Least-Cost Rations and Optimum Marketing Weights for Turkeys

Production Functions, Gain Isoquants, Substitution Ratios, Least-Cost Rations and Optimum Marketing Weights for Turkeys Fed Corn and Soybean Oilmeal in a Fortified Ration

by Earl O. Heady, Stanley Balloun
and Gerald W. Dean

Department of Economics and Sociology
Department of Poultry Husbandry

AGRICULTURAL EXPERIMENT STATION, IOWA STATE COLLEGE

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SUMMARY

Turkey production has become increasingly competitive and specialized in recent years. United States turkey production has nearly doubled since 1940, while the number of farms raising turkeys has steadily decreased. An increasing number of producers are looking toward savings in feed costs and better marketing methods as important aids in maintaining or increasing profits. The objective of this study is to provide turkey producers with useful predictions of least-cost rations and most profitable, or optimum, marketing weights for a wide range of price relationships.

Empirical data were obtained from Experiment 322 conducted by the Department of Poultry Husbandry in the summer of 1955. In this experiment, 600 turkeys were fed rations of (a) 21 to 31 percent protein from 0 to 6 weeks of age, (b) 15 to 25 percent protein from 6 to 12 weeks of age and (c) 10 to 20 percent protein from 12 to 24 weeks of age. Various types of regression equations, predicting gain as a function of the corn and soybean oilmeal fed, were fitted to the data for each of the three time intervals. These production functions were then used in predicting gain isoquants, marginal rates of substitution of soybean oilmeal for corn and feed input-gain output relationships for various rations. Economic analysis applied to these physical relationships allowed prediction of least-cost rations and optimum marketing weights.

Cobb-Douglas functions i, ii and iii were used, respectively, in predicting least-cost rations over three weight intervals of 0.11 to 2.44 pounds, 2.44 to 6.93 pounds and 6.93 pounds to finished weight (these weight ranges are based on observations from the 0-6, 6-12 and 12-24 week periods of the experiment). The least-cost combination of corn and soybean oilmeal for a given gain is determined by finding, along the gain isoquant, the marginal rate of substitution of soybean oilmeal for corn which equals the prevailing soybean oilmeal/corn price ratio. Since the isoclines of the Cobb-Douglas function are linear and pass through the origin of the feed plane, a constant rate of substitution exists along any particular ration line (i. e., the isoclines coincide with the ration lines). Hence, for a given soybean oilmeal/corn price ratio, a single ration which "averages least in cost" is predicted for each weight interval. As expected from nutritional theory, it was found that soybean oilmeal substitutes for less corn in the ration as the poults increase in weight. Accordingly, the predicted least-cost rations contain less protein in each of the three succeeding higher weight intervals. For example, with a soybean oilmeal/corn price ratio of 2.0, the least-cost rations for the three weight intervals contain, respectively, 22.0 percent protein, 20.0 percent protein and 15.0 percent protein.

$$\text{(i)} \quad Y = 1.7167\, C^{0.4422}\, S^{0.3647}$$

$$\text{(ii)} \quad Y = 1.7291\, C^{0.4997}\, S^{0.2531}$$

$$\text{(iii)} \quad Y = 1.0764\, C^{0.5108}\, S^{0.2517}$$

If the producer wishes to change the ration several times in the third or upper weight interval, square root function iv provides a basis for such predictions. Curvilinear isoclines for square root function iv allow marginal rates of substitution to change along ration lines in this weight interval. Hence, the least-cost rations predicted from function iv also change as weight increases, i. e., these rations are slightly higher in protein for the first part of the upper weight interval and slightly lower in protein for the latter part of the interval than the least-cost rations predicted from Cobb-Douglas function iii. To illustrate, substitution rates predicted from square root function iv are shown in table A for turkey gains of 4.50, 9.00 and 12.75 pounds, starting from a weight of 6.93 pounds (the average poult weight at 12 weeks).

$$\text{(iv)} \quad Y = -2.8884 + 0.0450C - 0.2966S + 0.9894\sqrt{C} + 2.4592\sqrt{S} + 0.1284\sqrt{CS}$$

With soybean oilmeal priced at 4.3 cents per pound and corn at 2.5 cents per pound, the price ratio of 4.3/2.5 or 1.72 specifies that, for 4.50-pound gains in the third weight interval (approximately a 11.43-pound total weight per turkey), the least-cost ration contains 18 percent protein. However, for this price ratio and a gain of 12.75 pounds in the third weight interval, a ration with somewhat more than 14 percent protein represents the least-cost feed combination. If the soybean oilmeal/corn price ratio falls to 1.33, a 20-percent protein ration is lowest in cost for gains of 4.50 pounds; approximately a 16-percent protein ration is lowest in cost for 12.75 pounds of gain (i. e., from an initial weight of 6.93 pounds at 12 weeks of age). Data presented in tables and graphs of the text allow specification of least-cost rations for a large number of bird weights and price ratios.

Having determined the least-cost ration for the various weight intervals, an important remaining question is one of estimating the optimum marketing weights. Optimum marketing weights are predicted by equating the marginal productivity of the feed for a particular ration with the feed/turkey price ratio. Square root function iv above was used in predicting optimum marketing

TABLE A.

Percent protein in ration	Marginal rates of substitution of soybean oilmeal for corn* for the following gains, from a starting weight of 6.93 lbs.		
	4.50 lbs.	9.00 lbs.	12.75 lbs.
12	5.39	4.42	3.86
14	3.23	2.43	1.95
16	2.28	1.56	1.13
18	1.72	1.05	0.64
20	1.33	0.70	0.30

*Pounds of corn replaced by 1 pound of soybean oilmeal.

weights shown in the text for many alternative turkey/feed price ratios. The figures in table B show optimum marketing weights for a few selected rations and price ratios. Under various turkey/feed price ratios of the past 15 years, it was found that optimum marketing weights may vary from 15 to 24 pounds (the practical marketing weight range for flocks containing half toms and half hens). Since feed consumption is particularly great as maturity approaches, considerable increases in profits may result from selling the turkey flock at the optimum marketing weight.

TABLE B

Turkey/feed price ratio	Percent protein in ration			
	14	16	18	20
6.0	15.2	15.4	15.1	14.6
6.4	16.3	16.3	15.9	15.2
6.8	17.4	17.2	16.6	15.8
7.2	18.5	18.2	17.4	16.3
7.6	19.7	19.1	18.1	16.9
8.0	20.9	20.1	18.8	17.4
8.8	23.6	22.1	20.3	18.5

Least-Cost Rations and Optimum Marketing Weights for Turkeys[1]

PRODUCTION FUNCTIONS, GAIN ISOQUANTS, SUBSTITUTION RATIOS, LEAST-COST RATIONS AND OPTIMUM MARKETING WEIGHTS FOR TURKEYS FED CORN AND SOYBEAN OILMEAL IN A FORTIFIED RATION

BY EARL O. HEADY, STANLEY BALLOUN AND GERALD W. DEAN

In recent years turkey production has become an enterprise of increasing importance in the United States. Annual output increased from 33,572,000 birds produced in 1940 to 63,066,000 birds produced in 1955. In the same span of time, Iowa turkey production more than doubled, with approximately 4,449,000 birds produced in 1955. Generally, the turkey enterprise is operated under highly specialized conditions with efficient management of great importance in both production and marketing. Where farm turkey production is on a relatively large scale, small savings in production costs or small increases in marketing margins per bird can have important effects on total profits.

A major problem in turkey production is to attain the least-cost ration. Because of the relatively heavy marketing weights for turkeys, as compared with chickens, the cost of the ration becomes of particular importance. While supplementation of turkey rations with vitamins and minerals is extremely important, these ingredients contribute relatively little to the total feed cost. By far the greatest portion of the cost of these rations is made up of (1) carbohydrates derived mainly from grains such as corn and (2) protein derived mainly from sources such as soybean oilmeal. With a relatively large feed input per bird, plus the fact that some turkeys approach maturity before marketing, there is considerable opportunity for substitution between feed categories toward the end of the production period.

LOGIC AND OBJECTIVES

This study is the second in a series dealing with production functions, feed substitution rates and least-cost rations in poultry production. The first study dealt with chicken broilers and included an explanation of the logic in determining least-cost rations.[2] Basically, the determination of a least-cost ration requires, first, estimation of the production function and, second, estimation of gain isoquants[3] and marginal rates of substitution between the major feed categories. Once marginal rates of substitution between feeds have been estimated, these quantities are compared with the inverse feed price ratio to determine the least-cost feed combination. With the marginal rate of substitution of soybean oilmeal for corn defined as the derivative, dC/dS along a gain isoquant, the minimum cost feed combination is attained for the particular gain under the condition of equation 1. The symbol C refers to pounds of corn and S to pounds of soybean oilmeal; P_s refers to the price per pound of soybean oilmeal and P_c to the price per pound of corn.[4] Since the details of these conditions are outlined in the earlier study on chicken broilers, they will not be repeated here.[5]

$$(1) \quad -\frac{dC}{dS} = -\frac{P_s}{P_c}$$

The basic objectives of this study are to predict least-cost rations and most profitable, or optimum, marketing weights for turkeys. As necessary information in attaining this final objective, the first step is one of predicting the feed-gain production function. From the production function, in turn, it is necessary to predict the gain isoquants showing the possible combinations of feed which will produce turkeys of a specified

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[1] Project 1135, Iowa Agricultural Experiment Station.

[2] See: Heady, Earl O., Balloun, Stanley and McAlexander, Robert. Least-cost rations and optimum marketing weights for broilers. Iowa Agr. Exp. Sta. Res. Bul. 442. 1956.

[3] A gain isoquant is an equal-gain or isogain contour, i.e., a gain isoquant represents the various combinations of corn and soybean oilmeal which will produce a particular gain.

[4] Since the gain isoquants and the price ratio lines are negatively sloping, a minus sign is attached to each side of equation 1.

[5] Heady, Balloun and McAlexander. Least-cost rations and optimum marketing weights for broilers. op. cit.

weight. From the gain isoquants can be predicted marginal substitution rates between the two major feed categories included in this study. Isoclines may also be computed, which, when related to feed price ratios, allow indication of rations which produce each pound of gain at lowest cost.[6] Finally, marginal rates of transformation of feed into gain can be used to predict optimum marketing weights.

The emphasis of this study is on feed substitution and least-cost rations in the latter part of the production period for turkeys. Emphasis is placed on this portion of the production period because daily and total feed intake is greatest then; major savings in cost can be made during this period when feed consumption is largest. Also, it is during this period that the greatest opportunity exists for substitution between feed categories with least restriction in the growth of poults. Accordingly, the experiment was designed mainly to allow prediction of the production surface for the upper weight range (based on observations in the 12-24 week portion of the turkey production period). Though only a limited number of observations were obtained at lighter weights, it was also possible to predict production surfaces for weight intervals based on observations in the 0-6 week and 6-12 week periods of production. The authors look upon the estimates for lighter weights as having limitations which do not attach to the predictions for heavier weights. However, it is believed that these estimates are generally of more value for feeding recommendations than data which heretofore have been available.

Production functions and feed substitution possibilities involve corn and soybean oilmeal as the central resources of decision. However, opportunities for feed substitution exist primarily when the ration is properly fortified with the vitamins, minerals and trace ingredients explained later.

EXPERIMENT AND BASIC DATA

This study is based on Experiment 322 conducted by the Department of Poultry Husbandry in which 600 Bronze turkey poults were fed on alternative rations for a 24-week period from June 10, 1955 to Nov. 25, 1955. At the start of the experiment, the 600 turkeys were randomly allotted to 48 different pens of 12 or 13 birds each; individual pens contained approximately half males and half females. Eight pens (or replicates) of birds were fed on each of six protein rations (21, 23, 25, 27, 29 and 31 percent) for the first 6 weeks of the experiment.[7] At 6 weeks, the 600 birds were completely re-randomized into 24 pens of 24 to 25 birds each, with four pens of birds fed on each of six protein rations (15, 17, 19, 21, 23 and 25 percent) for the 6-12 week period. At the end of 12 weeks, the 600 birds were again re-randomized into 24 pens, with four pens fed on each of six protein rations (10, 12, 14, 16, 18 and 20 percent) for the 12-24 week period of the experiment.

Previous experiments with broilers and hogs indicated that there were no important cumulative or "carry-over" effects of previous protein rations in meat production. That is, a bird or animal fed one percentage of protein in an early period and a different percentage of protein in a later period tended, after a short adjustment period, to perform in the later period as if it had received the second protein level throughout the entire production period.[8] For example, in the broiler study cited, statistical analysis indicated no significant difference occurred in gains for later periods between (1) birds carried through the entire production period on a single ration and (2) birds changed to this ration from one containing another percentage of protein. A comparable analysis for the present study was not possible, since no turkeys were fed on one protein ration for the entire 24-week period. However, it is not expected that the switch in rations causes outcomes to differ in later periods.

The average weight per poult at the start of the experiment was 0.11 pound, with each bird weighed thereafter at 3, 6, 12, 16, 20 and 24 weeks of age. The average gain per bird and the corresponding average feed inputs per bird were computed for each treatment and pen; these quantities provide the observations used in the regression analysis which follows.

Table 1 indicates the ingredients included in the various protein rations used for the 0-6 week period. Corn and soybean oilmeal were combined in various proportions with a fixed "basic" ration of other ingredients to provide protein levels ranging from 21 to 31 percent. Table 2 shows the rations used for the 6-12 week period; the "basic" ration remained the same, while the quantities of

TABLE 1. QUANTITIES OF VARIOUS INGREDIENTS REQUIRED FOR 100 POUNDS OF DIFFERENT PROTEIN RATIONS.*

Ingredients	Percent protein in ration					
	21	23	25	27	29	31
Corn	45	40	35	30	25	20
Wheat middlings	10	10	10	10	10	10
Bran	5	5	5	5	5	5
Soybean oilmeal	15	20	25	30	35	40
Fish meal	5	5	5	5	5	5
Meat scraps	5	5	5	5	5	5
Alfalfa meal	5	5	5	5	5	5
Dried whey	5	5	5	5	5	5
Minerals	4	4	4	4	4	4
Vitamin mix	1	1	1	1	1	1

*These rations were fed to turkeys from 0 to 6 weeks of age.

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[6] An isocline is a line passing along the production surface which indicates equal marginal rates of substitution between corn and soybean oilmeal for different weights. That is, an isocline connects the points on successive gain isoquants which indicate a given substitution rate.

[7] Four of the eight replicates for each protein level received 0.1 percent of lysine in addition to the specified ration. There is some indication that lysine has a significant influence on gains. However, for the purposes of this study, all eight replicates are treated as if fed on exactly the same ration.

[8] See: Heady, Balloun and McAlexander, op cit. Also, see: Heady, Earl O., Woodworth, Roger, Catron, Damon V. and Ashton, Gordon C. New procedures in estimating feed substitution rates and in determining economic efficiency in pork production. Iowa Agr. Exp. Sta. Res. Bul. 409. 1954.

TABLE 2. QUANTITIES OF VARIOUS INGREDIENTS REQUIRED FOR 100 POUNDS OF DIFFERENT PROTEIN RATIONS.*

Ingredients	Percent protein in the ration					
	15	17	19	21	23	25
Corn	60	55	50	45	40	35
Wheat middlings	10	10	10	10	10	10
Bran	5	5	5	5	5	5
Soybean oilmeal	0	5	10	15	20	25
Fish meal	5	5	5	5	5	5
Meat scraps	5	5	5	5	5	5
Alfalfa meal	5	5	5	5	5	5
Dried whey	5	5	5	5	5	5
Minerals	4	4	4	4	4	4
Vitamin mix	1	1	1	1	1	1

*These rations were fed to turkeys from 6 to 12 weeks of age.

TABLE 3. QUANTITIES OF VARIOUS INGREDIENTS REQUIRED FOR 100 POUNDS OF DIFFERENT PROTEIN RATIONS.*

Ingredients	Percent protein in the ration					
	10	12	14	16	18	20
Corn	78	72	66	60	54	48
Wheat middlings	10	10	10	10	10	10
Soybean oilmeal	0	6	12	18	24	30
Meat scraps	2.5	2.5	2.5	2.5	2.5	2.5
Alfalfa meal	2.5	2.5	2.5	2.5	2.5	2.5
Minerals	6	6	6	6	6	6
Vitamin mix	1	1	1	1	1	1

*These rations were fed to turkeys from 12 to 24 weeks of age.

corn and soybean oilmeal were varied to provide rations ranging from 15 to 25 percent in protein. Table 3 shows the composition of the rations used for the 12-24 week period. The "basic" ration was changed in this interval, with corn and soybean oilmeal combined to provide protein levels ranging from 10 to 20 percent in the ration.

DERIVATION OF PRODUCTION FUNCTIONS

The production functions used for later predictions are regression equations for each interval over which specific rations were fed. Because of the way in which birds were reallocated to different rations, it was impossible to predict a single over-all production function. There was opportunity only for predicting either gain isoquants or production functions based on observations for each of the age intervals of 0-6, 6-12 and 12-24 weeks. Practical use of substitution data is consistent with estimation of production functions over particular intervals because (1) rations which average lowest in cost for the total gain in the weight interval (but which do not necessarily represent the lowest cost for each ounce of gain) can be so predicted and (2) producers prefer to change the ration only a few times over the total production period (i. e., one ration is selected for an interval of time, then a shift is made to another ration to be used for some time, etc.). Because of the restricted number of weighings and the fact that there was little difference in gains among the rations over the lighter weight ranges, an alternative method was devised as a check on the accuracy of the production functions fitted to the 0-6 and 6-12 week observations. However, a greater number of weighings and considerable difference in rates of gain and total gain among rations caused this check to be unnecessary for functions fitted to the 12-24 week observations.

A problem of autocorrelation arises in estimating the production function within each weight interval where several measurements were taken from each pen of birds. To have independent observations along a particular ration line, it would be necessary to feed different pens of birds on each ration, with each pen being weighed and used only once as an observation showing gains forthcoming from particular levels of feed input (in contrast to the method used whereby the same pen was employed in prediction of gains associated with several levels of feed input within an interval). The autocorrelation presents problems mostly for probability statements and fiducial limits, rather than in prediction of mean gains and substitution rates. That is, the presence of autocorrelation does not present problems of predicting the relationship between the dependent and independent variables, but does introduce problems in making tests of significance. The effect of autocorrelation is to reduce the effective number of independent observations; the number of degrees of freedom which can be used for tests of significance in uncorrelated series is greater than it is when autocorrelation is present. Hence, a problem exists in specifying the number of degrees of freedom upon which probability statements should be based. Calculation of the autocorrelation coefficient and approximation of the effective number of degrees of freedom can be avoided by basing significance tests on a minimum number of observations (to which the series would be reduced by calculating the autocorrelation coefficient). Since the observations on different pens are independent, the number of noncorrelated observations generally is equal to the number of pens. Where a null hypothesis is rejected using this minimum number of degrees of freedom, it would certainly be rejected for the greater number of degrees of freedom represented by all observations in the series.[9]

INTERVAL FUNCTIONS

Because each pen of birds was fed on a constant protein ration from 12 to 24 weeks, it was possible to fit a production surface to this particular interval with greatest confidence. The 12-24 week interval is one in which feed consumption is great and is also the relevant period for marketing the birds. Thus, estimates were made of optimum marketing weights, as well as of least-cost rations,

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[9] The null hypotheses mentioned are of the following type: "The independent variable is not significant in predicting the value of the dependent variable." If this hypothesis is rejected for a given number of degrees of freedom, it would always be rejected for a greater number of degrees of freedom.

using interval functions fitted to the observations in the 12-24 week period.

A limited amount of information was available for the period up to 12 weeks of age. Only two weighings per pen (at 3 and 6 weeks) were made before the birds were re-randomized and the protein levels changed at the end of 6 weeks; only the 12-week weighing was made in the 6-12 week interval before the birds were re-randomized and the protein levels changed again at 12 weeks. Hence, because of the limited number of observations available at lighter weights, two alternative methods were used in obtaining estimates for the 0-6 week and 6-12 week periods. The first method attempted to predict, in the usual manner, the entire production surface for each interval; from these surfaces isoquants and marginal rates of substitution between corn and soybean oilmeal were obtained. However, since the available observations tended to be "clustered," a second or alternative method was devised as a check on the production surfaces. With this alternative procedure, gain isoquants were computed for the average turkey weights at 3, 6 and 12 weeks, i. e., the isoquants were computed directly from the adjusted data, rather than being derived from a previously estimated production surface. Marginal rates of substitution between corn and soybean oilmeal were then obtained along the "directly computed" isoquants. The check procedure simply involved comparing particular gain isoquants derived by the two methods for (1) consistency of slopes or marginal rates of substitution and (2) consistency with respect to the various feed combinations required to produce the specified gains.

A limitation of the alternative procedure involving direct estimation of the gain isoquants is in deciding whether to minimize sums of squares relative to corn or protein. Generally, corn inputs have been derived as a function of protein inputs where direct estimation of gain isoquants is involved. Since the alternative procedure is not used for predictive purposes, but only as a check on the reasonableness of the production surface, this limitation is not particularly serious. The data for the 12-24 week period are adequate for obtaining a reliable estimate of the production surface. Hence, the alternative procedure described above is not used for this period.

BASIS FOR SELECTION OF FUNCTION

The primary consideration in the selection of a production function is that the mathematical characteristics or restrictions of the function must fit the biological relationships involved. It is well known, for example, that as turkeys (and other birds and animals) increase in size, a greater percentage of the feed consumed is used for maintenance and a smaller percentage is used for growth. Thus, one of the requirements of the production function is that it must allow decreasing productivity to each unit of feed input, whether the feed unit is a combination of feeds or one feed alone. It is also known that as more protein is included in the ration, each pound of protein replaces less corn. Thus, a second requirement of the production function is that it must allow diminishing marginal rates of substitution between corn and soybean oilmeal. Another biological fact is that, for rapid gains, young birds require a greater percentage of protein in the ration than do older birds, i. e., the protein requirements of the birds change with increased size. Therefore, a third requirement of the production function is that it must allow substitution rates between feeds to change as the birds gain in weight and must result in isoclines which either are curved or which are linear and do not pass through the origin.

Production functions of the types 2 and 3 below fulfill all of the mathematical requirements stated in the preceding paragraph. The Cobb-Douglas function 4 permits decreasing productivity to the feeds and diminishing rates of substitution between feeds but does not allow substitution rates to change, for a particular ration, as birds increase in weight.

$$(2)\quad Y = a + b_1C + b_2S + b_3C^2 + b_4S^2 + b_5CS$$

$$(3)\quad Y = a + b_1C + b_2S + b_3\sqrt{C} + b_4\sqrt{S} + b_5\sqrt{CS}$$

$$(4)\quad Y = a\,C^{b_1}\,S^{b_2}$$

In equations 2, 3 and 4 and in all equations throughout the text, the symbols C, S and Y refer to pounds of corn, soybean oilmeal and gain per bird, respectively, within a specified weight interval. Each of the functions 2, 3 and 4 may have certain advantages over the other functions depending upon the particular problem being considered. For example, a turkey producer may wish to feed only one ration which "averages lowest in cost" over the entire production period (however, such predictions are not made in this study). The Cobb-Douglas function is appropriate for this purpose because the substitution rates between feeds along any ration line are then constant, i. e., the isoclines are linear passing through the origin of the feed plane.[10] Equating the feed price ratio with the slope of the gain isoquants gives one "average" least-cost ration for the entire growing period. Separate functions (with new origins for the feed plane) can be used for the different weight intervals to indicate the "average" least-cost ration for each particular interval. Also, use of the interval functions largely eliminates the problem of "overprediction" by the Cobb-Douglas function for large feed inputs. If the producer is interested in frequent changes of rations during the production period, however, equations 2 and 3 allow more precise estimates of the least-cost rations than the Cobb-Douglas function.

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[10] For added details on this point, see: Heady, Balloun and McAlexander, op. cit. Also, see: Heady, Earl O., Catron, Damon V., McKee, Dean E., Ashton, Gordon C. and Speer, Vaughn C. New procedures in estimating feed substitution rates and in determining economic efficiency in pork production. Iowa Agr. Exp. Sta. Res. Bul. to be published.

REGRESSION EQUATIONS FOR THE 0-3 WEEK INTERVAL

Only the observations obtained from the 3-week weighing were available for use in predictions over the 0-3 week interval. At this early stage in the development of the bird, very little difference (absolute or relative) occurred in the gains for birds fed on various protein rations. A regression equation which predicts gain as a function of the two categories of feed inputs gives a low coefficient of determination (R value).[11] Therefore, the alternative procedure of fitting gain isoquants directly to the data was used. With this procedure, all gains for the 0-3 week interval were adjusted to the average 3-week gain of 0.57 pound; total feed quantities associated with each gain were then adjusted in the same proportion and direction as the gain adjustment. Regression equations 5 and 6 were fitted to this adjusted data, where corn is expressed as a function of soybean oilmeal. These equations predict (for 0.57 pound of gain) the quantity of corn consumed as a function of the quantity of soybean oilmeal fed. In equations such as 5 and 6, where corn is predicted as a function of soybean oilmeal consumption, the symbols C and S refer to the pounds of corn and soybean oilmeal consumed relative to a given gain (0.57 pound in this case). In later equations, where gain is predicted as a function of the feed inputs, the symbols C and S refer to total quantities of corn and soybean oilmeal required to produce any specified gain.

$$C = 0.1671\ S^{-0.7413} \tag{5}$$

$$C = 0.8141 - 1.5800\ S + 0.9757\ S^2 \tag{6}$$

Table 4 shows the coefficient of determination (R), multiple correlation coefficient (R²) and the Student-t values for the regression coefficients in equations 5 and 6. The R and Student-t values for both equations are significant at the 1-percent level, and little difference occurs in the R values for the two equations. Given the limitations mentioned previously, either equation may be used, on a probability basis, for predicting substitution rates between corn and soybean oilmeal along the 0.57-pound gain isoquant. In addition to statistical "fit," the logic of nutritional requirements and practicality of feeding operations become the basis on which selection of a function is made.

TABLE 4. STUDENT-t, R AND R² VALUES FOR REGRESSION EQUATIONS 5 AND 6, USING ADJUSTED 3-WEEK GAIN AND FEED DATA.

Equation	Value of R	Value of R²	Student-t values for regression coefficients in the order shown in equations 5 and 6	
5	0.9648†	0.9308	24.93†	
6	0.9705‡	0.9420	6.56‡	2.85‡

†Significant at the 1-percent level with 46 degrees of freedom.
‡Significant at the 1-percent level with 45 degrees of freedom.

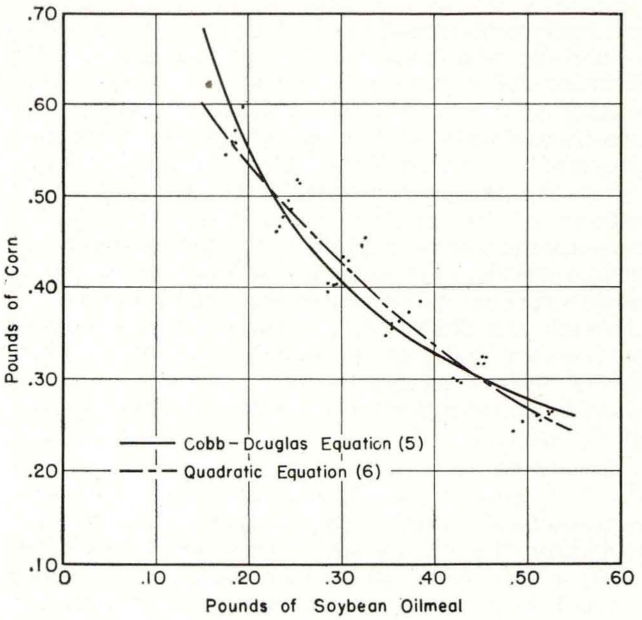

Fig. 1. Comparison of 0.57-pound gain isoquants, predicted by regression equations 5 and 6.

Figure 1 shows the 0.57-pound gain isoquants computed from regression equations 5 and 6 plotted against the adjusted observations. It should be pointed out that the dots of the scatter diagram in fig. 1 represent the adjusted feed quantities required to produce 0.57 pound of gain for the various rations and pens. In this case both equations, which predict corn consumption as a function of soybean oilmeal consumption, have been fitted to these adjusted observations. Thus, the comparison of "closeness of fit" of the isoquants to the dots in fig. 1 is relevant. However, in later cases where a function is fitted as a production surface, the isoquants computed from this surface cannot be compared directly with the adjusted dots.

REGRESSION EQUATIONS FOR THE 0-6 WEEK INTERVAL

Production functions 7, 8 and 9 were fitted to the accumulated feed and gain quantities for the 0-6 week period. Since the average gain per poult at 6 weeks of age varied considerably with the protein ration fed, it was possible to fit a production surface indicating gain as a function of the corn and soybean oilmeal fed. Equation 7 is a Cobb-Douglas function of this type. Functions 8 and 9 were fitted by the same process explained for the 0.57-pound gain isoquants at 3 weeks of age, i. e., the gain and feed quantities used were adjusted to the mean gain of 2.33 pounds for the 6-week period.

$$Y = 1.7167\ C^{0.1422}\ S^{0.3647} \tag{7}$$

$$C = 1.9512\ S^{-0.7506} \tag{8}$$

$$C = 3.3915 - 1.6659\ S + 0.2668\ S^2 \tag{9}$$

Table 5 presents the R, R² and Student-t values

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[11] The function $Y = 0.9162\ C^{0.2005}\ S^{0.2620}$, which was obtained for the 0-3 week interval, has a low R value of 0.7230.

for regression equations 7, 8 and 9. While the R values for equations 8 and 9 are larger than the R value for equation 7, it should be noted that equation 7 is a production surface computed from unadjusted data while equations 8 and 9 are gain isoquants computed from adjusted data. For equation 8 the proper interpretation of R is that 96.56 percent of the sum of squares of the adjusted corn quantities is explained by the soybean oilmeal variable. For equation 7, however, R should be interpreted as meaning that 92.95 percent of the sum of squares of the true gains is explained by the corn and soybean oilmeal variables. Thus, the R values of regression equations 8 and 9 should not be compared directly with the R value of equation 7.

Regression equations 8 and 9 predict directly the 2.33-pound gain isoquants (i. e., isoquants representing a total turkey weight of 2.44 pounds, including the 0.11-pound initial weight). However, when using production function 7, which represents a surface or family of isoquants, an isoquant equation must be derived for predicting the 2.33-pound gain isoquant. Isoquant equation 10 is derived from production function 7; the 2.33-pound gain isoquants resulting from equations 8,

$$(10) \quad C = \left[\frac{Y}{1.7167\, S^{0.3647}} \right]^{2.2614}$$

9 and 10 are presented in fig. 2. While the isoquant computed from equation 10 does not appear to fit the observations as well as the isoquants of equations 8 and 9, the scatter dots shown in fig. 2 have been adjusted to a constant gain of 2.33 pounds and thus functions 8 and 9 are partially "forced into a better fit." Hence, the isoquant computed from equation 10 should not be compared, in "closeness of fit to the scatter dots," with the isoquants from equations 8 and 9. The relevant comparison for equation 10 would be that of a family of gain isoquants compared with a set of unadjusted observations. It is expected that the 2.33-pound gain isoquant (i. e., an isoquant representing a total weight of 2.44 pounds, including the 0.11-pound initial weight) derived from equation 10 fits the unadjusted data better than either equation 8 or 9. Since equation 7 is a production surface based on unadjusted data, it will be used for predictive purposes; equations 8 and 9 provide some check on the reliability of this surface. The high degree of consistency between the 2.33-pound gain isoquants fitted by the three different equations (fig. 2) provides a basis for increased confidence in using equation 7 for predicting marginal rates of substitution and least-cost rations for the 0-6 week interval.

TABLE 5. STUDENT-t, R AND R^2 VALUES FOR REGRESSION EQUATIONS 7, 8 AND 9, USING 6-WEEK GAIN AND FEED DATA.*

Equation	Value of R	Value of R^2	Student-t values for regression coefficients in the order shown in equations 7, 8 and 9	
7	0.9295†	0.8640	12.44†	16.83†
8	0.9656‡	0.9324	24.95‡	
9	0.9727†	0.9461	6.94†	3.21†

*Equation 7 is computed from the actual quantities of feed and gain for the 0-6 week period; equations 8 and 9 are computed from feed and gain quantities which have been adjusted to a gain of 2.33 pounds.

†Significant at the 1-percent level with 45 degrees of freedom.

‡Significant at the 1-percent level with 46 degrees of freedom.

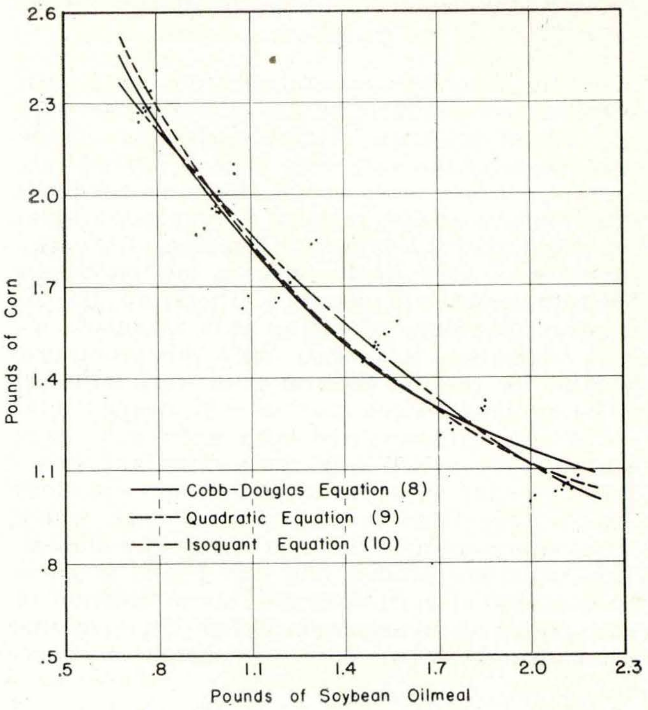

Fig. 2. Comparison of 2.33-pound gain isoquants, predicted by regression equations 8 and 9 and by isoquant equation 10.

REGRESSION EQUATIONS FOR THE 6-12 WEEK INTERVAL

Regression equations 11 and 12 were fitted to the data obtained at the 12-week weighing, i. e., to the gains and feed quantities for the 6-12 week

$$(11) \quad Y = 1.7291\, C^{0.4998}\, S^{0.2531}$$

$$(12) \quad C = 9.6199 - 3.3851\, S + 0.5219\, S^2$$

interval. Thus, predicted gains for the 6-12 week period are measured from a starting weight of 2.44 pounds, the average turkey weight at 6 weeks. Equation 11 predicts a production surface for the 6-12 week period computed from unadjusted data, with gain the dependent variable and corn and soybean oilmeal the independent variables. Equation 12, however, was computed by the alternative check procedure of deriving a gain isoquant directly from adjusted data, i. e., the equation uses observations adjusted to an average gain of 4.45 pounds over the 6-12 week interval. (The 4.45-pound gain isoquant for the 6-12 week interval represents a total turkey weight of approximately 6.89 pounds; 2.44 pounds weight at 6 weeks plus 4.45 pounds gain.) Again, equation

TABLE 6. STUDENT-t, R AND R^2 VALUES FOR REGRESSION EQUATIONS 11 AND 12, USING FEED AND GAIN DATA FOR THE 6-12 WEEK PERIOD.*

Equation	Value of R	Value of R^2	Student-t values for regression coeffcients in the order shown in equations 11 and 12	
11	0.9664†	0.9340	5.64†	14.50†
12	0.9812‡	0.9628	11.59‡	5.44‡

*Equation 11 was fitted to actual data for the 6-12 week period; equation 12 was fitted to feed and gain quantities adjusted to a constant gain of 4.45 pounds for the 6-12 week period.

†Significant at the 1-percent level with 17 degrees of freedom.

‡Significant at the 1-percent level with 21 degrees of freedom.

12 does not predict a production surface, but only a 4.45-pound gain isoquant in the 6-12 week period. Comparison of this isoquant with the 4.45-pound gain isoquant derived from equation 11 provides a check on the production surface predicted from equation 11.

Observations for four pens of birds fed on a 15-percent protein ration were omitted in fitting the Cobb-Douglas function 11. The four observations were not used because, at the 15-percent protein level, the quantity of soybean oilmeal in the ration is zero, i. e., the observation points fall directly on the corn axis. One of the mathematical restrictions of the Cobb-Douglas function is that the gain isoquants cannot intersect either the corn or the soybean oilmeal axis, i. e., the isoquants must be asymptotic to both axes.[12] The quadratic function 12 allows the 4.45-pound gain isoquant (4.45 pounds gain in the 6-12 week period) to intersect the corn and soybean oilmeal axes. This equation, then, using all of the observations for the 6-12 week period (including those for the 15-percent ration) adjusted to a common gain of 4.45 pounds was computed as a check on function 11. The Student-t, R and R^2 values for the regression equations 11 and 12 are given in table 6.

Equation 13 is the isoquant equation derived from production function 11. The 4.45-pound gain isoquants (average gain from 6 to 12 weeks) derived from isoquant equation 13 and directly from equation 12 are plotted in fig. 3. These two contours have quite consistent slopes except at the extreme upper ends (for protein rations of 15 to 17 percent). The influence of the adjusted observations for the 15-percent protein ration (falling on the corn axis) forces the upper portion of the isoquant from function 12 down, relative to the isoquant from equation 13. However, the divergence is probably exaggerated because gains are extremely low on the 15-percent protein ration. Hence, considerable inaccuracy may arise in the method of adjusting the gain and feed data for this ration to a common gain of 4.45 pounds for the 6-12 week period. For example, assume that 5.0 pounds of corn (with no soybean oilmeal) is required to produce 2.225 pounds of gain on the 15-percent ration. Using the adjustment procedure, 10.0 pounds of corn are then assumed to produce 4.45 pounds of gain, a doubtful conclusion. Diminishing returns to corn are more consistent with nutritional theory than the constant returns used in the above adjustment. Thus, the observations for the 15-percent ration should probably fall at greater quantities on the corn axis, forcing the upper portion of the isoquant from equation 12 to become more consistent with the isoquant from equation 13. Remember that the dots of the scatter diagram in fig. 3 are not the observations to which equation 11 is fitted; these dots represent only the adjusted means to which equation 12 was fitted. Observations for equation 11 would have, if they could be presented simply, a scatter more consistent with the 4.45-pound gain contour derived from isoquant equation 13. Or, again, the relevant comparison for equation 13 would be a family of contours related to the set of unadjusted observations. Because an entire production surface for the 6-12 week period is given by equation 11, this function will serve as a basis for prediction in the 6-12 week interval.

$$(13) \quad C = \left[\frac{Y}{1.7291\, S^{0.2531}} \right]^{2.0008}$$

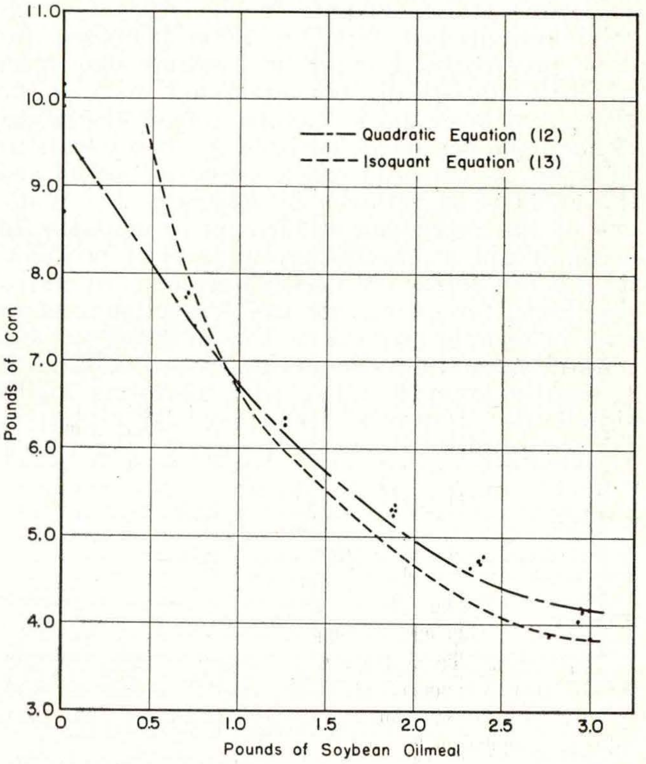

Fig. 3. Comparison of 4.45-pound gain isoquants (gains on birds weighing 2.44 pounds), predicted by quadratic equation 12 and by isoquant equation 13.

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[12] An alternative method was devised in an attempt to use the 15-percent protein ration observations. A very small quantity of soybean oilmeal (1-percent of the ration) was assumed for the 15-percent ration in order that no observation points would fall directly on the corn axis. Because the observation points were extremely close to the corn axis, however, the shape of the gain isoquants was distorted when these observations were used.

REGRESSION EQUATIONS FOR THE 12-24 WEEK INTERVAL

The data for the 12-24 week period is adequate for estimation of an interval production surface, with gain as a function of the two feed categories. Hence, the simple contour equations estimated as a check procedure for the 0-6 week and 6-12 week periods have not been computed for the 12-24 week interval. Three different types of functions were fitted to the gain and feed data for the 12-24 week interval; a Cobb-Douglas function (14), a square root quadratic function (15) and a quadratic crossproduct function (16). The gains predicted are those beyond the average 12-week weight of 6.93 pounds.

$$(14)\quad Y = 1.0764\, C^{0.5108}\, S^{0.2517}$$

$$(15)\quad Y = -2.8884 + 0.0450C - 0.2966S + 0.9894\sqrt{C} + 2.4592\sqrt{S} + 0.1284\sqrt{CS}$$

$$(16)\quad Y = 0.0148 + 0.1838C + 0.8837S + 0.0001C^2 - 0.0214S^2 - 0.0040CS$$

The 12-24 week observations for the 10-percent protein ration (with no soybean oilmeal included) are not used in computing the Cobb-Douglas function (14) for the reason given previously; use of observations falling on the corn axis distorts the gain isoquants. However, all the data are used in computing functions 15 and 16.

Table 7 shows the Student-t, R and R^2 values for regression equations 14, 15 and 16. Equations 14 and 15 will be used in predicting economic quantities at later points in the study. Quadratic crossproduct function 16 is not used for later predictions because it contains one term which is statistically non-significant even at the 50-percent level and it explains less of the deviations of the dependent variable, Y, than equations 14 or 15, as shown by the R values of table 7 and the analysis of variance in Appendix A. While one of the regression coefficients in equation 15 is significant at the 20-percent level of probability, all five terms are used, on grounds of nutrition logic, for estimating the production surface and for making predictions Dropping the non-significant term and re-computing the equation gives a slightly lower R value, with all terms highly significant. However, predictions differ only by minute quantities when the term is or is not used.

TABLE 7. STUDENT-t, R AND R^2 VALUES FOR REGRESSION EQUATIONS 14, 15 AND 16, USING FEED AND GAIN DATA FOR THE 12-24 WEEK PERIOD.

Equation	Value of R	Value of R^2	Student-t values for regression coefficients in the order shown in equations 14, 15 and 16				
14	0.9918†	0.9838	29.70†	21.10†			
15	0.9968‡	0.9936	1.40§	9.12‡	3.80‡	15.23‡	2.87‡
16	0.9864‡	0.9730	4.96‡	14.71‡	0.16††	7.36‡	1.67§

†Significant at the 1-percent level with 17 degrees of freedom.
‡Significant at the 1-percent level with 18 degrees of freedom.
§Significant at the 20-percent level with 18 degrees of freedom.
††Non-significant at the 50-percent level with 18 degrees of freedom.

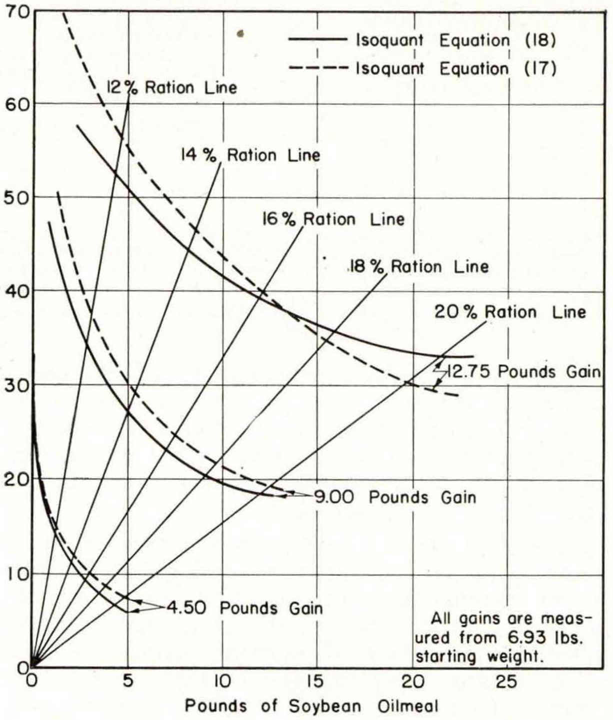

Fig. 4. Comparison of gain isoquants, predicted by isoquant equations 17 and 18.

Gain isoquant equations for the three production functions 14, 15 and 16 are shown, respectively, in equations 17, 18 and 19. Equations 17

$$(17)\quad C = \left(\frac{Y}{1.0764\, S^{0.2517}}\right)^{1.9577}$$

$$(18)\quad C = \left(-10.9933 - 1.4267\sqrt{S} \pm 11.1111\sqrt{0.0699\, S - 0.1886\sqrt{S} + 0.1800\, Y + 1.4988}\right)^2$$

$$(19)\quad C = -712.5504 + 15.4961\, S \pm 3875.9690\sqrt{0.000027\, S^2 - 0.0019\, S + 0.0005\, Y + 0.0338}$$

and 18 were used in predicting the gain isoquants of fig. 4, which shows three pairs of isoquants for the average turkey gain (in the 12-24 week interval) at 16, 20 and 24 weeks of age. The contours shown in fig. 4 are for gains starting from an average weight of 6.93 pounds at 12 weeks of age.

In connection with the isoquants of fig. 4, it should be remembered that the Cobb-Douglas function requires equal slopes for all isoquants along any straight line through the origin (ration line). Hence, this function tends to "average out" fluctuations over the input-output surface. The square root and quadratic functions are not subject to the restriction of constant slope along ration lines. Consequently, these types of functions provide a closer "fit" to data which are not consistent with the assumptions of constant slopes of isoquants at the points where they are intersected by any one ration line. The above restriction on the Cobb-Douglas function helps explain the difference in slopes along the two 12.75-pound gain isoquants shown in fig. 4. At high protein levels, to conform to the above-mentioned restriction, the isoquant computed from the Cobb-Douglas function is "pulled down" relative to the isoquant computed from the square root function. The slopes of contours from the two functions are quite similar at the lower protein levels; least-cost rations predicted from them would also be similar. If interest is in predicting a least-cost ration which changes with increasing weight within the 12-24 week interval, the square root function should be used since it allows the slope of the isoquants to change along a ration line. Too, it expresses, as is generally believed to be the case, lower rates of substitution of soybean oilmeal for corn as the bird approaches maturity.

Comprehensive tables of least-cost rations computed from Cobb-Douglas function 14 are given in a following section since it is believed that the majority of turkey producers are interested in a single "average" least-cost ration to be fed for the entire 12-24 week interval. However, because turkey production is becoming more and more a specialized enterprise, an increasing number of producers are interested in changing rations more frequently to obtain small savings in feed costs per bird. Appendix table B-1 shows substitution rates computed from square root function 15 for each isoquant level of fig. 4. It should serve as a guide to producers interested in changing the corn/soybean oilmeal proportions of the ration three times within the 12-24 week interval. The instructions accompanying Appendix table B-1 indicate the least-cost rations for the various gains are predicted by locating the substitution rates which most nearly equal the soybean oilmeal/corn price ratio.

The degree of conformity of the three functions 14, 15 and 16 to the data is suggested in figs. 5 through 9; input-output curves computed from the three regression equations are plotted against the data for protein rations of 12, 14, 16, 18 and 20 percent. The input-output curve for Cobb-Douglas function 14 falls below the data for 24 weeks of age on the 12-percent protein ration (see fig. 5).[13] However, for the 16-, 18- and 20-percent protein rations, the input-output curves for equation 14 predict greater gains than are shown by the dots representing the 24-week data (see figs. 7, 8 and 9). The input-output curves for quadratic function 16 fall below the 16-, 20- and 24-week observations along the 12- and 14-percent protein ration lines (see figs. 5 and 6). With 18- and 20-percent protein rations, however, equation 16 tends to overestimate at the 20-week observations and underestimate at the 24-week observa-

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[13] The three clusters of dots on each of figs. 5 through 9 represent the observations at the 16-, 20- and 24-week weighing dates.

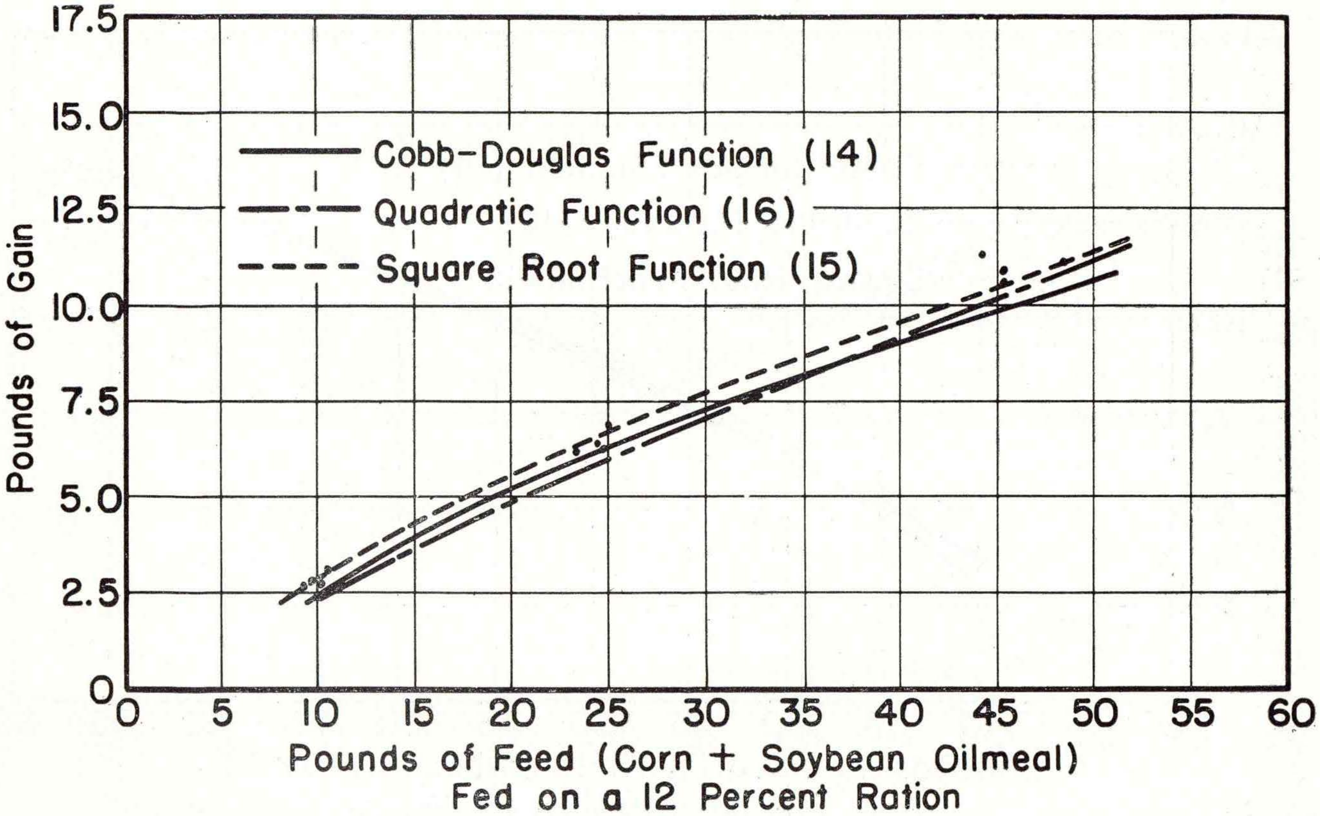

Fig. 5. Comparison of input-output curves for turkeys on a 12-percent protein ration, predicted by regression equations 14. 15 and 16.

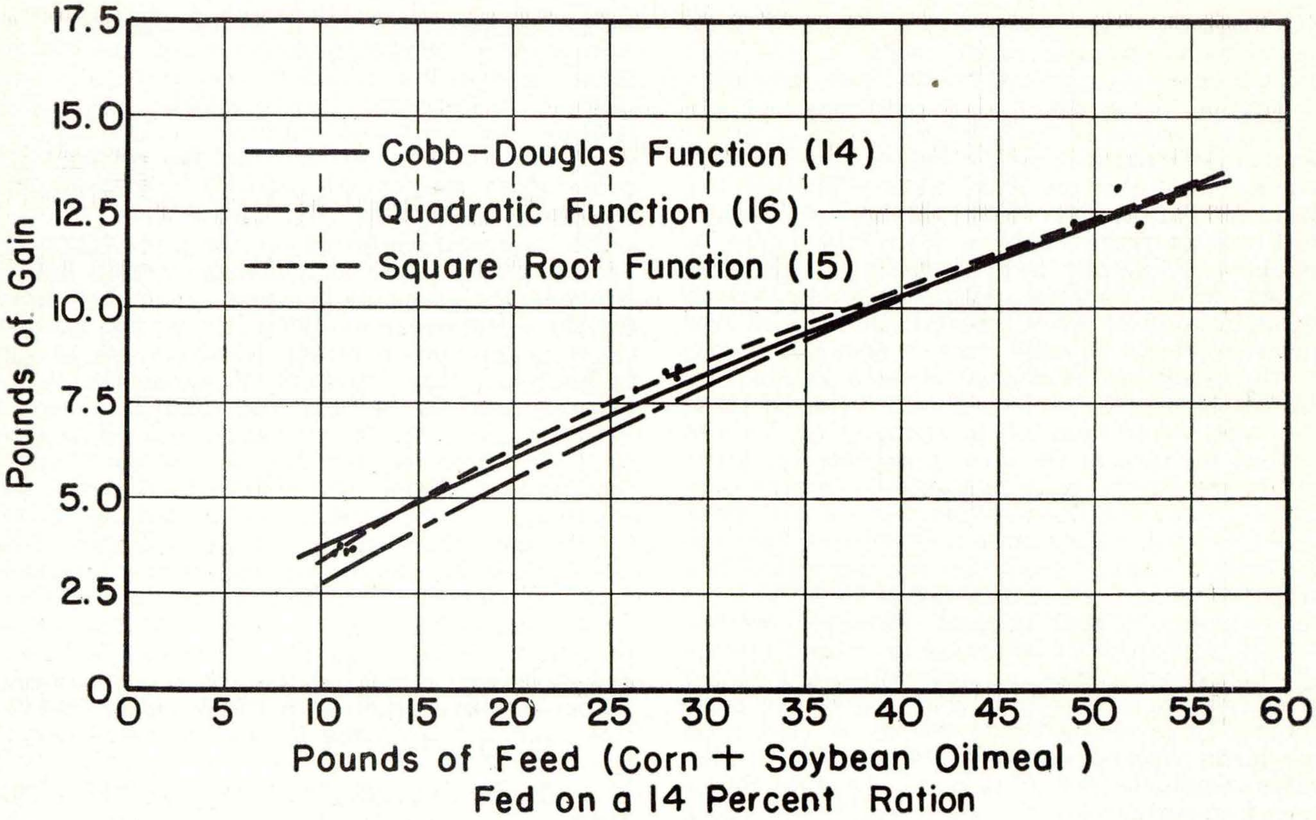

Fig. 6. Comparison of input-output curves for turkeys on a 14-percent protein ration, predicted by regression equations 14, 15 and 16.

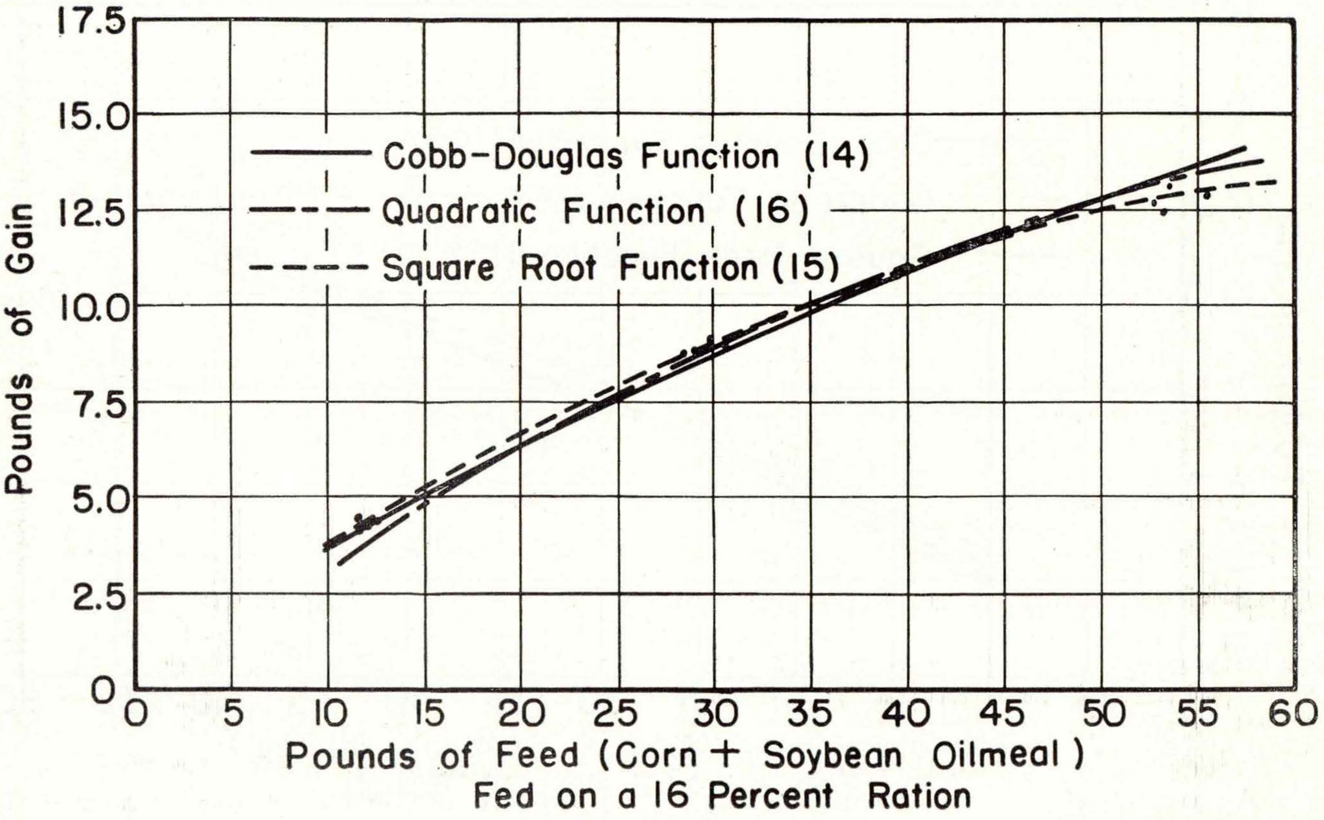

Fig. 7. Comparison of input-output curves for turkeys on a 16-percent protein ration, predicted by regression equations 14, 15 and 16.

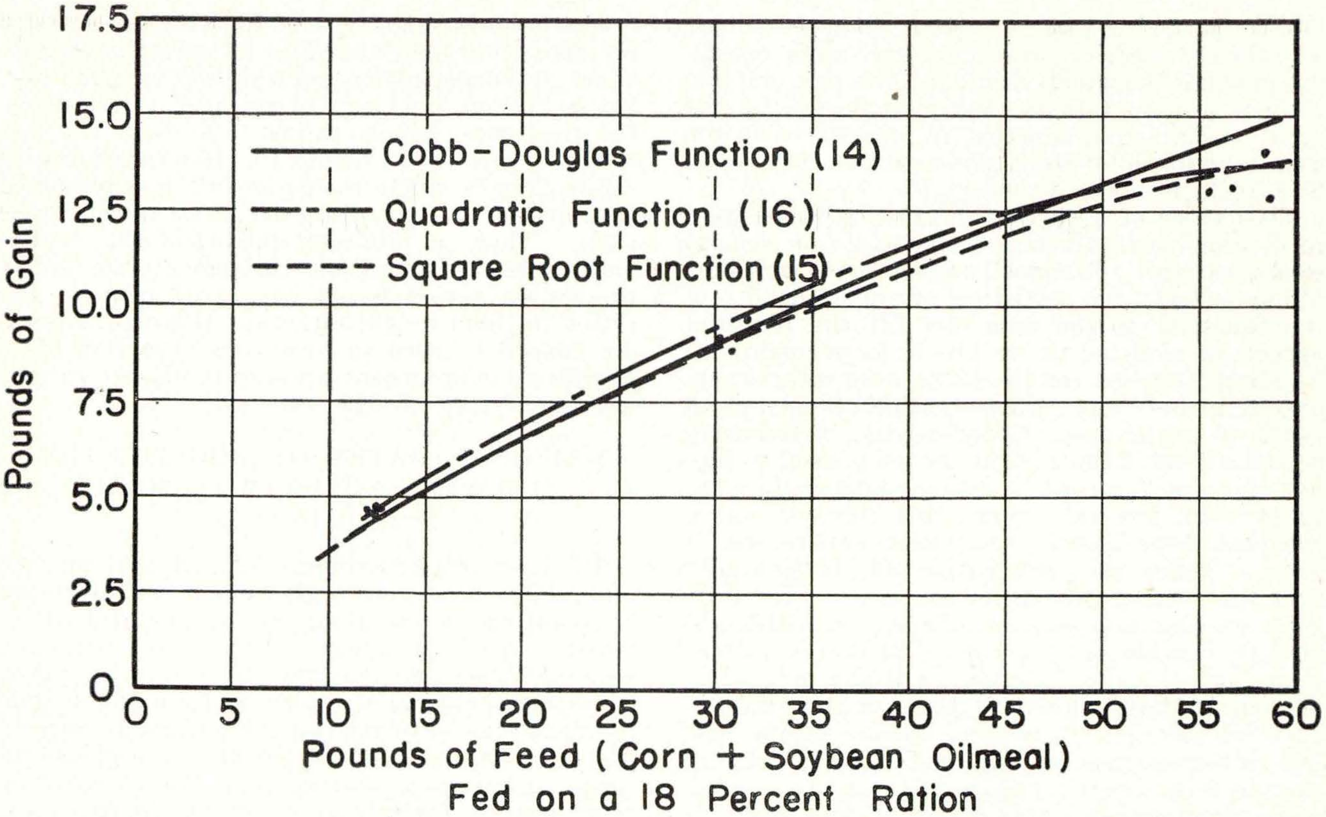

Fig. 8. Comparison of input-output curves for turkeys on an 18-percent protein ration, predicted by regression equations 14, 15 and 16.

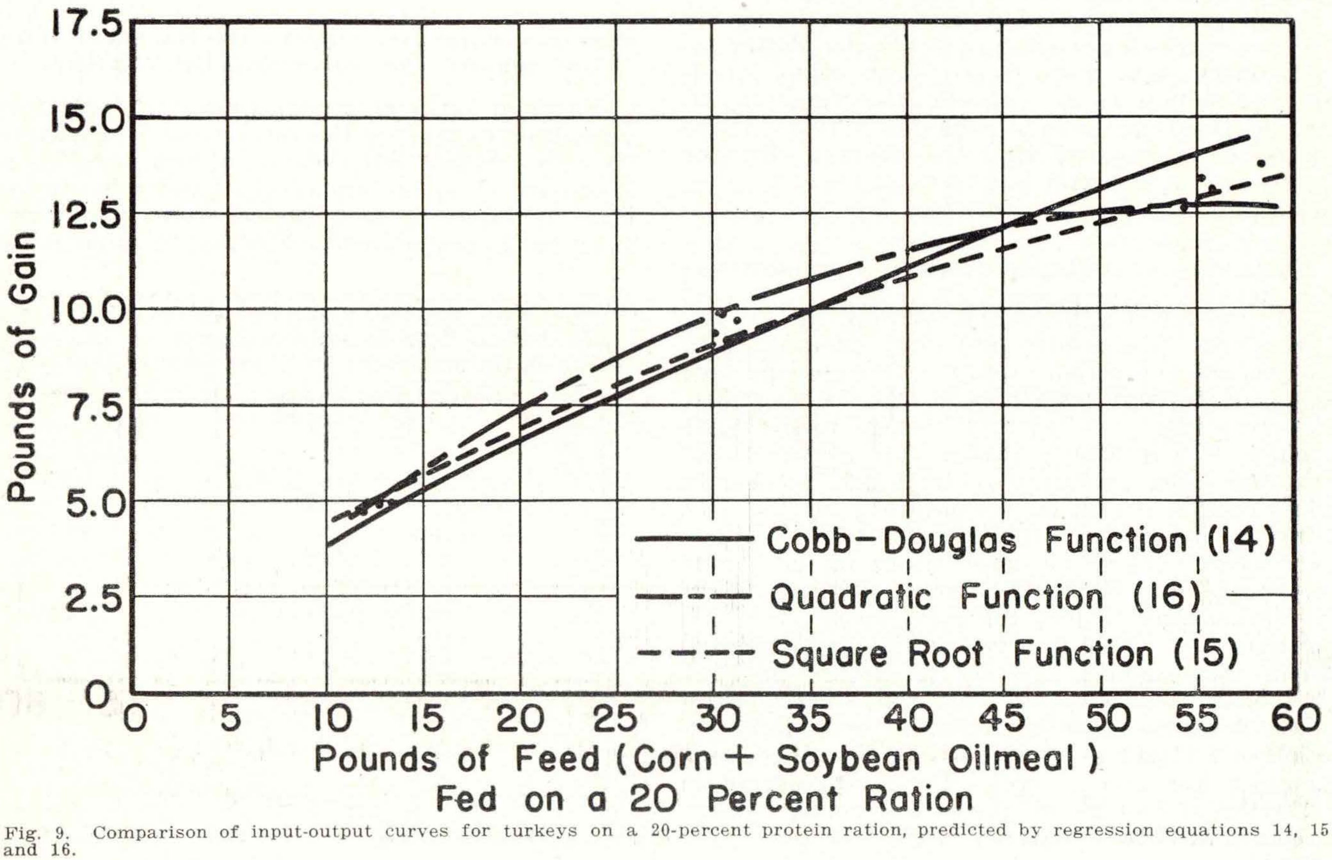

Fig. 9. Comparison of input-output curves for turkeys on a 20-percent protein ration, predicted by regression equations 14, 15 and 16.

tions (see figs. 8 and 9). Thus, it appears that equation 16 predicts more curvature (more rapidly diminishing marginal gains) at the 18- and 20-percent protein rations than is indicated by the data. Square root function 15 appears to fit the data best for all of the protein rations from 12 to 20 percent (see figs. 5 through 9).

The choice of a particular function to be used in making various estimates is based on several considerations. Foremost among these considerations are (1) the statistical and plotted "fit" of the functions to the data and (2) the practical aspects of applying the results to farm conditions. No single function for the 12-24 week interval appeared to best meet these considerations for all types of predictions. Consequently, in following sections, Cobb-Douglas function 14 is used to predict the best "average" least-cost rations over the 12-24 week interval. Since this function has a constant slope along a particular ration line, it gives a single least-cost ration, as an "average" over the feeding period, for any given price ratio between corn and soybean oilmeal. Functions 15 and 16 provide more accurate least-cost rations than Cobb-Douglas function 14 if the percent of protein is changed several times within the 12-24 week interval. However, because of the cost and inconvenience of frequently adjusting the protein level within a relatively short time period, many producers probably prefer to feed only one ration in the upper weight range.

While Cobb-Douglas function 14 is used to predict least-cost rations in the 12-24 week interval, square root function 15 is used in predicting optimum marketing weights: It fits the plotted input-output data more closely than either quadratic function 16 or Cobb-Douglas function 14. While the Cobb-Douglas function appears to be satisfactory in predicting the average slope or curvature of the gain isoquants, it tends to overestimate the slope for large feed inputs.

SUMMARY OF FUNCTIONS USED FOR PREDICTIONS

As explained in preceding sections, the production period was divided into three intervals, with a production function fitted to the gains and feed quantities within each interval. The three weight intervals used are (a) from initial weight (0.11 pound) to 2.44 pounds (based on observations for the 0-6 week period), (b) from 2.44 pounds to 6.93 pounds (based on observations for the 6-12 week period) and (c) from 6.93 pounds to marketing weight (based on observations for the 12-24 week period). It should be noted that the weight intervals used do not, for every ration, conform exactly to the time intervals of the production period. For example, on a low (21-percent protein) ration in the first weight interval, slightly more than 6 weeks would be required to produce turkeys of 2.44 pounds liveweight. On the other hand, slightly less than 6 weeks would be required to produce an average weight of 2.44 pounds with a 31-percent protein ration.

The Cobb-Douglas type of function is employed for predicting marginal rates of substitution and, hence, least-cost rations within each weight interval; equations 7, 11 and 14 are used, respectively, for the three weight ranges. Because the isoclines for the Cobb-Douglas function coincide with ration lines, a single least-cost ration is predicted within each weight interval for a given price ratio. Thus, a producer following the recommended least-cost ration throughout the entire production period would use three rations—one ration for each weight interval. However, Appendix table B-1, based on square root function 15, is provided for producers wishing to change rations within the upper weight interval.

MARGINAL RATES OF SUBSTITUTION BETWEEN SOYBEAN OILMEAL AND CORN

The least-cost combination of corn and soybean oilmeal for producing a given gain is determined by equating the marginal rate of substitution between feeds with their inverse price ratio. In general terms, the marginal rate of substitution of soybean oilmeal for corn along a given gain isoquant may be defined as the pounds of corn replaced by the addition of 1 pound of soybean oilmeal. Specifically, the marginal rate of substitution between feeds is given by the slope of the gain isoquant, or, by the first derivative of the regression equation used in predicting the gain isoquant.

FEED QUANTITIES FOR ISOGAINS AND MARGINAL RATES OF SUBSTITUTION FOR PARTICULAR WEIGHTS

Marginal rates of substitution and total feed quantities to produce the total gains over each of the three weight intervals are shown in tables 8, 9 and 10. Isoquant equation 10 and substitution rate equation 20, both derived from production function 7, provide the estimates in table 8 for

TABLE 8. COMBINATIONS OF CORN AND SOYBEAN OILMEAL REQUIRED TO PRODUCE TURKEYS WEIGHING 2.44 POUNDS, AND MARGINAL RATES OF SUBSTITUTION BETWEEN FEEDS IN THIS WEIGHT RANGE.*

Percent protein in the ration	Lbs. of feed to produce turkeys weighing 2.44 pounds		Marginal rates of substitution of soybean oilmeal for corn
	Corn	Soybean oilmeal	
21	2.40	0.80	2.47
22	2.18	0.90	2.00
23	2.00	1.01	1.65
24	1.84	1.10	1.37
25	1.70	1.21	1.15
26	1.57	1.33	0.97
27	1.46	1.46	0.82
28	1.35	1.60	0.70
29	1.25	1.75	0.59
30	1.16	1.93	0.49
31	1.07	2.14	0.41

*The figures in this table are derived from Cobb-Douglas function 7, $Y = 1.7167\, C^{0.4422}\, S^{0.3647}$, computed from observations for the 0-6 week period.

TABLE 9. COMBINATIONS OF CORN AND SOYBEAN OILMEAL REQUIRED TO INCREASE TURKEYS FROM 2.44 POUNDS TO 6.93 POUNDS LIVEWEIGHT, AND MARGINAL RATES OF SUBSTITUTION BETWEEN FEEDS IN THIS WEIGHT RANGE.*

Percent protein in the ration	Lbs. of feed to increase turkey weight from 2.44 pounds to 6.93 pounds		Marginal rates of substitution of soybean oilmeal for corn
	Corn	Soybean oilmeal	
16	10.11	0.43	11.65
17	7.86	0.72	5.57
18	6.75	0.97	3.55
19	6.03	1.21	2.53
20	5.50	1.45	1.92
21	5.08	1.69	1.52
22	4.73	1.95	1.23
23	4.43	2.23	1.01
24	4.17	2.50	0.84
25	3.93	2.81	0.71

*The figures in this table are derived from Cobb-Douglas function 11, $Y = 1.7291\, C^{0.4997}\, S^{0.2531}$, computed from observations for the 6-12 week period.

TABLE 10. COMBINATIONS OF CORN AND SOYBEAN OILMEAL REQUIRED TO INCREASE TURKEYS FROM 6.93 POUNDS TO 19.84 POUNDS LIVEWEIGHT, AND MARGINAL RATES OF SUBSTITUTION BETWEEN FEEDS IN THIS WEIGHT RANGE.*

Percent protein in the ration	Lbs. of feed to increase turkey weight from 6.93 to 19.84 pounds		Marginal rates of substitution of soybean oilmeal for corn
	Corn	Soybean oilmeal	
11	74.03	2.96	12.32
12	58.18	4.85	5.91
13	50.17	6.54	3.78
14	44.89	8.16	2.71
15	41.09	9.79	2.07
16	38.06	11.42	1.64
17	35.58	13.11	1.34
18	33.44	14.86	1.11
19	31.57	16.71	0.93
20	29.88	18.68	0.79

*The figures in this table are derived from Cobb-Douglas function 14, $Y = 1.0764\, C^{0.5108}\, S^{0.2517}$, computed from observations for the 12-24 week period.

the first weight interval. Isoquant equation 13 and substitution rate equation 21, both derived from production function 11, provide the estimates in table 9 for the second weight interval. Isoquant equation 17 and substitution rate equation 22, both derived from production function 14, provide the estimates in table 10 for the third weight interval. The marginal rate of substitution between soybean oilmeal and corn for a particular ration and weight interval can be found by substituting the proportionate feed quantities

$$(20) \qquad -\frac{dC}{dS} = -\frac{0.3647\ C}{0.4422\ S} = -0.8247\frac{C}{S}$$

$$(21) \qquad -\frac{dC}{dS} = -\frac{0.2531\ C}{0.4997\ S} = -0.5065\frac{C}{S}$$

$$(22) \qquad -\frac{dC}{dS} = -\frac{0.2417\ C}{0.5108\ S} = -0.4928\frac{C}{S}$$

for that ration in the appropriate substitution equation, i. e., equation 20, 21 or 22. Using the Cobb-Douglas function, equal rates of substitution along a ration line are predicted for each level of gain within a weight interval, e. g., the substitution rate along a ration line in the second weight interval is found by multiplying the particular corn/soybean oilmeal ratio of the ration by the constant —0.5065 of substitution equation 21. Constant rates of substitution along ration lines within each weight interval result in constant least-cost rations within these intervals.[14]

As is expected from nutritional logic and previous knowledge, predictions from the interval functions show that, for a given ration, the marginal rates of substitution of soybean oilmeal for corn decline as the bird increases in weight. This point is shown in substitution equations 20, 21 and 22 where the constants are —0.8247, —0.5065 and —0.4928 for the three successive weight intervals. A pound of soybean oilmeal replaces 2.47 pounds of corn for turkeys fed on a 21-percent protein ration in the first weight interval (table 8); on this same ration a pound of soybean oilmeal replaces only 1.52 pounds of corn for turkeys in the second weight interval (table 9). Tables 9 and 10 indicate that with 18 percent of protein in the ration, 1 pound of soybean oilmeal replaces 3.55 pounds of corn for birds in the second weight interval, but replaces only 1.11 pounds of corn for birds in the third weight interval.[15] These results occur because the bird requires more protein relative to carbohydrates in the early growing stages and more carbohydrates relative to protein as maturity and the finishing period approaches.

Too, as each of tables 8, 9 and 10 show, the marginal rate of substitution of soybean oilmeal for corn declines as relatively more protein is included in the ration for a particular level of gain. (Or, conversely, the marginal rate of substitution of corn for soybean oilmeal declines as the ration contains relatively less protein and relatively more carbohydrates.) Since the marginal rates of substitution between the two feed inputs are diminishing, unique rations can be found which minimize the cost of feed for a particular level of gain.

Table 11 illustrates the concept of diminishing marginal gains per pound of feed as birds reach

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[14] Substitution rate equation 23 below is derived from square root function 15. Since the terms of equation 23 are nonlinear,

$$(23)\ -\frac{dC}{dS} = -\left[\frac{-0.2996 + 1.2296\ S^{-0.5} + 0.0642\ C^{0.5}\ S^{-0.5}}{0.0450 + 0.4947\ C^{-0.5} + 0.0642\ S^{0.5}\ C^{-0.5}}\right]$$

the slopes of gain isoquants along a ration line are allowed to change for the various turkey weights in the upper weight interval. Hence, the alternative rations suggested in Appendix table B-1 can be used by producers who wish to feed changing protein levels within the third or upper weight interval.

[15] Some of the difference in substitution rates for a given protein ration between the second and third weight interval (see tables 9 and 10) is due to the fact that the "basic" ration (ingredients other than corn and soybean oilmeal) is changed at the higher weight interval.

TABLE 11. COMBINATIONS OF CORN AND SOYBEAN OILMEAL REQUIRED TO PRODUCE 1 POUND OF GAIN ON TURKEYS WEIGHING 2.44, 6.93 AND 19.84 POUNDS.

Percent protein in the ration	Lbs. feed for 1 lb. gain on 2.44 lb. turkeys*		Lbs. feed for 1 lb. gain on 6.93 lb. turkeys†		Lbs. feed for 1 lb. gain on 19.84 lb. turkeys‡	
	Corn	Soybean oilmeal	Corn	Soybean oilmeal	Corn	Soybean oilmeal
11	—	—	—	—	7.71	0.31
12	—	—	—	—	6.06	0.50
13	—	—	—	—	5.22	0.68
14	—	—	—	—	4.68	0.85
15	—	—	—	—	4.28	1.02
16	—	—	3.12	0.13	3.96	1.19
17	—	—	2.43	0.22	3.71	1.37
18	—	—	2.09	0.30	3.48	1.55
19	—	—	1.86	0.37	3.29	1.74
20	—	—	1.70	0.45	3.11	1.94
21	1.34	0.45	1.60	0.52	—	—
22	1.21	0.50	1.46	0.60	—	—
23	1.11	0.56	1.37	0.68	—	—
24	1.02	0.61	1.29	0.77	—	—
25	0.95	0.68	1.21	0.87	—	—
26	0.88	0.74	—	—	—	—
27	0.81	0.81	—	—	—	—
28	0.75	0.89	—	—	—	—
29	0.70	0.98	—	—	—	—
30	0.65	1.08	—	—	—	—
31	0.59	1.19	—	—	—	—

*Feed quantities predicted from Cobb-Douglas function 7.
†Feed quantities predicted from Cobb-Douglas function 11.
‡Feed quantities predicted from Cobb-Douglas function 14.

higher weights. For example, with a 22-percent protein ration, only 1.71 pounds of feed (1.21 pounds of corn and 0.50 pound of soybean oilmeal) are required to produce an additional pound of gain on birds weighing 2.44 pounds. However, to produce an additional pound of gain on birds weighing 6.93 pounds using a 22-percent protein ration, 2.06 pounds of feed (1.46 pounds of corn and 0.60 pound of soybean oilmeal) are required. Greater quantities of feed per pound of additional gain are required as birds reach 19.84 pounds liveweight. Since a diminishing input-output relationship exists between feed and gain, it is possible to equate the slope of particular input-output curves with the feed-turkey price ratio to predict optimum marketing weights.

LEAST-COST RATIONS

The data of tables 12, 13 and 14, predicted from the substitution equations of the preceding section, provide estimates of rations which "average" least in cost (for various corn and soybean oilmeal prices) over each of the three specified weight intervals. The figures of table 12, predicted from substitution equation 20, are estimates of the least-cost rations in the 0.11-pound to 2.44-pound weight interval. Substitution equations 21 and 22 are used, respectively, to provide the least-cost estimates given in tables 13 and 14 for the 2.44-pound to 6.93-pound interval and the 6.93-pound to finished weight interval. The least-cost ration in each of these weight intervals is found by equating the marginal rate of substitution over the appropriate weight range with the inverse price ratio of the feeds, then solving for the ratio of corn to soybean oilmeal in the ration. For example, with the price of corn at $1.68 per bushel (3 cents per pound) and the price of soybean oilmeal at $4.00 per hundred pounds (4 cents per pound), the inverse price ratio is —4/3 or —1.33.[16] With this price ratio, the least-cost ration for the first weight interval is found, as shown in equations 24 and 25, by setting the substitution rate of equation 20 equal to —1.33 and solving for the proportion C/S = 1.61. The 24.0-percent protein ration most nearly contains a corn/soybean oilmeal proportion of 1.61 in the first weight interval, and hence, is the least-cost ration estimate shown in table 12 for the feed prices assumed in the example. Least-cost rations for the second and third weight intervals are determined in a similar manner, using the appropriate substitution equations. Interval Cobb-Douglas functions, which specify equal marginal rates of substitution along a given ration line, are used in the predictions of tables 12, 13 and 14; hence, only one "average" least-cost ration is given for each price ratio within a weight interval.

$$(24) \quad -0.8247\frac{C}{S} = -1.33$$

$$(25) \quad \frac{C}{S} = \frac{-1.33}{-0.8247} = 1.61$$

The estimate of tables 12, 13 and 14 may be used by turkey producers as follows: With corn at $1.23 per bushel (2.2 cents per pound), and soybean oilmeal at $4.50 per hundred pounds (4.5 cents per pound), the inverse price ratio is —4.5/2.2 or —2.05. With this price ratio, the predicted least-cost ration contains 22.0 percent protein for the first weight interval (table 12), 19.5 percent protein for the second weight interval (table 13) and 15.0 percent protein for the third weight interval (table 14). In the first weight interval, the producer might choose to feed a slightly higher level of protein than given by the least-cost ration. From a practical standpoint, savings by a least-cost ration in the first weight interval are small, and the producer might not want to risk slower gains from a low protein ration. However, in the second weight interval, and particularly in the third weight interval, substantial savings in feed costs may be realized by using a least-cost ration rather than one which produces faster gains. Under certain price relationships, of course, a least-cost ration may also produce the most rapid gains.

The producer may wish to make further adjustments within the third weight interval to reduce feed costs. For example, during the first few weeks of the third weight interval, the producer may wish to feed a slightly higher protein level than prescribed by the least-cost ration; he may wish to decrease this protein level as the birds

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[16] As pointed out previously, a negative sign is attached to the price ratio because the price line (on a typical two-dimensional diagram) is negatively sloping.

TABLE 12. PERCENT PROTEIN IN RATIONS WHICH ARE LEAST-COST FOR TURKEYS FROM 0.11 POUND TO 2.44 POUNDS, WITH VARIOUS CORN AND SOYBEAN OILMEAL PRICES.*

Price of corn in cents per pound	Price of soybean oilmeal in cents per pound														
	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00	6.25	6.50
1.4	22.0	21.5	21.5	21.0	—	—	—	—	—	—	—	—	—	—	—
1.6	22.5	22.0	21.5	21.5	21.0	—	—	—	—	—	—	—	—	—	—
1.8	23.0	22.5	22.0	22.0	21.5	21.5	21.0	—	—	—	—	—	—	—	—
2.0	23.5	23.0	22.5	22.5	22.0	21.5	21.5	21.0	21.0	—	—	—	—	—	—
2.2	24.0	23.5	23.0	23.0	22.5	22.0	22.0	21.5	21.5	21.0	21.0	—	—	—	—
2.4	24.5	24.0	23.5	23.5	23.0	22.5	22.5	22.0	22.0	21.5	21.5	21.0	21.0	—	—
2.6	25.0	24.5	24.0	24.0	23.5	23.0	23.0	22.5	22.0	22.0	21.5	21.5	21.5	21.0	21.0
2.8	25.5	25.0	24.5	24.5	24.0	23.5	23.0	23.0	22.5	22.5	22.0	22.0	21.5	21.5	21.5
3.0	26.0	25.5	25.0	24.5	24.0	24.0	23.5	23.5	23.0	23.0	22.5	22.5	22.0	22.0	21.5
3.2	26.0	26.0	25.5	25.0	24.5	24.5	24.0	23.5	23.5	23.0	23.0	22.5	22.5	22.0	22.0
3.4	26.5	26.0	25.5	25.5	25.0	24.5	24.5	24.0	23.5	23.5	23.0	23.0	22.5	22.5	22.0
3.6	27.0	26.5	26.0	25.5	25.0	25.0	24.5	24.5	24.0	23.5	23.5	23.0	23.0	23.0	22.5
3.8	27.5	27.0	26.5	26.0	25.5	25.0	25.0	24.5	24.5	24.0	23.5	23.5	23.0	23.0	23.0
4.0	27.5	27.0	26.5	26.5	26.0	25.5	25.0	25.0	24.5	24.5	24.0	24.0	23.5	23.5	23.0

*Computed from substitution equation 20, $-dC/dS = -0.8247\ C/S$.

TABLE 13. PERCENT PROTEIN IN RATIONS WHICH ARE LEAST-COST FOR TURKEYS FROM 2.44 POUNDS TO 6.93 POUNDS, WITH VARIOUS CORN AND SOYBEAN OILMEAL PRICES.*

Price of corn in cents per pound	Price of soybean oilmeal in cents per pound														
	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00	6.25	6.50
1.4	20.0	19.5	19.5	19.0	19.0	18.5	18.5	18.5	18.0	18.0	18.0	18.0	17.5	17.5	17.5
1.6	20.0	20.0	19.5	19.5	19.0	19.0	18.5	18.5	18.5	18.5	18.0	18.0	18.0	18.0	17.5
1.8	20.5	20.5	20.0	19.5	19.5	19.5	19.0	19.0	18.5	18.5	18.5	18.5	18.0	18.0	18.0
2.0	21.0	21.0	20.5	20.0	20.0	19.5	19.5	19.5	19.0	19.0	18.5	18.5	18.5	18.5	18.5
2.2	21.5	21.0	21.0	20.5	20.0	20.0	19.5	19.5	19.5	19.0	19.0	19.0	19.0	18.5	18.5
2.4	22.0	21.5	21.0	21.0	20.5	20.5	20.0	20.0	19.5	19.5	19.5	19.0	19.0	19.0	19.0
2.6	22.5	22.0	21.5	21.0	21.0	20.5	20.5	20.5	20.0	20.0	19.5	19.5	19.5	19.0	19.0
2.8	22.5	22.5	22.0	21.5	21.5	21.0	20.5	20.5	20.5	20.0	20.0	20.0	19.5	19.5	19.5
3.0	23.0	22.5	22.5	22.0	21.5	21.5	21.0	21.0	20.5	20.5	20.0	20.0	20.0	19.5	19.5
3.2	23.5	23.0	22.5	22.5	22.0	21.5	21.5	21.0	21.0	20.5	20.5	20.5	20.0	20.0	20.0
3.4	23.5	23.5	23.0	22.5	22.0	22.0	21.5	21.5	21.0	21.0	20.5	20.5	20.5	20.0	20.0
3.6	24.0	23.5	23.0	23.0	22.5	22.5	22.0	21.5	21.5	21.5	21.0	21.0	20.5	20.5	20.5
3.8	24.5	24.0	23.5	23.0	23.0	22.5	22.0	22.0	21.5	21.5	21.0	21.0	21.0	20.5	20.5
4.0	24.5	24.0	24.0	23.5	23.0	23.0	22.5	22.0	22.0	21.5	21.5	21.5	21.0	21.0	20.5

*Computed from substitution equation 21, $-dS/dC = -0.5065\ C/S$.

TABLE 14. PERCENT PROTEIN IN RATIONS WHICH ARE LEAST-COST FOR TURKEYS FROM 6.93 POUNDS TO FINISHED WEIGHT, WITH VARIOUS CORN AND SOYBEAN OILMEAL PRICES.*

Price of corn in cents per pound	Price of soybean oilmeal in cents per pound														
	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00	6.25	6.50
1.4	15.0	15.0	14.5	14.5	14.0	14.0	13.5	13.5	13.5	13.5	13.0	13.0	13.0	13.0	12.5
1.6	15.5	15.0	15.0	14.5	14.5	14.0	14.0	13.5	13.5	13.5	13.5	13.0	13.0	13.0	13.0
1.8	16.0	15.5	15.5	15.0	14.5	14.5	14.5	14.0	14.0	14.0	13.5	13.5	13.5	13.5	13.0
2.0	16.5	16.0	15.5	15.5	15.0	15.0	14.5	14.5	14.5	14.0	14.0	14.0	13.5	13.5	13.5
2.2	17.0	16.5	16.0	16.0	15.5	15.5	15.0	15.0	14.5	14.5	14.5	14.0	14.0	14.0	13.5
2.4	17.5	17.0	16.5	16.5	16.0	15.5	15.5	15.0	15.0	15.0	14.5	14.5	14.5	14.0	14.0
2.6	18.0	17.5	17.0	16.5	16.5	16.0	16.0	15.5	15.5	15.0	15.0	15.0	14.5	14.5	14.5
2.8	18.0	18.0	17.5	17.0	16.5	16.5	16.0	16.0	15.5	15.5	15.0	15.0	15.0	15.0	14.5
3.0	18.5	18.0	17.5	17.5	17.0	17.0	16.5	16.0	16.0	16.0	15.5	15.5	15.0	15.0	15.0
3.2	19.0	18.5	18.0	18.0	17.5	17.0	17.0	16.5	16.0	16.0	16.0	15.5	15.5	15.5	15.0
3.4	19.5	19.0	18.5	18.0	17.5	17.5	17.0	17.0	16.5	16.5	16.0	16.0	15.5	15.5	15.5
3.6	19.5	19.5	19.0	18.5	18.0	17.5	17.5	17.0	17.0	16.5	16.5	16.0	16.0	16.0	15.5
3.8	20.0	19.5	19.0	18.5	18.0	18.0	17.5	17.5	17.0	17.0	16.5	16.5	16.0	16.0	16.0
4.0	20.0	20.0	19.5	19.0	18.5	18.5	18.0	17.5	17.5	17.0	17.0	16.5	16.5	16.5	16.0

*Computed from substitution equation 22, $-dC/dS = -0.4928\ C/S$.

increase toward marketing weight. Appendix table B-1 provides a basis for decision-making by producers who wish to change rations within the third or upper weight interval.

Referring again to tables 12, 13 and 14, if the price of soybean oilmeal should rise to 5.5 cents per pound, with corn remaining at 2.2 cents per pound, the price ratio becomes —2.50. Least-cost rations then contain 21.0 percent protein for the first interval, 19.0 percent protein for the second interval and 14.5 percent protein for the third interval. With a price of 2.0 cents per pound for corn and 5.0 cents for soybean oilmeal, the least-cost rations also would contain 21.0, 19.0 and 14.5 percent protein since the price ratio is still —2.50.

Graphic illustration of changes in "average" least-cost rations between weight intervals for a price ratio of —2.0 is produced in fig. 10. The line passing through the origin extending to and intersecting the 2.44-pound weight isoquant represents a 22-percent protein ration line. Using the intersection point on the 2.44-pound isoquant as a new origin (circled), the least-cost ration for the second weight interval contains 20 percent

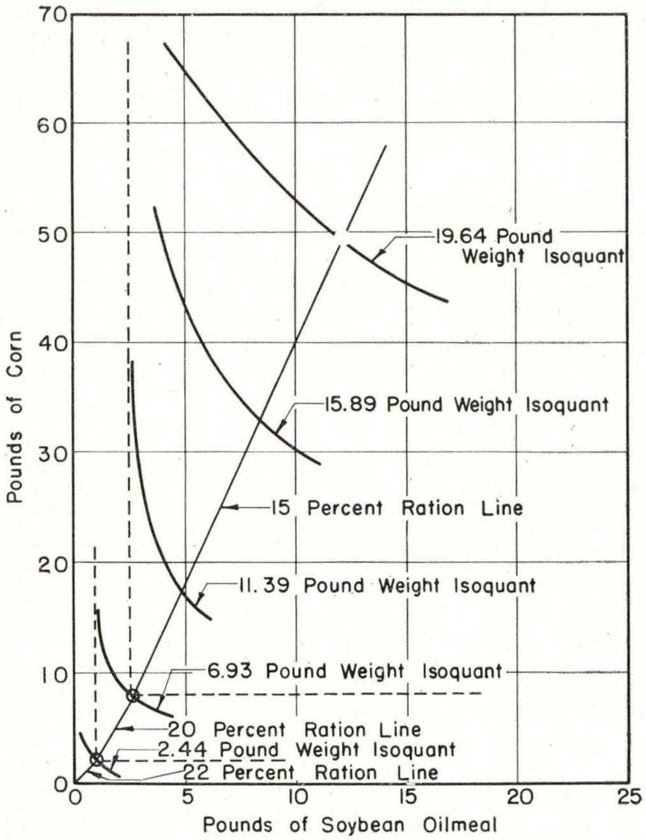

Fig. 10. Least-cost rations for three weight intervals, predicted by Cobb-Douglas functions 7, 11 and 14 with a soybean oilmeal/corn price ratio of 2.0.

protein. Again using the intersection point of the 20-percent ration line with the 6.93-pound isoquant as a new origin (circled), the least-cost ration for the third weight interval contains 15 percent protein.[17] Figure 11 illustrates the least-cost ration path (expansion path) for the entire feeding period for various price ratios. As expected from nutritional logic, the percentage of protein in the ration consistently decreases with each higher weight interval, regardless of the existing price ratio.

SIMPLE GRAPHICAL INDICATION OF LEAST-COST RATIONS

Figures 12, 13 and 14 have been included to allow simple graphical selection of least-cost rations over the three weight intervals. These graphs assume "linear segments" along gain isoquants and indicate least-cost rations for price ratios falling within the diagonal "price rays" shown. Figures 12, 13 and 14 may be used as follows: Suppose the price of corn is 2.5 cents per pound and the price of soybean oilmeal is 4.1 cents per pound. These prices are located at point A in fig. 12. Following to the right of the diagram between the two diagonal lines, it is found that the least-cost ration over the first weight interval contains 23.0 percent protein. One hundred pounds of the 23.0-percent ration may be formulated by mixing 40.0 pounds of corn, 20.0 pounds of soybean oilmeal and 40.0 pounds of the "basic" ingredients shown in table 1. The above feed prices are also found at point B, fig. 13 and specify a least-cost ration containing 20.5 percent protein for the second weight interval. One hundred pounds of a 20.5-percent ration contains 46.25 pounds of corn, 13.75 pounds of soybean oilmeal and 40.00 pounds of other "basic" ingredients shown in table 2. Point C, fig. 14, indicates a least-cost ration of only 16.0 percent protein for the third weight interval, with the corn and soybean oilmeal prices assumed. One hundred pounds of the 16.0-percent protein ration is composed of 60.0 pounds of corn, 18.0 pounds of soybean oilmeal and 22.0 pounds of other "basic" ingredients shown in table 3. The recommended rations resulting from use of figs. 12, 13 and 14 are identical with those of tables 12, 13 and 14 and are included only as a simple alternative method of presenting the same results.

Throughout the analysis, the criterion for selecting rations has been one of minimum cost. However, a ration other than the least-cost ration

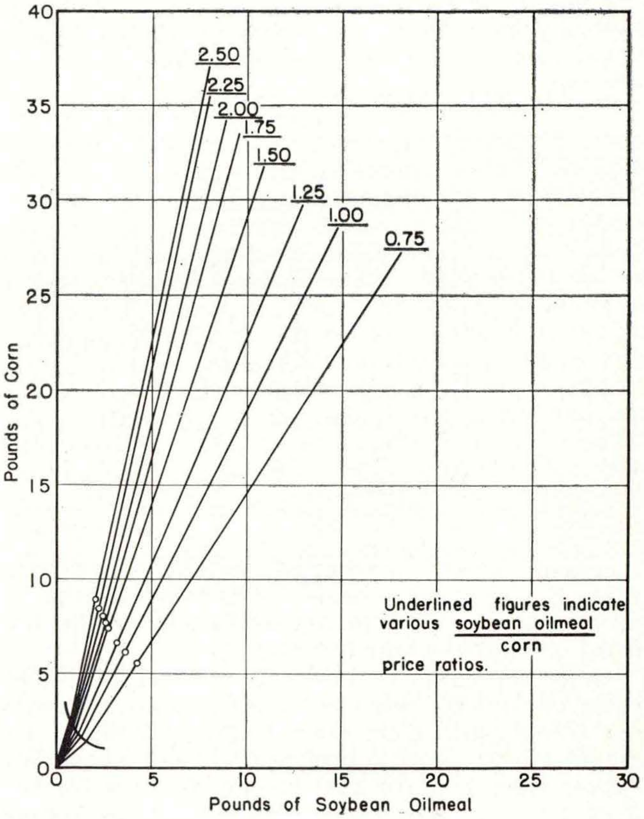

Fig. 11. Least-cost rations for three weight intervals, predicted by Cobb-Douglas functions 7, 11 and 14 with various soybean oilmeal/corn price ratios.

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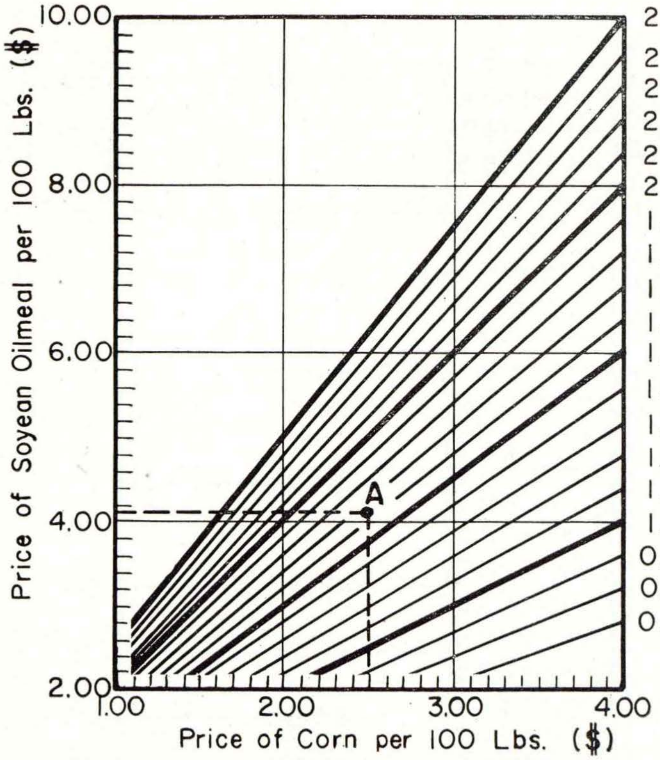
[17] The reason the slope of the 15-percent ration line in fig. 10 is not steeper relative to the 20- and 22-percent ration lines is as follows: Some of the high protein ingredients of the "basic" ration are reduced in quantity or removed entirely at the start of the third weight interval, thus requiring more soybean oilmeal relative to corn for a given percentage of protein in the ration.



Soybean Oilmeal-Corn Price Ratio	Optimum Percent Protein in the Ration	Estimated Quantities of Corn and Soybean Oilmeal per 100 Lbs. of Feed for Various Protein Rations: Corn (pounds)	Soybean Oilmeal (pounds)
2.5–2.4	21.0	45.00	15.00
2.4–2.3	21.0		
2.3–2.2	21.5	43.75	16.25
2.2–2.1	21.5		
2.1–2.0	22.0	42.50	17.50
2.0–1.9	22.0		
1.9–1.8	22.5	41.25	18.75
1.8–1.7	22.5		
1.7–1.6	23.0	40.00	20.00
1.6–1.5	23.5	38.75	21.25
1.5–1.4	23.5		
1.4–1.3	24.0	37.50	22.50
1.3–1.2	24.5	36.25	23.75
1.2–1.1	25.0	35.00	25.00
1.1–1.0	25.5	33.75	26.25
1.0–0.9	26.0	32.50	27.50
0.9–0.8	27.0	30.00	30.00
0.8–0.7	27.5	28.75	31.25

Fig. 12. Least-cost rations for the 0.11-pound to 2.44-pound weight interval, predicted by Cobb-Douglas function 7 with various corn and soybean oilmeal prices.

for prevailing prices may be better suited for producing the most rapid gains over a given weight range. For example, if the producer anticipates a fall in turkey prices, he may be interested in getting the poults to market weight as rapidly as possible, rather than in minimizing feed cost for a given gain.

OPTIMUM MARKETING WEIGHTS

The preceding sections provided estimates of the least-cost combinations of corn and soybean oilmeal over the three weight intervals. Once the least-cost ration has been determined, the next question is one of finding the most profitable, or optimum marketing weight for the turkeys. The marketing weight which maximizes returns above feed costs is determined by equating the marginal product of feed for the least-cost ration with the feed/turkey price ratio. In other words, the most profitable marketing weight above feed costs is attained under the condition of equation 26, where dY/dR is the marginal product of the particular ration, showing the amount added to gain by each small added quantity of the ration, i. e., dY/dR is the derivative of gain with respect to feed inputs predicted from the production function. In equation 26, P_t is the price per pound of turkeys and P_r is the price per pound of the ration.

$$(26) \qquad \frac{dY}{dR} = \frac{P_r}{P_t}$$

For practical purposes, it is supposed that the least-cost ration will be determined for each of the three weight intervals by the methods of the previous sections. In the third weight interval, the marginal products for the least-cost ration will be used in determining the optimum or most profitable marketing weight. Alternative methods of predicting optimum marketing weights and optimum rations for the last small increments of gain are available but are not used here since emphasis is on practical uses of the data.[18]

Square root function 15 is used in predicting

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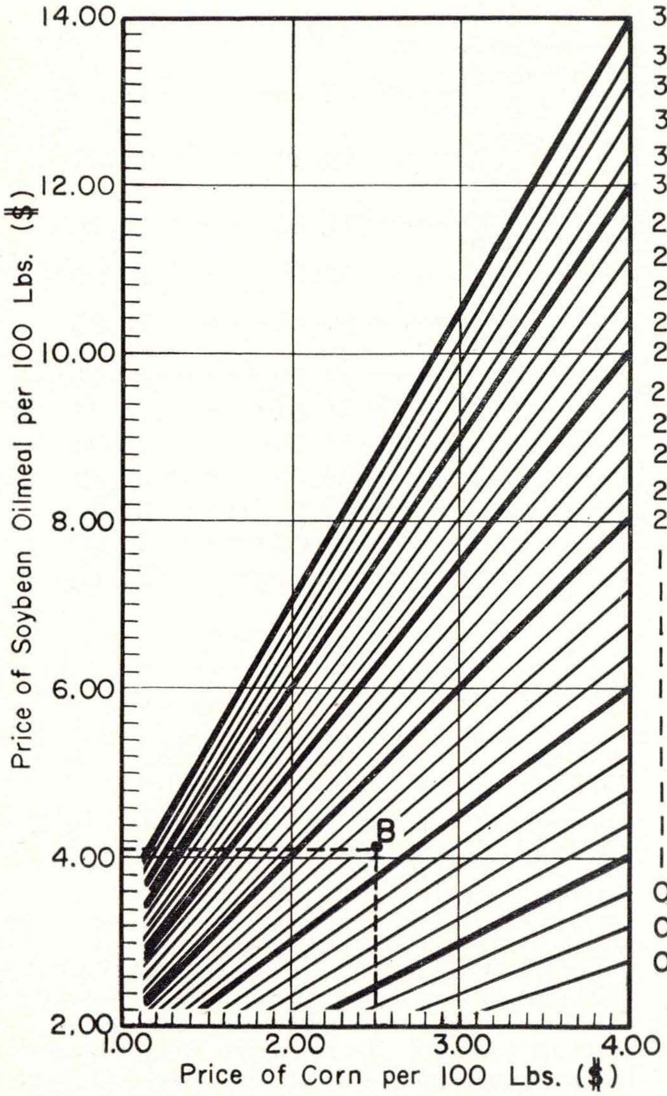
[18] The two quantities can be determined simultaneously by use of equations 27 and 28, where the terms on the left are partial derivatives with respect to the two feed categories and the terms on the right are the respective price ratios. By setting the partial derivatives equal to the particular price ratios, and solving the equations simultaneously, the optimum rations and marketing weights (i. e., from total feed inputs) can be predicted.

$$(27) \qquad \frac{\partial Y}{\partial C} = \frac{P_c}{P_t}$$

$$(28) \qquad \frac{\partial Y}{\partial S} = \frac{P_s}{P_t}$$



Soybean Oilmeal–Corn Price Ratio	Optimum Percent Protein in the Ration	Estimated Quantities of Corn and Soybean Oilmeal per 100 Lbs. of Feed for Various Protein Rations: Corn (pounds)	Soybean Oilmeal (pounds)
3.5–3.4	18.0	52.50	7.50
3.4–3.3	18.0		
3.3–3.2	18.5	51.25	8.75
3.2–3.1	18.5		
3.1–3.0	18.5		
3.0–2.9	18.5		
2.9–2.8	18.5		
2.8–2.7	18.5		
2.7–2.6	19.0	50.00	10.00
2.6–2.5	19.0		
2.5–2.4	19.0		
2.4–2.3	19.5	48.75	11.25
2.3–2.2	19.5		
2.2–2.1	19.5		
2.1–2.0	19.5		
2.0–1.9	20.0	47.50	12.50
1.9–1.8	20.0		
1.8–1.7	20.5	46.25	13.75
1.7–1.6	20.5		
1.6–1.5	21.0	45.00	15.00
1.5–1.4	21.0		
1.4–1.3	21.5	43.75	16.25
1.3–1.2	22.0	42.50	17.50
1.2–1.1	22.5	41.25	18.75
1.1–1.0	23.0	40.00	20.00
1.0–0.9	23.5	38.75	21.25
0.9–0.8	24.0	37.50	22.50
0.8–0.7	24.5	36.25	23.75

Fig. 13. Least-cost rations for the 2.44-pound to 6.93-pound weight interval, predicted by Cobb-Douglas function 11 with various corn and soybean oilmeal prices.

optimum marketing weights for turkeys fed on different rations, under a wide range of feed/turkey price ratios (the feed/turkey price ratio is the reciprocal of the turkey/feed price ratio, table 15). The square root function is used for these predictions because, as was mentioned previously, it fits the input-output observations for the various rations more closely than the other functions for the third weight interval. Table 15 indicates, for each ration and price ratio, the marketing weight which maximizes returns above feed costs. The practical marketing weight range for female birds is 12 to 18 pounds; for males the range is about 18 to 30 pounds.[19] Thus, in a mixed or "straight run" flock for which the predictions of this study apply, the practical marketing weight range is from approximately 15 to 24 pounds. Separate production functions (computed from observations on all males or all females) would be required to provide a guide to optimum marketing weights for the producer feeding a flock of predominately one sex. How-

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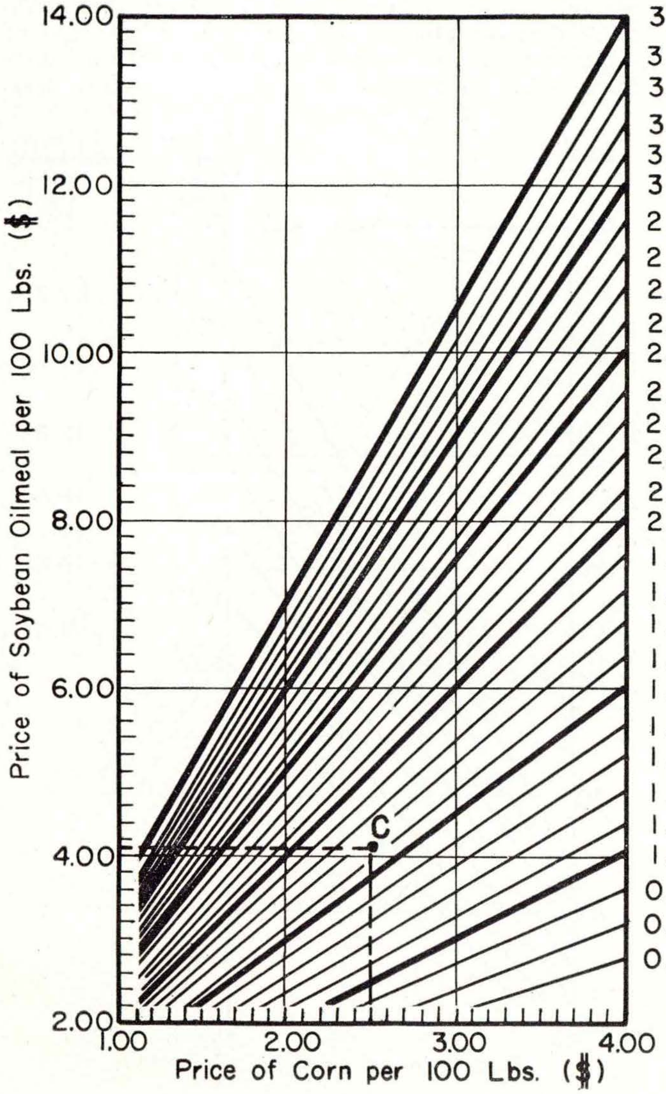
[19] Turkeys may also be sold as "broilers" in the 6- to 9-pound weight range. However, since this type of marketing program comprises a small part of the total turkey market, it is not considered in this study.

ever, the figures in table 15 should be relevant for that majority of producers who feed "straight run" flocks.

Before using table 15, the least-cost ration for the third weight interval is determined from table 14 or fig. 14 in the previous sections. Table 15 can then be used to predict, for any particular least-cost ration, the marketing weight which is optimum for a given price per pound of the ration and of the finished turkeys. Suppose that the least-cost ration for the third weight interval (predicted from table 14) contains 15.0 percent protein. If the price of turkeys is 32 cents per pound, and the price of the 15.0-percent protein ration is 4 cents per pound (a turkey/feed price ratio of 8.0), the predicted optimum marketing weight for birds on this ration is 20.5 pounds (table 15). If the turkey/feed price ratio is only 7.0, the optimum marketing weight is reduced to 17.8 pounds (table 15).

Turkey/feed price ratios outside of the 6.0 to 10.0 range shown in table 15 predict optimum marketing weights which do not fall within the practical marketing weight range for mixed flocks. However, the turkey/feed price ratios of the past 15 years have been characterized by wide fluctuations, resulting in ratios which have frequently been higher than the upper range of 10.0



Soybean Oilmeal-Corn Price Ratio	Optimum Percent Protein in the Ration	Estimated Quantities of Corn and Soybean Oilmeal per 100 Lbs. of Feed for Various Protein Rations: Corn (pounds)	Soybean Oilmeal (pounds)
3.5–3.4	13.5	67.50	10.50
3.4–3.3	13.5		
3.3–3.2	13.5		
3.2–3.1	13.5		
3.1–3.0	13.5		
3.0–2.9	13.5		
2.9–2.8	14.0	66.00	12.00
2.8–2.7	14.0		
2.7–2.6	14.0		
2.6–2.5	14.0		
2.5–2.4	14.5	64.50	13.50
2.4–2.3	14.5		
2.3–2.2	14.5		
2.2–2.1	15.0	63.00	15.00
2.1–2.0	15.0		
2.0–1.9	15.5	61.50	16.50
1.9–1.8	15.5		
1.8–1.7	15.5		
1.7–1.6	16.0	60.00	18.00
1.6–1.5	16.5	58.50	19.50
1.5–1.4	16.5		
1.4–1.3	17.0	57.00	21.00
1.3–1.2	17.5	55.50	22.50
1.2–1.1	18.0	54.00	24.00
1.1–1.0	18.5	52.50	25.50
1.0–0.9	19.0	51.00	27.00
0.9–0.8	19.5	49.50	28.50
0.8–0.7	20.0	48.00	30.00

Fig. 14. Least-cost rations for the 6.93-pound to finished weight interval, predicted by Cobb-Douglas function 14 with various corn and soybean oilmeal prices.

TABLE 15. TURKEY MARKETING WEIGHTS FOR MAXIMUM PROFITS WITH VARIOUS PROTEIN RATIONS AND TURKEY/FEED PRICE RATIOS.*

Turkey/feed price ratio	Feed/turkey price ratio	Percent protein in the ration															
		12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5	17.0	17.5	18.0	18.5	19.0	19.5	20.0
6.0	0.167	14.2	14.6	15.0	15.2	15.3	15.4	15.4	15.4	15.4	15.3	15.2	15.1	15.0	14.9	14.7	14.6
6.2	0.161	14.7	15.2	15.5	15.7	15.8	15.8	15.8	15.9	15.8	15.7	15.6	15.5	15.4	15.2	15.0	14.9
6.4	0.156	15.3	15.7	16.0	16.3	16.3	16.3	16.3	16.3	16.3	16.2	16.0	15.9	15.7	15.6	15.4	15.2
6.6	0.152	15.8	16.2	16.5	16.8	16.8	16.8	16.8	16.8	16.7	16.6	16.4	16.3	16.1	15.9	15.7	15.5
6.8	0.147	16.4	16.8	17.1	17.4	17.3	17.3	17.3	17.2	17.1	17.0	16.8	16.6	16.4	16.2	16.0	15.8
7.0	0.143	17.0	17.4	17.7	17.9	17.9	17.8	17.8	17.7	17.6	17.4	17.2	17.0	16.8	16.5	16.3	16.0
7.2	0.140	17.5	18.0	18.2	18.5	18.4	18.3	18.2	18.2	18.0	17.8	17.6	17.4	17.1	16.8	16.6	16.3
7.4	0.135	18.2	18.6	18.8	19.1	19.0	18.9	18.7	18.6	18.4	18.2	18.0	17.7	17.4	17.2	16.9	16.6
7.6	0.132	18.8	19.2	19.4	19.7	19.5	19.4	19.2	19.1	18.9	18.6	18.4	18.1	17.8	17.5	17.2	16.9
7.8	0.128	19.4	19.9	20.1	20.3	20.1	19.9	19.8	19.6	19.3	19.0	18.8	18.5	18.1	17.8	17.5	17.2
8.0	0.125	20.1	20.6	20.8	20.9	20.7	20.5	20.3	20.1	19.8	19.5	19.2	18.8	18.5	18.1	17.8	17.4
8.2	0.122	20.8	21.2	21.4	21.6	21.3	21.0	20.8	20.6	20.3	19.9	19.6	19.2	18.8	18.4	18.1	17.7
8.4	0.119	21.6	22.0	22.1	22.2	21.9	21.6	21.3	21.1	20.7	20.3	20.0	19.6	19.2	18.8	18.4	18.0
8.6	0.116	22.3	22.7	22.8	22.9	22.6	22.2	21.9	21.6	21.2	20.8	20.3	20.0	19.5	19.1	18.7	18.2
8.8	0.114	23.1	23.5	23.6	23.6	23.2	22.8	22.4	22.1	21.7	21.2	20.8	20.3	19.8	19.4	18.9	18.5
9.0	0.111	23.9	24.2	24.3	24.3	23.8	23.4	23.0	22.6	22.2	21.6	21.2	20.7	20.2	19.7	19.2	18.8
9.2	0.109	—	—	—	—	—	—	23.5	23.1	22.6	22.1	21.5	21.0	20.5	20.0	19.5	19.0
9.4	0.106	—	—	—	—	—	—	—	23.7	23.1	22.5	22.0	21.4	20.8	20.3	19.8	19.3
9.6	0.104	—	—	—	—	—	—	—	—	23.6	22.9	22.4	21.8	21.2	20.6	20.1	19.5
9.8	0.102	—	—	—	—	—	—	—	—	—	23.4	22.8	22.2	21.5	20.9	20.4	19.8
10.0	0.100	—	—	—	—	—	—	—	—	—	23.8	23.2	22.5	21.8	21.2	20.6	20.0

*Computed from square root function 15; $Y = -2.8884 + 0.0450\,C - 0.2966\,S + 0.9894\sqrt{C} + 2.4592\sqrt{S} + 0.1284\sqrt{CS}$.

shown in table 15. For example, the average turkey/feed price ratio for the United States in 1949 was 11.0, with a January high of 16.9 in Iowa. With such extreme ratios, the producer should sell his birds at the practical maximum marketing weight; about 24 pounds in the case of a mixed flock. Of course, a producer with all male birds could profitably increase this average mar-

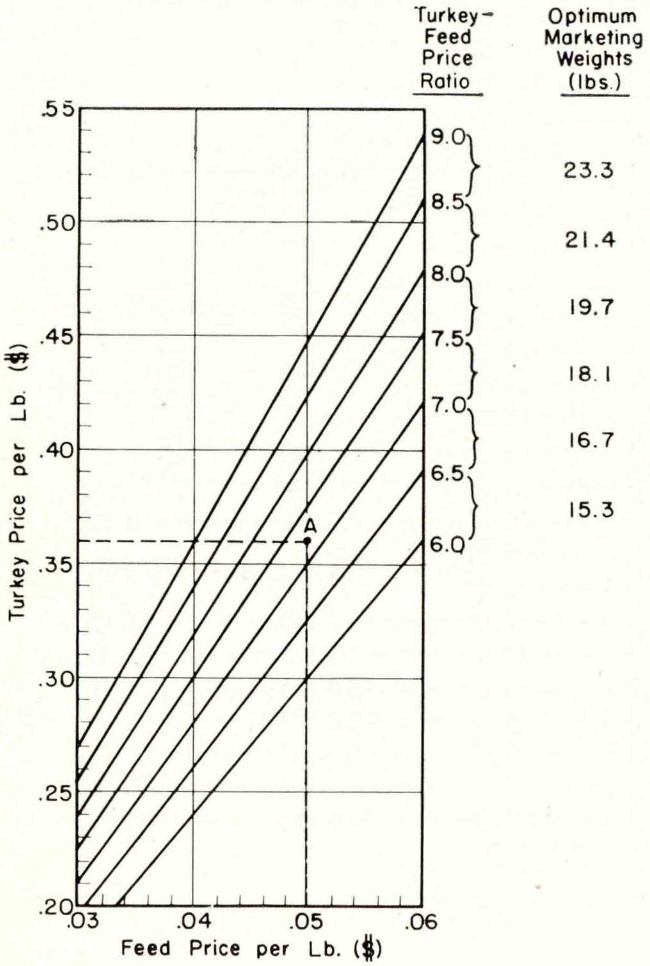


Fig. 15. Optimum marketing weights for turkeys fed on a 13-percent protein ration, predicted by square root function 15.

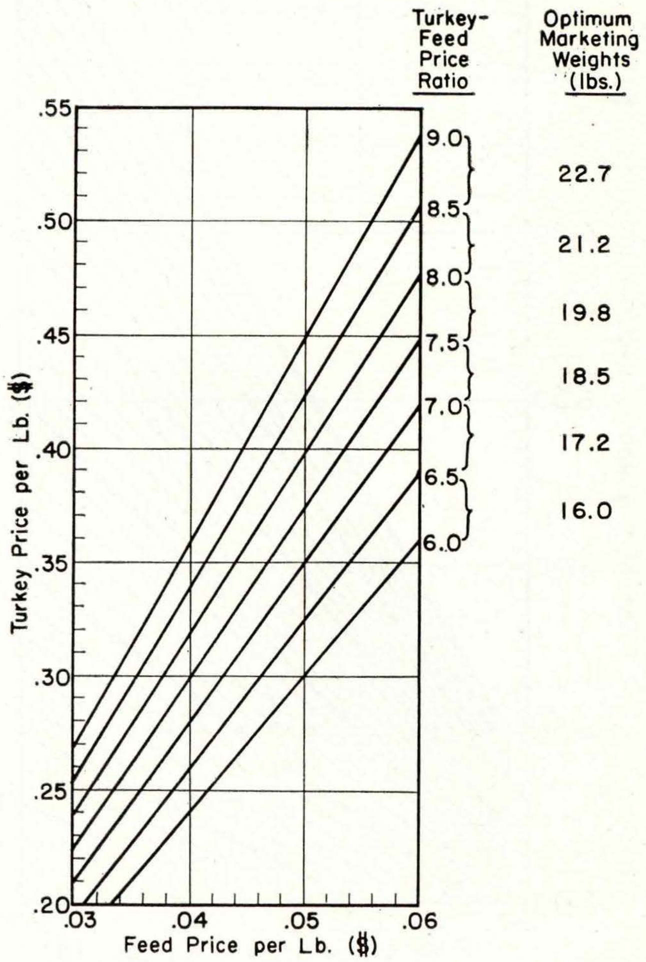


Fig. 16. Optimum marketing weights for turkeys fed on a 15-percent protein ration, predicted by square root function 15.

keting weight for his flock. On the other hand, turkey/feed price ratios below 6.0 have seldom been realized in the United States in the past 15 years, and then for very short periods of time. However, the average turkey/feed price ratio for the Pacific Coast states in 1954 was 6.4, with a low of 6.0 in some months. Assuming a turkey/feed price ratio of 6.0, the optimum marketing weight varies from 14.2 pounds on a 12.5-percent protein ration to 15.4 pounds on a 16.0-percent protein ration (table 15). Thus, given the highly flexible turkey/feed price ratios of recent years, the most profitable marketing weight above feed costs may fluctuate over the entire practical marketing weight range of the birds.

The optimum marketing weights of table 15 apply to situations in which (a) capital and labor are non-limitational, (b) risk and uncertainty are not considered and (c) returns above feed costs are maximized. Turkey producers with limited capital and labor supplies would likely market their birds previous to the weight at which marginal cost equals marginal return; use of these limited resources in the latter phases of turkey production may provide lower returns than their use in some other alternative. The risk and uncertainty associated with future prices and disease problems may also prompt the marketing of birds before returns above feed costs are maximized. However, the data of table 15 present the optimum marketing weights where the producer expects neither a serious disease problem nor a price break. A final qualification of table 15 is that the marketing weights predict maximum returns above feed costs. However, providing the resources used in turkey production do not have opportunities for higher returns, the producer is interested in maximizing returns above all costs (not merely feed costs) incurred by the turkey enterprise. Table 15 may also be used in making this latter estimate. However, for this case, it is necessary to replace the simple turkey/feed

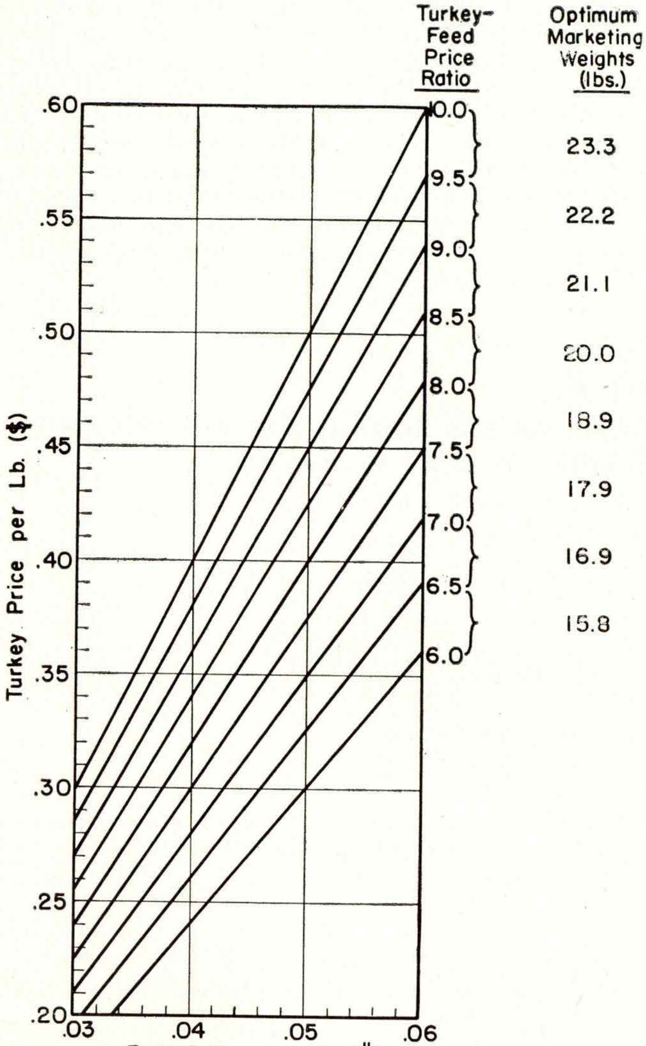


Fig. 17. Optimum marketing weights for turkeys fed on a 17-percent protein ration, predicted by square root function 15.

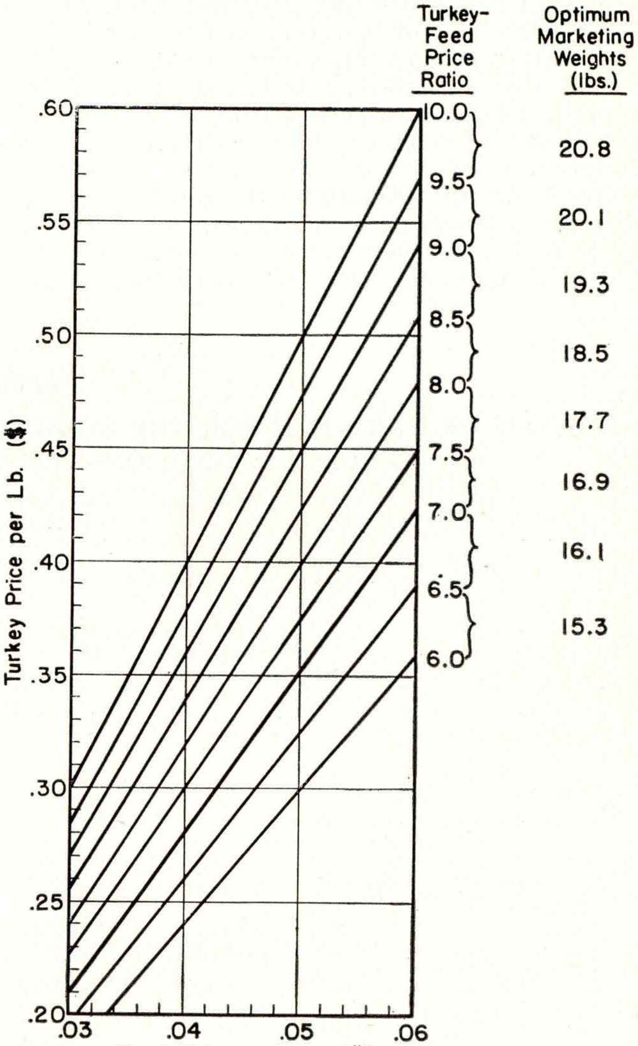


Fig. 18. Optimum marketing weights for turkeys fed on a 19-percent protein ration, predicted by square root function 15.

price ratio with the following ratio: turkey price per pound divided by the variable costs associated with feeding 1 pound of the ration (including the cost of the feed itself). The tangency of this latter price ratio with the input-output curve for a given ration indicates the point of maximum profits above all costs for the turkey enterprise.

SIMPLE GRAPHICAL PRESENTATION OF OPTIMUM MARKETING WEIGHTS

The data from table 15 are used in deriving figs. 15 through 18, which provide convenient graphical approximations of the marketing weights which maximize returns above feed costs for various rations and turkey/feed price ratios. Figures 15 through 18 are used as follows: First, find the figure (from figs. 15 through 18) for which the protein level corresponds most closely with that for the least-cost ration predicted for the third weight interval. Second, locate the intersection of the turkey and feed prices on the graph. Third, follow along between the diagonals to the right side of the graph and read off the approximate optimum marketing weight. For example, assume a least-cost ration of 13 percent protein, an expected turkey price of 36 cents per pound and a feed price of 5 cents per pound. The intersection of the two feed prices is found at point A, fig. 15. Moving to the right side of the diagram between the two diagonals which bracket point A, the optimum marketing weight is found to be 18.1 pounds. Only the marketing weights for rations containing alternate whole percentages of protein are included in figs. 15 through 18 because very slight differences occur for 1 percent protein changes in the rations. While figs. 15 through 18 provide a quick approximation of the optimum marketing weight under various conditions, more precise estimates are found in table 15.

SUGGESTIONS FOR FUTURE RESEARCH

Past research work in turkey production has seldom been subjected to economic analysis. To the knowledge of the authors, the procedures used in this study, although not new, have not heretofore been applied to turkey data. Use of these procedures resulted in predictions of least-cost rations and optimum marketing weights which may serve as guides to turkey growers.

It is suggested that future research might investigate more extensively the time considerations in turkey production. For example, it would be valuable to compare the time involved in raising birds to a given weight under various rations; a producer might be interested in getting the birds to marketing weight as rapidly as possible, rather than in feeding the lowest cost ration. Because of the difference in feed-gain relationships for tom and hen turkeys, further study might also be directed toward predicting least-cost rations and marketing weights separately for the two sexes. Perhaps different rations and marketing procedures for toms and hens would substantially increase profits.

APPENDIX A

ANALYSIS OF VARIANCE FOR THE COBB-DOUGLAS, SQUARE ROOT AND QUADRATIC CROSS-PRODUCT FUNCTIONS FOR THE THIRD WEIGHT INTERVAL.

TABLE A-1. ANALYSIS OF VARIANCE FOR COBB-DOUGLAS FUNCTION 14.

Source of variation	Degrees of freedom	Sum of squares	Mean square
Total	59	2.7941	0.0474
Due to regression	2	2.7487	1.3743
Deviations from regression	57	0.0454	0.0008

$$F = \frac{1.3743}{0.0008} = 1{,}717.91$$

TABLE A-2. ANALYSIS OF VARIANCE FOR SQUARE ROOT FUNCTION 15.

Source of variation	Degrees of freedom	Sum of squares	Mean square
Total	69	1,153.4413	16.7165
Due to regression	5	1,146.0593	229.2119
Deviations from regression	64	7.3820	0.1153

$$F = \frac{229.2119}{0.1153} = 1{,}987.9610$$

TABLE A-3. ANALYSIS OF VARIANCE FOR QUADRATIC CROSSPRODUCT FUNCTION 16.

Source of variation	Degrees of freedom	Sum of squares	Mean square
Total	69	1,153.4413	16.7165
Due to regression	5	1,122.3445	224.4689
Deviations from regression	64	31.0968	0.4859

$$F = \frac{224.4689}{0.4859} = 461.9747$$

APPENDIX B

ALTERNATIVE LEAST-COST RATIONS FOR THE THIRD WEIGHT INTERVAL, PREDICTED FROM SQUARE ROOT FUNCTION 15.

Table B-1 summarizes the marginal rates of substitution and combinations of corn and soybean oilmeal required for various gains in the third weight interval, as predicted from square root function 15. Columns (7), (8) and (9) in table B-1 show marginal rates of substitution of soybean oilmeal for corn along gain isoquants of 4.50, 9.00 and 12.75 pounds in the third weight interval. Since curvature is allowed in the isoclines of the square root function, the substitution rates along particular ration lines change as weight increases. The least-cost ration for attaining each of the three levels of gain in table B-1 is determined by locating the marginal rates of substitution in columns (7), (8) and (9) which most nearly equal the soybean oilmeal/corn price ratio. Thus, the least-cost ration for a particular price ratio may change three times as weight increases over the upper weight range. For example, with a soybean oilmeal/corn price ratio of 2.00, the least-cost ration for producing 4.50 pounds of gain in the third weight interval contains 17 percent protein since, in column (7), the marginal rate of substitution closest in value to the price ratio is 1.98 for the 17-percent protein ration. Using the same price ratio of 2.00, the least-cost ration for 9.00 pounds of gain contains 15 percent protein and the least-cost ration for 12.75 pounds of gain contains 14 percent protein. See columns (8) and (9) for the marginal rates of substitution used.

The marginal rates of substitution, and hence the least-cost rations, predicted from the square root function are constantly changing as weight increases. However, the data of table B-1 provide only three separate rations to be fed over the upper weight range, with a ration change occurring about every 4 weeks. If the producer also used separate least-cost rations for the first and second weight intervals, a total of five different rations would be fed over the entire production period. The authors believe that few turkey growers would actually change rations more frequently.

TABLE B-1. MARGINAL RATES OF SUBSTITUTION AND COMBINATIONS OF CORN AND SOYBEAN OILMEAL FOR VARIOUS GAINS IN THE THIRD WEIGHT INTERVAL.*

Percent protein in the ration	Lbs. of corn and soybean oilmeal for 4.50 lbs. gain†		Lbs. of corn and soybean oilmeal for 9.00 lbs. gain†		Lbs. of corn and soybean oilmeal for 12.75 lbs. gain†		Marginal rates of substitution along gain isoquants of:		
	Corn	Soybean oilmeal	Corn	Soybean oilmeal	Corn	Soybean oilmeal	4.50 lbs.†	9.00 lbs.†	12.75 lbs.†
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
10	34.7	0	74.5	0	113.4	0	—	—	—
11	18.2	0.7	41.0	1.6	64.2	2.6	8.41	7.29	6.62
12	14.8	1.2	34.1	2.8	54.3	4.5	5.39	4.42	3.86
13	12.8	1.7	30.1	3.9	48.5	6.3	4.03	3.16	2.66
14	11.4	2.1	27.2	5.0	44.5	8.1	3.23	2.43	1.95
15	10.2	2.4	25.0	6.0	41.7	9.9	2.70	1.93	1.47
16	9.4	2.8	23.3	7.0	39.1	11.7	2.28	1.56	1.13
17	8.6	3.2	21.8	8.0	37.2	13.7	1.98	1.28	0.86
18	8.0	3.6	20.6	9.2	35.5	15.8	1.72	1.05	0.64
19	7.3	3.9	19.4	10.3	34.3	18.2	1.52	0.86	0.45
20	6.8	4.3	18.5	11.5	33.3	20.8	1.33	0.70	0.30

*Computed from square root function 15.
†Gains measured from an average weight of 6.93 pounds at 12 weeks.