Least-Cost Rations and Optimum Marketing Weights for Broilers

Production Functions, Gain Isoquants, Substitution Ratios, Least-Cost Rations and Optimum Marketing Weights for Broilers Fed Corn and Soybean Oilmeal in a Fortified Ration

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RESEARCH BULLETIN 442 **OCTOBER, 1956** **AMES, IOWA**

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SUMMARY

This study was designed to allow prediction of a broiler production surface for corn and soybean oilmeal. From the basic production function developed, it was possible to predict (a) gain isoquants, (b) marginal rates of substitution of soybean oilmeal for corn, isoclines indicating ration paths for different substitution and price ratios and (c) other quantities related to the broiler production surface. Prediction of these physical quantities allowed attainment of the objectives of the study: (1) to predict least-cost rations for broilers of various weights with varying prices for corn and soybean oilmeal; (2) to predict optimum marketing weights for various rations with different prices for broilers and feed; and (3) to predict least-time rations in relation to least-cost rations.

The basic experiment was conducted in the winter and spring of 1955. It included 600 New Hampshire chicks fed on rations of 16, 18, 20, 22, 24 and 26 percent protein. Several forms of algebraic equations were fitted to the original observations. The production function selected for predicting optimum weights is the quadratic form I. The two interval functions used for predicting least-cost rations are logarithmic equations II and III. Equation IV with square root terms was used in predicting time relationships. In these equations, G refers to gain per bird (over the entire production period for equation I and in the respective gain intervals for equations II and III), C refers to corn intake per bird, S refers to soybean oilmeal intake per bird and T refers to time elapsed to consume a given quantity of feed.

$$\text{(I)}\quad G = 0.0331 + 0.4823C + 0.6415S - 0.0183C^2 - 0.0497S^2 - 0.0232CS$$

$$\text{(II)}\quad G = 1.0754C^{0.5245}\,S^{0.3838}\ \text{(gain to 1.23 lbs.)}$$

$$\text{(III)}\quad G = 0.7021C^{0.6463}\,S^{0.2944}\ \text{(gain over 1.23 lbs.)}$$

$$\text{(IV)}\quad T = 0.6735 + 4.7974C + 9.4575S + 21.4617\sqrt{C} + 13.6188\sqrt{S} - 12.0287\sqrt{CS}$$

Gain isoquants derived from equations I, II, III and IV show that corn and soybean oilmeal substitute at diminishing rates for a particular level of gain. Accordingly, different rations provide least-cost gains as the relative prices of corn and soybean oilmeal change. The gain isoquants also show that the marginal rate of substitution of soybean oilmeal for corn declines along a ration line as the bird progresses in weight. Hence, the least-cost ration under given prices for corn and soybean oilmeal for small birds is not the same as the least-cost ration for heavier birds. Least-cost rations for various feed price ratios have been determined by equating the marginal rate of substitution of soybean oilmeal for corn with the SBOM/corn price ratio. Using this procedure, the least-cost rations for the growth interval up to 1.32 pounds of liveweight are as follows: 16.5 percent protein for soybean oilmeal and corn prices of 6 cents and 2 cents, respectively; 17.5 percent protein for prices of 6 cents and 2.5 cents; 18.5 percent protein for prices of 4 cents and 2 cents; and 21.5 percent protein for prices of 4 cents and 3 cents. For broilers over 1.32 pounds, least-cost rations for the same respective prices for soybean oilmeal and corn are 15.0, 15.5, 16.5 and 18.5 percent protein.

Marginal rates of substitution of soybean oilmeal for corn are given in table A as an average over one weight interval. Data in the study provide similar information for other weight levels or intervals. These figures on substitution rates are related to price ratios in determining least-cost rations.

TABLE A. MARGINAL RATES OF SUBSTITUTION OF SOYBEAN OILMEAL FOR CORN FOR GAINS TO 1.23 POUNDS FOR BROILERS.

Percent protein in ration	Marginal rate of substitution of soybean oilmeal for corn, dC/dS
16	3.35
17	2.72
18	2.26
19	1.90
20	1.62
21	1.39
22	1.21
23	1.05
24	0.92

These data provide a basis for, first, determining the least-cost ration and, second, determining the optimum marketing weight. Optimum marketing weights are predicted by equating the marginal productivity of feed for a particular ration with the feed/broiler price ratio. While the ration to be selected depends on the prices of corn and soybean oilmeal and while numerous rations could be used, table B shows marketing weights and time required for particular prices of broilers and feed for 20- and 22-percent protein rations.

The time function shows that the least-cost ration is not identical with the least-time ration under normal price relationships. Data from the

TABLE B. OPTIMUM MARKETING WEIGHT, POUNDS OF FEED FOR OPTIMUM MARKETING WEIGHT AND TIME REQUIRED FOR 20- AND 22-PERCENT PROTEIN RATIONS.

Broiler/feed price ratio	Optimum marketing weight (lbs.)	Pounds feed required	Time required (weeks)
20-percent protein ration			
3.5-4.0	2.60	6.84	9.4
4.0-4.5	2.90	8.04	10.3
4.5-5.0	3.12	8.98	10.9
5.0-5.5	3.27	9.74	11.4
5.5-6.0	3.39	10.36	11.8
6.0-6.5	3.47	10.89	12.1
6.5-7.0	3.54	11.34	12.4
22-percent protein ration			
3.5-4.0	2.59	6.76	9.4
4.0-4.5	2.89	7.89	10.2
4.5-5.0	3.09	8.79	10.8
5.0-5.5	3.23	9.51	11.2
5.5-6.0	3.34	10.10	11.6
6.0-6.5	3.42	10.60	11.9
6.5-7.0	3.49	11.03	12.2

production function and time function can be used to predict the profitability of using the least-cost ration, or in feeding to speed time and move marketings ahead of price declines. The time function shows the amounts of time indicated in table C to attain a marketing weight of 3.25 pounds for various rations.

Over the entire production period, a 21-percent protein ration results in the most rapid gains. Of course, when the total production period is broken into intervals, a slightly higher percentage of protein gives most rapid gains for small weights while lower percentages of protein result in quickest gains over heavier weights. Under normal price relationships for corn and soybean oilmeal, the rations which give the lowest cost gains include somewhat less protein than least-time rations.

TABLE C. NUMBER OF DAYS AND AMOUNT OF FEED REQUIRED FOR 3.25-POUND MARKET WEIGHT UNDER VARIOUS RATIONS.

Percent protein	No. days to 3.25-lb. weight	Pounds feed required
16	82.6	9.82
17	80.1	9.41
18	78.7	9.29
19	77.6	9.17
20	77.0	9.11
21	76.7	9.09
22	76.8	9.11
23	77.3	9.18
24	78.1	9.28
25	79.3	9.44

Least-Cost Rations and Optimum Marketing Weights For Broilers[1]

Production Functions, Gain Isoquants, Substitution Ratios, Least-Cost Rations and Optimum Marketing Weights for Broilers Fed Corn and Soybean Oilmeal in a Fortified Ration

BY EARL O. HEADY, STANLEY BALLOUN AND ROBERT MCALEXANDER

The major cost item in broiler production is feed. Previous studies indicate that feed costs generally constitute 65-75 percent of the total cost of producing broilers. Hence, one of the major opportunities for increasing profits from broiler production is to minimize the costs of producing birds of a given weight. Great progress has been made in recent years in developing high energy feeds and feeding formulas which lessen the total pounds of feed required in producing a bird of a given weight. However, even though high energy, rapid gain formulas have been developed, the problem of how major sources or categories of feeds should be combined to minimize costs of gains still remains.

Ordinarily, broiler feeds are made up of two major categories of feeds, along with the proper vitamins and minerals. These two categories include feeds high in carbohydrate such as corn and feeds high in protein such as soybean oilmeal. If prices of these feeds did not change, the least-cost ration determined at one point in time also would be the least-cost ration at all later points in time. However, the prices of these major feed sources do change. In recent years the price of corn has been as low as 1.8 cents per pound with soybean oilmeal as high as 4.5 cents per pound, a SBOM/corn price ratio of 2.5; in other years the price of corn has been as high as 4.5 cents per pound with soybean oilmeal as low as 3.5 cents per pound, a SBOM/corn price ratio of 0.8. The ration or combination of these two feeds which minimizes costs of gains under one of these price ratios will not also minimize costs under the other ratio.

The least-cost ration can be determined by relating the prices of the feed sources to the rates of substitution of the feeds. For example, suppose that, beginning from a particular ration, 1 more pound of protein feed will substitute for or replace 2 pounds of grain; a second pound of protein feed will replace 1 pound of grain, with broiler gains remaining constant. If protein feed costs less than twice as much as grain, costs can be lessened by using the additional pound of protein feed since its "substitution rate" is twice that of grain (i. e., 1 pound of protein feed replaces 2 pounds of corn).

It is not the level of prices but the ratio of prices which is important. The substitution of the added pound of protein feed should take place if it costs 3 cents while grain costs 2 cents; 1 pound of protein feed with a value of 3 cents will replace 2 pounds of grain with a value of 4 cents. The substitution also should take place if protein feed costs 1.5 cents and grain costs 0.8 cent per pound; the 1.5 cents invested in protein feed will replace 1.6 cents invested in grain. However, the second pound of protein feed should not be used in either of these cases. Since it replaces only 1 pound of grain, 3 cents invested in it will save only 2 cents in grain under the first price situation; 1.5 cents invested in protein feed will save only 0.8 cent in grain under the second price situation. Even the first added pound of protein feed will not lessen costs if it costs more than twice as much as grain. For example, the first pound of protein feed worth 3 cents will replace only 2 cents in grain if the price of grain drops from 2 cents to 1 cent per pound.

Obviously, then, the least-cost ration can change as price ratios for various feeds change. Least-cost rations can be specified only if marginal rates of feed substitution are known. No previous study has been directed at predicting substitution rates in broiler production and rations which minimize costs under alternative prices of feed ingredients. This study is directed towards this end. However, since it is the first study of its particular kind, it is necessarily concerned with estimating other relationships which relate to, or are basic for, determining substitution rates. Since feed costs and rates of growth for different rations affect optimum marketing weights, this decision-making problem also is included in the analysis.

OBJECTIVES

The specific purposes of this study are: to predict the broiler production surface (function) of gains in relation to two feed categories; to predict input-output relationships of gain in terms of a fixed combination of the two feeds; to predict gain or growth isoquants indicating the possible combinations of two feeds which will result in a fixed gain level; to predict the marginal rates at which high-carbohydrate and high-protein feeds substitute for each other in producing a

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[1] Project 1135 of the Iowa Agricultural Experiment Station.

particular level of gain; to predict isoclines indicating feed combinations for particular gain levels which have the same rate of substitution; to indicate rations which will minimize costs of gains under various price relationships; to predict rations which minimize the time required for attaining a particular marketing weight and to indicate marketing weights which will maximize profits above feed costs for various price relationships.

Prediction of the numerous quantities and relationships mentioned above is dependent upon establishment of the basic production function or surface for the two major categories of feed used in this study. From this surface the marginal quantities necessary for specifying least-cost rations and maximum profit marketing weights can be predicted. While quantities and relationships such as these have been estimated for livestock and crops, they have not been predicted for broilers.[2] Predictions of optimum marketing weights have been made for single rations, but have not been made previously for both optimum rations and marketing weights.[3] It is hoped that the empirical quantities of this study will serve as a fundamental basis upon which related investigations will be built. Finally, it is expected that the results on minimum cost rations and optimum marketing weights can serve as the basis for decision by persons providing mixed feed to the broiler industry, by persons or firms mixing their own feed for large broiler operations and by producers concerned with rations and marketing weights which will maximize profits.

The empirical study which follows is based upon analytical models from production economics. These concepts for design and analysis of the experiment are outlined in some detail in the following section. This is done to provide the reader with a better concept of the quantities and relationships involved in determining least-cost rations and optimum marketing weights. The empirical counterpart of each of the concepts presented is used in this study; each of the basic economic principles is applied in determining optimum rations and marketing weights. The concepts are presented particularly as an aid to poultry nutrition research workers who may wish to adapt them to further experiments.

BASIC CONCEPTS

Broiler production results from the use of numerous categories of resources. In this study, however, there are two variable categories of feed that are of interest—grain and soybean oilmeal. Therefore, the general production function or surface to be estimated is of the form of equation 1 where G refers to gain or growth per bird, C refers to corn intake per bird and S refers to soybean oilmeal intake per bird.

$$(1) \quad G = f(C, S)$$

The specific function for this growth process is to be estimated in this study. Once the production function, which can be represented as a surface such as fig. 4, has been derived, the optimum quantity of a particular ration to be fed per bird (i. e., the optimum marketing weight) can be predicted. In other words, once the over-all or two-feed function or surface in equation 1 has been predicted, a function expressing the relationship between gain per bird, G, and the amount of a fixed ration (i. e., corn and soybean oilmeal held in constant proportions to give a fixed protein level), R, fed per bird can be expressed as in equation 2. This function, which can be derived from equation 1, provides the basis for predicting (a)

$$(2) \quad G = f(R)$$

total weight per bird associated with various amounts of a given ration and (b) the most profitable marketing weight. The optimum marketing weight can be established only if gain, G, increases at a decreasing rate for a particular ration and is determined by equation 3a. There, the ratio $\Delta G/\Delta R$ indicates the marginal product

$$(3a) \quad \frac{\Delta G}{\Delta R} = \frac{P_r}{P_g}$$

$$(3b) \quad (\Delta G)(P_g) = (\Delta R)(P_r)$$

or increase in gain, ΔG, associated with each small increase of the specific ration, ΔR. (The marginal product is the slope of the single-ration, input-output curve at the particular feed input.) The optimum marketing weight for a particular ration is attained when the marginal product is equal to the ratio $\frac{\text{price of ration}}{\text{price of broilers}}$ or P_r/P_g. When equation 3a is attained, the equivalent in equation 3b is attained: The value of the added feed, ΔR, is just equal to the value of the added gain, ΔG. If the marginal product is greater than the price ratio, as in equation 4a, the value of the added gain is greater than the value of the added feed. Profit per bird can be increased by feeding the bird to heavier weights until the marginal product ratio is driven down to equal the price ratio. The opposite is true in equation 4b, and birds should be sold at lighter weights. Once the specific production function corresponding to equation 1

$$(4a) \quad \frac{\Delta G}{\Delta R} > \frac{P_r}{P_g}$$

$$(4b) \quad \frac{\Delta G}{\Delta R} < \frac{P_r}{P_g}$$

and the particular input-output curve correspond-

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[2] See: Heady, Earl O., et al. New procedures in estimating feed substitution rates and in determining economic efficiency in pork production. Iowa Agr. Exp. Sta. Res. Bul. 409. 1954; Heady, Earl O., Pesek, John T. and Brown, William G. Crop response surfaces and economic optima in fertilizer use. Iowa Agr. Exp. Sta. Res. Bul. 424. 1955.

[3] See: Baum, M. T. and Fletcher, H. B. Application of profit maximizing techniques to commercial fryer enterprises. Poult. Sci. 32:415-23. 1953; Judge, George F. and Fellows, Irving F. Economic interpretations of broiler production problems. Storrs Agr. Exp. Sta. Bul. 302. Univ. of Conn. 1953.

ing to equation 2 have been computed, the marginal product can be estimated as the derivative of gain in respect to ration, as dG/dR; it is derived from equation 2.

However, before the optimum marketing weight can be determined, it is necessary to specify the least-cost ration. In other words, there are many ration functions such as represented in equation 2, and a prior task is to derive the one of these which allows a given gain with a minimum of cost. A first step in determining the least-cost ration for a particular gain is to derive a family of gain isoquants for the gain surface. The gain isoquant shows all possible combinations of the two feeds which will permit attainment of a particular gain. A gain isoquant can be looked on as a contour of a particular height around the gain surface and is of the general form of equation 5 where corn requirement is expressed as a function of the amount of soybean oilmeal fed, with gain constant at a specific level. The gain isoquant of equation 5 is derived from the production function or surface of equation 1.

$$(5) \quad C = f(S)$$

Once the isoquant equation has been determined, the isoquant for each level of gain can be determined, and the least-cost ration for a particular gain can be specified. The least-cost ration for a particular gain level is the one which results in the condition specified by equation 6a.[4] Here the marginal rate of substitution between

$$(6a) \quad -\frac{\Delta C}{\Delta S} = -\frac{P_s}{P_c}$$

$$(6b) \quad (-\Delta C)\,(P_c) = -\,(P_s)\,(\Delta S)$$

corn and soybean oilmeal, $-\Delta C/\Delta S$, is defined as the decrease in quantity of corn, $-\Delta C$, associated with each small increase in the quantity of soybean oilmeal, ΔS. When the substitution ratio is equal to the ratio of the price of soybean oilmeal, P_s, divided by the price of corn, P_c, the minimum cost ration has been determined. This is true since, as indicated in equation 6b, the value of the corn replaced is then equal to the value of the soybean oilmeal added. In case the marginal rate of substitution of soybean oilmeal for corn is greater than the price ratio, as in equation 7a, the value of the corn replaced will be greater than the value of the soybean oilmeal added; costs can

$$(7a) \quad -\frac{\Delta C}{\Delta S} > -\frac{P_s}{P_c}$$

$$(7b) \quad (-\Delta C)\,(P_c) > -(\Delta S)\,(P_s)$$

be lessened by substituting soybean oilmeal for corn until the equality of equation 6a is attained. If the marginal rate of substitution of soybean oilmeal for corn is less than the price ratio as in equation 8a, the value of the corn replaced is less than the value of the soybean oilmeal added; c
can be decreased by increasing the proportio
corn relative to soybean oilmeal until the sub
tution ratio is increased to the magnitude i
cated in equation 6a.

$$(8a) \quad -\frac{\Delta C}{\Delta S} < -\frac{P_s}{P_c}$$

$$(8b) \quad (-\Delta C)\,(P_c) < -\,(\Delta S)\,(P_s)$$

The equation for the substitution ratios is
rived from isoquant equation 5. It is, in this
ticular case, the derivative of corn in respec
soybean oilmeal, dC/dS. If this were a const
the least-cost ration would be composed of
bean oilmeal alone or corn alone. In other wo
the marginal rate of substitution of one feed
the other must decline as proportions of feeds
changed if the least-cost ration is to include s
combination of the two feeds. The marginal
of substitution represents the slope of the
isoquant for a particular feed combination (i
the substitution ratio is the derivative at a
ticular point on the iso-gain contour). Hence,
gain isoquants must be curved, rather
straight lines, if (a) the marginal rate of sub
tution is to change as the proportions of the f
change and (b) the least-cost ration is to inc
more than a single feed.

Decisions on the least-cost ration and the m
profitable marketing weight must be made si
taneously for birds which approach market
dition. Determination of these two quantities
be made through setting the partial derivat
of gain with respect to corn and gain with res
to soybean oilmeal to equal the respective
ratios as in equations 9 and 10. From these e
tions, the quantities of corn and soybean oil

$$(9) \quad \frac{\partial G}{\partial C} = \frac{P_c}{P_g}$$

$$(10) \quad \frac{\partial G}{\partial S} = \frac{P_s}{P_g}$$

can be predicted to indicate the ration w
gives lowest feed costs for market weight bi
the market weight to maximize profit also ca
determined.

The logic outlined above indicates the typ
data needed to specify the optimum ration
feeding level or period for any type of poult
livestock. The equations and quantities indic
have been derived from the basic experiment
plained in the next section.

DESIGN OF EXPERIMENT AND FEEDI METHODS

Data for this study were obtained from a
periment conducted by the Department of Po
Husbandry. Six hundred New Hampshire c
were used in the experiment. These chicks
randomly assigned to 30 pens (batteries) wi

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[4] Since the sign associated with the marginal rate of substitution is negative, a negative sign must also be adopted for the price ratio.

restriction of having 10 cockerels and 10 pullets per pen. The broilers were self-fed on six different rations consisting of 16, 18, 20, 22, 24 and 26 percent protein levels. The experiment was designed so that there were at least two replicates on each ration. Twelve groups of broilers were fed rations with fixed proportions of corn to protein for the entire period. In other words, two pens each of the birds were fed the entire period on the 16, 18, 20, 22, 24 and 26 percent rations. The other 18 pens of birds were fed up to a weight of approximately 1.32 pounds per bird, then changed to lower protein rations for the remainder of the feeding experiment as shown in table 1. The birds were weighed each week and corresponding feed inputs were determined to provide observations for regression analysis. The birds were taken off the experiment at the end of 11 weeks. The experimental unit was a pen, with each weighing becoming an observation.

Corn was the main source of carbohydrates and soybean oilmeal was the main source of protein. The soybean oilmeal contained approximately 45 percent crude protein while the corn contained approximately 8.4 percent (see table 2).

ESTIMATION OF PRODUCTION FUNCTIONS

Three over-all production functions were fitted to the experimental data as one step in estimating the broiler production surfaces. These functions are based on the 12 pens (two pens per ration), of rations which were continued from initiation of the experiment up to an average liveweight of 3.13 pounds, with 11 to 13 weighings per pen for a total of 146 observations.[5] The functions for the over-all production surface estimates are as follows:

$$G = 0.0331 + 0.4823C + 0.6415S - 0.0183C^2 - 0.0497S^2 - 0.0232CS \tag{11}$$

$$G = 10.1730 + 0.2300C + 0.1775S + 0.3314\sqrt{C} + 0.5004\sqrt{S} + 0.0200\sqrt{CS} \tag{12}$$

$$G = 0.9922C^{0.5537} S^{0.3371} \tag{13}$$

G refers to gain in pounds per broiler, C refers to pounds of corn per bird and S refers to pounds of soybean oilmeal per bird. Statistics for these three equations of the over-all production surfaces are presented in table 3. Aside from the interaction term in the square root equation, the regression coefficients are all highly significant. A problem of autocorrelation arises in estimating the regression coefficients of the over-all production surface for this reason: The 12 and 13 observations for each pen are not independent. Hence, it can be claimed that the total degrees of freedom (df) is something less than the 140 remaining after estimating the regression coefficients. However, if the total number of degrees of freedom remaining after estimation of regression coefficients is considered to be only 6 (i. e., to correspond to 12 pens or independent observations), the regression coefficients for the quadratic, 11, and logarithmic, 13, equations are still acceptable at the 0.01 and 0.05 probability levels. In all of the equations, the feed inputs account for over 99 percent of the variance in gains. As is indicated later, both the over-all quadratic and logarithmic equations are used for estimates of this study.

TABLE 1. DESIGN OF EXPERIMENT FOR BROILER STUDY.

Pen-numbers Replicates I	Pen-numbers Replicates II	Percent protein rations fed broilers from weight of 0.09 to 1.32 lbs.	Percent protein rations fed broilers from weight of 1.32 lbs. to end of feeding period
22	25	16	16
27	20	18	18
16	29	18	16
2	10	20	20
3	8	20	18
28	19	20	16
16	24	22	22
17	5	22	18
6	13	22	16
18	14	24	24
11	9	24	20
7	30	24	16
1	4	26	26
12	23	26	22
21	15	26	18

TABLE 2. POUNDS OF INGREDIENTS USED PER 100 POUNDS OF FEED IN BROILER EXPERIMENT.

Ingredients	Percent protein in ration					
	16	18	20	22	24	26
Ground yellow corn	71.0	65.5	59.6	53.9	48.2	42.5
Wheat middlings	5.0	5.0	5.0	5.0	5.0	5.0
Dehyd. alfalfa meal (17%)	2.5	2.5	2.5	2.5	2.5	2.5
Soybean oilmeal	15.0	20.5	26.0	31.5	37.0	42.5
Fishmeal	2.5	2.5	2.5	2.5	2.5	2.5
Steamed bonemeal	2.0	2.0	2.0	2.0	2.0	2.0
Ground oyster shells	0.5	0.5	0.5	0.5	0.5	0.5
Iodized salt	0.5	0.5	0.5	0.5	0.5	0.5
C-2054 (premix)	1.0	1.0	1.0	1.0	1.0	1.0
Soybean oil	—	0.2	0.4	0.6	0.8	1.0

TABLE 3. VALUES OF R AND t FOR EQUATIONS 11, 12 AND 13.

Equation	R	Values of t for regression coefficients in order shown in equation				
		b_1	b_2	b_3	b_4	b_5
11	0.9990*	39.82*	29.69*	7.42*	7.44*	3.28†
12	0.9986*	6.68*	3.01‡	6.10*	6.89*	0.24§
13	0.9979*	43.72*	26.97*	—	—	—

* $p < 0.01$ for both 140 and 6 df
† $p < 0.01$ for 140 df and $p < 0.02$ for 6 df
‡ $p < 0.01$ for 140 df and $0.01 < p < 0.05$ for 6 df
§ $p > 0.5$ for 140 df

These particular functions were selected for estimating a broiler production surface because of their logical basis. Other studies for meat production indicate that output tends to increase at a decreasing rate (i. e., each additional input of feed usually results in less and less gain in weight), and feeds tend to substitute for each

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[5] The number of observations were not the same for each pen since observations at 600 and 1,300 gram weights also were obtained for all pens; for some pens these weights occurred at the same time as the regular weekly weighings, resulting in fewer observations in these particular pens.

other at diminishing rates.[6] These conditions are expected for broilers. As birds increase in size, more and more of each pound of feed is required for maintenance. Also, as broilers increase in size, the composition of the body changes. The changes in body composition are expected to cause changes in the rate of substitution between feeds. The structural portions of the body and the organs develop first, followed in order by muscle, tissue and fat.[7] Thus, rations of a higher protein content are required during the early periods of growth. As birds increase in size and body weight, less protein is required, and feeds containing more carbohydrates may be substituted for high protein feeds as the fattening stage is approached.

Production functions which permit estimation of the above relationships were desired. That is, they should permit (a) decreasing productivity per pound of a fixed ration (the two feeds increased in fixed proportion) as well as diminishing productivity of either feed alone, (b) diminishing rates of substitution along a particular gain isoquant and (c) changing substitution rates along a ration line (i. e., changing substitution ratios for a particular ration) as the bird progresses in weight and higher gain isoquants are attained. Functions 11 and 12 meet all of these qualifications. Production function 13 does not permit substitution rates along a ration line to change as the broilers increase in size. That is, it does not account for the fact that protein, in relation to carbohydrates, is of greater value to the young birds than to older birds. It "forces" into the analysis the relationship that the rate of substitution must be constant along a ration line.

The difference between the quadratic and logarithmic equations with respect to conditions of substitution rates along a fixed ration line can be illustrated by figs. 1, 2 and 3. (The relationships in these illustrations are assumed and do not represent actual predictions in this study.) In fig. 1, the negatively sloped curves are gain isoquants or contours, indicating all of the possible combinations of the two feeds which produce, respectively, a 1.0-, 2.0- and 3.0-pound gain on broilers. The solid and positively sloped (straight) lines are isoclines. They indicate the points on successively higher isoquants where the substitution rates are constant. The points at which the positively sloped straight line or isocline labelled 2.0 intersects the gain isoquants indicates the feed combinations at which 1 pound of protein feed substitutes for 2 pounds of carbohydrate feed. The line 2.0 indicates, for each particular level of gain, the path of feed combinations over which the substitution rates are equal to 2.0. The positively sloped curve 2.5 indicates the feed combination for each possible level of gain, where 1 pound of protein feed substitutes for 2.5 pounds of carbohydrate feed.

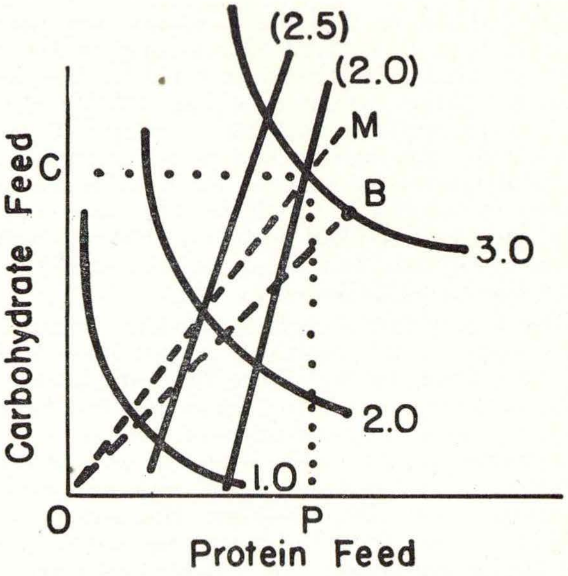

Fig. 1. Isoclines for an over-all quadratic function (assumed).

The isoclines are expansion paths showing the path of rations which should be followed as a bird gains in weight if profits are to be maximized (i. e., if the least-cost ration is to be used for each particular level of gain). Thus, if the price of protein feed were twice the price of carbohydrate feed, the price ratio would be 2.0. Using equation 3a to express the necessary condition, the rations (proportions of the two feeds) along isocline 2.0 should be followed in fig. 1. Since the isoclines in fig. 1 do not intersect the origin, a different ration (i. e., a different proportion of the two feeds as read off the two axes) would be required for each fractional pound of change in gain. This path is biologically logical since the proportion of carbohydrates to protein feed should increase with weight. (Rations higher in the plane along the isoclines of fig. 1 include a greater proportion of carbohydrates).

However, producers cannot practically change rations with each fractional pound of gain. Generally they feed the same ration, or change it only once, throughout the production period. If the optimum ration for a 3.0-pound gain were selected, through equating the substitution ratio with the price ratio as in equation 3a, the optimum ration would include OC of the carbohydrate feed and OP of the protein feed. If the ration with the proportion of feeds at OC/OP, were fed throughout, the "feed path" would be OM.[8] This line does not indicate the least-cost

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[6] See: Heady, et al., New procedures in estimating feed substitution rates and in determining economic efficiency in pork production, op. cit., pp. 922-924; Kehrberg, Earl W. Adaptation of economic production logic to feed utilization by livestock. Unpublished Ph. D. thesis. Iowa State College Library, Ames, Iowa. 1953. pp. 109-113; Heady, Earl O. et al. Milk production functions, hay/grain substitution rates and economic optima in dairy cow rations. Iowa Agr. Exp. Sta. Res. Bul., to be published; Baum and Fletcher, op. cit., pp. 415-422.

[7] Jull, Morley A. Poultry nutrition. McGraw-Hill Book Co., Inc., New York. 1938. p. 263.

[8] A diagonal line could be drawn from each point when the 2.0 isocline intersects the 1.0-, 2.0- and 3.0-pound isoquants to the origin. The slopes would indicate rations which could be fed in each of the three weight ranges: 0 to 1.0, 1.01 to 2.0 and 2.01 to 3.0 pounds, respectively. These rations would be "averages" for the intervals and would not equate substitution and price ratios for each gain isoquant within an interval.

other at diminishing rates.[6] These conditions are expected for broilers. As birds increase in size, more and more of each pound of feed is required for maintenance. Also, as broilers increase in size, the composition of the body changes. The changes in body composition are expected to cause changes in the rate of substitution between feeds. The structural portions of the body and the organs develop first, followed in order by muscle, tissue and fat.[7] Thus, rations of a higher protein content are required during the early periods of growth. As birds increase in size and body weight, less protein is required, and feeds containing more carbohydrates may be substituted for high protein feeds as the fattening stage is approached.

Production functions which permit estimation of the above relationships were desired. That is, they should permit (a) decreasing productivity per pound of a fixed ration (the two feeds increased in fixed proportion) as well as diminishing productivity of either feed alone, (b) diminishing rates of substitution along a particular gain isoquant and (c) changing substitution rates along a ration line (i. e., changing substitution ratios for a particular ration) as the bird progresses in weight and higher gain isoquants are attained. Functions 11 and 12 meet all of these qualifications. Production function 13 does not permit substitution rates along a ration line to change as the broilers increase in size. That is, it does not account for the fact that protein, in relation to carbohydrates, is of greater value to the young birds than to older birds. It "forces" into the analysis the relationship that the rate of substitution must be constant along a ration line.

The difference between the quadratic and logarithmic equations with respect to conditions of substitution rates along a fixed ration line can be illustrated by figs. 1, 2 and 3. (The relationships in these illustrations are assumed and do not represent actual predictions in this study.) In fig. 1, the negatively sloped curves are gain isoquants or contours, indicating all of the possible combinations of the two feeds which produce, respectively, a 1.0-, 2.0- and 3.0-pound gain on broilers. The solid and positively sloped (straight) lines are isoclines. They indicate the points on successively higher isoquants where the substitution rates are constant. The points at which the positively sloped straight line or isocline labelled 2.0 intersects the gain isoquants indicates the feed combinations at which 1 pound of protein feed substitutes for 2 pounds of carbohydrate feed. The line 2.0 indicates, for each particular level of gain, the path of feed combinations over which the substitution rates are equal to 2.0. The positively sloped curve 2.5 indicates the feed combination for each possible level of gain, where 1 pound of protein feed substitutes for 2.5 pounds of carbohydrate feed.

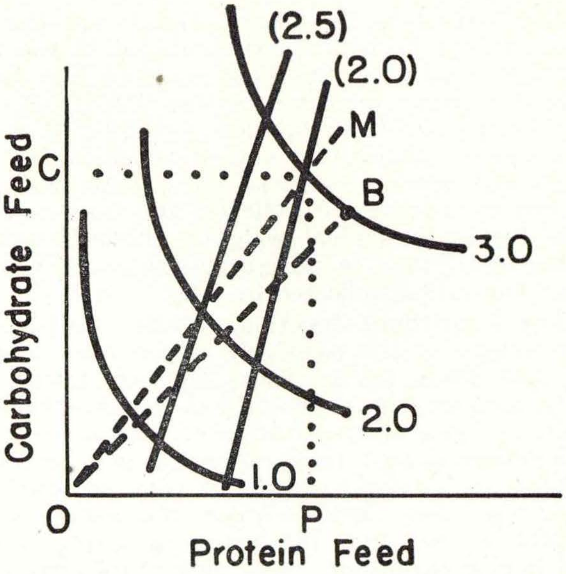

Fig. 1. Isoclines for an over-all quadratic function (assumed).

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ration. (Isocline 2.0 does.) Hence, another ration such as B, with feeds combined in the proportions read from the axes, could be less costly than the ration indicated by line OM, if fed over the entire production period. (Ration B would not equate substitution and price ratios for any particular levels of gain.) Thus, while the path traced by isocline 2.0 indicates the least-cost ration for all individual weights, the ration indicated by OB may be more practical and less costly than the ration indicated by OM.

The logarithmic function provides isoclines of the nature of the positively sloped lines labeled 2.5 and 2.0 in fig. 2. Since they are linear and pass through the origin, these isoclines suggest that the rates of substitution of the two feeds do not change along a fixed ration line as a bird progresses to heavier weights. If they are used for decisions, they indicate that the same ration should be used from the beginning to the end of the production period. Biologically, it is expected that rates of substitution do change along a ration line. Therefore, a linear isocline should not pass through the origin. However, while the logarithmic equation may not provide the greatest degree of biological accuracy, it may provide a practical basis for selecting the one "average" ration to be fed over the entire production period for producers who use this method of feeding. Hence, with a price of protein feed twice as great as the price of carbohydrate feed, the single ration (proportion of the two feeds) indicated by isocline 2.0 in fig. 2 would be fed over the entire production period. With a price of protein feed two and one-half times as great as the price of carbohydrate feed, the ration would be that indicated by isocline 2.5 in fig. 2.

An alternative combining the advantages of the quadratic and logarithmic functions also has been used in this study. It includes the estimation of logarithmic functions for different weight intervals or segments of the production function. The effect on isoclines is that shown in fig. 3. The logarithmic functions of the various weight intervals provide isoclines for a particular segment of the production surface. They indicate the one best "average" ration to be fed over the particular weight interval; the "average" ration for the interval can be determined by equating the price and substitution ratios. The isoclines for each interval can be combined to give isoclines with linear segments as in fig. 3. In this case, with a price ratio of 2.0, the lower linear segment of isocline 2.0 in fig. 3 indicates the "average," least-cost ration to be fed to 2 pounds of weight, and the middle segment indicates the "average" least-cost ration to be fed between 2 and 3 pounds of weight. Since the second segment, based on the second interval function, has a greater slope than the first segment, a ration containing a greater proportion of carbohydrate feed would be used for heavier weights.[9] Hence, on practical grounds, the producer could use two or more rations over the production period (without changing feed proportions for each fraction of gain as indicated along the isoclines of fig. 1).

For the reasons outlined above, the over-all logarithmic function is used to predict optimum feed quantities when a single ration is to be fed over the entire production period. Two interval logarithmic equations, presented later, have been used to provide data allowing one change in the ration where this practice is preferred. (Producers seldom use more than two rations.) However, since they assume constant elasticity over the entire production surface, the over-all logarithmic equations tend to overestimate the gains associated

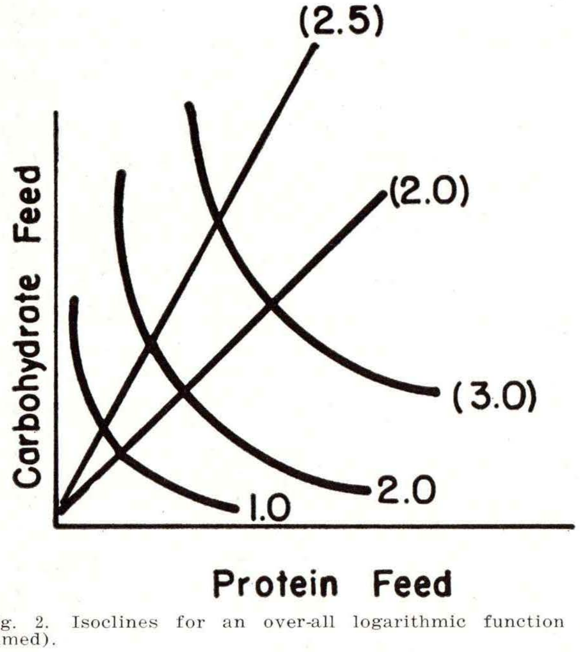

Fig. 2. Isoclines for an over-all logarithmic function (assumed).

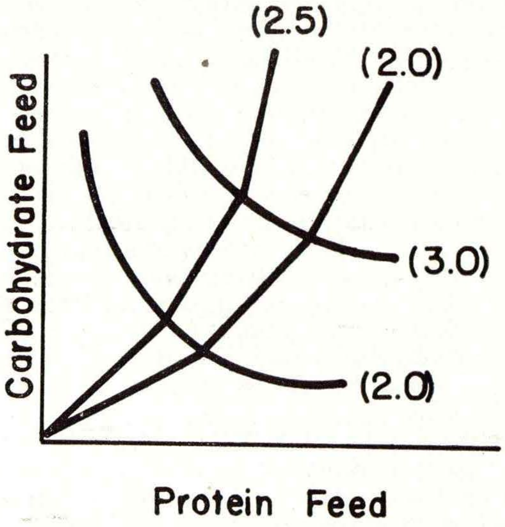

Fig. 3. Isocline segments for interval functions (assumed).

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[9] Actually all segments of the "combined isocline" in fig. 3 originate at zero for their particular weight interval. However, they can be spliced together as indicated to represent total gains and contours over the entire production surface, rather than gains within a single interval.

with total feed consumption during the latter part of the production period. Hence, they tend to overestimate the optimum marketing weight for a particular broiler price. For this reason the quadratic equation is used in predicting most profitable marketing weights. In effect, this procedure is one of using logarithmic functions to predict "average" rations to be used as practical alternatives up to weights of nearly 3 pounds. Beyond this weight, the quadratic function can be used to specify (a) "exact" feed combinations as marketing time approaches and (b) optimum marketing weights.

PRODUCTION SURFACES FROM OVER-ALL FUNCTIONS

Production surfaces based on equations 11 and 13 are presented, respectively, in figs. 4 and 5. Lines OD and OB in the feed plane are ration lines indicating 16- and 26-percent protein rations, respectively. Curve OE above the 16-percent ration line of OD indicates gain levels when various amounts of this particular ration are fed per bird. Curve OC above the 26-percent ration line of OB indicates gain levels when various amounts of a 26-percent ration are fed. Other ration lines (i. e., 18, 20, 22, etc.) such as OD and OB could be drawn in the feed plane, and each would have above it an input-output curve for the particular ration. These quantities, corresponding to equation 2, provide the basis for determining the optimum marketing weight when a particular ration is fed; they do not provide the basis for predicting the optimum ration.

The contours on the surfaces, indicated by pound quantities in figs. 4 and 5, are gain isoquants and show all of the possible feed combinations (measured in the feed plane) which will produce a pound of gain at the particular broiler weight. For both functions, the gain contours or isoquants are curved (i. e., are not straight lines) indicating that, as a greater proportion of one feed is used, the amount replaced of the other feed declines. The nature of the surfaces indicates diminishing marginal productivity (i. e., each pound of feed adds less to total weight than the previous pound) for particular rations. Horizontal slices through these surfaces provide the gain isoquants while vertical slices provide the input-output curve for a particular ration when these two relationships are presented in graphs of two dimensions. Actually the surfaces presented in figs. 4 and 5 represent wedges out of a larger surface since the experiment included rations ranging only from 16 to 26 percent protein.

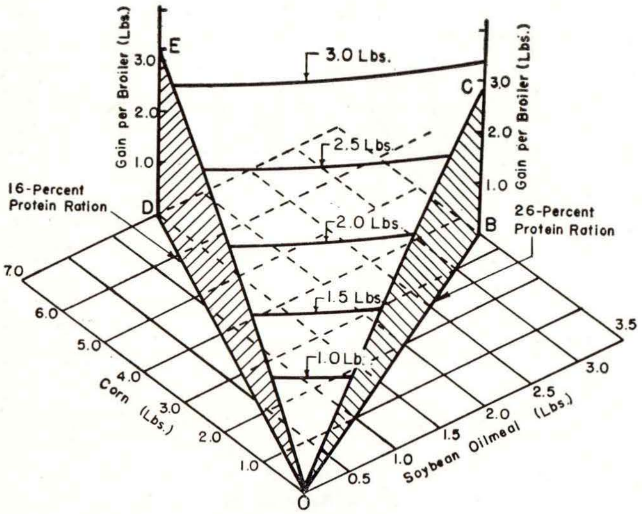


Fig. 4. Production surface showing feed-gain relationships predicted from over-all logarithmetic function 11 for broilers fed 16- to 26-percent protein rations.

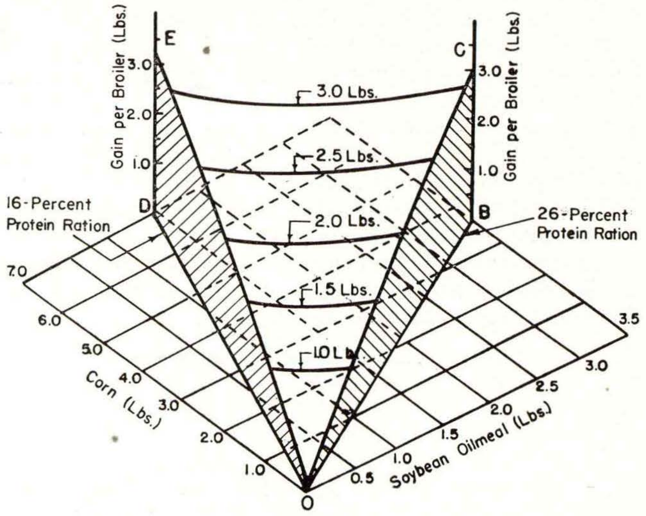


Fig. 5. Production surface showing feed-gain relationships predicted from over-all quadratic function 13 for broilers fed 16- to 26-percent protein rations.

INDIVIDUAL GROWTH FUNCTIONS

It has been suggested that under certain conditions input-output curves and isoquants derived from a single equation estimate of the production surface might be spurious. Supposedly, this situation might occur where one portion of the surface drops discretely down to a ledge, or a "canyon" exists on one part of the function. The over-all equation would be affected equally by all observations over the surface and would not allow prediction of this discrete depression in gains. Also, if the joint relationships involved were greatly complicated, estimation by simultaneous equations might be required. As a basis of comparison of the predictions made from the over-all functions, single-variable equations were estimated for each ration included in the study. An input-output curve for each ration, independent of those for all other rations, was then predicted from the single-variable equations and compared with a similar estimate from the over-all function.

The single-variable functions have been derived with gain, G, as the dependent variable and corn, C, as the independent variable. This procedure can be used since the proportion of soybean oilmeal to corn is fixed for any one of the six different protein levels. Not only does the single-variable equation express gain as a function of corn and soybean oilmeal, but it expresses gain as a function of all feed since each pound of feed for a particular protein level represents a fixed combination of corn, soybean oilmeal and the other feed ingredients indicated in table 2.

The individual regression functions derived are of the polynomial and logarithmic types. The derived polynomial functions, hereafter designated as quadratic single-variable functions for the indicated percent protein levels, are:

(14) $G = -0.0296 + 0.5984C - 0.0244C^2$ (16%)
(15) $G = 0.0370 + 0.6886C - 0.0323C^2$ (18%)
(16) $G = 0.0444 + 0.7183C - 0.0305C^2$ (20%)
(17) $G = 0.0256 + 0.8726 - 0.0520C^2$ (22%)
(18) $G = 0.0319 + 1.0030C - 0.0680C^2$ (24%)
(19) $G = 0.0377 + 1.0983C - 0.0868C^2$ (26%)

where G is pounds gain in weight per broiler, and C represents the pounds of corn fed in the various broiler rations. Implicit in each pound of C are other feed inputs as described earlier.

Single-variable ration functions of the logarithmic type for the various protein levels are:

(20) $G = 0.5878C^{0.9029}$ (16%)
(21) $G = 0.6669C^{0.8905}$ (18%)
(22) $G = 0.7240C^{0.8786}$ (20%)
(23) $G = 9.7997C^{0.9172}$ (22%)
(24) $G = 0.9422C^{0.8624}$ (24%)
(25) $G = 1.0048C^{0.8723}$ (26%)

The input-output curves for particular rations derived from the single-variable ration functions were all plotted on scatter diagrams for comparison with their respective over-all functions. Comparisons of input-output curves derived from the single-variable and over-all quadratic functions are shown in figs. 6-11 for the six rations. The similarity of these two sets of curves indicates that the over-all function does not give spurious results for any particular level of protein. Similar comparability existed for estimates from single-variable and over-all logarithmic equations.

INTERVAL FUNCTIONS

The interval functions of the logarithmic type used for predictions of "average" least-cost rations over two weight intervals (see earlier discussion of figs. 1-3) are provided in equations 26 and 27. Equation 26 has been fitted to observations in the weight interval of 1.3 pounds (600 grams) or less while equation 27 has been fitted to observations in the interval of weights greater than 1.3 pounds.

(26) $G = 1.0754C^{0.5425}\,S^{0.3838}$ (up to 1.3 pounds liveweight)

(27) $G = 0.7021C^{0.6163}\,S^{0.2944}$ (over 1.3 pounds liveweight)

The exponents in these equations are the elasticities of production, indicating the percentage

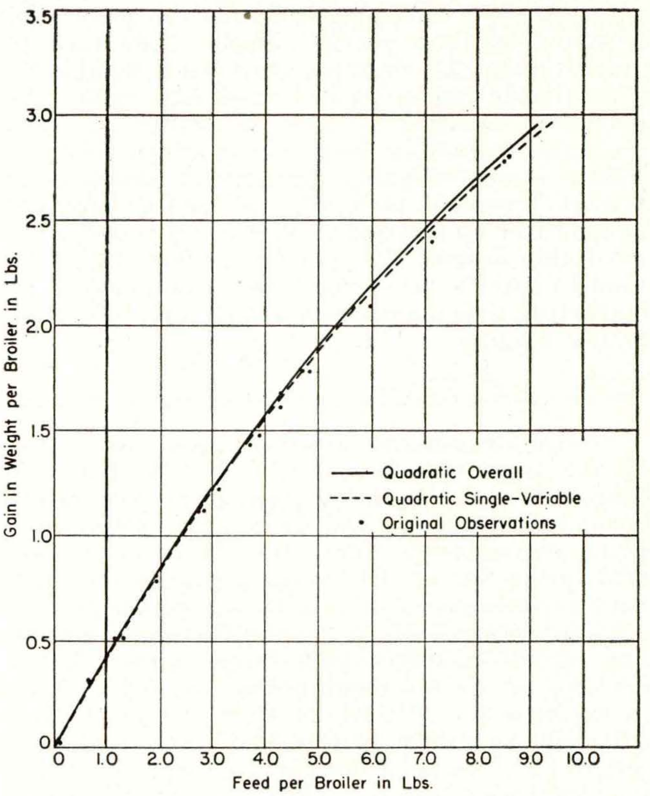


Fig. 6. Comparison of input-output curves for broilers on a 16-percent protein ration as predicted by quadratic over-all function 11 and quadratic single-variable function 14.

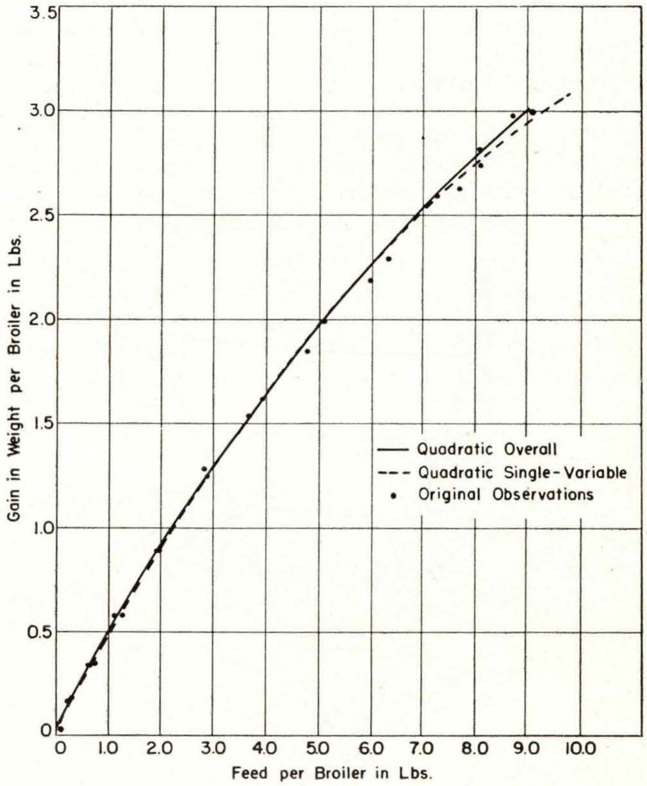


Fig. 7. Comparison of input-output curves for broilers on an 18-percent protein ration as predicted by quadratic over-all function 11 and quadratic single-variable function 15.

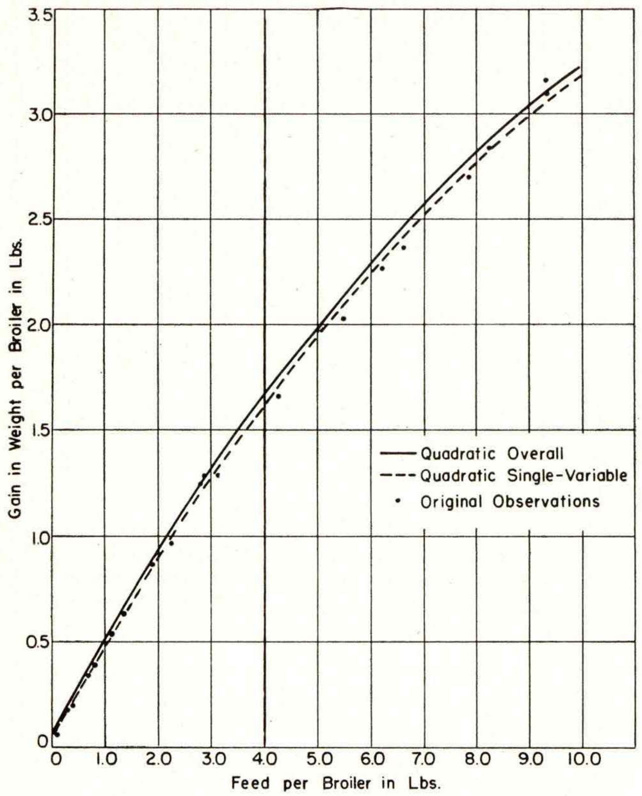


Fig. 8. Comparison of input-output curves for broilers on a 20-percent protein ration as predicted by quadratic over-all function 11 and quadratic single-variable function 16.

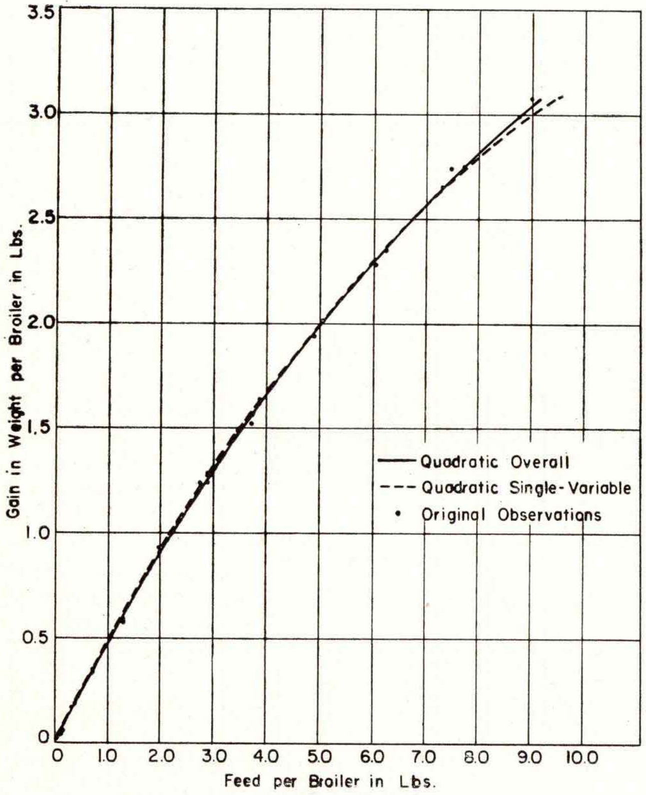


Fig. 9. Comparison of input-output curves for broilers on a 22-percent protein ration as predicted by quadratic over-all function 11 and quadratic single-variable function 17.

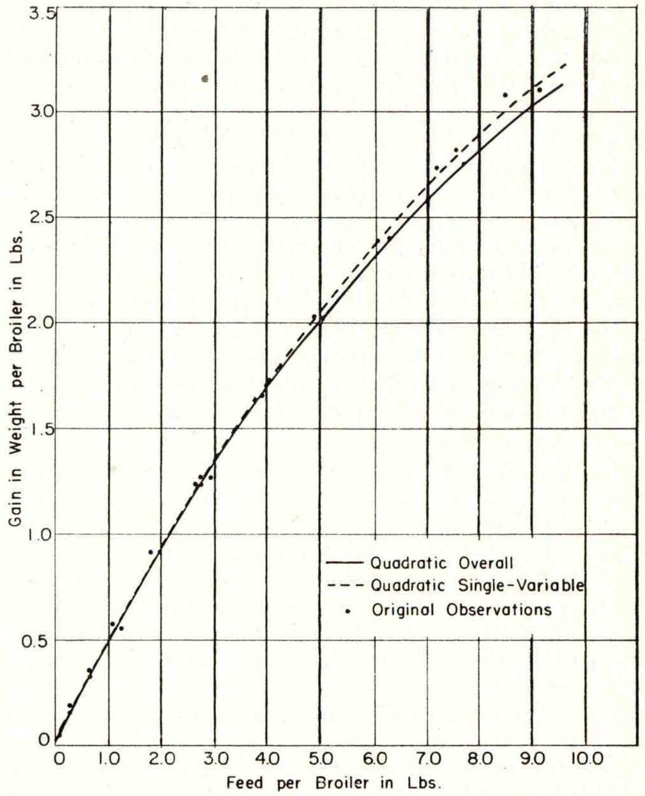


Fig. 10. Comparison of input-output curves for broilers on a 24-percent protein ration as predicted by quadratic over-all function 11 and quadratic single-variable function 18.

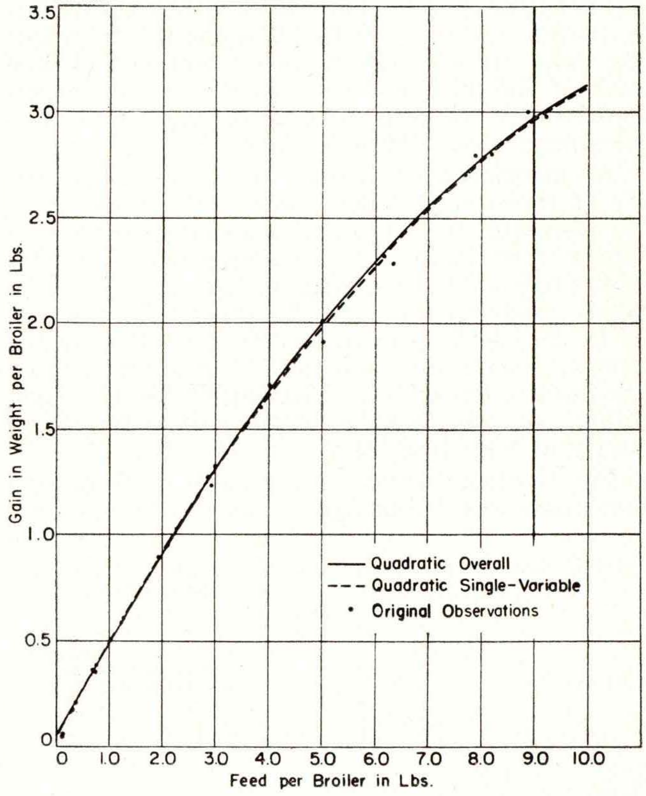


Fig. 11. Comparison of input-output curves for broilers on a 26-percent protein ration as predicted by quadratic over-all function 11 and quadratic single-variable function 19.

increase in gain associated with each 1-percent increase in consumption of the particular feed. Since each elasticity is less than 1.0, diminishing returns hold true for each feed. Also, since the sums of the elasticities are less than 1.0, decreasing returns hold true for both feeds increased in a fixed proportion. It is of interest to note that the elasticity of soybean oilmeal declines from 0.3838 for the first interval to 0.2944 for the second interval while the elasticity for corn increases from 0.5425 to 0.6463. These differences are in line with the nutritional requirements of the growing bird: Larger proportions of protein are needed for tissues, organs and muscles in early stages of growth while a larger proportion of carbohydrates is required as maturity and fattening are approached.

Statistics for the two interval functions are given in table 4. Even for a number of individual pens (rather than of pens x number of weighings) the statistics are significant at the 1-percent level of probability. As mentioned earlier, some pens were fed the same ration throughout the experiment. (These are the observations upon which the over-all functions and the single-variable functions are based.) Since some pens were switched to different rations at a liveweight level of 1.3 pounds, an analysis of variance was made for gains of birds in the second interval in relation to gains on rations fed in the first interval. It was found that gains in the second period did not differ significantly in terms of the ration fed in the first period. Gains in the second period did not appear to be associated with protein level in the first period. Hence, data for the "straight through" and "switched" pens were pooled, and each of the interval functions is based on observations for 30 pens averaging slightly over 6 weighings each (189 observations).

As mentioned previously, the constant elasticity of the over-all logarithmic function causes it to overestimate the gains associated with particular feed inputs as birds approach maturity. This tendency is illustrated in figs. 12, 13 and 14 where comparison is made for input-output curves of 18, 20 and 22 percent protein derived from the over-all logarithmic function 13 and the over-all quadratic function 11. The curves for the logarithmic over-all function fit the gain observations poorly at high feed inputs.

Input-output curves for the same three protein levels are provided in figs. 15, 16 and 17 when the

TABLE 4. MULTIPLE CORRELATION COEFFICIENTS AND VALUES OF t FOR INTERVAL LOGARITHMIC FUNCTIONS 27 AND 28.

Interval	R values	Value of t for regression coefficients in order given in equation	
		b_1	b_2
0.09-1.32 lbs. liveweight	0.9956*	33.10*	46.49*
Above 1.32 lbs. liveweight	0.9885*	38.09*	17.95*

* $p < 0.01$ for 186 or 27 degrees of freedom.

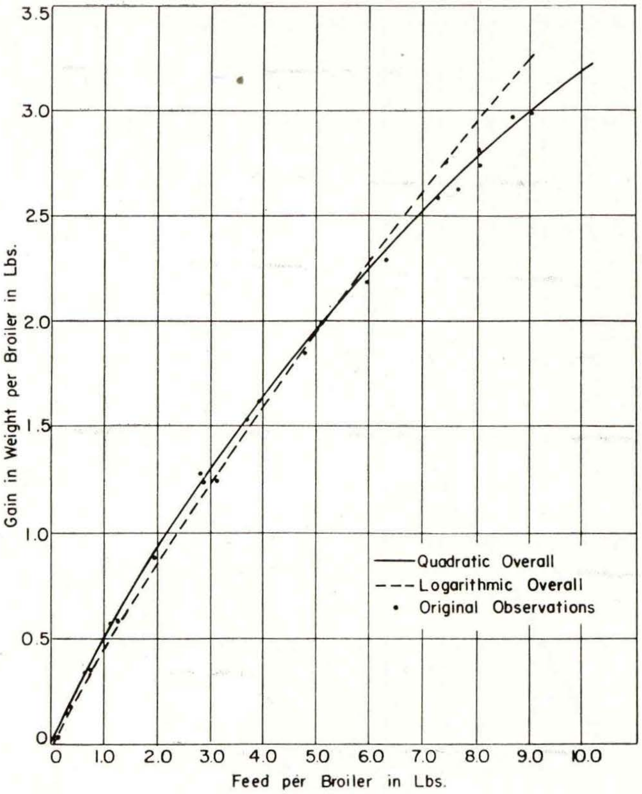


Fig. 12. Comparison of various over-all functions for estimating input-output curves for an 18-percent protein ration.

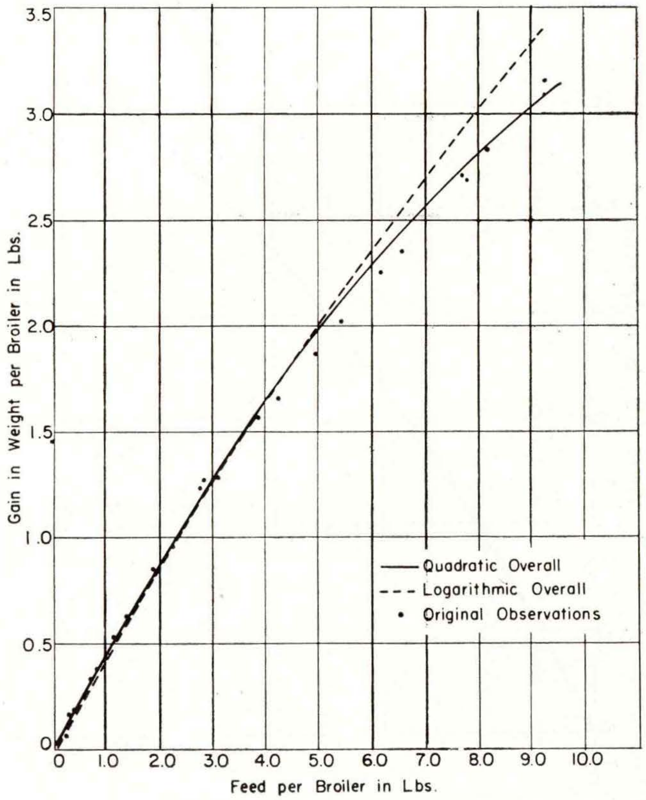


Fig. 13. Comparison of various over-all functions for estimating input-output curves for a 20-percent protein ration.

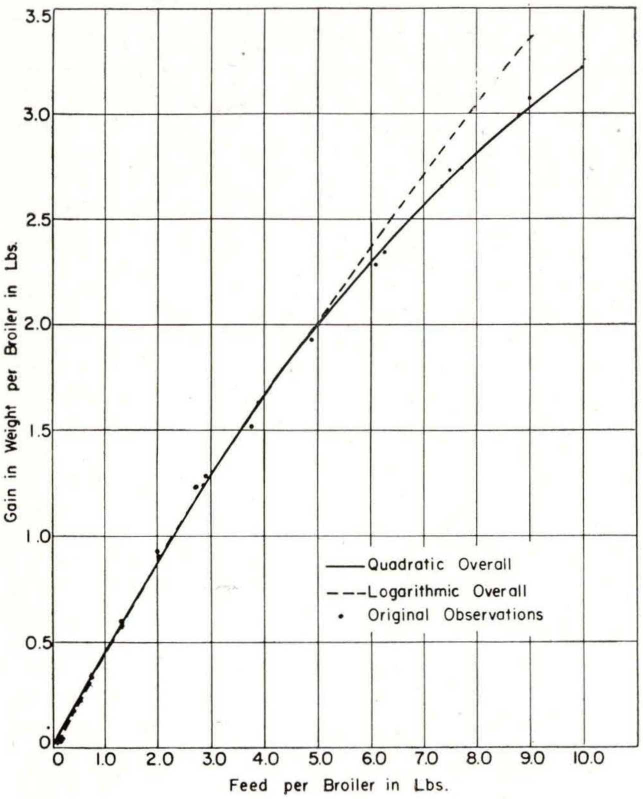


Fig. 14. Comparison of various over-all functions for estimating input-output curves for a 22-percent protein ration.

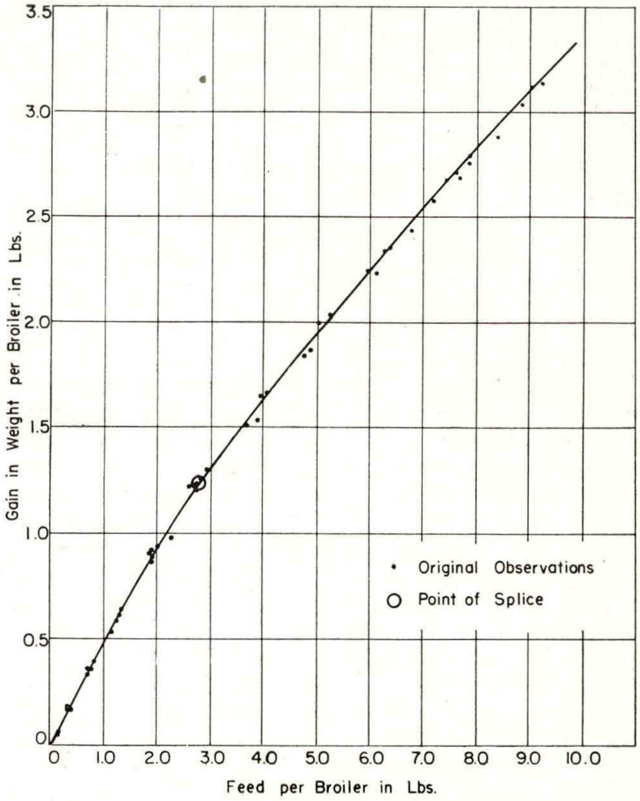


Fig. 16. "Spliced" input-output curve for broilers fed a 20-percent protein ration predicted from logarithmic interval functions 26 and 27.

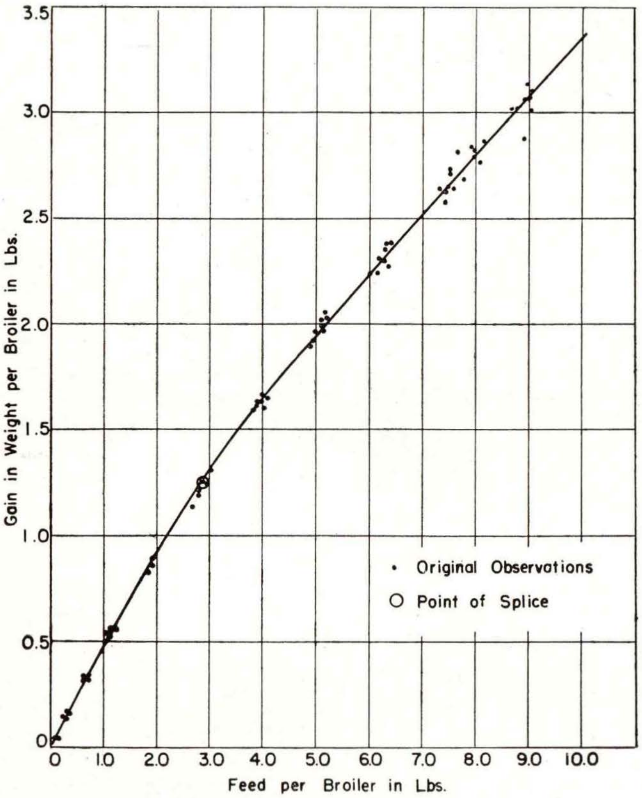


Fig. 15. "Spliced" input-output curve for broilers fed an 18-percent protein ration predicted from logarithmic interval functions 26 and 27.

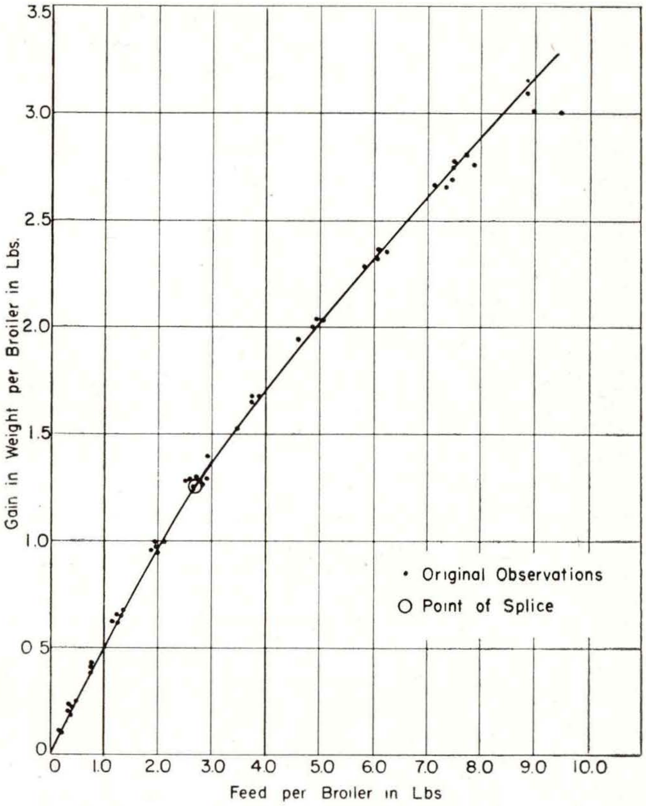


Fig. 17. "Spliced" input-output curve for broilers fed a 22-percent protein ration predicted from logarithmic interval functions 26 and 27.

estimates are made by "splicing" together the two interval logarithmic equations. The "spliced" input-output curves represent the portion for the second interval added (at the end of the first interval) to the portion for the first interval. Obviously, the problem of overestimation through use of logarithmic function has been lessened by splicing together the two interval functions; "average" least-cost rations can be estimated as practical measures for the two intervals without a problem of overestimating gains.

TOTAL AND MARGINAL GAINS

Production functions of the previous sections can be used to predict total weight associated with various levels of intake per bird of particular rations. They also can be used to predict the marginal productivity of (a) each unit of feed of a particular ration when corn and soybeans are held in fixed proportions to provide a constant percentage of protein or (b) each unit of one type of feed when the other feed is held fixed and the percent of protein changes. As indicated previously, the marginal product is the amount added to total gain for each small unit increase in feed intake per bird. Marginal products can be computed as derivatives from the production surface equation. Marginal product functions with corn and soybean oilmeal fixed are listed in equations 28 and 29, respectively, for the over-all quadratic production surface and in equations 30 and 31, respectively, for the over-all logarithmic production surface.

$$(28)\quad \frac{\partial G}{\partial C} = 0.4823 — 0.0364C - 0.0232S$$

$$(29)\quad \frac{\partial G}{\partial S} = 0.6415 - 0.0994S - 0.0232C$$

$$(30)\quad \frac{\partial G}{\partial C} = 0.5494C^{-0.4463}\,S^{0.3371}$$

$$(31)\quad \frac{\partial G}{\partial S} = 0.3345C^{0.5537}\,S^{-0.6729}$$

Table 5 includes total weights per bird and marginal gains per pound of feed when total feed input per bird is at specified levels for various rations. Diminishing productivity of feed is indicated in total weights; the amount added to total weight for each added pound of feed declines with total feed inputs. The maximum weight attained with 9 pounds of feed is with a 22-percent protein ration. Rations with a greater percentage include relatively too much protein for greatest nutritional efficiency at heavier weights; rations with a smaller percentage include relatively too much carbohydrate for greatest efficiency at low weights. If extrapolations are used, the 20-percent ration gives a maximum weight for 11 pounds of feed. Of course, the ration which gives maximum weight for a given total input of feed need not be the most profitable ration. The value of the greater gain from the particular ration must be compared with the prices of the two feeds and the quantity of each used in the ration.

The marginal gains per combined pound of feed for different rations again reflect the relative nutritive importance of the two feeds at different bird weights and total feed inputs. Up to a total feed input of 3 pounds, the marginal productivity of feed is greatest for a 26-percent protein ration; between 3 and 5 pounds of total feed input, marginal products are greatest for a 22-percent ration; between 6 and 8 total pounds of feed, a 20-percent ration has the largest marginal products while for total feed inputs of 9 or more pounds, the 16-percent ration has the greatest marginal productivity. These shifts in marginal productivity as feed inputs become greater parallel the total weights shown in the left-hand portion of the table. The fact that marginal gains per pound of feed are greatest for (a) higher protein rations at light weights and (b) lower protein rations at low weights, is illustrated graphically in fig. 18.

GAIN ISOQUANTS

The production functions of equations 11 and 13 are used to derive functions describing the va-

TABLE 5. TOTAL WEIGHT PER BIRD AND MARGINAL GAINS FOR VARIOUS LEVELS OF FEED INPUTS PER BIRD, WITH FEED IN FIXED PROPORTIONS FOR SPECIFIED RATIONS (OVER-ALL QUADRATIC FUNCTION 11).

Pounds of feed	Total weight (pounds)						Marginal gains (pounds)*					
	Percent protein levels						Percent protein levels					
	16	18	20	22	24	26	16	18	20	22	24	26
1	0.547	0.556	0.562	0.569	0.576	0.582	0.415	0.421	0.427	0.434	0.440	0.445
2	0.947	0.964	0.967	0.989	0.999	0.010	0.392	0.395	0.400	0.405	0.409	0.412
3	1.322	1.346	1.363	1.380	1.394	1.406	0.368	0.369	0.374	0.377	0.379	0.379
4	1.671	1.702	1.723	1.742	1.757	1.768	0.344	0.343	0.347	0.349	0.348	0.346
5	1.994	2.032	2.056	2.077	2.091	2.097	0.321	0.317	0.320	0.320	0.318	0.313
6	2.292	2.336	2.363	2.383	2.393	2.394	0.297	0.291	0.293	0.292	0.288	0.280
7	2.565	2.614	2.642	2.660	2.666	2.657	0.273	0.265	0.266	0.264	0.257	0.247
8	2.811	2.866	2.894	2.910	2.907	2.888	0.250	0.239	0.239	0.235	0.227	0.214
9	3.032	3.091	3.120	3.131	3.120	3.085	0.226	0.213	0.212	0.207	0.196	0.181
10	3.228	3.291	3.319	3.323	3.300	3.250	0.203	0.186	0.185	0.178	0.166	0.148
11	3.397	3.464	3.491	3.487	3.451	3.382	0.179	0.160	0.158	0.150	0.136	0.115

* Marginal gains are computed as a derivative of the over-all quadratic function and represent the marginal physical products at the feed quantities shown in the first column.

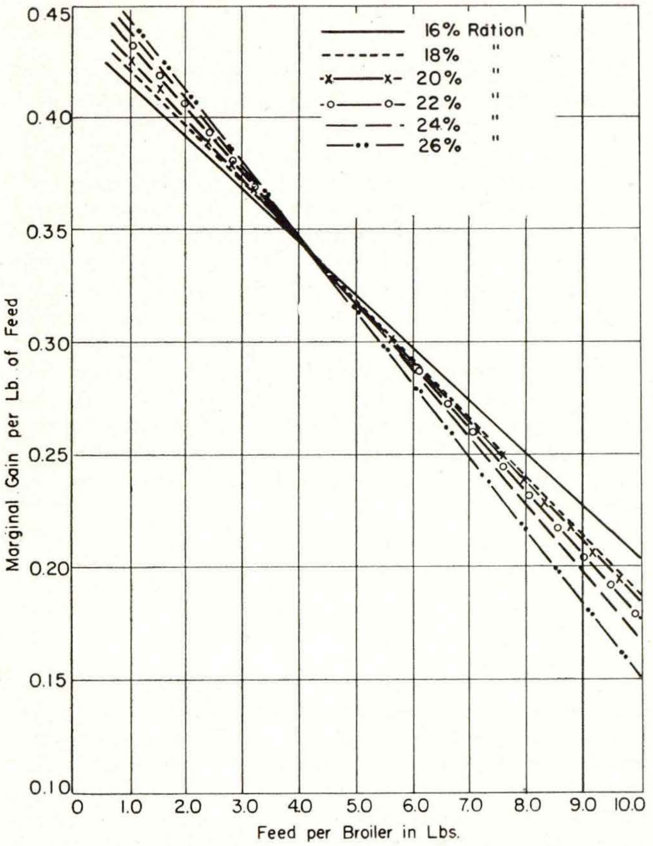


Fig. 18. Marginal (additional) gains per pound of feed for broilers on various protein rations. Derived from over-all quadratic function 11.

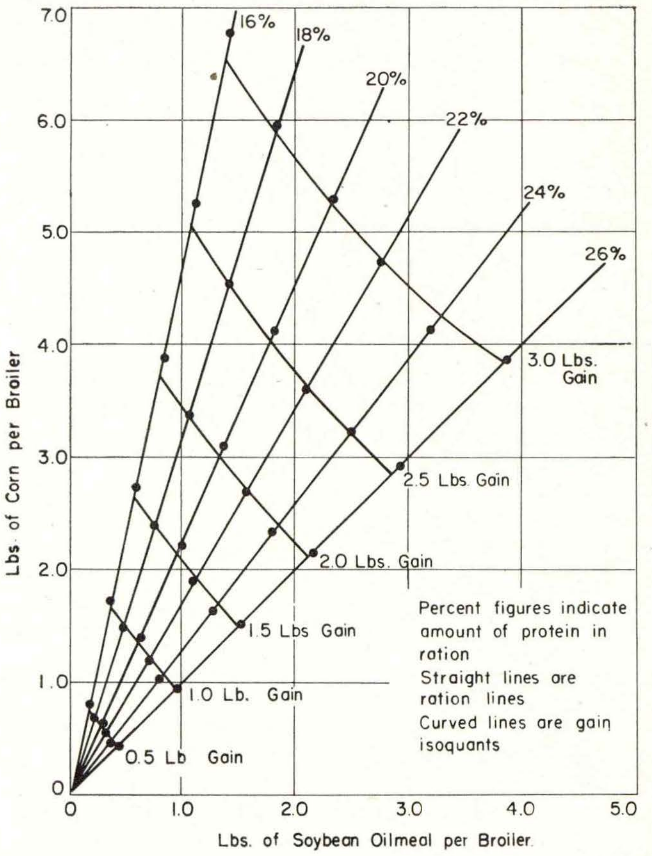


Fig. 19. Gain isoquants predicted from over-all quadratic function 11. Dots show feed quantities required for same gains when predictions are based on quadratic single-variable equations for particular rations.

rious combinations of the two feeds which will produce a given level of gain. These iso-gain relationships are expressed in equations 32 and 33 for the over-all quadratic and logarithmic functions, respectively.

$$(32)\quad C = 13.1959 - 0.6350S \pm 27.3581\sqrt{0.2351 + 0.0245S - 0.00310S^2 - 0.0731G}$$

$$(33)\quad C = \left(\frac{G}{0.9922S^{0.3371}}\right)^{1.8032}$$

The gain isoquants derived from equations 32 and 33 are presented in figs. 19 and 20. The contours from both equations for a given gain fall at about the same location in the feed plane for lower gains. However, for greater gains, the location of isoquants for the logarithmic function fall higher in the feed plane. (It was mentioned previously that the over-all logarithmic function tends to overestimate gains for large feed inputs or weights per bird.) The figuration of the isoquants is most accurate for the quadratic function. However, since the slope of the isoquants along a line of given percentage protein is the same for the logarithmic function, it can serve in the practical manner mentioned earlier (i. e., it can be used to suggest the "average" least-cost over the entire growth period, although it is not best for indicating the least-cost ration for a particular increment of gain). The dots in figs. 19 and 20 indicate the feed combinations and quantities necessary to give the specified gains when predictions are provided by the single-variable equations representing particular rations.

Isoquants for the lower and upper interval functions (logarithmic-type) are given in equations 34 and 35, respectively. It should be remembered that within each gain interval each

$$(34)\quad C = \left(\frac{G}{1.0754S^{0.3838}}\right)^{1.8403}$$

$$(35)\quad C = \left(\frac{G}{0.7021S^{0.2944}}\right)^{1.5473}$$

member of the family of gain isoquants will have the same slope along a fixed ration line for predictions from equations 34 and 35. Hence, the predictions provide the basis for the practical recommendation of the "average" least-cost ration within the particular interval.

SUBSTITUTION RATES FOR CORN AND SOYBEAN OILMEAL

Prediction of the substitution rates of soybean oilmeal for corn along the isoquants is necessary in specifying least-cost feed combinations for par-

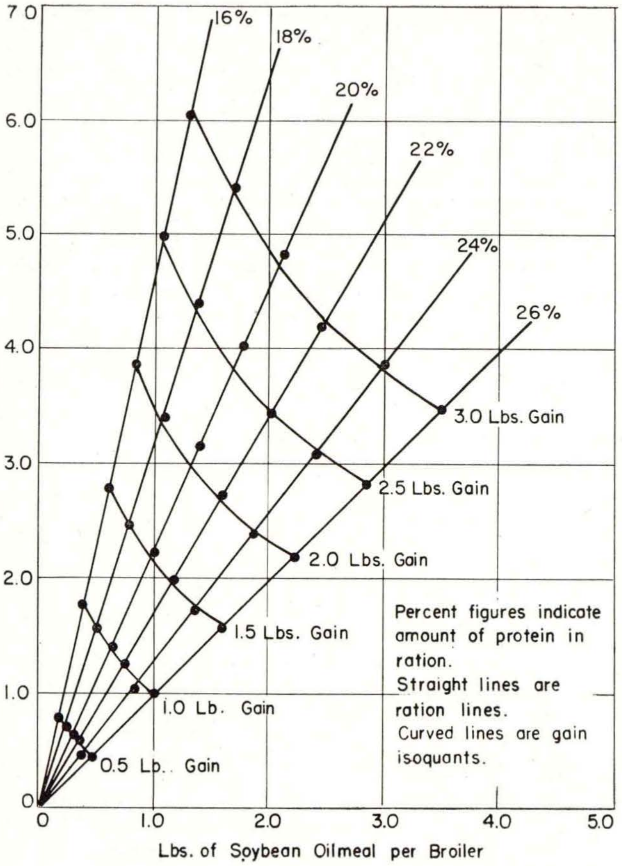


Fig. 20. Gain isoquants predicted from logarithmic function 13. Dots show feed quantities required for same gains when predictions are based on logarithmic single-variable equations for particular rations.

ticular gains (i. e., to specify the optimum percentage of protein in the ration). The marginal substitution rate is the slope of the iso-product curve at a particular point or for a particular feed combination. The equation for the marginal rates of substitution of soybean oilmeal for corn from the over-all logarithmic function is 36.

$$(36) \quad \frac{dC}{dS} = 0.6088 \frac{C}{S}$$

This equation expresses the amount of corn replaced by the addition of one unit of soybean oilmeal for a particular level of gain when the two feeds are combined in the proportions indicated. This substitution rate changes along the iso-product contours but remains the same along ration lines when the logarithmic function is used for estimation. For producers who want to use only one ration during the production period, the substitution rates from the above equation would provide the "average" basis of ration selection. Where they desire to feed two rations during the production period, equations 37 and 38 can be used to express "average" substitution rates within the lower and upper interval, respectively. (They are based, respectively, on equations 34 and 35.)

$$(37) \quad \frac{dC}{dS} = 0.7075 \frac{C}{S}$$

$$(38) \quad \frac{dC}{dS} = 0.4555 \frac{C}{S}$$

Data in table 6 derived from equations 34 and 35 show the various combinations of the two feeds which will produce 1 pound of gain at broiler weights of 1.32 and 3.09 pounds liveweight. As mentioned previously, the slope of the gain isoquant at any particular point denotes the marginal rate of substitution of one feed for another. Columns 4 and 7 provide the substitution quantities in tabular form and are derived from equations 37 and 38. Since the data in table 6 are for logarithmic functions, the substitution ratio will not change between isoquants within a gain interval (i. e., for other broiler weights) when the feeds are combined in a fixed proportion to result in a given percentage of protein in the ration. In other words, 1 pound of soybean oilmeal substitutes for 1.62 pounds of corn when 0.58 pound of soybean oilmeal, a total of 1.91 pounds,

TABLE 6. COMBINATIONS OF CORN AND SOYBEAN OILMEAL FOR PRODUCING A POUND OF GAIN AND MARGINAL SUBSTITUTION RATES FOR BROILERS OF 1.32 AND 3.09 POUNDS LIVEWEIGHT. (ESTIMATES BASED ON INTERVAL LOGARITHMIC FUNCTIONS 26 AND 27.)

Percent protein in ration	Lbs. feed to produce 1 lb. of gain*		Marginal rate of substitution of soybean oilmeal for corn‡	Lbs. feed to produce 1 lb. of gain†		Marginal rate of substitution of soybean oilmeal for corn‡
	Corn	Soybean oilmeal		Corn	Soybean oilmeal	
16	1.790	0.378	3.349	2.456	0.519	2.749
17	1.609	0.418	2.720	2.301	0.598	2.233
18	1.521	0.476	2.260	2.171	0.680	1.856
19	1.417	0.527	1.903	2.087	0.775	1.562
20	1.326	0.578	1.622	1.957	0.854	1.331
21	1.285	0.631	1.396	1.867	0.946	1.146
22	1.174	0.686	1.211	1.786	1.044	0.994
23	1.109	0.744	1.054	1.710	1.148	0.866
24	1.049	0.805	0.922	1.650	1.259	0.756
25	0.994	0.871	0.807	1.576	1.381	0.662
26	0.940	0.940	0.708	1.509	1.510	0.581

* Derived from equation 34, lower weight interval.

† Derived from equation 35, upper weight interval.

‡ The marginal rate of substitution of soybean oilmeal for corn is obtained from the appropriate Cobb-Douglas interval functions. Marginal rate of substitution or dC/dP refers to the pounds of corn replaced by a pound of soybean oilmeal at the indicated weights. Rates for 1.32-pound weights are derivatives from equation 37 while those for 3.09-pound weights are from equation 38.

is combined with 1.33 pounds of corn into a 20-percent protein ration for a pound of gain on birds weighing 1.32 pounds; it will substitute at the same rate for corn when feeds are combined in the same proportions for other weights up to 1.32 pounds. However, it will require less of the feeds in this fixed proportion to produce a pound of gain when broilers are at weights lighter than 1.32 pounds; it will take more at heavier weights. This difference in feed requirements per pound of gain, while feeds are held in fixed proportions to give a constant substitution rate, comes about because of a decline in the rate at which feed is transformed into gain. Comparisons of feed quantities to produce a pound of gain at weights of 1.32 pounds and 3.09 pounds illustrate this fact. A pound of gain for birds at the latter weight, with a 20-percent protein ration, requires 1.96 pounds of corn and 0.85 pound of soybean oilmeal, a total of 2.81 pounds.

Diminishing marginal rates of substitution of soybean oilmeal for corn are evidenced at each weight. As additional soybean oilmeal is added in the ration, for each weight level, each pound replaces less and less corn. For 1.32-pound broilers on a 16-percent protein ration, 1 pound of soybean oilmeal replaces 3.349 pounds of corn; with an 18-percent ration, 1 pound of soybean oilmeal replaces 2.260 pounds of corn and with a 22-percent ration, the rate is only 1.211. A similar decline holds true for broilers at the heavier weight, except that the substitution rates decline more rapidly.

Substitution rates for corresponding rations are lower for 3.09-pound than for 1.32-pound broilers; a pound of soybean oilmeal replaces less corn for heavier birds than for light birds when fed the same ration. For 1.32-pound broilers on a 20-percent protein ration, a pound of soybean oilmeal replaces 1.62 pounds of corn, but it replaces only 1.33 pounds of corn for 3.09-pound broilers. This relationship conforms to the nutritional needs of broilers at different weights: At low weights, protein is relatively more important for growth and corn is a less efficient substitute for soybean oilmeal than at heavier weights where maturity is approached.

One-pound gain isoquants for broilers of 1.32- and 3.09-pound liveweights based on the data of table 6 are shown in fig. 21. The isoquants in this figure are to be interpreted differently than the conventional isoquant maps such as shown in figs. 19 and 20. The lower and upper curves in fig. 21 show the combinations of corn and soybean oilmeal required for 1 pound of gain when broilers have liveweights of 1.32 and 3.09 pounds, respectively. Conventional isoquant maps show accumulated gains (or weights) and feed inputs rather than feed inputs for a pound of gain at a specified weight. The gain isoquants shown in fig. 21 illustrate graphically the preceding discussion on diminishing substitution rates between rations and between weights. The slopes of the curves decline as the ration contains a greater proportion of soybean oilmeal, indicating diminishing substitution rates of soybean oilmeal for corn. Conversely, as greater amounts of corn are used, the slopes of the curves become increasingly steep indicating decreasing substitution rates of corn for protein. The fact that along a fixed ration line the 1-pound isoquant for a 3.09-pound weight has less slope than for a 1.32-pound weight indicates that soybean oilmeal substitutes at a lower rate at the heavier weight.

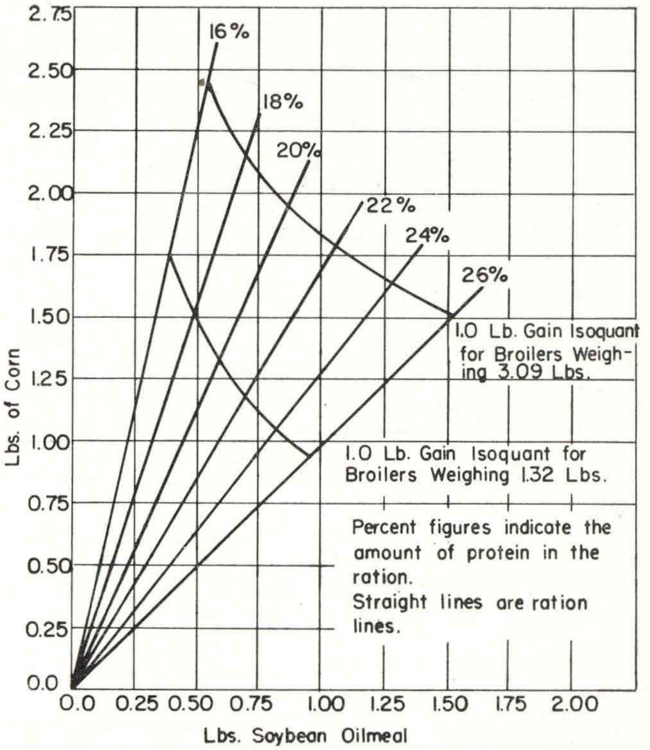


Fig. 21. One-pound gain isoquants for broilers at 1.32 and 3.09 pounds liveweight as determined from logarithmic interval functions 34 and 35, respectively.

LEAST-COST RATIONS

Quantities such as those derived in the preceding section provide the basis for specifying the optimum combinations of corn and protein (soybean oilmeal in this study) for broilers. The least-cost rations can be determined by equating the marginal rate of substitution with the inverse price ratios. The isoclines, or points of equal substitution rates, lie on a straight line passing through the origin for a logarithmic function. These lines are also ration lines for the particular type of function. Thus, where the need is to predict one ration which "averages" least-cost over the entire feeding period, equating substitution rates from the over-all logarithmic function with the price ratio will provide such a ration. Where the need is to change rations between two growth periods, equating the price ratio with substitution rates from interval equation 37 provides the average least-cost ration for the first 6-7 weeks; equating the price ratio with equation 38 provides the least-cost average ration in the latter part of the feeding period. As mentioned earlier, these procedures are used in this study as practical measures since most broiler producers feed the same ration throughout, or change it only once during the production period. While the quadratic function provides "biologically more accurate" isoclines, it is less practical in the sense that the isoclines do not indicate average rations to be fed over an interval of weight gains.

Data in table 7 provide substitution rates which can be used to indicate least-cost rations as averages over two weight intervals or over the entire production period. For example, if the price of soybean oilmeal is 3 cents per pound and the price of corn is 2 cents per pound, the price ratio is 3/2 or 1.5; a 20.5-percent protein ration gives the least-cost ration as an average over the first weight interval; for this price ratio, the least-cost ration falls between 17.5 and 18.0 percent protein for the second weight interval. If the same ration were to be fed over the entire production period, the best "average" ration is 19.5 percent protein. These are the rations where the marginal rates of substitution of soybean oilmeal for corn most nearly approximate the soybean oilmeal/corn price ratio of 1.5. If the price of soybean oilmeal increases to 4 cents, with corn remaining at 2 cents per pound, the price ratio becomes 2.0. An 18.5-percent protein ration then averages least-cost for the first weight interval. This ration would be fed for a total gain of 1.23 pounds (1.32 pounds liveweight), and then a ration of 16.5 percent protein would be fed through the second interval.

It is of interest to note that the substitution rates as averages for the over-all production period (based on the over-all logarithmic function) fall between those for the two intervals. For example, with a 20-percent protein ration, the rate of substitution of soybean oilmeal for corn is 1.622 for the first interval, 1.396 for the over-all period or function and 1.044 for the second interval. In other words, if the ration which averages least-cost over the entire production period is fed, it includes less protein for the first interval and more protein for the second interval than would be fed if separate rations averaging least-cost over the two weight ranges were used. Hence, the cost of gains to marketing would be greater for a single ration than for two different rations over the growth period. This difference must be compared to the equipment, labor and general practicality of feeding one ration throughout the period, or of shifting the ration to conform with changes in substitution rates with broiler growth.

Tables 8, 9 and 10 provide figures showing the least-cost rations, respectively, (a) throughout the production period, (b) for the first interval of growth and (c) for the second interval of growth when logarithmic functions are used as the basis for predicting "average" rations over the particular periods. Hence, with a "low" price for corn at 1.7 cents and a "high" price for soybean oilmeal at 6 cents per pound, the least-cost ration to be fed over the entire period includes 15.0 percent protein. With corn at 2 cents and soybean oilmeal at 4 cents, the least-cost ration in the first interval is 18.5 percent protein; the least-cost ration for the second interval is 16.5

TABLE 7. MARGINAL RATES OF SUBSTITUTION OF SOYBEAN OILMEAL FOR CORN FOR SPECIFIED GAINS AS ESTIMATED BY THE LOGARITHMIC OVER-ALL AND INTERVAL FUNCTIONS.

	1.23 lbs. gain (1.32 lbs. liveweight)		3.0 lbs. gain (3.09 lbs. liveweight)	
Percent protein in ration	Substitution rates for single ration over entire production period*	Substitution rates for first interval†	Substitution rates for single ration over entire production period	Substitution rates for second interval‡
15.0	3.666	4.259	3.666	2.742
15.5	3.234	3.758	3.234	2.420
16.0	2.882	3.349	2.881	2.156
16.5	2.589	3.008	2.589	1.937
17.0	2.341	2.720	2.341	1.751
17.5	2.129	2.474	2.130	1.593
18.0	1.945	2.261	1.945	1.455
18.5	1.782	2.071	1.782	1.333
19.0	1.638	1.903	1.638	1.225
19.5	1.510	1.755	1.510	1.130
20.0	1.396	1.622	1.396	1.044
20.5	1.294	1.504	1.294	0.968
21.0	1.202	1.397	1.202	0.899
21.5	1.118	1.299	1.118	0.837
22.0	1.042	1.210	1.042	0.779
22.5	0.972	1.129	0.962	0.727
23.0	0.908	1.054	0.908	0.679
23.5	0.848	0.986	0.848	0.634
24.0	0.793	0.922	0.793	0.593
24.5	0.742	0.862	0.742	0.555
25.0	0.695	0.807	0.695	0.520
25.5	0.650	0.756	0.650	0.487
26.0	0.609	0.708	0.609	0.456

* Derivatives for over-all logarithmic function covering both weight intervals. Substitution rates do not change in the different weight intervals when the over-all function is used (see earlier discussion on logic of estimation).

† Derivatives for logarithmic function in first weight interval.

‡ Derivatives for logarithmic function in second weight interval.

TABLE 8. RATIONS PROVIDING LEAST-COST COMBINATIONS OF CORN AND SOYBEAN OILMEAL WITH DIFFERENT FEED PRICES. (FOR BROILERS FED A FIXED PERCENTAGE OF PROTEIN THROUGHOUT THE FEEDING PERIOD. LOGARITHMIC OVER-ALL FUNCTION 13 USED AS A BASIS OF FEED COMBINATIONS.)

Price of corn in cents per pound*	Price of soybean oilmeal in cents per pound†							
	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5
1.6	18.0	17.5	16.5	16.0	15.5	15.5	—	—
1.8	19.0	18.0	17.5	16.5	16.0	15.5	15.5	15.0
2.0	19.5	18.5	18.0	17.0	16.5	16.0	16.0	15.5
2.2	20.0	19.0	18.5	17.5	17.0	16.5	16.5	16.0
2.4	20.5	19.5	19.0	18.0	17.5	17.0	16.5	16.5
2.6	21.0	20.0	19.5	18.5	18.0	17.5	17.0	16.5
2.8	22.0	20.5	20.0	19.0	18.5	18.0	17.5	17.0
3.0	22.5	21.0	20.5	19.5	19.0	18.5	18.0	17.5
3.2	22.5	21.5	20.5	20.0	19.5	18.5	18.0	18.0
3.4	23.0	22.0	21.5	20.5	19.5	19.0	18.5	18.0
3.6	23.5	22.5	21.5	20.5	20.0	19.5	19.0	18.5
3.8	24.0	23.0	22.0	21.0	20.5	20.0	19.0	18.5
4.0	24.5	23.5	22.5	21.5	20.5	20.0	19.5	19.0

* The price for corn includes the cost of grinding, mixing and a proportionate share of the other feed ingredients included in the feed mixture other than soybean oilmeal.

† The price of soybean oilmeal includes a charge for mixing along with a proportionate share of the other feed ingredients included in the feed mixture other than corn.

TABLE 9. RATIONS PROVIDING LEAST-COST COMBINATIONS OF CORN AND SOYBEAN OILMEAL WITH DIFFERENT FEED PRICES. (FOR BROILERS FED A FIXED PERCENTAGE OF PROTEIN FROM 0.09 TO 1.32 POUNDS LIVEWEIGHT. LOGARITHMIC INTERVAL FUNCTION 26 USED AS A BASIS OF FEED COMBINATIONS.)

Price of corn in cents per pound*	Price of soybean oilmeal in cents per pound†							
	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5
1.6	19.0	18.0	17.5	17.0	16.5	16.0	15.5	15.0
1.8	20.0	19.0	18.0	17.5	17.0	16.5	16.0	15.5
2.0	20.5	19.5	18.5	18.0	17.5	17.0	16.5	16.0
2.2	21.0	20.0	19.5	18.5	18.0	17.5	17.0	16.5
2.4	22.0	20.5	20.0	19.0	18.5	18.0	17.5	17.0
2.6	22.5	21.0	20.5	19.5	19.0	18.5	18.0	17.5
2.8	23.0	22.0	21.0	20.0	19.5	19.0	18.5	18.0
3.0	23.5	22.5	21.5	20.5	20.0	19.0	18.5	18.0
3.2	24.0	22.5	21.5	21.0	20.0	19.5	19.0	18.5
3.4	24.5	23.0	22.0	21.5	20.5	20.0	19.5	19.0
3.6	25.0	23.5	22.5	22.0	21.0	20.5	20.0	19.5
3.8	25.0	24.0	23.0	22.0	21.5	21.0	20.0	19.5
4.0	25.5	24.5	23.5	22.5	22.0	21.0	20.5	20.0

* The price for corn includes the cost of grinding, mixing and a proportionate share of the other feed ingredients included in the feed mixture other than soybean oilmeal.

† The price of soybean oilmeal includes a charge for mixing along with a proportionate share of the other feed ingredients included in the feed mixture other than corn.

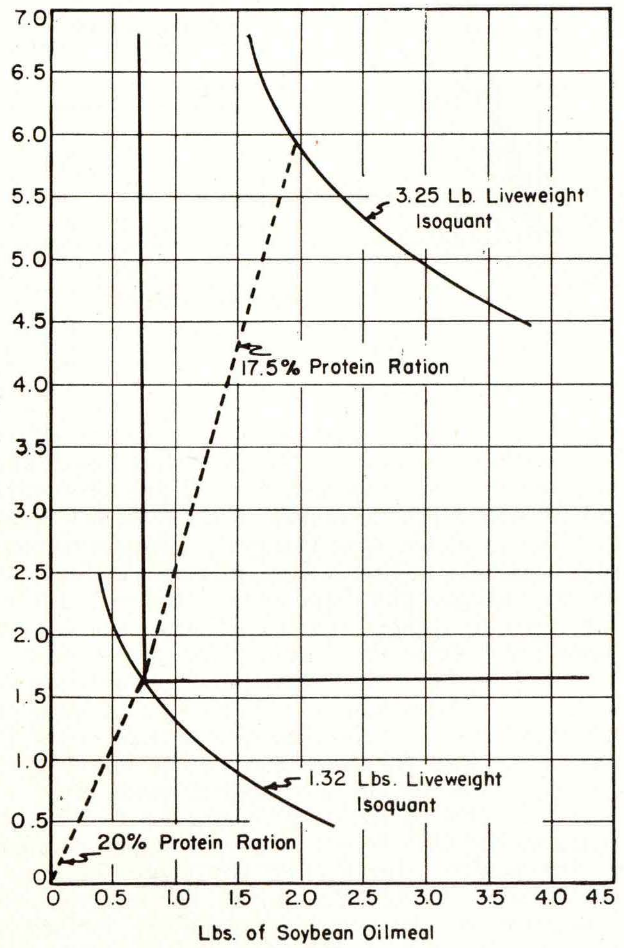


Fig. 22. Least-cost rations for two weight intervals based on logarithmic functions 34 and 35 with a soybean oilmeal/corn price ratio of 1.6.

percent protein. Hence, tables 8, 9 and 10 can be used to determine the percentage of protein in the ration which gives lowest feed costs per pound of gain for any of the combinations of the prices shown. The rations indicated in the cells of the table are those where the marginal rate of substitution of soybean oilmeal for corn is equal to the price ratio obtained by dividing the soybean oilmeal price at the top by the corn price in the left-hand column of the table. Table 11 indicates

TABLE 10. RATIONS PROVIDING LEAST-COST COMBINATIONS OF CORN AND SOYBEAN OILMEAL WITH DIFFERENT FEED PRICES. (FOR BROILERS FED A FIXED PERCENTAGE OF PROTEIN FOR ALL WEIGHTS ABOVE 1.32 POUNDS. LOGARITHMIC INTERVAL FUNCTION 27 USED AS A BASIS OF FEED COMBINATIONS.)

Price of corn in cents per pound*	Price of soybean oilmeal in cents per pound†							
	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5
1.6	16.5	16.0	15.5	—	—	—	—	—
1.8	17.0	16.5	16.0	15.5	—	—	—	—
2.0	18.0	17.0	16.5	16.0	15.5	—	—	—
2.2	18.5	17.5	17.0	16.0	16.0	15.5	15.0	—
2.4	19.0	18.0	17.5	16.5	16.0	15.5	15.5	15.0
2.6	19.5	18.5	17.5	17.0	16.5	16.0	15.5	15.5
2.8	20.0	19.0	18.0	17.5	17.0	16.5	16.0	15.5
3.0	20.5	19.5	18.5	18.0	17.5	17.0	16.5	16.0
3.2	20.5	19.5	19.0	18.0	17.5	17.0	16.5	16.5
3.4	21.0	20.0	19.5	18.5	18.0	17.5	17.0	16.5
3.6	21.5	20.5	19.5	19.0	18.5	17.5	17.5	17.0
3.8	22.0	21.0	20.0	19.0	18.5	18.0	17.5	17.0
4.0	22.5	21.0	20.5	19.5	19.0	18.5	18.0	17.5

* The price for corn includes the cost of grinding, mixing and a proportionate share of the other feed ingredients included in the feed mixture other than soybean oilmeal.

† The price of soybean oilmeal includes a charge for mixing along with a proportionate share of the other feed ingredients included in the feed mixture other than corn.

TABLE 11. ESTIMATED CORN AND SOYBEAN OILMEAL REQUIREMENTS PER 100 POUNDS OF FEED FOR VARIOUS PROTEIN RATIONS.*

Percent protein in ration	Corn (pounds)	Soybean oilmeal (pounds)
15.0	73.75	12.25
15.5	72.38	13.62
16.0	71.00	15.00
16.5	69.62	16.38
17.0	68.25	17.75
17.5	66.88	19.12
18.0	65.50	20.50
18.5	64.02	21.88
19.0	62.55	23.25
19.5	61.08	24.62
20.0	59.60	26.00
20.5	58.18	27.38
21.0	56.75	28.75
21.5	55.32	30.12
22.0	53.90	31.50
22.5	52.48	32.88
23.0	51.05	34.25
23.5	49.62	35.62
24.0	48.20	37.00
24.5	46.78	38.38
25.0	45.35	39.75
25.5	43.92	41.12
26.0	42.50	42.50
26.5	41.08	43.80
27.0	39.65	45.25

* These estimates are based on (a) ground yellow corn containing 8.4 percent crude protein and soybean oilmeal containing 45 percent crude protein and (b) a constant amount of other feeds consisting of 5 lbs. wheat middlings, 2.5 lbs. alfalfa meal, 2.5 lbs. fishmeal, 2.0 lbs. bonemeal, 0.5 lb. oyster shells, 0.5 lb. salt and 1.0 lb. of a premix including vitamins and antibiotics.

the amount of corn and soybean oilmeal needed for 100 pounds of a feed mixture containing the indicated percentages of protein.

Graphic illustrations of the average least-cost rations for two different price ratios of soybean oilmeal and corn are given in figs. 22 and 23 for the two weight intervals. Under a situation with a price ratio of soybean oilmeal to corn of 1.6, (e. g., $4.00 and $2.50 per hundred pounds, respectively, for the two feeds), the least-cost rations for the two periods are as shown in fig. 22. A 20-percent protein ration provides the "average" least-cost ration until a weight of about 1.32 pounds is attained; then a 17.5-percent ration provides the "average" least-cost ration for the remainder of the feeding period. An increase in the price ratio to 1.875—which could be caused by (a) an increase in soybean oilmeal prices, (b) a decrease in corn price or (c) a combination of (a) and (b)—would cause a new set of rations to become lowest in cost, as shown in fig. 23. The "average" least-cost rations now include 19.0- and 16.5-percent protein levels for the first and second periods. The time required for these gains may be of importance to the broiler producer. Time considerations are discussed in a succeeding section. Figure 24 shows the nature of *ration paths* over the two intervals when one change is made in feed combinations over the production period. The *break* in slopes of the isoclines comes at the end of the first interval. The corresponding isocline for the second interval is "spliced" on to indicate the "average" least-cost rations for the two weight ranges. Hence, the isocline labeled 2.0 would be followed for prices such as 4 cents for soybean oilmeal and 2 cents for corn; 3 cents for soybean oilmeal and 1.5 cents for corn; 2.5 cents for soybean oilmeal and 1.25 cents for corn, etc. The 2.5 isocline would be followed for all price combinations giving this value, etc.

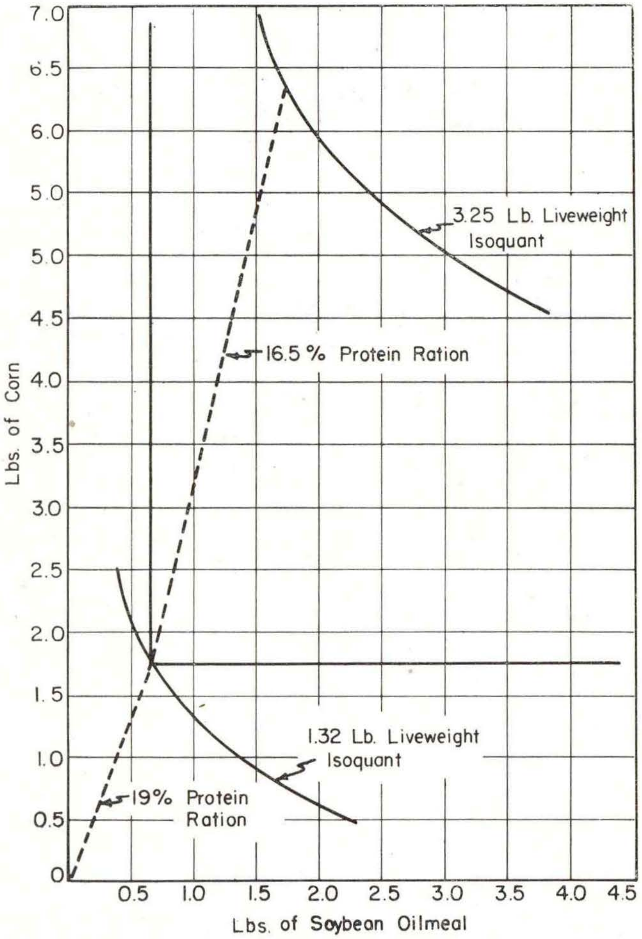


Fig. 23. Least-cost rations for two weight intervals based on logarithmic functions 34 and 35 with a soybean oilmeal/corn price ratio of 1.875.

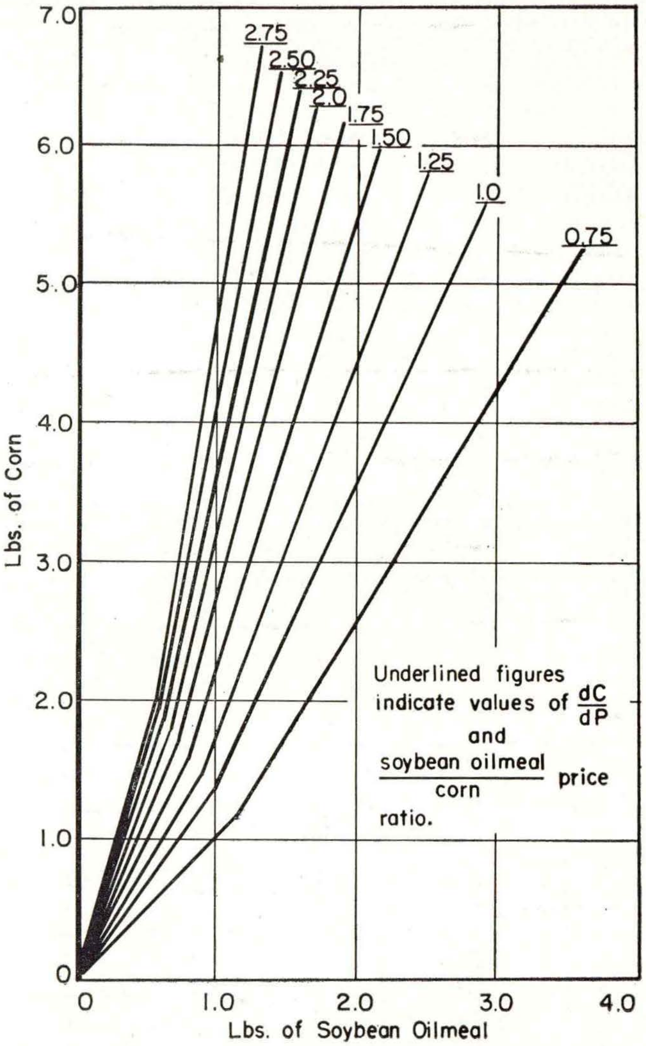


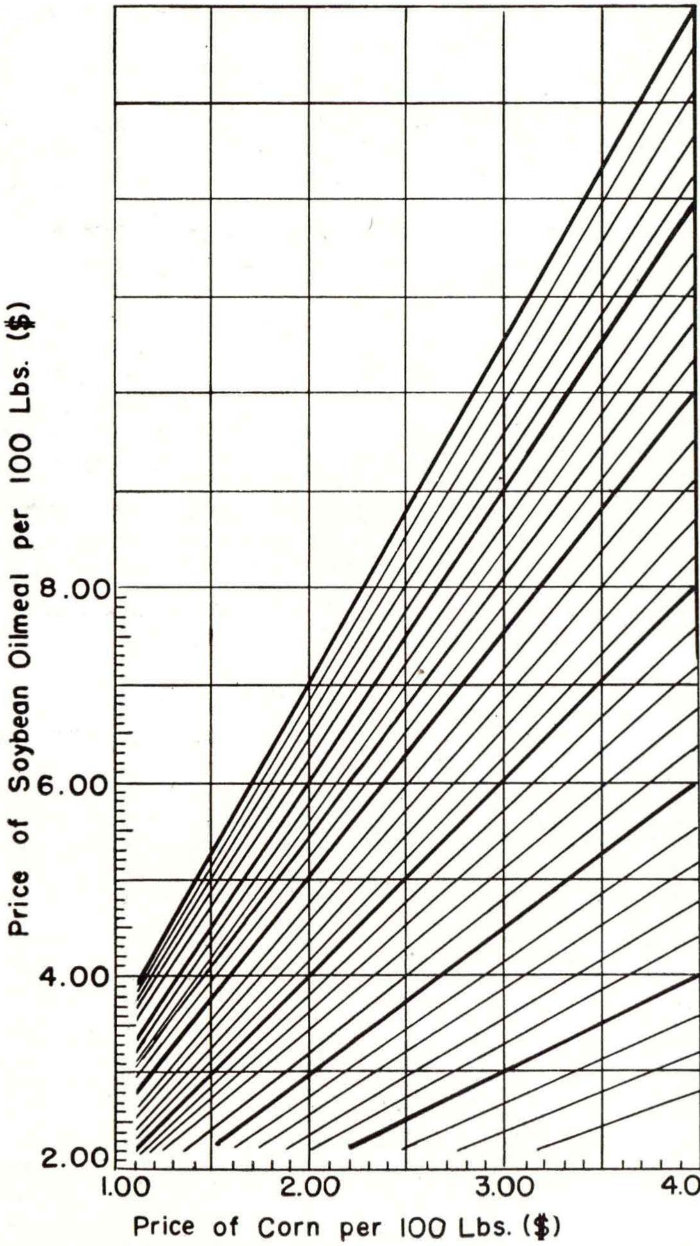
Fig. 24. "Spliced" isoclines for two weight intervals showing path of least-cost rations when feed combinations are changed once during the production period. Based on logarithmic functions 34 and 35.

In interpreting figs. 22, 23 and 24 it should be remembered that the slope of the upper segment of the isocline starts from the origin of a new feed plane. In figs. 22 and 23, for example, the boundaries of the new feed plane are formed by the two lines which intersect at the "splice" in the isocline. The scale for these new axes starts from zero and feeds are measured accordingly.

Feeds for the rations in the second interval are not measured in respect to the original axes for the first interval. To measure feeds for the second interval on the original axes for the first interval would result in changing rations for each bird weight, since the upper portion of "spliced" isocline, if it were extended to the axis, would intersect the soybean oilmeal axes (whereas, it intersects the origin for axes to which it refers). The same statement applies to the "points of splices" in fig. 24. Although the "new axes" are not shown because of space limitations, a new origin actually occurs at the point of splice for each pair of segments forming an isocline, and feeds must be measured accordingly.

SIMPLIFIED DETERMINATION

Figure 25 provides a simplified basis for estimating the least-cost ration in either weight in-



SBOM-Corn Price Ratio	Opt. Single Ration	Opt. Interval Rations	
		1st Int.	2nd Int.
3.5 3.4	15.0	16.0	—
3.4 3.3	15.5	16.0	—
3.3 3.2	15.5	16.0	—
3.2 3.1	15.5	16.5	—
3.1 3.0	15.5	16.5	—
3.0 2.9	16.0	16.5	—
2.9 2.8	16.0	17.0	—
2.8 2.7	16.0	17.0	15.0
2.7 2.6	16.5	17.0	15.0
2.6 2.5	16.5	17.5	15.5
2.5 2.4	16.5	17.5	15.5
2.4 2.3	17.0	18.0	15.5
2.3 2.2	17.0	18.0	16.0
2.2 2.1	17.5	18.5	16.0
2.1 2.0	17.5	18.5	16.0
2.0 1.9	18.0	19.0	16.5
1.9 1.8	18.5	19.0	16.5
1.8 1.7	19.0	19.5	17.0
1.7 1.6	19.0	20.0	17.5
1.6 1.5	19.5	20.5	17.5
1.5 1.4	20.0	20.5	18.0
1.4 1.3	20.5	21.0	18.5
1.3 1.2	21.0	22.0	19.0
1.2 1.1	21.5	22.5	19.5
1.1 1.0	22.0	23.0	20.0
1.0 0.9	23.0	24.0	20.5
0.9 0.8	23.5	24.5	21.5
0.8 0.7	24.5	25.5	22.5

Fig. 25. "Average" least-cost protein rations for broilers fed (a) a single ration and (b) two different rations during the production period for various prices of soybean oilmeal and corn. Logarithmic functions 36, 37 and 38 are used as a basis for ration selections.

terval, or for the total production period. While it has been devised by relating substitution ratios to price ratios, it considers only discrete points on the production surface and specifies a single optimum ration for small ranges of price ratios. For example, it indicates a 24.5-percent ration over the entire growth period for soybean oilmeal/corn price ratios between 0.7 and 0.8; for price ratios between 2.4 and 2.5, the optimum single ration over the entire production period is 16.5 percent protein.

The graph can be used as follows: Suppose the price of soybean oilmeal is 6 cents per pound ($6 per cwt.) while corn is 3 cents per pound ($3 per cwt.). Follow across the horizontal "$6 line" for soybean oilmeal until it intersects the "$3 line" for corn. Then follow the diagonal line passing through this point of intersection to find the least-cost ration. It will include 18 percent protein if a single ration is fed; it will include 19 percent for the first interval and 16.5 percent for the second interval if one change in rations is made during the growing period.

MOST PROFITABLE WEIGHTS FOR BROILERS

While the procedure and tables outlined above allow specification of the optimum ration, they do not indicate the total amount of feed to be fed per broiler and, hence, the optimum marketing weight. However, after the least-cost ration has been determined, it is possible to use the input-output equations to determine the optimum level of feeding and the most profitable marketing weight. The optimum marketing weight is determined, as outlined earlier, by equating the derivative of the gain-feed function for a particular protein ration with the feed/broiler price ratio. In other words, by equating the marginal physical products from feed with the feed/broiler price ratio, the optimum weight of broilers can be obtained.

The quadratic function 11 has been used for obtaining the optimum weights. The over-all function has been used to express gains as a function of feed inputs for fixed proportions of corn and soybean oilmeal (i. e., rations containing a given percentage of protein) for protein levels from 15 to 27 percent. Rations below 16 percent and above 26 percent protein are extrapolations outside the observations of the study. A comparison of total weights for broilers estimated for these various protein levels is shown in table 12. Again it is noticeable that from a physical efficiency standpoint,[10] rations high in protein provide the greatest gains per unit of feed used for low weights; then, as feed intake increases, rations lower in protein content are more efficient. The marginal quantities in table 13 illustrate this relationship more clearly. For the first few pounds of feed consumed, the marginal or additional gains per unit of feed input are highest at the 27-percent protein level. As more feed is consumed, the rations giving the highest additional gains per unit of feed consumed are those with lower protein levels.

Tables 14 and 15 indicate, respectively, the optimum amount of feed per bird and the optimum marketing weight for various ratios of feed and broiler prices. The least-cost ration would be determined first in tables 8, 9 and 10. Then tables 14 and 15 should be used to predict the total amount of the particular ration and the optimum marketing weight per broiler. By equating the derivative of each function representing gain along a ration line (isocline) with the feed-broiler

TABLE 12. TOTAL LIVEWEIGHT PER BROILER FOR INDICATED POUNDS OF ACCUMULATED FEED INPUTS WHEN FED VARIOUS PROTEIN RATIONS.*

Feed inputs in pounds	Percent protein in ration												
	15.0	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0	25.0	26.0	27.0
0.5	0.335	0.337	0.339	0.342	0.343	0.345	0.347	0.349	0.350	0.352	0.354	0.356	0.358
1.0	0.544	0.547	0.551	0.556	0.559	0.562	0.566	0.569	0.572	0.576	0.579	0.582	0.585
1.5	0.744	0.750	0.757	0.763	0.768	0.772	0.777	0.782	0.787	0.792	0.796	0.800	0.805
2.0	0.938	0.947	0.956	0.964	0.970	0.976	0.982	0.989	0.994	0.999	1.005	1.010	1.015
2.5	1.127	1.138	1.148	1.158	1.166	1.173	1.180	1.188	1.194	1.201	1.207	1.212	1.217
3.0	1.309	1.322	1.334	1.346	1.355	1.363	1.372	1.380	1.387	1.394	1.400	1.406	1.411
3.5	1.485	1.500	1.514	1.527	1.537	1.546	1.556	1.565	1.572	1.579	1.586	1.591	1.596
4.0	1.654	1.671	1.687	1.702	1.713	1.723	1.733	1.742	1.750	1.757	1.763	1.768	1.772
4.5	1.817	1.836	1.854	1.870	1.882	1.893	1.904	1.913	1.921	1.928	1.933	1.937	1.939
5.0	1.973	1.994	2.014	2.032	2.045	2.056	2.067	2.077	2.085	2.091	2.095	2.097	2.098
5.5	2.123	2.147	2.168	2.187	2.201	2.213	2.224	2.233	2.241	2.246	2.249	2.250	2.249
6.0	2.267	2.292	2.315	2.336	2.351	2.363	2.376	2.383	2.389	2.393	2.395	2.394	2.390
6.5	2.404	2.432	2.456	2.478	2.493	2.506	2.517	2.525	2.531	2.533	2.533	2.530	2.524
7.0	2.535	2.565	2.591	2.614	2.630	2.642	2.653	2.660	2.665	2.666	2.663	2.657	2.648
7.5	2.659	2.691	2.719	2.743	2.759	2.771	2.782	2.789	2.791	2.790	2.785	2.777	2.764
8.0	2.777	2.811	2.841	2.866	2.882	2.894	2.904	2.910	2.911	2.907	2.900	2.888	2.871
8.5	2.889	2.925	2.956	2.982	2.999	3.011	3.020	3.024	3.023	3.017	3.006	2.991	2.970
9.0	2.994	3.032	3.064	3.091	3.108	3.120	3.128	3.131	3.128	3.120	3.105	3.085	3.060
9.5	3.095	3.133	3.167	3.194	3.212	3.223	3.230	3.230	3.225	3.213	3.196	3.172	3.142
10.0	3.186	3.228	3.263	3.291	3.308	3.319	3.324	3.323	3.315	3.300	3.279	3.250	3.214
10.5	3.272	3.306	3.352	3.381	3.398	3.408	3.412	3.409	3.473	3.379	3.353	3.320	3.279
11.0	3.352	3.397	3.435	3.464	3.481	3.491	3.493	3.487	3.473	3.451	3.420	3.382	3.334

* Total liveweights obtained by adding initial weight of 0.09 pound for chicks, to gains estimated from quadratic over-all function 11.

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[10] Physical efficiency is used as the unit of gain per unit of feed input without regard to costs or returns.

TABLE 13. MARGINAL GAINS (LBS. GAIN PER ADDED LB. OF FEED) FROM SPECIFIED FEED INPUTS PER BROILER ON VARIOUS PROTEIN RATIONS (ESTIMATED FROM QUADRATIC OVER-ALL FUNCTION 11).*

Feed input in pounds	Percent protein in ration												
	15.0	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0	25.0	26.0	27.0
0.5	0.4215	0.4259	0.4302	0.4344	0.4376	0.4408	0.4444	0.4479	0.4513	0.4546	0·4579	0·4612	0.4643
1.0	0.4087	0.4131	0.4173	0.4213	0.4244	0.4274	0.4306	0.4337	0.4366	0.4395	0.4421	0.4447	0.4471
1.5	0.3960	0.4003	0.4044	0.4083	0.4112	0.4139	0.4168	0.4195	0.4220	0.4243	0.4263	0.4282	0.4299
2.0	0.3832	0.3875	0.3915	0.3953	0.3980	0.4005	0.4030	0.4053	0.4073	0.4091	0.4105	0.4117	0.4126
2.5	0.3704	0.3747	0.3786	0.3822	0.3848	0.3870	0.3892	0.3911	0.3927	0.3939	0.3947	0.3953	0.3954
3.0	0.3576	0.3619	0.3657	0·3692	0·3716	0.3736	0.3755	0.3769	0.3789	0.3787	0.3789	0.3788	0.3782
3.5	0.3448	0.3491	0.3528	0.3561	0.3584	0.3601	0.3617	0.3628	0.3634	0.3635	0.3631	0.3623	0.3610
4.0	0.3320	0.3363	0.3399	0.3431	0.3451	0.3467	0.3479	0.3486	0.3487	0.3483	0.3473	0.3458	0.3438
4.5	0.3193	0.3235	0.3270	0.3300	0.3319	0.3332	0.3341	0.3344	0.3341	0.3331	0.3315	0.3294	0.3266
5.0	0.3065	0.3107	0.3141	0.3170	0.3187	0.3198	0.3203	0.3202	0.3194	0.3179	0.3157	0.3129	0.3093
5.5	0.2937	0.2978	0.3013	0.3039	0.3055	0.3063	0.3065	0.3060	0.3048	0.3027	0.2999	0.2964	0.2921
6.0	0.2809	0.2850	0.2884	0.2909	0.2923	0.2929	0.2928	0.2918	0.2901	0.2875	0.2841	0.2799	0.2749
6.5	0.2681	0.2722	0.2755	0.2778	0.2791	0.2794	0.2790	0.2776	0.2754	0.2723	0.2683	0.2635	0.2577
7.0	0.2553	0.2594	0.2626	0.2648	0.2659	0.2660	0.2652	0.2635	0.2608	0.2525	0.2525	0.2470	0.2405
7.5	0.2426	0.2466	0.2497	0.2517	0.2526	0.2525	0.2514	0.2493	0.2461	0.2419	0.2367	0.2305	0.2232
8.0	0.2298	0.2338	0.2368	0.2387	0.2394	0.2391	0.2376	0.2351	0.2315	0.2268	0.2209	0.2140	0.2060
8.5	0.2170	0.2210	0.2239	0.2256	0.2262	0.2256	0.2238	0.2209	0.2168	0.2116	0.2051	0.1976	0.1888
9.0	0.2042	0.2082	0.2110	0.2126	0.2130	0.2122	0.2101	0.2067	0.2022	0.1964	0.1893	0.1811	0.1716
9.5	0.1914	0.1954	0.1981	0.1995	0.1998	0.1987	0.1963	0.1926	0.1875	0.1812	0.1735	0.1646	0.1544
10.0	0.1786	0.1826	0.1852	0.1865	0.1866	0.1853	0.1825	0.1784	0.1729	0.1660	0.1577	0.1481	0.1371
10.5	0.1659	0.1698	0.1723	0.1734	0.1734	0.1718	0.1687	0.1670	0.1612	0.1538	0.1419	0.1317	0.1200
11.0	0.1531	0.1570	0.1594	0.1604	0.1601	0.1584	0.1549	0.1500	0.1436	0.1356	0.1261	0.1152	0.1027

*Figures in body of table indicate added lbs. of gain from each 1-pound added unit of feed, starting from the total feed inputs shown in the first column.

TABLE 14. POUNDS OF FEED REQUIRED FOR OPTIMUM WEIGHTS UNDER VARIOUS BROILER AND FEED PRICE RATIOS (PREDICTED FROM EQUATION 11).

Broiler/ feed price ratio	Feed/ broiler price ratio	Percent protein in ration												
		15.0	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0	25.0	26.0	27.0
3.6	0.278	6.12	6.29	6.41	6.50	6.55	6.56	6.54	6.50	6.42	6.32	6.20	5.97	5.92
3.8	0.263	6.69	6.86	6.98	7.06	7.10	7.14	7.07	7.01	6.92	6.80	6.66	6.42	6.34
4.0	0.250	7.21	7.38	7.49	7.57	7.60	7.59	7.55	7.48	7.37	7.24	7.08	6.82	6.72
4.2	0.238	7.67	7.84	7.95	8.02	8.05	8.04	7.98	7.89	7.77	7.63	7.46	7.27	7.07
4.4	0.227	8.10	8.27	8.37	8.44	8.46	8.44	8.38	8.28	8.14	7.98	7.80	7.60	7.38
4.6	0.217	8.48	8.65	8.75	8.82	8.83	8.81	8.73	8.62	8.48	8.31	8.11	7.90	7.67
4.8	0.208	8.84	9.00	9.10	9.16	9.18	9.14	9.06	8.94	8.79	8.61	8.40	8.17	7.93
5.0	0.200	9.16	9.34	9.43	9.48	9.49	9.45	9.36	9.24	9.07	8.88	8.66	8.43	8.18
5.2	0.192	9.46	9.64	9.72	9.78	9.78	9.74	9.64	9.51	9.34	9.13	8.91	8.66	8.40
5.4	0.185	9.74	9.91	10.00	10.05	10.05	10.00	9.90	9.76	9.58	9.37	9.13	8.88	8.60
5.6	0.179	10.00	10.17	10.25	10.30	10.30	10.25	10.14	9.99	9.80	9.59	9.34	9.08	8.80
5.8	0.172	10.24	10.41	10.50	10.54	10.54	10.48	10.37	10.21	10.02	9.79	9.54	9.26	8.98
6.0	0.167	10.47	10.64	10.72	10.70	10.75	10·69	10.57	10.41	10.21	9.98	9.72	9.44	9.14
6.2	0.161	10.68	10.85	10.93	10.97	10.96	10.89	10.77	10.60	10.40	10.15	9.89	9.60	9.30
6.4	0.156	10.88	11.05	11.12	11.16	11.15	11.98	10.95	10.78	10.57	10.32	10.05	9.75	9.44
6.6	0.152	11.06	11.23	11.31	11.34	11.33	11.25	11.12	10.95	10.73	10.48	10.20	9.90	9.58
6.8	0.147	11.24	11.40	11.48	11.51	11.50	11.42	11.29	11.10	10.88	10.62	10.34	10.03	9.65
7.0	0.143	11.40	11.47	11.64	11.67	11.65	11.58	11.44	11.25	11.03	10.76	10.47	10.16	9.83

TABLE 15. WEIGHTS (POUNDS) FOR MAXIMIZING RETURNS ABOVE FEED COSTS FOR BROILERS ON VARIOUS PROTEIN RATIONS WITH SPECIFIED BROILER/FEED (FEED/BROILER) PRICE RATIOS (PREDICTED FROM EQUATION 11).

Broiler/ feed price ratio	Feed/ broiler price ratio	Percent protein in ration												
		15.0	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0	25.0	26.0	27.0
3.6	0.278	2.30	2.38	2.43	2.48	2.51	2.52	2.53	2.52	2.51	2.48	2.45	2.39	2.37
3.8	0.263	2.46	2.53	2.59	2.63	2.66	2.67	2.67	2.66	2.65	2.61	2.58	2.51	2.48
4.0	0.250	2.58	2.66	2.72	2.76	2.78	2.80	2.80	2.78	2.76	2.72	2.68	2.61	2.58
4.2	0.238	2.70	2.77	2.83	2.87	2.89	2.90	2.90	2.89	2.86	2.82	2.78	2.72	2.66
4.4	0.227	2.80	2.87	2.93	2.97	2.99	3.00	2.99	2.97	2.95	2.90	2.86	2.80	2.74
4.6	0.217	2.89	2.96	3.01	3.05	3.07	3.08	3.07	3.05	3.02	2.98	2.92	2.87	2.80
4.8	0.208	2.96	3.03	3.08	3.13	3.15	3.15	3.14	3.12	3.08	3.04	2.98	2.92	2.86
5.0	0.200	3.02	3.10	3.15	3.18	3.21	3.21	3.20	3.18	3.14	3.10	3.04	2.98	2.91
5.2	0.192	3.08	3.15	3.20	3.25	3.27	3.27	3.26	3.23	3.19	3.15	3.08	3.02	2.95
5.4	0.185	3.14	3.21	3.26	3.30	3.32	3.32	3.31	3.28	3.24	3.19	3.13	3.06	2.99
5.6	0.179	3.18	3.26	3.31	3.35	3.37	3.36	3.35	3.32	3.28	3.23	3.17	3.10	3.02
5.8	0.172	3.22	3.30	3.35	3.39	3.41	3.40	3.39	3.36	3.32	3.27	3.20	3.13	3.06
6.0	0.167	3.27	3.34	3.39	3.42	3.44	3.44	3.42	3.40	3.35	3.29	3.23	3.16	3.09
6.2	0.161	3.30	3.37	3.42	3.46	3.48	3.47	3.46	3.42	3.38	3.32	3.26	3.19	3.11
6.4	0.156	3.33	3.41	3.45	3.49	3.51	3.50	3.48	3.45	3.41	3.35	3.29	3.21	3.13
6.6	0.152	3.36	3.43	3.48	3.52	3.54	3.53	3.51	3.48	3.44	3.37	3.31	3.23	3.15
6.8	0.147	3.38	3.46	3.51	3.54	3.56	3.56	3.54	3.50	3.46	3.40	3.33	3.25	3.17
7.0	0.143	3.41	3.48	3.53	3.57	3.58	3.58	3.56	3.52	3.48	3.42	3.35	3.27	3.19

ratio, (column 2 of table 14) the optimum quantity of feed for a particular protein ration is obtained. (The feed/broiler ratio is the inverse of the broiler/feed ratio.) Broiler/feed price ratios (column 1 of table 15) from 3.6 to 7.0 are used as a basis for determining optimum feed quantities for the various rations. These ratios extend slightly beyond the relevant ranges of broiler/feed ratios in the U. S. during the past 5 years. (Broiler/feed ratios for Iowa have ranged between 3.8 and 6.1 during the past 5 years.) Once the optimum quantity of feed is obtained, the corresponding amount of gain is found by substituting the feed quantity into the appropriate ration function. Adding the initial weight of the chick, or about 0.09 pound, provides the optimum marketing weights for the various broiler and feed price combinations.

The predicted optimum marketing weights for broilers on rations of protein levels ranging from 15 to 27 percent with various broiler and feed prices are shown in table 15. These predicted weights are for situations where (a) capital is not limiting, (b) the weights provide maximum returns (or minimum losses) above feed costs, (c) risk and uncertainty are not considered, (d) time required for attaining optimum weights is not considered and (e) the same ration is fed throughout the feeding period.

Using the broiler/feed price ratio of 4.08 (the average Iowa broiler/feed price ratio, 1953-1954, was 4.9), the optimum marketing weights for broilers according to data in table 15 would range from 2.86 to 3.15 pounds, depending on the ration fed. With a 20-percent protein ration, the least-cost ration over this same period, a weight of 3.15 is optimum. Data from tables 14 and 15 have been used to develop graphical guides for determining optimum weights and corresponding feed inputs, as shown in figs. 26-32. These graphs provide only seven selections in choice of alternatives of marketing weights relative to price ratios. Hence, they do not include the degree of refinement in tables 14 and 15. However, the choice of alternatives may be sufficient for most practical decisions. This procedure can be used: First, select the proper broiler price on the vertical axis. Second, move across the diagram horizontally until the broiler price line intersects the appropriate vertical feed price line. Third, from this point of intersection, follow upward diagonally along the broiler/feed price ratio lines to the right side of the diagram. The optimum weight and feed inputs are denoted on the right side of the chart for the various ratios. As an illustration in fig. 26, assume that the expected price for broilers is 25 cents per pound and feed cost is $4.50 per hundred pounds or 4.5 cents per pound. The intersection of the horizontal "25-cent price line" and the vertical "4.5-cent feed line" occurs at point A. This represents a broiler/feed price ratio of between 5.5 and 6.0. Following up along the diagonals toward the upper right-hand corner of the page, the optimum average weight per broiler for this broiler/feed ratio is found to be approximately 3.2 pounds; the estimated feed consumption is about 10.12 pounds of feed. Graphs for selection of optimum weights are presented only

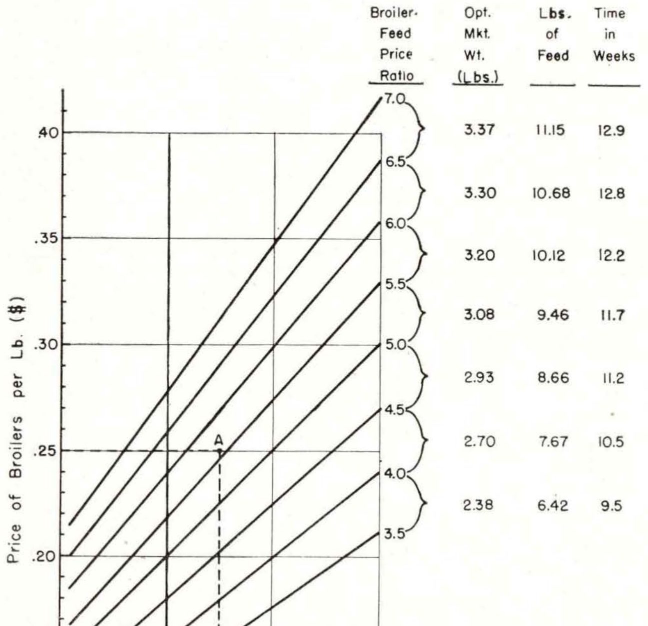


Fig. 26. Predicted optimum weights with corresponding feed and time requirements for various broiler-feed price ratios when broilers are fed a 15-percent protein ration.

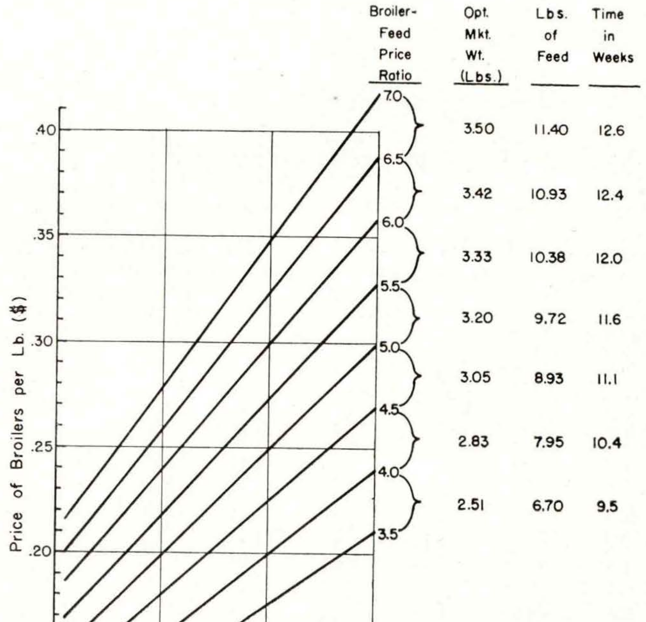
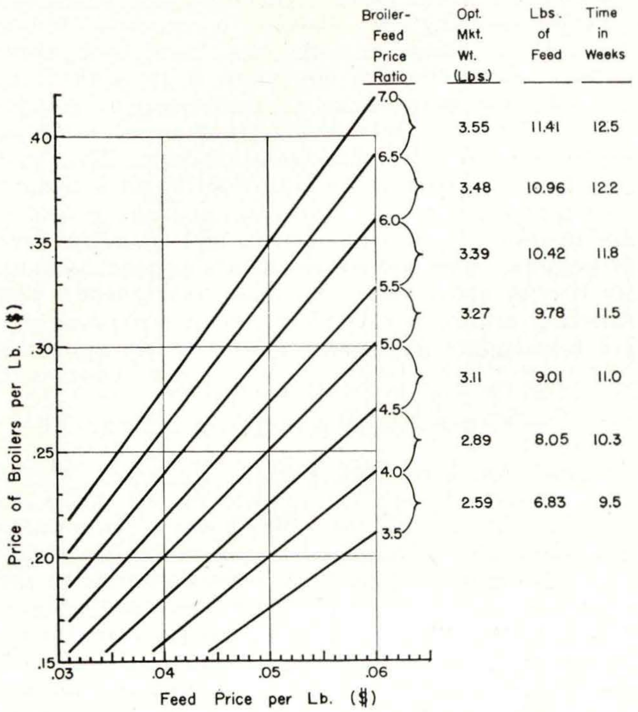
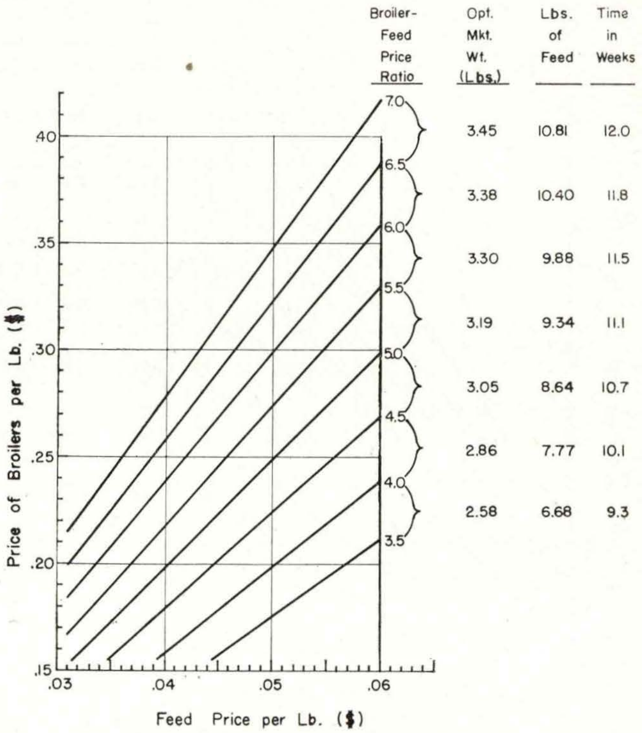


Fig. 27. Predicted optimum weights with corresponding feed and time requirements for various broiler-feed price ratios when broilers are fed a 17-percent protein ration.



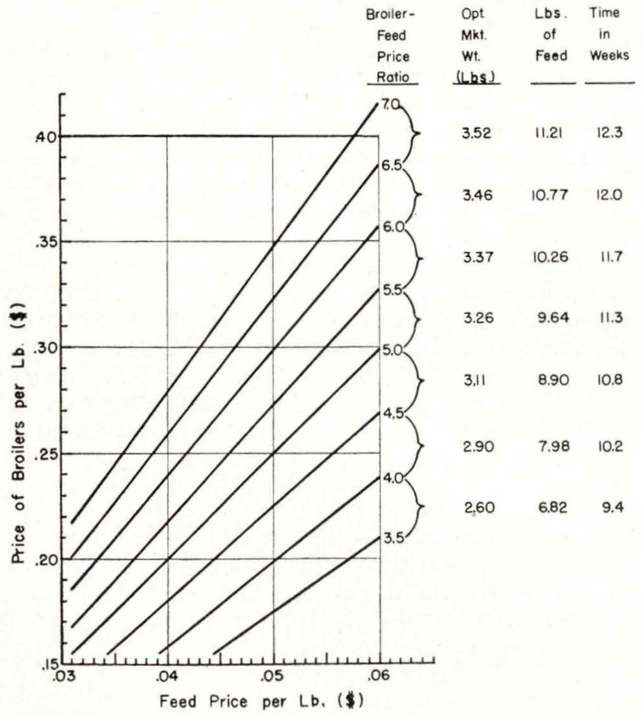
Broiler-Feed Price Ratio	Opt. Mkt. Wt. (Lbs.)	Lbs. of Feed	Time in Weeks
7.0–6.5	3.55	11.41	12.5
6.5–6.0	3.48	10.96	12.2
6.0–5.5	3.39	10.42	11.8
5.5–5.0	3.27	9.78	11.5
5.0–4.5	3.11	9.01	11.0
4.5–4.0	2.89	8.05	10.3
4.0–3.5	2.59	6.83	9.5

Fig. 28. Predicted optimum weights with corresponding feed and time requirements for various broiler-feed price ratios when broilers are fed a 19-percent protein ration.



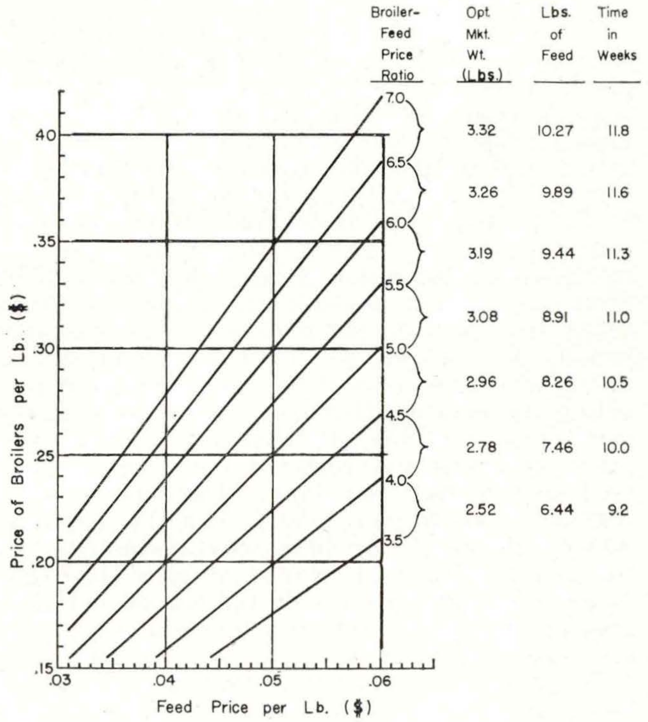
Broiler-Feed Price Ratio	Opt. Mkt. Wt. (Lbs.)	Lbs. of Feed	Time in Weeks
7.0–6.5	3.45	10.81	12.0
6.5–6.0	3.38	10.40	11.8
6.0–5.5	3.30	9.88	11.5
5.5–5.0	3.19	9.34	11.1
5.0–4.5	3.05	8.64	10.7
4.5–4.0	2.86	7.77	10.1
4.0–3.5	2.58	6.68	9.3

Fig. 30. Predicted optimum weights with corresponding feed and time requirements for various broiler-feed price ratios when broilers are fed a 23-percent protein ration.



Broiler-Feed Price Ratio	Opt Mkt. Wt. (Lbs.)	Lbs. of Feed	Time in Weeks
7.0–6.5	3.52	11.21	12.3
6.5–6.0	3.46	10.77	12.0
6.0–5.5	3.37	10.26	11.7
5.5–5.0	3.26	9.64	11.3
5.0–4.5	3.11	8.90	10.8
4.5–4.0	2.90	7.98	10.2
4.0–3.5	2.60	6.82	9.4

Fig. 29. Predicted optimum weights with corresponding feed and time requirements for various broiler-feed price ratios when broilers are fed a 21-percent protein ration.



Broiler-Feed Price Ratio	Opt. Mkt. Wt. (Lbs.)	Lbs. of Feed	Time in Weeks
7.0–6.5	3.32	10.27	11.8
6.5–6.0	3.26	9.89	11.6
6.0–5.5	3.19	9.44	11.3
5.5–5.0	3.08	8.91	11.0
5.0–4.5	2.96	8.26	10.5
4.5–4.0	2.78	7.46	10.0
4.0–3.5	2.52	6.44	9.2

Fig. 31. Predicted optimum weights with corresponding feed and time requirements for various broiler-feed price ratios when broilers are fed a 25-percent protein ration.

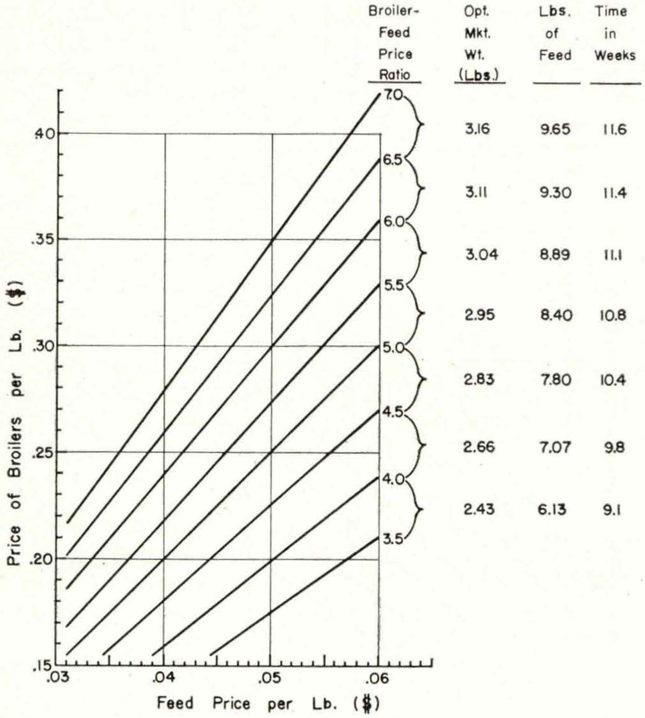


Fig. 32. Predicted optimum weights with corresponding feed and time requirements for various broiler-feed price ratios when broilers are fed a 27-percent protein ration.

for every second protein level since weight and feed consumption differences are small for changes of 1 percent protein. More precise estimates can be obtained from tables 14 and 15.

TIME REQUIREMENTS

Previous analysis dealt only with the cost of alternative rations. However, the broiler producer also is interested in the time required for gains. If he is faced with a seasonal or cyclical decline in broiler price, he may wish to use the least-time ration, rather than the least-cost combination of feeds. If he is faced with the possibility of rising broiler prices, he will undoubtedly want to use the least-cost ration. In other words, the least-time ration (i. e., the ration which will make a given marketing weight possible in the shortest time span) need not be identical with the least-cost ration. The two will be identical under price situations where the cost of the feed ingredient providing the greatest timeliness is low, so that the two types of rations are identical. Under other price situations the costs of the "time-saving" feed may be relatively high. The least-time ration will then be more costly than the least-cost ration. Hence, the producer must decide whether the gain in broiler price from using a least-time ration (and getting birds on the market early) is greater than the savings in feed costs from using the least-cost ration.

To provide information aiding these types of decisions, time functions have been computed from the basic experimental data. Previous equations provide predictions of the total weights associated with the various rations. An additional function showing time elapsed to consume various amounts of these rations has then been computed. From the two equations it is possible to compute time elapsed for a specific gain or weight level. Various algebraic functions were tried as expressions of the time relationship. The best function appeared to be equation 39 with square root terms where T is time in days and S and C are pounds of soybean oilmeal and corn per bird in pounds. The properties which appear to qualify it over other functions tried were these: The function allows a relatively sharp curvature for low feed inputs but tends to more nearly approach linearity for high feed inputs. In other words, it is consistent with the growth of the bird's digestive capacity at the outset when proportionately less time is required to consume a given additional quantity of feed; it is consistent with the tendency for a limit in growth of the bird's digestive capacity as the bird approaches maturity. At heavy weights, digestive capacity is limited, and a bird consumes about a constant amount per day (i. e., each additional pound of feed is consumed in about the same period of time as the bird approaches maturity). All of the coefficients for this time function are acceptable at a 1-percent level of probability, and 99 percent of the variance in time required to consume various quantities of feed is explained by the variables in the equation.

$$(39)\quad T = 0.6735 + 4.7974C + 9.4576S + 21.4617\sqrt{C} + 13.6188\sqrt{S} - 12.0287\sqrt{CS}$$

Analysis of variance also was used to test the significance in differences in rate of gain up to 1.23 of total gain and up to a total of 3.0 pounds of gain for the six rations of the experiment. These tests showed the differences to be significant at the 5-percent level of probability. However, there was no significant difference in rate of gain for the six rations between a total gain of 1.23 and 3.0 pounds. Evidently, the main effect of rations on rate of gain is in the earlier growing period.

Figures 33, 34 and 35 show the predicted relationship between feed consumption and time for three rations of the study. Time curves for other rations are similar. Table 16 indicates the predicted amount of time required to attain weights of 1.32 pounds and 3.25 pounds for rations of different protein levels. For the lower weight range, a ration of slightly over 23 percent protein is predicted to give a 1.32-pound liveweight in the minimum amount of time. For the entire weight range to 3.25 pounds liveweight, a protein percentage of slightly over 21.0 percent is predicted to give most rapid gains. Rations containing a greater percentage of protein are predicted to give somewhat less rapid gains. From the data of substitution rates in earlier tables, it is obvious that these rations which give the most rapid gains do not also give the least-cost rations under normal price relationships. However, the

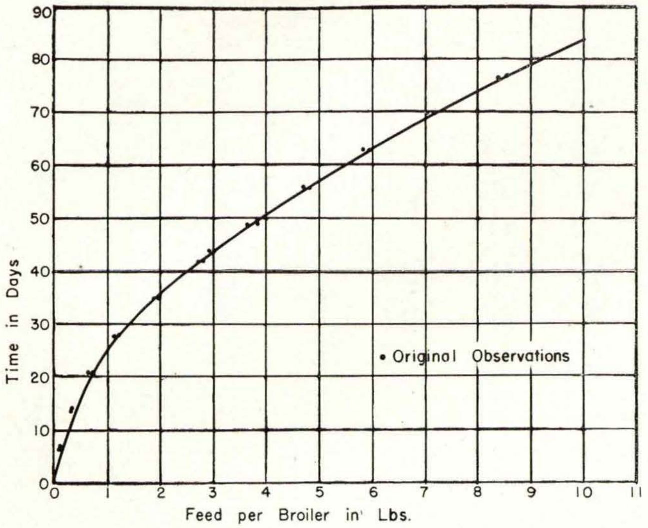


Fig. 33. Estimated number of days required to consume various quantities of feed for broilers fed a 16-percent protein ration. Time estimates are based on square root function 39.

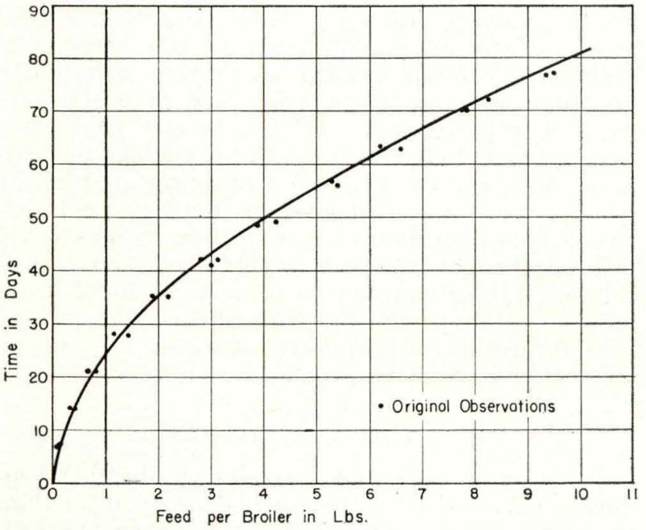


Fig. 34. Estimated number of days required to consume various quantities of feed for broilers fed a 20-percent protein ration. Time estimates are based on square root function 39.

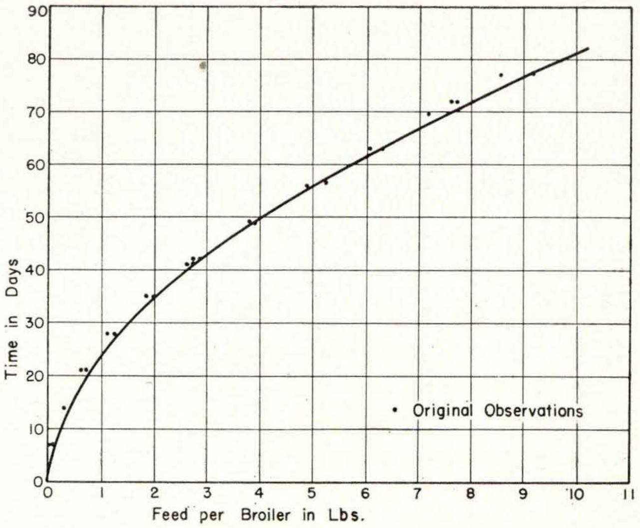


Fig. 35. Estimated number of days required to consume various quantities of feed for broilers fed a 24-percent protein ration. Time estimates are based on square root function 39.

cost of the least-time ration is only slightly above the least-cost ration when the price of soybean oilmeal is low relative to the price of corn. In cases where the price of soybean oilmeal is relatively high, the broiler producer needs to compare the savings in feed from use of the least-cost ration with any possible gain in broiler price obtained from getting to market sooner under the least-time ration. The absolute differences in profits from least-cost and least-time rations will be very small for a few birds but can be quite large for a large operation when corn is low in price compared to soybean oilmeal.

Table 17 shows the predicted number of days for broilers to reach the optimum marketing weights shown in table 15. The data of the two tables allow prediction of the weight which will allow maximum profit above feed costs. However, the proper sequence in considering ration costs, marketing weights and time to market would be this: (1) Select the least-cost ration in table 8. (2) Select the most profitable marketing weight for the least-cost ration from table 15. (3) Examine table 17 for the time involved and, after considering the prospects for prices, determine whether or not feeding plans should be altered to fit prospects for price increases or decreases. Table 17 can be interpreted thus: With a broiler/feed price ratio of 4.4, and with an 18-percent protein ration being the least-cost one shown in table 8, the optimum marketing weight of 2.97 pounds shown in table 15 can be obtained in 74.7 days. If broilers increase in price so that the broiler/feed price ratio becomes 5.0, with an 18-percent ration giving lowest feed costs, the optimum marketing weight is 3.18 pounds (table 15) and the time required is 79.5 days (table 17). If broilers are high enough in price relative to feed to give

TABLE 16. ESTIMATED FEED REQUIREMENTS AND NUMBER OF DAYS PER BROILER FOR SPECIFIED WEIGHTS WHEN FED VARIOUS PROTEIN RATIONS.

Percent protein in ration	Starting to 1.32-lb. weight		Starting to 3.25-lb. weight	
	Pounds feed*	No. days†	Pounds feed‡	No. days†
15.0	3.31	46.3	10.27	86.1
16.0	3.11	44.5	9.82	82.6
17.0	2.96	43.4	9.41	80.1
18.0	2.86	42.5	9.29	78.7
19.0	2.79	41.6	9.17	77.6
20.0	2.74	41.1	9.11	77.0
21.0	2.71	40.7	9.09	76.7
22.0	2.69	40.5	9.11	76.8
23.0	2.68	40.4	9.18	77.3
24.0	2.68	40.5	9.28	78.1
25.0	2.70	40.8	9.44	79.3
26.0	2.74	41.0	9.62	80.7

* Predicted from equation 26.
† Predicted from equation 39.
‡ Predicted from equations 26 and 27.

TABLE 17. PREDICTED TIME IN DAYS REQUIRED FOR BROILERS TO ATTAIN OPTIMUM MARKETING WEIGHTS SHOWN IN TABLE 15 (ESTIMATED FROM SQUARE ROOT TIME FUNCTION 39).

Broiler/ feed price ratio	Feed/ broiler price ratio	Percent protein in ration												
		15.0	16.0	17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0	25.0	26.0	27.0
		(days)	(days)	(days)	(days)	(days)	(days)	(days)	(days)	(days)	(days)	(days)	(days)	(days)
3.6	0.278	64.7	64.9	65.0	65.1	64.9	64.7	64.5	64.3	64.0	63.6	63.2	62.7	62.3
3.8	0.263	67.9	68.0	68.0	67.0	67.8	67.5	67.2	66.9	66.5	66.1	65.6	65.2	64.7
4.0	0.250	70.8	70.7	70.6	70.5	70.2	69.9	69.6	69.2	68.8	68.3	67.8	67.3	66.8
4.2	0.238	73.2	73.1	72.9	72.7	72.4	72.0	71.6	71.2	70.8	70.3	69.8	69.2	68.6
4.4	0.227	75.4	75.2	75.0	74.7	74.3	73.9	73.5	73.0	72.5	72.0	71.5	70.9	70.3
4.6	0.217	77.4	77.1	76.8	76.5	76.0	75.6	75.1	74.6	74.1	73.6	73.0	72.4	71.8
4.8	0.208	79.2	78.8	78.5	78.1	77.6	77.1	76.6	76.1	75.6	75.0	74.4	73.8	73.2
5.0	0.200	80.8	80.4	79.9	79.5	79.0	78.5	77.9	77.4	76.8	76.3	75.6	75.0	74.4
5.2	0.192	82.3	81.8	81.3	80.9	80.3	79.7	79.2	78.6	78.0	77.4	76.8	76.2	75.5
5.4	0.185	83.6	83.1	82.6	82.0	81.5	80.9	80.3	79.7	79.1	78.5	77.9	77.2	76.5
5.6	0.179	84.9	84.3	83.7	83.2	82.6	81.9	81.3	80.7	80.1	79.5	78.8	78.2	77.5
5.8	0.172	86.0	85.4	84.8	84.2	83.5	82.9	82.3	81.7	81.0	80.4	79.7	79.1	78.4
6.0	0.167	87.0	86.4	85.7	85.1	84.4	83.8	82.2	82.5	81.9	81.2	80.5	79.9	79.2
6.2	0.161	88.0	87.3	86.6	86.0	85.3	84.6	84.0	83.3	82.7	82.0	81.3	80.6	79.9
6.4	0.156	88.9	88.2	87.5	86.8	86.1	85.4	84.7	84.0	83.4	82.7	82.0	81.3	80.6
6.6	0.152	89.8	89.0	88.3	87.6	86.8	86.1	85.4	84.7	84.1	83.4	82.7	82.0	81.3
6.8	0.147	90.6	89.7	89.0	88.3	87.5	86.8	86.1	85.4	84.7	84.0	83.3	82.6	81.9
7.0	0.143	91.3	90.5	89.7	88.9	88.2	87.4	86.8	86.0	85.3	84.6	83.9	83.2	82.5

a 6.0 price ratio, the optimum marketing weight of 3.42 pounds (table 15) will be attained in 85.1 days (table 17). With a broiler/feed price ratio as low as 3.6, the optimum marketing weight would be 2.48 pounds (table 15) attained in 65.1 days (table 17), with an 18-percent ration resulting in least-cost gains (table 8). With a 20-percent ration as the lowest cost one, a broiler/feed price ratio of 4.4 would give an optimum marketing weight of 3.00 pounds (table 15) in 73.9 days (table 17); with a broiler/feed price ratio of 6.0, the respective figures are 3.44 pounds and 83.8 days.

NUMBER OF FLOCKS PER YEAR

A considerable variation exists in the number of flocks of birds raised by broiler producers each year. Some part-time poultrymen raise only a flock or two each year. However, poultrymen who have broiler production as their major source of income usually raise at least three or more flocks each year. For producers who raise only three or less groups per year, it usually is possible to carry the broilers to their optimum weights without any time conflict. For broiler producers who desire to raise four groups per year, there would be no conflict on time for broilers fed on any of the rations when the broiler/feed price ratio is 5.5 or less. Raising four groups under the above price ratios would permit at least a week between each flock, depending on the broiler/feed price ratio and the protein ration being fed. Under the conditions of this study and using the average broiler/feed price ratio of 5.1 for Iowa[11] for the period 1951-54, four flocks could be carried to their optimum weights each year for any of the rations shown in table 15. For example, with a price for broilers of 25.5 cents per pound and feed at 5 cents per pound, the optimum average weight per broiler would be 3.24 pounds interpolated when fed a 19-percent ration (table 15). Nearly 80 days time is estimated for each group to reach this optimum weight. This permits slightly over 10 days between groups. When the broiler/feed price ratio is above 5.5, birds on the lower protein rations would require marketing at slightly less than optimum weights if a four flock schedule were rigidly followed. With a broiler/feed price ratio of 6.0, four flocks could be carried to optimum weights on rations containing a 19.5 or greater percentage of protein with a week between flocks; birds on lower protein rations could not be carried to optimum weights without a time conflict. Data in table 15 or in figs. 26-32 could be used to determine whether birds could be held until optimum weights are attained if four or more flocks are to be produced each year.

RISK AND UNCERTAINTY

Because of risk and uncertainty, broiler producers may not hold their birds until they attain the optimum market weights. The uncertainty of expected prices and death losses due to disease and other hazards may result in earlier marketing. However, modern techniques for prevention and treatment of diseases have done much to reduce this type of uncertainty. Also, insurance against hazards such as fire tends to reduce risk for the poultrymen. Prices usually provide the greatest source of uncertainty except where the producers have some type of forward pricing.

Examination of data in table 18 illustrates the effect of selling broilers at less than optimum weights for a particular situation. This example is illustrated using a 20-percent protein ration, 25.5 cents for broilers and 5 cents a pound for feed, (i. e., broiler/feed price ratio of 5.1). The calculated optimum weight for this situation is 3.24 pounds which provides an average return above feed costs per bird of $0.3463, or $346.30 for 1,000 broilers. If the broilers are marketed at a 3-pound weight, the return above feed costs would be about $342.55 for 1,000 birds or only

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[11] Broiler/feed ratio of 5.1 is based on data from: Crops and markets. USDA, Agricultural Marketing Service, 1952-55.

TABLE 18. COMPARISON OF RETURNS ABOVE FEED COSTS AND TIME REQUIREMENTS BETWEEN BROILERS CARRIED TO OPTIMUM WEIGHT AND THOSE MARKETED AT LESS THAN OPTIMUM WEIGHTS WHEN FED A 20-PERCENT CRUDE PROTEIN RATION.*

Average broiler weight (lbs.)	Average feed requirement (lbs.)	Gross return ($)	Feed costs ($)	Gross return less feed costs ($)	Return above feed costs 1,000 broilers ($)	Average time requirement (days)
2.89	8.0	0.73695	0.40000	0.33695	336.95	71.8
3.01	8.5	0.76755	0.42500	0.34255	342.55	74.2
3.12	9.0	0.79560	0.45000	0.34560	345.60	76.4
3.17	9.25	0.80835	0.46250	0.34585	345.85	77.6
3.25 (Opt.)	9.598	0.82620	0.47990	0.34630	346.30	79.1

* Returns computed with a price of 25.5 cents per pound for broilers and 5 cents a pound for feed. Input-output data on gain and feed requirements attained from quadratic over-all function 11 and time requirements based on square root function 39.

$3.75 less than that for 1,000 of the 3.24-pound broilers. About 74 days are required for attaining a weight of 3 pounds and 79 days for attaining a weight of 3.24 pounds. To many producers, the discounted return for the extra 5 days required for attaining the optimum weight of 3.24 pounds might be greater than the $3.75 return above feed costs, leading to earlier marketing. Some producers might even desire to market their broilers a week ahead of the time required for reaching optimum weights. Data in table 18 show that marketing the birds a week earlier than required for reaching optimum weights would lower returns above feed costs by only $9.35 ($346.30-$336.95) per 1,000 birds, or less than 1 cent per bird. Thus, many producers might find that the uncertainty involved in keeping the birds until they reached 3.24 pounds would not be worth the additional 1 cent per bird. That is, the discounted marginal returns may be less than the marginal feed costs for keeping the birds the extra week. The above example has been worked out for a single situation. However, other price situations and ratios would provide similar relationships.

Obviously, the decision of the best marketing weight depends on many factors including (a) input-output relationships, (b) the previous commitments, i. e., contractual arrangements, (c) number of flocks per year, (d) price expectations and (e) risk preference of the individual poultryman. However, the data provided in this study on input-output data, selection of rations, estimation of optimum market weights and corresponding time required for attaining optimum weights provides information for reducing much of the uncertainty in broiler production.

APPENDIX

Table A-1 shows the total weight per bird predicted from various total inputs of corn and soybean oilmeal. The numerous feed combinations shown represent many percentages of protein within the ranges used in this study. Table A-2 includes the predicted marginal productivities of corn and soybean oilmeal for the feed quantities shown in the left-hand column on the same row and at the head of the column. The upper figure

TABLE A-1. TOTAL LIVEWEIGHTS FOR VARIOUS ACCUMULATED INPUTS OF CORN AND SOYBEAN OILMEAL INDICATED IN ROWS AND COLUMNS.*

Pounds of corn	Pounds of soybean oilmeal							
	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
0	0.090	0.559	0.713	0.971	1.205	1.414	1.598	1.757
1	0.585	0.882	1.154	1.401	1.623	1.820	1.993	2.151
2	1.013	1.298	1.558	1.793	2.004	2.190	2.350	2.510
3	1.404	1.677	1.926	2.149	2.348	2.520	2.671	2.831
4	1.758	2.020	2.257	2.469	2.656	2.819	2.957	3.116
5	2.076	2.326	2.552	2.752	2.928	3.079	3.205	3.365
6	2.357	2.596	2.810	2.999	3.163	3.302	3.416	3.576
7	2.602	2.829	3.031	3.208	3.361	3.489	3.592	3.751

* Total weights were obtained by finding total gains for indicated feed combinations from equation 11, then adding the initial weight of chicks of 0.09 pound.

TABLE A-2. MARGINAL GAINS (POUNDS OF GAIN PER POUND OF FEED) FOR COMBINATIONS OF CORN AND SOYBEAN OILMEAL INDICATED IN ROWS AND COLUMNS. UPPER FIGURE REFERS TO SOYBEAN OILMEAL; LOWER FIGURE REFERS TO CORN.*

Pounds of corn	Pounds of soybean oilmeal							
	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
0	—— ——	0.592 ——	0.542 ——	0.492 ——	0.443 ——	0.393 ——	0.343 ——	0.293 ——
1	—— 0.446	0.568 0.434	0.519 0.423	0.469 0.411	0.419 0.399	0.370 0.388	0.320 0.376	0.270 0.365
2	—— 0.409	0.545 0.398	0.496 0.386	0.446 0.374	0.396 0.363	0.346 0.351	0.297 0.340	0.247 0.328
3	—— 0.373	0.522 0.361	0.472 0.349	0.423 0.338	0.373 0.326	0.323 0.315	0.274 0.303	0.224 0.291
4	—— 0.336	0.499 0.325	0.449 0.313	0.399 0.301	0.350 0.290	0.300 0.278	0.250 0.267	0.201 0.255
5	—— 0.300	0.476 0.288	0.426 0.276	0.376 0.265	0.327 0.253	0.277 0.242	0.227 0.230	0.177 0.218
6	—— 0.263	0.453 0.251	0.403 0.240	0.353 0.228	0.303 0.217	0.254 0.205	0.204 0.193	0.154 0.182
7	—— 0.226	0.429 0.215	0.380 0.203	0.330 0.192	0.280 0.180	0.230 0.168	0.181 0.157	0.131 0.145

* These figures are derivatives of gains with respect to each of the feeds from equation 11, with soybean oilmeal and corn fixed at the quantities shown at the top of the columns or the left side of the table.

of each cell is the marginal productivity of soybean oilmeal with corn considered to be fixed; the lower figure is the marginal productivity of corn with soybean oilmeal considered to be fixed.

Table A-3 includes the relevant statistics for single-variable equations 14 to 25 in the text.

Tables A-4, A-5 and A-6 show analysis of variance tests for determining whether significant differences existed in the number of days required for broilers fed six different rations in attaining specified gains of (1) 1.23 pounds, (2) 3.0 pounds and (3) 1.23 to 3.0 pounds. The six rations contained 16, 18, 20, 22, 24 and 26 percent crude protein. Tests of significance show that there is a highly significant difference in the number of days required for an average of 1.23 pounds of gain per broiler when fed the different protein rations; there is a significant difference in the length of time required for broilers to attain gains of 3.0 pounds; however, there is no significant difference in the amount of time required for going from a gain of 1.23 pounds to a 3.0-pound gain when fed the different rations.

TABLE A-3. NUMBER OF OBSERVATIONS, COEFFICIENTS OF DETERMINATION, MULTIPLE CORRELATION COEFFICIENTS AND STUDENT-t VALUES FOR SINGLE-VARIABLE RATION FUNCTIONS OF QUADRATIC AND LOGARITHMIC TYPES.*

Equation	No. observations	R^2	R	t_c	t_{c^2}	t_{b_1}
14	24	0.9969	0.9984	30.25	7.38	——
15	24	0.9996	0.9993	18.16	4.99	——
16	24	0.9990	0.9995	48.67	11.47	——
17	24	0.9994	0.9997	69.27	19.72	——
18	25	0.9986	0.9993	46.87	13.51	——
19	25	0.9985	0.9992	43.04	13.16	——
20	24	0.9986	0.9993	——	——	126.93
21	24	0.9962	0.9981	——	——	76.03
22	24	0.9989	0.9994	——	——	140.04
23	24	0.9958	0.9979	——	——	72.30
24	25	0.9975	0.9987	——	——	92.95
25	25	0.9971	0.9986	——	——	87.33

* All regression coefficients significant at a probability level of less than 0.01.

TABLE A-4. ANALYSIS OF VARIANCE OF DAYS REQUIRED FOR 1.23 POUNDS OF GAIN FOR BROILERS ON SIX DIFFERENT PROTEIN RATIONS.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F value
Among rations	5	19.0835	3.8167	9.558**
Within rations	24	9.5832	0.3993	——

**Significant at probability level of 0.01.

Comparisons of feed consumption for 0.9-pound gain per broiler for birds on different rations beyond a 1.32-pound weight when (a) fed a single ration during entire feeding period and (b) fed a lower percentage of protein than during the initial part of the feeding period are given in table A-7. Analysis of variance in table A-8 indicates that the differences are not significant.

TABLE A-5. ANALYSIS OF VARIANCE OF DAYS REQUIRED FOR 3 POUNDS OF GAIN FOR BROILERS ON SIX DIFFERENT RATIONS.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F value
Among rations	5	53.4074	10.6815	4.57*
Within rations	6	14.0402	2.3400	——

*Significant at probability level of 0.05.

TABLE A-6. ANALYSIS OF VARIANCE OF DAYS REQUIRED FOR GAINS FROM 1.23 TO 3 POUNDS FOR BROILERS ON SIX DIFFERENT RATIONS.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F value
Among rations	5	5.8927	1.1785	1.283†
Within rations	6	5.5121	0.9187	——

† Not significant at probability level of 0.05.

TABLE A-7. FEED CONSUMPTION FOR FOUR RATIONS OVER TWO WEIGHT PERIODS.

Rations (percent protein)	Broilers fed single ration for entire period		Broilers fed lower protein rations after 1.3-lb. weight	
	Number of pens	Feed consumption (lbs.)	Number of pens	Feed consumption (lbs.)
16	2	6.06	8	23.98
18	2	5.68	6	17.19
20	2	5.89	2	5.73
22	2	5.50	2	5.69
Total	8	23.13	18	52.59

TABLE A-8. ANALYSIS OF VARIANCE FOR DATA IN TABLE A-7.*

Source of variation	Degrees of freedom	Sum of squares	Mean square	F value
Between rations	7	0.17801	0.02543	1.90†
Residual	18	0.24145	0.01341	
Total	25	0.41946		

* Analysis of variance based on test for unequal numbers in subclasses by Kendall, M. G. Advanced theory of statistics. Vol. 2. Charles Griffin, London. 1946. pp. 221-225.

† Non-significant at probability level of 0.05.