Department of

Civil Engineering

College of Engineering
The University of Iowa
Iowa City, Iowa

Loss of Prestress and Camber of Non-Composite and Composite Prestressed Concrete Structures

by
D. E. Branson
K. M. Kripanarayanan

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LOSS OF PRESTRESS AND CAMBER OF NON-COMPOSITE AND COMPOSITE PRESTRESSED CONCRETE STRUCTURES

by

D. E. Branson and K. M. Kripanarayanan, respectively, Professor and Research Associate, Department of Civil Engineering, University of Iowa, Iowa City, Iowa, U.S.A.

ABSTRACT

A systematic procedure for predicting the material behavior of different weight concretes and the time-dependent structural response of non-composite and composite prestressed concrete structures is presented. Continuous time functions are provided for all needed parameters, so that the general equations presented for predicting loss of prestress and camber readily lend themselves to computer solution.

Results computed by the material parameter equations are compared with representative data in the literature for normal weight, sand-lightweight, and all-lightweight concrete. Results computed by the loss of prestress and camber equations are compared with experimental data for a sand-lightweight composite bridge, and with data in the literature for non-composite and composite structures constructed of normal weight, sand-lightweight, and all-lightweight concrete. Both laboratory specimens and actual structures are included in these comparisons. Ranges of variation are also shown for the material behavior, loss of prestress, and camber results. Sample calculations are also included.

The procedures in this paper for predicting time-dependent material and structural behavior represent a nominal approach for design purposes, and not definitive or statistical results by any means. Probabilistic methods are needed for the accurate estimate of variability of behavior, etc.

STRENGTH AND ELASTIC PROPERTIES, CREEP AND SHRINKAGE

Strength and elastic properties

A study of concrete compressive strength versus time for the data of Ref. 1 - 6 (88 specimens) indicates an appropriate general equation in the form of Eq. (1) and average-value Eqs. (2) - (5) for predicting strength at any time[6,7,8].

$$(f'_c)_t = \frac{t}{a + bt} (f'_c)_{28d} \qquad (1)$$

Moist cured concrete, type I cement

$$(f'_c)_t = \frac{t}{4.00 + 0.85t} (f'_c)_{28d}; \text{ or } (f'_c)_{7d}=0.70(f'_c)_{28d},\ (f'_c)_u=1.18(f'_c)_{28d} \qquad (2)$$

Moist cured concrete, type III cement

$$(f'_c)_t = \frac{t}{2.30 + 0.92t} (f'_c)_{28d}; \text{ or } (f'_c)_{7d}=0.80(f'_c)_{28d},\ (f'_c)_u=1.09(f'_c)_{28d} \qquad (3)$$

Steam cured concrete, type I cement

$$(f'_c)_t = \frac{t}{1.00 + 0.95t} (f'_c)_{28d}; \text{ or } (f'_c)_{2.5d}=0.74(f'_c)_{28d},\ (f'_c)_u=1.05(f'_c)_{28d} \qquad (4)$$

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$$(f'_c)_t = \frac{t}{0.70 + 0.98t} (f'_c)_{28d}; \text{ or } (f'_c)_{2.5d}=0.80(f'_c)_{28d},\ (f'_c)_u=1.02(f'_c)_{28d} \qquad (5)$$

where a and b are constants, $(f'_c)_{28d}$ = 28-day strength, for example, t is age of concrete in days, and $(f'_c)_u$ refers to an ultimate (in time) value.

Eqs. (2) - (5) are compared in Fig. 1 with the data from Ref. 1 - 6, which includes different weight concretes, both moist and steam curing, and types I and III cement. The ranges of variation in the data, and effect of type of curing and cement type can be seen in Fig. 1. All of these data fall within about 20% of the values given by Eqs. (2) - (5). These curves were found to be equally applicable for normal weight, sand-lightweight, and all-lightweight aggregate concrete.

Eq. (6) is considered satisfactory in most cases for computing modulus of elasticity of different weight concretes[9,10].

$$E_c = 33\, w^{1.5} \sqrt{f'_c}\ \text{, psi;} \quad w \text{ in pcf and } f'_c \text{ in psi} \qquad (6)$$

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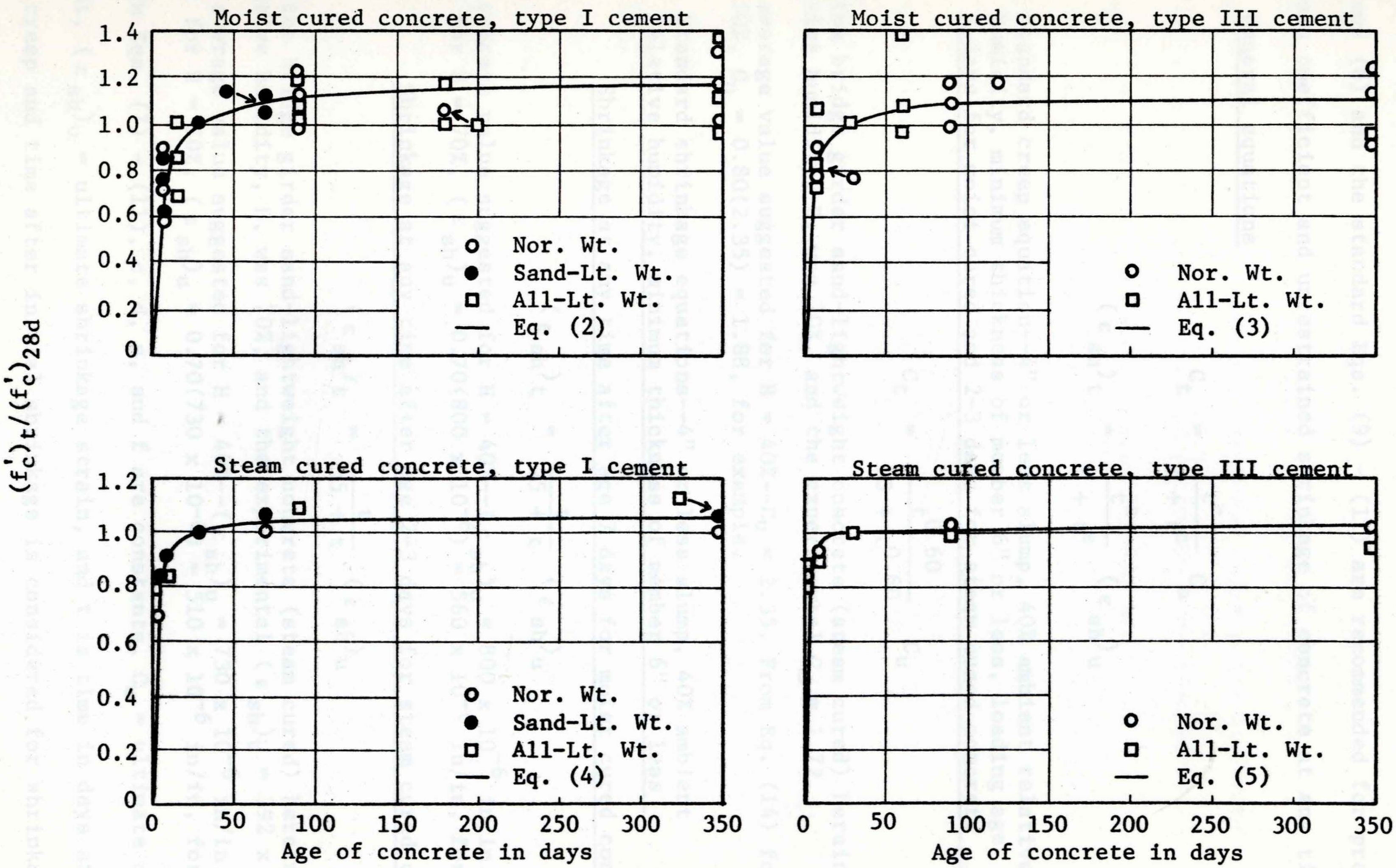

Fig. 1--Concrete strength versus time curves using Eqs. (2) - (5), and comparison with experimental data from References 1 through 6 for different weight concretes (using both moist and steam curing, and types I and III cement). Where three data points are shown for a given age, they refer to the upper and lower limits and the average value for a given set of data

Creep and shrinkage parameters

Based largely on information from Ref. 3 - 6 and 11 - 19, the general Eqs. (7) and (8) and the standard Eqs. (9) - (11) are recommended for predicting a creep coefficient and unrestrained shrinkage of concrete at any time[6,7,8].

General equations

$$C_t = \frac{t^c}{d + t^c} C_u \tag{7}$$

$$(\varepsilon_{sh})_t = \frac{t^e}{f + t^e} (\varepsilon_{sh})_u \tag{8}$$

Standard creep equation--4" or less slump, 40% ambient relative humidity, minimum thickness of member 6" or less, loading age 7 days for moist cured and 2-3 days for steam cured concrete

$$C_t = \frac{t^{0.60}}{10 + t^{0.60}} C_u \tag{9}$$

For the bridge girder sand-lightweight concrete (steam cured) herein--the relative humidity, H, was 70%, and the experimental $C_u = 1.72$.

The average value suggested for H = 40%--$C_u = 2.35$. From Eq. (14) for H = 70%, $C_u = 0.80(2.35) = 1.88$, for example.

Standard shrinkage equations--4" or less slump, 40% ambient relative humidity, minimum thickness of member 6" or less

Shrinkage at any time after age 7 days for moist cured concrete

$$(\varepsilon_{sh})_t = \frac{t}{35 + t} (\varepsilon_{sh})_u \tag{10}$$

The average value suggested for H = 40%--$(\varepsilon_{sh})_u = 800 \times 10^{-6}$ in/in. From Eq. (15) for H = 70%, $(\varepsilon_{sh})_u = 0.70(800 \times 10^{-6}) = 560 \times 10^{-6}$ in/in, for example.

Shrinkage at any time after age 2-3 days for steam cured concrete

$$(\varepsilon_{sh})_t = \frac{t}{55 + t} (\varepsilon_{sh})_u \tag{11}$$

For the bridge girder sand-lightweight concrete (steam cured) herein--the relative humidity, H, was 70%, and the experimental $(\varepsilon_{sh})_u = 392 \times 10^{-6}$ in/in.

The average value suggested for H = 40%--$(\varepsilon_{sh})_u = 730 \times 10^{-6}$ in/in. From Eq. (15) for H = 70%, $(\varepsilon_{sh})_u = 0.70(730 \times 10^{-6}) = 510 \times 10^{-6}$ in/in, for example.

In Eqs. (7) - (11), c, d, e, and f are constants, C_u = ultimate creep coefficient, $(\varepsilon_{sh})_u$ = ultimate shrinkage strain, and t is time in days after loading for creep and time after initial shrinkage is considered for shrinkage.

Eqs. (9) - (11) are compared with representative data (120 creep and 95 shrinkage specimens) in Figs. 2, 4 and 5, in which upper and lower limits and average values are shown. Normal weight, sand-lightweight, and all-lightweight concrete (using both moist and steam curing, and types I and III cement) is included. No consistent variation was found between the different weight concretes, etc., for either creep or shrinkage. The average values of C_u and $(\varepsilon_{sh})_u$ for these data (given with Eqs. 9 - 11, and shown in Figs. 2, 4 and 5) should be used only in the absence of specific creep and shrinkage data for local aggregates and conditions. However, the "time-ratio" part (right-hand side, except for C_u and $(\varepsilon_{sh})_u$) of Eqs. (9) - (11) appear to be applicable quite generally.

Values from the standard Eqs. (9) - (11) of C_t/C_u and $(\varepsilon_{sh})_t/(\varepsilon_{sh})_u$ are:

	1 mth	3 mths	6 mths	1 yr	5 yrs
C_t/C_u, Eq. (9)--	0.44	0.60	0.69	0.78	0.90
$(\varepsilon_{sh})_t/(\varepsilon_{sh})_u$, Eq.(10)--	0.46	0.72	0.84	0.91	0.98
$(\varepsilon_{sh})_t/(\varepsilon_{sh})_u$, Eq.(11)--	0.35	0.62	0.77	0.87	0.97

Correction factors[6,7,8]

All correction factors are applied to ultimate values. However, since creep and shrinkage for any period in Eqs. (9) - (11) are linear functions of the ultimate values, the correction factors in this procedure may be applied to short-term creep and shrinkage as well.

For loading ages later than 7 days for moist cured concrete and later than 2-3 days for steam cured concrete, use Eqs. (12) and (13) for the creep correction factors. These results can also be seen in Fig. 3.

$$\text{Creep } (C.F.)_{LA} = 1.25\, t_{LA}^{-0.118} \quad \text{for moist cured concrete} \tag{12}$$

$$\text{Creep } (C.F.)_{LA} = 1.13\, t_{LA}^{-0.095} \quad \text{for steam cured concrete} \tag{13}$$

where t_{LA} is the loading age in days. For example,

when t_{LA} =	Eq. (12)--$(C.F.)_{LA}$ =	Eq. (13)--$(C.F.)_{LA}$ =
10 days	0.95	0.90
20	0.87	0.85
30	0.83	0.82
60	0.77	0.76
90	0.74	0.74

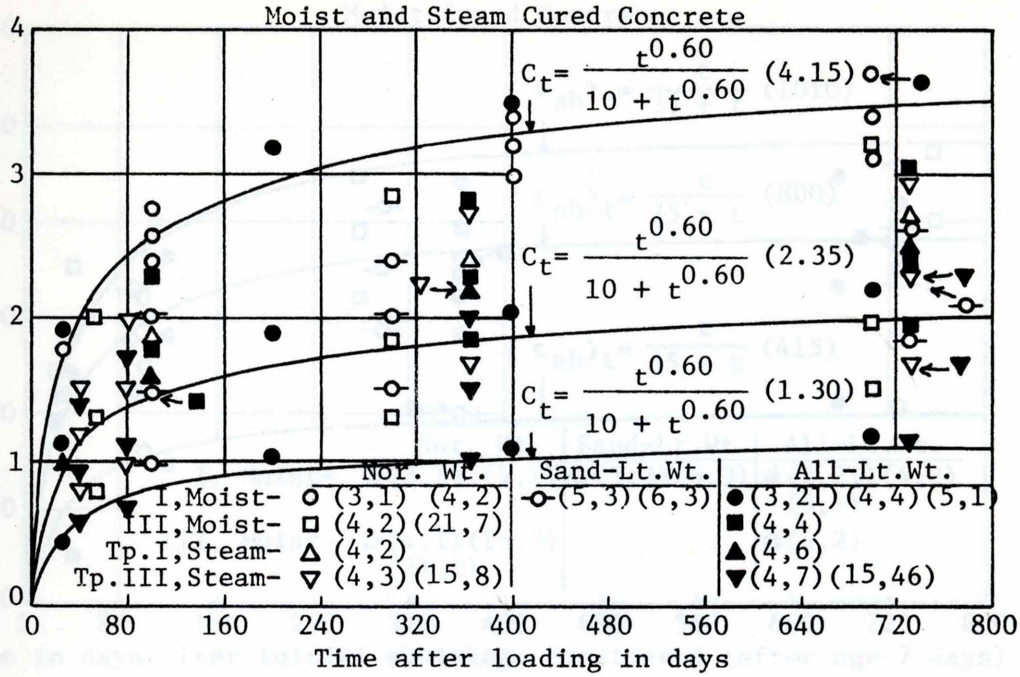

Fig. 2--Creep coefficient versus time curves using Eq. (11), and comparisons with the data from Refs. 3 - 6, 15, 21. For each condition indicated, upper and lower limits and average values are plotted. All data were reduced to "standard conditions" using the corr. factors herein. For each set of data, (3,21) refers to Ref. 3, and 21 data pts., for example

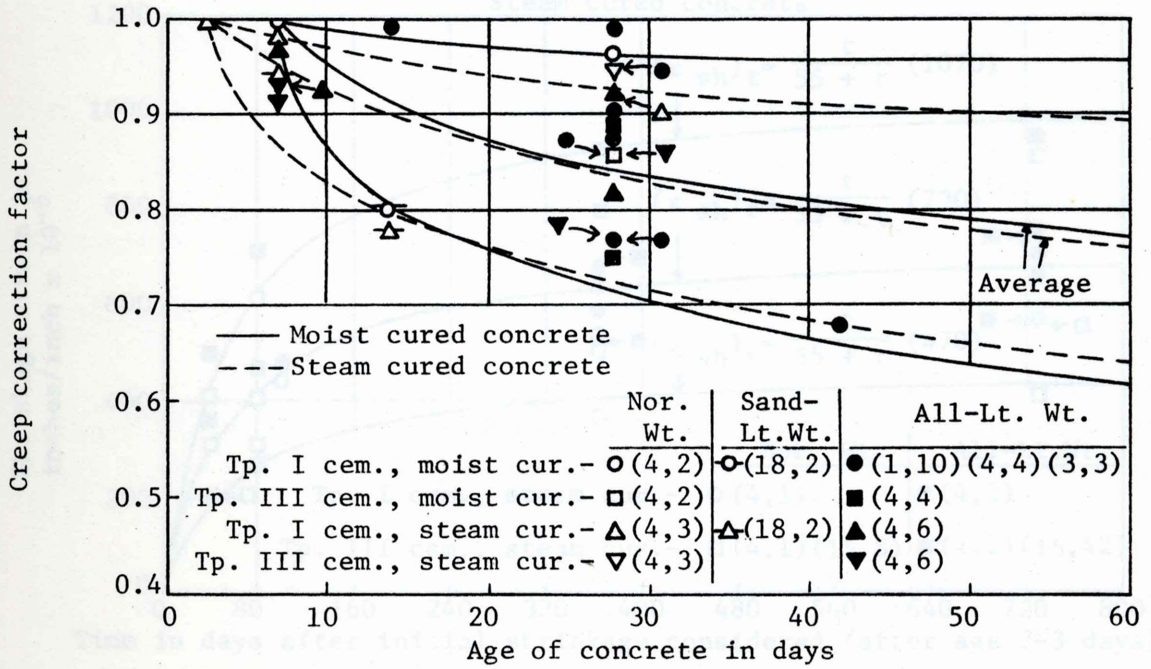

Fig. 3--Creep correction factor for time of initial loading, based on loading ages of 7 d. for moist cured and 2-3 d. for steam cured concrete. For each set of data, (4,2) refers to Ref. 4, and 2 data pts., for example. Upper and lower limit and average value curves are shown

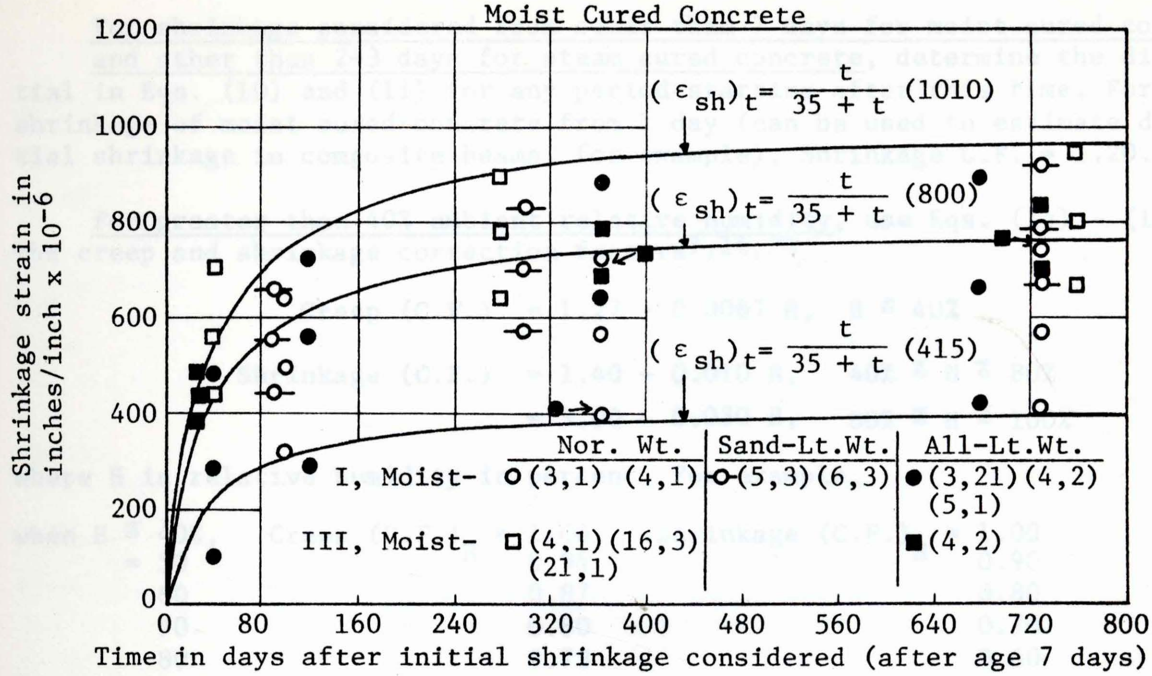

Fig. 4--Shrinkage strain versus time curves using Eq. (12), and comparisons with the data from Refs. 3 - 6, 16, 21. For each condition indicated, upper and lower limits and average values are plotted. All data were reduced to "standard conditions" using the corr. factors herein. For each set of data, (3,21) refers to Ref. 3, and 21 data pts., for example

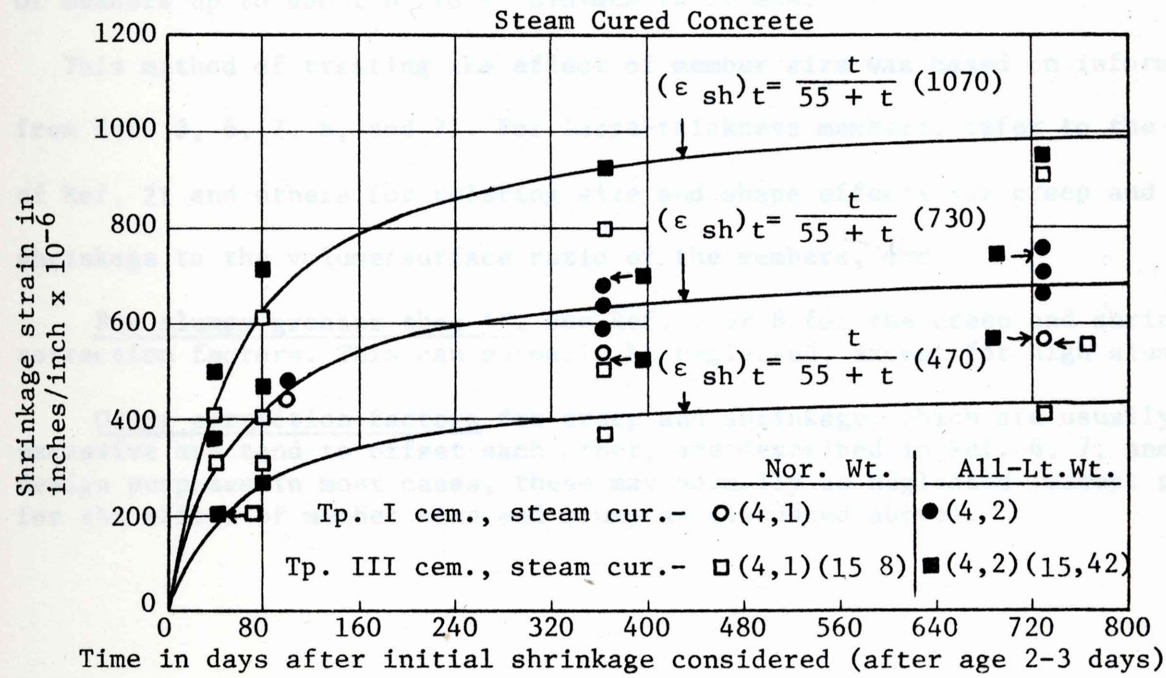

Fig. 5--Shrinkage strain versus time curves using Eq. (13), and comparisons with the data from Refs. 4 and 15. For each condition indicated, upper and lower limits and average values are plotted. All data were reduced to "standard conditions" using the corr. factors herein. For each set of data, (15,8) refers to Ref. 15 and 8 data pts., for example

For shrinkage considered from other than 7 days for moist cured concrete and other than 2-3 days for steam cured concrete, determine the differential in Eqs. (10) and (11) for any period starting after this time. For shrinkage of moist cured concrete from 1 day (can be used to estimate differential shrinkage in composite beams, for example), Shrinkage C.F. = 1.20.

For greater than 40% ambient relative humidity, use Eqs. (14) - (16) for the creep and shrinkage correction factors[7,16,20].

$$\text{Creep (C.F.)} = 1.27 - 0.0067\,H, \quad H \geqq 40\% \tag{14}$$

$$\text{Shrinkage (C.F.)} = 1.40 - 0.010\,H, \quad 40\% \leqq H \leqq 80\% \tag{15}$$

$$= 3.00 - 0.030\,H, \quad 80\% \leqq H \leqq 100\% \tag{16}$$

where H is relative humidity in percent. For example,

when H	Creep $(\text{C.F.})_H$	Shrinkage $(\text{C.F.})_H$
$\leqq$ 40%,	= 1.00,	= 1.00.
= 50	0.94	0.90
60	0.87	0.80
70	0.80	0.70
80	0.73	0.60
90	0.67	0.30
100	0.60	0.00

For minimum thickness of members greater than 6", see Ref. 7 or 8 for the creep and shrinkage correction factors, as a function of loading period and length of drying period. For most design purposes, this effect can be neglected for creep of members up to about 10" to 12" minimum thickness, and for shrinkage of members up to about 8" to 9" minimum thickness.

This method of treating the effect of member size was based on information from Ref. 3, 6, 7, 8, and 21. For large-thickness members, refer to the method of Ref. 21 and others for relating size and shape effects for creep and shrinkage to the volume/surface ratio of the members, etc.

For slumps greater than 4", see Ref. 7 or 8 for the creep and shrinkage correction factors. This can normally be neglected, except for high slumps.

Other correction factors for creep and shrinkage, which are usually not excessive and tend to offset each other, are described in Ref. 6, 7, and 8. For design purposes in most cases, these may normally be neglected (except possibly for the effect of member size and slump as discussed above).

LOSS OF PRESTRESS AND CAMBER

Computed loss of prestress and camber[6,22-31]

Non-composite beams at any time, including ultimate values

The loss of prestress, in percent of initial tensioning stress, is given by Eq. (17).

$$PL_t = \left[\overbrace{(n\,f_c)}^{(1)} + \overbrace{(n\,f_c)C_t(1 - \frac{\Delta F_t}{2\,F_o})}^{(2)} + \overbrace{(\varepsilon_{sh})_t\,E_s/(1 + npk_s)}^{(3)} + \overbrace{\frac{f_{si}}{100}\,1.5\mathrm{Log}_{10}\,t}^{(4)}\right]\frac{100}{f_{si}} \tag{17}$$

where:

Term (1) is the prestress loss due to elastic shortening = PL_{el}.

$f_c = \frac{F_i}{A_t} + \frac{F_i e^2}{I_t} - \frac{M_D\,e}{I_t}$, and n is the modular ratio at the time of prestressing. Frequently F_o, A_g, and I_g are used as an approximation instead of F_i, A_t, and I_t, where $F_o = F_i(1 - n\,p)$. Only the first two terms for f_c apply at the ends of simple beams. For continuous members, the effect of secondary moments due to prestressing should also be included.

Term (2) is the prestress loss due to concrete creep. The expression, $C_t(1 - \frac{\Delta F_t}{2\,F_o})$, was used in Ref. 22 and 25 to approximate the creep effect resulting from the variable stress history. See the section on required calculations and summary of general parameters for approximate values of $\Delta F_t/F_o$ (in form of $\Delta F_s/F_o$ and $\Delta F_u/F_o$) for this secondary effect (expression in parenthesis) at 3 wks to 1 mth, 2 to 3 mths, and ultimate values.

Term (3) is the prestress loss due to shrinkage[31]. The expression, $(\varepsilon_{sh})_t\,E_s$, somewhat (approx. 1% loss differential for the bridge girder ultimate value in the example herein) overestimates (on safe side) Term (3). The denominator represents the stiffening effect of the steel.

Term (4) is the prestress loss due to steel relaxation. This assumes a Max. value = 7.5% (at or above 10^5 hrs = 11.4 yrs)[6,29,30]. In this term, t is time after initial stressing in hours. This expression applies only when f_{si}/f_y is greater than or equal to 0.55, in which f_y is the 0.1%-offset yield strength.

The camber is given by Eq. (18). It is suggested than an average of the end and midspan loss of prestress be used for straight tendons and 1-pt. harping, and the midspan loss of prestress for 2-pt. harping (bridge girders herein)[6].

$$\Delta_t = \overbrace{(\Delta_i)_{F_o}}^{(1)} - \overbrace{(\Delta_i)_D}^{(2)} + \overbrace{\left[-\frac{\Delta F_t}{F_o} + (1 - \frac{\Delta F_t}{2\,F_o})C_t\right](\Delta_i)_{F_o}}^{(3)} - \overbrace{C_t(\Delta_i)_D}^{(4)} - \overbrace{\Delta_L}^{(5)} \tag{18}$$

where:

Term (1) is the initial camber due to the initial prestress force after elastic loss, F_o. See Appendix B for common cases of prestress moment diagrams, with formulas for computing camber, $(\Delta_i)_{F_o}$. Here $F_o = F_i(1 - n\,f_c/f_{si})$, where f_c is determined as in Term (1) of Eq. (17). For continuous members, the effect of secondary moments due to prestressing should also be included.

Term (2) is the initial dead load deflection of the beam. $(\Delta_i)_{F_o} = K\,M\,L^2/E_{ci}\,I_g$. I_g is suggested instead of I_t for practical reasons. See Notation for K and M formulas.

Term (3) is the creep (time-dependent) camber of the beam due to the prestress force. This expression includes the effects of creep and loss of prestress; that is, the creep effect under variable stress. ΔF_t refers to the total loss at any time minus the elastic loss. It is noted that the term, $\Delta F_t/F_o$, refers to the steel stress or force after elastic loss, and the prestress loss in percent, PL (as used herein), refers to the initial tensioning stress or force. The two are related as: $\frac{\Delta F_t}{F_o} = \frac{1}{100}(PL_t - PL_{el})\frac{f_{si}}{f_o}$, and can be closely approximated by $\frac{\Delta F_t}{F_o} = \frac{1}{100}(PL_t - PL_{el})\frac{1}{1 - n\,p}$.

Term (4) is the dead load creep deflection of the beam.

Term (5) is the live load deflection of the beam.

<u>Unshored and shored composite beams at any time, including ultimate values</u>

Subscripts 1 and 2 are used to refer to the slab (or effect of the slab such as under slab dead load) and precast beam, respectively.

The loss of prestress, in percent of initial tensioning stress, for unshored and shored composite beams is given by Eq. (19).

$$PL_t = \Bigg[\overbrace{(n\,f_c)}^{(1)} + \overbrace{(n\,f_c)C_{s_2}\left(1 - \frac{\Delta F_s}{2\,F_o}\right)}^{(2)} + \overbrace{(n\,f_c)(C_{t_2} - C_{s_2})\left(1 - \frac{\Delta F_s + \Delta F_t}{2\,F_o}\right)\frac{I_2}{I_c}}^{(3)}$$

$$+ \overbrace{(\varepsilon_{sh})_t\,E_s/(1 + npk_s)}^{(4)} + \overbrace{\frac{f_{si}}{100}\,1.5\mathrm{Log}_{10}t}^{(5)} - \overbrace{(m\,f_{cs})}^{(6)} - \overbrace{(m\,f_{cs})C_{t_1}\frac{I_2}{I_c}}^{(7)} - \overbrace{PG_{DS}}^{(8)}\Bigg]\frac{100}{f_{si}} \qquad (19)$$

where:

Term (1) is the prestress loss due to elastic shortening. See Term (1) of Eq. (17) for the calculation of f_c.

Term (2) is the prestress loss due to concrete creep up to the time of slab casting. C_{s_2} is the creep coefficient of the precast beam concrete at the time of slab casting. See Term (2) of Eq. (17) for comments concerning the reduction factor, $\left(1 - \frac{\Delta F_s}{2\,F_o}\right)$.

Term (3) is the prestress loss due to concrete creep for any period following slab casting. C_{t_2} is the creep coefficient of the precast beam concrete at any time after slab casting. The reduction factor, $\left(1 - \frac{\Delta F_s + \Delta F_t}{2\,F_o}\right)$, with the incremental creep coefficient, $(C_{t_2} - C_{s_2})$, estimates the effect of creep under the variable prestress force that occurs after slab casting. The reduction factor term was modified from previous references here. The expression, I_2/I_c, modifies the initial value and accounts for the effect of the composite section in restraining additional creep curvature (strain) after slab casting.

Term (4) is the prestress loss due to shrinkage. See Term (3) of Eq. (17) for comment.

Term (5) is the prestress loss due to steel relaxation. In this term t is time after initial stressing in hours. See Term (4) of Eq. (17) for the maximum value and limitations.

Term (6) is the elastic prestress gain due to slab dead load, and m is the modular ratio at the time of slab casting. $f_{cs} = \frac{M_{S,Di}\, e}{I_g}$, $M_{S,Di}$ refers to slab or slab plus diaphram dead load, and e, I_g refer to the precast beam section properties for unshored construction and the composite beam section properties for shored construction.

Term (7) is the prestress gain due to creep under slab dead load. C_{t1} is the creep coefficient for the slab loading, where the age of the precast beam concrete at the time of slab casting is considered. For shored construction, drop the term, I_2/I_c.

Term (8) is the prestress gain due to differential shrinkage. $PG_{DS} = m\, f_{cd}$, where $f_{cd} = Q\, y_{cs} e_c / I_c$, and f_{cd} is the concrete stress at the steel c.g.s. See Notation for additional descriptions of terms. Since this effect results in a prestress gain, not loss, and is normally small (see Table 2), it may usually be neglected.

The camber of unshored and shored composite beams is given by Eqs. (20) and (21), respectively.

Unshored construction:

$$\Delta_t = \overbrace{(\Delta_i)_{F_o}}^{(1)} - \overbrace{(\Delta_i)_2}^{(2)} + \overbrace{\left[-\frac{\Delta F_s}{F_o} + \left(1 - \frac{\Delta F_s}{2\,F_o}\right) C_{s_2}\right](\Delta_i)_{F_o}}^{(3)}$$

$$+ \overbrace{\left[-\frac{\Delta F_t - \Delta F_s}{F_o} + \left(1 - \frac{\Delta F_s + \Delta F_t}{2\,F_o}\right)(C_{t_2} - C_{s_2})\right](\Delta_i)_{F_o} \frac{I_2}{I_c}}^{(4)} - \overbrace{C_{s_2}(\Delta_i)_2}^{(5)}$$

$$- \overbrace{(C_{t_2} - C_{s_2})(\Delta_i)_2 \frac{I_2}{I_c}}^{(6)} - \overbrace{(\Delta_i)_1}^{(7)} - \overbrace{C_{t_1} (\Delta_i)_1 \frac{I_2}{I_c}}^{(8)} - \overbrace{\Delta_{DS}}^{(9)} - \overbrace{\Delta_L}^{(10)} \qquad (20)$$

where:

Term (1) is the initial camber due to the initial prestress force after elastic loss, F_o. See Term (1) of Eq. (18) for additional comments.

Term (2) is the initial dead load deflection of the precast beam. $(\Delta_i)_2 = K\, M_2\, L^2 / E_{ci}\, I_g$. See Term (2) of Eq. (18) for additional comments.

Term (3) is the creep (time-dependent) camber of the beam, due to the prestress force, up to the time of slab casting. See Term (3) of Eq. (18) and Terms (2) and (3) of Eq. (19) for additional comments.

Term (4) is the creep camber of the composite beam, due to the prestress force, for any period following slab casting. See Term (3) of Eq. (18) and Terms (2) and (3) of Eq. (19) for additional comments.

Term (5) is the creep deflection of the precast beam up to the time of slab casting due to the precast beam dead load. C_{s_2} is the creep coefficient of the precast beam concrete at the time of slab casting.

Term (6) is the creep deflection of the composite beam for any period following slab casting due to the precast beam dead load. See Term (3) of Eq. (19) for additional comments.

Term (7) is the initial deflection of the precast beam under slab dead load. $(\Delta_i)_1 = K\, M_1\, L^2/E_{cs}\, I_g$. See Notation for K and M formulas. When diaphrams are sued, add to $(\Delta_i)_1$: $(\Delta_i)_{1D} = \frac{M_{1D}}{E_{cs}\, I_g}\left(\frac{L^2}{8} - \frac{a^2}{6}\right)$, where M_{1D} is the moment between diaphrams, and a is L/4, L/3, etc., for 2 symmetrical diaphrams at the quarter points, third points, etc., respectively.

Term (8) is the creep deflection of the composite beam due to slab dead load. C_{t_1} is the creep coefficient for the slab loading, where the age of the precast beam concrete at the time of slab casting is considered. See Term (3) of Eq. (19) for comment concerning I_2/I_c.

Term (9) is the deflection due to differential shrinkage. For simple spans, $\Delta_{DS}=Q\, y_{cs} L^2/8E_{cs}I_c$, where $Q=D\, A_1E_1/3$. See Notation for additional descriptions of terms. The factor 3 provides for the gradual increase in the shrinkage force from day 1, and also approximates the creep and varying stiffness effects[27]. This factor 3 is also consistent with the data herein and elsewhere. See Table 3 for numerical values herein. In the case of continuous members, differential shrinkage produces secondary moments (similar to the effect of prestressing but opposite in sign--normally) that should be considered[32].

Term (10) is the live load deflection of the composite beam, in which the gross-section flexural rigidity, $E_c\, I_c$, is normally used.

Shored construction:

$$\Delta_t = \text{Eq. (20), with Terms (7) and (8) modified as follows:} \qquad (21)$$

Term (7) is the initial deflection of the composite beam under slab dead load. $(\Delta_i)_1 = K\, M_1\, L^2/E_{cs}\, I_c$. See Notation for K and M formulas.

Term (8) is the creep deflection of the composite beam under slab dead load = $C_{t_1}(\Delta_i)_1$. The composite-section effect is already included in Term (7).

It is suggested that the 28-day modulii of elasticity for both slab and precast beam concretes, and the gross I (neglecting the steel), be used in computing the composite moment of inertia, I_c, in Eqs. (19) - (21).

<u>Special case of "ultimate" loss of prestress and camber</u>

For computing ultimate values of loss of prestress and camber, Eqs. (22) - (26) correspond term by term to Eqs. (17) - (21), respectively.

Loss of prestress for non-composite beams, as per Eq. (17):

$$PL_u = \left[\overbrace{(n\, f_c)}^{(1)} + \overbrace{(n\, f_c)C_u\left(1 - \frac{\Delta F_u}{2\, F_o}\right)}^{(2)} + \overbrace{(\varepsilon_{sh})_u\, E_s/(1 + npk_s)}^{(3)} + \overbrace{0.075\, f_{si}}^{(4)}\right]\frac{100}{f_{si}} \qquad (22)$$

Camber of non-composite beams, as per Eq. (18):

$$\Delta_u = \underbrace{(\Delta_i)_{F_o}}_{(1)} - \underbrace{(\Delta_i)_D}_{(2)} + \underbrace{\left[-\frac{\Delta F_u}{F_o} + \left(1 - \frac{\Delta F_u}{2F_o}\right)C_u\right](\Delta_i)_{F_o}}_{(3)} - \underbrace{C_u(\Delta_i)_D}_{(4)} - \underbrace{\Delta_L}_{(5)} \qquad (23)$$

Loss of prestress for unshored and shored composite beams, as per Eq. (19):

$$PL_u = \left[\underbrace{(n\,f_c)}_{(1)} + \underbrace{(n\,f_c)(\alpha_s C_u)\left(1 - \frac{\Delta F_s}{2F_o}\right)}_{(2)} + \underbrace{(n\,f_c)(1 - \alpha_s)C_u\left(1 - \frac{\Delta F_s + \Delta F_u}{2F_o}\right)\frac{I_2}{I_c}}_{(3)}\right.$$

$$\left. + \underbrace{(\varepsilon_{sh})_u E_s/(1 + npk_s)}_{(4)} + \underbrace{0.075\,f_{si}}_{(5)} - \underbrace{(m\,f_{cs})}_{(6)} - \underbrace{(m\,f_{cs})(\beta_s C_u)\frac{I_2}{I_c}}_{(7)} - \underbrace{PG_{DS}}_{(8)}\right]\frac{100}{f_{si}} \qquad (24)$$

Camber of unshored composite beams, as per Eq. (20):

$$\Delta_u = \underbrace{(\Delta_i)_{F_o}}_{(1)} - \underbrace{(\Delta_i)_2}_{(2)} + \underbrace{\left[-\frac{\Delta F_s}{F_o} + \left(1 - \frac{\Delta F_s}{2F_o}\right)\alpha_s\,C_u\right](\Delta_i)_{F_o}}_{(3)}$$

$$+ \underbrace{\left[-\frac{\Delta F_u - \Delta F_s}{F_o} + \left(1 - \frac{\Delta F_s + \Delta F_u}{2F_o}\right)(1 - \alpha_s)C_u\right](\Delta_i)_{F_o}\frac{I_2}{I_c}}_{(4)} - \underbrace{\alpha_s C_u(\Delta_i)_2}_{(5)}$$

$$- \underbrace{(1 - \alpha_s)C_u(\Delta_i)_2\frac{I_2}{I_c}}_{(6)} - \underbrace{(\Delta_i)_1}_{(7)} - \underbrace{\beta_s C_u(\Delta_i)_1\frac{I_2}{I_c}}_{(8)} - \underbrace{\Delta_{DS}}_{(9)} - \underbrace{\Delta_L}_{(10)} \qquad (25)$$

Camber of shored composite beams, as per Eq. (21):

Δ_u = Eq. (25), except that the composite moment of inertia is used in Term (7) to compute $(\Delta_i)_1$, and the ratio, I_2/I_c, is eliminated in Term (8). (26)

It is noted that Eqs. (17) - (26) could be greatly shortened by combining terms and substituting the approximate parameters given in the next section, but are presented in the form of separate terms in order to show the separate effects or contributions to the behavior (such as due to the prestress force, dead load, creep, shrinkage, etc., that occur both before and after slab casting).

Grossly approximate equations:

Non-composite beams-- $\Delta_u = \Delta_i + \Delta_i C_u\left(1 - \frac{\Delta F_u}{2F_o}\right)$, $\Delta_i = (\Delta_i)_{F_o} - (\Delta_i)_D$ (27)

Composite beams--$PL_u = \left[n\,f_c\left(1 + \frac{C_u}{2}\right) - n\,f_{cs} + (\varepsilon_{sh})_u E_s + 0.075\,f_{si}\right]\frac{100}{f_{si}}$ (28)

$\Delta_u = \Delta_i + \Delta_i\,C_u\,(I_2/I_c)$, $\Delta_i = (\Delta_i)_{F_o} - (\Delta_i)_2 - (\Delta_i)_1$ (29)

Required calculations and summary of general (average) parameters

Continuous time functions are provided for all needed material parameters (and for different weight concretes, moist and steam cured), so that the equations herein readily lend themselves to computer solution. Certain other read-in data (such as for the effect of behavior before and after slab casting-- α_s, β_s, m, and $\Delta F_s/F_o$) are also included. The parameters related to material properties are summarized below, so that for composite beam hand calculations for example; in addition to the section properties, prestress force, F_o, and concrete stresses, f_c, f_{cs}, the only calculations needed for computing prestress loss and camber are the initial camber, deflections -- $(\Delta_i)_{F_o}$, $(\Delta_i)_2$, $(\Delta_i)_1$, and Δ_{DS}, Δ_L.

The following loss of prestress ratios at the time of slab casting and ultimate are suggested for most calculations:

$\Delta F_s/F_o$ for 3 wks to 1 mth between prestressing and slab casting = 0.11 for Nor. Wt., 0.13 for Sand-Lt. Wt., 0.15 for All-Lt. Wt.

$\Delta F_s/F_o$ for 2 to 3 mths between prestressing and slab casting = 0.15 for Nor. Wt., 0.18 for Sand-Lt. Wt., 0.21 for All-Lt. Wt.

$\Delta F_u/F_o$ = 0.22 for Nor. Wt., 0.25 for Sand-Lt. Wt., 0.29 for All-Lt. Wt.

Note that these are defined as the total loss (at slab casting and ultimate) minus the initial elastic loss divided by the prestress force after elastic loss. The different values for the different weight concretes are due primarily to different initial strains (because of different E's) for normal stress levels.

The following average modular ratios are based on f'_{ci} = 4000 to 4500 psi for both moist cured (M.C.) and steam cured (S.C.) concrete and type I cement; up to 3-mths f'_c = 6360 to 7150 psi (using Eq. 2) for moist cured and 3-mths f'_c = 5610 to 6310 psi (using Eq. 4) for steam cured, and for both 250K and 270 K prestressing strands:

		Modular Ratio	Nor. Wt. (w = 145) M.C.	Nor. Wt. (w = 145) S.C.	Sand-Lt. Wt. (w = 120) M.C.	Sand-Lt. Wt. (w = 120) S.C.	All-Lt. Wt. (w = 100) M.C.	All-Lt. Wt. (w = 100) S.C.
At release of prestress		n =	7.3	7.3	9.8	9.8	12.9	12.9
For the time bet-	= 3 weeks,	m =	6.1	6.3	8.1	8.4	10.7	11.0
ween prestressing	1 month,		6.0	6.3	8.0	8.3	10.5	10.9
and slab casting:	2 months,		5.9	6.2	7.9	8.2	10.2	10.8
	3 months,		5.8	6.2	7.7	8.1	10.2	10.8

$E_s = 27 \times 10^6$ psi for 250 K strands, $E_s = 28 \times 10^6$ psi for 270 K strands,

α_s refers to the part of the total creep that takes place before slab casting ($\alpha_s = \frac{t^{0.60}}{10 + t^{0.60}}$, as per Eq. 9), and β_s (= the avg. Creep $(C.F.)_{LA}$ from

Eqs. 12 and 13) is the creep correction factor for the precast beam concrete age when the slab is cast (under slab dead load). See Eqs. (9) - (11), and the correction factors herein, for suggested values of C_u and $(\varepsilon_{sh})_u$.

The following may be substituted for normal weight, sand-lightweight, and all-lightweight concrete (using both moist and steam curing, and types I and III cement), based on the standard conditions given with Eq. (9):

For the time between prestressing and slab casting:	= 3 weeks,	α_s = 0.38,	β_s = 0.85
	1 month,	0.44,	0.83
	2 months,	0.54,	0.78
	3 months,	0.60,	0.75

Sample calculations

The following numerical substitutions for ultimate loss of prestress at midspan, using Eqs. (24), (28), and ultimate midspan camber, using Eqs. (25), (29), with the general parameters given herein, are made for the sand-lightweight, steam cured, composite bridge girders (with moist cured slab) of this paper[6,33].

Parameters and terms for interior girders

Span = 86 ft, girder spacing = 7 ft, 2-point harping at 0.4L-pt. from end, e (midspan) = 14.3 in, e (end) = 6.2 in, f_{si} = 190,000 psi, F_i = 867 kips, A_s = 4.56 in^2, A_g = 520 in^2, p = 0.00883, I_g = 108,000 in^4, M_D (precast beam) = 410 ft-k, I_c = 334,100 in^4, (using slab width divided by a factor of E_{stem}/E_{slab}= 3.42/3.41 = 1.00), $M_{S,Di}$ (slab plus diaphram moment at midspan) = 630 ft-k.

Modulii of elasticity (using Eqs. 2, 4, and 6 for concrete):

$E_s = 28 \times 10^6$ psi, as suggested for 270 K grade strands herein.

Slab $E_c = 3.41 \times 10^6$ psi, for $f_c' = 3500$ psi, w = 145 pcf.

Precast beam -- (see description of m and n in the section on parameters for the concrete properties)

$E_{ci} = E_s/n = 28 \times 10^6/9.8 = 2.86 \times 10^6$ psi.

$E_{cs} = E_s/m = 28 \times 10^6/8.2 = 3.42 \times 10^6$ psi.

Using F_i, A_t, and I_t, as per Term (1) of Eq. (17) or (19) or (24), f_c = 2567 psi. As per Term (6) of Eq. (19) or (24), f_{cs} =1006 psi. These concrete stresses refer to the midspan section. As per Term (1) or Eq. (19) or (20) or (25), for camber calculations, $F_o = F_i(1 - n\, f_c/f_{si})$ = 758 kips, using f_c=2467psi.

From the parameter section: $n = E_s/E_{ci} = 9.8$; for 2 months period between prestressing and slab casting -- $m = E_s/E_{cs} = 8.2$, $\alpha_s = 0.54$, $\beta_s = 0.78$, $\Delta F_s/F_o = 0.18$; $\Delta F_u/F_o = 0.25$.

From Eqs. (9) and (11), for H = 70%, $C_u = 1.88$, $(\varepsilon_{sh})_u = 510 \times 10^{-6}$ in/in.
From Eq. (10), for diff. shrinkage, $(\varepsilon_{sh})_u = 1.2(560) = 670 \times 10^{-6}$ in/in.

Initial camber and deflection, and differential shrinkage deflection:

$(\Delta_i)_{F_o}$ = 4.09 in, as per Term (1) of Eq. (18) or (20) or (25).
$(\Delta_i)_2$ = 1.74 in, as per Term (2) of Eq. (18) or (20) or (25).

$(\Delta_i)_1 = 2.26$ in, as per Term (7) of Eq. (20) or (25). This deflection is due to the slab and diaphram dead load.

$\Delta_{DS} = 0.49$ in, as per Term (9) of Eq. (20) or (25).

Solutions for interior girders

Ultimate loss of prestress at midspan using Eq. (24):

$$PL_u = \overset{(1)}{12.7} + \overset{(2)}{11.7} + \overset{(3)}{2.8} + \overset{(4)}{6.5} + \overset{(5)}{7.5} - \overset{(6)}{4.3} - \overset{(7)}{2.0} - \overset{(8)}{1.6} = 33.3\%.$$

Ultimate midspan camber using Eq. (25) minus Δ_L:

$$\Delta_u = \overset{(1)}{4.09} - \overset{(2)}{1.74} + \overset{(3)}{3.05} + \overset{(4)}{0.80} - \overset{(5)}{1.77} - \overset{(6)}{0.48} - \overset{(7)}{2.26} - \overset{(8)}{1.06} - \overset{(9)}{0.49} = 0.14 \text{ in.}$$

Ultimate loss of prestress at midspan using the approximate Eq. (28):

$$PL_u = 24.6 - 5.2 + 7.5 + 7.5 = 34.4\%.$$

Ultimate midspan camber using the approximate Eq. (29):

$$\Delta_u = 0.09 + 0.05 = 0.14 \text{ in}, \quad \text{where } \Delta_i = 4.09 - 1.74 - 2.26 = 0.09 \text{ in.}$$

The prestress loss and camber results by the more reliable Eqs. (17) - (20) and (22) - (25), and the approximate Eqs. (27) - (29), are also tabulated in Table 1 for all 5 of the bridge girders, and using both experimental parameters and general or average parameters. Although the agreement above is good (note the camber is near zero due to the slab effect) by these methods, the approximate method may be suitable in many cases for rough calculations only. Also, the calculations needed by the approx. methods are not significantly fewer than by the other methods. The more reliable equations should be preferable for computer use.

Experimental results for a sand-lightweight unshored composite bridge (Fig. 6)[6,33]

The measured and computed (using the general Eq. 20 with experimental parameters) midspan camber versus time curves for 5 bridge girders are shown in Fig. 7. The results are reasonably good, but not precise, and probably indicate the nature of the correlation that might be expected, at best, for this type of behavior.

Computed ultimate loss of prestress and camber results by the general Eqs. (19), (20), with experimental parameters (see footnotes of Tables 1 - 3), the ultimate-value Eqs. (24), (25), with general parameters (given herein), and the approximate Eqs. (28), (29), with general parameters are shown in Table 1. These results, along with the more extensive comparisons summarized in the next section, serve to substantiate the generalized procedure presented for predicting loss of prestress and camber of non-composite and composite prestressed structures. The approximate equations may be suitable for rough calculations only in some cases.

The computed (using the general Eqs. 19, 20, with experimental parameters) ultimate values of loss of prestress and camber are shown term by term in Tables 2 and 3 as an illustration of the separate contributions to the total effect.

The ultimate loss of prestress for the sand-lightweight concrete bridge girders was 29% to 31% (see Fig. 7 and Table 1). It was determined that loss percentages for bridges under similar conditions using normal weight concrete will normally be of the order of 25%; and using all-lightweight concrete will normally be of the order of 35% or higher. Higher losses for the lighter concretes, for example, are due primarily to the lower modulus of elasticity (higher elastic strains for a given stress level), and not, necessarily, to greater creep and shrinkage behavior.

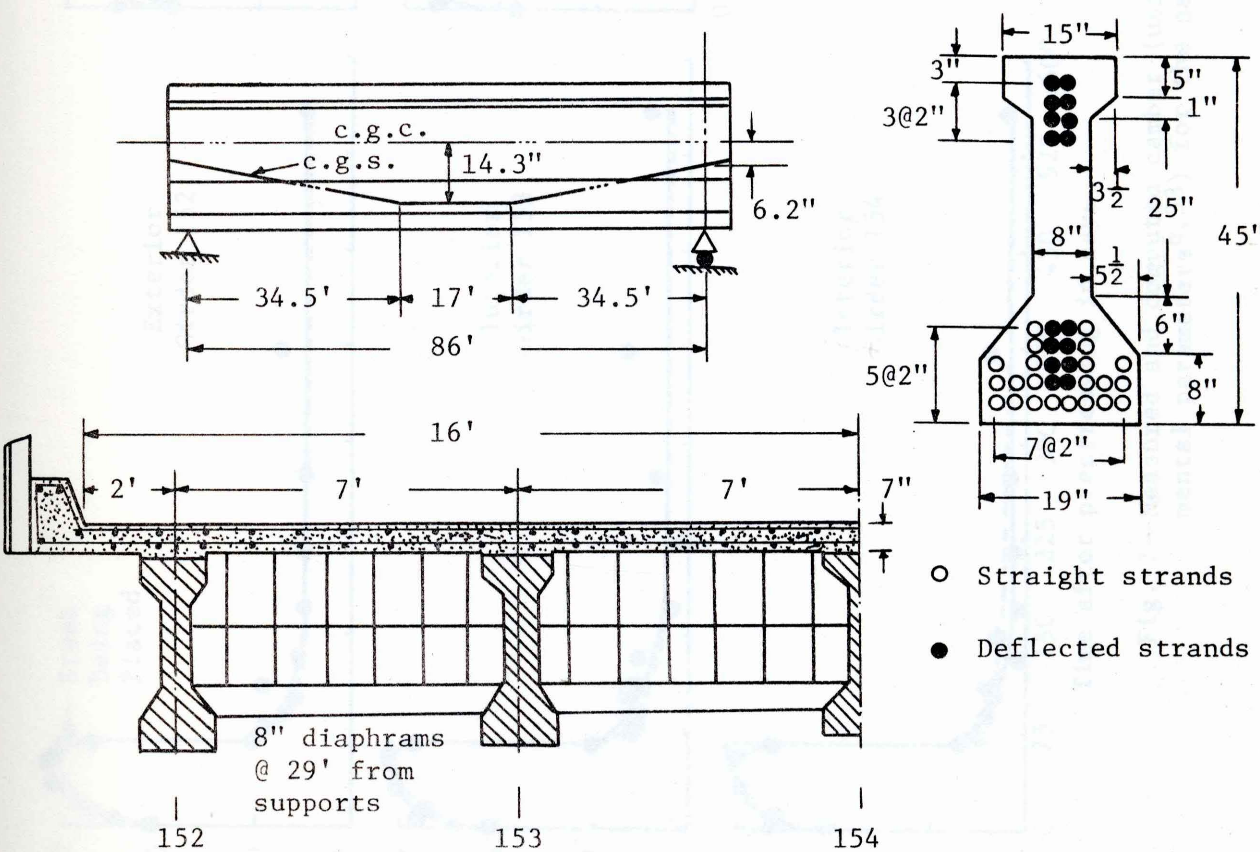

Fig. 6--Sand-lightweight concrete composite prestressed bridge girders with normal weight concrete deck slab[6, 33]

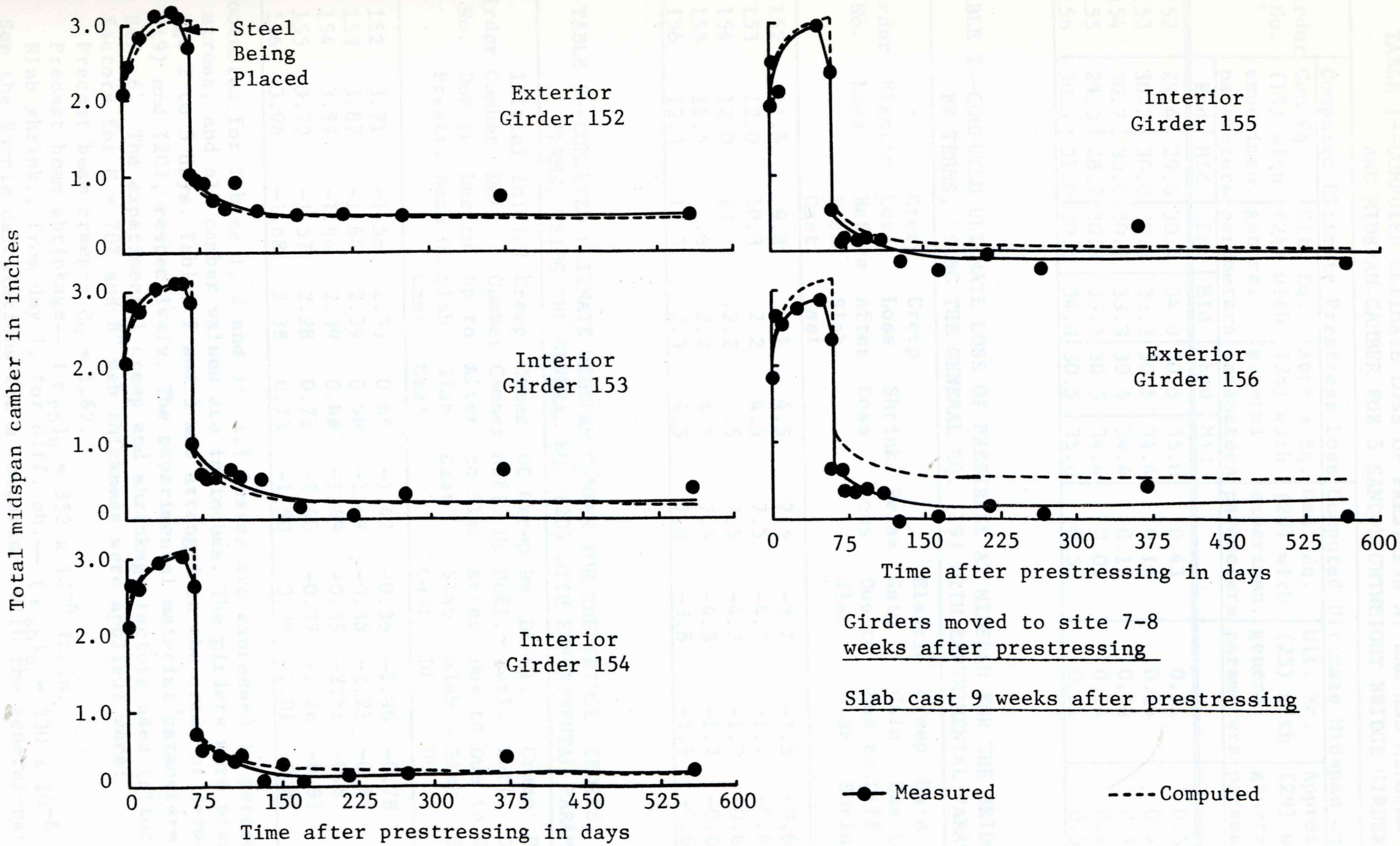

Fig. 7--Measured and computed camber (using Eq. 20 herein with experimental parameters[6,33]) for the sand-lightweight bridge girders

TABLE 1--COMPUTED ULTIMATE LOSS OF PRESTRESS AT END AND MIDSPAN AND MIDSPAN CAMBER FOR 5 SAND-LIGHTWEIGHT BRIDGE GIRDERS

Girder No.	Computed Ultimate Prestress Loss						Computed Ultimate Midspan Camber		
	Gen Eq. (19) with experimen. parameters		Ult. Eq. (24) with general parameters		Approx.Eq. (28) with general parameters		Gen. Eq. (20) with experimen. parameters	Ult. Eq. (25) with general parameters	Approx.Eq. (29) with general parameters
	End	Mid	End	Mid	End	Mid			
152	29.4	29.6	30.4	34.0	30.5	35.0	0.43	0.51	0.53
153	30.2	30.0	30.3	33.3	30.5	34.4	0.16	0.14	0.14
154	30.2	30.0	30.3	33.3	30.5	34.4	0.16	0.14	0.14
155	29.3	28.7	30.3	33.3	30.5	34.4	0.01	0.14	0.14
156	30.5	31.0	30.4	34.0	30.5	35.0	0.50	0.51	0.53

TABLE 2--COMPUTED ULTIMATE LOSS OF PRESTRESS AT MIDSPAN FOR THE BRIDGE GIRDERS, BY TERMS, USING THE GENERAL EQ. (19) WITH EXPERIMENTAL PARAMETERS

Girder No.	Elastic Loss	Creep Loss Before Slab Cast	Creep Loss After Slab Cast	Shrink Loss	Relax Loss	Elastic Gain Due to Slab	Creep Gain Due to Slab	Gain Due to Diff Shrink	Total Loss, Eq. (19)
152	11.5	9.8	2.1	4.5	7.5	-3.7	-1.5	-0.6	29.6
153	12.0	10.3	2.2	4.5	7.5	-4.2	-1.7	-0.6	30.0
154	12.0	10.3	2.2	4.5	7.5	-4.2	-1.7	-0.6	30.0
155	11.5	9.6	2.2	4.5	7.5	-4.3	-1.7	-0.6	28.7
156	12.3	10.3	2.3	4.5	7.5	-3.8	-1.5	-0.6	31.0

TABLE 3--COMPUTED ULTIMATE MIDSPAN CAMBER FOR THE BRIDGE GIRDERS, BY TERMS, USING THE GENERAL EQ. (20) WITH EXPERIMENTAL PARAMETERS

Girder No.	Initial Camber Due to Prestr.	Initial Defl. Due to Beam DL	Creep Camber Up to Slab Cast	Creep Camber After Slab Cast	DL Defl. to Slab Cast	Creep Defl. Up to Slab Cast	Bm. DL Defl. After Slab Cast	El. Defl. Due to Slab DL	Creep Defl. Due to Slab DL	Defl. Due to Diff. Shrink	Total Camber, Eq. (20)
152	3.71	-1.56	2.33	0.65	-1.42		-0.36	-1.96	-0.78	-0.18	0.43
153	3.87	-1.64	2.39	0.68	-1.49		-0.38	-2.21	-0.87	-0.19	0.16
154	3.87	-1.64	2.39	0.68	-1.49		-0.38	-2.21	-0.87	-0.19	0.16
155	3.72	-1.57	2.28	0.71	-1.40		-0.37	-2.26	-0.91	-0.19	0.01
156	3.96	-1.68	2.38	0.73	-1.50		-0.39	-2.01	-0.81	-0.18	0.50

Footnotes for Tables 1, 2 and 3: All losses are expressed in percent of initial stress, and all camber values are in inches. The girders were prestressed at age 2 to 3 days. Tables 2 and 3 are arranged in the order of terms in Eqs. (19) and (20), respectively. The experimental material parameters are given in Ref. 6. The experimental creep and shrinkage factors used (after correction factors for H = 70% and 8" web thickness were applied) were:

Precast beam creep-- $C_u = 1.62$.

Precast beam shrinkage-- $(\epsilon_{sh})_u = 352 \times 10^{-6}$ in/in.

Slab shrink., from day 1, for diff. sh.-- $(\epsilon_{sh})_u = 330 \times 10^{-6}$ in/in.

See the Sample calculations for a comparison with the general-parameter results.

Additional comparisons with data from four other studies[22,23,26,6 with 34]

For each of the four studies cited, the midspan loss of prestress and camber predicted by Eqs. (17) - (20) at various times, and using both experimental material parameters from the papers and general (average) parameters given herein, are compared with the experimental prestress loss and camber from the papers. These tests and comparisons are described in Figs. 8, 9, Table 4, and below.

The University of Florida[22] tests involved ten post-tensioned normal weight concrete laboratory beams of 19' - 6" spans. The cross-sections were 8" by 12" with 5 composite slabs, 2' - 2" by 3", cast on half of the ten beams. The test period was 5 months. The experimental creep and shrinkage parameters were slightly larger than the general creep and shrinkage parameters.

The University of Illinois[23] specimens consisted of two pretensioned non-composite rectangular beams (4" by 6") of normal weight concrete and 6' spans. The beams were observed for two years under laboratory conditions. The experimental creep and shrinkage parameters were somewhat larger than the corresponding general parameters. This is reflected in the results in Figs. 8, 9, and Table 4. The measured modulus of elasticity was also greater than the computed value (based on the compressive strength), and this tended to compensate for the smaller general creep and shrinkage parameters when used to obtain computed results.

The Texas A & M University[26] tests involved five non-composite pretensioned Type B Texas Highway Department bridge girders (4 lightweight and 1 normal weight) of 38' to 45' spans. The girders were observed in the field for a period of one year. The experimental creep and shrinkage parameters were slightly smaller than the general creep and shrinkage parameters.

The University of Iowa[6,34] specimens consisted of fifteen pretensioned laboratory beams (6" by 8") of 15' spans. Twelve were sand-lightweight concrete and three were all-lightweight concrete. Nine of the beams were non-composite and six were composite (slabs 20" by 2" and 20" by 3"). The test period was 6 months for 12 of the beams and 1 year for 3 beams. The experimental creep and shrinkage parameters were slightly smaller than the corresponding general parameters.

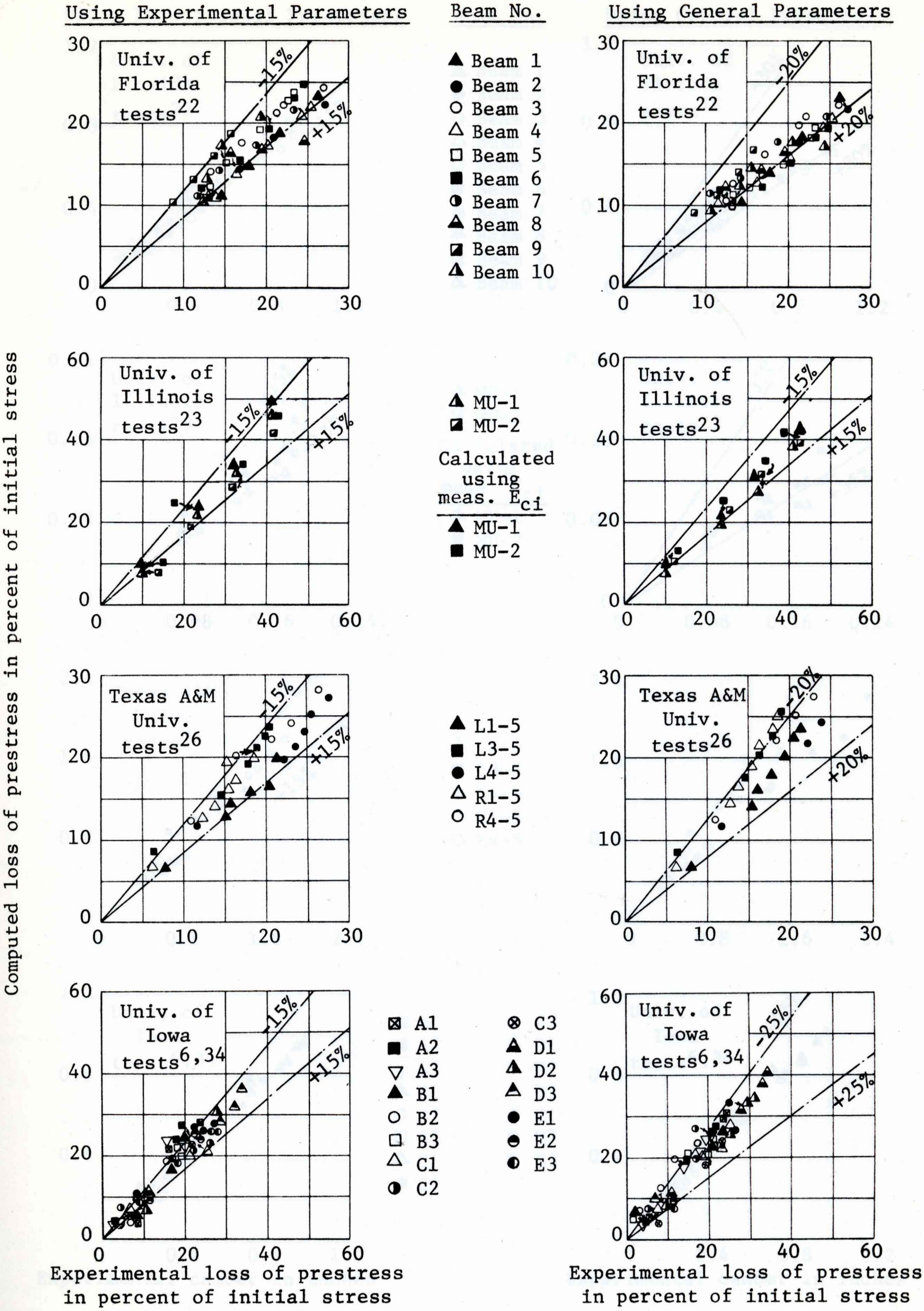

Fig. 8--Comparison of experimental and computed (using Eqs. 17 and 19 with both experimental and general or average parameters) midspan loss of prestress at various ages, for several studies in the literature

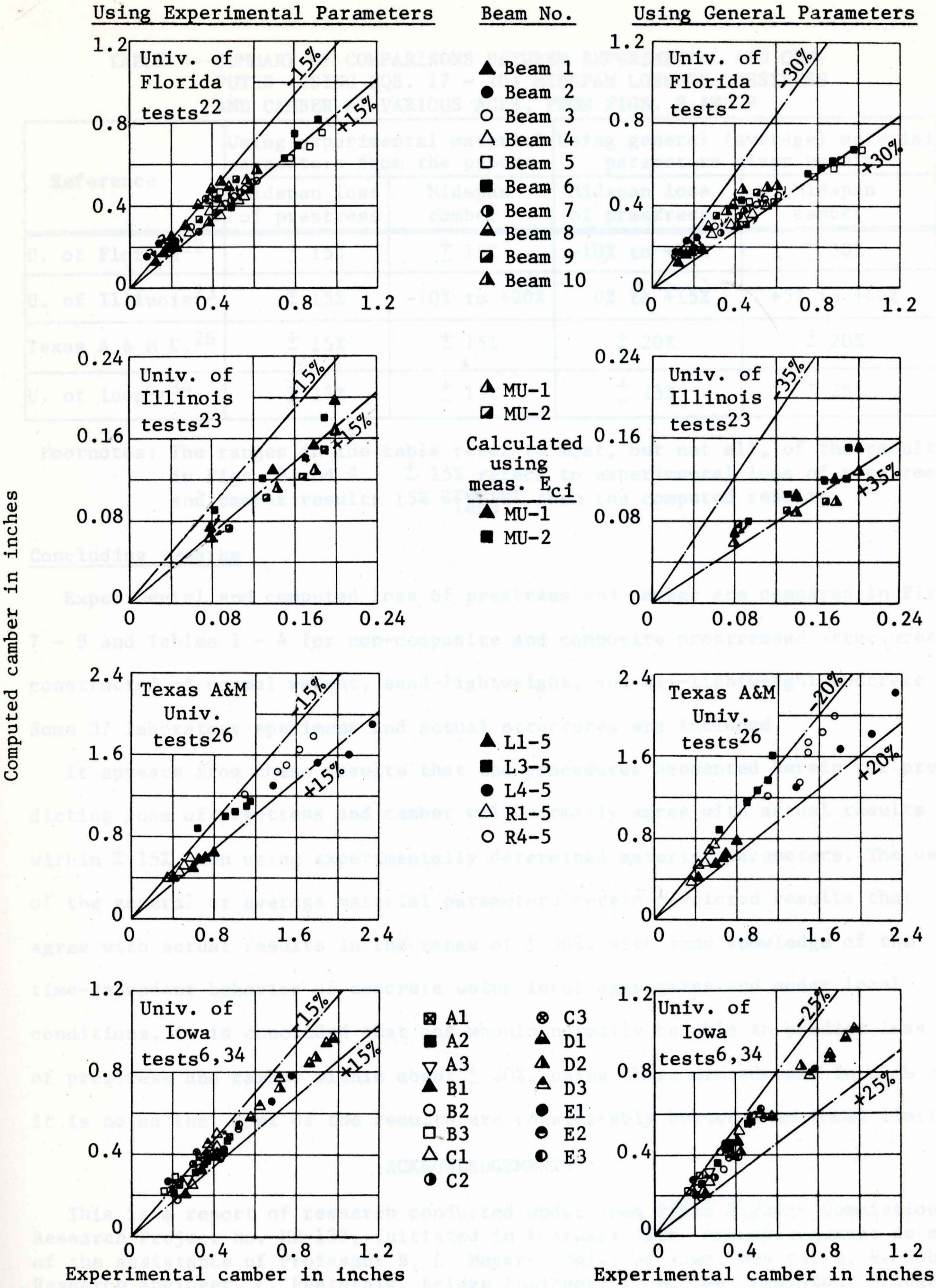

Fig. 9--Comparison of experimental and computed (using Eqs. 18 and 20 with both experimental and general or average parameters) mid-span camber at various ages, for several studies in the literature

TABLE 4--SUMMARY OF COMPARISONS BETWEEN EXPERIMENTAL AND COMPUTED (USING EQS. 17 - 20) MIDSPAN LOSS OF PRESTRESS AND CAMBER AT VARIOUS AGES, FROM FIGS. 8 AND 9

Reference	Using experimental material parameters from the papers		Using general (average) material parameters given herein	
	Midspan loss of prestress	Midspan camber	Midspan loss of prestress	Midspan camber
U. of Florida[22]	± 15%	± 15%	-10% to +25%	± 30%
U. of Illinois[23]	± 15%	-10% to +20%	0% to +15%	+5% to +40%
Texas A & M U.[26]	± 15%	± 15%	± 20%	± 20%
U. of Iowa[6,34]	± 15%	± 15%	± 25%	± 25%

Footnotes: The ranges in the table refer to most, but not all, of the results in Figs. 8 and 9. ± 15% refers to experimental loss of prestress and camber results 15% greater/less than the computed results.

Concluding remarks

Experimental and computed loss of prestress and camber are compared in Figs. 7 - 9 and Tables 1 - 4 for non-composite and composite prestressed structures constructed of normal weight, sand-lightweight, and all-lightweight concrete. Some 37 laboratory specimens and actual structures are included.

It appears from these results that the procedures presented herein for predicting loss of prestress and camber will normally agree with actual results within ± 15% when using experimentally determined material parameters. The use of the general or average material parameters herein predicted results that agree with actual results in the range of ± 30%. With some knowledge of the time-dependent behavior of concrete using local aggregates and under local conditions, it is concluded that one whould normally be able to predict loss of prestress and camber within about ± 20%, using these procedures. In each case, it is noted that most of the results are considerably better than these limits.

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Appendix A

Notation

1 = subscript denoting cast-in-place slab or the effect of the slab

2 = subscript denoting precast beam

A_g = area of gross section, neglecting the steel

A_s = area of prestressing steel

A_t = area of transformed section

C = creep coefficient defined as ratio of creep strain to initial strain

C.F. = correction factor

C_s = creep coefficient at time of slab casting

C_t = creep coefficient at any time

C_{t_1} = creep coefficient of the composite beam under slab dead load

C_{t_2} = creep coefficient due to precast beam dead load

C_u = ultimate creep coefficient

D = differential shrinkage strain. Also used to denote dead load

DS = subscript denoting differential shrinkage

E_{ci} = modulus of elasticity of concrete at the time of transfer of prestress

E_{cs} = modulus of elasticity of concrete at the time of slab casting

E_s = modulus of elasticity of prestressing steel

e_c = eccentricity of steel at center of beam--see Appendix B. Also used to denote eccentricity of steel in composite section

e_o = eccentricity of prestressing steel at end of beam--see Appendix B

F = prestress force after losses

F_i = initial tensioning force

F_o = prestress force at transfer (after elastic loss)

ΔF = loss of prestress due to time-dependent effects only (such as creep, shrinkage, etc.). The elastic loss is deducted from F_i, to obtain F_o

ΔF_s = total loss of prestress at slab casting minus the initial elastic loss

ΔF_t = total loss of prestress at any time minus the initial elastic loss

ΔF_u = total ultimate loss of prestress minus the initial elastic loss

f_c = concrete stress such as at steel c.g.s. due to prestress and precast beam dead load in the prestress loss equations

f_{cd} = concrete stress at steel c.g.s. due to differential shrinkage

f_{ci} = concrete stress at the time of transfer of prestress

f_{cs} = concrete stress at steel c.g.s. due to slab dead load (plus diaphram, etc., dead load when applicable)

f_{si} = initial or tensioning stress in prestressing steel

$(f'_c)_t$ = compressive strength of concrete at any time

$(f'_c)_u$ = ultimate (in time) compressive strength of concrete

H = relative humidity in percent. Also subscript denoting relative humidity

I_1 = moment of inertia of slab

I_2 = moment of inertia of precast beam

I_c = moment of inertia of composite section with transformed slab. The slab width is divided by E_{c_2}/E_{c_1}

I_g = moment of inertia of gross section, neglecting the steel

I_t = moment of inertia of transformed section

K = deflection coefficient. For example, for beams of uniform section and uniformly loaded:

cantilever beam, $K = 1/4$, Also (-) $M = q\ L^2/2$
simple beam, $K = 5/48$, (+) $M = q\ L^2/8$
hinged-fixed beam (one end continuous), $K = 8/185$, (-) $M = q\ L^2/8$
fixed-fixed beam (both ends continuous), $K = 1/32$, (-) $M = q\ L^2/12$

k_s = $1 + e^2/r^2$, where e is the steel eccentricity and $r^2 = I_g/A_g$

L = span length. Also used as a subscript to denote live load

LA = subscript denoting loading age

M = bending moment. When used as the numerical maximum moment, for beams of uniform section and uniformly loaded, see K above for values

M_1 = maximum bending moment under slab dead load

M_2 = maximum bending moment under precast beam dead load

m = modular ratio, E_s/E_{cs}, at the time of slab casting

n = modular ratio, E_s/E_{ci}, at release of prestress

PG = prestress gain in percent of initial tensioning stress or force

PL = total prestress loss in percent of initial tensioning stress or force

PL_{el} = prestress loss due to elastic shortening

PL_t = total prestress loss in percent at any time

PL_u = ultimate prestress loss in percent

p = steel percentage, A_s/A_g

Q = differential shrinkage force = $D\ A_1\ E_1/3$. The factor 3 provides for the gradual increase in the shrinkage force from day 1, and also approximates the creep and varying stiffness effects[6,27]

t = time in general, time in hours in the steel relaxation equation, and time in days in other equations herein

w = unit weight of concrete in pcf

y_{cs} = distance from centroid of composite section to centroid of slab

α_s = ratio of creep coefficient at the time of slab casting to

β_s = creep correction ratio for the precast beam concrete age when slab cast

Δ = maximum camber (positive) or deflection (negative)

$(\Delta_i)_1$ = initial deflection under slab dead load

$(\Delta_i)_2$ = initial deflection under precast beam dead load

$(\Delta_i)_{F_o}$ = initial camber due to the initial prestress force, F_o

Δ_t = total camber, deflection, at any time

Δ_u = ultimate camber, deflection

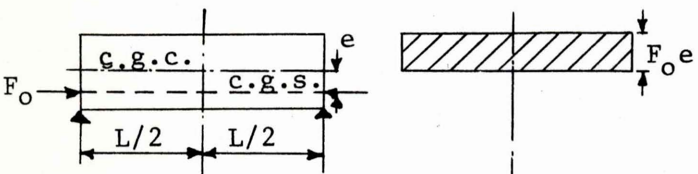
$(\varepsilon_{sh})_t$ = shrinkage strain in inches/inch or cm/cm, etc., at any time

$(\varepsilon_{sh})_u$ = ultimate shrinkage strain in inches/inch or cm/cm, etc.

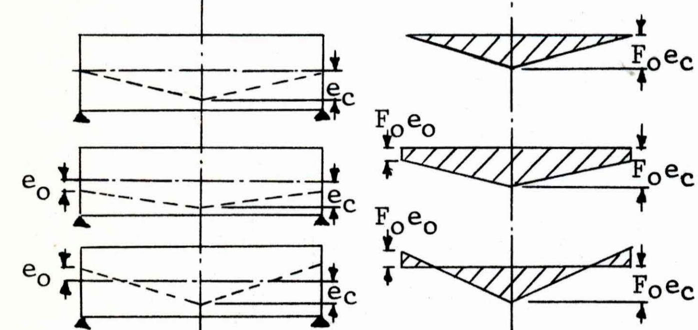
Appendix B

COMMON CASES OF PRESTRESS MOMENT DIAGRAMS WITH FORMULAS FOR COMPUTING CAMBER

Prestressed Beam	F_o e Moment Diagram	Midspan Camber Due to F_o e Moments



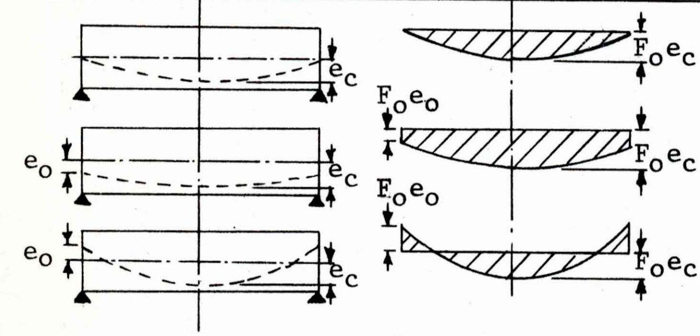
$$(\Delta_i)_{F_o} = F_o e\, L^2/8\, E_{ci} I_g$$



$$(\Delta_i)_{F_o} = F_o e_c L^2/12\, E_{ci} I_g$$

$$(\Delta_i)_{F_o} = \frac{F_o(e_c - e_o)L^2}{12\, E_{ci}\, I_g} + \frac{F_o e_o L^2}{8 E_{ci} I_g}$$

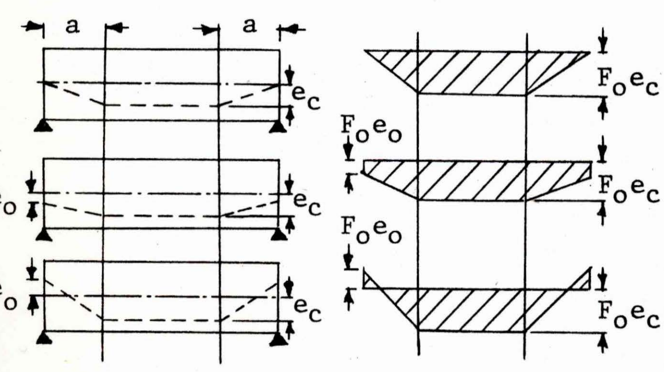
$$(\Delta_i)_{F_o} = \frac{F_o(e_c + e_o)L^2}{12\, E_{ci}\, I_g} - \frac{F_o e_o L^2}{8 E_{ci} I_g}$$



$$(\Delta_i)_{F_o} = 5\, F_o e_c\, L^2/48\, E_{ci}\, I_g$$

$$(\Delta_i)_{F_o} = \frac{5F_o(e_c - e_o)L^2}{48\, E_{ci}\, I_g} + \frac{F_o e_o L^2}{8 E_{ci} I_g}$$

$$(\Delta_i)_{F_o} = \frac{5F_o(e_c + e_o)L^2}{48\, E_{ci}\, I_g} - \frac{F_o e_o L^2}{8 E_{ci} I_g}$$



$$(\Delta_i)_{F_o} = \frac{F_o\, e_c}{E_{ci} I_g}\left(\frac{L^2}{8} - \frac{a^2}{6}\right)$$

$$(\Delta_i)_{F_o} = \frac{F_o(e_c - e_o)}{E_{ci}\, I_g}\left(\frac{L^2}{8} - \frac{a^2}{6}\right) + \frac{F_o e_o L^2}{8 E_{ci} I_g}$$

$$(\Delta_i)_{F_o} = \frac{F_o(e_c + e_o)}{E_{ci}\, I_g}\left(\frac{L^2}{8} - \frac{a^2}{6}\right) - \frac{F_o e_o L^2}{8 E_{ci} I_g}$$

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