Optimum Combinations of Competitive Crops at Particular Locations

(Application of Linear Programming: 1)

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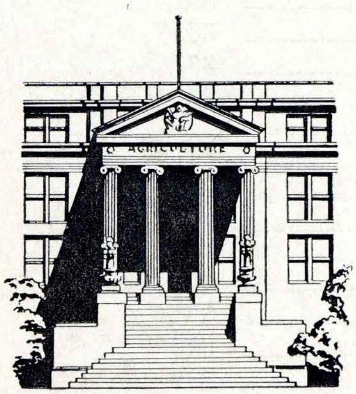

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CONTENTS

SUMMARY

The main objective of this study has been the determination of the crop production program which maximizes total profit for different soil areas in Iowa and for different levels of resource ownership. The analysis was carried out by the use of the Simplex method of the linear programming technique. Accordingly, an optimum (revenue maximizing) crop program was determined for 14 main soil associations. Six resource quantities were considered for each soil association, and the analysis was repeated for a second price period. In all, 168 situations were considered. Only one production technique was used in the analysis. That is, only one combination of resource inputs per unit of product was considered for each crop and each area. The analytical method lends itself well to considering several production techniques. But, the lack of yield data reflecting the results of different techniques accurately made this extension inadvisable.

The analysis has substantiated the hypothesis that the optimum plan will differ from farm to farm, even on the same soil type, if the quantity of resources available for production is different. The optimum plan will vary among areas due to relative differences in crop yields. Changes in price ratios over time may cause the optimum production plan of one period to be relatively less favorable in another price period. Therefore, the second price period considered was chosen to show the widest variation in relative crop prices during recent years.

The results obtained in Logan Township demonstrate the findings of the study. When capital was severely limitational, corn and flax were found to be the most profitable combination of crops for part of the land (1948-52 average price levels were used) while part of the acreage was not planted. As the quantity of available capital was increased, part of the unused acreage and part of the flax acreage were planted to corn. With unlimited capital, the most renumerative use of resources was found to be in the production of corn exclusively. The use of 1941-44 price ratios (oats and soybeans were relatively higher priced than corn compared to 1948-52) caused soybeans to be included in the optimum plan in only one area, Washington Township. However, their inclusion was associated with low available capital. As the capital quantity was increased, soybeans were replaced by corn, in spite of the relatively favorable price of the former crop.

Optimum Combinations of Competitive Crops at Particular Locations

(Application of Linear Programming: 1)[1]

BY BERNARD BOWLEN AND EARL O. HEADY

An ever-present problem facing farmers is this: How should crop and livestock enterprises be combined, considering the amount of labor and capital available, to produce maximum profits on a given acreage? The answer to this problem is not uniform among farms, even though they have the same soil type. The best combination of crops depends on the amount of capital available throughout the year; it also depends on the total labor supply and the availability of labor in each month. One farmer may have access to enough capital to devote his entire acreage to a single high-profit crop. Capital, then, will not limit his selection of a cropping program.

Labor may be a limiting resource, however, and cause him to select a cropping program which differs from the one which is best for his capital. But the labor of a particular month, such as May or October, rather than total labor over the year, may actually be the critical or limiting resource for choices among several cropping plans. Hence, the final plan must consider (1) the quantity of all resources used by the farmer, (2) their distribution throughout the year and (3) their "interactions" as limiting means of production. These "interactions" differ for farmers who have different quantities and proportions of capital and labor resources available at different times of the year.

OBJECTIVES

The major objective of this study is to apply linear programming techniques to determine optimum cropping plans for farms with different quantities and proportions of labor and capital. This objective is accomplished within the restrictions of given techniques or methods for producing the crops specified. The restriction is applied in this manner: While several different competitive crops are included in the possibilities of choice, only a single method of producing each is considered. The method or technique used is that which is an "average" or "typical" situation for each locality considered.

The technique assumed applies generally to farmers using an average amount of capital. However, it may not apply to farmers with smaller amounts of capital. They might use a different technique and plant all of their land to a low-income crop, such as oats or corn continuously, rather than let part of it fall into the disposal category outlined later.

Varying amounts of capital are used, with the specified technique, however, to illustrate how the optimum plan may differ with the amount of funds available. In applying the solutions of this study to farms, those situations which have real application to farms are mainly those where (1) land and labor are limited but production capital is unlimited or (2) land is limited and labor and production capital is unlimited. Situations where all resources are limited also have application where the same technique would be used. However, in some limited capital situations, hay might be allowed to go unharvested while more production capital is applied to producing the most profitable competitive crop.

Within these limitations, the linear programming method has been used to specify optimum crop combinations for a given set of techniques. Subsequent studies will deal with selection of the best cropping program when different techniques or methods of production are considered. Linear programming can be used as a "time saver" in specifying enterprise choices for farmers. It is a method whereby the one optimum plan can be selected from among hundreds or thousands of alternative plans. Considering the amount of resources available and the enterprises or practice for which they can be used, all farmers have a multitude of choices open to them.

Consider the farmer with $5,000 in capital and two crops, corn and soybeans: He can have 5,000 different combinations of the two crops if he considers all the possibilities of allocating "whole dollars" between them (e.g., he can use $1 for corn and $4,900 for soybeans, $1,999 for corn and $3,001 for soybeans, $4,950 for corn and $50 for soybeans, etc.). If he has 3,320 hours of his own labor which can be allocated between the two crops, without regard to capital allocation, he has

[1] Project 1135, Iowa Agricultural Experiment Station.

3,320 x 5,000 or 16,600,000 different ways to use the two resources, capital and labor, for the two crops.[2]

If we consider labor by months, instead of years, the number of possibilities is even greater; use of six rather than two crops "mushrooms" the number of alternative uses of resources even further.

SITUATIONS USED FOR STUDY

This study is an extension of the series dealing with particular locations and soil situations in Iowa. At a previous time, 14 townships were selected to typify particular soil and climatic situations of Iowa. These townships, and the soil-climate situation which they represent, are to serve as a basis for future studies dealing with farm organization and farming practices. They are "benchmark" situations selected from several hundred possible soil and climatic situations in Iowa. The number of townships has been limited to allow coverage of a wider range of farm organization problems. The townships, rather than other geographic units, were selected to typify soil situations because historic data are available for the townships.

A previous study using these townships analyzes crop combinations to minimize risk.[3] The current study examines optimum cropping programs in the 14 townships for farmers with different amounts of capital and labor. The next study will relate optimum rotation and livestock programs for the same soil situations.

The group of townshsips used in this study is a judgment sample selected with the intention of including areas which are representative of homogeneous soil types.[4] Each major soil type of the state is represented by one township. An exception is made for Clarion-Webster soil. To allow climatic differentials to be shown, it is represented by Harrison Township, Kossuth County, and Lincoln Township, Polk County. Harrison Township, Benton County, and Oakland Township, Louisa County, represent sandy loam and bottomland soils, respectively. Table 1 includes a list of the selected townships, the county location and the soil type represented by each. The county location is also shown in fig. 1.

The crops considered in this study are those normally grown in the townships selected. Corn, oats and soybeans are included in all townships. Flax is included in Harrison Township, Kossuth County; Logan Township, Lyon County; and Reading Township, Sioux County. Wheat is included in Cedar Township, Lee County; Jordan Township, Monona County; Lincoln Township, Montgomery County; Lincoln Township, Polk County; and Sheridan Township, Scott County.

The problem analyzed is not one of determining the optimum rotation wherein hay serves as the rotation-purpose crop.[5] The grain crop yields included for each township are averages based upon the quantity of hay grown over a number of years. (Hay contributes to fertility, organic matter and erosion control). Hence, the amount of hay grown previously in the townships is taken as the minimum to be allowed by the programming systems. Beyond this "minimum" quantity of hay, our question is: What proportion of the remaining cropland should be planted to corn, oats, soybeans, wheat or flax if profits from crops are to be maximized?

TABLE 1. TOWNSHIPS AND SOIL TYPES REPRESENTED.*

Township	County	Soil
1. Washington	Appanoose	Shelby-Seymour-Edina
2. Harrison	Benton	Sandy loam soils
3. Troy	Clarke	Grundy-Haig-Shelby
4. Grand Meadow	Clayton	Tama-Downs
5. Saratoga	Howard	Carrington-Clyde
6. Harrison	Kossuth	Clarion-Webster-Nicollet
7. Cedar	Lee	Grundy-Haig
8. Oakland	Louisa	River bottom soils
9. Logan	Lyon	Moody
10. Jordan	Monona	Ida-Napier-Monona
11. Lincoln	Montgomery	Marshall
12. Lincoln	Polk	Clarion-Webster-Nicollet
13. Sheridan	Scott	Tama-Muscatine-Garwin
14. Reading	Sioux	Galva-Primghar-Sac

* Henceforth only the township name will be used.

QUANTITIES OF RESOURCES

Capital

One purpose of this study is to show the optimum cropping program (for the technique used) for farmers who have different amounts of capi-

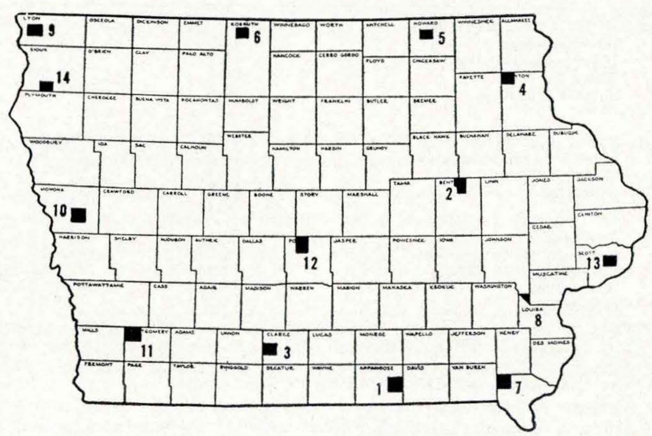

Fig. 1. Outline map of Iowa showing location of sample townships.

[2] In the programming techniques used later, however, certain postulates are used for the proportions of capital and labor. These are explained in detail in a subsequent section.

[3] Heady, Earl O., Kehrberg, Earl W. and Jebe, Emil H. Economic instability and choices involving income and risk in primary or crop production. Iowa Agr. Exp. Sta. Res. Bul. 404. 1954.

[4] The sample was selected by A. A. Aandahl, Agronomy Dept., Iowa State College and Division of Soil Survey, U.S. Dept. Agr.

[5] A rotation-purpose crop is one grown for a particular contribution or input for other crops. See: Heady, Earl O. and Jensen, Harald R. Farm management economics. Ch. 6. Prentice-Hall, Inc., New York. 1954.

tal and labor available to them. Hence, several levels of resource ownership were studied for each location. Three capital situations were used and include (1) capital for annual production expenses on crops limited to $1,893.51, (2) capital increased to 150 percent of this amount or $2,840.27 and (3) unlimited capital.[6] Not all of these quantities are available for competitive crops (crops produced beyond the average quantity of hay grown in the past). Priority on capital and other resources is first given to the "minimum" quantity of hay to be grown per farm and the oats acreage needed to seed the "minimum" hay acreage; the remainder is considered to be availabe for other crops.

All figures have been computed on the basis of 154 crop acres. Table 2 shows the "minimum" hay acreage per farm for each location, the amount of annual capital expense necessary to produce the "minimum" hay acreage and the oats necessary as a nurse crop for its seeding, and the amount available for other competitive crops under the three situations. The capital quantities for hay are those required for planting, harvesting and baling. Under limited capital, not many farmers would use a large part of their funds for hay while acreages for cash grain crops went unused. However, this procedure is used to illustrate the outcome where (1) farm capital is limited to specified levels and (2) minimum quantities of hay are produced.

Land

In determining the amount of land available for competitive crops, the acreage necessary for the minimum hay (hay is used to denote land used for rotation pasture as well as hay for harvest) was first subtracted from the 154 acres. Next, the acreage of oats used as a nurse crop in establishing the minimum hay acreage was subtracted from the remainder.[7] The final remainder is the acreage available for use in competitive crops and for which optimum enterprises or crop combinations have been computed. The available crop acreages used in the programs or plans which follow are shown in table 3. The oats acreage shown is the minimum quantity necessary for

[6] The quantity $1,893.51 is the estimated average capital used by all Iowa farms in 1952. This estimate is based on surveys conducted in 1951, adjusted by changes in the cost index.

[7] This acreage for oats was established as follows: One acre was included for each acre of red-clover timothy, one-half acre for each acre of alfalfa-brome and one-half acre for each acre of rotated pasture. It was indicated, by the county chairmen of some of the Agricultural Stabilization and Conservation offices, that rotated pasture was down more than 2 years in some townships and less in others. This called for appropriately smaller or larger acreages of oats.

TABLE 3. CROP ACREAGES FOR MINIMUM HAY AND OATS (FOR SEEDING MINIMUM HAY) AND COMPETITIVE CROPS.

Township	Minimum hay acreage	Oats for minimum hay acreage	Acreage available for competitive crops (including oats beyond nurse crop level)
	(acres)	(acres)	(acres)
Washington	45.5	41.1	67.4
Harrison, Benton Co.	61.2	24.5	68.3
Troy	38.0	36.5	79.5
Grand Meadow	68.7	44.6	40.7
Saratoga	43.2	36.1	74.7
Harrison, Kossuth Co.	23.6	13.9	116.5
Cedar	27.8	22.2	104.0
Oakland	38.0	24.8	91.2
Logan	20.1	11.4	122.5
Jordan	31.1	15.9	107.0
Lincoln, Montgomery Co.	36.0	24.0	94.0
Lincoln, Polk Co.	30.4	17.3	106.3
Sheridan	33.5	17.6	102.9
Reading	15.2	9.8	129.0

TABLE 2. "MINIMUM" HAY ACREAGE, ANNUAL CAPITAL EXPENSE FOR "MINIMUM" HAY ACREAGE AND CAPITAL AVAILABLE FOR OTHER CROPS UNDER THREE CAPITAL SITUATIONS.

Township	"Minimum" hay acreage*	Annual capital for "minimum" hay acreage and oats to seed "minimum" hay acreage†	Annual capital for competitive crops		
			Level 1 $1893.51‡	Level 2 $2840.27§	Level 3 Unlimited**
	(acres)	($)	($)	($)	($)
Washington	45.5	1613.24	280.27	1227.03	Unlimited
Harrison, Benton Co.	61.2	942.77	950.74	1897.50	Unlimited
Troy	38.0	1372.26	521.25	1468.01	Unlimited
Grand Meadow	68.7	1545.67	347.84 329.83**	1294.60 1278.59**	Unlimited
Saratoga	43.2	1120.47	773.04	1719.80	Unlimited
Harrison, Kossuth Co.	23.6	527.63	1365.88	2312.64	Unlimited
Cedar	27.8	846.47	1047.04	1993.80	Unlimited
Oakland	38.0	764.87	1128.64	2075.40	Unlimited
Logan	20.1	466.48	1427.03	2373.79	Unlimited
Jordan	31.1	696.44	1197.07	2143.83	Unlimited
Lincoln, Montgomery Co.	36.0	885.05	1008.46	1955.22	Unlimited
Lincoln, Polk Co.	30.4	691.34	1202.17	2148.93	Unlimited
Sheridan	33.5	768.63	1124.88	2071.64	Unlimited
Reading	15.2	398.26	1495.25	2442.01	Unlimited

* This is the average hay acreage grown over the period of years for which crop yields have been obtained. The supposition of this study is: The yields for competitive crops could be obtained only with this amount of hay as a minimum.

† Amount of annual expense to (1) produce and harvest "minimum" hay acreage listed in first column and (2) produce and harvest the oats needed for seeding this acreage.

‡ Amount of capital available after amount shown in column 2 is subtracted from $1831.51.

§ Amount of capital available after amount shown in column 2 is subtracted from $2840.27.

** July labor requirement in Grand Meadow Township was more than could be supplied by the operator alone. In the situation where no family labor was available, an additional hired labor charge was made which reduced the quantity of capital available.

seeding the hay. Beyond this quantity, oats is considered to be competitive with other crops. If oats is more profitable than other crops, the acreage will be increased.

Labor

Three labor situations were used in determining the optimum plan for competitive crops. The labor situations are as follows:

Situation I: The total supply of labor is the amount available from the operator's time, a total of 260 hours per month.[8] The amount of labor used on livestock in the particular locality was first subtracted from this 260 hours. (An "average" livestock system was used for each locality.) The remainder is the total amount of labor available for (1) the "minimum" hay acreage in each locality, (2) the amount of oats required for seeding the "minimum" hay acreage and (3) the competitive crops from which an optimum plan is to be selected. However, not all of the labor beyond that committed to livestock, hay and nurse crop production is available for growing competitive grain crops. Some days are not suitable for field work because of weather. Also, the minimum acreage of hay and the oats for seeding it require some of the labor available in days suitable for field work. Adjustments were made accordingly and the labor available for competitive crops was calculated in the manner shown in table 4 for Appanoose County. The labor supplies for competitive crops in each township are shown in table 5. In addition to this residual labor for competitive crops, it is supposed that labor can be hired during the peak harvesting months. (In the programming computations hired labor has been treated as a capital expense.)

Situation II: The total labor supply is the amount available from the operator's time plus 130 hours of family labor for crops in June, July and August. The amount necessary for the "average" livestock program, for the "minimum" hay acreage and for the oats seeded as a nurse crop was deducted as under Situation I. Adjustments were then made for weather, with hired labor considered to be available during harvesting. The resulting figures are shown in table 6.

Situation III: Labor is assumed to be unlimited. That is, labor can be hired in any quantity desired in each month. It is treated as a capital expense in the programs or plans of later sections.

[8] The operator's time is estimated on the basis of 10 hours per day for 26 working days per month. The farmer may, of course, work 18 hours on some days and 4 or 5 on others to total approximately 260 hours per month. This differential in number of hours worked has not been included in the analysis which follows.

TABLE 4. METHOD OF COMPUTING OPERATOR'S LABOR AVAILABLE BY MONTHS FOR COMPETITIVE CROP PRODUCTION IN WASHINGTON TOWNSHIP.

Item	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.
1. Total working hours per month	260	260	260	260	260	260	260	260	260
2. Labor required for livestock*	81.5	73.6	60.6	50.9	51.3	54.9	52.4	59.3	63.9
3. Remainder†	178.5	186.4	199.4	209.1	208.7	205.1	207.6	200.7	196.1
4. Hours favorable weather‡	26.5	183.8	207.5	201.3	237.6	232.0	232.0	241.0	167.5
5. Remainder or hours available for crop production§	26.5	183.8	199.4	201.3	208.7	205.1	207.6	200.7	167.5
6. Hours committed to production of "minimum" hay and oats acreage**	14.6	36.8	—	95.6	155.1	85.4	48.1	—	—
7. Remaining hours available for competitive crops††	11.9	147.0	199.4	105.7	53.6	119.7	159.5	200.7	167.5

* Labor required for the average livestock program of locality computed from assessor statistics.

† Line 1 minus line 2.

‡ Computed on the basis of rainy and stormy days. For details, see: Bowlen, B. Product planning of crops for Iowa farms using linear programming. Unpublished Ph.D. thesis. Iowa State College Library, Ames. 1954.

§ This line is the smaller of lines 3 and 4. If line 3 is smaller than line 4 the farm has fewer hours of labor available than the amount of time allowed by weather; if line 4 is smaller than 3, unfavorable weather prevents use of all labor available for crops.

** Acres of hay and oats multiplied by labor required per acre.

†† Line 5 minus line 6.

TABLE 5. OPERATOR'S LABOR IN HOURS PER MONTH AVAILABLE FOR COMPETITIVE CROPS.

Township	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.
Washington	11.9	147.0	199.4	105.7	53.6	119.7	159.5	200.7	167.5
Harrison, Benton Co.	17.8	119.4	158.8	118.2	82.4	120.1	142.0	165.7	158.8
Troy	13.5	134.6	184.6	129.6	75.0	117.3	168.1	188.5	167.5
Grand Meadow	10.7	45.6	105.6	49.3	—22.6	27.2	77.8	110.3	106.3
Saratoga	13.7	82.5	132.9	114.3	53.0	69.2	133.1	136.8	132.1
Harrison, Kossuth Co.	21.6	142.1	167.7	143.3	123.1	148.7	155.4	173.4	167.5
Cedar	18.6	162.4	193.7	155.6	123.8	154.5	179.5	197.3	167.5
Oakland	17.7	141.1	178.8	158.4	119.8	143.1	173.0	187.9	167.5
Logan	22.5	112.8	140.1	118.1	102.2	127.7	128.8	146.2	141.7
Jordan	20.9	169.6	198.8	138.5	123.1	174.6	161.5	202.9	167.5
Lincoln, Montgomery Co.	18.0	134.3	170.9	131.1	95.9	132.9	153.4	177.9	167.5
Lincoln, Polk Co.	20.4	168.3	197.5	155.6	134.2	169.4	173.9	200.3	167.5
Sheridan	20.3	114.4	146.1	100.6	77.3	120.9	120.2	152.1	147.0
Reading	23.0	128.7	152.2	136.0	122.3	141.6	146.0	158.4	154.5

TABLE 6. OPERATOR'S AND FAMILY'S LABOR IN HOURS PER MONTH AVAILABLE FOR COMPETITIVE CROPS (SECOND LABOR SITUATION).

Township	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.
Washington	11.9	147.0	199.4	235.7	183.6	249.7	159.5	200.7	167.5
Harrison, Benton Co.	17.8	119.4	158.8	248.2	212.4	250.1	142.0	165.7	158.8
Troy	13.5	134.6	184.6	259.6	205.0	247.3	168.1	188.5	167.5
Grand Meadow	10.7	45.6	105.6	179.3	107.4	157.2	77.8	110.3	106.3
Saratoga	13.7	82.5	132.9	244.3	183.0	199.2	133.1	136.8	132.1
Harrison, Kossuth Co.	21.6	142.1	167.7	273.3	253.1	278.7	155.4	173.4	167.5
Cedar	18.6	162.4	193.7	285.6	253.8	284.5	179.5	197.3	167.5
Oakland	17.7	141.1	178.8	288.4	249.8	273.1	173.0	187.9	167.5
Logan	22.5	112.8	140.1	248.1	232.2	257.7	128.8	146.2	141.7
Jordan	20.9	169.6	198.8	268.5	253.1	304.6	161.5	202.9	167.5
Lincoln, Montgomery Co.	18.0	134.3	170.9	261.1	225.9	262.9	153.4	177.9	167.5
Lincoln, Polk Co.	20.4	168.3	197.5	285.6	264.2	299.4	173.9	200.3	167.5
Sheridan	20.3	114.4	146.1	230.6	207.3	250.9	120.2	152.1	147.0
Reading	23.0	128.7	152.2	266.0	252.3	271.6	146.0	158.4	154.5

PER-ACRE RESOURCE REQUIREMENTS OR INPUTS

The linear programming technique requires estimates of input-output coefficients of each resource used in the crops being considered for the production plan. An input-output coefficient can be defined as the quantity of resource required to produce one unit of a specified crop. Input-output coefficients are required for each crop for the three resources—labor, capital expenses and land. Annual capital expenses have not been broken down between individual items such as seed, tractor fuel, fertilizer, etc.

As a step in establishing these input-output coefficients, it was necessary to establish labor and capital requirements, or inputs per acre. In linear programming, these inputs are taken to be constants per acre of land and, hence, per unit of crop product. The capital requirements per acre for the several crops are shown in table 7. Total capital requirements include a "fixed" cost, which does not vary with per-acre yield, and a "variable" cost which does vary with yield.

Labor for each month is considered as a separate resource from the standpoint of planning the optimum crop program. It is necessary to establish labor input-output coefficients accordingly. The monthly labor requirements for each crop are shown in table 8. The same per-acre inputs of la-

TABLE 7. PER-ACRE CAPITAL REQUIREMENTS FOR CROPS.

Township	Corn	Oats	Soy-beans	Flax	Wheat	Red clover-timothy*	Alfalfa-brome	Rotated pasture
	($)	($)	($)	($)	($)	($)	($)	($)
Washington	19.49	14.73	17.98	—	—	21.13	27.92	8.13
Harrison, Benton Co.	21.61	16.51	19.19	—	—	23.63	30.42	9.45
Troy	20.21	15.27	18.55	—	—	21.50	28.29	8.50
Grand Meadow	21.64	16.25	18.68	—	—	23.98	30.30	8.86
	22.24†	17.74†	19.21†	—	—	—	—	—
Saratoga	19.80	15.57	18.32	—	—	22.38	28.70	8.67
Harrison, Kossuth Co.	21.00	15.99	18.72	17.05	—	23.02	29.10	8.84
Cedar	20.71	15.35	18.50	—	17.56	22.21	29.23	8.50
Oakland	21.43	16.28	19.46	—	—	23.66	29.98	9.48
Logan	20.48	15.89	18.62	17.03	—	23.45	30.00	8.80
Jordan	20.90	15.82	18.96	—	17.55	22.13	29.39	9.13
Lincoln, Montgomery Co.	21.10	15.79	19.11	—	18.01	23.15	29.47	9.21
Lincoln, Polk Co.	23.28	17.72	20.65	—	19.20	24.74	31.53	10.56
Sheridan	22.58	16.87	19.64	—	18.53	24.39	31.66	9.51
Reading	20.88	15.72	18.74	16.97	—	23.64	30.20	8.76

* Capital service inputs for hay are on a county basis.

† The July labor requirement in Grand Meadow Township, Clayton County, was more than could be supplied by the operator alone. In the situations where no family labor was available additional labor had to be hired which increased the capital service input.

TABLE 8. LABOR REQUIREMENTS IN HOURS PER ACRE BY MONTHS AND TOTAL FOR YEAR FOR CROPS.*

Crop	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
Alfalfa-brome	—	—	—	4.52	3.85	—	3.25	—	—	—	11.6
Red clover-timothy	—	—	—	1.52	1.204	0.228	0.532	—	—	—	3.5
Corn	—	0.826	1.540	0.917	0.749	—	0.140	1.036	1.428	0.364	7.0
Oats	0.355	0.895	—	—	1.875	1.875	—	—	—	—	5.0
Soybeans	—	0.588	1.458	0.870	0.666	—	0.174	2.244	—	—	6.0
Flax	0.355	0.895	—	—	1.875	1.875	—	—	—	—	5.0
Wheat	—	—	—	—	3.810	0.762	1.428	—	—	—	6.0

* Based on a report by United States Department of Agriculture, Iowa Agricultural Experiment Station and Iowa Agricultural Extension Service cooperating. Iowa wartime maximum agricultural capacity. Unpublished report. Iowa State College, Ames, Iowa.

bor have been used for each township. These figures do not include hired labor needed for help during harvest.

PRICES AND YIELDS

Prices

Determination of optimum plans for competitive crops is based on two different price situations. The first price period includes the 5 years 1948-52. Average prices for each crop were obtained for this period. The capital costs or expenses per acre shown previously also refer to the 1948-52 period. This period was one in which corn had a favorable price relative to soybeans and oats; corn was less favorably priced than wheat and flax in the 1948-52 period. Then, to examine whether the optimum cropping plan would differ under other prices, a second price situation was selected. The relative crop prices of the period 1941-44 were used for this situation. Wheat and flax prices were lower relative to corn in 1941-44 than during 1948-52; soybeans and oats had more favorable prices relative to corn in 1941-44 than in 1948-52. The price ratios rather than absolute prices of 1941-44 were used. In other words, the prices were increased to the general level of 1948-52, but, relative to each other, the prices were adjusted to give the same ratio as in 1941-44 for corn as compared to the other crops. These prices for Iowa are indicated in table 9.

Capital costs or expenses were left at the 1948-52 level for both product price situations. This procedure was followed since cost ratios between crops remain almost constant over short periods of time. Also, the goal is to determine how different crop price ratios alter the optimum plan. The two sets of price ratios used are the extremes found for the last 25 years. Hence, if the same crop plan is optimum under each price situation, we can be certain that the same plan will be optimum for any other set of price ratios falling between these extremes.

Yields

The yields used for establishing input-output coefficients are averages for the period 1917-52, adjusted to current techniques. Adjustment was made by running a regression of yield agaist time. Where these regression coefficients were significant, the average predicted yield of the last 5 years was used. Where the regression coefficient was not significant, the average of the entire period was used. Corn yields also were adjusted for differences between open-pollinated and hybrid varieties. The 1917-52 base period (instead of a shorter more recent period) was used since (1) the yields of the longer period could be adjusted to the present by regression and (2) a second step in the programming analysis will include plans which consider variability of yield. The current study and the one following can then be related for recommendations on farmers' decisions. Table 10 shows the yields used.

TABLE 9. AVERAGE PRICE PER BUSHEL FOR CROPS FOR PERIODS 1948-52, 1941-44 ACTUAL AND 1948-52 ADJUSTED TO 1941-44 RATIOS.

Crop	1948-52*	1941-44 actual*	1948-52 adjusted to 1941-44 ratios
Corn	$1.43	$0.89	$1.43
Oats	0.76	0.57	0.92
Soybeans	2.54	1.74	2.80
Flax	3.97	2.44	3.92
Wheat	2.03	1.22	1.96

* Source: Iowa Crop and Livestock Reporting Service.

Number of Resources, Price Situations

An optimum cropping program was developed for the several quantities of labor and capital and the two price situations in each of the 14 locations. The total number of optimum plans to be computed is thus 12 × 14 or 168. The main objective is to (1) determine specifically the optimum use of resources for each of the 168 situations, (2) determine how different quantities of resources and different price situations cause the optimum cropping plan to differ on a single farm with a particular soil type and (3) determine how the optimum cropping program varies between locations or soil types. Table 11 lists the 12 resource and price situations for each location. These are the situations referred to in subsequent sections of this study.

LOGIC AND TECHNIQUE OF LINEAR PROGRAMMING

Logic

Linear programming obtains its name from the propositions used in respect to production coefficients. Application of the system supposes the production coefficients are constant or that the input-output curve or production function is linear. Constant rather than diminishing marginal products or substitution rates are employed.

TABLE 10. AVERAGE ADJUSTED YIELD PER ACRE IN BUSHELS BY TOWNSHIP 1917-52.*

Township	Corn†	Oats	Soy-beans	Flax	Wheat	Red clover-timothy‡	Aflalfa-brome‡
Washington	31	26	15	—	—	1.0	1.8
Harrison, Benton Co.	49	33	14	—	—	1.5	2.3
Troy	38	31	17	—	—	1.0	1.8
Grand Meadow	61	43	15	—	—	1.9	2.5
Saratoga	37	31	11	—	—	1.3	1.9
Harrison, Kossuth Co.	50	37	15	12	—	1.5	2.0
Cedar	49	31	19	—	18	1.3	2.2
Oakland	45	28	16	—	—	1.5	2.1
Logan	40	39	14	13	—	1.7	2.4
Jordan	40	23	15	—	17	1.0	2.0
Lincoln, Montgomery Co.	47	27	15	—	17	1.4	2.0
Lincoln, Polk Co.	62	40	21	—	20	1.5	2.3
Sheridan	67	45	23	—	24	1.8	2.8
Reading	42	35	18	9	—	1.8	2.5

* Source: Adjusted from average yields reported by Iowa Crop and Livestock Reporting Service.

† Open pollinated corn yields were corrected to hybrid basis.

‡ United States Census of Agriculture, 1944 and 1949. Average yields are for counties rather than townships.

TABLE 11. RESOURCE-PRICE SITUATIONS FOR EACH LOCATION.

Situation	Prices	Labor available	Capital available
1	1948-52	Operator's time	$1893.51
2	1948-52	Operator's time	$2840.27
3	1948-52	Operator's time	Unlimited
4	1948-52	Operator plus family	$2840.27
5	1948-52	Operator plus family	Unlimited
6	1948-52	Unlimited	Unlimited
7	1941-44	Operator's time	$1893.51
8	1941-44	Operator's time	$2840.27
9	1941-44	Operator's time	Unlimited
10	1941-44	Operator plus family	$2840.27
11	1941-44	Operator plus family	Unlimited
12	1941-44	Unlimited	Unlimited

For our study, the term "linear" refers to constant resource requirements per acre or constant yields for each additional acre, hour or dollar of resources used for different crops. The system supposes that if we produce 1 acre of corn yielding 55 bushels with 7 hours of labor and $15 capital expense, 110 bushels will be forthcoming from 2 acres, 14 hours and $30; 550 bushels will be forthcoming from 10 acres, 70 hours and $150. The same assumptions are used for all crops, up to the limit of 154 acres of cropland. The current study employs only one technique of production for each crop in each locality. Later studies will analyze optimum methods of production (different techniques) for given crops or combinations of crops.

In the process of linear programming, it is necessary to set up a "tableau" or "computation sheet" which allows "automatic" solution for the crops that should be produced and the amount of resources to be used for each. For this "tableau" or "computational sheet," we must have the prices of the crop products and the amount of each resource used to produce a unit (bushel or ton) of each. The amounts of each resource used per unit of product is the input-output coefficients. Once these quantities are obtained we can set up the computational sheet and quickly solve for the best or most profitable plan, with due consideration to the quantity of each resource and the manner in which it may limit production. These input-output quantities are computed from the yields and resource requirements outlined on previous pages.

Hence, if 55 bushels of corn can be produced with 1 acre of land, 2 hours of labor in June and $15 in capital, the input-output coefficients are $1 \div 55 = 0.018$ for land, $2 \div 55 = 0.036$ for June labor and $\$15 \div 55 = 0.273$ for capital. These numbers can be inserted in the tableau or computational sheet (also called a matrix) along with prices, and the optimum plan can be obtained. But, because certain other conditions must be met, the technical conditions are outlined below and are followed by sections on computations.

In the terminology of linear programming, each different enterprise (kind of crop) or method of production, is called an activity. Hence, the term "activity" is used synonymously with "crop enterprise" in the sections which follow. Three basic postulates are used in the linear programming:

1. The production or cropping opportunities of a farm are defined by, and limited to, the resources available and the crops which can be grown. Both the amount of one or more resources and the number of crops are limited. Choice must be made within these constraints.

2. An activity (enterprise) may be carried on at any positive level. However, a negative quantity of a crop cannot be grown. The output of product and the use of resources will be proportional to the level of activity (amount of crop produced) since linear conditions are used.

3. Several activities may be carried on simultaneously (the optimum use of resources can include a combination of enterprises). The quantity of resources used is the sum of the quantities necessary for the separate activities; output is the sum of production of the several enterprises or activities.

The basic concepts of linear programming which have been discussed in this section may be clarified by a geometric presentation. For example, a farmer in Washington Township, Appanoose County, may choose among various combinations of three competitive crops—corn, oats and soybeans. He may decide to grow any one, some combination of two of them or some combinations of all three. Each crop requires resource inputs in different proportions; 0.02416 hours of July labor and $0.62875 capital expense are required per bushel of corn while 0.07211 hours of July labor and $0.56652 capital expense are required per bushel of oats in this area. The production of a bushel of soybeans requires 0.04440 hours of July labor and $1.19873 capital expense. These relationships are illustrated in fig. 2. Only two resources, July labor and capital are considered. Other resources are necessary in the production of these crops but only two can be shown in a two dimensional drawing.

The production possibility curve for each pair of crops consists of one or more linear segments. Each segment is a part of an iso-resource curve for a resource used in both crops. A constant marginal rate of substitution is specified by the segment of each iso-resource curve. The segment of each iso-resource curve, defining the production possibility curve, indicates the extent to which the particular resource limits production of either crop. It also indicates the marginal rate at which one crop substitutes for the other along the particular segment. For example, if one more bushel of oats were to be grown in Washington Township, the July labor requirement for this expansion would necessitate the giving up of $\frac{0.07211}{0.02446} = 2.98$ bushels of corn or $\frac{0.07211}{0.04440} = 1.62$ bushels of soybeans.[9] Likewise, if another bushel of oats were grown, 2.98 bushels of corn or 1.62 bushels of soybeans would have to be given up.

[9] The substitution rates are obtained by dividing the July labor requirements of oats by the July labor requirements of corn and soybeans, respectively.

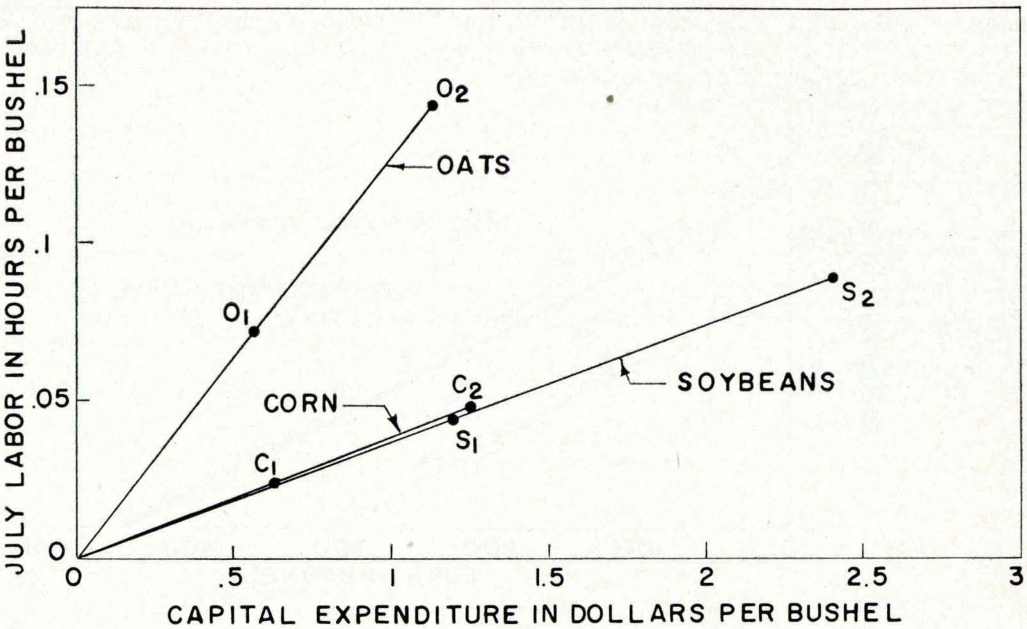

Fig. 2. Proportions of July labor and capital required in production of oats, corn and soybeans in Washington Township.

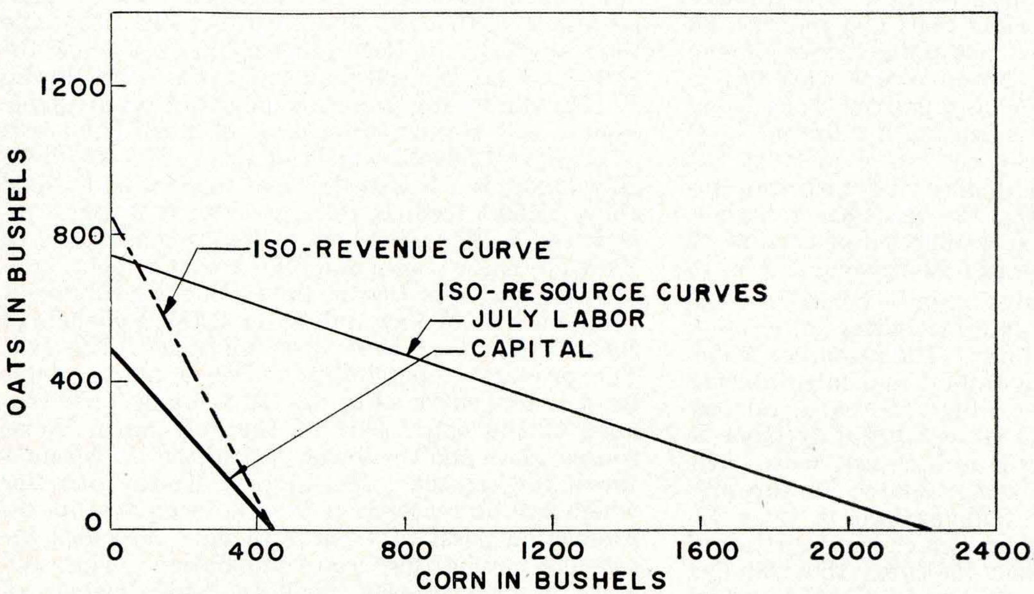

Fig. 3. Optimum combination of corn and oats under Situation 1, Washington Township.

In fig. 3 the iso-resource curves are shown for July labor and capital in the production of oats and corn. An iso-resource curve for capital in the production of soybeans and corn is shown in fig. 4. Other iso-resource curves could have been included for other resources used in producing each pair of crops. However, the two shown, July labor and capital in fig. 3 and capital in fig. 4, are the most restrictive in production. The extent of the restriction imposed on each crop by the resources considered is shown in table 12. In fact, July labor need not have been included in fig. 3 since capital is the most limiting resource to each activity or crop in this instance. That is, enough July labor is available to produce 2,216.5 bushels of corn but the available capital limits production to 445.8 bushels. Capital limits the production of oats to 494.7 bushels, even though July labor is sufficient to produce 742.6 bushels. Soybean production is limited to 233.8 bushels by the available capital while July labor is adequate for 1,206.0 bushels.

TABLE 12. NUMBER OF BUSHELS OF EACH CROP WHICH MAY BE GROWN WITH THE RESOURCES AVAILABLE, WASHINGTON TOWNSHIP.

Resource	Corn	Oats	Soybeans
Capital	445.76	494.72	233.81
July labor	2,216.50	742.60	1,206.00

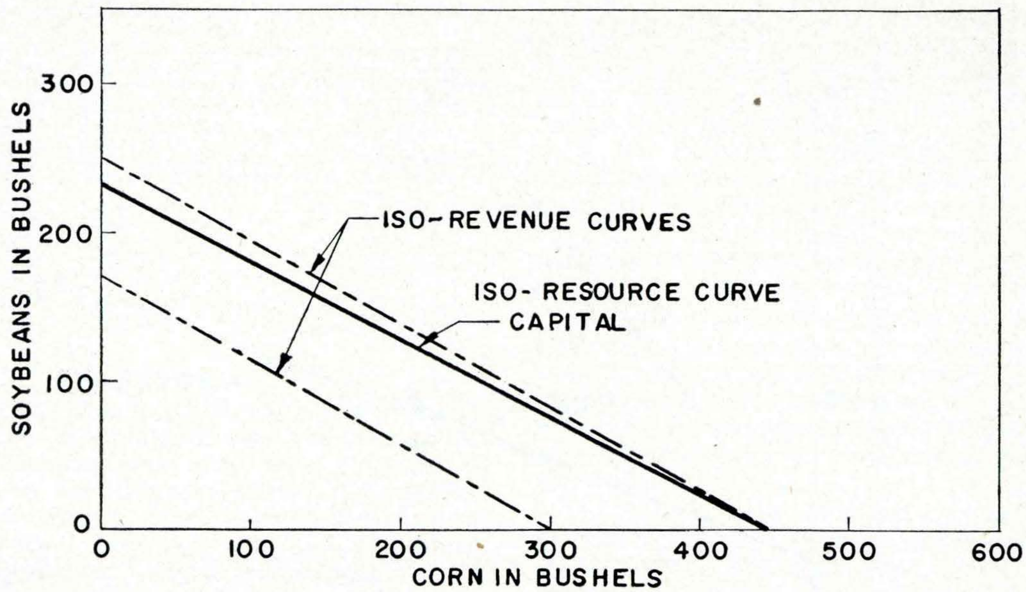

Fig. 4. Optimum combination of corn and soybeans under Situation 1, Washington Township.

Figures 3 and 4 can be used to indicate the most profitable combination of activities. Iso-revenue curves are shown as broken lines and indicate all combinations of the two crops which provide the same revenue.[10] The iso-revenue curves are drawn parallel to each other for any pair of crops being considered, each curve indicating a different level of revenue. The farther the curve is from the origin the larger is the quantity of each crop involved and, consequently, the revenue is higher. In figs. 3 and 4 note that production of corn alone permits reaching the highest iso-revenue curve.

The same type of analysis can be considered for Logan Township with four activities or crops—corn, oats, soybeans and flax. The resources which may be limitational are capital and labor during March, July and October. Flax and oats are compared in fig. 5. It is observed that July labor is the most limiting resource to both activities. The limitation imposed by each resource on the production of each crop is summarized in table 13. The highest iso-revenue line which can be attained in a flax-oats comparison indicates that all flax and no oats should be produced. Corn and soybeans are compared in fig. 6 and it is seen that growing all corn will maximize revenue. October labor is the most restricting resource in soybean production while capital limits corn most severely.

The analysis above indicates that flax is more profitable than oats, and corn is more profitable than soybeans in Logan Township.[11] The profitability of flax relative to corn is considered in fig. 7. This figure and table 13 indicate that available capital will permit production of 2,787.2 bushels of corn or 1,089.2 bushels of flax. The available July labor was less restrictive to corn and would allow 5,460.5 bushels to be grown; it is more restrictive to flax since only 708.7 bushels can be grown. These two resources are the most limitational and specify the production maximum as 708.7 bushels of flax (labor) or 2,787.2 bushels of corn (capital) or some combination of the two. The production possibility curve is not a single iso-resource curve as in figs. 3, 4, 5 and 6 but consists of the upper part of the July labor iso-resource curve and the lower part of the iso-resource curve for capital. The highest iso-revenue line which can be reached on the two segments of the production possibility curve in fig. 7 specifies the revenue maximizing crop combination. The optimum production plan includes 1,460 bushels of corn and 520 bushels of flax.

In each case illustrated (figs. 3, 4, 5, 6 and 7) the objective was the same, maximizing profit (reaching the highest possible iso-revenue curve) with a given set of resources. Resources are com-

[10] The quantities maximized in the linear programming computations of this study are gross revenues. However, when maximum gross revenue has been attained in this particular study, maximum net revenue is also attained. This identity exists because: (1) some resources are limited, (2) the capital costs per unit of crop, even where it is not subtracted as a cost to give net price, are always less than the price per unit of crop (3) the net price ratios have the same rank as the market price ratios used and (4) the cost per unit of crop is constant, up to the amount which can be produced with the limitational resources. Hence, optimum plans can be defined in terms of either gross revenue or net profits; the two terms are used synonymously in the text. In later tables, however, net profits have been computed by subtracting fixed and variable costs from total revenue. In these instances the term *net profit* is used. (Maximization of total revenue and net profit are attained simultaneously only under conditions of linearity and the price ratio conditions such as those of this study. Maximum total or gross revenue need not result in maximum net profit under conditions of diminishing returns.)

[11] The reader is again reminded that maximization of total revenue is identical with specification of the maximum profit plan under the conditions of this study. See footnote 10 for detailed reasons.

TABLE 13. NUMBER OF BUSHELS OF EACH CROP WHICH MAY BE GROWN WITH THE RESOURCES AVAILABLE, LOGAN TOWNSHIP.

Resource	Corn	Oats	Soybeans	Flax
Capital	2,787.17	3,502.17	1,072.93	1,089.24
March labor	—	2,467.03	—	822.04
July labor	5,460.47	2,126.04	2,148.83	708.68
October labor	5,646.33	—	912.35	—

bined (consistent with crop opportunities) in such a way that the crops produced allow the highest iso-revenue curve to be reached.

Algebraic Technique

The geometric presentation above provides the basic logic of linear programming. However, it does not explain or illustrate the mathematical technique of planning optimum programs under the linear technique. The notes which follow are presented to illustrate the procedure. Data, as well as algebraic equations, are used to illustrate the technique. The notes which follow are simple and do not attempt to include all conditions and equations, which otherwise make the procedure appear overly complex.

An algebraic presentation of the technique of linear programming can be outlined as follows:[12]

P_j, which refers to a crop enterprise or activity, is a column vector in which a_{ij}[13] denotes the amount of the *i*th scarce resource used in the *j*th activity.

[12] Dorfman, Robert. Application of linear programming to the theory of the firm. pp. 24-27. Univ. of California Press, Berkeley. 1951.

[13] The coefficient, a_{ij}, is the quantity of the particular resource used to produce 1 bushel of the crop being considered. These coefficients are readily computed from tables 7, 8 and 10. The reciprocal of the yield is the land requirement per bushel. The capital service requirement per acre divided by the yield per acre is the capital service requirement per bushel. The several labor requirements per bushel can be found in the same way.

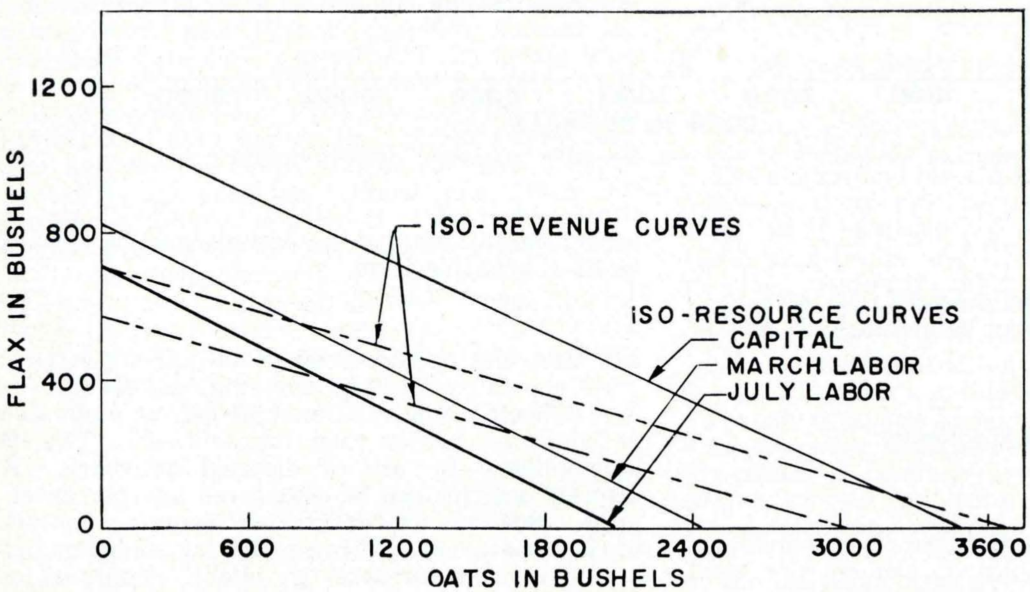

Fig. 5. Optimum combination of oats and flax under Situation 1, Logan Township.

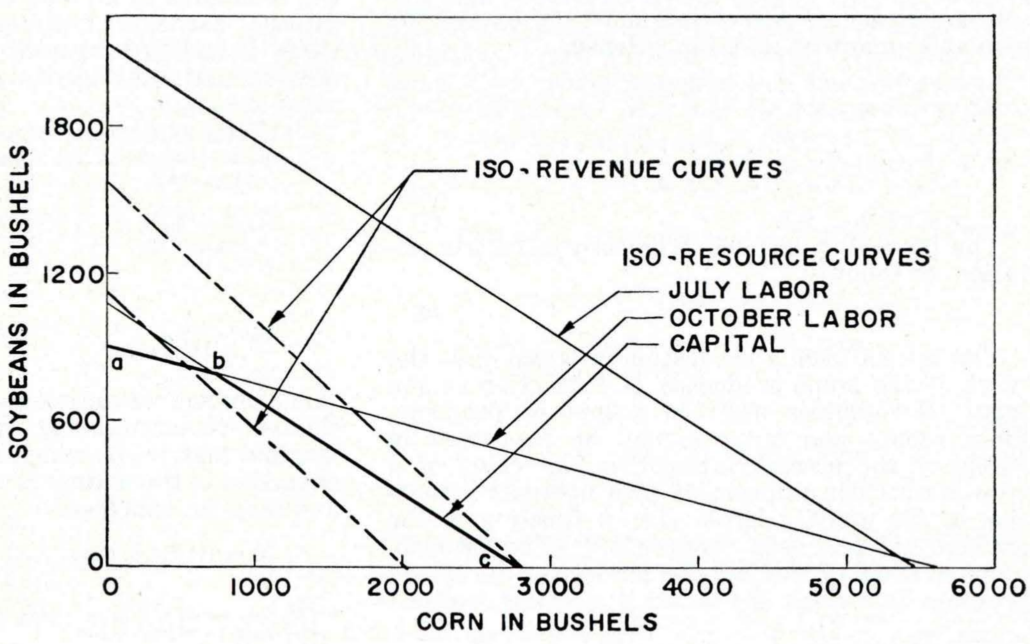

Fig. 6. Optimum combination of corn and soybeans under Situation 1, Logan Township.

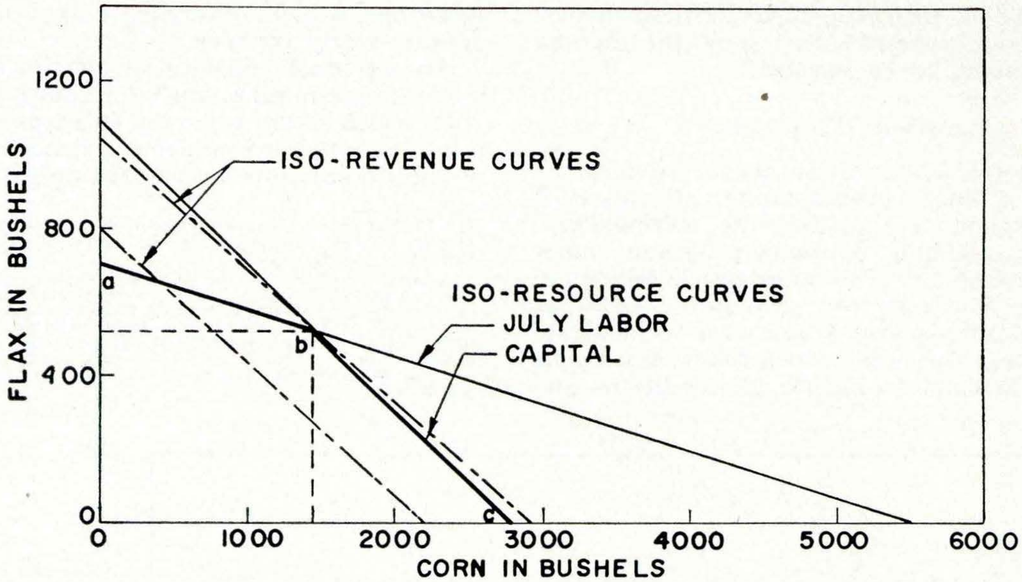

Fig. 7. Optimum combination of corn and flax under Situation 1, Logan Township.

$$P_j = (a_{1j}a_{2j} \ldots a_{nj}). \quad (1)$$
$$j = 1, 2, \ldots k$$

There are k activities and n scarce resources.

The column vectors may be arranged in matrix form:

$$A = (P_1P_2 \ldots P_k), \quad (2)$$

there being n rows and k columns.

The column vector,

$$X = (x_1x_2 \ldots x_k), \quad (3)$$

expresses the activity intensities (the amount of each crop produced) since x_j denotes the level at which the *j*th activity is carried on.

The quantity of each resource available is expressed by S_i. Consumption or use of each resource for all crop enterprises must not exceed the available quantity of each. Hence,

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1k}x_k \leqq S_1, \quad (4)$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2k}x_k \leqq S_2,$$
$$\ldots\ldots\ldots\ldots$$
$$a_{n1}x_1 + a_{n2}x_2 + \ldots a_{nk}x_k \leqq S_n.$$

$$S = (S_1 S_2 \ldots S_n). \quad (5)$$

The inequality may be expressed in matrix notation as follows:

$$AX \leqq S. \quad (6)$$

The return from a production program (i.e., the value of the crops produced) is a function of the input of resources and the output of products. Since inputs and outputs may be expressed in terms of the process intensities, the value of a crop production program or plan becomes a function of the activity levels (i.e., a function of the amount of each crop produced).[14] The problem becomes that of finding the production level and program for which the value is greatest, subject to the limitations that no activity can be carried on at a negative level,

$$X \geqq 0, \quad (7)$$

and that the resource cannot be exceeded (6).[15]

As shown by Dantzig, the empirical solution is less difficult if the relationships can be expressed as equalities rather than inequalities.[16] This is accomplished by use of disposal activities. A disposal activity can be considered another enterprise. But it is an "enterprise" denoting non-use of the resource (i.e., it means letting some amount of a particular resource "go idle"). Thus, in instances where less than the total quantity of a resource is required for all of the crops produced, the remainder is allowed to go to waste through a disposal activity. Consequently, one disposal activity is included for each scarce resource. These disposal activities may be represented by the variables $x_{k+1}\,x_{k+2} \ldots x_{k+n}$. The inequalities (4) may be written as equalities now.

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1k}x_k + 1x_{k+1} + 0x_{k+2} + \ldots + 0x_{k+n} = S_1 \quad (8)$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2k}x_k + 0x_{k+1} + 1x_{k+2} + \ldots + 0x_{k+n} = S_2$$
$$\ldots\ldots\ldots\ldots$$
$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nk}x_k + 0x_{k+1} + 0x_{k+2} + \ldots 1x_{k+n} = S_n$$

The numbers of activities is increased to k + n, the additional n being disposal activities. The original matrix, A, which had k columns has been expanded to the matrix B which has k + n columns and may be expressed:

$$B = (P_1 P_2 \ldots P_n \ldots P_{k+n}) = (A\, I). \quad (9)$$

[14] Ibid., p. 20.

[15] The bracketed numbers which appear in the text refer to the equations in this section.

[16] Ibid., p. 25.

where I is the identity matrix of n rows and columns.

The matrix B may be expressed in the following manner:

$$\begin{matrix} a_{11}\, a_{12} \ldots a_{1k}\; 1\; 0 \ldots 0 \\ a_{21}\, a_{22} \ldots a_{2k}\; 0\; 1 \ldots 0 \end{matrix} \quad (10)$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$a_{n1}\, a_{n2} \ldots a_{nk}\; 0\; 0 \ldots 1 = (A\ I). \quad (11)$$

The resource restrictions become:

$$X \geqq 0 \quad (12)$$

and

$$BX = S. \quad (13)$$

Actual data from Logan Township can be used to illustrate the above statements and the method. In this case, corn (P_1) is the first activity or enterprise, oats (P_2) is the second, soybeans (P_3) is the third and flax (P_4) is the fourth or *k*th activity or enterprise. Four resources are possibly limitational; they are capital, March labor, July labor and October labor. Labor during the other months is not considered limitational since the quantity available, relative to the requirements in crop production, is considerably greater than for the months considered. The iso-resource curve for each of the months not included would lie well above the iso-resource curves for other factors used in producing the crops under consideration. October labor is the *n*th scarce resource.

A matrix similar to (A) mentioned above is set up:

	P_1 (corn)	P_2 (oats)	P_3 (soy-beans)	P_4 (flax)
Capital ($)	0.51200	0.40740	1.33003	1.31012
March labor (hrs.)	0	0.00910	0	0.02731
July labor (hrs.)	0.01872	0.04808	0.04757	0.14424
October labor (hrs.)	0.02590	0	0.16029	0

(14)

The n rows specify the resources which may be limitational, and the k columns represent the activities (crop enterprise) which may be included in the production plan. The resources—capital, March labor, July labor and October labor—are written at the left of the matrix and identify the rows. The activities—P_1 (corn), P_2 (oats), P_3 (soybeans) and P_4 (flax)—are written across the top of the matrix. The figures in the matrix are the input-output coefficients mentioned previously and correspond to the a_{ij}'s (1). The matrix (15) is formed by adding to the matrix (14) a column P_0 [which corresponds to S in equation (5) above] specifying the quantity of each scarce resource available for competitive crop production in Logan Township. The rows in the matrix (15) are in the same order as in (14); capital is the top row and October labor is the bottom row.

P_0	P_1	P_2	P_3	P_4
1427.03	0.51200	0.40740	1.33003	1.31012
22.45	0	0.00910	0	0.02731
102.22	0.01872	0.04808	0.04757	0.14424
146.24	0.02590	0	0.16029	0

(15)

The matrix may be related to the geometric and algebraic presentation of the preceding sections. An equation can be written which describes each of the iso-resource curves in fig. 7. The iso-resource curve for capital is described by equation (16):

$$(0.51200)\ x_1 + (1.31012)\ x_4 = 1{,}427.03. \quad (16)$$

This equation specifies the capital requirement per bushel of corn produced, \$0.51,[17] multiplied by the number of bushels, x_1, plus the capital requirement per bushel of flax produced, \$1.31, multiplied by the number of bushels, x_4, equals \$1,427.03, the total capital available for annual expenses. Equations may be written for the other iso-resource curves as follows:

$$\text{March labor} \quad (0)\ x_1 + (0.02731)\ x_4 = 22.45. \quad (17)$$

$$\text{July labor} \quad (0.01872)\ x_1 + (0.14424)\ x_4 = 102.22. \quad (18)$$

$$\text{October labor} \quad (0.02590)\ x_1 + (0)\ x_4 = 146.24. \quad (19)$$

The similarity between the matrix (15) and the four equations (16 through 19) may be noted. If the four equations were grouped together, the right hand terms would be identical with column P_0 (15). Similarly, the input coefficients used in corn production form the elements of the column a_{i1} for activity P_1 (corn production). The columns P_2 (oats production) and P_3 (soybean production) have been excluded or placed at zero level. The analysis in figs. 5 and 6 indicated that oats and soybeans were less profitable than flax and corn. Therefore, to facilitate presentation in fig. 7 only the latter two crop enterprises were considered. The figures associated with the variable x_4 in the set of equations (16 to 19) comprise the coefficients of the different resources used in flax production (P_4).

To fulfill the condition imposed by each equation simultaneously, would require that all of each resource be used entirely. This, of course, is not necessary. Such a condition would be expressed in fig. 7 by a point at which all four iso-resource curves intersect. The use of any resource must not exceed the supply, but it may be less. Therefore, the relationships must be expressed as inequalities as in (4).[18]

$$(0.51200)x_1 + (0.40740)x_2 + (1.33003)x_3 + (1.31012)x_4 \leqq 1{,}427.03.$$

$$0x_1 + (0.00910)x_2 + 0x_3 + (0.02731)x_4 \leqq 22.45$$

$$(0.01872)x_1 + (0.04808)x_2 + (0.04757)x_3 + (0.14424)x_4 \leqq 102.22.$$

$$(0.02590)x_1 + 0x_2 + (0.16029)x_3 + 0x_4 \leqq 146.24. \quad (20)$$

In instances where the consumption is less than the supply, some portion of the resource is un-

[17] This is the capital input coefficient and has been rounded for convenience. Other input coefficients considered in the discussion will also be rounded for convenience in presentation.

[18] It was discovered in the geometric solution that oats, x_2, and soybeans, x_3, would not occur in the optimum plan. However, these opportunities must be included in the original matrix which was considered for a linear programming solution.

used. The unused portion will go into disposal or waste as was indicated in (8). One variable must be added for each disposal activity, x_5 represents the unused capital, x_6, x_7 and x_8 the unused labor in March, July and October, respectively. The four disposal activities expressed in rows and columns correspond to the "I" matrix which was referred to above.

The relationships may now be expressed as equations.

$$(0.51200)x_1 + (0.40740)x_2 + (1.33003)x_3 + (1.31012)\,x_4 + x_5 = 1{,}427.03.$$

$$0x_1 + (0.00910)x_2 + 0x_3 + (0.02731)x_4 + x_6 = 22.45.$$

$$(0.01872)x_1 + (0.04808)x_2 + (0.04757)x_3 + (0.14424)x_4 + x_7 = 102.22.$$

$$(0.02590)x_1 + 0x_2 + (0.16029)x_3 + 0x_4 + x_8 = 146.24 \qquad (21)$$

The completed matrix corresponding to B is as follows:

P_0	P_5	P_6	P_7	P_8	P_1	P_2	P_3	P_4
1,427.03	1				0.51200	0.40740	1.33003	1.31012
22.45		1			0	0.00910	0	0.02731
102.22			1		0.01872	0.04808	0.04757	0.14424
146.24				1	0.02590	0	0.16029	0

(22)

The price per bushel for each crop enterprise is listed in table 9; corn is \$1.43, oats \$0.76, soybeans \$2.54 and flax \$3.97 per bushel. With prices given, the problem is finding the activity levels, x_1, x_2, x_3 and x_4 (which are the numbers of bushels of corn, oats, soybeans and flax, respectively) which maximize the revenue function $(1.43)x_1 + (0.76)x_2 + (2.54)x_3 + (3.97)x_4$ (the price per bushel multiplied by the quantities of each crop).

The following optimum solution was obtained:

$$x_1 = 1{,}457.96 \qquad x_5 = 0$$
$$x_2 = 0 \qquad x_6 = 8.26$$
$$x_3 = 0 \qquad x_7 = 0$$
$$x_4 = 519.47 \qquad x_8 = 108.47$$

It is observed that the results are the same as those obtained in the geometric solution, figs. 5, 6 and 7. In both solutions, capital, x_5, and July labor, x_7 are found to be limitational. The production possibility curve in fig. 7 indicates that corn and flax production are extended as far as the capital and July labor resources permit, while in the algebraic solution $x_5 = 0$ and $x_7 = 0$ indicates that neither capital nor July labor was allowed to go to waste. In both solutions, it may be observed that something less than the available March and October labor is used.

Table 14 was set up from the matrix of coefficients (22) computed for crop production in Logan Township. Plan 1 of table 14 is essentially the same as the starting matrix. However, several rows and columns were added. The top row specified as c_j lists the prices which may be obtained for each unit of the activities included in the production plan.[19] The first plan, however, consists of the activities P_5, P_6, P_7 and P_8 which are the disposal or non-use of capital and labor in March, July and October. The price of disposal is zero by assumption, therefore, zeros appear in the c_i column. The P_0 column specifies the level or intensity (in the case of crop enterprises it is the number of bushels produced) of the activity in that row. Thus, in Plan 1[20] it may be seen that the level of disposal (non-use) is the complete stock of each resource. Zero profits are expected, therefore, from the production program specified in Plan 1.

[19] The c_j in most examples found in linear programming literature is a net price, that is, the cost of each input required per unit of product has been deducted. This is necessary when all the inputs are purchased in the market. In this problem, however, only the capital service input has a cost. In the case of labor it is assumed that the farmer does not have an opportunity to divert an hour's labor from the production of corn, for example, and work in the nearby factory. Thus no opportunity cost per hour can be considered for his labor. The same is true for land—he has all of the farm under his management and is concerned, therefore, with maximizing profits from the total acreage.

[20] Plan 1 specifies the first production program, Plan 2 the second program and so forth.

TABLE 14. A LINEAR PROGRAMMING SOLUTION BY THE SIMPLEX METHOD FOR FOUR ACTIVITIES WITH FOUR LIMITATIONAL RESOURCES IN LOGAN TOWNSHIP (SITUATION 1).

c_j		Disposal activities					1.43 Corn	0.76 Oats	2.54 Soybeans	3.97 Flax	
Plan 1											
c_i	Vector	P_0	P_5	P_6	P_7	P_8	P_1	P_2	P_3	P_4	R
	Cap. P_5	1,427.03	1	0	0	0	0.51200	0.40740	1.33003	1.31012	1,089.2
	Mar. P_6	22.45	0	1	0	0	0	0.00910	0	0.02731	822.0
	July P_7	102.22	0	0	1	0	0.01872	0.04808	0.04757	0.14424	708.7
	Oct. P_8	146.24	0	0	0	1	0.02590	0	0.16029	0	
z_j											
z_j-c_j							—1.43	—0.76	—2.54	—3.97	
Plan 2											
	P_5	498.57418	1	0	— 9.08292	0	0.34197	—0.02930	0.89795	0	555.2
	P_6	3.09595	0	1	— 0.18934	0	—0.00354	0	—0.00901	0	
3.97	P_4	708.67998	0	0	6.93289	0	0.12978	0.33333	0.32980	1	2,148.8
	P_8	146.24000	0	0	0	1	0.02590	0	0.16029	0	912.3
z_j		2,813.45952	0	0	27.52357	0	0.51523	1.32332	1.30931	3.97	
z_j-c_j		2,813.45952	0	0	27.52357	0	—0.91477	0.56332	—1.23069	0	
Plan 3											
2.54	P_3	555.23602	1.11365	0	—10.11517	0	0.38083	—0.03263	1	0	1,458.0
	P_6	8.09863	0.01003	1	— 0.28048	0	—0.00011	—0.00029	0	0	
3.97	P_4	525.56314	—0.36728	0	10.26887	0	—0.00418	0.34409	0	1	125,732.8
	P_8	57.24122	—0.17851	0	1.62136	1	—0.03514	0.00523	0	0	
z_j		3,496.78516	1.37057	0	15.07488	0	0.98390	1.28316	2.54	3.97	
z_j-c_j		3,496.78516	1.37057	0	15.07488	0	—0.44610	0.52316	0	0	
Plan 4											
1.43	P_1	1,457.96292	2.92427	0	—26.56085	0	1	—0.08568	2.62584	0	
	P_6	8.25901	0.01035	1	— 0.28340	0	0	—0.00030	0.00029	0	
3.97	P_4	519.46885	—0.37950	0	10.37989	0	0	0.34445	—0.01098	1	
	P_8	108.47404	—0.07575	0	0.68801	1	0	0.00222	0.09227	0	
z_j		4,147.17831	2.67509	0	3.22615	0	1.43	1.24494	3.71136	3.97	
z_j-c_j		4,147.17831	2.67509	0	3.22615	0	0	0.48494	1.17136	0	

Two rows appear at the bottom of Plan 1, table 14. The z_j row ($z_j = \Sigma x_{ij} c_i, j = 1, 2 \ldots 8$) specifies costs in an opportunity sense. The quantities in this row indicate the amount of revenue which would have to be sacrificed from the present program to permit the inclusion of 1 unit (bushel) of the *j*th crop in the program. In Plan 1, the z_j values are all zero which means that the addition of 1 bushel of either corn, oats, soybeans or flax involves no loss of revenue due to the giving up of some other crop enterprise. The explanation of this is expanded in the discussion of Plan 2.

Each quantity in the z_j-c_j row is the marginal revenue (expressed as a negative quantity) of one unit of the *j*th activity or crop enterprise. That is, the addition to total revenue resulting from the production of 1 additional bushel of the particular crop. It is noted in the corn enterprise (P_1) of Plan 1 that the marginal revenue is $1.43 per bushel. Since the z_1 value is zero, it is realized that increasing the production of corn by 1 bushel does not involve giving up the production of any other crop, nor, consequently, the revenue therefrom. Thus, the full price of the bushel of corn, $1.43, is added to total revenue. The reason why the quantities in the z_j-c_j row appear as negative values is considered below.

The total revenue from any production plan or program is the price per unit (bushel, in this example) multiplied by the number of units produced. The quantity of each crop produced is referred to as the intensity or level of the crop enterprise. Since the level of each crop enterprise (activity) in a production plan is given in column P_0 and the price per unit of each activity is given in the c_i column, the total revenue may be found by summing the products of prices and quantities produced. The total revenue for the production program specified in Plan 1 is the following:

$(0 \times 1{,}427.03) + (0 \times 22.45) + (0 \times 102.22) + (0 \times 146.24) = 0$, which is shown in the P_0 column, row z_0.

The production plan in which nothing is produced may be considered a feasible[21] starting point in the process of deciding the optimum allocation of resources. However, since the objective is the maximization of revenue, a new plan is introduced in which some production is carried on and consequently some revenue is added. The crop enterprise with the greatest "net marginal revenue," that is the crop which adds the most to total revenue per unit, is included in the plan to the extent that the available resources permit. It is seen in the z_j-c_j row of Plan 1, table 14 that flax has the largest "net marginal revenue" in absolute terms. Therefore, the production of flax is increased as much as possible in the new plan. How much can it be increased from the present zero level? Since $1.31 capital expense is required per bushel, the available capital, $1,427.03, permits the production of 1,089.24 bushels (column R). The March labor coefficient is 0.027. Consequently, the available March labor restricts flax production to 822.04 bushels. However, July labor is most limiting, being sufficient for the production of only 708.7 bushels. The decision to produce 708.7 bushels of flax exhausts the supply of July labor. The activity (P_7) denoting the disposal or non-use of this resource (July labor) may be removed from the production plan and the crop enterprise, flax (P_4), used to replace it. The production of flax has now been made an "active" crop enterprise and is included in Plan 2. All other activities are expressed in terms of it.

Flax production (P_4) appears in Plan 2 in the row previously occupied by P_7, the disposal or non-use of July labor. The level of the P_4 enterprise (production of 708.7 bushels of flax) appears in the P_0 column. It has been indicated that this number (708.7) was obtained by dividing the available July labor (102.22 hours) by the requirement per bushel of flax (P_4) produced. All other quantities in the P_7 row are divided by the same number (0.14424) to obtain the figures in the P_4 row. Returning briefly to Plan 1, the figures in the columns P_1, P_2, P_3 and P_4 of the P_7 row specify the requirement of July labor for the production of each bushel of corn, oats, soybeans and flax respectively. When these quantities are divided by the July labor requirement per bushel of flax, the resulting numbers specify the quantity of flax which has to be given up to allow 1 bushel of corn, oats, soybeans or flax to be produced. This is the meaning of the numbers which appear in row P_4 under P_1 (0.12978), P_2 (0.33333), P_3 (0.32980) and P_4 (1). The numbers are marginal rates of substitution[22] of corn, oats and soybeans, respectively, for flax as specified by the requirements of each for July labor. The last number, P_4, is obvious; one bushel of flax has to be given up to permit the production of another bushel of flax when the most limiting resource in flax production is being used to capacity. The meaning of the numbers under the other column headings, P_5 through P_8, in row P_4 may be explained in the same way. For example, the quantity 6.93 in column P_7 indicates the quantity of flax which would have to be given up if 1 hour of July labor were put in disposal (not used).

Row P_4, flax production, is now in the program. It has been decided to use all of the July labor available in this crop enterprise. Other resources have to be used in the proportions indicated by the production coefficients. Plan 2, table 14, specifies the new production program. It includes the production of flax and the disposal (non-use) of capital and labor in March and October.

The numbers appearing in the rows and columns of the Plan 2 section are completely changed from those in Plan 1.

Two formulae have been used to complete this transformation.

[21] The program is feasible in the sense that the use of resources does not exceed the supply.

[22] The marginal rate of substitution of corn for flax is the quantity of flax which has to be given up to allow the production of 1 bushel of corn ($\triangle F/\triangle C$).

$$a'_{kj} = a_{rj}/a_{rk}. \quad (23)$$

$$a'_{ij} = a_{ij} - (a_{rj}/a_{rk})\, a_{ik}. \; (i \neq k) \quad (24)$$

The subscript k indicates the crop enterprise (flax) coming into the production plan, r is the activity (disposal of July labor) being removed; j stands for any one of the column headings and i stands for the number of the row heading. The "prime" indicates that the number to which it is attached belongs to the new section being formed (i.e., the new program).

Equation (23) expresses the quantities which were discussed in the new row, P_4, of Plan 2. The other numbers appearing in Plan 2 were found by using equation (24). As an example, the quantity in the column P_0 row P_5 of Plan 2 is considered:

$$a'_{ij}\,(498.57) = a_{ij}\,(1{,}427.03) - \left[\frac{a_{rj}\,(102.22)}{a_{rk}\,(0.14424)}\right] a_{ik}\,(1.31012).$$

The meaning of the number thus obtained is important. The quantity 498.57 is the amount of capital which is unused in the second production plan. It is the original quantity, $1,427.03, less the amount required to produce 708.7 bushels of flax. The numbers in column P_0, rows P_6 and P_8 in Plan 2 are the quantities of March and July labor, respectively, which remain unused in the second production plan.

How can the optimum plan be identified? The rows z_j and z_j-c_j contain the answer to this question. The quantities in the z_j row ($j = 1, 2, \ldots 8$) are opportunity costs. That is, in the P_1 column the figure 0.52 is the value of the flax which has to be given up from the production plan if 1 more bushel of corn is grown.[23] However, the revenue from 1 bushel of corn is $1.43. Therefore, the "net marginal revenue" (considering the opportunity cost) of 1 bushel of corn is the opportunity cost minus the price.[24] Where the price is greater than the opportunity cost the "net marginal revenue" will be expressed as a negative number. The meaning is somewhat the same as the usual concept of marginal revenue; it is the addition to total revenue, in absolute terms, resulting from a 1-unit change in production. If 1 bushel of corn, worth $1.43, is added to the production plan, $0.52 worth of flax must be given up. The increase in total revenue will be $0.91. The opportunity cost of growing an additional bushel of oats, on the other hand, is $1.32, while the value of a bushel of oats is only $0.76. The value of the flax given up to permit the production of 1 bushel of oats exceeds the value of the oats by $0.56. Only $1.31 worth of flax must be given up to allow 1 bushel of soybeans to be produced. Since the bushel of soybeans is worth $2.54, the "net marginal revenue" of this crop enterprise, as given by z_3-c_3, is $1.23.

It can be seen that the optimum plan has not been reached until it is no longer possible to add to total revenue by substituting one crop for another. That is, until no negative quantities remain in the $z_j - c_j$ row. The optimum plan is reached by successive approximations. If a plan is found to be non-optimum (negative $z_j - c_j$ values appear), a new plan is made. The crop enterprise which had the largest "net marginal revenue" in the previous plan is increased as far as resources permit. Plan 2, table 14, does not specify an optimum plan since the addition of either soybeans (P_4) or corn (P_1) would increase total revenue. Soybeans have the larger "net marginal revenue" and will be included in the next program as specified in Plan 3. To what extent can soybeans be produced? The resources which are not being used appear in the P_0 column of Plan 2. In this plan $498.57 capital, 3.096 hours of March labor, zero July labor and 146.24 hours of October labor go to disposal (non-use). But July labor is required in the production of soybeans as well as flax, therefore, the production of flax must be reduced before any soybeans can be included in the new program.

To determine the extent to which soybeans may be added to the program, each number in column P_3 of Plan 2 is divided into the corresponding number in the P_0 column. The results, which are the restrictions imposed by each resource on production of soybeans appear in column R. The meaning of each number in the P_0 column (Plan 2) is clear but those in the P_3 column will bear further consideration. Consider the first one, 0.89795. It concerns the limitation imposed by available capital on the introduction of soybeans into a new production plan. According to equation (24) the number, 0.89795, results from the following calculation:

$$1.33003 - \frac{0.04757}{0.14424}\,(1.31012) = 0.89795$$

or

$$1.33003 - (0.32980)\; 1.31012 = 0.89795. \quad (25)$$

The first number in equation (25) is the capital expense per bushel of soybeans produced (table 14). The quantity in brackets (0.32980) is the marginal rate of substitution of soybeans for flax as specified by the relative requirement of each crop for July labor. The last quantity in equation (25) is the capital expense required per bushel of flax. When the latter two quantities are multiplied, the product is the amount of capital which is released from flax production as each bushel of soybeans is added to the production plan.[25] This quantity is subtracted from the capital require-

[23] It was indicated that 0.12978 bushels is the quantity of flax which has to be given up if 1 bushel of corn is added to the production plan. Since the price of flax is $3.97 per bushel, the opportunity cost is $3.97 x 0.12978 = $0.51523.

[24] If variable costs were subtracted from price to give net price, our figure might best be termed "net marginal profit." In both cases, the meaning deviates from the usual concept of marginal revenue where we refer to the gross amount added to total revenue as one more unit of resource is used for producing a particular product (without subtracting from the output of another product).

[25] The rate of substitution is specified by the relative requirement of flax and soybeans for July labor, 0.32980 bushels of flax being given up for each bushel of soybeans added to the production program.

ment per bushel of soybeans (1.33003) to give the "net" expenditure out of available capital ($498-.57) per bushel of soybeans to a new program. Capital will restrict soybean production to 555.24 bushels as seen in column R. March labor is not required in soybean production and therefore imposes no limitation.

The July labor requirement permits 2,148.82 bushels of soybeans to be produced. This requires that all flax production be given up since 0.32980 is the quantity of flax which is given up as each bushel of soybeans is added to the plan. If 0.32980 bushel of flax is given up, 1 bushel of soybeans may be added. Or, if all flax is given up, 708.7/0.32980 = 2,148.81 bushels of soybeans can be grown. The quantity 708.7/0.32980 may also be written as follows:

$$\frac{102.22/0.14424}{0.04757/0.14424} = \frac{102.22}{0.04808} = 2{,}148.81740.$$

This shows that the total July labor at the outset, divided by the requirement per bushel of soybeans, will permit the indicated production of soybeans. The statement is equivalent to saying that the total possible production of flax divided by the rate at which soybeans substitute for flax in production will give the maximum of soybeans which can be produced with the available quantity of the limiting resources (July labor).

October labor is not used in flax production so the quantity available in the original resource supply, 146.24 hours, is available for soybean production in a new plan. Since 0.16029 hour is required per bushel, October labor limits production to 912.35 bushels.

The limitations imposed on the production of soybeans in a new plan have been considered. It is found that the limitation imposed by each resource is at a different level of production. That is, capital will allow 555.24 bushels to be produced, March labor imposes no restriction. July labor is sufficient for the production of 2,148.82 bushels of soybeans. However, this soybean production requires that all flax be given up. October labor limits production to 912.35 bushels of soybeans. Each limitation was determined by dividing the quantities in the P_0 column of Plan 2 by the corresponding number in the P_3 column of Plan 2.

A new production program is set up in Plan 3. Both flax and soybeans are included. Capital is the most restricting resource in soybean production, therefore, P_3 replaces P_5, the disposal (nonuse) of capital. Equations (23) and (24) are used to complete the transformation.

The optimum program is found in Plan 4 with the production of 1,457.96 bushels of corn (P_1) and 519.47 bushels of flax. The total revenue from this production plan is $4,147.18. No other combination of crops can be produced which yields as much profit with the available resources.[26]

[26] Again, we remind the reader that maximization of total revenue also results in maximization of net profits under the conditions of this study (see footnote 10 for details). Maximization of total revenue does not result in maximization of net profit under conditions of diminishing returns, if unit cost of production exceed unit prices or if net price differs greatly from gross price ratios.

The allocation of resources among the several crop enterprises under Plan 4 is presented in table 15.

PRESENTATION OF RESULTS FOR TOWNSHIPS

The method outlined above has been used for all townships and all the resource-price situations outlined previously. The results of the study are considered under the subheadings, Situations 1 to 12. Situations 1 to 6 involve 1948-52 prices while 1941-44 price ratios are considered in Situations 7 to 12. The resource quantities differ in each of the Situations 1 to 6 but are repeated in Situations 7 to 12. That is, the same resources are available in Situation 7 as in Situation 1, etc. Prices and resource quantities available in each situation are summarized in table 11; the acreage available for competitive crops in each area is listed in table 3. An optimum or revenue maximizing program was determined for each situation and for each location included in the study.

Method of Presentation

The production plans and the allocation of resources for each area, under the several resource situations considered, are shown in tables 16 through 29, excluding table 21. In most situations the optimum plan for Logan Township includes two crops whereas the other townships include only one. Consequently, the results obtained in Logan Township, for Situations 1 to 6, are considered in the discussion. The results obtained for Washington Township are used to demonstrate the effect of changes in price ratios.

A "minimum" hay acreage, including rotated pasture,[27] was fixed for each township. The possibility of expanding the hay acreage beyond the "minimum" was not included as a crop opportunity. Therefore, the "minimum" acreage appears in each table. A minimum oats acreage was specified as nurse crop for the grass being seeded. The oats acreage could, however, be increased beyond this minimum amount if the crop proved sufficiently profitable. That is, oats is a competitive crop opportunity in each location. However, the analy-

[27] The pasture referred to in the tables includes only rotated pasture.

TABLE 15. ALLOCATION OF RESOURCES AMONG THE FOUR ACTIVITIES IN LOGAN TOWNSHIP (SITUATION 1).

	Corn (P_1)*	Oats (P_2)	Soybeans (P_3)	Flax (P_4)	Total
Acres	36.45	0	0	39.96	76.41
Capital	746.48	0	0	680.57	1,427.04
Mar. labor	0	0	0	14.19	14.19
July labor	27.29	0	0	74.93	102.22
Oct. labor	37.76	0	0	0	37.76

* The resource quantities devoted to each activity are determined by multiplying the activity level, specified in the optimum plan, by the requirement per unit of that activity for each resource. Thus the capital requirement for corn is (1,457.96292 bushels of corn specified in table 14) multiplied by the capital requirement per bushel of corn (0.51200 in Logan Township) equals $746.48.

sis indicates that the oats acreage should not be increased beyond the minimum in any location since the resources can be used to better advantage in producing other crops. Thus the same acreage figures (the "minimum" oats acreage required as a nurse crop in each area) appear in each of the tables of results (tables 16 to 29, excluding 21).

The acreages of corn, other crops and unused land specified in the tables vary with the quantities of resources available for production in each situation. Where capital and labor are severely limited, the crop acreage tends to be small; the non-used land would have to be "rented out" to some other person with more capital or the operator "might move to a smaller farm." Where capital and labor resources are in adequate supply, crop production can be extended over the entire farm acreage. The columns in the tables specifying "labor used" in June, July and October include the requirements of the crop program indicated during these 3 months. The labor requirements in these months, relative to the supply, tend to be more critical than in other months. The hours of labor which remained in disposal (non-use) are listed as "labor unused" in the tables. The net profit column in each table includes the gross value from grain, hay and pasture less all capital expenses involved in their production.[28]

1948-52 Price Ratios

Situation 1

Based on the available labor quantities shown in table 5 and the available capital and land quantities shown in tables 2 and 3, respectively, a revenue maximizing solution for each township included in the study was found. The results are shown in table 16. Substantial portions of farm acreage in each township are "unused" under Situation 1. Labor is underemployed in most townships and revenue is low. It will be observed below that the severe limitation imposed by capital in Situation 1 is responsible for these results.

The unused acreage caused by capital limitation probably represents a substantial proportion of land ordinarily falling in the category of "plowable pasture." Capital limitation may cause many farmers to adopt a "less than optimum" (less than profit maximizing) production plan. For example, they may plant a larger than optimum acreage to oats to avoid leaving land unused. That is, they "spread" their capital over as much land as possible by growing a crop with a relatively low capital requirement per acre. The results of this study show, however, that a smaller acreage of corn may be more profitable than planting part of the land to oats "just to use the land." A different technique than the one chosen in this study also may be adopted by some farmers. Where capital is severely limited, farmers may combine relatively small amounts of capital with land and labor. A change in the proportions of inputs used per unit of product constitutes a different technique as defined above. Again, this is an attempt to "spread" the available capital, and the consequences to profit have not been considered. Unused land or plowable pasture generally can be rented out although the return may be low. In any case, either society or the farmer, or both, suffer as programs are limited by severe capital shortages.

The resource-use pattern in Logan Township is typical of all areas in Situation 1. The solution presented in table 14 indicated that maximum profit for Logan Township is obtained by planting 36.5 acres to corn, 40 acres to flax, no more than the minimum 11.4 acres to oats and no soybeans. Forty-six acres available for competitive crops are not used. As table 16 shows, some land would go unused in each of the other townships, because of limited capital in all cases and also because of limited July labor in Grand Meadow and Logan Townships.

Situation 2

The only change in the resource supply in Situation 2 as compared to Situation 1 is the use of the second capital level as specified in table 2. The increased level of capital permits a greater acreage to be planted to crops. The greater acreage is possible since the proportion of capital to land and labor in Situation 2 is nearly equal to the proportions in which these resources are required as inputs in

[28] The quantities of capital used and unused were not included in the tables since capital was a limitational resource (used entirely) in most cases. The few exceptions will be referred to specifically.

TABLE 16. ACTIVITY LEVELS AND NET PROFIT UNDER RESOURCE SITUATION 1.

Township	Acres planted to:					Acres unused	Labor used (hours)			Labor unused (hours)			Net profit ($)
	Pasture	Hay	Corn	Oats	Other		June	July	Oct.	June	July	Oct.	
Washington	—	45.5	14.4	41.1	0	53.0	109	166	15	92	43	186	586
Harrison, Benton Co.	39.8	21.4	44.0	24.5	0	24.3	94	123	46	78	50	120	3,482
Troy	—	38.0	25.8	36.5	0	53.7	91	142	27	106	55	162	1,116
Grand Meadow	38.8	29.9	14.8	44.6	0	25.9	87	156	15	36	0	95	3,271
Saratoga	21.3	21.9	39.0	36.0	0	35.7	69	124	40	79	24	97	1,948
Harrison, Kossuth Co.	13.1	10.5	69.0	13.9	0	47.5	98	107	71	80	71	102	4,095
Cedar	5.4	22.4	50.6	22.2	0	53.4	92	127	52	109	76	145	2,697
Oakland	28.2	9.8	52.7	24.8	0	38.5	79	112	55	110	80	133	3,037
Logan	10.3	9.8	36.5	11.4	40.0	46.0	69	154	38	85	0	108	3,348
Jordan	16.8	14.3	57.3	15.9	0	49.7	115	126	59	86	80	144	2,629
Lincoln, Montgomery Co.	16.8	19.2	47.8	24.0	0	46.2	95	123	50	87	60	128	2,760
Lincoln, Polk Co.	17.5	12.9	51.6	17.3	0	54.7	93	110	53	108	95	147	4,186
Sheridan	15.9	17.6	49.8	17.6	0	53.1	104	119	52	55	40	100	4,928
Reading	6.1	9.1	71.6	9.8	0	57.4	94	96	74	70	68	84	3,259

crop production.[29] Corn acreage is greater in each location and unused acreage occurs in only 10 townships and to a much smaller extent than in Situation 1 (table 16). A fuller use of labor is approached or achieved in most areas and net revenue is higher under Situation 2 than under Situation 1. The optimum crop production program for each area is listed in table 17.

Attention is focused on Logan Township (row 9, table 17) for a more detailed consideration of Situation 2. The increased corn acreage in this township under Situation 2, is accompanied by a reduction in both flax and unused acreage as compared to Situation 1. In fact, land was considered a limitational resource in Logan Township under Situation 2. Thus, the system of equations which specified the limitations imposed by available resources includes land. The equations are as follows:[30]

Land
$$(0.02500)x_1 + (0.02564)x_2 + (0.07143)x_3 + (0.07692)x_4 + x_5 = 122.5$$

Capital
$$(0.51200)x_1 + (0.40740)x_2 + (1.33003)x_3 + (1.31010)x_4 + x_6 = 2{,}373.8$$

July labor
$$(0.01872)x_1 + (0.04808)x_2 + (0.04757)x_3 + (0.14424)x_4 + x_7 = 102.2$$

October labor
$$(0.02590)x_1 + (0)x_2 + (0.16029)x_3 + (0)x_4 + x_8 = 146.2$$

These equations were converted to matrix form and the optimum solution obtained by the method demonstrated in table 14. The solution includes the production of 4,226.4 bushels of corn and 160.2 bushels of flax. The acreages associated with these yields are 105.7 of corn and 12.3 of flax. For a farmer with more capital than under Situation 1, corn rather than flax now becomes the main competitive crop with greatest profit. The particular shortages of resources, and the interaction of their shortages, causes some flax also to be profitable.

The reason for the increase in corn and reduction in flax acreage, relative to Situation 1, is readily apparent in a comparison of figs. 7 and 8.[31] The iso-resource curves for capital and July labor form the production possibility curve in each figure. However, the larger quantity of available capital in Situation 2 than Situation 1 causes the capital iso-resource curve to be farther from the origin in fig. 8 than in fig. 7. Thus, as indicated in fig. 8, 708.7 bushels of flax or 4,636.3 bushels of corn, or various combinations of both, can be produced in Logan Township under Situation 2. It is indicated in fig. 7 that 708.7 bushels of flax or only 2,787.2 bushels of corn, or various combinations of both can be produced under Situation 1.

A maximum of 708.7 bushels of flax can be produced in either situation. If the maximum flax is produced, the marginal rate of substitution of corn for flax is a constant, 0.1298.[32] However, 0.3602 bushel of flax is required to equal the value of 1 bushel of corn. Therefore, revenue can be increased by shifting resources from production of flax to corn as long as the marginal rate of substitution of corn for flax is 0.1298 (ab in each figure). The marginal rate of substitution becomes a different constant, 0.3908, if production of corn is extended beyond 1,458 bushels in Situation 1 (fig. 7) and beyond 4,226.4 bushels in Situation 2 (fig. 8) (bc in each figure). Capital is the most limiting resource if corn production is extended beyond these levels. The marginal rate of substitution becomes the relative requirement per bushel of each crop for capital, 0.51200/1.31010 = 0.3908. That is, 0.3908 bushel of flax must be

[29] It has been assumed in the disposal activity technique that unused resource quantities go costlessly to waste. In the four instances where land is a limitational resource some quantity of both labor and capital go into disposal (non-use). It would not be realistic to assume that capital goes to waste. Although in some cases it may be true that capital which is not urgently needed in the production program is used for household expenditures, it is considered that some part of the available capital is made up of bank credit. When not needed, the credit is not drawn.

[30] The quantity of corn, oats, soybeans and flax included in the program are expressed by x_1, x_2, x_3 and x_4, respectively. The disposal (non-use) of any resource is represented by the additional variable in that equation. For example, the disposal of land (acres not used) is expressed by x_5 or capital not used by x_6.

[31] Figure 7 represents Situation 1 and fig. 8, Situation 2.

[32] The marginal rate of substitution of corn for flax is the quantity of flax which would have to be given up from the production plan for each bushel of corn which is added. The rate, 0.1298, is specified by the relative requirement per bushel of each crop for July labor.

TABLE 17. CROP ACREAGES PRODUCED, PATTERN OF RESOURCE USE AND NET PROFIT UNDER SITUATION 2 REPRESENTING SECOND LEVEL OF CAPITAL.

Township	Acres planted to:					Acres unused	Labor used (hours)			Labor unused (hours)			Net profit ($)
	Pasture	Hay	Corn	Oats	Other		June	July	Oct.	June	July	Oct.	
Washington	—	45.5	63.0	41.1	0	4.4	153	202	65	48	6	136	1,793
Harrison, Benton Co.	39.8	21.4	68.3	24.5	0	0	116	141	71	56	31	95	4,658
Troy	—	38.0	72.6	36.5	0	6.9	134	177	75	63	20	114	2,715
Grand Meadow	38.8	29.9	40.7	44.6	0	0	111	175	42	12	0	68	4,953
Saratoga	21.3	21.9	70.7	36.1	0	4.0	98	147	73	50	0	64	2,995
Harrison, Kossuth Co.	13.1	10.5	116.5	13.9	0	0	142	143	121	36	35	52	6,551
Cedar	5.4	22.4	96.3	22.2	0	7.7	134	161	100	67	42	97	4,758
Oakland	28.2	9.8	91.2	24.8	0	0	114	120	94	75	52	94	4,692
Logan	10.3	9.8	105.7	11.4	12.3	4.5	133	154	110	21	0	36	4,934
Jordan	16.8	14.3	102.6	15.9	0	4.4	157	160	106	44	46	97	4,273
Lincoln, Montgomery Co.	16.8	19.2	92.7	24.0	0	1.3	136	157	96	46	25	82	4,828
Lincoln, Polk Co.	17.5	12.9	92.3	17.3	0	14.0	130	140	96	71	65	104	6,845
Sheridan	15.9	17.6	91.8	17.6	0	11.1	142	151	95	17	8	57	7,997
Reading	6.1	9.1	117.0	9.8	0	12.0	136	130	121	28	34	37	5,035

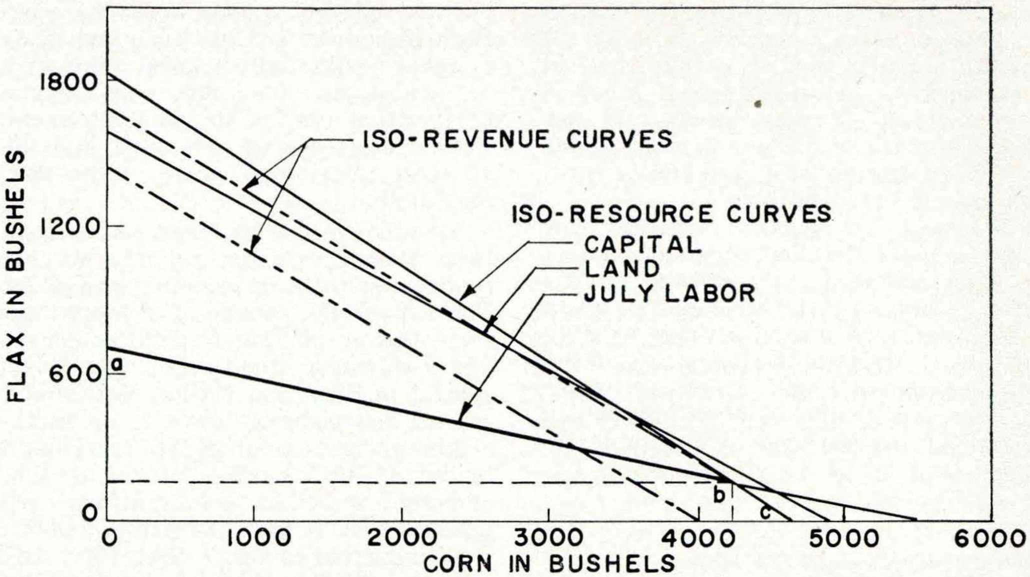

Fig. 8. Optimum combination of corn and flax under Situation 2, Logan Township.

given up for each bushel of corn added to the production program. However, the price ratio, 0.3602, does not change; 0.3602 bushel of flax is equal in value to 1 bushel of corn. Therefore, revenue will be decreased by adding more than 1,458 bushels of corn in Situation 1 or more than 4,226.4 bushels in Situation 2. Corn should be substituted for flax as long as this condition is true: The marginal rate of substitution of corn for flax is less than the quantity of flax required to equal the value of 1 bushel of corn, or the inverse price ratio of flax for corn.

Situation 3

Capital is unlimited in Situation 3.[33] The same quantities of labor and land are assumed as in Situations 1 and 2. The acreages available for competitive crops are listed in table 3 and the quantities of the operator's labor available in each month are specified in table 5. In review, the severe capital limitation in Situation 1 results in a substantial acreage in each area being unused. Capital is limitational in most areas in Situation 2. In Situation 3, however, the only limitations on production are imposed by land and labor.[34]

The combination of crops which maximizes revenue in each township is shown in table 18. Land is limitational in all but Saratoga Township. In the townships where land is limitational, the acreage available for competitive crops is planted to corn. This program is optimum in all but Saratoga Township because of the following relationships: In comparing corn and any other crop opportunity, the iso-resource curve for land is part of the production possibility curve. The marginal rate of substitution of corn for each other crop considered with it, is specified by the production possibility curve for the two crops. In each case the marginal rate of substitution is less than the inverse price ratio of the other crop and

[33] Unlimited capital means that a farmer can obtain as much money as needed in his production program. Net prices are used in solving for the optimum program. The price minus the capital expense involved in producing a bushel of grain is the net price per bushel.

[34] The possibility of limited labor in an unlimited capital situation may seem unrealistic. However, this situation concerns farms which are unable to hire the desired quality of labor or are unwilling to hire labor other than at harvest time.

TABLE 18. ACTIVITY LEVELS AND NET PROFIT UNDER RESOURCE SITUATION 3.

Township	Acres planted to:					Acres unused	Labor used (hours)			Labor unused (hours)			Net profit ($)
	Pasture	Hay	Corn	Oats	Other		June	July	Oct.	June	July	Oct.	
Washington	—	45.5	67.4	41.1	0	0	157	206	70	44	3	131	1,903
Harrison, Benton Co.	39.8	21.4	68.3	24.5	0	0	116	141	71	56	31	95	4,658
Troy	—	38.0	79.5	36.5	0	0	140	182	83	57	15	106	2,949
Grand Meadow	38.8	29.9	40.7	44.6	0	0	111	175	42	12	0	68	4,953
Saratoga	21.3	21.9	70.7	36.1	0	4.0	98	147	73	50	0	64	2,995
Harrison, Kossuth Co.	13.1	10.5	116.5	13.9	0	0	142	143	121	36	35	52	6,551
Cedar	5.4	22.4	104.0	22.2	0	0	141	167	108	60	36	89	5,107
Oakland	28.2	9.8	91.2	24.8	0	0	114	140	94	75	52	94	4,692
Logan	10.3	9.8	122.5	11.4	0	0	148	143	127	6	11	19	5,126
Jordan	16.8	14.3	107.0	15.9	0	0	161	163	111	40	43	92	4,434
Lincoln, Montgomery Co.	16.8	19.2	94.0	24.0	0	0	137	158	97	45	24	81	4,891
Lincoln, Polk Co.	17.5	12.9	106.3	17.3	0	0	143	151	110	58	54	90	7,761
Sheridan	15.9	17.6	102.9	17.6	0	0	153	159	107	6	0	45	8,812
Reading	6.1	9.1	129.0	9.8	0	0	147	139	134	17	25	24	5,507

corn. Thus, revenue is maximized by increasing the corn acreages as much as resources will permit.

The production possibility curve in Saratoga Township includes the iso-resource curves for July labor and land. The former is the more limitational. Thus, 4 acres of land are not planted. Land and July labor are exhausted simultaneously in Sheridan Township. Therefore, they are equally limitational in corn production.

The analysis of Situation 3 is demonstrated by a geometric presentation of the resource-price relationships in Logan Township (fig. 9). The iso-resource curves for land and July labor specify the production possibility curve for flax and corn. Two marginal rates of substitution are specified in the production possibility curve, 0.1298 where July labor is limitational (ab) and 0.3250 where land is limitational (bc). Thus, 1 bushel of corn may be added to the production program for each 0.3250 bushel of flax given up in the range where land is limitational to corn production (bc). Since 0.3451 bushel of flax is required to equal the value of 1 bushel of corn, profit will be maximized in Logan Township, as was true for all areas in Situation 3, by using the available resources for corn production.

The determination of the optimum program for Logan Township by the Simplex method involved the following three equations:

Land
$$(0.02500)x_1 + (0.02564)x_2 + (0.07143)x_3 + (0.07692)x_4 + x_5 = 122.5$$

July labor
$$(0.01872)x_1 + (0.04808)x_2 + (0.04747)x_3 + (0.14424)x_4 + x_6 = 102.2$$

October labor
$$(0.02590)x_1 + (0)x_2 + (0.16029)x_3 + (0)x_4 + x_7 = 146.2$$

A matrix was formed and the solution followed the method outlined in table 14. The results appear in table 18.

Situation 4

The second level of capital specified in table 2 is availabe for competitive crop production in Situation 4 and the labor supply considered in the previous situations is increased by 130 hours of family labor. Only the increase in available labor causes the resource supply to be different in Situation 4 than Situation 2 (table 6).

The optimum crop program for each area, with the resource quantities of Situation 4, is presented in table 19. With the exception of one township, corn is the only competitive crop to be grown in any location. However, less than the available acreage is planted in several townships. With the exception of Saratoga and Logan Townships, unused acreage occurs in Situation 4 (table 19) in the same locations and to the same extent as in Situation 2 (table 17). Use of the entire available acreage is prevented in Saratoga and Logan Townships in Situation 2 by lack of labor. The additional family labor in Situation 4 allows all the land available for competitive crops to be used in these two townships. The unused acreage in other than Saratoga and Logan Townships in both Situations 2 and 4 results from capital limitation. The optimum program in the eight locations where unused acreage does occur (tables 17 and 19) specifies the planting of corn on as many acres as the limited capital will permit. The remainder of the land is not used. The possibility was suggested above that some farmers may prefer to "spread" their limited capital over the entire acreage by planting a crop involving lower capital expense per acre. Less than the maximum profit again results from this practice.

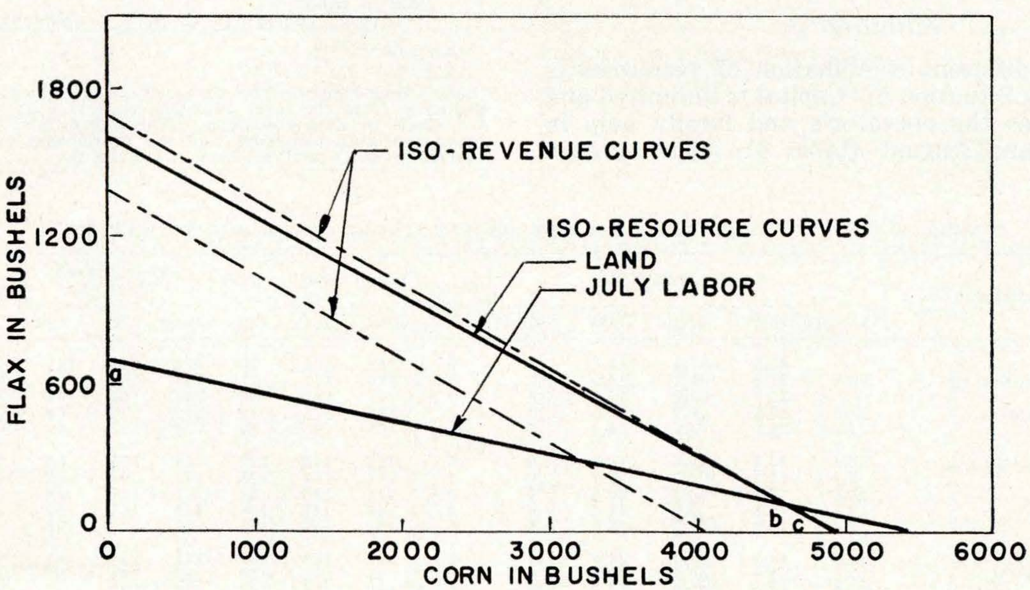

Fig. 9. Optimum combination of corn and flax under Situation 3, Logan Township.

TABLE 19. ACTIVITY LEVELS AND NET PROFIT UNDER RESOURCE SITUATION 4.

Township	Acres planted to:					Acres unused	Labor used (hours)			Labor unused (hours)			Net profit ($)
	Pasture	Hay	Corn	Oats	Other		June	July	Oct.	June	July	Oct.	
Washington	—	45.5	63.0	41.1	0	4.4	153	202	65	178	136	136	1,793
Harrison, Benton Co.	39.8	21.4	68.3	24.5	0	0	116	141	71	186	161	95	4,658
Troy	—	38.0	72.6	36.5	0	6.9	134	177	75	193	150	114	2,715
Grand Meadow	38.8	29.9	40.7	44.6	0	0	111	175	42	142	77	68	4,977
Saratoga	21.3	21.9	74.7	36.1	0	0	102	150	77	176	127	60	3,128
Harrison, Kossuth Co.	13.1	10.5	116.5	13.9	0	0	142	143	121	166	165	52	6,551
Cedar	5.4	22.4	96.3	22.2	0	7.7	134	161	100	197	172	97	4,758
Oakland	28.2	9.8	91.2	24.8	0	0	114	140	94	205	182	94	4,692
Logan	10.3	9.8	83.3	11.4	39.2	0	112	188	86	172	96	60	5,042
Jordan	16.8	14.3	102.6	15.9	0	4.4	157	160	106	174	176	97	4,273
Lincoln, Montgomery Co.	16.8	19.2	92.7	24.0	0	1.3	136	157	96	176	155	82	4,828
Lincoln, Polk Co.	17.5	12.9	92.3	17.3	0	14.0	130	140	96	201	195	104	6,845
Sheridan	15.9	17.6	91.8	17.6	0	11.1	142	151	95	147	138	57	7,997
Reading	6.1	9.1	117.0	9.8	0	12.0	136	130	121	158	164	37	5,035

The method of solving for the optimum program in Logan Township is typical of each location in Situation 4. Five resources appeared to be limitational in this township. The limitations which these resources impose on the four crop opportunities, corn, oats, soybeans and flax are expressed in the following equations:

Land
$$(0.02500)x_1 + (0.02564)x_2 + (0.07143)x_3 + (0.07692)x_4 + x_5 = 122.5$$

Capital
$$(0.51200)x_1 + (0.40740)x_2 + (1.33003)x_3 + (1.31010)x_4 + x_6 = 2{,}373.8$$

March labor
$$(0)x_1 + (0.00910)x_2 + (0)x_3 + (0.02731)x_4 + x_7 = 22.5$$

July labor
$$(0.01872)x_1 + (0.04808)x_2 + (0.04757)x_3 + (0.14424)x_4 + x_8 = 232.2$$

October labor
$$(0.02590)x_1 + (0)x_2 + (0.16029)x_3 + (0)x_4 + x_9 = 146.2$$

The activity levels which appear in table 19 for Logan Township were obtained by a solution which followed the method used in table 14.

Situation 5

Again, a different combination of resources is considered in Situation 5. Capital is unlimited and labor includes the operator's and family help in June, July and August (table 6). The acreage available is the same as in the previous examples. Situation 5 corresponds closely in resource availability to Situation 3 except that only the operator's labor is available in the latter case.

A revenue maximizing crop program was determined for each area with the resources specified in this situation. The results are listed in table 20. The production program and net revenue figures in table 20 are different from table 18 (Situation 3) in only Grand Meadow and Saratoga Townships. The large dairy enterprises which are found in these areas allow the additional labor in Situation 5, as compared to Situation 3, to be used profitably.[35] The analysis indicates that labor is not a serious limitation to crop production on Iowa farms. Exceptions are found where a large livestock enterprise, particularly dairy, is found.

The optimum program for Logan Township is typical of all areas. Four resource limitations are expressed in the following four equations for Logan Township:

Land
$$(0.02500)x_1 + (0.02564)x_2 + (0.07143)x_3 + (0.07692)x_4 + x_5 = 122.5$$

March labor
$$(0)x_1 + (0.00910)x_2 + (0)x_3 + (0.02731)x_4 + x_6 = 22.5$$

[35] Labor requirements for the average livestock program were deducted from the total available to arrive at labor available for crops in each township. The added family labor makes more available for crops and the production program can be increased until land becomes limitational.

TABLE 20. ACTIVITY LEVELS AND NET PROFIT UNDER RESOURCE SITUATION 5.

Township	Acres planted to:					Acres unused	Labor used (hours)			Labor unused (hours)			Net profit ($)
	Pasture	Hay	Corn	Oats	Other		June	July	Oct.	June	July	Oct.	
Washington	—	45.5	67.4	41.1	0	0	157	206	70	174	133	131	1,903
Harrison, Benton Co.	39.8	21.4	68.3	24.5	0	0	116	141	71	186	161	95	4,658
Troy	—	38.0	79.5	36.5	0	0	140	182	83	187	145	106	2,949
Grand Meadow	38.8	29.9	40.7	44.6	0	0	111	175	42	142	77	68	4,977
Saratoga	21.3	21.9	74.7	36.1	0	0	102	150	77	176	127	60	3,128
Harrison, Kossuth Co.	13.1	10.5	116.5	13.9	0	0	142	143	121	166	165	52	6,551
Cedar	5.4	22.4	104.0	22.2	0	0	141	167	108	190	166	89	5,107
Oakland	28.2	9.8	91.2	24.8	0	0	114	140	94	205	182	94	4,692
Logan	10.3	9.8	122.5	11.4	0	0	148	143	127	136	141	19	5,126
Jordan	16.8	14.3	107.0	15.9	0	0	161	163	111	170	173	92	4,434
Lincoln, Montgomery Co.	16.8	19.2	94.0	24.0	0	0	137	158	97	175	154	81	4,891
Lincoln, Polk Co.	17.5	12.9	106.3	17.3	0	0	143	151	110	188	184	90	7,761
Sheridan	15.9	17.6	102.9	17.6	0	0	153	159	107	136	130	45	8,812
Reading	6.1	9.1	129.0	9.8	0	0	147	139	134	147	155	24	5,507

July labor
$(0.01872)x_1 + (0.04808)x_2 + (0.04757)x_3 + (0.14424)x_4 + x_7 = 232.2$

October labor
$(0.02590)x_1 + (0)x_2 + (0.16029)x_3 + (0)x_4 + x_8 = 146.2$

The solution followed the method demonstrated in table 14 and the results appear in table 20.

Situation 6

Both capital and labor are unlimited in Situation 6, thus land is the only limitational resource in each area. The crop selected for each area will depend on the relationship between the capital expense involved in production and the price per unit of each crop opportunity. But this is the quantity expressed by $z_j - c_j$ (table 14). The solution of the optimum program is obvious when only one resource is considered; however, the demonstration of the technique is useful.

The limitation imposed by land on the crop opportunities in Logan Township is expressed as follows:

$$(0.02500)x_1 + (0.02564)x_2 + (0.07143)x_3 + (0.07692)x_4 + x_5 = 122.5$$

This equation is presented in matrix form in table 21. Three alternative production plans are considered by the Simplex method. Plan 1 specifies the disposal (non-use) of all land. The negative quantities in the $z_j - c_j$ row indicate that profit can be increased by including production of corn, oats, soybeans or flax in the production program. The largest increase in profit per unit (bushel) is found in flax production. Therefore, it is included in Plan 2 to the extent that resources (land only in this case) permit. That is 1,592.6 bushels. The negative quantity in the $z_1 - c_1$ column of Plan 2 indicates that the optimum program has not been found. In Plan 2 the opportunity cost of adding corn is the value of the flax which would be foregone ($z_1 = \$1.29$) for each bushel of corn added. However, each bushel of corn is worth \$1.43. Therefore, corn is included in Plan 3. An optimum is indicated by all positive quantities appearing in the $z_j - c_j$ row.

The optimum program was found for each area with unlimited labor and capital. The results appear in table 22. Planting the available acreage to corn maximizes revenue in each location. Net profit is, therefore, less than in Situation 5, since the cost of labor has been deducted.[36]

Conclusions From Situations 1 to 6

The addition of family labor to the operator's labor affected the production program in only two areas. It is assumed throughout that hired labor is available at harvest time in all situations. Other resource quantities remaining the same, unlimited labor did not change the production program from the operator plus family labor situations. The conclusion may be drawn that labor supply on the

[36] The average cost by season of farm labor in Iowa without board for the period 1948-52 was used to make the appropriate deduction. Farm Labor Situation, United States Department of Agriculture.

TABLE 21. A LINEAR PROGRAMMING SOLUTION BY THE SIMPLEX METHOD FOR FOUR ACTIVITIES (CROPS) WITH ONE LIMITATIONAL RESOURCE (SITUATION 6) IN LOGAN TOWNSHIP

	c_j	Disposal activities		1.43 Corn	0.76 Oats	2.54 Soybeans	3.97 Flax	
	Vector	P_0	P_5	P_1	P_2	P_3	P_4	
Plan 1	Land P_5	122.5	1	0.02500	0.02564	0.07143	0.07692	1,592.6
	z_j	0	0	0	0	0	0	
	z_j-c_j	0	0	—1.43	—0.76	—2.54	—3.97	
Plan 2	3.97 P_4	1,592.6	13.0005	0.32501	0.33333	0.92863	1	4,900.0
	z_j	6,322.5	51.612	1.290	1.323	3.687	3.97	
	z_j-c_j	6,322.5	51.614	—0.14	0.563	1.147	0	
Plan 3	1.43 P_1	4,900.0	40.000	1	1.026	2.857	3.077	
	z_j	7,007.1	57.20	1.43	1.467	4.086	4.400	
	z_j-c_j	7,007.1	57.20	0	0.707	1.546	0.430	

TABLE 22. ACTIVITY LEVELS AND NET PROFIT UNDER RESOURCE SITUATION 6.

Township	Pasture	Acres planted to:				Acres unused	Net profit ($)
		Hay	Corn	Oats	Other		
Washington	—	45.5	67.4	41.1	0	0	1,529
Harrison, Benton Co.	39.8	21.4	68.3	24.5	0	0	4,283
Troy	—	38.0	79.5	36.5	0	0	2,508
Grand Meadow	38.8	29.9	40.7	44.6	0	0	4,751
Saratoga	21.3	21.9	74.7	36.1	0	0	2,714
Harrison, Kossuth Co.	13.1	10.5	116.5	13.9	0	0	5,904
Cedar	5.4	22.4	104.0	22.2	0	0	4,529
Oakland	28.2	9.8	9.1	24.8	0	0	4,184
Logan	10.3	9.8	122.5	11.5	0	0	4,446
Jordan	16.8	14.3	107.0	15.9	0	0	3,840
Lincoln, Montgomery Co.	16.8	19.2	94.0	24.0	0	0	4,369
Lincoln, Polk Co.	17.5	12.9	106.3	17.3	0	0	7,152
Sheridan	15.9	17.6	102.9	17.6	0	0	8,245
Reading	6.1	9.1	129.0	9.8	0	0	4,792

"family farm" is adequate for the crop opportunities and the farm size considered in this study.

Capital limitation is more serious. Shortage of capital prevents the full use of land and labor; consequently net profit is low. Capital limitation not only causes land and labor to remain in disposal (non-use) but causes crops which are not the most profitable to be included in the program. A comparison of the crop plan in Logan Township under Situation 2 (table 17) and Situation 3 (table 18) demonstrates this point. The available resources differ only in the quantity of capital. Flax is included in the former plan in which capital is limited. In the latter plan all acreage is planted to corn resulting in higher profit per acre.

1941-44 Price Ratios

Iso-revenue curves which involve corn and one of the other crops mentioned in this section have a different slope when based on the 1941-44 price ratios than when based on 1948-52 ratios.[37] Since the same resource quantities considered in Situations 1 to 6 are involved in Situations 7 to 12, respectively, the production possibility curve for each area in the corresponding resource situation is the same. The results which were obtained in Situations 7 to 12 are compared with those obtained in Situations 1 to 6. Thus, the effect of changes in the price ratios may be seen.

[37] Provided crop prices are sufficiently high to yield a positive return, the level of prices will not influence the extent to which the production program is carried on, nor the combination of crops included in the program.

Situation 7

The optimum crop programs under the new prices appear in table 23. A comparison of table 23 with table 16 (Situation 1) reveals that the changes in price ratios cause the optimum plan to differ in only Washington Township. Corn production maximizes profit in this township in Situation 1 but soybean production represents the most profitable crop in Situation 7.

The marginal rate of substitution of corn for soybeans is 0.5245 in both situations. The inverse price ratio in Situation 1 is 0.5630 which indicates that profit is maximized by shifting all resources to corn production (0.5630 bushel of soybeans is required to equal the value of 1 bushel of corn, while the giving up of the production of 1 bushel of corn will permit the addition of only 0.5245 bushel of soybeans). In Situation 7 the inverse price ratio is 0.5107 which means that 0.5107 bushel of soybeans equals the value of 1 bushel of corn, while the giving up of the production of 1 bushel of corn still permits the addition of 0.5245 bushel of soybeans. Thus, soybeans are a more profitable crop than corn under the new price ratios.

Situation 8

A comparison of Situation 8 in table 24 and Situation 2 in table 17 reveals that the optimum

TABLE 23. ACTIVITY LEVELS AND NET PROFIT UNDER RESOURCE SITUATION 7.

Township	Acres planted to:					Acres unused	Labor used (hours)			Labor unused (hours)			Net profit ($)
	Pasture	Hay	Corn	Oats	Other		June	July	Oct.	June	July	Oct.	
Washington	—	45.5	0	41.1	15.6	51.8	109	166	35	92	43	166	1,221
Harrison, Benton Co.	39.8	21.4	44.0	24.5	0	24.3	94	123	46	78	50	120	4,732
Troy	—	38.0	25.8	36.5	0	53.7	91	142	27	106	55	162	1,639
Grand Meadow	38.8	29.9	14.8	44.6	0	25.9	87	156	15	36	0	95	4,957
Saratoga	21.3	21.9	39.0	36.1	0	35.7	69	124	40	79	24	97	2,719
Harrison, Kossuth Co.	13.1	10.5	69.0	13.9	0	47.5	98	107	71	80	71	102	4,569
Cedar	5.4	22.4	50.6	22.2	0	53.4	92	127	52	109	76	145	3,189
Oakland	28.2	9.8	52.7	24.8	0	38.5	79	112	55	110	80	133	3,818
Logan	10.3	9.8	36.5	11.4	40.0	46.0	69	154	38	85	0	108	3,796
Jordan	16.8	14.3	57.3	15.9	0	49.7	115	126	59	86	80	144	3,225
Lincoln, Montgomery Co.	16.8	19.2	47.8	24.0	0	46.2	95	123	50	87	60	128	3,423
Lincoln, Polk Co.	17.5	12.9	51.6	17.3	0	54.7	93	110	53	108	95	147	4,876
Sheridan	15.9	17.6	49.8	17.6	0	53.1	104	119	52	55	40	100	5,819
Reading	6.1	9.1	71.6	9.8	0	57.4	94	96	74	70	68	84	3,617

TABLE 24. ACTIVITY LEVELS AND NET PROFIT UNDER RESOURCE SITUATION 8.

Township	Acres planted to:					Acres unused	Labor used (hours)			Labor unused (hours)			Net profit ($)
	Pasture	Hay	Corn	Oats	Other		June	July	Oct.	June	July	Oct.	
Washington	—	45.5	10.0	41.1	57.4	0	155	201	139	46	8	62	2,473
Harrison, Benton Co.	39.8	21.4	68.3	24.5	0	0	116	141	71	56	31	95	5,908
Troy	—	38.0	72.6	36.5	0	6.9	134	177	75	63	20	114	3,238
Grand Meadow	38.8	29.9	40.7	44.6	0	0	111	175	42	12	0	68	6,639
Saratoga	21.3	21.9	70.7	36.1	0	4.0	98	147	73	50	0	64	3,767
Harrison, Kossuth Co.	13.1	10.5	116.5	13.9	0	0	142	143	121	36	35	52	7,025
Cedar	5.4	22.4	96.3	22.2	0	7.7	134	161	100	67	42	97	5,249
Oakland	28.2	9.8	91.2	24.8	0	0	114	140	94	75	52	94	5,473
Logan	10.3	9.8	105.7	11.4	12.3	4.5	133	154	110	21	0	36	5,400
Jordan	16.8	14.3	102.6	15.9	0	4.4	157	160	106	44	46	97	4,870
Lincoln, Montgomery Co.	16.8	19.2	92.7	24.0	0	1.3	136	157	96	46	25	82	5,492
Lincoln, Polk Co.	17.5	12.9	92.3	17.3	0	14.0	130	140	96	71	65	104	7,534
Sheridan	15.9	17.6	91.8	17.6	0	11.1	142	151	95	17	8	57	8,888
Reading	6.1	9.1	117.0	9.8	0	12.0	136	130	121	28	34	37	5,393

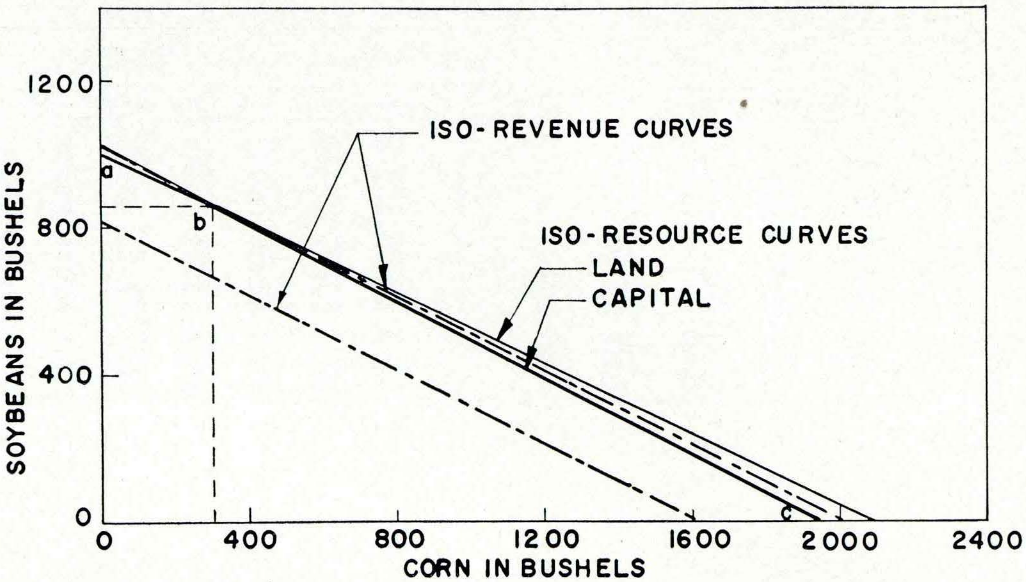

Fig. 10. Optimum combination of corn and soybeans under Situation 8, Washington Township.

program is the same in each area (under the two sets of prices) except Washington Township. Soybeans are included with corn to maximize revenue in Situation 8, whereas corn alone yielded the greatest return in Situation 2. The reason for the change is indicated in fig. 10.

The production possibility curve for soybeans and corn consists of two segments formed by the iso-resource curves for land and capital. Thus, land and capital are the most limitational resources. A marginal rate of substitution of corn for soybeans is specified by each resource, 0.4839 in the range where land is limitational (ab) and 0.5245 where capital is limitational (bc). The inverce price ratio in Situation 8, 0.5107, falls between the two substitution rates. Revenue is increased by expanding corn acreage throughout the range in which the marginal rate of substitution of corn for soybeans is 0.4839. Revenue decreases if corn production is extended into the range in which the marginal rate of substitution of corn for soybeans exceeds 0.5107. Thus 311.5 bushels of corn and 860 bushels of soybeans maximize profit in Situation 8 since the marginal rate of substitution changes from a constant, 0.4839, to a new constant, 0.5245, at this level of production. In Situation 2 the inverse price ratio is 0.5630. Therefore, revenue is increased by expanding corn acreage as far as resources permit.

Situations 9 to 12

The optimum program for Situations 9 to 12 are presented in tables 25 to 28. The advantage enjoyed by soybeans over corn in Washington Township disappears when capital is unlimited. This result might be forecast from an examination of fig. 10. If the iso-resource curve for capital is removed, the iso-resource curve for land becomes the production possibility curve. The highest iso-revenue curve is reached if all resources are used in corn production.

The 1941-44 period was selected for these situations since the price of soybeans was more favorable in these years relative to corn, than in any recent period. In spite of this advantage, soybeans occur in the optimum program only when capital or labor limitation prevents the entire available acreage being planted to corn.

TABLE 25. ACTIVITY LEVELS AND NET PROFIT UNDER RESOURCE SITUATION 9.

Township	Acres planted to:					Acres unused	Labor used (hours)			Labor unused (hours)			Net profit ($)
	Pasture	Hay	Corn	Oats	Other		June	July	Oct.	June	July	Oct.	
Washington	—	45.5	67.4	41.1	0	0	157	206	70	44	3	131	2,520
Harrison, Benton Co.	39.8	21.4	68.3	24.5	0	0	116	141	71	56	31	95	5,908
Troy	—	38.0	79.5	36.5	0	0	140	182	83	57	15	106	3,472
Grand Meadow	38.8	29.9	40.7	44.6	0	0	111	175	42	12	0	68	6,639
Saratoga	21.3	21.9	70.7	36.1	0	4.0	98	147	73	50	0	64	3,767
Harrison, Kossuth Co.	13.1	10.5	116.5	13.9	0	0	142	143	121	36	35	52	7,025
Cedar	5.4	22.4	104.0	22.2	0	0	141	167	108	60	36	89	5,598
Oakland	28.2	9.8	91.2	24.8	0	0	114	140	94	75	52	94	5,473
Logan	10.3	9.8	122.5	11.4	0	0	148	143	127	6	11	19	5,600
Jordan	16.8	14.3	107.0	15.9	0	0	161	163	111	40	43	92	5,030
Lincoln, Montgomery Co.	16.8	19.2	94.0	24.0	0	0	137	158	97	45	24	81	5,555
Lincoln, Polk Co.	17.5	12.9	106.3	17.3	0	0	143	151	110	58	54	90	8,451
Sheridan	15.9	17.6	102.9	17.6	0	0	153	159	107	6	0	45	9,702
Reading	6.1	9.1	129.0	9.8	0	0	147	139	134	17	25	24	5,865

TABLE 26. ACTIVITY LEVELS AND NET PROFIT UNDER RESOURCE SITUATION 10.

Township	Acres planted to:					Acres unused	Labor used (hours)			Labor unused (hours)			Net profit ($)
	Pasture	Hay	Corn	Oats	Other		June	July	Oct.	June	July	Oct.	
Washington	—	45.5	10.0	41.1	57.4	0	155	201	139	176	138	62	2,473
Harrison, Benton Co.	39.8	21.4	68.3	24.5	0	0	116	141	71	186	161	95	5,908
Troy	—	38.0	72.6	36.5	0	6.9	134	177	75	193	150	114	3,239
Grand Meadow	38.8	29.9	40.7	44.6	0	0	111	175	42	142	77	68	6,664
Saratoga	21.3	21.9	74.7	36.1	0	0	102	150	77	176	127	60	3,900
Harrison, Kossuth Co.	13.1	10.5	116.5	13.9	0	0	142	143	121	166	165	52	7,025
Cedar	5.4	22.4	96.3	22.2	0	7.7	134	161	100	197	172	97	5,249
Oakland	28.2	9.8	91.2	24.8	0	0	114	140	94	205	182	94	5,473
Logan	10.3	9.8	83.3	11.4	39.2	0	112	188	86	172	96	60	5,491
Jordan	16.8	14.3	102.6	15.9	0	4.4	157	160	106	174	176	97	4,870
Lincoln, Montgomery Co.	16.8	19.2	92.7	24.0	0	1.3	136	157	96	176	155	82	5,492
Lincoln, Polk Co.	17.5	12.9	92.3	17.3	0	14.0	130	140	96	201	195	104	7,534
Sheridan	15.9	17.6	91.8	17.6	0	11.1	142	151	95	147	138	57	8,888
Reading	6.1	9.1	117.0	9.8	0	12.0	136	130	121	158	164	37	5,393

TABLE 27. ACTIVITY LEVELS AND NET PROFIT UNDER RESOURCE SITUATION 11.

Township	Acres planted to:					Acres unused	Labor used (hours)			Labor unused (hours)			Net profit ($)
	Pasture	Hay	Corn	Oats	Other		June	July	Oct.	June	July	Oct.	
Washington	—	45.5	67.4	41.1	0	0	157	206	70	174	133	133	2,520
Harrison, Benton Co.	39.8	21.4	68.3	24.5	0	0	116	141	71	186	161	95	5,908
Troy	—	38.0	79.5	36.5	0	0	140	182	83	187	145	106	3,472
Grand Meadow	38.8	29.9	40.7	44.6	0	0	111	175	42	142	77	68	6,664
Saratoga	21.3	21.9	74.7	36.1	0	0	102	150	77	176	127	60	3,900
Harrison, Kossuth Co.	13.1	10.5	116.5	13.9	0	0	142	143	121	166	165	52	7,025
Cedar	5.4	22.4	104.0	22.2	0	0	141	167	108	190	166	89	5,598
Oakland	28.2	9.8	91.2	24.8	0	0	114	140	94	205	182	94	5,473
Logan	10.3	9.8	122.5	11.4	0	0	148	143	127	136	141	19	5,600
Jordan	16.8	14.3	107.0	15.9	0	0	161	163	111	170	173	92	5,030
Lincoln, Montgomery Co.	16.8	19.2	94.0	24.0	0	0	137	158	97	175	154	81	5,555
Lincoln, Polk Co.	17.5	12.9	106.3	17.3	0	0	143	151	110	188	184	90	8,451
Sheridan	15.9	17.6	102.9	17.6	0	0	153	159	107	136	130	45	9,702
Reading	6.1	9.1	129.0	9.8	0	0	147	139	134	147	155	24	5,865

TABLE 28. ACTIVITY LEVELS AND NET PROFIT UNDER RESOURCE SITUATION 12.

Township	Pasture	Acres planted to:				Acres unused	Net profit ($)
		Hay	Corn	Oats	Other		
Washington	—	45.5	67.4	41.1	0	0	2,147
Harrison, Benton Co.	39.8	21.4	68.3	24.5	0	0	5,533
Troy	—	38.0	79.5	36.5	0	0	3,031
Grand Meadow	38.8	29.9	40.7	44.6	0	0	6,437
Saratoga	21.3	21.9	74.7	36.1	0	0	3,486
Harrison, Kossuth Co	13.1	10.5	116.5	13.9	0	0	6,378
Cedar	5.4	22.4	104.0	22.2	0	0	5,020
Oakland	28.2	9.8	91.2	24.8	0	0	4,965
Logan	10.3	9.8	122.5	11.4	0	0	4,920
Jordan	16.8	14.3	107.0	15.9	0	0	4,436
Lincoln, Montgomery Co.	16.8	19.2	94.0	24.0	0	0	5,033
Lincoln, Polk Co.	17.5	12.9	106.3	17.3	0	0	7,842
Sheridan	15.9	17.6	102.9	17.6	0	0	9,135
Reading	6.1	9.1	129.0	9.8	0	0	5,149