Fertilizer Production Functions for Corn and Oats;

Including an Analysis of Irrigated and Residual Response

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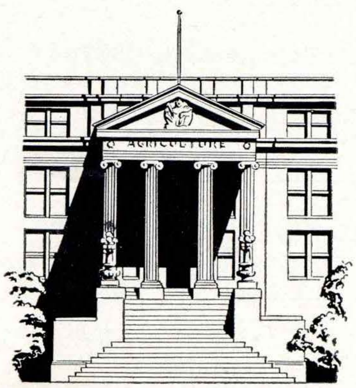

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SUMMARY

This study includes predictions of fertilizer production functions for four experiments. The experiments on Clarion and McPaul soils contain predictions of total and marginal yields, isoquants and marginal rates of substitution, isoclines and economic optima. Similar analyses for the other two experiments, conducted on Carrington soil during successive growing seasons, were not warranted because insufficient rainfall limited yield responses. The experiments included in this study were based on partially replicated factorial designs.

The experiment with corn on Clarion silt loam in 1954 included nitrogen, phosphorus and potash as variable nutrients. The production function, isoquants and isoclines are represented by equations a, b, c, d, e, f and g. In these equations (and equations listed below), N, P and K denote the pounds per acre of nitrogen, P_2O_5 and K_2O, respectively. Isoquant and isocline equations were derived for each pair of nutrients. Hence, there are three of each of these equations. The $\propto$ in the isocline equations represents a constant price or substitution ratio.

The production surfaces predicted for Clarion silt loam show large yield responses to initial nutrient applications. As input applications increase, however, the surfaces appear relatively flat. Decreasing total products are predicted for each nutrient. The largest yield responses resulted from nitrogen applications, and examination of the isoquant maps shows that all but the highest yields can be obtained with nitrogen applications alone. Profit maximizing nutrient combinations were derived for various nutrient and corn prices. These combinations included, in general, large applications of nitrogen and relatively small applications of P_2O_5 and K_2O. Profits resulting from the use of optimum fertilizer combinations ranged from \$10 to \$29 per acre. Confidence limits were derived for yields resulting from predicted optimum input combinations.

The experiment on McPaul silt loam included nitrogen and phosphorus as variables. The resulting corn yields, obtained under irrigated conditions, were high. In an attempt to determine the type and magnitude of differences in estimates derived from varying mathematical forms of regression equations, both a square root and a quadratic equation were fitted to the McPaul data. The quadratic equation with the isoquant and isocline equations derived therefrom are presented in equations h, i and j, respectively; the square root equation and resulting isoquant and isocline equations are represented by equations k, l and m.

Production surfaces for McPaul silt loam show steady yield increases as nutrient applications in-

(a) $\hat{Y} = 56.1272 + 2.6917\sqrt{N} + 1.5926\sqrt{P} + 0.8816\sqrt{K} - 0.0956N - 0.1184P - 0.0633K$

(b) $P = \left[6.7255 \pm 4.2230\sqrt{2.5363 - 0.0453N + 1.2748\sqrt{N} - 0.4736\ (Y-56.127)}\right]^2$

(c) $K = \left[6.9663 \pm 7.8985\sqrt{0.7772 - 0.0242N + 0.6816\sqrt{N} - 0.2532\ (Y-56.127)}\right]^2$

(d) $K = \left[6.9640 \pm 7.8985\sqrt{0.7772 - 0.0300P + 0.403\sqrt{P} - 0.2532\ (Y-56.127)}\right]^2$

(e) $P = \left[\dfrac{0.7962}{\propto(-0.0955+1.3458N^{-1/2})+0.1183}\right]^2$

(f) $K = \left[\dfrac{0.4408}{\propto(-0.0955+1.3458N^{-1/2})+0.0633}\right]^2$

(g) $K = \left[\dfrac{0.4408}{\propto(-0.1184+0.7963P^{-1/2})+0.0633}\right]^2$

(h) $\hat{Y} = 33.1614 + 0.7842S + 0.0273N + 0.1638P - 0.000149N^2 - 0.000565P^2 + 0.000243NP$

(i) $P = 145.113100 + 0.215509N \pm 885.739593\ (0.208265 + 0.000141N - 0.00000028N^2 - 0.002258Y)^{1/2}$

(j) $P = \dfrac{-0.027296 + 0.00298N + \propto(0.163833 + 0.000234N)}{0.000243 + \propto(0.001129)}$

(k) $\hat{Y} = 30.1751 + 0.8268S + 0.539309\sqrt{N} + 1.256664\sqrt{P} - 0.0014N - 0.0610P + 0.088035\sqrt{N}\sqrt{P}$

(l) $P = 10.292594 + 0.721043\sqrt{N} \pm 8.190410\ (0.352952\sqrt{N} - 0.007230N + 21.088729 - 0.002258Y)^{1/2}$

(m) $P = \Big[-3.062997 + (0.693432\propto - 0.696976)\sqrt{N} \pm 11.358990 \left(\left[(0.061047\propto - 0.061359)\sqrt{N} + 0.269654\right]^2 + 0.176072\left[\propto\sqrt{N}\ (0.628332 + 0.044018\sqrt{N})\right]\right)^{1/2}\Big]^2$

crease. Yield responses to nutrient applications predicted by the square root equation are initially larger but increase less rapidly than predictions by the quadratic equation. Also, the square root equation predicts a maximum yield at input levels above those at which the quadratic equation attains a maximum. Interaction causes yield responses to increase when the nutrients are applied simultaneously. Hence, sloping ridgelines are predicted by both equations. Isoclines and ridgelines are linear for the quadratic equations and curved for the square root equations. Because of the high level of fertility found throughout the experimental plots, optimum or profit maximizing input combinations recommended by the equations contain unusually small amounts of nutrients. In general, the recommendations of the square root equation require more nutrient inputs and result in greater gains as compared with the quadratic equation. Profits, predicted by either equation, are small and range from 0 to $8 per acre for the quadratic equation and from $2 to $9 per acre for the square root equation. Confidence limits were derived for predicted yields resulting from optimum inputs.

The experiments on Carrington silt loam represent 2 years of yield data from the same experimental plots; the second year represents an attempt to measure residual response. Nitrogen, phosphorus and potash were included as variable nutrient inputs in these experiments. As mentioned, an economic analysis was not carried out for these data. The functions are presented, however, to suggest the types of results which confront farmers in particular years. Predicted yields and marginal products are presented for the 1955 experiment with corn and the 1956 oats experiment. Standard errors for predicted yields are presented to illustrate the uncertainty of predictions made from data of years of limited rainfall. In addition, 2-year costs and returns are considered by transforming the physical production functions into value terms. Following this procedure, it was found that discounted 2-year returns failed to exceed fertilizer costs in any of the situations considered.

Fertilizer Production Functions for Corn and Oats; Including an Analysis of Irrigated and Residual Response[1]

BY JOHN P. DOLL, EARL O. HEADY AND JOHN T. PESEK

Research in crop response to fertilization has three fundamental objectives as a basis for aiding decision making by farmers. First, the type of response function within individual years must be estimated as accurately as possible. This involves fitting a logical mathematical equation for each experiment. Second, the probability distribution of production functions should be predicted. Even if logical and appropriate models for yearly yield responses are known, variations among years will affect the amounts of fertilizer which farmers should use as well as the strategy they select in making decisions. Production functions for years of small yield responses are as important to farmers as those years when responses are large. Finally, residual responses to fertilizers need to be evaluated. Relevant questions arising in the phase of research are: In what manner do (discounted) future returns from fertilizer affect present decisions to use fertilizer? How do small or large responses in the initial year affect subsequent yields? Factors such as yearly rainfall, type of crop, loss of nutrients through leaching and related variables obviously may affect total response over a period of years. Thus residual responses in years following the one of fertilizer application may well be important in determining optimum rates of fertilization.

This study, fourth in a series of studies considering the above problems, includes a production function analysis for four experiments.[2] The first experiment, with corn, is on Clarion silt loam and includes nitrogen, P_2O_5 and K_2O as nutrient inputs. It was conducted in Polk County during 1954. The second experiment, with irrigated corn, is for nitrogen and P_2O_5 on McPaul silt loam. This experiment was carried out in Mills County in 1955. Predictions of two different mathematical equations are compared for this experiment. The last two experiments represent 2 years of yield data from the same experimental plots, with the second experiment representing an attempt to measure residual response. These experiments, located on Carrington silt loam in Delaware County, include the nutrient inputs, nitrogen, P_2O_5 and K_2O. The initial experiment in 1955 was with corn; in 1956 oats were planted to determine the residual response.[3]

The four experiments reported in this study are based on similar designs. All treatment combinations of a complete factorial nature appear once in each experiment. In addition, treatment combinations falling within the range of expected optimum fertilizer recommendations were replicated. Hence, all treatment combinations occur once in each experiment and certain combinations occur twice. This design was used to reduce the size of the soil area needed for each experiment; the objective being that of decreasing yield variations due to soil differences. However, because of the partially replicated design, estimation of experimental error by analysis of variance is impossible.[4]

Rainfall was limited in the area and year of each of these experiments, and stands were not as high as desirable on any of the locations. As mentioned, however, knowledge of the response function in dry years is important in determining the distribution of production functions over time. Further, productivity coefficients become small in relation to their standard errors when rainfall

[1]Projects 1189, 1193 and 1293 of the Iowa Agricultural and Home Economics Experiment Station.

[2]Past studies are: Heady, Earl O., Pesek, John T. and Brown, William G. Corn response surfaces and economic optima in fertilizer use. Iowa Agr. Exp. Sta. Res. Bul. 424. 1955; Brown, William G., Heady, Earl O., Pesek, John T. and Stritzel, Joseph A. Production functions, isoquants, isoclines and economic optima in corn fertilization for experiments with two and three variable nutrients. Iowa Agr. Exp. Sta. Res. Bul. 441. 1956; Pesek, John T., Heady, Earl O. and Doll, John P. Production surfaces and economic optima for corn yields in respect to stand and nitrogen levels. Unpublished Research, 1957; and Heady, Earl O. and Pesek, John T. Some methodological considerations in the Iowa-TVA research project on economics of fertilizer use. In Baum, E. L., Heady, E. O., Pesek, J. T. and Hildreth, C. G., eds. Economic and technical analysis of fertilizer innovations and resource use. pp. 144-167. Iowa State College Press, Ames, Iowa, 1957.

[3]The experiment on Clarion silt loam was conducted on the Ankeny experimental farm. Farmer cooperators for the experiments on the McPaul and Carrington soils were Frank Beda, Pacific Junction, Iowa and John Wacker, Masonville, Iowa, respectively.

[4]In past experiments of this general nature, mean squares for deviations from regression and mean squares for experimental error have been found to be remarkably similar. See: Pesek, John T. Agronomic problems in securing fertilizer response data desirable for economic analysis. In Baum, E. L., Heady, E. O. and Blackmore, J. eds. Methodological procedures in the economic analysis of fertilizer use data. pp. 108-110. Iowa State College Press, Ames, Iowa. 1956.

is limiting. Hence, predictions made from data of this type are surrounded by more uncertainty than for years of high rainfall. This is exactly the decision-making environment in which farmers make investment choices. Hence, while response functions derived for dry years may not be as "mathematically neat" as those for years with favorable moisture, they are equally important from the standpoint of managerial choices.

The functions reported are for nutrients added to the soil. Later studies will contain analysis of functions which include nutrients already available in the soil, moisture and soil type as variables. Inclusion of these variables will reduce unexplained variance in yields. To conserve space in this report, only summary indication is given of the degree of uncertainty involved in the predicted yield quantities and in nutrient ratios and levels which represent economic optima.

EXPERIMENT ON CLARION SILT LOAM IN 1954

The experiment with corn on Clarion silt loam in 1954 was a completely randomized 5x4x4 factorial with five rates of nitrogen (N) and four each of phosphorus (P_2O_5) and potash (K_2O). Yields are given in table 1.

Yield response to the first 40-pound application of nitrogen or potash was relatively high. Table 1 shows a response of 19 bushels per acre to the first 40 pounds of nitrogen, 12 bushels to the first 40 pounds of K_2O and 5 bushels to the first 40 pounds of P_2O_5. However, in this year of limited rainfall, marginal yields diminished quickly, and decreasing total yields were evidenced at the highest rates of application. The highest yield differential between a fertilized plot and the check plot was 43.8 bushels per acre.

Part of the lack of yield response at the higher rates of fertilization may have been due to stand.

TABLE 1. CORN YIELDS ON CLARION SILT LOAM AT ANKENY IN 1954 (BUSHELS PER ACRE).

Pounds of P_2O_5 per acre	Pounds of K_2O per acre	Pounds of nitrogen per acre: 0	40	80	160	320
0	0	49.4 52.3	69.3 70.5	76.8 71.8	76.8	74.7 73.4
	40	60.5 64.7	75.1 67.5	75.7 68.1	74.3	70.5 69.9
	80	54.1 55.8	73.4 71.3	79.0 79.8	81.3	85.2 84.7
	160	53.9	71.7	78.3	77.1	67.3
40	0	49.6 52.9	74.4 76.9	73.0 70.0	68.2	80.3 85.9
	40	65.5 65.1	75.3 74.6	86.4 78.2	86.4	81.4 67.7
	80	64.3 63.4	77.0 78.7	82.5 81.9	88.3	77.0 75.8
	160	58.7	70.8	70.3	90.8	88.0
80	0	61.8 65.9	70.8 70.6	76.1 76.9	81.2	84.1 77.0
	40	63.9 65.0	66.5 76.2	92.0 89.4	85.2	87.3 94.8
	80	74.4 64.0	72.0 79.9	70.2 83.5	75.9	94.2 95.2
	160	50.8	73.2	72.7	79.7	79.2
160	0	58.4	87.0	73.4	79.3	73.3
	40	51.7	74.3	62.8	76.3	86.9
	80	60.1	62.8	75.9	86.3	84.5
	160	57.9	69.4	77.7	76.9	75.9

TABLE 2. VALUES OF t FOR THE COEFFICIENTS OF EQUATION 1.

Coefficient	Value of t	Probability level*
S	7.83	0.001
N	11.50	0.001
P	4.61	0.001
K	2.97	0.01
N^2	10.67	0.001
P^2	5.27	0.001
K^2	2.98	0.01
NP	0.12	0.90
NPK	0.20	0.90
NK	0.40	0.70
PK	0.28	0.80

*Probability of drawing a t value as large or larger by chance, given the null hypothesis.

At high rates of fertilization, the number of plants per acre replaced fertilizer as the resource limiting yield.

Regression Analysis

Two types of equations were fitted to the corn data from Clarion silt loam: a quadratic function with crossproduct terms and a square root function using only those terms shown to be significant in the quadratic equation.

Equation 1 is the quadratic function where $\hat{Y}$ is the predicted corn yield expressed as bushels per acre, S is stand (coded, plants per acre = 174S) while N, P and K are the pounds per acre of nitrogen, phosphorus and potash, respectively. The value of R^2 is 0.7816 for this equation. Probability levels of the t values for each coefficient are given in table 2.

$$(1)\quad \hat{Y} = 18.5975 + 0.8429S + 0.2116N + 0.14211P + 0.0920K - 0.000511N^2 - 0.000853P^2 - 0.000487K^2 + 0.000013NP - 0.00000026NPK - 0.000045NK - 0.000061PK$$

The stand term is included to increase the precision of the yield estimate by accounting for the variations in yield caused by the variations in stand. The t value for stand (7.83) indicates that the yield estimate is considerably improved by including this term. However, lack of interaction terms for stand and nutrients does not allow determination of joint effects, nor was the experiment designed for measuring such effects.

Significance levels of the interaction terms in equation 1 are low (see table 2). For the NP and NPK terms, a t value as large or larger might have been obtained nine out of ten times in samples drawn at random from the t distribution. It is possible, however, that when an input variable is included in several regression terms, none of these terms will be significant even if the total effect of the input variable is significant. Each fertilizer input occurs in five terms for equation 1. Hence, examination of reduction in sums of squares was first made for the interaction terms NK, PK and NPK. By deleting the NK, PK and NPK terms from equation 1, the R^2 value was lowered from 0.7816 to 0.7793. In the analysis of variance of the reduction in sums of squares due

TABLE 3. ANALYSIS OF VARIANCE OF DELETION OF TERMS FROM EQUATION 1.

A. Deletion of NK, PK and NPK terms.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	115	11,640.09		
Due to regression on all variables	11	9,098.57		
Regression on all variables except NK, PK and NPK	8	9,071.52		
Difference	3	27.05	9.02	0.37
Deviations from regression	104	2,541.52	24.44	

B. Deletion of NP, NK, PK and NPK terms.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	115	11,640.09		
Due to regression on all variables	11	9,098.57		
Regression on all variables except NP, NK, PK and NPK	7	9,071.35		
Difference	4	27.22	6.81	0.28
Deviations from regression	104	2,541.52	24.44	

to regression (table 3-A), the F value is not significant. Next, an analysis was made of dropping the NP term. This step reduced the R^2 from 0.779334 to 0.779320. Table 3-B gives the resulting analysis of variance, and the F value again is not significant. Hence, equation 2 resulted when the NP, NK, PK and NPK interaction terms were removed from the original equation.

$$(2)\quad \hat{Y} = 20.0717 + 0.8248S + 0.2088N + 0.1388P + 0.0825K - 0.000511N^2 - 0.000859P^2 - 0.000499K^2$$

A square root function was then fitted to the Clarion experimental data, with the interaction terms excluded from the equation. Equation 3 resulted where the symbols are those previously defined. The probability levels of the t values for

$$(3)\quad \hat{Y} = 18.9682 + 0.7921S + 2.691717\sqrt{N} + 1.592583\sqrt{P} + 0.881608\sqrt{K} - 0.0956N - 0.1184P - 0.0633K$$

the coefficient in equation 3 are given in table 4. Since the t values of the coefficients are all highly significant and the R^2 (0.8063) is slightly higher than that of the quadratic form, the square root function is used for subsequent economic analysis.

TABLE 4. VALUES OF t FOR THE SQUARE ROOT EQUATION 3.

Coefficient	Value of t	Probability level*
S	8.38	0.001
N	8.25	0.001
P	4.90	0.001
K	2.61	0.025
$\sqrt{N}$	12.30	0.001
$\sqrt{P}$	5.43	0.001
$\sqrt{K}$	2.99	0.010

*Probability of drawing a t value as large or larger by chance, given the null hypothesis.

TABLE 5. ANALYSIS OF VARIANCE FOR SQUARE ROOT EQUATION 4.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	115	11,640.08		
Due to regression	7	9,385.40	1,340.77	64.21
Deviations from regression	108	2,254.68	20.88	

The square root function also allows curved isoclines, a property thought to be logical for the particular soil and experimental data. An analysis of variance of this regression is presented in table 5. The F value is highly significant.

Stand was included to increase the precision of the yield estimate but is not regarded as a variable in the following analysis. By substituting the mean of stand for all plots of the experiment into equation 3, equation 4 was obtained.

$$(4)\quad \hat{Y} = 56.1272 + 2.691717\sqrt{N} + 1.592583\sqrt{P} + 0.881608\sqrt{K} - 0.0956N - 0.1184P - 0.0633K$$

PRODUCTION SURFACES AND PREDICTED YIELDS

Table 6 includes the *predicted* yields for various combinations of fertilizer inputs, based on equation 4. The highest predicted yield in table 6 is 83 bushels per acre, 27 bushels more than the lowest predicted yield. (When comparing tables 1 and 6, it should be remembered that stand has been set at its mean in the latter, while it is variable in the former.) Decreasing total returns are evidenced for each of the inputs (input levels at which yield is predicted to be a maximum are derived in later sections).

The predicted yields in table 6 represent a multi-faceted surface. However, since this surface cannot be depicted as a three-dimensional figure, surfaces for each combination of two fertilizer inputs are illustrated separately. Actually, for any two nutrients considered to be variable, there are many surfaces, depending upon the level at which the third nutrient is held constant. However, the difference between these many surfaces is in height and not in slope, since there is no interaction between any of the input categories.

Figures 1, 2 and 3 illustrate geometrically the production surfaces predicted by equation 4 for varying rates of nutrient pairs. In each figure, three surfaces were obtained by varying two input categories while holding a third constant at 0, 80 or 160 pounds per acre.

The response to nitrogen is illustrated in figs. 1 and 2 (where nitrogen is a variable input) and fig. 3 (where nitrogen is the fixed input). These surfaces illustrate high marginal products of the first 40 pounds of nitrogen. Beyond 80 pounds, however, predicted corn yield response to nitrogen flattens out and diminishes slightly at the highest input levels. Because of this, fig. 3-B presents a surface only slightly different from the one obtained when nitrogen is applied at 40 pounds per acre. Also, fig. 3-C differs very little from the surfaces resulting from fixed nitrogen inputs ranging from 120 to 320 pounds per acre.

Figures 1 and 3 depict responses to P_2O_5 as a variable input, while it is the constant input of fig. 2. As with nitrogen, P_2O_5 exhibits the highest marginal response to the first 40-pound input. Beyond that, yields level off and decrease slightly.

TABLE 6. PREDICTED CORN YIELDS FOR CLARION SILT LOAM (BUSHELS PER ACRE).

Pounds of P_2O_5 per acre	Pounds of K_2O per acre	Pounds of nitrogen per acre								
		0	40	80	120	160	200	240	280	320
0	0	56.13	69.33	72.56	74.15	74.89	75.08	74.89	74.41	73.70
0	40	59.17	72.37	75.60	77.19	77.93	78.12	77.93	77.45	76.74
0	80	58.95	72.15	75.38	76.96	77.71	77.90	77.71	77.23	76.52
0	120	58.19	71.39	74.62	76.21	76.95	77.14	76.95	76.47	75.76
0	160	57.15	70.35	73.58	75.17	75.91	76.10	75.91	75.43	74.72
40	0	61.46	74.66	77.89	79.48	80.22	80.42	80.23	79.75	79.03
40	40	64.51	77.71	80.94	82.52	83.26	83.46	83.27	82.79	82.08
40	80	64.28	77.48	80.71	82.30	83.04	83.24	83.05	82.57	81.85
40	120	63.52	76.72	79.95	81.54	82.28	82.48	82.29	81.81	81.09
40	160	62.49	75.69	78.92	80.50	81.24	81.44	81.25	80.77	80.06
80	0	60.90	74.10	77.33	78.92	79.66	79.85	79.66	79.18	78.47
80	40	63.94	77.14	80.37	81.96	82.70	82.90	82.71	82.23	81.51
80	80	63.72	76.92	80.15	81.74	82.48	82.67	82.48	82.00	81.29
80	120	62.96	76.16	79.39	80.98	81.72	81.91	81.72	81.24	80.53
80	160	61.92	75.12	78.35	79.94	80.68	80.88	80.69	80.21	79.49
120	0	59.36	72.56	75.79	77.38	78.12	78.32	78.13	77.65	76.93
120	40	62.41	75.61	78.84	80.43	91.17	81.36	81.17	80.69	79.98
120	80	62.19	75.39	78.62	80.20	80.94	81.14	80.95	80.47	79.76
120	120	61.43	74.66	77.86	79.44	80.18	80.38	80.19	79.71	79.00
120	160	60.39	73.59	76.82	78.41	79.15	79.34	79.15	78.67	77.96
160	0	57.33	70.53	73.76	75.35	76.09	76.28	76.09	75.61	74.90
160	40	60.37	73.57	76.80	78.39	79.13	79.32	79.13	78.65	77.94
160	80	60.15	73.35	76.58	78.17	78.91	79.10	78.91	78.43	77.72
160	120	59.39	72.59	75.82	77.41	78.15	78.34	78.15	77.67	76.96
160	160	58.35	71.55	74.78	76.37	77.19	77.30	77.11	76.63	75.92

Thus, in fig. 2-B, the surface obtained is similar to that which would result from 40 or 120 pounds of P_2O_5.

The smallest yield response resulted from the K_2O applications. In figs. 2 and 10, where K_2O is varied, small responses are predicted for the initial 40-pound application of K_2O. Yields level off and decrease only slightly for the input quantities ranging from 80 to 160 pounds. This causes fig. 3-B to be identical to the surfaces obtainable when K_2O is held at either 80 or 120 pounds.

Again, for any of these production surfaces, if one fertilizer input is held constant while the other is varied, a single input production function is obtained. Input-output curves of this nature are shown for N, P_2O_5 and K_2O in figs. 4, 5 and 6, respectively. The slope of any of these curves is not changed by the amount of the fixed inputs present. That is, a change in the amount of the fixed nutrient changes total yield (i.e., the height of the surface) for any given level of the variable input, but does not change the marginal physical products (i.e., the slope of the surface) of the variable nutrients. This is true since there are no interaction terms in the over-all production function.

In fig. 4, the response curve for nitrogen is lower with P_2O_5 and K_2O constant at 160 pounds per acre than when the latter input is at an 80-pound level. This is due to the negative marginal products of K_2O and P_2O_5 when they are used in larger quantities. Similarly, in figs. 5 and 6, curves for the variable input are higher when the levels of the constant nutrients are at intermediate levels. No attempt is made to illustrate optimum quantities of the inputs in these figures. This step is accomplished in a later section.

Figures 7 to 12 represent response curves when two of the nutrients are varied in fixed proportions of each other while the third is held constant. In fig. 7, for example, nitrogen and K_2O are varied in the ratio of 1 pound of nitrogen to 1 pound of K_2O for one curve and 1 pound of nitrogen to 2 pounds of K_2O for the other. The input of P_2O_5 was held constant at 80 pounds for both curves, however. In terms of the production surfaces, these curves represent a vertical slice through the origin and over the nutrient plane for the variable input categories. The maximum points on these curves suggest the extent to which a given ratio of fertilizer nutrients can be used without depressing yields. Since some of the curves cross, they indicate that while one nutrient ratio may give higher yields for small inputs of a particular nutrient ratio, another nutrient ratio may allow yields to be taken to a higher level if fertilizer use is extended sufficiently high. In other cases, one curve lies above the other over all ranges of inputs, indicating that a particular ratio gives (1) higher yields at low inputs and (2) a higher maximum yield. However, more efficient methods of suggesting optimum input ratios are illustrated in later sections.

Marginal Physical Products

Marginal physical products of the three inputs can be predicted from the partial derivative equations 5, 6 and 7.

$$(5)\ \frac{\partial \hat{Y}}{\partial N} = -0.0956 + \frac{1.3459}{\sqrt{N}}$$

$$(6)\ \frac{\partial \hat{Y}}{\partial P} = -0.1184 + \frac{0.7963}{\sqrt{P}}$$

$$(7)\ \frac{\partial \hat{Y}}{\partial K} = -0.0663 + \frac{0.4408}{\sqrt{K}}$$

Table 7 lists the marginal physical products obtained by substituting input values into these equations. The marginal physical products represent the slope of the production function at the given nutrient level. Since the production function

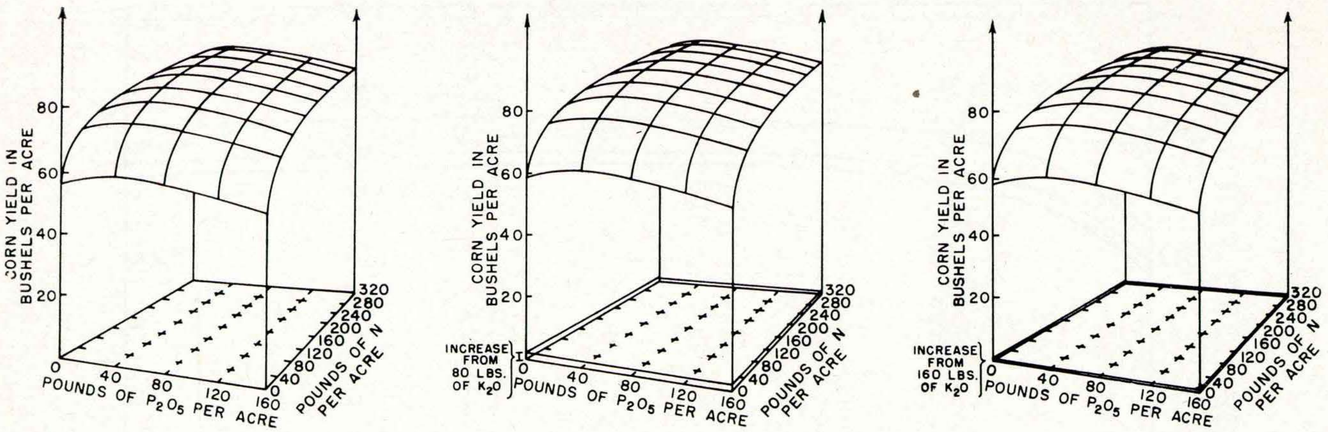

Fig. 1. Corn yield response to nitrogen and P_2O_5 (K_2O constant at three levels) on Clarion silt loam.

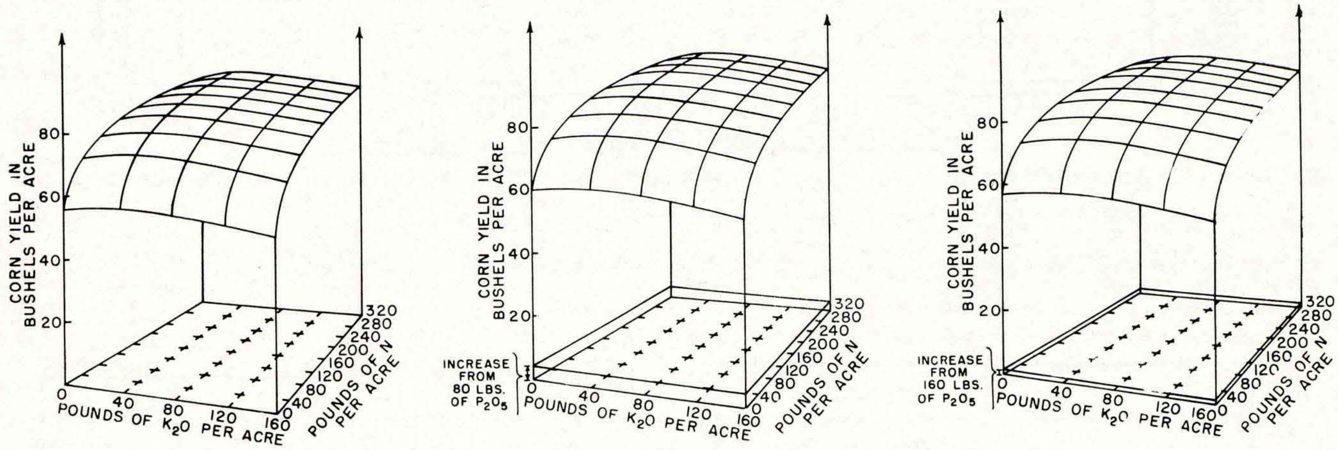

Fig. 2. Corn yield response to nitrogen and K_2O (P_2O_5 constant at three levels) on Clarion silt loam.

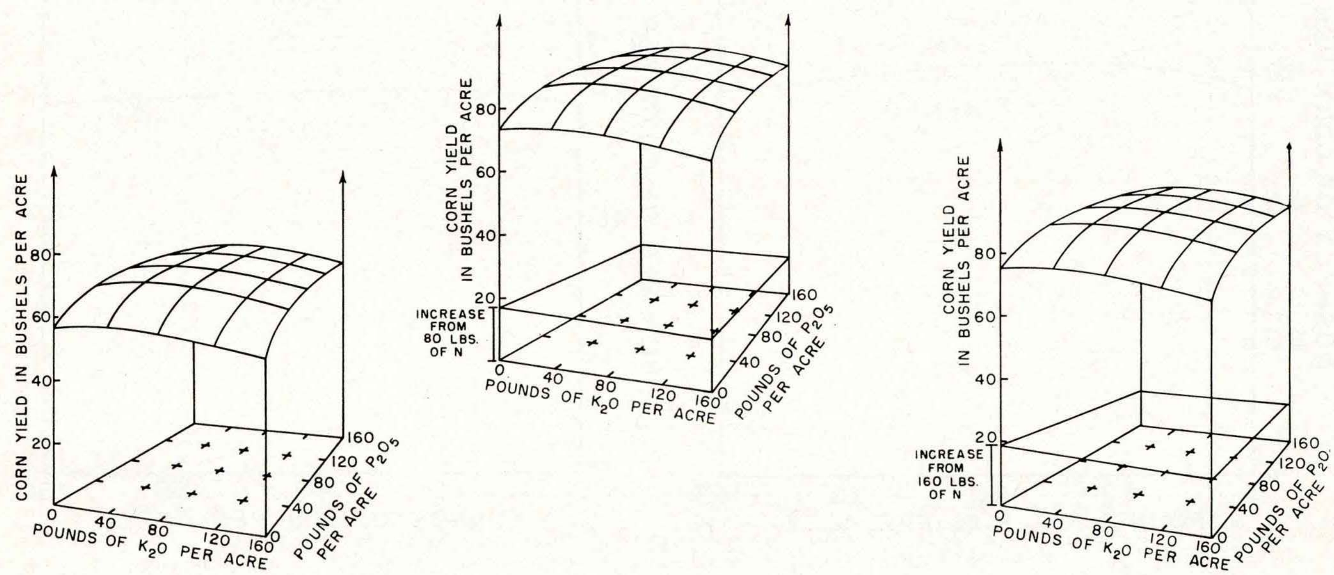

Fig. 3. Corn yield response to P_2O_5 and K_2O (nitrogen constant at three levels) on Clarion silt loam.

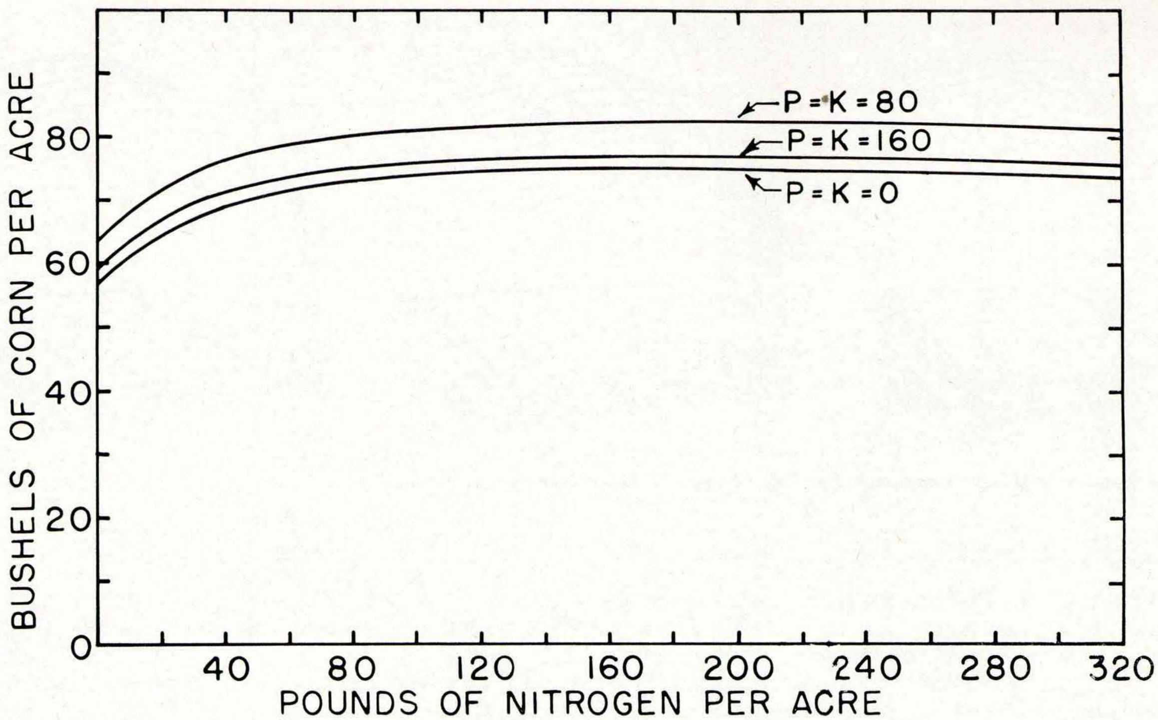

Fig. 4. Corn yield response to nitrogen (P_2O_5 and K_2O held constant) on Clarion soil.

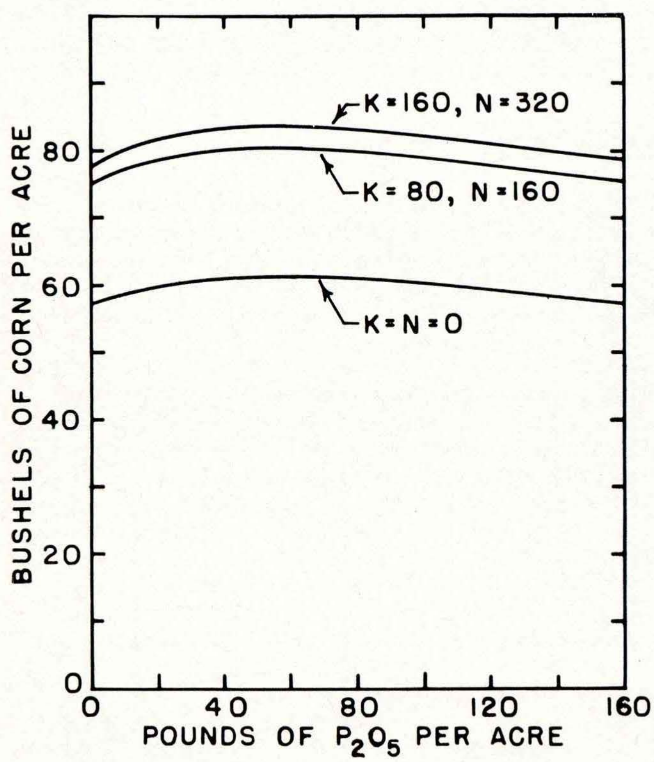

Fig. 5. Corn yield response to P_2O_5 (nitrogen and K_2O held constant) on Clarion soil.

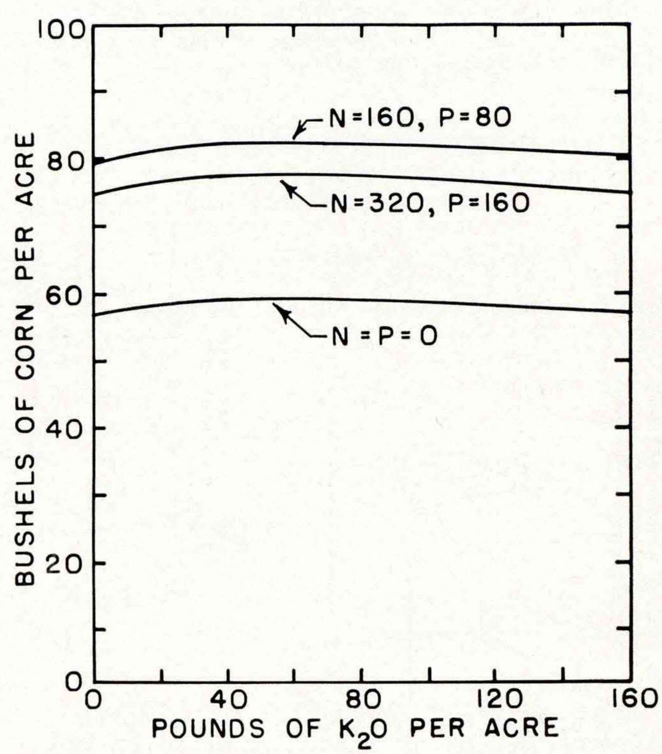

Fig. 6. Corn yield response to K_2O (nitrogen and P_2O_5 held constant) on Clarion soil.

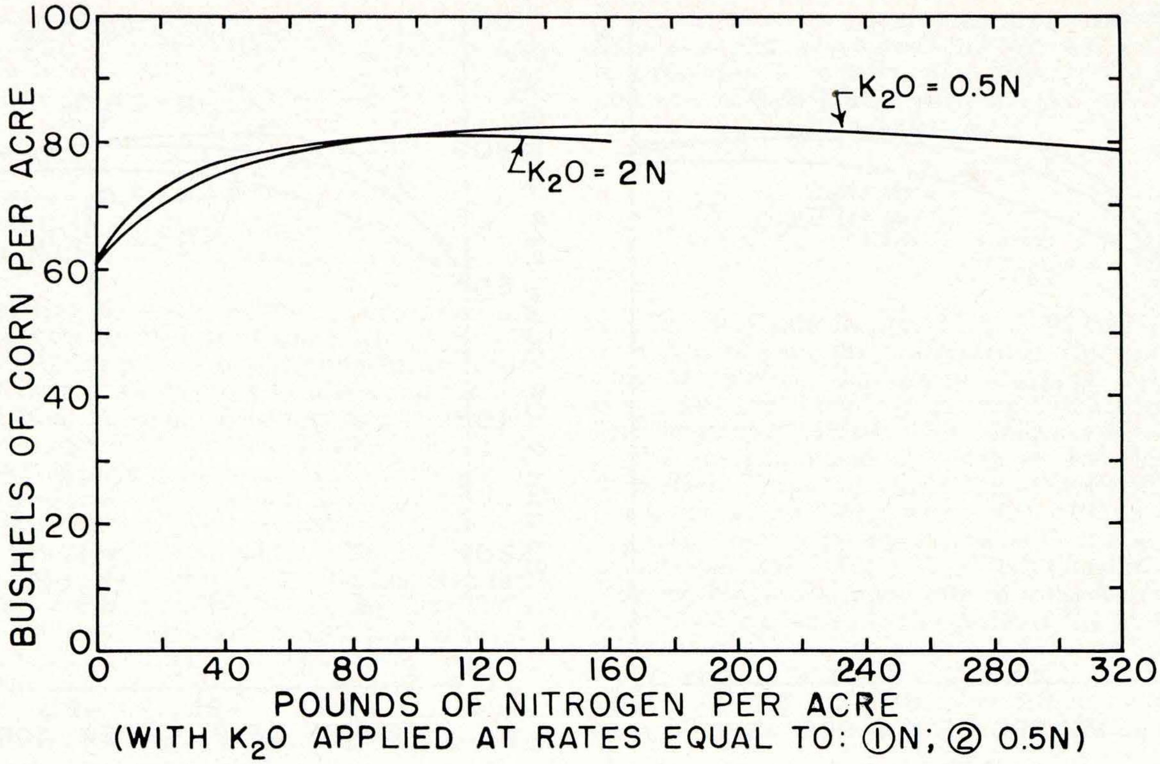

Fig. 7. Corn yield response to aggregated inputs of nitrogen and K_2O (P_2O_5 held constant at 80 pounds) on Clarion soil.

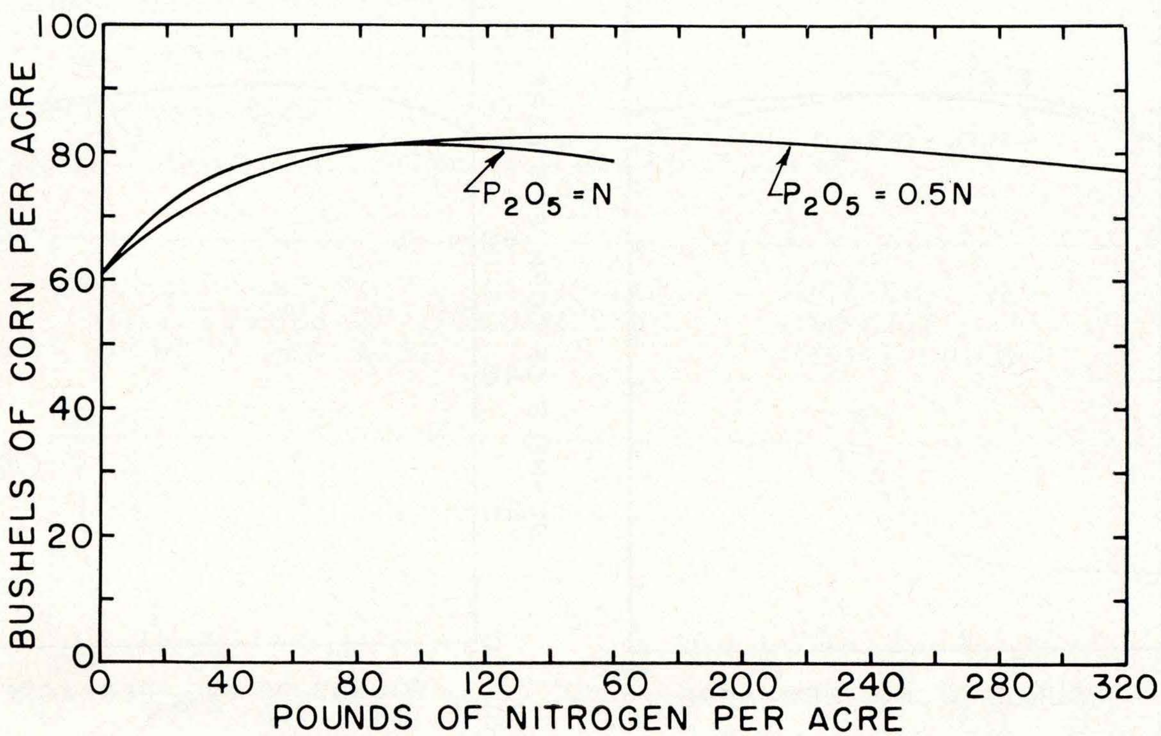

Fig. 8. Corn yield response to aggregated inputs of nitrogen and P_2O_5 (K_2O held constant at 80 pounds) on Clarion soil.

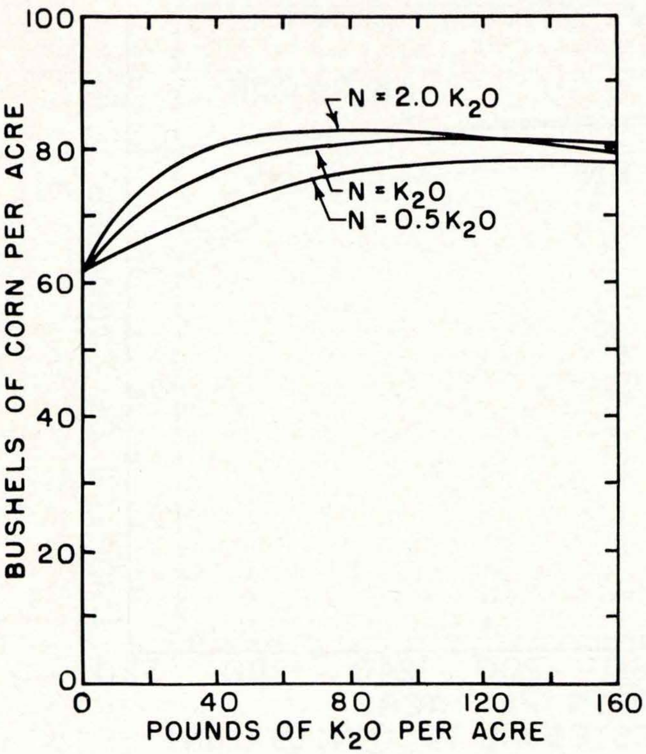

Fig. 9. Corn yield response to aggregated inputs of K_2O and nitrogen (P_2O_5 held constant at 80 pounds) on Clarion soil.

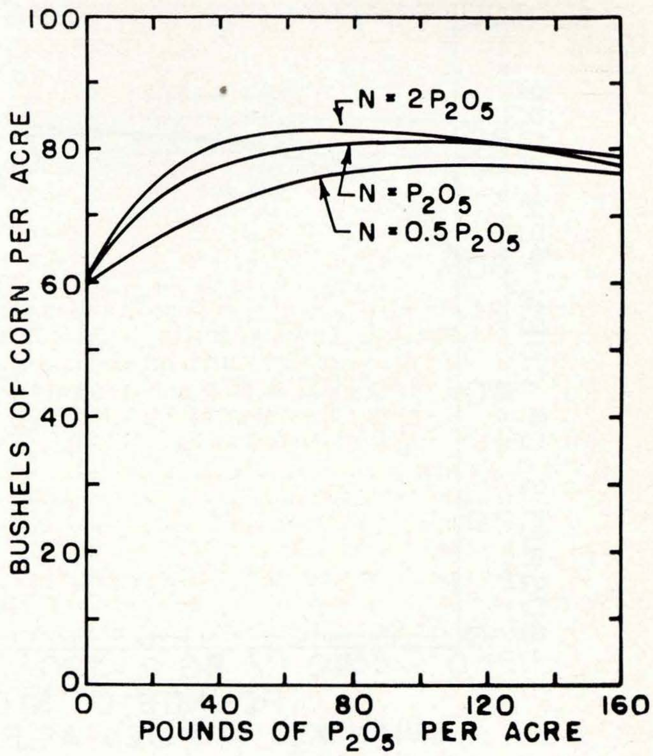

Fig. 11. Corn yield response to aggregated inputs of P_2O_5 and nitrogen (K_2O held constant at 80 pounds) on Clarion soil.

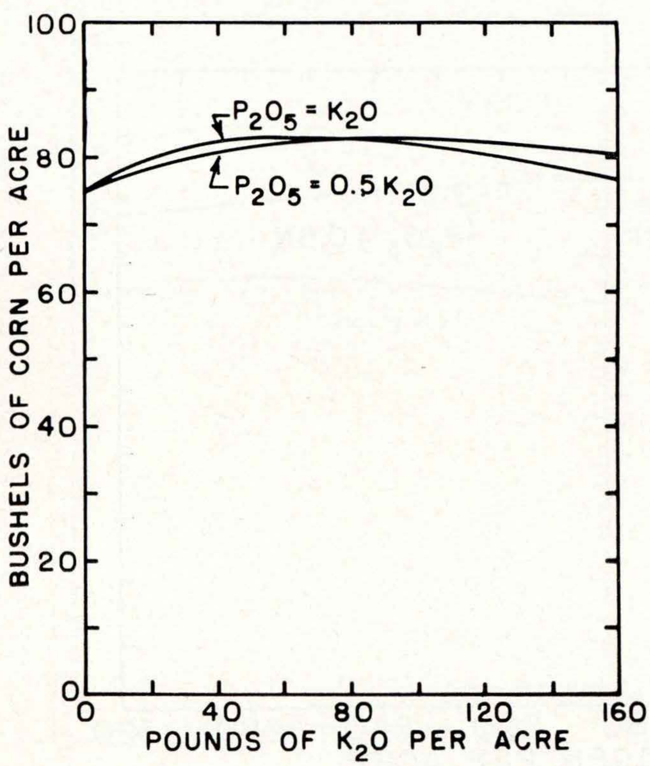

Fig. 10. Corn yield response to aggregated inputs of K_2O and P_2O_5 (nitrogen held constant at 160 pounds) on Clarion soil.

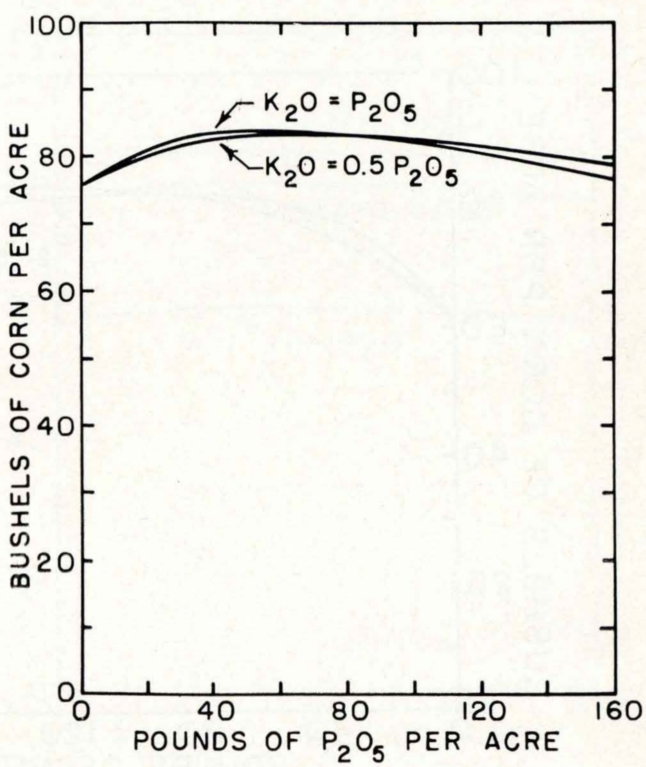

Fig. 12. Corn yield response to aggregated inputs of P_2O_5 and K_2O (nitrogen held constant at 160 pounds) on Clarion soil.

TABLE 7. MARGINAL PHYSICAL PRODUCTS OF NITROGEN, P_2O_5 AND K_2O (BUSHELS PER ACRE).

Inputs of fertilizer (pounds per acre)	MPP of nitrogen	MPP of P_2O_5	MPP of K_2O
10	0.330	0.134	0.073
20	0.206	0.060	0.032
40	0.117	0.008	0.003
80	0.055	—0.029	—0.017
120	0.027	—0.046	—0.032
160	0.011	—0.056	—0.350

lacks interaction terms, these marginal products are independent of the quantities of other nutrients.

By setting the equation for the marginal physical products equal to zero and solving the equations for the magnitude of the nutrient, the quantities of inputs where yield is at a maximum can be predicted. Marginal products of all nutrients are zero, and yield is predicted to be a maximum with 198.2 pounds per acre of N, 45.2 pounds per acre of P_2O_5 and 48.5 pounds per acre of K_2O. With these inputs, yield is predicted to be 83.5 bushels per acre. The standard error of estimate for this predicted yield is 0.9 bushel per acre. Maximum yields resulting when one nutrient is varied and the others are constant at zero are 75.1, 61.5 and 59.2 bushels per acre for N, P_2O_5 and K_2O, respectively, when each input is used at the quantity listed above. The standard errors of estimate for these yields are 1.1, 1.2 and 1.3 bushels per acre, respectively.

Yield Isoquants and Minimum Cost Nutrient Combinations

Since three resources are included in the production function for Clarion silt loam, three isoquant equations, one for each possible combination of the three inputs, can be derived from equation 4. Isoquant equation 8 expresses P_2O_5 as a function of nitrogen supposing that K_2O is fixed at zero or any other level (since the production function lacks interaction terms). The marginal rate of substitution between the two inputs can be predicted by equation 9.

$$(9)\ \frac{\partial \hat{Y}/\partial P}{\partial \hat{Y}/\partial N} = \frac{-0.1184 + \frac{0.7963}{\sqrt{P}}}{-0.0956 + \frac{1.3459}{\sqrt{N}}}$$

The isoquants for 70, 75 and 80 bushels per acre of corn yield are illustrated in fig. 13. Since they are curved, decreasing marginal rates of substitution hold true for the two nutrient inputs. The ridgelines (dotted lines) indicate the range in the nutrient plane in which substitution can take place. These ridgelines fall at nutrient combinations where marginal products of nutrients are zero (45.2 pounds per acre of P_2O_5 and 198.2 pounds per acre of N). This relationship holds true since the marginal rate of substitution between two resources is the ratio of the marginal physical products. Isoquant schedules and marginal rates of substitution of P_2O_5 for N for various yield levels are given in table 8. The marginal rates of substitution predicted by equation 9 are

TABLE 8. COMBINATIONS OF P_2O_5 AND NITROGEN NEEDED TO PRODUCE A GIVEN CORN YIELD AND CORRESPONDING MARGINAL RATES OF SUBSTITUTION.

70 bushels			75 bushels		
Pounds of nitrogen	Pounds of P_2O_5	MRS $\left(\frac{\partial N}{\partial P_2O_5}\right)$	Pounds of nitrogen	Pounds of P_2O_5	MRS $\left(\frac{\partial N}{\partial P_2O_5}\right)$
14	45.23	0.00	43	45.23	0.00
20	9.26	0.75	50	17.89	0.74
30	1.97	3.00	60	9.34	1.83
40	0.19	14.56	70	5.45	3.45
45	0.00	0.00	80	3.19	6.00
			100	1.06	17.01

$$(8)\ P = \left[6.725548 \pm 4.223044 \sqrt{2.536321 - 0.045256N + 1.274776\sqrt{N} - 0.473592\ (Y-56.127)}\right]^2$$

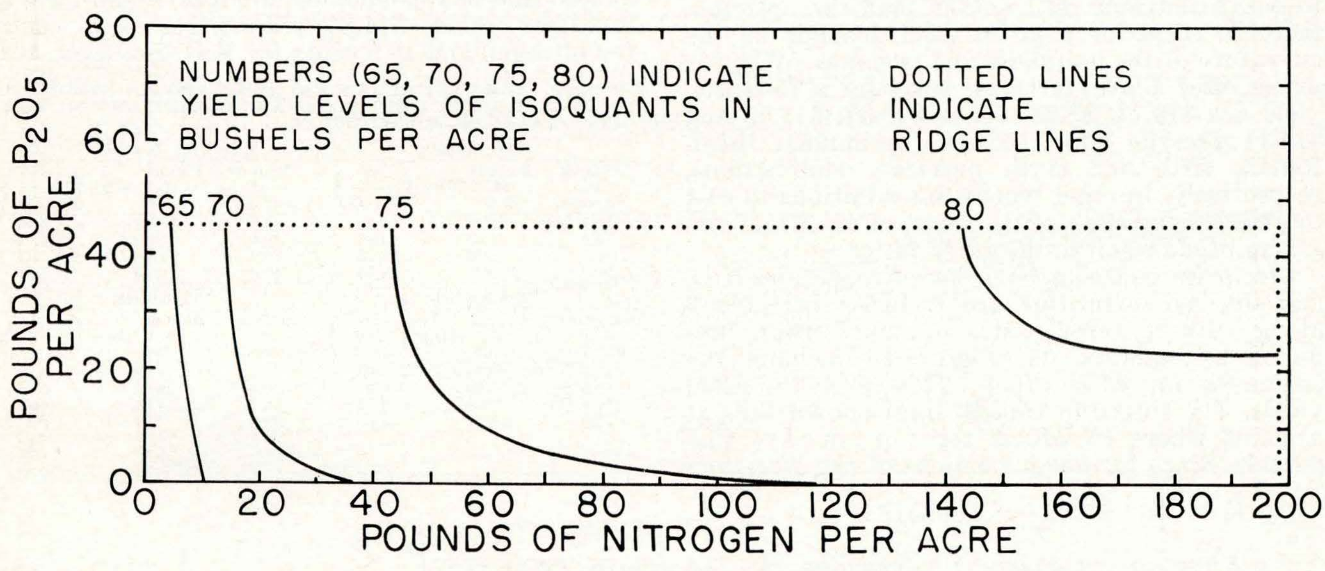

Fig. 13. Corn yield isoquants for nitrogen and P_2O_5 (K_2O held constant at zero) for Clarion soil.

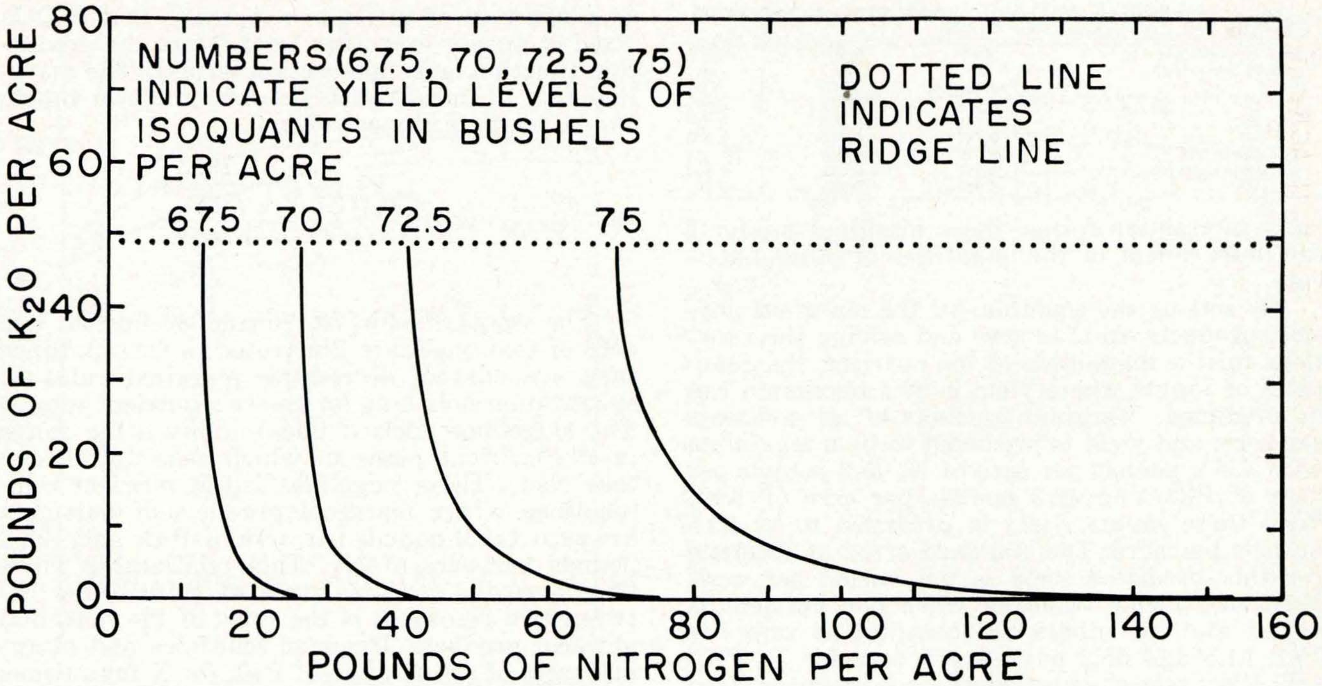

Fig. 14. Corn yield isoquants for nitrogen and K_2O (P_2O_5 held constant at zero) for Clarion soil.

suggestive of nutrient ratios which will allow attainment of a given yield with minimum fertilizer costs.

Since the nutrient ratio which gives minimum fertilizer costs for a particular yield is defined by equating the marginal rate of substitution with the inverse price ratio, a cost of purchasing and applying nutrients equal to 14 cents per pound of nitrogen and 10.5 cents per pound of P_2O_5 gives a P_2O_5/N price ratio of 0.74. The optimum nutrient ratio for a 75-bushel corn yield is then 50 pounds of nitrogen and 17.89 pounds of P_2O_5; a ratio roughly equivalent to 56:20. For a 70-bushel yield, the ratio of N to P_2O_5, to minimize fertilizer costs, would approach 56:25. The extent to which nutrient ratios other than the optimum increase costs for a given yield depends on the curvature of the isoquants and isoclines. With the prices cited above, fertilizer costs for a 75-bushel yield are $10.77, $8.88, $9.38, $10.37, $11.53 and $14.11 for the first, second (optimum), third, fourth, fifth and sixth nutrient combinations, respectively. In other words, the additions to cost increase as the slope of the isoquant deviates from a magnitude equal to the price ratio.

Isoquant equation 10 is for nitrogen and K_2O, and derived quantities are included in table 9 along with marginal rates of substitution predicted by equation 11. Figure 14 includes the isoquants for 67.5-, 70.0-, 72.5- and 75-bushel yields. The ridgeline (dotted line) shown falls at a point where $\partial Y/\partial K$ is zero; namely, 48.5 pounds. Since the lower portions of the isoquants

$$(11)\quad \frac{\partial \hat{Y}/\partial K_2O}{\partial \hat{Y}/\partial N} = \frac{-0.0663 + \frac{0.4408}{\sqrt{K}}}{-0.0956 + \frac{1.3459}{\sqrt{P}}}$$

have slopes differing considerably from common K_2O/N price ratios, nutrient combinations on these parts of the curves give per-acre fertilization costs substantially greater than those falling lower on the isoquants. A 75-bushel yield, for example, has fertilization costs of $12.73 for 80 pounds of nitrogen and 14.6 pounds of K_2O but $15.63 for 110 pounds of nitrogen and 2.16 pounds of K_2O. The same yield produced with 48.5 pounds of K_2O and 69.5 pounds of nitrogen would have a fertilizer cost of $14.83, with prices of 14 cents for nitrogen and 10.5 cents for K_2O. However, the

TABLE 9. COMBINATIONS OF K_2O AND NITROGEN NEEDED TO PRODUCE A GIVEN CORN YIELD AND CORRESPONDING MARGINAL RATES OF SUBSTITUTION.

67.5 bushels			70 bushels		
Pounds of nitrogen	Pounds of K_2O	MRS $\left(\frac{\partial N}{\partial K_2O}\right)$	Pounds of nitrogen	Pounds of K_2O	MRS $\left(\frac{\partial N}{\partial K_2O}\right)$
12.9	48.52	0.00	26.0	48.52	0.00
16.0	9.73	0.35	30.0	8.12	0.61
20.0	2.55	1.13	35.0	2.86	1.49
25.0	0.13	6.69	40.0	0.65	4.12

72.5 bushels			75 bushels		
Pounds of nitrogen	Pounds of K_2O	MRS $\left(\frac{\partial N}{\partial K_2O}\right)$	Pounds of nitrogen	Pounds of K_2O	MRS $\left(\frac{\partial N}{\partial K_2O}\right)$
41.0	48.52	0.00	69.4	48.52	0.00
50.0	9.53	0.84	70.0	39.63	0.10
60.0	2.61	2.72	80.0	14.61	0.96
70.0	0.42	9.58	90.0	7.51	2.11
			100.0	4.06	4.05
			110.0	2.16	7.25
			120.0	1.12	12.99

$$(10)\quad K = \left[6.966339 \pm 7.898519 \sqrt{0.777232 - 0.024196N + 0.681576\sqrt{N} - 0.253212\ (Y\text{-}56.127)}\right]^2$$

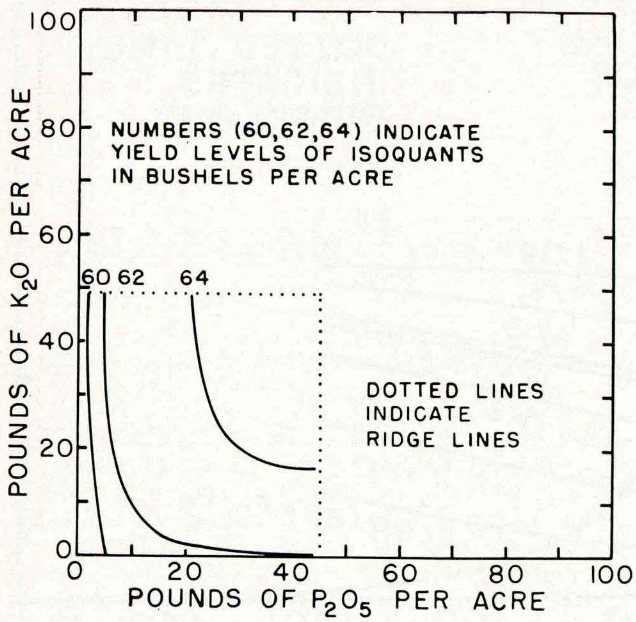

Fig. 15. Corn yield isoquants for K_2O and P_2O_5 (nitrogen held constant at zero) for Clarion soil.

75-pound yield need not necessarily represent the most profitable level of fertilization, as shown in a later section.

Equation 12 is the isoquant equation for K_2O and P_2O_5; equation 13 gives the marginal rates of substitution between these two inputs. The isoquants and marginal rates of substitution resulting from these equations are presented in fig. 15 for corn yields of 62 and 64 bushels per acre. The ridgelines are located at 48.5 pounds per acre of K_2O and 45.2 pounds per acre of P_2O_5. Table 10 lists the marginal rates of substitution of K_2O for P_2O_5.

TABLE 10. COMBINATIONS OF K_2O AND P_2O_5 NEEDED TO MAINTAIN A GIVEN CORN YIELD LEVEL (WITH THE CORRESPONDING MARGINAL RATES OF SUBSTITUTION).

62 bushels			64 bushels		
Pounds of K_2O	Pounds of P_2O_5	MRS $\left(\frac{dp}{dk}\right)$	Pounds of K_2O	Pounds of P_2O_5	MRS $\left(\frac{dp}{dk}\right)$
4.7	48.53	0.00	20.9	48.53	0.00
5.0	29.00	0.08	25.0	26.73	0.55
10.0	8.35	0.67	30.0	20.52	1.26
15.0	3.83	1.86	35.0	17.71	2.56
20.0	1.99	4.17	40.0	16.34	6.09
25.0	1.13	8.58	45.0	15.83	158.00

$$(13)\quad \frac{\partial\hat{Y}/\partial K}{\partial\hat{Y}/\partial P} = \frac{-0.0663 + \frac{0.4408}{\sqrt{K}}}{-0.1183 + \frac{0.7963}{\sqrt{P}}}$$

Because the production function 4 does not contain interaction terms, isoquants for any two nutrients can be derived independently of the third. However, while the rate of application of the third input does not affect the marginal rates of substitution between the two allowed to vary, it does affect the absolute yield level. Thus, the distance of the isoquant from the origin depends on the level (pounds per acre) at which the third input is fixed. Isoquants in the above tables and figures were derived when the third input was set at zero.

Yield Isoclines

Isoclines for nutrient pairs have been derived from equations 14, 15 and 16 and are presented graphically in figs. 16, 17 and 18. The $\propto$ in the

$$(12)\quad K = \left[6.964030 \pm 7.898519 \sqrt{0.777232 - 0.029976P + 0.403260\sqrt{P} - 0.253212\ (Y\text{-}56.127)}\right]^2$$

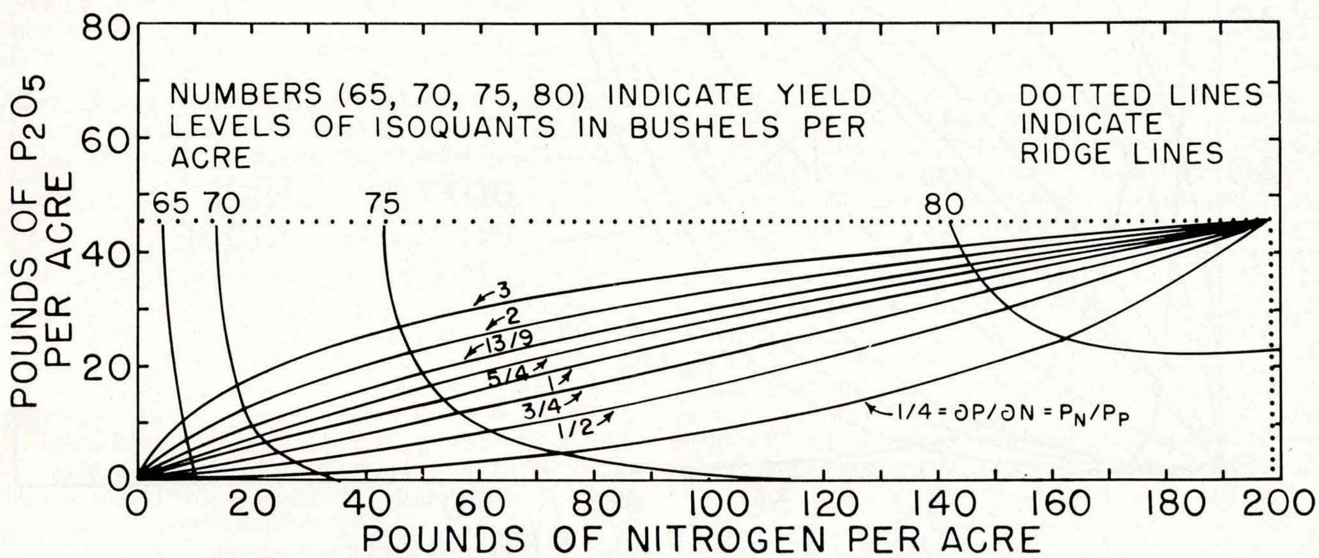

Fig. 16. Corn yield isoclines for P_2O_5 and nitrogen (K_2O held constant at zero) for Clarion soil.

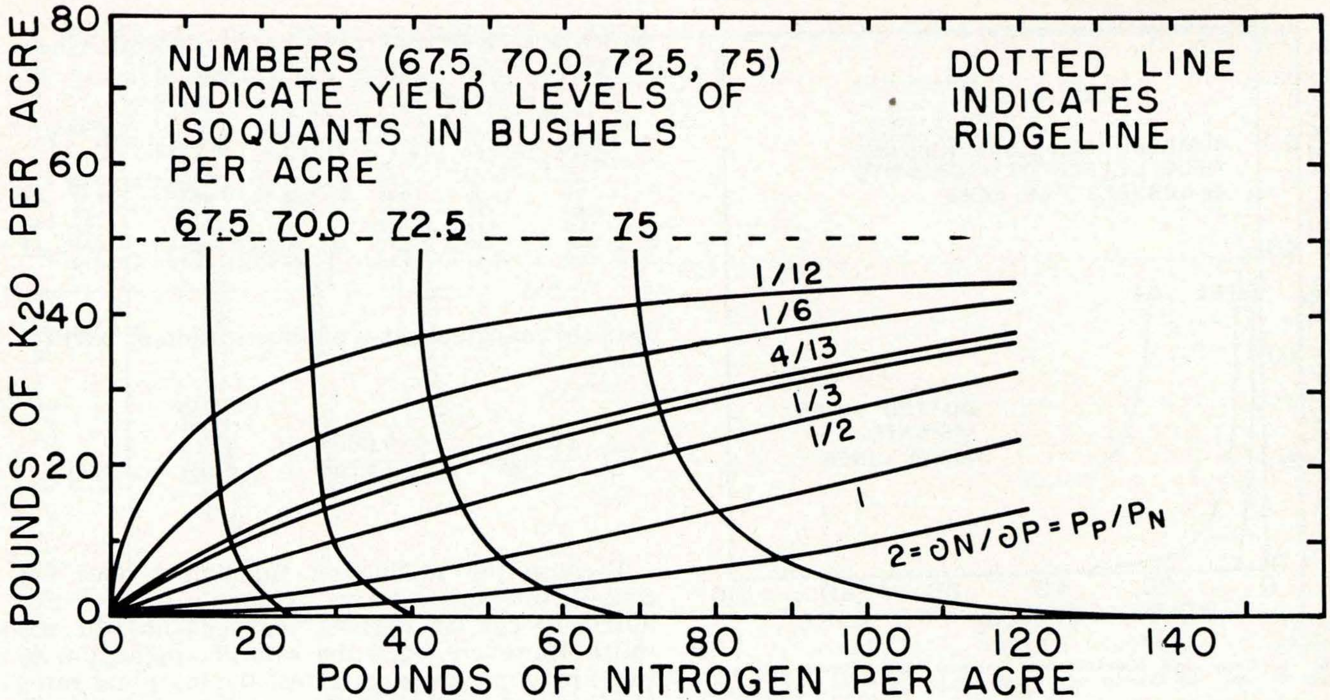

Fig. 17. Corn yield isoclines for K_2O and nitrogen (P_2O_5 held constant at zero) for Clarion soil.

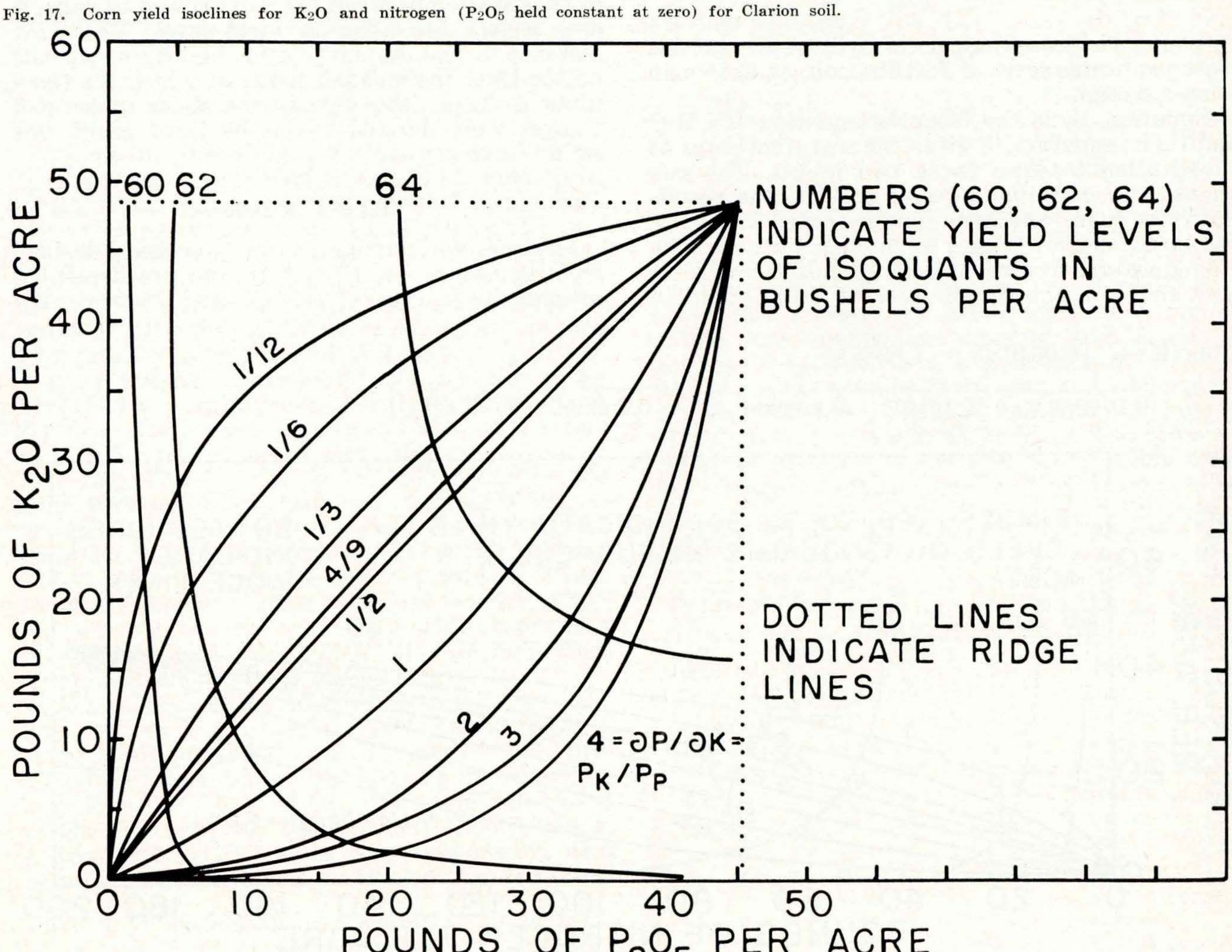

Fig. 18. Corn yield isoclines for P_2O_5 and K_2O (nitrogen held constant at zero) for Clarion soil.

equations represents a constant substitution or price ratio. The isocline represents the least-cost expansion path for any given fertilizer price ratio.

$$(14)\quad P = \left[\frac{0.7962}{\propto(-0.955 + 1.3458N^{-1/2}) + 0.1183}\right]^2$$

$$(15)\quad K = \left[\frac{0.4408}{\propto(-0.0955 + 1.3458N^{-1/2}) + 0.0633}\right]^2$$

$$(16)\quad K = \left[\frac{0.4408}{\propto(-0.1183 + 0.7963P^{-1/2}) + 0.0633}\right]^2$$

The isoclines converge with maximum yield—the point at which the ridgelines intersect. The yield at this point in fig. 16 for nitrogen and P_2O_5 when K_2O is applied at 48.5 pounds is 83.5 bushels per acre. When K_2O is zero, the yield for convergence is 80.4 bushels. For K_2O and nitrogen, the isoclines converge (a) at the maximum yield of 83.5 bushels per acre when P_2O_5 is used at 45.2 pounds per acre and (b) at the maximum yield of 78.2 bushels per acre when P_2O_5 is zero. For K_2O and P_2O_5 the yield where the ridgelines intersect and the isoclines converge is (a) 83.5 bushels per acre when nitrogen is fixed at 198.2 pounds and (b) 64.5 bushels when nitrogen is zero. For each isoquant map, one particular isocline most nearly approaches a straight line. However, this need not be an isocline representing a substitution ratio of one, as in fig. 16 for nitrogen and P_2O_5. While the isoclines in fig. 17 for nitrogen and K_2O do not converge as drawn, they would do so if the graph were extended sufficiently along the nitrogen axis.

Economic Optima and Confidence Limits

Since there are no interaction terms in the original production function in this experiment, the optimum combinations for a given yield from any two of the inputs can be found independently of the third. That is, the level at which the third input is fixed will not affect the ratio of the other two nutrients which minimize cost for a given yield. The level at which the third input is fixed will, however, affect the optimum amounts (but not the ratios) of the other two nutrients.

Least-cost combinations for two nutrients have been derived, with the third nutrient constant at a zero rate per acre, and are presented in table 11 for specified yield levels. For the price ratios indicated, the nutrient combinations are those which minimize costs for specified yields. The apparent discrepancies which occur occasionally in table 11, such as in table 11-A where 9 pounds of nitrogen are recommended for three price ratios and three different input rates of P_2O_5, are caused by rounding errors. The 13/9 ratio most closely approximates the present N/P_2O_5 price ratio while the ratios of 4/13 and 4/9 correspond to the K_2O/N and K_2O/P_2O_5 price ratios, respectively (prices which would be paid for a "single-element" dry fertilizer).

TABLE 11. COMBINATIONS OF NUTRIENTS WHICH MINIMIZE COSTS FOR GIVEN YIELDS AND VARYING RESOURCE PRICE RATIOS.

A. Combinations of P_2O_5 and nitrogen given $K_2O = 0$.

Yield level bushels per acre	Price ratio: P_N/P_P	Optimum pounds per acre of nitrogen	Optimum pounds per acre of P_2O_5
65	1/2	11	1
70	1/2	28	3
75	1/2	61	9
80	1/2	153	30
65	3/4	10	2
70	3/4	25	5
75	3/4	56	12
80	3/4	149	33
65	1	9	3
70	1	22	7
75	1	53	15
80	1	147	36
65	5/4	9	4
70	5/4	21	8
75	5/4	51	17
80	5/4	146	37
65	13/9	9	5
70	13/9	20	9
75	13/9	50	19
80	13/9	146	38

B. Combinations of K_2O and nitrogen given $P_2O_5 = 0$.

Yield level bushels per acre	Price ratio: P_K/P_N	Optimum pounds per acre of nitrogen	Optimum pounds per acre of K_2O
67.5	4/13	16	11
70.0	4/13	28	16
72.5	4/13	44	21
75.0	4/13	72	28
67.5	1/3	16	9
70.0	1/3	28	14
72.5	1/3	45	20
75.0	1/3	73	27
67.5	1/2	17	8
70.0	1/2	29	10
72.5	1/2	46	15
75.0	1/2	75	22
67.5	1	20	3
70.0	1	33	4
72.5	1	52	8
75.0	1	81	14

C. Combinations of K_2O and P_2O_5 given nitrogen $= 0$.

Yield level bushels per acre	Price ratio: P_K/P_P	Optimum pounds per acre of P_2O_5	Optimum pounds per acre of K_2O
60	1/3	4	8
62	1/3	8	13
64	1/3	23	31
60	4/9	4	6
62	4/9	8	11
64	4/9	24	28
60	1/2	4	5
62	1/2	9	10
64	1/2	25	27
60	1	5	2
62	1	12	6
64	1	28	22

Since an isocline may be regarded as a fertilizer ratio line, the loss of efficiency (departure from least-cost combinations) resulting from use of mixed fertilizers containing fixed ratios of nutrients can be determined by comparing the fixed ratio to optimum ratios predicted by the isoclines. If the ratio of inputs denoted by the isocline is constant for all yield levels, a mixed fertilizer containing that ratio will minimize the costs of producing any yield. Accordingly, for given nutrient prices (costs), the size of the deviations from a constant nutrient ratio will determine the amount of losses incurred from use of mixed fertilizers containing that fixed ratio. In table 11, the N/P_2O_5 ratios for yields of 65, 70, 75 and 80 bushels per acre are 1.8, 2.2, 2.6 and 3.8, respectively, when the price ratio is 13/9. For N/K_2O nutrient ratios, 1.5, 1.8, 2.1 and 2.6 are those which will minimize the cost of producing 67.5, 70.0, 72.5 and 75.0 bushels with a price ratio of 4/13. When the price ratio is 4/9, the yields of 60, 62 and 64 bushels of corn can be produced at minimum cost when K_2O/P_2O_5 nutrient ratios of 1.5, 1.4 and 1.2, respectively, are used. Hence, cost minimizing ratios predicted in this study cannot, for any pair of nutrients, be approximated by a constant ratio of nutrients. However, K_2O/P_2O_5 ratios could be more closely approximated by a fixed ratio than N/P_2O_5 ratios since the former have a smaller variance than the latter.

By setting the partial derivative of the production function equal to the nutrient-corn price ratios in the equations below (1) the ratios of all three nutrients to minimize costs of the optimum yield and (2) the optimum level of fertilization can be determined. The lack of interaction terms in the production function allows the optimum amount of each input to be obtained independently of the other inputs.

$$(17) \quad \frac{\partial \hat{Y}}{\partial N} = -0.0956 + 1.345859N^{-1/2} = \frac{P_N}{P_C}$$

$$(18) \quad \frac{\partial \hat{Y}}{\partial P} = -0.1184 + 0.796292P^{-1/2} = \frac{P_P}{P_C}$$

$$(19) \quad \frac{\partial \hat{Y}}{\partial K} = -0.0633 + 0.44084K^{-1/2} = \frac{P_K}{P_C}$$

Table 12 lists profit maximizing combinations of nutrients for alternate corn and fertilizer prices. The required amounts of nitrogen, P_2O_5 and K_2O were derived using equations 17, 18 and 19, respectively, where P_C is the price of corn and all other symbols are as defined previously. Confidence limits for the predicted yields of table 12 are presented in table 13. Profit from fertilizer, in table 12, was computed by subtracting the cost of the fertilizer from the value of the added product. The prices in table 12 are used to illustrate possible changes in profit maximizing combinations of nutrients when nutrient and corn prices increase, decrease or vary relative to each other. When purchased in single element fertilizers, nitrogen, P_2O_5 and K_2O prices are currently about \$0.13, \$0.09 and \$0.04 per pound, respectively, while the cost of these nutrients in mixed fertilizer increases about \$0.02 per pound in each case. Nitrogen presently costs about \$0.08 per pound in anhydrous ammonia. Using these nutrient prices as base points, the prices in table 12 were derived.

As would be expected from examination of production surfaces and individual response curves, nitrogen has the largest effect on yield. Therefore, the optimum combinations require more nitrogen than other inputs, even when the price of nitrogen is high relative to other input prices (compare situations 7-9 to 10-12). Conversely, the amounts of K_2O and P_2O_5 vary in relation to each other as their relative prices change (compare situations 1-3 with 9-12).

Small shifts in nutrient prices (\$0.02 per pound) cause correspondingly small shifts in the predicted optimum amounts. Usage of nitrogen, the one of the three nutrients which is the most

TABLE 12. INPUT COMBINATIONS WHICH MAXIMIZE PROFITS FOR VARIOUS NITROGEN, POTASH, PHOSPHORUS AND CORN PRICES.

Situation number	Price per unit				Optimum input in pounds per acre			Predicted corn yield in bushels per acre	Profit from use of optimum inputs*
	Corn per bushel	N per pound	P per pound	K per pound	N	P	K		
1	\$1.60	\$0.17	\$0.13	\$0.08	44.45	15.91	15.13	76.77	\$22.19
2	1.30	0.17	0.13	0.08	36.53	13.29	12.47	75.46	16.20
3	1.00	0.17	0.13	0.08	25.68	10.28	9.46	73.32	10.73
4	1.60	0.15	0.11	0.06	50.57	18.11	19.12	77.71	23.81
5	1.30	0.15	0.11	0.06	40.74	15.39	16.23	76.36	17.53
6	1.00	0.15	0.11	0.06	30.04	12.16	12.78	74.47	11.73
7	1.60	0.13	0.09	0.04	57.97	20.80	24.92	78.70	25.71
8	1.30	0.13	0.09	0.04	47.36	18.02	21.99	77.49	19.12
9	1.00	0.13	0.09	0.04	35.60	14.60	18.20	75.75	12.95
10	1.60	0.08	0.09	0.04	85.49	20.80	24.92	80.47	29.24
11	1.30	0.08	0.09	0.04	73.43	18.02	21.99	79.54	22.06
12	1.00	0.08	0.09	0.04	58.77	14.60	18.21	78.11	15.24

*Profit figured on increase in yield over 56.13 bushels per acre.

TABLE 13. CONFIDENCE LIMITS FOR YIELDS PREDICTED IN TABLE 12.

Situation number	Predicted corn yield in bushels per acre	95% confidence limits for predicted yields in bushels per acre		90% confidence limits for predicted yields in bushels per acre	
		Lower	Upper	Lower	Upper
1	76.77	75.03	78.51	75.31	78.23
2	75.46	73.76	77.16	74.03	76.89
3	73.32	71.66	74.98	71.93	74.71
4	77.71	75.94	79.48	76.23	79.19
5	76.36	74.63	78.09	74.91	77.81
6	74.47	72.78	76.16	73.06	75.88
7	78.70	76.90	80.50	77.20	80.20
8	77.49	75.72	79.26	76.01	78.97
9	75.75	74.02	77.48	74.30	77.20
10	80.47	77.53	83.41	78.01	82.93
11	79.54	77.74	81.34	78.04	81.04
12	78.11	76.34	79.88	76.63	75.59

sensitive to price changes, decreases by approximately 7 pounds (between situations 7 and 4) when its price increases $0.02. For a similar price change between situations 4 and 1, nitrogen usage decreases by only 6 pounds. The profit resulting from these situations, decreasing only slightly, is $25.71, $23.81 and $23.81 for situations 7, 4 and 1, respectively.

Corn prices have more effect on optimum nutrient inputs and the resulting profits than do prices of other nutrients. Compare, for instance, situations 7, 8 and 9: Each $0.30 decrease in corn price causes a decrease of approximately 10, 3 and 3 pounds per acre in the predicted optimum inputs for nitrogen, P_2O_5 and K_2O, respectively. Profits resulting from these situations are $25.71, $19.12 and $12.95, respectively. The decreases in nutrient inputs caused by corn price reductions are slightly smaller (as are the resulting profits) when nutrient prices rise as in situations 1, 2 and 3. Contrariwise, the input decrements and the resulting profits are larger when nutrient prices fall as in situations 10, 11 and 12.

Effects of using a nutrient ratio and level of fertilization other than the optimum one can also be illustrated from the data in table 12. For situation 7, use of the optimum fertilizer combination gives a return of $25.71 over costs. Using the optimum combination of situation 3 for the prices of situation 7, the return is $22.86 over fertilizer costs. If the combination of situation 10 is used for the prices of situation 7, the gain over cost is $24.96; only slightly less than if the optimum combination for situation 7 is used under the prices of situation 7.

EXPERIMENT ON IRRIGATED McPAUL SILT LOAM IN 1955

The experiment on McPaul silt loam included nitrogen (N) and phosphorus (P_2O_5) as variables. The 9 x 9 factorial design had each nutrient applied in rates of 0, 20, 40, 60, 80, 120, 160, 200 and 240 pounds per acre. A completely randomized design was used. The check plots were replicated 23 times in an attempt to gain an accurate estimate of the yield intercept value and to allow for a residual response study. Also, the 20-, 40- and 80-pound rates of each nutrient and their combinations were replicated twice. The corn yields, obtained under irrigated conditions, are presented in table 14.

TABLE 14. CORN YIELDS ON IRRIGATED McPAUL SILT LOAM EXPERIMENTAL PLOTS, 1955 (BU. PER ACRE).

Check plots: N = P_2O_5 = 0.

76.2	90.4	81.7	89.3	75.5	58.4
87.3	70.4	89.6	84.0	81.5	70.0
85.4	97.0	84.5	66.4	63.8	59.3
83.3	95.6	72.1	74.2	81.9	

Fertilized plots:

Pounds of nitrogen per acre	Pounds of P_2O_5 per acre 0	20	40	60	80	120	160	200	240
0	check plots	45.1 82.4	87.5 87.1	85.5	79.7 89.5	92.7	92.8 99.7	73.1	77.3
20	97.2 60.6	102.6 63.1	107.3 88.6	82.9	97.5 83.9	94.7	90.1	89.8	88.0
40	76.3 69.8	94.2 90.0	94.7 100.0	91.8	113.0 96.2	87.6	94.2	83.0	95.9
60	96.9	89.9	91.7	75.0	80.2	97.9	101.2	101.2	101.8
80	109.7 66.9	51.8 94.6	101.9 83.4	99.6	99.1 82.3	95.6	88.3	109.6	103.8
120	58.0	109.7	74.7	96.4	97.1	110.2	98.3	104.6	98.7
160	53.9	83.7	84.7	91.9	103.6	75.8	101.4	97.0	108.4
200	81.4	73.0	88.0	90.4	93.2	92.7	98.7	89.4	91.8
240	103.9	76.8	78.4	101.0	99.3	114.2	102.6	103.4	96.6

The yields in table 14 are highly variable. This variation is due, at least in part, to uneven applications of the irrigation water. Hence, definite responses to the nutrients are difficult to obtain from examination of table 14. Upward trends are indicated as the nutrient inputs increase, and interaction seems to be present. The general level of yields is high. The highest yield is 114.2 bushels per acre; an increase of 69.1 bushels over the lowest yield.

REGRESSION ANALYSIS

Of the equations considered for the regression analysis, the square root function or the quadratic function has been found, in general, to be most satisfactory.[5] It appears logical to assume, however, that the production surface estimated from a given set of experimental results, as well as the optimum quantities derived therefrom, will vary depending upon the mathematical form chosen for the regression equation. As an exploratory attempt to determine the type and magnitude of the differences of estimates derived from varying mathematical forms, both a square root and a quadratic equation were fitted to the present data. Accordingly, an economic analysis was derived from each equation.

$$(20)\quad \hat{Y} = 33.1614 + 0.7842S + 0.0273N + 0.1638P - 0.000149N^2 - 0.000565P^2 + 0.000243NP$$

$$(21)\quad \hat{Y} = 30.1751 + 0.8268S + 0.539309\sqrt{N} + 1.256664\sqrt{P} - 0.0614N - 0.0610P + 0.088035\sqrt{N}\sqrt{P}$$

Equation 20 is a quadratic form fitted to the data in table 14, and equation 21 is a square root transformation fitted to the same data. In these functions, $\hat{Y}$ is the predicted bushels per acre of corn, N and P denote the pounds per acre of nitrogen and P_2O_5, respectively, and S is the stand in inputs of 218 corn plants per acre. The quadratic equation gives an R^2 of 0.3918 while the square root equation yields an R^2 of 0.4131. Values of t for the partial regression coefficients of the two equations are given in table 15. Table 16 includes an analysis of variance of regression for each equation.

The coefficient of determination, R^2, is similar for both equations, i.e., each equation explains approximately the same amount of the variance

[5]See: Heady, Earl O., Pesek, John T. and Brown, W. G., **op. cit.**; and Brown, W. G., Heady, Earl O., Pesek, John T. and Stritzel, Joseph A., **op. cit.**

TABLE 15. VALUES OF t FOR THE PARTIAL REGRESSION COEFFICIENTS OF EQUATIONS 20 AND 21.

A. t values for the coefficients of quadratic equation 20.

Regression coefficient for:	Value of t	Probability level*
S	4.47	0.0001
N	0.57	0.50
P	3.44	0.0001
N^2	0.71	0.50
P^2	2.69	0.01
NP	1.37	0.20

B. t values for the coefficents of square root equation 21.

Regression coefficient for:	Value of t	Probability level*
S	4.82	0.0001
$\sqrt{N}$	1.37	0.20
$\sqrt{P}$	1.34	0.20
N	0.81	0.50
P	1.88	0.10
$\sqrt{N}\sqrt{P}$	2.16	0.05

*Probability of drawing a t value as large or larger by chance, given the null hypothesis.

TABLE 16. ANALYSIS OF VARIANCE OF REGRESSION FOR EQUATIONS 20 AND 21.

A. The analysis of variance of quadratic equation 20.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Total	118	22,586.36		
Due to regression	6	8,849.96	1,474.99	12.03
Deviations from regression	112	13,736.40	122.65	

B. The analysis of variance of the square root equation 21.

Source of variation	Degrees of freedom	Sums of squares	Mean square	F
Total	118	22,586.36		
Due to regression	6	9,330.44	1,555.07	13.14
Deviations from regression	112	13,255.92	118.36	

of the corn yield.[6] Hence, the error mean square is almost the same for both equations (see table 16). Values of t for the coefficients of the two equations, with the exception of the stand terms and nitrogen power terms, are not similar. Since the error variance is the same for both equations, the difference in t values may be due to either the algebraic differences in the equations or to correlations between the "independent" variables. Thus, terms deleted on the basis of t tests would differ for each equation.

The N and N^2 terms might be omitted from the quadratic equation since their t values have low probabilities. However, the crossproduct term of equation 20 has a probability level which warrants its retention; and since N is included in the crossproduct, it seems reasonable to take account of its linear and quadratic effects in the economic analysis even if they are not significant at the usual probability levels. The only term which would be omitted from the square root equation on the basis of the t tests is the linear term for nitrogen. Again, it does not seem logically appropriate to delete only the linear term. Thus, all terms were retained to facilitate the comparison of the two equations in the following analysis.

As described in previous studies, the linear stand terms are inserted in the regression equations in place of a conventional covariance analysis. When the mean value for stand is substituted into equations 20 and 21, equations 22 and 23 result. These equations will be used for the economic analysis. The yield intercept values of equations 22 and 23 are similar to each other and to the average of the 23 check plots (79.03).

$$(22)\quad \hat{Y} = 80.3472 + 0.0273N + 0.1638P - 0.000149N^2 - 0.000565P^2 + 0.000243NP$$

$$(23)\quad \hat{Y} = 79.9242 + 0.539309\sqrt{N} + 1.25664\sqrt{P} - 0.0614N - 0.0610P + 0.088035\sqrt{P}\sqrt{N}$$

Production Surfaces and Predicted Yields

Yields predicted by equations 22 and 23 are included in table 17. These yields were calculated by substituting the combinations of nitrogen and P_2O_5 listed in the table into the production functions. It should be noted that yields in table 17 have been adjusted to the average stand level, while stand is a random variable in table 14.

The yield intercept values for the two equations differ very little because of the large number of replications of the check plots. Differences in magnitude of yield response do occur, however. When there is no interaction (that is, one of the inputs is applied at a zero rate) the quadratic function predicts larger yields and yield decreases less rapidly, as compared with predictions by the square root form. When the nutrients are applied simultaneously and interactions are present, the square root function predicts yields which are initially higher, but which increase at a less rapid rate as compared with predictions by the quad-

TABLE 17. PREDICTED CORN YIELDS FOR McPAUL SILT LOAM (BU. PER ACRE).

A. Corn yields predicted by quadratic equation.

Pounds of P_2O_5 per acre	Pounds of nitrogen per acre						
	0	40	80	120	160	200	240
0	80.35	81.20	81.58	81.47	80.90	79.84	78.31
40	86.00	87.24	88.01	88.29	88.10	87.44	86.29
80	89.84	91.47	92.63	93.30	93.50	93.23	92.47
120	91.88	93.90	95.44	96.51	97.10	97.21	96.84
160	92.11	94.52	96.45	97.91	98.89	97.39	99.41
200	90.53	93.33	95.66	97.50	98.87	99.76	100.17
240	87.15	90.34	93.05	95.29	97.04	98.32	99.13

B. Corn yields predicted by square root equation.

Pounds of P_2O_5 per acre	Pounds of nitrogen per acre						
	0	40	80	120	160	200	240
0	79.92	80.88	79.83	78.46	76.39	75.27	73.55
40	85.43	89.90	90.62	90.07	89.47	88.66	87.68
80	86.28	92.51	93.23	93.44	93.24	92.77	92.11
120	86.36	93.42	94.90	95.47	95.57	95.35	94.93
160	86.05	94.05	95.92	96.80	97.14	97.15	96.93
200	85.48	94.31	96.54	97.66	98.23	98.44	98.40
240	84.74	94.20	96.86	98.22	98.99	99.38	99.68

[6]The square root function usually has a higher R^2 than a quadratic equation fitted to the same data if equations include similar terms. See: Heady, E. O., Pesek, J. T. and Brown, W. G., **op cit.**, and the Clarion experimental data included earlier in this bulletin.

ratic form. This fact is apparent in table 17. By comparing parts A and B of table 17, we see that for rows where $P_2O_5=40$ or the columns where $N=40$, the square root function predicts higher yields. Examination of the rows or columns where $N=P_2O_5=80$ shows the predictions of the two equations to be similar. The quadratic equation predicts higher yields for input combinations between 120 and 200 pounds. Hence, the quadratic equation reaches a point of maximum yield at lower input levels than does the square root function.

For nitrogen rates of 80 pounds or more, the square root equation does not indicate decreasing total yields resulting in the range of P_2O_5 applications listed in table 17. The quadratic, however, predicts decreasing returns to P_2O_5 at all rates of nitrogen. Contrariwise, both equations predict a diminishing total product for nitrogen, differing only in predicted level of P_2O_5 input at which these negative marginal products occur. However, the yield level at which decreasing total returns shift to increasing returns, because of change in the level of a "fixed" nutrient, is similar in both cases.

Production surfaces for the quadratic and square root functions are depicted graphically in figs. 19 and 20, respectively. Figure 20 clearly illustrates the large increases in yield caused by the initial fertilizer applications predicted by the square root function. Beyond the 80-pound rates of each nutrient, however, the surface in fig. 20 appears to increase at a near-constant rate. In contrast, fig. 19 depicts a function which rises smoothly, but has a "more curved appearance" over the complete surface. Interaction between the nutrients causes the surfaces to slope slightly downward on the far corner of the graphs.

The response of each nutrient and the interaction between nutrients can also be shown by two-dimensional graphs. Figures 21 to 24 depict curves resulting from parallel slices taken along the input axis of the production surfaces (figs. 19 and 20). That is, the slice is parallel to the axis of the variable nutrient. Figures 21 and 22 illustrate corn yield response to nitrogen, when P_2O_5 is held constant at three levels, predicted by the quadratic and square root equations, respectively. In the same order, figs. 23 and 24 show the response to P_2O_5 predicted by the equations when nitrogen is constant at three levels. These figures show clearly the predictional differences discussed above.

A slice through the production surfaces and passing through the origin on the input plane ($N=P_2O_5=O$) depicts the corn yield resulting from the application of a constant ratio of the two nutrients. Hence, the curves in figs. 25 to 28 result from applications of an aggregate input where, for any given curve, this aggregate input is composed of constant proportions of the nutrients, i.e., $2N=P$, $N=P$, etc. While the curves illustrate, in general, the considerations previously discussed, it should be noted that extreme extrapolations are made for the input ratios ($2N=P$) and ($2P=N$). For the curve labelled $P=2N$ in

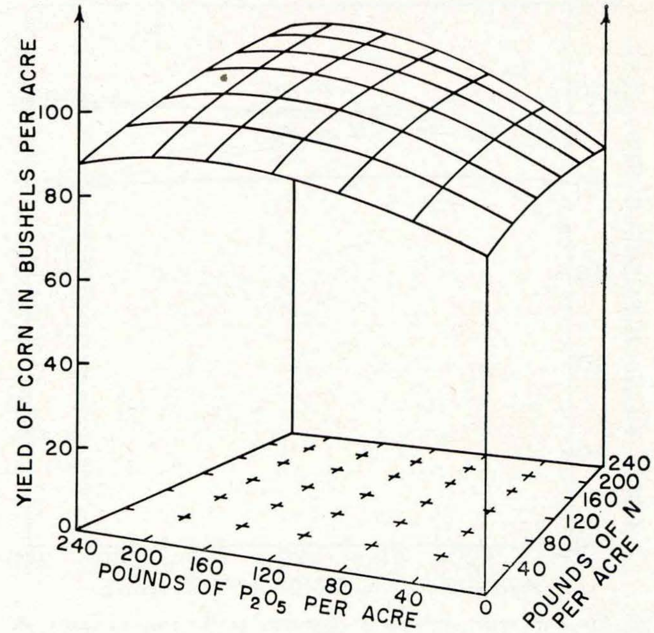

Fig. 19. Production surface for McPaul silt loam predicted by quadratic equation.

fig. 27, P_2O_5 equals 480 pounds when nitrogen equals 240 pounds. The curves are included here, however, because they illustrate interesting differences between the two mathematical forms. Comparing the curves ($2N=P$) in figs. 27 and 28, the quadratic form attains a maximum at much lower input levels than the square root form. In fact, the latter reaches a maximum yield at

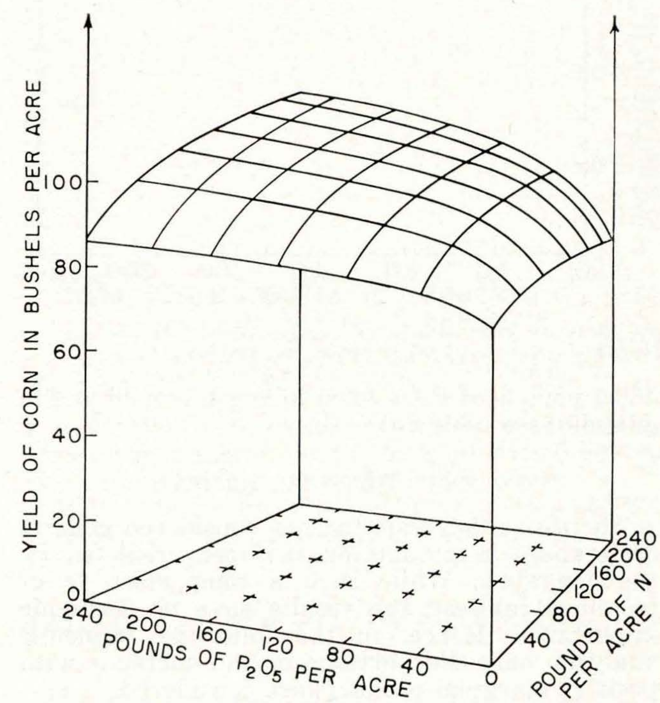

Fig. 20. Production surface for McPaul silt loam predicted by square root equation.

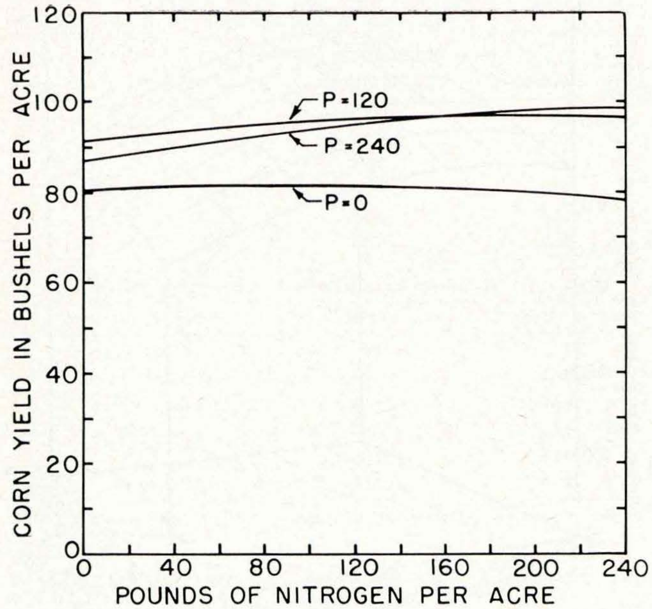

Fig. 21. Corn yield response to nitrogen (P_2O_5 held constant) on McPaul silt loam—predicted by quadratic equation.

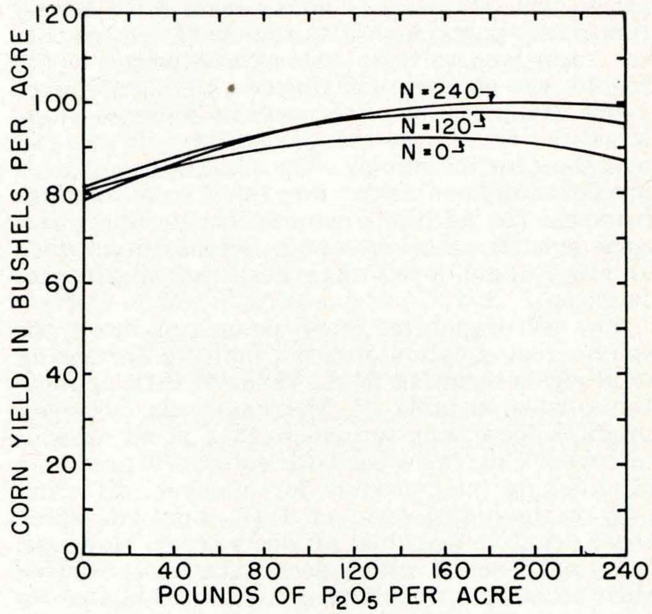

Fig. 23. Corn yield response to P_2O_5 (nitrogen held constant) on McPaul silt loam—predicted by quadratic equation.

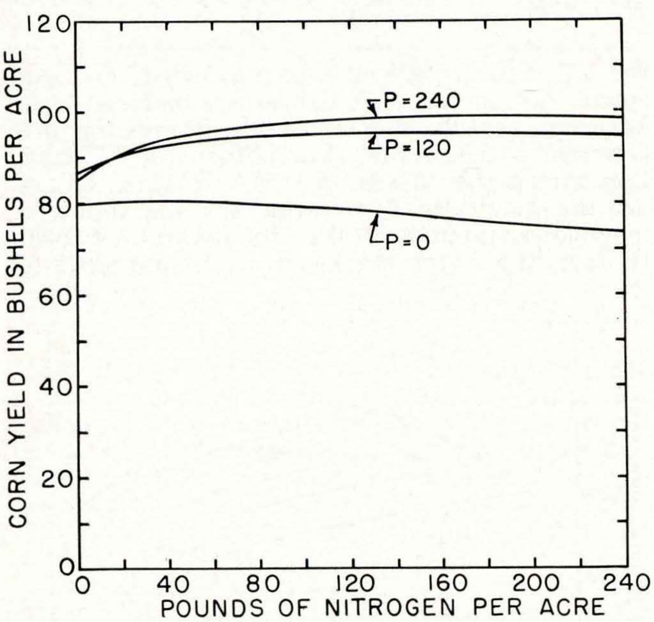

Fig. 22. Corn yield response to nitrogen (P_2O_5 held constant) on McPaul silt loam—predicted by square root equation.

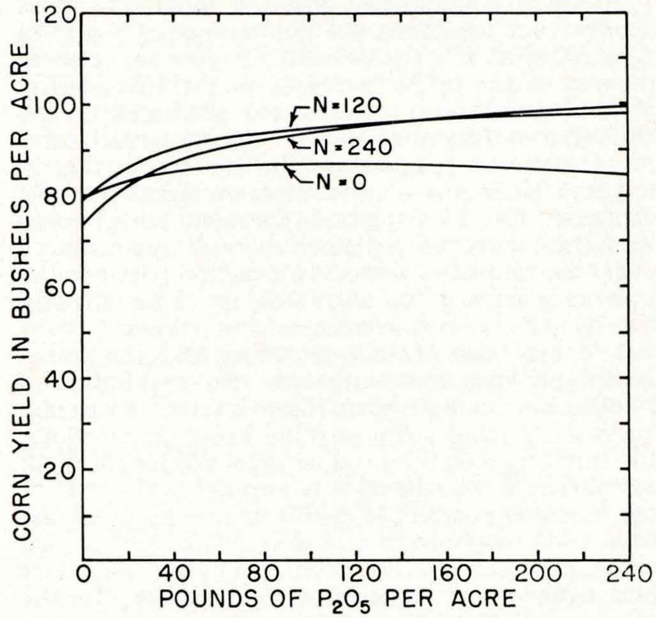

Fig. 24. Corn yield response to P_2O_5 (nitrogen held constant) on McPaul silt loam—predicted by square root equation.

460.1 pounds of P_2O_5 when nitrogen is held at 240 pounds (see table 20).

Marginal Physical Products

The discussion thus far has considered general differences in production surfaces predicted by the equations. While such a comparison is of technical interest, the results have no economic importance. Hence, in the following economic analysis, only the portions of the surfaces with positive marginal products are considered.

Equations 24 and 25 are the marginal product equations of nitrogen derived from the quadratic (22) and square root (23) equations, respectively. The marginal products of nitrogen, with P_2O_5

$$(24)\ \frac{\partial \hat{Y}}{\partial N} = 0.0273 - 0.0003N + 0.000243P$$

$$(25)\ \frac{\partial \hat{Y}}{\partial N} = -0.0614 + 0.269654N^{-1/2} + 0.044018P^{1/2}N^{-1/2}$$

fixed at levels of 0, 120 and 240 pounds per acre, are presented in fig. 29. The marginal product equation of the square root equation is curved while that of the quadratic is linear.

The marginal physical products of nitrogen,

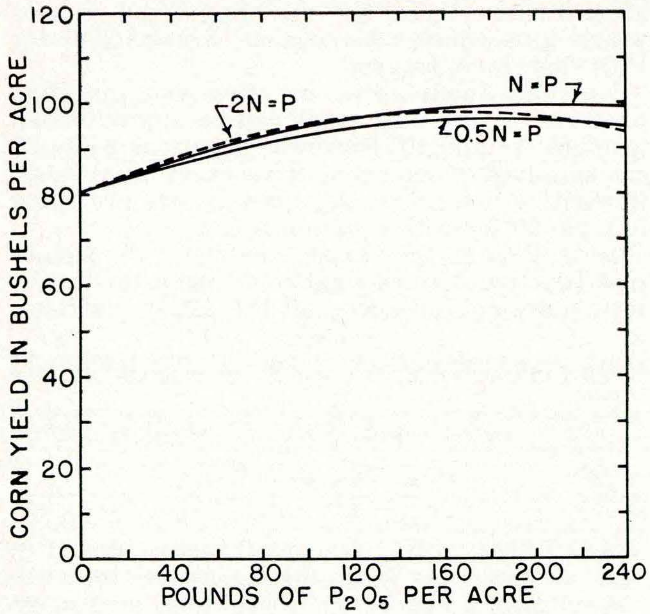

Fig. 25. Corn yield response to aggregated inputs of P_2O_5 and nitrogen on McPaul silt loam—predicted by quadratic equation.

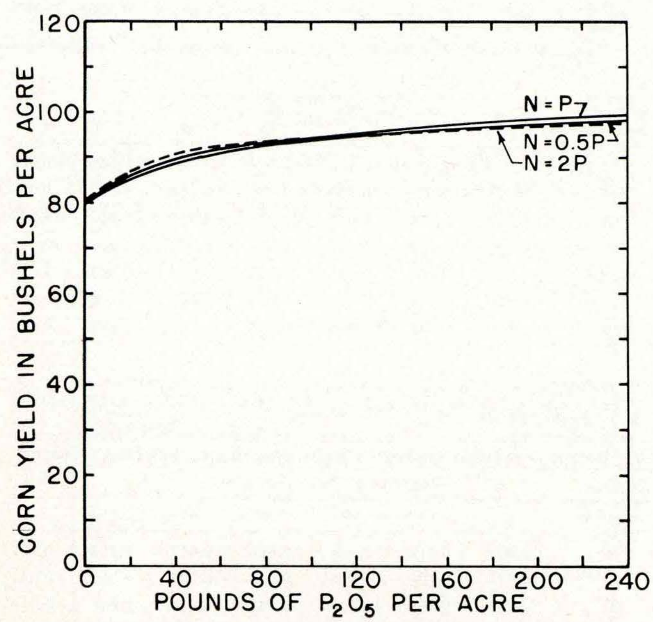

Fig. 26. Corn yield response to aggregated inputs of P_2O_5 and nitrogen on McPaul silt loam—predicted by square root equation.

predicted by the two equation forms, are most similar when nitrogen inputs range between 40 and 60 pounds. For smaller inputs those predicted from the square root equation are larger; for larger inputs those of the quadratic function are larger. Whether or not recommended rates of nitrogen differ between equations depends on the ratio of nitrogen costs to corn prices. If this ratio were approximately 0.07, 0.08 or 0.02 (when P_2O_5 is fixed at 0, 120 or 240 pounds, respectively) predictions from the two equations would result in about equal nitrogen inputs. The quantities of nitrogen would be about 10, 34 or 50

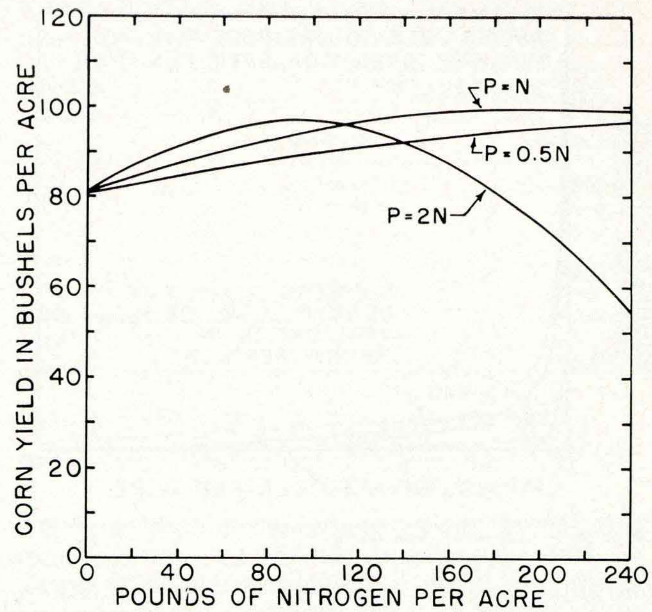

Fig. 27. Corn yield response to aggregated inputs of P_2O_5 and nitrogen on McPaul silt loam—predicted by quadratic equation.

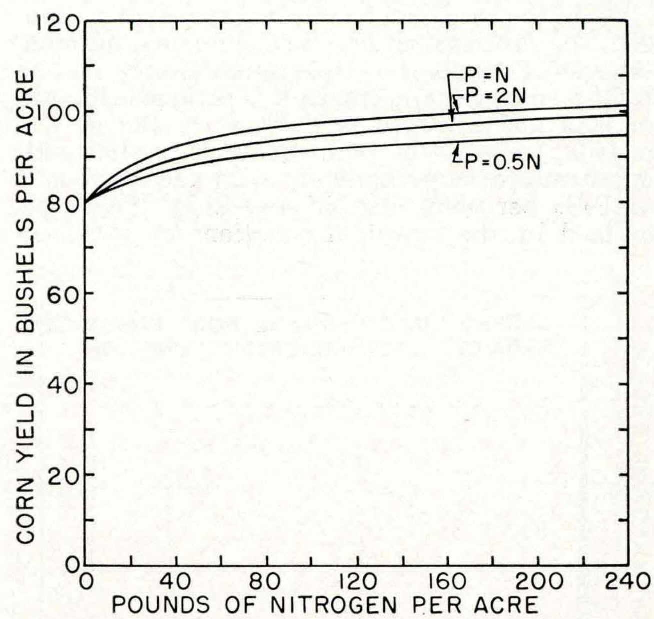

Fig. 28. Corn yield response to aggregated inputs of P_2O_5 and nitrogen on McPaul silt loam—predicted by square root equation.

pounds per acre, respectively. As price ratios vary from these, optimum rates predicted by the two equations take on greater differences.

Equations 26 and 27 provide predictions of the marginal physical products of P_2O_5 for the quadratic and square root equations, respectively. The

$$(26)\ \frac{\partial \hat{Y}}{\partial P} = 0.1638 - 0.0011P + 0.000243N$$

$$(27)\ \frac{\partial \hat{Y}}{\partial P} = -0.0610 + 0.628332P^{-1/2} + 0.044018N^{1/2}P^{-1/2}$$

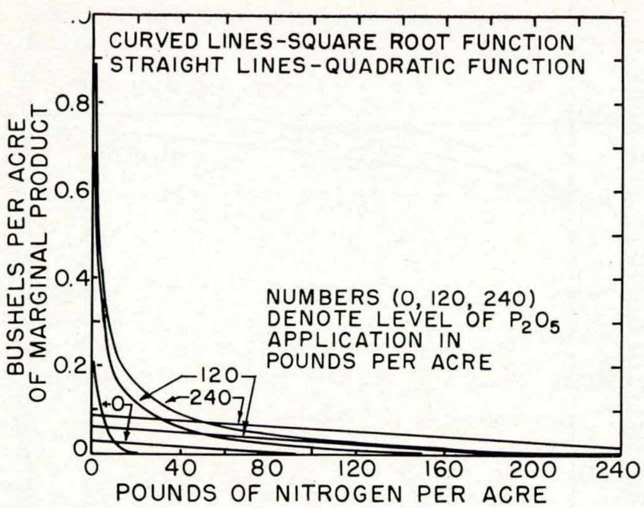

Fig. 29. Marginal physical product of nitrogen on McPaul silt loam (when P_2O_5 is fixed at three levels).

marginal products predicted by these equations are presented in table 19 for selected rates of nutrient inputs. Equations for the marginal product of P_2O_5 when nitrogen is held at three levels are pictured in fig. 30.

As in the case of the marginal curves for nitrogen, the discrepancy between predicted optimum rates of P_2O_5 for the equations depends on the P_2O_5-corn price ratio. When this ratio is 0.16, 0.17 or 0.19 and nitrogen is fixed at 0, 120 or 240 pounds, respectively, both equations predict optimum rates of approximately 8, 24 and 26 pounds of P_2O_5 per acre, respectively. Since the slopes defined by the marginal equations of P_2O_5 are

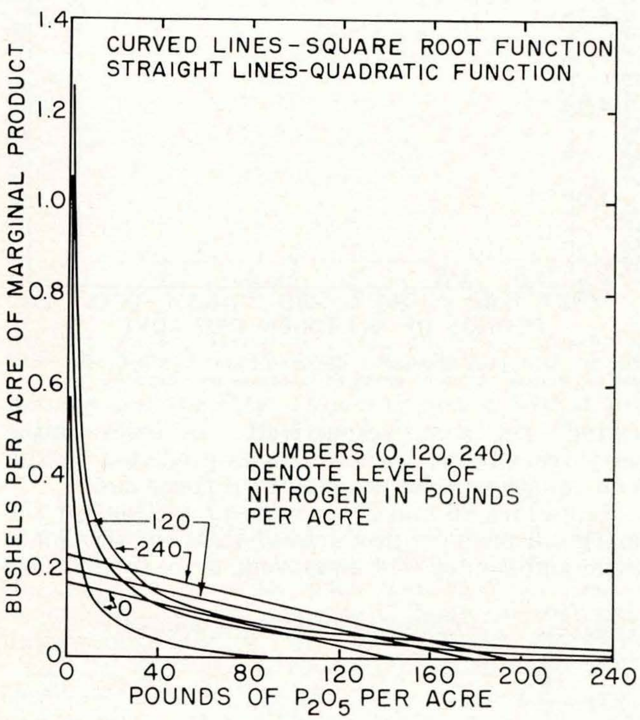

Fig. 30. Marginal physical product of P_2O_5 on McPaul silt loam (when nitrogen is fixed at three levels).

greater than those for nitrogen, input ranges which give comparable optima are smaller for P_2O_5 than for nitrogen.

Nutrient input rates denoting the marginal products of zero magnitude can be approximated in tables 18 and 19. However, equations 24 to 27 can be solved to determine these exact input rates. Quantities, determined in this way, are presented in table 20 for both equation forms.

With P_2O_5 as the variable nutrient, the square root function defines smaller nitrogen inputs for zero marginal products of the latter nutrient.

TABLE 18. MARGINAL PHYSICAL PRODUCT OF NITROGEN AT SEVEN LEVELS OF P_2O_5 FOR McPAUL SILT LOAM (BU. PER ACRE).

A. Marginal physical product of nitrogen predicted by quadratic equation.

Pounds of P_2O_5 per acre	Pounds of nitrogen per acre						
	0	40	80	120	160	200	240
0	0.027	0.015	0.003	—0.009	—0.020	—0.032	—0.044
40	0.037	0.025	0.013	0.001	—0.011	—0.023	—0.035
80	0.047	0.035	0.023	0.011	—0.001	—0.013	—0.025
120	0.057	0.045	0.036	0.021	0.009	—0.003	—0.015
160	0.066	0.054	0.042	0.030	0.019	0.007	—0.005
200	0.076	0.064	0.052	0.040	0.028	0.016	0.004
240	0.086	0.074	0.062	0.050	0.038	0.026	0.014

B. Marginal physical product of nitrogen predicted by square root equation.

Pounds of P_2O_5 per acre	Pounds of nitrogen per acre						
	0	40	80	120	160	200	240
0	0.208	—0.019	—0.031	—0.037	—0.040	—0.042	—0.044
40	0.477	0.025	—0.000	—0.011	—0.018	—0.023	—0.026
80	0.602	0.044	0.013	—0.001	—0.009	—0.015	—0.019
120	0.690	0.058	0.023	0.007	—0.002	—0.008	—0.013
160	0.765	0.069	0.031	0.014	0.004	—0.003	—0.008
200	0.831	0.080	0.038	0.020	0.009	0.002	—0.004
240	0.890	0.089	0.045	0.026	0.014	0.006	0.000

TABLE 19. MARGINAL PHYSICAL PRODUCT (MPP) OF P_2O_5 AT SEVEN LEVELS OF NITROGEN FOR McPAUL SILT LOAM (BU. PER ACRE).

A. Marginal physical product of P_2O_5 predicted by quadratic equation.

Pounds of nitrogen per acre	Pounds of P_2O_5 per acre						
	0	40	80	120	160	200	240
0	0.164	0.119	0.074	0.028	—0.017	—0.062	—0.107
40	0.174	0.128	0.083	0.038	—0.007	—0.052	—0.097
80	0.183	0.138	0.093	0.048	0.003	—0.043	—0.088
120	0.193	0.148	0.103	0.058	0.012	—0.033	—0.078
160	0.203	0.158	0.112	0.067	0.022	—0.023	—0.068
200	0.213	0.167	0.122	0.077	0.032	—0.013	—0.059
240	0.222	0.177	0.132	0.087	0.042	—0.004	—0.049

B. Marginal physical product of P_2O_5 predicted by square root equation.

Pounds of nitrogen per acre	Pounds of P_2O_5 per acre						
	0	40	80	120	160	200	240
0	0.567	0.038	0.009	—0.004	—0.012	—0.017	—0.021
40	0.846	0.082	0.040	0.022	0.010	0.003	—0.003
80	0.961	0.101	0.053	0.032	0.020	0.011	0.005
120	1.049	0.115	0.063	0.040	0.027	0.018	0.011
160	1.124	0.126	0.071	0.047	0.033	0.023	0.015
200	1.190	0.137	0.079	0.053	0.038	0.027	0.020
240	1.249	0.146	0.085	0.057	0.043	0.032	0.024

TABLE 20. RATES OF INPUTS AT WHICH THE MARGINAL PHYSICAL PRODUCT OF ONE INPUT IS ZERO, GIVEN VARIOUS RATES OF THE SECOND INPUT.

Pounds of P_2O_5 per acre	Pounds per acre at which marginal physical product of nitrogen is zero		Pounds of nitrogen per acre	Pounds per acre at which marginal physical product of P_2O_5 is zero	
	Quadratic	Square root		Quadratic	Square root
0	91.49	19.27	0	145.11	105.88
40	124.12	79.74	40	153.73	220.02
80	156.74	116.64	80	162.35	279.89
120	189.36	150.06	120	170.97	330.05
160	221.98	181.17	160	179.59	378.36
200	254.60	211.11	200	188.21	419.43
240	287.22	240.25	240	196.84	460.10

However, excluding the nitrogen rate of zero, the reverse is true for inputs of P_2O_5. Equating functions 24 and 26 to zero, the quadratic equation predicts a maximum yield at 200.0 and 254.5 pounds per acre of P_2O_5 and nitrogen, respectively. The corresponding yield is 100.2; 0.6 bushel less than the maximum yield predicted by the square root equation for the same nutrient levels. Equating functions 25 and 27 to zero, the square root equation predicts a maximum yield at 777.3 and 594.9 pounds for P_2O_5 and nitrogen quantities far beyond the observations of the study. The corresponding yield is 104.0 bushels.

Yield Isoquants

Isoquants derived from the quadratic equation 22 and the square root equation 23 are presented in figs. 31 and 32, respectively.

$$(28)\quad P = 145.113100+0.215509N \pm 885.739593 (0.208265+0.000141N-0.00000028N^2 -0.002258Y)^{0.5}$$

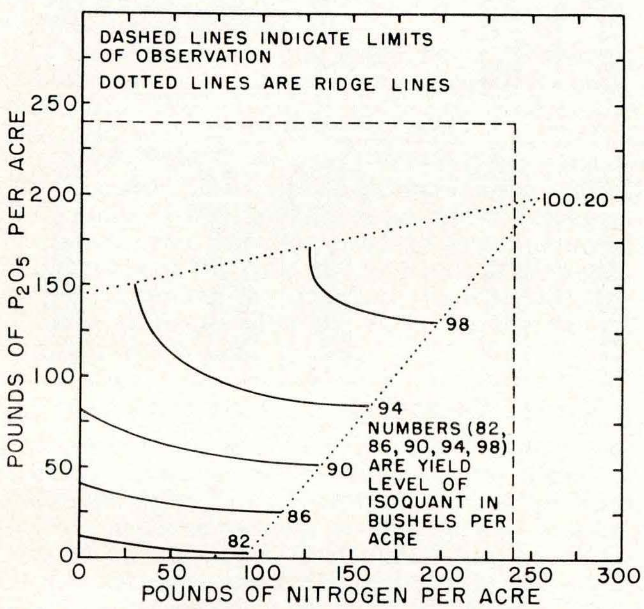

Fig. 31. Corn yield isoquants for P_2O_5 and nitrogen on McPaul silt loam—predicted by quadratic equation.

$$(29)\quad P = 10.292594+0.721043\sqrt{N} \pm 8.190410 (0.352952\sqrt{N}-0.007230N+21.088729 -0.244188Y)^{0.5}$$

In the isoquant equations 28 and 29, derived from equations 22 and 23, respectively, P_2O_5 is expressed as a function of nitrogen and yield. The slope of the isoquants can be predicted from equation 30, derived from the quadratic function, and equation 31 from the square root equation.

$$(30)\quad \frac{\partial \hat{Y}/\partial N}{\partial \hat{Y}/\partial P} = \frac{0.027296-0.000298N+0.000243P}{0.163833-0.001129P+0.000243N}$$

The same general type of isoquant is associated with both equations (figs. 31 and 32). The

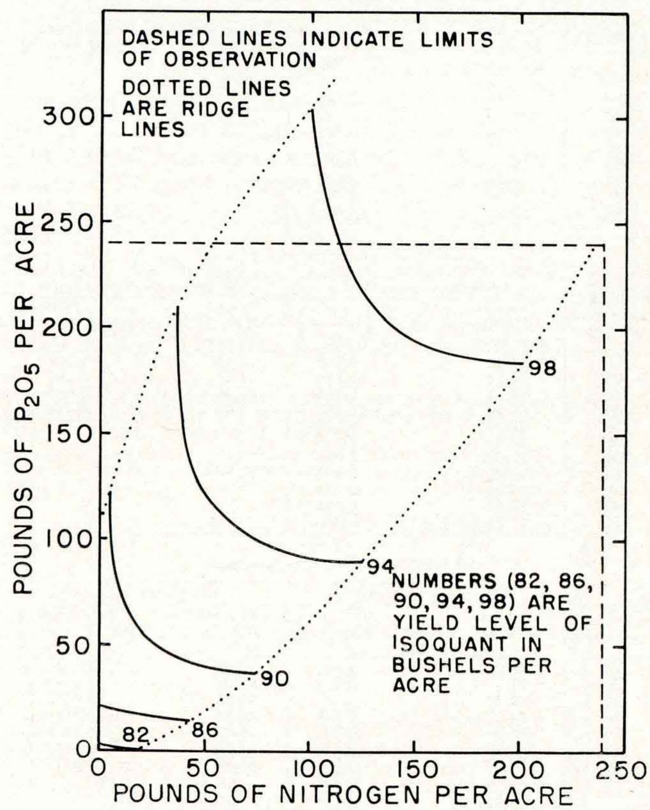

Fig. 32. Corn yield isoquants for P_2O_5 and nitrogen on McPaul silt loam—predicted by square root equation.

$$(31)\quad \frac{\partial \hat{Y}/\partial N}{\partial \hat{Y}/\partial P} = \frac{-0.061359 + 0.269654N^{-1/2} + 0.044017P^{1/2}N^{-1/2}}{-0.061047 + 0.628332P^{-1/2} + 0.044017N^{1/2}P^{-1/2}}$$

$$(32)\quad P = \frac{-0.027296 + 0.00298N + \propto (0.163833 + 0.000234N)}{0.000243 + \propto (0.001129)}$$

$$(33)\quad P = \Big[-3.062997 + (0.693432\propto - 0.696976)\sqrt{N} \pm 11.358990 \Big(\Big[(0.061047 \propto - 0.061359)\sqrt{N} + 0.269654\Big]^2 + 0.176072 \Big[\propto \sqrt{N}(0.628332 + 0.044018\sqrt{N}\Big]\Big)^{1/2}\Big]^2$$

isoquants for each equation are convex to the origin and, for equal yield increments, are successively farther apart. The position and slope of the isoquants differ between functions, however. Of the isoquants shown, those representing the 94-bushel yield are most comparable. For lower yields, the quadratic form requires greater inputs than the square root form; for yields above 94 bushels, the reverse is true.

The ridgelines, dotted lines in figs. 31 and 32, differ considerably for the equations. The square root function allows wider ranges of substitution for high yield levels (e.g., the 98-bushel isoquants). At low yield levels, the quadratic equation allows wider ranges of substitution (e.g., the 82- and 76-bushel isoquants).

Table 21 presents isoquant schedules, derived from equations 28 and 29, and marginal rates of substitution, derived from equations 30 and 31. To produce 82 bushels of corn when 10 pounds are used, the quadratic equation predicts 8.64 pounds of P_2O_5 and a marginal substitution rate of 0.1688. The square root equation predicts 0.37 pound and a marginal rate of substitution of 0.1143. While combinations of inputs are similar for the 94-bushel yield, corresponding marginal rates of substitution are different. For higher or lower yield levels, input combinations and marg-

TABLE 21. COMBINATIONS OF P_2O_5 AND NITROGEN NEEDED TO PRODUCE GIVEN CORN YIELDS AND CORRESPONDING MARGINAL RATES OF SUBSTITUTION.

Yield = 82 bushels per acre

Pounds of nitrogen per acre	Quadratic equation: Pounds of P_2O_5 per acre	Quadratic equation: MRS $\left(\frac{\partial P_2O_5}{\partial N}\right)$	Square root equation: Pounds of P_2O_5 per acre	Square root equation: MRS $\left(\frac{\partial P_2O_5}{\partial N}\right)$
10	8.64	0.169	0.37	0.114
20	7.10	0.144	0.29	0.012
30	5.78	0.112		
40	4.70	0.098		
50	3.81	0.078		
60	3.14	0.058		
70	2.66	0.040		
80	2.35	0.023		
90	2.21	0.005		

Yield = 86 bushels per acre

Pounds of nitrogen per acre	Quadratic equation: Pounds of P_2O_5 per acre	Quadratic equation: MRS $\left(\frac{\partial P_2O_5}{\partial N}\right)$	Square root equation: Pounds of P_2O_5 per acre	Square root equation: MRS $\left(\frac{\partial P_2O_5}{\partial N}\right)$
10	37.14	0.268	13.82	0.466
20	34.66	0.230	11.41	0.187
30	32.54	0.195	10.35	0.065
40	30.74	0.164	9.95	0.014
50	29.24	0.136		
60	28.01	0.110		
70	27.03	0.086		
80	26.22	0.065		
90	25.74	0.043		
100	25.41	0.023		
110	25.28	0.004		

Yield = 90 bushels per acre

Pounds of nitrogen per acre	Quadratic equation: Pounds of P_2O_5 per acre	Quadratic equation: MRS $\left(\frac{\partial P_2O_5}{\partial N}\right)$	Square root equation: Pounds of P_2O_5 per acre	Square root equation: MRS $\left(\frac{\partial P_2O_5}{\partial N}\right)$
10	76.26	0.541	78.06	5.1019
20	71.41	0.439	60.99	1.6989
30	67.40	0.365	45.96	0.6332
40	64.05	0.306	40.36	0.3120
50	61.26	0.255	38.87	0.1739
60	58.92	0.212	37.45	0.0845
70	57.00	0.174	36.96	0.0277
80	55.43	0.142		
90	54.11	0.109		
100	53.23	0.081		
110	52.55	0.055		
120	52.06	0.031		
130	51.93	0.008		

Yield = 94 bushels per acre

Pounds of nitrogen per acre	Quadratic equation: Pounds of P_2O_5 per acre	Quadratic equation: MRS $\left(\frac{\partial P_2O_5}{\partial N}\right)$	Square root equation: Pounds of P_2O_5 per acre	Square root equation: MRS $\left(\frac{\partial P_2O_5}{\partial N}\right)$
30	149.81	20.347		
40	122.77	1.290	155.50	4.281
50	112.63	0.814	122.98	1.941
60	105.70	0.594	108.78	1.026
70	100.48	0.457	100.80	0.622
80	96.06	0.360	96.16	0.396
90	93.08	0.286	92.73	0.247
100	90.63	0.227	90.82	0.147
110	88.61	0.177	89.68	0.076
120	87.06	0.133	89.30	0.049
130	85.74	0.094		
140	85.14	0.617		
150	84.68	0.298		
160	84.45	0.001		

Yield = 98 bushels per acre

Pounds of nitrogen per acre	Quadratic equation: Pounds of P_2O_5 per acre	Quadratic equation: MRS $\left(\frac{\partial P_2O_5}{\partial N}\right)$	Square root equation: Pounds of P_2O_5 per acre	Square root equation: MRS $\left(\frac{\partial P_2O_5}{\partial N}\right)$
110			245.23	3.536
120			210.83	1.557
130	149.94	0.953	207.93	1.038
140	142.99	0.556	199.65	0.700
150	138.45	0.368	193.76	0.469
160	135.40	0.250	189.61	0.314
170	133.34	0.164	187.41	0.207
180	132.04	0.097	185.77	0.121
190	131.06	0.040	185.09	0.055
200				

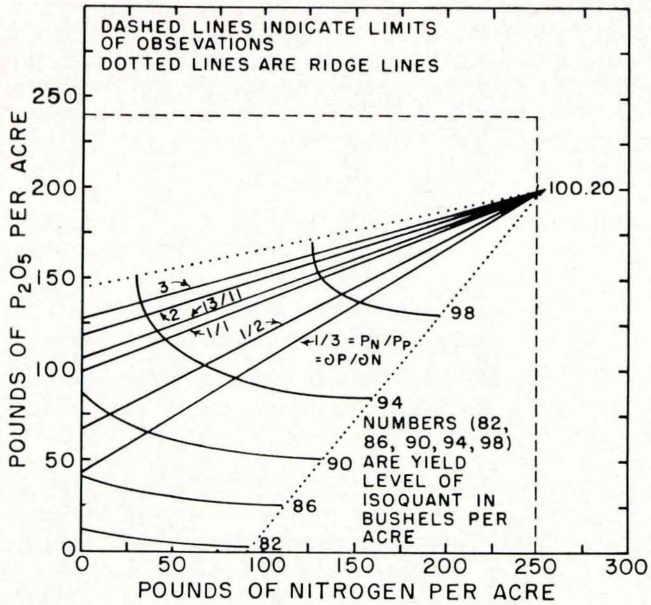

Fig. 33. Yield isoclines for P_2O_5 and nitrogen on McPaul silt loam—predicted by quadratic equation.

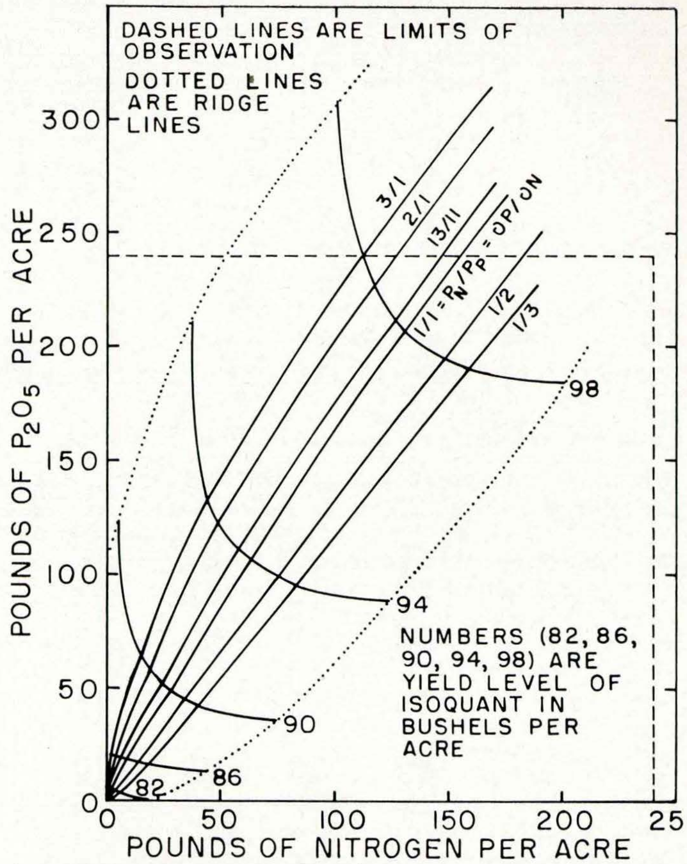

Fig. 34. Yield isoclines for P_2O_5 and nitrogen on McPaul silt loam—predicted by square root equation.

inal rates of substitution differ considerably between equation forms.

Yield Isoclines

Equations 32 and 33 are the isocline equations for the quadratic and square root functions, respectively, where $\propto$ is the price ratio (P_N/P_P). For selected values of $\propto$, isoclines from these equations are presented in figs. 33 and 34. The isoclines of the quadratic equation are linear (fig. 33); those of the square root are curvilinear (fig. 34). Too, all isoclines of the square root equation converge toward the origin. (Actually, the square root isocline equation is discontinuous when nitrogen is zero.)

The isoclines, also the ridgelines (see above), of both equations converge at the point of maximum yield. The maximum yields predicted by the quadratic and square root equation are 100.20 and 104.02 bushels per acre, respectively. To attain this yield, the quadratic function requires 200 pounds of P_2O_5 and 254.5 pounds of nitrogen. The square root function predicts a maximum at 77.3 pounds of P_2O_5 and 594.9 pounds of nitrogen. At the maximum, the quadratic requires a P_2O_5-nitrogen ratio of 0.79; the ratio for the square root function is 1.31.

Economic Optima and Confidence Limits

Isoclines are least-cost expansion paths, derived by equating the marginal rate of substitution to the price ratio. Hence, comparing isoclines of the equations permits comparisons of predictions in dollar values. Cost minimizing combinations of nitrogen and P_2O_5 predicted by the equations are presented in table 22 for selected yield levels and price ratios. With per-pound prices of \$0.10 for nitrogen and \$0.30 for P_2O_5, the quadratic equation provides estimates indicating that the 90-, 94- and 98-bushel yield isoquants can be attained at the minimum costs of \$23.00, \$36.80 and \$56.30 per acre, respectively. For the square root equation, these costs are \$16.60, \$36.50 and \$72.80. When prices are \$0.13 for nitrogen and \$0.11 for P_2O_5, the quadratic predicts attainment of the three yield levels at costs of \$9.02, \$18.64 and \$33.71. The square root predictions, for the same yields, are \$8.84, \$19.86 and \$39.83. For

TABLE 22. QUANTITIES OF NITROGEN AND P_2O_5 WHICH WILL MINIMIZE COSTS OF PRODUCING SPECIFIED YIELDS OF CORN FOR VARYING RESOURCE PRICES.

Corn yield	Price ratio P_N/P_P	Optimum number of pounds of nitrogen per acre		Optimum number of pounds of P_2O_5 per acre	
		Quadratic	Square root	Quadratic	Square root
90	1/3	35	40	65	42
90	1/2	14	34	75	45
90	1/1	—*	27	—*	49
90	13/11	—	24	—	52
90	2/1	—	18	—	58
90	3/1	—	15	—	64
94	1/3	83	83	95	94
94	1/2	77	75	102	97
94	1/1	44	63	117	107
94	13/11	41	58	121	112
94	2/1	35	49	130	123
94	3/1	32	44	137	134
98	1/3	152	158	137	190
98	1/2	143	148	142	194
98	1/1	131	132	151	207
98	13/11	129	127	154	212
98	2/1	127	118	159	227
98	3/1	126	112	164	239

*To attain a 90-bushel yield, the quadratic equation would use no nitrogen and 82.25 pounds of P_2O_5 for price ratios other than 1/2 and 1/3.

TABLE 23. OPTIMUM RATES AND COMBINATIONS OF P_2O_5 AND NITROGEN FOR SPECIFIED CORN AND INPUT PRICES.

Situation number	Price per unit			Quadratic equation				Square root equation			
	Corn per bushel	Nitrogen per pound	P_2O_5 per pound	Optimum inputs in pounds per acre		Predicted corn yield in bushels per acre	Profit per acre from use of optimum combination	Optimum inputs in pounds per acre		Predicted corn yield in bushels per acre	Profit per acre from use of optimum combination
				Nitrogen	P_2O_5			Nitrogen	P_2O_5		
1	$1.60	$0.17	$0.13	0	19.56	83.33	$2.33	6.55	27.13	86.06	$5.18
2	1.30	0.17	0.13	0	0	80.35	0	6.01	20.89	85.96	4.11
3	1.00	0.17	0.13	0	0	80.35	0	3.50	13.84	85.16	2.84
4	1.60	0.15	0.11	0	44.04	87.31	6.29	10.63	35.66	88.07	7.51
5	1.30	0.15	0.11	0	8.02	81.30	0.35	7.91	26.66	87.09	5.28
6	1.00	0.15	0.11	0	0	80.35	0	4.64	17.87	85.82	3.23
7	1.60	0.13	0.09	0	59.66	88.11	7.06	16.08	47.12	89.27	8.62
8	1.30	0.13	0.09	0	29.20	84.65	2.97	10.82	35.05	88.04	6.00
9	1.00	0.13	0.09	0	0	80.35	0	6.43	23.99	86.68	3.76
10	1.60	0.08	0.09	0	87.00	90.35	8.14	28.41	54.17	90.45	9.70
11	1.30	0.08	0.09	0	62.96	88.43	4.84	19.92	40.10	89.09	6.71
12	1.00	0.08	0.09	0	24.39	84.01	1.47	12.40	26.89	87.55	4.21

TABLE 24. CONFIDENCE LIMITS FOR YIELDS PREDICTED IN TABLE 23.

Situation number	Quadratic equation					Square root equation				
	Predicted corn yield in bushels per acre	95% confidence limits for predicted yields in bushels per acre		90% confidence limits for predicted yields in bushels per acre		Predicted corn yield in bushels per acre	95% confidence limits for predicted yields in bushels per acre		90% confidence limits for predicted yields in bushels per acre	
		Lower	Upper	Lower	Upper		Lower	Upper	Lower	Upper
1	83.33	79.81	86.85	80.39	86.27	86.06	82.49	89.63	83.07	89.05
2	80.35	76.41	84.29	77.05	83.65	85.96	82.50	89.42	83.06	88.86
3	80.35	76.41	84.29	77.05	83.65	85.16	81.77	88.55	82.32	88.00
4	87.31	83.48	91.14	84.11	90.51	88.07	84.45	91.69	85.04	91.10
5	81.30	77.63	84.97	78.22	84.38	87.09	83.56	90.62	84.13	90.05
6	80.35	76.41	84.29	77.05	83.65	85.82	82.38	89.26	82.94	88.70
7	88.11	83.88	92.34	84.57	91.65	89.27	85.61	92.93	86.20	92.34
8	84.65	81.09	88.21	81.67	87.63	88.04	84.43	91.65	85.02	91.06
9	80.35	76.41	84.29	77.05	83.65	86.68	83.16	90.20	83.73	89.63
10	90.35	85.44	95.26	86.24	94.46	90.45	86.77	94.13	87.37	93.53
11	88.43	84.11	92.75	84.82	92.04	89.09	85.45	92.73	86.04	92.14
12	84.01	80.49	87.53	81.06	86.96	87.55	84.05	91.05	84.62	90.48

prices of $0.30 for nitrogen and $0.10 for P_2O_5, minimum cost nutrient quantities for the yields 90, 94 and 98 bushels are predicted to cost, in the same order, $8.23, $34.40 and $59.30 for the quadratic and $4.50, $26.60 and $57.50 for the square root function.

Quantities of nutrients which maximize profits per acre are presented in table 23 for several price situations. Confidence limits for predicted yields in table 23 are presented in table 24. The amounts of nutrients listed in table 23 represent movements along an isocline, until marginal revenue becomes zero, They were obtained for the quadratic equation by setting equations 24 and 26 equal to appropriate nutrient-product price ratios and solving simultaneously; results for the square root equation were obtained from equations 25 and 27. Prices in table 23 are comparable to present prices. Nitrogen prices range from $0.08 (approximate cost of liquid forms) to $0.17 (slightly higher than costs of mixed fertilizers). Prices of P_2O_5 vary from $0.09 to $0.13 per pound; prices representative of P_2O_5 purchased in straight fertilizer or in mixed fertilizers, respectively.

The quadratic equation does not allow prediction of nitrogen levels. However, the square root allows prediction for all price situations. Except for situations 7, 10, 11 and 12, however, quantities of nitrogen recommended by the square root function are small. For high corn prices and low P_2O_5 prices, the quadratic equation predicts higher P_2O_5 inputs (situations 4, 7 and 10). For all other situations, P_2O_5 predictions by the square root function are higher.

In general, profits from the predicted optimum mix of nutrients are small for both equations; mainly because of the high initial level of fertility in the experimental plots. Corn yields are predicted at about 80 bushels when no fertilizers are applied.

Because corn response was generally small, the high initial marginal products predicted by the square root function result in the optimum nutrient quantities in table 23. When nutrient-corn price ratios become smaller (i.e., corn prices rise or nutrient prices fall) profit and yield predictions by the equations become comparable. This is true because low price ratios cause the most profitable level of nutrients to be higher. Marginal and total yields predicted by the equations are similar at high input levels.

Except for a few limited ranges, predictions from the two algebraic forms of equations result in (a) least-cost combinations of nutrients and (b) profit maximizing quantities of inputs which differ considerably. Clearly, further research is needed to determine the mathematical form which is most appropriate for the soil and moisture conditions represented by a given experiment.

EXPERIMENTS ON CARRINGTON SILT LOAM IN 1955 AND 1956

Rainfall limited yield response on Carrington soil during both the 1955 and 1956 growing seasons. However, corresponding rainfall and response data occur over time and, while the response functions may not be as "neat," they are of as much practical importance as data derived for years of more favorable weather. Hence, data for these 2 years of limited rainfall are summarized briefly in the current section of this report. The estimates are included to indicate results which face farmers in particular years and which must be considered in decision making.

1955 Experiment With Corn on Carrington Silt Loam

The experimental design on the Carrington soil was a 5x4x4 completely randomized factorial with 0-, 40-, 80-, 120- and 240-pound levels of nitrogen and 0, 40, 80 and 160 pounds each of K_2O and P_2O_5. In addition, the 0-, 40- and 80-pound rates were replicated, providing a 3x3x3 replication. A total of 15 check plots were used to provide a more precise estimate of the yield intercept.

Corn yield observations for 1955 are presented in table 25. The average yield responses to the nutrients were low; the plots receiving 40 pounds of nitrogen average approximately 3 bushels more per acre than those receiving no nitrogen. Similar figures for P_2O_5 and K_2O are 4 and 5 bushels, respectively. Also, the yields in table 25 are highly variable.

Equation 34 is the first equation fitted to the data of table 25. The symbols are as defined previously, except that stand (S) now refers to 218 stalks per acre. This equation has an R^2 of 0.5944. The t values and their probability levels are listed in table 26.

TABLE 25. CORN YIELDS OBTAINED ON CARRINGTON SILT LOAM EXPERIMENTAL PLOTS, 1955 (BU. PER ACRE).

A. Check plots: $N = P_2O_5 = K_2O = 0$

77.8	69.2	78.0	61.8	67.2
68.9	82.5	74.5	61.4	68.6
77.5	59.7	73.1	62.1	77.9

B. Fertilized plots:

Pounds of K_2O per acre	Pounds of P_2O_5 per acre	Pounds of nitrogen per acre				
		0	40	80	120	240
0	0	check plots	69.8 77.3	61.6 76.6	62.6	68.3
0	40	72.9 81.7	69.5 76.8	78.3 80.4	64.6	78.4
0	80	85.6 80.9	71.1 74.5	82.9 85.0	81.5	87.2
0	160	83.8	77.3	87.3	80.1	77.3
40	0	65.6 80.6	76.7 70.8	86.2 76.3	66.1	79.6
40	40	62.6 83.9	84.7 81.5	90.1 85.7	90.4	74.1
40	80	84.3 75.2	67.0 71.0	65.5 94.0	82.9	84.1
40	160	82.5	82.7	87.3	76.5	82.0
80	0	51.2 65.5	88.7 78.3	86.3 89.8	76.1	82.9
80	40	90.5 76.3	88.0 79.1	68.9 77.0	79.0	66.1
80	80	88.9 79.5	83.2 79.4	80.8 69.6	78.3	71.9
80	160	73.8	87.7	82.0	84.7	74.5
160	0	73.1	69.3	73.8	72.9	63.7
160	40	60.7	81.2	80.3	84.7	70.0
160	80	86.0	61.1	72.6	75.2	81.1
160	160	82.4	94.3	79.9	82.3	86.4

TABLE 26. VALUES OF t FOR COEFFICIENTS OF EQUATION 34.

Coefficient	Value of t	Probability level*
S	10.13	0.001
N	0.54	0.60
P	2.23	0.05
K	0.77	0.50
N^2	0.31	0.80
P^2	0.74	0.50
K^2	1.03	0.40
NP	0.80	0.50
NK	0.001	0.90
PK	0.51	0.70
NPK	0.12	0.90

*Probability of drawing a t value as large or larger by chance, given the null hypothesis.

$$(34)\quad \hat{Y} = 22.2377 + 0.9404S + 0.0130N + 0.0720P + 0.0251K - 0.000028N^2 - 0.000140P^2 - 0.000196K^2 - 0.000142NP - 0.00000023NK + 0.000125PK - 0.00000025NPK$$

The significance levels of the coefficients of equation 34 are low. Only one nutrient coefficient (P) has a t value significant at the usual 5-percent probability level; contrariwise, the stand coefficient is highly significant (the correlation coefficient between yield and stand was 0.69). The low rainfall of the year caused nitrogen to have a yield response which was even more limited than the response for the other nutrients (see table 26). Also, negative signs on interaction terms including nitrogen indicate that the joint effects of nitrogen with K_2O or P_2O_5 may be to lower yields during a growing season similar to 1955.

Equation 35, obtained by deleting the nitrogen terms from equation 34, has an R^2 of 0.5858, only 0.0086 less than that of equation 34. Also, t values for the coefficient of equation 35, contained in table 27, are similar to those for like coefficients of equation 34. An analysis of variance of regression of equation 35 is presented in table 28.

$$(35)\quad \hat{Y} = 22.7216 + 0.939S + 0.0673P + 0.0300K - 0.000177P^2 - 0.000213K^2 + 0.000080PK$$

In this experiment, as in those described above, stand was not a predetermined variable. However, because it did vary among plots, a linear

TABLE 27. VALUES OF t FOR THE COEFFICIENTS OF EQUATION 35.

Coefficient	Value of t	Probability level*
S	10.41	0.001
P	2.23	0.05
K	0.99	0.40
P^2	0.96	0.40
K^2	1.15	0.30
PK	0.49	0.70

*Probability of drawing t value as large or larger by chance, given the null hypothesis.

TABLE 28. ANALYSIS OF VARIANCE FOR EQUATION 35.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	119	8,480.30		
Due to regression	6	4,967.76	827.96	26.64
Deviation from regression	113	3,512.54	31.08	

TABLE 29. PREDICTED CORN YIELDS AND THEIR STANDARD ERRORS IN PARENTHESES ON CARRINGTON SILT LOAM, 1955 (BU. PER ACRE).

Pounds of P_2O_5 per acre	Pounds of K_2O per acre				
	0	40	80	120	160
0	73.65 (1.07)	74.51 (0.97)	74.68 (1.19)	74.18 (1.36)	72.99 (1.90)
40	76.06 (0.97)	77.04 (0.77)	77.35 (0.93)	76.97 (0.95)	75.92 (1.40)
80	77.90 (1.19)	78.50 (0.93)	79.45 (1.05)	79.20 (1.05)	78.27 (1.42)
120	79.17 (1.36)	80.42 (0.96)	80.98 (1.06)	80.26 (1.30)	80.06 (1.61)
160	79.89 (1.90)	81.26 (1.41)	81.95 (1.43)	81.95 (1.61)	81.28 (2.52)

stand coefficient was included in the regression equations. In fact, the t value and the correlation coefficient between stand and yield indicate that the stand coefficient explains a large part of the total variation associated with the regression equations. When the mean stand of plants is substituted in equation 35, equation 36 results.

$$(36)\quad \hat{Y} = 73.648110 + 0.067306P + 0.029995 - 0.000177P^2 - 0.000213K^2 + 0.000080PK$$

Corn yields predicted by equation 36 are presented in table 29. In general, as might be expected, predicted response from the use of fertilizer is low. The highest yield in table 29 is 81.95 bushels; 8.83 bushels more than the lowest yield (73.65 bushels). Decreasing total yields occur after the 80-pound rate of K_2O but do not occur even at the 160-pound rate of P_2O_5. Response to variable amounts of one input when the other input is held constant is depicted in figs. 35 and 36. The small response obtained in this experiment is shown clearly by these curves.

Because of the relatively large standard errors of the regression coefficients, standard errors for the predicted yields are included in parentheses in table 29. The 95-percent confidence limits for any predicted value can be determined by (a) multiplying its standard error by two (approximate t value for the 95-percent level), then (b) alternatively adding and subtracting the resulting product from the predicted yield. For example, when P_2O_5 and K_2O are 160 pounds, the 95-percent confidence limits for the predicted yield are 76.3 to 86.3 bushels per acre (81.28±2x2.5). Standard errors of the predicted yields are smallest for the input levels closest to the average input level of the experiment.

Equations 37 and 38 express the marginal products of K_2O and P_2O_5, respectively. Marginal

$$(37)\quad \frac{\partial \hat{Y}}{\partial K} = 0.0300 - 0.0004K + 0.000080P$$

$$(38)\quad \frac{\partial \hat{Y}}{\partial P} = 0.0673 - 0.0004P + 0.000080K$$

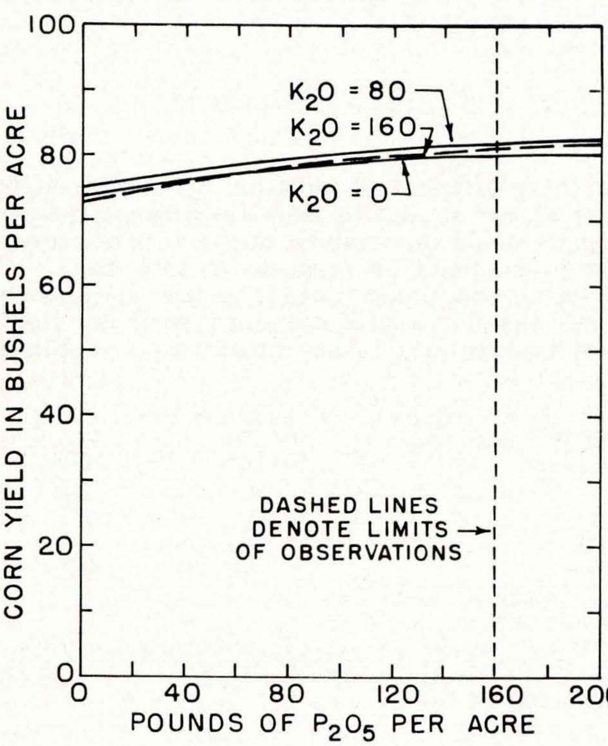

Fig. 35. Corn yield response to P_2O_5 (K_2O held constant) on Carrington silt loam, 1955.

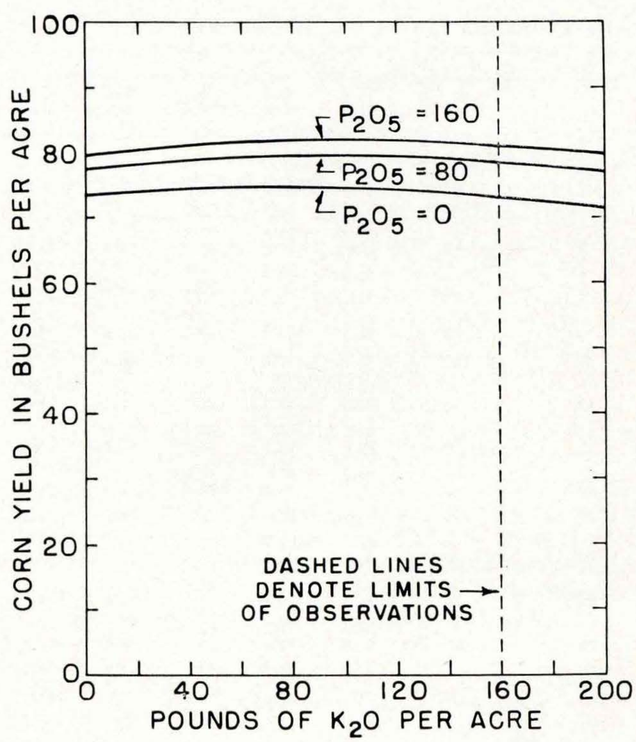

Fig. 36. Corn yield response to K_2O (P_2O_5 held constant) on Carrington silt loam, 1955.

TABLE 30. MARGINAL PHYSICAL PRODUCTS OF P_2O_5 AND K_2O FOR CORN ON CARRINGTON SILT LOAM, 1955 (BU. PER ACRE).

A. Marginal physical product of P_2O_5

Pounds of K_2O per acre	Pounds of P_2O_5 per acre				
	0	40	80	120	160
0	0.067	0.053	0.039	0.025	0.011
40	0.071	0.056	0.042	0.028	0.014
80	0.073	0.060	0.045	0.031	0.017
120	0.077	0.063	0.049	0.034	0.020
160	0.080	0.066	0.052	0.038	0.023

B. Marginal physical product of K_2O

Pounds of P_2O_5 per acre	Pounds of K_2O per acre				
	0	40	80	120	160
0	0.030	0.013	—0.005	—0.022	—0.038
40	0.033	0.016	—0.001	—0.018	—0.035
80	0.036	0.019	0.002	—0.015	—0.032
120	0.040	0.023	0.006	—0.012	—0.029
160	0.043	0.026	0.009	—0.008	—0.025

physical products predicted by these equations are presented in table 30. If corn were \$1.30 per bushel and K_2O were \$0.06 a pound ($P_K/P_C = 0.046$), average prices for present conditions, K_2O would not be applied even if P_2O_5 were free. Likewise, with the same corn price and P_2O_5 costing \$0.11 per pound ($P_P/P_C = 0.085$), no P_2O_5 would be used if K_2O were free. Large increases in corn prices or large decreases in nutrient costs would cause fertilizers to be used. But for prices of recent levels, use of the nutrients would not be recommended under the experimental conditions.

By equating equations 37 and 38 to zero and solving, the maximum yield of 82.6 bushels of corn is found to occur at 215.2 pounds of P_2O_5 and 110.8 pounds of K_2O. This yield is 8.9 bushels more than the yield intercept value. However, the 215.2 rate of P_2O_5 is extrapolated beyond the limits of the experiment.

1956 EXPERIMENT WITH OATS ON CARRINGTON SILT LOAM

In 1956, oats were seeded on the Carrington experimental plots. Hence, data presented here represent yield response to fertilizers applied to the plots the previous year. This procedure allows an estimate of residual response in a second year of low rainfall, but it does not allow comparison of residual and initial (first year) responses which might occur during growing seasons similar in respect to rainfall. Experimental work now being carried on will enable comparisons of the latter type to be made. Results of this work will be included in later reports.

Residual oat yields obtained during the 1956 season are contained in table 31. Experimental design for the 2 years is, of course, the same except that the 1956 corn experiment has five less check plots. Yields in table 31 are highly variable and while yield trends are difficult to establish, the over-all response is small. A 31.7-bushel yield differential exists between the highest and lowest yielding plots; a small difference for an experiment of this size. Both of these plots occur where K_2O is zero.

TABLE 31. OAT YIELDS OBTAINED ON CARRINGTON SILT LOAM EXPERIMENTAL PLOTS, 1956 (BU. PER ACRE).

A. Check plots: N = P_2O_5 = K_2O = O

46.4	42.6	43.2	42.9	52.8
48.5	44.0	41.3	39.2	42.6

B. Fertilized plots:

Pounds of K_2O per acre	Pounds of P_2O_5 acre	Pounds of nitrogen per acre				
		0	40	80	120	240
0	0	check plots	51.5 55.5	40.0 43.3	47.0	34.5
0	40	43.2 56.3	45.2 48.8	52.0 45.8	45.3	51.1
0	80	42.7 46.5	51.5 41.7	57.8 59.1	51.1	54.1
0	160	39.0	49.4	66.2	47.5	63.1
40	0	44.6 44.4	54.6 54.1	42.1 42.5	47.0	45.5
40	40	45.7 49.4	56.8 52.4	48.9 59.2	49.0	55.2
40	80	56.6 52.1	57.5 52.4	53.8 58.8	58.1	55.0
40	160	51.2	51.1	62.7	50.5	41.7
80	0	41.7 41.7	54.0 52.7	61.5 53.2	62.2	54.1
80	40	54.8 49.6	44.6 48.0	39.9 66.6	49.1	42.6
80	80	53.5 47.5	45.7 49.1	51.2 51.5	59.7	53.3
80	160	47.3	49.3	52.8	53.2	62.7
160	0	46.2	47.0	48.4	54.3	55.3
160	40	37.9	41.3	55.9	45.7	56.6
160	80	46.0	45.7	49.9	61.5	52.0
160	160	44.2	49.7	46.4	54.1	59.2

Quadratic equation 39 fitted to the residual oat yields gave an R^2 of 0.3168. (Symbols are as previously defined.) The linear, quadratic and third

$$(39)\quad \hat{Y} = 45.4148 + 0.0446N + 0.0547P + 0.0541K - 0.000276N^2 - 0.000275P^2 - 0.000453K^2 + 0.000371NP + 0.000451NK - 0.000051PK - 0.000003NPK$$

order interaction term (NPK) have the same sign as in equation 34 for corn in 1955. Signs of the second order coefficients are reversed for the equations, however. Futhermore, t values for the 1956 equation are, except for the PK coefficient, significant at levels suggesting that all terms be retained in the equation (see table 32). This is true even though the fit (R^2) of the 1956 equation is smaller than for the 1955 equation. An analysis of variance of regression for equation 39 is presented in table 33.

Residual oat yields predicted by equation 36

TABLE 32. VALUES OF t FOR THE COEFFICIENTS OF EQUATION 39.

Coefficient	Value of t	Probability level*
N	1.87	0.10
P	1.65	0.10
K	1.63	0.10
N^2	3.05	0.01
P^2	1.45	0.20
K^2	2.39	0.02
NP	2.05	0.05
NK	2.50	0.02
PK	0.21	0.90
NPK	1.22	0.30

*Probability of drawing a t value as large or larger by chance, given the null hypothesis.

TABLE 33. ANALYSIS OF VARIANCE FOR EQUATION 39.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	114	4,858.68		
Due to regression	10	1,539.23	153.92	4.81
Deviation from regression	104	3,319.45	32.01	

TABLE 34. PREDICTED RESIDUAL OAT YIELDS AND THEIR STANDARD ERRORS ON CARRINGTON SILT LOAM, 1956 (BU. PER ACRE).

Pounds of K_2O per acre	Pounds of P_2O_5 per acre	Pounds of nitrogen per acre			
		0	80	160	240
0	0	45.42 (2.00)	47.22 (1.77)	45.49 (2.59)	40.23 (5.32)
0	80	48.03 (2.12)	52.21 (1.44)	52.85 (2.47)	49.97 (5.39)
0	160	47.13 (3.19)	53.68 (2.51)	56.70 (4.63)	56.19 (8.18)
80	0	46.85 (2.12)	51.53 (1.69)	52.69 (2.23)	50.32 (4.83)
80	80	49.14 (1.63)	55.62 (1.29)	56.66 (2.25)	55.12 (5.21)
80	160	47.91 (2.53)	54.27 (2.15)	57.11 (4.35)	56.41 (7.82)
160	0	42.48 (3.19)	50.05 (2.27)	54.10 (2.90)	54.61 (5.60)
160	80	44.44 (2.53)	51.32 (1.65)	54.66 (2.66)	54.48 (5.59)
160	160	42.89 (3.59)	49.07 (2.74)	51.71 (4.91)	50.83 (8.98)

are presented in table 34. Yield response to fertilization is low. When other inputs are zero, the yield increase due to the first 80 pounds of nitrogen is 1.80 bushels. For similar situations, the yield increases for P_2O_5 and K_2O are 2.61 and 1.43 bushels, respectively. The highest yield (57.11) in table 34 is only 11.69 bushels more than the lowest yield (45.42). The response to one nutrient when the others are constant at various levels is shown in figs. 37 to 39.

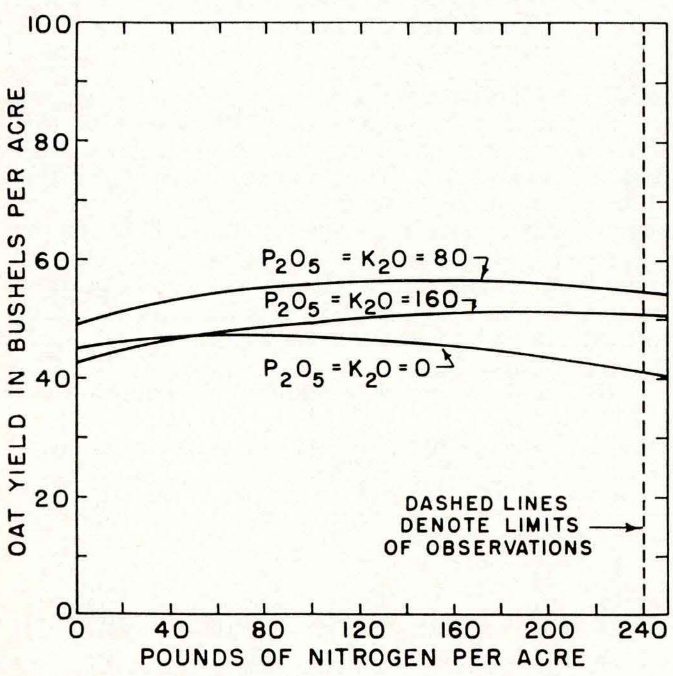

Fig. 37. Oat yield response to nitrogen (P_2O_5 and K_2O held constant) on Carrington silt loam, 1956.

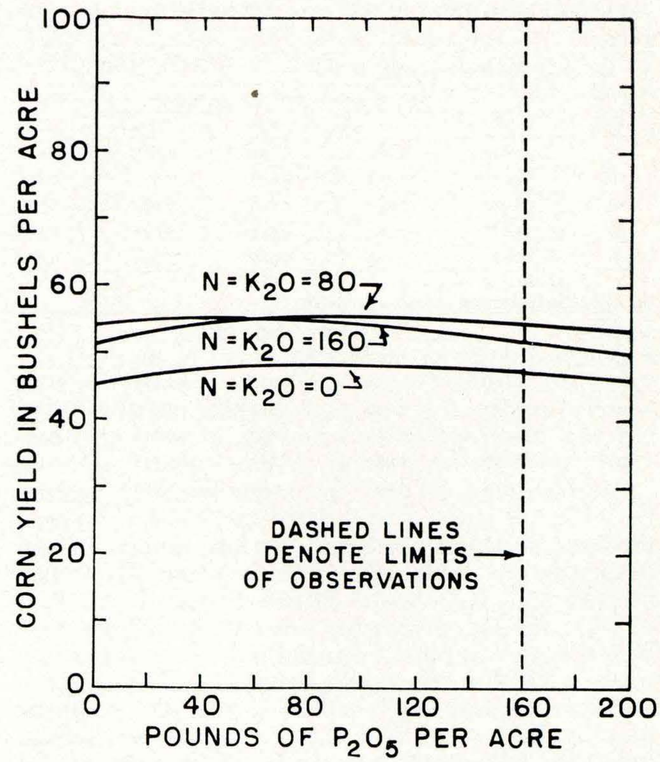

Fig. 38. Oat yield response to P_2O_5 (nitrogen and K_2O held constant) on Carrington silt loam, 1956.

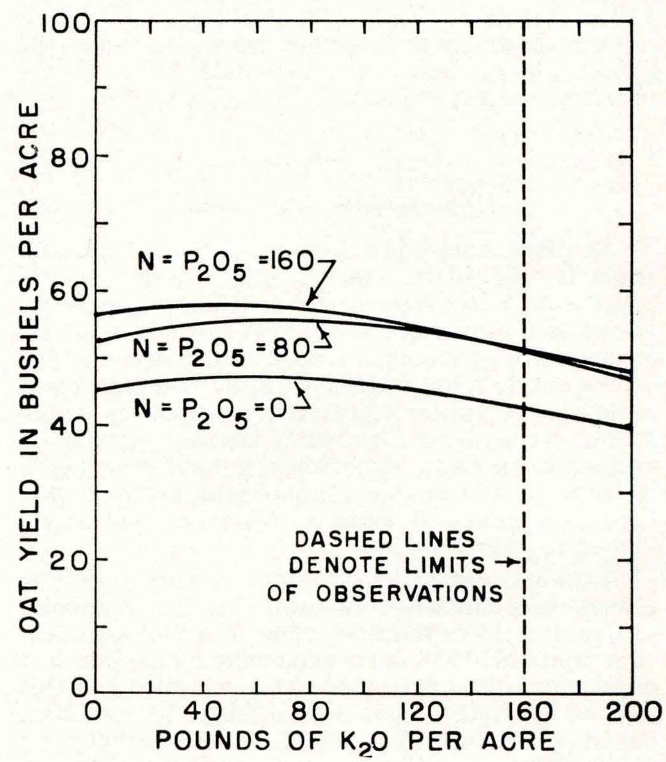

Fig. 39. Oat yield response to K_2O (nitrogen and P_2O_5 held constant) on Carrington silt loam, 1956.

The standard errors of the predicted yields are included within brackets in table 34. They are included to demonstrate the uncertainty associated with predictions made from these data of a year with limited rainfall. Confidence limits for predicted yields can be calculated as explained above. For example, 95-percent confidence limits for the highest yield in the table (57.11) are 48.4 to 65.8 bushels; for the lowest yield (45.42), they are 41.4 to 49.4 bushels.

Marginal products of nitrogen, P_2O_5 and K_2O can be predicted by equations 40, 41 and 42, respectively. Quantities predicted by these equations

$$(40) \quad \frac{\partial \hat{Y}}{\partial N} = 0.044616 - 0.000552N + 0.000371P + 0.000451K - 0.000003PK$$

$$(41) \quad \frac{\partial \hat{Y}}{\partial P} = 0.054711 - 0.000550P + 0.000371N - 0.000051K - 0.000003NK$$

$$(42) \quad \frac{\partial \hat{Y}}{\partial K} = 0.054121 - 0.000906K + 0.000451N - 0.000051P - 0.000003NP$$

are presented in table 35. Again, if corn were \$1.30 and nitrogen, P_2O_5 and K_2O were \$0.15, 0.11 and 0.06 per pound, respectively, the price ratios would be 0.115, 0.085 and 0.046, in the same order. If other nutrients were free, nitrogen would not be applied with less than 160 pounds of K_2O; P_2O_5 would be applied with somewhat more than 80 pounds of nitrogen, and K_2O might be used with zero rates of the other nutrients, still supposing zero prices of the other nutrients. The negative PK coefficient causes applications of one of these elements to decrease marginal products of the second. The negative NPK coefficient has a similar effect when the three nutrients are combined.[7]

By solving the marginal product equations as above, the maximum yield is found to occur at 177.1, 126.3 and 87.5 pounds of nitrogen, P_2O_5 and K_2O per acre. The yield at this input level, 57.3 bushels, is 11.9 bushels more than the yield when no fertilizer is applied.

TABLE 35. MARGINAL PHYSICAL PRODUCTS OF NITROGEN, P_2O_5 AND K_2O FOR OATS ON CARRINGTON SILT LOAM, 1956 (BU. PER ACRE).

A. Marginal physical products of nitrogen

Pounds of K_2O per acre	Pounds of P_2O_5 per acre	Pounds of nitrogen per acre			
		0	80	160	240
0	0	0.045	0.001	-0.044	-0.088
0	80	0.074	0.030	-0.015	-0.058
0	160	0.104	0.060	0.016	-0.029
80	0	0.081	0.037	-0.008	-0.052
80	80	0.091	0.047	0.003	-0.042
80	160	0.102	0.057	0.013	-0.031
160	0	0.117	0.073	0.028	-0.016
160	80	0.108	0.064	0.020	-0.025
160	160	0.099	0.55	0.011	-0.034

B. Marginal physical products of P_2O_5

Pounds of nitrogen per acre	Pounds of K_2O per acre	Pounds of P_2O_5 per acre		
		0	80	160
0	0	0.055	0.011	-0.034
0	80	0.051	0.007	-0.037
0	160	0.047	0.003	-0.041
80	0	0.084	0.040	-0.004
80	80	0.061	0.017	-0.027
80	160	0.038	-0.006	-0.050
160	0	0.114	0.070	0.026
160	80	0.072	0.028	-0.016
160	160	0.029	-0.015	-0.059
240	0	0.114	0.099	0.056
240	80	0.082	0.038	-0.006
240	160	0.020	-0.024	-0.068

C. Marginal physical products of K_2O

Pounds of nitrogen per acre	Pounds of P_2O_5 per acre	Pounds of K_2O per acre		
		0	80	160
0	0	0.054	-0.018	-0.091
0	80	0.050	-0.022	-0.095
0	160	0.046	-0.027	-0.099
80	0	0.090	0.018	-0.055
80	80	0.067	-0.006	-0.078
80	160	0.044	-0.029	-0.101
160	0	0.126	0.054	-0.019
160	80	0.083	0.011	-0.061
160	160	0.041	-0.031	-0.104
240	0	0.162	0.090	0.017
240	80	0.101	0.028	-0.044
240	160	0.039	-0.033	-0.106

Two-Year Costs and Returns on Carrington Silt Loam

The economic feasibility of fertilization which provides responses in 2 years must consider jointly the two sets of responses. Clearly, returns over the complete time period must exceed costs of the complete period if a profit is to be realized. The present section includes an analysis which relates to this problem. The techniques used here are not necessarily the best (or only) ones available for residual analysis. However, they allow results which are sufficient for the data at hand.

Because different crops were grown each of the years, the individual equations cannot be summed directly to obtain a cumulative equation. However, physical products can be transformed to value products when equations are multiplied by product prices. The resulting equations then can be summed. The result is a production surface representing total value.

Equations 43 and 44 are value equations for the 1955 corn crop and the 1956 oat crop, respectively. Equation 43 was obtained by multiplying equation 34 by \$1.30, the approximate value of a bushel of corn under present conditions. To obtain

$$(43) \quad \$ \text{ of product} = 95.2917 + 0.0168N + 0.0936P + 0.0326K - 0.000036N^2 - 0.0000182P^2 - 0.000255K^2 - 0.000185NP - 0.0000003NK + 0.000163PK - 0.0000003NPK$$

$$(44) \quad \$ \text{ of product} = 23.6157 + 0.0232N + 0.0285P + 0.0281K - 0.000144N^2 - 0.000143P^2 - 0.000236K^2 + 0.000193NP + 0.000235NK - 0.000027PK - 0.000002NPK$$

[7]Two points should be noted here: (1) Optimum recommendations for nutrient levels giving rise to residual responses cannot be discussed independently of initial (first year) response because the two are not independent. (2) The marginal yields, as well as total yields, are subject to large variability because of climatic conditions.

equation 44, the oats price, \$0.65 a bushel, was used. Because of the uncertainties involved in future returns, the oats price was discounted by 20 percent and equation 39, the physical product equation, was multiplied by \$0.52.

The cumulative value equation (45) is the sum of equations 43 and 44. All of the two-factor interaction terms are positive in this equation (although this was not true for the individual

(45)

$$\$ \text{ of product} = \$118.9075+0.0400N +0.1221P+0.0611K-0.000180N^2 -0.000325P^2-0.000491K^2+0.000008NP +0.000235NK+0.000136PK-0.000002NPK$$

equations). Other terms have the same signs in all three equations. Table 36 contains prediction of gross increases in revenue from fertilization in the 2-year period. These values, predicted by equation 45, were obtained by subtracting the value of product resulting from the use of no fertilizer (\$118.91) from the equation. Contrariwise, quantities within the brackets in table 36 are total costs of each fertilizer combination when nitrogen, P_2O_5 and K_2O cost \$0.15, \$0.11 and \$0.06 per pound, respectively. These prices approximate those existing currently for mixed fertilizer.

Gross revenue does not exceed fertilizer costs for any of the combinations in table 36. Hence, fertilization was not profitable for climatic conditions surrounding these experiments. When considering the experiments separately, it appeared that small amounts of fertilizers might have been recommended for the 1956 crop of oats. After the

TABLE 36. GROSS INCREASE IN VALUE OF PRODUCT FOR 2 YEARS PRODUCTION ON CARRINGTON SILT LOAM PREDICTED BY EQUATION 45—COSTS OF FERTILIZER COMBINATIONS INCLUDED WITHIN PARENTHESES.

Pounds of K_2O per acre	Pounds of P_2O_5 per acre	Pounds of nitrogen per acre: 0	80	160	240
0	0	\$ 0.00 (0.00)	\$ 2.05 (12.00)	\$ 1.80 (24.00)	\$ 0.76 (36.00)
0	80	7.68 (8.80)	9.79 (20.80)	9.58 (32.80)	7.08 (44.80)
0	160	11.21 (17.60)	13.36 (29.60)	13.21 (41.60)	10.76 (53.60)
80	0	1.71 (4.80)	5.26 (16.80)	6.52 (28.80)	5.46 (40.80)
80	80	10.27 (13.60)	13.49 (25.60)	13.13 (37.60)	11.11 (49.60)
80	160	14.67 (22.40)	16.28 (34.40)	15.58 (46.40)	12.58 (58.40)
160	0	—2.85 (9.60)	2.21 (21.60)	4.96 (33.60)	5.41 (45.60)
160	80	6.58 (18.40)	9.64 (30.40)	10.40 (42.40)	8.85 (54.40)
160	160	11.84 (27.20)	12.91 (39.20)	11.67 (51.20)	8.13 (63.20)

price of oats was discounted and losses from the corn crop were considered, over-all contributions of fertilizer were unprofitable. Prices used will, of course, affect results. Product prices could be increased or nutrient prices decreased until some profits are shown. While prices used were selected to closely approximate existing market conditions, the product prices may be slightly high relative to those expected over the next few years.