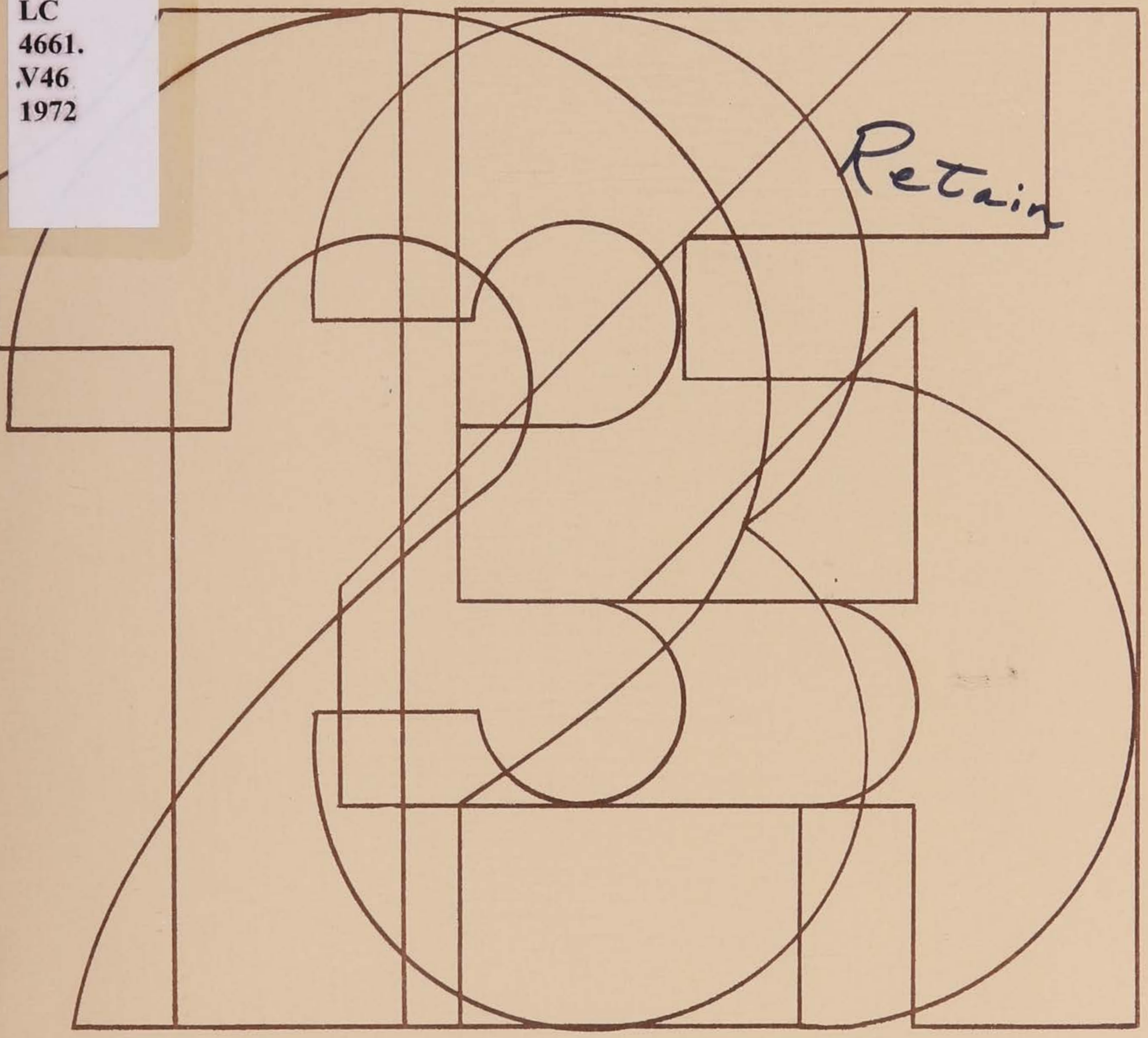

quality math experiences

EDUCATION CURRICULUM DEVELOPMENT CENTER
RVICE TRAINING APPROACH . . . . .

ATIVE PROGRAM INVOLVING THE IOWA STATE DEPARTMENT
C INSTRUCTION AND THE UNIVERSITY OF IOWA

quality math experiences

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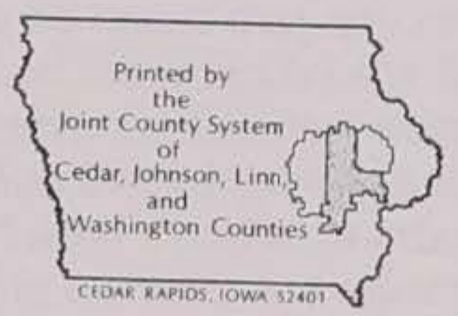
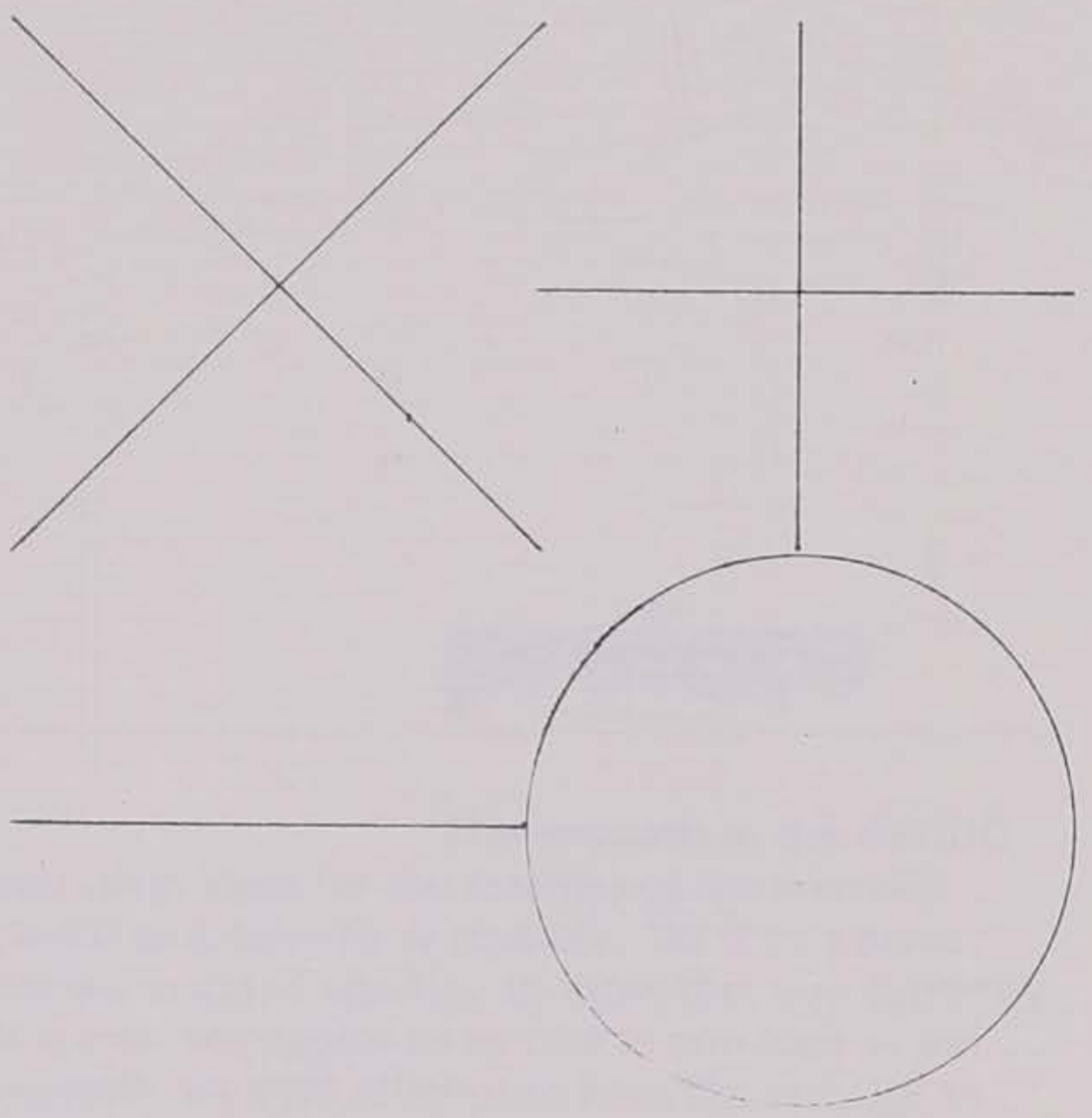




The Special Education Curriculum Development Center has as its main objective the operation of a statewide in service training program for teachers of the mentally retarded. Twenty special class teachers from different geographic areas of Iowa serve as consulting teachers. They attend training sessions in Des Moines and then return to their home area to conduct field sessions. All materials prepared for SECDC are intended for dissemination through the field sessions conducted by the consulting teachers. These materials are prepared by the SECDC staff in response to the suggestions of special class teachers. Persons who use SECDC materials but do not attend the field sessions should keep in mind that the purpose of the material is to serve as a starting point for in service training and that the publications themselves are not end products.

It should also be noted that any reference to commercially prepared materials by the Special Education Curriculum Development Center does not constitute a recommendation or endorsement for purchase. The consideration of such material is intended solely as a means of assisting teachers and administrators in the evaluation of materials.

preface

The emphasis in this SECDC document will be to present math ideas for the teachers of the mentally retarded or slow learning child as succinctly as possible. We shall attempt to give you ideas and materials to use in addition to those that you currently use in your classes. While it may not appear to be true as you look at the first few pages of this document, we shall attempt to keep the verbiage to a minimum while we provide you with examples and samples of math ideas and materials. Sometimes, the ideas may seem a little incredible and a certain amount of justification will be necessary to encourage you, the classroom teacher, to try the ideas in your classrooms.

While the ideas will generally be developed from simple to complex, there will be no attempt to provide a grade level label to the different ideas presented. The classroom teacher must be the judge of when certain types of educational experience will be met with a desired amount of success by each child. The most difficult parts of any instructional program is to know the teachable moment for each child for a specific skill and the desirable sequence of experiences necessary for optimum learning. The answers to both problems must be found in the classroom with each child. However, some of the ideas, examples and samples may be incorporated into your program and help you sequence some learning experiences for some of your children. That certainly is our intent as we labor to put this document into a usable form.

B. V.
T. P.

one

statement of philosophy

While there is not complete concurrence among the writers as to degree, there is general agreement that the retarded child, at least in the educable range, is a slow learner rather than a dull child. We need to do more than go slowly and speak loudly. Rather, we need to sequence properly and react carefully, even tenderly, to the child and his problems. The retarded child, like any child, learns best by experiencing and finds satisfaction and reinforcement in successful endeavors. When a child fails to do well on an assigned task, it is unjust to blame the child for his failure, since at best, the teacher should share in the child's failure and at worst the teacher is the one who fails by requiring too much and perhaps teaching too little. Furthermore, placing blame accomplishes no constructive purpose for the child or the teacher and that should guide our thinking and our behavior. We believe the retarded child is hurt by unrealistic expectation early in life and during the first few years of formal education. This is especially true in math. That statement is not intended to place blame, because teachers and parents cannot know about all the degrees and kinds of retardation and how each individual is affected. While in the early years the expectations tend to be too high, the continual failure will quite naturally lead the student to low self-esteem and unrealistically low teacher expectations of the student in the later years which will affect, in a negative way, learning and a productive life.

It has often been stated in recent years that the retarded have poor, short-term memory, and this may well be true for the trainables, but we believe that the educables (as do we all) simply have trouble remembering concepts and skills they really have not learned. We do not believe the educables need more drill, but they do need varied ways to practice their new skills. When proper and varied practice is not provided, retention for many retarded children becomes impossible. To sum up our philosophy, it takes longer, but it is possible.

two

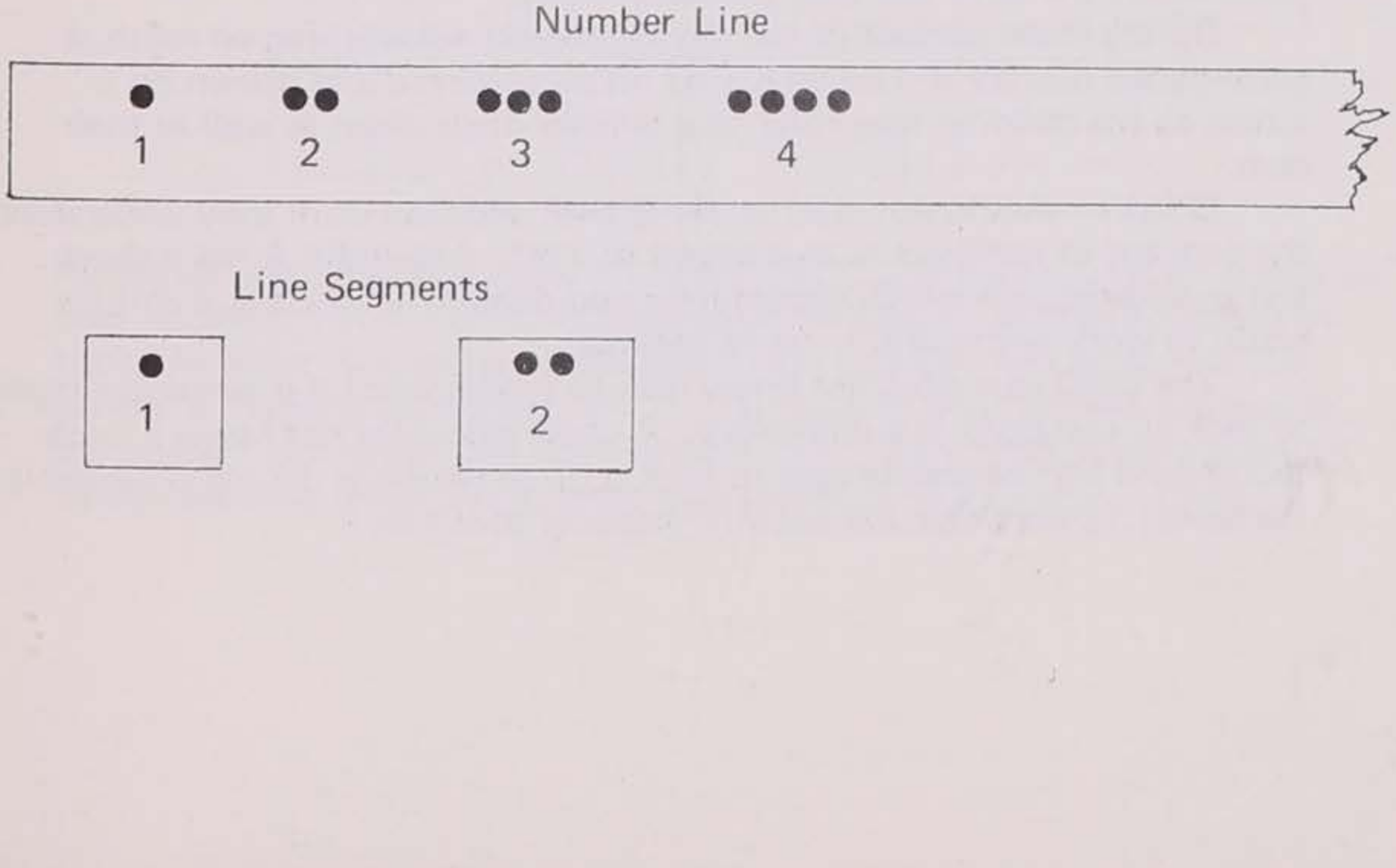
fundamental operations in mathematics

Be slow to destroy myths. If a child believes brown-eyed children have sharper vision and blonds have more fun, don't get overly excited unless you are a redhead with blue eyes. *However, always discourage a child from guessing about matters of fact.* If a child does not know that two plus two equals four, he should have a number of methods available to him to find the truth. *Do not reward random guessing.* When a child looks up a math fact from available sources or computes the correct response to simple number facts, a smile, a touch or comment is in order.

ADDITION

When children are learning addition facts, they should have ready access to a number line. It should start with zero and go to twenty. The number line should be constructed of heavy tag board or lightweight wood, and if possible should be secured to the top part of the desk. It should imply both the ordinal (order) and cardinal sense (how much) of the number system. Color coded line segments should be given to each child after he knows the counting numbers, and these may be given out in order of difficulty.

FIGURE 1



A child should have at least two of each color-coded line segments so that he might put one and one together or two and two. It would help if he had a special box in which to keep his line segments and other math treasures.

As children progress, they should be allowed to develop a color-coded addition chart using stick-on tabs that follow the same coding used in the color-coded line segments. For example:

FIGURE 2

White 0	Blue 1	Pink 2	Red 3	Violet 4	Green 5	Yellow 6
Blue 1	Pink 2	Red 3	Violet 4	Green 5	Yellow 6	
Pink 2	Red 3	Violet 4	Green 5	Yellow 6		
Red 3	Violet 4	Green 5	Yellow 6			
Violet 4	Green 5	Yellow 6				
Green 5	Yellow 6					
Yellow 6						

These charts should be developed progressively as the child masters the addition facts and the actual color rather than the name for the color should be used as background for the numeral. If you plan to utilize Cuisenaire Rods (see Appendix B for listing) at some point in the math program all color coding should follow that dictated by the Cuisenaire Rod color coding. As charts are developed, they should be kept in a place where the child has easy access to them.

During those periods of the day when you are working on math, a color-coded overlay should be placed on an overhead and shown on a screen so the children may refer to a teacher-made chart as well as their own.

Children should not start working with addition until they understand the concept of numbers as an element of a set. Appendix A has a short test with directions which should help you determine whether a child is ready to work with number combinations.

The child who does not know how to spell a word is often encouraged to look up the word in a dictionary. A child who does not know a math fact should also be encouraged to look it up in whatever source is available. We repeat: *Discourage the habit of guessing about facts.*

Over stimulation in modern America may be a problem for some of the citizens, but most agitated behavior arises when the environment presents problems that appear to lack solutions. The inability to cope brings on inappropriate behavior, often of a random nature, seemingly unrelated to environmental pressures. The solutions come by providing many clues that help provide correct solutions. Fasten flash cards around the room with the answers visible to the child when they start learning the number combinations. It may be wise to keep one wall rather drab and when some child seems to be over reacting to environmental pressures, turn his desk to that wall. Otherwise, use your classroom space to present problems and answers in a form the child will come to understand.

While it is very helpful to learning if the child has a number line and number charts and the teacher displays number charts and flash cards with answers for the child to see, it also helps if there is a large number line above the blackboard or bulletin board for the children to observe and use. It should be used often for demonstration purposes and have the same color coding and cardinal number concept (how many) that the child would have on his own desk top number line.

A sample of a worksheet (Appendix A, Sample Worksheet No. 2) demonstrates, at least in part, how we would develop worksheets for a primary grade child who is just beginning to work with addition. First, the worksheets should not be typed and second, the writing should be large and neat. Third, it is doubtful that coloring objects in with crayons adds much to number knowledge, although it does help the child use up his day. This might be better accomplished with a clean sheet of paper and crayons during a period of the day set aside for art. It would be a shame if a child failed to learn the fundamental operations because he lacked the ability to color in boxes. Fourth, arithmetic periods may happen two, even three times in one day, but the periods themselves should always be short. In the primary grades, fifteen minutes is long enough and therefore, assignments or worksheets should reflect the time allotments. It also helps if you work with a single concept and have sample problems completed on the worksheet.

You should often arrange work in patterns to give children clues. For instance, working with odd or even numerals one day and then changing it the next will help the slow child pattern his thinking. See Appendix A, Sample Worksheets No. 3 and No. 4 for examples.

FIGURE 3

odd + odd = even

even + even = even

odd + even = odd

Note that odd plus odd equals an even number and even plus even equals an even number. When a child discovers this he is ready to start counting by twos. Another type of patterned exercise in addition can be seen in Appendix A, Sample Worksheet No. 5. The answers may become obvious to the student, but seeing the relationships helps rather than hinders learning. Design learning that reveals rather than conceals relationships. Children

will learn math best when success is fairly well assured. One hundred percent, day after day, means the material is too easy. Seventy percent on a rather constant basis indicates the lessons are too difficult. Work for scores somewhere near ninety percent and you are doing the children and your profession a good service.

A Count-A-Ladder is available commercially that is a vertical number line and has pegs that can be pushed through the stand-up wooden number line. It has many uses and is very helpful in learning the basic operations. The one significant thing about it is the higher numerals are actually higher up and this can be helpful to a child who has trouble conceptualizing more and/or less. It can also be very helpful in learning how to count by twos, threes, fours and fives. Information on where to obtain this type of teaching aid is listed in Appendix B under Edukaid of Ridgewood. This company also has a film, primarily designed for teachers, that demonstrates how their number line can be used in the classroom. It can be ordered for viewing by groups of teachers for the price of the postage.

Long before multiplication is introduced as a new or separate operation, children should learn how to count and/or add by the twos, threes, fours and fives. Teachers realize, however, that adding is a binary operation (two numerals or numbers at a time). In figure four you can see some examples of problems of addition that will help when multiplication becomes an important learning exercise.

FIGURE 4

(2 + 2) + 2 =	4 + (4 + 4) =
(3 + 3) + 3 =	5 + (5 + 5) =

Number sentences, like a regular English sentence, are more understandable if proper punctuation is used.

The operations shown in figure four are considerably more difficult for a slow learning child than that of adding or combining two numerals to make a number, expressed as a numeral and labeled the answer. In the process listed above, the child combines two numerals to produce a number to add to a numeral to create a number that must be expressed as a numeral which may be the correct answer. If this language confuses you, think what those numerals do to the children. Despite the confusion, it is still better in our collective judgment to have children learn, first how to count, and then how to combine, numerals of like value before multiplication is introduced as a new operation. There are limits imposed by the very nature of language and number as to how far one can go in the development of problems that involve the counting or combining of numerals of like value between one and five. However, by going to the vertical rather than the horizontal lay-out for problem development, you do eliminate some of the need for extraneous stimuli found in the punctuation. Figure five shows the same algorithm using the horizontal and vertical form.

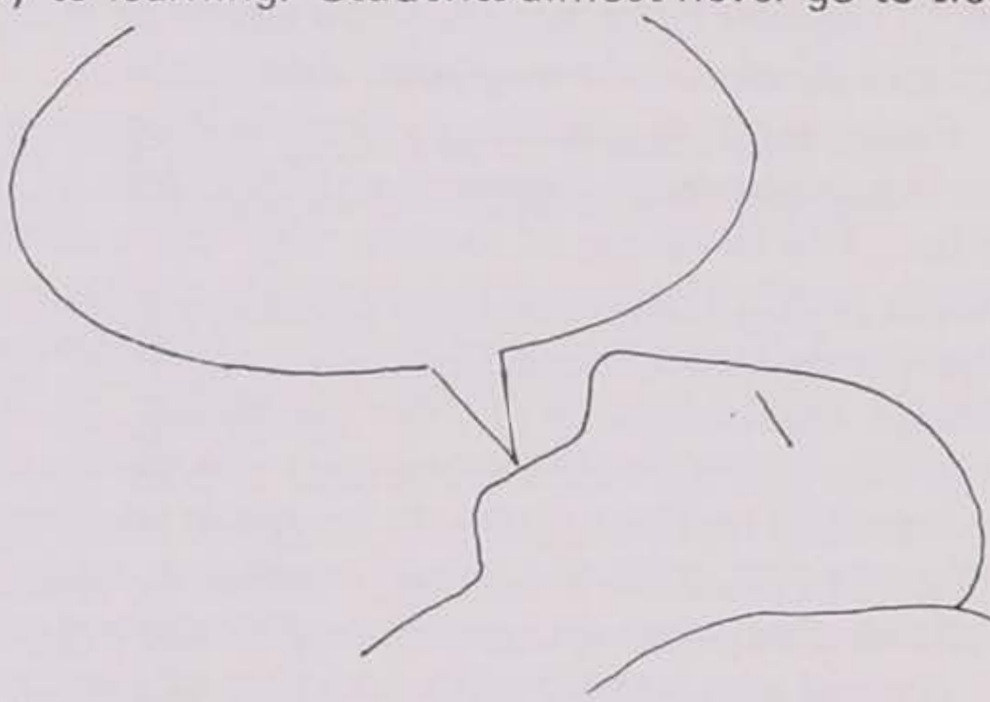
FIGURE 5

(a) $(5 + 5) + (5 + 5) + 5 = 25$

(b)
$$\begin{array}{r} 5 \\ 5 \\ 5 \\ 5 \\ \underline{5} \\ 25 \end{array}$$

In example (a), figure five, the problem looks difficult and somewhat complex for the slow learner, while example (b) appears on the surface to be a much simpler problem, and it very well may be. In example (a), combining numerals of like value calls for addition which is one of the basic operations of math. Example (b), on the contrary, may be simply a counting operation which is fundamental to the four basic operations but a lower level of thinking is involved. Many young children might think of twenty-five as something older, rather than something more. Often, the young child will even turn this around and believe the tallest man in the neighborhood is also the oldest. The cardinal concept of number (how much) is not easy to comprehend.

To find out what may be going on in the mind of each child, you might develop a worksheet similar to the one shown in Appendix A, Sample Worksheet No. 6. If the student does equally well with both types of algorithms, it is time to move on to new fields of endeavor. If the child does column B well and not the other, he may be demonstrating the ability to count by two, three, four and five, but also that he lacks the ability to quantify data or it may simply be that too much data (punctuations) over-stimulates and/or disturbs the child. If he does better with column A than column B the child may be indicating that arithmetic is easier to comprehend when all the sentences are read from left to right or that combining is more interesting than counting or even something else that is beyond our scope of understanding. Participation is the key to learning. Students almost never go to sleep when they are talking.



Students do not really understand basic facts until easy verbalization can be elicited. Children should be given an opportunity to demonstrate that they know that two plus three equals five. Work with real pennies to show that two plus three equals five. Real money holds their interest and if you want to work with a large group you can place the coins directly on an overhead

projector and let some of the students observe the shadows while others work with the substance. Do not use nickels or other coins when working with the basic facts dealing with units. You can relate the learning of the basic facts to one penny, one dime and one dollar, but you are likely to cause confusion if you bring into the discussion nickels and quarters. Our money system is set up on a decimal basis, but the slow learner is almost certain to be confused by those coins that do not fit the pattern of units, tens and hundreds.

Our number system is additive, that is, we add one to ten and get eleven. We write it as one ten and one one. Because the system is additive the most important facts a child should learn, and learn well, are the addition facts. Subtraction, as one of the four fundamental operations, should be delayed as long as possible for it not only undoes addition, it also may undo some valuable learning. The truth is, we believe that learning multiplication should follow addition and after that the students should learn subtraction and division. We understand that this is not in accord with most, if not all, curricula designed and in use; however, we believe it to be pedagogically sound, and even more important, it will make the student feel more socially acceptable. By the time most of the mentally retarded or slow learning children have mastered the addition facts, their age mates have completed addition and subtraction and are working with multiplication. When a slow learning nine-year-old is learning multiplication facts along with age mates, a feeling of well being and power is likely to develop in the child and act as a powerful agent in the teaching-learning process.

MULTIPLICATION

The rationale for teaching multiplication after addition and before subtraction was developed after observing adults learn other number bases. Multiplication involving regrouping appears to be easier to learn than subtraction which also involves regrouping. Using the language of traditional math, it appears to be significantly easier to carry than to borrow. That point will be discussed in greater detail later.

Problems in Appendix A, Sample Worksheet No. 7, are developed to show the relationship between ones and tens as they relate to dimes and pennies. The language of money may cause confusion so children should probably work together on worksheets similar to the one suggested in Appendix A (Sample Worksheet No. 7). Readiness, or at least the check for readiness, would involve directed questions, such as, *If 2 plus 5 equals 7, what does 2 tens plus 5 tens equal?* If a child knows, write it down and have the child write it down on his paper as something he knows. Constantly attempt, with the use of questions, to develop relationships between dimes and tens, ones and pennies and between ten pennies and one dime.

A child should stay with addition of numerals until he has mastered all the basic facts -- that includes addition through nine plus nine and also addition of any single digit numeral to any numeral such as ten, twenty, thirty up to ninety. He should also learn to add any set of two numerals that end in zero up to ninety plus ninety. The one concept that you should try to avoid is regrouping except when you place a one in the tens column or a one in the hundred column when adding numerals similar to nine plus nine or ninety plus ninety. A slow learning child should not be expected to hold a number in mind for regrouping purposes while adding a set of numerals until he is at least nine years old and for many of the educables, this should be delayed until a child is ten or older. For the so-called average

or normal learner, it is safe to state that numerals confront their senses and numbers confound their minds during the first eight years of life. The retarded child is almost certain to know too much frustration if he is expected to learn a large number of math relationships prior to age eight. After age eight, at least no later than age nine, your expectation of learning by the child should increase sharply. This needs to be demonstrated, in part, by removal of some of the charts and problem and answer cards (flash cards) that the child may have come to rely on. You can start with the easy ones and as part of the daily ritual, block out one more section of the teacher-made addition chart and remove a flash card from the bulletin board. In effect, you will force the child to internalize data that prior to this time he had available outside of himself. After a few days of this, you might have a chart that appears somewhat like the one seen in figure six.

FIGURE 6

1	2	3	4	5
2				7
3				8
4				9
5	7	8	9	10
6	8	9	10	11

However, never take it all away in a short period of time and never let a child resort to the guessing game. Let him keep whatever learning aids he may need.

As you slowly remove one set of learning aids, you might just as slowly introduce a second set. One of the first things a child must learn about math is that there are four fundamental operations. If you proceed from addition to multiplication as we think you should, you can introduce the symbols, or signs for addition and multiplication, speak of likenesses and differences and demonstrate simple problems on the number line. Problems, as shown in Appendix A, Sample Worksheet No. 8, will give you an activity that the children can do prior to real understanding of multiplication that simply calls for the child to identify the operation. However, the process is not a simple one. It will elicit a new type of thinking from most of the children. First, it involves matching a number (an abstraction) with a numeral to see if the sum or product is correct. Second, it involves a process where a child jumps ahead in his thinking and then moves backward for his response and in the case of multiplication, he may be responding inadequately because of

a lack of trust. The ability to reverse yourself is often difficult for the young child or the mentally retarded.

FIGURE 7

One, five is five

1 X 5 = 5

One, eight is eight

1 X 8 = 8

Many slow learning children have trouble understanding the concept that one is the identity numeral for multiplication. To multiply a numeral by one does not change its identity. You might state that, *One cow is how many cows,* and then go on to ask, *If you have one five, how many fives do you have?* The child has trouble with this because of what he knows. One plus five equals six, therefore, one times five really ought to be more than five. Encouraging a child to verbalize problems helps him to order his life and his world. Many children cannot know what they think until they say it and, like anything else, saying it once is often insufficient practice.

The communtative principle of multiplication should be employed early in the learning process. See Appendix A, Sample Worksheet No. 9, for an example of a worksheet that might be used.

In new math, the precision of language has made it possible to teach many ideas that were formerly considered too difficult for the elementary age child. However, the slow learner fails at this point, because he cannot understand the language, no matter how precise you are in your choice of words. You cannot depend on explanation; you need to use examples and demonstrations. You also should understand that there are words that need to be used and many that are best left unspoken. You can teach a child about the communtative properties of addition and multiplication without eluding to the word commutative. For words related to math that you believe you must use in teaching, proceed with caution, provide examples, demonstrate and discuss. Figure eight shows an example of math language that you might place on a bulletin board or up in some corner of the chalk board. But, don't place it there and believe you have now found a solution, you may have merely changed the nature of the problem by your efforts.

FIGURE 8

Language of Math

+ Addend / Addend	X Multiplicand / Multiplier
Sum	Product

Jerome S. Bruner, in his book *Process of Education,** indicates that you can teach almost any concept to any child in some understandable way if you can only find the proper language. The situation you face when you work with the very young, the retarded or slow learning child is that of not having a proper language to deal with. The solution is found in finding the proper way to behave, primarily at a non-verbal level, without frustration, to bring meaning and joy into the lives of the children. Do it with deeds.

The normal order of learning the multiplication facts should be revised somewhat for the slow learning child. The first ideas to keep in mind is that the zero (undoubtably one of the most important inventions of all time) causes more trouble than it should. That is because it seems too simple and we tend to pass it off lightly. Find ways to let the child comprehend that no eights equal zero and no dogs cannot bark. If that language bothers you, you may ask the child how much barking can you hear from no dogs. However you do it, it is important that the child learns the significance of zero before he goes too far in what may start to appear like a formal math program. The child should learn how to count by ones to twenty, by twos to twenty, by threes to thirty, by fours to forty and by fives to fifty while he is beginning to learn the significance of the multiplication process or operation. This can best be done with daily practice using a chart similar to the one illustrated in figure nine.

FIGURE 9

0	5	10	15	20	25	30	35 →
0	4	8	12	16	20	24	28 →
0	3	6	9	12	15	18	21 →
0	2	4	6	8	10	12	14 →

There are other types of charts that will be discussed later. However, before we discuss them we would like to point out that anything that appears to be drill should be used frequently and be of short duration.

The child will probably find multiplying by five easier than multiplying by three or four, hence we believe you should start with zero and one, then two and then jump to the fives. After that you should work with the threes and fours. You and the children should make multiplication charts. These should be developed ahead of planned learning activities. For instance, the first chart displayed might look like the example shown in figure ten. The next addition should probably include all the multiplication facts through nines.

---

*Bruner, Jerome S. *Process of Education,* New York: Vintage Books, A Division of Random House, 1960.

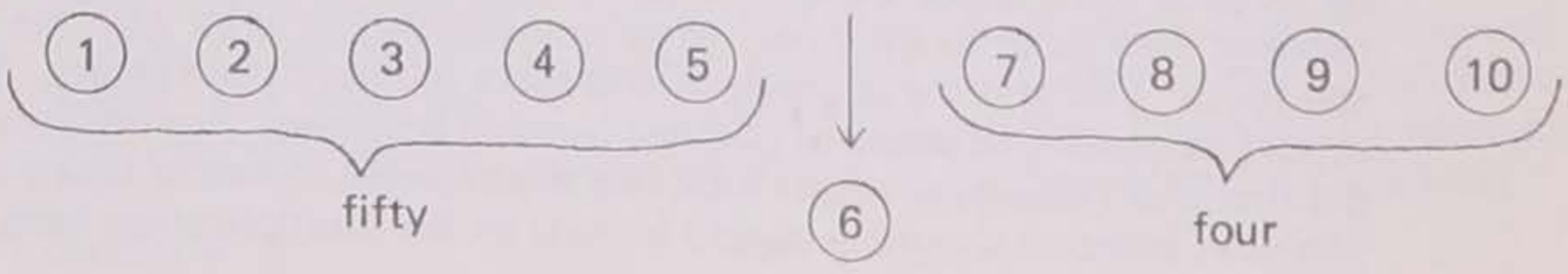
FIGURE 10

0	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

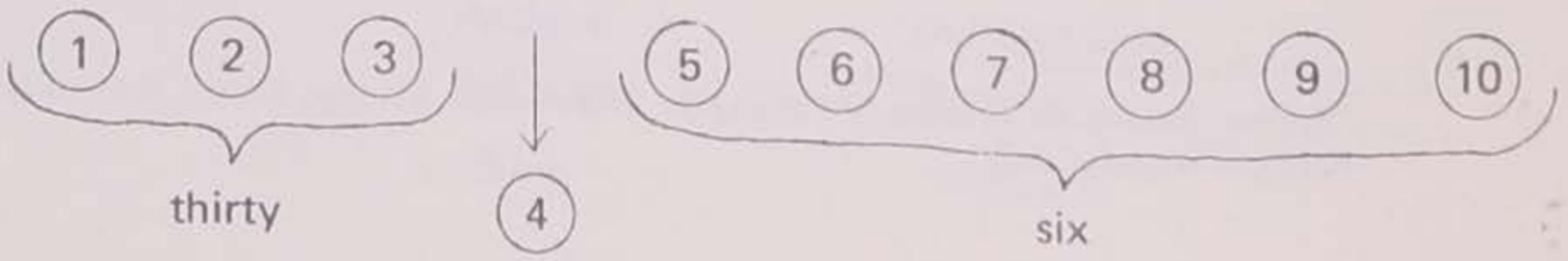
After a child has learned the multiplication facts for the fives, fours, threes, twos and ones, he should be encouraged to learn the nines next. This is not in line with current curriculum design, but it does make sense if one tries to go from simple to complex. For the sheer joy of learning, the nines have it. Without too much effort on your part, you can develop ten small squares of tag board or ten poker chips with numerals from one to ten on them. You can get stick-on vinyl numerals at an art store to simpify your task and follow the example shown in figure eleven and let the child teach himself. We must not be offended if he learns better when we teach less.

FIGURE 11

9 X 6 = 54



9 X 4 = 36



Most children are delighted with this discovery. You can even put tape over their fingernails and number them from left to right. That way, the child can practice his nines wherever he may be.

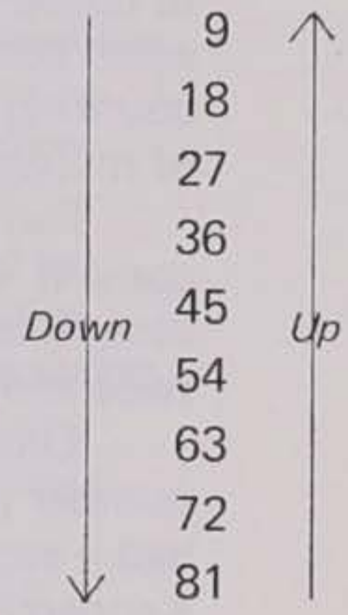
There are many times when drill appears to be necessary but it is almost always more effective if you disguise what is being done and make it appear like fun. The nines help. By the time a child starts working with multiplication, he should be encouraged to look for patterns and relationships in number and numeral situations.

For example, in figure twelve there are certain patterns demonstrated with the nines that will deserve the title of relationships when children are caused to discover them.

FIGURE 12

9 X 1 = 9	=	9
9 X 2 = 18	=	1 + 8 = 9
9 X 3 = 27	=	2 + 7 = 9
9 X 4 = 36	=	3 + 6 = 9
9 X 5 = 45	=	4 + 5 = 9
9 X 6 = 54	=	5 + 4 = 9
9 X 7 = 63	=	6 + 3 = 9
9 X 8 = 72	=	7 + 2 = 9
9 X 9 = 81	=	8 + 1 = 9

Note:



Because of our collective desire to cling to the tried and true, you will surely want to resist teaching the nines (or better yet, letting the children learn them) after they have completed the fives. However, we want you to know that as far as we have been able to ascertain, there is no research anywhere that indicates the sixes must follow the fives. We do believe we can convince you that nines are easier to learn than sixes and that most learning should proceed from the simple to the complex. Therefore, we will encourage you and your children to develop number charts similar to the one pictured in figure thirteen.

FIGURE 13

0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5				9
2	2	4	6	8	10				18
3	3	6	9	12	15				27
4	4	8	12	16	20				36
5	5	10	15	20	25				45
6									54
7									63
8									72
9	9	18	27	36	45	54	63	72	81

The first advantage we see in this type of chart is that the multiplier and multiplicand stand out more clearly and hence are easier to define, at least in terms of function. Secondly, when the chart is developed for the nines, the children come to feel that the facts up through five times five should be mastered. Your implied expectations along with their own developing self concept are the two best allies you have in a learning environment. Third, the empty space represents a frontier and all of us, at least part of the time, enjoy that type of a challenge.

In a normal learning situation, the child is caused to learn the multiplication facts, more or less, through the fives as a one shot deal. In the process, he normally learns that five nines equals forty-five. The child would probably learn this more rapidly, which is very important for a slow learner, if he could learn it as nine fives equals forty-five. As it is, learning to count by two, three, four and five is something of an inverse operation when compared with learning multiplication facts. To state it another way, counting by five and multiplying by five employs the commutative properties of multiplication in the learning process which makes it difficult for the child.

Perhaps, there is no way to make it easy, but a partial solution would come if you reversed the process at least in part, by teaching the child that nine times five equals forty-five before he learns all the fives, fours, threes, twos and ones.

Learning number combinations is difficult when there is no discernable number patterns or relationships to rely on. For example, President Kennedy had a secretary named Lincoln and President Lincoln had a secretary named Kennedy. Knowing that has little or no relationship to your survival, but because of the relationship of names, it is going to be very difficult for you to forget that fact. Now you may forget it, because you read it and never wrote it or we might also say you listened but never told it. What is needed is active rather than passive participation in a learning program where the development of relationships between and among facts is a common practice.

For instance, if one looks for patterns in multiplication facts that might act as an aid in helping a child learn, then the nines really are the easiest of all and the sixes and threes have much in common and should be related in the learning process.

FIGURE 14

9 X 1 = 9
9 X 2 = 18* 1 + 8 = 9
9 X 3 = 27* 2 + 7 = 9
9 X 4 = 26* 3 + 6 = 9
9 X 5 = 45* 4 + 5 = 9

6 X 1 = 6
6 X 2 = 12* 1 + 2 = 3
6 X 3 = 18* 1 + 8 = 9
6 X 4 = 24* 2 + 4 = 6
6 X 5 = 30* 3 + 0 = 3

3 X 1 = 3
3 X 2 = 6
3 X 3 = 9
3 X 4 = 12* 1 + 2 = 3
3 X 5 = 15* 1 + 5 = 6

**Add digits in product*

When you deal in magic you can attract a crowd. When children can add and yet need more practice, encourage them to add the digits of the products of a multiplication problem. You will be amazed with the number of relationships and patterns that can be developed.

You may question some of the suggestions that have been made and others that will follow, and it is proper that you do question and examine the activities that take place in your classroom regardless of whether you follow any suggestion put forth in this SECDC publication.

However, we believe we should try to justify some of the ideas that have been and will be put forth here. We believe that even many of our gifted students do a rather poor job of learning their arithmetic facts. Learning the multiplication facts is very much like learning the names for eighty new people that you meet for brief periods of time each week, and there is probably not much of an emotional attachment for them as persons. Because of that you could probably care less that Betty has red hair and John has a big nose. Without some emotional involvement we cannot care enough to really see, and without awareness concern is lacking. Relationships involving math and people that interest us, motivate us to become aware of likenesses and differences so that people become persons and number combinations become number facts.

Our responsibility as we see it, is to help children become aware of as many interesting relationships as is possible inherent in the number system. No child should have to memorize all the addition, multiplication, subtraction and division facts, but all who are able should be permitted to know them. The child should spend more time using data in his environment rather than reacting right or wrong to the data available.

There is something of interest about every one of the multiplication facts. This type of data does not make the facts easier to memorize but it does give a child something he can work on and become interested in. For example, three-fourths of the answers to multiplication problems are likely to be even numerals.

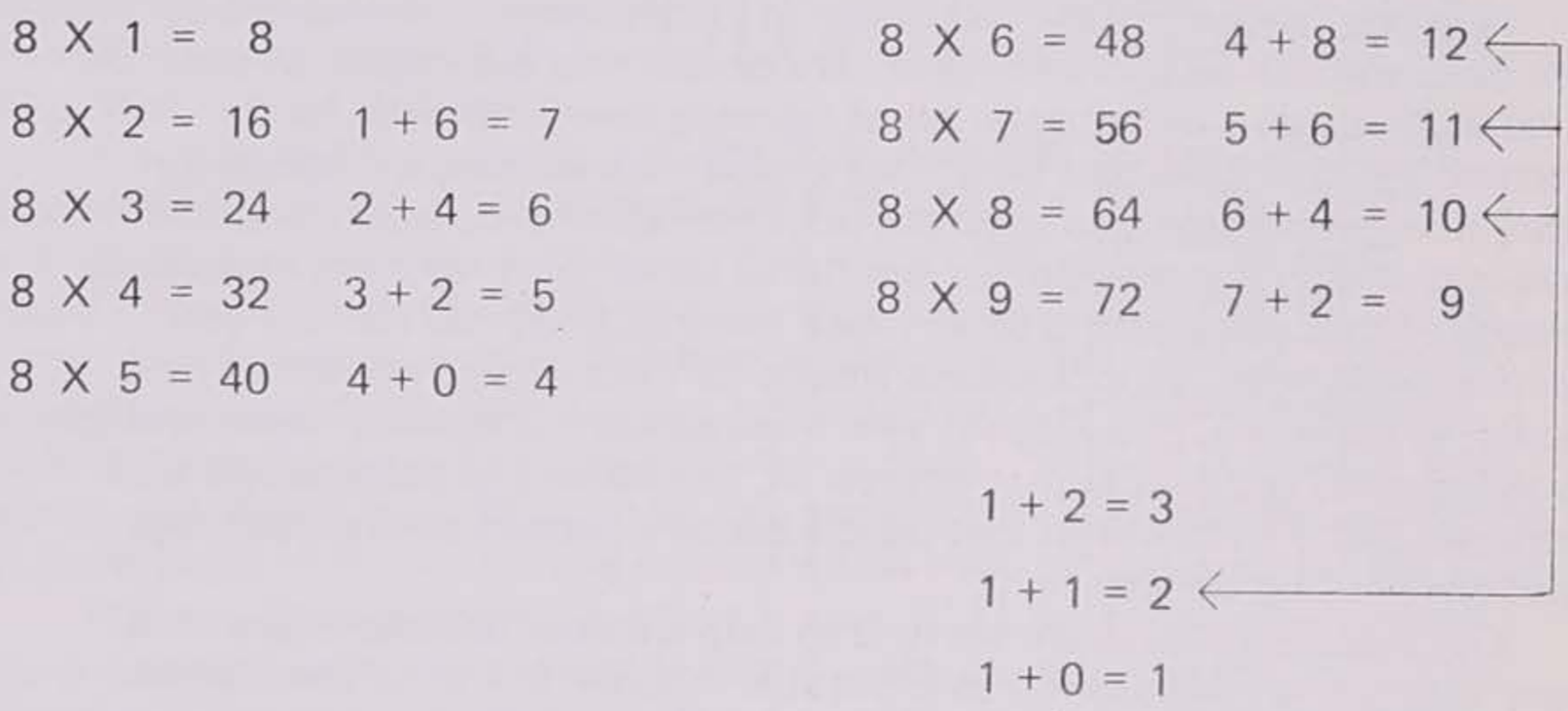
FIGURE 15

odd x odd = odd
even x even = even
odd x even = even
even x odd = even

This type of data can be shared with children without placing on them the obligation to know what is implied. When the children are ready, they will try to figure it out simply because it tends to be novel, rather than by using conventional wisdom. When children feel inadequate, and most of them do, the novel rather than the conventional excites them.

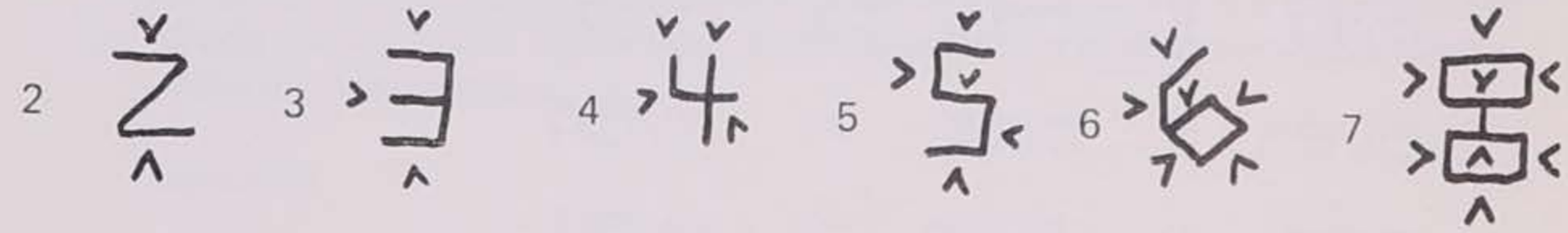
It was mentioned earlier that the order of learning the multiplication facts could be altered to the benefit of the child. For instance, the threes, sixes, and nines have much in common and the relationship among them might help the child put it all together. The other facts also have some interesting patterns that may interest at least some of your children.

FIGURE 16



When teachers help children see relationships, learning is enhanced. When teachers fail to do this children will generate their own. For instance, when adding, children will visualize certain attributes in the numeral that may or may not be present that aid in the counting process or operation. Figure seventeen will indicate how some children apply visual clues when other aids are not permitted.

FIGURE 17



Seven and nine are often extra difficult in addition. It might be better for some children if we distorted the shape of the numerals to enhance the child's chance of success.

Other patterns that can be demonstrated with the basic multiplication facts are illustrated in figure eighteen. Remember, a child should not be expected to know the facts before he learns about the beauty in the patterns that can be developed. We certainly cannot expect the child to know about the basic facts of art, which include line and form, harmony and contrast and color before he learns to appreciate a work of art. Speak of beauty and order and children will listen.

Ask your children to memorize a poem and only a few will meet with real success. Let the poem become lyrics for a song that you teach and almost all your children will soon learn the words. More pre-schoolers have learned the alphabet by setting it to music than was ever thought possible by the traditional first grade teacher who used to spend hours trying to pound those twenty-six letters into our tortured heads in something of a sequential manner. Those who write the songs for the nation may have greater control over our destiny than any of us can possibly comprehend.

The secret to learning is more than relationships, such as, relating lyrics and song. There must also be joy in accomplishment. The words work and/or play do not apply, but the words hard or easy should have significance for both teacher and child. Without pride in accomplishment there can be no joy. To accomplish something viewed as too easy diminishes pride. The child must feel new power when he learns. The good teacher should seek out methods to make all learning easy and yet the child should be made to feel that all accomplishments come through hard work. To fail at something easy is equivalent to being twice defeated.

It is very difficult to make some activities and learning appear hard when they are introduced late in the life of a child. A high school student just beginning to learn addition and multiplication facts is usually aware that these facts are learned by most students while they are young children. To cause a child to view such a task as difficult and worthy of pride in accomplishment is not easy and yet the longer one waits beyond the teachable moment to learn a new skill, the more difficult it becomes. If a child has the ability to learn understandable speech before formal education begins, the ability to learn math combinations will surely be present before the child is twelve. Just about all French speaking people would agree that it is easier to learn basic math than a foreign language such as English. Of course, they learn to speak French so well because of constant usage. All ideas and languages are foreign until we are exposed to them.

There are different types of charts that can be helpful to the student as he learns multiplication; and, as the student grows in ability it may help him internalize, not just the facts, but the meaning of multiplication. The charts in figure nineteen can be extended and developed for each family of facts.

FIGURE 19

Number Charts

(Six) 6
1 - 6
2 - 12
4 - 24
8 - 48

(Seven) 7
1 - 7
2 - 14
4 - 28
8 - 56

(Eight) 8
1 - 8
2 - 16
4 - 32
8 - 64

For instance, if you double eight you get sixteen; if you double sixty-four you get one hundred twenty-eight. Therefore, sixteen times eight equals one hundred twenty-eight. If the child needs to know what six times eight equals, he can learn to add the product of two eights and four eights. In the three charts presented, there are many answers that a child can find with the use of addition.

Number line segments, or color factor rods, or Cuisenaire Rods should also be used with multiplication. A child can match up three fours and demonstrate the commutative property of multiplication even if he doesn't use the language to explain the process.

There are certain multiplication processes that are not generally taught or learned well in conventional math programs that can be a great help to the learner if introduced and made a regular part of the math program. Children should be encouraged to multiply by ten and double and possibly redouble. When a child learns how to multiply by ten and twenty, it becomes a simple matter for him to learn how to handle one hundred and two hundred with proper verbal encouragement. This type of activity should be at the verbal level for many days before you formalize it with written assignments. If you will look again at figure twenty, you can see that twelve times forty is relatively easy to compute while eight times forty is slightly

FIGURE 20

8 X 10 = 80	80 X 2 = 160	8 X 20 = 160
10 X 12 = 120	120 X 2 = 240	12 X 20 = 240

more difficult. When you double and redouble from one hundred twenty to two hundred forty to four hundred eighty, there is no regrouping process needed. However, when you go from eighty to one hundred sixty to three hundred twenty, regrouping is needed.

When a child has memorized his facts he can write or state that four times eight equals thirty-two. When he really knows his number combinations, he should be able to state that forty times eight equals three hundred and twenty. It may seem mechanical at first, but it is very useful later. If you write five times six on the board and the children know it equals thirty, add a zero to the multiplier and help them see the answer as three hundred. With enough practice and discussion, this extension of the basic facts is learned and can become particularly helpful later with division problems.

Always try to use simple numerals to teach difficult concepts. As your children grow beyond the basic facts program the teaching of multiplication should be done carefully. In figure twenty-one, there are two multiplication problems that look somewhat similar and yet the first is much easier to do than the second because of the problems implicit in regrouping.

FIGURE 21

32	46
X 4	X 5

One method you can use to help the child over this difficult area is that of providing part of the solution and direct his activities as he responds to the many practice problems he will need to do.

FIGURE 22

$$\begin{array}{r} 32 \\ \times\ 4 \\ \hline 1\_\ \_ \end{array} \qquad \begin{array}{r} 46 \\ \times\ 5 \\ \hline 2\_\ \_ \end{array}$$

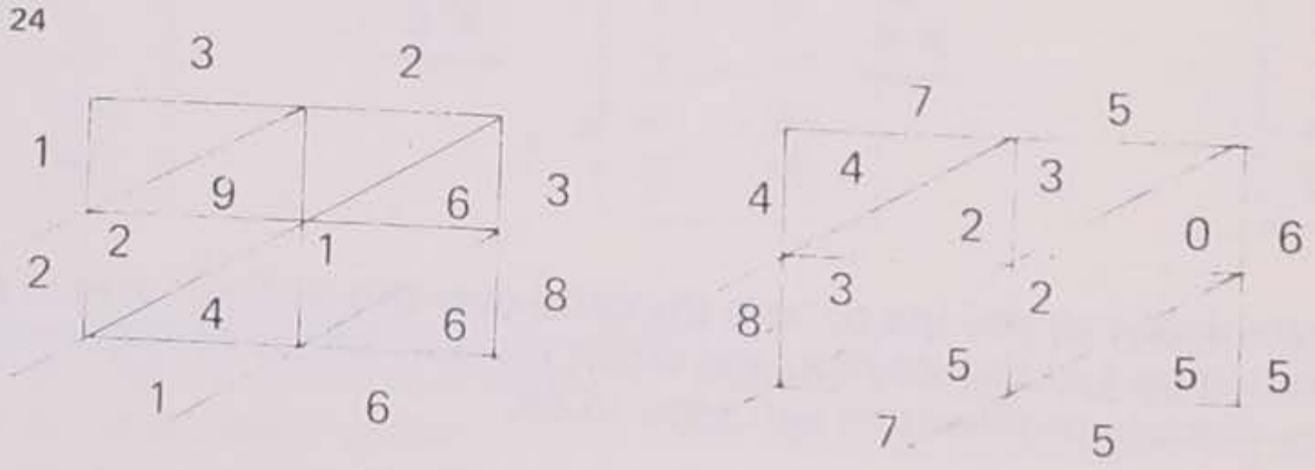
In each case in figure twenty-two, one of the three digits needed in the answer is provided. In the first problem it really does not matter which digit is provided but in the second the three in the tens column is definitely the most difficult part of that problem. If the three place answer were not directed by the dash numeral dash, many children would tend to respond with an answer of two thousand and thirty. Letting the children fill in spaces for product or later on for partial product and product will increase the likelihood of success and help provide that courage necessary to move on to greater things.

When regrouping is first taught it should probably come with column addition. To add another numeral on in an adding operation is probably much easier than adding on a numeral during the multiplication operation. The second problem in figure twenty-two is rather complex to a child. For children who have a particular problem with this type of regrouping there is a method that can be employed where the regrouping is all in the addition operation and none is carried on in the multiplication operation. This method has been used, with success, with children who have had difficulty with multiplication.

FIGURE 23

In each of the two examples shown in figure twenty-three, there is a one numeral multiplier and two place multiplicand. In both cases regrouping would have been necessary in the multiplying operation using the traditional method and yet it was avoided here. In other problems similar to the two shown in figure twenty-three, regrouping in the adding operation might be necessary, but never as part of the multiplication operation.

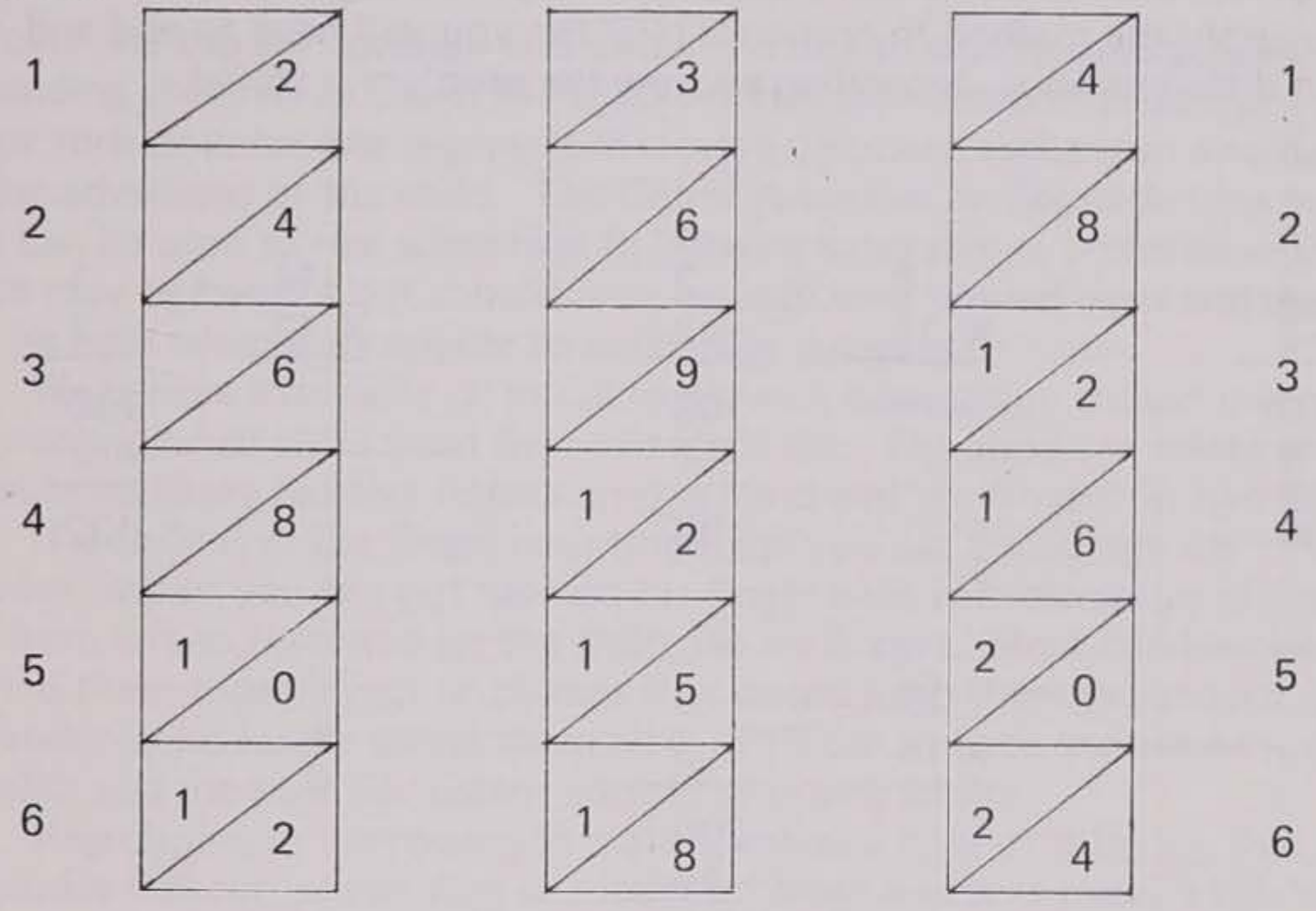
FIGURE 24



In the first problem shown in figure twenty-four the multiplicand is the numeral thirty-two and the multiplier is the numeral thirty-eight. The answer is 1,216 and there is some regrouping in the adding process. In the second problem there *was* no need to regroup or carry as we use to say. All the adding with this method is done on a diagonal.

The Lattice method of multiplication can be expanded considerably and be a real help to a child that has trouble using the conventional aids such as a number line or charts.

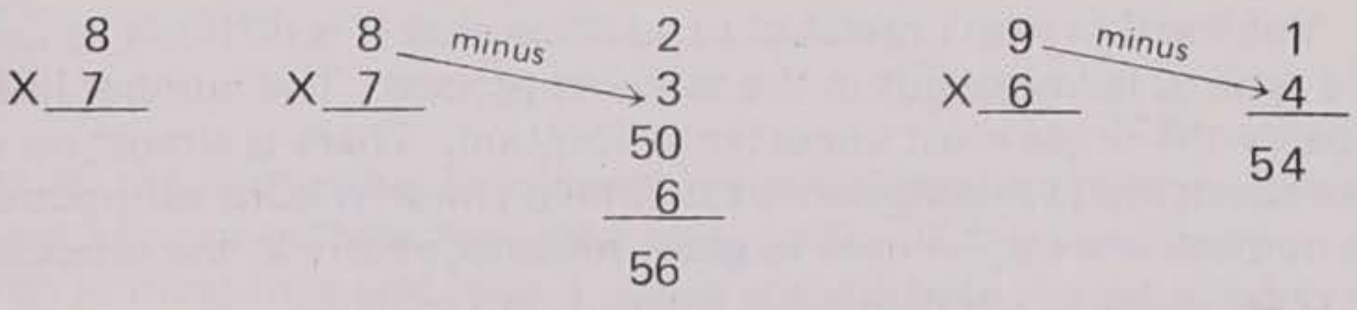
FIGURE 25



In figure twenty-five you can see the start of three bars for the twos, threes and fours. You can readily see that one times 234 equals 234, and two times 234 equals 468, but three times 234 is a bit more difficult to see since you must add on the diagonal and regroup. Since the one and nine equals ten you carry one and have an answer of 702. If you will look again you can see that five times 234 equals 1,170. For a quick and accurate answer this method is hard to beat. However, it works best when the bars are side to side, without space between them.

A junior or senior high age child who has been unable to learn the basic multiplication facts will often find regular drill and practice with the basic facts demeaning. It is sometimes necessary to find methods that look more complex even though the problems can be solved more readily by the student in order to help the student save face. A *method* that can be used is demonstrated below in figure twenty-six.

FIGURE 26



This type of problem (figure twenty-six) looks complex, and therein lies its virtue. The first problem, seven times eight, is solved by putting 8 plus 2 together for ten, and seven plus three together for ten. Multiply the three times two for the six in the units or ones column and then subtract the three from eight to achieve the five for the tens column. The second problem, six times nine, is done using the same method. The four subtracted from nine equals fifty, and the four times one equals four. The answer for six times nine if fifty-four. There are many variations for this type of problem but it works best with the sixes, sevens, eights and nines. If you apply it to numerals of less than five you will end up working with negative numerals and if you apply the method to numerals over ten you will need to add and subtract in different ways, depending on how the problem is stated.

FIGURE 27

```
12  -  2          8  +  2          11  +  1
X 8    2        X 12    2        X 12    2
---   --        ---   --        ---   ---
 100              100              130
 - 4              - 4              + 2
---              ---              ---
 96               96              132
```

```
15  +  5
X 15   5
---   --
 200
 + 25
---
 225
```

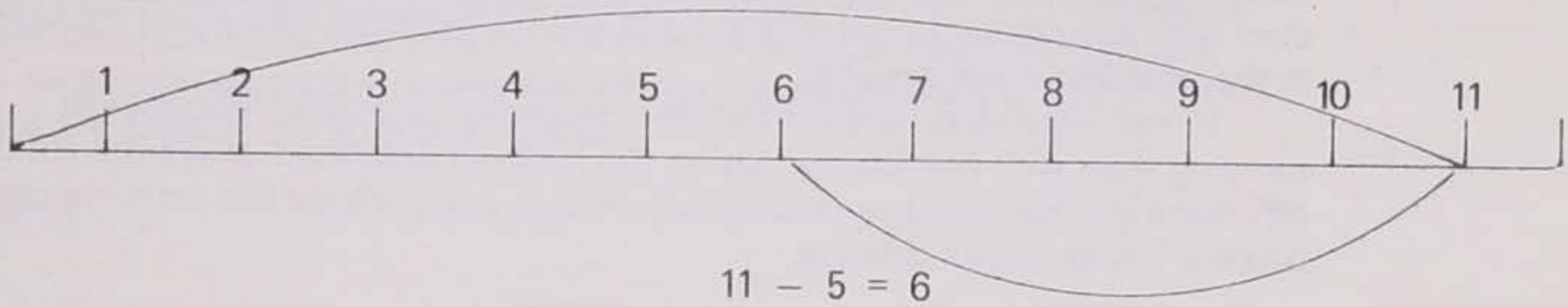
If you cannot figure out the variations without the help of further instructions, it is probably best to forget and not try to teach them to your students. However, the first set suggested in figure twenty-six can be very helpful to children in the higher grades who have not mastered the more difficult basic facts.

It was suggested earlier that multiplication should follow addition in the teaching-learning process. Students can apply the commutative associative, and distributive properties to multiplication and addition but not to subtraction and division. Addition and multiplication are related and multiplication was invented in order to quantify equal sets a given number of times without going to the problem of adding long columns of numerals.

SUBTRACTION

Subtraction is so unrelated to addition that it is difficult to use the same type of learning aids in the teaching process. The number line is probably the single most important exception. There is almost no way to make charts that we are aware of that help children learn subtraction facts. The number line can be used to great advantage early in the process and one possible help is illustrated in figure twenty-eight.

FIGURE 28



Go over the top for addition and under the line to signify subtraction. Keep reminding children to count the spaces rather than the points on the line. Color rods or color line segments that were discussed earlier can also be used to the advantage of the child. The Count-A-Ladder or Count-A-Line teaching aids can be used to real advantage in learning subtraction. Practice with flash cards may be helpful but should only be used on a limited basis and should not be used when they appear to encourage guessing.

We believe that early or initial work with subtraction should involve subtracting small sums from five, and from ten. The child can relate to this because he likely has five fingers on one hand and ten fingers on two hands.

If a child is at the finger counting stage you can encourage use of a number line or you can put tape on his finger nails with numerals of one to ten from left to right and let the child use his fingers. Most children leave behind them those things or objects that would keep them dependent when dependency no longer serves them well. This can include the use of fingers in math and the need for parent control as young adults.

Regrouping or borrowing in math involves a type of thinking that often precludes real comprehension or meaning. Most adults in today's world do not actually comprehend the true meaning of place and face value of numerals, and without that understanding borrowing is and has been essentially a mechanical process or operation. If it truly is a mechanical operation for the average adult it is probably best to treat it as a mechanical operation for the retarded child or for any young child. You probably should attempt to teach some meaning but there is much about conventional regrouping in subtraction that is difficult for the young child to comprehend. The zero that becomes a nine during the regrouping process is always difficult to understand. A method that might help at least some of your students is illustrated below in figure twenty-nine.

FIGURE 29

```
  3,000          4,233          3(9)(9)(10)
    900        - 1,564          4,2 3  3
     90                        - 1,5 6 4
  +  10                        ---------
  -----                          2,6 6 9
  4,000
```

In the problem, all the regrouping (borrowing) is completed as one operation. The four thousand becomes three thousand plus nine hundred plus ninety plus ten. The child must then add ten and three and subtract four; add nine and three and subtract six; add nine and two and subtract five and finally

subtract one from three. This method may appear cumbersome and mechanical at first but more meaning can be generated with this technique than with the conventional method and the bothersome zero when included in the problem is not an issue.

In the regrouping process children generally have less trouble with carrying than with borrowing. Using the method described above the children can make the borrowing a mechanical process and perhaps the carrying can become a meaningful process.

FIGURE 30

```
  5,000          6,372          5 (1)(9)(9)(10)
    900        - 2,456          6, 3  7  2
     90                       - 2, 4  5  6
+    10                       ------------
------                          3, 9  1  6
6,000
```

In figure thirty, all the borrowing process is achieved when 6,000 becomes 5,000 plus 900 plus 90 plus 10. However, it was necessary to regroup again when working the problem. It was necessary to carry one, a process that appears to be easier for children than that of borrowing.

There are many teaching aids that will help your children learn basic operations. The idea of using cards or sticks placed in pockets representing ones, tens and hundreds can be helpful in learning to subtract. The simple abacus should also be used if available. The Japanese abacus is much too complex for most mentally retarded children and should be avoided. During the initial learning of subtraction, students can use poker chips and either physically move or cover that number to be subtracted to determine the difference. Working at this level the child can find answers without going to the trouble of regrouping.

FIGURE 31

15 − 6 = 9

Slow learning children should have a great deal of practice crossing out circles or squares on a paper or moving poker chips around on a desk top to generate problems and answers. Work with arrays can be very helpful in understanding math. While it is wrong to put in too much data at any one time with children, the array pictured above can be used to show that three times five equals fifteen, or that two-fifths of fifteen equals six, or that three-fifths of fifteen equals nine. Arrays have all types of use in math and when poker chips or cardboard disks are used children are permitted to participate at an active level and learning is enhanced. Using arrays, the teacher can create different problems rapidly and allow children to discover the problem or the answers. An overhead projector and a number of coins will work very well for this type of teacher-directed activity.

DIVISION

Division is generally thought of as long or short division and is related to subtraction. Division like subtraction is difficult to learn with the use of number charts but number lines can be used with initial learning exercises. Flash cards with answers exposed can also be placed around the room and will aid the child in his efforts to know.

FIGURE 32

2 times	3 times	4 times
10 = 20	10 = 30	10 = 40
20 = 40	20 = 60	20 = 80
40 = 80	40 = 120	40 = 160
100 = 200	100 = 300	100 = 400
50 = 100	50 = 150	50 = 200

```
2 | 334 | 200 | → 100
  | 134 | 100 | →  50
  |  34 |  20 | →  10
  |  14 |  14 | →   7
  |   0 |     |     167

3 | 445 | 300 | → 100
  | 145 | 120 | →  40
  |  25 |  15 | →   5
  |  10 |   9 | →   3
  |   1 |     |     148   r = 1

4 | 274 | 200 | →  50
  |  74 |  60 | →  15
  |  14 |  12 | →   3
  |   2 |     |      68   r = 2
```

When children first learn long division you should avoid using a trial divisor method which is nothing more than another form of guessing, a practice that should not be encouraged. If you will develop for each of the first ten numbers multiplication charts similar to those shown on figure thirty-two, the children can learn to treat division as a form of serial subtraction that undoes multiplication. The charts make it possible for a child to devote his mental energy to the serial subtraction or division aspect of the problems to be completed. This method improves accuracy and leads to a more experimental approach to long division.

FIGURE 33

5 times	6 times	7 times
10 = 50	10 = 60	10 = 70
20 = 100	20 = 120	20 = 140
40 = 200	40 = 240	40 = 280
100 = 500	100 = 600	100 = 700
50 = 250	50 = 300	50 = 350

In figure thirty-three, the basic multiplication charts for five, six and seven are shown. You are requested to notice that each of the charts uses doubling, for instance, ten times five equals fifty, therefore, twenty times five is twice the value and forty times five is four hundred or twice the value of twenty times five. Halving is also used and consequently if we know that one hundred times five equals five hundred, half that amount is two hundred and fifty or fifty times five. If the children can help the teacher generate this type of data for the first ten numerals, they are ready to do long division using the first ten numerals as divisors. If the children cannot do that, then doing long division using the charts will be largely a meaningless operation.

When children divide in the long division operation, they must multiply and subtract, the number of times necessary, to partition the dividend according to the dictates of the divisor. This is using two operations to accomplish a third operation and the process is complex and confusing to most children. If the operations are separated and applied slowly as the children prove themselves ready, your joy will be increased.

FIGURE 34

8 times

10 = 80
20 = 160
40 = 320
100 = 400
50 = 400

(a) 8 X 30 = 80 + 160 or 240
(b) 8 X 60 = 160 + 320 or 480
(c) 8 X 24 = 160 + 32 or 192
(d) 8 X 15 = 80 + 40 or 120

As children grow, they can be encouraged to find answers similar to those shown in figure thirty-four by using the data available on the charts. In problem (a), if ten time eight equals eighty and twenty times eight equals one hundred sixty, then thirty times eight equals two hundred forty. In problem (d), if fifty times eight equals four hundred, then five times eight equals forty and add that to the product of ten times eight and you have the product of fifteen times eight or one hundred twenty. It is not simple but it can become meaningful and the use of charts in division will reduce error and guessing, and will, in some cases, stop the feeling of hopelessness that some children feel.

The secret to success in using this method for long division will come when we learn that long division should be made as easy as possible during the learning process and more than that, it should be academically honest. The traditional method used by most adults does not help children truly understand the meaning implied by the process.

three

time

A listing of a variety of activities for learning about time, money and measurement are available in an earlier SECDC publication entitled *Planning an Arithmetic Curriculum,* published November, 1968.

Any listing of this nature should not limit your imagination and search for meaningful activities. The one reason we did not create an extensive list of activities for you to refer to as you look for methods to use in teaching is related to the problem of dependency. Teachers that are constantly informed about means of achieving specific ends fail to develop the one characteristic that leads to future success -- an open, trusting and imaginative mind.

Please remember, if there is information you want the students to learn and use, make this information available to them in the form of data sheets they can keep in their desks or on wall charts they may refer to when needed.

Telling time is very important in the world beyond the school and some students will never learn how to tell time unless it is considered important within the classroom. Perhaps the best way to demonstrate the importance of knowing how to tell time is to select students to stop or start a new activity by watching for the correct time on a wall clock or a watch that the student is permitted to wear. When the classroom teacher always determines that it is time to go to recess or lunch or put away the reading books and get out the spelling workbooks, the child is not encouraged to know or be concerned about time. There is probably even good reason to cover the wall clock and allow only a few selected students to wear watches and inform the others when scheduling changes should take place. Once you establish the importance of students knowing what time it is, learning will likely take place very rapidly. However, to teach children how to tell time before most of them can count to sixty or count by fives to fifty or sixty and before they comprehend the concept of less than and more than, as well as before and after, may be an exercise in futility. On the other hand, the social demands placed on most children by age eight are such that learning how to read a clock at least in

terms of fifteen and thirty minutes after the hour and fifteen minutes before the hour has some significance.

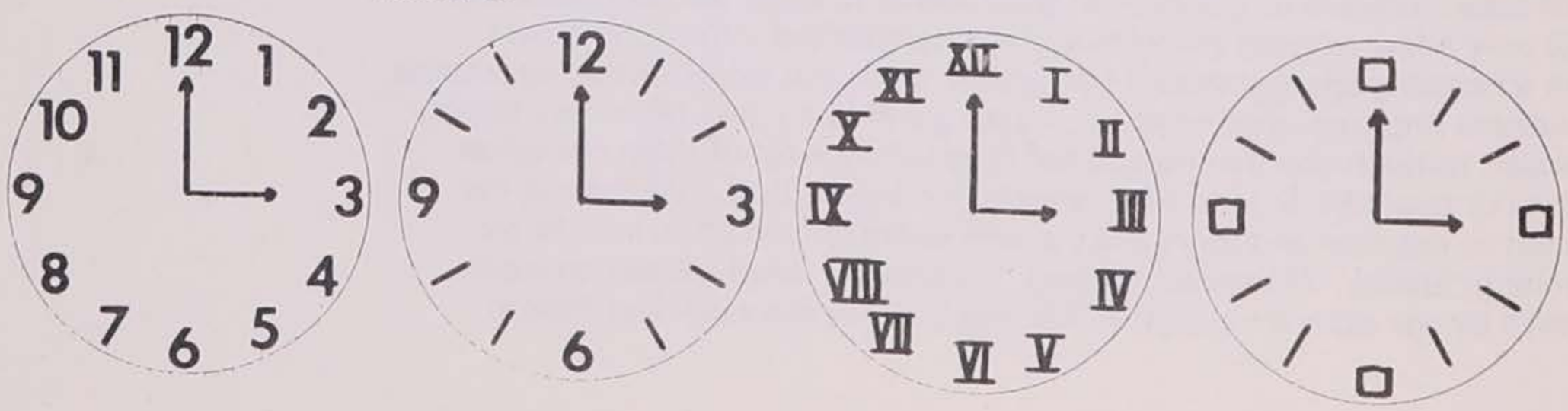
There is a good deal of versatility in the English language that allows us to read fifteen minutes to ten as nine forty-five, and this must be a source of confusion to the young child. From what we have been able to ascertain from looking at other languages, as well as our own, it is probably best to use the half hour as the dividing line and speak of time as so many minutes after or before the hour. Initially, speak of fifteen minutes *before* ten rather than fifteen minutes to ten and this too should help with comprehension. The key to success in learning time may be found by listening to children carefully and attempting to hear precisely how they discuss time.

Most of us have some problems with time long after we have learned how to read the face of a clock. Boring or painful periods in our lives appear to literally slow down the sweep of the hands on the clock while at the same time our longer units of time such as weeks, months and years may appear to move around us more rapidly. The happy child and adult should find greater achievement and success in the present and his short blocks of time should move rapidly while his years stretch out to increase his moments of joy.

To help a child gain a perspective on time you can teach him to count seconds to sixty without the benefit of a sweep hand. A system that has been used is that of counting one thousand one, one thousand two, etc., preceding each counting number by the words one thousand. This forces the student to use up a full second for each of the counting numbers and helps the child to conceptualize the meaning of time passing in constant units. Early in the lives of children, time passing can be related to short or long television programs. Even distance as it relates to a trip should be described in terms of time units rather than miles and a one hundred-mile trip in a car can be discussed as something that is as long as two long (two one-hour) television programs.

Other ways to encourage children to become aware of time and the passage of time is to encourage them to guess units of time such as five- or ten-minute periods. You can ask the children to tell you when the next ten minutes have passed and see which child can come the closest to giving a correct response. The more mature children are much more likely to be able to wait the full ten minutes to respond or indicate that time is up. Rewarding the child that comes the closest to the correct time may help all the children gain a better appreciation for the meaning of time and consequently makes learning how to read a clock more meaningful. You can also ask questions such as how many more minutes do we have before we break for lunch or go to the playground. Knowing how to tell time should not be essential for entering into such activities.

FIGURE 35



There are many types of clock and watch faces to learn. The slow learning child is sure to be disturbed if too many different varieties of clocks are introduced during the initial learning periods. It is probably best during the elementary years to use only the face with the twelve numerals the child can easily recognize and understand. Later when various types of clock faces might be used or discussed, transfer of training will be greatly enhanced if all the clocks will have minute and hour hands in approximately the same position. When real transfer takes place additional instruction is not necessary.

CONSIDERATIONS

The concept of time is abstract and difficult to comprehend for most slow learners. It involves the use of specific devices and language such as the calendar, clock, watch, month, day, hour, A.M., P.M., and many more.

Initially, it will probably be easier for students to understand the concepts presented if you are consistent in the use of auditory, visual and tactile materials. Use one type of clock face and set of numerals to avoid confusion. Eventually, you will be able to introduce clock faces with different types of numerals or those that show no numerals.

Do not skip from one concept to another. Be sure that students have mastered skills in a sequential order before presenting additional information.

Traditionally, the concept of time is taught over a three-year period so do not feel that you are compelled to cover this concept in three or four lessons.

The calendar should be taught in a separate unit that sequences the components of day, week, month and year.

PREREQUISITE SKILLS

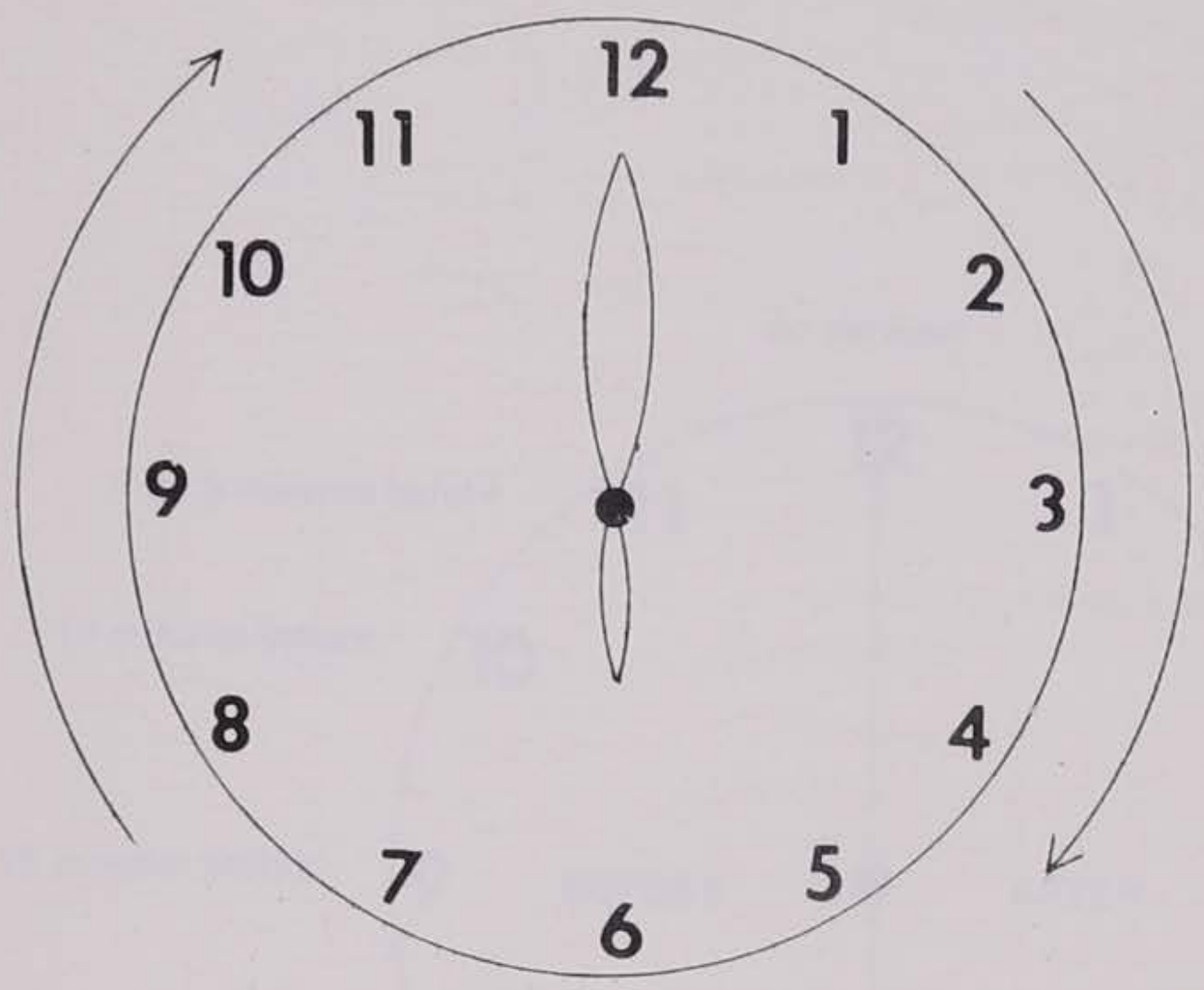
If students have mastered the following list of skills, it will make learning the concept of time easier.

1. Count by 1's to 60, with or without the use of a number line.
2. Count by 5's to 60, with or without the use of a number line.
3. Be able to distinguish between before and after.
4. Be able to distinguish between long and short.
5. Know fractional parts of ¼ and ½.

The suggested learning sequence, activities and reinforcements are by no means complete but are merely examples that we hope will assist you in developing lessons on this topic.

Suggested Learning Sequence	Suggested Activities	Suggested Reinforcements
1. Distinguish between long and short hands on the clock.	All students should have clocks to manipulate as these concepts are presented.	Give students information from Sample Date Sheet No. 1.
2. Direction the hands move on the clock	Develop worksheets that enable children to practice concepts. See Sample Worksheet; for other exercises use ditto masters available from Continental Press, Inc., or make your own.	Clock faces with missing numbers, covered with clear contact paper, can be marked.
3. Distinguish between units of time: day, hour, minute, A.M., P.M.		
4. Reading hours and express time in written form.		Sample Bulletin Board, *Our Day*
5. Identify the lines that indicate minutes and be able to count minutes to the long hand.		
6. Reading hour and minutes together and express time in written form.	Develop a worksheet that expresses time in written form below clock faces without hands. Have students accurately locate the hands on clocks.	Develop a learning center with several clocks set at different times. Have students record the times on paper. Change the times when all have had the opportunity to try this activity.
7. Reading minutes (long hand) by 5's.		Give students information from Sample Data Sheet No. 2.
8. Distinguish whether minutes are before or after the hour.	Develop activities that will demonstrate the concept of before and after. Make a clock face on a piece of tagboard complete with numbers. Draw a line through the center of the clock and color each half a different color. On the left side of the clock write the word *before* and on the right side write the word *after*. Post the clock where all students can see it and discuss the concept of before and after the hour.	Have one or two students be responsible for informing the class when it is time for certain activities, i.e., recess, lunch, etc.

Suggested Learning Sequence	Suggested Activities	Suggested Reinforcements
9. Introduce half hour.		
10. Introduce quarter 'till and quarter after the hour.	When this concept is learned a line can be drawn from the 9 to the 3 which will lead to the introduction of the quarter hour.	Ask parents to have children tell them what time it is when various activities are going on at home, i.e., supper time, bed time, etc.
11. Introduce different types of clocks: - clocks without minute lines - clocks without hour numerals	Bring and discuss different types of clocks and watches. Have children demonstrate their ability to use these clocks by accurately telling the correct time.	
12. Introduce concept of second.	Discuss differences between hour hand, minute hand, and second hand (also called sweephand). Time game activities in the classroom and discuss uses of the second as a unit of time.	



VOCABULARY

minute hand
hour hand
A.M.
P.M.
morning
afternoon
noon
midnight
o'clock
clockwise

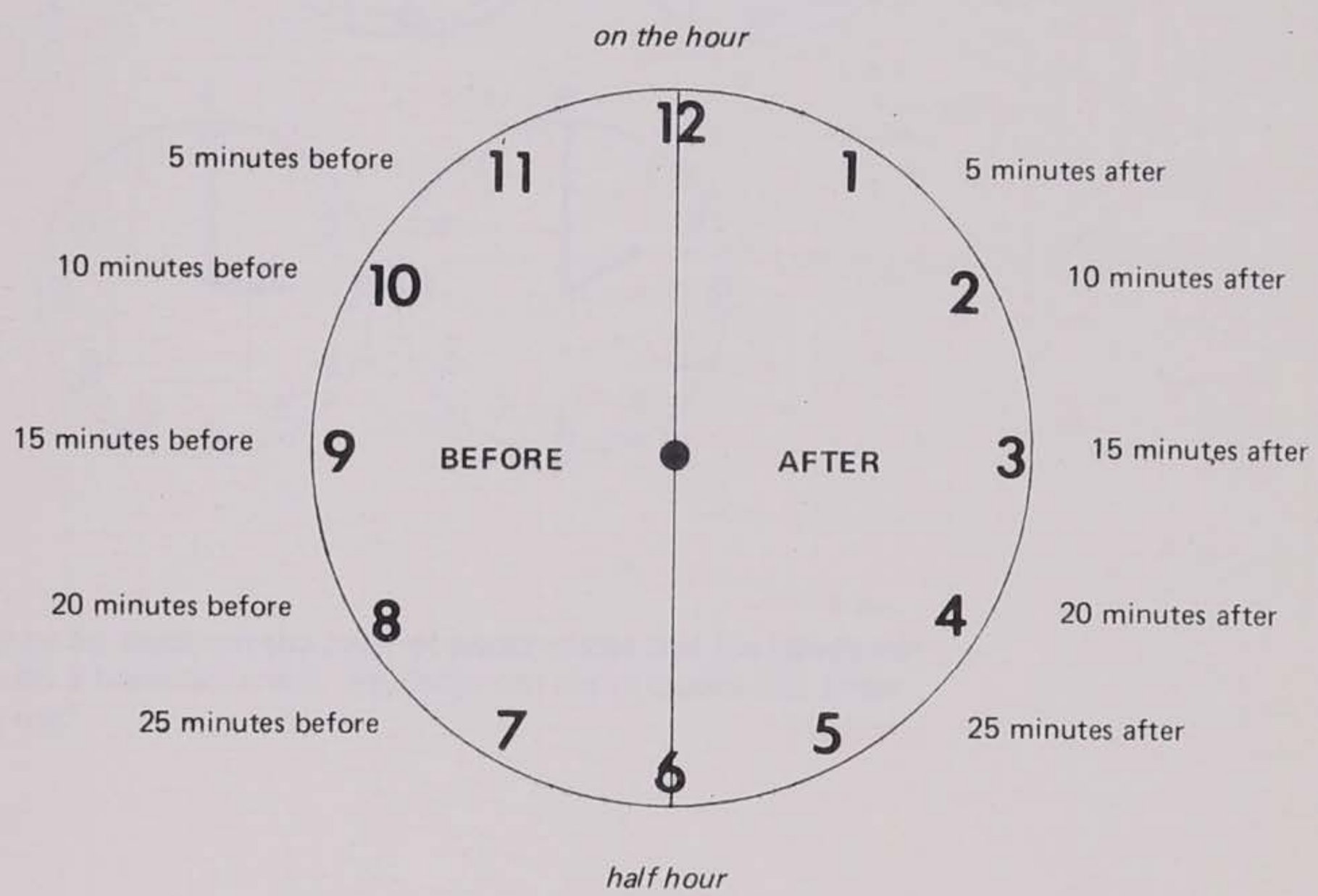
Little hand tells the hour.
Big hand tells the minutes.

1 day = 24 hours (little hand goes around the clock 2 times each day)

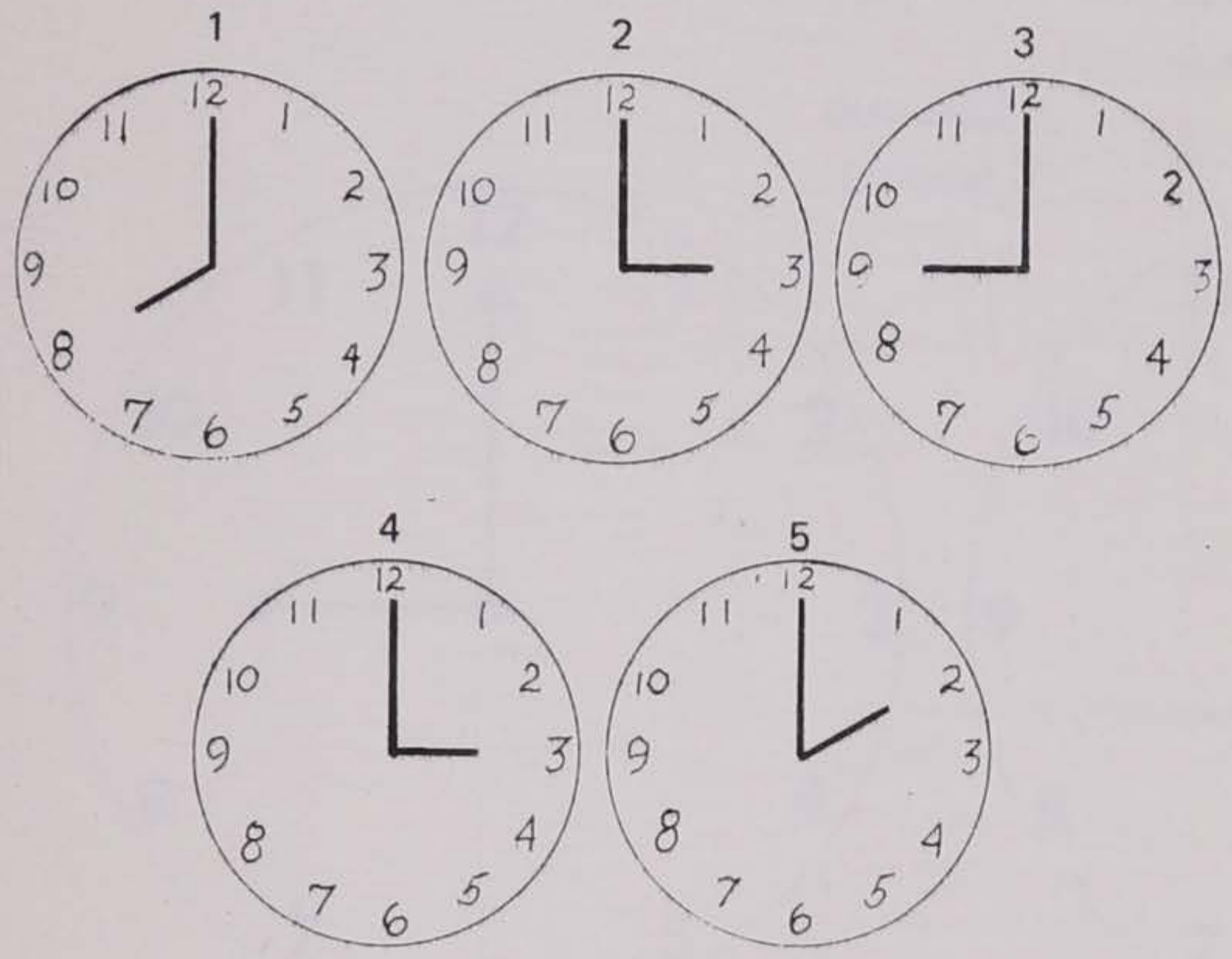
1 hour = 60 minutes (the big hand goes around the clock 24 times each day)

12 o'clock at night until 12 o'clock noon is called A.M.

12 o'clock noon until 12 o'clock midnight is called P.M.



You may wish to use practice sheets available from the Continental Press, Inc., or make your own which will specifically relate to the tasks and concepts you have introduced. Either way, remember that a great deal of practice is necessary.



Clock faces may be made on the back of paper plates and the hands can be attached with a brass fasterner. Students can make clocks like these for their own use.

Sample record sheet for learning center.

1. _____ o'clock
2. _____ o'clock
3. _____ o'clock
4. _____ o'clock
5. _____ o'clock

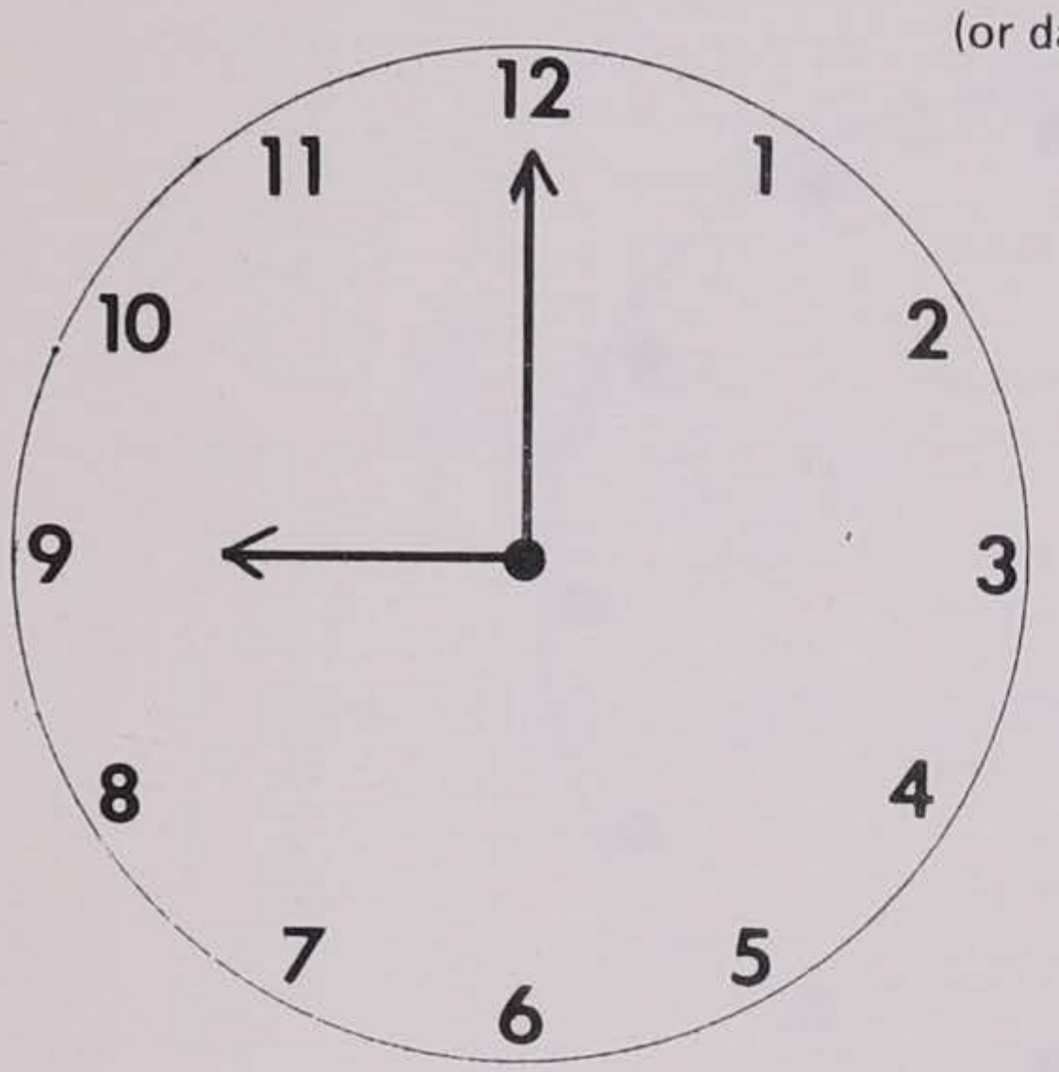
Name _____

Date _____

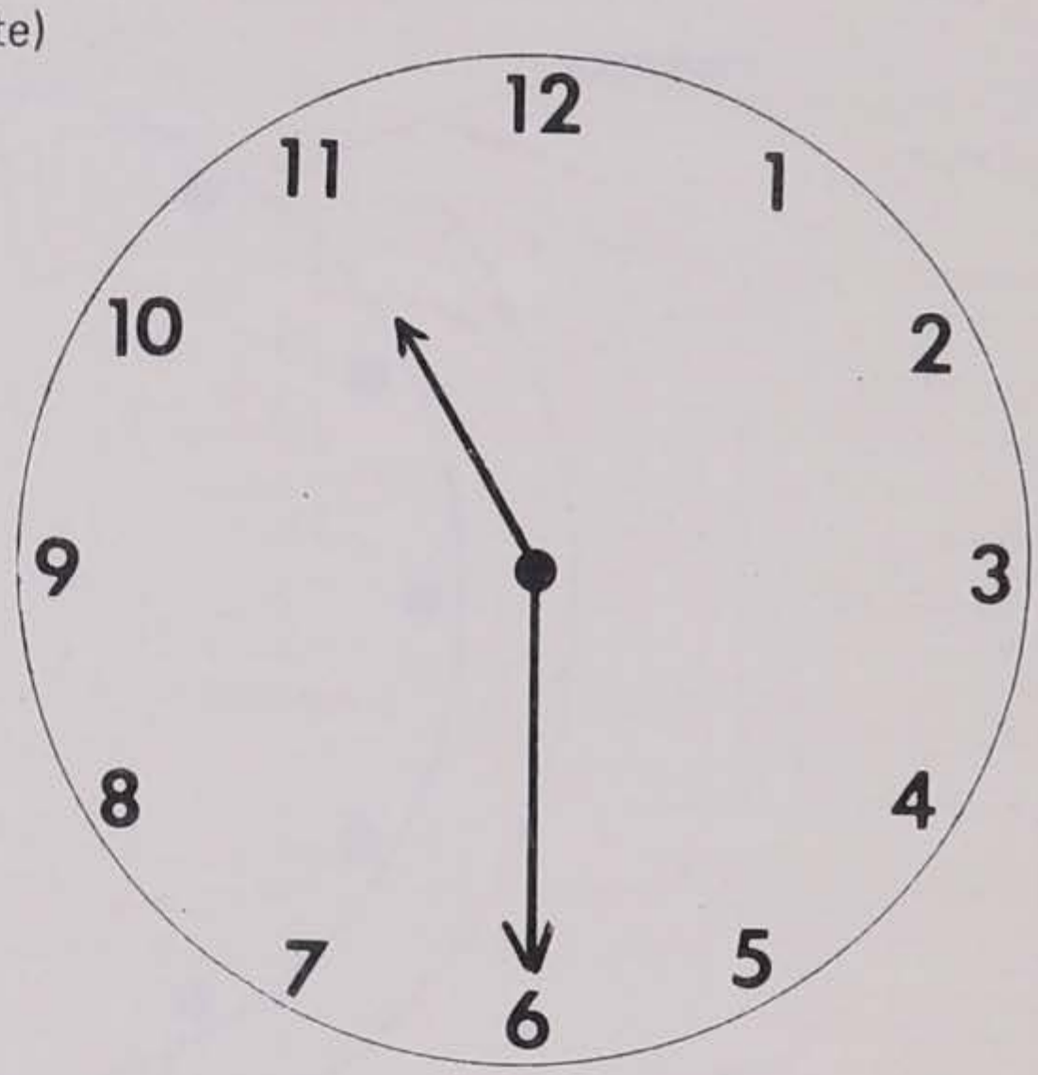
Activity No. 1

You can use this record sheet to keep track of how students are progressing.

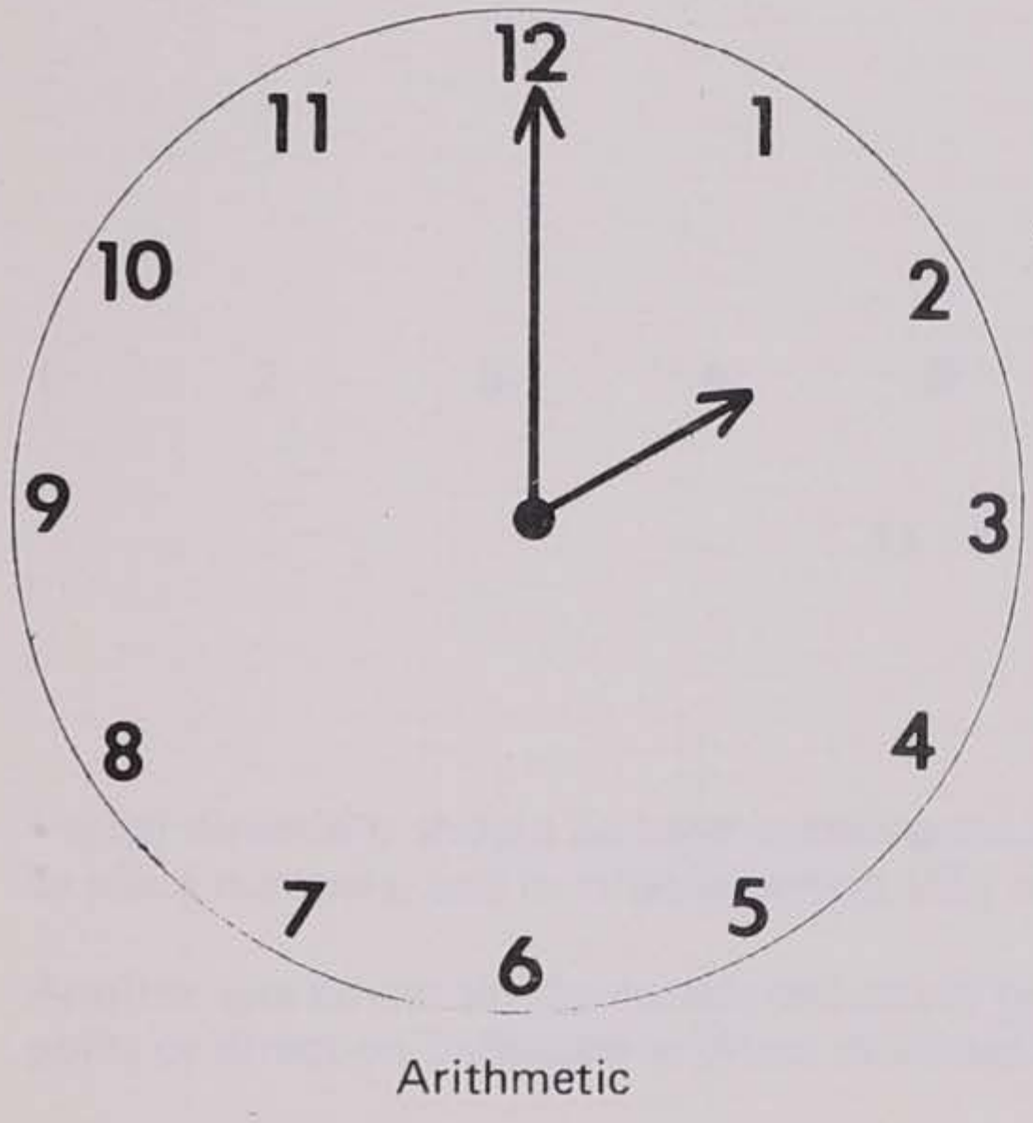
OUR DAY
(or date)



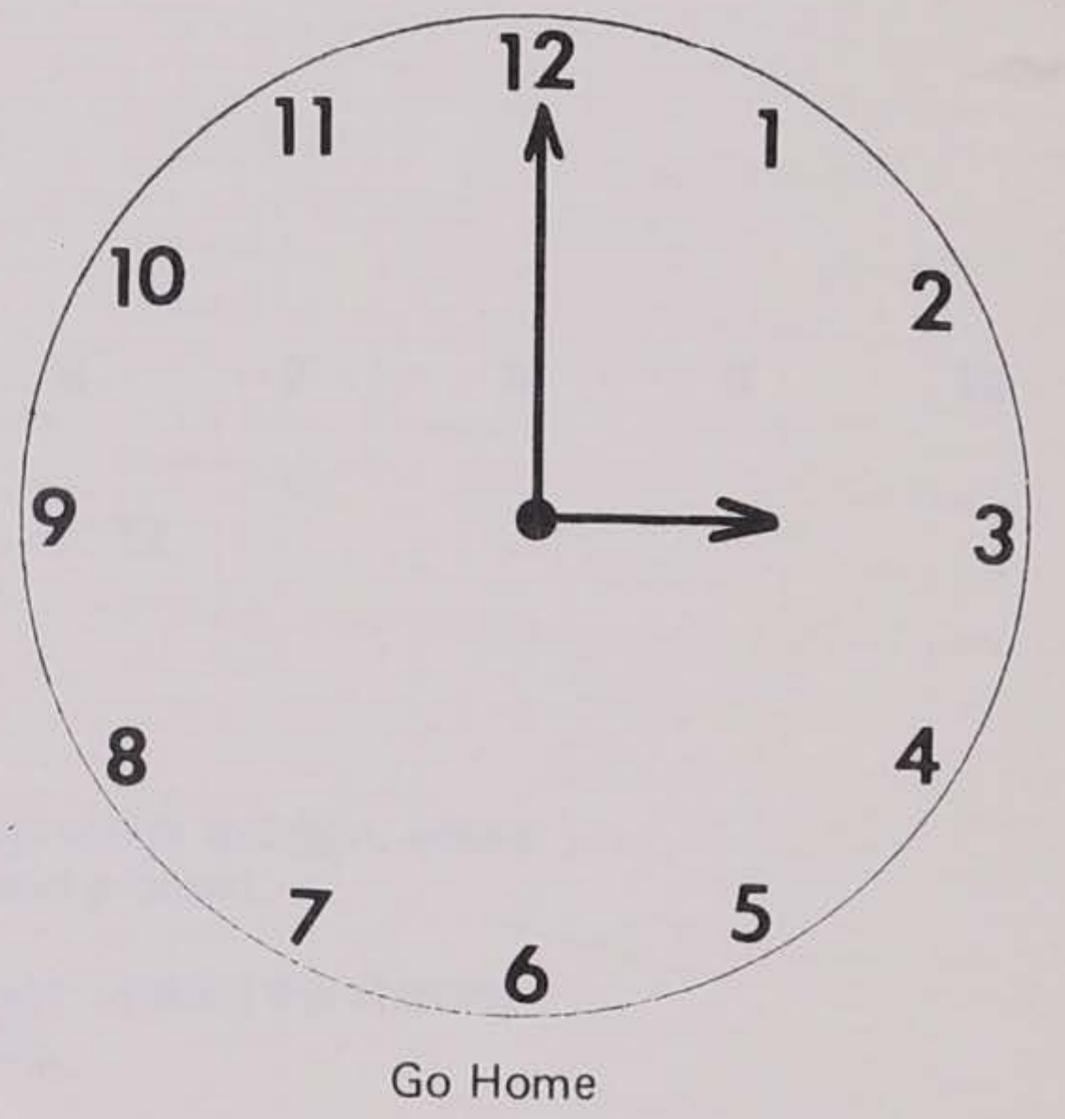
Reading



Lunch



Arithmetic

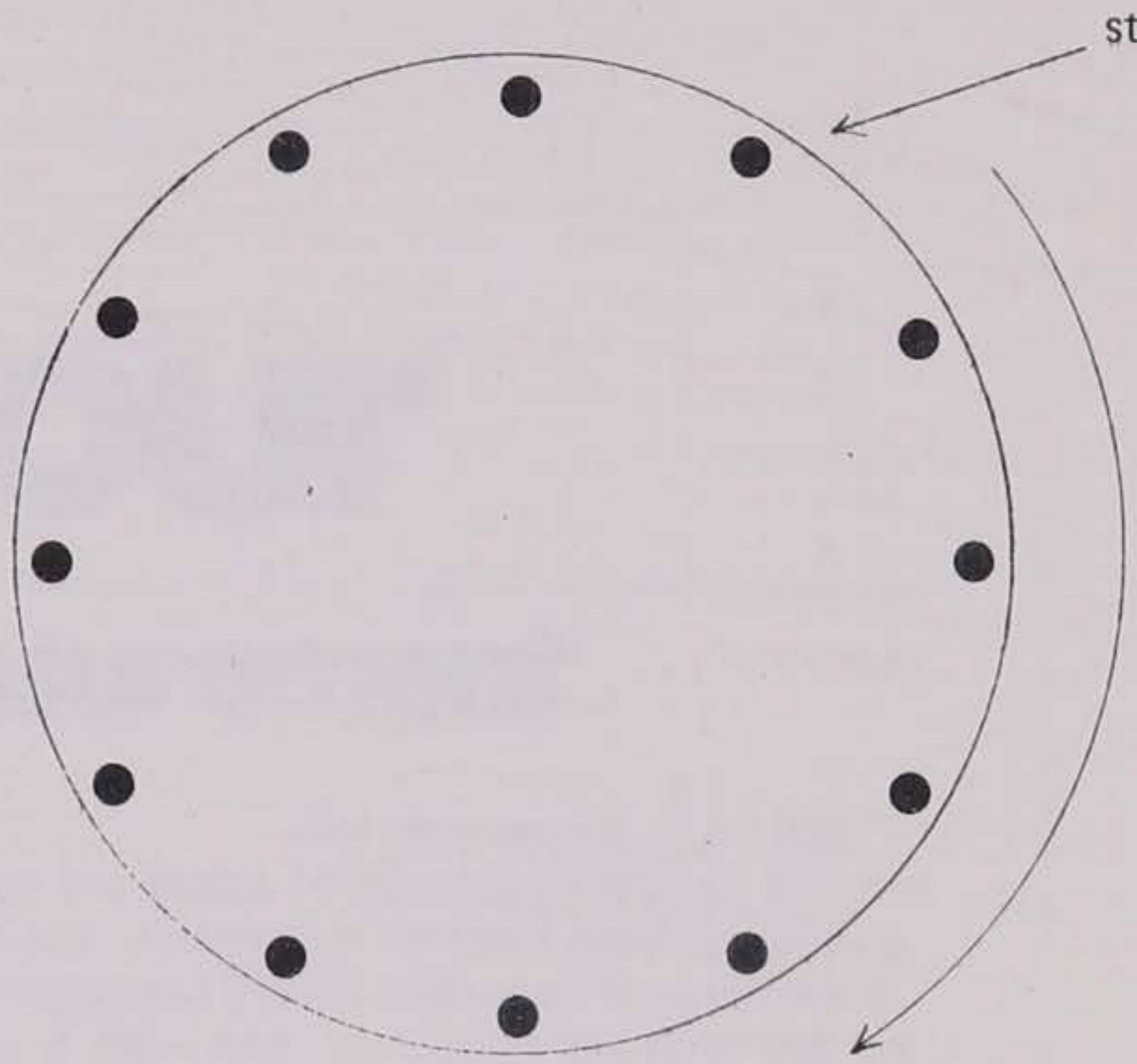


Go Home

1. Teacher should discuss and place hands on appropriate settings until students have developed the skills necessary for them to assume this responsibility.

2. Moveable hands.

3. Changed every morning in planning.



1 2 3 4 5 6 7 8 9 10

11 12

Verbal directions should be used in telling students where to begin, where to place numbers, and in what direction they should proceed.

Another worksheet similar to this one could be used without the starting point or direction indicated to check students' skills.

four

measurement

Our lives would all be less complex if this country were on the metric measurement system, but this is not to be, at least for some time. Hence, our children need to develop some skills working with the antiquated British system of measurement. Your work would be simplified if there were ten inches in a foot and ten ounces in a pound, but that too cannot be, so your job is difficult and the scope and sequence of experience in the teaching of measurement is very important.

Before we share with you a few ideas that may help as you attempt to teach measurement, we would like to share with you a bit of data that has been generated in our classes at the university. Most young adults forget a majority of the specific facts that are taught about measurement in the first eight or nine years of formal education. Apparently, much that is presented has little relevance in the lives of the students and is not retained very long after the examination covering the unit on measurement. The secret to success in learning measurement will be found when you don't teach it, but instead use it. Bake a cake, map out a city block and build it to a scale, determine the weight of the air in the room, or any one of a hundred different projects involving measurement.

Some of the first work with measurement can be that of collecting data on outside temperature and plotting it on a weather chart. You can do this at two or three different times during the day and permit different children to read the clock and the thermometer. Permitting children to place different color marks on a chart for different times of the day you will have created many different experiences in one activity. The children will need to know the day of the month, the temperature outside, the time of the day and the proper color for marking.

It is probably best to avoid spending too much time discussing the differences between liquid and dry measures since nearly all children forget this before they reach the age of seventeen, but children should be allowed to discover how many cups in a pint, how many pints in a quart and how many quarts in a gallon. This they can do in or around a school building

when the proper size containers are provided and a hose is connected to a water outlet. To keep the children from throwing water on one another you might have them water the trees and plants in the area around the school with all the water they place in the various containers after they have discovered how many cups of water are needed to fill a gallon container. You can also have the children learn that a pint of water weighs sixteen ounces or a pound by having them state: *A pint's a pound the world around.*

Children can learn that water boils at 212° F. and a cooking thermometer will stay right at 212° F. unless you use a pressure cooker. You can also freeze a thermometer in a block of ice and take it to school and observe the temperature as the ice melts.

To teach linear measurement the children can be permitted to cut strips of paper to various lengths and weave different size paper mats. The ten- to twelve-year-old child can be taught to take steps approximately equal to two feet in length and pace off city blocks or distances within a classroom or building and develop scale models. This helps the child do work with the ruler and gives the child a real sense for distance.

If you want to build buildings to scale you can use tag board or lightweight cardboard. Children might like developing log cabins and this can be done by gluing pretzels (the straight variety) with white glue or paste and building one wall at a time. When you get four walls glued together put on a tag board roof, smear it with glue and sprinkle on coffee grounds and you will have a log cabin. Develop a little frontier village to scale and your children will learn a great deal about cooperation and the use of linear measurement.

If a child never learns that there are 5,280 feet in a mile he can still survive and do well in the larger society. However, we should develop some feeling for the meaning of a mile, and a walk four times around a city block is a rough approximation of that. Let the children walk the distance and estimate the number of miles one could walk in one hour, and you will add to their health and knowledge and the experience is real. When math experts speak of the meaning theory of mathematics or of discovery as a method to employ, they do not mean to infer that all math has real meaning or that children are likely to rediscover all that is known. With slow learning children, often the best that can be hoped for is that many of their math experiences will have an emotional impact and be retained because of the joy of accomplishment that comes into the lives of the children. It is always better when we can do something with those concepts they are learning. With measurement, that is not only possible, it is probably necessary.

CONSIDERATIONS

In the preceding narrative, it was mentioned that most facts about measurements are forgotten once the unit is completed. This was in no way meant to deemphasize the importance of measurement concepts. It does, however, emphasize the importance of practical application of these skills when they are being taught.

Keep in mind that students in the regular classroom learn these skills over a period of several years. Do not be as concerned about how many concepts are presented as how well the students are able to appropriately use those they are able to perceive.

PREREQUISITE SKILLS

If students are capable of appropriately applying the following skills and concepts, it will be easier for them to comprehend concepts dealing with measurements.

1) Use of the number line.
2) More and less; smaller and larger; long and short; as well as heavier and lighter.

Suggested Learning Sequence	Suggested Activities	Suggested Reinforcements
Linear Measure 1. Introduce 12-inch ruler and identify inch marks.	Incorporate terms length, width, depth and height throughout unit.	Give students information on Sample Data Sheet.
2. Measure inches.	Discuss uses of measuring devices and reasons for measuring throughout unit.	
3. Express inches in written form (inches, ", in.).	For all linear measures, develop exercises that will permit students to practice using a ruler, yard stick and tape measure. For example: Have students measure objects familiar to them, such as books, desks, themselves, classroom, etc.	
4. Introduce concept; 12 inches equals one foot.		
5. Measure feet with 12 inch ruler and express in written form.		Develop an art project that requires the use of various lengths of yarns or string to make mobiles or geometric designs.
6. Measure feet and inches and express in written form.	Develop exercises that incorporate the concepts of width, depth, more than and less than.	
7. Conversion of: inches to feet feet to inches	Have students estimate the length and width of objects and then measure to determine how accurate their estimates were.	Set up a learning center with objects of various sizes. Have students measure and compare the size of these objects with other objects in the room.
8. Introduce concept: 36 inches equals a yard and 3 feet equals a yard; and introduce a yard stick.		
9. Measure yards and express in written form.	Initial objects to me measured should be exactly 1 yard, 2 yards, etc.	
10. Measure into feet marks on yard stick. Measure yards and feet and express in written form.		

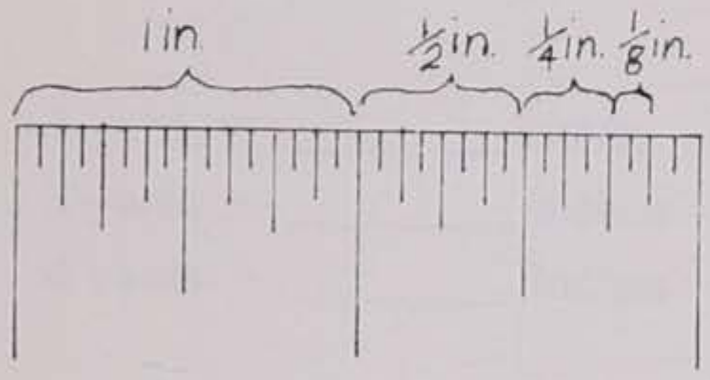
Suggested Learning Sequence	Suggested Activities	Suggested Reinforcements
11. Measure yards, feet and inches and express in written form.	Divide class into groups and have them measure and record specified areas with yard stick and tape measure. Then have them compare measurements. If great discrepencies arise, find out why.	
12. Conversion of: - inches to yards - feet to yards - yards to inches - yards to feet - feet to yards and feet - inches to yards & feet - inches to yard & feet, and inches	Sample Worksheet No. 1 will give you an idea of some exercises that may be used. Several variations can be made from this worksheet. The entire class should be exposed to this type of work together.	Construct a map of the classroom to scale 1" = 3' or 1" - 2 yards.
13. Introduce units of measure less than an inch (½, ¼, 1/8), and these markings on a 12-inch ruler and yard stick.		
14. Measure less than one inch. (Sequence the learning)		
15. Introduce concept of block.	Have students measure their stride. Take them for a walk and have them estimate how many feet there are in a block, how many blocks in a mile.	
16. Introduce concept of mile.		Tell students to watch the odometer on the family car and record how many miles they travel between two points.
Weight Measure 1. Introduce pound scale, markings and concept of pound.	Begin weighing objects on a scale that shows only the pound markings.	

Suggested Learning Sequence	Suggested Activities	Suggested Reinforcements
Weight Measure, continued		
2. Measure pounds with a scale.	Provide many experiences using the scale before you have them attempt to estimate weights.	
3. Expressing pounds in written forms (pounds and lb.)		
4. Introduce concept that 16 oz. equals a pound.	You can introduce a scale that shows oz. as well as pounds.	
5. Introduce oz. scale, marking an oz. scale and concept of oz.		
6. Measure ounces with a scale.		
7. Express ounces in written forms (ounce and oz.).	Have students weigh several items and record weight in pounds and ounces.	
8. Weigh pounds and ounces and express in written form.	Have a number of containers with labels that indicate various weights, i.e., soup can, etc. Have students group containers according to weights, more than a pound, less than a pound.	See Sample Learning Center Activity. This activity may be used when students have had a great deal of practice weighing objects on each type of scale. Ask parents to take students shopping and have them weigh fruits, vegetables or candy. They could incorporate the cost in relation to weight.
9. Conversion of: - lb. to oz. - oz. to lb. - oz. to lb. and oz.	For an example of appropriate practice, see Sample Worksheet No. 1.	
Liquid and Dry Measure		
1. Introduce the following containers: container cup, pint, quart, gallon as a tool to measure.	Attach appropriate labels to each container.	

Suggested Learning Sequence	Suggested Activities	Suggested Reinforcements
Liquid and Dry Measure, continued		
2. Relationships of standard cup, pint, quart and gallon.	Experiment with relationships by giving students a worksheet which required them to actually determine how many cups in a pint. See Sample Worksheet No. 2. Introduce a half-gallon container in the experiment, i.e., milk carton.	Use experience chart for feedback and review.
3. Introduce written expressions, i.e., cup and C.	If you feel that any of the activities we have suggested are forcing you to bark up the wrong tree, then find your own tree.	
4. Measure liquid quantities.		
5. Introduce graduated markings on measuring cup and written expression of markings. (Also individual measuring cups as 1/2, 1/3, 1/4)	This concept may be better understood if you would introduce a set of kitchen measuring cups. Have students determine how many 1/4 cups it takes to fill a graduated measuring cup, etc.	
6. Introduce teaspoon, tablespoon, 1/2 teaspoon, 1/4 teaspoon and 1/8 teaspoon, their relationships and written expressions.	Develop an activity that incorporates the use of dry and liquid measures, such as cooking. This will offer you the opportunity to evaluate the students' ability to recognize the written expressions of measurement in recipes. You can also make your own modeling clay which would require measuring.	Ask parents to allow students to assist in activities that require the use of liquid and dry measures--give examples to parents.
Temperature Measure		
1. Types of thermometer.	Sequence this topic using preceding outline.	
2. Use of thermometer.		
3. Have students read indoor, outdoor and oral thermometer.	Have students read indoor, outdoor and oral thermometers.	

Linear Measures

12 inches (in.) = 1 foot (ft.)
36 inches (in.) = 1 yard (yd.)
3 feet (ft.) = 1 mile (mi.)
5,280 feet (ft.) = 1 mile (mi.)
1,760 yards (yd.) = 1 mile (mi.)



Weight Measures

16 ounces (oz.) = 1 pound (lb.)
2,000 pounds (lb.) = 1 ton (T.)

Liquid and Dry Measures

2 measuring cups (C.) = 1 pint (pt.)
2 pints (pt.) = 1 quart (qt.)
4 quarts (qt.) = 1 gallon (gal.)
3 teaspoons (tsp.) = 1 tablespoon (tbsp.)
4 tablespoons (tbsp.) = 1/4 cup (C.)

(a) 36 inches = _____ yards
72 inches = _____ yards
108 inches = _____ yards

(b) 3 feet = _____ yards
6 feet = _____ yards
12 feet = _____ yards

(c) 1 yard = _____ inches
3 yards = _____ inches
4 yards = _____ inches

(d) 1 yard = _____ feet
3 yards = _____ feet
5 yards = _____ feet

(e) 4 feet = _____ yards and _____ feet
7 feet = _____ yards and _____ feet
10 feet = _____ yards and _____ feet

(f) 48 inches = _____ yards and _____ feet
60 inches = _____ yards and _____ feet

(g) 53 inches = _____ yards, _____ feet and _____ inches
76 inches = _____ yards, _____ feet and _____ inches

Item	Type of Scale
1. rock	pd. scale
2. your weight	nurse's scale
3. your weight	bathroom scale
4. eraser	ounce scale
5. book	pd. & oz. scale
6. your pencil	ounce scale
7. your teacher	bathroom scale

Student Record Sheet

1. rock	_____ lb.	
2. your weight	_____ lb.	_____ oz.
3. your weight	_____ lb.	
4. eraser		_____ oz.
5. book	_____ lb.	_____ oz.
6. your pencil		_____ oz.
7. your teacher	_____ lb.	

Set up stations with objects and appropriate scale.

Instruct students to weigh objects indicated and record weights on their record sheet as indicated above.

_____ cups = 1 pint

_____ cups = 1 quart

_____ pints = 1 quart

_____ cups = 1 gallon

_____ pints = 1 gallon

_____ quarts = 1 gallon

Use actual containers and let students fill in the blanks by measuring water.
Give directions verbally and demonstrate to students.

five

money

Money may not be the root of all evil, but for children who never possess any, and never have an opportunity to spend any, it must be a cause of much distress when they must learn how to do something they have not done and are not likely to do for some time to come. Learning about money is often unreal.

Play money and playing store may be fine for the average child but by the time a slow learning child is ready to handle money and make change, play money and playing store may be the type of activity that will turn the child off. Most children get their most meaningful experiences with money through home and store activities. It would appear that children will probably learn best about money if the real item is used and the child is allowed to make change of all types while working in small groups with close supervision.

The school could make a real contribution to the child's comprehension of money if they would permit children to purchase certain small items for some activities in the classroom. The key to understanding money, as is true with time and measurement, is that the experiences must be as real as possible. It is impossible to do math without symbols; it is unnecessary to do many math-related activities with symbols. To attempt to teach new concepts before the so-called teachable moment leads to frustration and avoidance behavior. Your task as we view it is one of constant discovery about what each child is ready to do and then encouraging each child to expand his perceptual world by doing, and keep in mind that each child is ready to learn something by participation in his own learning.

CONSIDERATIONS

The previous narrative suggests that real money should always be used in presenting, practicing and reinforcing the concepts to be learned. This procedure is the ideal and may not be feasible in your situation. If not, at least obtain the type of play money that most closely resembles real cash. Be sure, however to provide as many experiences as possible with real money.

If you are like most of us, you have not *developed a real understanding of the value of money,* so do not be discouraged if your students find it

difficult to comprehend this nebulous concept. Be specific about what you want them to learn and practice. Proceed through the learning sequence at a rate commensurate with the student's comprehension and mastery of skills.

For most of us, it takes a great deal of practice in order to become accurate in manipulating and exchanging money. Please provide your students with as many realistic experiences as possible.

PREREQUISITE SKILL

If the students have mastered the following skill, they will probably find it easier to learn about money:

1. Counting by 1's, 5's, 10's, to 100, with or without a number line.

2. A functional use of the concept: more than and less than.

Suggested Learning Sequence	Suggested Activities	Suggested Reinforcements
1. What is money?	A discussion of what students know about money. Exhibit real money (coins & bills) and let students examine to determine similarities and differences. (Note: *The purpose of this exercise is to determine how much students know, not how much you can tell them.)*	Learning center exhibit of all coins. Cut holes in tagboard to accommodate coins--label all coins, then cover both sides with clear contact paper. Students can flip card over to see either side of coins. We suggest that you have several of these cards available at the learning center.
2. *Coins* a. Penny - recognition: size, shape, color, markings - value and purchasing power - counting - express in written form: penny, cent, - reading the written expression	Give each student a penny. Have them examine the penny and discuss as a group, identifying characteristics peculiar to this coin. Have the students discuss what they could purchase with a penny. Provide students with different quantities of pennies to count and record.	The Continental Press, Inc. has a series of worksheets on money that would be appropriate for use throughout the learning sequence.
b. Nickel - recognition: size, shape, color, markings - value and purchasing power - counting - express in written form: nickel, 5 cents, 5¢ - reading the written expressions - introduce concept that combinations of coins with less value are equal to coins of greater value. (5 pennies = 1 nickel) - counting nickels and pennies	Students will probably have more success in learning about money if you use a constant approach in the introduction and practice of each coin. If you use the suggested activity above for the penny, use the approach throughout. Provide items that are values at five cents. Have the students determine how these items could be purchased if only pennies were available. Place different combinations of nickels and pennies in small boxes. Have students count coins and determine the amount in the box. Box A - 1 nickel and 3 pennies; Box B - 3 nickels and 2 pennies, etc.	Give students information similar to that on Sample Data Sheet No. 1. Develop a learning center using cards with amounts of money expressed in different ways and have students put coins on cards that equal the amount shown. See Sample Learning Center Activity.

Suggested Learning Sequence	Suggested Activities	Suggested Reinforcements
Coins, continued		
c. Dime	Develop a similar learning sequence for dime, quarter and half dollar.	
d. Quarter		
e. Half dollar	Use various items worth different amounts of money. Have students purchase items by combining coins of different value to equal the cost of the item.	Give each student the same amount of money, but a variety of coins to equal that amount. Have students trade coins, for example, one student may give another a dime and he then must be sure that the other student gives him coins that equal ten cents, i.e., 1 nickel & 5 pennies. At the end of the exercise, the student must have the amount equal to that with which they started the activity. Plan a field trip to nearby store and have students purchase small items.
3. *Dollars*		
a. Relationship to coins	Develop activities that will help students visualize relationships (4 quarters = 1 dollar; 10 dimes = 1 dollar; etc.).	Give student information similar to that on Sample Worksheet. A wall chart with this information on it would also be appropriate.
b. Recognition of bills: 1; 5; 10; etc.	Show various bills on opaque projector and have students identify similarities and differences. Bills with denominations of 2 and greater than 10 could be seen on a field trip to bank.	
c. Counting and equal value concept (i.e., 5 ones = 1 five).	Provide practice of these items using real money and worksheet.	
d. Express in written form: 1 dollar; $1.00; etc.	Demonstrate how to count dollars and cents and how to express it in written form. Provide a great deal of opportunity for practice.	

Suggested Learning Sequence	Suggested Activities	Suggested Reinforcements
Dollars, continued e. Reading written expression f. Purchasing power		
4. *Dollars and Cents* a. Counting b. Express in written form c. Read written expression of dollars and cents.	Demonstrate how to read the written expressions. Provide money numerals to read; i.e., use flash cards of $12.00, $1.98, $10.27, etc.	Have the students discuss the many ways that they spend money for fun and necessities. Counting and written expression could easily be incorporated. Supply students with a variety of price markings as actual price tags, catalogs, grocery and sundry items, ads, service bills, receipts, etc.
5. *Purchasing* a. Reading price marks.	See reinforcement activity above.	
b. Selective purchasing	Ask a home economist or extension worker to discuss selective purchasing with the class.	Ask parents to take students on shopping trips. Tell them to discuss why items were purchased, i.e., comparative values, etc. Field trip to supermarket and have manager emphasize cost difference of brands of similar producers.
c. Arithmetic skill using money symbols. - Addition - Subtraction	Demonstrate use of symbols in adding and evaluating money problems. Refer to examples on Sample Worksheet.	
d. Sales tax	Sales tax charts can be obtained from local merchants or banks. Explain sales tax and practice using chart.	
e. Making change	Begin making change using coins, progress to dollars. Practice - - - Practice - - - Practice - - -	

From this point you can introduce learning activities centered around budgeting, banking, paychecks, taxes, etc.

Penny		One cent 1 cent -- 1 -- $ .01
Nickel		Five cents 5 cents -- 5 -- $ .05
Dime		Ten cents 10 cents -- 10 -- $ .10
Quarter		Twenty-five cents 25 cents -- 25 -- $ .25
Half dollar		Fifty cents 50 cents -- 50 -- $ .50

The Continental Press, Inc. has a variety of ditto masters that may be used to help develop these concepts and provide meaningful practice in recognizing the names, markings, sizes and ways of expressing the values of coins.

100 pennies = 1 dollar

20 nickels = 1 dollar

10 dimes = 1 dollar

4 quarters = 1 dollar

2 half dollars = 1 dollar

Illustrated data sheet may be helpful to some students.

Instruct the students in a variety of the ways that each value may be expressed by different combinations of coins.

35¢

twenty-two cents

$7.16

55¢

forty-three cents

$5.09

seventeen cents

80 cents

Make cards out of heavy tagboard. Be sure they are large enough to accommodate all the combinations of coins possible.

Addition

Type A: Problems without remembering (carrying).

$ .13 + .54	$5.73 +.22	$14.06 +3.91	$ .31 .15 + .52

Type B: Problems with remembering in ones column.

$ .73 + .17	$1.29 +.53	$16.58 +1.27	$12.25 1.45 + .21

Type C: Problems with remembering only in tens column.

Type D: Problems with remembering only in hundreds column.

Type E: Problems with remembering in ones and tens columns.

Type F: Mixed types of problems.

Subtraction

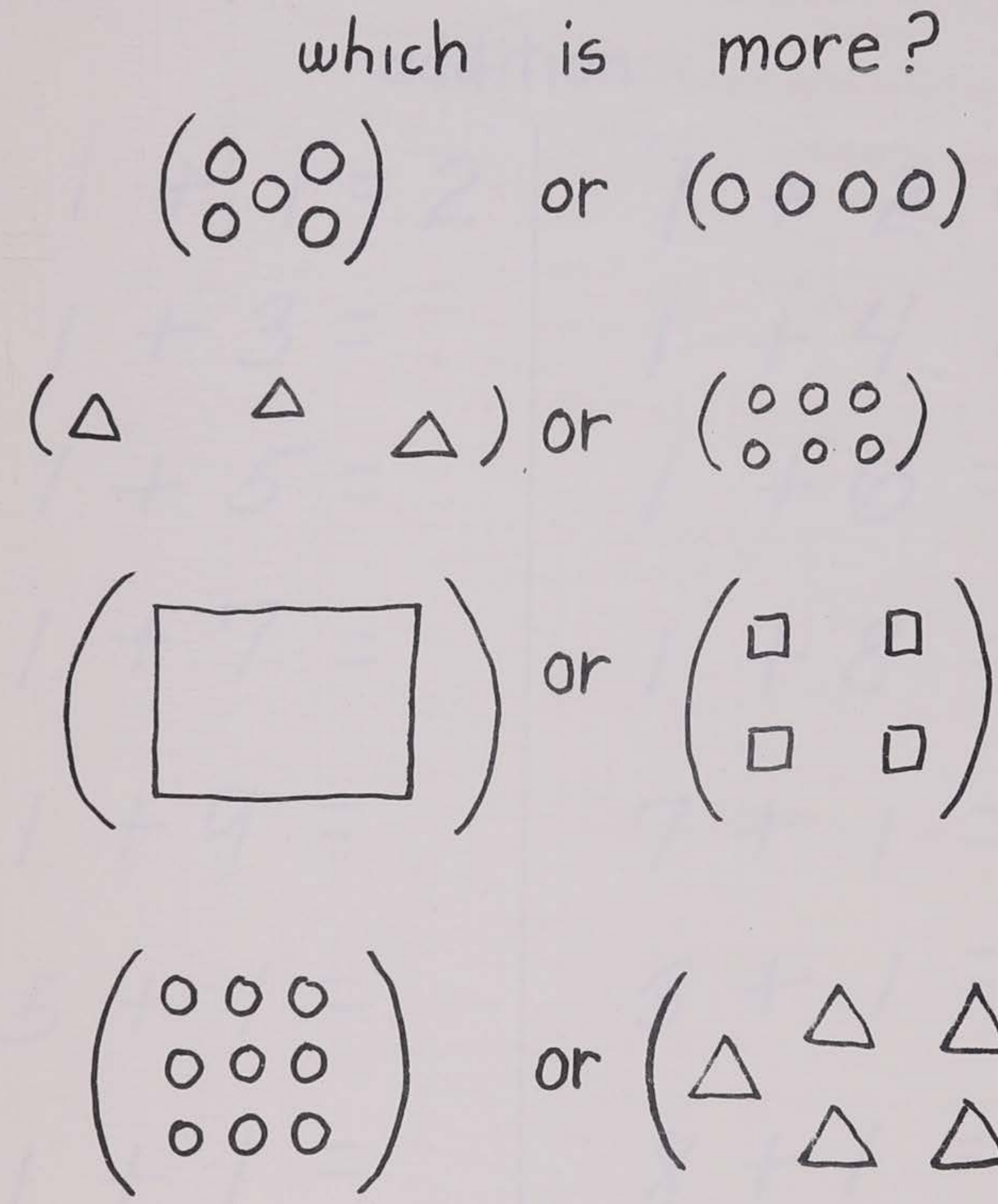
Sequence subtraction problems.

summary

What is presented in this SECDC publication is not a curriculum guide or a course of study. It is not related to any scope and sequence chart that we are aware of. It was our intent to give readers ideas that might have some practical use in some classrooms for a limited time during the academic year. We are cognizant that some children may be involved in certain types of activities discussed in this publication at age ten and others will not be able to replicate the activity and learning prior to age fifteen. The classroom teacher working with the slow learning child should remain sensitive to ability as well as needs. We need to understand that children want to improve their ability to cope with life's problems and when avoidance behavior is used, the proper sequencing of experience has been overlooked resulting in fragmented skill development. We also believe that when a child has failed to learn a specific skill, when the teacher has used a rather specific method, it is wrong to remind the child of his inadequacy by going back over the same ground a second or third time. We must look for new ways of teaching or reteaching concepts when the child has resorted to avoidance behavior.

We wish you luck; we know your task is difficult.

appendix A
worksheets



Directions: Ask the child which is more. Do not show approval or disapproval; if space seems to mean more than number go back and ask the child to count the numbers in each set and tell you again which is more. If after counting the child still indicates space is more important than number he will not be ready to work with number combinations. Remember, no matter how the child responds he is correct. If you retest at a later time it is probably best to use solid objects arranged in different patterns with clearly discernable space differences. If the child is unable to count, you might encourage and arrange for him to view Sesame Street.

addition

1 + 1 = 2	1 + 2 =
1 + 3 =	1 + 4 =
1 + 5 =	1 + 6 =
1 + 7 =	1 + 8 =
1 + 9 =	7 + 1 =
5 + 1 =	3 + 1 =
1 + 1 =	8 + 1 =
6 + 1 =	4 + 1 =
2 + 1 =	9 + 1 =

addition
even addends

2+4=	2+6 = 8
2+2 =	4+2 =
6+4 =	6+6 =
8+2 = 10	8 +4 =
6 + 8 =	2 + 8 =
6 + 2 =	4 + 6 =
0 + 6 =	8 + 6 =

Addition
odd addends

1 + 5 =	3 + 7 =
5 + 9 =	9 + 7 =
7 + 5 =	5 + 3 =
3 + 1 =	5 + 1 =
7 + 3 =	9 + 5 =
7 + 9 =	1 + 3 =
3 + 5 =	5 + 7 =

Addition

1 + 9 =	2 + 1 =
2 + 8 =	4 + 3 =
3 + 7 =	6 + 5 =
4 + 6 =	8 + 7 =
5 + 5 = 10	1 + 9 = 10
6 + 4 =	3 + 2 =
7 + 3 =	5 + 4 =
8 + 2 =	7 + 6 =
9 + 1 =	9 + 8 =

Addition of Like Numerals

A

(2+2)+2 =

(2+2)+(2+2) =

3+(3+3) =

(4+4)+(4+4) =

4+(4+4)+(4+4) =

(4+4)+4 =

5+(5+5) =

(5+5)+(5+5) =

B

$$\begin{array}{r} 2 \\ 2 \\ +\,2 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ 2 \\ 2 \\ +\,2 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ 3 \\ +\,3 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ 4 \\ 4 \\ 4 \\ +\,4 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ 4 \\ 4 \\ +\,4 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ 4 \\ 4 \\ 4 \\ +\,4 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ 5 \\ 5 \\ +\,5 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ 5 \\ 5 \\ 5 \\ +\,5 \\ \hline \end{array}$$

Addition
ones, pennies, tens, and dimes

1	2	1 dime	2 dimes
+3	+5	+3 dimes	+5 dimes
5	4	5 dimes	4 dimes
+7	+6	+7 dimes	+6 dimes
3 ones	3 tens	6 ones	6 tens
+8 ones	+8 tens	+5 ones	+5 tens

---

3 tens	3 dimes	30 pennies
+4 tens	+4 dimes	+40 pennies
2 tens	2 dimes	20 pennies
+5 tens	+5 dimes	+50 pennies

Operations
Put in the signs

3 − 2 = 5

4 ? 3 = 12

5 ? 2 = 10

6 ? 1 = 7

2 ? 2 = 4

1 ? 5 = 5

2 ? 4 = 6

8 ? 1 = 8

3 − 2 = 6

4 ? 3 = 7

5 ? 2 = 7

6 ? 1 = 6

2 ? 2 = 4

1 ? 5 = 6

4 ? 2 = 8

? 8 = 0

multiplication

0 x 1 = 0	1 x 1 = 1
1 x 2 =	2 x 1 =
1 x 3 =	3 x 1 =
1 x 4 =	4 x 1 =
1 x 5 =	5 x 1 =
1 x 6 =	6 x 1 =
1 x 7 =	7 x 1 =
1 x 8 =	8 x 1 =
1 x 9 =	9 x 1 =

appendix B
resource materials

Company and Item	Description	Number	Price
Academic Therapy Publications 1539 Fourth Street San Rafael, California 94901			
A Number Game	Simple arithmetic solves problems posed by geometric dice. Increased math facility.	Reckon	$ 3.85
Book-Lab, Inc. 1449 - 37th Street Brooklyn, New York 11218			
Math Projects: Map-making	Experiments and projects include reading landmarks, using the compass, and mapping the outdoors. For grades 4 - 6 or for retarded readers in grades 6 - 10. 1968 copyright. 48 pp.	Order by name	$ 1.50
Cuisenaire Company of America, Inc. 12 Church Street New Rochelle, New York 10805			
Cuisenaire Cubes, Squares, and Rods	More than 350 wooden solids in 28 different sizes have been combined into sets with instructional literature to help children investigate concepts of algebra, arithmetic and geometry. A natural extension of the world-famous Cuisenaire rods, the Cubes and Squares, exactly match the color of the rods and offer the aid of an additional model for exploring mathematics. In addition to matching the Cuisenaire colors, the Cubes and Squares offer the same finish and accuracy for children to exactly model additional relationships. They have no unit markings on their faces to limit their use. Instead, a transparent plastic centimeter grid sheet is included with each set for measuring the faces of each of the different solids	CX-11	$39.95
Developmental Learning Materials 3505 N. Ashland Avenue Chicago, Illinois 60657			
Add or Subtract Box	The two troughs of this box serve as set holders. In addition, the first addend (set) is formed in the upper trough and the second addend in the lower; the second addend is then joined to the first in the upper	W 167	$15.50

Company and Item	Description	Number	Price
Developmental Learning Materials, cont			
Add or Subtract Box, continued	trough, and the answer is read from the upper trough. In subtraction, the sum is formed in the upper trough, the given addend is then formed in the lower trough by using cubes from the upper trough, and the answer is read from the upper trough. The mutually inverse relationship of addition and subtraction is readily demonstrated.		
Counting Picture Cards	Within the counting picture card series, there are five sets, and each has from one to six colorful items. The groupings are: ice cream cones, pennies, cars, pears, and clocks. The cards measure 4 1/2" x 2 3/4" each.	P 126	$ 1.00
Deci-cued Colored Cubes	For the teacher frustrated by the restrictions of a 6-color assortment of cubes upon arithmetic presentations in a base 10 system, these one-inch cubes of finished wood in colors: red, yellow, orange, green, blue, purple, pink, brown, black, and white. The colors are non-toxic and the edges have been rounded for safe handling. The use of 10 different colors is especially desirable in developing the number line concept with the Add or Subtract Box: in introducing the concept of sets to be joined in the Join or Separate Box and in dramatizing the presentation of multiplication as successive addition in the Multiply or Divide Box, especially in conjunction with the Measure Bars.	W 171	
Inch Cubes, Primary Colors	This box of 96 cubes, 16 of each of the six colors: red, yellow, orange, green, blue and purple, has a total of 160 cubes.	W 110	$ 4.50
Join or Separate Set Box	The box provides for 10 physically defined set holders, each capable of holding 10 one-inch cubes. Alternately through use of a movable stop, two such holders may be converted to a single 20-cube set holder. It is used in introducing multiplication as successive addition of equal addends and division as successive subtraction of the same number. Division is not abstract but very concrete if posed as *Put 6 cubes in the long holder* (20-cube set holder). *Take 2 of these cubes and put them in a small holder. Now another 2. Now another 2. They're all gone. How many holders did you use?*	W 168	$20.50

Company and Item	Description	Number	Price
Developmental Learning Materials, cont.			
Multiply or Divide Box	This box is intended for the student who has already learned multiplication as successive addition of equal addends and division as successive subtraction of the same number. In this box, sets are formed in vertical columns or in horizontal rows, rather than in physically outlined set holders. Through this box, the mutually inverse relationship of multiplication and division is readily shown. Included with the box are four 10" x 10" cardboard inserts representing bars of 1" width and of length 1 to 10 inches, for use with 1" cubes or the measure bars. Also, there are two removable clue strips, ruled in one inch squares and numbered consecutively 1 through 10 for indicating the numerical value of the 2 factors.	W 169	$17.00
Numbero Cubes Set I	This set is composed of 10 different dice specifically manufactured to present both the figure and configuration in developmental sequence. One half of the set begins with the values of 1 and 2 dots and progresses with each additional die to 3, 4, 5 and 6 dots. The other half of the set has comparable value progressions, but in numbers. Beginning with the simple dots and progressing to the more advanced numbers, the student throws each die separately at first, and upon recognition of the dots and then the number value, progresses to the next higher die. This affords a connection of value to the number. The dice are 1" square, easy-to-handle, of top quality material and come in their own durable box.	M 120	$ 9.00
Numer-Cubes Set II	This set includes 4 dice, designed as an appealing and fundamental approach to the development and reinforcement of number meaning and concept. Two of the dice contain dots, each 1 to 6; and the other two dice contain the numbers from 1 to 6.	M 121	$ 3.75
The Abacus	This abacus bears little resemblance to ordinary ones. For instance, the wire span is about 3 times the space occupied by ten beads. The purpose of this is to clearly show a left and right and definate space in between to a person for whom figure and background tend to fuse. To facilitate the use of the abacus in teaching both number concept and number manipulation, the rods and the beads are adjustable by	W 132	$13.50

Company and Item	Description	Number	Price
Developmental Learning Materials, cont			
Add or Subtract Box, continued	trough, and the answer is read from the upper trough. In subtraction, the sum is formed in the upper trough, the given addend is then formed in the lower trough by using cubes from the upper trough, and the answer is read from the upper trough. The mutually inverse relationship of addition and subtraction is readily demonstrated.		
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Company and Item	Description	Number	Price
Developmental Learning Materials, cont.			
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Company and Item	Description	Number	Price
Developmental Learning Materials, cont.			
The Abacus, continued	removing a side. The beads may be restrung in a desired sequence making possible many different approaches to number concepts. Unique also about this abacus is the fact that it has ten rods with ten colors of beads. Other approaches may be taken by using only some of the ten rods. This versatile produce measures approximately 15" x 18½" and is made of the finest materials available.		
Today's Date Box	This simple manipulative date box has been designed to aid in a better understanding of the calendar process. Four sets of color-cued and sized cards fit into an attractive box, ideal for desk or counter-top. The object of this calendar is to allow a student or students to arrange the four sets of cards (months, weekdays, dates, and years) making the necessary daily adjustments, in order to realize that the passing of time on a daily, weekly, monthly and yearly basis is a perpetual progression. The wood-grained-covered box measures 9 1/4" x 6 1/8" x 1 1/4" and the cards are printed on heavy card stock.	P 178	$ 4.75
Sensanumber Track	By walking on this uniquely different and colorful SENSANUMBER TRACK, the child receives a multi-sensory reinforcement of number concepts and of basic number facts. The 18" by 12 foot yellow runner of sturdy plastic is printed with 3 colors and accurately ruled at 12-inch intervals so that it may be used along with standard rulers, yardsticks, and tape measures in teaching linear measure. The child may comfortably read the large numerals while walking forward or backward, either *up* or *down* the number sequence. Since his normal step length must be shortened when counting by ones and lengthened when counting by twos, he readily learns the distinction between these two important modes of counting -- both forward and backward. He is reinforced by observing the color coding which also serves to classify numbers as odd or even. By moving his body through the distances representing the numbers presented to him for number fact acquisition, he physically experiences the relative sizes of numbers -- and the fundamental difference between addition and subtraction. Similarly, he feels the concept of fractions, such as 1/2 of 10 or 1/3 of 6. The manual of suggested applications includes exercises in multiplication and division.	M 194	$ 9.75

Company and Item	Description	Number	Price
Developmental Learning Materials, cont.			
Sequential Calendar	The continuity of time, showing the days following each other in a continuing clockwise path down the three-dimensional spiral surface on a seven-sided vertical column. The cap has the name of the day placed on each of the seven surfaces and is movable so that it can properly designate the correct day for the start of each month. A platform, bearing removable *month* and *year* cards, rotates freely on the cap. The path is printed with 31 days (plus a blank for the last day of the preceding month) and may be converted to a month of any length through the use of blanks. A marker bearing the word TODAY is placed on today's date. To establish and reinforce an understanding of time sequence, markers for yesterday and tomorrow are included. Also included are markers for American national holidays, the principal days of Christian and Jewish religious observances, and 3 *birthday cake* markers, to identify approaching birthdays of class members and the teacher, month and year cards and blanks for covering unneeded numbers on the column. The calendar stands 22 inches high and is 8½" in diameter.	P 149	$19.50
Edukaid of Ridgewood 1250 East Ridgewood Ave. Ridgewood, New Jersey 07450			
Count A-Ladder	See page 6 of text.	1 - 10 1 - 20 1 - 40	$ 4.95 $ 6.25 $ 7.85
Count-A-Line	See page 23 of text. 0 - 20 by ones 0 - 100 by fives 0 - 100 by tens 1 - 200 by tens	4 - 15	$ 9.95
The Instructor Corporation Paoli, Pennsylvania 19301			
Arithmetic Readiness	Charming *Old Woman in the Shoe* theme. Different sizes of individual children, groups of children, windows, trees, illustrate such mathematical concepts as: many, few, more, less, big, little, and others. Grades K-2. For Flannel Board use.	27	$ 1.95

Company and Item	Description	Number	Price
The Instructor Corporation, cont.			
Beginning New Math	Develops sets and operations and the meaning of numbers. Promotes an understanding of fractions and geometrice shapes, fractional parts, numerals, new math symbols and printed numeral names. Grades 1-6.	212	$ 4.95
Desk Tapes	Provides a roll of 40 individual, self-adhesive desk tape number lines. Each child works with his own personal number line at his seat. Except for Fractional Number Line, all Desk Tapes are printed at 1'' intervals, supplying a beginning measuring instrument.		
	Desk Tape Number Line (0-10)	1125	$ 1.95
	Desk Tape Number Line (1-20)	1126	$ 2.95
	Desk Tape Positive-Negative Number Line (-10 through +10)	1127	$ 2.95
	Desk Tape Fractional Number Line (0 to 1 2/3) Shows 1/8, 1/6, 1/4, 1/3 and 1/2;	1129	$ 2.95
Discovering Fractions	To develop the concept of fractions and the meaning of halves, thirds and quarters. Many experiences are provided with interlocking plastic circles, felt squares and realistic illustrations of everyday objects that are commonly divided into fractions. Exploration and manipulation provide a concrete base for later learnings. Contains ten wholes and 24 sets of fractional parts.	1072	$ 4.50
Enlarged U.S. Coins	Teaches money value and equivalents. Children can make purchase, put money in bank, make change. Includes piggy bank, 25 pennies, large assortment of other coins, two dollars, numerals, coin names, plus objects to buy and sell. Grades K-3. For Flannel Board use.	246	$ 2.50
Flannel Board Clock	Large demonstration clock face with extra perimeter ring showing the 60 minutes. All necessary terms supplied, which can be placed adjacent to clock on flannel board. Moveable hands. Roman numerals included. Big enough to be seen from back of classroom.	48	$ 1.95
Fractional Number Line	For teaching addition and subtraction of fractions. Demonstrates visually how to change an improper fraction to a mixed number, and a mixed number to an improper fraction. Also shows what happens to fractions when they are added, subtracted, multiplied, and divided. Grades 3 - 6.	251	$ 2.50

Company and Item	Description	Number	Price
The Instructor Corporation, cont.			
Fractional Parts (Circles)	Eight seven-inch felt circles, assorted colors. Two whole circles and one circle each divided into halves, thirds, fourths, etc. Use one whole circle for comparison, place other whole circle over fractional ones to prove parts equal whole. Arithmetic signs included. All grades. For Flannel Board use.	220	$ 1.25
Fractional Parts (Squares)	Seven seven-inch felt squares, assorted colors. Two whole squares, plus three ways of dividing into quarters, two ways of dividing into halves. A departure from *pie* concept. Arithmetic signs included. All grades. For Flannel Board use.	222	$ 1.25
Geometric Shapes	Helps to identify geometric shapes and major characteristics by manipulating and comparing outlines and perimeter strips. Includes 8 shapes plus strips. Grades 2 - 6.	245	$ 2.50
Hundred Chart	Develops basic understanding of our number system. Can be used for counting by 1's, 2's, 3's, 5's, 10's, etc. Aids in understanding of carrying and borrowing (bridging). Leads to discovery of number patterns. Chart separates into strips for presentation of sets of tens, Grades 1 - 6.	248	$ 2.95
Introduction to Fractions	Illustrations of bread, cookies, pie, candy bar, and other familiar objects which can be divided into halves, quarters, and thirds. Helps differentiate between equal and unequal parts. For Flannel Board use. Grades K-3.	225	$ 2.95
Kinesthetic Numeral Cards and Counting Discs	Consists of 11 cards with flocked numerals 0 - 10, and matching black die-cut felt numerals, which adhere to white flocked cards. 55 counting discs are also supplied. Flocked numerals can be traced with fingers (kinesthetic-tactile approach), then felt numerals matched and place on blank flocked cards (visual approach). Counting discs can be used on flocked cards for showing *how many* for each numeral. Large numerals are 4½" high.	1227	$ 4.50

Company and Item	Description	Number	Price
The Instructor Corporation, cont.			
Know 'N' Show Beginning Math	Child participates in answering questions by selecting appropriate cards from holder and placing them in *answer slot* at top. Useful for recognition of numerals, learning meaning of numbers and other beginning math learnings, including operations and equations. Numerals for 10 children include 10 slotted holders, 10 sets of numeral cards, dot pattern cards, operation signs, and number words. Through *flashing* of answers to teacher, group evaluation is quick and simple yet individual participation is stressed. Kit for 10 students.	1249	$ 6.95
Measurement	Pieces can be manipulated to help discover relationships. Includes pints, quarts, gallon; dozen eggs and dozen cookies; inch, foot, yard, plus objects to be measured on the flannel board. Grades 1-4.	226	$ 2.50
Modern Math Numerals and Symbols	Contains over 200 red plastic numerals (1½" in size) and white symbols. Special beveled edges designed to be easily picked up by little fingers. Provides numerals and symbols for counting, building equations, and many other learnings. These are basic manipulatives for individual and group explorations.	1070	$ 2.95
New Math Readiness	Aids children in discovering sets as groups of related or unrelated items. Includes set containers, felt geometric shapes and numerals; realistic illustrations of dogs, dog houses, bones, and wagon to count, compare, match, and make into sets. Grades K-1.	211	$ 2.95
Ones and Tens	Develops an understanding of number combinations involving ones and tens. Provides one felt grid, numerals, column headings and counters for exploring to ten. Illustration shows 2 uses of felt grid supplied. (Use horizontally or vertically). Grades K-3. For Flannel Board use.	217	$ 1.50
Place Value - Expanded Notation	Develops place value and expanded notation with concrete examples. Build and take apart numerals in strips of 1's, 10's, 100's and 1,000's. Additional pieces include place value labels, process signs for addition and subtraction and brackets for expanding numerals. Grades 2 - 6.	216	$ 2.50

Company and Item	Description	Number	Price
The Instructor Corporation, cont.			
Reading Whole Numbers	Helps understanding of place value, facilitates the reading of whole numbers through billions, and shows how to write large numbers. Also for demonstrating that the position of any numeral in a group determines its value. Grades 3 - 8.	253	$ 1.95
Roman Numerals	Includes 90 Roman numerals and 55 Arabic numerals, three inches high, pre-cut felt. Arithmetic signs included. Comes in compartmented box. Grades 3 - 8.	249	$ 2.95
Set Dominoes	Giant 3" x 6" rubber composition dominoes. Domino patterns are modern math sets, ranging from empty sets through sets of five. Many patterns for each number are provided to emphasize matching by number property of sets. Many uses including beginning meaning of sets, equivalence, and non-equivalence, meaning of numbers, etc. 36 dominoes.	1120	$ 4.95
Study of Angles	Contains 18-inch protractor and materials to construct and demonstrate angles. Complete with set of angle names and vocabulary necessary for complete demonstrations. Grades 5 - 8.	235	$ 1.95
Study of Graphs	Complete set for constructing and teaching bar, line, picture, and circle graphs. Contains felt grid and a variety of terms to place on graph. Includes 18-inch calibrated circle and separate radii. Saves time, and enables teacher to construct clear demonstration size graphs quickly. Grades 4 - 9.	230	$ 3.95
Ten Frame	Basic teaching tool for developing understanding of our number system. Used for counting, fundamental processes, grouping, discovering patterns, place value, number families, etc. Grades K 6.	214	$ 2.95
Walk On Sets	Beginning mathematical concepts become clear and understandable with total physical involvement. For example, walking forward to *higher numbers* and stepping backward to *subtract* provides a type of participation that leads to genuine understanding. Each Stepping Stone set contains ten 6 3/4" squares made to nonskid rubber composition. No need to attach them to the floor.		

Company and Item	Description	Number	Price
The Instructor Corporation, cont.			
Walk-On Sets, continued	*Stepping Stones - Number Patterns* Two each of domino-like number patterns 1 through 5, can show number patterns through 10.	1116	$ 4.95
	Stepping Stones - Numerals Ten squares numbered from 1 to 10. Can be arranged at random or in a straight line.	1117	$ 4.95
	Walk-On Number Line (0-10) Made of durable vinyl, 10 feet long, easily taped to floor and easily cleaned. Bright red background with large black numbers.	1115	$ 2.50
	Walk-On Number Line (0-20) Same construction as 0-10 number line.	1118	$ 4.50
We Learn to Count	A kit that provides many, varied counting experiences for discoveries by individuals and small groups. Includes picture counter (1 through 10) to be placed in counting cards interlocking printed numeral cards, two sets of counting strips, and a ten frame. Over 90 pieces.	1071	$ 3.50
Milton Bradley Company Springfield, Mass. 01101			
Arithmetic Quizmo	Educational Lotto for valuable practice in number combinations. Each set gives double learning value and contains cards for the entire class.		
	Add-Subtract	9309	$ 2.25
	Multiply-Divide	9310	$ 2.25
Classroom Counting Frame	A versatile and very important device to express number facts in visual terms. Can be used in Kindergarten through third grade. Ten rows of beads on plated steel rods. High quality wood construction. (On stand)	9370	$17.50
Classroom Thermometer	Individual reusable thermometers (box of 12). In addition to the introduction and study of temperatures, the Classroom Thermometer has practical applications in demonstrating negative numbers. Red and white elastic tape *mercury bar* moves easily to show temperature. Centigrade and Fahrenheit scales for comparison on back, 21½" x 6½"	7628	$ 1.00

Company and Item	Description	Number	Price
Milton Bradley Co. cont.			
Clock Dial	Front has large 11½" diameter clock dial, large numerals and adjustable metal hands. Reverse displays special clock face, shaded for easy learning.	8062	$ 1.50
Count Your Change Game	Object of the game is to make change for a dollar. Teaches children the various coin combinations that equal one dollar. 2 to 8 players.	7635	$ 3.00
Cubical Counting Blocks	Beginning with simple exercises in number grouping, these blocks aid the child in the development of number concepts. Edges are rounded for safe handling. Permanent non-toxic colors	8039	$ 5.25
Day by Day Calendar	A perennial calendar in full color, with easy to change date cards. Special full color picture cards for holidays, special events changes of season, moon phases, etc. Durable 21" by 20" chipboard frame.	9374	$ 4.00
Educational Toy Money	For use in nursery, kindergarten and primary grades for number practice and making change. 72 crisp bills from *ones* to *twenties* and 190 realistic coins on metal foil.	9315	$ 1.50
Fractions Are As Easy As Pie	A learning game for fractional parts. Actually two games in one. The first shows the difference in sizes of fractional parts. The second helps older children to add fractions. Colorful spinner with metal parts.	9504	$ 1.25
Hickory Dickory Dock Mouse Counters	100 Plastic mice in assorted colors that will serve a variety of uses. They can be used for sorting and counting; for discovering simple number concepts; for color discrimination; and for developing fine muscle control. Unbreakable plastic, 2" in overall length. Recommended grade levels, K-3.	7639	$ 3.25
Individual Clock Dials	Durable Clock Dials with a plastic stand, printed on heavy cardboard. Metal hands with brass eyelet. Dial has both conventional and Roman Numerals.	7620	$ 1.90

Company and Item	Description	Number	Price
Milton Bradley Co., cont.			
Large Beads and Laces	One hundred 1" beads, 6 plastic tipped laces. Non-toxic assorted colors, smooth wax finish. Plastic container.	470F	$ 7.00
Colored Beads	Bright colorful non-toxic beads make seatwork number experiences interesting and fun. 144, ½ in. cubes, spheres and cylinders. 6 standard colors with smooth lustrous wax finish.	470	$ 2.20
	Above, but with 1000 beads	473	$13.00
	Bead laces (per dozen)	471	$ .80
Link Numbers	An Arithmetic aid that provides an interesting method for the primary grade child to build number facts and play number games. Die-cut numbers link together.	9518	$11.25
Math Blocks and Trays	4 clear trays with 1" grid card inserted in the bottom. Trays hold 4, 9, 16 and 25 blocks. They provide comparison and measurement experiences at the primary level as well as advanced concepts of perimeter, area and volume. Directions for use included. K-6.	7638	$ 5.50
Mathfacts Games	A chest of 5 self-instructional self-checking games that deal with the learning of basic additional subtraction number facts. Each of these pupil centered games covers a different set of facts and is graded in levels of increasing complexity. Small group or solitaire. Grades 1-3 and remedial beyond.	7642	$15.00
Number Concept Cards	Large cards easily seen by whole class. Supplements the textbook for drill in numbers, numerals and number facts through ten. Clearly defined pictures and patterns on white tag.	7600	$ 2.25
Plastic Counters	Counting discs are another tool to develop the number sense through work with concrete objects. Suggested uses include making a number staircase number pictures and the like. Assorted ¼" discs in red, white, blue and yellow.	7616	$ 1.00
Plastic Pegboard	Beginners use the plastic peg board in developing a number sense by combining 100 plastic pegs in colorful patterns or number groups. In higher grades it helps the child understand the teaching of fractions and decimals.	7803	$ 3.00

Company and Item	Description	Number	Price
Milton Bradley Co., cont.			
Primary Number and Geometry Board	Designed for use by individual students in experimental and self evaluative situations. Reinforces the concepts of numbers 1 through 10. Basic number combinations and geometric shapes. Includes mark and erase pegboard, 100 pegs and 20 template cards. When guide cards are exposed or placed over the pegboard certain holes in the pegboard are exposed allowing the child to build numbers and shapes.	7633	$ 2.00
Symbols for the New Mathematics	All new Mathematics symbols sized in proportion to 3" letters and numerals. Over 40 pieces. Supplements other New Math materials.	7810	$ 1.25
Teddy Bear Counters	100 plastic *Teddy Bears* in assorted colors. Designed as a kinesthetic aid to the introduction of numbers. *Teddy Bear* molded in unbreakable plastic, over 1" tall. Useful in traditional and new math programs.	7632	$ 3.25
Ten-Tens Counting Frame	For individual or small group work. Complements the large classroom counting frame. Can be used also as an abacus. Sturdy, plastic frame with colorful plastic beads and wire stand.	9316	$ 3.50
Tick-Tock Primary Clock	Teaches time by color. *Past* side of clock is blue; *Of* is gray; minute hands and marks are red; hour hand and numerals are black. Disc turns to reveal red minute numerals. Over 20" square.		
Understanding Numbers Flash Cards	With this flash card learning aid the child learns 25 basic addition facts. Cards show like quantities by pictures, perception dots, units, and counting numbers. Helps to make the first steps in mathematics easy and enjoyable.	9517	$ 1.50
Walk on Number Squares	20 Washable non-skid plastic squares, approximately 1 foot square, individually imprinted with number symbols 1 through 20. Walking and hopping games can be played to teach number facts and time telling. For primary grades. Directions for use included.	7637	$ 4.25

Company and Item	Description	Number	Price
Milton Bradley Co., cont.			
Weather Station	An excellent device to introduce the science of weather to the early grades. Includes red marking crayon to mark on plastic laminated wipe-off surface. Movable clock hands. Provides experiences in time, temperature and graph making. Measures 21" x 20".	7700	$ 4.00
Milton Bradley: Playskool Manufacturing Company Springfield, Mass. 01101			
Great Triangles Game	Ages 4-8. Game teaches equivalence and identity. The triangle is used as the basic geometric construction element. Placement of the small triangle within the large triangle constructs five different quadrilaterals and several trilaterals. Consists of 1 inlaid puzzle board 12 13/16" x 12 13/16", 25 wood right isosceles triangles; 3 sets of cue cards. Instruction booklet.	598	$10.00
Science Research Assoc., Inc. 2590 Devon Avenue Des Plaines, Illinois 60018			
Math Discovery Board	K-3. Mathematics becomes an active exercise for the energetic primary grader through the manipulation of colorful squares and sticks supplied with the board.	3-3730	$ 6.70
	Teachers Handbook	3-3720	$ 1.05
Ten Frame	K-3. This plastic frame and its ten different sized bars enable the youngster to visualize numerical relationships.	3-613	$ 3.05
Teaching Resources Corp. 100 Boylston Street Boxton, Mass. 03116			
Quality and Number Relationships	(Ruth Cheves - Part 5 - Program 1) These materials develop and reinforce many of the basic arithmetic skills; concepts of more than one; relationships of parts of whole; differientiation by visual comparison; sets; numberical relationships; and sequential order.	10-150	$ 9.95

Company and Item	Description	Number	Price
Teaching Resources Corp., cont.			
Quality and Number Relationships, continued	*Configuration Cards* (4" x 3"), 108 in 3 sets, with configuration patterns from 1 to 6. The first set is colorcued. The second set has only single color stimulus. The third set is black and white relating to the symbols of the printed page. Number and quantity concepts are established by visual comparison and spatial relationships within a framework of form perception. The structured steps in the presentation establish the relationship of increase in quantity to increase in space and numerical progression. The pattern cards can also be used extensively later in addition and subtraction processes *Ordinal Placement Board* A slotted, 15" cardboard desk stand and 2 sets of tabbed numerical cards (1-6) are used to introduce numerical notations. Five procedures provide the child numerous experiences with quantity increase and decrease to establish the concept of numerals as notations that represent quantity. Left to right progression is reinforced. Complete instructions for all procedures and extended learnings.	10-150	$ 9.95
Guessing Game	Put many beans (or counters tickets drinking straws tongue depressors) in a clear plastic or glass container. Beside it place a card *How many?* Children record their names and numbers (of guess) As an independent activity, children can count objects into sets of 10s, putting each set into plastic bag. Then count 10s plus 1s for total.		5 cents
Hand Counting Chart	Pairs of cutouts of children's hands may be mounted on a long strip of wrapping paper (or wall paper) for counting by 10s. Later, count by 5s.		Paper
Number Name Chart	Draw pictures or used gummed seals or cut outs on tagboard. Extend the chart as each subsequent number is studied. Each new entry should be made during class discussion.		Paper, tagboard, gummed seals (of used

Company and Item	Description	Number	Price
Teaching Resources Corp., cont.			
Paper-hand Cutouts	On drawing paper, show (help) children draw around one outspread hand. Cut out the drawing. (Color both sides -- or use 2 pieces of different colored paper glued together). Let children show how many fingers are left standing when one of five fingers of a paper hand is bent down; two are bent down; three; four; five.		Paper
Play Store	Set up a play store, letting children make price tags. Discuss a sensible price for each article. Play storekeeper and customers using money (play) for buying and making change. *Store* can be made of cardboard carton and empty cans, bottles and boxes.		Play or real money used