Optimum Combinations of Livestock Enterprises and Management Practices on Farms Including Supplementary Dairy and Poultry Enterprises

(An Application of Linear Programming)

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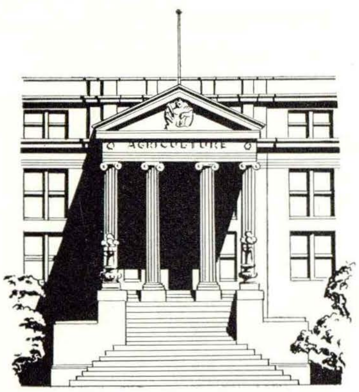

AGRICULTURAL EXPERIMENT STATION, IOWA STATE COLLEGE

RESEARCH BULLETIN 437 **FEBRUARY, 1956** **AMES, IOWA**

CONTENTS

SUMMARY

The object of this study is to determine (1) how scarce feed and other resources should be allocated between livestock enterprises and (2) which management practices or levels should be selected on farms producing a given feed supply. The situation selected for study is an average of 160-acre farms in northeast Iowa which have supplementary dairy and poultry enterprises (i.e., where these two enterprises are not on a large-scale or commercial basis). The cropping program on these farms results in production of 2,652 bushels of corn, 1,230 bushels of oats, 120 bushels of soybeans and 68 tons of forage from pasture and hay land. In addition, optimum programs have been worked out with only 48 tons of forage to determine the effects of restriction in this resource on enterprise combinations. Soybeans are considered to be sold for cash, while grain can be either fed to livestock or sold.

Linear programming techniques are used in determining the most profitable management practices and resource allocations or enterprise combinations. In the major solutions, 43 activities or investment opportunities were included: dairy cows of above-average ability, average ability and below-average ability, each fed five different hay-grain combinations and using competitive labor; spring pigs of above-average, average and below-average efficiency; fall hogs with these same levels of efficiency; poultry with these three levels of efficiency, using competitive labor; beef cows; dairy cows as outlined above but using supplementary labor; and poultry as outlined above but using supplementary labor. Several different capital situations also were included in the optimum solutions.

With capital limited to very small amounts, the most profitable farm organization included 27 litters of spring pigs of above-average efficiency. This enterprise could out-compete all other enterprises for the use of scarce funds. Excess grain would be sold for cash. Hay would be sold or go unused. With 48 tons of forage and $2,500 or more in capital, the optimum program would include two dairy cows and 37 litters of spring pigs. With $2,500 or more in capital and 68 tons of forage, the optimum plan included five dairy cows and 37 litters of spring pigs. With part of the labor set aside as a supplementary supply for dairy and poultry enterprises, the plan should include four dairy cows, 25 litters of spring pigs, four litters of fall pigs and 176 laying hens, plus chickens raised for replacement.

Regardless of the resource situations studied, the most profitable plan never included livestock or poultry of average or below-average efficiency. Even with the most severe restrictions on resource supplies, the most profitable plan always included above-average dairy cows, hogs and poultry. However, the optimum level of grain feeding for dairy cows did vary between resource situations because of the competition of the various enterprises for grain and other resources.

With risk considerations forced into the linear programming solutions, the following enterprise combinations resulted: (1) eight dairy cows, 12 spring litters, 12 fall litters and 100 hens, with a complementary number of chickens raised for replacement, were included under the situation with $2,500 capital where dairy and poultry were "forced" into the solutions to give greater income stability; (2) four dairy cows, 16 spring litters, 16 fall litters and 174 hens were included where fall and spring litters were "forced" into a 1:1 ratio to spread price risks. With space limitations on hogs, the optimum enterprise combination included six dairy cows, 19 spring litters, eight fall litters and as many as 287 hens. Plans with (a) restrictions on hog capacity or (b) risk precautions to meet uncertainty result in income of $1,000 to $1,500 less from the same resources.

The analysis of this study shows how resource limitations cause the best enterprise combination and the best management practices to differ—depending on the supply of grain, forages, capital, building space and competitive or supplementary labor. It shows that there is not an optimum set of livestock enterprises or management practices (i.e., level of grain feeding) for all farms, but that recommendations should differ between farms depending on their capital and labor situations, as well as on their ability to stand risks.

Optimum Combinations of Livestock Enterprises and Management Practices on Farms Including Supplementary Dairy and Poultry Enterprises[1]

(An Application of Linear Programming)

BY EARL O. HEADY AND J. C. GILSON

How feed resources should be allocated between different classes of livestock is a problem in all areas of Iowa. However, it is a problem of particular importance in northeast Iowa. Previous investigations show that this is the main feed-importing area of the state. Normally, the so-called northeast dairy area uses a third more feed grain than it produces. This is in contrast to other areas of the state which export feed grains (i.e., produce more feed grains than are required for livestock produced in the area). Yet, not all farms purchase feed grains in northeast Iowa. The greatest number of farms in this area tend to center their livestock production around the feeds grown on the farm. This question then arises: What proportions of various feeds grown on the farm should be allocated to the different classes of livestock to maximize profit on farms where dairying is a supplementary enterprise?

Numerous hypotheses have been proposed as possible answers to this question. It is sometimes suggested that since forage crops grow rather abundantly, while grain acreage is limited in northeast Iowa, farm management plans should center around dairy cows to consume a maximum of forage—with as much grain as possible going to hogs as more efficient converters of this type of feed. A further hypothesis along this same line is: Where forages are abundant and grains are limited, the dairy cows might well be those with less inherent ability. Cows of this nature might compare favorably with better cows in utilization of roughages on low-grain rations, but would require less capital and, hence, allow a greater swine enterprise.

A few farmers have suggested that profits might be maximized if the farm plan includes two main livestock enterprises, dairy and hogs. The milk cows would be fed no grain, with the entire corn production being fed to hogs and chickens. At the other extreme, it is sometimes suggested that optimum utilization of given feed stock can be accomplished best by high-grade dairy cows. This hypothesis arises because the maintenance requirements are similar for cows, regardless of the amount of milk produced.

Hence, with maintenance as a cost, should a farm with given feed stock allocate a large proportion of the grain to high-producing cows, with the hog enterprise curtailed accordingly? How should livestock enterprises be combined, and what management practices should be adopted to maximize profit from given resources? How do price changes, such as a relative lowering of dairy support prices, affect these answers?

OBJECTIVES

The main objectives of this study are to provide answers to the questions posed above. The study attempts to determine the combination of livestock enterprises and patterns of feed allocation which result in maximum profits for a typical farm with typical feed supplies in northeast Iowa.

FARM SITUATION AND PROCEDURE

The major farm situation selected for analysis includes 160 acres with the following average annual feed supplies: corn, 2,652 bushels; oats, 1,230 bushels; soybeans, 120 bushels; rotated hay, 68 tons used as hay or pasture (of which permanent pasture produces the equivalent of 20 tons of hay). Fifteen acres of land were considered to be devoted to farmstead, lots, roads, fences, trees and other non-crop uses. The figures are based on assessors' records for 160-acre farms in seven counties: Mitchell, Floyd, Chickasaw, Butler, Bremer, Black Hawk and Buchanan. In addition to the feed and land resources outlined above, labor and capital resources were considered as they relate to the optimum farm management plan.

OPPORTUNITY COST PRINCIPLE

Fundamental consideration in this study is given to management practices within each of several alternative livestock enterprises. However, overall farm production decisions ultimately should be based on more than partial analyses of this type. The farmer formally or informally must decide the optimum combination of livestock enterprises for

[1] Project 1135, Iowa Agricultural Experiment Station.

his farm. Several complex, interrelated factors influence this decision.

The opportunity cost principle is perhaps the greatest single consideration in the choice of an optimum livestock combination. This principle implies that a farmer should, if he wishes maximum profits, use each unit of scarce resources in those enterprises yielding the greatest return. Given a limited quantity of grain, for example, would it pay the farmer to invest all the grain in pigs or should he feed the grain to both pigs and dairy cows? The final choice, of course, depends on the relative returns from the two enterprises.

The same allocation principle applies to other scarce resources, such as labor or capital. It usually is not practical to consider the opportunity cost principle for any one resource in isolation. Rather, an optimum choice requires that the opportunity cost principle be applied simultaneously to the multitude of scarce resources.

One basic aspect of the choice of any optimum livestock combination is that of enterprise interrelationships. The three basic types of inter-relationships are: (1) competitive, (2) supplementary and (3) complementary. On most farms livestock enterprises are competitive with respect to available grain. Yet a dairy enterprise fed entirely on forage would not necessarily conflict with a hog enterprise fed on grain. Generally, a small farm poultry flock is considered supplementary to other enterprises in use of labor. Quite frequently, only the time of the housewife is invested in the poultry enterprise. On some farms both dairy and poultry are considered supplementary enterprises with respect to available labor. Hog enterprise emphasis may be in the spring when there is a peak demand on labor; or it may be a supplementary enterprise in the fall when labor otherwise would remain unused.

LINEAR PROGRAMMING APPLICATION AND MULTIPLICITY OF PLANS

The farm with several categories of feeds which are limited in supply, a given amount of labor in each month and limited capital has many plans available. For example, the farm with 3,000 bushels of grain can, using a bushel as the unit, allocate this feed in 3,000 different ways between two enterprises, such as hogs and dairy cows. All of the grain can be used for hogs, all for dairy cows, half for each enterprise or any other possible combination. If the same farm has 200 hours of labor available for livestock in June, the time (if the unit is 1 hour) can be used in 200 different ways. If we consider grain and June labor together, there are 200 × 3,000 or 600,000 different ways to allocate or combine these two resources for these two enterprises.

Hence, considering the fact that the farmer usually has more than two enterprises and more than two resources, he has a very great number of possible plans available. The one plan which maximizes profits can be determined—subject to the techniques considered, the supply of resources available and the prices for products and resources—by a mathematical procedure called linear programming. The linear programming technique allows the limitations of each resource to be considered in specifying the optimum plan.

Farms with the same soil type may differ in the crop and livestock plan which will maximize profits because these farms have different quantities of labor, capital funds or managerial skills. A farmer with very limited capital may find that funds (rather than labor, feeds or building space) limit his program. In this case, the program which gives greatest profit will be the one which gives highest returns on capital.

However, if sufficient capital is available, the enterprises combination may revolve around the limited labor supply of one particular month. One plan or enterprise may be expanded until the labor of this month becomes exhausted; the plan then will need to expand enterprises which use more labor from another month. This process may continue until plans are fitted to the labor supply of each month, with the possibility that the most profitable plan calls for labor to go unused in some months.

If the supply of labor also is sufficient, land or building space may provide the limiting resource around which plans must be built. If hogs use corn most profitably, this enterprise may be expanded until the corn supply is exhausted. However, the availability of forage may cause use of some grain by dairy cows to be profitable, rather than using the entire supply for hogs. When labor of a particular month is sufficiently limited, profits may be maximized by using a portion of the grain for poultry; if the poultry uses labor in other, non-limitational months.

LIVESTOCK ENTERPRISES, MANAGERIAL LEVELS AND RESOURCE SITUATIONS

To answer the question of whether it might be profitable to keep cows of low ability to consume mostly forage, and an efficient hog enterprise for grain utilization, several different productivity (or management) levels were considered for the several enterprises. These different levels of productivity or management also were used to determine production levels for the most profitable enterprises. In addition to considering three levels of management for dairy cows, hogs and chickens, five levels of grain feeding were considered for dairy cows. These several levels of grain feeding were used to determine whether or not the most profitable farm organization is one with most of the grain going to hogs and forage going mainly to dairy cows. In this way, the procedure allowed various levels of grain feeding to be considered as possibilities in the most profitable plan.

In the main analysis, hog, poultry and dairy enterprises were considered the competitive enterprises for all resources. However, for some situations poultry was considered a supplementary enterprise in the use of housewife labor if housewife labor is set aside for poultry. In another part of the study, the dairy enterprise also is considered supplementary in the use of some labor. (It is noncompetitive with other enterprises for a particular amount of this resource.) An enterprise can, of course, be supplementary only to the limits of the resource for which it is noncompetitive. Finally, a beef cow enterprise has been added in one part of the analysis. This enterprise was introduced to examine this possibility: If hogs can use all of the grain profitably, can beef cows use the remaining forage, labor and capital more efficiently than dairy cows?

COMPETITIVE DAIRY ACTIVITIES

Considering (1) the several levels of management (productivity) for the different enterprises and (2) the various levels of grain feeding for dairy cows, the linear programming analysis includes a total of 43 activities or investment alternatives. The dairy enterprises are:

P_1: Above-average dairy cow fed a high forage ration of 8.45 tons of hay equivalent and 520 pounds of grain. Milk production of 6,128 pounds. Each unit of output is considered to be 100 pounds of milk, 2.77 pounds of veal calf and 5.30 pounds of cull cow.

P_2: Above-average dairy cow fed a high-forage ration of 7.76 tons of hay equivalent and 1,036 pounds of grain. Milk production per cow of 6,950 pounds. Unit output includes 100 pounds milk, 2.44 pounds veal calf and 4.67 pounds cull cow.

P_3: Above-average dairy cow fed 7.11 tons of hay equivalent and 1,940 pounds of grain. Milk production of 7,650 pounds. Unit output includes 100 pounds milk, 2.22 pounds veal calf and 4.24 pounds cull cow.

P_4: Above-average dairy cow fed 7.01 tons of hay equivalent and 2,374 pounds of grain. Milk production of 8,300 pounds. Unit output includes 100 pounds milk, 2.04 pounds veal calf and 3.91 pounds cull cow.

P_5: Above-average dairy cow fed 6.80 tons of hay equivalent and 2,894 pounds of grain. Milk production of 8,850 pounds. Unit output includes 100 pounds milk, 1.92 pounds veal calf and 3.67 pounds cull cow.

P_6: Average dairy cow fed 7.38 tons of hay equivalent and 520 pounds of grain. Milk production of 5,200 pounds. Unit output includes 100 pounds milk, 3.26 pounds veal calf and 5.53 pounds cull cow.

P_7: Average dairy cow fed 6.73 tons of hay equivalent and 963 pounds of grain. Milk production of 5,700 pounds. Unit output includes 100 pounds milk, 2.98 pounds veal calf and 5.05 pounds cull cow.

P_8: Average dairy cow fed 6.46 tons of hay equivalent and 1,734 pounds of grain. Milk production of 6,150 pounds. Unit output includes 100 pounds milk, 2.76 pounds veal calf and 4.68 pounds cull cow.

P_9: Average dairy cow fed 6.22 tons of hay equivalent and 2,145 pounds of grain. Milk production of 6,500 pounds. Unit output includes 100 pounds milk, 2.61 pounds veal calf and 4.43 pounds cull cow.

P_{10}: Average dairy cow fed 6.11 tons of hay equivalent and 2,264 pounds of grain. Milk production of 6,800 pounds. Unit output includes 100 pounds milk, 2.50 pounds veal calf and 4.23 pounds cull cow.

P_{11}: Below-average dairy cow fed 6.87 tons of hay equivalent and 520 pounds of grain. Milk production of 4,200 pounds. Unit output includes 100 pounds milk, 4.04 pounds veal calf and 5.95 pounds cull cow.

P_{12}: Below-average dairy cow fed 6.46 tons of hay equivalent and 937 pounds of grain. Milk production of 4,550 pounds. Unit output includes 100 pounds milk, 3.73 pounds veal calf and 5.49 pounds cull cow.

P_{13}: Below-average dairy cow fed 6.17 tons of hay equivalent and 1,315 pounds of grain. Milk production of 4,800 pounds. Unit output includes 100 pounds milk, 3.54 pounds veal calf and 5.20 pounds cull cow.

P_{14}: Below-average dairy cow fed 5.95 tons of hay equivalent and 1,635 pounds of grain. Milk production of 5,000 pounds. Unit output includes 100 pounds milk, 3.40 pounds veal calf and 5.00 pounds cull cow.

P_{15}: Below-average dairy cow fed 5.92 tons of hay equivalent and 1,813 pounds of grain. Milk production of 5,150 pounds. Unit output includes 100 pounds milk, 3.30 pounds veal calf and 4.85 pounds cull cow.

The feed, labor and other input items per head cited above, and the coefficients shown later in table 4, include resources required for replacement and breeding stock. The grain per dairy cow thus includes feed for young replacement stock as well as feed for milk production. Feed per unit of poultry product includes feed for the hen plus that for (a) young chicks to be marketed and (b) young chicks used for replacement stock. The 120 pounds of forage input per 100 pounds of pork, shown later in the input-output table, includes pasture (a) actually consumed by the pig and (b) wasted in using pasture ground for sanitation purposes.

Input-output coefficients shown later include requirements for replacement stock. Some use of near-waste feeds (cornstalks, etc.) is assumed, with a greater proportion of the feeds for lower producing cows. Only forage represented as regular hay and pasture is included in the above figures, with the assumption that some of this actually is wasted but that it has to be charged against the class of livestock and subtracted from the total feed supply. The unit outputs shown are those for which prices are quoted later. For the entire dairy herd, the product sold includes veal calf and cull cow as well as milk or butterfat. Protein feed requirements, which increase with milk level, are included in the cash expenses shown in the later table of input-output coefficients.

COMPETITIVE HOG ACTIVITIES

The hog enterprises or activities are:

P_{16}: Above-average spring hogs fed 305 pounds grain and supplement per 100 pounds pork produced; 6.6 pigs saved per litter. Pigs raised on pasture. Unit output includes 81 pounds market hog and 19 pounds sow.

P_{17}: Average spring hogs fed 390 pounds grain and supplement per 100 pounds pork produced; 6.5 pigs saved per litter. Pigs raised on pasture. Unit output includes 80 pounds market hog and 20 pounds sow.

P_{18}: Below-average spring hogs fed 493 pounds grain and supplement per 100 pounds pork produced; 5.9 pigs saved per litter. Pigs raised on pasture. Unit output includes 79 pounds market hog and 21 pounds sow.

P_{19}: Above-average fall hogs fed 345 pounds grain and supplement per 100 pounds pork produced; 6.7 pigs saved per litter. Pigs raised in drylot. Unit output includes 81 pounds market hog and 19 pounds sow.

P_{20}: Average fall hogs fed 430 pounds grain and supplement per 100 pounds pork produced; 6.5 pigs saved per litter. Pigs raised in drylot. Unit output includes 80 pounds market hog and 20 pounds sow.

P_{21}: Below-average fall hogs fed 538 pounds grain and supplement per 100 pounds pork produced; 6.0 pigs saved per litter. Pigs raised in drylot. Unit output includes 79 pounds market hog and 21 pounds sow.

Marketing weight used for market hogs is 225 pounds; this is the most common marketing weight. It was assumed that spring pigs used the equivalent, in pasture, of 120 pounds hay per 100 pounds of pork produced. While the hogs may not consume this amount, the rest would be lost to hay production.

COMPETITIVE POULTRY ACTIVITIES

The competitive poultry activities are as follows:

P_{22}: Above-average poultry with 195 eggs per hen; 12 percent death loss for hens and 14 percent for replacement flock; 125 chicks purchased per 100 hens and 11 percent of pullets culled. Unit output includes 10 dozen eggs and 2.74 pounds meat.

P_{23}: Average poultry with 165 eggs per hen; 15 percent death loss for hens and 18 percent for replacement flock; 125 chicks purchased per 100 hens and 7 percent of pullets culled. Unit output includes 10 dozen eggs and 3.01 pounds meat.

P_{24}: Below-average poultry with 125 eggs per hen; 15 percent death loss of hens and 22 percent for replacement flock; 125 chicks purchased per 100 hens and 3 percent of pullets culled. Unit output includes 10 dozen eggs and 3.80 pounds meat.

BEEF ACTIVITY

The beef cow and calf enterprise is as follows:

P_{25}: Average beef cows with calves sold as good to choice feeders at 400 pounds; replacement of cows every 8 years and 90 percent calf crop. Unit output includes 67 pounds feeder calf and 33 pounds cull cow.

SUPPLEMENTARY ACTIVITIES FOR POULTRY AND DAIRY

For situations where supplementary labor is used for (1) poultry and (2) poultry and dairy, the following enterprises or activities also are included:

P_{26}: Above-average poultry as denoted above except this poultry enterprise is now considered supplementary for the labor listed as type B in table 2. It is used in planning only for situations S_{36}, S_{37} and S_{38} in table 3.

P_{27}: Same as P_{26} except it is average poultry for situations S_{36}, S_{37} and S_{38}.

P_{28}: Same as P_{26} except it is below-average poultry for situations S_{36}, S_{37} and S_{38}.

P_{29} through P_{43}: These 15 dairy enterprises are, in numerical order, identical with the 15 dairy enterprises listed above as P_1 through P_{15}, respectively. They do not, however, compete with other enterprises for labor but use the labor supply denoted under type C of table 2. Hence, this dairy enterprise is considered supplementary in the use of labor listed under type C. Activities P_{29} through P_{43} are used in solutions, along with activities P_{26}, P_{27} and P_{28} above, as well as the previous 25 activities, in situations S_{39}, S_{40} and S_{41} of table 3.

PRICES USED

The above 43 alternative activities or enterprises (P_1 through P_{43}) were allowed in some or all situations as alternatives to which given resources could be allocated in determining the maximum profit plan. Maximum profit plans were worked out for several price situations. The price situations used were: (1) 1953 prices with milk sold as grade A, (2) the average of 1949-53 prices with milk sold as grade A, (3) the average of 1949-53 with grade B milk sold at condensory prices and (4) the average of 1949-53 prices with milk priced at 75 percent of parity for grade A. The net unit prices (market prices per unit less cash variable costs per unit) used under these situations are supplied in table 1. The magnitude of the net unit prices (see earlier discussion of activities and unit outputs) depends on the composition of the unit and the magnitude of the variable costs. For example, the net unit price of an **average** dairy product is higher than for the **above-average** dairy prouct because the former includes a greater amount of cull cow and veal calf with each 100 pounds of milk sold.

RESOURCE SITUATIONS

In determining optimum plans for farmers who have different quantities and combinations of resources, the resource situations listed below are those used in the linear programming or planning computations. In each case, the quantity of capital refers to that above the requirements for crop production. It refers to capital beyond that required for land, permanent buildings, machinery and annual crop expenses. Remember that land is constant at 160 acres and feed resources include the equivalent of 3,267 bushels of corn (including oats converted to corn) and 68 tons of hay (including hay harvested and the hay equivalent, in tons, of the pasture). For a few resource situations, hay and pasture supply was limited to 48 tons of hay

TABLE 1. NET UNIT PRICES USED IN DETERMINING MOST PROFITABLE ALLOCATION OF RESOURCES (MARKET PRICE PER UNIT OUTPUT MINUS VARIABLE CASH EXPENSE PER UNIT OUTPUT).*

Enterprise or activity	Price level: 1953; grade A milk	1949-53; grade A milk	1949-53; grade B milk	1949-53; 75% parity for milk
P_1 dairy	3.83	4.69	3.84	3.99
P_2 dairy	3.67	4.48	3.63	3.78
P_3 dairy	3.70	4.46	3.61	3.76
P_4 dairy	3.69	4.40	3.55	3.70
P_5 dairy	3.67	4.37	3.52	3.67
P_6 dairy	3.93	4.84	3.99	4.14
P_7 dairy	3.76	4.62	3.77	3.92
P_8 dairy	3.76	4.59	3.74	3.89
P_9 dairy	3.77	4.56	3.71	3.86
P_{10} dairy	3.75	4.52	3.67	3.82
P_{11} dairy	4.02	5.02	4.17	4.32
P_{12} dairy	3.86	4.82	3.97	4.12
P_{13} dairy	3.87	4.80	3.95	4.10
P_{14} dairy	3.89	4.78	3.93	4.08
P_{15} dairy	3.88	4.75	3.90	4.05
P_{16} hogs	17.79	14.15	14.15	14.15
P_{17} hogs	17.66	13.90	13.90	13.90
P_{18} hogs	17.28	13.23	13.23	13.23
P_{19} hogs	17.06	13.79	13.79	13.79
P_{20} hogs	15.34	13.76	13.76	13.76
P_{21} hogs	16.49	13.47	13.47	13.47
P_{22} poultry	2.54	2.12	2.12	2.12
P_{23} poultry	2.29	1.86	1.86	1.86
P_{24} poultry	2.08	1.66	1.66	1.66
P_{25} beef	17.87	19.87	19.87	19.87

* Labor costs have not been subtracted in computing these net unit prices. However, in the linear programming computations with unlimited capital and unlimited labor, the value of labor has been treated as a cash expense and subtracted from unit prices to determine net unit prices.

TABLE 2. HOURS OF LABOR PER MONTH FOR THE THREE TYPES OF AVAILABLE LIVESTOCK LABOR SITUATIONS.

Month	Type A Available labor	Type B Available labor		Type C Available labor		
	All competitive enterprises	Competitive enterprises	Supplementary poultry	Competitive enterprises	Supplementary poultry	Supplementary dairy
January	301.0	270.0	31.0	192.5	77.5	31.0
February	298.0	270.0	28.0	200.0	70.0	28.0
March	284.2	237.7	46.5	160.2	77.5	46.5
April	203.9	158.9	45.0	83.9	75.0	45.0
May	195.1	133.1	93.0	55.6	77.5	62.0
June	295.9	235.9	90.0	160.9	75.0	60.0
July	244.3	182.3	62.0	104.8	77.5	62.0
August	360.7	298.7	62.0	221.2	77.5	62.0
September	299.6	254.6	45.0	179.6	75.0	45.0
October	194.1	147.6	46.5	70.1	77.5	46.5
November	159.8	129.8	30.0	54.8	75.0	30.0
December	268.2	237.2	31.0	159.7	77.5	31.0

equivalent. This step was taken to examine the effect of different ratios of farm-raised feed grain and hay on optimum livestock plans. The capital-labor situations used are:

1. Capital limited to $1,500 for livestock and labor as follows: 270 hours per month for the operator; 140 hours during June, July and August for another family member and 1½ hours of housewife labor per day except for November, December, January and February when only 1 hour per day would be available. From this total labor supply, the amount needed to produce the crops on the farm was subtracted. The remainder was considered available for livestock. For some situations, the poultry enterprise was considered supplementary to other enterprises in use of housewife labor and some additional family labor. For other planning situations, all labor including that of the housewife was added together and all enterprises considered to be competitive. In a few cases, some labor was "held out" of the total supply to allow consideration of dairying as a supplementary enterprise in use of labor. Hence, the total labor by months becomes that in table 2. Under type A, all enterprises are competitive for labor. Under type B, poultry is supplementary for the labor listed; housewife and some additional family labor cannot be used for other enterprises, but poultry can compete for labor listed for other competitive enterprises. Under type C, some operator and family labor is available as supplementary labor for the dairy enterprise. The dairy enterprise can use other labor but other enterprises cannot use the supplementary labor available for dairying. Under type C, poultry again has some supplementary labor for which other enterprises do not compete.

2. Capital limited to $2,500 for livestock and labor as outlined above.

3. Capital limited to $4,000 for livestock and labor as outlined above.

4. Capital limited to $6,000 for livestock and labor as outlined above.

5. Capital unlimited and labor unlimited.

In addition, a few other combinations were used where labor or capital was, or was not, limited. Feed supplies are limited in the quantities mentioned previously, even when labor and capital are not limited. A total of 41 linear programming solutions were completed for these resource-price situations which do not include space limitations for hogs. In addition, optimum programs were computed for 16 more situations with hog space limited or "risk restraints" placed on the enterprise combinations. The linear programming solutions thus total 57.

The resources for the first 41 situations are listed in table 3. Each of the first 41 situations is denoted as "S" with an appropriate subscript to facilitate later identification. In cases of a zero in the supplementary poultry and dairy columns, all enterprises compete for the labor shown in the third column (which is the monthly average for type A labor in table 2). In cases where a figure occurs in the supplementary poultry column, all enterprises compete for labor in the third column, but, in addition, poultry alone can use the housewife labor in the fourth column (i.e., poultry is a supplementary enterprise for this labor which is a monthly average of the housewife labor of type B in table 2). In cases where a figure occurs in the fifth column, all enterprises compete for the labor in the third column; poultry alone can use the labor of the fourth column, and dairy alone can use the labor in the fifth column.

Hence, the 43 situations or solutions can be classified as follows: Situations S_1 through S_{32} include 15 dairy enterprises, three fall hog enterprises, three spring hog enterprises and three poultry enterprises—all of which compete for the labor supply shown. Situations S_{33} through S_{35} include the 24 enterprises mentioned above plus the beef cow enterprise, P_{25}. Situations S_{36}, S_{37} and S_{38} include the same 24 competitive enterprises as under S_1 through S_{32} but also include three poultry enterprises (P_{26}, P_{27} and P_{28}) which alone use the supplementary housewife labor. In other words, poultry can compete with the other enterprises for operator labor, but it alone uses housewife labor. Situations S_{39}, S_{40} and S_{41} include the same 27 enterprises as situations S_{36} through S_{38} and also include 15 dairy enterprises (P_{29} through P_{43}) having some supplementary family labor for which other enterprises do not compete. On the basis of these resource situations and activities, some of the solutions include 43 activities and 39 resource supplies. The majority of solutions, however, involve only 25 activities and 15 resource supplies.

The quantities of capital refer to annual costs or expense capital. They do not refer to capital investment in livestock, supplies or buildings. The figures do, however, include the annual capital expense for these items. For example, the annual capital expense includes depreciation on cows, veterinary fees, breeding fees, taxes, insurance and all other items for the dairy enterprise, including replacement stock. It also includes the annual depreciation and other costs of all equipment, buildings and materials used in production. The investment in these same assets would be considerably greater than the annual expense or production capital used in the resource situations. This classification of capital was used to facilitate computations and because the assets themselves provide the chattel basis for obtaining investment capital. Also, it is supposed, for the purposes of this study, that the limits on investment capital are proportional to the limits on production or expense capital.

In each instance where an enterprise is listed as supplementary, it is supplementary only with respect to the labor supply shown. Thus, supplementary poultry and dairy enterprises compete for capital and labor only beyond the quantity shown under the supplementary column of table 3. Of course, poultry and fall hogs are supplementary in the use of forage; they use no forage. The beef cow enterprise is included as an alternative only when "yes" appears in the beef column of table 3. It is then competitive for all resources.

While the average amount of labor available per month is shown for complementary and supplementary enterprises in table 3, labor supplies and labor requirements have actually been computed for each individual month. In table 3 when no labor is shown for supplementary enterprises, poultry and dairy are considered to be competitive for use of this resource; they must compete with other enterprises for use of this resource.

RESOURCE REQUIREMENTS OF INPUT-OUTPUT COEFFICIENTS BY ACTIVITIES

The amounts of each resource required to produce one unit of output for each activity are shown in table 4. These input-output coefficients are

TABLE 3. SITUATIONS OF RESOURCE SUPPLIES FOR WHICH MOST PROFITABLE PLAN WAS DETERMINED BY LINEAR PROGRAMMING.

Situation	Capital	Average hours labor per month: Competitive enterprises	Supplementary poultry	Supplementary dairy	Grain (bu.)	Hay (ton)	Beef	Prices
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
S_1	1,500	258	0	0	3,267	48	no	1953; grade A milk
S_2	2,500	258	0	0	3,267	48	no	1953; grade A milk
S_3	4,000	258	0	0	3,267	48	no	1953; grade A milk
S_4	6,000	258	0	0	3,267	48	no	1953; grade A milk
S_5	1,500	258	0	0	3,267	68	no	1953; grade A milk
S_6	2,500	258	0	0	3,267	68	no	1953; grade A milk
S_7	4,000	258	0	0	3,267	68	no	1953; grade A milk
S_8	6,000	258	0	0	3,267	68	no	1953; grade A milk
S_9	2,500	258	0	0	3,267	48	no	1953; grade A milk
S_{10}	4,000	258	0	0	3,267	48	no	1949-53; grade A milk
S_{11}	unlimited	unlimited	0	0	3,267	48	no	1949-53; 75% parity milk
S_{12}	2,500	258	0	0	3,267	68	no	1949-53; grade A milk
S_{13}	4,000	258	0	0	3,267	68	no	1949-53; grade A milk
S_{14}	6,000	258	0	0	3,267	68	no	1949-53; grade A milk
S_{15}	unlimited	258	0	0	3,267	68	no	1949-53; grade A milk
S_{16}	unlimited	unlimited	0	0	3,267	68	no	1949-53; grade A milk
S_{17}	4,000	unlimited	0	0	3,267	68	no	1949-53; grade A milk
S_{18}	6,000	unlimited	0	0	3,267	68	no	1949-53; grade A milk
S_{19}	2,500	258	0	0	3,267	68	no	1949-53; 75% parity milk
S_{20}	4,000	258	0	0	3,267	68	no	1949-53; 75% parity milk
S_{21}	6,000	258	0	0	3,267	68	no	1949-53; 75% parity milk
S_{22}	unlimited	258	0	0	3,267	68	no	1949-53; 75% parity milk
S_{23}	unlimited	unlimited	0	0	3,267	68	no	1949-53; 75% parity milk
S_{24}	2,500	unlimited	0	0	3,267	68	no	1949-53; 75% parity milk
S_{25}	4,000	unlimited	0	0	3,267	68	no	1949-53; 75% parity milk
S_{26}	6,000	unlimited	0	0	3,267	68	no	1949-53; 75% parity milk
S_{27}	2,500	258	0	0	3,267	68	no	1949-53; grade B milk
S_{28}	4,000	258	0	0	3,267	68	no	1949-53; grade B milk
S_{29}	6,000	258	0	0	3,267	68	no	1949-53; grade B milk
S_{30}	unlimited	258	0	0	3,267	68	no	1949-53; 75% grade B milk
S_{31}	unlimited	unlimited	0	0	3,267	68	no	1949-53; 75% grade B milk
S_{32}	4,000	unlimited	0	0	3,267	68	no	1949-53; 75% grade B milk
S_{33}	2,500	258	0	0	3,267	68	yes	1949-53; grade A milk
S_{34}	4,000	258	0	0	3,267	68	yes	1949-53; grade A milk
S_{35}	6,000	258	0	0	3,267	68	yes	1949-53; grade A milk
S_{36}	2,500	213	56	0	3,267	68	no	1949-53; grade A milk
S_{37}	4,000	213	56	0	3,267	68	no	1949-53; grade A milk
S_{38}	6,000	213	56	0	3,267	68	no	1949-53; grade A milk
S_{39}	2,500	136	76	46	3,267	68	no	1949-53; grade A milk
S_{40}	4,000	136	76	46	3,267	68	no	1949-53; grade A milk
S_{41}	6,000	136	76	46	3,267	68	no	1949-53; grade A milk

TABLE 4. BASIC TABLEAU SHOWING RESOURCE REQUIREMENTS (INPUT-OUTPUT COEFFICIENTS) TO PRODUCE ONE UNIT OF OUTPUT FOR THE VARIOUS ENTERPRISES OR ACTIVITIES.

Resource	Above-average dairy					Average dairy					Below-average dairy				
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}
Corn (lb.)	8.48	16.9	24.9	28.6	32.7	10.0	23.7	28.2	33.0	34.4	12.4	20.6	27.4	32.7	35.2
Hay (lb.)	277	222	186	174	163	284	236	210	193	189	327	284	257	238	230
Capital ($)	1.23	1.22	1.09	1.03	0.98	1.34	1.29	1.19	1.13	1.10	1.46	1.39	1.30	1.24	1.20
Labor (hours)															
Jan.	0.282	0.261	0.242	0.238	0.241	0.309	0.295	0.275	0.266	0.264	0.349	0.341	0.323	0.319	0.310
Feb.	0.269	0.249	0.231	0.227	0.230	0.295	0.281	0.262	0.254	0.252	0.333	0.326	0.309	0.304	0.296
March	0.282	0.261	0.242	0.238	0.241	0.309	0.295	0.275	0.266	0.264	0.349	0.341	0.323	0.319	0.310
April	0.243	0.225	0.209	0.205	0.208	0.267	0.255	0.238	0.230	0.228	0.301	0.294	0.279	0.276	0.268
May	0.192	0.178	0.165	0.162	0.164	0.211	0.201	0.188	0.182	0.180	0.238	0.232	0.220	0.218	0.211
June	0.154	0.142	0.132	0.130	0.131	0.169	0.161	0.150	0.145	0.144	0.190	0.186	0.176	0.174	0.169
July	0.154	0.142	0.132	0.130	0.131	0.169	0.161	0.150	0.145	0.144	0.190	0.186	0.176	0.174	0.169
Aug.	0.166	0.154	0.143	0.140	0.142	0.183	0.174	0.162	0.157	0.156	0.206	0.201	0.191	0.188	0.183
Sept.	0.154	0.142	0.132	0.130	0.131	0.169	0.161	0.150	0.145	0.144	0.190	0.186	0.176	0.174	0.169
Oct.	0.192	0.178	0.165	0.162	0.164	0.211	0.201	0.188	0.182	0.180	0.238	0.232	0.220	0.218	0.211
Nov.	0.218	0.201	0.187	0.184	0.186	0.239	0.228	0.212	0.206	0.204	0.269	0.264	0.250	0.246	0.240
Dec.	0.256	0.237	0.220	0.216	0.219	0.281	0.268	0.250	0.242	0.240	0.317	0.310	0.294	0.290	0.282

Resource	Spring hogs			Fall hogs			Poultry			Beef cows
	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}
Corn (lb.)	305	390	493	345	430	538	56.9	66.7	89.4	81.6
Hay (lb.)	120	120	120	0	0	0	0	0	0	1,291
Capital ($)	3.57	4.82	5.41	4.52	5.98	6.75	2.05	2.36	2.73	3.16
Labor (hours)										
Jan.	0.114	0.134	0.156	0.176	0.210	0.242	0.098	0.116	0.154	0.443
Feb.	0.114	0.134	0.156	0.138	0.164	0.190	0.098	0.116	0.154	0.443
March	0.137	0.162	0.188	0.127	0.151	0.175	0.106	0.125	0.166	0.492
April	0.144	0.170	0.198	0.098	0.117	0.135	0.126	0.150	0.198	0.332
May	0.131	0.155	0.180	0.086	0.102	0.118	0.195	0.231	0.305	0.166
June	0.119	0.141	0.164	0.095	0.112	0.130	0.135	0.161	0.212	0.166
July	0.119	0.141	0.164	0.091	0.108	0.125	0.106	0.125	0.166	0.166
Aug.	0.119	0.141	0.164	0.149	0.177	0.205	0.198	0.166	0.154	0.166
Sept.	0.114	0.134	0.156	0.237	0.281	0.325	0.094	0.112	0.147	0.166
Oct.	0.114	0.134	0.156	0.226	0.268	0.310	0.075	0.089	0.117	0.166
Nov.	0.113	0.133	0.154	0.198	0.235	0.272	0.084	0.099	0.131	0.218
Dec.	0.102	0.121	0.140	0.198	0.235	0.272	0.075	0.089	0.117	0.312

those required to produce the combination of products explained earlier and for which net prices are shown in table 1 (e.g., the figures for above-average dairy cows receiving the most forage and least grain, P_3, show the amount of each resource required to produce a unit of product composed of 100 pounds of milk, 2.77 pounds of veal calf and 5.30 pounds of cull cow). In comparing resource requirements or input-output coefficients, remember that a unit of output contains different proportions of products for the same enterprise at different levels of managerial efficiency. For the dairy enterprises, each unit of output represents a greater proportion of milk and less of beef sales within each of the feeding ranges—P_1 through P_5; P_6 through P_{10}; and P_{11} through P_{15}. Also, an above-average dairy enterprise has a greater proportion of milk and a smaller proportion of meat than an average or below-average dairy activity receiving about the same level of grain-forage feeding. Poultry, hogs and cattle are represented by output units of entirely different magnitudes (dozen eggs, cwt. of meat, etc.), and input-output coefficients differ accordingly.

LINEAR PROGRAMMING COMPUTATIONS

The solutions for the most profitable use of resources have been worked out by the simplex method.[2] They have been computed for the 41 different resource and price situations. This section deals with the mathematical computations and procedures. Readers who are not interested in discussion of the methodology may wish to turn to the section following which deals with results.

In the procedure illustrated below, we determine the optimum program or use of resources in the resource situation including fixed supplies of 3,267 bushels of corn equivalent and 48 tons of hay equivalent. This situation is the one in which capital and labor are unlimited and prices are at the 1953 level except that milk is at 75 percent of parity (Situation S_{11} in table 3). All enterprises are competitive for labor (type A labor in table 2), and enterprises P_1 through P_{24} from table 1 are included. That is, all enterprises and levels of livestock management, except beef cows, are included in this solution selected to illustrate the method.

Two disposal activities, P_{25} and P_{26}, are added to allow non-use of the two limitational resources, grain and hay, respectively. The expenses include all cash outlays for protein feed, veterinary fees, breeding fees, insurance, housing, etc. and have been subtracted from unit prices to give **net unit prices.** These **net unit prices** are those listed in the C_j row of the matrix which follows.

SIMPLEX SOLUTION FOR SITUATION S_{11}

Enterprises P_1 through P_{24} represent the feasible enterprises of the matrix. The disposal activities (or enterprises) are designated as P_{25} and P_{26}. Column P_0 represents the amounts of limitational resources, corn and hay. The figures within the body of the first tableau of the matrix system which follows are the input-output coefficients of the respective enterprises, P_1 through P_{24} at unit

[2] See Charnes, A., et al. An introduction to linear programming. Wiley, New York. 1953; and Heady, Earl O. Simplified presentation and logical aspects of linear programming technique. Jour. Farm Econ. 36: 1035-1048. 1954.

level output. For example, in column P_1, the figures 8.48 and 277 represent the pounds of corn and hay, respectively, to produce one unit of the enterprise P_1.[3]

Algebraically, the corn and hay isoquants may be represented by equations. For the corn iso-resource curve the equation is:

$$8.48X_1 + 16.9X_2 + 24.9X_3 + 28.6X_4 + 32.7X_5 + 10.0X_6 + 23.7X_7 + 28.2X_8 + 33.0X_9 + 34.4X_{10} + 12.4X_{11} + 20.6X_{12} + 27.4X_{13} + 32.7X_{14} + 35.2X_{15} + 302X_{16} + 390X_{17} + 493X_{18} + 345X_{19} + 430X_{20} + 538X_{21} + 56.9X_{22} + 66.7X_{23} + 89.4X_{24} + 1X_{25} = 182{,}952$$

where the X_j's represent the level of output of each of the respective enterprises. The feasible enterprises, P_1 through P_{24}, may or may not use all the corn in the final solution. If the corn is entirely used up, then the disposal activity P_{25} is zero, and corn is a limitational factor in the livestock production program. If the corn is not entirely used by the active enterprises, the disposal process P_{25} is at some positive level, and corn is no longer a limiting factor of production. The equation for the hay iso-resource curve is:

$$277X_1 + 222X_2 + 186X_3 + 174X_4 + 163X_5 + 284X_6 + 236X_7 + 210X_8 + 193X_9 + 189X_{10} + 327X_{11} + 284X_{12} + 257X_{13} + 238X_{14} + 230X_{15} + 120X_{16} + 120X_{17} + 120X_{18} + 0X_{19} + 0X_{20} + 0X_{21} + 0X_{22} + 0X_{23} + 0X_{24} + 1X_{26} = 96{,}000$$

For the hay iso-resource equation, enterprises P_1 through P_{24} represent feasible activities. The X_j's correspond to those of the corn iso-resource equation. Disposal activity P_{26} is at a positive level when hay is not a limitational factor and, conversely, equals zero when the active enterprises utilize all the hay.

The solution concerns itself with finding the activity or enterprise levels, X_1 through X_{24}, which maximize total profit. The problem is to determine which enterprises are to be selected for the optimum program. At what level should the selected enterprises be operated? In practical terms, these questions imply the determination of the number of cows, hogs, poultry or some combination of the 24 possible enterprises and management levels which will yield a maximum total profit.

The coefficients of the X_j's of this equation are the net unit prices shown in the C_j row of the simplex tableau which follows. The net unit prices used in table 5 differ somewhat from the net unit prices shown in table 1: The value of labor has been subtracted, in the current case of unlimited capital-unlimited labor, to determine net unit prices. (This alternative procedure is used only for situations of unlimited labor and capital.) The P_0 column in this tableau (table 5) indicates which particular enterprises are the active ones of the program. For example, Plan 1 of the program is composed entirely of the disposal activities P_{25} and P_{26}. Column C_i indicates the prices of the activities or enterprises used in a particular plan of the program. The disposal activities P_{25} and P_{26} have no prices by assumption. Thus, Plan 1 yields no profit.

The $Z_j - C_j$ row in Plan 1 contains all negative numbers. Hence, there is an opportunity to improve the production plan or to increase profits. The next step is to introduce one of the feasible enterprises, P_j ($j=1 \ldots 24$), as an active enterprise in the program. Accordingly, the enterprise with the largest marginal profit per unit of product in Plan 1 is selected as the active enterprise. Enterprise P_{16} with the largest marginal profit, \$14.81, is chosen to be included in Plan 2. Now, two questions arise: What enterprise in Plan 1 will be replaced by P_{16}? At what level is the enterprise P_{16} to be operated? The answers are found in column R of Plan 1. The figures in column R are found by dividing each of the quantities in the P_0 column of Plan 1 by the corresponding coefficients in column P_{16}. For example, the amount of available corn, 182,952 pounds, is divided by 305, the corn requirement per unit output of P_{16}. This division gives the figure 599.84262, the number of units of P_{16} (spring hogs of above-average management level) that can be produced if all the corn were used by this enterprise. Similarly, when the 96,000 pounds of available hay are divided by 120, the hay requirement per unit of P_{16} produced, the resulting figure, 800, indicates the maximum total amount of P_{16} that could be produced from the hay supply. The level of enterprise P_{16} is most limited by corn. Thus, above-average spring pigs, enterprise P_{16}, will be chosen to replace the disposal activity of P_{25} in Plan 2.

In summary, Plan 2 contains the active enterprise P_{16}, tentatively operated at a level of 599.843 units (i.e., 59,984 pounds of pork). All available corn is used by this enterprise. The maximum profit for Plan 2 is found by multiplying the level of output by the unit price (599.843 × \$14.81). The resulting profit figure, \$8,884, is obviously larger than the zero profit of Plan 1.[4] ("Profit," as used at this point, is the net above variable costs from which fixed costs must yet be subtracted.) Thus, the change from Plan 1 to Plan 2 represents a definite improvement in the production program.

The general formula used in all numbers in row P_{16} of Plan 2 is:

$$a'_{kj} = \frac{a_{rj}}{a_{rk}}$$

where the subscript k identifies the livestock enterprise (P_{16}) coming into the program, r is the activity (P_{25}) being removed, j indicates any one of the column headings and i stands for any row.

[3] The corn input reflects the grain ration of the dairy replacement stock. The dairy cow herself for P_1 actually received no corn.

[4] The fixed costs referred to here are those such as taxes or crop expenses which do not differ between livestock plans. For "fixed" inputs on livestock which vary with organization, costs have been subtracted to obtain the profit figures of the text.

TABLE 5. LINEAR PROGRAMMING SOLUTION BY THE SIMPLEX METHOD FOR 24 FEASIBLE LIVESTOCK ENTERPRISES WITH TWO LIMITATIONAL RESOURCES—SITUATION S_{11}.

	C_j	....	....	....	1.61	1.58	1.71	1.69	1.63	1.53	1.43	1.57
	C_i	P_0	P_{25}	P_{26}	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
PLAN 1												
Corn equivalent	P_{25}	182,952	1	0	8.48	16.9	24.9	28.6	32.7	10.0	23.7	28.2
Hay equivalent	P_{26}	96,000	0	1	277	222	186	174	163	284	236	210
	Z_j	....	....	....	....	....	....	....	....	....	....	....
	Z_j-C_j	....	....	....	−1.61	−1.58	−1.71	−1.69	−1.63	−1.53	−1.43	−1.57
PLAN 2												
14.81	P_{16}	599.84262	0.00328	0	0.027803	0.05541	0.081639	0.09377	0.10721	0.03279	0.07770	0.09246
	P_{26}	24,018.88560	−0.39360	1	273.66360	215.35080	176.20330	162.74760	150.13440	280.06560	226.67540	198.90490
	Z_j	8,883.66923	0.4858	0	0.41176	0.82062	1.2097	1.38873	1.58782	0.48558	1.15081	1.36932
	Z_j-C_j	8,883.66923	0.4858	0	−1.19824	−0.75938	−0.50093	−0.30127	−0.04218	−0.04442	−0.27919	−0.20068
PLAN 3												
14.81	P_{16}	597.40267	0.00332	−0.00010	0	0.03353	0.06374	0.07724	0.09196	0.00434	0.05468	0.07225
1.61	P_1	87.76792	−0.00144	0.00365	1	0.078692	0.64387	0.59470	0.54861	1.02339	0.82830	0.72682
	Z_j	8,988.83989	0.04690	0.00440	1.61	1.76352	1,98062	2.10139	2.24519	1.71193	2.14337	2.24020
	Z_j-C_j	8,988.83938	0.04690	0.00440	0	0.18352	0.27062	0.41139	0.61519	0.18193	0.71337	0.67020

	C_j	1.61	1.59	1.37	1.24	1.37	1.38	1.43	14.81	12.32	11.39	12.10
	C_i	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}
PLAN 1												
Corn equivalent	P_{25}	33.0	34.4	12.4	20.6	27.4	32.7	35.2	305	390	493	345
Hay equivalent	P_{26}	193	189	327	284	257	238	230	120	120	120	0
	Z_j	....	....	....	....	....	....	....	....	....	....	....
	Z_j-C_j	−1.61	−1.59	−1.37	−1.24	−1.37	−1.38	−1.43	−14.81	−12.32	−11.39	−12.10
PLAN 2												
14.81	P_{16}	0.10820	0.11279	0.04066	0.06754	0.08986	0.10721	0.11541	1	1.27869	1.61639	1.13115
	P_{26}	180.01640	175.46560	322.12130	275.89510	246.21970	225.13440	216.15080	0	−33.44270	−73.96720	− 135.73780
	Z_j	1.60240	1.67038	0.60212	1.00028	1.33047	1.58782	1.70922	14.81000	18.93738	23.93878	16.75230
	Z_j-C_j	−0.00760	0.08038	−0.76788	−0.23972	−0.03953	0.20782	0.27922	0	6.61738	12.54878	4.65230
PLAN 3												
14.81	P_{16}	0.08991	0.90496	0.00793	0.03951	0.06482	0.08434	0.09345	1	1.28209	1.162391	1.14494
1.61	P_1	0.65780	0.64117	1.17707	1.00815	0.89972	0.88267	0.78984	0	−0.12220	−0.27029	−0.49600
	Z_j	2.39603	2.43864	2.01253	2.20836	2.40853	2.57357	2.65564	14.81	18.79101	23.61494	16.15800
	Z_j-C_j	0.78063	0.84864	0.64253	0.96826	1.03853	1.19357	1.22564	0	6.47101	12.22494	4.05800

	C_j	11.75	11.15	0.92	0.44	0.22	R
	C_i	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}	....
PLAN 1							
Corn equivalent	P_{25}	430	538	56.9	66.7	89.4	599.84262
Hay equivalent	P_{26}	0	0	0	0	0	800.00000
	Z_j	....	....	....	....	....	....
	Z_j-C_j	−11.75	−11.15	−0.92	−0.44	−0.22	....
PLAN 2							
14.81	P_{16}	1.40984	1.76393	0.186557	0.21869	0.29312	21,574.74453
	P_{26}	−169.18030	−211.67210	−22.38680	−26.24270	−35.17380	87.76792
	Z_j	20.87967	26.12386	2.76291	3.23878	4.34103	....
	Z_j-C_j	9.72967	14.97386	1.84291	2.79878	4.56103	....
PLAN 3							
14.81	P_{16}	1.42702	1.78544	0.18883	0.22135	0.29669	....
1.61	P_1	−0.61821	−0.77348	−0.08180	−0.09589	−0.12853	....
	Z_j	20.13885	25.19706	2.66487	3.12381	4.18705	....
	Z_j-C_j	8.38885	14.04706	1.74487	2.68381	4.40705	....

The prime mark (′) indicates that the number belongs to the new program. The numbers derived from the formula describe marginal rates of substitution. For example, when 8.48 in row P_{25} and column P_1 is divided by 305, it gives the figure (0.027803); multiplied by the unit price ($14.81), it gives the opportunity cost ($0.41176) of producing one more unit of enterprise P_1. All other opportunity costs in the Z_j row of Plan 2 are derived in a similar manner.

A second formula is used to derive the numbers in the P_{26} row of Plan 2.

$$a'_{ij} = a_{ij} - \left(\frac{a_{rj}}{a_{rk}}\right) a_{ik}$$

As an example, the quantity in the P_0 column in Plan 2 is:

$$24{,}018.88560 = 96{,}000 - \left(\frac{182{,}952}{305}\right)(120)$$

The quantity 24,018.88560 gives the amount of hay which is unused after supplying the necessary quantity of hay to enterprises P_{16}. It is the original quantity of hay, 96,000 pounds, minus the amount needed to produce 599.84262 units of enterprise P_{16}.

Plan 2 is not an optimum program because negative quantities still exist in the Z_j-C_j row. Total profit can be increased further by introducing several other enterprises, such as P_1, P_2, P_3 etc. Enterprise P_1, above-average dairy cows fed the highest forage ration, yields the largest marginal profit ($1.19824) and should be added as an active enterprise in Plan 3. Thus, a new production program is determined in Plan 3. This program includes both enterprises P_1 and P_{16}. Enterprise P_1 replaces activity P_{26} (non-use of hay) of Plan 2. Plan 3 indicates that enterprise P_{16} was reduced somewhat to permit enterprise P_1 to come into the program. Enterprise P_1 used the 24,018.88560 pounds of hay residual from Plan 2 plus the amount of corn released when enterprise P_{16} was reduced from 599.84262 in Plan 2 to 597.40267 units in Plan 3.

On checking the Z_j-C_j row in Plan 3, it is found that no negative numbers exist. Thus an optimum program is attained with production of 597.40267 units of P_{16} and 87.76792 units of P_1. The total "unadjusted" profit is $8,988.84. (Fixed costs still need to be subtracted from the amount to determine net profits.)[5] This plan returns $100 more than the previous plan. (Sometimes an optimum plan gives so little additional profit that a "previous" plan may be selected on the basis of (1) greater diversification or (2) personal preference.)

RESULTING PLANS

The plans which give maximum profits for the first 41 resource-price situations outlined earlier are presented in this section. (These results do not put capacity limitations on hogs; plans with limited hog capacity are presented later.) An initial point of interest with respect to the results is this: Only **above-average** dairy cows, hogs and poultry are included in **all** of the optimum plans. In other words, either capital, labor or feed requirements were sufficiently high for the average and below-average levels of management so that these enterprises never come into the plan which maximizes profit. Hence, one of the questions posed earlier (should low capacity cows be used to consume surplus forage while limited grain and capital supplies are used for hogs), has already been answered.

While dairy, hogs, beef and poultry enterprises must be combined in various proportions to make the best use of available resources, livestock representing above-average management conditions alone should be used in the program. The level at which above-average dairy cows should be fed grain, or the manner in which grain should be allocated between dairying and pork production, should vary depending on the amount of capital and labor available. However, even for a farm with a very small amount of capital, investment should not be made in average or below-average dairy cows.

The solutions or optimum programs are listed in table 6. The terms "hogs" and "poultry" always refer to **above-average** enterprises of these types. The term "dairy" always refers to **above-average** dairy cows, but the figure in parentheses indicates the level of hay and grain feeding: (1) refers to cows or activity P_1 fed 8.4 tons of hay and 520 pounds of grain in the lactation period; (2) refers to cows or activity P_2 fed 7.8 tons of hay and 1,036 pounds of grain; (3) refers to cows or activity P_3 fed 7.1 tons of hay and 1,904 pounds of grain; (4) refers to cows or activity P_4 fed 7.0 tons of hay and 2,374 pounds of grain; (5) refers to cows or activity P_5 fed 6.8 tons of hay and 2,894 pounds of grain.

"Beef" refers to beef cows of the average productivity levels outlined previously. Under the heading of limiting resources, a month such as "May" or "November" refers to competitive labor of the particular month. Under this same heading, "February poultry" means that supplementary labor in February for poultry limited this enterprise as a supplementary one. The notation "February dairy" indicates that, when considered as a supplementary enterprise, labor in February limited the size of this enterprise. The notations in the column, "limiting resources," indicate the specific resources which have restricted the particular plan to the one shown. The units of output are those mentioned earlier in the text. While grain supply is not listed under the table heading, it always amounts to 3,267 bushels.

PLANS IN RELATION TO LIMITING RESOURCES

While solutions were completed for 41 price-re-

[5] Fixed costs refer to real estate taxes and similar items which do not vary with the plan. Also, crop production costs are "fixed" for the livestock plans, since livestock are fitted to a single cropping plan. Fixed costs which do vary with livestock plans have been subtracted to give the profit figures cited.

TABLE 6. RESOURCE SUPPLIES, LIMITING RESOURCES AND AMOUNT AND COMBINATIONS OF LIVESTOCK PRODUCED UNDER VARIOUS RESOURCE-PRICE SITUATIONS.

Situation	Capital *	Labor	Hay (tons)	Limiting resources	Enterprise and units produced †				
					Dairy	Spring hogs	Fall hogs	Poultry	Beef ‡
S_1	$1,500	A	48	Capital	0	420	0	0	....
S_2	2,500	A	48	Corn, hay	(3) 136	589	0	0	....
S_3	4,000	A	48	Corn, hay	(3) 136	589	0	0	....
S_4	6,000	A	48	Corn, hay	(3) 136	589	0	0	....
S_5	1,500	A	68	Capital	0	420	0	0	....
S_6	2,500	A	68	Corn, hay	(3) 363	570	0	0	....
S_7	4,000	A	68	Corn, hay	(3) 363	570	0	0	....
S_8	6,000	A	68	Corn, hay	(3) 363	570	0	0	....
S_9	2,500	A	48	Corn, hay	(3) 136	589	0	0	....
S_{10}	4,000	A	48	Corn, hay	(3) 136	589	0	0	....
S_{11}	no limit	no limit	48	Corn, hay	(3) 136	589	0	0	....
S_{12}	2,500	A	68	Corn, hay	(3) 363	570	0	0	....
S_{13}	4,000	A	68	Corn, hay	(3) 363	570	0	0	....
S_{14}	6,000	A	68	Corn, hay	(3) 363	570	0	0	....
S_{15}	no limit	A	68	Corn, hay	(3) 363	570	0	0	....
S_{16}	no limit	no limit	68	Corn, hay	(3) 363	570	0	0	....
S_{17}	4,000	no limit	68	Corn, hay	(3) 363	570	0	0	....
S_{18}	6,000	no limit	68	Corn, hay	(3) 363	570	0	0	....
S_{19}	2,500	A	68	Corn, hay	(3) 363	570	0	0	....
S_{20}	4,000	A	68	Corn, hay	(3) 363	570	0	0	....
S_{21}	6,000	A	68	Corn, hay	(3) 363	570	0	0	....
S_{22}	no limit	A	68	Corn, hay	(3) 363	570	0	0	....
S_{23}	no limit	no limit	68	Corn, hay	(3) 363	570	0	0	....
S_{24}	2,500	no limit	68	Corn, hay	(3) 363	570	0	0	....
S_{25}	4,000	no limit	68	Corn, hay	(3) 363	570	0	0	....
S_{26}	6,000	no limit	68	Corn, hay	(3) 363	570	0	0	....
S_{27}	2,500	A	68	Corn, hay	(3) 363	570	0	0	....
S_{28}	4,000	A	68	Corn, hay	(3) 363	570	0	0	....
S_{29}	6,000	A	68	Corn, hay	(3) 363	570	0	0	....
S_{30}	no limit	A	68	Corn, hay	(3) 363	570	0	0	....
S_{31}	no limit	no limit	68	Corn, hay	(3) 363	570	0	0	....
S_{32}	4,000	no limit	68	Corn, hay	(3) 363	570	0	0	....
S_{33}	2,500	A	68	Corn, hay	(3) 363	570	0	0	0
S_{34}	4,000	A	68	Corn, hay	(3) 363	570	0	0	0
S_{35}	6,000	A	68	Corn, hay, May	(3) 363	570	0	0	0
S_{36}	2,500	B	68	Corn, hay, Nov.	(3) 340§	574	0	55	....
S_{37}	4,000	B	68	Corn, hay, Nov.	(3) 340§	574	0	55	....
S_{38}	6,000	B	68	Corn, hay, Nov.	(3) 340§	574	0	55	....
S_{39}	2,500	C	68	May, Nov. (Feb. poultry)	(2) 303	388	55	286	....
S_{40}	4,000	C	68	May, Nov. (Feb. poultry)	(2) 303	388	55	286	....
S_{41}	6,000	C	68	May, Nov. (Feb. poultry)	(2) 303	388	55	286	....

* "No limit" refers to unlimited labor or capital.

† In some cases two levels of feeding were indicated for dairying. The figures in parentheses indicate the feeding level which predominates.

‡ Beef cows were included in the problem only for situations S_{33}, S_{34} and S_{35}.

§ This average is brought down below that for four cows under S_{39}, S_{40} and S_{41}, because it includes approximately one-fourth of the year when feeding would be at the lowest level of grain feeding (P_1).

source situations of table 6, without the capacity limitations for hogs explained later, only five different plans resulted. This is because grain and hay were limiting in the majority of situations.

The situations which resulted in the identical plans are shown in table 7. For example, situations S_1 and S_5 result in the same plan. These two situations both include $1,500 capital, competitive labor averaging 258 hours per month and 1953 prices with grade A milk. The only difference is that S_1 includes 48 tons of hay equivalent while S_5 includes 68 tons. Capital is more limiting than any other single resource. Consequently, it alone determines the most profitable plan; hay goes unused in both situations, and, therefore, the plans are the same with 48 (S_1) and 68 (S_5) tons.

TABLE 7. NUMBERS OF ANIMALS OR BIRDS FOR 41 DIFFERENT RESOURCE-PRICE SITUATIONS.

Situation	Number of animals or birds			
	Dairy cows	Spring litters	Fall litters	Hens
S_1, S_5	0	27	0	0
S_2, S_3, S_4, S_9, S_{10}, S_{11}	2	37	0	0
S_6, S_7, S_8, S_{12}, S_{13}, S_{14}, S_{15}, S_{16}, S_{17}, S_{18}, S_{19}, S_{20}, S_{21}, S_{22}, S_{23}, S_{24}, S_{25}, S_{26}, S_{27}, S_{28}, S_{29}, S_{30}, S_{31}, S_{32}, S_{33}, S_{34}, S_{35}	5	37	0	0
S_{36}, S_{37}, S_{38}	4	37	0	34
S_{39}, S_{40}, S_{41}	4	25	4	176

Spring hogs is the only enterprise entering into the plan for S_1 and S_5 since this enterprise gives the largest return per $1 input of working or operating capital. Even under S_1, with 48 tons of hay equivalent, the forage would go unused or be sold. Diverting part of the capital to dairy or beef cows, to allow utilization of the forage, would result in smaller profits than use of all capital for the 27 litters of spring pigs. The plan shown applies to 1953 prices. With the possibility of fluctuating prices, a farmer might, of course, use some capital for dairy or poultry enterprises to diversify as a safeguard against price uncertainty (see later section where dairy and poultry are included in the plan to meet these conditions). Also, if hog prices were sufficiently low, a different management plan might be best. Under S_1 and S_5, 980 bushels of corn would be available for cash sales.

With spring hogs as the only livestock enterprise, most of the farm income would be forthcoming in August and September. Addition of poultry or dairy enterprises would allow income in each month. However, choice of such a plan to give a more even flow of income would be made at the expense of total profits where all labor is considered as competitive.

Situations S_2, S_3, S_4, S_9, S_{10} and S_{11} have an optimum plan which includes two dairy cows and 37 litters of spring pigs. Corn and hay are the limiting resources for all of these situations which include 3,267 bushels of corn equivalent, 48 tons of hay equivalent and competitive labor of type A in table 2. (Poultry competes directly with other enterprises for labor. In other solutions where labor for poultry is non-competitive, as it is on most Iowa farms, a poultry enterprise is included.)

While these situations vary in capital from $2,500 to unlimited funds, feeds restrict the program before available capital is used up in each case. Hence, differences between situations in capital do not cause variations in the optimum plan. Aside from the optimum or most profitable plan of two dairy cows and young stock to go with them, many other combinations of dairy cows and hogs would simply exhaust the feed supplies.

However, none of these alternatives would give profits as great as for the plan indicated. Again, risk or "even income flow" considerations might cause addition of some poultry or a larger dairy enterprise. But these goals would be attained, given the prices used, at a reduction in profits. (In a later section, income from an organization of enterprises as a risk precaution is compared with an organization which maximizes profit.)[6]

Situations S_6, S_7, S_8, and S_{12} consecutively through S_{35} include 3,267 bushels of corn equivalent, 68 tons of hay equivalent and capital ranging from $2,500 to unlimited. Prices include all three sets explained earlier. However, price and capital differentials are not great enough to offset the limiting effects of grain and hay in all of the situations. These two resources together specify an optimum plan which includes five dairy cows, 37 litters of spring pigs and no poultry. With hay and capital supplies allowing five dairy cows and 37 spring litters, labor and feed are not available for fall pigs or poultry. While spring pigs are more profitable under the three price situations, certain farmers might choose some fall pigs to spread price risks. Or, as is illustrated later, space restrictions on spring litters might require some fall pigs.

All of the situations explained above include competitive labor of type A in table 2. Situations S_{36}, S_{37} and S_{38}, which include some supplementary labor (type B of table 2) for poultry, have an optimum plan of four dairy cows, 37 spring litters and 34 hens. Limiting resources are grain, hay and competitive labor in November. Situations S_{39}, S_{40} and S_{41}, which include supplementary labor (type C of table 2) for both dairy and poultry, have an optimum plan of four dairy cows, four fall litters, 25 spring litters and 176 hens. The availability of supplementary labor for poultry increases the ability of this enterprise to compete for grain.

In previous situations and solutions, housewife or other labor which might be used for poultry was classed as competitive labor (type A of table 2), and this enterprise had to compete with other enterprises for labor.[7] In situations S_{39}, S_{40} and S_{41}, however, supplementary labor has been subtracted from the total stock of labor (i.e., to give type B in table 2). With less total labor for competitive enterprises (the same amount is available as previously except part of it now is set aside for supplementary poultry and dairy enterprises and cannot be used by competitive enterprises), spring hogs are restricted by supplies of competitive labor. With spring hogs restricted, some grain becomes available for fall hogs. Also, fall hogs come into the plan to spread use of competitive labor. Not only does grain and hay restrict the program under situations S_{39}, S_{40} and S_{41}, but also competitive November and May labor is limiting, along with supplementary February labor for poultry.

LEVEL OF GRAIN FEEDING

The optimum level of grain feeding for dairy cattle is level 3 (P_3 in the earlier description of activities), except for situations S_{39} through S_{41} inclusive. Under unlimited capital and feed, level 5 would be profitable in the sense of added returns which are greater than added costs. With limited grain supplies, however, the guiding principle in the use of grain resources is to use each bushel of grain where it will bring the greatest return. For all situations through S_{38}, grain brings a greater return if fed to hogs or poultry than if used to increase grain feeding from level 3 to level 4 or 5. Conversely, feeding dairy cows at level 3 (P_3) is more profitable than restricting feed to level 1 for dairy cows in order to raise more hogs or keep a poultry flock.

As is explained in more detail later, the optimum ration for one type of livestock cannot be determined apart from the organization of the farm as a whole when feed and capital resources are limited. Diminishing returns cause one rather than another dairy ration to give returns from grain as high for the milk cow enterprise as for the hog enterprise. Since, as table 8 illustrates, the return from additional grain declines with the level of grain feeding, level 3 of this study allows competition of dairy cows with hogs for grain under situations S_1 through S_{35}. Returns on the added grain for levels 4 and 5 are too low, and profits are greater if this amount of grain goes to hogs or poultry.

Diminishing returns also are encountered in pork production. The added gain forthcoming

[6] Where two different price situations result in the same farm organization, profits would still differ between plans—even though numbers of livestock and units of production are the same.

[7] In situations S_{39}, S_{40} and S_{41}, competitive poultry and dairy enterprises were included as activities, along with supplementary poultry and dairy enterprises. However, only dairy and poultry enterprises using supplementary labor came into the programs.

TABLE 8. EFFECT OF DIFFERENT LEVELS OF GRAIN FEEDING ON INCREMENT MILK PRODUCTION FROM ADDED FEED.

Total pounds T.D.N. fed	Total pounds milk produced	Feed added from previous level (lbs.)	Milk added from previous level (lbs.)	Added lbs. milk per added lb. feed
2,860	8,500	....	....	....
9,110	9,530	850	1,030	1.21
9,910	10,270	800	740	0.93
10,690	10,840	780	570	0.73
11,430	11,270	740	430	0.59

Source: Einer Jensen et al. Input-output relationships in milk production. USDA Tech. Bul. 815.

from each pound of grain declines as hogs are fed to heavier weights.[8] Hence, at hog weights heavier than 225 pounds, dairy cows fed at levels 4 and 5 might give highest returns on grain. This aspect of farm management has not been included in this study since a standard marketing weight for hogs of 225 pounds is included in the definition of activities.

In table 6, grain level 2 is optimum for dairy cows when supplementary labor is available for the poultry enterprise. The lower level of grain feeding now is the most profitable because of labor considerations. (Availability of supplementary labor for poultry allows this enterprise to compete with dairy cows for some of the grain.)

FEED ALLOCATION

The optimum or most profitable plan results in the allocation of grain and forage resources shown in table 9. From these figures, it is again apparent that maximum profits are forthcoming with most of the grain going to hogs. The majority of the feed goes to this enterprise under each of the three price situations for dairy products.

Hence, one hypothesis stated at the outset is refuted by the figures, while the other is supported. (1) Dairy cows of low ability to utilize forage are not profitable; but (2) even with better cows, hogs give greatest profit for the major part of the grain. The computations also show that poultry cannot profitably compete with hogs for the use of grain, labor and capital.

In situations where the hog and poultry enterprises compete for all three of these resources, poultry either was not included in the optimum plan or did not exceed 34 hens. Poultry represents a profitable enterprise, as compared with hogs, only where supplementary labor is available to the poultry enterprise and where hogs are restricted by lack of other resources or facilities.

Grain is available for cash sales under situations S_1 and S_2, where capital restricts the plan to 27 litters of spring pigs. Also, 12 percent of the total amount is available for cash sales under situations S_{39}, S_{40} and S_{41}. Under the latter situations, diversion of some labor to supplementary categories for dairy and poultry enterprises provides a situation where not all grain can be fed. Some hay also would go unused under these two sets of situations. Capital would not be available for buying beef cows to utilize this hay under situations S_1 and S_5. However, under S_{39}, S_{40} and S_{41}, the hay could be used to support four beef cows if arrangements could be made to get around the labor restrictions in May and November.

The beef enterprise was included as an activity in the problem only for situations S_{33}, S_{34} and S_{35}. However, the enterprise did not come into the final plan or solution. There was no grain or hay left over in these three situations. Hence, the conclusion, in these three situations where beef cows were allowed, is that the other enterprises give greater profits on scarce feed resources than do beef cows. However, in situations S_{39}, S_{40} and S_{41} (situations where beef cows were not included in the programming), where capital is not limiting, it is likely that beef cows could be added profitably to utilize the excess forage.

Many farmers are not limited on funds and buy grain for all enterprises. Under the prices and techniques included in this study, the most profitable plan for these farmers would be to feed dairy cows at or above the highest level of grain feeding included in this study. However, this study fo-

[8] See Earl O. Heady et al. New procedures in estimating substitution rates in pork production. Iowa Agr. Exp. Sta. Res. Bul. 409.

TABLE 9. PERCENTAGE ALLOCATION OF GRAIN AND HAY BETWEEN LIVESTOCK ENTERPRISES.

Situation	Percent grain used by:					Percent hay used by:*				
	Dairy cows	Spring hogs	Fall hogs	Poultry	Cash sales	Dairy cows	Spring hogs	Fall hogs	Poultry	Not used
S_1, S_5	0	70	0	0	30	0	52	0	0	48
S_2, S_3, S_4, S_9, S_{10}, S_{11}	2	98	0	0	0	26	74	0	0	0
S_6, S_7, S_8, S_{12}, S_{13}, S_{14}, S_{15}, S_{16}, S_{17}, S_{18}, S_{19}, S_{20}, S_{21}, S_{22}, S_{23}, S_{24}, S_{25}, S_{26}, S_{27}, S_{28}, S_{29}, S_{30}, S_{31}, S_{32}, S_{33}, S_{34}, S_{35}	5	95	0	0	0	50	50	0	0	0
S_{36}, S_{37}, S_{38}	4	93	0	3	0	49	51	0	0	0
S_{39}, S_{40}, S_{41}	4	65	10	9	12	49	34	0	0	17

* Some of the plans which are duplicated, under different situations, used different amounts of hay. The percent distribution of the hay refers to the situations which include 68 tons of hay. The reader can compute the percent distribution for the situations which include only 48 tons but have the same livestock plan.

cused its findings on the greater number of farmers who limit their livestock program to about the level of feed supplies produced on the farm.

Again, this point should be emphasized: Under the prices and techniques used for this study, the hypothesis of low-grade dairy cows to use forage (see introduction) must be rejected. While the level of grain feeding varies with labor and capital availability, the optimum plan for each situation includes the above-average dairy cows. The feed to maintain a good dairy cow does not cost a great deal more than the feed to maintain an average or poor cow. The added milk from the good dairy cow more than offsets her added costs, even though she may be fed a high ratio of forage in relation to grain.

LIMITED RESOURCES AND PRICE VARIATIONS

Since dairying is largely a supplementary enterprise in the use of part of the forage and a portion of the labor in some months, changes from grade A to grade B price or to 75 percent of parity do not alter the most profitable use of resources. In other words, the enterprise combination is one coming at the corner of a production possibility curve. Price changes for milk are not great enough to cause the iso-revenue curve to shift enough to cause another **corner combination** of enterprises to be most profitable.[9]

With a change in price relationships, other products remaining at the stated levels and milk dropping to a 75-percent of parity level, enterprise combinations should be left as previously. Profits will still be at a maximum, as compared with the same resource situation under a higher milk price, but they will not be as great as previously. Of course, a sufficient change in price relationships would call for a different enterprise combination. Price changes used in this study were small. Under other situations, price changes of sufficient magnitude would cause enterprise combinations to shift between corner combinations of the opportunity curve.

[9] See Heady, Earl O. Economics of production and resource use. Prentice-Hall, New York. 1952. pp 254-258; and Bowlen B. and Heady, Earl O. Optimum combinations of competitive crops. Iowa Agr. Exp. Sta. Res. Bul. 426.

LABOR ALLOCATION

While labor is a limiting resource under several situations, seldom does the labor of more than one or two months restrict the optimum plan. But even the labor supply of a single month can become crucial. While December may be a month of labor surplus, May or November may be labor-deficit months. The allocation of labor in these two months is shown in table 10. Remember that, while one enterprise may use the major part of the labor in one month shown, different seasonal requirements may cause a second enterprise to be the major user of labor in another month.

As mentioned previously, labor limitations in particular months have both direct and indirect effects on the enterprise and management practice to be selected for the most profitable plan. Where May and November labor are sufficiently limited, they restrict the size of the hog enterprise (an enterprise which gives a higher return on grain than high-level feeding of dairy cows or poultry). With the hog enterprise restricted, enough grain is available to include 176 hens and some fall litters under situations S_{39}, S_{40} and S_{41}. If more labor were to be hired for these months, or if it were available from other members of the family, hog and poultry enterprises could be larger.

PLANS WITH LIMITATIONS ON HOG CAPACITY

Solutions of the optimum plans explained earlier did not include building space or disease limitations for any class of livestock. Most farms adapted to supplemental dairying have enough barn space to accommodate more cows than were included in any of the plans outlined above. On many farms, however, the size of the hog enterprise may be limited by building capacity or disease hazard.

TABLE 10. PERCENTAGE ALLOCATION OF COMPETITIVE MAY AND NOVEMBER LABOR AMONG COMPETITIVE ENTERPRISES.*

Situation	Percent May labor used by:					Percent November labor used by:				
	Dairy cows	Spring hogs	Fall hogs	Poultry	Unused	Dairy cows	Spring hogs	Fall hogs	Poultry	Unused
S_1, S_5	0	28	0	0	72	0	30	0	0	70
S_2, S_3, S_4, S_9, S_{10}, S_{11}	12	40	0	0	48	16	42	0	0	42
S_6, S_7, S_8, S_{12}, S_{13}, S_{14}, S_{15}, S_{16}, S_{17}, S_{18}, S_{19}, S_{20}, S_{21}, S_{22}, S_{23}, S_{24}, S_{25}, S_{26}, S_{27}, S_{28}, S_{29}, S_{30}, S_{31}, S_{32}, S_{33}, S_{34}, S_{35}	31	38	0	0	31	43	40	0	0	17
S_{36}, S_{37}, S_{38}	43	55	0	2	†	50	50	0		0
S_{39}, S_{40}, S_{41}	0‡	91	9	0‡	0	0‡	80	20	0‡	0

* Percentage distributions are not shown for situations S_{36} through S_{41} since they also included supplementary labor for poultry and/or dairy. However, all November labor was used for S_{36}, S_{37} and S_{38} while all November and May labor was used for S_{39}, S_{40} and S_{41}.

† Less than 1 percent of total competitive May labor goes unused.

‡ The dairy and poultry enterprises which come into the program use supplementary labor and, therefore, do not make claims on competitive labor in May or November.

Many farmers are sufficiently skilled in hog production so that the size of the enterprise can be increased greatly before any particular disease hazard or diseconomy of scale arises. Other farmers, perhaps because of limited facilities, believe that increasing the hog enterprise causes it to encounter greater disease problems or other factors giving rise to important diseconomies to scale (increasing costs).

To determine the effect which scale limitations for hogs might have on the optimum combination of enterprises and the most profitable management practices, linear programming solutions have been completed for eight situations with space restrictions.

The number of spring pigs has been restricted to 19 litters by including space limitations in the beginning matrix. The space input-output coefficient is set at 1.0 per output unit and the "space supply" has been set at 297 units. Similarly, fall pigs have been restricted to eight litters by including a space input-output coefficient of 1.0 and a "space supply" of 130 units.

Linear programming solutions then have been completed to determine the most profitable plan for 11 additional situations. These 11 situations are Z_{12}, Z_{13}, Z_{14}, Z_{15}, Z_{19}, Z_{20}, Z_{21}, Z_{22}, Z_{36}, Z_{37} and Z_{38}. They are exactly the same, except for the space limitations on hogs, as the situations denoted by the same subscript and by S in table 3. Hence, Z_{12}, is the same situation as S_{12}, except that space limitations for hogs are included in Z_{12}. Similarly Z_{13} and Z_{14} parallel S_{13} and S_{14}, etc. Situations Z_{12}, Z_{13}, Z_{14} and Z_{15} all include 1949-53 prices with grade A milk, 68 tons of hay equivalent, 3,267 bushels of corn equivalent and the competitive labor under type A of table 2.

The capital for these four situations is $2,500, $4,000, $6,000 and unlimited, respectively. Situations Z_{36}, Z_{37} and Z_{38} are similar except that they include supplementary labor for poultry and do not include unlimited capital. Aside from space limitations for hogs, they are the same as situations S_{36}, S_{37} and S_{38}. Situations Z_{19}, Z_{20}, Z_{21} and Z_{22} are the same as Z_{12}, Z_{13}, Z_{14} and Z_{15}, except that milk is priced at 75 percent of parity. The optimum program for each of these situations is included in table 12. Output units are the same as those mentioned earlier.

The added situations outlined immediately above have been planned with only dairy, hog and poultry enterprises. Beef cattle were left out since they were not included in previous plans.

OPTIMUM PLANS AND LEVEL OF GRAIN FEEDING

Space limitations cause different plans to be optimum. Since spring pigs are always the most profitable enterprise under the prices used, they first enter the plan up to the space limitations of 19 litters. Fall pigs then come into the plan up to the limit of eight litters. The sizes of spring and fall hog enterprises are the same under all of the resource and price situations used for tables 11 and 12, since the two hog enterprises always come into the plan up to the limits of the space available. Third in level of profitability and in order of "coming into the plan" is the dairy enterprise. It is now included under each of the 11 situations in tables 12 and 13. The size of the dairy enterprise is about the same under these situations as in the parallel situations where restrictions were not put on the size of the hog enterprise. Note, however, that the intensity of grain feeding is one level higher (i.e., level 4 in table 11) than for the parallel situation without hog restrictions.

TABLE 11. MOST PROFITABLE PLAN FOR 11 SITUATIONS WTH RESTRICTIONS ON SIZE OF HOG ENTERPRISE (UNITS OF OUTPUT).*

Situation	Capital level ($)	Number of units produced				Limiting resources
		Dairy†	Spring hogs	Fall hogs	Poultry	
		1949-53 prices; grade A milk				
Z_{12}	2,500	(4) 462	297	130	183	Capital, hog space, Nov. labor
Z_{13}	4,000	(4) 333	297	130	467	Hog space, May and Nov. competitive labor.
Z_{14}	6,000	(4) 333	297	130	467	Same as Z_{13}
Z_{15}	unlimited	(4) 333	297	130	467	Same as Z_{13}
Z_{36}	2,500	(4) 379	297	130	225	Capital, hog space
Z_{37}	4,000	(4) 333	297	130	467	Hog space, May and Nov. competitive labor, Feb. poultry labor.
Z_{38}	6,000	(4) 333	297	130	467	Same as Z_{37}
		1949-53 prices; 75 percent parity milk				
Z_{19}	2,500	(4) 462	297	130	183	Same as Z_{12}
Z_{20}	4,000	(4) 333	297	130	467	Same as Z_{13}
Z_{21}	6,000	(4) 333	297	130	467	Same as Z_{13}
Z_{22}	unlimited	(4) 333	297	130	467	Same as Z_{13}

* The explanation of earlier tables also apply to tables 12 and 13 (see earlier text and footnote to tables). In the situations of tables 12 and 13, all labor is competitive (type A of table 2) except for situations Z_{36}, Z_{37} and Z_{38} where supplementary labor (type B of table 2) is included.

† The number (4) refers to the fourth level (P_4) of grain feeding (see earlier discussion of activities).

TABLE 12. UNITS OF LIVESTOCK FOR 11 SITUATIONS WITH RESTRICTIONS ON SIZE OF HOG ENTERPRISE.

Situation	Capital level ($)	Number animals or birds				Percent farm feeds sold or unused	
		Dairy cows	Spring litters	Fall litters	Hens	Corn	Hay
1949-53 prices; grade A milk							
Z_{12}	2,500	6	19	8	112	13.1	14.6
Z_{13}	4,000	4	19	8	287	6.3	30.8
Z_{14}	6,000	4	19	8	287	6.3	30.8
Z_{15}	unlimited	4	19	8	287	6.3	30.8
Z_{36}	2,500	5	19	8	139	13.1	22.7
Z_{37}	4,000	4	19	8	298	6.3	30.8
Z_{38}	6,000	4	19	8	298	6.3	30.8
1949-53 prices; 75 percent parity milk							
Z_{19}	2,500	6	19	8	112	13.1	14.6
Z_{20}	4,000	4	19	8	287	6.3	30.8
Z_{21}	6,000	4	19	8	287	6.3	30.8
Z_{22}	unlimited	4	19	8	287	6.3	30.8

TABLE 13. COMPARISON OF PROFIT ABOVE FIXED COSTS FOR (1) SITUATIONS WITHOUT AND (2) SITUATIONS WITH, SPACE LIMITATIONS FOR HOGS.

Situations compared	With restrictions on hogs (Z situations)	Without restrictions on hogs (S situations)
Z_{13} vs. S_{13}	$6,326	$7,828
Z_{14} vs. S_{14}	6,326	7,828
Z_{15} vs. S_{15}	6,326	7,828
Z_{37} vs. S_{37}	6,213	7,802
Z_{38} vs. S_{38}	6,213	7,802
Z_{19} vs. S_{19}	6,277	7,574
Z_{20} vs. S_{20}	6,277	7,574
Z_{21} vs. S_{21}	6,277	7,574

The level of grain feeding increases because space limitations curtail the ability of hogs to compete for grain. Six dairy cows are included in the plan for situations Z_{12} and Z_{19} while five cows are included for Z_{36}. (As in previous plans, additional young stock is included on the farm; the numbers in table 13 refer only to the cows being milked.) The remainder of the situations include only four dairy cows since the poultry enterprise is larger and restricts the amount of labor available for milk cows.

All of these situations include a poultry enterprise while the parallel situations without restrictions on hogs did not always include poultry—or they included a considerably smaller poultry enterprise. In other words, if hogs are not restricted by space or other considerations, they are able to outcompete poultry on a profit basis for grain, labor and capital.

For situations presented in table 12, capital limits the enterprise combination only for situations Z_{12}, Z_{19} and Z_{36}. Even in these cases, however, it is not the sole limiting resource, and hog space and/or November labor interact with it to determine the plans shown under the three situations. Under situations Z_{13}, Z_{14}, Z_{15}, Z_{20}, Z_{21} and Z_{22}, with only competitive labor (type A of table 2), May labor also is a limiting resource and helps determine the final, most profitable plan. Under the remaining situations, Z_{37} and Z_{38}, February labor for poultry also is limiting. Grain and hay were the main limiting resources under the parallel situations without restrictions on the hog enterprise. However, with space limitations for hogs, the changed combination of enterprises draws on labor supplies of months which limit the plan before feed supplies become limitational.

A considerable portion of hay supplies would go unused under the situations of tables 12 and 13. However, if beef cows had been allowed to come into the plan without capital restrictions, the following number of cows for raising beef calves would have been included: Z_{12} and Z_{19}, two cows; Z_{36}, three cows; all other situations, four cows.

Space limitations for hogs require plans which give lower profits than when restrictions on the size of this enterprise do not exist. As table 13 shows, plans with restrictions on the hog enterprise average about $1,500 less than those without restrictions. (Fixed costs, which are the same for all situations and do not vary with livestock programs, still need to be subtracted from these figures to give net profits.)

EFFECT OF RESOURCE ALLOCATION AND RATIONS ON PROFIT

The data above show that, if profits are to be maximized, even such decisions as the optimum grain ration for dairy must be related to the farm as a whole. In this case, whether or not a particular ration for dairy cows is optimum depends on the availability of space for hogs. If more or less space is available, hogs profitably use more or less, respectively, of the grain which might be used for a heavier concentrate feeding of dairy cows. It is possible, of course, for profits to be only slightly greater under a maximum profit plan than under some other organization where a particular ration is fed. For example, profit may be lowered slightly if the farmer feeds grain level 2 (P_2) or level 4 (P_4) rather than level 3 (the optimum level in most of the solutions presented in tables 6 and 7). To examine these possibilities, programs were completed for several situations where a non-optimum grain ration is used for dairy cows.

Table 14 includes the figures for Situation S_{12}. The first line is for the original maximum profit solution in table 6. The other lines include situations with the same resources and prices as S_{12}, except that level 1 of grain feeding was "forced" into the solution for line 2; level 2 was "forced" into the solution for line 3, etc. If grain level 4 is used, rather than 3, profits are depressed by $228. Grain levels 1, 2 and 5 cause somewhat greater sacrifices in profits. These figures apply, of course,

TABLE 14. RELATION OF ALTERNATIVE DAIRY RATIONS ON PROFIT. 1949-53 PRICES AND GRADE A MILK.

Situation	Level of grain feeding for dairy cows *	Animals in optimum plan		Profit above fixed costs
		Dairy cows	Litters of pigs	
S_{12} and grain level 3	P_3	5	37	$7,828
S_{12} and grain level 1	P_1	5	37	7,524
S_{12} and grain level 2	P_2	5	37	7,581
S_{12} and grain level 4	P_4	5	37	7,600
S_{12} and grain level 5	P_5	5	37	7,417

* P_i notations refer to the grain levels and dairy management situations outlined earlier in the text.

to price situations used, namely 1949-53 prices with grade A milk prices.

Differences would be smaller or larger as (1) hog prices were respectively lower or higher or (2) dairy product prices were respectively lower or higher. Other situations in table 6 have dairy product prices which are lower relative to hog prices than those used for the 1949-53 comparisons of table 14. Hence, profit sacrifices from using rations other than grain level 3 (P_3) would be even greater than those shown.

PROFIT EFFECTS OF PLANS TO MEET RISK

The plans outlined above are those which, given the prices and input-output coefficients used, maximize profits under the several resource situations. Farmers may not use plans corresponding exactly to the results because: (1) They have different price or input expectations; (2) they attempt to combine their enterprises to minimize risk and income variability. The first point is not of particular concern since linear programming can be used to solve the most profitable plan for any set of prices or input-output coefficients relevant for a particular farm.

The procedure does not provide a direct basis for comparing the advantages of one plan with another from the standpoint of risk. This section has been included, however, to compare returns and plans which include diversification "forced into the plan" to spread risks.

Two approaches are used in accomplishing this objective: First, enterprises including eight cows and 100 hens (with the auxiliary young or replacement stock as explained in description of activities) are "forced" into the plan as **diversified enterprises to meet risk.** Spring and fall hogs then are allowed to come into the plan in a manner to maximize profits, given the restraints of using resources for a minimum of eight dairy cows and 100 hens.

Second, spring and fall hogs are combined into a single enterprise or activity including equal proportions of the two. This amounts to saying that one litter of fall pigs will be farrowed for each litter of spring pigs to spread price risks. This **combination hog activity** is included, along with poultry and dairy cows, in the initial matrix and the optimum program is determined by linear programming.

PLANS WITH RESTRAINTS OF A MINIMUM OF EIGHT COWS AND 100 HENS

Table 15 shows the number of animals and birds included in the best plan when resources must be used for a minimum of eight cows and 100 hens (of course, more cows and hens are allowed in the plan if they can use resources more profitably than hogs).

The two situations, designated as X_{12} and X_{13}, are the same as situations S_{12} and S_{13} except that they require that resources be "saved out" for a minimum of eight cows and 100 hens. Situation X_{12} includes 3,267 bushels of corn, 68 tons of hay equivalent, type A competitive labor, $2,500 in operating capital and 1949-53 prices. Situation X_{13} is the same, except that it includes $4,000 of operating capital.

TABLE 15. OPTIMUM LIVESTOCK ORGANIZATION AND RELATED ITEMS WITH MINIMUM OF EIGHT DAIRY COWS AND 100 HENS.

Item	Situation X_{12}	Situation X_{13}
Number dairy cows	8	8
Litters spring pigs	12	12
Litters fall pigs	12	19
Number hens	100	100
Bushels corn sold	678	0
Percent grain used by:		
Dairy cows	8	8
Spring hogs	31	31
Fall hogs	35	56
Poultry	5	5
Sold	21	0
Percent hay used by:		
Dairy cows	84	84
Hogs	16	16
Unused	0	0
Profit above fixed costs	$6,557	$7,139

With operating capital limited to $2,500 under X_{12}, the plan includes eight dairy cows, 100 hens and 12 litters each of fall and spring hogs. The plan is the same under X_{13} with $4,000, except that the number of litters of fall hogs moves up to 19. Poultry and dairying are not, of course, increased beyond the original restraints explained above. Spring hogs are limited in both cases by forage and pasture. However, a farmer might reorganize his labor supplies and shift some of the fall hogs back to spring—using less pasture per litter. Capital limits the size of the fall hog enterprise under X_{12}, while grain limits it under X_{13}.

This organization of enterprises would give a less variable income than the organization outlined in tables 6 and 7 for situations S_{12} and S_{13} which do not include the **diversification restraints for spreading risks.**[10]

Income under S_{12} is $7,115, or $558 greater than for X_{12}; under S_{13}, it is $7,828, or $689 greater than for X_{13}. Since these differences are not extremely great, it is possible that many farmers would choose the plans of X_{12} and X_{13} over the plans for S_{12} and S_{13}. The latter are more specialized and more "risky" with a large spring hog enterprise, no fall hogs and less reliance on dairy and poultry. Undoubtedly, much of the dairy and poultry products comes from farms that have selected these enterprises, of a moderate size, to help minimize risk.

PLANS WITH RESTRAINTS OF SPRING AND FALL LITTERS IN EQUAL PROPORTIONS

Table 16 provides the optimum plans where the restraint on use of resources is that fall and spring litters must be produced in equal numbers to

[10] This statement is substantiated by the publication: Brown, William G. and Heady, Earl O. Economic instability and choices involving income and risk in livestock and poultry production. Iowa Agr. Exp. Sta. Res. Bul. 431. 1955.

TABLE 16. OPTIMUM LIVESTOCK ORGANIZATION AND RELATED ITEMS WITH SPRING AND FALL LITTERS IN EQUAL NUMBERS.

Item	Situation R_{aa}	Situation R_{36}	Situation R_{37}
Number dairy cows	7	3	4
Litters spring pigs	8	17	16
Litters fall pigs	8	17	16
Number hens	0	15	174
Bushels corn sold	1,670	0	0
Units of production:			
Dairy	497	245	286
Spring hogs	118	270	245
Fall hogs	118	270	245
Poultry	0	24	285
Profit above fixed costs	$5,069	$6,227	$6,530

spread price risk. Two situations, R_{36} with $2,500 and R_{37} with $4,000 of capital, are the same as S_{36} and S_{37} except for this restriction.[11] Situation R_{aa}, is the same as S_{12}, except that it includes only $1,500 in working capital.

With only $1,500 in operating capital (R_{aa}), eight litters each of fall and spring pigs are included in the optimum plan. Limiting resources are capital and November labor. Since (1) there is an ample amount of labor in other months and since (2) hogs use only a small portion of the forage, seven dairy cows and no poultry are included in this plan. A total of 1,670 bushels of corn is available for cash sale; some hay is left over.

With $2,500 in capital ($R_{36}$), the most profitable plan includes three dairy cows, 17 litters each of fall and spring pigs and 15 hens. The limiting resources are corn, capital and November labor.

[11] They also are the same as S_{12} and S_{13} except that R_{36} and R_{37} include supplementary labor (type B in table 2) for poultry (and S_{12} and S_{13} do not put restrictions on producing spring and fall pigs in fixed proportions).

With more capital, hogs compete with dairy and cause the latter enterprise to contract. Capital, however, is still too limited to allow much poultry. To use more capital for poultry would lower profits.

Under R_{37} with $4,000, operating capital is not limiting, and poultry comes into the plan up to 174 hens (and 125 chickens raised per 100 hens; see earlier discussion on units of output). Limiting resources are now corn, November labor and February supplementary labor for poultry.

The costs of diversifying fall and spring litters to spread risks (i.e., producing them in equal proportions) can now be examined. Under S_{36} without this restriction, profits above fixed costs are $7,520; they are $6,227 under R_{36}. Under S_{37}, profits are $7,802, as compared with $6,230 under R_{37}.

Selecting a farm organization to spread risks causes a sacrifice of $1,293 in the first instance and $1,272 in the second. These quantities may seem quite large as the costs of selecting a farm organization to spread risks.

The farmer in a secure financial position would not likely use these risk precautions. He could weather price setbacks of individual years, or could use farm outlook information to specialize within the year to maximize profits relative to changing price ratios. However, the farmer with less capital or a smaller equity and less ability to use farm outlook information might well prefer one of these "risk spreading" alternatives. Again, it is likely true that the organization of the majority of farms includes some of this precaution, rather than revolving entirely around profit maximization.