Informed Control Over Inputs and Extent of Industrial Processing

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Abstract

Stylized facts regarding the industrial process include emphases on obtaining information about and control over the quality of raw materials. We provide a model that establishes conditions under which informed control involves ensuring uniformity in inputs and increased uniformity encourages more extensive processing. We show when the Boltzmann-Shannon entropy statistic is an appropriate measure of uniformity.

Keywords: entropy, homogeneous inputs, industrialization, information technologies, sorting.

JEL classification: D2; L1

Introduction

Most industries using raw materials allocate considerable resources to understanding and controlling both the raw materials themselves and details about the processing environment. This observation is old; see Braverman 1974. It is famously true for micro-chips and pharmaceutical drugs. It is also true for some franchise food chains (Schlosser 2001; Ritzer 2004) and is increasingly true in confined agriculture (Nicolson 1998; Lacey 1999; Hennessy, Miranowski, and Babcock 2004). With the intent of understanding and controlling, companies detail production practices in contracts with input suppliers, and buyers make judgments based on years of cumulated experience. Diagnostic tests are widely used to identify attributes and impurities.

The demand for control may be due to a large number of motives, some of which will be specific to an industry. One motive is to promote product consistency so that transaction costs in the output market are reduced, branding and marketing can be better targeted, and pricing power is strengthened. In addition, internal handling costs will be reduced because humans and machines can be better conditioned to operate on the raw materials. Motivated by the emphasis placed in the technical literature (Shell and Hall 2000; Tamime and Law 2001) on knowing the raw material to be processed, the intent of this note is to model a consequence of being able to better condition operations to accommodate raw materials that are better understood. We identify conditions such that enhanced information on, and subsequent control over, inputs through increasing input homogeneity will increase profits from processing and the extent of processing. We also show when entropy is an appropriate statistic for measuring how uniformity in raw materials affects profit.

Model

A firm handling a single input can engage in n processing steps at cost $C(n)$, an increasing, twice continuously differentiable and strictly convex function. We have in mind the situation in which an additional step might involve turning wooden planks into furniture, cutting blown glass for fine crystal, aging wine, or fitting finished product for a more demanding export market. We allow the number of processing steps to assume any non-negative real value.

Benefits per unit of successfully processed raw material amount to $B(n)$, an increasing, twice continuously differentiable and strictly concave function. The raw material can come in M types, with shares $s_m \geq 0$, $\sum_{m\in\Omega_M} s_m = 1$, and $m \in \Omega_M = \{0,1, \ldots, M-1\}$. The types could be distinguished by physical, biological, or chemical attributes, for example, heat content, genetic origin, or acidity. Collectively, the share simplex coordinates are described as $S = (s_0, s_1, \ldots, s_{M-1})$.

Because of comparative familiarity with types, the number of things that can go wrong during a processing step when processing a given type decreases with the share of that type in the raw material. We index the number of things that can go wrong with type m by $g(s_m)$, a positive and decreasing function that may be considered to be an index of scale efficiency in learning-by-doing. Thus, processing will be better geared toward accommodating the *m*th type if s_m is comparatively large. The probability that one unit of type m in the lot is affected by one of these potential failure sources is $\omega \in (0,1)$. Each failure source is independent so that, in share form, the probability the unit does not fail at a step is $(1-\omega)^{g(s_m)}$ and the probability it does fail is $1-(1-\omega)^{g(s_m)}$.

With $\varepsilon = 1-\omega$ and upon aggregating over types, the unconditional (i.e., not type conditioned) probability that a failure occurs on a unit at a given step is

$$\sum_{m\in\Omega_M} s_m \times \left[1-\varepsilon^{g(s_m)}\right] = 1 - \sum_{m\in\Omega_M} s_m \varepsilon^{g(s_m)}, \tag{1}$$

and the unconditional probability a unit does not fail the step is

$$I(S;\omega) = \sum_{m\in\Omega_M} s_m \varepsilon^{g(s_m)}; \qquad \varepsilon = 1-\omega \tag{2}$$

where $I(\cdot) \in [0,1]$ and $I(S;\omega)\,|_{\omega=0} = 1$. Product is tested at the end of n steps only so that one cannot terminate the process early in the event of a failure. Failures are independent events across steps so that the unconditional probability a unit survives the entire process is

$$\left[I(S;\omega)\right]^{n}. \tag{3}$$

Failed product has value 0 so that expected profit per unit is

$$V(n;S,\omega)=B(n)\left[I(S;\omega)\right]^{n}-C(n)=B(n)e^{n\mathrm{Ln}[I(\cdot)]}-C(n). \tag{4}$$

The first- and second-order conditions with respect to the extent of processing are

$$B_{n}(n)e^{n\mathrm{Ln}[I(\cdot)]}+B(n)\mathrm{Ln}[I(\cdot)]e^{n\mathrm{Ln}[I(\cdot)]}-C_{n}(n)=0; \tag{5.1}$$

$$B_{nn}(n)e^{n\mathrm{Ln}[I(\cdot)]}+2B_{n}(n)\mathrm{Ln}[I(\cdot)]e^{n\mathrm{Ln}[I(\cdot)]}+B(n)\left(\mathrm{Ln}[I(\cdot)]\right)^{2}e^{n\mathrm{Ln}[I(\cdot)]}-C_{nn}(n)\leq 0. \tag{5.2}$$

Insert (5.1) into (5.2) to obtain

$$B_{nn}(n)e^{n\mathrm{Ln}[I(\cdot)]}+\left(B_{n}(n)e^{n\mathrm{Ln}[I(\cdot)]}+C_{n}(n)\right)\mathrm{Ln}[I(\cdot)]-C_{nn}(n). \tag{6}$$

This is certainly negative, so that the second-order condition is satisfied at any interior optimum. Because of continuity, any interior optimum must therefore be unique. Henceforth we assume an interior solution and label it as $n=n^{*}$, and as $n=n^{*}(S)$ when the emphasis is needed.

The cross-derivative of (5.1) with respect to $\mathrm{Ln}[I(\cdot)]$ is

$$\begin{aligned}&n^{*}B_{n}(n=n^{*})e^{n^{*}\mathrm{Ln}[I(\cdot)]}+B(n=n^{*})e^{n^{*}\mathrm{Ln}[I(\cdot)]}+n^{*}B(n=n^{*})\mathrm{Ln}[I(\cdot)]e^{n^{*}\mathrm{Ln}[I(\cdot)]}\\&=\left(n^{*}B_{n}(n=n^{*})+B(n=n^{*})+n^{*}B(n=n^{*})\mathrm{Ln}[I(\cdot)]\right)e^{n^{*}\mathrm{Ln}[I(\cdot)]}\\&=n^{*}C_{n}(n=n^{*})+B(n=n^{*})e^{n^{*}\mathrm{Ln}[I(\cdot)]}>0,\end{aligned} \tag{7}$$

where (5.1) has been employed. Consequently, $d^{2}V(n;S,\omega)/dndI(\cdot)\geq 0$ when $n=n^{*}$. We will use this observation shortly.

We seek to understand how the share simplex allocation vector S affects the incentive to process. To this end the concept of majorization is relevant.

DEFINITION 1. (Marskall and Olkin, 1979, pp. 10 and 59) *Vector* $Q'\in\mathbb{R}^{n}$ *is majorized by* $Q''\in\mathbb{R}^{n}$ *(written as* $Q'\prec Q''$*) if* $\sum_{i=0}^{k}q'_{(i)}\geq\sum_{i=0}^{k}q''_{(i)}$ $\forall k\in\Omega_{n}$ *and* $\sum_{i=0}^{N-1}q'_{(i)}=\sum_{i=0}^{N-1}q''_{(i)}$

where the $q_{(i)}$ *are defined as order statistics,* $q_{(0)} \le q_{(1)} \le \ldots \le q_{(N-1)}$. *A Schur-convex function* $U(Q): \mathbb{R}^n \to \mathbb{R}$ *satisfies the statement:* $U(Q') \le U(Q'')$ *whenever* $Q' \prec Q''$.

To illustrate, let $S' = (0.1, 0.3, 0.6)$ and $S'' = (0.65, 0.05, 0.3)$. Then the cumulants under S' are 0.1, 0.4, and 1, while the cumulants under S'' are 0.05, 0.35, and 1. Since $0.05 \le 0.1$, $0.35 \le 0.4$, and $1 \le 1$, the definition asserts that $S' \prec S''$. The definition captures the idea of more homogeneity/uniformity because S'' is more concentrated. As it happens, S'' concentrates on s_0 but the definition is symmetric in placing no preferences for any coordinate, that is, quality type. The concept has been used extensively in the literature on income inequality because it is an alternative presentation of the Lorenz curve in discrete form (Dasgupta, Sen, and Starrett 1973; Shorrocks 1983).

PROPOSITION 1. *If* $S' \prec S''$ *and* $g(\cdot)$ *is concave, then firm profits and extent of processing are larger under* S'' *than under* S'.

Proof of Proposition 1. Statistic $\left[I(S;\omega)\right]^n$ is larger under S'' than under S' (i.e., is Schur-convex) if $I(S;\omega) = \sum_{m\in\Omega_M} s_m \varepsilon^{g(s_m)} \left(= \sum_{m\in\Omega_M} s_m e^{g(s_m)\mathrm{Ln}(\varepsilon)} \right)$ is larger under S'' than under S'. From Marshall and Olkin (1979, p. 11), this is true for all majorizing vectors if and only $h(s_m) = \; s_m e^{g(s_m)\mathrm{Ln}(\varepsilon)}$ is convex in s_m. The derivatives are

$$\begin{aligned}
&h_{s_m}(s_m) = e^{g(s_m)\mathrm{Ln}(\varepsilon)} + s_m g_{s_m}(s_m)\mathrm{Ln}(\varepsilon) e^{g(s_m)\mathrm{Ln}(\varepsilon)};\\
&h_{s_m,s_m}(s_m) = \left[2g_{s_m}(s_m) + s_m \left(g_{s_m}(s_m) \right)^2 \mathrm{Ln}(\varepsilon) + s_m g_{s_m,s_m}(s_m) \right] \mathrm{Ln}(\varepsilon) e^{g(s_m)\mathrm{Ln}(\varepsilon)} \\
&\overset{sign}{=} -2g_{s_m}(s_m) - s_m \left(g_{s_m}(s_m) \right)^2 \mathrm{Ln}(\varepsilon) - s_m g_{s_m,s_m}(s_m) \ge 0.
\end{aligned} \tag{8}$$

Since $h(s_m)$ is convex, we have $V(n^*(S'); S', \omega) \le V(n^*(S'); S'', \omega) \le V(n^*(S''); S'', \omega)$ after re-optimizing. Finally, from (7), an increase in the value of $\mathrm{Ln}[I(S;\omega)]$, that is, of $I(S;\omega)$, due to $S' \to S''$ increases the marginal value of processing and so increases the extent of processing when processing and input homogeneity are complements in production. ■

Notice that $s_m g(s_m)$ concave suffices to ensure that $h_{s_m,s_m}(s_m) \geq 0$. The condition that $g(\cdot)$ be concave is reasonable in that it may be interpreted as diminishing marginal gains from learning-by-doing.

Entropy and Information Content of Raw Materials

Relation $S' \prec S''$ is partial, not being able to rank all pairs of weighting coordinates on the unit simplex. A specific function form on $g(s_m)$, rather than, say, the set of decreasing and concave functions as given previously, would be necessary in order to completely rank the weighting coordinates in terms of their implications for processor profits and decisions. Summary statistics, such as (higher) variance or (lower) entropy rather than $S' \prec S''$, provide a complete ordering on data. But that complete ordering will provide an inappropriate level of exactness when there is only limited knowledge about the context being studied, as was the case with $g(s_m)$.

The capacity to control inputs is predicated upon information and technologies using that information. Most overtly, one might receive signals about raw material and then use the information to physically sort the materials into homogeneous lots. Alternatively, as in Chalfant et al. 1999, the information may be embedded in the technology where a sieve or grader sorts existing raw materials. In other cases, technology can be used during the production of the raw materials to endow the raw materials with information regarding the extent of order or homogeneity on the raw materials. This is the case in the manufacture of steel, and when genetics are used to control the nature of the beast to be born. Sorting mechanisms can employ economic incentives, as when providing agents with a menu of contracts in order to ensure that agents with private information deliver similar (say, higher quality) inputs.

The most widely used statistic intended to depict order in a system, be it regarding energy flows or information flows, is the Boltzmann-Shannon entropy statistic. Theoretical motivation is provided by Weitzman (2000) concerning its use as a measure of ecological diversity. For our purposes we write it

$$\text{as } E(S) = -\sum_{m\in\Omega_M} s_m \mathrm{Ln}(s_m); \qquad s_m \geq 0 \ \forall m \in \Omega_M; \qquad \sum_{m\in\Omega_M} s_m = 1. \tag{9}$$

It is concave in the s_m, measuring the extent of disorder rather than the extent of order in the system. Note that $S' \prec S''$ implies $-E(S') \le -E(S'')$. We ask whether a reasonable technology and circumstances on the nature of uncertainty exist such that our index $I(S;\omega)$ and the (inverse) entropy index $-E(S)$ are essentially the same. The answer is in the affirmative when $\omega \to 0$, that is, when failure due to a given cause and at a given step is rare and when $g(\cdot)$ takes the form $g(\cdot) = k - \mathrm{Ln}(s_m)$.[1]

PROPOSITION 2. *If* $\omega \sim 0$ *and* $g(\cdot) = k - \mathrm{Ln}(s_m)$*, then firm profits are increasing in the extent of raw materials uniformity as measured by the negative of Boltzmann-Shannon entropy.*

Proof of Proposition 2. Bearing in mind that $e^x - e^0 \approx 1 + x - 1 = x$ in the neighborhood of $x = 0$, take a first-order Taylor series expansion of $e^{g(s_m)\mathrm{Ln}(1-\omega)}$ near $\omega = 0$ to obtain the approximate change in value as $1 + g(s_m)\mathrm{Ln}(1-\omega) - 1 = g(s_m)\mathrm{Ln}(1-\omega)$ plus terms of order two and higher so that

$$\begin{aligned}&\sum\nolimits_{m\in\Omega_M} s_m e^{g(s_m)\mathrm{Ln}(1-\omega)} - 1 \approx \mathrm{Ln}(1-\omega)\sum\nolimits_{m\in\Omega_M} s_m g(s_m)\\ &= \mathrm{Ln}(1-\omega)\sum\nolimits_{m\in\Omega_M} s_m \times \big(k - \mathrm{Ln}(s_m)\big) = k\mathrm{Ln}(1-\omega) - \mathrm{Ln}(1-\omega)\sum\nolimits_{m\in\Omega_M} s_m \mathrm{Ln}(s_m).\end{aligned} \tag{10}$$

Now take a first-order Taylor series expansion of $B(n)\left[I(S;\omega)\right]^n$ near $\omega = 0$;

$$\begin{aligned}&B(n)\left[I(S;\omega)\right]^n \approx B(n) + nB(n)\left[I(S;\omega)\right]^{n-1}|_{\omega\sim 0}\Big(\mathrm{Ln}(1-\omega)\sum\nolimits_{m\in\Omega_M} s_m g(s_m)\Big)\\ &= B(n) + nB(n)\left[I(S;\omega)\right]^{n-1}|_{\omega\sim 0}\,\mathrm{Ln}(1-\omega)\Big(k - \sum\nolimits_{m\in\Omega_M} s_m \mathrm{Ln}(s_m)\Big).\end{aligned} \tag{11}$$

But $\mathrm{Lim}_{\omega\downarrow 0}\, nB(n)\left[I(S;\omega)\right]^{n-1} = nB(n)$ so that

$$B(n)\left[I(S;\omega)\right]^n \approx B(n) + nB(n)\mathrm{Ln}(1-\omega)\big(k + E(S)\big). \tag{12}$$

This is decreasing in the value of $E(S)$. ■

Notice that $g(\cdot) = k - \text{Ln}(s_m)$ is convex and so the conclusions in Proposition 1 do not follow. However, apply the last line in (8) to $g(\cdot) = k - \text{Ln}(s_m)$ and obtain

$$-2g_{s_m}(s_m) - s_m\left(g_{s_m}(s_m)\right)^2 \text{Ln}(\varepsilon) - s_m g_{s_m,s_m}(s_m) = \frac{1}{s_m}\left(1 - \text{Ln}(\varepsilon)\right) \geq 0. \tag{13}$$

Upon reconsidering the proof of Proposition 1, it can be seen that the assertion applies for $g(\cdot) = \ k - \text{Ln}(s_m)$ also. However, we cannot replace $S' \prec S''$ with $-E(S') \leq -E(S'')$ in the proposition when not in the neighborhood of $\omega = 0$.

Conclusion

A related issue not addressed here regards the possible roles of input control on the rate and nature of automation activities in manufacture. Capital is generally less flexible than labor in accommodating heterogeneities. When capital substitutes for labor in an industrial process, then it is likely that demand for information inputs will grow. When the supply of information on input composition increases (as with the advent of a new test), then it is likely that capital will substitute for labor in the processing of that input. The hypothesis that capital and information on raw materials complement should be testable if an acceptable index of information content of raw materials can be settled upon.

Endnote

1. Weitzman posed a problem that was broadly similar, and our proof is similar to that for his theorem (p. 255).

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