Iowa Engineering Experiment Station

special report

THE INFLUENCE OF SLENDERNESS RATIOS ON TRIAXIAL SHEAR TESTING

by

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INTRODUCTION

Throughout the literature, it is assumed that, apart from certain end effects, a homogeneous state of stress is produced in the triaxial shear test. This point of view appears to have carried over from earlier studies of rocks by Haar and von Karman in 1909. Effects of end plate restraint are a major deviation from the basic assumption of a homogeneous state of stress in a specimen tested by triaxial compression, and appear to constitute a controlling factor in the determination of slenderness ratios[a]. Particle size and the absolute dimensions of test specimens are practical factors which also influence the determination of slenderness ratios.

This paper is an effort to summarize existing significant literature applicable to potential standardization of the slenderness ratio of undisturbed and remolded soil specimens for triaxial shear testing.

THEORETICAL CONSIDERATIONS

Determinations of the effect of the slenderness ratio on the results of triaxial testing depend theoretically on the boundary conditions induced by (a) shape of the test specimen, (b) manner of the transmission of the external load, and (c) deformations, as presented by Balla (3).

Stress conditions of the triaxial test are in axial symmetry, as manifested by the cylindrical shape of the test specimen and the character

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[a] Slenderness ratio is herein defined as the ratio of height to diameter of a cylindrical test specimen.

of the maximum principal stresses; i.e., lateral pressure and axial stress, acting respectively perpendicular to the cylindrical surface and parallel with the longitudinal axis. Tangential stresses do not act on the mantle surface of the specimen. Top and bottom surfaces of the test specimen are restrained by radial shear on rigid loading plates and are thus not deformed, but lateral surfaces can undergo arbitrary deformations. The solution of the stress conditions must satisfy all boundary conditions.

The triaxial test has been analyzed against the background of plastic theory by Haythornthwaite (9) against the background of elastic theory by Balla (3). Haythornthwaite points out that the problem posed by the test situation is statically indeterminate because only the total thrust on the end plates, not the pressure distribution, is given as a boundary condition. Some assumption must therefore be made, or a certain boundary condition must be introduced, before the stress condition can be solved. Haythornthwaite assumes that the minor and intermediate principal stresses are equal, which appears reasonable in view of its correctness in the elastic range at locations remote from the ends of the specimen.

Balla (3) approaches the above problem by introducing a boundary condition of roughness of the loading plate. He further states that this is not a close-limit condition but a disputable one, and as such, is only an approximation because experimental results have not been published concerning conditions of roughness of the loading plate and the relative displacement occurring on it.

Balla (3) further states that the theoretical portion of soil mechanics relies heavily on the theory of elasticity and thus justifies the applicability of an elastic analysis of the triaxial test. He also points out that it is less important to obtain numerically accurate values, than to get acquainted with the character of the stress distribution and deformation, compute their approximate order of magnitude, and obtain an idea of the influence of the various factors, admitting that such an analysis is only a first approximation. The major value of Balla's analysis is that it offers some opportunity for the consideration of roughness of the loading plates for the entire range from full constraint to frictionless loading. Balla (2) demonstrated that the absolute dimensions of the cylinder are of little significance, but the stresses are inversely dependent on the slenderness ratios; i.e., compressive strength decreased with increased slenderness ratio.

PRACTICAL CONSIDERATIONS

Descriptive Analysis

From a practical point of view enough length of cylindrical specimen should be available to develop two complete cones of failure, and the length of the specimen should equal the diameter times the tangent of $(45 + \varphi/2)^{\circ}$, where φ is the angle of internal friction. Carmany (5) reported that a tall specimen was assumed to fail along a plane which was dependent on the angle of internal friction, whereas a short specimen was forced to fail along a plane developed from corner to corner of the specimen. Endersby (7) stated that if a bituminous stabilized specimen of low slenderness ratio is dissected after deformation, the end cones will be found blunted and the particles on the ends will show stripping and breakage of the asphaltic materials. Thus for non-cohesive as well as cohesive materials, Endersby assumes that excess resistance, greater than that by friction only, is concentrated in the central area of the specimen and is suggestive of true arch action. For specimens with a large slenderness ratio where the cones have not interfered, shearing appears to have occurred without a central concentration of resistance (blunting of the cones).

When a specimen has a moderate slenderness ratio, moderate cone interference develops, columnar action increases, and high arch action is attained. The magnitude of deformation begins to have an effect. As columnar action progressively increases, the excess stress necessary to break up the original formation of particles in the cones is beginning to contribute to the measureable shearing resistance.

When the slenderness ratio is small, strong cone interference develops, columnar action becomes very strong, but arch action weakens because of the low ratio of height to diameter. At the same time, however, increased force is necessary to rupture the original arrangement of particles in the cones, because the movement of particles has become perpendicular to the applied vertical force. Under this condition the arch resistance is increasingly dependent upon the friction between specimen and end plates, and upon the presence of cohesion in a cohesive material.

Endersby (7) points out that, in general, asphaltic compounds produce two opposite effects on stability, reducing it by acting as a lubricant and increasing it by acting as a cohesive material.

Complex effects in triaxial shear test analyses result from this paradox, and rate of axial loading and temperature during testing affect the test results.

Smith (13) shows that the various elements resisting failure of a specimen loaded vertically in a triaxial compression test are (a) lateral or confining pressure, (b) cohesion of the material, (c) confining stresses resulting from friction against the testing heads at the ends of the specimen, and (d) the internal friction of the material. It is not possible to separate items (b) and (c) mathematically or otherwise. However, the confining stresses at the testing head surfaces can be reduced to zero by using a test specimen having a slenderness ratio of approximately 2. These statements by Smith appear to be the main argument in favor of a tall specimen. Since a short specimen tends to fail from corner to corner, use of the tall specimen would appear essential if both frictional resistance and cohesive properties are to be measured in a single test without columnar action, arch action, and end restraint, and yet take place within the unrestrained portion of the specimen.

Akroyd (1) points out two very practical factors that control the slenderness ratios: (a) with specimen lengths greater than three times the diameter there is a danger of side buckling; (b) with lengths less than one-and-a-half times the diameter, the whole specimen is restrained by the friction of the end loading plates.

Effect of End Restraint

Roughness of end loading plates undoubtedly exerts some influence on the stresses and deformations within a triaxial test specimen. Friction and cohesion between the ends of the specimen and the rigid loading plates restricts lateral deformation adjacent to these surfaces.

Tests carried out by Taylor (16) using special fittings to eliminate end restraint, with specimens having different ratios of length to diameter, indicate that no significant error occurs in the strength measurement, provided the slenderness ratio is about 2. Rutledge (15) reviewed the cooperative triaxial research program of the Corps of Engineers of which Taylor's work was a part. Neither publication is available at Iowa State University, but the impression conveyed by the numerous references in the literature to these two articles is that the work was performed on sands. A slenderness ratio of 2 has been supported by tests run on sandstone by Baushinger as reported in Upton's "Materials of Construction".

Bishop and Henkel (4) consider the end restraint effects under three headings: strength, volume change, and pore-pressure characteristics. They state that a slenderness ratio range of 1.5 to 2.5 is permissible, though the range is dependent on the soil type and on the freedom of movement of the top cap.

No direct quantitative measurements of effect of end restraint on volume change have apparently been undertaken. However, Bishop and Henkel (4) have observed that the specimen diameter does not appear to decrease under a confining lateral pressure at ends of the cylinder. On subsequent application of the axial load, the diameter increases near the central portion of the specimen while the diameter at and near the ends appears to be unchanged. They also observed that a non-uniformity of pore water pressure was likely to occur within the specimen due to end restraint.

In the triaxial testing of bituminous mixes, McLeod (11) recommends a slenderness ratio of 2 to avoid the effects of friction between the end plates and the test specimen, and the direct transfer of load between the two plates. If the angle of internal friction is likely to approach 40°, McLeod suggests the ratio be increased to 2.5. Nyboer (12) uses a slenderness ratio of 3 for asphaltic concrete mixes and a ratio of 2.5 for sand-asphalt, including sheet asphalt mixtures.

Effect of Particle Size

Endersby (7) reported the influence of particle size differences in a test specimen is slight at high slenderness ratios and great at low ratios. Such is probably true for any soil sample that is tested over a wide range of slenderness ratios. Hall (8) stated that for gravels a specimen diameter of five times the largest particle size is technically desirable. This ratio of particle size to specimen diameter appears to have been originally developed for sands by Taylor (16) in 1941, and reviewed by Rutledge (15) in 1947.

In Converse's discussion of Hall's paper (8) he stated there is no theoretical guide to the correct limiting ratio of particle size to specimen diameter. He further points out that since the shearing resistance of a granular mixture of irregular-shaped particles depends on the particles arrangement, there should be more variation in shear test results when only a few particles are along the shearing surface than when there were many. Uniformity of results will also be dependent on such factors as the grading of the material, and the angularity of the particles. The degree of permissible variability appears to be under the control of the analyzer only, though quantitative variability can be determined by several tests on specimens of similar material, all tested under similar conditions.

Endersby (7) stated that lack of compaction acts like reducing the maximum particle size. The influence of changes of density during axial loading may also have a similar effect.

The United States Bureau of Reclamation (6) proposes the tabulation below as a guide for selection of specimen size as based on maximum particle size of soil:

Specimen size, inches	Remolded or undisturbed	Type of soil	Maximum particle size
1 3/8 by 3[a]	Undisturbed	Fine grained	No. 40 sieve
3 1/4 by 9	Remolded	Fine and coarse grained	No. 4 sieve
6 by 15	Remolded	Coarse grained	No. 4 to 3/4 inches
9 by 22.5	Remolded	Very coarse grained	3/4 to 3 inch

In testing bituminous mixes, the California Research Corporation uses a specimen approximately four inches in diameter and eight inches in height in order that all aggregates normally encountered in bituminous paving can be handled. Materials with particle sizes not exceeding one inch can be tested with excellent reproducibility, while mixes having particles up to two inches in diameter can be tested with sufficient accuracy and reproducibility for most design and control purposes (13). Nyboer (12) however, preferred that the diameter of the bituminous test

[a]First number is diameter, second is height.

specimen should be at least six times the diameter of the largest particle; the cross-sectional area of a single large particle is thus only about three per cent of the cross-sectional area of the test specimen.

For fine grained soil the Waterways Experiment Station, U. S. Army Corps of Engineers, generally uses a specimen diameter of 1.4 inches with a slenderness ratio of 2.5 (10).

Balla (3) has shown that, based on an elastic analysis, the absolute dimensions of the cylinder are of no significance and that the stresses depend only on the slenderness ratio. Endersby (7) points out that the results on the same material using equipment of different dimensions may be far apart. He reasons that this fact may be attributed to the complex relations between slenderness ratios of individual columns within the material and the mass column formed by the whole specimen. He interprets this fact to indicate (a) that the results between investigators are not comparable unless methods are standardized, (b) that results of present testing methods must be correlated on a dimensional basis for correct application to field conditions, and (c) particularly that any method of testing bituminous paving materials without correlating dimensional effects to road conditions is far afield.

SUMMARY

Determination of the effect of the slenderness ratio on the results of the triaxial shear test depends, theoretically, on the boundary conditions induced (a) by the shape of the test specimen, (b) manner of the transmission of the external load, and (c) by the deformations. There apparently has been no study conducted to determine the relationship between the conditions encountered in the field and the slenderness ratios to be used in the laboratory test to reproduce field conditions; the results of such a study would be especially valuable when designing foundations to be set on relatively thin bedded geologic formations. The analysis of the triaxial test against the background of the plastic theory by Haythornthwaite (9) and against the background of the elastic theory by Balla (3) appear to be major steps in placing the triaxial shear test on a firm theoretical footing.

From a practical point of view, adequate length should be available to develop two complete cones of failure, and the length of the specimen should equal the diameter times the tangent of $(45 + \varphi/2)^{\circ}$. When a specimen has a slenderness ratio greater than 3 there is a danger of side buckling. When a specimen has a moderate slenderness ratio, moderate cone interference, column action has become rather strong and strong arch action is coming into play. When the slenderness ratio is small, strong cone interference, column action has become very strong, and the arch action is weakening.

A ratio of particle size to specimen diameter of 1 to 5 has been developed for sands. This ratio may also be applicable to gravels. The lack of compaction acts like reducing the maximum particle size also.

Balla (3) has shown that, based on an elastic analysis, the absolute dimensions of the cylinder are of no significance and that the stresses depend only on the slenderness ratio. Endersby (7) points out that, on the basis of experience in the laboratory, the results on the same material with equipment of various dimensions may be quite diverse.

Most research personnel working with triaxial testing of soils apparently accept a slenderness ratio between 1.5 and 3.0. Most workers in the field of triaxial testing of bituminous paving mixes also accept this range of slenderness ratios. However, it appears that a ratio of height to diameter of 2.0 could be established as a more common laboratory triaxial test specimen size, with appropriate regard to exact dimensions based on a maximum particle size to diameter ratio of about 1 to 5.

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