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Distribution of Loads in Beam-and-Slab Bridges

by

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Prepared by the

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Distribution of Loads in Beam-and-Slab Bridges

by

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ABSTRACT

A new procedure for predicting the strains and deflections of the beams in simple-span beam-and-slab bridges of the usual proportions has been developed. It divides the calculations into two primary steps:

1. Temporary reactions are assumed at the beams to prevent deflections of the beams, and the loads are distributed to these reactions by the slab acting as a continuous beam.

2. The temporary reactions are removed and the consequent effects on the beams are computed.

Since no deflections or moments are produced in the beams in step 1, the entire effect on the beams is found in step 2. This effect on a beam is assumed to be that of a loading consisting of:

1. a concentrated or narrowly distributed force, the temporary reaction reversed, and

2. a widely distributed force produced by the resistance of the slab to deformation.

Part 2 of the beam loading has been assumed to be sinusoidal, but any other form could be assumed. For the bridges tested the effects of part 2 are relatively small; so the precision of the predictions of maximum strains and deflections is not sensitive to changes in the form assumed.

It is suggested that, pending further study, the use of the procedure be limited to bridges having a ratio of span to beam spacing of 2 or more, and also a ratio of beam to slab stiffness, H, of 2 or more.

To obtain checks on the predictions by the proposed procedure, by the present (1953) AASHO specifications, and by the tentative revisions (T-15-50), four bridges were tested. Two are full-size bridges in use on a highway; their spans are 41.25 ft. and 71.25 ft., and their roadways are 30 ft. wide. The other two were built in a laboratory. They include crown, curbs, and diaphragms; their spans are 10 ft. and 25 ft., and their roadways are 10 ft. wide. Each of the four bridges has four beams equally spaced, has the interior beams larger than the exterior, and is of composite construction. Among the four bridges the span to beam spacing ratio varied from 3.1 to 7.8, and the beam stiffness to slab stiffness ratio varied from 3.0 to 10.7. The loads on the laboratory bridges were either single-axle or tandem-axle trucks; either one truck, alone, or, two, side by side. The load on the highway bridges was a single semi-trailer truck having tandem rear axles.

Strains and deflections were measured at a number of locations at each bridge for various positions of the loads. Of these test results, those

of most interest to designers and those directly comparable to the predictions under the specifications are the maximum strains caused by a given loading when it may be placed in any position. Comparing the predicted maximum strains with those observed, the ranges in percent of error for all the beams, both interior and exterior are:

proposed procedure	+ 11 to −10
AASHO	+87 to − 8
T-15	+106 to +5

It is concluded that the proposed procedure provides improved predictions under a much wider range of conditions than does either specification method. To understand and use it requires no special training, and the time required for its use is only about one hour per analysis; so it should be practical for practicing engineers to use it. It lends itself readily to refinement through further research, and a number of subjects for further research are recommended.

It is further concluded that the present AASHO specifications provide what may be regarded as acceptable predictions of the effects of two trucks side by side, +30 to −8 percent error, but may be grossly in error in predicting the effects of a single truck, +87 to +5 percent error. The tentative revisions are grossly in error in predicting the maximum effects of two trucks on an exterior beam, +106 to +51 percent, as well as in predicting the effects of a single truck, +90 to +34 percent error.

I. INTRODUCTION

A. Description of the Type of Bridge Considered

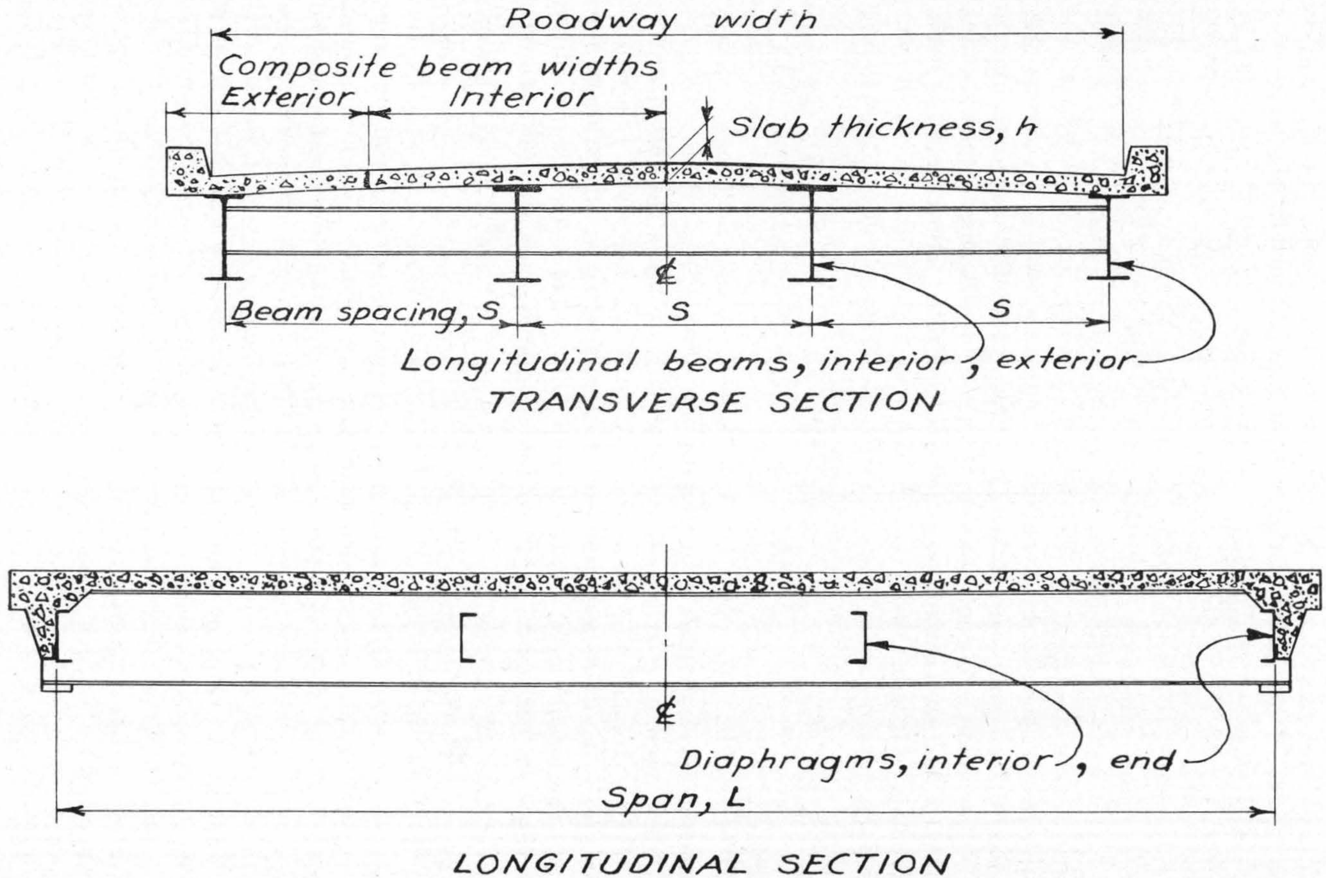
Highway bridges composed of longitudinal steel beams or stringers carrying a reinforced concrete slab are widely used for individual spans, L[1], of from 20 to 100 ft. as well as for the floor systems of truss bridges of longer spans. In these "beam-and-slab bridges" the slab is supported by the beams and is continuous across them. It may, also, be supported at the ends by the abutments. The beams, in turn, are supported at the abutments or at floor-beams transverse to the roadway. The beams are often essentially simply supported, but may be continuous over several spans. They may in theory consist of the steel sections alone, assuming a frictionless surface between the steel and concrete. However, in practice it is found there is substantial bonding between the steel and concrete even when no special attempt is made to secure it. This results in a composite beam composed of the steel and of the concrete contiguous to the steel. In many bridges "shear lugs" are provided at the common face to insure the occurrence of this composite action, and the beams are designed as composite sections.

The number and spacing of longitudinal beams in a bridge varies, of course. One common current (1956) practice is to use a spacing of nearly 10 ft. The corresponding number of beams for a two-lane roadway is three or four. Many designers use a smaller spacing, down to about five ft, and a correspondingly greater number of beams. Also, some of the older bridges still in service may be found to have even smaller spacings. The spacing of the beams is ordinarily the same throughout the width of the bridge. The edge beams may be smaller than the interior beams or they may be made the same size as the interior ones.

The longitudinal beams are usually connected by crossbeams, commonly called "diaphragms," at the ends and at intermediate points. These are likely to be of much smaller section than the longitudinal beams and the tops of the interior diaphragms are often below the concrete; hence they are not composite beams. The end diaphragms are likely to be composite and much stronger than the interior ones, the slab being turned down at the ends and partially encasing the steel. The diaphragms are often provided primarily for temporary use during construction. When they are left in place, as they usually are, they also affect the behavior of the bridge in use.

The reinforced concrete slab is from 6 to 10 in. thick, either uniform

[1]Definitions of symbols are repeated in the Glossary, p. 143, for convenient reference.



TYPICAL SECTIONS
FIG. 1

or variable in thickness. The use of a uniform thickness requires the minimum amount of concrete; hence provides the minimum dead load on the structure. If the slab is uniform in thickness, however, the transverse "crown" and longitudinal "camber" of the roadway must be provided by proper fabrication of the steel. Some designers, therefore, provide crown and camber by using a minimum thickness of concrete at the corners and increasing the thickness in both directions toward the center. Even when this is done the usual practice is to assume a uniform thickness equal to the minimum in computing the strength of the slab or beam.

The concrete is usually reinforced by steel bars in both directions near both faces of the slab. The primary reinforcement is perpendicular to the longitudinal beams and provides for positive moment (tension in the bottom) in the center portions of the panels between beams and for negative moment over the beams.

The general features described above are illustrated in Fig. 1 which shows, in particular, the type of bridge tested as part of the project reported herein. These are characterized by four equally spaced beams, by minimum curbs, by exterior beams smaller than the interior, by slab thicknesses that vary so little they may be assumed to be uniform, and by heavy end diaphragms.

B. Loads

The vertical loads that must be considered in the design of bridge floors are the weight of the structure itself, the dead load; and the forces arising from the passage of highway traffic, the live load and impact.

The determination of the magnitude of the total dead load is relatively easy after a preliminary design has been set up for analysis. Its effects on the individual beams, however, are not readily determined because the construction procedure is not the same for all bridges. One common construction procedure includes supporting the wet concrete on forms built up from the bottom flanges of the steel sections. These forms act as simple beams spanning between the longitudinal beams, with the form for the curb cantilevered from the edge beam. In the construction of the shorter spans the concrete may be placed in such a short time that the last of it is placed before the first part placed has gained appreciable strength. Under these conditions the dead loads carried by the beams are often assumed to be simply the reactions of the simple-beam forms. Also, under these conditions the dead load is assumed to be supported by the steel beams alone, with no composite action possible because the concrete is wet. These assumptions would be entirely correct except for the subsequent effects of such things as shrinkage and plastic flow.

Alternate methods of construction involving partial or complete shoring of the forms or placing the concrete in segments over a period of days or weeks would lead to much more complicated behavior under dead load. Some designers arbitrarily divide the total dead load equally among all the beams to avoid this complication. A study of the many possibilities arising under differing construction procedures is beyond the scope of the project reported herein.

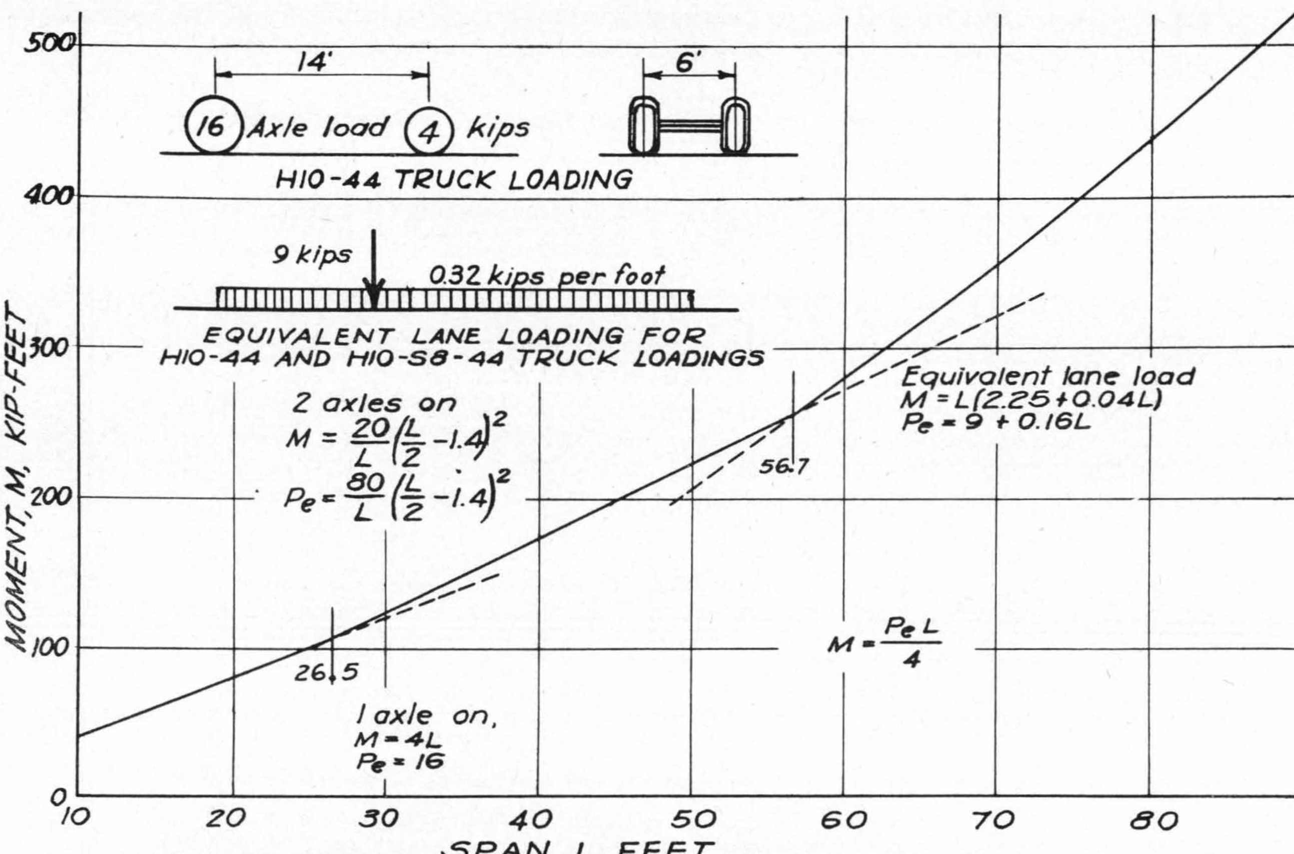
The actual live load occurring on bridges consists of a mixture of vehicles of various sizes and weights, having different characteristics, traveling at different speeds at variable spacings. The determination of the proper static and dynamic loads to use on bridges of different spans is another problem beyond the scope of the current project. Current design practice as established by the specifications of the American Association of State Highway Officials (AASHO) is to design for either (1) a representative heavy truck in each lane, or (2) an equivalent lane load consisting of a uniform load plus a transverse line load in each lane (1, pp 159-163).

The individual design trucks can be regarded as composed of pairs of wheel loads, each pair constituting an axle load. Thus, the three basic types of loading are: (1) axle loads, (2) uniform lane loads, and (3) transverse line loads. Whatever future changes may be made in the specified magnitudes, arrangements, and combinations of these, it seems likely the three basic types will remain. Hence, a generally useful method of analysis must be adaptable to all three types.

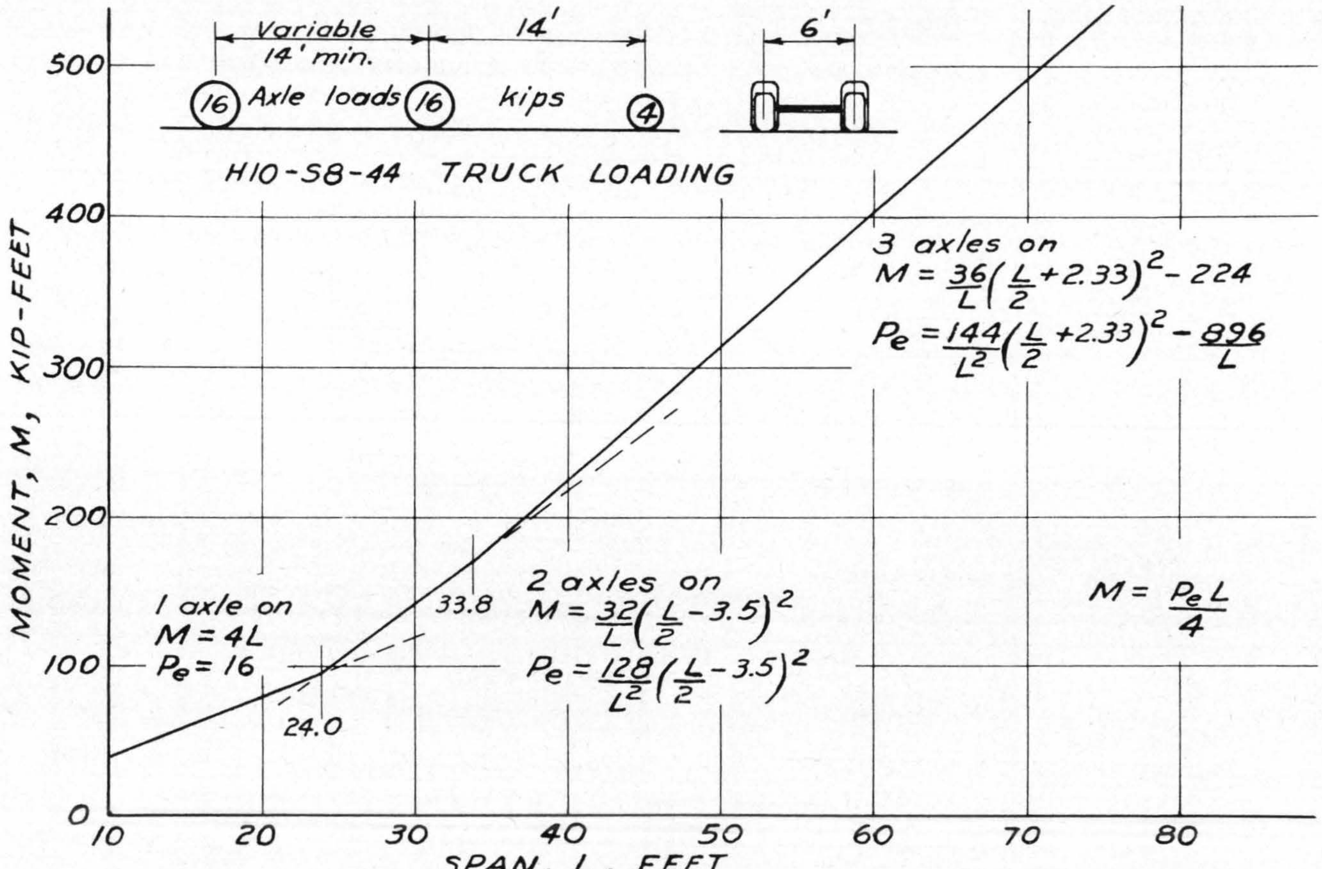
Current specifications provide for two different arrangements of axle loads in the individual design trucks and for a choice of magnitudes for each arrangement. The "HN_1-44" trucks consist of two axles 14 ft. apart having a total weight of "N_1" tons, of which $.2N_1$ tons are on the front axle and $.8N_1$ tons are on the rear. The number, 44, indicates the year, 1944, in which this particular loading was first adopted. The "HN_1-SN_2-44" trucks are the same, but they have another axle load of $N_2 = .8N_1$ tons following the rear axle. These are shown diagrammatically in Figs. 2 and 3 for H10-44 and H10-S8-44 trucks, respectively. Trucks of other weights have the same arrangements. The axle loads, moments, etc., are increased proportionately as the total weight is increased.

In computing the maximum moment, M, caused in a simple beam by these trucks the number of wheels on the span changes as the span increases. The resulting maximum moment curve for the "H10-44" loading is shown in Fig. 2, and that for the "H10-S8-44" loading is shown in Fig. 3. It will be noted that the "equivalent lane load" governs spans above 56 ft. when the "H" trucks are used. The equivalent lane load does not govern until a span of 140 ft. is reached when "H-S" trucks are used.

Further consideration of Figs. 2 and 3 will reveal that the same maxi-



MAXIMUM MOMENTS CAUSED BY H10-44 LOADING
PER LANE, SIMPLE SPAN
Fig. 2



MAXIMUM MOMENTS CAUSED BY H10-S8-44 LOADING

FIG. 3

mum moment versus span diagrams could be obtained by using a single axle-load at the center of the spans and varying the magnitude of this load as the span varies. Formulas for this variable load, P_e, are shown in the figures. The use of such a variable "equivalent" concentrated load in place of the currently specified axle arrangements would probably simplify the work of the designer, particularly when relatively complex analyses are undertaken.

C. Design and Analysis Problems

For complex structures a direct design procedure is not to be expected. Instead, it is customary to arrive at a final design by a series of successive trials. A trial design is set up, analyzed to find the critical stresses and stress distribution, and revised in the light of these stresses. Analysis and revision are repeated until satisfactory stress patterns are obtained. Thus, if methods for analyzing the structure are available, a design is possible. The remainder of this discussion will, therefore, be concerned only with analysis, with the understanding it is to be used as part of the design procedure.

In the analysis of simple span beam-and-slab bridges the critical stresses are normally those in the bottom flanges of the beams at or near the centerline. The variation of stress along the beams is of interest only if the size of the beams is to be varied; as with cover plates. The maximum shear stress in the web of the beam near the supports must be investigated, but experience teaches that it seldom controls the design, particularly when steel stringers are used. Similarly, the maximum compressive stresses in the concrete are of some interest but are seldom critical.

As indicated in the discussion of loads, the usual design loading consists of a truck or lane loading in each lane. The basic problem in analysis is to find the maximum stresses caused as these loads are moved laterally and longitudinally on the bridge.

In addition to determining the effects of the standard loading it is often necessary to determine the effects of a single non-standard vehicle; in particular, of an overweight one. These do occur, even though illegal, and it must be assumed they will pass through the positions at which their effects will be greatest. There are, also, the legal "permit" loads operating under controlled conditions. They can be required to cross bridges in the most favorable lateral position; for instance, along the centerline.

D. Present Methods of Analysis

In the analysis of beam-and-slab bridges it has been the convenient practice to consider the slab and the beams as separate members even though the material of the slab may also be part of the material of the beams. When composite action is assumed, each beam is considered to be composed of a steel section and of the contiguous concrete within specified limits (1, p. 250). Curbs and sidewalks may or may not be included as part of exterior beams depending on their dimensions, the construction procedure, and the judgment of the designer. Actually, of course, the structure acts monolithically with no division into parts as assumed. However, analysis as a monolith seems to be a desirable goal beyond practical reach at present. Therefore, the practice of referring to the "beams" and the "slab" as though they were discrete entities is continued herein. Assuming such separate action, the basic problem in the analysis of a beam becomes that of determining the load that will be carried by the beam or of determining some equivalent load known to cause essentially the same effects as the true load. Once the load is determined; the moments, stresses, and other effects can be computed by ordinary methods.

When a load is applied to one of these bridges the slab distributes most of it to the beams but may carry part of it directly to the abutments. As the load is distributed to the beams it is spread longitudinally as well as laterally so that the effect on a beam becomes that of a non-uniform distributed load even when the applied loads is uniformily distributed or is concentrated. Under the assumption of separate slab and beam action, the loads on the beams are the reactions of the slab. The division of the total load among the beams and the distribution along the beams would seem to depend on many different variables. Among these are the:

arrangement of the applied loads,
longitudinal location of the applied loads,
lateral location of the applied loads,
span of the beams,
number of beams,
spacing of the beams,
elastic constants, EI, of the beams,
variation of these constants along the beams,
thickness of the slab,
variation of slab thickness, if any,
elastic properties of the materials used in the slab,
width of the roadway,
number of lanes of traffic assumed,
dimensions of the curb, and
size, spacing, and manner of connecting the diaphragms.

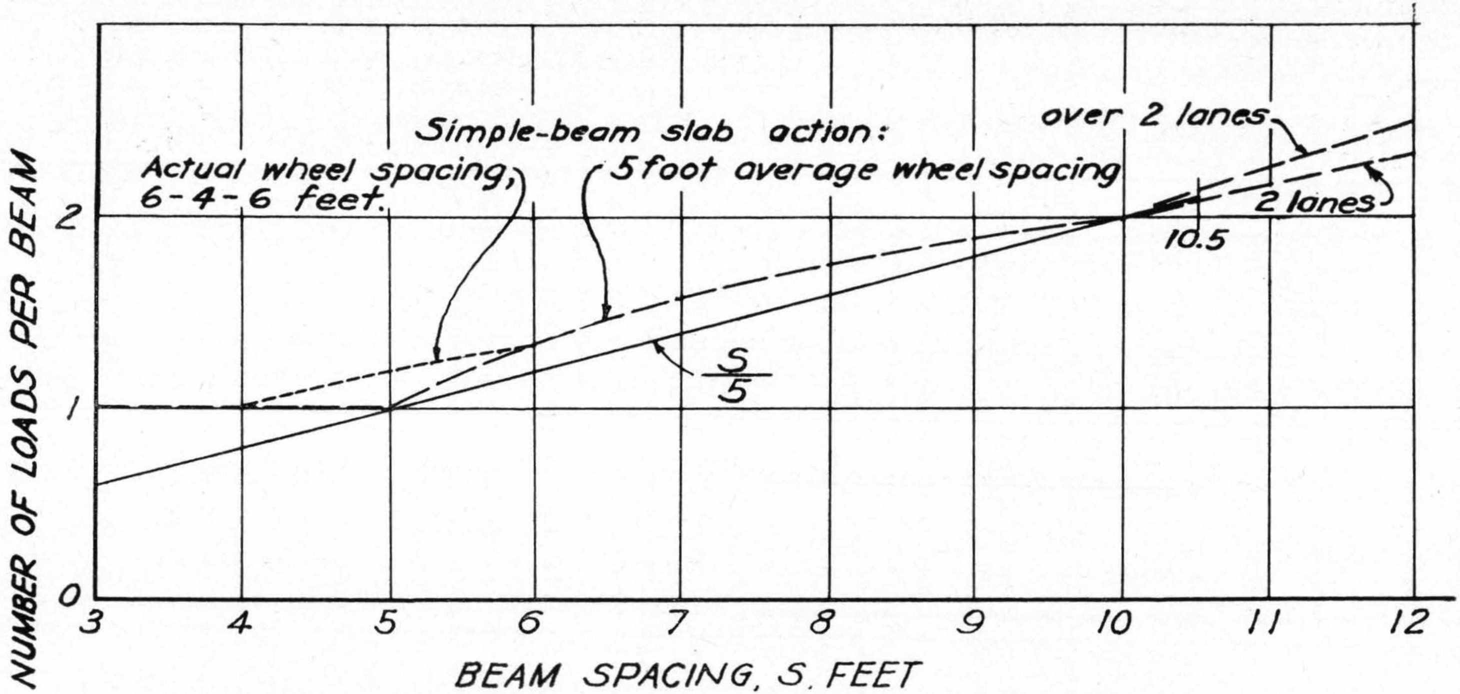
In spite of the many variables involved, the current AASHO specifications provide for determining the loads on beams by greatly simplified procedures. Both interior and exterior beams are to be loaded with concentrated loads in the same longitudinal pattern as those in the standard trucks. For interior beams the amounts of these loads are specified as S/5 times the standard truck wheel load if two or more lanes of traffic are acting, S being the average beam spacing in feet. If only one lane is acting, the formula is changed to S/6. For exterior beams the amounts are to be determined as the reactions of the slab when it is assumed to be simply supported by the beams and longitudinal bending in the slab is ignored. No provision is made for unusual loadings, for loads restricted to a particular lateral position, for changes in the relative size of the beams, or for variations in the slab thickness. Nothing is said about the position of the curb with respect to the outer beam.

It seems that little is gained by having different rules for the interior and exterior beams. As shown in Fig. 4 the S/5 formula gives results that differ only slightly from those that would be obtained by assuming simple beam action for the interior beams, as is done for the exterior.

No discussion of the reasons for either of the regulations is given in the specifications. Presumably the simple-beam provision for exterior beams follows the old rule that if something is designed as though statically determinate and then built continuous it will be safe. Presumably, also, the S/5 provision recognizes that in a fully loaded bridge of infinite width the average load per beam would be S/5 wheel loads because the average lateral spacing of wheels of the standard trucks is 5 ft. This, of course, ignores longitudinal bending and torsion in the slab.

Applying the AASHO provisions to bridges having the dimensions of those tested (Figs. 14 and 15) results in designing the exterior beams for approximately 55 percent as much live load as that for which the interior ones are designed. A recently proposed revision of the specifications (unpublished) would require that the exterior beams be designed for at least as much live load as are the interior ones. The required size of the exterior beams would thereby be increased without a compensating decrease in the size of the interior ones.

As an alternative to the simplified analysis provided by the specifications the general differential equations of flexure of elastic slabs and beams are available. As discussed in more detail in the chapter reviewing the literature, solutions of these equations for some particular conditions are available. These solutions, in general, yield results in the form of equations that are complex and cumbersome even though simplifying assumptions are made in their derivation. A general method for obtaining numerical solutions to particular problems is also available. It, also, seems to be too cumbersome for ordinary use in that it has not been adopted in engineering practice or even to any great extent as a research tool.



LIVE LOAD ON INTERIOR BEAMS
BY AASHO SPECIFICATIONS AND
BY SIMPLE-BEAM SLAB ACTION
FIG. 4

E. Objectives of the Project

As indicated in the foregoing discussion there is not available at present a generally satisfactory method of analysis for the beams of beam-and-slab bridges. A method is needed that is simple and brief enough for understanding and use by busy practicing engineers having no special training and that yields results with a precision consistent with that of the loads, dimensions, and the properties of the materials. Ideally the method should be capable of taking into account most of the variables found in these bridges without undue complication. It should permit refinement through increased time and labor in calculations and as experience, judgment, or future research provide added information concerning the effects of any assumptions made.

The primary objective of the project reported herein was to discover such a method of analysis. Since no truly exact method of analysis is available as a standard, the value of a suggested method or procedure must be decided on the basis of comparisons between predicted strains and deflections and those actually observed in tests. Such tests, then, became a necessary part of the project, and were made.

A secondary objective was to determine through tests whether or not bridges of the particular type tested are safe and well proportioned when designed according to the current AASHO specifications. Another secondary objective was to determine whether or not the proposed revisions of the specifications would provide a better prediction of the behavior of these bridges than do the current specifications.

F. Outline of the Project

The accomplishment of these objectives has been attempted through the following main steps that were, in general, not distinct and separate.

1. A review of the pertinent literature.
2. The development and refinement of the proposed new analysis procedure.
3. Applications of the method in analyses of the bridges tested.
4. Field tests of highway bridges 30 ft. wide.
5. Laboratory tests of bridges 10 ft. wide.
6. An analysis of the data from the tests.
7. Comparisons between predicted and measured results.
8. The preparation of this report of these activities.

The report developed in the subsequent pages will be seen to consist of the following principal divisions.

1. A review of the literature.
2. The proposed analysis procedure.

3. A description of the tests made.
4. The results of these tests including comparisons between predicted and measured values.
5. The conclusions that may be drawn from the foregoing.
6. Recommendations for further research.
7. A list of references.

II. REVIEW OF LITERATURE

The investigation of load distribution to the beams supporting a slab constitutes a part of the over all study of slab behavior. This study is said to have begun with Euler around 1766, and to have been advanced by such savants as Lagrange, Navier, and Poisson. The history of this early development of the subject has been reported by various writers including Todhunter and Pearson (2), and Love (3). It was summarized in 1921 by Westergaard (4) as part of a paper in which further advances were also presented. Study of the latter reference reveals that at the time of its publication the status of the problem was briefly as follows.

1. A theoretical foundation had been developed.
2. The general slab equations had been applied in a limited number of specific cases and solutions yield-numerical results obtained.
3. A limited amount of physical testing had been carried on to yield empirical equations for use in analyzing slabs of the particular types tested.

While the development of the subject had not yet progressed to the point where results useful to the present study were obtained, Westergaard did point out the limitations of the theory (4, p. 423). These limitations are equally applicable to the results obtained up to 1921 and to those that have been obtained since or may be obtained in the future. They are, in brief, as follows.

1. The plates are medium-thick, that is, they are not so thick in proportion to the span that vertical stresses (shears, tensions, and compressions) absorb an appreciable part of the energy of deformation, nor so thin that the tension and compression in the middle plane are significant.
2. The plates are homogeneous and of uniform thickness.
3. Hooke's law applies to the horizontal strains, and the modulus of elasticity is the same for tension and compression.
4. A straight line drawn vertically through the plate before bending remains straight after bending.

Following the discussion of the theory summarized above. Westergaard and Slater (4) presented numerical results obtained from the theory as applied to slabs supported on four sides and to slabs supported on column capitals. They also reported and analyzed extensive load tests of such slabs. While none of the detailed results reported is directly related to the present subject, one observation made is of general interest in slab analysis and testing as follows (4, p. 512):

> The tests of slabs supported on four sides indicate that when the deformations increase, certain redistributions of moments and stresses take place with the result, in general, that the larger co-

efficients of moments are reduced. The ultimate load is found to be, in general, larger, and in some cases much larger, than would be estimated on the basis of the theoretical moment coefficients and the known strength of beams with the same ratio of steel.

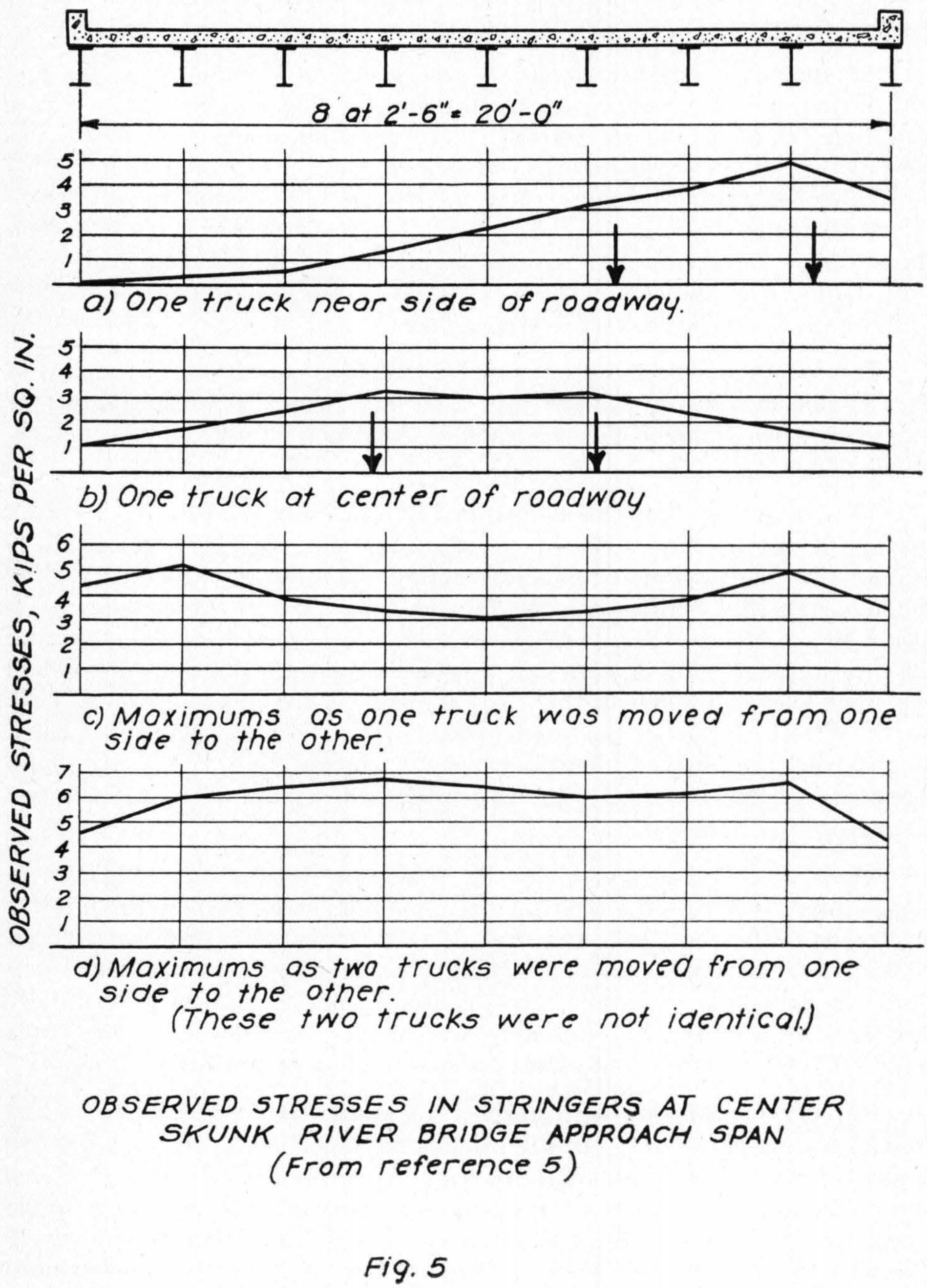
Although the preliminary theoretical developments had been made, many years were to pass before application of the theory to the solution of the present problem were attempted. Meanwhile, however, physical testing of bridge floors for the purpose of obtaining some immediate answers of practical value had begun. Probably the first tests to determine load distribution to the stringers of bridge floors were reported by Agg and Nichols of Iowa State College in 1919 (5). These tests were conducted on bridges having steel stringers but having timber floors loaded with flat steel wheels. They are, therefore, primarily of historical interest, The detailed results are of little value now, but the testing procedures and analyses of the data set the pattern for subsequent test programs.

Strains were measured along the lower flanges of the steel beams for various positions of the loads. The strains were converted to equivalent moments, and the moments were converted to equivalent numbers of wheel loads per stringer. This was probably the first time the load carried by a stringer was expressed in this way. One interesting observation was that the strain increased uniformly from the ends of a beam toward the load as it would in a theoretical simple beam. It, was, thus concluded that the longitudinal distribution of the load was negligible. It will be shown subsequently that the assumption of negligible longitudinal distribution has been carried in specifications to the present, and it is continued as a first approximation in the method of analysis later presented herein.

The project at Iowa State College was continued under the direction of Fuller, Caughey, and others (6, 7). Full-scale bridges were tested with the primary objective of determining impact factors for the various components of then typical bridges. As a part of the overall project, a study of the static load distribution to the stringers was made, also.

The bridges tested had either timber or reinforced concrete floors supported on steel stringers. Among those having concrete floors the stringer spacing was constant within each bridge, and was either 28.5, 29.5, 30, or 36 inches. The spans were 14 ft. to 32 ft. 8 in., and the slab thickness was either 6 in. or 8 in. Some spans had interior and exterior stringers of the same size; others had exterior stringers somewhat smaller than the interior ones.

Various loads were used, but among them was a loaded truck closely approximating an "H15-44" standard truck. The results from tests in which it was used are of the greatest present interest. The trucks used hard rubber tires and were different in other ways from modern trucks.



OBSERVED STRESSES IN STRINGERS AT CENTER
SKUNK RIVER BRIDGE APPROACH SPAN
(From reference 5)

Fig. 5

While these differences probably had a major effect in the impact tests, it seems likely they were of little significance in the static load tests.

Strains were measured with an assortment of mechanical gages and with a then newly developed electrical telemeter that had been reported by McCollum, Burton, and Peters (8). One mechanical gage, the West extensometer, and the telemeter were judged most useful and adopted for the major portion of the work. It is of interest to note that the operation of the telemeter depended on the variations in resistance of a pile of carbon plates. This variation was measured by means of a Wheatstone bridge circuit. Thus, this telemeter can be regarded as the predecessor of the now widely used electrical resistance strain gages.

The observed strains were converted to equivalent stresses by means of an assumed modulus of elasticity of 29,000,000 psi. The resulting stress data were then summed up by means of diagrams similar to the ones reproduced herein as Fig. 5.

The particular diagrams shown in Fig. 5 are for the 32 ft. 8 in. span. In this bridge all nine stringers were of the same size (15 in., 43 lb. I-beams) but the amount of concrete acting with the exterior beams was smaller than that acting with the interior ones. Thus, the composite exterior beams had smaller moments of inertia and section moduli than did the interior ones. Fig. 5 shows, a) a typical stress distribution curve when one truck was near a side of the roadway, b) a typical curve when one truck was at the center, c) the maximum stresses (strains) observed in each beam as one truck was moved laterally across the bridge, and d) the maximum stresses (strains) observed when two trucks side by side were moved laterally across the bridge.

These typical diagrams show that when two trucks side by side were moved laterally across the roadway the maximum stresses in all the interior beams were essentially the same, Fig. 5d. The maximum stresses developed in the exterior beams were substantially smaller than in the interior ones. They also show the maximum stress caused by one truck was substantially smaller than the maximum caused by two trucks. In addition, if the one truck could be kept near the centerline, the maximum was reduced still more.

Further conclusions with respect to static load distribution to the stringers were as follows. In the bridges having concrete floors, with two trucks side by side on the span, the maximum observed stress (strain) varied from 0.60 to 0.69 times the total observed stress attributable to a single wheel load. The 0.60 value was the average when the stringer spacing, S, was 28.5 in., and the 0.69 value was for the 36 in. spacing. These values are seen to be somewhat higher than the current AASHO specification of S/5 (8, p. 168) would yield. By this specification the values would be 0.48 and 0.60, respectively. However, it was further stated that the observed stresses were far below those predicted by "usual" methods. Various reasons for this difference were suggested, but data were not available for substantiating the hypotheses.

Another historically important observation was that the steel beams and concrete slab did act together as composite "T-beams" even though no special provision had been made to insure such action.

It is believed the tests described above formed the basis for the first formal specification as to distribution of wheel loads to stringers. This was indicated by Fuller in one of several papers reporting new tests on the same bridges after 25 years of service (9, 10, 11). He stated that "The present AASHO distribution of load is changed slightly from the original which (as far as the author knows) first appeared in the 1923 specifications of the Iowa Highway Commission." (9, p. 8). During this period there were, of course, many changes in the trucks in common use and in the type of bridge commonly built; thus, more extensive changes in the specifications might have been expected.

In the 1948 tests roughly the same procedures were followed as in the earlier ones except that relatively few data were taken. A modern truck having dual pneumatic tires and tandem axles was used, as were modern strain measuring instruments. The thickness of the slab had been increased from 6 in. in 1922 to 9 in. in 1948. Both visual observation and test results indicated that in 1948 the 32 ft. 8 in. approach span still retained full composite action but that the shorter span, part of the floor system of a truss bridge, had lost practically all composite action.

Because of these various changes the observed load distribution factors (maximum fraction of a wheel load carried by one stringer) changed between 1923 and 1948. However, it seems significant that the change was essentially the same for both spans even though one had retained full composite action and one had been reduced, very nearly, to separate beam and slab action. As a specific example, for the side position of the loads (W and Y) the averages were as shown in Table 1 (9, p. 7).

Table 1. Load distribution factors measured in 1925 and 1948

Bridge	Distribution factor in 1925	Distribution factor in 1948
West approach (32 ft. 8 in.) (Composite throughout)	.24	.19
West panel (18 ft. 9 in.) (Composite-noncomposite)	.245	.185

This is interpreted as an indication that the distribution factor is not sensitive to the presence or absence of composite action, hence, is not sensitive to the absolute size of the beams. As in 1923, the observed stresses were substantially lower than predicted by "usual" methods.

It should perhaps be emphasized that the procedure used in determining the foregoing factors was simply to divide the observed stress for a beam by the total of the observed stresses for all the beams. This, in effect, assumed that the longitudinal distribution of the load, hence of the moment and stress, was the same for every beam. This assumption undoubtedly introduced errors which could have been evaluated only by much more elaborate experimentation.

It was emphasized in the reports that "Practically all of the available information on the behavior of bridge floors has been obtained in situations where the load was inadequate to develop stresses which even approached design values." (9, p. 4), and that "Although these deductions may reflect correctly the small unit stresses developed by the available live load, no information is available for extending the results to fully loaded structures." (10 p. 402). This is a limitation that usually applies to present-day test results, also.

Another early testing project of some interest was described by Davis in 1927 (12). He reported extensive testing of two slabs simulating the then proposed floor for the Delaware River Bridge at Philadelphia. In those tests the behavior of the slab was the primary concern, but some deflection and strain measurements on the steel beams were made. The reactions of the steel beams were measured, also.

There were no shear connectors between the steel beams and the concrete; the only diaphragms were at the ends of the spans, and they were relatively flexible. Loading was limited to a single semi-concentrated load and only two positions of the load were studied. A feature of the tests was the repeated application of impact loads of various magnitudes. This impact loading caused, among other things, changes in the properties of the structures that were attributed to the breaking of the bond between the slab and beams with a consequent decrease in the T-beam action. This was in contrast with the behavior of one of the Iowa bridges re-tested after 25 years of service and found to have retained its composite action (9, 10, 11).

While the tests served the purpose for which they were intended, that is, to determine if the bridge floor was adequate as designed; they were too limited to support general conclusions, and no such conclusions were drawn. Considerable pioneer work was required in instrumenting the tests to obtain the data desired, and this work has undoubtedly benefitted subsequent investigators. Also, the recognition of the problems created by temperature changes, lapsed time during loading, and rotation of the steel beams about their longitudinal axes must have been helpful in later research.

The analytical study of bridge floors was advanced by Westergaard in 1930 (13). His paper included, again, the derivation of Lagrange's fundamental equation for slabs, and went on to develop formulas for various arrangements of concentated loads or of loads uniformly distributed over small circular areas. In general his analyses were for single span, infinitely long slabs simply supported on rigid supports. His analyses were directed primarily at the determination of moments in the slab and of "effective widths" for moment. He did, however, include the derivation of a formula for the distribution of the reaction along a supporting beam. It was shown that this theoretical distribution took the form of a sharply peaked bell shaped curve. The position of the resultant of one half of this curve was shown to vary only slightly as the position of the load varied, remaining near 0.2 times the span of the slab from the peak. If this condition can be taken as qualitatively indicative of the distribution when the slab is continuous over several beams, it provides some further indication that the error introduced by disregarding longitudinal distribution along the beam as small.

The analytical study was continued by Holl, who presented formulas in complex infinite series form for slabs of finite width having free edges (14). Except for the width, the conditions of his analyses were the same as for those of Westergaard. The indicated distribution of the reaction pressure remains essentially the same as that described above.

An experimental investigation of the reaction distribution has been reported by Spangler (15, 16). The slabs tested were simply supported, had free edges, and were of finite width. They ranged in thickness from 2½ to 12 in., in span from 3½ to 10 ft, and in width from 5 to 20 ft. The distribution of the reaction was measured for many different positions and magnitudes of applied load, and for a variety of sizes and types of contact area. The measurements indicated the same type of peaked reaction distribution indicated by theory. Of the conclusions reached, the one of present interest is that the effective width for shear at the edges (distribution of the reaction) is essentially the same for loads in all positions along a line perpendicular to the supports. This is, also, in accord with Westergaard's work described above.

Similar, more extensive tests conducted at the University of Illinois and including reaction measurements gave reaction distributions of the same general type (17). However, these results were not regarded as satisfactory because of excessive deflections within the reaction-measuring supports. It was shown both by the tests and by theory that this distribution is extremely sensitive to slight deflections of the slab support (17, p. 68). Spangler's tests mentioned above were less subject to this error because of his methods of measurement. On the other hand, the slab support in bridge floors is provided by relatively flexible beams, so the results of tests on rigid supports can only be qualitatively useful, at best.

A relatively extensive, long-range program of investigation of the behavior of slabs in general, including slabs supported by steel beams, was undertaken at the University of Illinois in 1936. It has been continued intermittently to the present (1956). Various phases of this program have been reported in one or more of a number of papers, to one of which reference has already been made (17). Those most pertinent to the present discussion are reviewed as follows.

In a paper (18) discussing all the work done to that time (1954) Newmark and Siess summarized the general method of attack thus: "First, analyses were made to establish the variables and to aid in the planning of tests. Next, tests were made on laboratory specimens, usually scale models of highway bridges. And finally, recommendations for design were developed, based on the results of both the analytical and experimental studies." (18, p. 32).

In the first analytical study reported, by Jensen, analyses were made by the classical procedure of obtaining solutions of Lagrange's differential equation for the deflection of a slab (19). Solutions for a number of special cases were derived. Of these the most complex was a symmetrical system composed of a slab and of three beams. The solution for this system might be used directly in the design of such a bridge. The solutions were, in general, presented in algebraic forms consisting of infinite series of varying complexity. Even though the cases studied were not extended to include structures of the usual complexity, it was stated that, ". . . . by their very cumbersomeness, (they) indicate that other methods of analysis should be applied . . . ."; and that, ". . . . the formulas are not suitable for direct use in design. . . ." (19, p. 9).

In the next paper in the series, Newmark described a broadly applicable method of analysis primarily useful for obtaining numerical results in particular problems rather than general formulas (20). "The essential features of the procedure are similar to, and derived from, the moment-distribution method of analysis . . . ." (20, p. 8). And, "The procedure . . . bears somewhat the same relation to other procedures and the formulas derived thereby as the moment-distribution procedure for continuous frames bears to the slope-deflection method . . ." (20, p. 8).

In this procedure the first step is to divide the design load into components varying sinusoidally along the longitudinal axis, that is, to express it as a Fourier series. It was shown that each component, each term of the series, can be handled separately and its effects found. The total effect, moment, reaction, or other function is, then, the sum of the component effects; as many components must be treated as is necessary to obatin the desired precision. The method is not applicable if the beams vary in section or if the slab thickness varies between beams.

The determination of the effects of each component is accomplished through a distribution procedure using factors resembling conventional stiffnesses, carry-over factors, and fixed-end moments. Derivations of these constants by application of ordinary slab theory to a single panel were presented, as were extensive tables of such constants for panels of various proportions. In each distribution procedure the number of "stiffness factors" needed for each panel is four, the number of "carry-over factors" needed is three. The paper included a detailed description of the procedure and numerical examples of its application.

A useful general relationship emphasized in the development of the method is that each sinusoidal load component produces moments, reactions, and deflections of the same sinusoidal form (20, p. 15).

In spite of the great ingenuity of the method described above and in spite of its potential value as a research tool, it does not seem to be suited to general use. This conclusion is indicated by the fact no instances of its application have been reported except in connection with the University of Illinois project. As part of the Illinois project it was used in the analysis of a series of 20 basically different bridges, as reported by Newmark and Siess (21). Even though the method remains subject to the usual limiting assumptions, it was stated that:

> The detailed calculations for the effect of concentrated loads in I-beam bridges are long and tedious, and would serve no useful purpose if given here. The calculations were made by means of infinite trigonometric series, with as many as 16 terms being considered for some of the structures analyzed . . . In certain cases the slow convergence of the series made it necessary to estimate the effect of the terms in the series that were neglected. (21, p. 9).

The bridges studied by Newmark and Siess were all alike in that they had five beams of the same size equally spaced, and in that the slabs were of uniform thickness. Beam spacing to span ratios of 0.1, 0.2, and 0.3 were used, and for each of these ratios several beam to slab stiffness ratios were analyzed. Moments were determined for each combination for many positions of a unit load, providing data for the "influence surface" for each moment. These influence values were then used to determine the maximum values of the various moments caused in each of over 50 structures each having a particular span and width and subject to the standard highway truck loading. The resulting maximum values were plotted and the plots used in arriving at simplified recommendations for design use. The influence of interior diaphragms between the beams was neglected. The edge beams were assumed to be at the edge of the slab, and the effects of curbs, sidewalks, and handrails were neglected.

A significant condition of the analyses was that the faces of the curbs were assumed to be at the edge beams, though some supplementary calculations were made with the curbs 2 ft. outside of the edge beams. This gave recognition to a variable that seems highly influential in determining the loads carried by the edge beams. Among the maximum moments calculated, none was found to occur in an edge beam unless the face of the curb was 2 ft. outside the edge beam, i. e., unless the outermost wheel load could be placed directly over the edge beam (21, pp. 38, 39).

Moments were computed at midspan instead of at the theoretical point of maximum moment, 1.4 ft. from midspan, and it was pointed out that the error thus introduced was negligible (21, p. 41). An important conclusion reached was that Poisson's ratio could be disregarded without serious error (21, p. 14).

The resulting design recommendations neglected the portion of the load that might be carried directly to the abutments by the slab, and were as follows. The fraction, k, of a wheel load to be carried by one beam when two or more lanes of traffic are present should be, when the outer load is:

more than 2 ft. inside the edge beam

$$k = \frac{S}{4.40 + .42L/(10\sqrt{H})}, \tag{1}$$

less than 2 ft. inside the edge beam,

$$k = \frac{S}{4.40 + .21L/(10\sqrt{H})}, \tag{2}$$

In these equations,

S = beam spacing in feet,

L = beam span in feet,

H = ratio of beam to slab stiffness, $\frac{EI}{LE_sI_s}$,

E = modulus of elasticity of the beams, lb. per in.^{2},

I = moment of inertia of the beam, in.^{4},

E_s = modulus of elasticity of the slab, lb. per in.^{2},

I_s = moment of inertia of a one foot wide strip of the slab, in.^{4} per ft.

These recommendations were based on the assumption that all the beams, composite or otherwise, would be of the same size. They also included the assumption that in T-beam structures the EI value for a beam would be determined from the transformed section consisting of the steel beam plus a full panel width of the concrete. It was emphasized that the EI values for both the slab and beams, but particularly for the slab are uncertain because of the usual variability of the modulus of concrete, because of the lack of homogeneity in a reinforced slab, and, in particular, because of the effect of hair cracks in the slab.

To supplement the analytical investigation, an extensive series of tests of model bridges was performed and reported by Newmark, Siess, and Penman (22). "A principal object of the tests was to determine whether the theoretical analysis, limited as it was by numerous assumptions, could be used to predict the behavior of the slab and beams in an I-beam bridge." (18, p. 41). Fifteen quarter-scale models were tested, the span being either 5 or 15 ft, the beam spacing 18 in., and the slab thickness $1\frac{3}{4}$ in. Shear connectors were used in some, omitted in others, and the natural bond was deliberately broken in still others. The diaphragms used were small compared to those in the bridges reported on herein. No crown was provided in the roadway. The edge of the slab was located at the outer edge of the flange of the outer beam, and no curb was provided. Thus, the amount of concrete acting as part of an edge composite beam was considerably smaller than that acting as part of an interior beam.

The loads were applied through steel disks cushioned by sponge rubber. Strains and deflections were measured both before and after cracking, the strains of most interest in the present discussion being the longitudinal strains along the bottoms of the steel beams. These were compared with those predicted by the analytical method, and it was found that "The distribution of moments to the several beams as determined from measured strains was in excellent agreement with the distribution predicted by the analysis." (18, p 41). The actual measured strains, however, were up to 39 percent greater than the computed (22, p. 115). These discrepancies were largely attributed to unpredictable cracking of the slab and to the fact that in composite bridges the calculations assumed beams of equal size whereas they were actually not equal, as noted above.

Cracking of the slab was found to have only a small effect on the distribution of the loads to the beams. And it was found that the effects of composite action were quite closely predicted using only the simple assumption that the beam stiffnesses were increased to those of the transformed areas.

The principal conclusions and, in particuar, the design recommendations from the Illinois project have been repeated in several other papers (23, 24, 25, 26). In one, the design recommendation previously given, Eq. 1, was further simplified to

$$k = S/5.5 \tag{3}$$

for both interior and exterior beams in composite bridges in which the outer wheel is assumed to come no closer than 2 ft from the edge beam (23,

p. 160). In another, Newmark reviewed the project and emphasized several points of present interest (24). One of these was that the slab acts as a very effective diaphragm, so that it is unnecessary to provide additional diaphragms, except for construction purposes (24, p. 1002). Another was that the agreement between measured and computed beam strains was much closer for the 15 ft models than for the 5 ft ones (24, p. 1003).

Also presented by Newmark was a discussion of the nature of the loading on each beam when a concentrated load, P, is placed over one beam. As shown in Fig. 6, the total load on the beam directly under the load consists of the concentrated load and an upward distributed load. The load on the other beams is distributed, only. It was stated that all the distributed loads are approximately sine curves (24, p. 1001).

Another paper reviewed the program with particular emphasis on the changes in the behavior of the bridges as their proportions were changed (25), and still another particularly emphasized the design of the composite type structure (26).

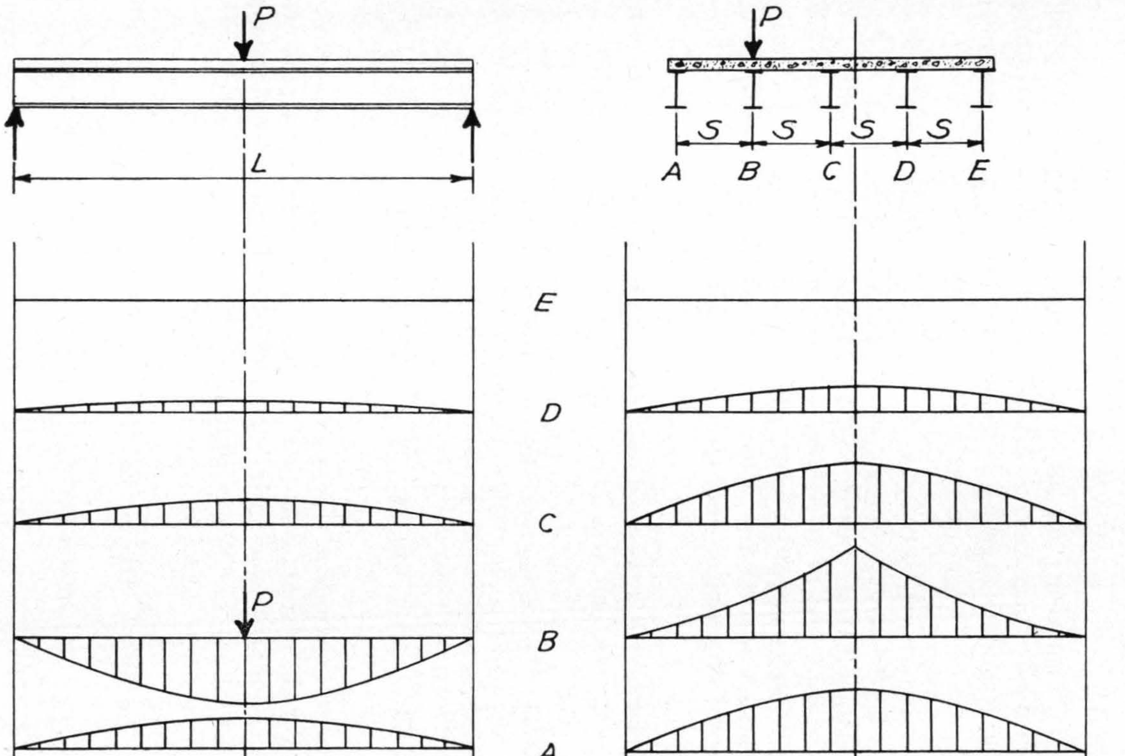
During the time since the University of Illinois project was started, a few limited investigations have been reported by others. Hindman and Vandegrift reported measurements of the deflections, only, of some full-scale bridges that were not typical of the type under discussion (27). They did emphasize the difficulties caused by temperature changes in actual bridges exposed to the vagaries of the weather.

Lin and Horonjeff in one paper (28) and Clough and Schaffey in another (29) reported tests on a full-scale three-girder bridge in which a center span was suspended from cantilevered side spans. This bridge was unusual in that when the outer wheel was placed 2 ft. from the curb it was actually 1 ft. outside the edge beam instead of 2 ft. inside it as assumed in the University of Illinois recommendations. Diaphragms were relatively small and were not in contact with the concrete slab.

Difficulties in determining the modulus of elasticity of the concrete were reported because it changed with the weather. Rapid changes of air temperature and changes in the radiant heating effects of the sun were found drastically to influence strain and deflection measurements so that it was necessary to take "no-load" and "load" readings within a few minutes of each other. Still another source of difficulty was that the local effects of concentrated loads distorted the readings of gages near the loads.

In reporting the analysis of the data it was stated (29, p. 941) that,

> The manner in which load is distributed to the girders by the slab and diaphragms is indicated by the relative magnitudes of the bending moments acting in the three girders at a given cross section.



a) Load distribution to beams b) Moment diagrams for beams

TYPICAL LOAD AND MOMENT DIAGRAMS FOR BEAMS
(From reference 24)

Fig. 6

This would seem to indicate that it was assumed the variation of load along all the girders was the same. As discussed previously and indicated in Fig. 6, Newmark has shown the variation of the load along different girders to be quite different.

It was concluded that the effect of the diaphragms was relatively small. It was also concluded that the AASHO specification method assuming simple-span slabs predicted the load distribution to the edge beams quite accurately.

Foster has reported measurements made on six different 60 ft. span bridges having a 28 ft. roadway supported by seven equally spaced stringers (30). The slab thickness varied both laterally and longitudinally. As result of this variation and of the typical difficulties encountered in full-scale field testing, no quantitative conclusions were justified. The major qualitative conclusion was to the effect that the type of diaphragm or even the absence of diaphragms had no discernible effect on the load distribution.

Similarly inconclusive tests have been reported by Wise (31). No data were published, but it was said (31, p. 180) that,

> The measured stresses were in excellent agreement with the theoretical stresses. The basic elementary theory used for the static stress analysis assumed that the diaphragms were rigid and distributed the load in any lane to all the girders. The diaphragms were found to be completely effective.

This result and this theory are both in complete disagreement with the results of all the other tests and analyses reviewed.

For the sake of convenient reference the complete AASHO specifications applying to load distribution to the beams are included as follows (1, p. 167-168). As noted previously, these sections of the specifications have remained essentially unchanged since around 1923. The section and paragraph numbering and lettering are from the specifications:

Section 3 - DISTRIBUTION OF LOADS

3.3.1. - DISTRIBUTION OF WHEEL LOADS TO STRINGERS AND FLOOR BEAMS.

(a) *Position of Loads for Shear*

In calculating end shears and end reactions in transverse floor beams and longitudinal beams and stringers, no lateral or longitudinal distribution of the wheel load shall be assumed for the wheel or axle load adjacent to the end at which the stress is being determined. For loads in other positions on the span, the distribution for shear shall be determined by the method prescribed for moment, except that the calculations of horizontal shear in rectangular beams shall be in accordance with article 3.4.14.

(b) *Bending Moment in Stringers*

In calculating bending moments in longitudinal beams or stringers, no longitudinal distribution of the wheel loads shall be assumed. The lateral distribution shall be determined as follows:

(1) *Interior Stringers*

Interior stringers shall be designed for loads determined in accordance with the following table:

Kind of floor	One traffic lane, fraction of a wheel load to each stringer	Two or more traffic lanes, fraction of a wheel load to each stringer
---	---	---
Concrete	$\frac{S}{6.0}$ If S exceed 6.0 ft. see footnote*	$\frac{S}{5.0}$ If S exceeds 10.5 ft. see footnote*
---	---	---

S = average spacing of stringers in feet.

* In this case the load on each stringer shall be the reaction of the wheel loads, assuming the flooring between stringers to act as a simple beam.

(2) *Outside Stringers*

The live load supported by outside stringers shall be reaction of the truck wheels, assuming the flooring to act as a simple beam between stringers.

(3) *Total Capacity of Stringers*

The combined load capacity of the beams in a panel shall not be less than the total live and dead load in the panel.

Section 9 - COMPOSITE BEAMS

3.9.2 - EFFECTIVE FLANGE WIDTH

In composite beam construction the assumed effective width of the slab as a T-beam flange shall not exceed the following:

(1) One-fourth of the span length of the beam.
(2) The distance center to center of beams.
(3) Twelve times the least thickness of the slab.

For beams having a flange on one side only, the effective flange width shall not exceed one-twelfth of the span length of the beam, nor six times the thickness of the slab, nor one-half the distance center to center of the next beam.

An unpublished tentative revision of the AASHO specifications designated as T-15-50 has been considered by the Bridge Committee of the AASHO. This revision would increase the denominators of the fractions in the preceding table; S/5 would be changed to S/5.5, and S/6 to S/7. It would also revise the article concerning outside stringers by requiring that they be designed for a live load not less than that specified in the table for interior stringers, i. e., S/5.5 or S/7. No provision was made in the tentative revision for variation of the load on the outside stringer in response to variations in the position of the curb face with respect to the stringer or to variations in any of the other seemingly significant quantities.

III. PROPOSED ANALYSIS PROCEDURE

As indicated previously, the primary objective of the investigation was to develop an analysis procedure more generally useful than those presently available. It was mentioned that such a procedure should:

1. take into account more of the significant variables,
2. be understandable to practicing engineers without special training,
3. be brief enough for practical use,
4. retain accuracy consistent with the accuracy of the data going into the analysis, and
5. Lend itself to future refinement.

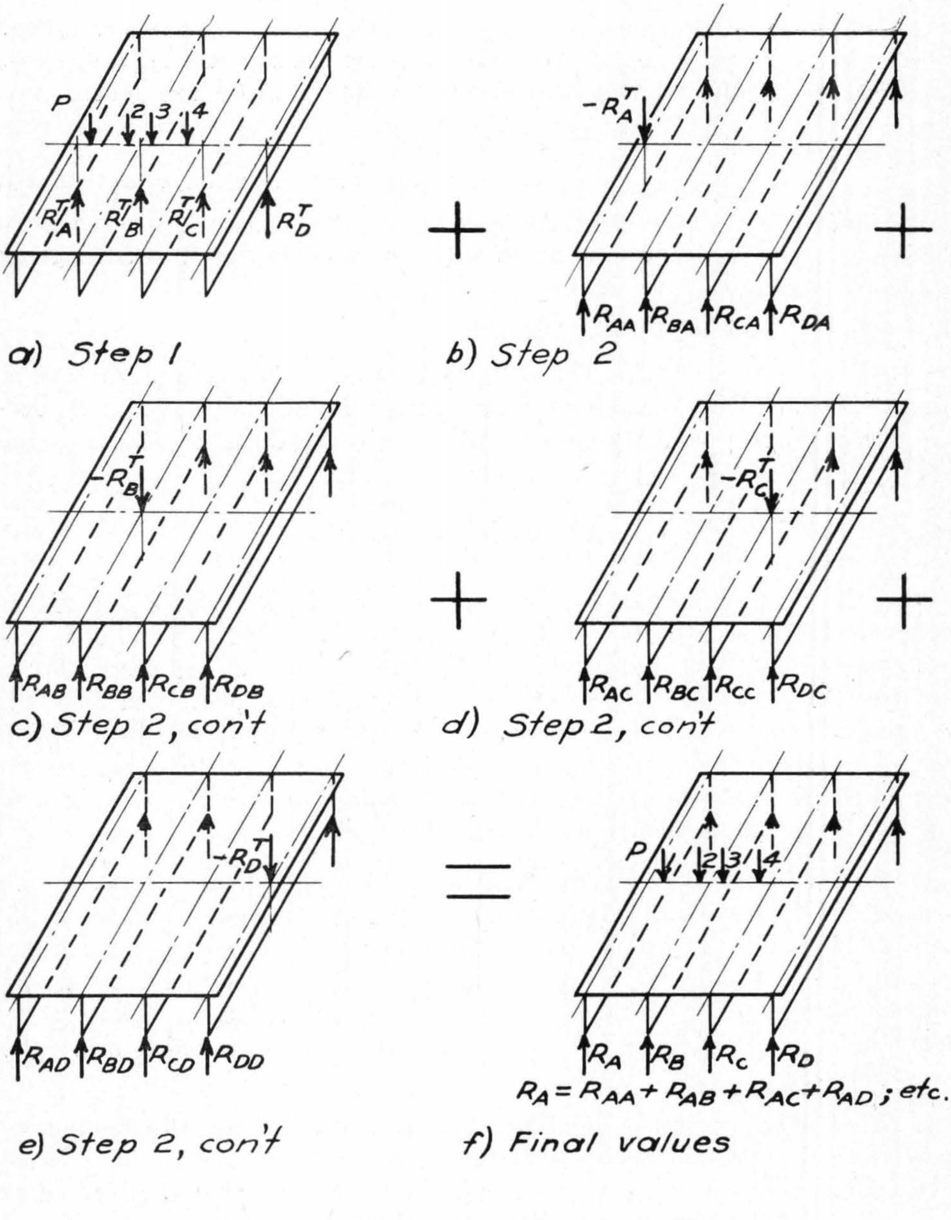
A new procedure has been developed that seems to meet all these requirements in the analysis of simple-span bridges, and that may be useful in the analysis of continuous bridges. It is, therefore, presented on the following pages and recommended for use.

A. Basic Procedure

If the usual assumption that superposition is permissible is made, it follows that when a bridge is loaded it may be regarded as passing to the fully loaded, stressed, and deformed condition in two distinct steps. First, the loads are applied while the beams are temporarily prevented from deflecting. This gives rise to forces transmitted to the beams by the slab and to temporary reactions under the beams that are everywhere equal and opposite to the forces acting on the beams, Fig. 7a. No net load acts on the beams and no moments are induced in them.

Second, the temporary reactions are removed and the effects on the beams of this removal are calculated. These effects constitute, then, the total effects of the original loads. The effects of removing a temporary reaction are assumed to be the same as those of applying an equal and opposite force. This entire procedure of superposition of effects is illustrated in Fig. 7 for a group of concentrated loads applied along the transverse centerline.

When concentrated loads are applied to the bridge, the temporary reactions, R^T, may be assumed to be concentrated, uniformly distributed over some arbitrary length, or distributed in any other way indicated by present knowledge or future developments. When transverse line loads are applied to the bridge the temporary reactions would probably be distributed in the same way as are those for concentrated loads. When a uniformly distributed load is applied to the bridge, the temporary reaction, W^T, would be assumed uniformly distributed along the entire length of the beams.



BASIC SUPERPOSITION PROCEDURE
FIG. 7

B. Assumptions

In expanding the basic procedure to the evaluation of moments and deflections, various initial simplifying assumptions are made. It will be seen that these are all either in accord with present practice or have been indicated by previous investigations described in the Review of Literature. In general, the assumptions are such that modifications can be made to improve results without changing the overall procedure. Future research and further experience with the method can be expected to provide the information on which to base modifications that will improve the assumptions.

The assumptions are as follows.

1. The beams and slab making up a bridge are regarded as separate entities even though some material may act both as part of a composite beam and part of the slab. The slab material included as part of a beam extends to the center of each adjacent space or to the edge of the bridge. Curbs are included.
2. When the beams are temporarily prevented from deflecting, step 1, the reactions of the slab are those of a continuous beam of uniform width on rigid supports. The temporary beam reactions are the same as the slab reaction.
3. There is no longitudinal distribution of the temporary reactions; when the applied loads are concentrated, the reactions are concentrated, also.
4. The effects of the diaphragms are neglected.
5. When a concentrated load is applied at one beam, the resulting distributed forces acting on all the beams are distributed sinusoidally, as was suggested by Newmark (24, p. 1001), Fig. 6, and by independent studies conducted as part of the current project. When a uniform load is applied along one beam, the distributed forces resulting on the other beams are also assumed to be distributed sinusoidally.
6. The EI values for the beams are those of the transformed composite sections.
7. The EI value for the slab is that of the gross concrete section, neglecting the reinforcement.
8. The slab carries no load directly to the abutments, i. e., longitudinal bending in the slab is neglected.
9. Torsion of the slab and of the beams is neglected.
10. The Poisson effect is ignored.
11. The maximum moment in a beam is assumed to be the maximum at the center of the beam.
12. The moment at the center caused by a load applied to the bridge

at some other point, y, is assumed equal to the moment at y caused by the load placed at the center.

13. To find the moment at y caused by a load at y, it is assumed that the moment diagram is composed of straight line segments one of which passes through zero at the end of the beam, passes through the value of the moment at the center, and is extended to the point y.

C. Limitations on the Use of the Procedure

The foregoing assumptions are believed to introduce relatively small errors in the analysis of a bridge having a span, beam spacing, and slab thickness within the usual ranges of these variables previously mentioned. The close agreement between the predicted and measured results reported subsequently tends to confirm this belief. However, unusual structures may occur in which one or more of the variables or combinations of the variables is substantially larger or smaller than usual. For some such structures the use of the proposed procedure based on the assumptions listed might yield analyses excessively in error. For others the use of the method might yield results of acceptable accuracy, but a simplified procedure might be found also to yield acceptable results. The ranges of the variables within which the use of the procedure is necessary and within which it yields acceptable results are by no means established. The extreme conditions can, however, be qualitatively identified.

At one extreme, as the span decreases or the beam spacing increases, thus as the ratio of the span to the spacing decreases, longitudinal bending in the slab must become significant, contrary to assumption 8. If this ratio should become one, for instance, roughly half the load would be carried directly to the abutments through longitudinal bending. Among the bridges tested and analyzed with good results the ratio was as low as 3. Also, in the design of slabs supported on four sides it is common to ignore bending in the long direction if the long side is as much as twice the length of the short side. It is suggested that the use of the proposed procedure be similarly limited to the analysis of bridges having slabs within the ordinary range of thickness and having a span to spacing ratio of 2 or more. A very thick slab, hence a low value of H, would cause longitudinal bending to become significant, also. Among the bridges tested the value of H was as low as 3, and the agreement between the analysis and test results remained good. Pending further study, it is suggested the proposed procedure not be used in analyzing bridges having values of H lower than 2.

It seems probable modifications of the procedure can be devised that will adapt it to the analysis of the unusual cases outside the limits suggested, but this has not yet been done.

At the other extreme, as the span to spacing ratio becomes very large, the effects of cross-bending of the slab between the beams must become insignificent and the assumption of a laterally rigid slab would be justified. The presence of a relatively thick slab in combination with the large ratio would intensify this effect. Conversely, when the slab becomes thin, particularly in combination with a small ratio of span to spacing the effects of the beam deflections should become negligible and sufficient accuracy be obtained by considering the effects of cross-bending, step 1, only. Only added experience with the procedure can establish the ranges of the variables within which these simplified assumptions could be used.

D. Expansion of the Procedure

1. *Sign convention*

Throughout this discussion upward forces and deflections will be considered positive; downward ones negative. Moments, therefore, will be positive when they cause compression in the top of a simple beam.

2. *The evaluation of temporary reactions*

Under assumptions 2 and 3 above, the temporary reactions (step 1) are simply those of a beam of uniform width continuous over rigid supports. They may be evaluated through the use of any of the methods applicable to the analysis of continuous beams. Moment distribution will probably be preferred in the general case; it is widely understood and used and is readily adaptable to beams in which the cross-section varies or in which the spans are unequal. On the other hand, reaction influence lines are most convenient when dealing with a group of bridges having closely similar proportions. For instance, all the bridges tested had slabs of constant thickness and had four beams equally spaced. Influence lines for the reactions were used in analyzing them, and are included in the Appendix for convenient reference. The corresponding table of influence values is included, also.

For uniform or line loads it becomes necessary to determine areas under the influence lines if they are to be used. The equations of the various segments of the lines for use in determining areas are also included in the Appendix.

3. *Concentrated loads applied at beams*

When the original loads are concentrated loads or line loads, step 2 of the basic procedure requires the determination of the effects of applying a concentrated load at each beam in turn. The method suggested is developed as follows.

a. *Preliminary considerations.* If y is used to designate a variable distance measured along a beam from one end, the ratio y/L appears frequently, and it is convenient to let

$$r = \frac{y}{L} \tag{4}$$

When a concentrated load, P, is applied at the center of a separate simple beam of uniform section, the deflection is given by

$$\Delta_r = \frac{PL^3}{48\ EI}(3r - 4r^3). \tag{5}$$

the maximum value, at $r=\frac{1}{2}$, being

$$\Delta_{max} = \frac{PL^3}{48\ EI}. \tag{6}$$

When a sinusoidally distributed load,

$$w_r = w_{max}\sin\pi r, \tag{7}$$

is applied to a separate beam of uniform section, the resulting moments and deflections are distributed sinusoidally, also.

$$M_r = -w_{max}\frac{L^2}{\pi^2}\sin\pi r, \tag{8a}$$

$$= M_{max}\ \sin\pi r, \tag{8b}$$

in which, at $r=\frac{1}{2}$,

$$M_{max} = -w_{max}\ \frac{L^2}{\pi^2} \tag{9}$$

And,

$$\Delta_r = w_{max}\ \frac{L^4}{\pi^4 EI}\sin\ \pi r, \tag{10a}$$

$$= \Delta_{max}\sin\ \pi r, \tag{10b}$$

in which,

$$\Delta_{max} = w_{max}\frac{L^4}{\pi^4 EI}. \tag{11}$$

It is convenient to let

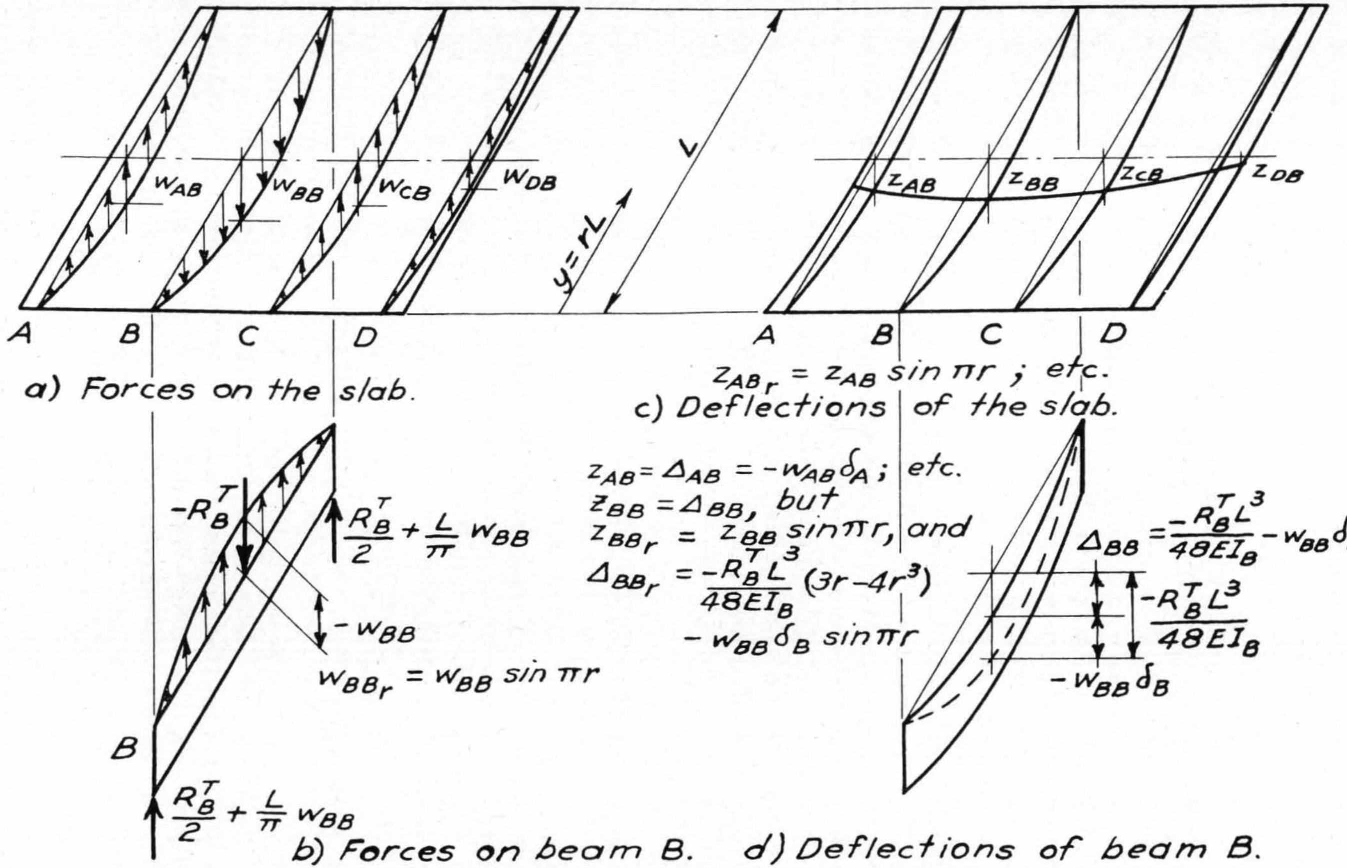
$$\frac{L^4}{\pi^4 EI} = \delta. \tag{12}$$

This quantity is the maximum deflection of a beam when it is acted on by a sinusoidally distributed load whose maximum value is unity. In general it will have a different value, δ_A, δ_B, etc., for each beam.

When considering the slab the following substitution will, also, be found convenient.

$$\delta_s = \frac{S^3}{E_s I_s}, \tag{13}$$

in which $E_s I_s$ represents the product of the elastic constants for a unit



FORCES AND DEFLECTIONS
CONCENTRATED FORCE AT CENTER OF BEAM B
FIG. 8

width of the slab. Under assumption 7, for a unit width

$$E_sI_s = \frac{E_sh^3}{12}, \tag{14}$$

in which h represents the thickness of the slab.

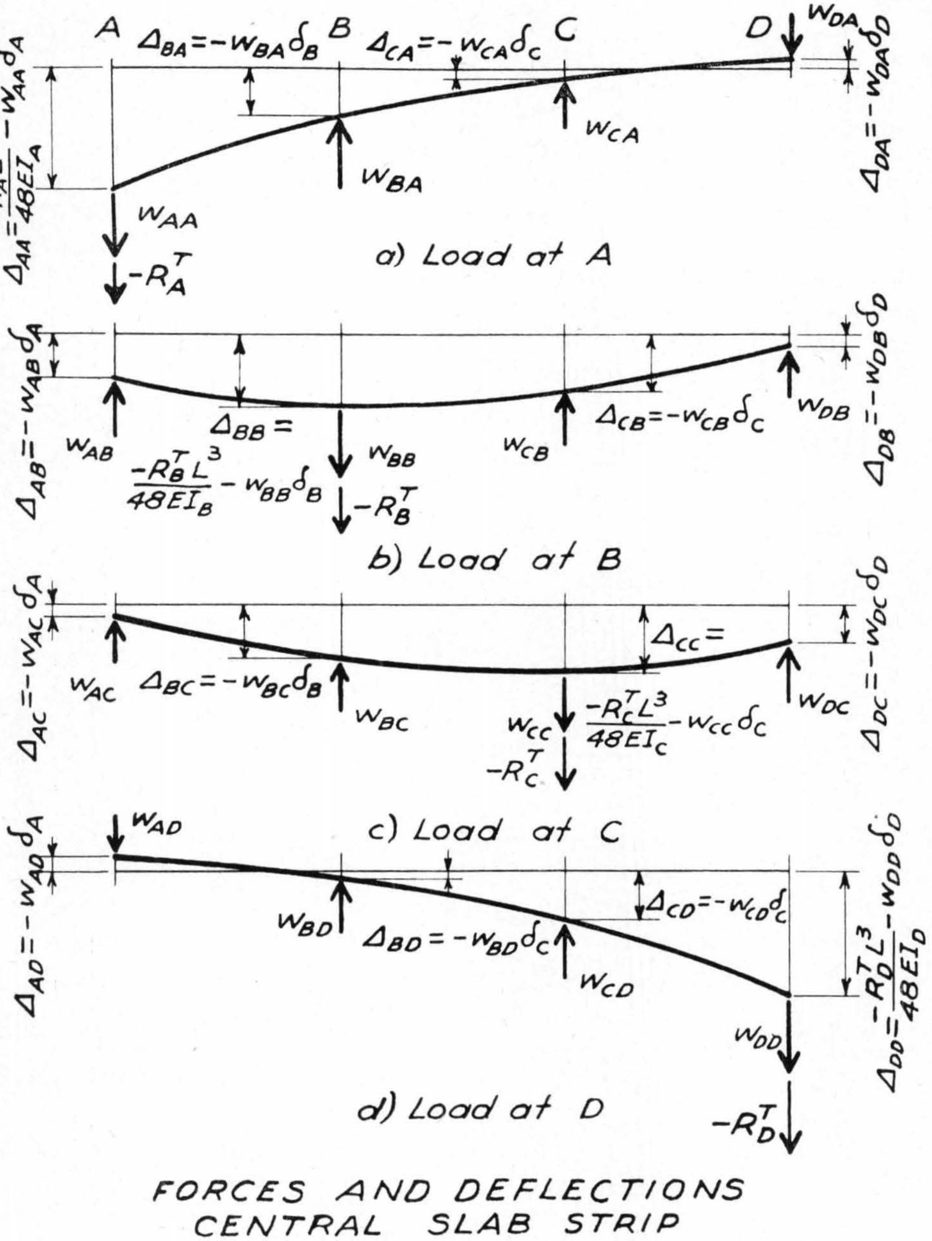
b. *General discussion.* When a concentrated load is applied at a beam in a beam-and-slab bridge the beam deflects and pulls the slab along with it. The resulting tensile force acting between the loaded beam and the slab is distributed sinusoidally, according to assumption 5. The application of this sinusoidal load to the slab induces reactions at the other beams that are, also, distributed sinusoidally as was illustrated in Fig. 6. These slab reactions constitute loads on the beams and cause moments and deflections that vary sinusoidally, in turn.

Typical forces and deflections involved, those occuring when a concentrated load is applied at beam B, are shown in Fig. 8. It will be noted in Fig. 8d that the final maximum deflection of the beam at which the load is applied, Δ_{BB}, is made up of two parts; whereas the corresponding deflection of the slab z_{BB}, is assumed to be purely sinusoidal. Thus, when the deflections of the beam and slab are made equal at the center they are not exactly equal at other points. This difference is an indication of the error introduced by the assumption of sinusoidally distributed forces.

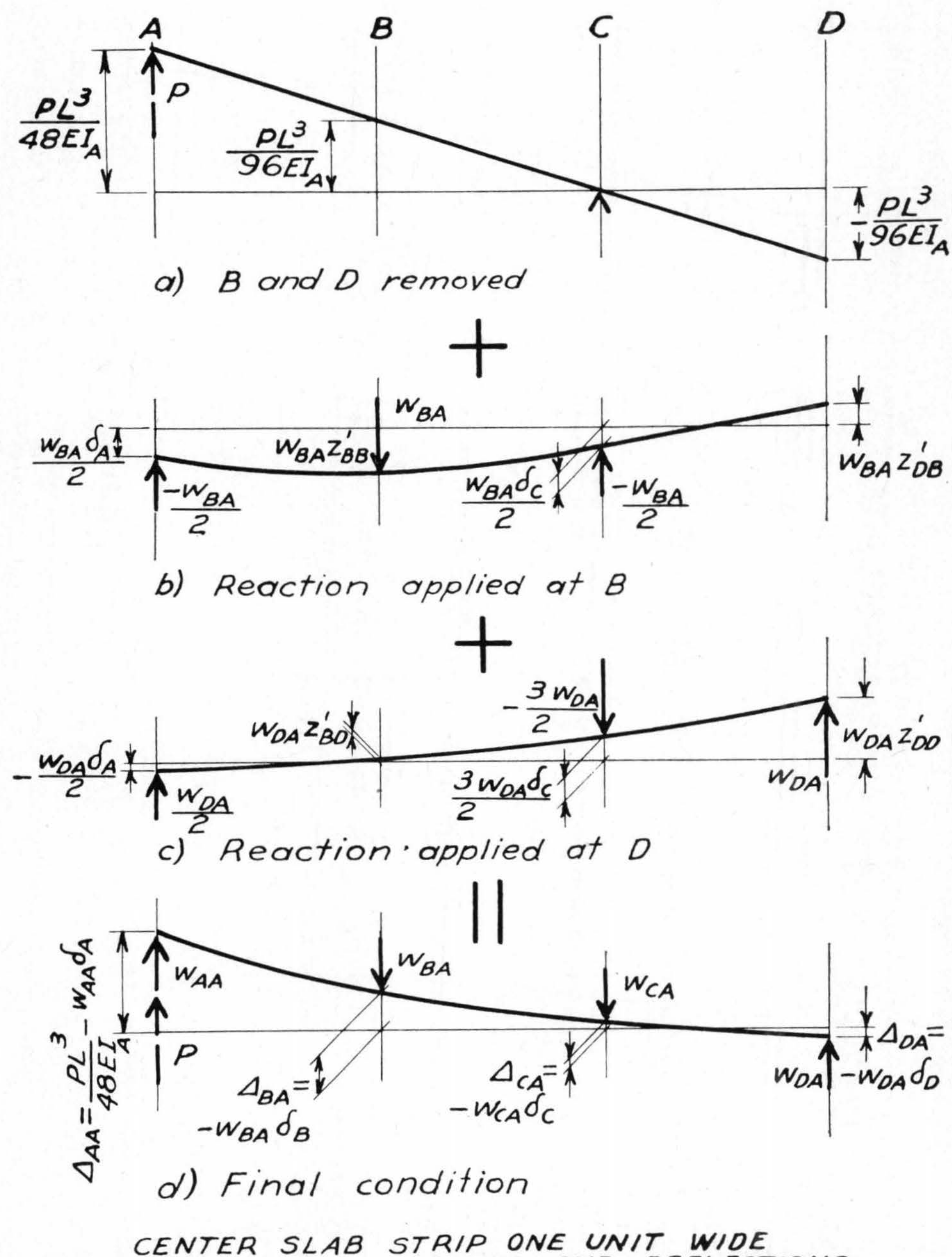
The typical system of Fig. 8 has, essentially, only two redundants. Under the assumptions previously listed, removal of any two of the sinusoidally distributed forces would leave a statically determinate arrangement. The loaded beam would simply deflect under the concentrated load without help from the slab. Since each such distributed force is removed is fully determined by a single constant, the determination of two constants renders the complete system statically determinate, also. Bridges having a larger number of beams would, of course, have more constants to be determined, two less than the number of beams, to be exact.

These constants can be evaluated through consideration of the central lateral strip of slab of unit width, Fig. 9. With the assumptions of no longitudinal bending and no torsion, each lateral strip must be in equilibrium under the action of the parts of the distributed forces acting on it. For the central strip these parts become the maximum values of the distributed forces, w_{AB}, w_{BB}, etc. The deflection of the central strip at each beam must be equal to the maximum beam deflection, Δ_{AB}, Δ_{BB}, etc. These forces and deflections, when the concentrated force is applied to each beam in turn, are fully identified in Fig. 9.

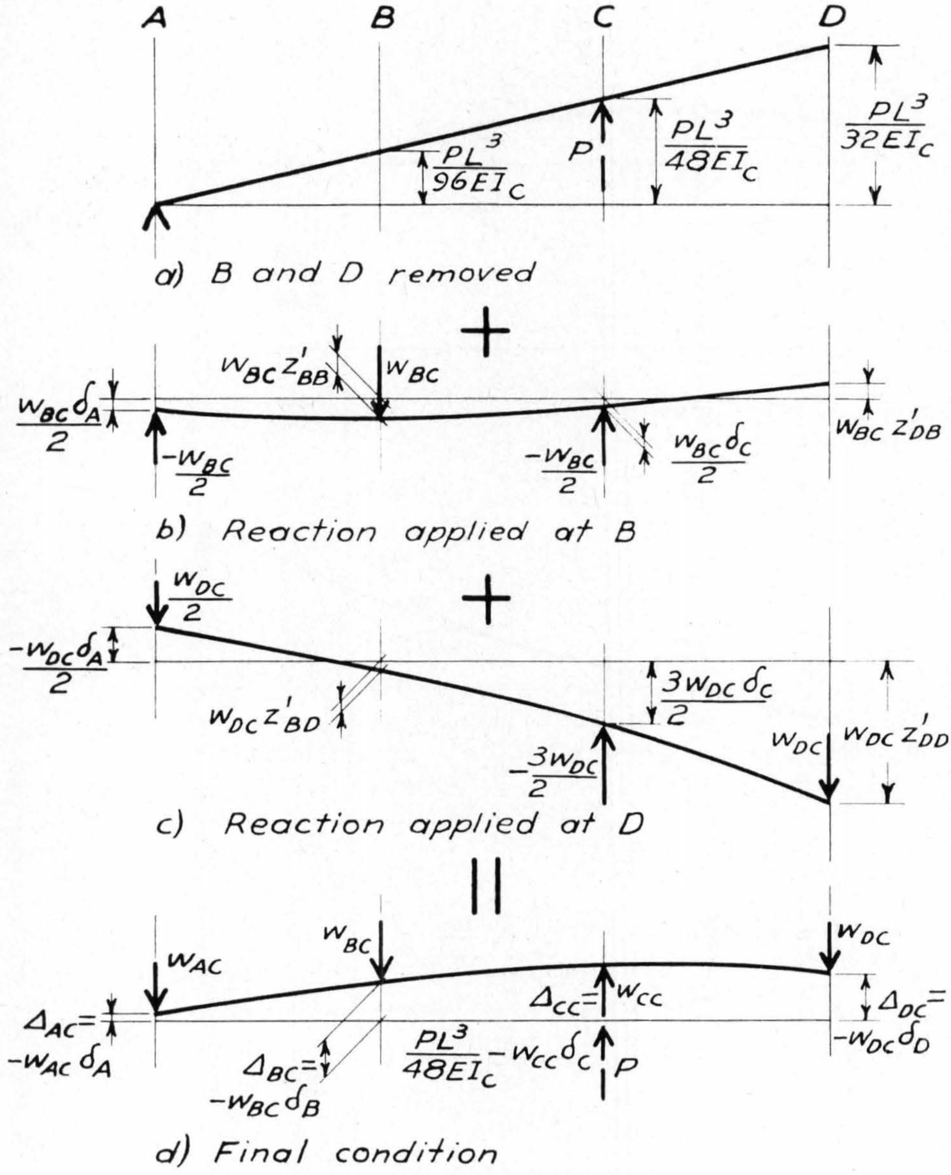
Any valid procedure for the analysis of continuous beams on elastic supports is applicable in the analysis of the slab strip. The use of relaxation procedures leading to numerical solutions of one problem at a time may be



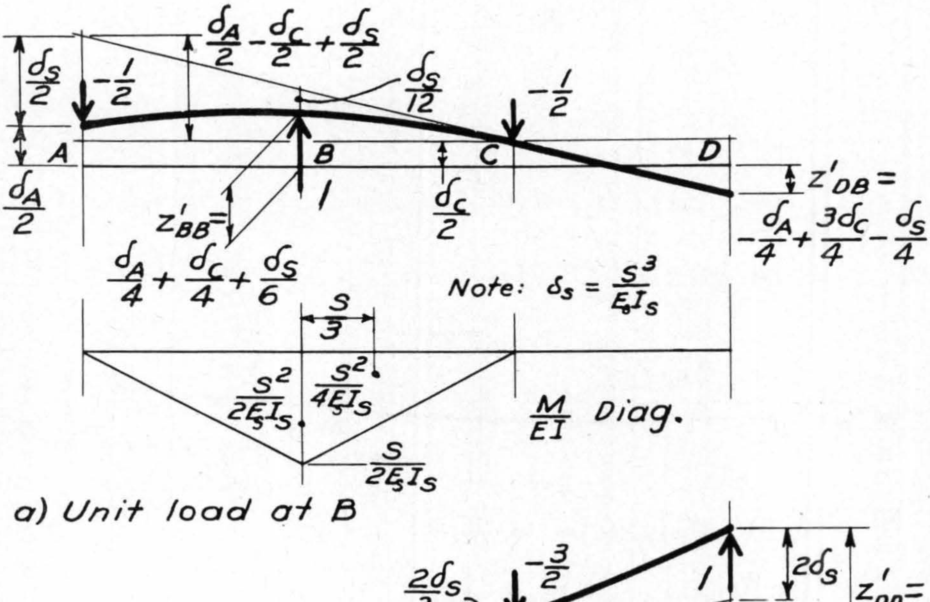
FORCES AND DEFLECTIONS
CENTRAL SLAB STRIP
FIG. 9



CENTER SLAB STRIP ONE UNIT WIDE
SUPERPOSITION OF FORCES AND DEFLECTIONS
CONCENTRATED FORCE AT CENTER OF BEAM A
FIG. 10



CENTER SLAB STRIP ONE UNIT WIDE
SUPERPOSITION OF FORCES AND DEFLECTIONS
CONCENTRATED FORCE AT CENTER OF BEAM C
FIG. 11



BEAMS B AND D REMOVED
UNIT LOADS ON CENTRAL STRIP
FIG. 12

preferred. At the other extreme it is theoretically possible to derive general equations for the desired values, but such equations were found to be unduly complex even for the relatively simple bridges considered.

For these bridges it is found convenient to carry the general derivations only part way, as shown below. Complete solutions for a particular bridge and loading are, then, obtained after the numerical values pertaining to the particular case have been substituted into the partial general solution.

c. *Partial general solution.* The analysis of the slab strip is made by reducing it first to a determinate condition by the temporary removal of two of the distributed forces. For instance, if B and D are removed and a concentrated force, P, is then applied to beam A it will deflect the full amount, $PL^3/48EL_A$, unrestrained by the slab, Fig. 10a. The deflection at C is zero, the slab is unstrained, and the deflections at B and D are as shown. Similarly, if the load is applied at beam C, the deflections will be shown in Fig. 11a.

Next, if a sinusoidal load whose maximum value is unity is applied to the slab at beam B, Fig. 12a, it is resisted only by beams A and C, at each of which the maximum reaction is $-\frac{1}{2}$. At A the resulting maximum deflection of the beam and the deflection of the slab strip is $\delta_A/2$ and at C it is $\delta_C/2$. Applying the moment-area principles, the deflection at B, z'_{BB}, and at D, z'_{DB}, are obtained as shown, Fig. 12a. Similarly, if the unit sinusoidal load is applied to the slab at beam D, Fig. 12 b, the force at A is $+\frac{1}{2}$ and at C it is $-3/2$. The resulting deflections, z'_{BD} and z'_{DD}, are as shown in Fig. 12b.

When the load P is applied at A, the final condition is as shown in Fig. 10d, with initially unknown values of w_{BA} and w_{DA} superimposed on the original condition, Fig. 10b, c. These values of w_{BA} and w_{DA} induce corresponding values of w_{AA} and w_{CA}. To evaluate w_{BA} and w_{DA} the results of the preceding analyses are superimposed in equations for the final deflections of B and D, as follows:

$$\Delta_{BA} = -w_{BA}\,\delta_B = \frac{PL^3}{96\ EI_A} + w_{BA}z'_{BB} + w_{DA}z'_{BD}, \tag{15}$$

$$\Delta_{DA} = -w_{DA}\,\delta_D = -\frac{PL^3}{96\ EI_A} + w_{BA}z'_{DB} + w_{DA}z'_{DD}. \tag{16}$$

Solving these equations simultaneously yields:

$$w_{BA} = -\frac{PL^3}{96\ EI_A}\;\frac{(z'_{DD} + \delta_D + z'_{BD})}{(z'_{BB} + \delta_B)(z'_{DD} + \delta_D) - z'_{DB}z'_{BD}}, \tag{17}$$

$$w_{DA} = \frac{PL^3}{96\ EI_A}\ \frac{(z'_{BB} + \delta_B + z'_{DB})}{(z'_{BB} + \delta_B)(z'_{DD} + \delta_D) - z'_{DB}z'_{BD}} \ . \tag{18}$$

By the reciprocal theorem, $z'_{BD}=z'_{DB}$. Making this substitution, and letting

$$(z'_{BB} + \delta_B)(z'_{DD} + \delta_D) - z'^2_{BD} = N, \tag{19}$$

$$w_{BA} = -P(\frac{L^3}{48\ EI_A})(\frac{z'_{DD} + \delta_D + z'_{BD}}{2N}), \tag{20}$$

$$w_{DA} = -P(\frac{L^3}{48\ EI_A})(\frac{-z'_{BB} - \delta_B - z'_{BD}}{2N}), \tag{21}$$

and, from Figs. 10 and 12,

$$w_{AA} = -\frac{w_{BA}}{2} + \frac{w_{DA}}{2}\ , \tag{22}$$

$$w_{CA} = -\frac{w_{BA}}{2} - \frac{3w_{DA}}{2}\ . \tag{23}$$

When the concentrated load is applied at C the superposition of deflections, Fig. 11, yields:

$$\Delta_{BC} = -w_{BC}\,\delta_B = \frac{PL^3}{96\ EI_C} + w_{BC}z'_{BB} + w_{DC}z'_{BD}\ , \tag{24}$$

$$\Delta_{DC} = -w_{DC}\,\delta_D = \frac{PL^3}{32\ EI_C} + w_{BC}z'_{DB} + w_{DC}z'_{DD}\ . \tag{25}$$

Solving and reducing, as before,

$$w_{BC} = -P(\frac{L^3}{48\ EI_C})(\frac{z'_{DD} + \delta_D - 3z'_{BD}}{2N})\ , \tag{26}$$

$$w_{DC} = -P(\frac{L^3}{48\ EI_C})\left[\frac{3(z'_{BB} + \delta_B) - z'_{BD}}{2N}\right]. \tag{27}$$

And,

$$w_{AC} = -\frac{w_{BC}}{2} + \frac{w_{DC}}{2}\ , \tag{28}$$

$$w_{CC} = -\frac{w_{BC}}{2} - \frac{3w_{DC}}{2} \tag{29}$$

When the concentrated force is applied at B the results are obtained by symmetry, as follows.

$$w_{AB} = w_{DC}\,, \tag{30}$$
$$w_{BB} = w_{CC}\,, \tag{31}$$
$$w_{CB} = w_{BC}\,, \tag{32}$$
$$w_{DB} = w_{AC}\,. \tag{33}$$

When it is applied at D,

$$w_{AD} = w_{DA}\,, \tag{34}$$
$$w_{BD} = w_{CA}\,, \tag{35}$$
$$w_{CD} = w_{BA}\,, \tag{36}$$
$$w_{DD} = w_{AA}\,. \tag{37}$$

Thus, all the sinusoidally distributed reactions acting on the slab and resulting from the application of a concentrated force to any beam are evaluated. The corresponding forces acting on the beams are, of course, opposite in sign. Also, in a bridge analysis the general force, P, is replaced by the appropriate reversed temporary reactions, $-R^T$, in turn.

4. *Uniform loads applied at beams*

When the original loads are uniform along the length of the bridge, step 2 of the basic procedure requires the determination of the effects of applying a uniform load, W, at each beam in turn. Following an analysis paralleling that for concentrated loads, corresponding formulas are obtained, as follows.

$$w_{BA} = -W\left(\frac{5L^3}{384\ EI_A}\right)\left(\frac{z'_{DD} + \delta_D + z'_{BD}}{2N}\right), \tag{38}$$

$$w_{DA} = =W\left(\frac{5L^3}{384\ EI_A}\right)\left(\frac{-z'_{BB} - \delta_D - z'_{BD}}{2N}\right), \tag{39}$$

$$w_{AA} = -\frac{w_{BA}}{2} + \frac{w_{DA}}{2}, \tag{40}$$

$$w_{CA} = -\frac{w_{BA}}{2} - \frac{3w_{DA}}{2}, \tag{41}$$

$$w_{BC} = -W\left(\frac{5L^3}{384\ EI_C}\right)\left(\frac{z'_{DD} + \delta_D - 3z'_{BD}}{2N}\right), \tag{42}$$

$$w_{DC} = -W\left(\frac{5L^3}{384\ EI_C}\right)\left[\frac{3(z'_{BB} + \delta_B) - z'_{BD}}{2N}\right], \tag{43}$$

$$w_{AC} = -\frac{w_{BC}}{2} + \frac{w_{DC}}{2}, \qquad (44)$$

$$w_{CC} = -\frac{w_{BC}}{2} - \frac{3w_{DC}}{2}. \qquad (45)$$

As in the equation for concentrated loads, in an analysis the general load, W, is replaced by the reversed temporary uniform reaction, $-W^T$, in place of $-R^T$, Fig. 7.

5. *Beams of varying section*

The preceding derivations are directly applicable only to bridges in which the beams are of constant cross-section. However, they can be extended to bridges whose beams vary in section quite easily by making the following substitutions.

1. In the preceding derivations the quantity $L^3/48EI$ is the deflection of a beam of uniform section caused by one pound acting at its midpoint. For beams of varying section this quantity is replaced in each instance by the appropriate numerical value of the deflection of the beam of varying section, also for one pound at its midpoint.

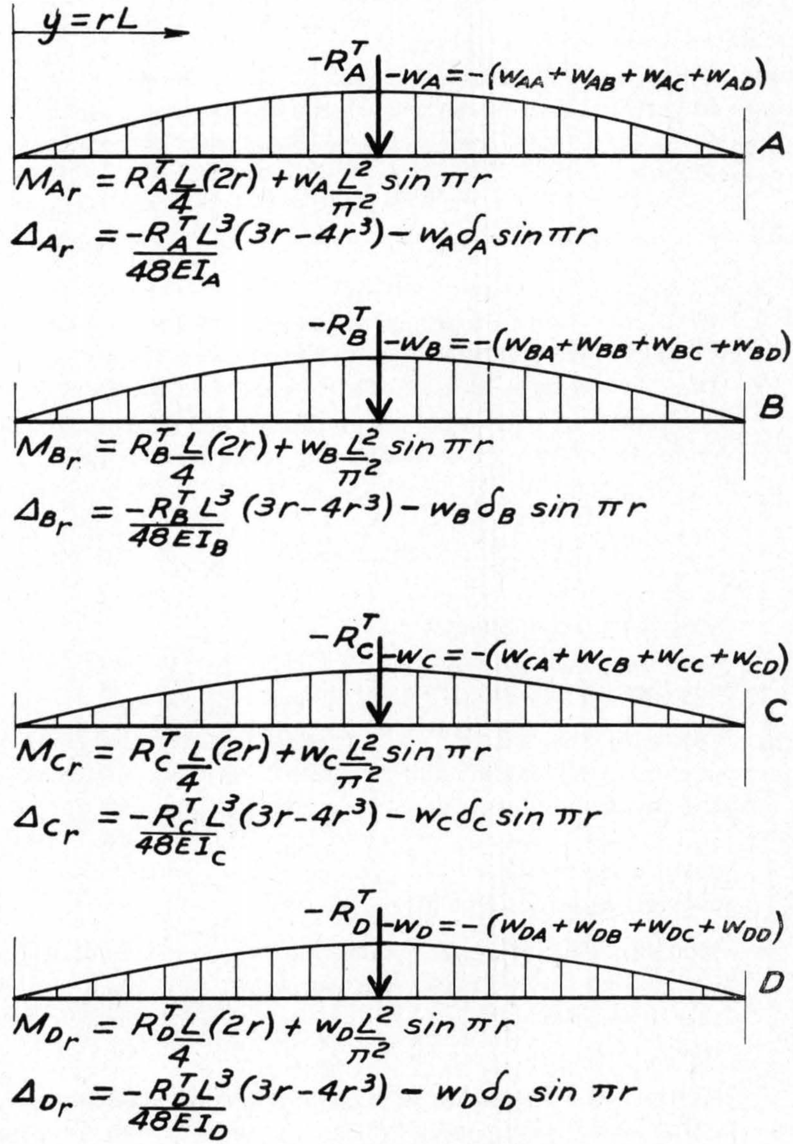
2. Similarly, the quantity $5L^3/384EI$ is the deflection of a beam of uniform section acted on by a uniformly distributed load of unit intensity. For beams of varying section it is replaced by the corresponding numerical values of the deflections of the beams of varying section under the same load.

3. The quantity δ is defined as the deflection of a beam acted on by a sinusoidally distributed load whose maximum intensity is one pound per unit of length. This definition is equally applicable to beams of uniform and varying section. The formula $\delta = L^4/\pi^4 EI$ for beams of uniform section is, of course, not applicable to beams of varying section.

Some of the bridges tested contained beams of varying section, and the substitutions listed have been made in analyzing them. In other words, the variations in the beam cross-sections have been taken into account.

6. *Final values*

The final load on each beam consists of the reversed temporary reaction for that beam, $-R^T$ or $-W^T$, and of a sinusoidally distributed force that is the sum of all such forces coming to the beam, Fig. 13. The moments in the beam and the deflections are those caused by this combination of forces.



FINAL LOADS ON BEAMS
CONCENTRATED OR TRANSVERSE LINE LOADS
FIG. 13

E. Detailed Procedure

Summarizing the foregoing discussion, the successive steps in analyzing a bridge are listed below. An actual analysis is presented as an example in the next section, section F, and the various steps in the example correspond to those listed below and are identified by corresponding numbers and letters.

1. *For each bridge, all loadings*

   a. Compute the various constants for the beams and slab, EI_A, δ_A, $L^3/48EI_A$, I_{st}/c_{st}, etc., Calculation Sheets 1 and 2.

   b. Compute the quantities occurring in the equations, $(z'_{BB} + \delta_B)$, z'_{BD}, $(z'_{BB} + \delta_B + z'_{BD})$, N, etc., Calculation Sheet 3.

   c. Combine the preceding to obtain values of the sinusoidal loads resulting from unit values of the concentrated force, P, or of the uniform load, W.

   $$w'_{BA} = -(1)\left(\frac{L^3}{48EI_A}\right)\left(\frac{z'_{DD} + \delta_D + z'_{BD}}{N}\right), \qquad (46)$$

   etc., Calculation Sheet 4.

2. *For each loading on a particular bridge*

   a. Calculate the location, x, of each concentrated load within its particular slab span; and calculate the ratios, x/S, needed in using the influence lines, tables, or equations included in the Appendix. Or, similarly locate line loads or uniform loads. Read (or compute) the influence value for each temporary reaction caused by each given load, Calculation Sheet 5.

   b. Multiply the influence values by the corresponding load values, and add all the resulting reaction values to get the total temporary reactions, concentrated or uniform, caused by all the loads, Calculation Sheet 5.

   c. Multiply the negative of each of the previously determined distributed load per pound values, w'_{BA}, etc., by the appropriate temporary reaction value reversed. Add all the resulting distributed load values w'_{BA}, etc., at each beam to get the total distributed load, w_A, etc., Calculation Sheet 6.

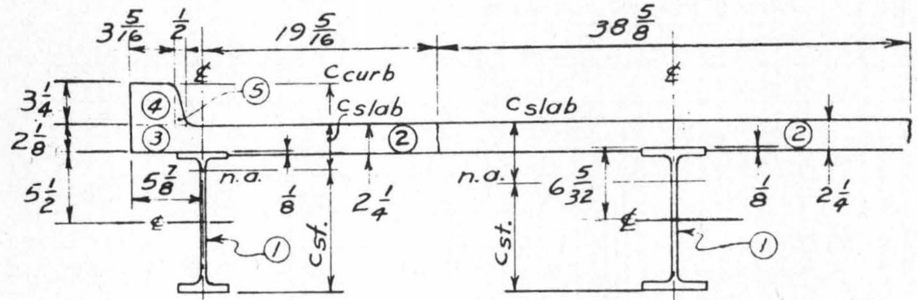
   d. Compute the maximum beam moments and deflections resulting from the combination loading, Calculation Sheet 6.

   e. Compute moments and deflections at points other than the center as needed, Calculation Sheet 7.

Calculation Sheet 1

Step 1a

Dimensions and properties of bridge: 25 ft



Steel:	Area 5.01	Area 6.75
	$I_O =$ 90.0	$I_O =$ 161.0
	Depth = 11.0	Depth = $12\ 5/16$

$L =$ 25 ft 0 in. = 300 in.; $L/4 =$ 75 in.; $L^3/48 = 56.25(10)^4$ in.^{3}

$5L^3/384 = 35.16(10)^4$ in^3; $L^2/\pi^2 = 9.119(10)^3$ in.^{2}; $L^4/\pi^4 = 83.15(10)^6$ in.^{4}

$S =$ 3 ft $2\ 5/8$ in. = 38.63 in.

$E = 29.4\ (10)^6$ psi.; $n =$ 8 ; $E_S = 3.68\ (10)^6$ psi

SLAB; 1 in. wide strip: $I_S = 1(2.25)^3/12 = 0.949$ in.^{3}

$E_S I_S = 3.68\ (10)^6(.949) = 3.49(10)^6$

$\delta_S = S^3/E_S I_S = (38.63)^3/3.49(10)^6 = 16.52(10)^{-3}$

INTERIOR BEAMS, B AND C

No.	DIMENSIONS	AREA	$\bar{y}$, ℄	$A\bar{y}$	$\bar{y}$, N.A.	$I_o + A\bar{y}^2$
	(161)(8)					1288
1	6.75(8)	54.0	0	0	4.41	1052
	$86.9\ (2.25)^2/12$					37
2	2.25(38.63)	86.9	7.16	622	2.74	653
TOTALS		140.9	4.41	622	$I_{conc.} =$	3030

$I_{st.} =$ 379

$EI_B = 3.68\ (10)^6(3030) = 11.14(10)^9$

$c_{slab.} =$ 3.87 in. $\qquad c_{st.} =$ 10.57 in.

$I_{conc.} =$ 783 in.^{3} $\qquad I_{st.}/c_{st.} =$ 35.8 in.^{3}

Calculation Sheet 2

1a, continued — Bridge: 25 ft

EXTERIOR BEAMS, A AND D

No.	DIMENSIONS	AREA	$\bar{y}$, ℄	$A\bar{y}$	$\bar{y}$, N.A.	$I_o + A\bar{y}^2$
	(90.0)8					720
1	(5.01)8	40.1	0	0	4.38	769
	$43.5(2.25)^2/12$					18
2	2.25(19.31)	43.5	6.50	283	2.12	195
	$12.5(2.13)^2/12$					5
3	2.13(5.88)	12.5	6.56	82	2.18	59
	$10.8(3.25)^2/12$					10
4	3.25(3.31)	10.8	9.25	100	4.87	256
	—					–
5	0.5(3.25)(0.5)	0.8	8.71	7	4.33	15
TOTALS		107.7	4.38	472	$I_{conc.}$	2047

$$I_{st.} = 256$$

$$EI_A = 3.68(10)^6(2047) = 7.52(10)^9$$

$c_{curb} = 6.50$ in. ; $c_{slab} = 3.25$ in. ; $c_{st.} = 9.88$ in.

$I_c/c_{curb} = 315.$ in.^{3} ; $I_c/c_{slab} = 630.$ in.^{3} ; $I_{st}/c_{st} = 25.9$ in.^{3}

$$L^3/48EI_A = 56.25(10)^4/7.52(10)^9 = 7.48(10)^{-5}$$

$$5L^3/384EI_A = 35.16(10)^4/7.52(10)^9 = 4.68(10)^{-5}$$

$$\delta_A = \delta_D = L^4/\pi^4 EI_A = 83.15(10)^6/7.52(10)^9 = 11.06(10)^{-3}$$

$$L^3/48EI_B = 56.25(10)^4/11.14(10)^9 = 5.05(10)^{-5}$$

$$5L^3/384EI_B = 35.16(10)^4/11.14(10)^9 = 3.16(10)^{-5}$$

$$\delta_B = \delta_C = L^4/\pi^4 EI_B = 83.15(10)^6/11.14(10)^9 = 7.46(10)^{-3}$$

Calculation Sheet 3

Bridge: 25 ft

Step 1b

Quantities occurring in equations 20-27

From Fig. 12:

$$\underline{\underline{z'_{BB}}} = \frac{\delta_A}{4} + \frac{\delta_C}{4} + \frac{\delta_S}{6} = \frac{11.06(10)^{-3}}{4} + \frac{7.46(10)^{-3}}{4} + \frac{16.52(10)^{-3}}{6} =$$
$$= (2.76 + 1.87 + 2.75)(10)^{-3} = \underline{\underline{7.38(10)^{-3}}}$$

$$\underline{\underline{z'_{BD}}} = -\frac{\delta_A}{4} + \frac{3\delta_C}{4} - \frac{\delta_S}{4} = -\frac{11.06(10)^{-3}}{4} + \frac{3(7.46)(10)^{-3}}{4} - \frac{16.52(10)^{-3}}{4} =$$
$$= (-2.76 + 5.59 - 4.13)(10)^{-3} = \underline{\underline{-1.30(10)^{-3}}}$$

$$\underline{\underline{z'_{DD}}} = \frac{\delta_A}{4} + \frac{9\delta_C}{4} + \delta_S = \frac{11.06(10)^{-3}}{4} + \frac{9(7.46)(10)^{-3}}{4} + 16.52(10)^{-3} =$$
$$= (2.76 + 16.78 + 16.52)(10)^{-3} = \underline{\underline{36.06(10)^{-3}}}$$

For subsequent use:

$$\underline{\underline{(z'_{DD} + \delta_D)}} = (36.06 + 11.06)(10)^{-3} = \underline{\underline{47.12(10)^{-3}}}$$

$$\underline{\underline{(z'_{BB} + \delta_B)}} = (7.38 + 7.46)(10)^{-3} = \underline{\underline{14.84(10)^{-3}}}$$

$$\underline{\underline{(z'_{DD} + \delta_D + z'_{BD})}} = (47.12 - 1.30)(10)^{-3} = \underline{\underline{45.82(10)^{-3}}}$$

$$\underline{\underline{(-z'_{BB} - \delta_B - z'_{BD})}} = (-14.84 + 1.30)(10)^{-3} = \underline{\underline{-13.54(10)^{-3}}}$$

$$\underline{\underline{(z'_{DD} + \delta_D - 3z'_{BD})}} = (47.12 + 3.90)(10)^{-3} = \underline{\underline{51.02(10)^{-3}}}$$

$$\underline{\underline{3(z'_{BB} + \delta_B) - z'_{BD}}} = (44.52 + 1.30)(10)^{-3} = \underline{\underline{45.82(10)^{-3}}}$$

By equation 19:

$$\underline{\underline{N}} = (z'_{DD} + \delta_D)(z'_{BB} + \delta_B) - z'^{2}_{BD} = [(47.12)(14.84) - (1.30)^2](10)^{-6} =$$
$$= (699.3 - 1.7)(10)^{-6} = \underline{\underline{697.6(10)^{-6}}}$$

$$\underline{\underline{2N}} = 2(697.6)(10)^{-6} = \underline{\underline{1395.2(10)^{-6}}}$$

Calculation Sheet 4

Bridge: 25 ft

Step 1c

Sinusoidal loads on slab* caused by P = 1 applied:

At beam A,

$$\underline{\underline{w'_{BA}}} = -\left(\frac{L^3}{48EI_A}\right)\frac{(z'_{DD}+\delta_D+z'_{BD})}{2N} = -\frac{7.48(10)^{-5}(45.82)(10)^{-3}}{1395(10)^{-6}} = \underline{\underline{-2.46(10)^{-3}}}$$

$$\underline{\underline{w'_{DA}}} = -\left(\frac{L^3}{48EI_A}\right)\frac{(-z'_{BB}-\delta_B-z'_{BD})}{2N} = -\frac{7.48(10)^{-5}(-13.54)(10)^{-3}}{1395(10)^{-6}} = \underline{\underline{+0.73(10)^{-3}}}$$

$$\underline{\underline{w'_{AA}}} = -\frac{w'_{BA}}{2}+\frac{w'_{DA}}{2} = \left(\frac{2.46}{2}+\frac{0.73}{2}\right)(10)^{-3} = (1.23+0.36)(10)^{-3} = \underline{\underline{+1.59(10)^{-3}}}$$

$$\underline{\underline{w'_{CA}}} = -\frac{w'_{BA}}{2}-\frac{3w'_{DA}}{2} = \left(\frac{2.46}{2}-\frac{2.19}{2}\right)(10)^{-3} = (1.23-1.09)(10)^{-3} = \underline{\underline{+0.14(10)^{-3}}}$$

At beam C,

$$\underline{\underline{w'_{BC}}} = -\left(\frac{L^3}{48EI_C}\right)\frac{(z'_{DD}+\delta_D-3z'_{BD})}{2N} = -\frac{5.05(10)^{-5}(51.02)(10)^{-3}}{1395(10)^{-6}} = \underline{\underline{-1.85(10)^{-3}}}$$

$$\underline{\underline{w'_{DC}}} = -\left(\frac{L^3}{48EI_C}\right)\left[\frac{3(z'_{BB}+\delta_B)-z'_{BD}}{2N}\right] = -\frac{5.05(10)^{-5}(45.82)(10)^{-3}}{1395(10)^{-6}} = \underline{\underline{-1.66(10)^{-3}}}$$

$$\underline{\underline{w'_{AC}}} = -\frac{w'_{BC}}{2}+\frac{w'_{DC}}{2} = \left(\frac{1.85}{2}-\frac{1.66}{2}\right)(10)^{-3} = (0.92-0.83)(10)^{-3} = \underline{\underline{+0.09(10)^{-3}}}$$

$$\underline{\underline{w'_{CC}}} = -\frac{w'_{BC}}{2}-\frac{3w'_{DC}}{2} = \left(\frac{1.85}{2}+\frac{4.98}{2}\right)(10)^{-3} = (0.92+2.49)(10)^{-3} = \underline{\underline{+3.41(10)^{-3}}}$$

At beam D,

$w'_{AD} = w'_{DA} = +0.73(10)^{-3}$

$w'_{BD} = w'_{CA} = +0.14(10)^{-3}$

$w'_{CD} = w'_{BA} = -2.46(10)^{-3}$

$w'_{DD} = w'_{AA} = +1.59(10)^{-3}$

At beam B,

$w'_{AB} = w'_{DC} = -1.66(10)^{-3}$

$w'_{BB} = w'_{CC} = +3.41(10)^{-3}$

$w'_{CB} = w'_{BC} = -1.85(10)^{-3}$

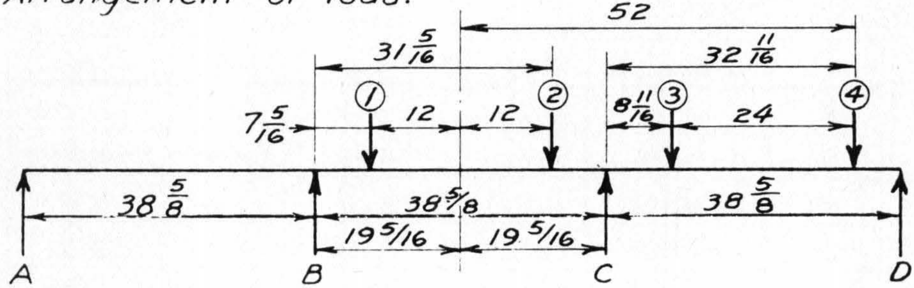
$w'_{DB} = w'_{AC} = +0.09(10)^{-3}$

* The corresponding loads acting on the <u>beams</u> are equal in magnitudes but opposite in signs.

Calculation sheet 5

Bridge: 25 ft
Load: 2-4000-1
a and d lines

Arrangement of load:



Step 2a
INFLUENCE ORDINATES

LOAD No	IN SPAN	x	x/S	INFLUENCE ORDINATE R_A	R_B	R_C	R_D
1	BC	7.31	.189	+.062	−.904	−.188	+.030
2	BC	31.31	.811	+.030	−.188	−.904	+.062
3	CD	8.69	.225	−.021	+.124	−.961	−.142
4	CD	32.69	.846	−.010	+.060	−.244	−.806

Step 2b
TEMPORARY REACTIONS

LOAD No	AMOUNT	R_A^T	R_B^T	R_C^T	R_D^T
1	−2000	−124	+1808	+ 376	− 60
2	−2000	− 60	+ 376	+ 1808	− 124
3	−2000	+ 42	− 248	+ 1922	+ 284
4	−2000	+ 20	− 120	+ 488	+ 1612
TOTALS	+8000	− 122	+ 1816	+ 4594	+ 1712

Calculation sheet 6

Bridge: 25 ft
Load: 2-4000-1
a and d lines

Step 2c

SINUSOIDAL FORCES ON BEAMS; ppi, MAX.

$-R^T$	AMOUNT	w_A	w_B	w_C	w_D
$-R_A^T$	+122	−0.19	+0.30	−0.02	−0.09
$-R_B^T$	−1816	−3.01	+6.19	−3.36	+0.16
$-R_C^T$	−4594	+0.41	−8.50	+15.67	−7.63
$-R_D^T$	−1712	+1.25	+0.24	−4.21	+2.72
TOTALS		−1.54	−1.77	+8.08	−4.84

Step 2d

MOMENTS, STRESSES, STRAINS, AND DEFLECTIONS

$M_1 = R^T L/4$; $M_2 = -wL^2/\pi^2$; $M = M_1 + M_2$

$f_{st} = M/I_{st}/c_{st}$; $\epsilon_{st} = f_{st}/E_s$

$\Delta_1 = -R^T L^3/48EI$; $\Delta_2 = +w\delta$; $\Delta = \Delta_1 + \Delta_2$

	BEAM A	BEAM B	BEAM C	BEAM D
$\times 10^3$ M_1 ip	− 9.1	+136.2	+344.6	+128.4
$\times 10^3$ M_2 ip	+ 14.0	+ 16.1	− 73.7	+ 44.1
$\times 10^3$ M ip	+ 4.9	+ 152.3	+ 270.9	+172.5
f_{st} psi	190	4250	7560	6660
ϵ_{st} $10^{-6}\times$	6	145	257	227
Δ_1 in.	+.009	−.092	−.232	−.128
Δ_2 in.	−.017	−.013	+.060	−.054
Δ in.	−.008	−.105	−.172	−.182

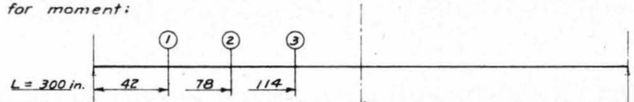
Calculation sheet 7

Step 2e

Bridge: 25 ft
Load: 2-4000-1
a and d lines

MOMENTS, STRESSES, AND STRAINS ALONG THE BEAMS

Location of points for moment:

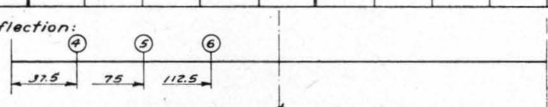


$M_r = M_1(2r) + M_2 \sin \pi r$

All moments in $(10)^3$ in.-lb. All stresses in psi. All strains in $(10)^{-6}$ in. per in.

Point	y	$r = y/L$	$2r$	$\sin \pi r$	Beam A $M_1(2r)$	Beam A $M_2 \sin$	Beam A M_r	Beam A $f_{st.}$	Beam A $\epsilon_{st.}$	Beam B $M_1(2r)$	Beam B $M_2 \sin$	Beam B M_r	Beam B $f_{st.}$	Beam B $\epsilon_{st.}$	Beam C $M_1(2r)$	Beam C $M_2 \sin$	Beam C M_r	Beam C $f_{st.}$	Beam C $\epsilon_{st.}$	Beam D $M_1(2r)$	Beam D $M_2 \sin$	Beam D M_r	Beam D $f_{st.}$	Beam D $\epsilon_{st.}$
℄	150	.500	1.000	1.000	-9.1	+14.0	+4.9	190	6	+136.2	+16.1	+152.3	4250	145	+344.6	-73.7	+270.9	7560	258	+128.4	+44.1	+172.5	6660	227
1	42	.140	.280	.426	-2.6	+ 6.0	+ 3.4	130	4	+ 38.2	+ 6.9	+ 45.1	1260	43	+ 96.5	-31.4	+ 65.1	1820	63	+ 36.0	+ 18.9	+ 54.8	2110	72
2	78	.260	.520	.729	-4.7	+10.2	+ 5.5	210	7	+ 70.9	+ 11.7	+ 82.6	2310	79	+ 179.2	-53.7	+ 125.5	3500	119	+ 66.8	+ 32.2	+ 99.0	3820	130
3	114	.380	.760	.930	-6.9	+ 13.0	+ 6.1	240	8	+ 103.7	+ 15.0	+118.7	3310	113	+261.9	-68.5	+ 193.4	5400	184	+ 97.5	+ 41.0	+ 138.5	53.50	182

Location of points for deflection:



DEFLECTIONS ALONG THE BEAMS

$\Delta_r = \Delta_1(3r - 4r^3) + \Delta_2 \sin \pi r$

All deflections in inches.

Point	y	$r = y/L$	$3r$	$4r^3$	$\alpha = 3r - 4r^3$	$\sin \pi r$	Beam A $\Delta_1 \alpha$	Beam A $\Delta_2 \sin$	Beam A Δ_r	Beam B $\Delta_1 \alpha$	Beam B $\Delta_2 \sin$	Beam B Δ_r	Beam C $\Delta_1 \alpha$	Beam C $\Delta_2 \sin$	Beam C Δ_r	Beam D $\Delta_1 \alpha$	Beam D $\Delta_2 \sin$	Beam D Δ_r
℄	150.	0.500	1.500	0.500	1.000	1.000	+0.009	-0.017	-0.008	-0.092	-0.013	-0.105	-0.232	+0.060	-0.172	-0.128	-0.054	-0.182
4	37.5	.125	.375	.008	.367	.383	+ .003	- .007	- .004	- .034	- .005	- .039	- .085	+ 0.023	- .062	- .047	- .021	- .068
5	75.	.250	.750	.062	.688	.707	+ .006	- .012	- .006	- .063	- .009	- .072	- .160	+ 0.042	- .118	- .088	- .038	- .126
6	112.5	.375	1.125	.211	.914	.924	+ .008	- .016	- .008	- .084	- .012	- .096	- .212	+0.055	- .157	- .117	- .050	- .167

F. Example Analysis

The actual calculations for one of the bridges tested, the 25 ft. bridge, for one particular loading, two 2000 pound axle loads side by side on the "a" and "d" lines defined in Fig. 20, are reproduced on the following pages as an example. Calculations for moments and deflections along the beams are included, Calculation Sheet 7, though in practice these would seldom be required. Also, it will be noted that the calculations on sheets 1 and 2 are essentially those that would be needed in the most simplified analysis. Thus, Calculation Sheets 3, 4, 5, and 6 include the calculations that would ordinarily be needed, and that are peculiar to the proposed method.

G. Effects of Crown and of Longitudinal Distribution

One effect of the crown of the roadway is to cause more than half the load on a truck to be carried by the outer wheels and less than half by the inner ones. While a relatively small effect, it is easily taken into account by adjusting the loads used in the calculations, and this has been done in the analyses for which the results are reported subsequently.

Assumption number 3, that there is no longitudinal distribution of the temporary reactions can, also, be improved upon rather easily. While the exact extent of longitudinal distribution of these reactions is not known, the assumption of a zero length seems to be at one extreme, and any reasonable value would be an improvement. As a first approximation, the effective slab width, L_E, currently specified for the design of slabs for moment has been used (1, p. 170). For the highway bridges, using a truck with tandem-axles,

$$L_E = .063S + 4.65. \tag{47}$$

For the laboratory bridges the equations become:

with a single axle,

$$L_E = .4S + 1.25, \tag{48}$$

with tamden axles,

$$L_E = .063S + 1.55. \tag{49}$$

In these equations S and L_E are to be measured in feet.

If the load, P, is assumed to be uniformly distributed over this length, L_E, the maximum moment becomes:

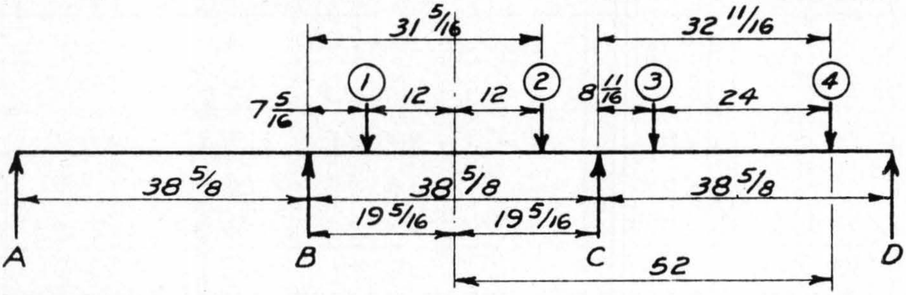
$$M_1 = \frac{PL}{4} \left(1 - \frac{L_E}{2L}\right) \tag{50}$$

The distribution has no significant effect on the beam deflection, hence has no effect on any of the calculations except that for M_1. This distribution has, also, been taken into account in the analyses.

Calculation sheet 8

Bridge: 25 ft
Load: 2-4000-1
a and d lines
Corrected for crown

Arrangement of load:



Step 2a
INFLUENCE ORDINATES

LOAD NO	IN SPAN	x	x/S	INFLUENCE ORDINATE R_A	R_B	R_C	R_D
1	BC	7.31	0.189	+0.062	-0.904	-0.188	+0.030
2	BC	31.31	.811	+.030	- .188	- .904	+ .062
3	CD	8.69	.225	-.021	+ .124	- .961	- .142
4	CD	32.69	.846	-.010	+ .060	- .244	- .806

Step 2b
TEMPORARY REACTIONS

LOAD NO	AMOUNT	R_A^T	R_B^T	R_C^T	R_D^T
1	-2000	-124	+1808	+376	-60
2	-2000	-60	+376	+1808	-124
3	-1900	+40	-236	+1826	+270
4	-2100	+21	-126	+512	+1693
TOTALS		-123	+1822	+4522	+1779

Calculation sheet 9

Bridge: 25 ft
Load: 2-4000-1

Step 2c Corrected for crown, a and d lines
SINUSOIDAL FORCES ON BEAMS; ppi, MAX.

$-R^T$	AMOUNT	w_A	w_B	w_C	w_D
$-R_A^T$	+ 123	- 0.20	+ 0.30	- 0.02	- 0.09
$-R_B^T$	-1823	-3.03	+ 6.22	- 3.37	+ 0.16
$-R_C^T$	-4522	+ 0.41	- 8.37	+ 15.42	- 7.51
$-R_D^T$	-1779	+ 1.30	+ 0.25	- 4.38	+ 2.83
TOTALS		- 1.52	- 1.60	+ 7.65	- 4.61

Step 2d
MOMENTS, STRESSES, STRAINS, AND DEFLECTIONS

$M_1 = R^T L/4^*$; $M_2 = -wL^2/\pi^2$; $M = M_1 + M_2$

$f_{st} = M/I_{st}/c_{st}$; $\epsilon_{st} = f_{st}/E_s$

$\Delta_1 = -R^T L^3/48EI$; $\Delta_2 = +w\delta$; $\Delta = \Delta_1 + \Delta_2$

	BEAM A	B	C	D
$\times 10^3$ M_1 ip	- 8.8	+129.8	+322.0	+126.7
M_2 ip	+ 13.9	+ 14.6	- 69.8	+ 42.0
M ip	+ 5.1	+144.4	+252.2	+168.7
f_{st} psi	200	4030	7050	6510
ϵ_{st} $10^{-6}\times$	7	137	239	222
Δ_1 in.	+0.009	-0.092	-0.228	-0.133
Δ_2 in.	- .017	- .012	+ .057	- .051
Δ in.	- .008	- .104	- .171	- .184

*
Correcting for distribution, $L_E = 30$ in.,

$$M_1 = \frac{R^T L}{4}\left(1 - \frac{L_E}{2L}\right) = R^T(75)\left(1 - \frac{30}{600}\right) = R^T(71.2)$$

When these two refinements are made in the preceding example, calculation sheets 5 and 6 are changed slightly, as shown on calculation sheets 8 and 9. Calculation sheet 7 would not be changed except for moments at points within the length L_E.

IV. TESTS

As indicated previously, no truly exact method of analysis of beam and slab bridges is available as a standard; hence the value of any proposed method can only be determined by comparing predicted strains and deflections with those measured in actual bridges. Such measurements have been made on four bridges, two full-size structures in use on a highway, and two one-third-size bridges in the laboratory. The highway bridges were designed and built before the testing project was conceived; hence they were not specially controlled. Also, the field tests were subject to difficulties and errors resulting from the distance to them, the necessity for setting up and taking down equipment each day, shortages of time and personnel, the size of the loads to be handled, traffic, rapid changes in temperature, and bad weather that could be eliminated or reduced in the laboratory. Consequently, the laboratory bridges were designed, built, and tested.

A. Descriptions of Bridges Tested

The bridges tested are all alike in some ways. Each has four longitudinal beams equally spaced, the centerlines of the edge beams being approximately 6 in. from the faces of the curbs in the highway bridges and 2 in. in the laboratory ones. All have shear connectors welded to the upper flanges of the beams to help produce composite action of the steel and concrete, and all have relatively massive composite end diaphragms. Intermediate diaphragms are relatively small and are not composite. Curbs are of essentially the minimum permissible size. They either have no handrails or relatively light handrails that are judged to have a negligible effect on the behavior of the bridge.

All the bridges were built of the usual materials, mild steel in the beams and reinforcing, and "class A" concrete in the slabs and integral curbs. The usual average modulus of elasticity for steel, 29,400,000 psi, has been used. A modular ratio, n, of 8 has been used, giving 3,680,000 psi as the modulus of elasticity of the concrete. For these materials it is common practice in design to use a value of 10, reflecting the 28 day strength of the concrete. The value of 8 was chosen because the concrete was much older, 6 months to 3 years, than 28 days when tested. Auxiliary analyses have shown that the predicted maximum strains are not sensitive to the assumed value of n.

1. *Highway bridges*

These two bridges have the same roadway width, 30 ft., the same curb dimensions, and the same crown, Fig. 14. The spans are 41.25 ft. and 71.25 ft., and the beam sizes are different, accordingly, Fig. 14. The beams rest on bearing plates that are curved to provide for rotation at the ends. The plates at one end can slide to provide for expansion and contraction. Partial length cover plates are welded to the lower flanges of the beams; so their moments of inertia are not constant.

The slabs vary slightly in thickness in the transverse sections. In the longitudinal sections the slab of the longer bridge is constant in thickness, but that of the shorter one is varied to compensate for dead load deflection, Fig. 14. An average thickness of 8.07 in. has been used throughout for the 71.25 ft. bridge. An average of 8.63 in. has been used in computing the δ_s quantities for the 41.25 ft. span, but the actual thicknesses have been used in computing the moments of inertia of the beams. The primary reinforcement of the slabs consists of $\frac{3}{4}$ in. round straight bars at 8.5 in. center to center in both the top and bottom. According to the design drawings these bars were to have been placed at an average of 2 in. from the surfaces to the centers of the bars. Limited exploration disclosed, however, that they are actually severely displaced in the completed bridges. Longitudinal reinforcement consists of 13 $\frac{3}{4}$ in. round bars in each space between beams. Of these, 7 are near the top surface and 6 are near the bottom.

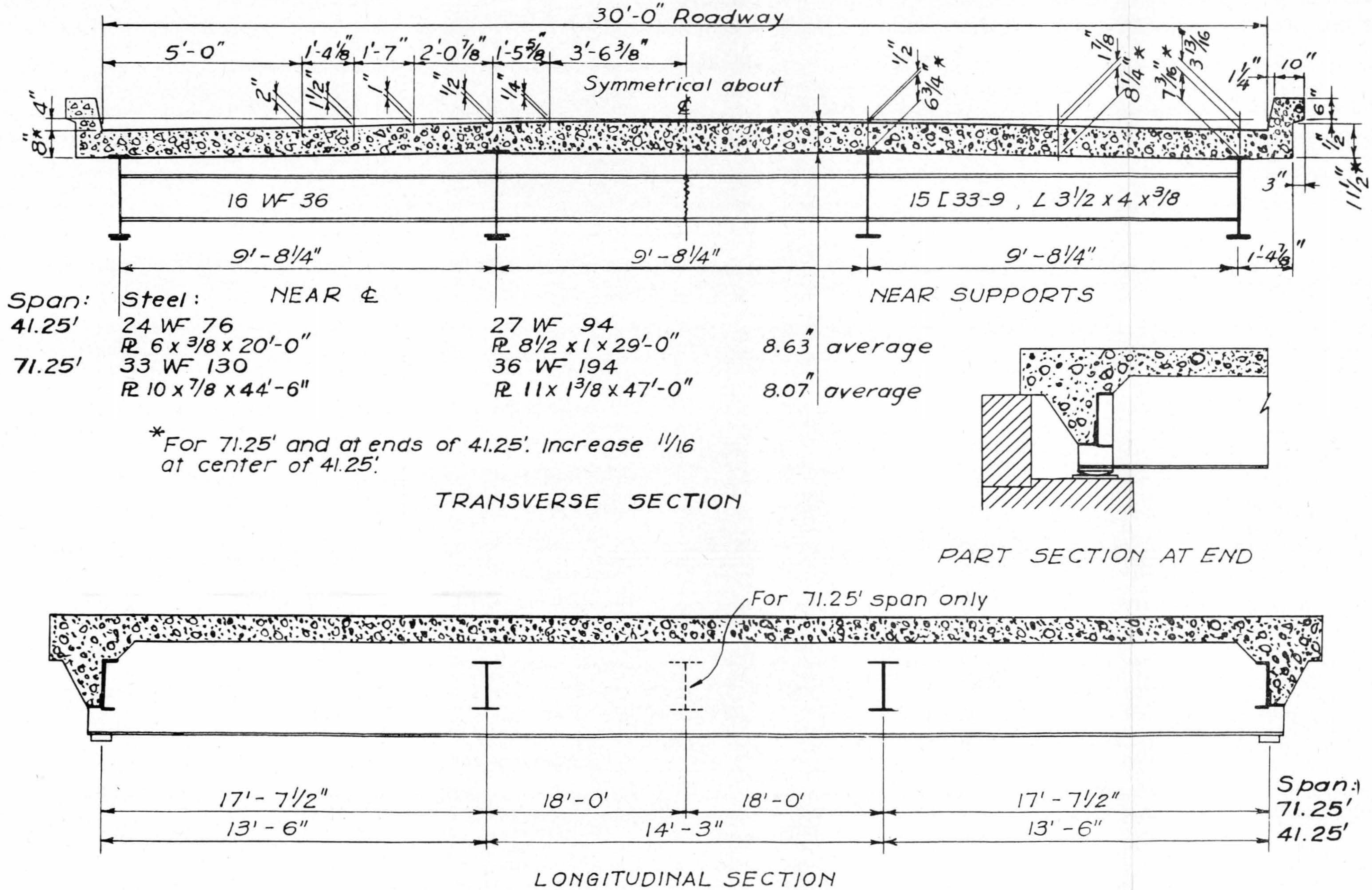
Visual inspection of the two bridges indicated a "built-in" condition at the supports resulting from expansion of the approach pavements and from pouring the concrete of the abutments against the edges of the bridges. This condition, along with the sliding plate supports, was expected to cause end restraint in the beams and consequent reversals of the bending moments.

The moments of inertia and other properties computed on the basis of the foregoing data and of the assumptions listed in the preceding chapter are given in Table 2. In this table the symbols used are those defined in the preceding chapter. Also included are the equivalent slab widths, L_E, computed by the AASHO specifications (1, p. 170).

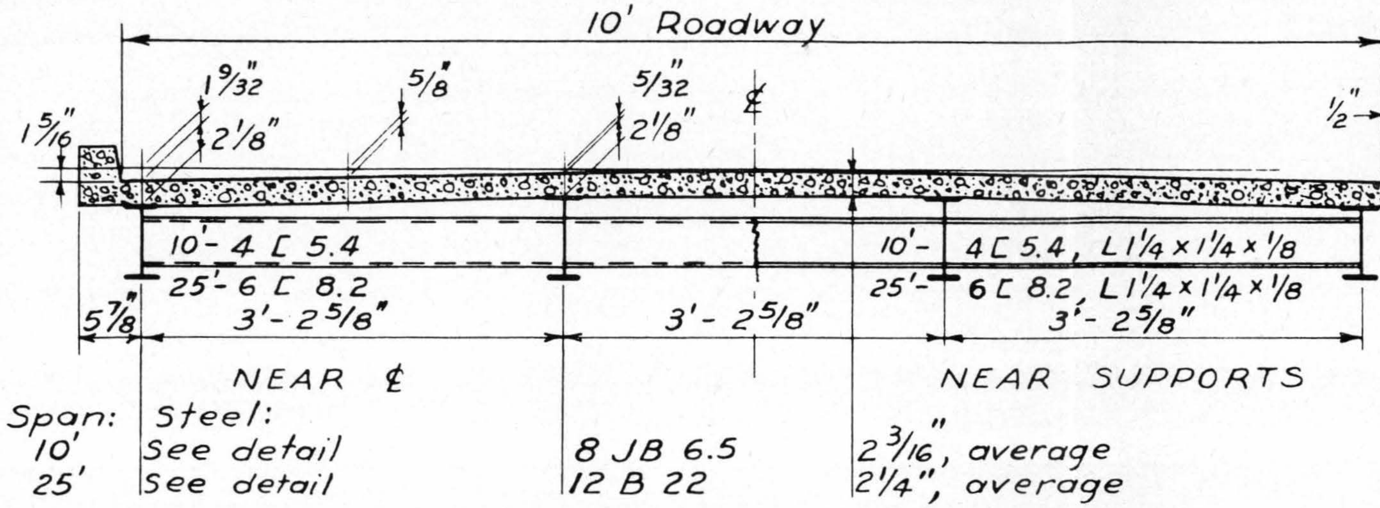
The original calculations and design drawings for these bridges are on file with the Iowa State Highway Commission, Ames, Iowa. The 41.25 ft. bridge is designated as design no. 3845, file 11744; the 71.25 ft. one is design 3645, file 11744.

2. *Laboratory bridges*

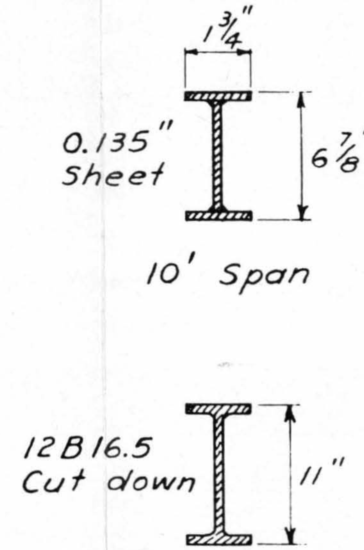
These two bridges have the same roadway width, 10 ft., the same curb dimensions, and the same crown, Fig. 15. These dimensions are one-



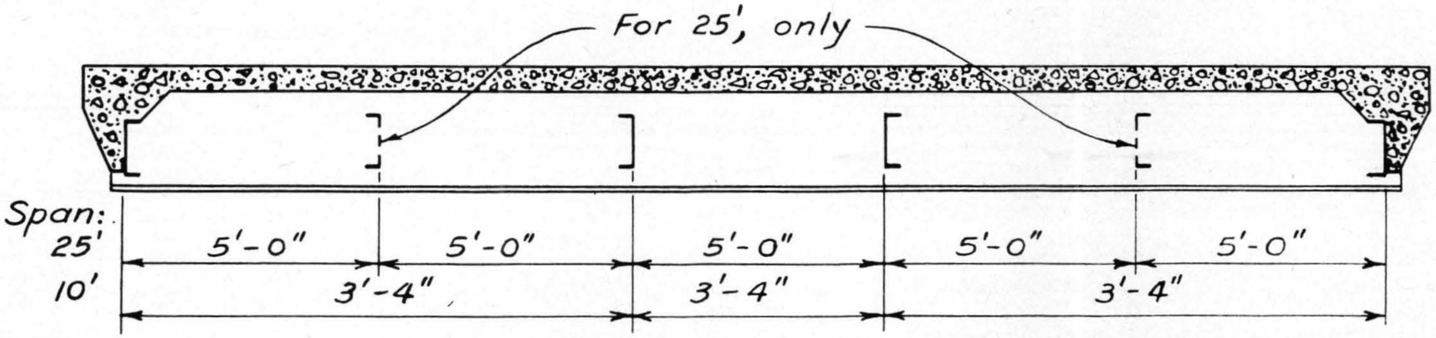
HIGHWAY BRIDGES
FIG. 14



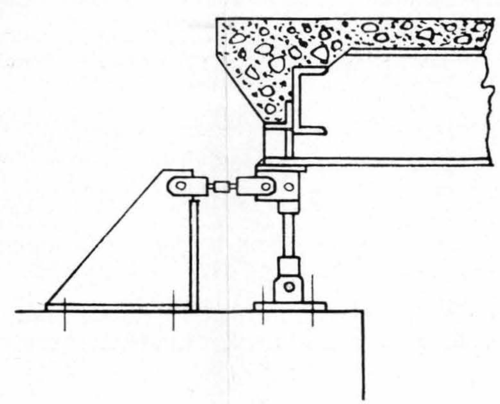
TRANSVERSE SECTION



25' Span
Details of Exterior Beams



LONGITUDINAL SECTION



SECTION AT END

LABORATORY BRIDGES
FIG. 15

Table 2. **Properties of bridges tested**

Bridge, span, ft.	10		25		41.25		71.25	
Span, L, in.	120		300		495		855	
Beam spacing, S, in.	38.63		38.63		116.25		116.25	
Equiv. width, L_E								
Single axle, in.	30		30		90		90	
Tandem axles, in.	42		42		126		126	
Slab thickness, h, in.	2.19		2.25		8.63[a]		8.07	
δ_s, = S^3/E_sI_s, $(10)^{-3}$ in.^{2}/lb	17.90		16.52		7.98		9.75	
Ratio, I int./I ext., b	1.33		1.48		1.68		1.65	
Beam	B Int.	A Ext.	B Int.	A Ext.	B Int.	A Ext.	B Int.	A Ext.
I at center, [c] in.^{4}	67.2	50.6	379	256	16,600	9,900	45,500	27,500
I at end, [c] in.^{4}	—	—	—	—	10,000	7,750	35,900	19,600
EI at center, $(10)^9$ lb.-in.^{2}	1.98	1.49	11.14	7.52	488	291	1,338	811
EI at end, $(10)^9$ lb.-in.^{2}	—	—	—	—	292	228	1,056	575
$I_{st.}/c_{st.}$ at center in.^{3}	8.20	6.54	35.8	25.9	620	395	1,450	928
$I_{st.}/c_{st.}$ at end in.^{3}	—	—	—	—	372	311	1,105	633
H = EI/LE_sI_s, [b]	5.1	3.9	10.7	7.2	5.0	3.0	9.7	5.9
Deflection caused by:								
1 lb. at center, $L^3/48EI$, $(10)^{-5}$in.	1.82	2.42	5.05	7.48	.527	.898	.989	1.64
(1)Sin πr lb./in., δ, $(10)^{-3}$in.	1.08	1.43	7.46	11.06	1.29	2.21	4.15	6.78

[a]At center, varies to 8.00 at ends.

[b]I at center.

[c]Equivalent all-steel section.

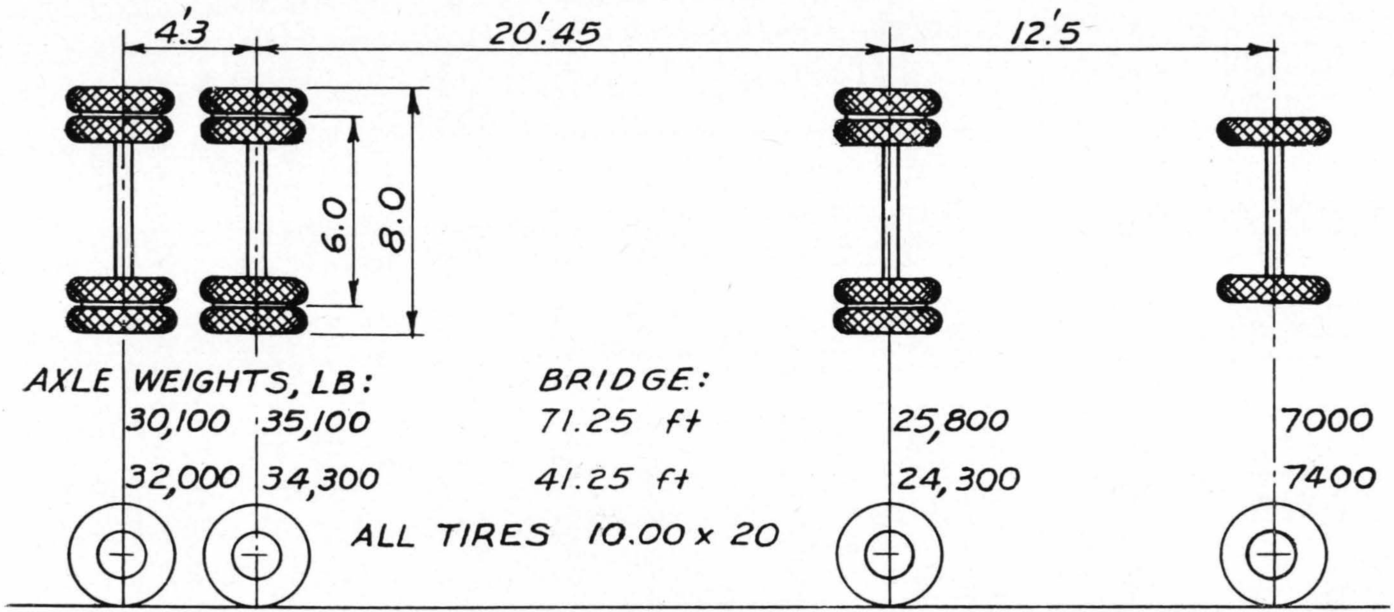
third the corresponding dimensions of the highway bridges. No other dimensions of the laboratory bridges is scaled from the full-size. Instead, they were independently designed for use as test specimens.

The two spans, 10 ft. and 25 ft., were chosen as being near the extremes for which this type of bridge might be used. The slabs were made relatively thin, 2-3/16 and 2-¼ in., in line with a trend toward the use of thinner slabs and to give a greater range of beam to slab stiffness ratios. The relative size of the interior and exterior beams was intended to be about the same as in the highway bridges, but the beams were made somewhat smaller than would be obtained by scale reduction. This was done in anticipation of the possible use of less conservative specifications and to increase the strains and deflections measured. The as-built sizes differed somewhat from design sizes. The as-built sizes are shown in Fig. 15, and the resulting properties are given in Table 2. These were, of course, used in the analyses.

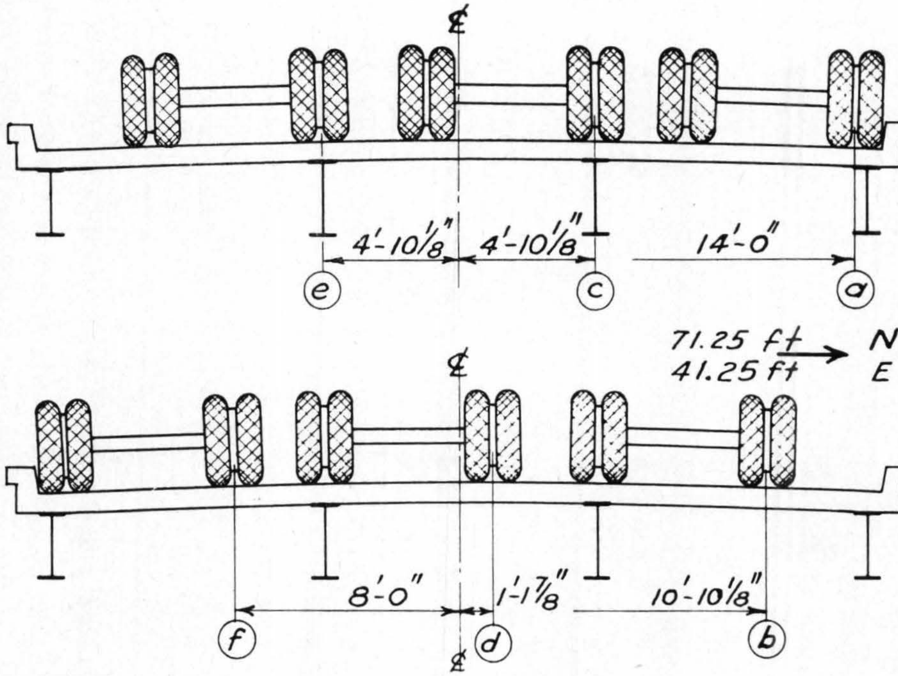
The primary slab reinforcement consists of 0.207 in. diameter smooth rods at 2 in. center to center for both positive and negative moment. Every third bar is straight in both the top and bottom. The two intermediate bars are trussed. Longitudinal reinforcement consists of 6 bars per panel, all near the bottom. The cover is 7/16 in. to the center of the primary reinforcement at both faces. This arrangements of the reinforcing uses only about one-half the weight of steel that would be required if it were simply scaled down from the full-size bridges tested.

The beams are constant in cross-section and are supported at the ends by vertical steel rods having machined clevises and ground steel pins 5/8 in. in diameter at each end. They are thus relatively free to rotate and expand without the accidental restraint of abutments and sliding plates. By placing strain gages on the rods the reactions can be determined. Longitudinal and lateral support are provided by similar hinged rods offering minimum resistance to deformation.

The weight of one-third-scale models is reduced to 1/27 of that of prototypes made of the same materials. To obtain the same dead load strains and to obtain dead load deflections reduced by the scale factor, the weight of the models should be 1/9 of that of the prototype. The models, therefore, are only 1/3 as heavy as they should be for similarity of these effects. Though no dead load effects were measured, the deficiency in the weight of the laboratory bridges was approximately made up by hanging concrete blocks from the slab. This was done to seat the reaction rods and to increase all initial gage readings so that slight reversals caused by the live load would not cause actual reversals but would leave each net strain or deflection always of the same sign.



HIGHWAY BRIDGE LOADS
DIMENSIONS AND AXLE WEIGHTS
FIG. 16



Wheel loads for 5 ft height to c. of g.

LOAD LINE	CROWN EFFECT, IN.	FRACTION OF AXLE LOAD		32,000 LB AXLE	
		INTERIOR	EXTERIOR	INTERIOR	EXTERIOR
c, d	0.44	.495	.505	15,840	16,160
b, e	1.87	.478	.522	15,300	16,700
a, f	2.32	.473	.527	15,140	16,860

HIGHWAY BRIDGES
LATERAL POSITIONS OF TRUCKS,
EFFECT OF CROWN ON WHEEL LOADS
FIG. 17

B. Loads and Positioning of Loads

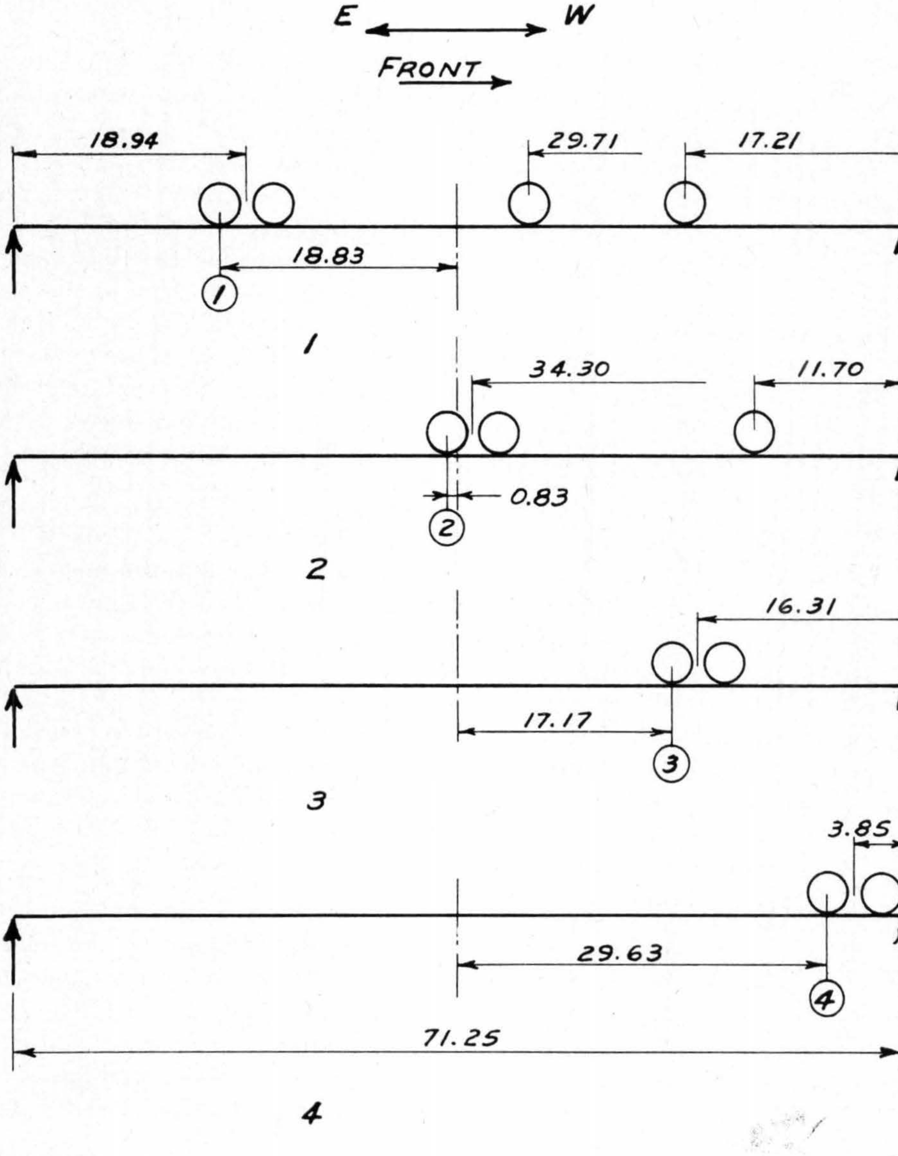
1. *Field test loads*

The test loads for the highway bridges consisted of a single commercial semi-trailer truck loaded with pig-iron to a total of 98,000 pounds. Because it was not possible to move the fully loaded truck over the highway, it was necessary to load and unload it at each bridge. As a result, the distribution of the weight was not the same for the two bridges. The total load was determined by beam scale weighings of the partially loaded truck and of another truck hauling pig-iron. Weighings of the fully loaded truck at each site had to be attempted, however, to determine the distribution to the axles. This was done with calibrated hydraulic jacks. The total loads obtained from the jacks, 100,000 lb. and 105,000 lb. did not agree with that obtained from the scale weighings. The jack readings were, then, reduced proportionately so that the totals did agree. The resulting axle loads and the critical dimensions of the truck are shown in Fig. 16.

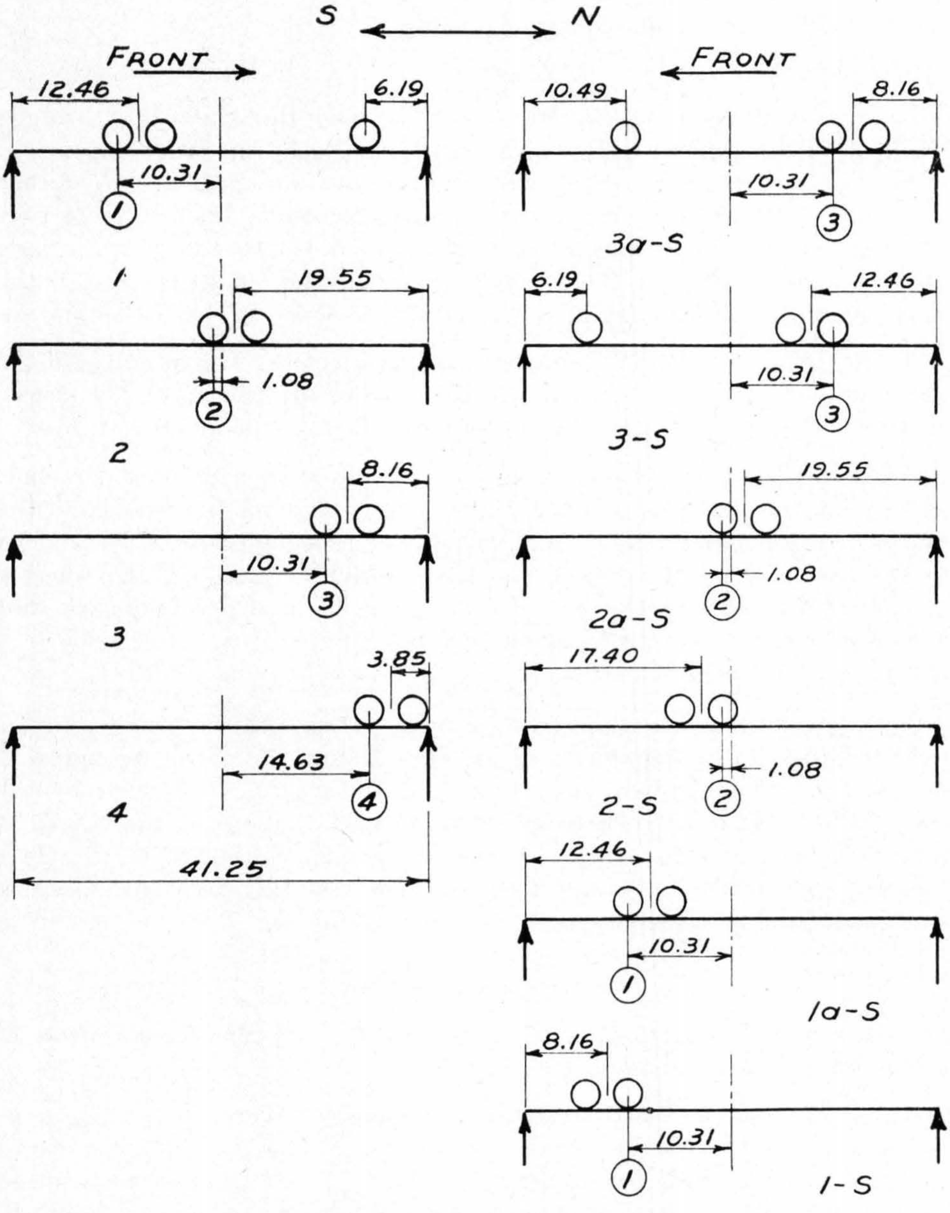
The truck was positioned on the bridges by means of systems of lettered and numbered lines painted on the roadway. The lettered lines were parallel to the direction of motion, thus they determined the lateral position of the truck. When it was moving along the "a" line its outer tires were tight against one curb, etc. The locations of these lines and of the truck when in position along each in turn are shown in Fig. 17. The numbered lines ran across the roadway and represented longitudinal positions at which the truck was stopped as it was moved along one of the lettered lines. Normally the truck was stopped with the rearmost axle at a numbered line. Hence, its position at any such stop is fully indicated by a letter and a number. For instance, "b-2" indicates the truck moving along the "b" line and stopped with the rearmost axle at the "2" line.

On the 41 ft. bridge a few stops were made with the front tandem axle at a line. These were indicated by the suffix "a", as "b-2a", etc. Also on the 41 ft. span, most of the runs were made with the truck headed North, but a few were made with it turned around. These were indicated by the suffix "S", as "b-2-S", etc. The locations of the numbered lines and of the various axles as the truck was stopped at each in turn are shown in Figs. 18 and 19.

Because of the crown of the roadway the truck was not level in any of the lateral positions used. Hence, more than half of each axle load was carried on the outer wheels and less than half on the inner wheels. The amount of the change from one-half depends on the difference in elevations at the two wheels and on the height of the center of gravity of the load. The theoretical differences in elevation have been determined from the design drawings, and the height of the center of gravity was estimated to



71.25 FT BRIDGE
LONGITUDINAL POSITIONS OF TRUCK
FIG. 18



41.25 FT BRIDGE
LONGITUDINAL POSITIONS OF TRUCK
FIG. 19

be 5 ft. The resulting divisions of the axle loads to the wheels in each lateral position are tabulated as part of Fig. 17.

2. *Laboratory test loads*

Simplified model trucks were used to load the laboratory bridges. Each of these trucks consists of a structural steel framework carried by either one or two axles. Each axle mounts four 4.00-8 tires, two at each end corresponding to the usual dual tire arrangement. Each of these tires is very nearly a true one-third-scale model of a 12:00-24 tire, a size used on very heavy trucks. The pressure used in the model tires is 100 psi, approximately the same as in the full-scale, as it should be for similarity.

The distance center to center of the dual tires is 2 ft., one-third of the usual full-size spacing of 6 ft. When two axles are used they are spaced 1 ft. 5 in. apart, one-third of the common full-size spacing of 4 ft. 3 in.

The model trucks were loaded by stacking steel bars (scraper blade edges) on the framework until the desired weight was obtained. The trucks were weighed empty and each bar was weighed as it was added to the load. The total weight was thus obtained by adding the weights of the truck and of the bars used. The capacity of the tires is such that a load of 4000 lb. per axle can be and was used. This corresponds to a full-scale axle load of 36,000 lb.

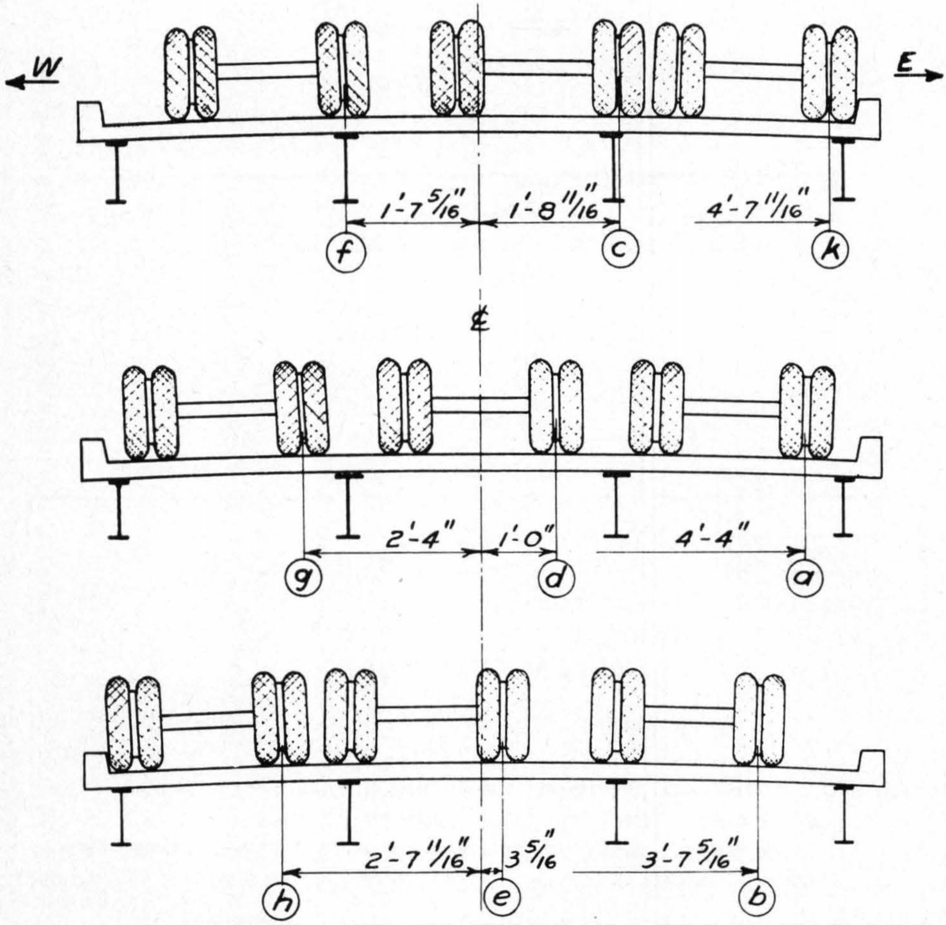
The model trucks were positioned by a system of lines entirely similar to those used on the highway bridges, Figs. 20 and 21. As in the highway bridges the crown caused more than half the load to be carried by the outer tires. When a truck is loaded to 4000 lb. (one axle) the center of gravity is estimated to be at 21 in. When it is loaded to 8000 lb. (2 axles) the center of gravity moves up to 24 in. The resulting distribution of the axle loads to the wheels is tabulated as part of Fig. 20.

C. Instrumentation

Strains and deflections were measured at a number of points in each bridge for each arrangement of loads.

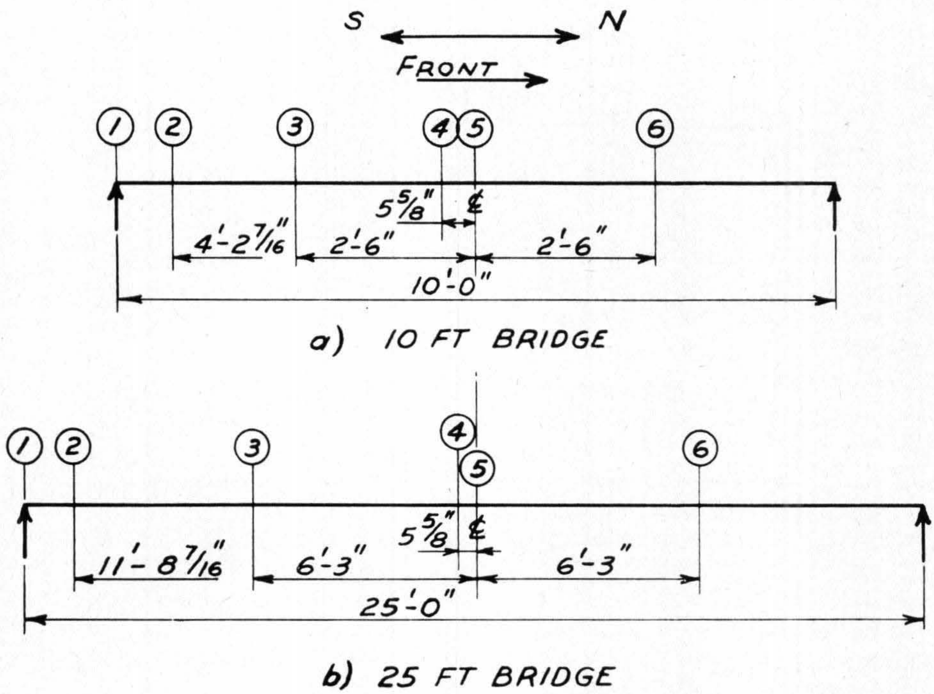
"SR-4", type A, electric resistance strain gages were used throughout. At each bridge these were assigned numbers; the locations of these gages by number are given in Figs. 22, 23, 24, 27 and 28. In the field tests speed in taking readings was essential, so most strain gage readings were obtained by means of a 48 channel automatic switching and recording unit. A few gages were read by means of the usual Baldwin-Southwark "K" unit. The numbers of these include the prefix "A", Figs. 22 and 23. In the laboratory all readings were made using a "K" unit.

Deflection gages were all of the dial type, independently supported



LOAD LINE	CROWN EFFECT IN.	WHEEL LOADS, 4000 LB AXLE			
		21 IN. HEIGHT		24 IN. HEIGHT	
		INTERIOR	EXTERIOR	INTERIOR	EXTERIOR
C, E	0.20	1970	2030	1970	2030
B, F	0.57	1920	2080	1900	2100
A, G	0.70	1900	2100	1880	2120
K, H	0.75	1890	2110	1870	2130

LABORATORY BRIDGES
LATERAL POSITIONS OF TRUCKS,
EFFECT OF CROWN ON WHEEL LOADS
FIG. 20



Notes: When a single axle was used it was stopped on the numbered lines.

When tandem axles were used the rear axle was stopped on the numbered lines.

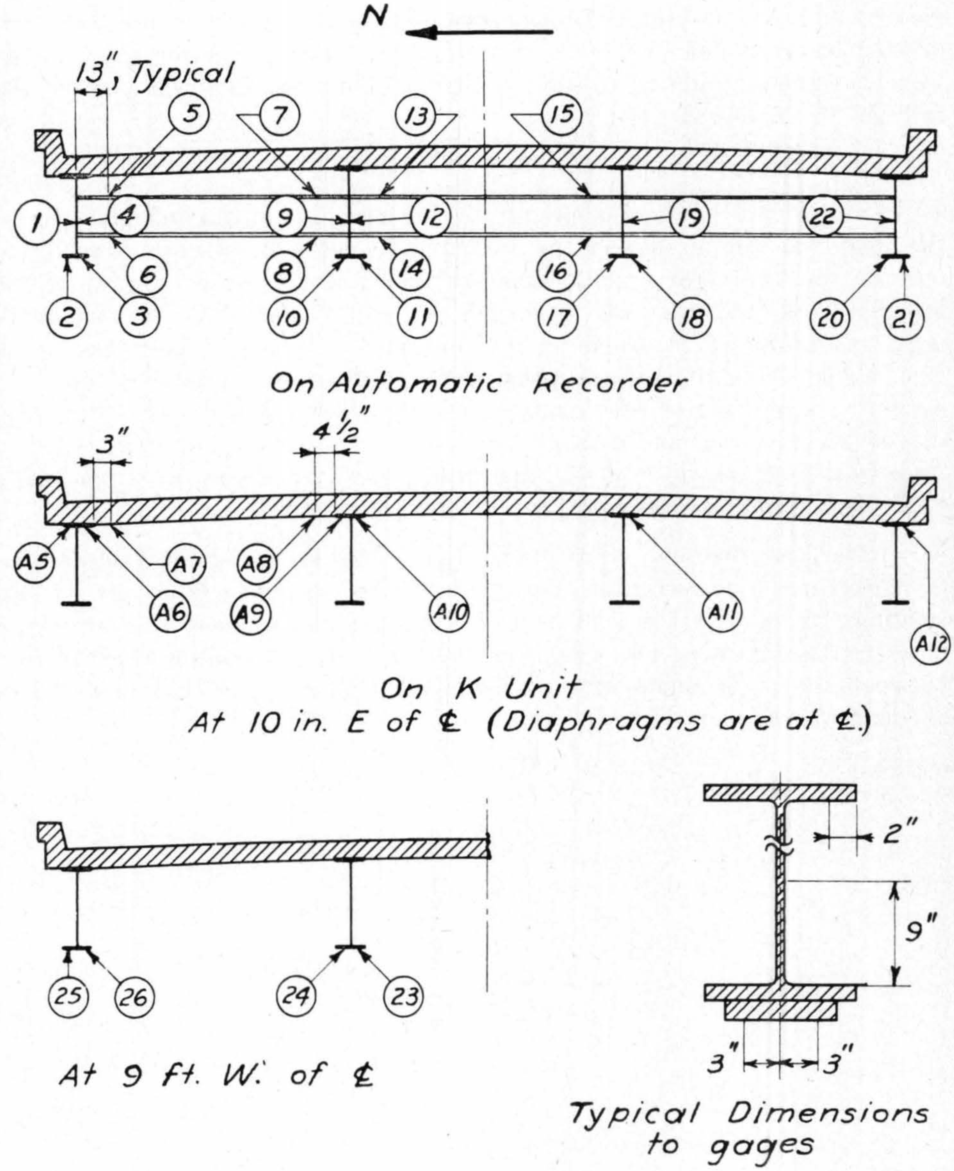
LABORATORY BRIDGES
LONGITUDINAL POSITIONS OF TRUCKS
FIG. 21

from the ground or floor. On those placed under the beams one dial division corresponds to 0.001 in. deflection, whereas on those under the slab each division corresponds to 0.0001 in. deflection. The deflection gages, also, were assigned numbers at each bridge. Their locations were as shown in Figs. 25, 26, and 29.

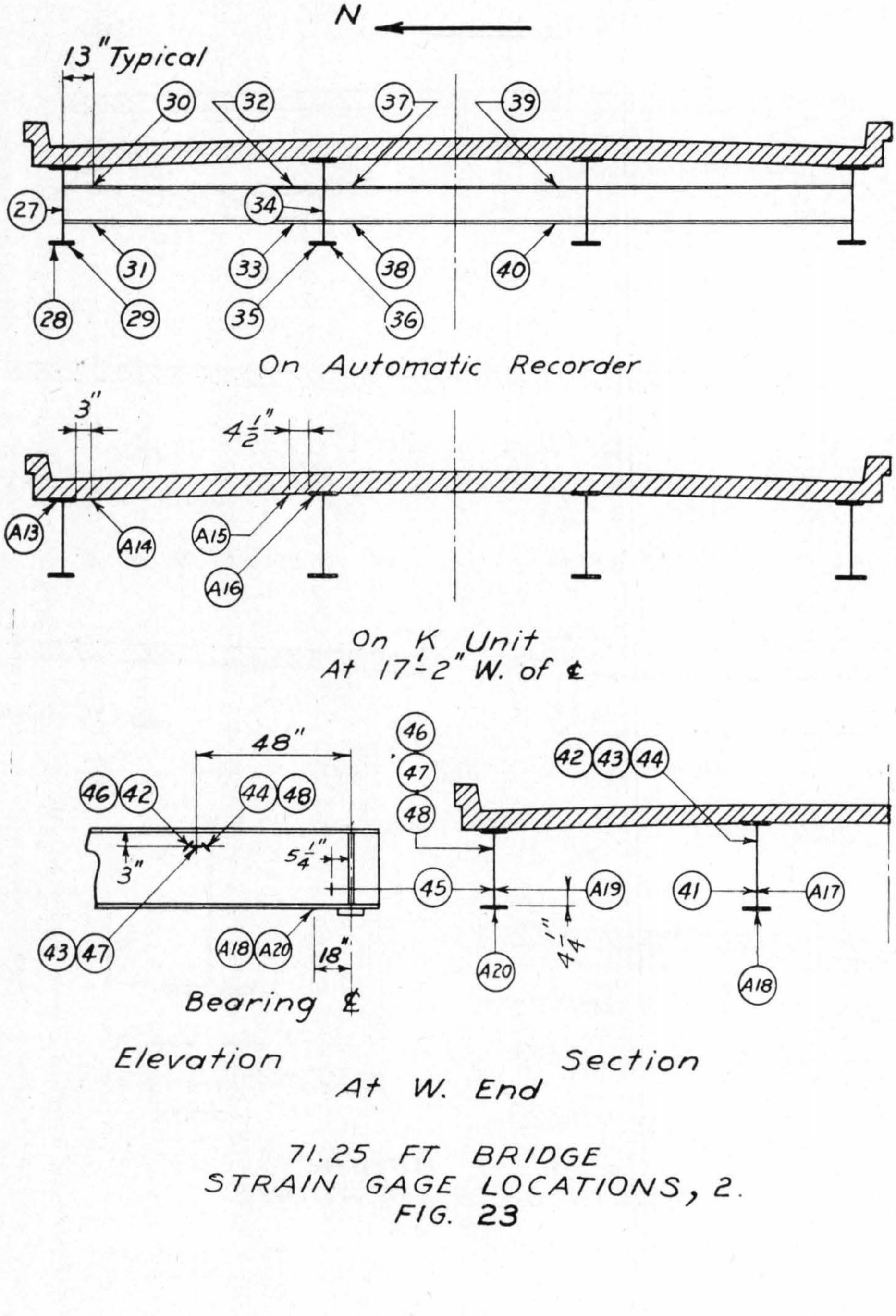
D. Test Procedure

The test procedure was essentially the same for all the bridges. After the load was prepared and the instruments were in place and ready to operate the truck (or trucks) was positioned along one (or more) of the lettered lines, but just off the span. A set of "zero" or no-load readings was taken or recorded on the automatic machine. Then the truck was moved along the line, stopped at the various numbered lines, and for each stop a new set of readings taken or recorded. Finally, the truck was moved off the span and a final set of no-load readings (sometimes called "re-zeros") was made. In the laboratory, conditions were so stable many of the zero readings were omitted.

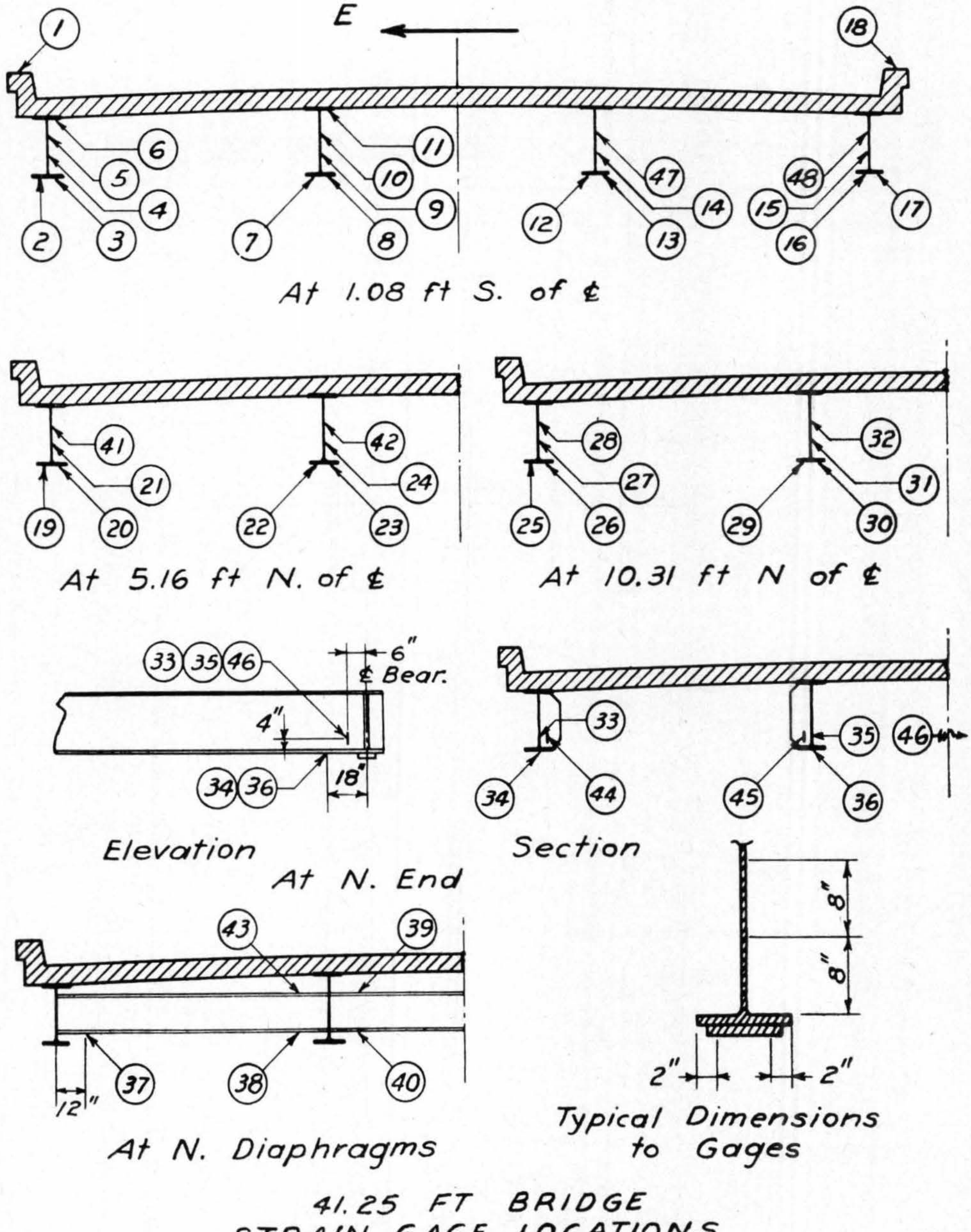
After the readings were made they were converted into net strains or deflections by subtracting the proper "zero" readings from the various readings taken with the load in place. On the charts from the automatic machine this subtraction was performed simply by scaling the distance between the mark made with no load on the bridge and that made when the load was in place.



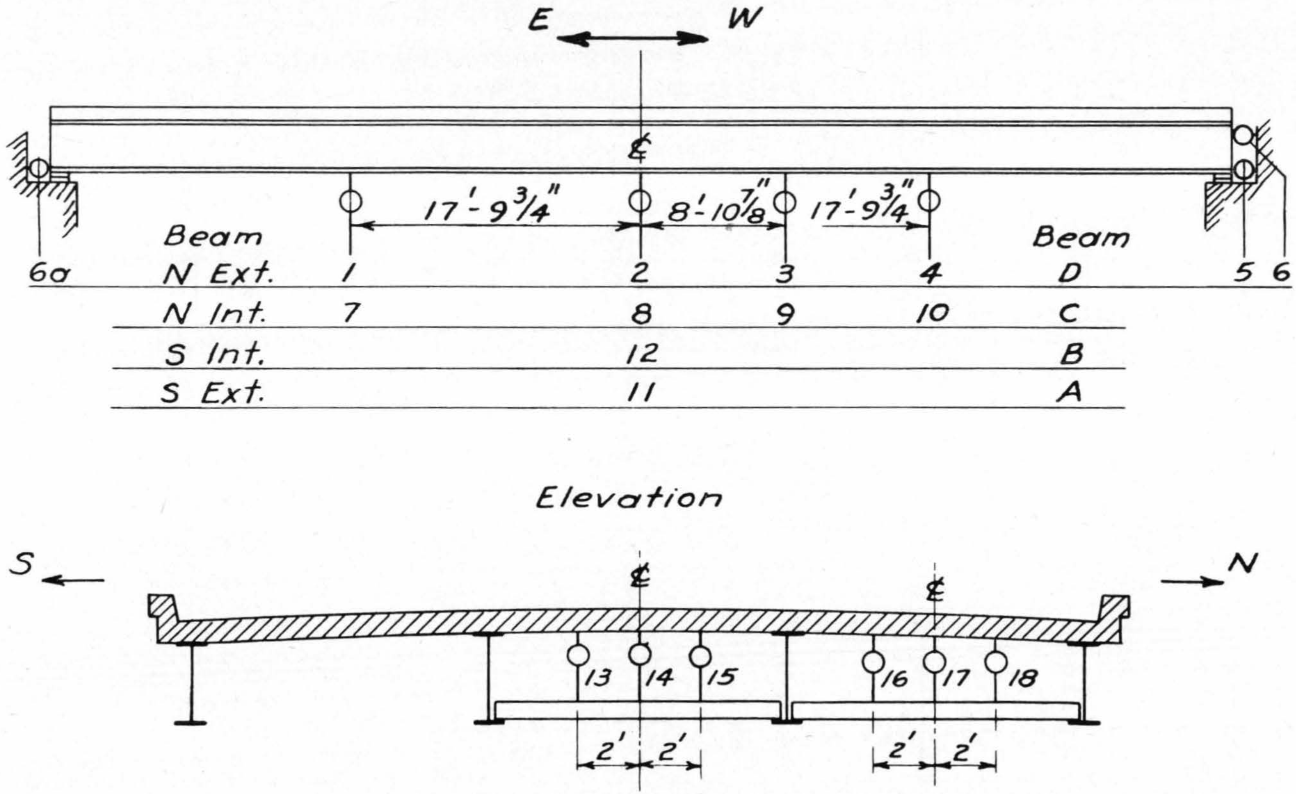
71.25 FT BRIDGE
STRAIN GAGE LOCATIONS, I.
FIG. 22



71.25 FT BRIDGE
STRAIN GAGE LOCATIONS, 2.
FIG. 23

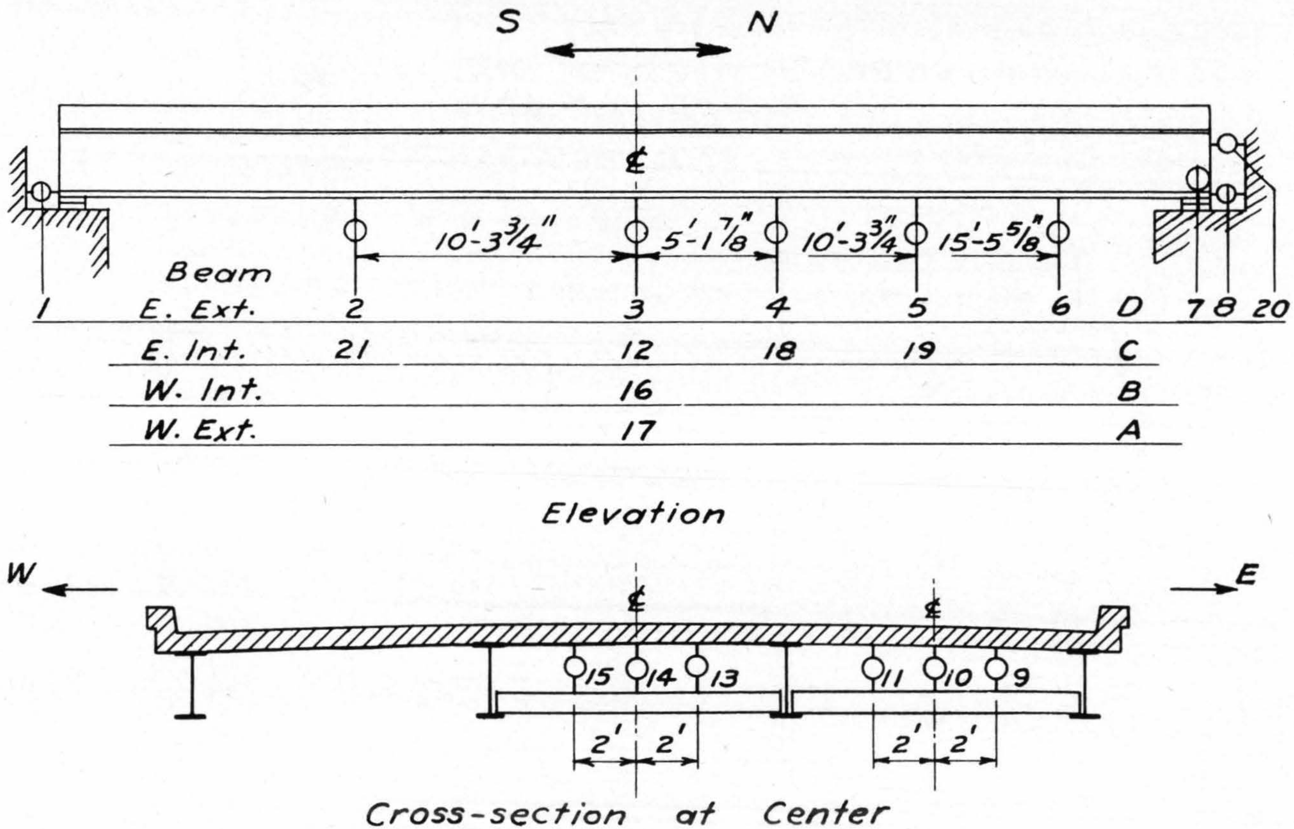


41.25 FT BRIDGE
STRAIN GAGE LOCATIONS
FIG. 24

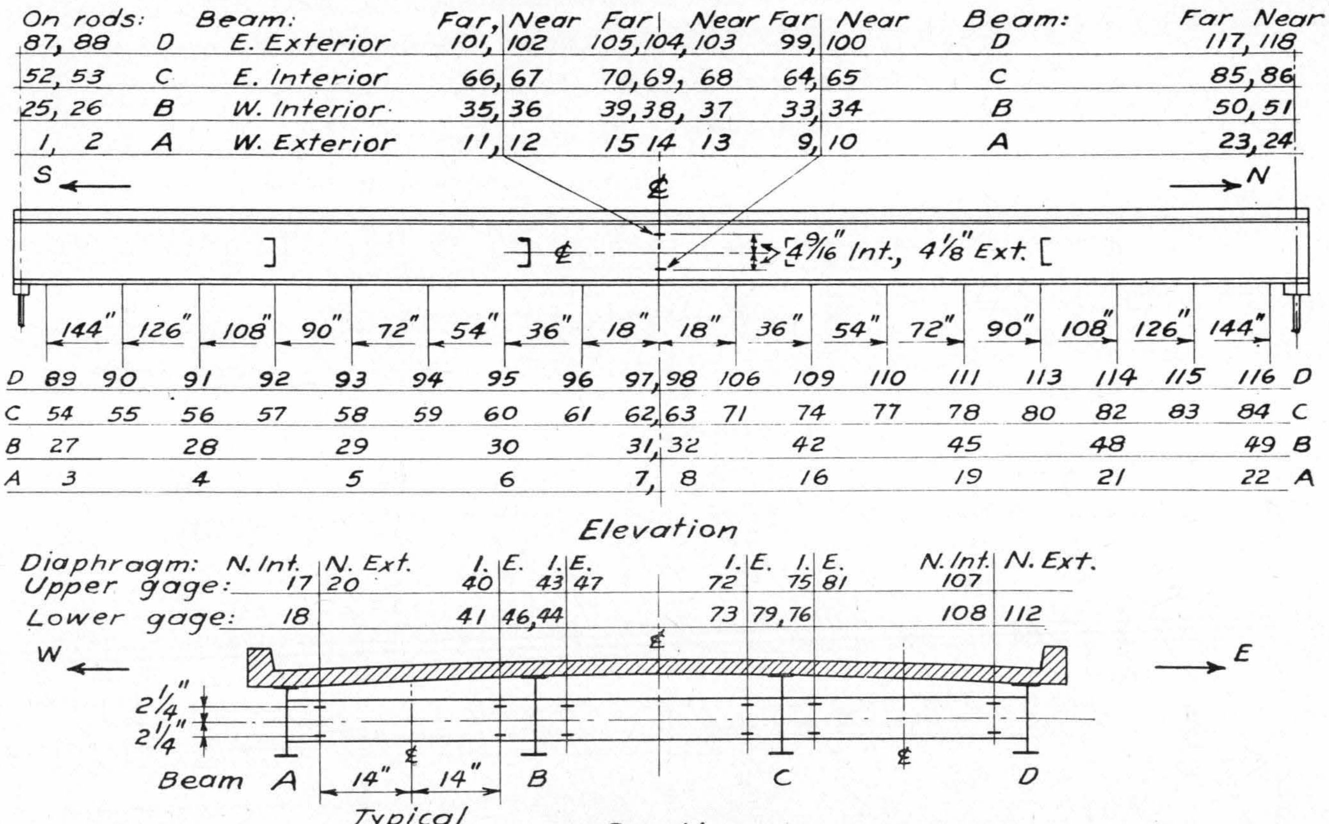


Cross-section 10 in. E. of Center

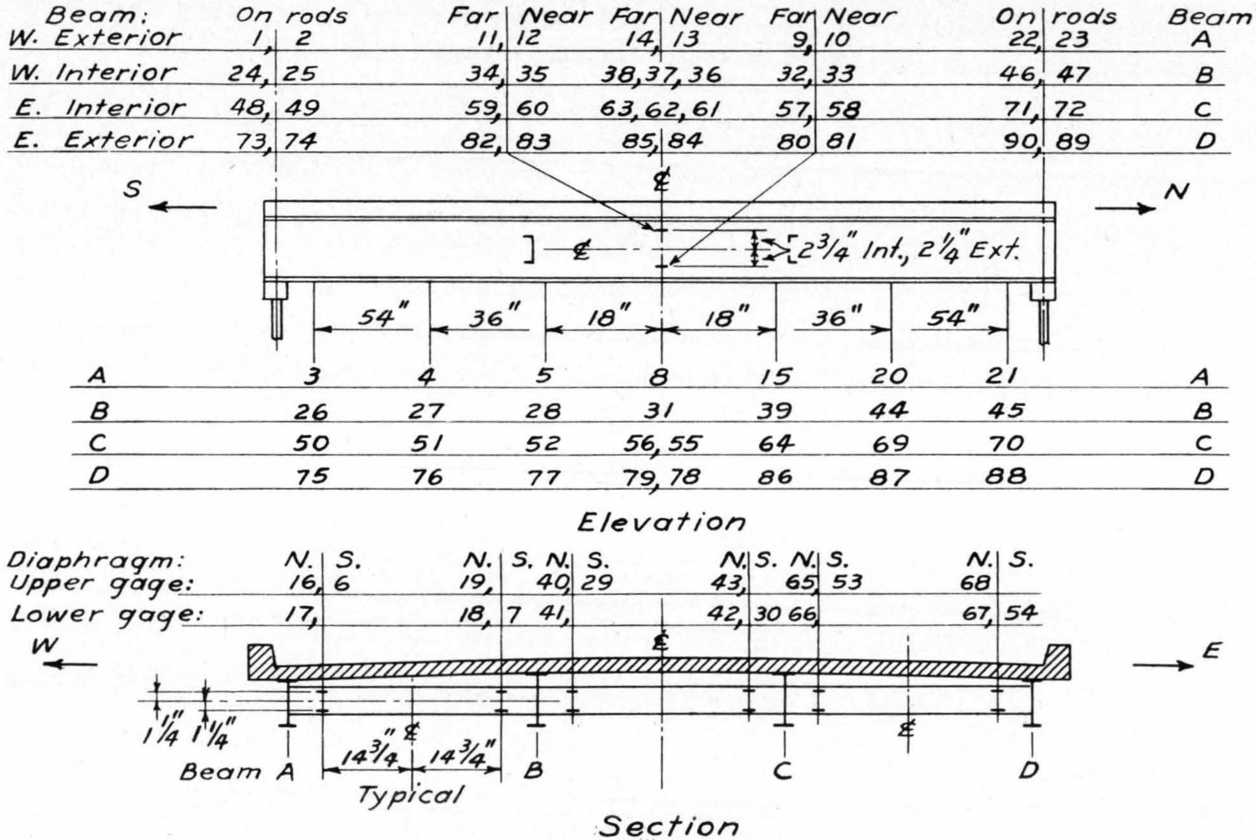
71.25 FT BRIDGE
DEFLECTION GAGE LOCATIONS
FIG. 25



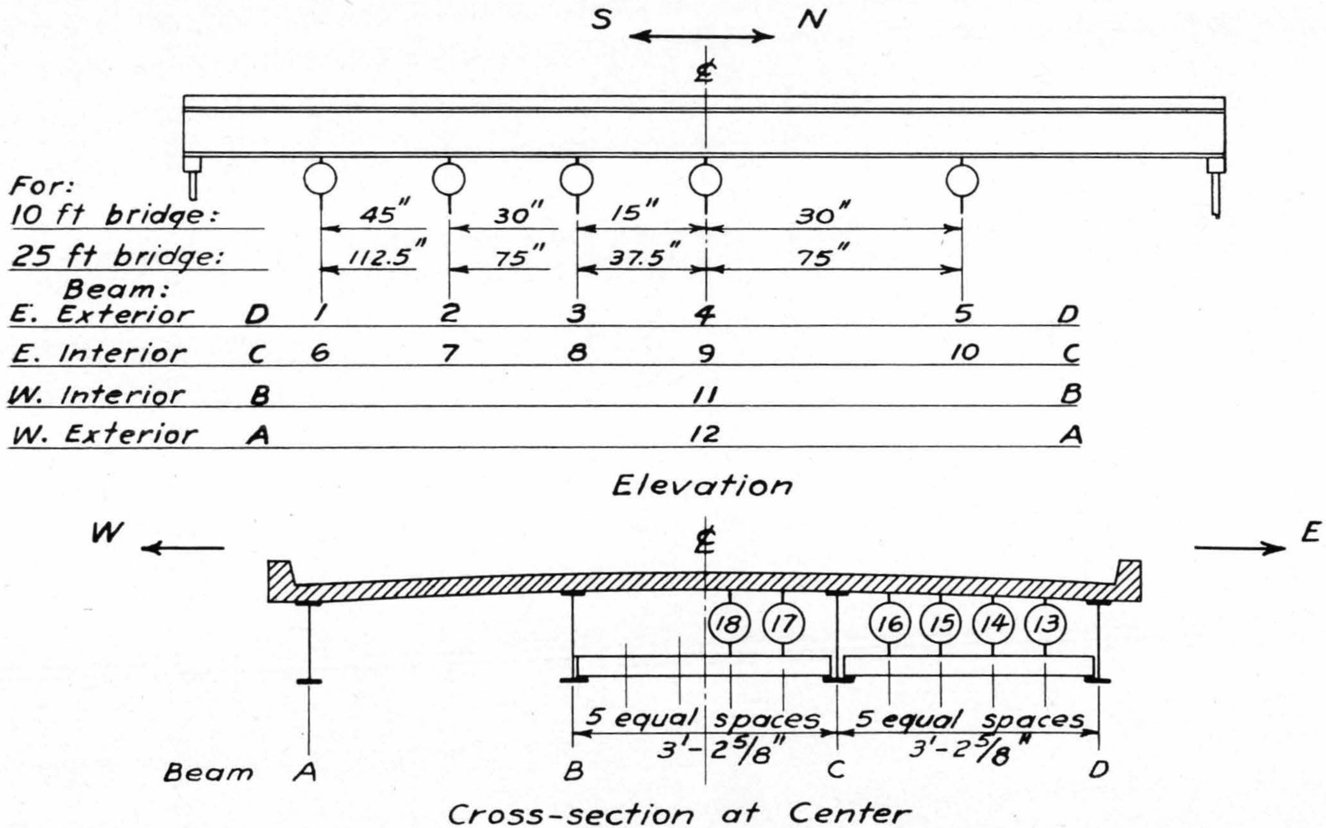
41.25 FT BRIDGE
DEFLECTION GAGE LOCATIONS
FIG. 26



25 FT BRIDGE
STRAIN GAGE LOCATIONS
FIG. 27



10 FT BRIDGE
STRAIN GAGE LOCATIONS
FIG. 28



MODEL BRIDGES
DEFLECTION GAGE LOCATIONS
FIG. 29

V. RESULTS

The results to be reported consist of the predicted values of strains and deflections and of the corresponding measured values. The two types of results are described separately in the following pages, but they are plotted together in the subsequent figures to facilitate comparisons. These figures are of three types as follows.

1. Influence lines for the strains and deflections at the centers of the spans. Each of these shows the variation of a particular strain or deflection as a particular loading is moved laterally across the bridge at or near mid-span.
2. Deflection diagrams each showing the simultaneous deflections at the center of the span of all the beams in a bridge when a load is in a particular lateral position at or near midspan.
3. Strain and deflection diagrams each showing the variation of the strain or deflection along a beam when two trucks of a particular type are side by side at or near midspan and in the AASHO specified lateral position.

In the AASHO specified position the two trucks are side by side, 10 ft. center to center with the outermost wheel 2 ft. inside the face of the curb on the full-size bridges, and 3 ft. 4 in. center to center with the outer wheel 8 in. from the curb on the laboratory bridges.

It should be noted that both the predictions and measurements include truck positions in which the outermost wheel is against the curb. These are outside the specified position and are not considered in subsequent comparisons.

In the figures a solid line is used to connect points predicted by the proposed method, a dashed line to show values predicted by the AASHO specifications, and a dotted line to show values predicted by the T-15 specifications. Points obtained from test data are circled, or if a correction has been applied, are circled and starred. Where curves have been drawn through observed points, a light solid line has been used.

A. Predicted Results

The proposed method of analysis described in Chapter III has been used to calculate the strains and deflections to be expected in each beam of each bridge tested for a number of different lateral positions of the loads. Each bridge and loading has, also, been analyzed according to the AASHO specifications and according to the T-15-50 tentative revision of these specifications. The results predicted by the proposed method are presented first in the form of influence lines each showing the variation of a particular strain or deflection at midspan as the load is moved laterally across the bridge in a position at or near the center of the span, Figs. 31-54. Analysis under the specifications does not, of course, provide results that vary as the lateral position of the load varies. It provides only a single value for each beam that is intended to be the maximum that can be expected in that beam for any lateral position of the load. Results predicted by the specifications are indicated in each figure by short dashes or dotted lines, or are written in parentheses if they are outside the range of a particular chart. These are comparable to the maximum values obtained by the proposed method or by test as long as the outer wheel is 2 ft. or more inside the curb on the highway bridges or 8 in. on the laboratory ones.

The 10 ft. and 25 ft. bridges have been analyzed and the influence lines drawn for the following loads. The load positions, as defined by line numbers and letters, are as shown in Figs. 20 and 21.

1. One single-axle truck weighing 4000 lbs. at line 5, Figs. 31, 32, 39, and 40.
2. Two 4000 lb. single-axle trucks side by side, 40 in. center to center, at line 5, Figs. 33, 34, 41, and 42.
3. One tandem-axle truck weighing 8000 lbs., at line 4, Figs. 35, 36, 43, and 44.
4. Two 8000 lb. tandem-axle trucks side by side, 40 in. center to center, at line 4, Figs. 37, 38, 45, and 46.

The 41.25 ft. and 71.25 ft. bridges were analyzed and influence lines drawn for the following loading conditions. The load positions, as defined by line numbers and letters, are as shown in Figs. 17, 18, and 19.

1. A single truck of the same dimensions and weights as the one used in testing, at line 2, Figs. 47, 48, 51, and 52.
2. Two such trucks side by side and 10 ft. center to center, at line 2, Figs. 49, 50, 53, and 54.

In addition to the above, each bridge was analyzed for symmetrically placed trucks, that is, for two trucks at various equal distances from the longitudinal center line. Under no condition did the symmetrical arrangement cause the largest strain or deflection; therefore the influence lines are not included.

Some of the predicted deflections are presented in center-of-span deflection diagrams, Figs. 55-62. Each of these shows the simultaneous midspan deflections of all the beams in a bridge caused by a load in a particular position. Analysis according to the specifications does not provide for changing the lateral position of the loads, as previously discussed; therefore no result predicted under the specifications is shown in these figures.

Finally, the variation of the strain and deflection along an interior and an exterior beam of each bridge is shown, Figs. 63-70. The results presented are those obtained when two trucks at or near midspan are in the AASHO specified lateral position with the outermost wheel 2 ft. inside the curb on the full-size bridges or 8 in. on the laboratory ones. Reference to the influence lines for two trucks shows that this position causes the maximum strain and deflection in the exterior beams in every case and causes either the maximum or very nearly the maximum in the interior beams.

The AASHO does not specify tandem axles in the design of beams; a single rear axle is assumed except in the design of the flooring. However, the loads used in obtaining the test data presented in Figures 63 through 70 did have tandem axles and the proposed method of analysis does make an allowance for the effects of tandem axles. Therefore, extra analyses have been made in which the specifications were assumed to be modified to include the effects of tandem axles. In this modification the distribution of the wheel loads to the beams remained the same, but the two tandem axle loads were not replaced by a single axle load equal to the sum of the two. The deflections computed according to this modification do not differ significantly from those computed under the present specifications for a single load, so only one "AASHO" deflection curve and one "T-15" is shown for each beam, Figs. 63, 65, 67 and 69. The strains computed according to this modification do differ significantly from those computed for a single load. Therefore, in each strain diagram the results of both analyses are shown, Figs. 64, 66, 68, and 70.

B. Test Results

Strains and deflections were measured at a number of points in each bridge when the loads were in each of a number of different positions, as described in Chapter IV. These measurements provided data in the form of inked charts from the automatic recording unit or of dial readings and strain gage readings. The original data from all the tests will be found on file with the Iowa State Highway Commission at Ames, Iowa.

The original data have been converted to usable form by scaling the distances between lines on the charts and by subtracting the proper "zero" readings from the readings taken with the loads in place. Of the resulting strain and deflection measurement, those appropriate have been plotted in the same figures in which the predicted values are presented. In the figures relating to the full-size bridges there are shown, also, points from the tests "as corrected." The "corrections" applied and some other things considered in using the test results are described as follows.

1. *Differences in strains at a cross-section*

On the tension flange of a beam, strain gages were, in general, placed along the longitudinal center line. However, at a number of cross-sections two gages were placed equidistant from the longitudinal centerline, Figs. 22, 23, 24, 27, and 28. The readings from such pairs of gages differ by as much as 15 percent of the average of the two readings. It has been assumed that these differences are caused by lateral bending of the tension flange, and that the average of the two values can be used. This lateral bending could be caused by initial crookedness of the tension flange. It could also be caused by twisting of the beams as the bridges deflect and the beams deflect different amounts, Figs. 55-62.

2. *Use of average values*

Two different circumstances occurring in the tests gave rise to sets of results that theoretically duplicated other such sets.

First, the bridges tested were supposedly symmetrical about both the longitudinal and transverse center lines. Thus, the results for a particular beam and loading should be duplicated for the symmetrically located beam and loading. In practice, of course, the theoretically equal results have been found unequal as a result of accidental errors in construction, in placing the loads, and in reading the instruments.

Second, ordinarily only one set of readings was taken with a given load in a given location on a bridge. However, a few tests were repeated giving results that should be equal to those previously obtained with the load in the same position. These theoretically equal values have been found in practice to differ somewhat, also.

Most of the differences between theoretically equal results of either of the two types described have been found to be so small that plotting separate points in the figures was impractical. Consequently, only the average value has been shown, except in a few cases. The results from the 71.25 ft. bridge include some such discrepancies that seem too large to average out, yet contain nothing to indicate which is more nearly correct. In these cases both such values have been plotted, Figs. 51, 52, 53, and 69.

3. *Corrections for end restraint*

In the 10 ft. and 25 ft. bridges the strain in the beams approaches zero at the ends as nearly as can be determined from strain diagrams such as those in Figs. 64 and 66. This is taken to indicate that the moments induced at the supports of these beams are negligible; the beams are essentially simply supported.

In the 41.25 and 71.25 ft. bridges the strain does not approach zero at the ends of the beams but reverses direction and reaches a substantial negative value, as in Figs. 68 and 70. The presence of these negative strains is taken to indicate end restraint, which was expected because of the sliding plate supports and because of the seemingly "built-in" condition observed at the ends of these bridges. As a result of the end restraint, the observed strains and deflections along these beams are assumed to be smaller than they would be if there were no restraint.

The analyses, both by the proposed method and according to the specifications, assume simple beam action, that is, a condition of no end restraint. For comparison with the results of the analyses, the test results have been corrected by a procedure in which the end moments are reduced to zero, as follows, Fig. 30.

a) The observed strains, Fig. 30b, were converted to moment diagrams, Fig. 30c, by multiplying each strain by the modulus of elasticity and by the section modulus of the beam at the section where the strain was measured. (The use of cover plates on these beams and the variation of the thickness of the slab cause changes in their properties.)

b) The resulting moment diagrams were extended to the support by continuing the straight line segment connecting the two points closest to the support. This yields an approximate value of the end moment, for instance: –860,000 in.-lb. in Fig. 30c. The same moment was assumed to exist at the opposite end.

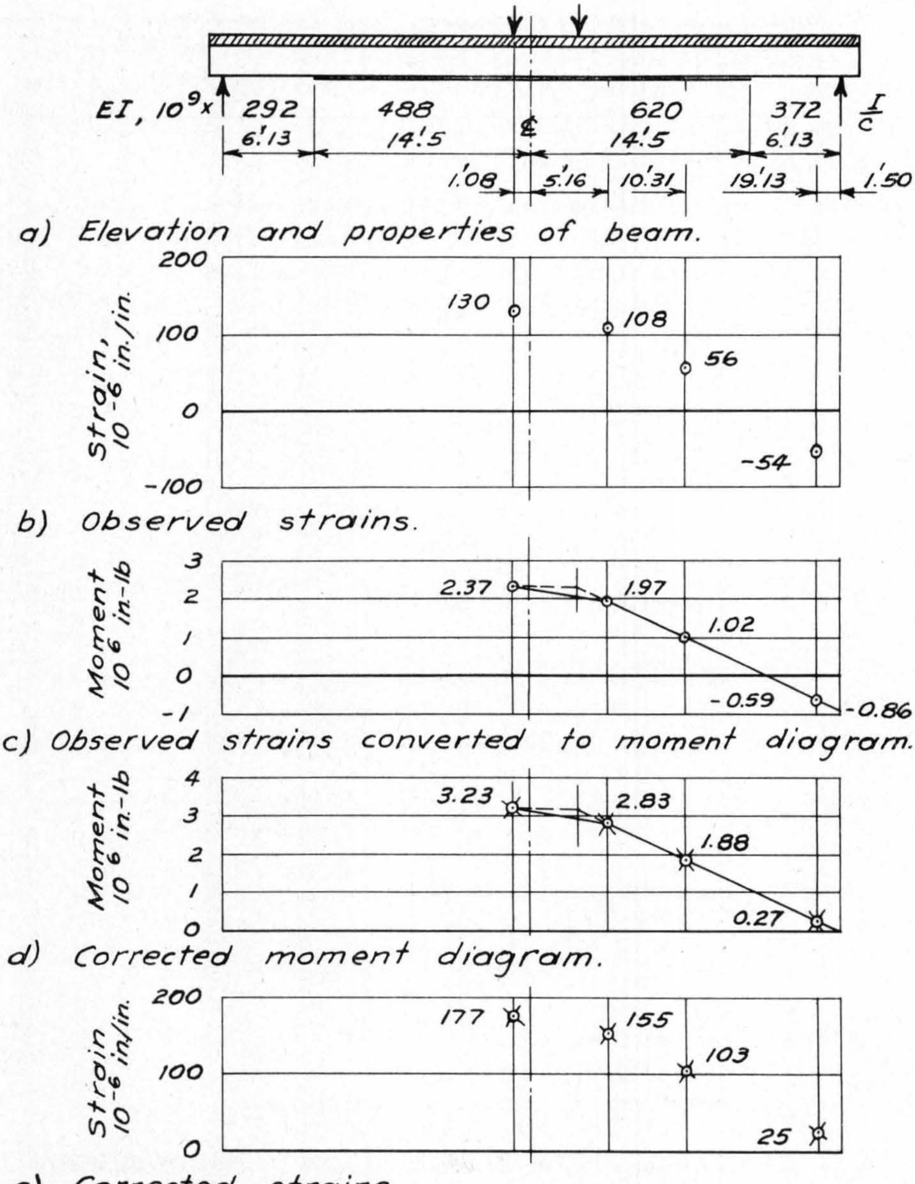
c) Corrected moment diagrams were constructed by moving the original diagram upward until the end moments were reduced to zero, Fig. 30d. By this operation each original moment was increased by the amount of the original end moment.

d) Corrected strains were computed by dividing the corrected moments by the appropriate section moduli and by the modulus of elasticity, Fig. 30e.

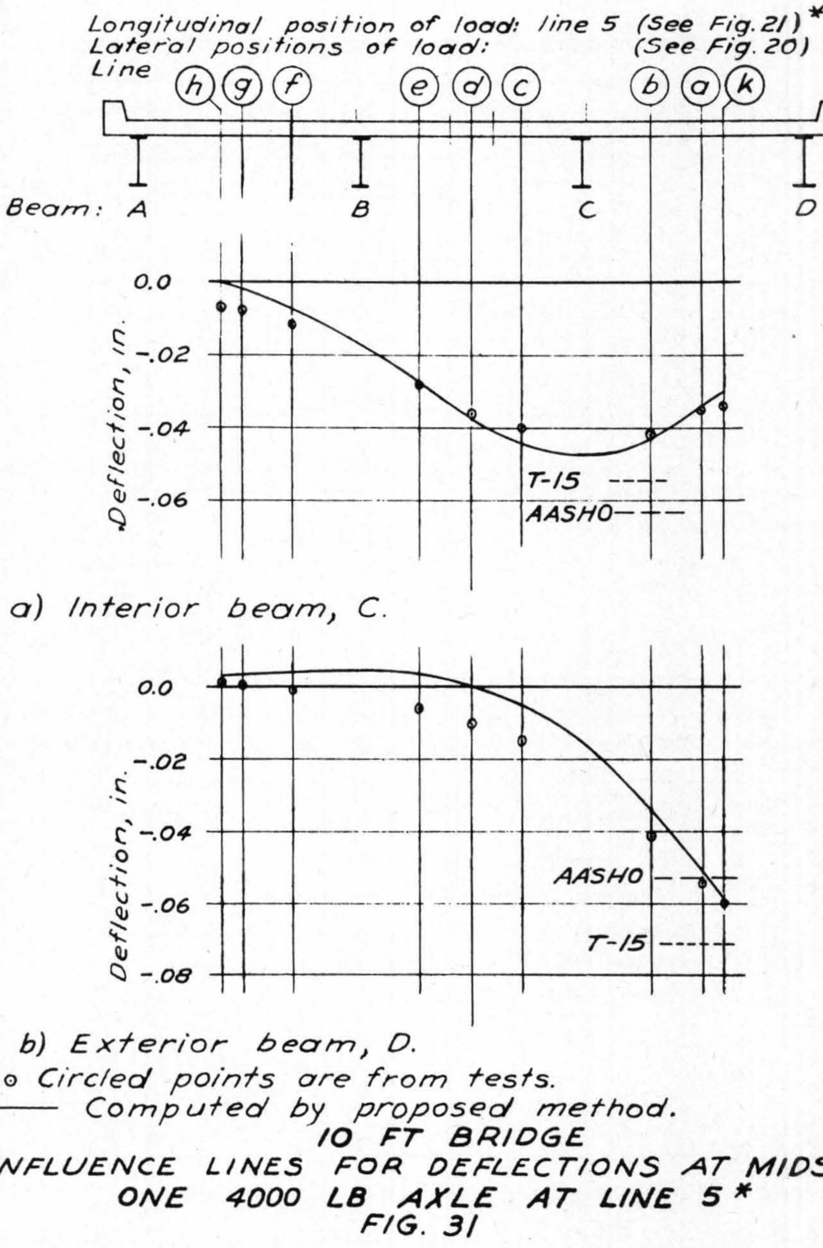
e) Corrected deflections were obtained by computing the deflections that would be caused by the end moments and by superimposing them upon the measured deflections.

4. *Superposition and interpolation*

In the field tests only one truck was available. To obtain "measured" strains and deflections reflecting the effects of two trucks side by side it was, therefore, necessary to assume that superposition was permissible. These results, then, have been computed by adding the two strains or deflections at each point caused by two different lateral positions of the truck. In testing the smaller bridges two trucks were available, but the tests with two trucks were incomplete, so superposition has been necessary in obtaining some of the results for two trucks on these bridges, also.



EXAMPLE OF CORRECTION OF OBSERVED STRAINS
41.25 FT BRIDGE. INTERIOR BEAM
TRUCK AT "a-2"
FIG. 30



a) Interior beam, C.

b) Exterior beam, D.

○ Circled points are from tests.

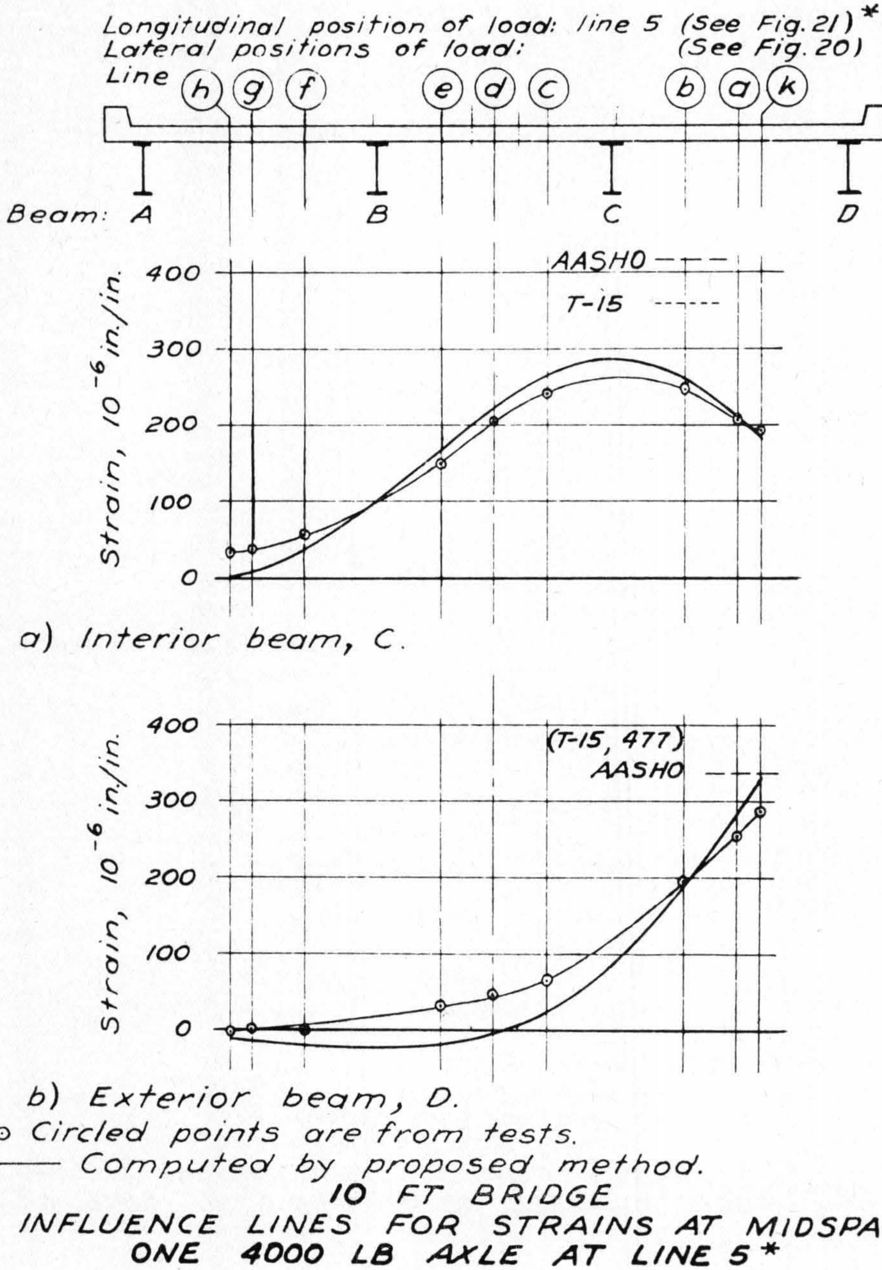
—— Computed by proposed method.

10 FT BRIDGE

INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN

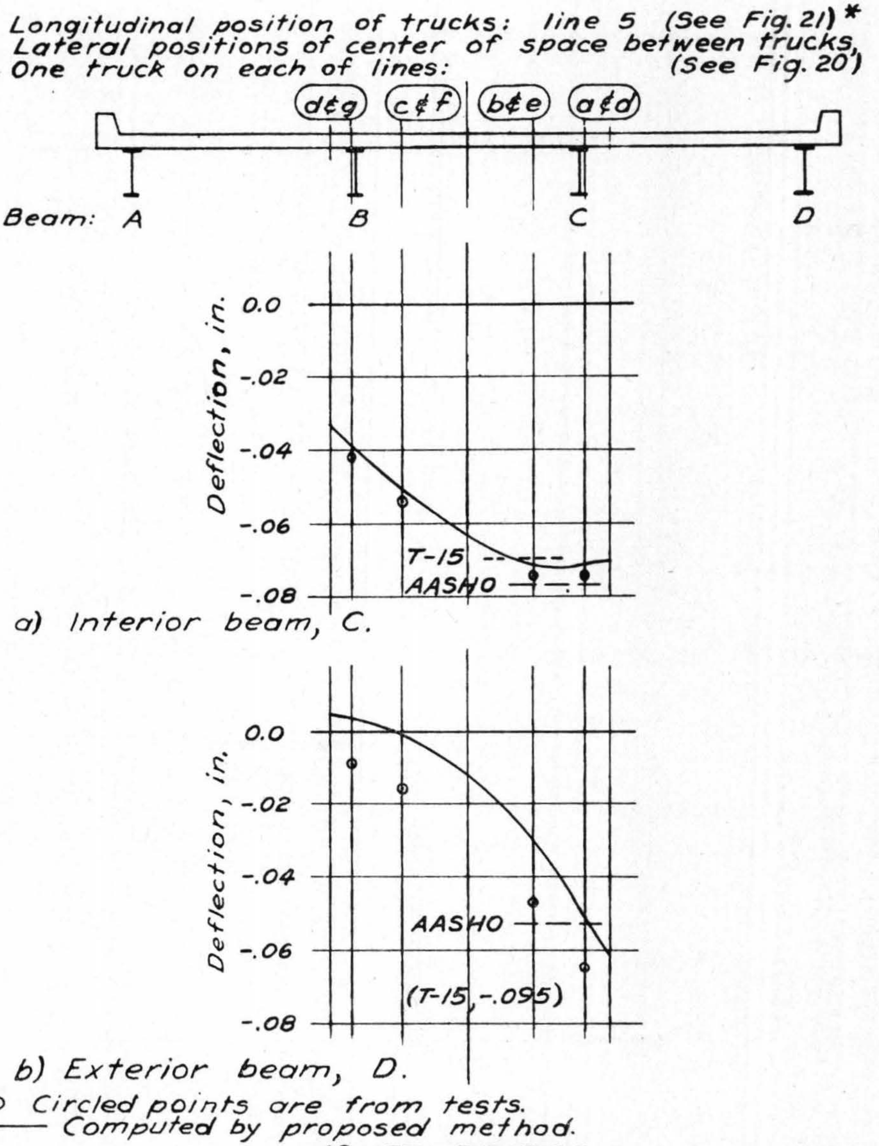
ONE 4000 LB AXLE AT LINE 5 *

FIG. 31



⊙ Circled points are from tests.
—— Computed by proposed method.

10 FT BRIDGE
INFLUENCE LINES FOR STRAINS AT MIDSPAN
ONE 4000 LB AXLE AT LINE 5*
FIG. 32

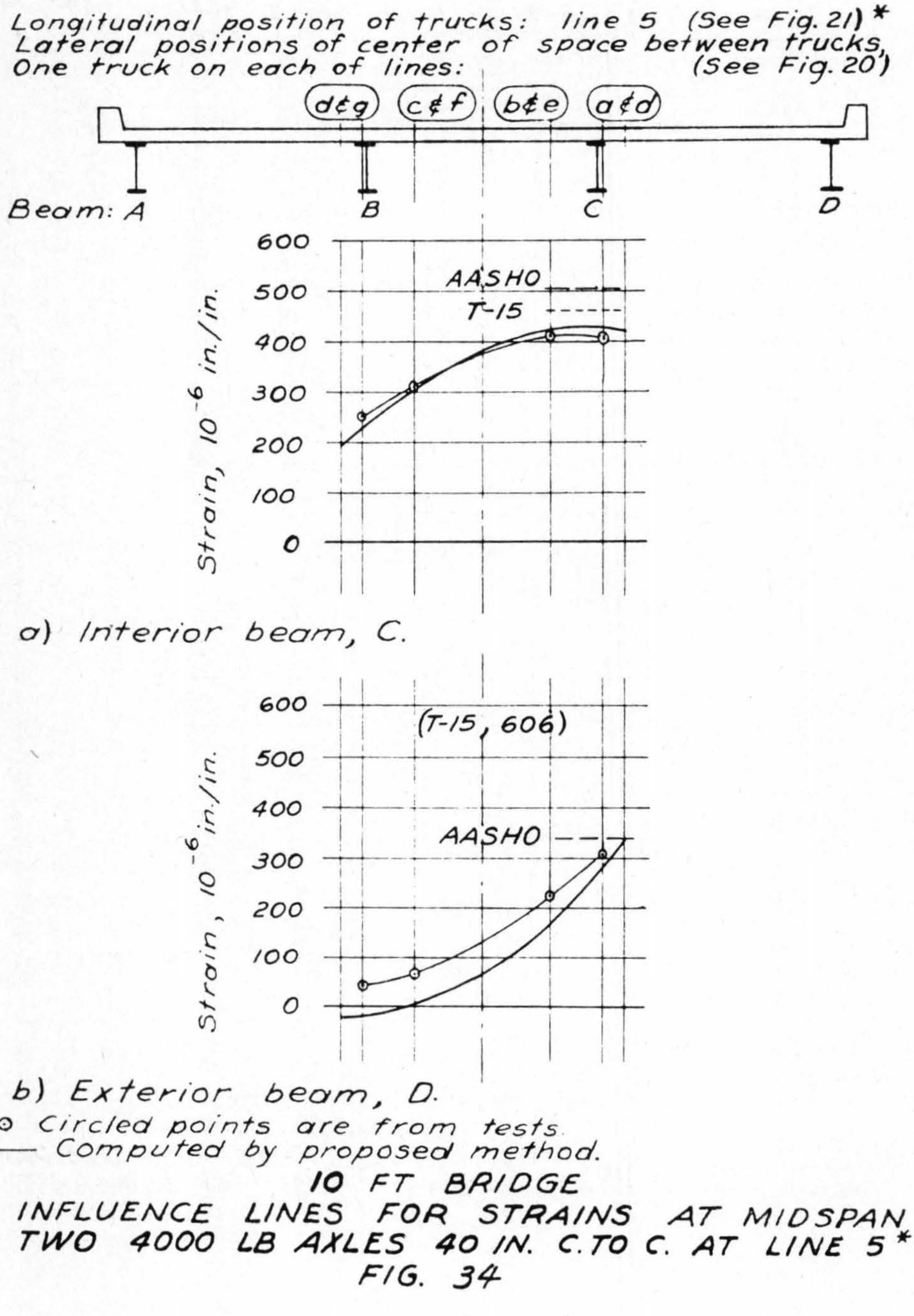


10 FT BRIDGE

INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN

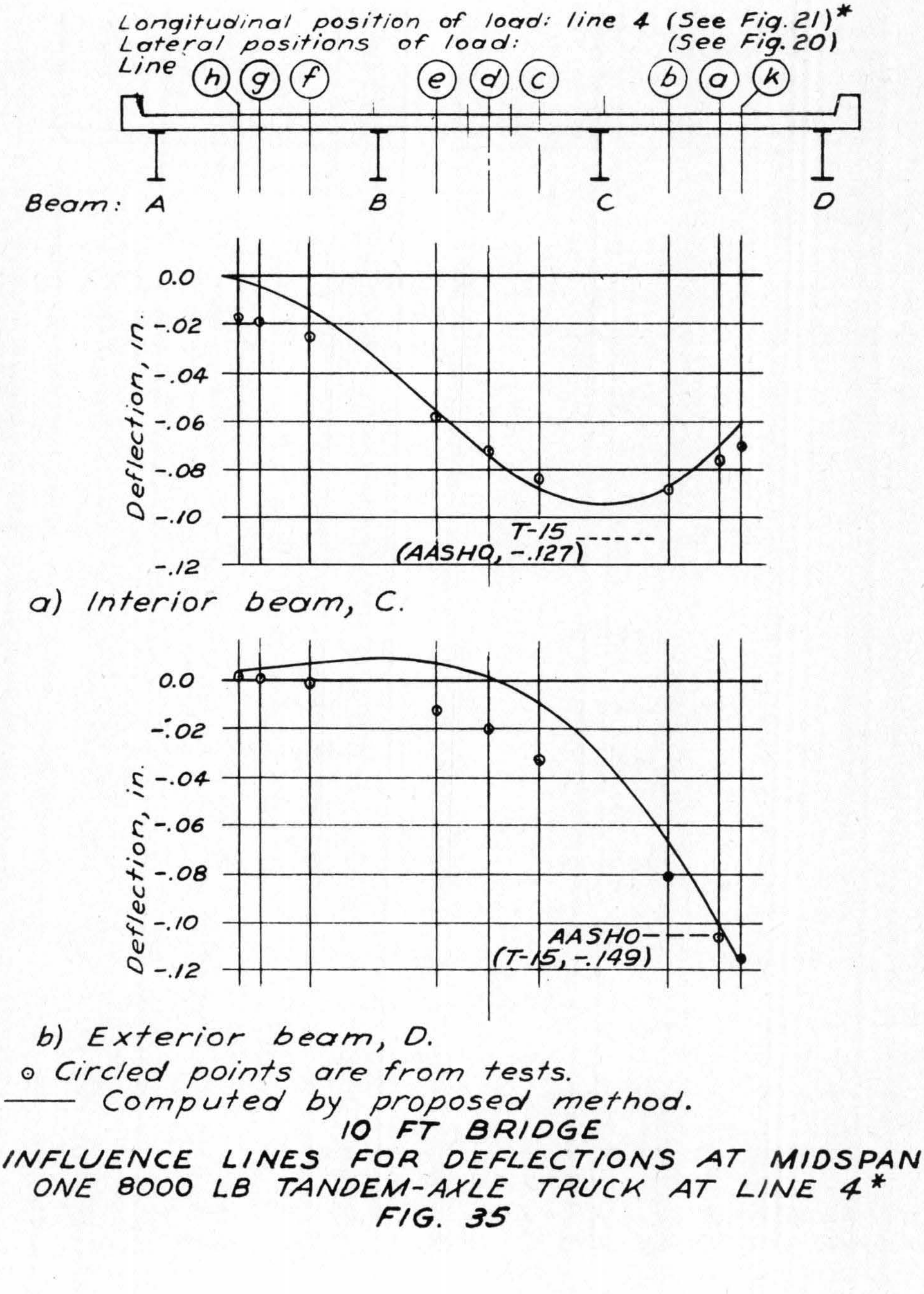
TWO 4000 LB AXLES 40 IN. C. TO C. AT LINE 5 *

FIG. 33

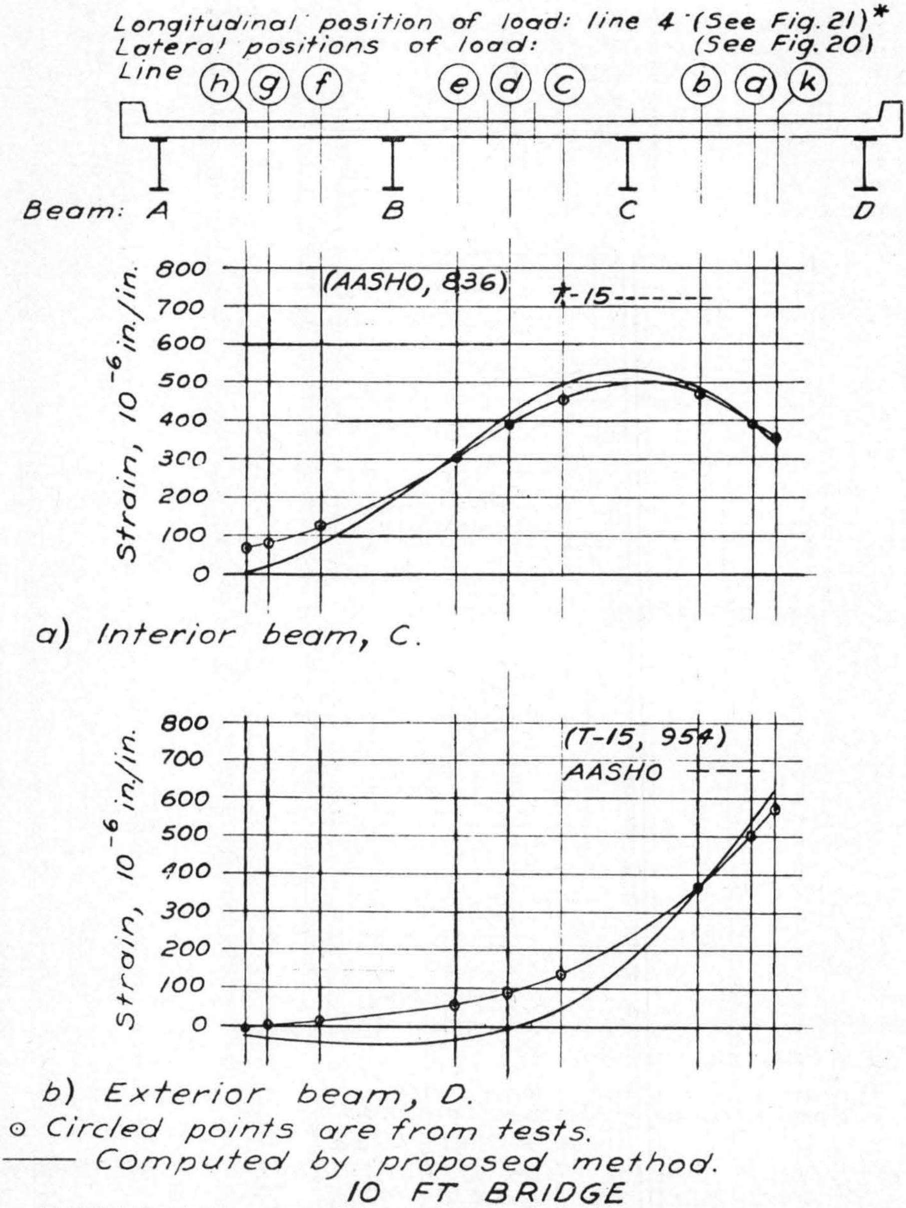


⊙ Circled points are from tests.
—— Computed by proposed method.

10 FT BRIDGE
INFLUENCE LINES FOR STRAINS AT MIDSPAN
TWO 4000 LB AXLES 40 IN. C. TO C. AT LINE 5*
FIG. 34



10 FT BRIDGE
INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN
ONE 8000 LB TANDEM-AXLE TRUCK AT LINE 4*
FIG. 35



⊙ Circled points are from tests.

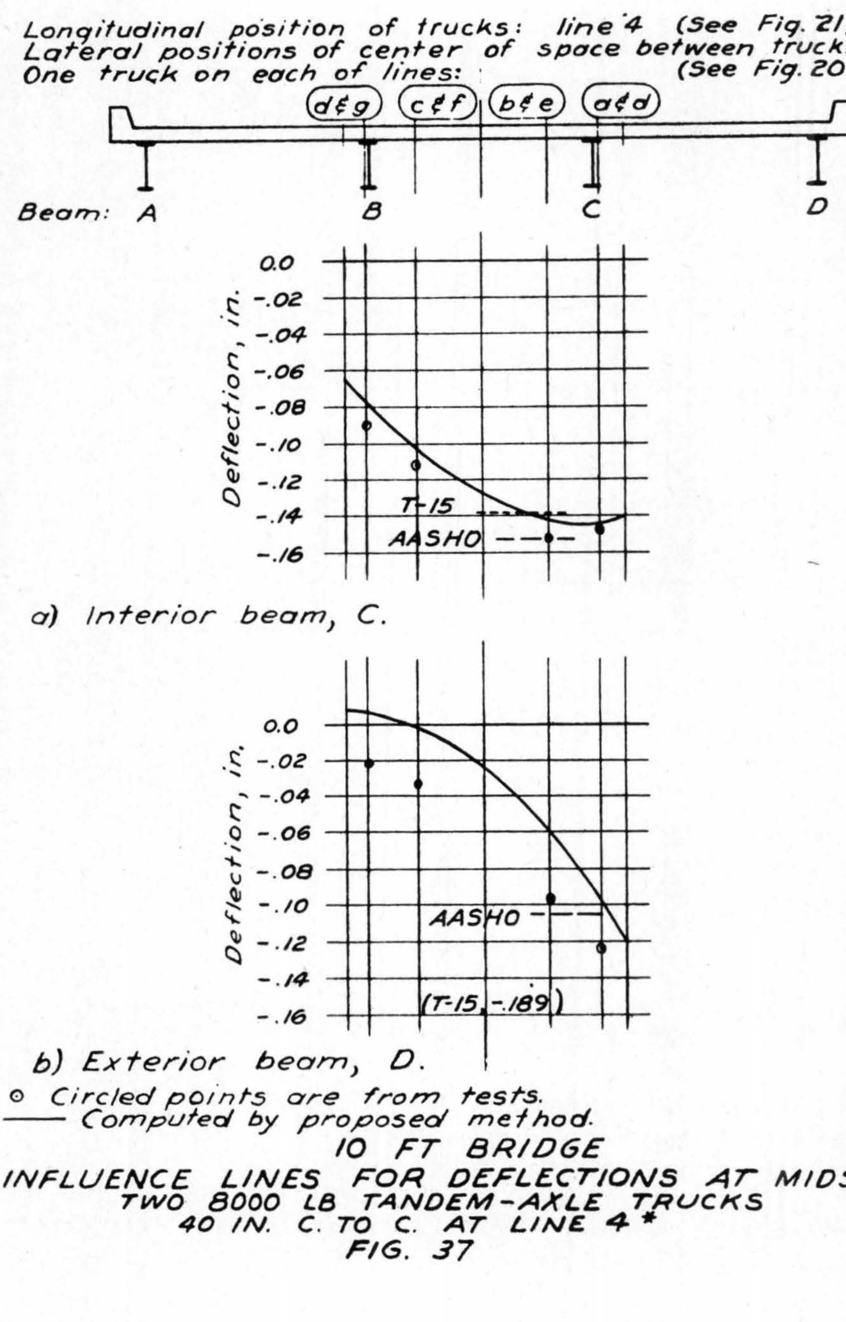
—— Computed by proposed method.

10 FT BRIDGE

INFLUENCE LINES FOR STRAINS AT MIDSPAN

ONE 8000 LB TANDEM-AXLE TRUCK AT LINE 4*

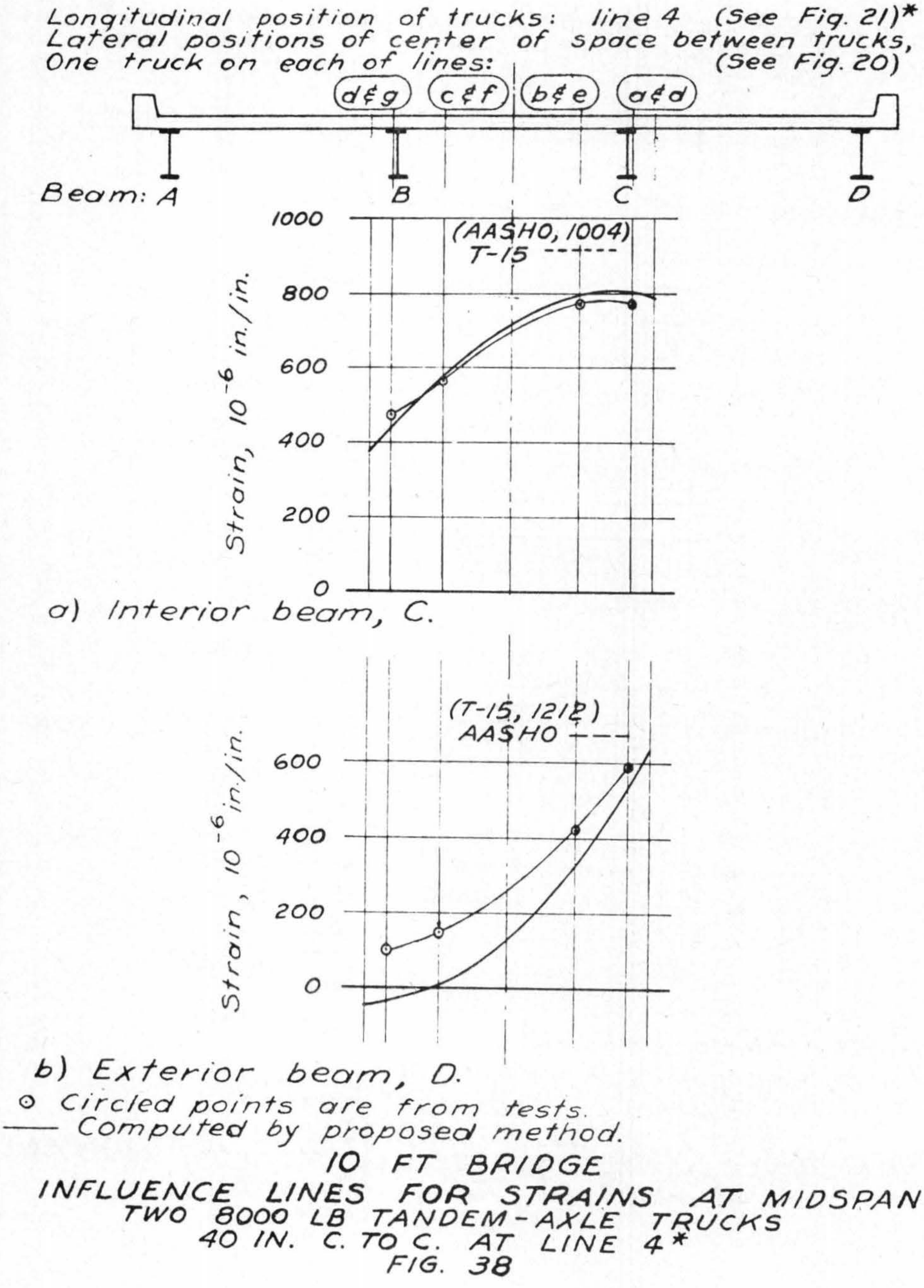
FIG. 36



10 FT BRIDGE

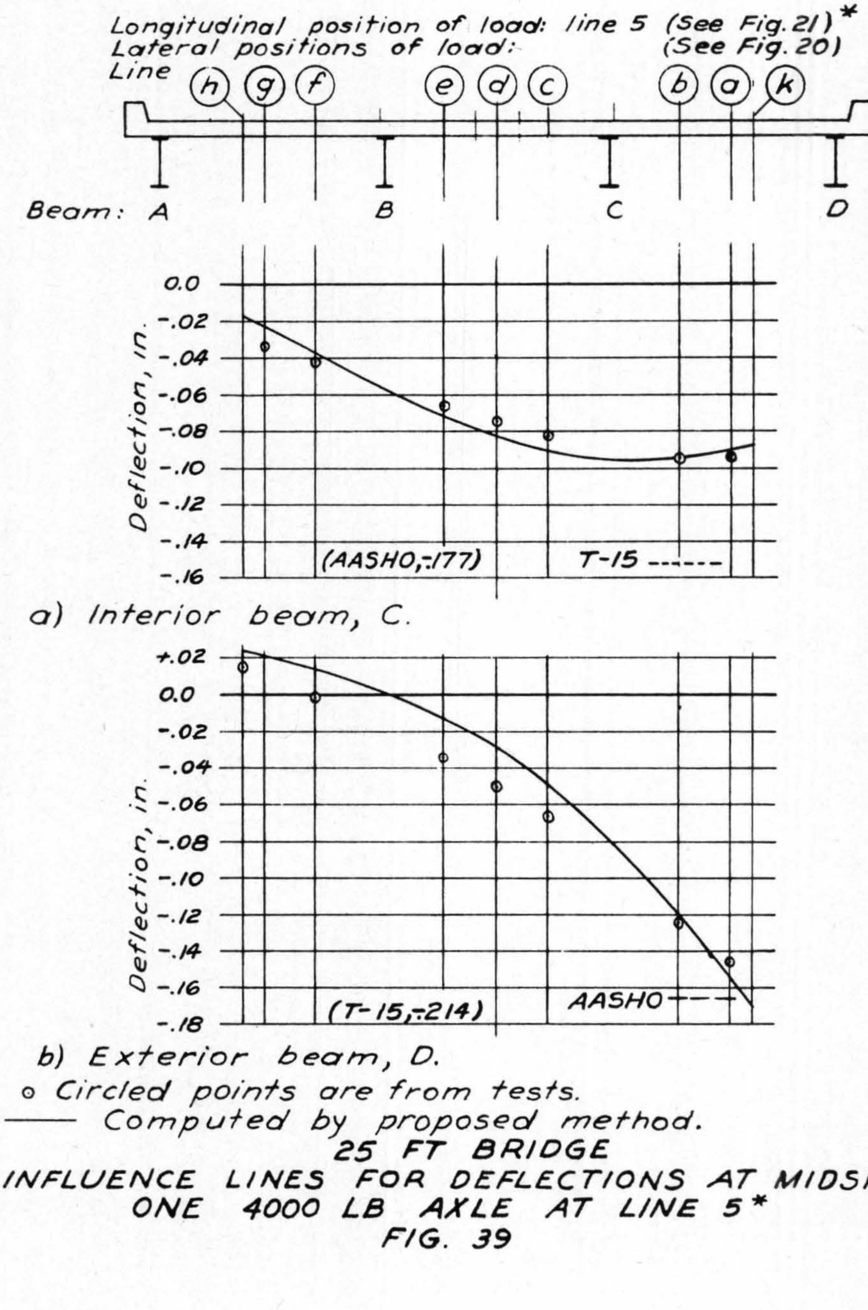
INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN
TWO 8000 LB TANDEM-AXLE TRUCKS
40 IN. C. TO C. AT LINE 4*

FIG. 37

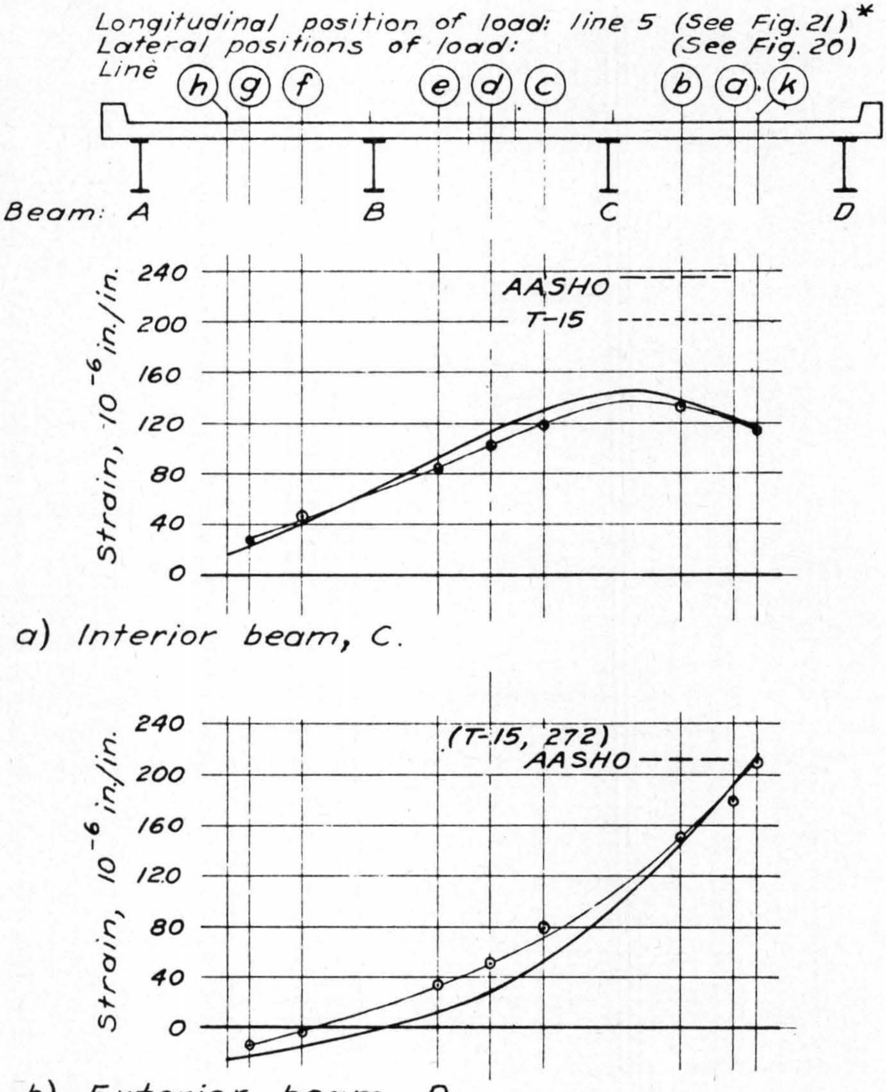


⊙ Circled points are from tests.
—— Computed by proposed method.

10 FT BRIDGE
INFLUENCE LINES FOR STRAINS AT MIDSPAN
TWO 8000 LB TANDEM-AXLE TRUCKS
40 IN. C. TO C. AT LINE 4*
FIG. 38

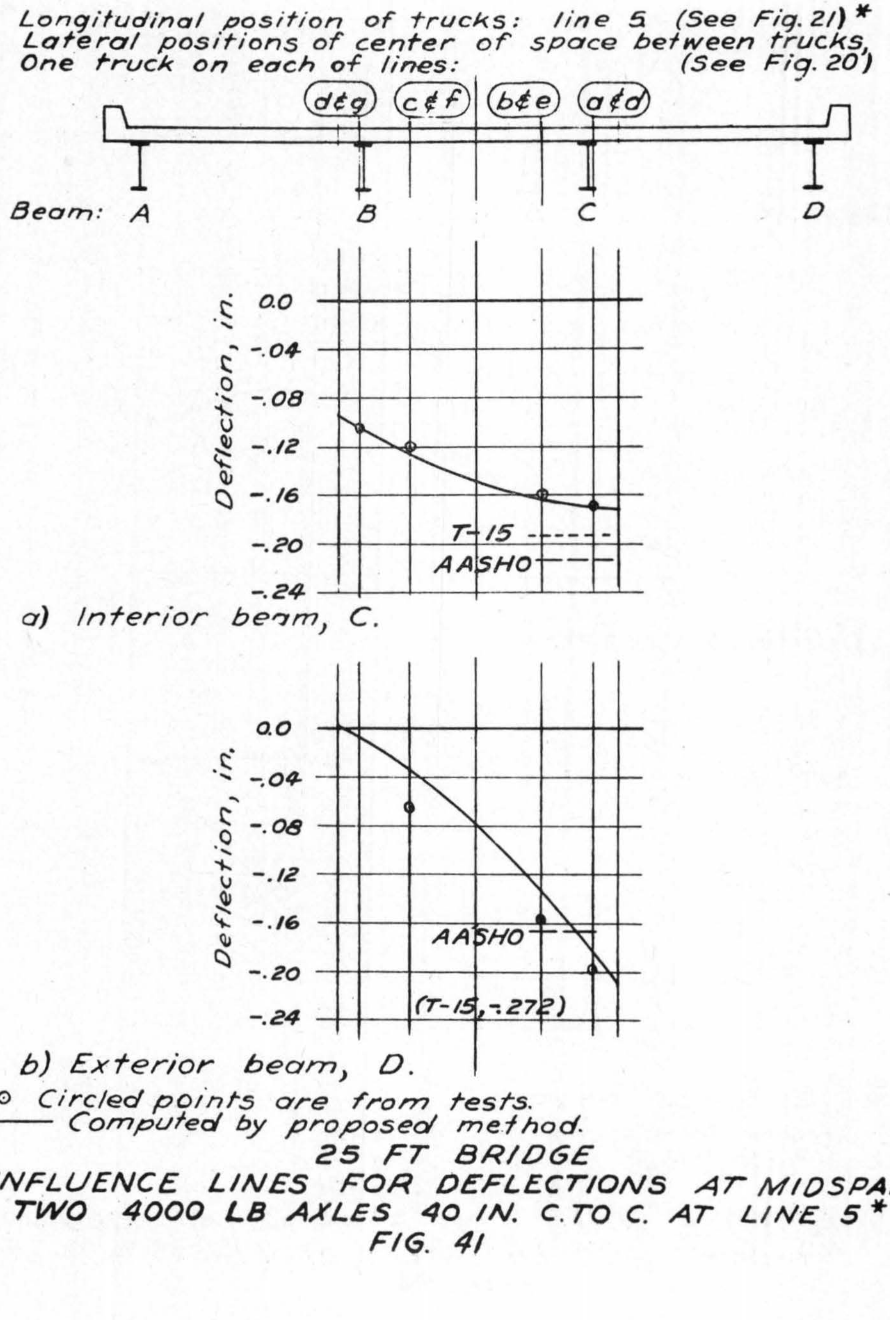


25 FT BRIDGE
INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN
ONE 4000 LB AXLE AT LINE 5*
FIG. 39

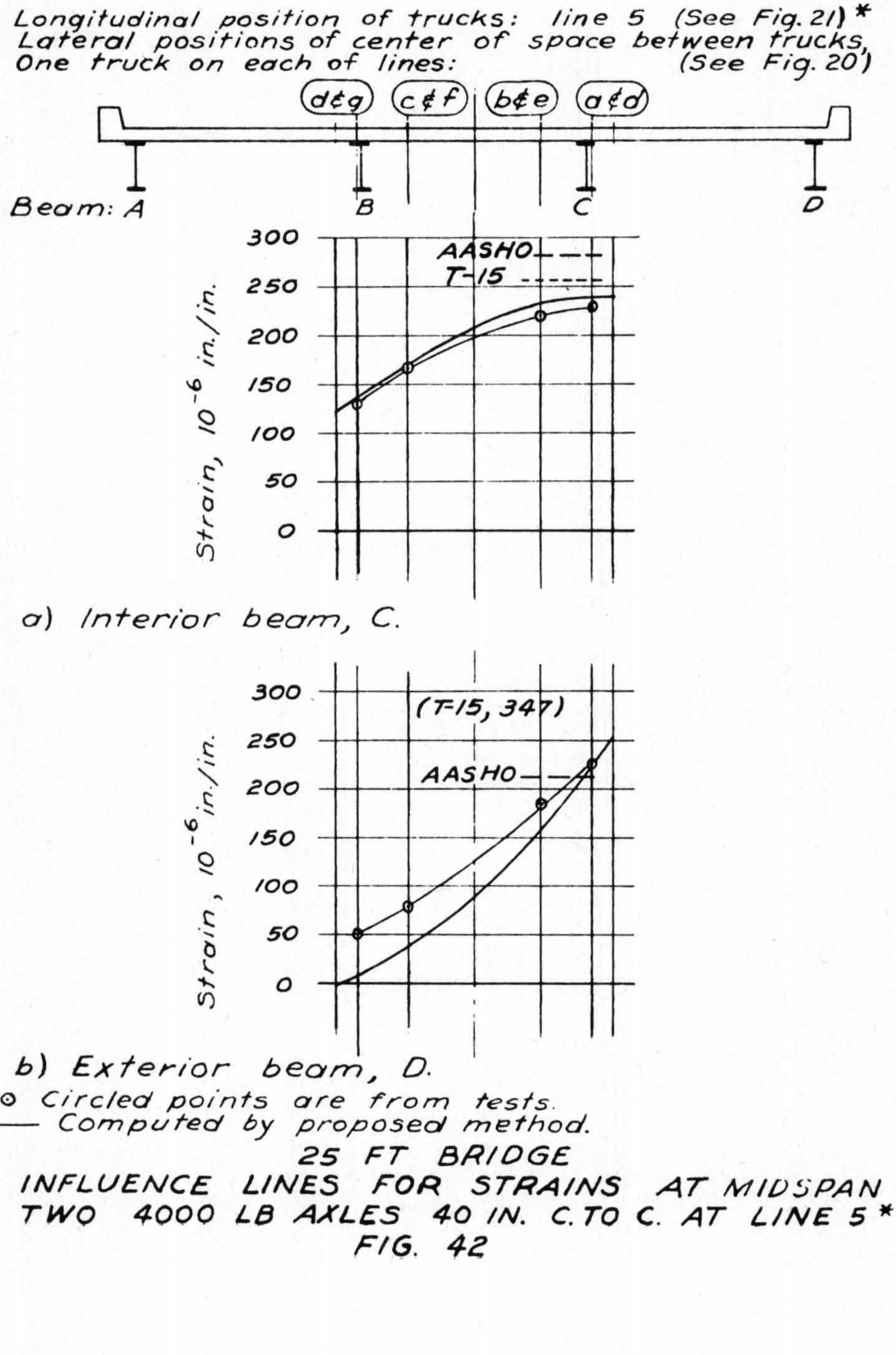


⊙ Circled points are from tests.
—— Computed by proposed method.

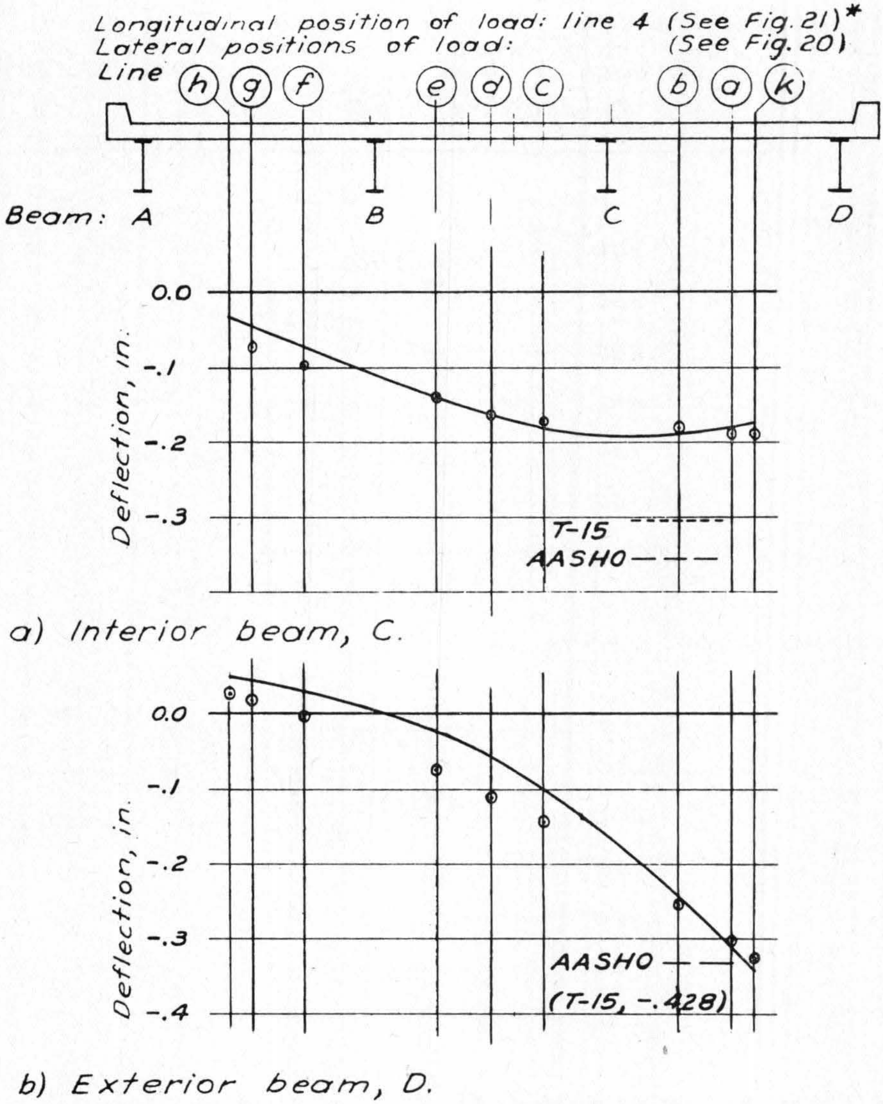
25 FT BRIDGE
INFLUENCE LINES FOR STRAINS AT MIDSPAN
ONE 4000 LB AXLE AT LINE 5*
FIG. 40



25 FT BRIDGE
INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN
TWO 4000 LB AXLES 40 IN. C. TO C. AT LINE 5*
FIG. 41

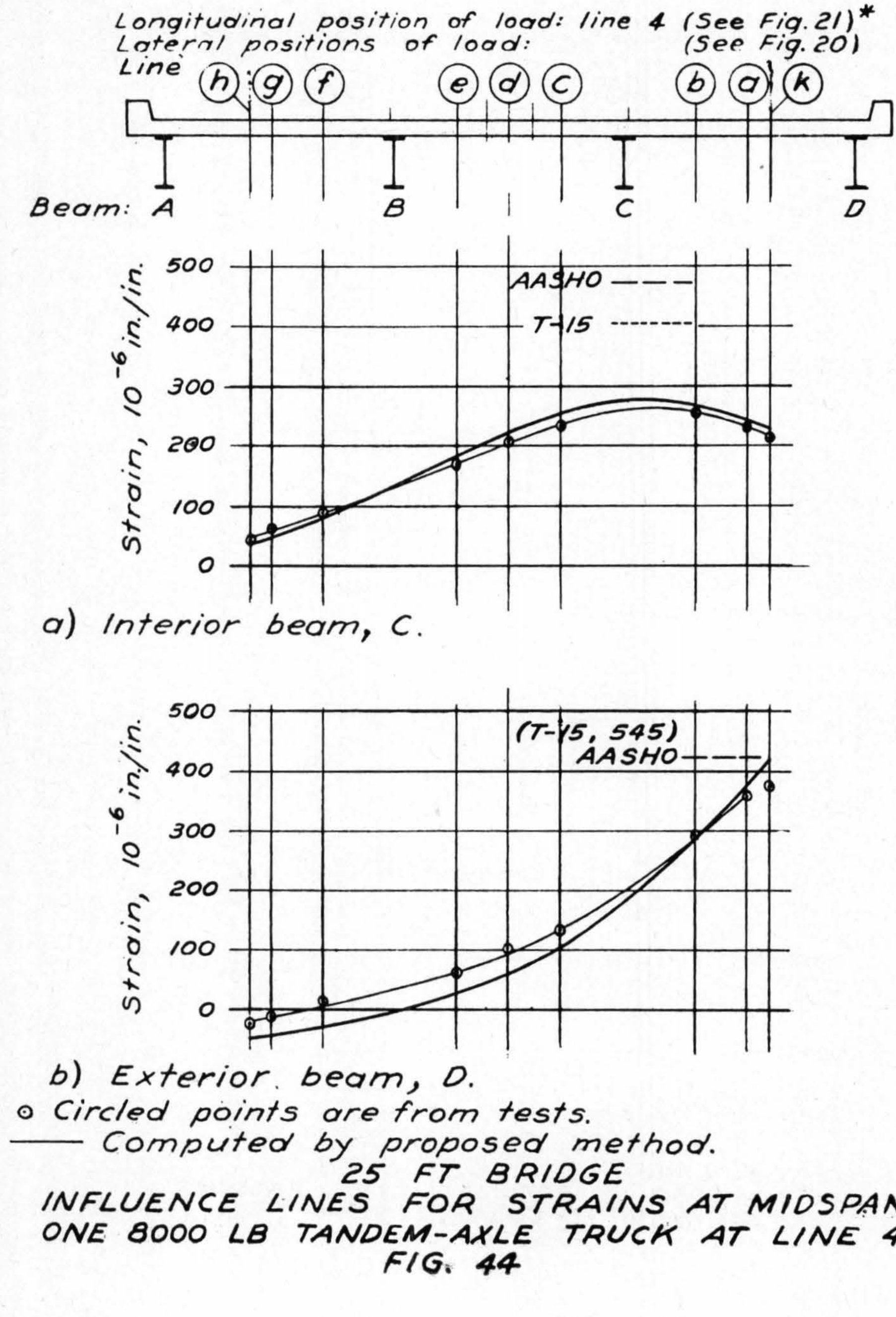


25 FT BRIDGE
INFLUENCE LINES FOR STRAINS AT MIDSPAN
TWO 4000 LB AXLES 40 IN. C. TO C. AT LINE 5 *
FIG. 42



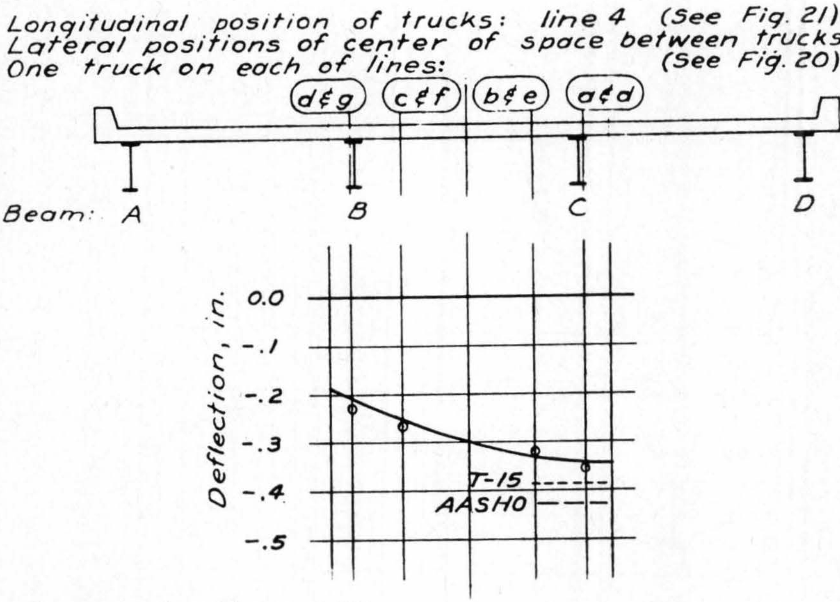
○ Circled points are from tests.
—— Computed by proposed method.

25 FT BRIDGE
INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN
ONE 8000 LB TANDEM-AXLE TRUCK AT LINE 4*
FIG. 43

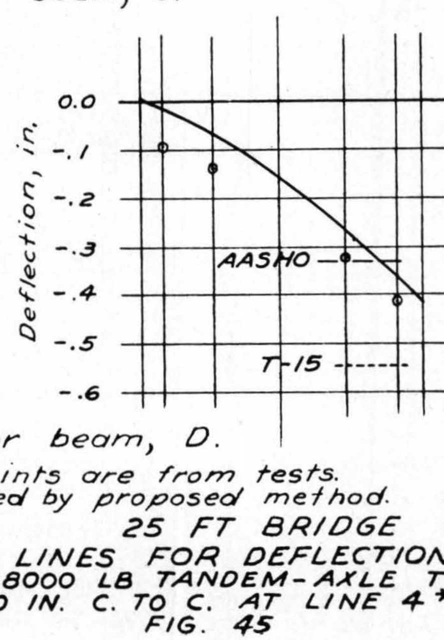


⊙ Circled points are from tests.
—— Computed by proposed method.

25 FT BRIDGE
INFLUENCE LINES FOR STRAINS AT MIDSPAN
ONE 8000 LB TANDEM-AXLE TRUCK AT LINE 4*
FIG. 44



a) Interior beam, C.

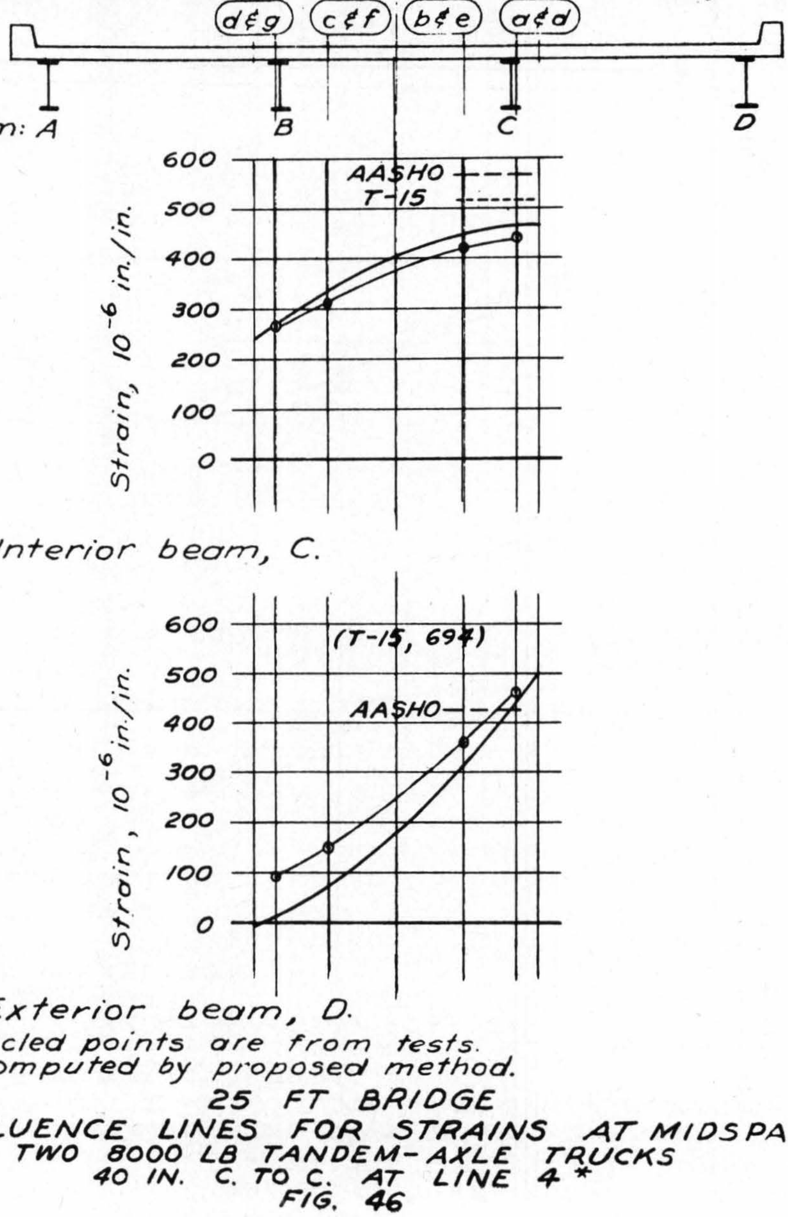


b) Exterior beam, D.

o Circled points are from tests.
—— Computed by proposed method.

25 FT BRIDGE
INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN
TWO 8000 LB TANDEM-AXLE TRUCKS
40 IN. C. TO C. AT LINE 4 *
FIG. 45

Longitudinal position of trucks: line 4 (See Fig. 21)*
Lateral positions of center of space between trucks,
One truck on each of lines: (See Fig. 20)

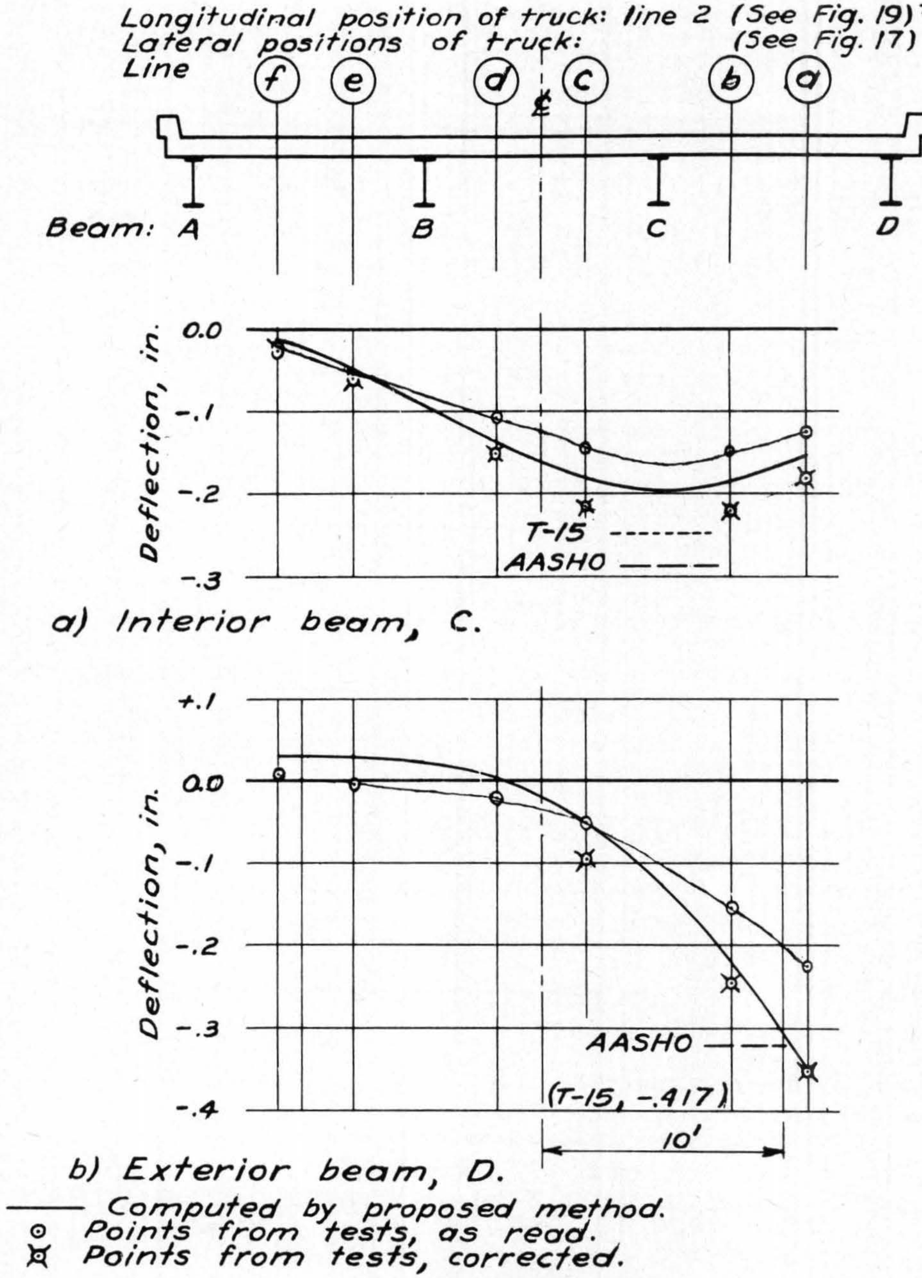


a) Interior beam, C.

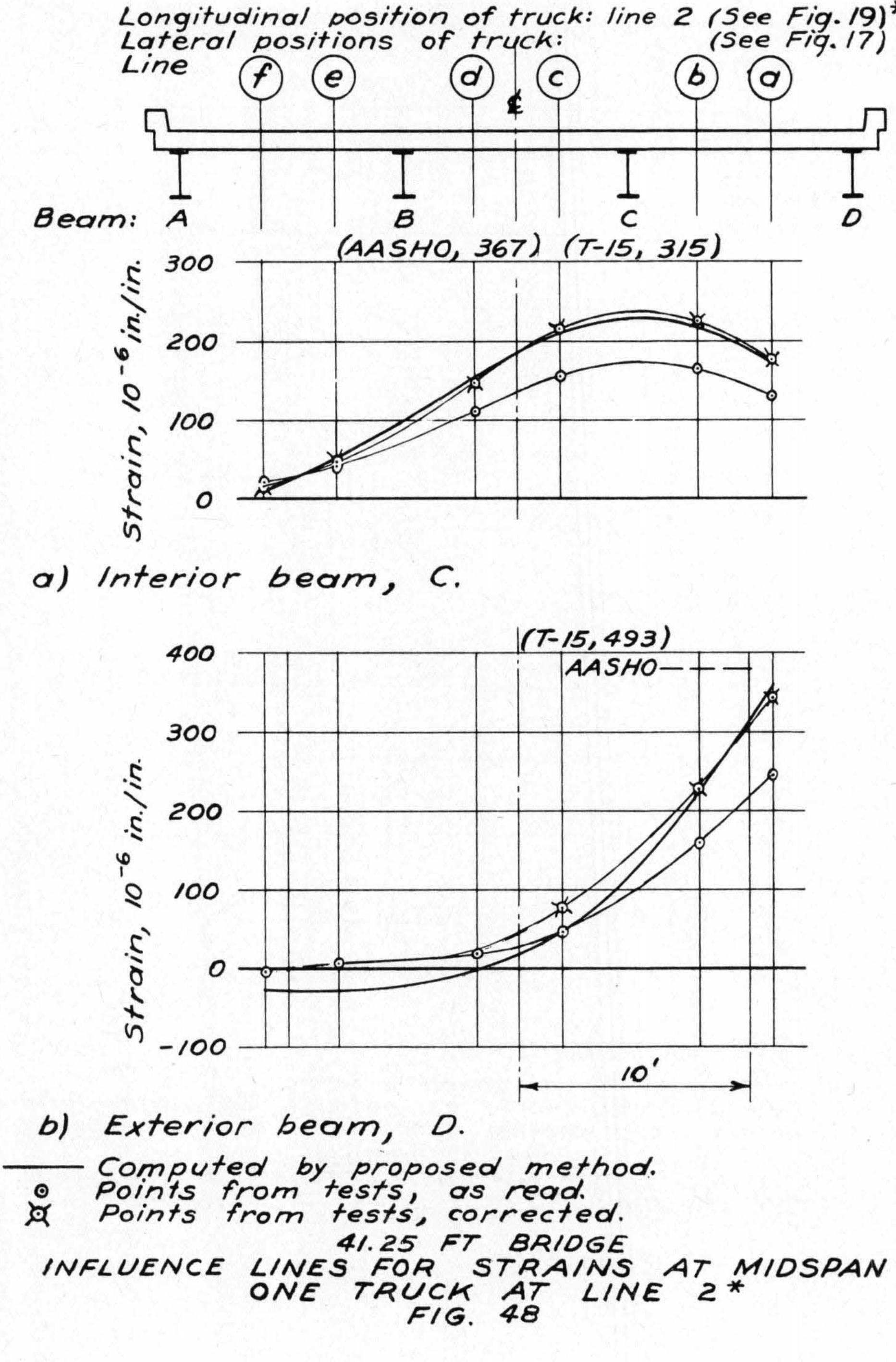
b) Exterior beam, D.

⊙ Circled points are from tests.
—— Computed by proposed method.

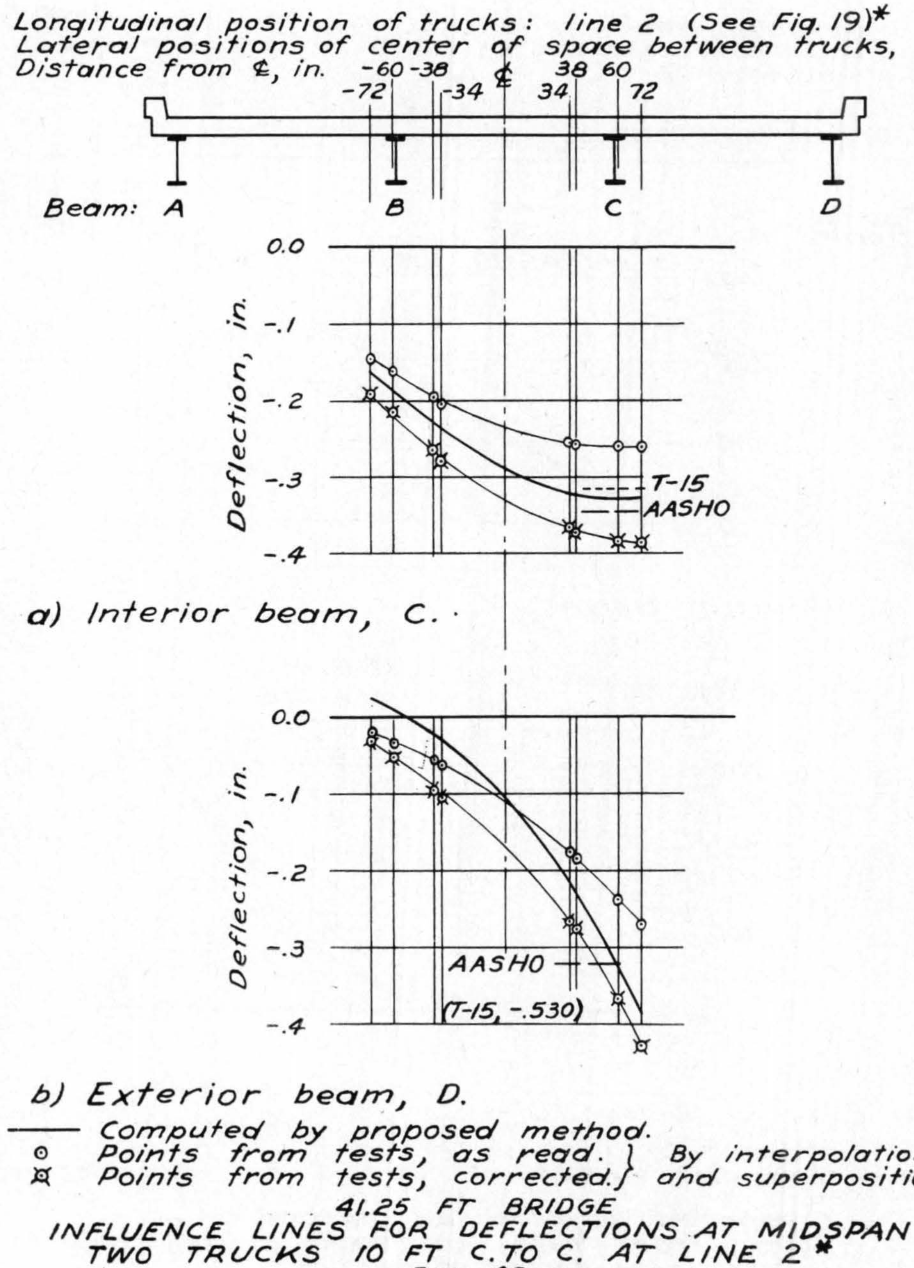
25 FT BRIDGE
INFLUENCE LINES FOR STRAINS AT MIDSPAN
TWO 8000 LB TANDEM-AXLE TRUCKS
40 IN. C. TO C. AT LINE 4 *
FIG. 46



41.25 FT BRIDGE
INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN
ONE TRUCK AT LINE 2 *
FIG. 47

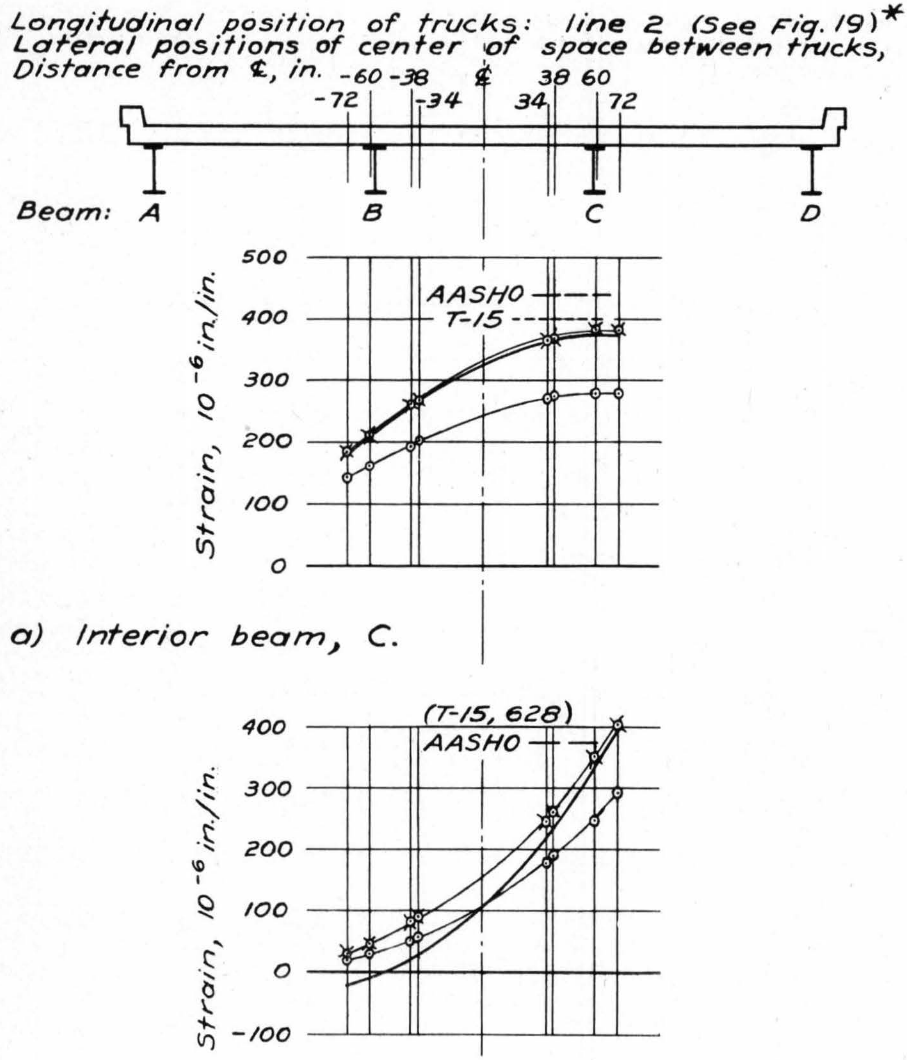


41.25 FT BRIDGE
INFLUENCE LINES FOR STRAINS AT MIDSPAN
ONE TRUCK AT LINE 2*
FIG. 48



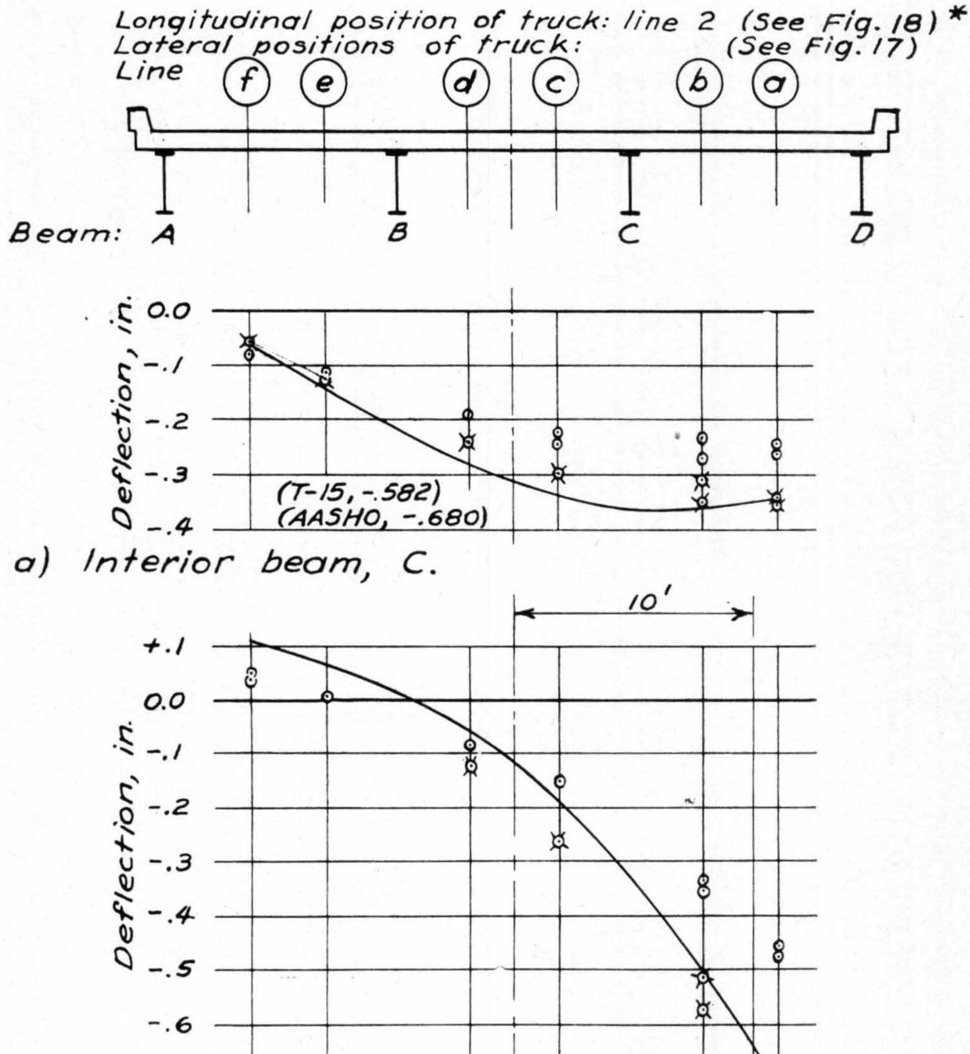
—— Computed by proposed method.
⊙ Points from tests, as read. } By interpolation
⌘ Points from tests, corrected. } and superposition.

41.25 FT BRIDGE
INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN
TWO TRUCKS 10 FT C. TO C. AT LINE 2 *
FIG. 49



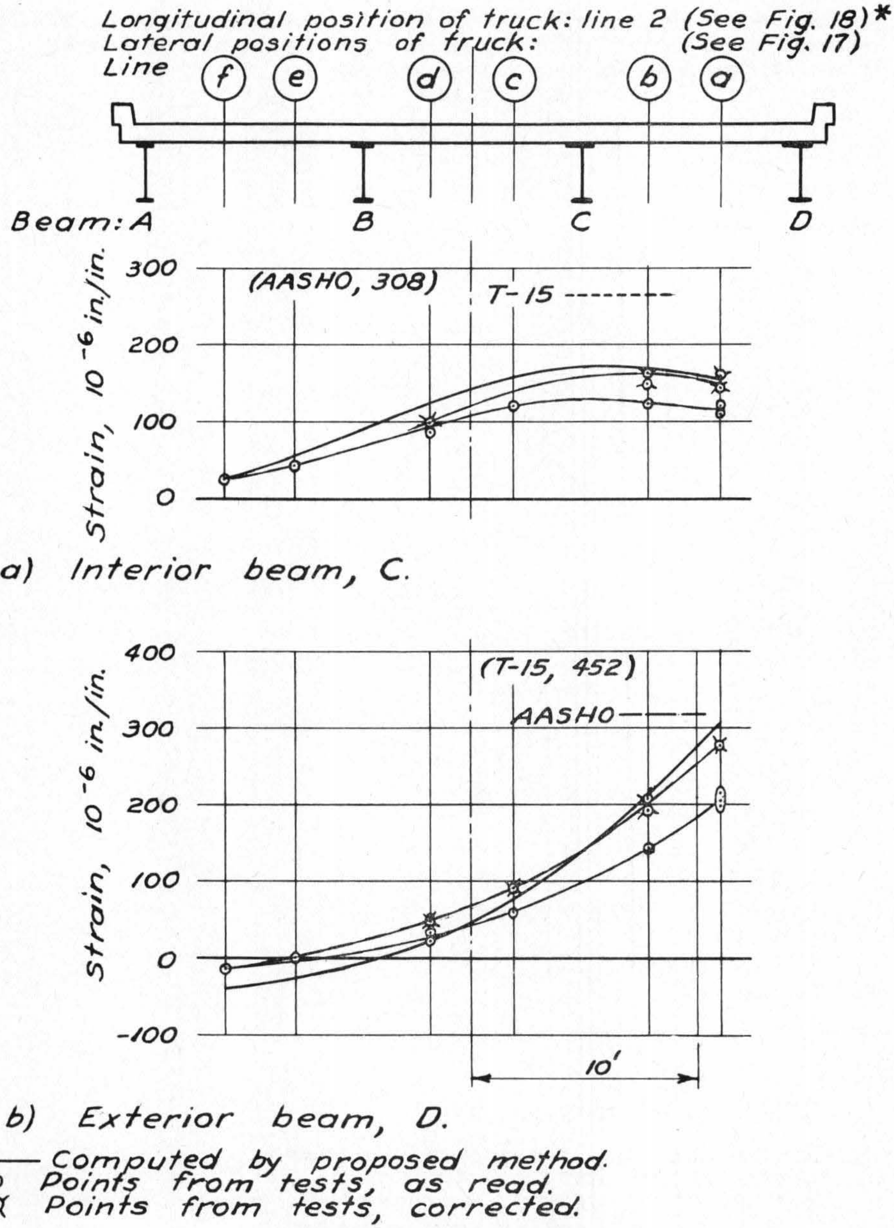
—— Computed by proposed method.
⊙ Points from tests, as read. } By interpolation
⌧ Points from tests, corrected. } and superposition.

41.25 FT. BRIDGE
INFLUENCE LINES FOR STRAINS AT MIDSPAN
TWO TRUCKS 10 FT C. TO C. AT LINE 2*
FIG. 50



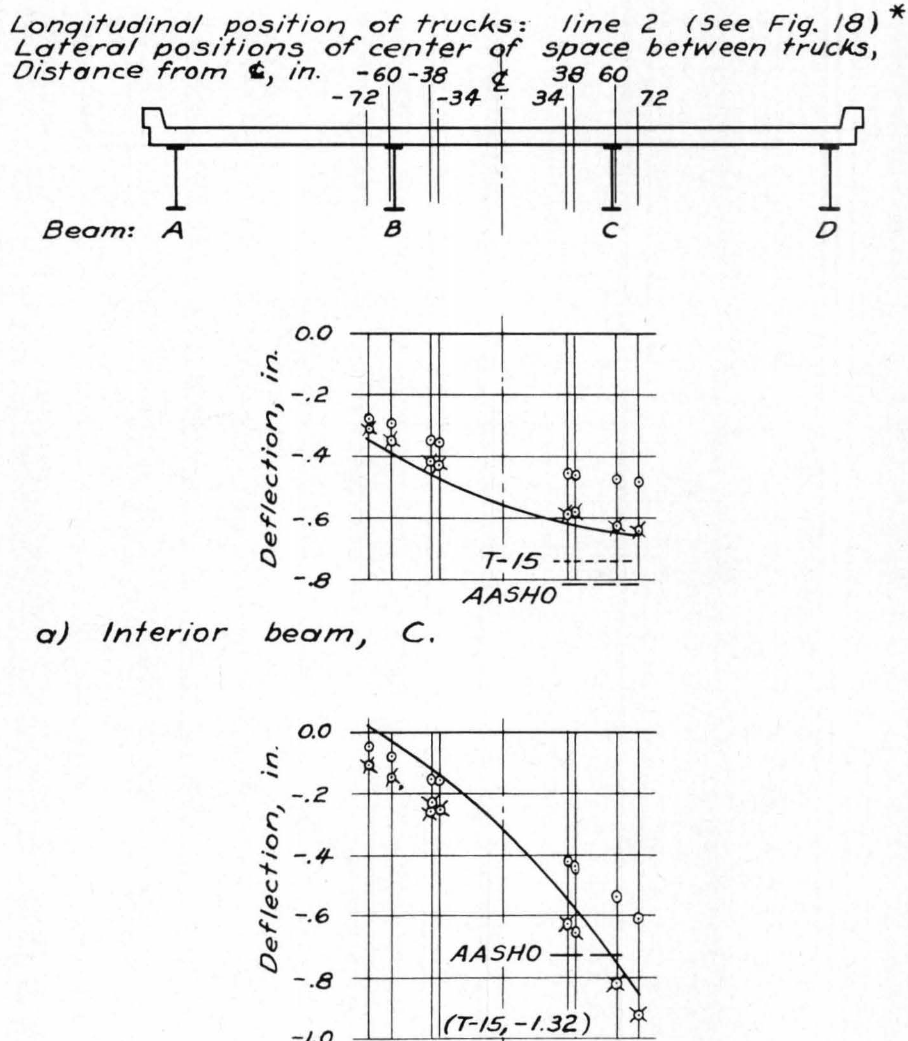
—— Computed by proposed method.
⊙ Points from tests, as read.
⊠ Points from tests, corrected.

71.25 FT BRIDGE
INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN
ONE TRUCK AT LINE 2 *
FIG. 51



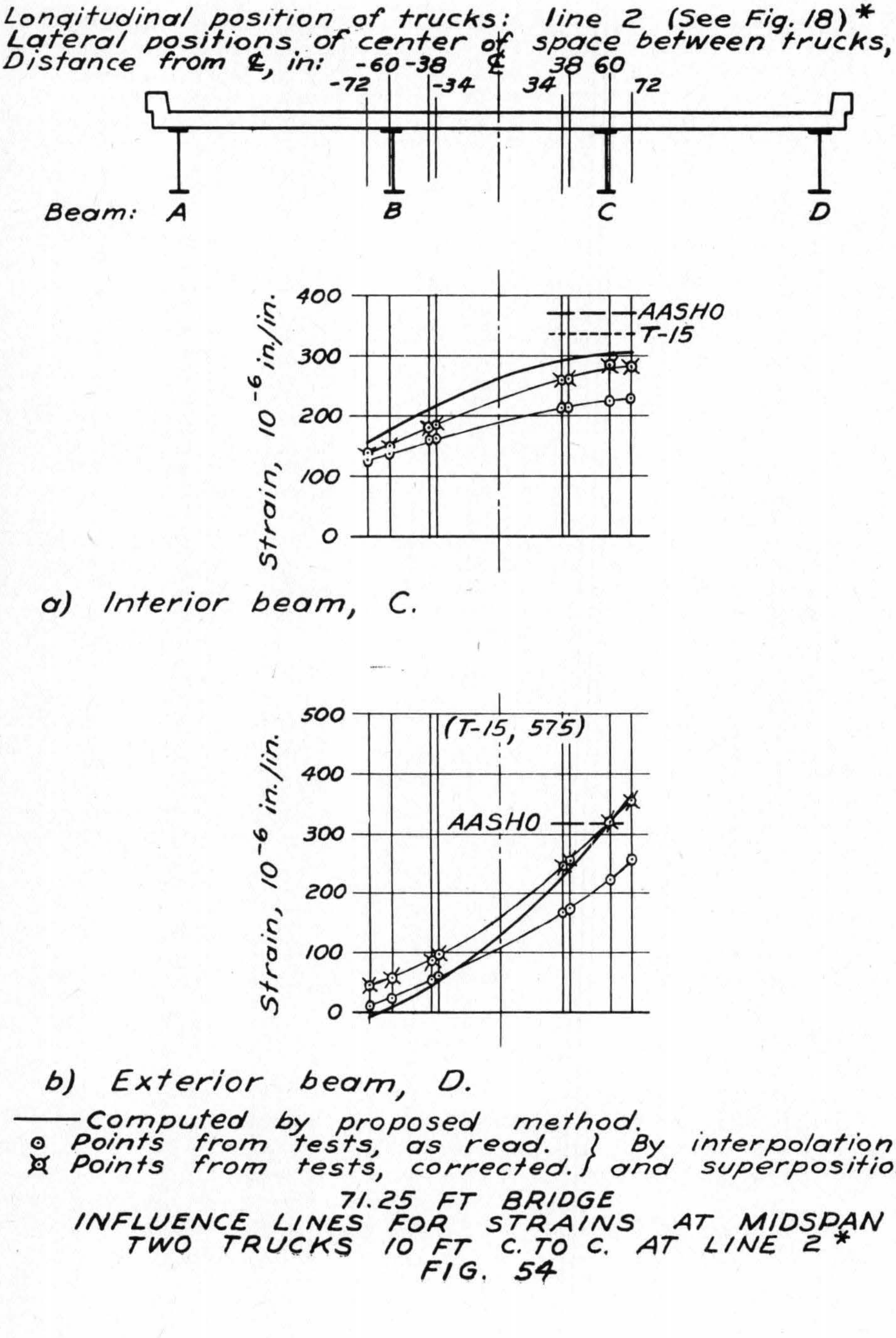
—— Computed by proposed method.
⊙ Points from tests, as read.
¤ Points from tests, corrected.

71.25 FT BRIDGE
INFLUENCE LINES FOR STRAINS AT MIDSPAN
ONE TRUCK AT LINE 2*
FIG. 52

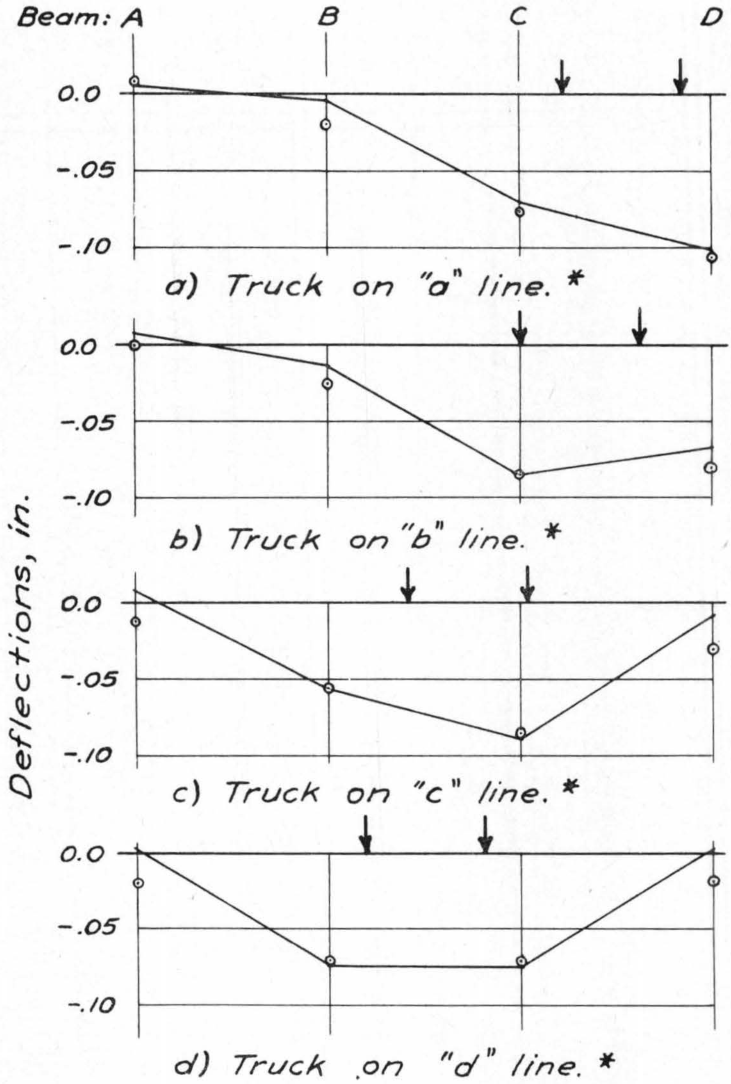


—— Computed by proposed method.
⊙ Points from tests, as read. } By interpolation
⌘ Points from tests, corrected. } and superposition.

71.25 FT BRIDGE
INFLUENCE LINES FOR DEFLECTIONS AT MIDSPAN
TWO TRUCKS 10 FT C. TO C. AT LINE 2 *
FIG. 53

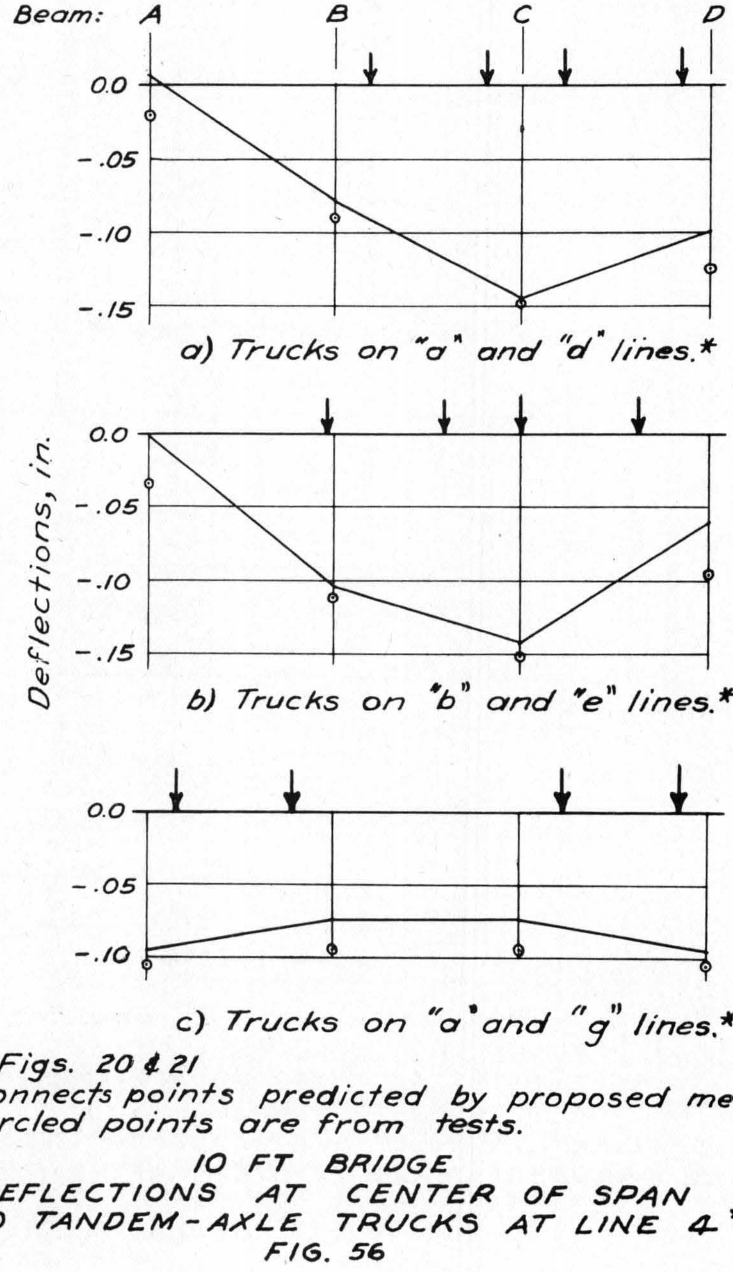


71.25 FT BRIDGE
INFLUENCE LINES FOR STRAINS AT MIDSPAN
TWO TRUCKS 10 FT C. TO C. AT LINE 2*
FIG. 54



—— Connects points predicted by proposed method.
⊙ Circled points are from tests.
* See Figs. 20 & 21

10 FT BRIDGE
DEFLECTIONS AT CENTER OF SPAN
ONE TANDEM-AXLE TRUCK AT LINE 4 *
FIG. 55

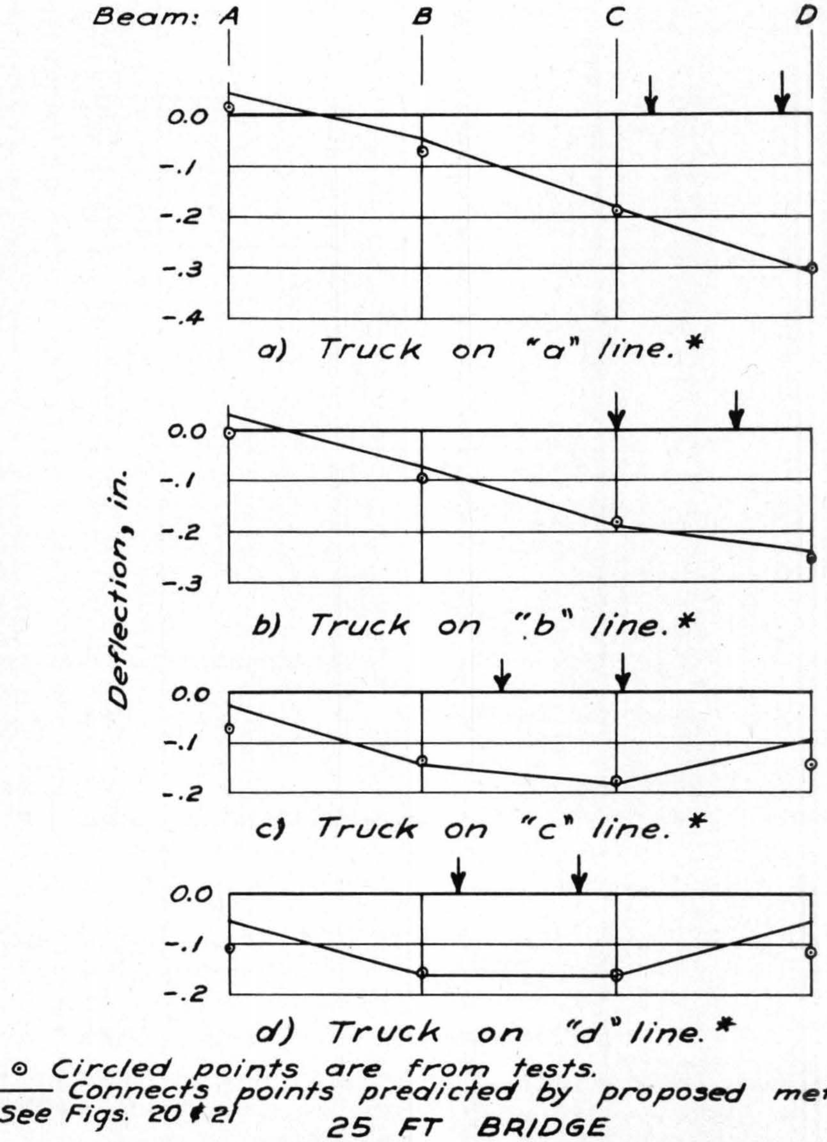


* See Figs. 20 & 21

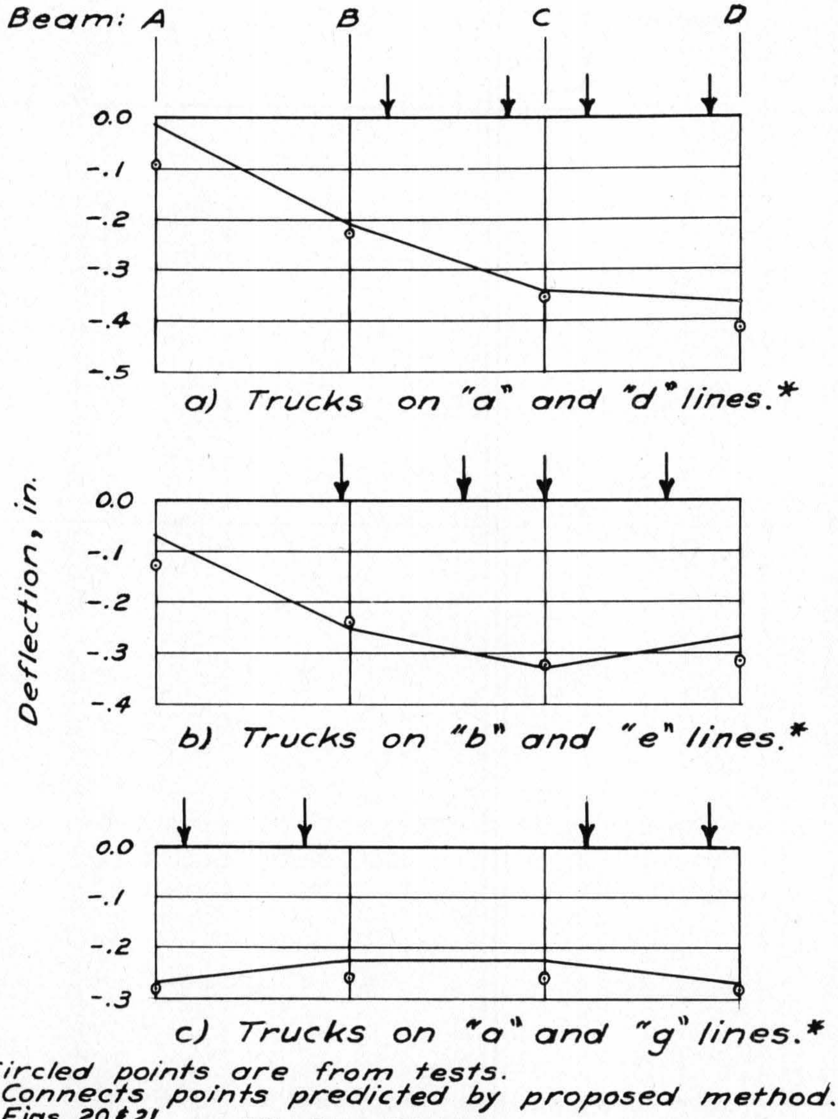
—— Connects points predicted by proposed method.

⊙ Circled points are from tests.

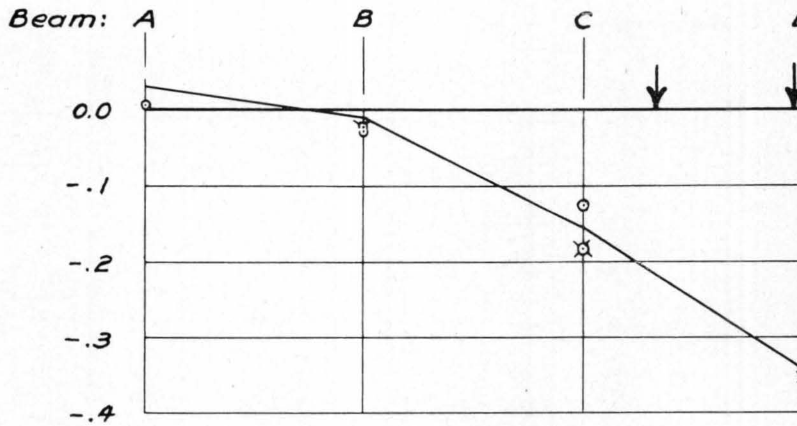
10 FT BRIDGE
DEFLECTIONS AT CENTER OF SPAN
TWO TANDEM-AXLE TRUCKS AT LINE 4*
FIG. 56



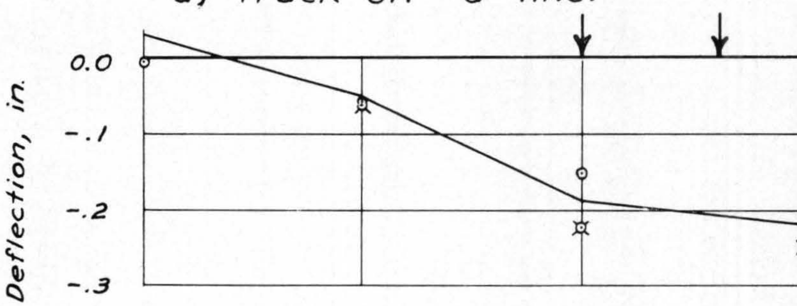
25 FT BRIDGE
DEFLECTIONS AT CENTER OF SPAN
ONE TANDEM-AXLE TRUCK AT LINE 4*
FIG. 57



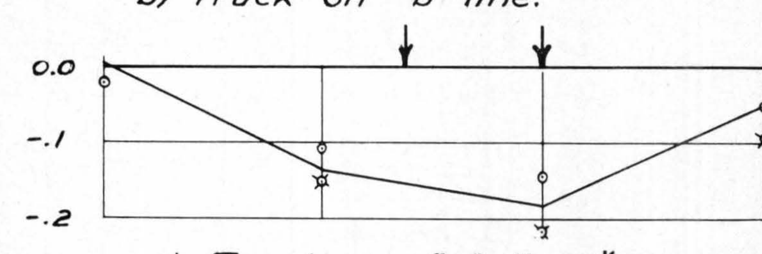
25 FT BRIDGE
DEFLECTIONS AT CENTER OF SPAN
TWO TANDEM-AXLE TRUCKS AT LINE 4*
FIG. 58



a) Truck on "a" line. *



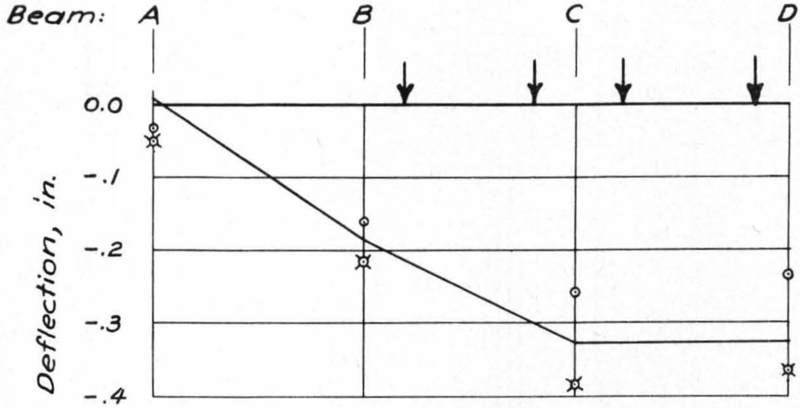
b) Truck on "b" line. *



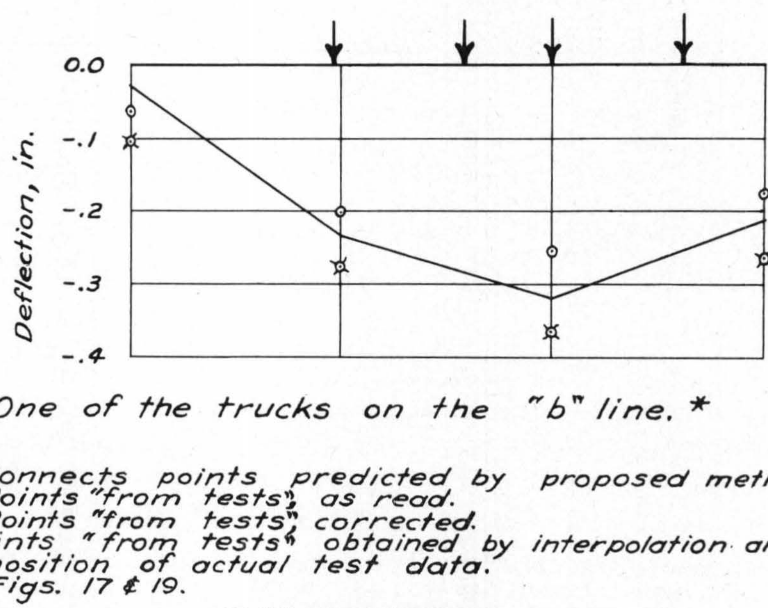
c) Truck on "c" line. *

—— Connects points predicted by proposed method.
⊙ Points from tests, as read.
⌑ Points from tests, corrected.
* See Figs. 17 & 19

41.25 FT BRIDGE
DEFLECTIONS AT CENTER OF SPAN
ONE TRUCK AT LINE 2 *
FIG. 59



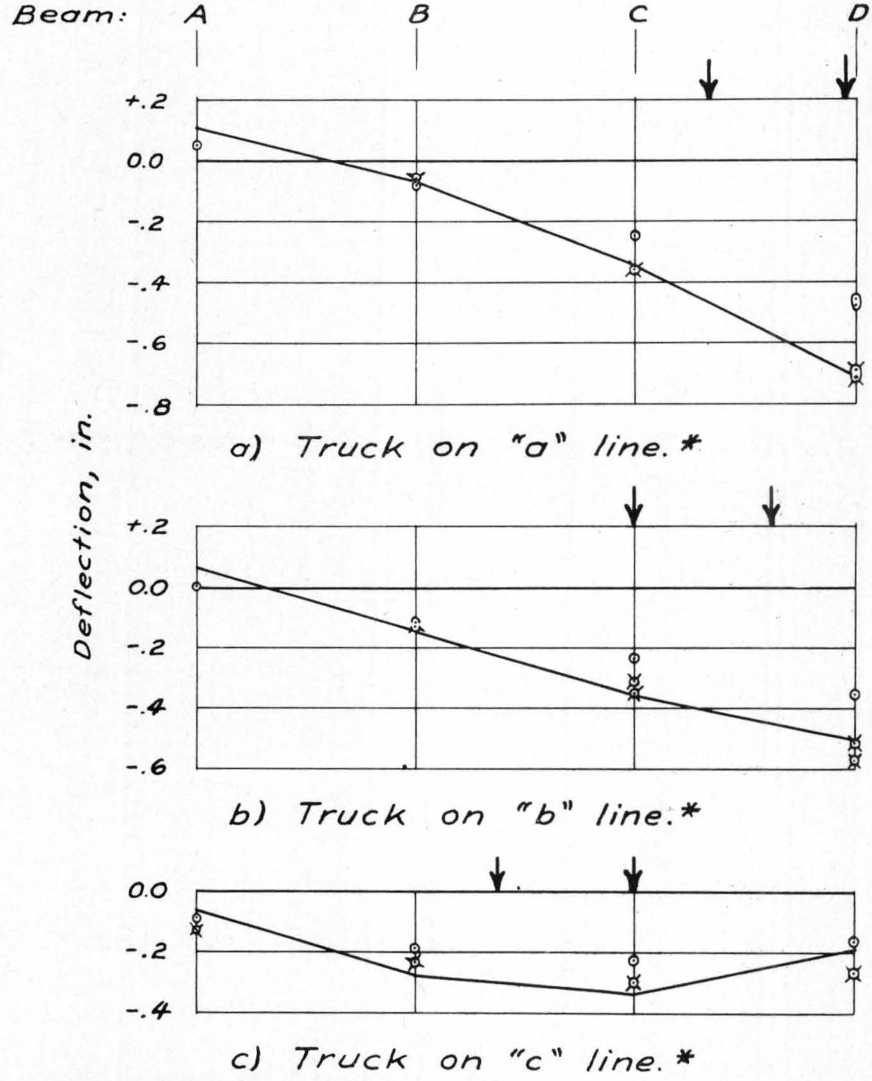
a) Trucks in AASHO specified design position, outer wheel 2 ft from curb.



b) One of the trucks on the "b" line. *

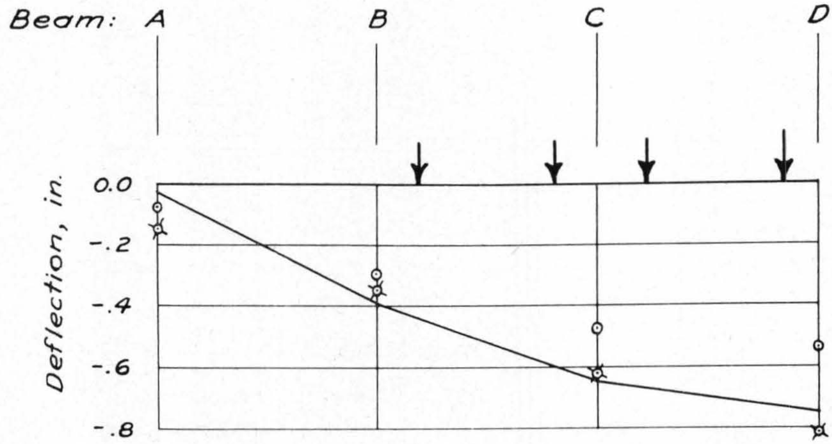
—— Connects points predicted by proposed method.
⊙ Points "from tests", as read.
¤ Points "from tests", corrected.
Points "from tests" obtained by interpolation and superposition of actual test data.
* See Figs. 17 & 19.

41.25 FT BRIDGE
DEFLECTIONS AT CENTER OF SPAN
TWO TRUCKS 10 FT C. TO C. AT LINE 2 *
FIG. 60

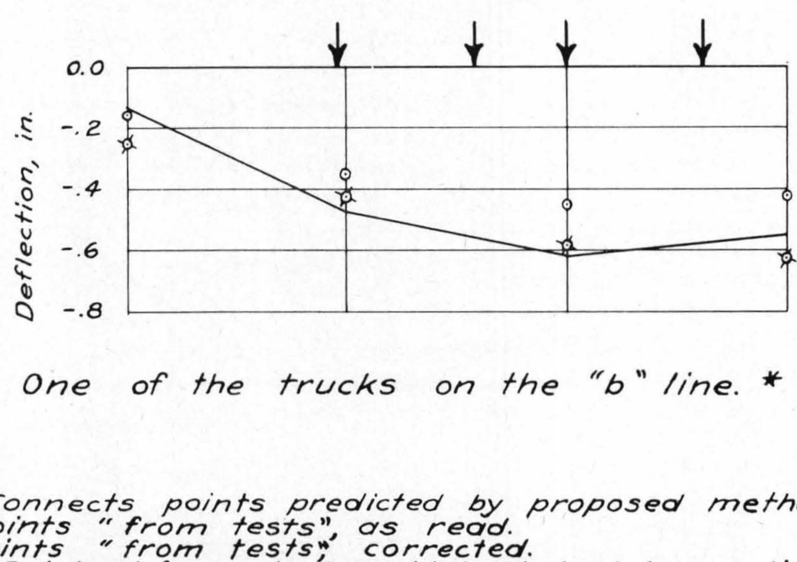


—— Connects points predicted by proposed method.
⊙ Points from tests as read.
¤ Points from tests, corrected.
* See Figs. 17 ¢ 18.

71.25 FT BRIDGE
DEFLECTIONS AT CENTER OF SPAN
ONE TRUCK AT LINE 2 *
FIG. 61



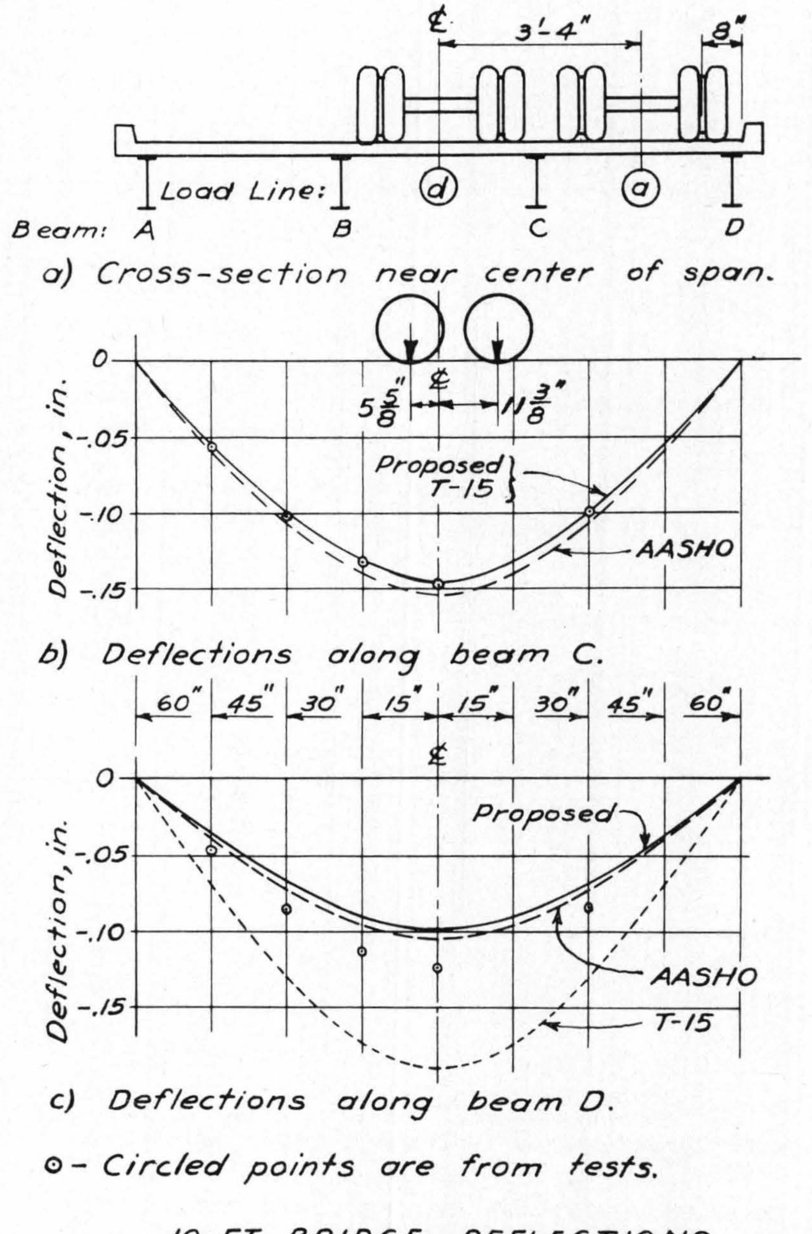
a) Trucks in AASHO specified design position, outer wheel 2 ft from curb.



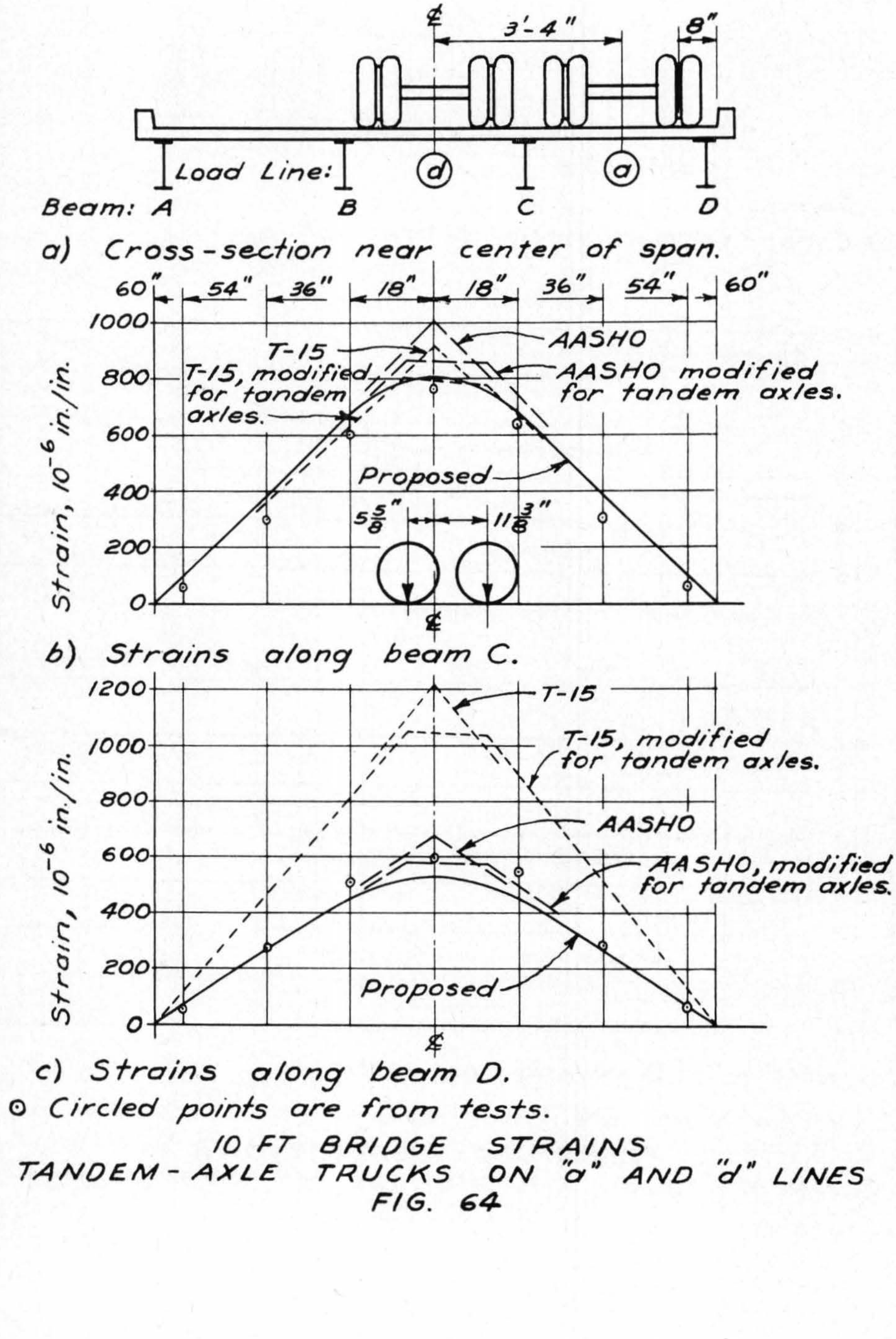
*b) One of the trucks on the "b" line. **

—— Connects points predicted by proposed method.
⊙ Points "from tests", as read.
⌧ Points "from tests", corrected.
Points "from tests" obtained by interpolation and superposition of actual test data.
* See Figs. 17 & 18.

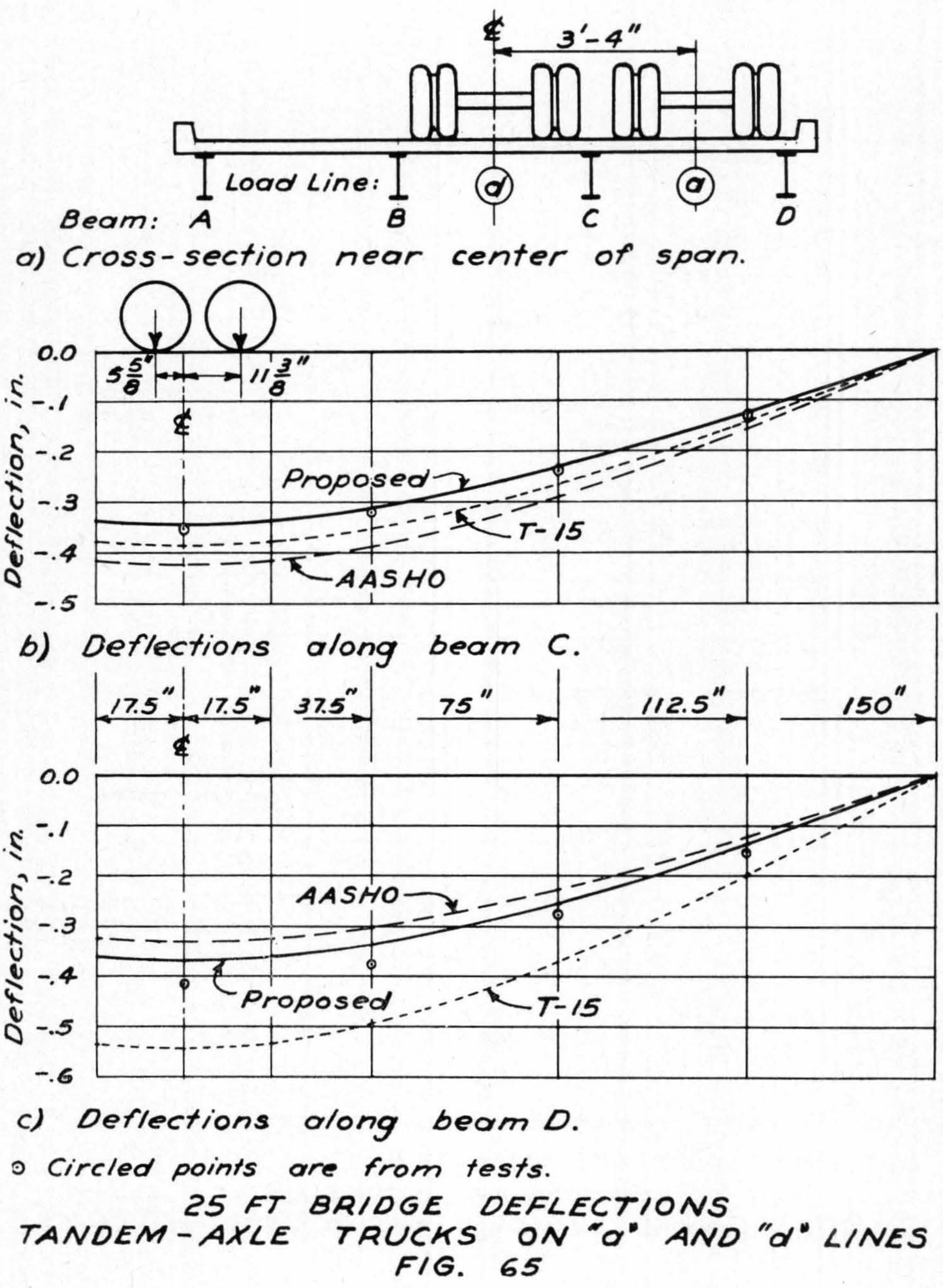
71.25 FT BRIDGE
DEFLECTIONS AT CENTER OF SPAN
TWO TRUCKS 10 FT C. TO C. AT LINE 2 *
FIG. 62



10 FT BRIDGE DEFLECTIONS
TANDEM-AXLE TRUCKS ON "a" AND "d" LINES
FIG. 63

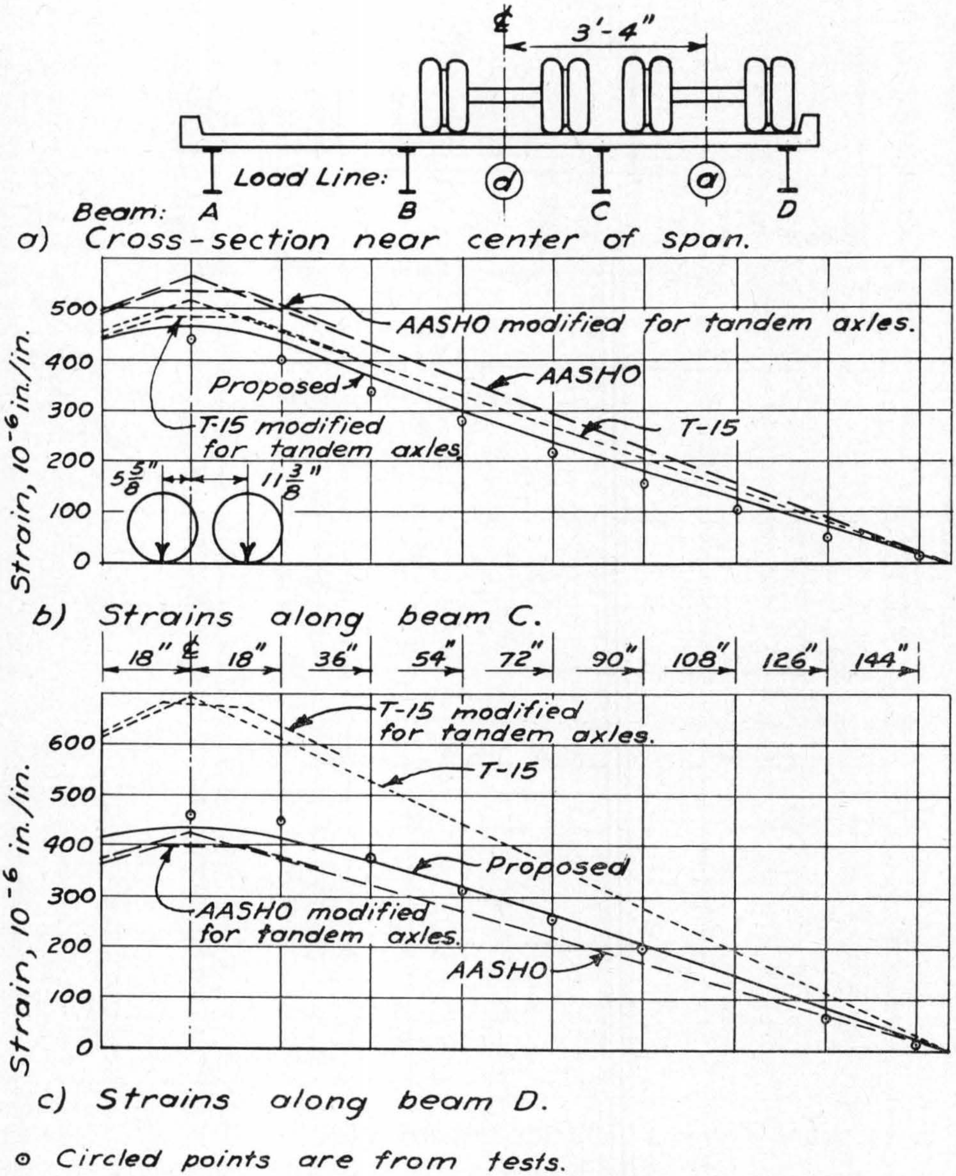


10 FT BRIDGE STRAINS
TANDEM-AXLE TRUCKS ON "a" AND "d" LINES
FIG. 64

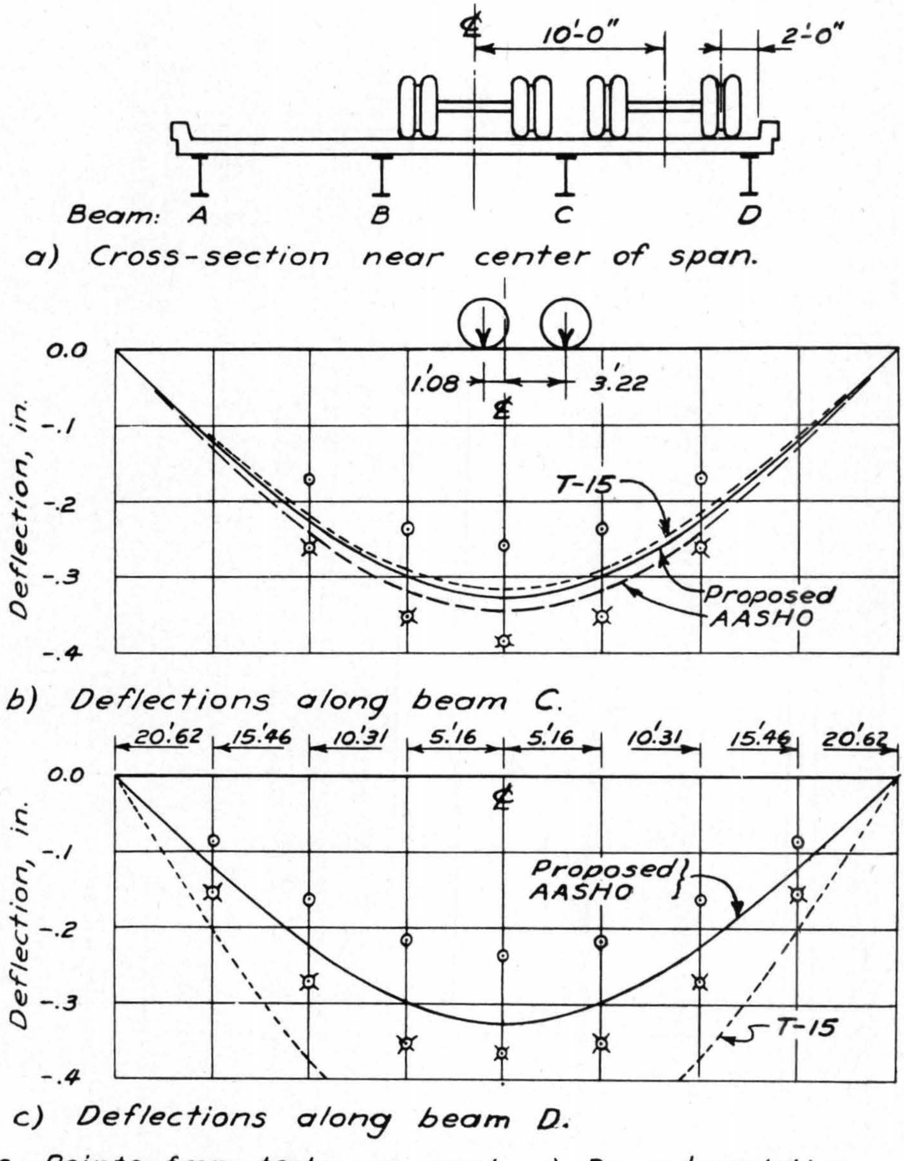


⊙ Circled points are from tests.

25 FT BRIDGE DEFLECTIONS
TANDEM-AXLE TRUCKS ON "a" AND "d" LINES
FIG. 65

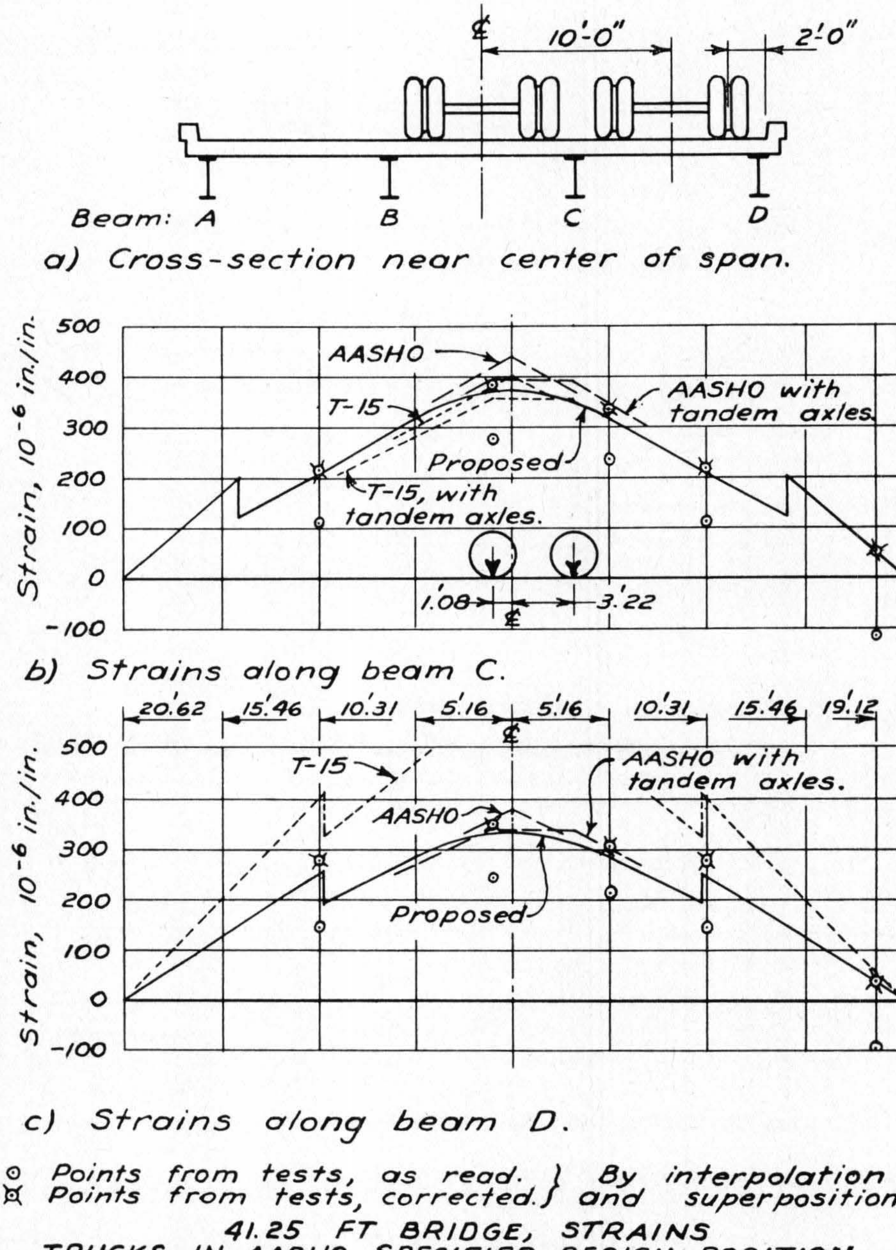


25 FT BRIDGE STRAINS
TANDEM-AXLE TRUCKS ON "a" AND "d" LINES
FIG. 66

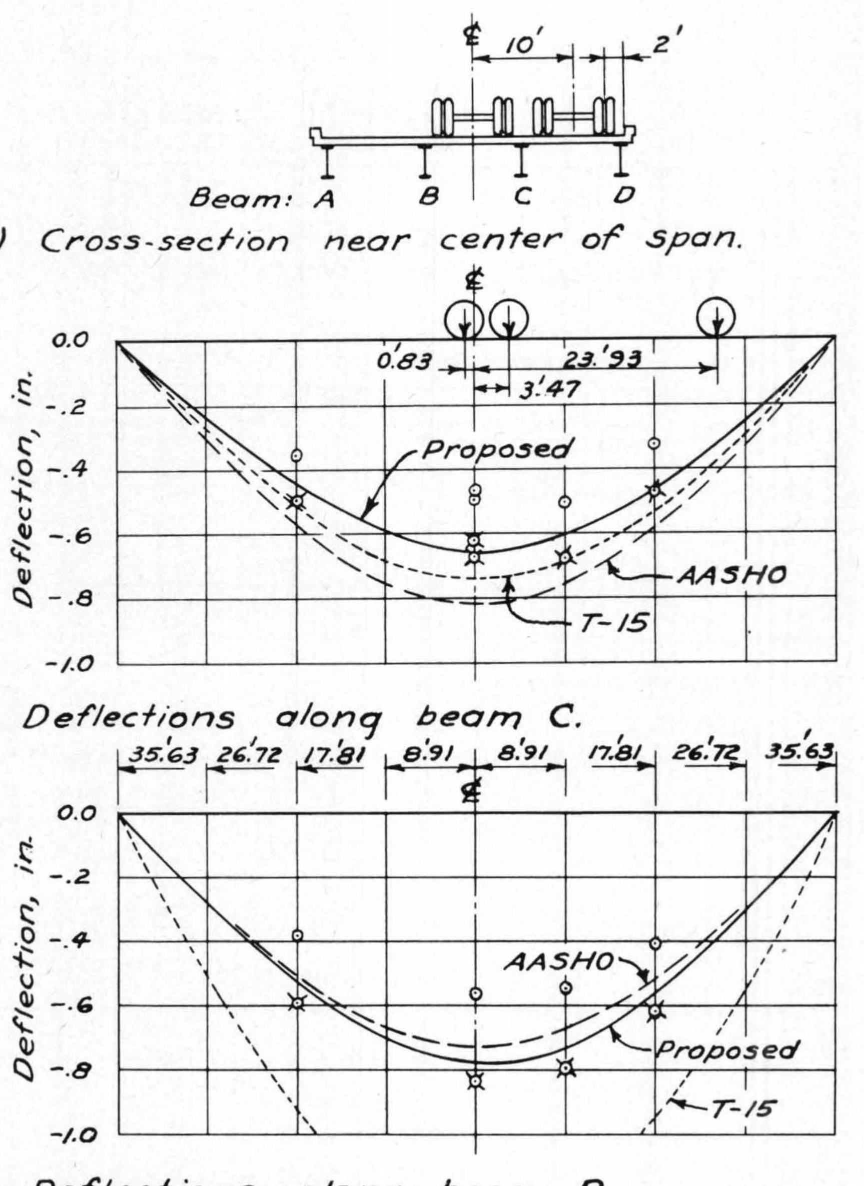


Points from tests, as read. } By interpolation
Points from tests, corrected. } and superposition.

41.25 FT BRIDGE, DEFLECTIONS
TRUCKS IN AASHO SPECIFIED DESIGN POSITION
FIG. 67

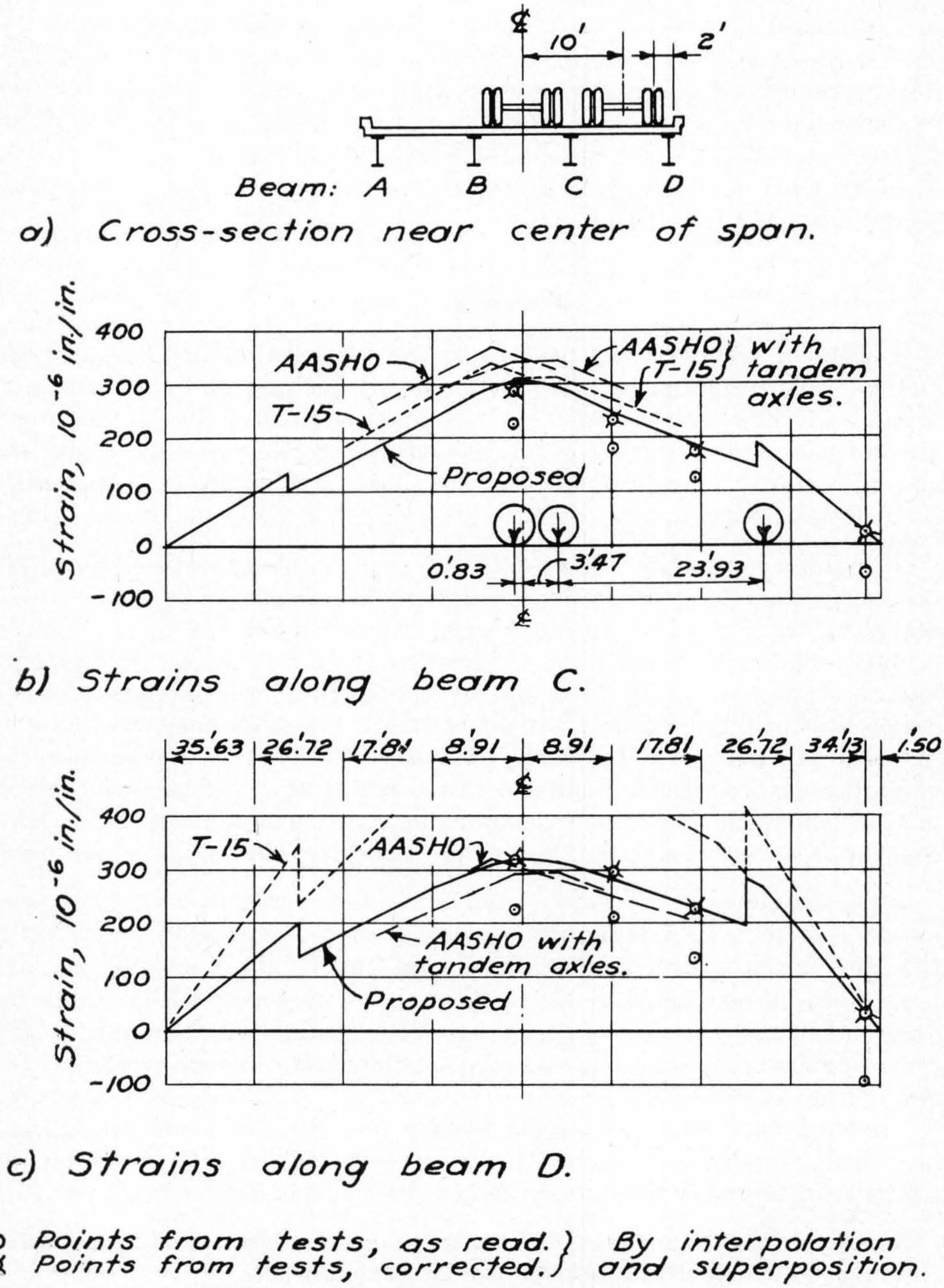


41.25 FT BRIDGE, STRAINS
TRUCKS IN AASHO SPECIFIED DESIGN POSITION
FIG. 68



⊙ Points from tests, as read. } By interpolation
¤ Points from tests, corrected. } and superposition

71.25 FT BRIDGE, DEFLECTIONS
TRUCKS IN AASHO SPECIFIED DESIGN POSITION
FIG. 69



71.25 FT BRIDGE, STRAINS
TRUCKS IN AASHO SPECIFIED DESIGN POSITION
FIG. 70

The AASHO specified lateral spacing of trucks is 10 ft. center to center, full scale. The locations of single trucks used did not include all the ones needed for the superposition process described above. It was, therefore necessary to interpolate between the points actually obtained by test to find values for other lateral positions of the loads. This was done in influence lines such as the ones already described, Figs. 31 to 54. Curves were drawn through the points from the tests and results at intermediate points were read from these curves.

C. Discussion of Results

Both the tests and the analysis by the proposed method provided numerical values of a great many different strains and deflections in each bridge. Of these values the ones of primary interest to bridge designers are the maximum strains (or the corresponding stresses). Other strains and all deflections are ordinarily of secondary importance. Consequently, the detailed comparison of results will be based on these maximum values.

In this comparison, Tables 3-8, the observed strains are either those caused by a single truck or those caused by two trucks side by side, 10 ft. center to center on the highway bridges or 3 ft. 4 in. on the laboratory bridges. The maximum values of these strains have been taken from the influence lines previously described by scaling the highest ordinate to the curves within the extreme positions in which the outermost wheel is correspondingly 2 ft. or 8 in. inside the curb. Still higher strains observed when the outer wheel was outside this position have not been considered in this comparison since they are not considered under the specifications. For the full-size bridges the "corrected" observed strains have been used.

The maximum strains predicted by the proposed method have been obtained from the predicted influence lines in the same manner and with the same limitations. The maximum strains predicted by the specifications have been computed and are the same as those represented in the influence diagrams by short dashed or dotted lines or by parenthetical notations. The error in each predicted value has been computed by subtracting from it the appropriate observed value, and the percent of error has been computed by dividing this error by the observed value. Finally, these calculations have been summarized by tabulating the percentages, only, including the averages of those for the interior and exterior beams, Tables 9 and 10.

Referring to the tables, it is seen that for all the conditions tested the ranges of the percent of error for the various methods are:

proposed method	+11 to −10,
AASHO	+87 to −8,
T-15	+106 to +5.

Within these ranges the median percentages are:

proposed method	+5,
AASHO	+24,
T-15	+52.

Table 3. **Comparison of maximum observed and predicted strains in stringers of laboratory bridges. One 4000 lb. truck at line 5**[a]

		10 ft. Bridge		25 ft. Bridge	
		Interior stringers	Exterior stringers	Interior stringers	Exterior stringers
		(Strains in $(10)^{-6}$in. per in.)			
Observed		262	255	136	192
Predicted by					
Proposed method	Amount	287	284	144	192
	Error	25	29	8	0
	Percent	10	11	6	0
AASHO	Amount	418	337	236	212
	Error	156	82	100	10
	Percent	60	32	74	5
T-15	Amount	360	477	202	272
	Error	98	222	66	80
	Percent	37	87	49	38

[a]See Fig. 21.

These comparisons would seem to indicate that the proposed method is superior to the specification methods on an overall basis. It is seen to be superior, also, on an individual percentage basis. In Tables 9 and 10, in no individual case does either of the specification methods provide a better prediction than does the proposed method.

In addition, the proposed method provides a means for predicting the strains caused by unusual loadings and by any load in any particular position. The specifications, however, provide predictions of the maximum effects, only, of trucks of a particular type.

Table 4. **Comparison of maximum observed and predicted strains in stringers of laboratory bridges. Two 4000 lb. trucks 40 in. center to center at line 5**[a]

		10 ft. Bridge		25 ft. Bridge	
		Interior stringers	Exterior stringers	Interior stringers	Exterior stringers
		(Strains in $(10)^{-6}$in. per in.)			
Observed		412	306	228	225
Predicted by					
Proposed method	Amount	430	278	239	221
	Error	18	–28	11	–4
	Percent	4	–9	5	–2
AASHO	Amount	502	337	283	212
	Error	90	31	55	–13
	Percent	22	10	24	–6
T-15	Amount	458	606	257	347
	Error	46	300	29	122
	Percent	11	98	13	54

[a]See Fig. 21.

Further examination of the percentages leads to more detailed conclusions concerning the specification predictions as follows.

1. The best predictions under the present AASHO specifications are those of the strains in the exterior beams when two trucks are acting,

Table 5. **Comparison of maximum observed and predicted strains in stringers of laboratory bridges. One 8000 lb. tandem-axle truck at line 4[a]**

		10 ft. Bridge		25 ft. Bridge	
		Interior stringers	Exterior stringers	Interior stringers	Exterior stringers
		(Strains in $(10)^{-6}$in. per in.)			
Observed		500	501	261	359
Predicted by					
Proposed method	Amount	530	537	276	378
	Error	30	36	15	19
	Percent	6	7	6	5
AASHO	Amount	836	674	472	424
	Error	336	173	211	65
	Percent	67	13	81	18
T-15	Amount	720	954	404	545
	Error	220	453	143	186
	Percent	44	90	55	52

[a]See Fig. 21.

but the tentative revisions are grossly in error for these conditions. The ranges are:

proposed method	+1 to −10,
AASHO	+14 to −8,
T-15	+106 to +51.

2. The best predictions under the tentative revisions are those of the strains in interior beams when two trucks are acting. For these conditions the revisions provide better predictions than do the present specifications.

Table 6. **Comparison of maximum observed and predicted strains in stringers of laboratory bridges. Two 8000 lb. tandem axle trucks 40 in. center to center at line 4**[a]

		10 ft. Bridge		25 ft. Bridge	
		Interior stringers	Exterior stringers	Interior stringers	Exterior stringers
		(Strains in $(10)^{-6}$in. per in.)			
Observed		780	589	440	460
Predicted by					
Proposed method	Amount	810	528	465	437
	Error	30	–61	25	–23
	Percent	4	–10	6	–5
AASHO	Amount	1004	674	566	424
	Error	224	85	126	–36
	Percent	29	14	29	–8
T-15	Amount	916	1212	514	694
	Error	136	623	74	234
	Percent	17	106	17	51

[a]See Fig. 21.

The ranges are:

proposed method +6 to −2,
AASHO +30 to +15,
T-15 +18 to +5.

3. Neither the present specifications nor the proposed revisions provide what might be considered satisfactory predictions of the strains in either beam when only one truck is acting, Table 9. The proposed method does provide predictions of these strains within the range between +11 and − 3 percent error.

In connection with the foregoing discussion, it should perhaps be noted that all the bridges tested were designed under the present specifications. If they had been designed under other rules the errors in the predictions under the specifications, present and revised, would have been distributed differently between the interior and exterior beams. For instance, if the bridges had been designed under the tentative revisions, the interior beams

Table 7. **Comparison of maximum observed and predicted strains in stringers of highway bridges. One truck at line 2**[a]

		41.25 ft. Bridge				71.25 ft. Bridge			
		Interior stringers		Exterior stringers		Interior stringers		Exterior stringers	
		As read	Corrected	As read	Corrected	As read	Corrected	As read	Corrected
		(Strains in $(10)^{-6}$in. per in.)							
Observed		171	236	218	307	130	165	183	250
Predicted by									
Proposed method	Amount	228	228	316	316	173	173	273	273
	Error	57	–8	98	9	43	8	90	23
	Percent	33	–3	45	3	33	5	49	9
AASHO	Amount	367	367	379	379	308	308	316	316
	Error	196	131	161	72	178	143	133	66
	Percent	115	56	74	23	137	87	73	26
T-15	Amount	315	315	493	493	264	264	452	452
	Error	144	79	275	186	134	99	269	202
	Percent	84	34	126	60	103	60	147	81

[a]See Figs. 18 and 19.

Table 8. **Comparison of maximum observed and predicted strains in stringers of highway bridges. Two trucks 10 ft. center to center at line 2**[a]

		41.25 ft. Bridge				71.25 ft. Bridge			
		Interior stringers		Exterior stringers		Interior stringers		Exterior stringers	
		As read	Corrected	As read	Corrected	As read	Corrected	As read	Corrected
		(Strains in $(10)^{-6}$in. per in.)							
Observed		277	382	246	351	226	285	223	318
Predicted by									
Proposed method	Amount	373	373	334	334	303	303	320	320
	Error	96	-9	88	-17	77	18	97	2
	Percent	35	-2	36	-5	34	6	43	1
AASHO	Amount	440	440	379	379	370	370	316	316
	Error	163	58	133	28	144	85	93	-2
	Percent	59	15	54	8	64	30	42	-1
T-15	Amount	400	400	628	628	336	336	575	575
	Error	123	18	382	277	110	51	352	257
	Percent	44	5	155	79	49	18	158	80

[a]See Figs. 18 and 19.

Table 9. **Errors in predicted maximum strains in stringers in percent of observed maximum strains[a]. One truck on each bridge**

		Interior stringers	Exterior stringers	Av.	Interior stringers	Exterior stringers	Av.
		Laboratory bridges					
		10 ft. Bridge			25 ft. Bridge		
One 4000 lb. single-axle truck at line 5[b]							
Method	Proposed	10	11	11	6	0	3
	AASHO	60	32	46	74	5	40
	T-15	37	87	62	49	38	44
One 8000 lb. tandem-axle truck at line 4[b]							
Method	Proposed	6	7	6	6	5	6
	AASHO	67	13	40	81	18	50
	T-15	44	90	67	55	52	54
		Highway bridges					
		41.25 ft. Bridge			71.25 ft. Bridge		
One truck at line 2[b]							
Method	Proposed	–3	3	0	5	9	7
	AASHO	56	23	40	87	26	56
	T-15	34	60	47	60	81	70

[a]As corrected, for highway bridges.
[b]See Figs. 18, 19 and 21.

Table 10. **Errors in predicted maximum strains in stringers in percent of observed maximum strains[a]. Two trucks side by side on each bridge.**

		Interior stringers	Exterior stringers	Av.	Interior stringers	Exterior stringers	Av.
		Laboratory bridges					
		10 ft. Bridge			25 ft. Bridge		
Two 4000 lb. single-axle trucks 40 in. center to center at line 5[b]							
Method	Proposed	4	–9	–3	5	–2	2
	AASHO	22	10	16	24	–6	9
	T-15	11	98	54	13	54	34
Two 8000 lb. tandem-axle trucks 40 in. center to center at line 4[b]							
Method	Proposed	4	–10	–3	6	–5	1
	AASHO	29	14	22	29	–8	10
	T-15	17	106	61	17	51	34
		Highway bridges					
		41.25 ft. Bridge			71.25 ft. Bridge		
Two trucks 10 ft. center to center at line 2[b]							
Method	Proposed	–2	–5	–4	6	1	4
	AASHO	15	8	12	30	–1	14
	T-15	5	79	42	18	80	49

[a]As corrected, for highway bridges.
[b]See Figs. 18, 19 and 21.

would have been somewhat smaller and the exterior beams considerably larger. For some conditions the predictions by the specification methods might be improved, for others impaired. Predictions by the proposed method should be equally good regardless of the design procedure.

In addition to the percentages of error discussed above, the various graphs presenting the results provide some general information concerning the different methods. Inspection of the influence lines for deflections and strains, Figs. 31-54, reveals that the proposed method correctly predicts the general shape of these lines as compared with lines drawn through the points from the tests. The lines drawn through observed points are typically less sharply curved than are the predicted lines, as in Fig. 36. For the exterior beams the observed lines tend to be higher near the center and lower at the edge, while for the interior beams the observed lines tend to be lower near the center and higher near both edges, Fig. 36, *et al.* In each case these differences demonstrate a greater transfer of load to the beams at a distance from the load than was predicted. That is, when a truck was in the center of the roadway, the actual load on the exterior beams was greater than predicted, and when the truck was near one edge the load on the interior beams was greater than predicted.

The increased transfer of load indicates greater lateral stiffness than was assumed in the analysis. This greater lateral stiffness can be explained, at least in part, as the effect of the diaphragms that were actually present in the bridges tested but were ignored in the analyses. The effects of the diaphragms can be seen, also, in the cross-sectional deflection diagrams, Figs. 55 to 62. Here, it is seen that for every condition, whether the deflections yielded a surface that was generally convex or concave upward, the actual deflections define a more nearly straight line than do the predicted ones. In every case the effect of the diaphragms is to increase the deflections of the less heavily loaded beams and relieve those of the most heavily loaded as compared with the predicted deflections. It is, therefore, inferred that neglecting the effect of the diaphragms in design is conservative.

The effects of the diaphragms can be observed, also, in the longitudinal strain diagrams, as in Fig. 66. For the conditions represented the cross-section is generally concave upward, Fig. 58a, therefore, the diaphragms must be adding load to the exterior beams and subtracting it from the interior ones. While the effect is slight, it will be seen that the strain diagram for the exterior beam is correspondingly raised and flattened near the center, Fig. 66c, while that for the interior beam is lower and more sharply peaked than would be expected, Fig. 66b.

The proposed method of analysis could be adapted to include the effect of intermediate diaphragms simply by increasing the number of simultaneous equations to be solved if the equivalent elastic constants of the diaphragms could be known. One of the aims of future investigations could well be the determination of these equivalent constants by suitable tests including tests with the diaphragms removed.

D. Time Required for Calculations by the Proposed Method

Since the proposed analysis procedure presented in Chapter III is intended for use by practicing engineers, some mention of the time required for typical calculations can be considered one part of the results of the investigation. This time, of course, depends on the amount of detail required in the analysis and on the amount of experience the designer may have had with the method.

Referring to the example analysis, Calculation Sheets 1 to 9, it will be seen that sheets 1 and 2 are used primarily to present the data and to compute the properties of the composite sections. Almost all of these calculations would be required for an analysis under the specifications; not over 10 minutes are required to make the added ones required by the proposed method. Calculation Sheet 7 is somewhat in the same category. If strains and deflections along the beams are needed, they would have to be calculated by the usual methods under the specifications and the calculations on sheet 7 may be taken as simply replacing these usual calculations.

Calculation Sheets 3, 4, 5 and 6, (or 3, 4, 8 and 9), then, are the ones peculiar to this method. Sheets 3 and 4, the calculation of certain constants of the structure, are prepared only once for each bridge; sheets 5 and 6 must be repeated for each different loading. Often a single arrangement of the loads such as that illustrated on Calculation Sheet 5 (and 8) is all that would be required.

It has been found that a designer having a fair degree of familiarity with the method, such as might be obtained through having used it several times before, can perform the operations on sheets 3, 4, 5, and 6 in as little as half an hour. Adding the extra 10 minutes required on sheets 1 and 2 and allowing for normal delays, the extra time required for a single analysis under the proposed method is only about one hour. Further, this time is not appreciably affected by unusual loads or arrangements of the loads. Other changes from the ordinary such as variable spacing of beams, dissymmetry of the bridge, an increase in the number of beams, or taking the diaphragms into account would, of course, increase the time required.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

In line with the original objectives of the investigation and as indicated by the presentation contained in the preceding chapters, the following conclusions have been reached.

1. An improved procedure for the analysis of the beams in simple-span beam and slab bridges has been developed. It has the following characteristics.

   a. Its development involves only those principles of mathematics and mechanics commonly studied; so practicing engineers without special training should be able to understand and use it.

   b. The extra time required for its use is on the order of one hour per analysis for ordinary conditions. Therefore, its routine use seems practical.

   c. Without changes in the basic steps, all the previously listed variables (p. 13) that affect the strains and deflections of the beams can be taken into account. However, the inclusion of some of them, such as the effects of the diaphragms, would increase the time required for an analysis.

   d. Also without changing the basic steps, the method can be refined and its predictions improved as a result of future research and improved judgment. Further, it facilitates future research because it breaks the computations into discrete steps that can be physically duplicated in the structure and studied separately.

2. Even without including such effects as those of the diaphragms, and without further refinement, the proposed procedure provides predictions of useful accuracy, superior to those under the specifications for every condition tested. The proposed method is especially superior in predicting the maximum effects of single trucks.

3. The present (1953) AASHO specifications provide what may be regarded as satisfactory predictions of the maximum strains caused by two trucks on the bridges tested, the range in the percent of error being from +30 to –8. Correspondingly, it is concluded that the design of such bridges under the present specifications may be regarded as acceptable.

4. The tentative revisions, T-15-50, of the specifications provide predictions of the maximum strains caused by two trucks that are:

   a. for the interior beams, somewhat superior to those under the present specifications, +18 to +5 percent in error, and

   b. for the exterior beams, grossly over-conservative, +106 to +51 in error.

Correspondingly, it is concluded that the tentative revisions are not acceptable for the design of bridges of the type tested.

5. Neither the present specifications nor the proposed revisions provide what can be regarded as satisfactory predictions of the maximum strains caused by one truck, +87 to +5 percent error.

B. Recommendations for Future Research

Further improvement of the predictions by the proposed procedure as well as greater confidence in its applicability within wider ranges of the variables can be achieved through continued investigation. Future research programs recommended, much of which could be carried out on the laboratory bridges already available, are as follows.

1. Tests to determine the actual properties of the beams in place. One suggested procedure for these tests is to load a bridge with concentrated loads, one load directly over each beam at its center, until the deflections of all the beams are the same. From the load, deflection, and strain readings the properties of the beams could, then, be determined free of the effects of cross-bending.

2. More detailed studies of the effects of diaphragms, including efforts to arrive at the equivalent properties of the diaphragms. In these studies it should be helpful to perform a number of tests both with the intermediate diaphragms in place and with them removed.

3. Tests to check the distribution of the loads to the beams and along the beams when they are prevented from deflecting, as in step 1 of the proposed method. These tests could include loading the bridge while the beams are supported by temporary reactions at the cross-section including the loads. Strain gages on these temporary supports could be converted into the forces acting on them for a check on the assumed distribution. Also with the beams temporarily supported, strain measurements in the slab might yield useful information as to the distribution of the temporary reactions along the beams. If temporary supports along the beams are used, they should be so placed that they introduce little or no resistance to the rotation of the beams because this rotation affects the distribution.

4. Studies of the effects of single concentrated loads applied directly over the beams as in step 2 of the proposed method. These studies should probably include unusually careful analyses of the deflection and strain diagrams to evaluate the errors introduced by the assumption of sinusoidal curves in step 2. Possibly some other assumption would be found to yield better predictions without excessive labor.

5. Further investigation of the effects of varying the cross-sections

of the beams. For these investigations, partial length cover plates could be welded or bolted to the lower flanges of the existing beams and such measurements repeated both with and without them as deemed necessary.

6. Checks of the accuracy of the proposed method within a wider range of the relative sizes of the beams. The necessary tests for these checks could be made possible by welding or bolting full-length cover plates to the existing beams, by knocking the curbs off the existing bridges, or otherwise.

Recommended programs which would require the testing of bridges other than the existing models are:

7. Studies of bridges having more than four beams.

8. Investigations of the possibilities of extending the proposed method to the analysis of continuous-beam bridges.

In addition to improving on the proposed method of analysis, it seems possible related studies could be facilitated through use of the method, the existing bridges, or both. Among these are:

9. The extension of the analysis to the determination of moments in the slab. The step-by-step procedure used in analyzing the beams might well be adapted to the analysis of the slab. The strains in the slab would be computed for the condition in which the beams are temporarily supported, the effects on the slab of the differential deflections of the beams determined, and the two effects superimposed. The predictions of both the component effects and the combined totals could be checked by tests.

10. Studies of the dynamic characteristics of the bridges. It should be possible to use the laboratory bridges for checking theoretical modes of vibration and natural frequencies and for obtaining information as to the damping characteristics of composite type bridges. These could be useful in developing an improved method of predicting impact effects.

11. Studies to determine the most economical number of beams, beam spacings, and relative sizes of beams.

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VIII. GLOSSARY

Units of quantities are as noted where used.

Term	Definition	See Page
A, B, C, D	Beam designations	25
A	Area of cross-section, used in computing moment of inertia	47
E	Modulus of elasticity of the beams	22
E_s	Modulus of elasticity of the slab	22
f_{st}	Maximum stress in the steel of a beam	52
h	Thickness of slab	2
H	AASHO truck load designation	4
H	Dimensionless ratio of beam stiffness to slab stiffness	22
I	Moment of inertia of a beam	22
I_A, etc.	Moment of inertia of Beam A, etc	35
$I_{conc.}$	Moment of inertia of a composite area transformed to concrete	47
I_o	Moment of inertia of a section about its centroid	47
I_s	Moment of inertia of a unit width of the slab	22
I_{st}	Moment of inertia of a composite area transformed to steel	47
k	Fraction of a wheel load to be carried by one beam	22
L	Span of beams	1, 2
L_E	Effective slab width over which a concentrated load is assumed to be distributed	54
M	Bending moment	4, 5, 6
M_A, etc.	Bending moment in beam A, etc	45
M_1	Maximum moment in beam caused by concentrated load at its center	52
M_2	Maximum moment in beam caused by sinusoidally distributed load	52
M_{max}	Maximum bending moment in a beam	34
M_r	Bending moment at r in a beam	34
N	Common factor occurring in equations derived	42
N_1	Total number of tons on an "H" truck	4

Term	Definition	See Page
N_2	Number of tons on the rear axle of an "H-S" truck	4
P, P_1, etc.	Concentrated force	24, 25, 30
P_e	Equivalent axle load	5, 6, 7
r	Ratio y/L	34, 35
R_A, etc.	Total final reaction of beam A, etc.	30
R_{AB}, etc.	Reaction of beam A caused by the application of the reversed temporary reaction at beam B, $R^T{}_B$, etc.	30
R^T	Temporary concentrated beam reaction	29, 30
$R^T{}_A$, etc.	Temporary concentrated beam reaction at beam A, etc.	30
S	AASHO truck-trailer designation	4
S	Spacing of beams	2
w_A, etc.	Total maximum intensity of distributed load acting on slab along line of beam A, etc.	45
w_{AB}, etc.	Maximum intensity of a distributed force on the slab along beam A, caused by a concentrated load applied at the center of beam B, etc.	35, 37
w_{ABr}, etc.	Intensity of r of a distributed force on the slab along beam A caused by a concentrated load applied at the center of beam B, etc.	35
w_{max}	Maximum value of a distributed load	34
w_r	Distributed load intensity at r	34
W	Total uniformly distributed force	43
W^T	Total temporary uniformly distributed beam reaction	29
z_{AB}, etc.	Maximum deflection of slab at beam A, caused by a concentrated load at center of beam B, etc.	35, 36
z_{ABr}, etc.	Deflection of the slab at 4 along beam A caused by a concentrated load at center of beam B, etc.	35
z'_{BD}, etc.	With beams B and D removed, the maximum deflection of the slab along B when a sinusoidal load of unit maximum intensity is applied to the slab along D	38, 40, 41
x	Distance perpendicular to beams measured from nearest beam to the left	46, 51
y	Distance along a beam measured from one end	32, 35

Term	Definition	See Page
$\bar{y}$	Vertical distance to an area, used in computing moment of inertia	47
∂	Maximum deflection of a beam when acted on by a sinusoidally distributed load whose maximum intensity is unity	34
∂_A, etc.	Maximum deflection of beam A when acted on by a sinusoidally distributed load whose maximum value is unity, etc.	34
∂_s	Slab constant $= S^3/E_sI_s$	34
Δ_{AB}, etc.	Maximum deflection of beam A, caused by a concentrated load applied at center of beam, B, etc.	35, 36, 37
Δ_{ABr}, etc.	Deflection at r in beam A caused by a concentrated load applied at the center of beam B, etc.	35
Δ_1	Maximum deflection of a beam caused by a concentrated load at its center	52
Δ_2	Maximum deflection of a beam caused by a sinusoidally distributed load	52
Δ_{max}	Maximum deflection of a beam	34
Δ_r	Deflection of a beam at r	34, 35
ε_{st}	Maximum strain in the steel of a composite beam	52

IX. ACKNOWLEDGEMENTS

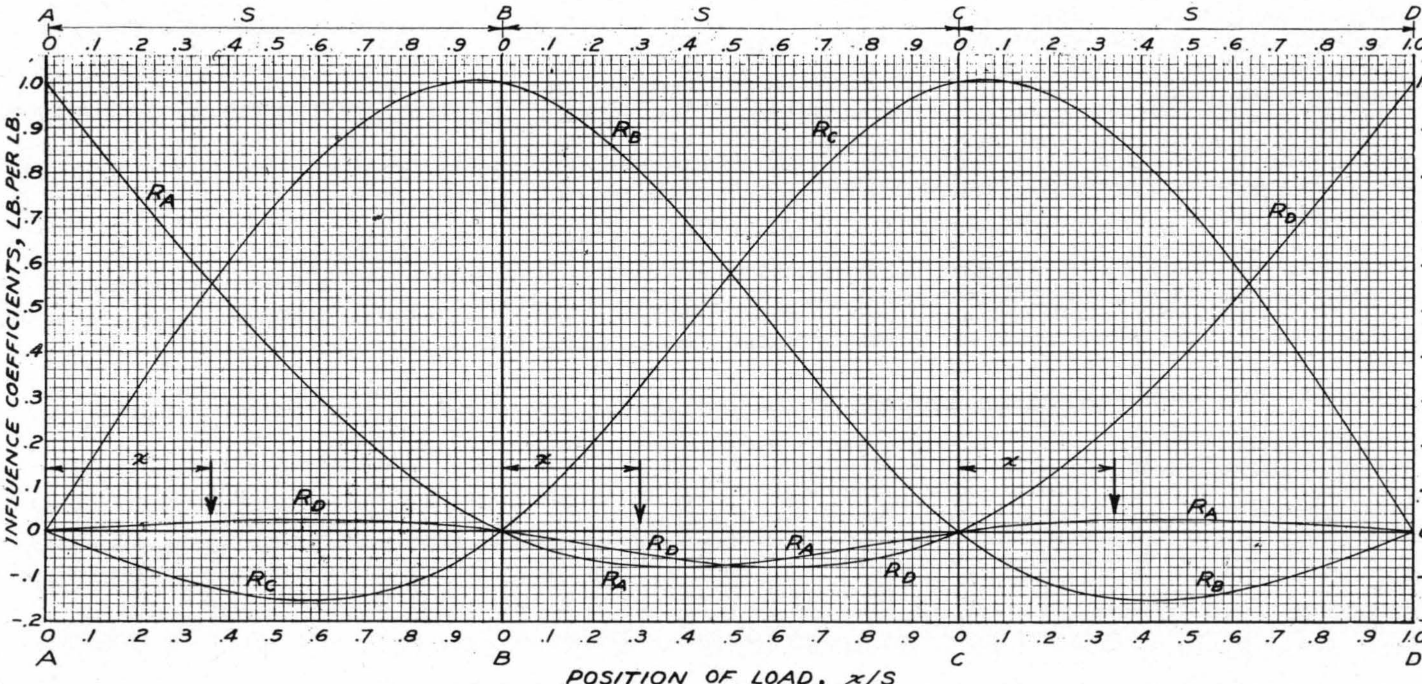
The investigation reported herein resulted from the recognition by Mr. E. W. Blumenschein, then Bridge Engineer for the Iowa State Highway Commission, of a need for renewed study of the distribution of loads to the beams of beam-and-slab bridges. His judgment, based on many years of experience and observation, lead him to believe that the designs evolved over the years by the Iowa State Highway Commission were essentially sound even though they were not in accord with then newly proposed revisions of the specifications governing their design. It is an enduring credit to his judgment and persistence that the investigation has proven he was correct. His contribution in stimulating thought on the subject and in setting in motion the administrative machinery to make the investigation possible is gratefully acknowledged.

The test results utilized in evaluating the proposed method of analysis were made available through the cooperation of the Iowa State Highway Commission and of the Iowa Engineering Experiment Station. The material assistance of these agencies was essential to the successful completion of the investigation. In bringing together the various individuals and agences involved in the testing program, and in perfecting the necessary arrangements among them Prof. Frank Kerekes, then Assistant Dean of the Divison of Engineering, deserves special mention. For the Experiment Station, Dr. George Town, Assistant Director, has been most helpful. For the Highway Commission, Mr. Bert Myers, Materials Engineer, with the cooperation of Mr. Mark Morris, Research Director, was responsible for providing the men, money, and equipment needed in making the tests.

Also working for the Highway Commission, Mr. Lyle Henry was immediately responsible for building and testing the laboratory bridges. His long-continued devotion to the project and his care, skill, judgment, and patience in performing the work are worthy of special note and are especially appreciated.

Finally, the helpful suggestions, patience, tolerance, and cordiality of the members of the author's present Graduate Committee are gratefully acknowledged. It is composed of Prof. R. A. Caughey, Chairman; and of Professors A. H. Fuller, J. L. Hinrichsen, C. L. Hulsbos, Glenn Murphy, G. R. Town, and M. G. Spangler.

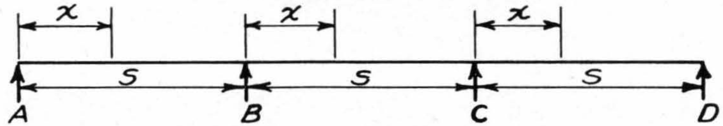
X. APPENDIX



INFLUENCE LINES FOR REACTIONS OF A THREE-SPAN CONTINUOUS BEAM
(Equal spans, constant section.)
FIG. 71

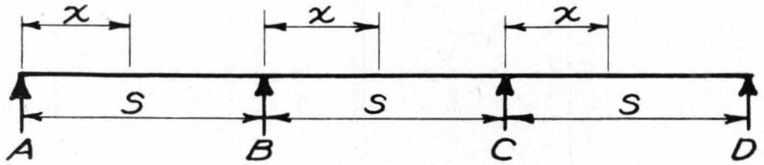
Table 11.

REACTION INFLUENCE TABLES
3-SPAN CONTINUOUS BEAM
EQUAL SPANS, CONSTANT SECTION



IN	SPAN AB				SPAN BC				SPAN CD			
x/S	R_A	R_B	R_C	R_D	R_A	R_B	R_C	R_D	R_A	R_B	R_C	R_D
0.00	1.0000	.0000	.0000	.0000	.0000	1.0000	.0000	.0000	.0000	.0000	1.0000	.0000
.05	.9367	.0799	-.0200	.0033	-.0214	.9856	.0429	-.0071	.0062	-.0371	1.0056	.0253
.10	.8736	.1594	-.0396	.0066	-.0390	.9630	.0910	-.0150	.0114	-.0684	1.0026	.0544
.15	.8109	.2380	-.0587	.0098	-.0531	.9329	.1436	-.0234	.0157	-.0944	.9915	.0871
.20	.7488	.3152	-.0768	.0128	-.0640	.8960	.2000	-.0320	.0192	-.1152	.9728	.1232
.25	.6875	.3906	-.0938	.0156	-.0719	.8531	.2594	-.0406	.0219	-.1313	.9469	.1625
.30	.6272	.4638	-.1092	.0182	-.0770	.8050	.3210	-.0490	.0238	-.1428	.9142	.2048
.35	.5681	.5343	-.1229	.0205	-.0796	.7524	.3841	-.0569	.0250	-.1502	.8752	.2499
.40	.5104	.6016	-.1344	.0224	-.0800	.6960	.4480	-.0640	.0256	-.1536	.8304	.2976
.45	.4543	.6653	-.1436	.0239	-.0784	.6366	.5119	-.0701	.0256	-.1535	.7802	.3477
.50	.4000	.7250	-.1500	.0250	-.0750	.5750	.5750	-.0750	.0250	-.1500	.7250	.4000
.55	.3477	.7802	-.1535	.0256	-.0701	.5119	.6366	-.0784	.0239	-.1436	.6653	.4543
.60	.2976	.8304	-.1536	.0256	-.0640	.4480	.6960	-.0800	.0224	-.1344	.6016	.5104
.65	.2499	.8752	-.1502	.0250	-.0569	.3841	.7524	-.0796	.0205	-.1229	.5343	.5681
.70	.2048	.9142	-.1428	.0238	-.0490	.3210	.8050	-.0770	.0182	-.1092	.4638	.6272
.75	.1625	.9469	-.1313	.0219	-.0406	.2594	.8531	-.0719	.0156	-.0938	.3906	.6875
.80	.1232	.9728	-.1152	.0192	-.0320	.2000	.8960	-.0640	.0128	-.0768	.3152	.7488
.85	.0871	.9915	-.0944	.0157	-.0234	.1436	.9329	-.0531	.0098	-.0587	.2380	.8109
.90	.0544	1.0026	-.0684	.0114	-.0150	.0910	.9630	-.0390	.0066	-.0396	.1594	.8736
.95	.0253	1.0056	-.0371	.0062	-.0071	.0429	.9856	-.0214	.0033	-.0200	.0799	.9367
1.00	.0000	1.0000	.0000	.0000	.0000	.0000	1.0000	.0000	.0000	.0000	.0000	1.0000

INFLUENCE ORDINATE FORMULAS
3-SPAN CONTINUOUS BEAM
EQUAL SPANS, CONSTANT SECTION



$$v = x/S$$

LOAD IN AB:

$R_A = 1 - 1.2667v + .2667v^3$
$R_B = 1.6v - .6v^3$
$R_C = -.4v + .4v^3$
$R_D = .0667v - .0667v^3$

LOAD IN BC:

$R_A = -.4667v + .8v^2 - .3333v^3$
$R_B = 1 - .2v - 1.8v^2 + v^3$
$R_C = .8v + 1.2v^2 - v^3$
$R_D = -.1333v - .2v^2 + .3333v^3$

LOAD IN CD:

$R_A = .1333v - .2v^2 + .0667v^3$
$R_B = -.8v + 1.2v^2 - .4v^3$
$R_C = 1 + .2v - 1.8v^2 + .6v^3$
$R_D = .4667v + .8v^2 - .2667v^3$