat. Production

[1]Project 1135, Iowa Agricultural Experiment Station.

is highly mechanized, and, as in northern Iowa, capital services represent by far the greatest input of all resources; land and labor provide less than one-third of the annual value of inputs used for production.

In addition to the 1950 data for the four areas, similar data from the two Iowa areas for 1939 have also been analyzed. Generally, these data are not discussed in the text since the nature of the information was not as refined or as exact as that for 1950. Certain estimates for the two Iowa areas with their relevant statistics are included mainly in Appendix D.

The Sample Areas

montana, 1950[2]

The sample area in Montana is shown in fig. 1. It is composed of two geographic areas, one in north-central Montana and the other in the northeast corner of the state. Production conditions for winter and spring wheat were considered to be more homogeneous than if a single contiguous area had been selected. The boundaries, except for some minor alterations, are those defined by the Bureau of Agricultural Economics for type of farming areas III, IV, VI and VII.[3]

northern and southern iowa, 1950

The two areas in Iowa were delineated along county lines using soil, type of farming, income and other supplementary data as guides. The areas correspond approximately to other designations of the "cash grain" (north) and "southern pasture" (south) areas of Iowa. They are indicated in fig. 2. The samples were drawn by the Statistical Laboratory, Iowa State College, using their designation of segments classified as open country. Farms of less than 30 acres were excluded from the universe.

alabama, 1950

The Alabama sample was drawn from the Piedmont area in Alabama. The approximate area is indicated in fig. 3. The sample was drawn by the Statistical Laboratory, North Carolina State College. The original sample was drawn for a general farm management and tenure study and included 330 farms. Data for the current study were included in the questionnaire for the original study, where added information was needed. This was a random sample of all farms in the area including owner-operator, tenant, share cropper and multiple unit farms. To reduce the number from the original sample (330 eligible) to a number better suited

[2]Hereafter the six samples are designated as Montana, 1950, northern Iowa, 1950, southern Iowa, 1950, Alabama, 1950, northern Iowa, 1939 and southern Iowa, 1939. They will be considered usually in this order.

[3]U.S. Department of Agriculture, Bureau of Agricultural Economics. Farm adjustments in Montana. (Graphic supplement). U.S. Dept. Agr., Washington, D. C. July 1940.

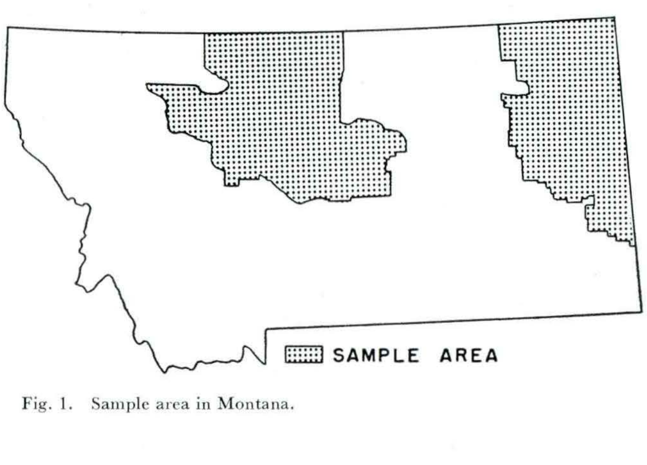

Fig. 1. Sample area in Montana.

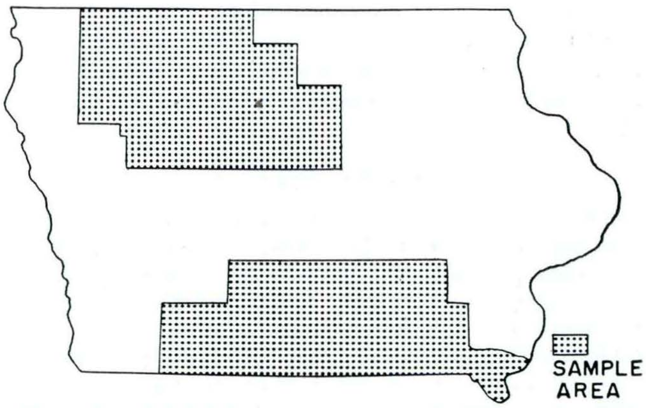

Fig. 2. Sample areas in Iowa.

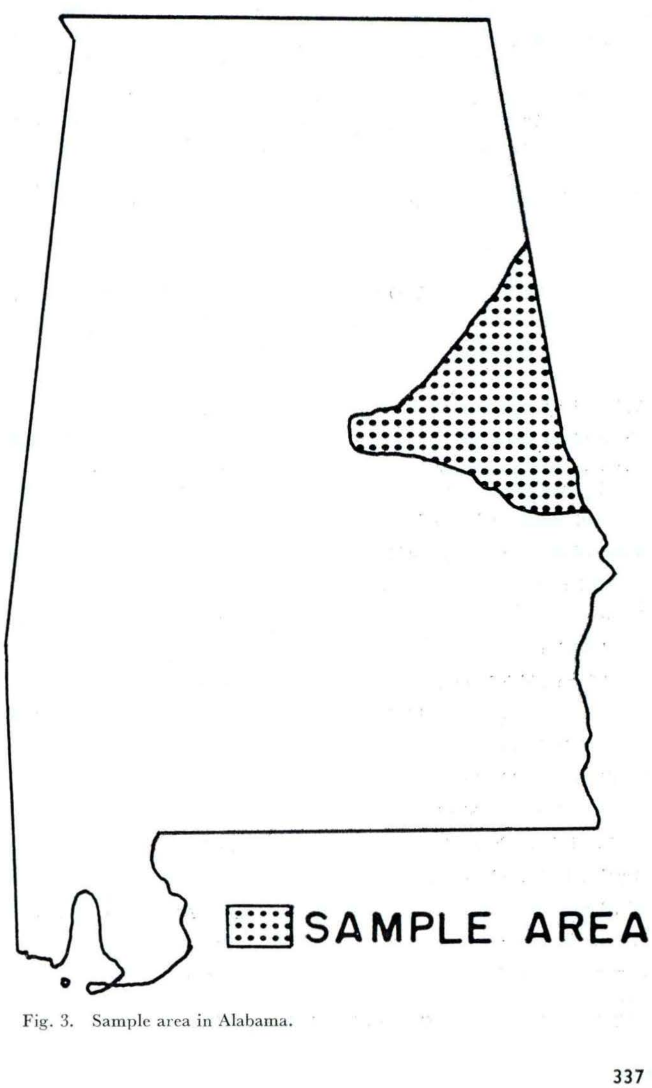

Fig. 3. Sample area in Alabama.

for this study, every second schedule (starting at number 2) was taken as a subsample. This procedure was used to obtain approximately the same distribution of the sample over the area but to reduce the sampling rate.

Data Enumeration

The 1950 data were enumerated in 1951 by teams working in each area. The usual farm record schedule was used and all data pertaining to production and resource use was obtained. The schedule was designed to furnish (1) the dollar value of output and (2) the quantities of the various resources employed in producing that output. These items included crop production, sales, purchases and inventories; livestock sales, purchases and inventories; miscellaneous receipts; machinery and equipment inventories, repairs, custom work, seeds, feeds, fuel, fertilizer, sprays, labor (family and hired) and other data necessary in computing input-output relationships.

CHARACTERISTICS OF FARMS IN THE AREAS STUDIED

Farms in the four sample areas differ greatly in the size of the total product and the kinds and quantities of resources employed. Census data providing descriptive characteristics of the farm population in the sample counties are given in table 1. The Montana and northern Iowa farms, respectively, produced a 1949 product, in value terms, 70 and 90 percent greater than southern Iowa farms; they produced value products more than 1,000 percent greater than the average of all Alabama farms in the sample counties. In both Iowa areas, the value of livestock sales was greater than the value of crops harvested; in Montana and Alabama, crops harvested exceeded livestock sales by 50 and 34 percent, respectively.

One reason for differences between areas in value of product produced is apparent in the figures showing quantities of resources employed in the four areas. As an average for all farms in the sample counties, land inputs in Montana (measured in acres) were roughly 10 times those of Iowa and 15 times those of Alabama. On the other hand, labor inputs (measured in man-years) were only slightly higher in northern Iowa than in Montana or southern Iowa but 20 percent higher than in Alabama. With labor inputs nearly as great but with capital inputs considerably smaller in the case of southern Iowa, and greatly smaller in the case of Alabama, it might be expected that (1) capital productivity and returns would be high and (2) labor productivity and returns would be low in these two areas.

Sample Data

Statistics characterizing the sample farms in the four areas are given in table 2. These statistics are averages for all farms in the respective samples. They are expected to differ from the census data of table 1 for these reasons: (1) The data refer to the production year 1950 while census data are for 1949; variations in weather and yields explain some of the differences, particularly for crops. (2) The samples were designed to include only commercial farms; units under 30 acres were excluded in northern and southern Iowa while part-time and subsistence units were not sampled in Alabama (with the exceptions mentioned elsewhere). Since all farms by census definition (any farm over 25 acres in size or having sales of $100 or more) are smaller than the more-

TABLE 1. SPECIFIED CHARACTERISTICS OF ALL FARMS IN THE FOUR SAMPLE AREAS, CENSUS AVERAGE PER FARM, 1949.*

Item	Unit	Montana	Northern Iowa	Southern Iowa	Alabama
Value of all crops harvested	$	6,707	6,294	3,394	777
Value of all crops sold	$	5,056	2,710	888	401
Value of all livestock and livestock products sold	$	3,421	7,013	4,171	298
Value of forest products sold	$	5	1	3	51
Value of all farm products sold	$	8,482	9,724	5,062	750
Selected row crops	acre	8.5	87.1	41.8	10.4
Cotton	acre	—	—	—	5.5
Small grains	acre	322.9	43.6	23.1	1.1
Land from which hay was cut	acre	40.1	11.6	15.6	1.6
Pasture and crops not specified	acre	1,392.1	39.0	87.2	94.5
All land in farms	acre	1,763.6	181.3	167.7	113.1
Family and/or hired workers	no.	1.5	1.6	1.6	1.4
Percent labor force hired	percent	18.1	13.1	7.9	8.2
Feed and livestock purchased	$	871	2,995	1,498	174
Repairs, fuel, seeds, etc.	$	1,789	1,263	671	83
Hired wage rate	$ per year	2,619	1,843	1,568	510
Percent farms rented	percent	16.09	49.07	25.58	34.86
Percent commercial farms	percent	93.4	96.3	87.1	38.6

*Tabulated from U. S. Census of Agriculture, 1950.

TABLE 2. SPECIFIED CHARACTERISTICS OF SAMPLE FARMS IN THE FOUR AREAS, SAMPLE AVERAGE PER FARM, 1950.*

Item	Unit	Montana	Northern Iowa	Southern Iowa	Alabama
Total production†	$	30,634	22,718	14,339	2,734
Crop production†	$	21,752	8,971	5,272	1,398
Livestock production	$	8,883	13,747	9,067	1,336
Cropland	acre	975	176.7	123.2	32.3
Pasture land	acre	1,350	26.0	70.8	15.1
Total land	acre	2,325	202.7	194.0	47.4‡
Labor on crops	mo.	13.7	9.4	8.7	10.4
Value of land services	$	2,994	2,175	983	123
Machine services	$	4,026	1,598	1,044	275
Crop services	$	1,181	570	377	278
All crop capital services	$	8,201	4,344	2,403	677
Value of crop labor	$	3,133	1,803	1,699	1,044
All crop services	$	11,334	6,147	4,102	1,721
Feed	$	2,400	6,616	4,542	597
Livestock input	$	3,946	5,260	2,758	359
Other livestock services	$	193	491	315	61
All livestock capital services	$	6,540	12,366	7,614	1,017
Labor on livestock	mo.	6.6	8.1	7.4	3.5
Value of livestock labor	$	1,499	1,549	1,445	353
All livestock services	$	8,038	13,915	9,059	1,370
Land investment	$	59,876	43,503	19,659	2,463
Machine investment	$	13,010	5,642	3,656	369
Improvement investment; crops	$	4,822	§	§	§
Total investment on crop production	$	77,709	49,145	23,315	2,832
Livestock investment	$	9,516	6,168	5,268	743
Improvement investment; livestock	$	3,172	6,058	4,153	534
Total livestock investment	$	12,688	12,226	9,421	1,277
Total all investment	$	90,396	61,371	32,736	4,109
Resource service inputs:					
Value all capital services used	$	14,741	16,710	10,018	1,694
Value all labor services used	$	4,632	3,352	3,144	1,396
Value all services used	$	19,372	20,062	13,162	3,090
Income:					
Residual over all costs, crops**	$	10,418	2,824	1,170	-322
Residual over all costs, livestock**	$	845	-168	8	-34
Residual over all costs, total**	$	11,263	2,656	1,178	-356

*Tabulated from sample data; for differences in areas and definitions see text.

†Includes value of pasture.

‡Does not include waste, woods pastured and woods not pastured; all additional land in farms was 108.0 acres.

§None allocated to crops but entirely to livestock production and storage activity.

**Computed by subtracting the value of all resource services (cost of feed, seed, repairs, fertilizer and other annual expenses, depreciation on buildings, machinery, livestock and rental value of land and market wage rate for all labor) from total value of production (including sales, home-used and inventory increases).

nearly commercial farms enumerated in the sample, the per-farm items of table 1 are considerably smaller than those in table 2. However, the same general differences between areas in resource-product relationships are reflected in the sample as in the census data. The value of the total crop and livestock product in 1950, a year of good wheat yields, was greatest in Montana with an average of $30,634 per farm. The $22,718 of northern Iowa was 60 percent greater than the per-farm output in southern Iowa and 732 percent greater than in Alabama. The greatest proportion of the total value product came from crops in Montana and from livestock in both Iowa areas. The contribution of crops and livestock was about equal in Alabama.

While input of land services, measured in acres, was greatest for crops in Montana, the value of land services, relative to the total of all crop inputs, was greatest in northern Iowa (see table 3). Montana, southern Iowa and Alabama followed in order, with land inputs measured in rental values for this resource. In relative terms, labor was the major input for crops in Alabama. Capital, with relatively large outlays for fertilizer was second in importance and land inputs were less than 10 percent of the total for crops. The high proportion of labor inputs for Alabama stems from (1) the type of main crop, cotton, with high labor requirements and (2) the small amount of capital used relative to labor resources. Southern Iowa has a relatively greater proportion of its inputs for crops represented by labor than Montana and northern Iowa for the second reason. Montana, an area highly mechanized for crop production, had the greatest proportion of total crop inputs represented by capital services. The major portion of inputs for livestock was represented by capital services for all areas.

In total farm production, Alabama and southern Iowa have less than 5 percent of their total inputs represented by land, and Alabama depends on labor for 45.2 percent of all resource services used. Nearly three-fourths of all inputs came from capital services in northern Iowa. Montana farms derived 71.3 percent of their income from crops while the Iowa areas obtained more than 60 percent of their income from livestock.

TABLE 3. RELATIVE INPUT OF DIFFERENT CATEGORIES OF RESOURCE SERVICES. SAMPLE AVERAGES, 1950.

Input category	Montana	Northern Iowa	Southern Iowa	Alabama
Inputs for crop production				
Land services (%)	26.4	35.4	24.0	7.2
Capital services (%)*	46.1	35.3	35.2	32.2
Labor services (%)	27.5	29.3	40.8	60.6
All crop services	100	100	100	100
Inputs for livestock production				
Capital services (%)†	81.3	88.9	83.8	74.2
Labor services (%)	18.7	11.1	16.2	25.8
All livestock services	100	100	100	100
Inputs for crops and livestock				
Land services (%)	15.5	10.8	3.9	4.4
Capital services (%)‡	60.6	72.4	69.0	50.4
Labor services (%)	23.9	16.8	27.1	45.2
All resource services	100	100	100	100
Sources of income				
Crops	71.3	39.5	36.8	51.1
Livestock	28.7	60.5	63.2	48.9
Crops and livestock	100	100	100	100

*Includes all machine and crop services.
†Includes feed, livestock and all other capital services.
‡Includes * and † above.

Investment and Residual Returns, Sample Averages

The contrast in capital investment per farm in both crops and livestock and in all forms of assets is clearly evident in table 2. Capital investment per farm was almost 50 percent greater in Montana than in northern Iowa, while northern Iowa was nearly double southern Iowa, and southern Iowa had nearly eight times as much capital per farm as Alabama. In capital per month of labor used, Montana farms had $4,404, northern Iowa, $3,507, southern Iowa, $2,033 and Alabama, $296. These differences in resource productivity might be expected among areas: labor returns are expected to be low and capital returns to be high under these capital/labor ratios.

The last three lines of table 2 show the magnitude of the residual per farm of product sales over the value of resource inputs. They can be looked upon as net profits per farm, above the cost of productive resource services. The residual figures show the returns above all costs (including a wage rate for operator and family labor and interest on the capital owned by the farmer but excluding taxes) going to the farm. They are not divided between farm owner and farm operator when the farm is rented.

A negative figure does not indicate zero returns. It does indicate, however, that if the family paid market prices on its own labor and capital, it would have had a loss. The value of productive services going into both crops or livestock exceeded the value of product produced for Alabama farms.

Livestock production, as an average, evidently "approximated competitive equilibrium" in all four areas. (By competitive equilibrium, we refer to the condition specified as the long-run, "bench-mark" or "stability conditions" suggested in economics; namely, the tendency for value of production to equal value of resource service inputs with certain restrictions in respect to relationships between resources, products and prices.) The surplus or deficit of value production relative to total value of resource inputs was at a maximum of 10 percent in Montana and a minimum of less than 1 percent in southern Iowa. This near-competitive equilibrium might have been expected in 1950 for livestock production; starting from completely new product demand and resource supply situations in 1945, the 5 years following the war likely gave sufficient time for prices (on the side of both products and factors) to approach a short-run equilibrium. A value of livestock product equal to the value of inputs does not, however, actually specify that "competitive equilibrium" has been attained. This condition might come about as farmers use too much of one resource and too little of another resource. Also, the average for the sample does not provide the basis for inferences to individual strata of farms in each area. As later sections show, differences between farms within an area can be very large.

Differences between value of output and value of resource inputs were much greater in all areas for crops (see bottom of table 2). Of course, fluctuations in weather can cause the production and return of any 1 year to differ greatly from the value of resource services used. This surplus of value of production over value of service inputs was greatest in Montana due especially to above-average wheat yields. The residual in production was almost 100 percent of the value of resource services used in crop production in Montana. It was around 46 percent in northern Iowa and 28 percent in southern Iowa; the deficit was 20 percent in Alabama. Not only did the Alabama farmers have a small amount of resources on which to earn a return, but also the return was so low that production did not cover explicit plus implicit resource costs. In a purely monetary sense, Alabama farmers who might have hired out all of their resources at market prices would have had greater incomes than were earned in the farming occupation. (Not all operators have these opportunities, however.) Too, some categories of farms in other areas are faced with the same situation. This will be illustrated later.

PRODUCTIVITY AND COMBINATION OF RESOURCES

On the following pages, various types of estimates have been prepared to suggest the nature of resource productivity in the various sample areas. These statistics involve different degrees of "refinement." Some involve arithmetic or tabular procedures of the conventional type used in the majority of studies which attempt to measure returns and productivity; others involve productivity figures derived from "formal" production functions or regression estimates. Both types of data are presented to (1) give a picture as complete as possible of resource combinations and resource returns in the several areas, (2) allow interpretation of the data by a greater number of persons, including those who more readily accept one or other type of estimate, (3) provide refinement where it is needed and a wider range of statistics where refinement is unnecessary and (4) point out the limitations of a particular method and the advantage of another where questions of logic and interpretation are required. The section below explains the basis of the production function estimates. More complete statistics on resource combination and average productivities are provided in later sections.

While the functions derived are not restricted to a single crop or livestock product, they are useful for estimates of the kind desired in this study. To the extent that they generally represent the "path of expansion" followed between products and techniques as farmers acquire more resources or the path which would be followed by present low-capital farms were they to acquire more information and resources, they serve the major objectives of this study.[4]

Regression Equations for Productivity Estimates

The regression equation or production function employed in deriving production coefficients (termed Cobb-Douglas in economic literature) is linear in the logs and is of the form below:

$$Y = \alpha X_1^{\beta_1} X_2^{\beta_2} X_3^{\beta_3} \ldots X_n^{\beta_n}$$

where Y refers to the value of output and X's refer to the inputs, or quantities used of the various resources. The β's are the regression coefficients for the equations in logarithmic form; they are the elasticities of production for the production functions or regression equations, in the form presented above, and singly indicate the percent increase in product for each 1-percent increase in input of the respective resources. The sums of the β's indicate the percentage by which the total value of product increases as all factors ($X_1, X_2, X_3 \ldots X_n$) are increased by 1 percent. Under the condition

$$\beta_1 + \beta_2 + \beta_3 \ldots + \beta_n = 1.0,$$

constant returns to scale hold true; a 1-percent increase in input results in a 1-percent increase in output, and constant productivity prevails as all resources are increased by the farm in constant proportions. If this sum is less than 1.0, diminishing returns to scale hold true, and marginal productivity declines as more of all resources is used, with proportions held constant; a sum greater than 1.0 indicates increasing returns to scale and increasing productivity. If the β or exponent for any one resource is less than 1.0, diminishing returns hold true; the productivity of the resource declines as more of it is used, with the quantity of other resources remaining fixed at some specified level. Regression coefficients or β values equal to or greater than 1.0 indicate constant and increasing returns to scale respectively of one factor, other factors remaining fixed in quantity.

The production function or regression equation outlined above can be used to estimate the marginal productivity of any one resource or of all resources taken together. Using three resources, X_1, X_2 and X_3, the marginal productivity of X_1 can be estimated as a derivative:

$$\frac{dY}{dX_1} = \beta_1 \left[\frac{\alpha X_1^{\beta_1} X_2^{\beta_2} X_3^{\beta_3}}{X_1} \right]$$

Here $\frac{dY}{dX_1}$ refers to the marginal product, the increase in value of output for each one-unit change in resource X_1, with other resources held constant.

Several production function equations and various groupings of resource inputs were tried for crops and livestock in each area. Of the three sets of functions estimated, the second one (II shown in Appendix A) was logically and statistically most acceptable. For these regression equations, three categories of resource inputs were used for crops — namely land services, capital (crop and machine) services and labor services. The two categories of resources used in the livestock equations included capital services and labor services. The classifications of variable (outputs and inputs) were as follows:

A. Crop functions in all areas:

Y_c is the value of crop production in the year. It includes the value of all crops produced on cropland whether sold, stored or used on the farm for feed, seed and home use.

D_c is the input of cropland services measured in acres. It has been computed, for later analysis, in dollar terms as the rental value of land used for crops. It does not include pasture land.

L_c is the input of labor services used on crops and is measured in months. It includes hired labor plus the labor by the operator and family members. (Local wage rates were used to compute the value of operator and family labor where it was needed for later analyses.)

C_c is the input of capital services used on crops, measured in dollar value. It includes seed, fertilizer, insecticides, seed treatment, tractor fuel, repairs, oil, grease, depreciation on machinery and all other capital items used directly or indirectly in producing crops.

B. Livestock functions in all areas:

Y_l is the output of livestock in the year, measured in dollars. It includes sales, home used products and inventory increases less purchases and inventory decreases for breeding stock. It also includes sales in the case of feeder cattle and sheep.

L_l is labor used on livestock measured in months. It includes operator, hired and family labor.

C_l is all capital inputs used for livestock measured in dollars. It includes the value of grains, hay, pasture, supplements and all other feeds; it includes livestock services represented by the depreciation on breeding stock and the purchase value of feeding stock plus veterinary costs, breeding fees and all similar items. It also includes the annual value of all building and equipment services used by livestock, computed as depreciation, repairs and similar items. (Details on other functions and resource classification are given in Appendix B.)

The production functions or regression equations estimated for use in the test are as follows: (These functions are those indicated as II in Appendix A.)

Crops:

Montana: $Y_c = 4.85\, D_c^{0.5032} L_c^{0.0394} C_c^{0.5804}$
Northern Iowa: $Y_c = 6.01\, D_c^{0.9124} L_c^{0.9756} C_c^{0.1647}$
Southern Iowa: $Y_c = 5.23\, D_c^{0.7948} L_c^{0.0875} C_c^{0.3930}$
Alabama: $Y_c = 14.13\, D_c^{0.3847} L_c^{0.3192} C_c^{0.4627}$

Livestock:

Montana: $Y_l = 1.8893\, L_l^{0.0839} C_l^{0.9370}$
Northern Iowa: $Y_l = 2.2893\, L_l^{0.0769} C_l^{0.9067}$
Southern Iowa: $Y_l = 1.1404\, L_l^{0.1166} C_l^{0.9820}$
Alabama: $Y_l = 5.4570\, L_l^{0.2334} C_l^{0.7431}$

[4]See: Heady, Earl O. Production functions from a random sample of farms. Jour. Farm Econ. 28:989-1004. 1946; Heady, Earl O. Use and estimation of input-output relationships or productivity coefficients. Jour. Farm Econ. 34:775-786. 1953; and Heady, Earl O. Productivity and income of labor and capital on Marshall silt loam farms in relation to conservation farming. Iowa Agr. Exp. Sta. Res. Bul. 401.

Significance and Scale Returns

Table 4 below includes the elasticity or regression coefficients (for the data in logarithmic form) for 1950, along with other statistics of interest in the analysis. All regression coefficients are significant at a probability level of 10 percent or greater; all but 4 of the 20 regression coefficients are significant at a probability level of 5 to 0.5 percent. The writers accept the four regression coefficients for labor which are significant at approximately the 8-percent probability level; the logic of production suggests no basis for dropping the labor resources from the production equation. It appears desirable to retain this variable (category of resources) in the production function but to qualify productivity statements in terms of fiducial limits relating to an 8- rather than a 5-, 1- or 0.5- percent probability level.

In testing returns to scale (i.e., the departure of the sums of the elasticities from 1.0), only the livestock equations of southern Iowa indicated increasing returns; the data for the other three areas provide, in a probability sense, only for the inference that the sum of the elasticities does not depart significantly from 1.0. In the case of crops, the sum of elasticities was significantly different from 1.0 at the 5-percent level for northern and southern Iowa, at approximately an 8-percent level for Montana and at a 5-percent level for Alabama. Declining costs and increasing returns to scale with greater outputs are expected particularly in crop production. The opportunities for producing a greater product exist especially because of the indivisibility of machinery and the ability to operate increasing quantities of land with one set of equipment (although this is not exactly the relationship of concern under a *true-scale* relationship).[5] Because of "hand and horse" methods of production, it might be expected that the tendency of constant returns would be greater in Alabama than in the other areas. However, because of the small size of many operating units using even these methods, some "saving of resources" is expected; even an increase of a two-mule over a one-mule unit has some cost and scale advantage. In the same way, a greater elasticity (sum of regression coefficients) can be expected on southern Iowa farms where crop acreage is smaller and a greater proportion of sample observations fall in a lower acreage range than in northern Iowa and Montana.

[5]See Heady, Earl O. Economics of agricultural production and resource use, Ch. 13, Prentice-Hall, New York, 1952, for further distinctions between scale economies and cost advantage as size of the farm firm is increased.

MARGINAL PRODUCTIVITIES

The next step in analysis is derivation of the marginal productivity of resources, with the quantity of all resources held constant at the per-farm mean of each sample.[6] Marginal productivity is a measure indicating the quantity by which the value of output (per farm in this case) is predicted to increase if one more unit of the particular resource were to be employed with (1) inputs of the specific resource at stated levels and (2) inputs of other resources held constant or increased by stated amounts. Table 5 indicates the returns which might be expected, as an average for the farm sample (or the "average" or "typical" farm in the sense of a normal distribution), if one more unit of each resource were to be used on crops or livestock while inputs of other resources are held constant at their arithmetic mean.

Marginal and "Gross Average" Productivities for Mean Resource Quantities and Inter-Area Comparisons

The quantities specifying the arithmetic means of resources employed and products produced are included in table 5. Also included are the "gross average" and the predicted marginal product per unit of each kind of

[6]The marginal productivities derived as means for the sample, represent only one marginal quantity from among large numbers of possible marginal quantities. There is no such thing as the marginal productivity of resources; instead there is a marginal product for each quantity of a single resource, with other resources "fixed" at one level. For each quantity of a single resource, its marginal product differs depending on the quantity of other resources with which it is used. Marginal product is a constant (a single value) only under a linear production function.

TABLE 4. REGRESSION COEFFICIENTS AND RELATED STATISTICS FOR THE FOUR SAMPLE AREAS, 1950.

Item	Montana	Northern Iowa	Southern Iowa	Alabama
Crop function				
Elasticities (regression coefficients):				
Cropland not pasture	0.5032*	0.9124*	0.7948*	0.3847*
Labor	0.0394‡	0.0756‡	0.0875†	0.3192*
Machine-crop services	0.5804*	0.1647*	0.3930*	0.4627*
Sum of elasticities	1.1230	1.1527	1.2753	1.1666
F-test for departure of sum of elasticities from 1.0	3.85 ‡	7.85 *	7.32 *	6.20 †
Livestock function				
Elasticities (regression coefficients):				
Labor	0.0839‡	0.0769†	0.1166*	0.2334*
All capital services	0.9370*	0.9067*	0.9820*	0.7431*
Sum of elasticities	1.0209	0.9836	1.0986	0.9765
F-test for departure of sum of elasticities from 1.0	0.35 §	0.21 §	4.34**	0.22 §

* Significant at probability level of 1 to 5 percent.
† Significant at probability level of 5 percent.
‡ Significant at probability level of approximately 8 percent.
§ Non-significant at an acceptable probability level.
** Significant at a 1-percent probability level.

TABLE 5. ARITHMETIC MEAN PER FARM, MARGINAL PRODUCT AND AVERAGE PRODUCT OF RESOURCE SERVICES USED IN PRODUCTION, ALL INPUTS AT THEIR ARITHMETIC MEANS, 1950.

Item	Montana	Northern Iowa	Southern Iowa	Alabama
Crop function				
Arithmetic mean of production and inputs: *				
Product; actual ($)	21,419	8,551	4,777	1,322
Predicted product ($) †	19,994	8,383	4,572	1,267
Cropland not pasture (acre)	975.0	166.6	114.9	23.8
Labor (mo.)	13.7	9.5	8.7	10.4
Machine-crop (capital) services ($)	5,207	2,168	1,420	553
Marginal product or return *				
Cropland not pasture ($/acre)	10.32	45.91	31.61	20.48
Labor ($/month)	57.33	67.09	45.98	38.73
Machine-crop (capital) services ($/$)	2.23	0.64	1.26	1.16
Average product or return *				
Cropland not pasture ($/acre) ‡	21.97	51.33	41.58	55.55
Labor ($/month) ‡	1,563	901	549	127
Machine-crop (capital) services ($/$) ‡	4.11	3.94	3.36	2.39
Livestock function				
Arithmetic mean of production and inputs: *				
Product; actual ($)	12,084	13,943	9,067	1,336
Predicted product ($) †	11,389	13,986	9,324	1,258
Labor (mo.)	8.9	8.2	7.3	3.5
All capital service inputs ($)	8,896	12,543	7,614	1,017
Marginal product or return *				
Labor ($/month)	106.86	130.65	148.46	83.18
Capital service inputs ($/$)	1.20	1.05	1.20	0.92
Average product or returns *				
Labor ($/month) ‡	1,357	1,700	1,245	382
All capital service inputs ($/$) ‡	1.36	1.11	1.19	1.31

*Units of measurement: product ($); cropland, cropland not pasture and pasture land (acres); labor (months); machine-crop, crop services and capital inputs on livestock ($). A marginal product for land of $10.32 in Montana means that "one more" acre of land adds $10.32 to value of product produced. The figure for labor means that 1 month adds $57.32 to total product while 1 more dollar of capital adds $2.30 to value of product produced. Interpretation is the same for other resources and other areas.

†Predicted from the regression equations for the mean resource quantities.

‡Gross value product (actual sample average) divided by mean quantity of each resource. The "gross average" product represents the total value of production divided by the mean quantity of each resource. The "average" resulting includes the product of all resources, and not simply the product attributable to the single resource. All marginal products are based on the total product predicted from the production function with inputs at their arithmetic means (rather than based on the total product as an arithmetic mean of the samples).

resource. The marginal productivity of land, with land "increased away from its mean" and all others constant at the arithmetic mean, follows an ordering expected in terms of soil type, rainfall and climatic conditions. It is highest in northern Iowa ($45.91 per acre) and followed by southern Iowa ($31.61 per acre), Alabama ($20.48 per acre) and Montana ($10.32 per acre).

These differences in marginal productivity of land do not cause concern about the allocation of this resource between different producing regions. It is an immobile resource and must be used in one location, even though productivities differ between regions. Problems do relate to the magnitude of the marginal product of land, however. One of these is an individual farm management question and concerns the extent to which the price of land or land services (the capitalized and discounted value of the marginal product in the case of land purchase or leasing rates in the case of rented farms) approximate the marginal value productivity of land. Individual farmers, in their investment or management decisions, will prosper or fail depending on the relationship between these two quantities.

LABOR PRODUCTIVITY ON CROPS

Marginal labor productivity on crops displays differentials expected from the capital/labor ratios and resource quantities of tables 1 and 2. For mean resource combinations, the marginal value products of labor are greatest in northern Iowa and Montana. Small farms and a smaller quantity of capital per worker undoubtedly provide the major explanation for a lower "mean" marginal productivity of labor in Alabama (a marginal return of $38.73 per month) and southern Iowa (a return of $45.98 per month). Because of relatively less capital per worker, marginal labor return in Alabama might have been expected to be lower than in southern Iowa.

The differences expressed in the marginal labor quantities are also shown in the average labor productivities. The average productivities are "gross" in this sense; they are computed by dividing the total product by the months of labor per farm. (No product is imputed to land or capital resources in computing the average labor productivities.) Montana has the highest gross average productivity and Alabama has the lowest. The magnitude of these average figures depends on (1) the productivity of the particular resource and (2) the amount of other resources used for which no product is imputed. Hence, Montana ranks above northern Iowa since both (1) its marginal productivity in table 5 is near that of northern Iowa and (2) farmers used a larger amount of capital (and none of the product is imputed to capital).[7]

CAPITAL PRODUCTIVITY ON CROPS

Greatest marginal productivity of capital, in the form of either machine services or crop resources, was found

[7]For further details on these imputational problems see: Heady, Earl O. Production functions from a random sample of farms. Jour. Farm Econ. 28:989-1004. 1946; and Heady, Earl O. Elementary models in farm production economics research. Jour. Farm Econ. 30:201-225. 1948.

in southern Iowa and Montana. In addition to sampling error, the relatively low returns in northern Iowa perhaps are best explained in the machine component of capital. This group of farms is about as highly mechanized, relative to the acreage and types of crops produced, as any other group in the nation. Added machine investment alone, as an average for all farms, would likely add less to value of annual production than the annual cost of the machines. Farmers have pushed machine investment to a high level to ease farm work and add to the living satisfactions of the family. A marginal return of less than $1 for each $1 in annual capital services for crops, with all inputs at the mean, does not mean that the return on all machinery and crop inputs is low. For smaller inputs, machine and other crop services are higher. They may be higher than the return on any other single category of resource. Without machinery no product would be forthcoming from seed and similar capital services for crops. The "gross average" product of crop capital services also suggests that the marginal productivity of small capital inputs on crops may be high in northern Iowa.

Returns on mean inputs of crop capital are high in Alabama. The marginal return is $1.16 for each $1 in input. The sample includes a large proportion of sharecroppers and other small units. These farmers have little capital and cannot borrow or hesitate to borrow because of equity and uncertainty considerations. Hence, a large gap is left between returns from capital used on crops and its cost or price in the form of interest.

Returns were even higher for Montana farms. This phenomenon is expected because of the structure of resources used in wheat production. Crop services include mainly seed. There is little opportunity to increase seed capital beyond the "standard rates." Use of more seed resources would add slightly to yield in some years but the return would decline rapidly. Similarly, machine inputs are "near complements" with land. They give high returns when used in "standard amounts."

In southern Iowa, farmers use less machinery and fertilizer or soil amendments than in northern Iowa. The difference between regions in marginal productivity of capital corresponds with the experiences of extension workers; namely, more capital and improved techniques can give returns as high in southern Iowa as in other parts of the state.

The marginal product figures do not indicate the magnitude of returns which might be earned on many individual farms if they used more resources and different techniques. Since the estimates are based on random samples of farms, they suggest "broad averages" of resource productivities. Use of more resources in new forms to represent different techniques would give high returns on many individual farms in all the sample areas.[8]

Resource Productivity in Livestock Production

Mean marginal labor productivity was higher for livestock than for crops in all areas. For Iowa, it was greatest in the southern area. These farmers had, on the average, extended capital investment less far than farmers in the northern area. Average labor products were highest in northern Iowa and Montana and somewhat lower in southern Iowa. Both marginal and average products for livestock labor were low in Alabama. Lower marginal products for livestock labor in southern Iowa and Alabama are to be expected. The capital/labor ratio is lower in these areas than in northern Iowa and Montana. The low level of returns on livestock in Alabama also may be explained by the techniques and practices used. The share-cropper tenant and other small units in the Alabama sample had low levels of production per head of livestock.

Marginal returns on livestock capital were high in both Iowa areas, as compared to the returns on crop capital. They were lower in Montana, partly due to the above average wheat yields in 1950. (High grass yields in favorable rainfall years give more feed for beef cows, the main type of livestock in the farms in the Montana sample. However, most farmers have herds of fixed size as they go into a grazing year. They can make no, or only partial, use of above average forage yields.)

Returns on capital for livestock were lower than for crop capital in Alabama. Numerous facets of farm production and decision-making may go to explain this difference, including the following two: (1) Skill required for using small quantities of capital resources on conventional crop techniques may not be as great as that required for livestock production on a more profitable basis. For the small quantities of resources used per farm in both lines, the small amount of capital does not restrict methods of production as much in crops as in livestock. (The $1,017 of total capital services used for livestock would not allow output levels or techniques as efficient as the $677 capital services used for crops, particularly in the light of scale returns.) (2) The estimating equations used, although allowing the productivity of one resource to depend on the amount of other resources, do not allow for conditions of strict technical complementarity between resources. Although returns on the small amount of livestock capital are predicted to be low, its use might be entirely profitable in this sense: Use of the small amount of capital allows some return on family labor which would otherwise be unemployed. The higher labor return, for livestock as compared to crops, thus justifies use of some "complementary capital" on livestock. This point is explained in more detail in a later section.

The marginal productivity figures presented above suggest that as an "average," any intra-area addition or reallocation of resources is expected to give greatest returns if used for livestock rather than crops. With the land area fixed, resource investments beyond the mean quantities are expected to give returns which diminish at a relatively rapid rate for crops. With space being less of a limitational factor and a smaller degree of fixity in any single resource, added inputs for livestock are not expected to have such a rapidly diminishing productivity. This situation holds true particularly if feed, as well as other resources, can be brought in from outside of each of the areas.

[8]Estimation of returns under these types of resource adjustments can be made only through the study of carefully defined farm strata and by (1) budget analysis, (2) the current technique applied to samples of homogeneous farms or (3) other refined methods. These steps are needed to give estimates of returns on more specific kinds and forms of resources than the categories included in this study.

Levels of Productivity Estimates

Some of the derived marginal productivities may appear to be low. These (apparently) low returns are explained partly in a later section where the following considerations are taken into account: (1) the nature of the marginal productivity concept as applied to a single resource, (2) the "accounting procedures" and price considerations which apply to farmer decisions in use of the resource quantities and (3) the effect of the quantities of particular resources on the residual and predicted productivity of other resources.

Other qualifications also apply to the predicted marginal products. Included are: (1) The functions and resource categories used may not sufficiently account for resource complementarity. (2) The weather and yields of a single year may provide some quirks in the production function which would not be found as an average over time. (3) Sampling errors may account for the magnitudes derived by using the "mean" regression coefficients. However, considering all of these possibilities, the relative levels of the productivity figures appear reasonable in terms of (1) the quantities and proportions of resources used in the four areas and (2) the comparisons made with productivity figures computed by residual and arithmetic procedures.

In evaluating the levels of the marginal productivities, we also must remember that they are computed for each input of each resource at the mean of the sample. Some farms use large amounts of labor and little capital; the productivities are expected to be low for labor and high for capital. Other farms use large amounts of both resources. Because the elasticities of labor are much less than 1.0, the farms with a large amount of labor and a low capital/labor ratio may have the effect of "pulling down" labor productivities computed at the mean input for all farms. This difficulty is overcome in a later section where productivities are computed at the input levels for groups of farms using entirely different resource quantities and combinations.[9]

Productivities at Geometric Means

Since geometric means were computed in estimation of regression coefficients in logarithmic form and since these statistics sometimes differ considerably from arithmetic means, productivity figures for resource inputs at the geometric means of the samples are included in table 6. The marginal and average productivity figures are of the same order and relative magnitude of those shown in table 5; inferences based on one table are generally the same as those based on the other set of data. All figures and estimates in later sections, unless specified otherwise, apply to arithmetic means.

Capital/Labor Ratios and Gross and Residual Resource Productivity

The figures of table 7 point up some of the reasons for the differences in mean marginal productivity of resources which are shown in tables 2 through 6. These figures again illustrate the very low ratio of capital to labor (or conversely, the high ratio of labor to capital) in the Alabama Piedmont area. They also indicate a relatively low capital/labor ratio in southern Iowa as compared to northern Iowa or the dry-land area of Montana.

Table 7 includes gross and residual productivity figures which can be used as alternative criteria in gauging the efficiency in use of resources. The gross productivity figures show the total amount of production divided by the units of labor, land, capital services or all resource services, as the case may be. The residual productivity

[9]The "pulling down" effect depends on the rate at which the marginal product is decreasing. If a single farm might use 1, 2, 3, 4 or 5 months of labor, the marginal product of the 3rd month need not be identical with the average of all five units computed separately.

TABLE 6. MEAN PRODUCTION AND RESOURCE INPUTS AND MARGINAL AND AVERAGE PRODUCTIVITY OF RESOURCES AT GEOMETRIC MEANS OF SAMPLES, 1950.

Item	Montana	Northern Iowa	Southern Iowa	Alabama
Crop function				
Geometric mean of inputs: *				
Cropland not pasture (acre)	774.6	153.8	97.6	21.0
Labor (mo.)	11.1	8.9	8.0	8.4
Machine-crop services ($)	4,320	1,988	1,181	440
Marginal product: *				
Cropland not pasture ($/acre)	10.29	45.34	31.38	18.55
Labor ($/mo.)	56.48	65.12	42.14	38.47
Machine-crop services ($/$)	2.13	0.63	1.28	1.07
Average Product: *				
Cropland not pasture ($/acre)	20.46	49.70	39.24	48.21
Labor ($/mo.)	1,432	861	480	120
Machine-crop services ($/$)	3.67	3.85	3.24	2.31
Livestock function				
Geometric mean: *				
Product ($)	5,355	10,524	6,928	772
Labor (mo.)	6.05	6.92	6.65	2.71
All capital services inputs	4,116	9,739	5,691	573
Marginal product: *				
Labor ($/mo.)	74.27	117.01	121.42	66.75
All capital services ($/mo.)	1.21	1.03	1.20	1.00
Average product: *				
Labor ($/mo.)	884.79	1,521.60	1,041.40	285.33
All capital services ($/mo.)	1.30	1.08	1.21	1.34

*Units of measurement and methods of calculation same as in table 5, aside from measurement of mean. Productivity figures based on predicted product for inputs at geometric mean, rather than geometric mean of product of each sample.

TABLE 7. RESOURCE RATIOS AND GROSS AND RESIDUAL PRODUCTIVITY OF RESOURCES FOR ARITHMETIC MEANS OF SAMPLES, 1950.

Item	Montana	Northern Iowa	Southern Iowa	Alabama
Crop production				
Percent labor on crops	67.6	53.8	54.0	74.7
Cropland per man-year (acres) *	851	224	169	37
Machine services (expenses) per man-year ($) *	2,845	2,030	1,436	316
Total crop investment per man-year ($) *	67,866	62,430	32,064	3,255
Total crop capital services (including land) per year ($) *	7,162	5,518	3,305	778
Gross crop product per man-year ($) *	19,641	10,479	6,589	1,462
Gross product per acre of cropland ($) †	21.96	51.33	41.58	55.55
Gross crop product per $1 all capital services ($) ‡	1.92	1.46	1.29	0.81
Average residual crop product per man-year ($) §	11,834	5,878	3,946	829
Average residual return on crop investment (%) **	17.3	10.2	9.2	-7.1
Livestock production				
Percent labor on livestock	32.4	46.2	46.0	25.3
Feed fed per man-year ($) *	4,381	9,782	7,342	2,029
Total livestock capital services per man-year ($) *	11,938	18,285	12,300	3,458
Total livestock investment per man-year ($) *	23,163	18,078	15,207	4,340
Gross livestock product per man-year ($) *	17,946	20,451	14,847	4,918
Gross livestock product per $1 capital services ($) ‡	1.36	1.11	1.19	1.31
Average residual livestock product per $1 all services ($) ‡	1.11	0.99	1.00	0.98
Average residual livestock product per man-year ($) §	4,278	2,042	2,348	1,084

*Computed by dividing the specified item by the number of man-years (i.e., each 12-month quantity of labor).

†Computed by dividing the total product by the number of crop acres.

‡Total value of production divided by annual value of non-labor services for crops or livestock. Land rent included with crop and machine services for crops.

§Gross product less (1) rent for land, (2) interest charge for capital and (3) annual capital inputs or expense, with the residual divided by the number of man-years of labor.

**Same as §, except labor return at market wage rates subtracted in place of land rent, with residual divided by total capital investment.

figures are computed by subtracting from total production an amount equal to the market return (i.e., the wage rate for labor, rental rate for land or interest rate for capital) for all resources except the one for which the productivity figure is to be computed. The remainder is then divided by the number of units of the particular resource to obtain the residual product as an average for each unit of the resource.

In a few instances, these simple estimates show relative productivities between areas which differ somewhat from the marginal quantities of previous tables. Therefore, the two sets of estimates may appear to be inconsistent. However, when differences in computational and accounting procedures are considered, they are not necessarily inconsistent. The Montana and northern Iowa figures for crop labor can be used as an example; marginal productivities of table 5 are lower for Montana than for northern Iowa while the gross and residual products in table 7 are higher for Montana. However, these comparisons are not inconsistent because (1) a greater quantity of capital resources is used per man in Montana and (2) the market charges for capital resources used in computing residual returns are less in both areas than their productivities.[10]

The gross product per unit of capital services used on livestock can be used as another example. Aside from Alabama, the ordering of the gross returns for capital services (the total value of livestock production divided by the amount of livestock services or expenses) in table 6 has the same ranks as the marginal productivity of livestock labor in table 5. On a gross basis, Alabama rises above the Iowa areas because, even though capital returns may be low considering the livestock techniques used, no part of the product is imputed to labor; the marginal productivity figures for capital of table 5 include a part of the product imputed to labor. Because of the large amount of labor used relative to capital in Alabama, the procedure (which does not impute any share of the product to labor) allows a large "gross product" figure for capital services. Residual productivity figures partly eliminate this "imputational" problem but do so entirely only if the market charges used for resources approach their "actual productivity," (an infrequent occurrence under the arithmetic procedures of table 7).

The same logic applies to the predicted marginal products (table 5) and the average residual products (table 7) for crop labor in Montana and Alabama. The charge for capital used in computing residual labor return is less than the marginal product of capital. Hence, a margin between the imputed return and the actual return of capital is imputed to labor. Montana uses much more capital per man on crops than northern Iowa. Consequently, the average residual product to labor, part of which is actually attributable to capital, is greater in Montana.[11]

Marginal Productivity of Labor With Various Quantities of Capital; Possibilities of Intra-Area Adjustments

To provide estimates of possible changes in marginal productivity of labor as it is used with different quantities of capital, the productivity figures of tables 8 and 9 have been derived. They show labor productivity when inputs of other resources are held "fixed" at various levels relative to the mean of each sample. All of the

10For further details relating to the manner in which computation procedures cause either gross or residual products to depart from the "actual productivity" of resources, see: Heady, Earl O. Elementary models in farm production economics research. Jour. Farm Econ. 30:201-225. May 1948; and Heady, Earl O. Economics of agricultural production and resource use. Chap. 13. Prentice-Hall, Inc., New York. 1952.

11Gross and residual productivity figures such as those in table 7 have their place in efficiency analyses and often lead to the same conclusions as more refined marginal productivitiy estimates. Their great limitations are to be found in the problems of product imputation outlined above and also in the fact that they imply constant productivity coefficients; when computed as averages for groups of farms, they are based on the assumption that the return for all units of resources is the same as the "computed" productivity figures regardless of the quantity or proportions of resources used. On the other hand, these arithmetic procedures may allow more flexibility in the form of relationships expressed in the data; they are less subject to the "quirks" that can arise from mathematical functions.

TABLE 8. PREDICTED MARGINAL PRODUCT OF CROP LABOR WITH DIFFERENT QUANTITIES OF LAND AND CAPITAL SERVICES, RELATIVE TO MEAN CAPITAL INPUTS FOR EACH AREA, 1950.

Months of Labor	Value of marginal product ($ per mo.) for labor with land and capital inputs at:			
	50% of mean	100% of mean	150% of mean	200% of mean
	Montana			
8	45.49	96.41	149.50	204.27
10	36.71	77.80	120.73	164.85
12	30.81	65.30	101.33	138.36
13.7 (arithmetic mean)		57.33		
14	26.58	56.32	87.39	119.33
16	23.37	49.53	76.85	104.95
	Northern Iowa			
6	48.37	102.06	157.97	215.26
8	37.08	78.24	121.10	165.02
9.4 (arithmetic mean)		67.09		
10	30.16	63.66	98.52	134.25
12	25.49	53.78	83.24	113.43
	Southern Iowa			
6	29.29	66.73	107.98	151.67
8	22.65	51.91	83.52	95.88
8.7 (arithmetic mean)		45.98		
10	18.48	42.00	68.13	81.18
12	15.65	35.64	57.69	70.53
	Alabama			
6	31.39	56.60	79.64	101.61
8	25.81	46.44	65.48	83.55
10	22.17	39.87	56.24	71.76
10.4 (arithmetic mean)		38.73		
12	19.58	35.22	49.66	63.39
14	17.63	31.72	44.71	55.91

capital/labor proportions presented fall within the range of observations actually found in the samples. In comparing these figures, it should be remembered that large capital differences are still present even where labor inputs are the same. For example, with capital services inputs (including land) equal to 200 percent of the mean or average on Montana crops, the capital service input is $16,402; 200 percent of the mean in Alabama gives a total capital service input (value of machine, crop and land services for the year) of only $1,354. (The absolute quantities of capital services shown in table 8 can be computed from table 2.)

Within the restrictions which must be placed on the particular method of analysis, the figures of tables 8 and 9 allow some predictions of changes in production on an intra-area and intra-farm basis. In table 8, for example, we might predict these things: Addition of another month of labor on a Montana farm, with labor input at 10 months and capital service input at 50 percent of the sample mean, is expected to add $36.71 to total product; the same labor added to a Montana farm, with 10 months of labor and capital input at 200 percent of the sample mean, is expected to add $164.85 to total product. Similarly, addition of a unit of labor on an Alabama farm, with labor input at 6 months and 50 percent of the sample mean, is predicted to give an added return of $31.39; for the same labor input but with capital input at 200 percent, an Alabama farm is predicted to have a marginal product of $101.61.

Figures of this nature are of interest in suggesting the effect of added capital on labor productivity within given farms. Starting with a northern Iowa farm having 8 months of labor and capital service inputs equal to 50 percent of the mean ($2,172 from table 2), doubling of the capital inputs (i.e., increasing them by $2,172) increases marginal labor productivity by $41.16. Another increment of capital by the same amount increases the marginal labor product by $42.86. Still another increment of capital increases the marginal product of labor by $43.92. Using the same procedure and starting with 8 months of labor and 50 percent of the mean capital, an increase in capital services in Alabama by $1,354 (i.e., an increase from 50 percent to 200 percent of the mean for the area) increases marginal labor productivity by $57.74 (from $25.81 to $83.55). The predictions show that $1,354 in capital services on crops added to a low capital farm in Alabama increases marginal labor productivity by a greater amount than $2,172 on a low capital farm in northern Iowa. Economic logic plus the form of function would lead to the statement that this differential response is due to (1) the interaction of capital on labor productivity and (2) the fact that the capital input on Alabama farms is so extremely low.

Predicted differences are just as great for capital services added to livestock production. Working with labor figures nearest to the mean input of this resource in each area (8 months in Montana, northern Iowa and southern Iowa and 4 months in Alabama) and moving consecutively between the 50 to 100, 100 to 150 and 150 to 200 capital intervals in table 9, we obtain the increases in labor marginal products in table 10. These estimates suggest that a small amount of capital services invested in livestock in Alabama farms (with an average of only $1,017 in table 2) increases the marginal product of a month's labor by more than a larger amount of capital in Montana or in either Iowa area. An increase in capital from 100 to 150 percent results in marginal product increases of $54.70, $59.62 and $67.19 in Montana, northern Iowa and southern Iowa, respectively; the increases in capital services for this increase

TABLE 9. MARGINAL PRODUCT OF LIVESTOCK LABOR WITH DIFFERENT QUANTITIES OF CAPITAL RELATIVE TO SAMPLE MEANS, 1950.

Months of Labor	Value of marginal product ($ per mo.) for labor with capital services inputs at:			
	50% of mean	100% of mean	150% of mean	200% of mean
	Montana			
4	116.66	223.30	326.52	427.47
6	80.46	154.03	225.22	294.85
8	61.82	118.35	173.05	226.55
8.9 (arithmetic mean)		106.86		
10	50.39	96.46	141.04	184.66
12	42.64	81.61	119.34	156.24
	Northern Iowa			
4	135.67	254.38	367.43	476.84
6	93.30	174.95	252.68	327.93
8	71.54	134.13	193.75	251.44
8.2 (arithmetic mean)		130.65		
10	58.23	109.18	157.70	204.65
12	49.21	92.27	133.27	172.96
	Southern Iowa			
4	128.26	253.31	377.26	500.30
6	89.63	177.04	263.67	349.66
7.3 (arithmetic mean)		148.46		
8	69.52	137.31	204.50	271.21
10	57.09	112.74	167.92	222.69
	Alabama			
2	76.76	128.60	173.76	215.20
3.5 (arithmetic mean)		83.18		
4	45.12	75.56	102.17	126.50
6	33.06	55.36	74.84	92.70

TABLE 10. EFFECT OF ADDED CAPITAL ON CHANGES IN THE PREDICTED MARGINAL PRODUCT OF LABOR USED ON LIVESTOCK.

Changes in capital relative to mean and change in labor marginal product	Montana	Northern Iowa	Southern Iowa	Alabama
Change from 50 to 100%				
Added capital ($)	$ 3,270	$ 6,183	$ 3,807	$ 509
Increase in labor marginal product ($/mo.)	56.53	62.59	67.79	30.44
Change from 100 to 150%				
Added capital ($)	$ 3,270	$ 6,183	$ 3,807	$ 509
Increase in labor marginal product ($/mo.)	54.70	59.62	67.19	26.61
Change from 150 to 200%				
Added capital ($)	$ 3,270	$ 6,183	$ 3,807	$ 509
Increase in labor marginal product ($/mo.)	53.50	57.69	66.71	24.33
Change from 50 to 200%				
Added capital ($)	$ 9,810	$18,549	$11,421	$ 1,526
Increase in labor marginal product ($/mo.)	164.73	179.90	201.69	81.38
Change from 100 to 200%				
Added capital ($)	$ 6,540	$12,366	$ 7,614	$ 1,017
Increase in labor marginal product ($/mo.)	108.20	117.31	133.90	50.94
Labor input (mo.) used for all calculations	8	8	8	4

in labor productivity are, respectively, $3,270, $6,183 and $3,807. In Alabama, however, an increase in capital services from 50 to 200 percent adds only $1,526 to capital service input but adds a predicted $81.38 to the marginal value output of labor. Again, production logic would lead one to expect these differences. The explanation is to be found in capital inputs. The capital/labor ratio for livestock is highest in northern Iowa and lowest in Alabama; Montana and southern Iowa fall between these two.

Adjustment of Capital Services to Level of Montana in Crop Production

To predict marginal productivity of labor if farms in the different areas had equal amounts of capital to go with labor, the figures of table 11 have been derived for crop production. They have been derived from the original production functions with the total dollar value of non-labor resource service inputs in each area set at the average of the Montana sample ($8,201). The figures in the bottom of table 11 indicate the amount of land and machine-crop services necessary in each area to give a total capital service per farm equal to the Montana average.[12] (Only the amounts of cropland and machine-crop services necessary to give inputs as great as in other areas are shown in table 11 for Alabama. These quantities lie too far outside the range of observation to allow "reasonable" predictions of marginal productivity quantities. Estimates for Alabama are made in a later table.) In this section, as well as in the preceding one, the concern is not whether farmers used resources in the proportions indicated, although the input levels used do fall within the range of sample observations. The main concern is with the manner in which changes in input levels of one resource, others remaining constant or at specified levels, change the predicted productivity of the resource in question. With labor input at the mean of each area, capital service inputs equal to those of Montana cause the predicted marginal product of labor in northern Iowa and southern Iowa to jump above that of Montana.

Since the quantity of capital services ($8,201) used in table 11 is considerably outside of the range of observations in Alabama, table 12 has been prepared to include productivity estimates for this area. Even using the "modest" capital service input of $2,718 and the investment of $11,328 (the amount of land and machine-crop capital necessary to give an annual input of capital services equal to 400 percent of the mean), the predicted marginal product of 8 months of labor in Alabama is $150.30. This figure is greater than the productivity estimate for 8 months of labor in Mon-

TABLE 11. MARGINAL PRODUCT OF CROP LABOR IN DOLLARS PER MONTH WITH LAND AND CAPITAL SERVICES INPUTS FOR ALL AREAS EQUAL TO MONTANA AVERAGE, 1950.

Months of Labor	Montana	Northern Iowa	Southern Iowa	Alabama
	Marginal product per mo. labor ($ per mo.)			
Labor at mean	57.33	133.19	204.42	320.48
6		202.62	286.92	
8	96.41	155.32	220.68	
10	77.80	126.36	180.02	
12	65.30	106.77	152.43	
14	56.32			
16	49.53			
Quantity of machine-crop service ($) *	5,207	4,098	4,828	6,697
Quantity cropland not pasture (acres) *	975.0	314.9	390.66	288.22
Value all capital services ($)	8,201	8,201	8,201	8,201
Adjusted investment ($) †	77,628	92,884	79,271	34,296
Capital services as percent of own mean (%)	100.0	189.0	340.0	1,211.0

* Quantity of capital services and land services (rental value) required to give input of all annual, non-labor resource services equal to Montana average (which serves as the basis for estimating labor productivity in top half of table).

† Investment in land and capital necessary to give annual, non-labor resource service inputs equal to Montana average.

[12]In making the adjustment to the Montana average, inputs of land services and machine-crop services were increased in the same proportions from the means of the other areas in this manner: The values of land services and machine-crop services were totaled for northern Iowa. Since the Montana average was 189 percent of the Iowa average, both land and machine-crop services were increased by 89 percent in Iowa for predicting labor productivities in table 11.

TABLE 12. MARGINAL PRODUCT OF LABOR IN DOLLARS PER MONTH IN ALABAMA CROP PRODUCTION WITH CAPITAL SERVICE INPUTS AT SPECIFIED LEVELS RELATIVE TO MEAN OF ALABAMA SAMPLE FARMS.

Months of labor	Marginal productivity of labor ($ per mo.) with capital input services increased to: twice mean	three times mean	four times mean
6	101.61	143.27	182.80
8	83.55	117.78	150.30
10	71.76	101.19	129.12
12	63.39	89.38	114.06
14	55.91	80.46	102.67
Amount all capital services	$ 1,354	$ 2,031	$ 2,718
Investment	$ 5,664	$ 8,496	$11,328

tana, with capital at a much higher level; it is nearly as high as for northern Iowa figures when capital input is at the Montana level. Table 13 provides further estimates of labor productivity in Alabama when labor and capital resources are combined in different proportions. With labor input at the arithmetic mean of 10.4 months and land and capital service increased to only 119.2 acres and $1,659 respectively, the predicted marginal productivity of labor is $119.60. This figure is roughly 300 percent greater than the $38.73 shown in table 5 for the same labor input with mean quantities of capital resources.

TABLE 13. MARGINAL PRODUCT OF RESOURCES IN CROP PRODUCTION IN ALABAMA WITH CAPITAL SERVICES AND LABOR INPUTS AT SPECIFIED LEVELS, 1950.

Quantity of resource or service: All capital services as percent of mean for Ala. sample	Cropland not pasture (acres)	Labor (mo.)	Machine-crop services ($)	Marginal product: Cropland not pasture ($ per acre)	Labor ($ per mo.)
100	23.8	8	553	18.81	46.44
200	47.6	8	1,106	16.92	83.55
300	71.4	8	1,659	15.91	117.78
400	95.2	8	2,212	15.22	150.30
100	23.8	10.4	553	20.48	38.73
*	95.2	10.4	1,659	14.51	109.75
†	119.2	10.4	1,659	12.65	119.60

* Cropland not pasture, 400 percent, and machine-crop services, 300 percent of mean.

† Cropland not pasture, 500 percent, and machine-crop services, 300 percent of mean.

Table 14 provides estimates of marginal productivities of land and machine-crop services in Montana and the Iowa areas when their input is adjusted to the levels of table 11. These figures differ only slightly from the corresponding estimates (showing land and machine-crop productivities when inputs are at the mean of each area) in table 5. Their magnitudes remain near the same level, even where labor is constant at the mean because (1) both resource categories are increased proportionately in table 14 and (2) the production elasticities are sufficiently high.

TABLE 14. MARGINAL PRODUCT OF CAPITAL SERVICE INPUTS IN DOLLARS. ALL CAPITAL SERVICES FOR EACH AREA EQUAL TO THE MONTANA AVERAGE, LABOR INPUTS AT 8 MONTHS AND AT SAMPLE MEANS, 1950.

Inputs	Montana	Northern Iowa	Southern Iowa
Quantities of factors except labor			
Cropland not pasture (acre) *	975.0	314.9	390.7
Machine-crop services ($) *	5,207	4,098	4,828
Marginal products with labor at mean			
Cropland not pasture ($/acre)	10.32	48.22	41.36
Machine-crop services ($/$)	2.23	0.70	1.65
Marginal products with labor at 8 months			
Cropland not pasture ($/acre)	10.10	47.62	41.05
Machine-crop services ($/$)	2.18	0.66	1.64

* Necessary to make input of all annual non-labor inputs equal to Montana average (see note for table 11).

Adjustment of Capital Services to Northern Iowa Average for Livestock Production

Estimates of marginal labor productivity, with capital services for livestock in Montana and southern Iowa adjusted to the mean levels of northern Iowa, are shown in table 15. These figures give a picture similar to those for crop production. With labor either at the mean input level of each area, or at parallel levels, the estimated marginal productivity of labor in the two areas becomes as great or greater than for northern Iowa. Table 16 gives estimates of the productivity of capital services used on livestock when capital input is equal to the northern Iowa average. On the basis of these estimates, an increase in capital services of southern Iowa and Montana to the northern Iowa level would

TABLE 15. MARGINAL PRODUCT OF LIVESTOCK IN DOLLARS PER MONTH WITH CAPITAL SERVICES INPUTS FOR ALL AREAS EQUAL TO NORTHERN IOWA AVERAGE, 1950.

Months of labor	Marginal product of labor ($ per month): Montana	Northern Iowa	Southern Iowa
Mean labor	147.44	130.65	242.39
4	308.12	254.38	413.55
6	212.52	174.95	289.03
8	163.29	134.13	224.19
10	133.10	109.18	184.08
Quantity of capital services ($) *	12,542	12,542	12,542

*Quantity of capital services necessary to equal northern Iowa average and to serve as basis for predicting marginal product of labor in top part of table.

TABLE 16. MARGINAL PRODUCT OF LIVESTOCK RESOURCES WITH CAPITAL SERVICE INPUTS FOR ALL AREAS EQUAL TO NORTHERN IOWA AVERAGE, LABOR AT AREA MEANS AND AT 6 MONTHS, 1950.

Area	Quantity: Labor	Capital	Marginal product of: Labor ($/mo.)	Capital ($/$)
Montana	8	12,542	163.29	1.163
Northern Iowa	8	12,542	134.13	1.090
Southern Iowa	8	12,542	224.19	1.204
Montana	8.9 (mean)	12,542	147.44	1.174
Northern Iowa	8.2 (mean)	12,542	130.65	1.101
Southern Iowa	7.3 (mean)	12,542	242.39	1.192

leave marginal returns greater than the cost of credit in the first two areas. The marginal return per $1 input, with labor at the mean level and capital services at the northern Iowa level, is $1.17 in Montana and $1.19 in southern Iowa.

Predicted marginal labor productivity for Alabama is included, within the range of observations, in tables 17 and 18 when capital is "fixed" at different levels relative to the mean of the Alabama sample. With capital services on livestock increased by four times, the input is only $4,068 (table 17). Still the marginal labor product for parallel inputs of labor, becomes nearly comparable to northern Iowa under a larger input of capital services. Table 17 gives comparisons when labor is held constant at specified levels and livestock capital is increased. Small amounts of capital again cause a relatively large increase in the predicted marginal product of labor. Even if added capital is considered to return only itself or to return a negative amount, i.e., necessitate a cost, the predicted increases in the marginal productivity of labor would cause use of more capital to be highly profitable on these farms with little capital and a large supply of family labor.

Estimated Marginal Products for Capital

The productivity figures shown in table 19 are for a "combined" unit of land and machine-crop services, with their input in a constant ratio and at a specified level relative to the mean of all farms in each sample area. While the procedure used in stratifying resource service categories supposedly took into account technical complementarity, we provide these estimates for two reasons: (1) If the classification of variables was not consistent with conditions of complementarity, the land and capital services now can be considered as "technical complements"; the marginal product figures can be looked upon as those associated with a combined land-capital unit. (2) The estimates provide predictions of dollar returns on capital investment of varying quantities of these two resources increased by the same proportions, even if land and capital are not complementary. The amounts of machine crop services shown in the bottom of table 19 are the average amounts used with each acre of cropland in the individual areas. In other words, the average Montana acre had capital costs (excluding taxes and other "non-production" expenses) of $5.34; an average of $13.01 was used per acre in the northern Iowa sample, and so forth.

With input at the level of 160 percent in Montana, the marginal return of a combined land-capital unit is $23.12. In each area the marginal return per "combined unit" of capital and land services for crop production is considerably greater than the cost of the resource services. Using the arbitrary rental rates for land (based on share rents for all cropland) of $9 in Montana, $23 in northern Iowa, $18 in southern Iowa and $8 in Alabama, the marginal return per dollar of "combined" capital service inputs (with input as high as 160 percent of the mean), we get these marginal returns per dollar of capital service inputs: Montana, $1.61; northern Iowa, $1.56; southern Iowa, $1.61; and Alabama, $1.34. Returns per "combined unit" of resource services with inputs at the mean (100 percent) are as follows: Montana, $1.54; northern Iowa, $1.51; southern Iowa, $1.74; and Alabama, $1.44. These returns were high relative to the cost of resource services in Alabama and especially so in the other areas. Part of the high returns undoubtedly are due to the fact that farmers were expecting declining prices in 1950; rental rates had held low because of this anticipation. As a result,

TABLE 17. MARGINAL PRODUCT OF LIVESTOCK LABOR IN DOLLARS PER MONTH IN ALABAMA WITH CAPITAL SERVICE INPUTS AT SPECIFIED LEVELS RELATIVE TO MEAN OF SAMPLE, 1950.

Months of labor	Marginal product of labor ($ per mo.) with capital inputs at:		
	twice mean	three times mean	four times mean
2	215.20	290.82	360.14
4	126.50	170.96	211.64
6	92.70	125.26	155.13
8	74.37	100.48	124.43
Quantity of capital services ($)	2,034	3,051	4,068

TABLE 18. MARGINAL PRODUCT OF LABOR USED ON LIVESTOCK IN ALABAMA, WITH CAPITAL AND LABOR INPUTS AT SPECIFIED LEVELS, 1950.

Labor input (months)	Capital input		Marginal product of labor
	($)	Percent of mean	($ per mo.)
8	1,017	50	44.40
8	2,034	100	74.37
8	3,051	150	100.48
8	4,068	200	124.43
3.5 (mean)	508	25	49.66
3.5 (mean)	1,017	50	83.18
3.5 (mean)	1,526	75	112.47
3.5 (mean)	2,034	100	139.25

TABLE 19. MARGINAL PRODUCT OF CAPITAL SERVICES IN DOLLARS PER COMBINED UNIT FOR CROP PRODUCTION WITH SPECIFIED LEVELS OF INPUT, LABOR AT MEAN QUANTITY OF EACH SAMPLE, 1950.

Input of land and machine-crop services as percent of mean for each sample	Montana *	Northern Iowa *	Southern Iowa *	Alabama *
60	21.32	52.08	44.64	48.77
80	21.83	53.28	46.99	46.68
100	22.83	54.20	49.06	45.11
120	22.55	54.95	50.83	43.87
140	22.84	55.59	52.31	42.85
160	23.12	56.20	53.70	41.99
Weights (combined unit of land and machine-crop services):				
Cropland not pasture (acres)	1	1	1	1
Machine-crop services ($)	5.34	13.01	12.36	23.24

* Interpret as marginal product resulting from the unit variation of land and machine-crop services in constant proportions by the quantities indicated at the bottom of the table. A marginal product per dollar of the jointly varied resources may be found by putting a value on land, adding machine-crop services indicated at the bottom of the table and dividing the result into the tabulated figure. In computing the figures of the table, the marginal product of land and capital were estimated with the input levels indicated at the left. The marginal product of $1 in capital was then multiplied by the number of dollars in the unit and added to the marginal product of an acre of land. In the case of southern Iowa with inputs of 160 percent, the $53.70 for the combined unit is the marginal product of $35.90 for an acre of land plus the amount of $1.44 (the marginal product of capital) times 12.36 units of capital. The marginal product of labor is now imputed to land and capital in this case.

the cost of land inputs was low relative to the productivity of this resource.[13]

Somewhat higher rental rates, and hence a lower return per unit of combined resource in Alabama, were likely due to (1) the greater number of farm families relative to cropland and (2) the relatively less efficient techniques found on these farms. The returns per combined unit of capital and labor may seem high for southern Iowa compared to northern Iowa, particularly since land in the two areas had marginal products of $31.61 and $45.91, respectively, in table 5. The combined unit has high returns in southern Iowa, however, because of the high productivity of the capital services used with the land (see table 5).

LIVESTOCK PRODUCTION

The figures in table 20 show predicted marginal returns for various quantities of capital services (feed, buildings, livestock, etc.) in livestock production, with labor inputs constant at the mean of each area. These predictions suppose that the proportions of the resources are variable rather than strictly of a complementary nature, over the combinations examined. They also suppose that the mean quantity of labor used on farms is great enough to handle larger quantities of capital services. Certainly this possibility holds true on the "downward" side of capital quantities; less livestock could always be handled with the same amount of labor. More capital could be handled with the same labor to the extent that labor on some farms is not fully employed (or if forms of mechanization can be added to substitute for labor). The maximum quantity of capital services included in table 20 was observed on some farms using no more labor than the mean quantity of each sample.

The marginal productivity figures show the dollar return for each $1 annual input of services (the value of the services used and not the investment in capital itself). Hence, the capital services used in livestock production were profitable in three areas. A $1 expense input gave a return of more than $1 in all areas except Alabama.

The relatively low returns for large capital inputs in northern Iowa may be explained in part by the presence of feeder cattle on some of the farms; cattle sold in the early part of the year gave low returns above feed and purchase price. Lower marginal products for capital inputs are expected for northern Iowa, however, because it uses more capital than the other areas. Also, the "mean elasticity" for capital services is considerably less than 1.0.

A small livestock enterprise in Alabama is supplementary with crops for labor on most farms; some labor of the operator or family ordinarily is unemployed for crops during the winter and quite often even during the summer. If it is not used for livestock production, it is "unemployed" and has a zero return (unless, of course, it can be used in off-farm employment). Hence, as long as a small amount of capital causes labor to have "any small return," use of the capital is profitable even though its return is less than its cost.[14]

TABLE 20. MARGINAL PRODUCT OF CAPITAL SERVICES USED IN LIVESTOCK PRODUCTION WITH SPECIFIED LEVELS OF INPUT, LABOR AT MEAN, 1950. (DOLLAR MARGINAL PRODUCT PER DOLLAR INPUT OF SERVICES).

Quantity of capital services ($)	Montana, 1950	Northern Iowa, 1950	Southern Iowa, 1950	Alabama, 1950
1,000	—	—	—	0.93
2,000	—	—	—	0.77
3,000	—	—	—	0.70
4,000	1.26	1.13	1.21	—
6,000	1.23	1.09	1.21	—
8,000	1.20	1.05	1.20	—
10,000	1.19	1.04	1.20	—
Mean labor (mo.)	8.9	8.2	7.3	3.5

PRODUCTIVITY OF RESOURCES BY LABOR AND CAPITAL STRATA

The estimates of previous sections provide comparisons of resource productivities on an intra-farm and inter-regional basis. On an intra-farm basis, the figures allow predictions of marginal productivities when one category of resource inputs is increased or decreased while the inputs of other categories are held constant. Inter-regional comparisons are provided by the productivity figures estimated at the mean inputs of each area and for parallel quantities of resources.

So that comparison of resource combinations and productivities might be provided between groups of farms within individual areas, the data and estimates of tables 21 to 28 have been prepared. These tables for crops are prepared for nine capital-labor groups in each of the areas. The nine strata were delineated in this manner: First, all farms were separated into three groups in terms of labor inputs for crop production and livestock production taken separately. Each labor group includes one-third of the farms in the sample area. Second, all farms were separated into three groups in terms of the annual input of capital services including land. (Stratification is not in terms of capital investment but in terms of the estimated value of all services used, i.e., the expenses attached to the resource on a "hired" or "purchased" basis, even where owned.) Each capital group includes one-third of the farms. Finally, the three capital groups have been kept separate under each of the three labor groups to give a total of nine labor-capital groups ranging from low-labor, low-capital to high-labor, high-capital. Data dealing with per-farm averages of resource inputs and capital/labor ratios are provided for each farm group. Also, gross resource productivities are computed and marginal productivities are estimated for the mean quantities of resources in each labor-capital group.

Input and Productivity of Crop Resources;

FARMS STRATIFIED BY CROP INPUTS ONLY

Tables 21 to 24 include resource inputs and resource ratios for crop production in the four areas. In grouping the data for these comparisons, farms were classified in terms of inputs for crops only; no attention was

[13]For other findings of this nature, see: Heady, Earl O. and Kehrberg, E. W. Relationship of crop-share and cash leasing systems to farming efficiency. Iowa Agr. Exp. Sta. Res. Bul. 386.

[14]Although (1) the physical production process is possible only by using labor with capital and (2) the total product is attributable to both capital and labor, the farmer's accounting procedure can allow him to "impute part of the capital product" to capital.

given to livestock production. A farm falling in one group for crops may fall in an entirely different group for livestock. (Stratification is in terms of capital services or the annual value of non-labor inputs and not in terms of capital investment.) The quantities of inputs represented by each labor-capital category vary greatly by areas. In Montana and southern Iowa, the percentage of farms in the high-labor, high-capital group (the southeast cell of the tables) was as great as for the low-labor, low-capital group. The high-labor, high-capital group included as large a proportion of the farms as the "intermediate" labor-capital groups in the three areas.

The capital service inputs of the high-labor, high-capital group in Alabama were smaller than the parallel inputs for the low-labor, low-capital group in the other three areas. Also, capital inputs of the low-labor, low-capital groups in Montana, northern Iowa and southern Iowa were greater than for the low-labor, high-capital group of Alabama. Again these figures indicate that even if labor productivity in Alabama is great for relatively large amounts of capital (in terms of the all-farm average in Alabama) the amount of income per farm must still be low. This is true because of the small total

TABLE 21. MONTANA SAMPLE STRATIFIED BY THIRDS IN TERMS OF LABOR AND CAPITAL INPUTS ON CROP PRODUCTION, SELECTED CHARACTERISTICS PER FARM, 1950. *

	Low capital	Medium capital	High capital
Low labor			
1. Percent farms	21.2	8.6	4.0
2. Crop acres	418	790	1,166
3. All land	774	1,634	5,654
4. Month labor	5.2	6.5	7.8
5. Machine services ($)	1,792	3,295	5,289
6. Crop services ($)	488	1,004	1,317
7. All capital services ($)	4,487	7,830	14,592
8. Crop acres per man	972	1,466	1,786
9. Investment per man	65,522	102,140	214,123
10. All crop services per man	7,809	11,817	19,411
Medium labor			
1. Percent farms	9.9	15.2	8.0
2. Crop acres	520	951	1,325
3. All land	893	2,508	2,039
4. Month labor	11.6	11.8	12.7
5. Machine services ($)	2,006	3,756	4,855
6. Crop services ($)	594	1,279	1,689
7. All capital services ($)	6,188	10,034	14,278
8. Crop acres per man	537	965	1,248
9. Investment per man	29,664	63,592	109,585
10. All crop services per man	3,752	7,448	10,660
High labor			
1. Percent farms	2.7	9.3	21.2
2. Crop acres	562	870	1,767
3. All land	773	1,766	4,618
4. Month labor	16.8	20.0	26.0
5. Machine services ($)	2,251	3,778	7,482
6. Crop services ($)	651	958	2,099
7. All capital services ($)	6,857	11,447	21,732
8. Crop acres per man	402	521	816
9. Investment per man	19,787	41,864	70,982
10. All crop services per man	2,626	4,334	7,169

* The figures shown are, starting at the top of each cell and reading down: (1) percent farms in sample falling in the particular capital and labor group for crops only, (2) acres of cropland, (3) total acres including pasture, (4) months labor used on crops, (5) value of machine services (fuel, repairs, depreciation, etc.) used on crops, (6) value crop services (seed, fertilizer, etc.) used on crops, (7) total value of land, machine and crop services used on crops, (8) cropland per 12 months of labor, i. e., per man-year (Cropland per man, like all other "per man" figures, is computed by dividing the number of acres by number of man-years, i. e., 12 months of labor is a man-year. It does not show crop acres per man on the farm during all or part of the year. A farmer operating 640 acres and using only 6 months of labor would have the equivalent of 1,280 crop acres per man-year. The same procedures apply to the "per man" figures below.), (9) total investment per 12 months labor, (10) total value of crop services (value land, machine and crop services) per 12 months labor.

TABLE 22. NORTHERN IOWA SAMPLE STRATIFIED BY THIRDS IN TERMS OF LABOR AND CAPITAL INPUTS IN CROP PRODUCTION, SELECTED CHARACTERISTICS PER FARM, 1950. *

	Low capital	Medium capital	High capital
Low labor			
1. Percent farms	14.8	14.1	4.9
2. Crop acres	111	160	221
3. All land	133	179	226
4. Month labor	5.6	6.5	6.3
5. Machine services ($)	1,120	1,308	1,686
6. Crop services ($)	291	538	884
7. All capital services ($)	3,743	5,131	6,990
8. Crop acres per man	237	294	423
9. Investment per man	60,566	81,109	133,867
10. All crop services per man	5,623	7,061	10,966
Medium labor			
1. Percent farms	11.3	12.0	9.9
2. Crop acres	133	161	246
3. All land	146	199	278
4. Month labor	8.7	9.1	9.0
5. Machine services ($)	1,053	1,488	2,323
6. Crop services ($)	344	453	931
7. All capital services ($)	4,397	5,609	8,088
8. Crop acres per man	183	211	329
9. Investment per man	41,072	55,974	95,341
10. All crop services per man	3,743	5,013	8,546
High labor			
1. Percent farms	7.8	7.0	8.3
2. Crop acres	115	153	266
3. All land	142	181	302
4. Month labor	13.1	12.9	13.7
5. Machine services ($)	1,053	1,659	2,407
6. Crop services ($)	329	416	920
7. All capital services ($)	4,881	6,185	9,517
8. Crop acres per man	105	143	233
9. Investment per man	22,164	37,391	72,365
10. All crop services per man	2,226	3,522	6,148

* See footnote for table 21 for more complete definitions of items.

TABLE 23. SOUTHERN IOWA SAMPLE STRATIFIED BY THIRDS IN TERMS OF LABOR AND CAPITAL INPUTS IN CROP PRODUCTION, SELECTED CHARACTERISTICS PER FARM, 1950. *

	Low capital	Medium capital	High capital
Low labor			
1. Percent farms	19.6	9.8	4.2
2. Crop acres	63	115	172
3. All land	119	203	246
4. Month labor	5.2	5.5	5.4
5. Machine services ($)	481	770	1,671
6. Crop services ($)	152	261	870
7. All capital services ($)	2,085	3,288	5,449
8. Crop acres per man	144	250	380
9. Investment per man	20,595	56,584	89,959
10. All crop services per man	2,292	4,701	9,614
Medium labor			
1. Percent farms	10.5	14.7	8.4
2. Crop acres	80	120	167
3. All land	145	191	217
4. Month labor	8.0	7.6	7.8
5. Machine services ($)	628	1,111	1,408
6. Crop services ($)	171	316	594
7. All capital services ($)	2,751	3,696	5,399
8. Crop acres per man	121	188	258
9. Investment per man	14,451	29,514	60,020
10. All crop services per man	1,766	3,379	5,692
High labor			
1. Percent farms	3.5	9.1	20.3
2. Crop acres	59	123	193
3. All land	114	205	278
4. Month labor	12.3	12.6	13.5
5. Machine services ($)	461	1,057	1,700
6. Crop services ($)	189	302	674
7. All capital services ($)	3,426	4,447	6,565
8. Crop acres per man	58	118	171
9. Investment per man	10,102	17,732	35,877
10. All crop services per man	1,067	2,032	3,638

* See footnote for table 21 for more complete definitions of items.

TABLE 24. ALABAMA SAMPLE STRATIFIED BY THIRDS IN TERMS OF LABOR AND CAPITAL INPUTS IN CROP PRODUCTION, SELECTED CHARACTERISTICS PER FARM, 1950. *

	Low capital	Medium capital	High capital
Low labor			
1. Percent farms	16.5	10.5	6.8
2. Crop acres	23	26	37
3. All land	37	54	54
4. Month labor	3.3	4.6	5.6
5. Machine services ($)	103	206	467
6. Crop services ($)	85	236	410
7. All capital services ($)	598	1,058	1,623
8. Crop acres per man	83	68	79
9. Investment per man	6,917	8,436	9,545
10. All crop services per man	1,021	1,544	2,290
Medium labor			
1. Percent farms	14.3	7.5	11.3
2. Crop acres	21	27	43
3. All land	30	34	61
4. Month labor	9.6	8.8	11.0
5. Machine services ($)	137	271	468
6. Crop services ($)	160	220	529
7. All capital services ($)	1,306	1,436	2,342
8. Crop acres per man	27	37	47
9. Investment per man	1,625	2,263	5,427
10. All crop services per man	445	770	1,310
High labor			
1. Percent farms	3.0	15.0	15.0
2. Crop acres	16	34	52
3. All land	20	47	71
4. Month labor	13.4	18.3	17.4
5. Machine services ($)	121	237	483
6. Crop services ($)	157	252	466
7. All capital services ($)	1,656	2,414	2,846
8. Crop acres per man	14	22	36
9. Investment per man	785	1,400	3,031
10. All crop services per man	284	385	778

* See footnote for table 21 for more complete definitions of items.

quantity of capital resources used; even with a low marginal product per month of labor in Iowa and Montana, income for family living can still be greater than on a high-capital Alabama farm because of the quantity of resources involved.

In the four areas, there is a large increase between capital groups (but within labor groups) in (1) the absolute acreage, (2) the quantity of machine services, (3) the quantity of crop services, (4) the total investment in resources used for crop production and (5) the total input of all capital services (the annual input or "computed expense" of machine and crop services and land rental value). With only one or two exceptions, the magnitudes increase from low-capital to high-capital strata. The total value of product produced per farm increases similarly. However, capital and product increase within a labor group by a much greater proportion than does labor. (Labor is free to "vary" only within the group limits.) The figures again suggest the effectiveness of greater quantities of capital in increasing the productivity of a given amount of labor. In general, input of the three categories of capital services (rental value of land and crop and machine services or expenses) increased in somewhat similar proportions from low- to high-capital strata, within a given labor stratum.

There was not a parallel increase in total product from low- to high-labor groups within a single capital stratum. While product increased slightly from one labor group to the next, the increase was relatively small. (See table 25 for differences in value of product for the different labor-capital groups.) These figures suggest that farms in the sample, with given labor resources, may be able to organize increased quantities of capital to produce a much greater product. In contrast, a given supply of capital allows only minor increases in value of product as labor is increased.

The stratification by capital and labor groups in the tables causes widely different ratios between capital inputs or investment and labor. In the Montana sample, the ratio of cropland varies from 402 acres (in the high-labor, low-capital group) per man-year to 1,786 acres (in the low-labor, high-capital group) per man-year. The range is from 105 acres to 423 acres in northern Iowa; 58 to 380 acres in southern Iowa and 14 to 83 acres in Alabama. Input of all capital services used for crops ranges from $2,626 to $19,411 per man-year in Montana, from $2,226 to $10,966 in northern Iowa, from $1,067 to $9,614 in southern Iowa and from $284 to $2,290 in Alabama. These differences in resource ratios give rise to the differences in productivity shown in table 25 for crops.

Table 25 includes two sets of productivity ratios. Line 1 represents the gross product per man-year.[15] Line 2 is the residual product per man-year.[16] The marginal products of the several resource services have been derived from the production function equations in the manner outlined earlier. Examination of the (1) gross residual productivity figures for labor or (2) predicted marginal products for capital services shows striking differences between farms in single areas depending on the capital or labor resources and their ratios. The relative differences within the Alabama sample are as great between labor-capital groups as within the other samples. However, the absolute level for any one capital-labor group in Alabama is far below that of the other three areas. The residual product per man in the low-labor, high-capital group of Alabama is as great as for the low-labor, low-capital group in southern Iowa. However, it is far below the low-labor, low-capital groups for Montana and northern Iowa.

The predicted marginal products show a relationship which is expected, partly because of the different capital/labor ratios of the various groups and partly because of the type of functions employed and the magnitude of the regression coefficients derived from the sample. Regression coefficients (production elasticities) of less than 1.0 specify that (1) the marginal productivity of any one resource category will decline as more of it is used, other resource inputs held constant, and (2) the marginal productivity of a given quantity of one resource will increase as the inputs of other resources used with it are increased in quantity. The magnitude of the regression coefficients specifies the first condition while the type of function partly specifies the second.[17]

15This has been computed by dividing the total value of product per farm by the total "man-years" of labor per farm. It does not impute any share of the product to capital services.

16This has been computed by first imputing the annual value of the input to capital resources (the rental value of land, the expense of crop and machine services including the market rate of interest on machine capital). Next the remaining value of product per farm has been divided by the man-years of labor.

17The interaction allowed by the fact that resource quantities are multiplied by each other in the function causes the productivity of one resource to increase as input of the other is increased. However, the use and acceptance of this type of function is not arbitrary. Any person acquainted with agriculture knows that, over fairly small changes in proportions of resources used, more of one resource will generally cause the productivity of another resource to change, even if the resources are represented by broad categories, such as labor and capital, or more specific categories, such as fertilizer and land or feed and animals. Given the existing logic of production and knowledge of actual production relationships in agriculture, one initial task was finding a function which allows these conditions but is flexible in allowing constant, diminishing or increasing productivity of one resource or all resources.

TABLE 25. PREDICTED MARGINAL PRODUCTIVITY AND AVERAGE PRODUCTIVITY (IN DOLLARS) OF RESOURCES FOR CAPITAL AND LABOR GROUPS SHOWN FOR CROPS IN TABLES 21 TO 24, 1950.

Labor group and item	Montana			Northern Iowa			Southern Iowa			Alabama		
	Low capital services	Medium capital services	High capital services	Low capital services	Medium capital services	High capital services	Low capital services	Medium capital services	High capital services	Low capital services	Medium capital services	High capital services
Low labor												
Gross product per man-year*	17,659	35,574	46,450	11,577	13,609	23,053	5,689	9,109	16,537	1,676	2,230	3,784
Average residual product per man-year†	9,850	23,757	27,039	5,954	6,548	12,087	1,477	2,031	3,125	640	658	1,419
Marginal productivity, land per acre‡	9.26	9.95	10.60	42.76	43.83	45.11	25.87	27.83	36.43	13.64	17.94	17.53
Marginal productivity, labor per month‡	59.33	95.14	123.59	67.35	84.27	117.06	34.50	64.06	127.88	38.80	52.74	81.60
Marginal productivity per $1 machine-crop services‡	1.98	2.11	2.16	0.59	0.65	0.62	1.27	1.54	1.22	0.98	0.80	0.75
Predicted product‡	7,779	15,621	24,570	5,010	7,273	9,710	2,050	4,027	7,893	397	760	1,426
Medium labor												
Gross product per man-year*	9,695	23,846	28,229	8,686	10,380	17,072	4,268	7,615	11,447	1,048	1,606	2,203
Average residual product per man-year†	5,944	16,398	17,569	4,944	5,367	8,525	1,670	2,699	3,737	595	819	845
Marginal productivity, land per acre‡	9.36	10.19	10.09	43.51	45.33	47.40	28.03	32.25	34.48	18.05	19.98	21.78
Marginal productivity, labor per month‡	32.77	64.08	82.12	51.69	62.53	103.97	30.86	56.06	81.59	27.38	39.13	56.61
Marginal productivity per $1 machine-crop services‡	2.16	2.22	2.36	0.70	0.64	0.63	1.39	1.34	1.43	0.94	1.02	0.91
Predicted product‡	9,672	19,249	26,558	5,952	7,551	12,348	2,821	4,869	7,274	821	1,075	1,953
High labor												
Gross product per man-year*	9,902	11,006	18,127	4,967	7,076	12,756	2,491	5,025	8,133	886	1,036	1,882
Average residual product per man-year†	7,267	6,672	10,958	2,741	3,554	6,608	1,454	3,138	5,072	597	645	1,076
Marginal productivity, land per acre‡	9.74	10.49	11.22	45.35	47.23	48.92	28.57	32.90	37.58	20.97	21.45	23.25
Marginal productivity, labor per month‡	25.55	35.69	59.73	31.17	44.03	73.56	15.08	35.36	59.13	19.59	26.20	42.00
Marginal productivity per $1 machine-crop services‡	2.18	2.22	2.39	0.64	0.60	0.66	1.28	1.47	1.51	1.37	1.42	1.12
Predicted product‡	10,879	18,138	39,391	5,398	7,496	13,323	2,120	5,092	9,125	822	1,500	2,291

* Average gross product per man for crops or livestock is the gross product divided by the number of man-years (i. e., by the number of 12-month units of labor).
† Average residual product is gross product less an imputed return (based on market prices) to capital items with the remainder divided by the number of man-years.
‡ Predicted with input of resources at the arithmetic mean of each capital-labor group.

The "general relationships" shown by the derived marginal productivities are also paralleled by the more simple, arithmetic procedures. The gross-residual productivities for labor, computed by simple arithmetic, serve as examples. They increase from left to right in table 25 between capital groups and within labor groups; increases in capital per worker cause the gross-residual productivity of labor to increase. A movement from low to high between labor groups within a capital group is paralleled by a decline in the gross-residual productivity of labor. These changes in productivity as capital/labor proportions change are even more striking when viewed in terms of the derived marginal products. (Marginal quantities always change at a faster rate than average quantities such as gross and residual products per man or per dollar of capital services.) Within labor groups and between capital groups, the consistent and relatively large increases in marginal productivity are for labor. Movements between capital groups but within labor groups in table 25 are equivalent to an increase in capital per man. Within the low-labor groups of Montana and northern Iowa, marginal labor productivity doubles between the low- and high-capital groups. It more than quadruples in southern Iowa and slightly more than doubles in Alabama. Similar increases in labor productivity between capital groups are to be found within the medium- and high-labor groups of farms. The level of increase in marginal labor productivity depends on the increment in capital services represented by one capital group as compared to another.

"Movement" from low- to high-labor groups within a given capital group causes the marginal products to decline. While capital is not entirely constant, it increases little from low- to high-labor groups. Consequently, the capital/labor ratios (including land services as well as crop-machine services in capital services) decline greatly (see tables 21 to 24). While the marginal products of labor (1) increase between capital groups within a labor group and (2) decrease between labor groups within capital groups, the marginal products of machine-crop services follow an opposite pattern. They increase as the input of labor increases relative to the quantity of capital. In "movements" from low- to high-capital groups within a labor stratum, however, the marginal products of machine and crop expenses increase or decrease depending on the relative quantity of land or labor. They increase from low- to medium-capital groups but decline from medium- to high-capital groups. This pattern occurs (even though the labor/capital ratio declines) because (1) the input of land increased by enough to more than offset the decline in the labor/capital ratio between the first two captial groups while (2) the decrease in the labor/capital ratio is more than enough to offset the increase in land inputs between the medium- and high-capital groups.

Changes in the marginal product of land generally are smaller, relative to the changes for other resources, either "across" capital strata or "down" labor strata. The most important changes in land productivity are "across" capital strata in Alabama and southern Iowa. In Alabama, the marginal product per acre increases by roughly 30 percent between low- and high-capital groups in the first labor stratum, by 22 percent in the second labor stratum and by 10 percent in the third labor stratum. The absolute input of capital services is low even in the "high" capital groups of Alabama. However, the paucity of capital in the "low" capital groups is so extreme that more capital on land gives very great rewards to land. Increases in the marginal product of land average about 10 percent "across" capital strata for the Montana and northern Iowa samples, and about 40 percent for southern Iowa.

Profitability of Using Resources for Crops in Relation to Marginal Productivities

If the marginal productivities are viewed together in table 25, the marginal returns for labor and machine-crop services in combination may appear low. The level of returns can appear to be low because of (1) the marginal productivity concept itself or (2) the accounting procedures used by (and the nature of resource pricing procedures open to) farmers, procedures which differ somewhat from the "marginal productivity accounting procedure." First, the marginal productivity concept, when referring to quantities obtained as derivatives, defines the increase in value of product for each "small change" in a particular resource. This increase is always smaller than increases in total product forthcoming when all resources were increased together. The interest here, however, is in the increment to production from one resource increased alone. An increase in labor which drives the marginal product of labor from $124 to $60 in Montana does not mean that the marginal product of all labor drops to $60. The first "added" month may have had a marginal product of $124, the second added month may have had a marginal product of $115 while the "next to last" added month may have had a marginal product of $70. The 11.6 added months between the low- and high-labor groups in Montana may add an average of $100 to total farm production.

Second, the farmer's accounting procedure does not include the degree of refinement used in our calculations. He usually can buy his resources or their services at a constant price. Consequently, he can add resources and simply figure whether "taken together, the added resources were profitable." (To apply profit maximizing principles in a refined manner he would also need to use our "marginal accounting procedures" applied with even more detail.) Alabama can be used as an example. For the low- and high-capital groups, respectively, in the low-labor stratum, the marginal product of labor is only $38.80 and $81.60 per month. The marginal product of machine-crop capital services is only $0.98 and $0.75 per $1 input. These figures are low, and use of the added capital and labor would be unprofitable if the farmer's accounting procedure and land leasing or pricing arrangement caused him to pay (or impute to land) the marginal product of land. He does not have to pay a price for each acre equal to its marginal product, however. Use of the added capital, labor and land is therefore profitable; a portion of the marginal product of land can be used to reward capital services and labor.[18]

[18]First, take as an example an owner-operator who has 54 acres of cropland and can decide to use 23 acres or all of it. (The 37 and 54 acres are the quantities shown in table 24 for the low- and high-labor groups.) If he increases acreage from 23 to 54 acres, the total product is increased by $779. If he adds 2.3 months of labor and $269 in machine-crop capital services, which have marginal products of $81.60 and $0.75 respectively (on the "last whole units"), the increase in the marginal product in land (brought about by operating all of it with more labor and capital) can be used to help reward labor and capital services; he does not have to "pay

Input and Productivity of Livestock Resources; farms stratified by livestock inputs only.

Data for livestock production stratified by labor and capital thirds are shown for the respective state samples in tables 26 to 29. As in the case of crops, the entire samples have now been stratified by labor- and capital-service thirds (the value of annual inputs used on livestock and not capital investment). These groups are not identical with the parallel groups shown previously for crops. A farm falling in the high-capital, high-labor crop group may fall in the low-labor, low-capital livestock group. Each cell in these tables includes the percent of farms falling in the particular capital and labor intervals, the per-farm quantity of various resources, the marginal productivity of labor and capital services (predicted for the mean inputs of each stratum), the gross product per man and the return per $1 of capital services. Differences in inputs between states for the same labor-capital stratum are as great as for crop production. The low-labor, low-capital farms in Alabama used an average of only $210 in capital services per farm. They had an investment in livestock and equipment of only $492. Northern Iowa farms used an average of $4,235 of capital services and had an investment of $6,995. At the other extreme, Montana farms in the high-labor, high-capital group used $28,190 in capital services and had an investment of $46,992; Alabama farms in the same labor-capital group used $2,766 services and had an investment of $2,647. The ratio of capital services and investment per man were generally greatest throughout all strata for northern Iowa followed by Montana, southern Iowa and Alabama.

The marginal products (for the mean quantity of resources in each cell) are shown as the next to last and last figures in the first column for labor- and capital-services respectively. The productivity figures for labor are generally greater than those in the corresponding labor-capital stratum for crops. The same situation is true for all capital services used for livestock as compared to machine-crop services used on crops. (These figures are not strictly comparable since the marginal product of land, also a resource which provides capital services, is computed separately for crops.)

These comparisons suggest that added capital and labor resources for any one labor-capital stratum can add more to total production when used for livestock rather than for crops. In other words, the elasticity coefficients are sufficiently high for any one resource taken by itself (although not necessarily for all resources taken together) that major increases in production can be made from using a unit of resource for livestock. This statement does not imply, of course, that all units of resources have a greater productivity in livestock than in crop production. (The figures shown refer only to the mean quantities of resources of each farm group. Resource inputs smaller than these "mean quantities" may have larger products for crops than for livestock.) Livestock production depends on feeds from crops. Farmers would not invest first in crops and second in livestock if they did not believe that small quantities of resources used for crops give greater returns than the same resources used for livestock. But for mean resource inputs, the marginal productivity of added resources is greatest for livestock. It is true, however, that capital representing a new method or technique can give returns in crops as high as in livestock, even though an increase in resources of the forms now in use may give lower returns for crops than for livestock.[19]

The within-area livestock data also suggests great opportunities between groups of farms for readjusting uses of resources to increase labor productivity and income. In southern Iowa, the figures within the low-labor strata and the differences between low- and high-capital groups show this: The marginal labor productivity increased by 400 percent and gross labor productivity increased by 500 percent with an increase of about $4,845 in investment per man equivalent. In Montana the differences between the medium- and high-capital strata within the high-labor group show an increase of slightly more than 350 percent in the marginal productivity of labor and of nearly 440 percent in the gross productivity of labor, with an increase of investment by $38,546; the marginal product of capital is still $1.21 at the higher investment level.

The types of group comparisons which are of interest to a large number of farmers again are these: (1) within a labor group but horizontally between capital groups for the tables and (2) diagonally from northwest to southeast over the cells of the table. A family with a given amount of labor with which more capital can be used is concerned with changes in labor and capital productivity as more capital services are used (a horizontal movement within a labor group). A family with some underemployed labor or one that can or is willing to hire more labor is interested in returns from and productivity of resources as more labor and capital are added (a diagonal movement from the upper left to lower right corners of the tables). The productivity figures again show an increase in the marginal productivity of labor and a decrease for capital for comparisons between capital groups within a labor stratum (i. e., for movements across the cells). For comparisons "down" the cells of the tables, the marginal productivities of labor decline and those of capital increase as labor is increased relative to capital inputs. Aside from a few exceptions due to sampling variations, the gross productivities of labor and capital (the first two figures in the second column of each cell) show similar changes in magnitude. With the alternative in accounting procedures which can be used by farm operators for decision-making (outlined in the previous section for crops), adjustments in quantities and proportions of resources reflected between labor and capital groups would generally be profitable. While labor productivity increases and, in most cases, capital pro-

anything to the land," even if he does have to pay for the added labor and capital services. Thus the added $1,022, when divided among the added labor and capital, gives high returns to the use of these resources, especially since the "last units" already have marginal products of $38.40 and $0.75 respectively to which the marginal product of land can be added. If he gave $150 per month to the 2.3 months of labor, he would still have $684 to allocate to the added $269 in machine-crop services, an average return of $2.65 for each dollar of inputs. This return is high especially since it allows labor to earn $150 per month when it would otherwise be unprofitable.

19Classification of resources into categories was not sufficiently refined to allow comparisons of this nature from the estimating techniques used. As explained elsewhere in the text, estimates of returns from small quantities of resources used as a new technique likely can be made best through budgeting methods where the new technique is represented by discrete and discontinuous resource inputs.

TABLE 26. MONTANA SAMPLE STRATIFIED BY THIRDS IN TERMS OF LABOR AND CAPITAL SERVICE INPUTS IN LIVESTOCK PRODUCTION, SELECTED CHARACTERISTICS AND PRODUCTIVITY FIGURES, 1950.*

		Low capital		Medium capital		High capital	
Low labor	1.	24.3%	$12,072	9.0%	$19,666	0	...
	2.	$ 629	$ 1.33	$ 1,568	$ 1.31	...	...
	3.	$ 724	$ 2,135	$ 2,250	$ 3,149	...	...
	4.	$ 1,820	$ 1,774	$ 4,658	$ 4,852	...	...
	5.	1.9 mo.		3.1 mo.		...	
	6.	$ 2,685		$ 7,491		...	
	7.	$ 9,075		$15,075		...	
	8.	$17,321		$28,733		...	
	9.	$ 80.02		$ 130.14		...	
	10.	$ 1.18		$ 1.16		...	
Medium labor	1.	9.0%	$ 3,946	17.1%	$ 9,565	7.2%	$24,869
	2.	$ 1,891	$ 1.26	$ 4,980	$ 1.36	$15,406	$ 1.19
	3.	$ 630	$ 384	$ 1,899	$ 1,823	$ 7,688	$ 2,318
	4.	$ 2,854	$ 2,078	$ 5,249	$ 4,802	$14,589	$15,944
	5.	5.8 mo.		6.2 mo.		4.4 mo.	
	6.	$ 4,011		$ 7,527		$20,377	
	7.	$ 3,143		$ 7,018		$20,904	
	8.	$ 8,371		$14,457		$32,893	
	9.	$ 30.31		$ 64.48		$ 179.94	
	10.	$ 1.29		$ 1.23		$ 1.15	
High labor	1.	0	...	7.2%	$ 4,607	26.1%	$19,970
	2.	...	...	$ 4,542	$ 1.22	$13,106	$ 1.40
	3.	...	...	$ 1,565	$ 455	$15,084	$ 4,255
	4.	...	...	$ 6,251	$ 5,152	$28,190	$30,733
	5.	...		11.8 mo.		20.0 mo.	
	6.	...		$ 7,446		$46,992	
	7.	...		$ 3,774		$14,306	
	8.	...		$ 7,553		$28,161	
	9.	...		$ 36.54		$ 128.76	
	10.	...		$ 1.30		$ 1.21	

*The items in each capital-labor cell are, reading from top to bottom, in the first column: (1) percent of farms in the group, (2) total value of feed inputs used during year, (3) total value of all livestock inputs during year, (4) value of all capital services including feed, livestock, building, veterinary fees, etc., (5) labor used on livestock, (6) total investment in livestock resources, (7) input of capital services per man-year (total value of services divided by man-year equivalent of labor), (8) investment per man-year of labor (6 ÷ 5), (9) computed marginal product of labor from production function, (10) computed marginal product of capital services from production function. Starting in the second column of each cell, the figures are: (1) gross value of product per worker (value of product divided by man-years of labor), (2) gross value of product per $1 of capital input services for livestock (value of product divided by total value of annual capital services including feed, livestock inputs, buildings, veterinary fees, etc.), (3) average residual product of labor per man-year (gross product less value of capital inputs divided by man-years of labor) and, (4) predicted product per farm. (All computed products and marginal quantities refer to the mean inputs of resources for each cell.)

TABLE 27. NORTHERN IOWA SAMPLE STRATIFIED BY THIRDS IN TERMS OF LABOR AND CAPITAL SERVICE INPUTS IN LIVESTOCK PRODUCTION, SELECTED CHARACTERITICS AND PRODUCTIVITY FIGURES, 1950.*

		Low capital		Medium capital		High capital	
Low labor	1.	20.0%	$17,584	8.6%	$31,574	5.0%	$55,321
	2.	$ 2,565	$ 1.26	$ 5,749	$ 1.06	$ 7,726	$ 1.08
	3.	$ 1,388	$ 2,469	$ 3,812	$ 331	$10,340	$ 1,964
	4.	$ 4,235	$ 4,900	$ 9,963	$10,740	$18,717	$19,156
	5.	3.6 mo.		4.0 mo.		4.4 mo.	
	6.	$ 6,995		$ 9,993		$15,632	
	7.	$13,964		$29,762		$51,219	
	8.	$23,064		$29,555		$42,778	
	9.	$ 103.55		$ 205.60		$ 335.86	
	10.	$ 1.05		$ 0.98		$ 0.93	
Medium labor	1.	10.0%	$10,222	15.0%	$18,110	8.6%	$30,256
	2.	$ 2,884	$ 1.23	$ 5,633	$ 1.16	$ 9,662	$ 1.05
	3.	$ 1,698	$ 1,274	$ 3,180	$ 1,605	$ 7,199	$ 244
	4.	$ 4,863	$ 5,852	$ 9,316	$10,563	$17,424	$18,664
	5.	7.0 mo.		7.1 mo.		7.3 mo.	
	6.	$ 7,328		$10,045		$15,811	
	7.	$ 8,320		$15,660		$28,709	
	8.	$12,537		$16,880		$26,051	
	9.	$ 64.16		$ 113.80		$ 197.07	
	10.	$ 1.09		$ 1.03		$ 0.97	
High labor	1.	3.6%	$ 6,004	10.0%	$10,089	19.3%	$25,673
	2.	$ 2,935	$ 1.27	$ 6,248	$ 1.19	$13,621	$ 1.08
	3.	$ 1,579	$ 790	$ 3,127	$ 1,173	$13,380	$ 841
	4.	$ 4,942	$ 7,281	$ 9,832	$11,679	$27,839	$30,019
	5.	12.6 mo.		14.0 mo.		14.0 mo.	
	6.	$10,502		$10,654		$22,487	
	7.	$ 4,714		$ 8,458		$23,868	
	8.	$10,017		$ 9,165		$19,279	
	9.	$ 44.51		$ 64.38		$ 164.94	
	10.	$ 1.34		$ 1.08		$ 0.99	

*For identification of figures see footnote for table 26.

TABLE 28. SOUTHERN IOWA SAMPLE STRATIFIED BY THIRDS IN TERMS OF LABOR AND CAPITAL SERVICE INPUTS IN LIVESTOCK PRODUCTION, SELECTED CHARACTERISTICS AND PRODUCTIVITY FIGURES, 1950.*

		Low capital		Medium capital		High capital	
Low labor	1.	12.6%	$10,613	14.0%	$17,184	7.0%	$50,624
	2.	$ 1,709	$ 1.19	$ 3,699	$ 1.13	$ 8,554	$ 1.12
	3.	$ 987	$ 827	$ 2,031	$ 1,110	$ 4,678	$ 3,278
	4.	$ 2,838	$ 3,279	$ 5,960	$ 6,962	$13,770	$15,392
	5.	3.8 mo.		4.7 mo.		3.7 mo	
	6.	$ 5,473		$ 6,826		$14,358	
	7.	$ 8,925		$15,204		$45,000	
	8.	$17,211		$17,413		$46,922	
	9.	$ 100.09		$ 172.53		$ 498.02	
	10.	$ 1.14		$ 1.15		$ 1.10	
Medium labor	1.	11.2%	$ 8,555	10.5%	$13,196	11.9%	$27,460
	2.	$2,038	$ 1.42	$ 4,108	$ 1.19	$ 7,445	$ 1.13
	3.	$ 1,173	$ 2,053	$ 1,920	$ 1,396	$ 5,066	$ 3,995
	4.	$ 3,399	$ 4,185	$ 6,314	$ 7,698	$12,909	$15,556
	5.	6.8 mo.		6.8 mo.		6.9 mo.	
	6.	$ 5,369		$ 8,472		$12,138	
	7.	$ 6,027		$11,058		$22,411	
	8.	$ 9,520		$14,837		$21,073	
	9.	$ 72.09		$ 131.09		$ 262.19	
	10.	$ 1.21		$ 1.20		$ 1.18	
High labor	1.	9.8%	$ 5,026	9.1%	$ 7,400	14.0%	$16,095
	2.	$ 2,118	$ 1.23	$ 3,792	$ 1.15	$ 7,973	$ 1.18
	3.	$ 888	$ 503	$ 2,060	$ 563	$ 5,817	$ 1,654
	4.	$ 3,174	$ 4,061	$ 6,140	$ 7,956	$14,404	$18,595
	5.	9.3 mo.		11.5 mo.		12.7 mo.	
	6.	$ 5,602		$ 9,223		$17,489	
	7.	$ 4,101		$ 6,416		$13,614	
	8.	$ 7,238		$ 9,637		$16,530	
	9.	$ 50.95		$ 80.77		$ 170.72	
	10.	$ 1.26		$ 1.27		$ 1.27	

*For identification of figures see footnote for table 26.

TABLE 29. ALABAMA SAMPLE STRATIFIED BY THIRDS IN TERMS OF LABOR AND CAPITAL SERVICE INPUTS IN LIVESTOCK PRODUCTION, SELECTED CHARACTERISTICS AND PRODUCTIVITY FIGURES, 1950.*

		Low capital		Medium capital		High capital	
Low labor	1.	21.1%	$ 3,034	6.0%	$ 5,177	6.8%	$16,840
	2.	$ 135	$ 1.59	$ 350	$ 1.10	$ 1,213	$ 1.02
	3.	$ 53	$ 891	$ 126	$ 9	$ 678	$ -378
	4.	$ 210	$ 310	$ 526	$ 614	$ 1,964	$ 1,662
	5.	1.3 mo.		1.3 mo.		1.4 mo.	
	6.	$ 492		$ 993		$ 1,830	
	7.	$ 1,913		$ 4,722		$16,444	
	8.	$ 4,484		$ 8,906		$15,325	
	9.	$ 54.81		$ 107.19		$ 270.70	
	10.	$ 1.10		$ 0.87		$ 0.63	
Medium labor	1.	12.8%	$ 2,350	13.5%	$ 3,540	6.8%	$ 7,508
	2.	$ 144	$ 1.87	$ 332	$ 1.30	$ 726	$ 1.23
	3.	$ 76	$ 917	$ 154	$ 598	$ 529	$ 1,014
	4.	$ 257	$ 415	$ 516	$ 685	$ 1,325	$ 1,427
	5.	2.4 mo.		2.3 mo.		2.6 mo.	
	6.	$ 727		$ 786		$ 1,750	
	7.	$ 1,259		$ 2,734		$ 6,089	
	8.	$ 3,563		$ 4,195		$ 8,042	
	9.	$ 39.62		$ 70.52		$ 127.56	
	10.	$ 0.80		$ 0.99		$ 0.80	
High labor	1.	...	...	13.5%	$ 1,695	19.6%	$ 5,643
	2.	...	...	$ 305	$ 1.37	$ 1,593	$ 1.35
	3.	...	...	$ 204	$ 327	$ 1,027	$ 1,263
	4.	...	...	$ 555	$ 884	$ 2,766	$ 3,197
	5.	...		5.4 mo.		7.9 mo.	
	6.	...		$ 1,141		$ 2,647	
	7.	...		$ 1,242		$ 4,180	
	8.	...		$ 2,551		$ 4,000	
	9.	...		$ 38.40		$ 93.95	
	10.	...		$ 1.18		$ 0.86	

*For identification of figures see footnote for table 26.

ductivity declines for these "across" and "diagonal" comparisons, the marginal productivity of capital is still sufficiently above its cost in Montana and southern Iowa to merit use of more of this resource; labor productivity increases materially.

Labor productivity increases by large amounts under these "across" and "diagonal" comparisons for Alabama and northern Iowa, but the marginal return for capital is less than its cost for farms using the extreme amounts of capital. Two types of phenomena may explain this decline in productivity for capital in Alabama and northern Iowa. The techniques used in livestock production in Alabama were "less efficient" (in a purely physical sense) than those of other areas; increased capital would still give low returns unless invested in new techniques. While the techniques were (physically) at a "higher level" in northern Iowa, the quantity of capital used, as an average in all farm groups, was relatively high; extended use of capital would be expected to accompany a lower return than for other areas. However, the pricing mechanisms require only that farmers pay the market wage rate for labor. Consequently, since unpaid and unemployed family labor may be on hand, an increase in use of capital which boosts labor productivity sufficiently can cause use of added capital to be profitable.

The type of accounting procedure allowing this inference supposes that a semi-complementary relationship exists between capital and labor; if more of one resource is used, more of the other may be used. Technical complementarity does exist if wide adjustments are made in capital ratios. However, the wide differences in ratios of capital and labor between groups displayed in tables 26 to 29 illustrate that these resources need not be used in combinations denoting technical complementarity.[20] Also, statistical tests did not denote "fixed proportions" in the use of capital and labor.

In a total economy, labor serves as a limiting resource in increasing the national product, measured either in civilian or defense goods or a combination of the two. Hence, interest may focus on comparison of farms falling in the low-labor, low-capital category with those of low- or medium-labor and high-capital. How much can the productivity of labor on small farms with a large amount of labor and a small amount of capital be increased as labor is withdrawn from farms and capital is added?[21] Adjustments of this general nature would allow large increases in either the marginal or gross productivity of labor. The returns for capital could remain at a high level. Northern Iowa is one of the agricultural areas where the relatively favorable capital and income situations of farmers have allowed them to accumulate capital and combine it with labor in a manner more nearly approximating the "stability" conditions of production than for other areas. Alabama represents the other extreme: Adjustment opportunities are great if added capital is in the form of known and improved techniques for producing livestock. Southern Iowa is somewhat representative of a broad area in the southern Corn Belt where addition of capital and changes in the capital/labor ratios can increase labor productivity in the manner suggested by comparison of the extremes of the southwest and northeast cells of table 28. Budgeting studies can be used to point out these alternatives.

Input and Productivity of Total Farm Resources; farms stratified separately by labor and capital services used on both crops and livestock.

The comparisons in the two previous sections showed resource inputs and productivity coefficients for farms classed by labor and capital services. Crops and livestock were considered separately. We now classify farms by thirds in terms of labor and capital services used both for crops and livestock. Inputs are added for the two products and classification is in terms of these totals. The resulting figures are presented for the nine labor-capital groups of each area in tables 30 to 33. These descriptive figures include only resource combinations and resource productivities computed by simple arithmetic methods; marginal productivities are not estimated for the resulting resource combinations.[22]

Aside from sampling variations, the capital/labor ratio increases across capital groups within a labor stratum and decreases across labor groups within a capital stratum. In Alabama, southern Iowa and northern Iowa, the residual product of labor increases between capital groups as the capital/labor ratio increases in magnitude. The same is true of Montana, except for the high-labor, high-capital group. This group evidently included ranches where (1) livestock was relatively more important as an income source than crops but (2) livestock returns were lower than for crops. The residual return to capital tends (although less clearly in Montana than in the other areas) to decline as more labor is added (i. e., within a capital group but between labor groups); these changes in proportions are expected to increase capital productivity.

The "computed" decline undoubtedly grows out of the use of the "conventional imputation procedure"; namely, subtracting a wage for labor and imputing the remainder to capital. With (1) a wage charge above the marginal productivity of labor and (2) a diminishing productivity of labor as more is used relative to capital, "use" of more labor leaves a diminishing quantity to be allocated to a given amount of capital even if its productivity is constant. It is difficulties such as these which give rise to the need for examining alternative procedures, such as the marginal analysis of this study, for estimating productivity coefficients.

20In this sense, the final task of the farm operation in maximizing profits, if optimum quantities and proportions of resources are to be used, is to add to each specific category of resource as long as its marginal value productivity is greater than its cost. As pointed out later, capital limitations and other considerations prevent this "complete adjustment."

21Another possibility is that both less labor and less capital can be used to produce the same or a greater product on many farms. This possibility exists where small units can be consolidated and, with two or more operated by one family, power units, machinery, buildings and equipment need not be duplicated. The capital otherwise needed for these things then can be used for more livestock, fertilizer, seed or resources representing improved techniques for crops or livestock.

22Generally, the relationships appearing in previous tables also appear in those immediately following. However, aggregation of crop and livestock production into one activity does cause some "loss of information." For example, we might use an area where added capital is associated (for ratios computed by arithmetic procedures) with an increased productivity of labor on crops but a decreased productivity of labor on livestock because of the prices used for "charging" one resource to compute the productivity of another. When we add the two enterprises together, the productivity figures may show either a zero return or a constant return on capital. The resulting inference might then be that no capital should be added to farms of the area. The more detailed figures might show, however, that capital should be added for crops but subtracted from livestock.

TABLE 30. RESOURCE INPUTS, RESOURCE RATIOS AND SELECTED PRODUCTIVITY FIGURES, CROPS AND LIVESTOCK. MONTANA SAMPLE STRATIFIED BY CAPITAL AND LABOR THIRDS, 1950.

Item*	Low labor			Medium labor			High labor		
	Low capital	Medium capital	High capital	Low capital	Medium capital	High capital	Low capital	Medium capital	High capital
1. Percent farms (%)	23.8	9.9	0	8.6	15.2	9.3	1.3	8.0	23.8
2. Total production ($)	12,070	31,104	....	10,045	26,261	35,646	17,374	29,161	58,513
3. Acres all land	727	1,493	....	875	1,874	2,210	1,300	2,020	5,286
4. Labor on crops (mo.)	6.3	9.4	....	11.4	14.4	12.0	16.0	19.4	22.0
5. Value land services for crops ($)	1,122	3,157	....	948	2,963	3,938	1,025	2,269	5,540
6. Value all crop capital services ($)	3,743	8,810	....	3,571	7,982	10,600	3,646	7,438	13,792
7. Value all crop services ($)	5,215	11,395	....	5,986	11,179	13,281	6,574	11,727	18,868
8. Value of livestock feed ($)	344	450	....	1,194	985	3,006	1,915	1,642	6,652
9. Value of livestock services ($)	408	778	....	1,165	1,070	4,844	1,214	1,417	12,292
10. Value all capital services for livestock ($)	782	1,253	....	2,451	2,141	8,171	3,182	3,195	19,453
11. Labor on livestock (mo.)	1.3	1.4	....	6.3	3.5	6.3	8.9	8.1	15.5
12. All services on livestock ($)	1,075	1,638	....	3,788	2,923	9,579	4,808	4,991	23,035
13. Land investment ($)	22,446	63,142	....	18,951	59,254	78,753	20,500	45,383	110,800
14. Machine investment ($)	6,083	11,608	....	6,091	13,491	16,085	5,975	14,002	21,576
15. Livestock investment ($)	742	1,829	....	2,608	4,433	11,039	3,305	5,076	28,469
16. Total farm investment ($)	31,138	79,175	....	30,821	83,979	120,519	32,555	71,037	177,898
17. Total labor (mo.)	7.6	10.8	....	18.7	18.0	18.3	24.8	27.5	37.6
18. Value all labor ($)	1,764	2,970	....	3,753	3,979	4,090	4,553	6,084	8,658
19. Value all capital services ($)	4,526	10,063	....	6,022	10,123	18,770	6,828	10,633	33,245
20. Value all services ($)	6,290	13,033	....	9,774	14,102	22,860	11,381	16,717	41,903
21. Investment per man ($)	49,230	87,979	....	20,838	56,065	79,091	15,731	31,013	56,828
22. Land per man (acre)	1,149	1,659	....	592	1,251	1,450	628	882	1,688
23. Value crops per acre cropland ($)	19.95	26.20	....	15.21	21.87	21.73	22.56	22.81	23.09
24. Product per ($) all services ($)	1.92	2.39	....	1.03	1.86	1.56	1.53	1.74	1.40
25. Gross product of labor ($)	11,927	23,380	....	2,720	10,774	11,075	5,096	8,089	8,072
26. Average residual product labor ($)	11,231	22,488	....	2,319	9,548	9,703	4,803	7,527	7,001
27. Average residual return on investment (%)	22.2	26.8	....	24.5	18.0	13.9	21.6	20.7	12.5

* Those items, which are not self explanatory in the table, are computed as follows: (5) rental value of land used for crops; (6) value (or annual expenses) for machinery repairs and depreciation, fuel, power, seed, fertilizer and all other annual services used for crops; (7) value of labor, land rental and capital services (expenses) used in crop production; (9) expenses for livestock including purchase of feeder animals and depreciation on breeding stock; (10) value of feed, livestock and all other capital services (expenses) used on livestock; (12) item 10 plus the value of labor used on livestock; (19) rental value of land, and all annual services (expenses included depreciation) for all forms of capital; (20) item 19 plus the wage value of labor; (24) total value of production divided by item 20; (25) total value of production divided by the man-years of labor; (26) total value of production less a rental charge for land and interest charges for capital divided by man-years of labor; (27) total value of production less a wage return to labor divided by total investment.

TABLE 31. RESOURCE INPUTS, RESOURCE RATIOS AND SELECTED PRODUCTIVITY FIGURES, CROPS AND LIVESTOCK. NORTHERN IOWA SAMPLE STRATIFIED BY CAPITAL AND LABOR THIRDS, 1950.

Item*	Low labor			Medium labor			High labor		
	Low capital	Medium capital	High capital	Low capital	Medium capital	High capital	Low capital	Medium capital	High capital
1. Percent farms (%)	17.6	12.7	3.5	12.0	12.7	8.5	4.2	7.8	21.1
2. Total production ($)	11,466	19,070	29,237	11,728	19,344	29,287	12,197	20,513	41,705
3. Acres all land	148	182	186	123	189	264	164	212	296
4. Labor on crops (mo.)	7.4	6.7	7.3	8.6	8.5	9.9	17.4	11.0	12.0
5. Value land services for crops ($)	1,635	1,944	1,636	1,171	1,876	2,790	1,928	2,016	3,464
6. Value all crop capital services ($)	3,265	3,760	3,327	2,650	3,823	5,413	3,966	3,988	6,812
7. Value all crop services ($)	4,780	5,106	4,748	4,334	5,431	7,509	7,019	5,933	9,030
8. Value of livestock feed ($)	2,260	5,690	6,834	3,151	5,831	8,965	2,598	6,068	13,263
9. Value of livestock services ($)	1,407	3,212	1,164	1,685	3,343	8,220	1,019	3,012	12,302
10. Value all capital services for livestock ($)	3,934	9,342	18,806	5,113	9,686	17,791	3,927	9,518	26,414
11. Labor on livestock (mo.)	3.8	5.7	4.3	7.0	7.7	6.6	9.0	14.7	13.1
12. All services on livestock ($)	4,724	10,490	19,645	6,496	11,144	19,123	5,164	12,115	28,835
13. Land investment ($)	32,706	38,885	32,720	23,423	37,526	55,795	38,551	40,318	69,276
14. Machine investment ($)	4,118	4,974	5,090	3,865	5,180	6,456	5,965	5,322	8,414
15. Livestock investment ($)	2,399	4,508	8,855	2,620	5,326	8,028	2,937	5,067	12,677
16. Total farm investment ($)	43,185	53,046	50,265	34,243	53,736	77,874	52,544	56,530	100,264
17. Total labor (mo.)	11.2	12.3	11.6	15.6	16.2	16.4	26.4	25.6	25.0
18. Value all labor ($)	2,305	2,494	2,260	3,067	3,066	3,329	4,290	4,543	4,639
19. Value all capital services ($)	7,199	13,102	22,133	7,762	13,509	23,204	7,893	13,506	33,226
20. Value all services ($)	9,504	15,596	24,393	10,829	16,574	26,532	12,183	18,048	37,865
21. Investment per man ($)	46,187	51,634	51,820	26,340	39,854	56,867	25,841	26,452	48,056
22. Land per man (acre)	159	177	192	95	141	193	80	99	142
23. Value crops per acre cropland ($)	50.38	49.73	47.93	45.79	49.74	47.90	53.77	46.54	55.11
24. Product per ($) all services ($)	1.21	1.22	1.21	1.08	1.17	1.10	1.00	1.14	1.10
25. Gross product of labor ($)	4,564	5,810	7,849	3,050	4,329	4,442	2,117	3,279	4,064
26. Average residual product labor ($)	4,004	5,123	6,586	2,635	3,728	3,637	1,773	2,900	3,322
27. Average residual return on investment (%)	8.3	10.2	13.2	6.1	8.7	7.1	3.7	7.9	7.3

* See footnote for table 30 for more complete definitions of items.

TABLE 32. RESOURCE INPUTS, RESOURCE RATIOS AND SELECTED PRODUCTIVITY FIGURES, CROPS AND LIVESTOCK. SOUTHERN IOWA SAMPLE STRATIFIED BY CAPITAL AND LABOR THIRDS, 1950.

Item*	Low labor			Medium labor			High labor		
	Low capital	Medium capital	High capital	Low capital	Medium capital	High capital	Low capital	Medium capital	High capital
1. Percent farms (%)	16.8	8.4	8.4	9.8	14.7	9.1	7.0	10.5	15.4
2. Total production ($)	6,876	11,994	20,671	6,622	12,154	22,399	7,832	12,654	26,649
3. Acres all land	118	185	208	137	175	213	154	230	210
4. Labor on crops (mo.)	5.8	6.9	6.1	7.6	7.6	8.0	11.1	12.7	12.8
5. Value land services for crops ($)	537	1,018	1,598	428	759	1,308	521	1,098	1,621
6. Value all crop capital services ($)	1,246	2,167	3,641	1,255	1,941	3,253	1,561	2,734	3,947
7. Value all crop services ($)	2,436	3,589	4,941	2,778	3,478	4,896	3,658	5,061	6,285
8. Value of livestock feed ($)	2,035	3,635	7,400	1,871	4,014	7,158	2,077	3,572	8,651
9. Value of livestock services ($)	1,013	2,060	3,123	1,228	2,151	5,121	1,080	1,852	6,380
10. Value all capital services for livestock ($)	3,196	5,943	10,991	3,245	6,444	12,713	3,329	5,707	15,637
11. Labor on livestock (mo.)	5.5	4.7	5.3	6.7	6.7	6.6	9.1	9.5	11.6
12. All services on livestock ($)	4,328	6,911	12,131	4,594	7,791	14,065	5,056	7,417	17,755
13. Land investment ($)	10,741	20,365	31,965	8,560	15,181	26,158	10,422	21,959	32,418
14. Machine investment ($)	1,499	3,052	5,425	1,596	3,438	5,270	3,316	4,428	5,566
15. Livestock investment ($)	2,729	3,195	6,071	3,020	4,260	8,864	2,732	4,990	10,343
16. Total farm investment ($)	17,178	30,069	49,378	15,172	26,872	45,076	19,094	35,815	41,551
17. Total labor (mo.)	11.4	11.6	11.4	14.3	14.3	14.6	20.2	22.2	24.3
18. Value all labor ($)	2,322	2,389	2,439	2,873	2,884	2,995	3,825	3,992	4,456
19. Value all capital services ($)	4,442	8,111	14,632	4,500	8,385	15,966	4,890	8,442	19,584
20. Value all services ($)	6,763	10,500	17,072	7,373	11,269	18,961	8,714	12,434	24,040
21. Investment per man ($)	18,162	31,039	51,827	12,713	22,618	36,913	11,348	19,382	27,455
22. Land per man (acre)	123	191	218	115	148	174	92	124	153
23. Value crops per acre cropland ($)	38.67	43.19	52.01	33.72	37.05	43.51	43.59	45.19	44.21
24. Product per ($) all services ($)	1.02	1.14	1.21	0.90	1.08	1.18	0.90	1.02	1.11
25. Gross product of labor ($)	2,574	4,009	6,388	1,778	3,172	5,268	1,748	2,279	3,484
26. Average residual product labor ($)	2,233	3,508	5,423	1,501	2,680	4,493	1,490	1,904	2,911
27. Average residual return on investment (%)	3.8	8.4	10.5	-2.1	6.1	10.5	-1.9	3.7	7.6

* See footnote for table 30 for more complete definitions of items.

TABLE 33. RESOURCE INPUTS, RESOURCE RATIOS AND SELECTED PRODUCTIVITY FIGURES, CROPS AND LIVESTOCK. ALABAMA SAMPLE STRATIFIED BY CAPITAL AND LABOR THIRDS, 1950.

Item*	Low labor			Medium labor			High labor		
	Low capital	Medium capital	High capital	Low capital	Medium capital	High capital	Low capital	Medium capital	High capital
1. Percent farms (%)	18.0	12.0	3.7	13.5	7.5	12.0	2.3	13.5	17.3
2. Total production ($)	837	1,741	3,121	1,352	2,063	4,248	2,091	2,446	5,948
3. Acres all land	34	41	59	27	44	53	28	47	80
4. Labor on crops (mo.)	4.2	4.6	4.6	10.4	10.0	10.1	16.2	20.7	13.9
5. Value land services for crops ($)	85	120	167	62	85	146	54	99	231
6. Value all crop capital services ($)	283	562	784	379	594	885	479	645	1,318
7. Value all crop services ($)	694	1,031	1,251	1,416	1,593	1,940	2,101	2,713	2,688
8. Value of livestock feed ($)	194	322	906	141	333	997	135	318	1,613
9. Value of livestock services ($)	68	214	671	70	114	715	102	176	960
10. Value all capital services for livestock ($)	288	587	1,626	240	475	1,796	243	522	2,735
11. Labor on livestock (mo.)	1.8	3.1	2.2	1.9	2.9	3.5	3.0	2.7	8.5
12. All services on livestock ($)	462	901	1,846	428	764	2,164	542	790	2,549
13. Land investment ($)	1,712	2,410	3,336	1,241	1,700	2,929	1,073	1,989	4,610
14. Machine investment ($)	79	137	965	118	335	378	64	178	1,097
15. Livestock investment ($)	296	580	894	297	445	966	235	585	1,804
16. Total farm investment ($)	2,359	3,674	5,595	1,967	2,812	5,094	1,424	3,063	8,636
17. Total labor (mo.)	6.0	7.7	6.9	12.3	12.9	13.7	19.2	23.3	22.4
18. Value all labor ($)	585	784	687	1,224	1,288	1,423	1,921	2,335	2,183
19. Value all capital services ($)	571	1,148	2,410	619	1,070	2,681	722	1,168	4,053
20. Value all services ($)	1,156	1,932	3,097	1,843	2,357	4,104	2,643	3,502	6,236
21. Investment per man ($)	4,719	5,713	9,781	1,916	2,609	4,471	888	1,574	4,687
22. Land per man (acre)	68	64	102	26	40	47	17	24	43
23. Value crops per acre cropland ($)	19.71	35.72	36.21	39.88	52.07	49.11	50.10	51.10	48.83
24. Product per ($) all services ($)	0.72	0.90	1.01	0.73	0.88	1.04	0.79	0.70	0.95
25. Gross product of labor ($)	704	1,108	1,534	774	1,000	1,504	887	708	1,154
26. Average residual product labor ($)	468	823	1,045	679	869	1,281	843	629	919
27. Average residual return on investment (%)	-9.9	-1.9	3.4	-21.8	-7.5	5.7	-35.0	-31.2	-0.7

* See footnote for table 30 for more complete definitions of items.

Input and Productivity of Total Farm Resources; farms stratified by labor alone and capital alone.

Tables 34 to 37 provide capital and labor comparisons on an even more aggregative basis than tables 30 to 33. In the tables which follow, farms are grouped by thirds in terms of labor alone and then in terms of capital alone; capital is allowed to "increase" with labor over the three labor strata while labor is allowed to "increase" with capital over the three capital strata. Actually, these figures represent the means of the various labor and capital groups from tables 30 to 33; labor groups are not subsorted by capital while capital groups are not subsorted by labor in tables 34 to 37.

Residual labor productivity declines between all labor groups in Montana, southern Iowa and northern Iowa but increases between the first and second groups in Alabama. Similarly, residual capital productivity declines between labor groups except for the same case in Alabama. Taken together these residual figures, derived by simple computational procedures, suggest "decreasing returns to scale" and a declining productivity of resources for the farm as a whole as total resource input increases. The productivity of capital between the first and second capital groups generally increases, however, suggesting that increasing returns to scale and increasing productivity hold true for farms with small capital even when crop and livestock production is aggregated into "farm production." The residual computational procedures and the pricing problems mentioned earlier cause the computed residual labor productivity to increase similarly even though diminishing productivity may actually hold true. Further suggestion of diminishing (1) "returns to scale" and (2) resource productivity for crops and livestock aggregated into "farm production" and with resource proportions deviating from a "true scale line" is reflected in the value of product per $1 input of all resources (with labor, land and capital interest returns included with annual expenses in computing "total services"). The figures (item 34 in the tables) decline between labor groups, aside from the Alabama exception.

The figures showing return per $1 input of all services (line 34 of the tables above) point up clearly the differences in "aggregative" productivity of resources relative to the existing market prices for the same services. In Montana the return for each $1 input of all resources was high in all three capital or labor groups; the same situation held true in northern Iowa. Returns were considerably lower for parallel labor or capital groups in southern Iowa. Resource costs were greater than resource returns in all Alabama groups.

Relative Inputs

The relative proportion of inputs coming from different categories of resources is suggested for the different capital strata of each area in table 38. These figures again emphasize how the proportioning of resources differs between farms, depending on their capital position. Farms with few funds use relatively more labor and less capital, either in the form of services from land or other items. This procedure is followed largely because labor is "cheap" and capital is "dear." The cost of credit, when it can be obtained, is not great to the low-capital farmer, but it is "dear" in terms of the rate at which he discounts returns because of his financial position. (He may also fail to use borrowed capital because of the kinds of values he attaches to "being in debt"; or credit may not be available to him due to his capital position.) Labor is "cheap" in the sense that the farmer, where off-farm employment opportunities are not favorable, has his own year-around labor and usually some from other members of the family to engage in production. Since labor has "no cost," it is profitable to substitute labor for capital even though labor has a low marginal product.

Capital-Labor Substitution

To obtain some notion of the rates at which capital services and labor substitute, the estimates in tables 39 and 40 have been derived from the production function equations mentioned in the first of this report. Figures have been derived for crops only. The opportunities for capital-labor substitution are, considering mechanization particularly, greater than for livestock. Fewer machine techniques have been developed for livestock than for crops. Since machines and capital representing biological techniques both can be substituted for labor, rates were computed between the capital category of machine-crop services and labor. While it considers both possibilities in substitution, this procedure gives "hybrid" or "average" replacement rates between capital services and labor. Either machinery increased alone or crop services increased alone might substitute for labor at rates differing from those shown. However, interest here is in substitution rates as "averages" for capital services in general rather than in substitution ratios for specific forms of capital.

The figures in columns 2 and 3 of table 39 show the quantity of capital services and labor which are predicted to produce the "average" product found in the farm samples, when labor inputs are at various levels relative to the mean quantities now used. With a 10-percent reduction in labor per farm in Montana (a labor input of 90 percent) $5,244 in capital services and 12.4 months of labor are expected to produce the same product as $5,206 in capital services and 13.8 months of labor, the mean quantities found in the sample. Given the production functions used, substitution is at diminishing rates for all areas. Increasing quantities of capital services are necessary to substitute for each additional month of labor. This fact is illustrated by the figures of column 5. The substitution quantities show the quantity of capital services necessary to substitute for the amount of labor replaced with production constant and labor and capital services combined in the proportions of columns 2 and 3. In northern Iowa, the amount increases from $107 between the combinations of (1) labor at 9.4 months and machine-crop services at $2,169 and (2) labor at 8.5 months and capital services at $2,276; it increases to $126 between the next two combinations. (The substitution rates shown refer to the differences between combinations in

TABLE 34. MONTANA SAMPLE GROUPED SEPARATELY BY LABOR THIRDS AND CAPITAL THIRDS FOR TOTAL PRODUCTION, SELECTED INPUTS AND PRODUCTIVITY RATIOS, 1950.

Item*	Labor group			Capital group			All farms
	low	medium	high	low	medium	high	
1. Percent of farms	33.8	33.1	33.1	33.8	33.1	33.1	100
2. Total product	17,668	24,673	49,882	11,762	28,410	52,110	30,634
3. Cropland (acre)	722.0	939.2	1,268.6	536.1	1,076.5	1,320.9	974.9
4. Pasture land (acre)	230.3	769.2	3,073.8	251.1	718.2	3,103.5	1,350.3
5. Total land (acre)	952.2	1,708.4	4,342.4	787.2	1,794.8	4,424.4	2,325.2
6. Labor on crops (mo.)	7.2	13.0	21.1	8.0	14.1	19.2	13.7
7. Value of land services ($)	1,721	2,712	4,574	1,074	2,855	5,091	2,994
8. Value of machine services ($)	2,632	3,729	5,746	2,004	3,986	6,129	4,026
9. Value of crop services ($)	881	1,127	1,541	618	1,259	1,677	1,181
10. Value of all crop capital services ($)	5,233	7,568	11,861	3,696	8,100	12,898	8,201
11. Value of all crop services including labor ($)	7,029	10,415	16,669	5,451	11,360	17,305	11,334
12. Feed ($)	375	1,605	5,260	622	982	5,631	2,400
13. Livestock input ($)	517	2,151	9,239	633	1,066	10,206	3,946
14. Other livestock input ($)	29	153	401	47	80	456	193
15. Value of all livestock capital services ($)	921	3,910	14,900	1,302	2,127	16,294	6,540
16. Labor on livestock (mo.)	1.3	5.0	13.5	2.8	4.0	13.0	6.6
17. Value of all livestock services ($)	1,244	5,014	17,968	1,926	3,049	19,265	8,038
18. Land investment ($)	34,416	54,235	91,488	21,479	57,092	101,826	59,876
19. Machine investment ($)	7,708	12,293	19,134	6,081	13,049	20,039	13,010
20. Livestock investment ($)	1,061	5,808	21,848	1,318	3,806	23,588	9,516
21. Building investment ($)	1,673	6,598	10,630	1,774	4,460	12,665	6,270
22. Total investment ($)	45,267	80,389	146,436	31,113	79,432	161,830	90,396
23. Total labor (mo.)	8.5	18.0	34.6	10.9	18.1	32.2	20.3
24. Value of all labor ($)	2,119	3,951	7,876	2,380	4,182	7,379	4,632
25. Value of all services ($)	8,273	15,429	34,637	7,378	14,409	36,571	19,372
26. Percent labor on crops	84.76	72.04	61.05	73.76	77.95	59.73	67.64
27. Return to investment at 5 percent ($)	2,263	4,019	7,322	1,556	3,972	8,092	4,520
28. Investment per man ($)	63,652	53,584	50,733	34,392	52,647	60,371	53,403
29. Crop capital services per man ($)	8,654	7,651	9,271	5,524	6,779	10,890	8,708
30. Total land per man (acre)	1,339	1,139	1,504	870	1,190	1,651	1,374
31. Crop product per cropland ($)	22.83	20.94	23.02	19.00	23.46	22.75	22.31
32. Product per $1 all services, crops ($)	2.345	1.889	1.752	1.868	2.223	1.736	1.919
33. Product per $1 all services, livestock ($)	0.953	0.998	1.147	0.818	1.036	1.145	1.105
34. Product per $1 all services, all products ($)	2.136	1.599	1.438	1,594	1.972	1.425	1.581
35. Average residual product of labor ($ per man-year)	15,432	7,925	7.038	6,941	11,309	7,429	8,487
36. Average residual return on investment (%)	24.56	14.87	13.49	17.54	21.22	12.75	15.77
37. Average marginal residual product of labor ($ return added per added month)	....	97	507	....	....	....	....
38. Average marginal residual return on investment ($ return added per added $ capital investment)	....	....	....	....	23.59	4.58	....

* Methods of computation have been explained in footnotes of previous tables for the last two items. Average marginal residual product of labor is computed thus: The added product, added labor input and added capital have been computed between labor groups. An interest charge for the added capital quantity has then been subtracted from the added product. The remaining product has been divided by the added months of labor, to give the "added product per added unit of labor." The average marginal return for capital has been computed similarly but with this exception: A charge for the added labor has been subtracted from the added product between capital groups; the remainder has then been divided by the added capital, between capital groups.

TABLE 35. NORTHERN IOWA SAMPLE GROUPED SEPARATELY BY LABOR THIRDS AND CAPITAL THIRDS FOR TOTAL PRODUCTION, SELECTED INPUTS AND PRODUCTIVITY RATIOS, 1950.

Items*	Labor group			Capital group			All farms
	low	medium	high	low	medium	high	
1. Percent of farms	33.8	33.1	33.1	33.8	33.1	33.1	100
2. Total product	16,185	19,128	32,979	11,650	19,513	37,226	22,718
3. Cropland (acre)	148.9	158.8	222.9	127.2	168.7	235.1	176.7
4. Pasture land (acre)	16.0	25.6	36.6	14.0	23.2	41.0	26.0
5. Total land (acre)	164.8	184.4	259.5	141.3	191.9	276.1	202.7
6. Labor on crops (mo.)	7.1	8.9	12.4	9.1	8.4	10.9	9.4
7. Value of land services ($)	1,751	1,854	2,929	1,507	1,935	3,097	2,175
8. Value of machine services ($)	1,223	1,501	2,077	1,199	1,461	2,142	1,598
9. Value of crop services ($)	482	449	782	428	441	845	570
10. Value of all crop capital services ($)	3,457	3,805	5,788	3,155	3,837	6,084	4,344
11. Value of all crop services including labor ($)	4,898	5,537	8,052	4,905	5,416	8,153	6,147
12. Feed ($)	4,023	5,662	10,218	2,618	5,833	11,482	6,616
13. Livestock input ($)	3,148	3,989	8,688	1,457	3,215	11,188	5,260
14. Other livestock input ($)	341	450	684	276	467	734	491
15. Value of all livestock capital services ($)	7,511	10,101	19,589	4,350	9,515	23,404	12,366
16. Labor on livestock (mo.)	4.6	7.2	12.7	5.4	8.6	10.5	8.1
17. Value of all livestock services ($)	8,441	11,502	21,897	5,403	11,128	25,386	13,915
18. Land investment ($)	35,024	37,090	58,578	30,148	38,700	61,947	43,503
19. Machine investment ($)	4,540	5,031	7,378	4,259	5,135	7,561	5,642
20. Livestock investment ($)	3,862	5,037	9,653	2,544	4,952	11,084	6,168
21. Building investment ($)	4,193	5,692	8,329	4,235	5,339	8,639	6,058
22. Total investment ($)	47,619	52,849	83,938	41,187	54,126	89,231	61,371
23. Total labor (mo.)	11.7	16.0	25.1	14.4	16.9	21.4	16.6
24. Value of all labor ($)	2,371	3,133	4,572	2,823	3,192	4,052	3,352
25. Value of all services ($)	13,339	17,039	29,950	10,308	16,545	33,539	20,062
26. Percent labor on crops	60.77	55.28	49.52	62.72	49.46	51.07	53.79
27. Return to investment at 5 percent ($)	2,381	2,642	4,197	2,059	2,706	4,462	3,069
28. Investment per man ($)	48,928	39,552	40,132	34,278	38,389	50,000	41,934
29. Crop capital services per man ($)	11,269	10,407	12,133	6,230	9,470	16,523	11,417
30. Total land per man (acre)	169	138	124	118	136	155	138
31. Crop product per cropland ($)	49.86	48.11	53.31	49.50	48.90	52.83	50.78
32. Product per $1 all services, crops ($)	1.515	1.380	1.476	1.284	1.523	1.523	1.459
33. Product per $1 all services, livestock ($)	1.038	0.999	0.963	0.991	1.012	0.977	0.988
34. Product per $1 all services, all products ($)	1.213	1.123	1.101	1.130	1.179	1.110	1.132
35. Average residual product of labor ($ per man-year)	4,714	3,320	3,027	3,006	3,823	3,571	3,493
36. Average residual return on investment (%)	9.65	7.46	7.10	6.92	9.06	7.60	7.87
37. Average marginal residual product of labor ($ return added per added month)	....	36	209	....	....	....	....
38. Average marginal residual return on investment ($ return added per added $ capital investment)	....	....	....	....	15.88	5.36	....

* Methods of computation have been explained in footnotes of previous tables for the last two items. Average marginal residual product of labor is computed thus: The added product, added labor input and added capital have been computed between labor groups. An interest charge for the added capital quantity has then been subtracted from the added product. The remaining product has been divided by the added months of labor, to give the "added product per added unit of labor." The average marginal return for capital has been computed similarly but with this exception: A charge for the added labor has been subtracted from the added product between capital groups; the remainder has then been divided by the added capital, between capital groups.

TABLE 36. SOUTHERN IOWA SAMPLE GROUPED SEPARATELY BY LABOR THIRDS AND CAPITAL THIRDS FOR TOTAL PRODUCTION, SELECTED INPUTS AND PRODUCTIVITY RATIOS, 1950.

Item*	Labor group			Capital group			All farms
	low	medium	high	low	medium	high	
1. Percent of farms	33.6	33.6	32.9	33.6	33.6	32.9	100
2. Total product ($)	11,604	13,315	18,179	7,001	12,270	23,948	14,339
3. Cropland (acre)	106.2	110.1	154.0	76.0	122.1	172.6	123.2
4. Pasture land (acre)	51.2	64.1	97.5	55.4	72.7	84.6	70.8
5. Total land (acre)	157.4	174.2	251.5	131.4	194.7	194.7	194.0
6. Labor on crops (mo.)	6.2	7.7	12.4	7.4	9.0	9.8	8.7
7. Value of land services ($)	923	811	1,220	502	930	1,527	983
8. Value of machine services ($)	811	984	1,343	612	1,019	1,511	1,044
9. Value of crop services ($)	341	301	490	201	297	638	377
10. Value of all crop capital services ($)	2,075	2,096	3,052	1,314	2,245	3,677	2,403
11. Value of all crop services including labor ($)	3,351	3,658	5,322	2,792	3,988	5,556	4,102
12. Feed ($)	3,776	4,241	5,631	1,996	3,781	7,919	4,542
13. Livestock input ($)	1,802	2,686	3,808	1,090	2,035	5,200	2,758
14. Other livestock input ($)	253	282	410	152	272	523	315
15. Value of all livestock capital services ($)	5,831	7,209	9,849	3,238	6,089	13,642	7,614
16. Labor on livestock (mo.)	5.3	6.7	10.4	6.6	7.1	8.6	7.4
17. Value of all livestock services ($)	6,924	8,558	11,754	4,556	7,453	15,301	9,059
18. Land investment ($)	18,452	16,223	24,400	10,038	18,595	30,571	19,659
19. Machine investment ($)	2,869	3,397	4,724	1,906	3,651	5,448	3,656
20. Livestock investment ($)	3,681	5,145	7,016	2,815	4,222	8,843	5,268
21. Building investment ($)	3,448	3,624	5,411	2,233	3,998	6,271	4,153
22. Total investment ($)	28,450	28,389	41,551	16,992	30,466	51,134	32,736
23. Total labor (mo.)	11.4	14.4	22.8	14.1	16.1	18.4	16.1
24. Value of all labor ($)	2,368	2,911	4,174	2,796	3,107	3,537	3,144
25. Value of all services ($)	10,275	12,216	17,075	7,348	11,441	20,857	13,162
26. Percent labor on crops	53.85	53.65	54.37	52.84	56.09	53.12	54.03
27. Return to investment at 5 percent ($)	1,422	1,419	2,078	850	1,523	2,557	1,637
28. Investment per man ($)	29,845	23,686	21,905	14,504	22,740	33,417	24,325
29. Crop capital services per man ($)	8,294	7,763	6,802	3,886	6,222	11,319	7,444
30. Total land per man (acre)	165	145	133	112	145	168	144
31. Crop product per cropland ($)	44.74	38.72	44.40	38.45	41.35	45.79	42.80
32. Product per $1 all services, crops ($)	1.418	1.165	1.285	1.047	1.266	1.422	1.285
33. Product per $1 all services, livestock ($)	0.990	1.058	0.965	0.895	0.969	1.049	1.001
34. Product per $1 all services, all products ($)	1.129	1.090	1.065	0.953	1.072	1.148	1.089
35. Average residual product of labor ($ per man-year)	3,356	2,837	2,329	1,793	2,497	3,660	2,725
36. Average residual return on investment (%)	7.92	6.73	5.59	0.91	5.77	9.04	6.60
37. Average marginal residual product of labor ($ return added per added month)	....	69	122	....	....	....	....
38. Average marginal residual return on investment ($ return added per added $ capital investment)	....	....	....	....	11.90	13.84	....

* Methods of computation have been explained in footnotes of previous tables for the last two items. Average marginal residual product of labor is computed thus: The added product, added labor input and added capital have been computed between labor groups. An interest charge for the added capital quantity has then been subtracted from the added product. The remaining product has been divided by the added months of labor, to give the "added product per added unit of labor." The average marginal return for capital has been computed similarly but with this exception: A charge for the added labor has been subtracted from the added product between capital groups; the remainder has then been divided by the added capital, between capital groups.

TABLE 37. ALABAMA SAMPLE GROUPED SEPARATELY BY LABOR THIRDS AND CAPITAL THIRDS FOR TOTAL PRODUCTION, SELECTED INPUTS AND PRODUCTIVITY RATIOS, 1950.

Item*	Labor group			Capital group			All farms
	low	medium	high	low	medium	high	
1. Percent of farms	33.8	33.1	33.1	33.8	33.1	33.1	100
2. Total product	1,412	2,567	4,253	1,127	2,102	5,009	2,734
3. Cropland (acre)	25.7	29.3	42.1	22.0	30.3	44.9	32.3
4. Pasture land (acre)	13.6	11.0	20.6	8.6	13.9	22.9	15.1
5. Total land (acre)	39.3	40.2	62.8	30.6	44.1	67.8	47.4
6. Labor on crops (mo.)	4.4	10.2	16.8	7.5	12.4	11.5	10.4
7. Value of land services ($)	107	98	165	74	104	193	123
8. Value of machine services ($)	175	257	396	122	257	450	275
9. Value of crop services ($)	156	257	425	138	243	457	278
10. Value of all crop capital services ($)	438	612	986	334	603	1,100	677
11. Value of all crop services including labor ($)	877	1,648	2,656	1,085	1,848	2,250	1,720
12. Feed ($)	318	496	983	169	323	1,309	597
13. Livestock input ($)	187	314	581	71	175	838	359
14. Other livestock input ($)	38	49	97	26	37	121	61
15. Value of all livestock capital services ($)	543	859	1,660	266	535	2,267	1,017
16. Labor on livestock (mo.)	2.3	2.7	5.6	1.9	2.9	5.8	3.5
17. Value of all livestock services ($)	771	1,134	2,217	445	823	2,854	1,370
18. Land investment ($)	2,140	1,959	3,297	1,481	2,076	3,845	2,463
19. Machine investment ($)	198	262	650	94	199	820	369
20. Livestock investment ($)	463	574	1,198	293	551	1,396	743
21. Building investment ($)	384	501	719	273	402	932	534
22. Total investment ($)	3,186	3,296	5,865	2,104	3,228	7,002	4,109
23. Total labor (mo.)	6.7	12.9	22.4	9.4	15.3	17.3	14.0
24. Value of all labor ($)	667	1,311	2,221	930	1,533	1,737	1,396
25. Value of all services ($)	1,648	2,782	4,873	1,530	2,671	5,104	3,090
26. Percent labor on crops	65.80	79.01	74.99	79.76	81.19	66.23	74.73
27. Return to investment at 5 percent ($)	159	165	293	107	161	350	205
28. Investment per man ($)	5,701	3,054	3,139	2,730	2,533	4,854	3,529
29. Crop capital services per man ($)	1,754	1,363	1,416	765	893	2,334	1,455
30. Total land per man (acre)	70	37	34	39	35	47	41
31. Crop product per cropland ($)	27.45	46.88	50.56	31.38	46.29	47.12	48.23
32. Product per $1 all services, crops ($)	0.806	0.833	0.802	0.637	0.759	0.941	0.813
33. Product per $1 all services, livestock ($)	0.915	1.053	0.957	0.980	0.851	1.013	0.975
34. Product per $1 all services, all products ($)	0.857	0.923	0.873	0.736	0.787	0.981	0.885
35. Average residual product of labor ($ per man-year)	680	954	792	630	712	1,028	823
36. Average residual return on investment (%)	-4.02	-3.55	-7.76	-15.37	-14.41	1.39	-5.67
37. Average marginal residual product of labor ($ return added per added month)	....	104	48	....	....	....	....
38. Average marginal residual return on investment ($ return added per added $ capital investment)	....	....	....	....	-12.50	14.89	....

* Methods of computation have been explained in footnotes of previous tables for the last two items. Average marginal residual product of labor is computed thus: The added product, added labor input and added capital have been computed between labor groups. An interest charge for the added capital quantity has then been subtracted from the added product. The remaining product has been divided by the added months of labor, to give the "added product per added unit of labor." The average marginal return for capital has been computed similarly but with this exception: A charge for the added labor has been subtracted from the added product between capital groups; the remainder has then been divided by the added capital, between capital groups.

TABLE 38. RELATIVE SOURCE OF INPUTS FROM DIFFERENT RESOURCE CATEGORIES BY CAPITAL GROUPS, 1950.*

	Montana			Northern Iowa			Southern Iowa			Alabama		
	low capital	medium capital	high capital	low capital	medium capital	high capital	low capital	medium capital	high capital	low capital	medium capital	high capital
Crop inputs												
Land services (%)	19.7	25.1	29.4	30.7	35.7	38.0	18.0	23.3	27.4	6.8	5.6	8.6
Capital services (%)	48.1	46.2	45.1	33.2	35.1	36.6	29.1	33.0	38.7	24.0	27.0	40.3
Labor services (%)	32.2	28.7	25.5	36.1	29.2	25.4	52.9	43.7	33.9	69.2	67.3	51.1
All resources (%)	100	100	100	100	100	100	100	100	100	100	100	100
Livestock inputs												
Capital services (%)	67.6	69.7	84.6	80.5	85.5	92.2	71.1	81.7	89.2	59.8	65.0	79.4
Labor services (%)	32.4	30.3	15.4	19.5	14.5	7.8	28.9	18.3	10.8	41.2	45.0	20.6
All resources (%)	100	100	100	100	100	100	100	100	100	100	100	100
All product output												
Percent from crops	86.6	88.9	57.7	44.1	62.3	33.4	41.7	41.2	33.0	61.3	66.4	42.3
Percent from livestock	13.4	11.1	42.3	55.9	37.7	66.6	58.3	58.8	67.0	38.7	33.6	57.7
All production	100	100	100	100	100	100	100	100	100	100	100	100
All product inputs												
Cropland services (%)	14.6	19.8	13.9	14.6	11.7	9.2	6.8	8.1	7.3	4.8	3.9	3.8
Capital services (%)	53.2	51.2	65.9	38.0	44.6	71.8	55.2	64.7	75.7	34.4	38.7	62.2
Labor services (%)	32.2	29.0	20.2	47.4	43.7	19.0	38.0	27.2	17.0	60.8	57.4	34.0
All resources (%)	100	100	100	100	100	100	100	100	100	100	100	100

*Proportions have been computed on the basis of annual inputs of resource services including the wage value of labor, the rental value of land, depreciation on breeding stock, machinery and buildings and the cost or value of feed, fertilizer, tractor fuel, etc. Pasture land (rental value) is included with capital service inputs for livestock.

TABLE 39. CAPITAL-LABOR SUBSTITUTION IN CROP PRODUCTION. OUTPUT AT PREDICTED LEVEL FOR MEAN RESOURCE INPUTS OF VARIOUS AREAS, 1950.

Labor input as percent of mean	Quantity of resource services for "mean" product*: Machine-crop services ($)	Quantity of resource services for "mean" product*: Labor (mo.)	Months labor replaced†	Dollars capital services necessary to replace labor‡
Montana				
100	5,206	13.8	......	......
90	5,244	12.4	1.4	38
80	5,286	11.0	1.4	42
70	5,335	9.6	1.4	49
Northern Iowa				
100	2,169	9.4	......	......
90	2,276	8.5	0.9	107
80	2,402	7.6	0.9	126
70	2,554	6.7	0.9	152
Southern Iowa				
100	1,420	8.7	......	......
90	1,454	7.8	0.9	34
80	1,494	6.9	0.9	40
70	1,532	6.0	0.9	48
Alabama				
100	553	10.4	......	......
90	595	9.4	1.0	42
80	645	8.4	1.0	50
70	707	7.4	1.0	62

* These quantities have been derived from the contour equations based on the original production function. This equation, to predict machine-crop capital services with labor input at the levels specified, is:

$$C = \left(\frac{\overline{Y}}{\alpha \overline{D}^{\beta_d} L^{\beta_l}} \right)^{\frac{1}{\beta_c}}$$

where C is the amount of capital services for crops to be predicted, $\overline{Y}$ is the "mean" value of crop production found in the sample, $\overline{D}$ is the mean cropland input found in the sample, L is the labor input on crops at the percentage levels of the table and β_d β_l and β_c are the elasticities for the resources indicated by the subscripts and α is the constant. "Exact" marginal rates of substitutions have been obtained from the derivitive of this equation.

† Arithmetical decrease in labor between combinations (equal within each sample except for rounding).

‡ Arithmetical decrease in capital between combinations.

TABLE 40. CAPITAL INVESTMENT NECESSARY TO REDUCE LABOR INPUTS BY 10, 20 AND 30 PERCENT, 1950.*

Labor input as percent of mean	Capital necessary to substitute for: labor replaced in table 38	Capital necessary to substitute for: each month of labor replaced	Capital necessary to substitute for: total labor replaced
Montana			
100	....	....	....
90	228	163	163
80	252	180	343
70	294	210	553
Northern Iowa			
100	....	....	....
90	651	723	651
80	781	866	1,432
70	942	1,047	2,374
Southern Iowa			
100	....	....	....
90	211	234	211
80	248	276	459
70	298	291	757
Alabama			
100	....	....	....
90	58	58	58
80	66	66	124
70	93	93	217

* The capital investment quantities in column 2 have been estimated from the machine investment and capital input figures found in different strata of the samples. The figures of column 3 have been obtained by dividing the parallel figures of column 2 by the amount of labor replaced and shown in table 37. The figures of column 4 are the cumulative totals of column 2 and represent the total capital necessary to replace 10, 20 and 30 percent of the crop labor.

columns 2 and 3, i. e., they are not derivatives.)[23] Starting from the mean quantity of resources used (labor input at 100 percent), the rate of substitution of capital for labor is lowest in northern Iowa; a larger amount of capital services is required to substitute for labor than in the other areas. The next lowest rate is in Alabama. A likely explanation for these rates is to be found in the existing high degree of mechanization in northern Iowa. Also, capital services in Alabama are composed of but little high-capacity machinery; they are mainly mule feed, and repairs on implements still requiring a large amount of labor.

[23]Figuring substitution rates at derivatives we get the following machine quantities "at exactly" the combinations indicated in table 39. These quantities show the value of machine-crop services replaced by 1 month of labor, or conversely, the amount of capital services required to substitute for 1 month of labor. They are not "simple differences" such as those of table 39, but are derivatives indicating substitution rates for changes approaching the limit zero.

Percent input of labor	Substitution rate ($ capital service to replace 1 month labor or vice versa)			
	Montana	Northern Iowa	Southern Iowa	Alabama
100	25	105	30	37
90	29	123	33	44
80	33	146	37	53
70	38	177	43	67

When converted to a capital investment basis, the quantities necessary to substitute for labor are greatest in northern Iowa and Montana, followed by southern Iowa and Alabama. Because of its large inputs of machine-crop services and the low rate of labor/capital substitution, the largest amounts of capital are required in the northern Iowa area; $2,374 in capital would be necessary for a 30-percent reduction in labor. In contrast, the low-capital, high-labor area of Alabama would require only $217 in capital for a 30-percent reduction in labor.[24]

[24]In evaluating the substitution quantities mentioned above, these points should be kept in mind: Great differences exist between samples in (1) the average level of production and resources used and (2) the average proportions of resources used, for any one level of output. For example, northern Iowa uses a large amount of machinery relative to labor and has a much greater output per producing unit than Alabama. If a northern Iowa farmer used as little capital as an Alabama farmer, substitution rates computed for the mean rates of substitution of machinery capital for labor would very likely (considering the types of machinery techniques and the size of farms in northern Iowa as compared to mule techniques in Alabama) be greater than for the Alabama Piedmont area. The substitution rates have been computed around the average of output and resource combinations. They would differ for different input combinations or different production levels.

APPENDIX A. COMPUTATION METHODS AND ALTERNATIVE FUNCTIONS

As mentioned in the text, several different production functions were estimated from the sample data. Those indicated in the text (denoted as II) were computed after the first set (I) was deemed not entirely satisfactory. Functions indicated as III include those where inputs were aggregated even more than under II. The functions indicated as II were used because they are more acceptable in a probability sense. The two separate capital categories used in the equations denoted as I are more nearly technical complements. (Production logic suggests that they should be aggregated and treated as a "single bundle" of resource services.) In outline form, the functions are as follows:

(a) *Crop function I*: $Y = \alpha X_1^a X_2^b X_3^c X_4^d X_5^e$

Where Y = crop product I ($), (or value of all crop product and miscellaneous receipts); X_1 = cropland (acres); X_2 = pasture land (acres); X_3 = crop labor (man-months); X_4 = machine services ($), (or custom work hired, fuel and lubrication; depreciation and repairs); X_5 = crop services ($), (or home-grown seed, purchased seed, fertilizer, lime and spray materials).

(b) *Crop function II*:[25] $Y = \alpha X_1^a X_2^b X_3^c$

Where Y = crop product II ($), (or value of all crop product plus miscellaneous receipts less value of all pasture); X_1 = cropland not pasture (acres); X_2 = crop labor (months); X_3 = machine-crop services ($), (or sum of variable X_4 and X_5 in crop function).

(c) *Crop function III*: $Y = \alpha X^a$

Where Y = crop product III ($), (or same as for crop function I); X = value of all crop inputs ($), (or value of all land services, crop labor services and machine-crop services).

(d) *Livestock function I*: $Y = \alpha X_1^a X_2^b X_3^c X_4^d$

Where Y = livestock product ($), (or value of non-breeding stock at end of year, non-breeding stock sold, products used in the household, livestock products sold and breeding stock raised); X_1 = feed fed ($), (or value of home-produced feed and purchased feed); X_2 = livestock labor (months); X_3 = livestock input ($), (or value of non-breeding stock at the beginning of the year, non-breeding stock purchased and breeding herd depreciation); X_4 = other inputs ($), (or value of building services, fences, veterinary supplies and equipment services, etc.)

[25]Same as functions in text where β's are used to indicate productivity coefficient.

(e) *Livestock function II*: $Y = \alpha X_1^a X_2^b$

Where Y = livestock product ($), (or same as livestock function I); X_1 = livestock labor (months); X_2 = all other inputs ($), (or sum of X_1, X_3 and X_4 of livestock function I).

(f) *Livestock function III*: $Y = \alpha X^a$

Where Y = livestock product ($), (or same as livestock function I); X = value of all livestock inputs ($), (or value livestock labor plus X_2 of livestock function II).

(g) *Aggregate Cobb-Douglas function A*: $Y = \alpha X_1^a X_2^b$

Where Y = total product ($)[26] (or crop product I plus livestock product I); X_1 = labor-capital services ($), (or value of all inputs used in crop production except land plus value of all inputs used in livestock production); X_2 = land (acres), (or cropland plus pasture land).

(h) *Aggregate polynomial functions*: Functions up to the third degree were fitted. These were:

(i) Cubic: $Y = a + bX + cX^2 + dX^3$

(ii) Quadratic: $Y = a + bX + cX^2$

(iii) Linear: $Y = a + bX$

Where Y = total product ($), (or same as for aggregate function I); X = value of all inputs ($), (or X_1 from aggregate function I plus value of land services); X^2 = the square of the individual farm aggregate input; X^3 = the cube of the individual farm aggregate input.

[26]Miscellaneous receipts were erroneously omitted in northern and southern Iowa, 1950. The sample averages per farm for these items for northern Iowa, 1950 and southern Iowa, 1950 were $195 and $150 respectively. This omission was not considered sufficiently serious to warrant recalculation of the functions involved.

APPENDIX B. STATISTICS FROM PRODUCTION FUNCTION ESTIMATES

Table B-1 presents the relative statistics for the first attempt to estimate crop production functions (I). Because of the large error of estimate, the low t value for the pasture regression coefficient and the high correlation between machine and crop services for some areas, the former was dropped while the latter two were combined for the function presented in the text. (Crop product was dropped from the second production function (II) of the text and was introduced into the livestock function with other feed.) In deriving the second production function (both for crops and livestock), numerous small errors in computations were removed from the data. While most of these had little importance for magnitude of coefficients, they were quite important in two or three cases. Statistics for the first livestock production function are presented in table B-2. Because of the low t values and the magnitude of the correlation coefficients between feed and livestock inputs, X_1, X_3 and X_4 were aggregated into a single category of resource inputs. Hence, the function presented in the text (function II) includes only labor and capital services as inputs. Statistics are shown in tables B-3 and B-4 following for the single-aggregate Cobb-Douglas production functions for crops and livestock (III). These estimates would suppose that inputs could not be separated accurately into non-complementary cate-

TABLE B-1. STATISTICS FOR FIRST ESTIMATE (WITH COEFFICIENTS REFERRING TO DATA IN LOGARITHMS) OF CROP PRODUCTION FUNCTION I, 1950.

Item and input	Montana	Northern Iowa	Southern Iowa	Alabama
Number of farms	151	142	143	133
Regression coefficients				
a (cropland)	-0.2242	0.9263	0.7116	0.2205
b (pasture land)	-0.0067	0.0037	-0.0005	0.0485
c (labor)	0.0653	0.0796	0.0697	0.2122
d (machine services)	0.3384	0.0041	0.1955	0.2173
e (crop services)	0.4690	0.0721	0.0668	0.3192
Sum of coefficients	1.0901	1.0858	1.0431	1.0177
Value of α (log form)	0.9297	1.5733	1.4037	1.2367
Value of t for coefficients				
a (cropland)	1.94	15.61	12.91	3.26
b (pasture land)	0.37	0.34	0.03	0.96
c (labor)	0.95	1.69	1.11	2.79
d (machine services)	3.18	0.32	3.87	3.30
e (crop services)	4.59	1.93	2.44	4.72
R^2	0.750	0.830	0.825	0.714

TABLE B-2. STATISTICS (WITH COEFFICIENTS REFERRING TO DATA IN LOGARITHMS) FOR ESTIMATION OF LIVESTOCK FUNCTION I, 1950.

Item and input	Montana	Northern Iowa	Southern Iowa	Alabama
Number of farms	111	140	143	134
Regression coefficients				
a (feed)	0.2640	0.4136	0.3741	0.3024
b (labor)	0.0873	0.0660	0.1155	0.2300
c (livestock)	0.5674	0.4473	0.5029	0.3697
d (miscellaneous)	0.0741	0.0316	0.0578	0.0458
Sum of coefficients	0.9928	0.9585	1.0503	0.9479
Value of α (log form)	0.7853	0.7791	0.6366	1.1508
Value of t for coefficients				
a (feed)	5.02	8.18	9.05	4.25
b (labor)	1.36	1.54	2.58	3.48
c (livestock)	13.37	13.66	15.41	8.47
d (miscellaneous)	1.73	0.73	1.67	1.30
R^2	0.922	0.904	0.905	0.834

TABLE B-3. CROP FUNCTION III: SUMMARY OF RELEVANT STATISTICS.

Item	Montana, 1950	Northern Iowa, 1950	Southern Iowa, 1950	Alabama, 1950	Northern Iowa, 1939	Southern Iowa, 1939
Regression coefficients (elasticity)*	1.0659	1.1308	1.1436	0.9785	0.9347	0.9516
Log of α	-0.0305	-0.3477	-0.4450	-0.0683	0.3475	0.2435
Value of α	0.9323	0.4490	0.3590	0.8544	2.2258	1.7517
Mean of log:						
Y	4.2067	3.9060	3.6386	3.0292	3.4288	3.0838
X	3.9750	3.7619	3.5709	3.1655	3.2967	2.9850
Geometric mean of:						
Y ($)	16,095	8,054	4,352	1,069	2,684	1,213
X ($)	9,440	5,779	3,723	1,464	1,980	966
Marginal product at the:						
Geometric mean ($/$)	1.82	1.58	1.34	0.71	1.27	1.19
Arithmetic mean ($/$)	1.96	1.70	1.36	0.70	1.19	1.11
Average product at the:						
Geometric mean ($/$)	1.70	1.39	1.17	0.73	1.36	1.26
Arithmetic mean ($/$)	1.84	1.51	1.19	0.72	1.27	1.17
r^2	0.6295	0.7081	0.6397	0.5984	0.7874	0.7772
Value of departure from 1.0†	0.9698	4.5467‡	3.9480‡	0.940	2.0274‡	1.0260
Standard error	0.1324	0.1212	0.1428	0.1386	0.0908	0.0948

* All regression coefficients significant at the 1-percent level of probability.

† Test for departure of elasticity from 1.0.

‡ Significant at 5-percent level of probability, or higher.

TABLE B-4. LIVESTOCK FUNCTION III: SUMMARY OF RELEVANT STATISTICS.

Item	Montana, 1950	Northern Iowa, 1950	Southern Iowa, 1950	Alabama, 1950	Northern Iowa, 1939	Southern Iowa, 1939
Regression coefficients (elasticity)*	1.0852	1.0163	1.1679	1.0145	0.9815	1.2309
Log of a	-0.3501	-0.0812	-0.6855	-0.1092	-0.0136	-0.9146
Value of a	0.4466	0.8297	0.2063	0.7777	0.9693	0.1217
Mean of log:						
Y	3.7288	4.0222	3.8407	2.8780	3.4941	3.3007
X	3.7587	4.0375	3.8755	2.9445	3.5738	3.4245
Geometric mean of:						
Y ($)	5,355	10,524	6,930	755	3,120	1,999
X ($)	5,737	10,902	7,508	880	3,748	2,658
Marginal product at the:						
Geometric mean ($/$)	1.12	0.98	1.087	0.87	0.87	0.93
Arithmetic mean ($/$)	1.19	0.99	1.25	0.90	0.86	1.18
Average product at the:						
Geometric mean ($/$)	0.93	0.97	0.93	0.86	0.89	0.75
Arthmetic mean ($/$)	1.10	0.98	1.07	0.89	0.88	0.96
r^2	0.9055	0.8983	0.8825	0.8153	0.7867	0.7674
Value of F for departure from 1.0†	6.7939‡	0.3130	21.9051‡	0.1184	0.1467	13.1262‡
Standard error	0.0648	0.0575	0.0709	0.0834	0.0956	0.1261

* All regression coefficients significant at the 1-percent level of probability.

† Test for departure of elasticity from 1.0.

‡Significant at 1-percent level of probability.

TABLE B-5. STATISTICS FOR CROPS (DATA IN LOGS) WITH AGGREGATE COBB-DOUGLAS PRODUCTION FUNCTION A.

Item	Montana, 1950	Northern Iowa, 1950	Southern Iowa, 1950	Alabama, 1950	Northern Iowa, 1939	Southern Iowa, 1939
Number of farms	151	142	143	134	114	115
Regression coefficients (of observations in logarithms)						
a (labor-capital)	0.6219	0.7424	1.0205	1.0109	0.6481	1.0893
b (land)	0.2510	0.3250	0.1867	0.1427	0.3822	0.1286
Sum of elasticities	0.8729	1.0674	1.2072	1.1536	1.0303	1.2179
Value of a (log form)	1.0187	0.4438	0.4582	-0.3395	0.5339	-0.5990
Means in logs						
X_1 (capital-labor)	4.0772	4.1769	4.0315	3.3903	3.7052	3.5331
X_2 (land)	3.1834	2.2704	2.2323	1.6204	2.2167	2.1708
Y (value of product)	4.3532	4.2829	4.0727	3.3190	3.7823	3.5286
Geometric means (original observations)						
X_1 (capital-labor)	11,945	15,030	10,753	2,456	6,057	3,378
X_2 (land)	1,526	186	171	42	165	148
Y (value of product)	22,553	19,181	11,821	2,084	6,057	3,378
Value of t for coefficients						
a (labor-capital)	6.97	20.59	19.67	20.17	14.04	13.06
b (land)	3.20	6.50	3.85	2.57	8.82	1.93
r^2	0.683	0.895	0.830	0.847	0.887	0.752

gories or that we need not concern ourselves with the form of resources but need only examine elasticities and productivities regardless of how the make-up of total inputs changes between resources as total inputs increase. In tables B-3 and B-4 all of the single regression coefficients are significant at an acceptable level of probability. Examination of these figures then can be in terms of "a bundle of inputs none of which need to be distinguished separately." On this basis, the marginal productivities per aggregate input of crop resources are still highest in Montana and lowest in Alabama; they are higher in northern Iowa than in southern Iowa.

Statistics are shown in table B-5 for the aggregate Cobb-Douglas function A where inputs were classed as land (X_2) measured in acres and all other inputs, including labor (X_1) measured in dollars. This logarithmic function was derived for the four areas for 1950 and for northern Iowa and southern Iowa for 1939. With these exceptions, all of the elasticities are significant at the 1-percent level of probability. Alabama land coefficient is significant at the 5-percent level and the southern Iowa labor-capital coefficient for 1939 is significant at the 10-percent level of probability. Use of this production function would be justified under logic which supposes labor and capital services to be technical complements to an extent that they should be grouped as a single resource.

The statistics in table B-6 were those derived for the aggregate polynomial functions with a single input category (i, ii and iii).

TABLE B-6. STATISTICS FOR ESTIMATION OF "AGGREGATIVE" POLYNOMIAL PRODUCTION FUNCTION AND "SINGLE" CATEGORY OF RESOURCE SERVICE. 1950.

Item	Montana	Northern Iowa	Southern Iowa	Alabama
Number of farms	151	142	143	134
(i) Regression coefficients: Equations with linear, squared and cubed terms				
b	2.42	0.73	1.52	0.24
c	-0.0335	0.0133	-0.0156	0.1337
d	0.00022	-0.00012	0.00020	0.5933
Value of a	-3.21	3.04	-3.67	0.59
Value of t for coefficients:				
b	7.07	5.18	5.80	1.16
c	3.75	3.23	1.33	3.51
d	3.85	3.74	1.42	2.76
(ii) Regression coefficients: Equations with linear and squared terms				
b	1.25	1.22	1.17	0.7627
c	0.0004	-0.0019	0.0008	0.0308
Value of a	5.54	-0.77	-1.45	0.04
Value of t for coefficients:				
b	7.61	21.23	12.27	7.93
c	0.23	2.70	0.39	3.94
(iii) Regression coefficients: Equations with linear terms:				
b	1.29	1.07	1.21	1.12
Value of a	5.13	1.01	-1.74	-0.69
Coefficient of determination:				
equation i	0.798	0.965	0.905	0.909
equation ii	0.778	0.952	0.903	0.902
equation iii	0.778	0.944	0.950	0.890

APPENDIX C. DEPARTURE OF MARGINAL PRODUCTIVITIES FROM FACTOR PRICES

The information in table C-1 indicates the probability level at which the estimated marginal productivities, computed at the mean, differed from the market price of the resource. In Montana, for example, the value of t computed for crop labor, testing the derived coefficient for capital against the market rate of interest rather than zero, was significant at the 1-percent level. (These data are for the functions used in the text.) In other words, the marginal return of crop capital, computed at the mean, differed significantly from the market interest rate. In northern Iowa, however, one can only say that farmers as an average were using an equilibrium amount of labor. The computed mean productivity of labor did not differ significantly from the interest rate at the 10-percent level of probability in any area except Alabama. Productivity of capital on livestock differed significantly from the market interest rate at the 1-percent level in Montana and southern Iowa and at the 10-percent level in Alabama and northern Iowa.

TABLE C-1. PROBABILITY LEVEL AT WHICH ESTIMATED MARGINAL PRODUCTIVITY DIFFERS FROM MARKET PRICE OF RESOURCE.

Item	Montana	Northern Iowa	Southern Iowa	Alabama
Crop function:				
Labor	13.8	1.41	2.84	4.09
Capital	2.66	8.28	5.60	1.66
Land	2.64	1.52	1.97	0.13
Livestock function:				
Labor	0.62	0.01	0.24	0.13
Capital	3.76	1.44	2.66	1.48

APPENDIX D. 1939 IOWA DATA

The data below are the statistics for the 1939 Iowa functions paralleling those used in the text (crop function II and livestock function II). These data, except for crops in southern Iowa, do not appear very useful for estimating productivity coefficients. The data were not originally obtained for these purposes and parts of the information appear incomplete.

TABLE D-1. STATISTICS FOR ESTIMATION OF CROP FUNCTION II AND LIVESTOCK FUNCTION II, NORTHERN AND SOUTHERN IOWA, 1939.

Item		Northern Iowa	Southern Iowa
Crop function			
Value of log		1.5905	7.9130
Elasticities:	labor	0.1120	5.6945
	capital	0.0396	0.3554
	land	0.7736	-5.0707
Values of t:	labor	0.12	2.68
	capital	0.49	4.84
	land	0.86	2.42
R		0.9023	0.8718
Livestock function			
Value of log		0.4401	0.0853
Elasticities:	labor	-0.0039	-0.0211
	capital	0.8872	0.9979
Value of t:	labor	0.11	0.47
	capital	25.61	30.63
R		0.9306	0.9451

APPENDIX E. LIMITATIONS OF METHODS.

Two systems of estimating resource returns were used in the study. One included tabular analysis to estimate gross average productivities and average residual production. The gross average productivities computed by dividing the sample average product by the sample average input of one resource with no share imputed to other resources is of limited value: The average gross productivity of a single resource will depend on the quantity and productivity of other resources with which it is used. Average gross productivity of labor will appear large on farms where much capital is used; it will appear small in types of agriculture that use little capital. The average residual product can serve as a fairly accurate predictor of marginal resource productivity only when (1) constant returns to scale hold true for each resource and (2) the prices applied to resources are equal to their marginal value products.[27]

Estimation of productivity coefficients through regression equations eliminates the difficulties outlined above but also involves certain limitations in method. One problem is the selection of the particular algebraic function. Agriculture involves a highly complex production process and it is doubtful that any single algebraic function can, considering limitational resources, discontinuity in factor supply and resources or products which can serve both as technical complements or rivals, accurately predict all of the relevant productivity coefficients. Also, while a function may allow estimates with small error over some ranges of the data, it may involve larger errors over other ranges of the data.[28] It is likely, for example, that the logarithmic functions employed in the text of this study provide reasonably accurate estimates of productivity coefficients for mean inputs of the resources but provide less satisfactory estimates for larger or smaller inputs of any one resource. In the logarithmic functions, we have been able to relate productivity of one resource to its quantity or input of other resources. In this single function, however, we may not have been able to account for discontinuities in all cases where two factors must be increased together as technical complements.

[27]For detailed discussion of these points, see: Heady, Earl O. Economics of agricultural production and resource use. Op. cit. Ch. 13; Heady, Earl O. Use and estimation of input-output relationships or productivity coefficients. op. cit.; Heady, Earl O. Production functions from a random sample of farms. op. cit.; and Heady, Earl O. Elementary models in farm production economics research. op. cit.

[28]For illustrations of this point, see: Heady, Earl O. Use and estimation of input-output relationships or productivity coefficients. op. cit.