SELF INSTRUCTIONAL MATH

HR - 324

Iowa Department of Transportation

January 1995

ABSTRACT

The Self Instructional Math course book is designed to provide a basic math knowledge for those involved in the planning, design, and construction of highways. It was developed in a manner to allow the student to take the course with minimal supervision and at times that the work schedule allows. The first version of the course was developed in the early 1970's and due to its popularity was revised in the early 1990's to reflect changes in the highway construction math needs.

The anticipated move to metric (System International) measurements by the highway industry has necessitated the need to change the math course problem values to metric units. The course includes the latest in Iowa DOT policy information relative to the selection and use of metric values for highway design, and construction. Each unit of the book contains instructional information, section quizzes and a comprehensive examination. All problem values are expressed in metric rather than dual (english and SI) units. The appendix contains useful conversion factors to assist the reader in making the change to metric.

PREFACE

This Self Instructional Math textbook is your personal property. It is designed to help you on your present job and future, more demanding work assignments. After you complete this course, the text should be a valuable reference book for you.

The test is written so that the problems can be solved on your own with minimum supervision. Blank work pages have been provided in the text for your work.

The text consists of three sections and several appendices. Each section is made up of several chapters and a comprehensive examination. Each chapter contains three parts: (a) an explanation of rules and formulas; (b) a series of exercises for practice; (c) a quiz consisting of problems related to actual situations you may encounter on your job. All of your work should be kept in the book for later reference. The Table of Contents has a complete listing of the material contained within the text and on which page the material can be found.

The comprehensive examination is the only work in the text that will be graded by your supervisor. However, you are free to work on it at home. You should do the work on a separate sheet of paper and turn in the work along with the answer to your supervisor. **Turning in just the answer alone is not acceptable.** The purpose of this exam is to advise your supervisor on how well you are understanding the material.

The appendices are found in the back of the book. They provide all the additional information you will need for this course. You should look through the material in the appendices and become familiar with it.

Unless your supervisor states otherwise, there is no specific time that each comprehensive examination is due. However, a good schedule to follow is to finish a section and a comprehensive exam every three weeks. The three exam scores will be averaged together.

At the end of the course you will be given a final examination in class. This will be an open book exam which will allow you to use any references you wish. The final will be similar in content to the pre-test exercise. The final exam score and the comprehensive examination average will be sent to the Training Officer in Ames by the supervisor and will be used to evaluate the course.

There is no passing or failing grade, but your supervisor may ask you to re-work a section(s) if your score is below 70%.

At the end of the test is a course evaluation sheet. Please complete the evaluation and submit it to your supervisor. Your suggestions and constructive criticism on how to improve the course will be welcome.

Good luck!

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APPENDICES

Introduction To Metric Measurements and Conversions

The United States is converting its transportation system to the metric system, and the date that the Federal Highway Administration has established is September 30, 1996. After that date, no highway projects may be paid for with Federal Lands Highway or Federal-aid Highway funds unless the **Plans, Specifications and Estimate (PS&E)** are all in metric units (System International or SI).

GUIDE TO COMMON METRIC UNITS AND CONVERSIONS

There are many metric units which can be applied to the various branches of science and engineering. No attempt will be made to define all of these units in this document. only those which are most commonly used in Civil and Structural Engineering will be discussed.

BASIC UNITS

There are only **five** metric **"basic units"** that are of primary concern in the field of roadway, traffic and bridge design and construction. The base units uniquely describe a property requiring measurement. The base units are **length, mass, time, temperature, electrical current, luminous intensity, and plane angles**. The units are dimensionally independent. Four of these base units are shown below

Quantity	Unit	Symbol
length	**meter**	**m**
mass (weight)	**kilogram**	**kg**
time	**second**	**s**
temperature	**Celsius**	**C**

The department will continue to use (degree, minutes, and seconds) rather than the metric unit --- metric radian.
A more detailed description of each of the base units is shown in the following paragraphs.

Length

One of the most common units in civil engineering and other disciplines is length. In **SI**, the base length unit is the **meter (m)**. The meter, and a few of its submultiples, are presented in figure below:

- The meter is slightly longer than a yard.
- Other common units of length with their proper prefix are the centimeter and the millimeter.
- The centimeter is 1/100 of a meter and is about the width of the nail on the smallest finger.

- The millimeter is 1/1000 of a meter and is about the the thickness of the wire of a paper clip.

- The centimeter is not a preferred unit on highway plans. The millimeter is the preferred unit.

The kilometer is a particularly important multiple of the meter for highway applications. The figure illustrates its magnitude.

- A Speed of 62 mph is equal to a speed of 100 km/h.

- The proper pronunciation is **KILL'-o-meter** and **NOT kil-AHM-eter**.

Mass

The second base unit relevant to highway applications is the **kilogram (kg)** which is a measure of mass. Mass is the measurement of inertia of an object. It may be thought of as the measure of an object's resistance to acceleration. Mass is illustrated in below:

- A liter of water (slightly more than a quart) has roughly one kilogram of mass.

- There are 1000 grams in a kilogram. There are a million milligrams in a kilogram.

- There are 1000 kilograms in a megameter. There 1,000,000 milligrams in a megameter.

- One milligram per kilogram is referred to as one part per million.

Time

A familiar **SI** base unit which is relevant to highway, and most other applications, is time. The **second (s)** is the **SI** unit of time. It is the same as in the English system.

- The second is approximately the duration of a slow heartbeat on a relaxed person. It is also the time it takes an apple to fall 5 m (16 feet) to the ground from the branch of a tree.

- One day is 86.4 ks.

- There have been many attempts at developing a.decimal hour and day. None have been adopted. The minute, hour, and day are not **SI** units, but are acceptable for use with **SI** units.

Temperature

Temperature is another base unit with usage in highway applications especially in material testing procedures and construction specifications. **Kelvin (K)** is the base **SI** temperature unit with the **Celsius scale (C)** being closely related, as shown below.

- The Celsius scale is the metric temperature scale that the public will learn, e.g., through weather reports. It is based on freezing and boiling points of water at atmospheric pressure.

- Consider that 100 degrees C is boiling, 50 degrees C is warm and 0 degrees C is freezing.

- The kelvin scale is used in science and engineering, and it is the **SI** temperature scale. There are no negative values.

TABLES OF METRIC UNIT NAMES, PREFIXES AND SYMBOLS

Following are some "**technical**" metric units and the symbols for each:

Name		Symbol
ampere	(electric current)	A
candela	(luminous intensity)	cd
hertz	(frequency)	Hz
joule	(energy, work)	J
lumen	(luminous flux)	lm
newton	(force)	N
pascal	(pressure, stress)	Pa
volt	(electric potential)	V
watt	(power)	W

Following are prefixes used in the metric system, along with the symbol and magnitude for each:

Prefix	Symbol		Magnitude	
micro	μ	0.000001	(millionth)	(10^{-6})
milli	m	0.001	(thousandth)	(10^{-3})
centi	c	0.01	(hundredth)	(10^{-2})
deci	d	0.1	(tenth)	(10^{-1})
deka	da	10	(ten)	(10^{1})
hecto	h	100	(hundred)	(10^{2})
kilo	k	1000	(thousand)	(10^{3})
mega	M	1,000,000	(million)	(10^{6})

The above table depicts the progression, by multiples of ten, from one prefix to the next. In practice, the prefixes deci, deka and hecto are rarely used. Also, the prefix centi is normally used only with meter. Preferred metric practice is to use units which represent multiples of 1000. Therefore, **use of the centimeter and square centimeter, while not disallowed, is discouraged.**

Another preferred practice is to choose a prefix so that the numerical value expressed is between 0.1 and 1000; e.g., 10 kilometers rather than 10,000 meters. Tables of numbers would be an exception to the practice.

The following is the **Iowa Department of Transportation's Policy and Procedure** on the use the Metric System.

POLICY

1. The Department adopts the international system of measurement known as the modernized metric system. This system is officially named ***Le Systeme International d'Unites***, or **SI**.

2. The Department also adopts **ASTM E 380-92**, ***Standard Practice for Use of the International System of Units SI)***, as its standard for use of the modernized metric system, with two exceptions:

   * The words **meter** and **liter** use the *er* ending (rather than *re*).

   * The U.S. customary practice of using the comma to separate whole numbers into groups of three may continue. Decimals need not be separated into groups of three.

RULES FOR WRITING METRIC UNIT NAMES AND SYMBOLS

1. Symbols

The short form of a metric unit name is called a symbol, not an abbreviation.

* Do **NOT** use *italics* for unit symbols. Print unit symbols in upright type regardless of the type style used in the surrounding text. *Italics* might be misread.

  Examples: g is the symbol for gram *g* indicates the acceleration of gravity

* Do **NOT** use a period after a unit symbol except when it occurs at the end of a sentence.

  Right: 12 g   **Wrong:** 12 g.

2. When to Use Symbols and Names

* Use unit symbols in technical writing, on business forms and in tables. However, use the unit name in an indefinite measurement.

  Examples: 8 km   several meters

* Use unit names in nontechnical writing. It is recommended that most **DOT** publications be judged nontechnical. Using unit names lessens the chance of misinterpretation of symbols.

3. Numbers

* Express numbers as figures (including those from 1 to 10) with either unit symbols or names.

  Right: 8 meters   **Wrong:** eight meters

  However, do not use a figure to begin a sentence -- re-arrange the sentence.

* Use a zero before the decimal point for values less than one.

  Right: 0.03 m   **Wrong:** .03 m

* Use decimals, not fractions.

  Right: 0.25 m   **Wrong:** 1/4 m

* Express whole numbers between 0 to 9999 without a comma or space.

  Right: 1234   **Wrong:** 1,234
  Right: 6789   **Wrong:** 6 789

* Express whole numbers larger than 9999 with the use of a comma after every 3 places.

  Right: 12,345   **Wrong:** 12 345
  Right: 56,789   **Wrong:** 56789

* Express decimal numbers without the use of commas or spacing

Right: 12,345.6789 **Wrong:** 12,345.6,789
Right: 56,789.0123 **Wrong:** 56 789.0 123
Right: 90,123.4567 **Wrong:** 90123.4567

4. Capitalization

* Do **NOT** capitalize prefix and unit symbols.

Exceptions:

* The symbol for liter is "**L**". Thus, the symbol for milliliter is "**mL**".

* Symbols for prefixes larger than are capitalized.

  Example: is "**M**" for mega, which is 10^6 or one million.

* The first letter of symbols for names derived from proper names are capitalized.

  Examples: are "**C**" (Celsius), "**Hz**" (hertz), "**N**" (newton), "**Pa**" (pascal), "**V**" (volt).

* Do **NOT** capitalize the first letter of a unit name, except at the beginning of a sentence or in capitalized material such as a title. Thus, even those metric names derived from proper names (e.g., hertz, newton) are printed in lower case.

  Exception: degrees Celsius.

5. Spacing

* Do **NOT** leave a space between a prefix and unit symbol or name.

  Right: km **Wrong:** km
  Right: kilometer **Wrong:** kilo meter

* **DO** leave a space between a number and a symbol.

  Right: 45 kg **Wrong:** 45kg
  Exception: 20°C

However, if a quantity is used as an adjective, use a hyphen instead of a space.

Examples: 35-mm film   3-meter pole

6. **Plurals**

* Do **NOT** add an *s* to a unit symbol to make it plural.

  Right: 54 mm   **Wrong**: 54 mms

* **DO** use the plural of unit names when required by the rules of English grammar.

  Example: 350 kilometers

7. **Area and Volume**

* The symbol for **square** is the **superscript** ^{2}

  Example: 10 square meters is 10 m^2

* The symbol for **cubic** is the **superscript** ^{3}

  Example: 5 cubic meters is 5 m^3

8. **Product and Quotient**

* With unit symbols, indicate the product of two or more units with a dot above the line. With unit names, use a space between the words.

  Examples: N•m   newton meter

* With unit symbols, indicate a quotient or rate with a forward slash (/). With unit names, use the word *per*.

  Examples: km/h   kilometers per hour

9. **Mixing Names and Symbols**

* Do **NOT** mix unit names and symbols.

  Right: km/h   **Wrong**: km/hour

METRIC CONVERSION FACTORS.

FORCE

In order to perform metric calculations properly it is extremely important that the distinction between mass **"kg"** and force **"N"** be understood. When working in the If **"inch-pound"** system, the quantities for **"weight"** and **"force"** are interchangeable. A block of concrete weighing 1000 pounds when placed on a beam produces a force of 1000 pounds. In the metric system there are separate units for mass **"kg"** and force **"N"**. The familiar law of physics applies. Force **"N"** mass = mass "g" x acceleration due to gravity. The metric acceleration is 9.807 m/s (32.2 ft/s^{2}x 0.3048 m/ft). Mass indicates the weight of an object. The mass must be converted to force (by multiplying by 9.81) before computing structural reactions, shears, moments, or internal stresses.

For example, a simply supported beam 10 meters long with a mass of 231 kg/m would weigh 2310 kg. However, the dead load of the beam used to calculate reactions, shears, moments, etc. would be 231 x 9.81 = 2266 kN/m. The distinction between mass and force in structural calculations is very important. Conversion factors to convert pounds directly to newtons are given in the section:

STRESS

The pascal is not universally accepted as the only unit of stress. Because steel section properties are expressed in millimeters, it is more convenient to express stress in a derivative of pascals - newtons per square millimeter (**1 N/mm = 1 MPa**).

ENERGY

Although the joule is a standard metric unit, it is typically not used in structural design. Moments are always expressed in terms of **N•m**, or the derivative **kN•m**.

OPERATIONS IN BASIC ARITHMETIC

Introductory Exercise

Errors which are made in simple arithmetic are usually due to lack of practice and care in performing the operation. The following problems will help you review your abilities and weak spots. Do these problems as rapidly as you can. Refer to answers only after you have completed all the problems.

ADD:

(1)	341	(2)	743	(3)	5087	(4)	7941
	767		808		4734		8086
	943		254		3462		7671
	758		666		7697		8031

(5)	2134	(6)	8061	(7)	3999
	6981		3		234
	2678		861		17
	1891		47622		94107

SUBTRACT:

(8)	4721	(9)	7171	(10)	5461	(11)	70886
	-1935		-5789		-3999		-54987
(12)	88254	(13)	18451	(14)	67291	(15)	65746
	-9874		-11627		-59873		-23857

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Operations in Basic Arithmetic Continued

MULTIPLY:

(16)	174 21	(17)	278 94	(18)	6700 487	(19)	2742 3617
(20)	9699 8654	(21)	21479 8677	(22)	57769 2984		

DIVIDE:

(23) $84\overline{)241248}$ (24) $48\overline{)768864}$

(25) $952\overline{)3852268}$ (26) $6873\overline{)41018064}$

(Refer to answers on Page 109)

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PRE-TEST EXERCISE

This exercise consists of problems in the various operations in mathematics which are contained in this textbook. The purpose of this exercise is to show you which sections of this book you need to study carefully and which sections you will be able to complete more rapidly.

After you have completed as many of these problems as you can, check your answers with those on Page 110. Notice that after each answer there is a reference to the section of the textbook which explains the operation. These references will alert you to the sections of the book you will need to study carefully.

(1) 31.34 + 647 + 0.297 + 4886.9 = ____________________

(2) 78.36 x 861.437 = ____________________

(3) 787.51 - 168.4971 = ____________________

(4) 0.08609153 ÷ 0.7639 = ____________________

(5) Add: 141° 47′ 51"
60° 31′ 22"
77° 40′ 38"

(6) 13,770,000 mm^3 = ____________________ m^3

(7) Subtract 86° 37′ 24" from 180° = ________________

(8) Multiply 2 hrs. 28 min. 40 sec. by 7 = ________________

(9) Divide 63.042 m by 7 = ____________________

(10) Write 0.375 as a percent ____________; as a fraction ________

(11) Write 1/8 as a decimal ___________; as a percent ___________

(12) 35% of 108 is ________________

(13) 36% of a number is 16.2. The number is ________________

(14) 1/4 + 2/3 + 5/8 - 7/18 = ________________

(15) Write 141/32 as a decimal number ________________

(16) 5/12 x 7/15 = ____________________

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Pre-Test Exercise Continued

(17) 17/48 - 51/64 = ________________

(18) 3 2/21 - 1 6/7 = ________________

(19) 204 kg of bituminous mix contains 51 kg of aggregate "A", 55 kg of aggregate "B", and 86 kg of aggregate "C". The rest is asphalt. What percentage of the total mass is asphalt? ________________

(20) Use Appendix C in the back of this book to solve:

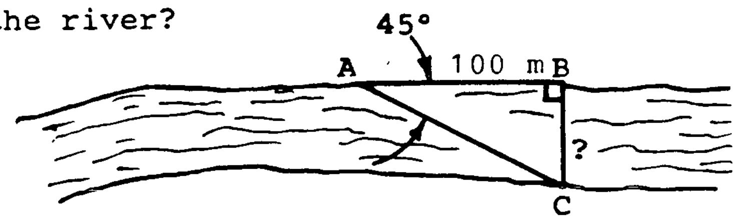
$\sqrt{336}$ $\sqrt{31.5}$ $\sqrt{80}$

(21) Find the area of a trapezoid with b = 14 m; b′ = 21 m; and h = 9 m ________________

(22) Find the circumference of a circle with radius = 8.5 m? ________________

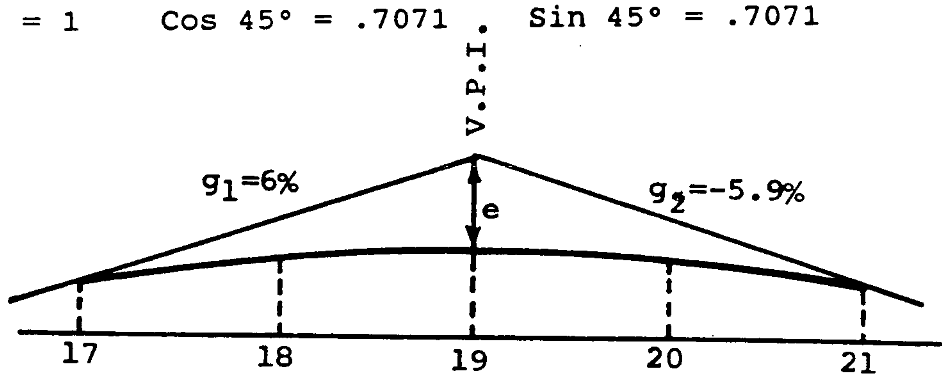
(23) Find the volume of a pipe whose inner diameter is 1800 mm and whose length is 36 m. ________________

(24) What is the width of the river?



Tan 45° = 1 Cos 45° = .7071 Sin 45° = .7071

(25)



L = 4 Stations

What is the vertical distance (e) from the VPI to the curve in meters? ________________

(Refer to answers on Page 110)

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SECTION I

Chapter 1 Numbers and Measurement

NUMBERS AND MEASUREMENT

In many instances, numbers are used together with a word which describes a unit of measurement. In highway inspection, there constantly appear such numbers. For instance in surveying, we speak of an angle of 32 **DEGREES** (symbol °), 20 **MINUTES** (symbol ′), 5 **SECONDS** (symbol ″). This would be written 32° 20′ 05″. Likewise, in computing distances, we add the terms **KILOMETERS, METERS, MILLIMETERS**; and in computing areas, **HECTARES, SQUARE KILOMETERS, SQUARE METERS, SQUARE MILLIMETERS**; in computing volumes, **CUBIC METERS, CUBIC MILLIMETERS**. Other measurement words are **LITERS, MILLILITERS, GRAMS, KILOGRAMS and MEGAGRAMS**.

> **NOTE:** Look now at the tables in Appendix A. In working the problems in this section, refer to these tables when you find it necessary.

Addition and Subtraction of Numbers Indicating Measurement

The most important thing to remember in these operations is to combine only the units which are alike. That is, you cannot add Newtons to meters, or subtract millimeters from liters.

1 minute (′) = 60 seconds
1 degree (°) = 60′

RULE: Adding numbers of measurement.

Step 1. Add all units of the same kind:

Step 2. Arrange the answer to correct measurement (Refer to example).

Example

Add	27°	20′	25″
	15°	31′	22″
	1°	18′	21″
	43	69′	68″

a. Beginning at the right column: 68″ = 1′ **08″**

b. Add the 1′ to its proper column: 69′ + 1′ = 70′

c. Remembering that 60′ = 1°, then 70′ = 1° **10′**

d. Add the 1° to its proper column: 43° + 1° = **44°**

e. Underlined numbers are the answer: **44° 10′ 08″**

EXERCISE NO. 1

Add the following and arrange the answer to correct measurement:

1.	2.	3.
16,004.600 m	28° 40′ 16"	4.400 liters
21,006.100 m	17° 09′ 21"	6.100 liters
15,007.100 m	114° 59′ 06"	2.200 liters

4.	5.
8,020.120 km^2	8,025.999 km^3
17,006.096 km^2	10,018.520 km^3

(Refer to answers on Page 111)

RULE: Subtracting numbers of measurement: **Example**

Subtract: 10,002.600 km
\- 602.900 km

Solution: Since 0.600 km is less than 0.900 km, borrow 1 km from the 2 km and write:

10,001.1600 km
\- 602. 900 km

Moving to the left, 1 km is less than 2 km
So: Borrow 10 km from the 10000 km and write:

	999	11.	1600	km
	- 60	2.	900	km
Now subtract:	939	9.	700	km
Answer:		**9399.700 km**		

EXERCISE NO. 2

Subtract the following:

1.	2.	3.
15° 21′ 00"	6001.900 km	179° 16′ 21"
-6° 25′ 10"	-3001.100 km	- 94° 20′ 49"

4.	5.
9007.190 km^2	51,420.290 km^3
-6008.860 km^2	-27,622.627 km^3

(Refer to answers on Page 111)

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Rule: Multiplying numbers of measurement:

Example: Multiply 2400.700 km by 5

Step 1. Multiply each unit by the number:

```
  2400.700 m
x        5
 12003.500 m
```

Step 2. Moving from the right, arrange the product to correct measurement:

a. 12,003.500 m = 12.0035 km

12003.500 m or 12.0035 km (Answer)

EXERCISE NO. 3

Multiply the following:

1. 25° 40′ 16" by 6 = ____________
2. 21,002.700 km by 22 = ____________
3. 60.750 L by 214 = ____________
4. 1787.940 km^2 by 74 = ____________
5. 86125.964 km^3 by 13 = ____________

(Refer to answers on Page 111)

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RULE: Dividing numbers of measurement: **Example**

Divide: 15° 19′ 46″ by 2

a. Divide the greatest unit first

```
   7°
2)15
  14
   1
```

b. Change any remainder to the next lower unit: 1° = 60′

c. Add this 60′ to the given 19′: 60′ + 19′ = 79′

d. Divide this unit:

```
  39'
2)79'
  6
  19
  18
   1'
```

e. Change any remainder to the next lower unit: 1′ = 60″

f. Add this 60″ to the given 46″: 60″ + 46″ = 106″

g. Divide this unit:

```
   53"
2)106"
  10
    6
    6
```

h. Combined answer: 7° 39′ 53″

<u>EXERCISE NO. 4</u>

Divide the following:

1. 45.300 L by 3 = ______________________

2. 142° 56′ 06″ by 3 = ______________________

Exercise No. 4 Continued

3. 24.110 m by 5 = ____________________

4. 285° 28′ 12″ by 6 = ____________________

5. 97.480 m by 4 = ____________________

(Refer to answers on Page 111)

WORK SPACE

SECTION I

Chapter 1 Quiz
(Use Appendix A)

1. Multiply this number of measurement:

   7016.800 mm by 3.5 = ____________________

2. Divide this number of measurement:

   168,049 kg by 7 = ______________________

3. Add the following numbers of measurement:

   99° 58′ 10"
   91° 14′ 33"
   75° 19′ 02"
   87° 43′ 28"
   = ________________

4. Calculate the number of kilograms of Type I cement necessary to mix with 834.5 kg of water (at 4° C). Assuming that 5 liters of water are needed per sack of cement. (Use page A-3, Appendix A). = ______________________

(Refer to answers on Page 111)

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SECTION I

CHAPTER 2 Decimals

The Decimal System and Decimal Fractions

Anyone who is able to make change for a dollar bill is familiar, in some degree, with the operation which goes by the name of "**decimals,**" or as part of the operation is sometimes called, **decimal** fractions.

The word "decimal" means "**by tens**", or "**by tenths**". For example, when the unit 2 is **multiplied by ten**, the result is two tens, or 20. when the two tens (20) is **multiplied by ten**, the result is two hundreds, or 200, and so on. In the same way when the unit 2 is **divided by ten**, the result is two tenths, or 0.2. when the two tenths (0.2) is **divided by ten**, the result is two hundredths, or 0.02. The methods involved in these operations will be explained in this chapter. The important thing to remember here is that the word "**decimal**" means some change involving "**tens**".

Basic highway mathematics involves no operation which is more common, more widespread than decimals. Decimals appear on every sheet of highway plans; in measurements, in stationing, in locatlons of items along or off the grade, and in the required lists.

The first thing to be learned is how to read and write decimals correctly. The student should learn well the names and places on both sides of the decimal point, as shown in the following table:

Prefix	Symbol		Magnitude	
micro	μ	0.000001	(millionth)	(10^{-6})
milli	m	0.001	(thousandth)	(10^{-3})
centi	c	0.01	(hundredth)	(10^{-2})
deci	d	0.1	(tenth)	(10^{-1})
deka	da	10	(ten)	(10^{1})
hecto	h	100	(hundred)	(10^{2})
kilo	k	1000	(thousand)	(10^{3})
mega	M	1,000,000	(million)	(10^{6})

Starting from the decimal point in the center of the table, it can be seen that every time a digit (number) is moved one place to the left, its value is increased ten times. (1 moved one place to the left becomes 10). In the same manner, starting again with the decimal point, every time a digit is moved one place to the **right**, its value is **decreased** to an amount equal to one tenth of its original value. (0.1 moved one place to the right becomes 0.01). To illustrate this last operation, 0.1 (one tenth) of a dollar is 1 cent; and 1 cent is just one tenth of 10 cents.

EXERCISE NO. 1

For practice, change the following strings of words to numbers:

1. Two million, six hundred seventy-one thousand, nine hundred eighty-six ________________

2. Nine million, two hundred one ________________

3. Eight hundred seven thousand ninety ________________

4. One hundred eighty-four thousand one hundred thirty millionths ________________

5. One thousand seven ten thousandths ________________

6. Twenty thousand six hundred thousandths ________________

(Refer to answers on Page 112)

Operations with Decimals

Turning back to the chart on the preceding page, notice the readings to the **right** of the decimal point. These represent what are called **decimal** fractions, because, like the fractions discussed in the preceding chapter, they represent **part** of a whole number.

In reading a decimal fraction, read the digits just as if it were a whole number, and then add the name of the digit which is farthest from the decimal point.

For example:	0.3	Three **tenths**
	0.14	Fourteen **hundredths**
	0.642	Six hundred forty-two **thousandths**
	0.00481	Four hundred eighty-one **hundred thousandths**

(Check the chart if you are not clear, and write the following fractions as decimals).

EXERCISE NO. 2

1. 181/1000 (In numbers) ____________ (In words) ____________

2. 61/1000 (In numbers) ____________ (In words) ____________

3. 17/100 (In numbers) ____________ (In words) ____________

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Exercise No. 2 Continued

4. 89576/100000 (In numbers) ____________ (In words) ___________

5. 989/1000 (In numbers) ____________ (In words) ___________

(Refer to answers on Page 112)

If there are digits both to the **left** and **right** of the decimal point, first read the digits to the **left** of the point as a whole number, then read the word **"and"**. Finally, read the digits to the **right** of the point as described in the preceding paragraph.

Example: 256.321 is read: Two hundred fifty-six **and** three hundred twenty-one thousandths

EXERCISE NO. 3

Convert the following improper fractions to decimals:

1. 161/100 ________________________

2. 8762/1000 ________________________

3. 14/10 ________________________

4. 17674/1000 ________________________

5. 876241/100000 ________________________

(Refer to answers on Page 112)

Addition of Decimals

The main thing to remember in adding decimals is that the numbers should be written so that all the decimal points are in line, one directly underneath the other. Then perform the addition as for whole numbers, and place the decimal point of the answer in line with all the other points.

Example: Add 26.94 0.027 7.2 0.0032

```
26.94
 0.027
 7.2
 0.0032
34.1702  (Answer)
```

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EXERCISE NO. 4

Add the following:

1. 263 + 0.21 + 6.491 + 1.0001 = ______________
2. 0.61 + 6.20 + 19.01 + 6.2 = ______________
3. 1 + 100.01 + 8.31 + 10.6842 = ______________
4. 61.1 + 3.642 + 7.20 + 18.02 = ______________
5. 8914.62 + 62101.024 + 391.01976 = ______________

(Refer to answers on Page 112)

Subtraction of Decimals

Follow the same directions as in addition by placing the decimal points in a vertical line, and subtract as you would whole numbers.

Example: Subtract 0.00871 from 0.632

$$\begin{array}{r} 0.63200 \\ \underline{0.00871} \\ \mathbf{0.62329} \end{array} \quad \textbf{(Answer)}$$

Note: In this example two zeros had to be added to 0.632 before subtraction.

The rule is that **there must be the same number of decimal places in all three sets of digits** when decimals are subtracted.

EXERCISE NO. 5

Subtract the following:

1. 0.61 from 1.7 = ______________
2. 0.004 from 11.62 = ______________
3. 6.2101 from 19.22 = ______________
4. 1461.291 from 24691.0014 = ______________
5. 14.921 from 862 = ______________

(Refer to answers on Page 112)

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Multiplication of Decimals

In multiplying decimals, forget about the decimal places until the answer is arrived at, and multiply as you would whole numbers. Then add up the number of decimal places in (a) the number to be multiplied, and (b) the multiplier. In the answer, starting at the right, mark off this total number of places:

Example: Multiply 8.341 by 2.46

```
   8.341     (3 decimal places)
    2.46     (2 decimal places)
   50046
  33364
 16682
20.51886     (Total: 5 decimal places)
```

EXERCISE NO. 6

Multiply the following:

1. 10.62 by 4.21 = ____________
2. 104.2 by 6.212 = ____________
3. 91.864 by 9.8 = ____________
4. 1024.1 by 1.0081 = ____________
5. 92.74 by 821.1 = ____________

(Refer to answers on Page 112)

Division of Decimals

The following rules may be applied in the division of decimals:

Rule 1: Change the decimal form of the number **which does the dividing** to a whole number. This is done by moving the decimal point to the right as many places as necessary.

Rule 2. Locate the decimal point in **the number to be divided.** Move this decimal point to the right the same number of places required in applying **Rule 1.**

Rule 3: If required, add zeros to the right of the last digit in the number to be divided. In some cases, this is necessary before the correct location of the decimal point can be made as described in **Rule 2**. These rules are applied in solving the following problems:

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Example: Divide 47.84257 by 2.83

Rule 1: 283.

Rule 2: In applying Rule 1, the decimal point was moved two places to the right.

Therefore: 4784.257

combining these two, we have

```
               16. 905
         ____________
    283.) 47  84. 257
          28   3
          19   54
          16   98
           2   56 2
           2   54 7
                1 557
                1 415
                  142  remainder
```

Notice the decimal point in the answer is located directly above the new location of the decimal point in the number which was divided.

Example 2: Divide 574 by 1.75

Rule 1: 175. (decimal point moved 2 places)

Rule 3: 574.000 (add zeros)

Rule 2: 57400.0 (decimal point moved 2 places)

Combine the above

```
          32 8.0
       _________
175. )5740 0.0
      525
       49 0
       35 0
       14 00
       14 00
           0
```

Example 3: Divide 0.048762 by 0.572

Rule 1: 0572.

Rule 2: 0048.762

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Therefore,

```
           0.085
0572.) 0048.762
         45 76
          3 002
          2 860
            142   (remainder)
```

EXERCISE NO. 7

Divide the following: (Carry out to four places)

1. 16.2147 by 2.16 = ________________
2. 174.2 by 8.18 = ________________
3. 1089.56492 by 24.6 = ________________
4. 0.64982 by 0.18 = ________________
5. 0.194 by 0.16542 = ________________

(Refer to answers on Page 112)

When To Use Decimals

Many times when making field measurements where precision is of importance, the use of decimals should be used. This is extremely important on structures work where only millimeters are involved.

For example, when converting a length of 750 mm to an equivalent length in meters, the decimal 0.750 could be used. The effect can best be seen by use of the following example problem.

A concrete pavement 7 m 750 mm wide by 900 m long is to be paid for at $100.00 per square meter.

Using the decimal of 0.750 m = 750 mm, the payment would be $100.00 x 900 x 7.750 = $697,500.00

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NOTE: Any fraction expressed in its lowest terms whose denominator, when factored, has an odd number other than 1, 5, or a multiple of 5 as one of the factors should never be converted to a decimal for exact calculations.

EXERCISE NO. 8

Can the following fractions be converted to exactly equal decimals?

		Yes	No
1.	1/2	______	______
2.	2/3	______	______
3.	10/32	______	______
4.	5/8	______	______
5.	4/17	______	______
6.	6/13	______	______
7.	97/100	______	______
8.	8/27	______	______
9.	48/49	______	______
10.	11/16	______	______

(Refer to answers on Page 112)

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SECTION I

Chapter 2 Quiz
(Use Appendices A and B-3)

1. If a borrow area had the dimensions 383 m by 79 m. What would be the area in square meters? ______________ m^2

2. A rectangular area to be seeded measures 60 m by 305 m. How many hectares is the area? ______________ ha

3. The width of a gravel surface roadway is 4.8 m. The slope from the crown (center of roadway) to the edge has a 0.02 m/m out. How much lower should the outside edge be than the crown? ______________ mm

4. Pit run gravel weighs 1350 kg/m^3 and the gravel compacted weighs 1750 kg/m^3. How many megagrams (Mg) of pit run gravel will be needed to lay a compacted base 2000 m long, 7.2 m wide, and 240 mm deep?

   Note: Use decimal form.

   ______________ Mg

(Refer to answers on Page 112)

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SECTION I

CHAPTER 3 Percentages and Averages

Percentage

Closely related to fractions and decimals is the arithmetic operation called percentage. It is used often in highway computations, in ordinary daily life, and even in figuring baseball batting averages.

In all problems in percentage, there are three items to be considered. These are:

1. The Whole
2. The Part
3. The Percent

In working a practical problem in percentage, when **any two** of the above items are given, the **third may be found.** To do this, the student should study the problem carefully and determine first of all, just what number is the whole, what the part, and what the percent. This will save time and prevent errors. **Remember that the whole, of which parts are taken, is always represented by 100%.**

The chapter which follows deals with the relationships of fractions, decimals, and percentages. All we will do in this chapter is to see the **three types** of percentage problems. They, and their formulas for solution, are as follows:

I.

Formula: The $\frac{\text{percent}}{100}$ x the whole = the part.

Example: A sample of bituminous mix contains 25.61% of aggregate A, and 0.17% of Aggregate B. How much of each aggregate would be found in one megagram (Mg) of the mix?

1 megagram = 10^6 grams **(the whole)**

$\frac{25.61}{100}$ x 1 Mg = 0.256 Mg Aggregate A **(the part)**

$\frac{0.17}{100}$ x 1 Mg = 0.0017 Mg Aggregate B **(the part)**

II.

Formula: $\frac{\text{The part}}{\text{The whole}} \times 100 = \text{the percent}$

Example: A shipment of 225 reinforcing bars was delivered to a construction site. The inspector accepted 189 of the bars. What percent was accepted? What percent was rejected?

225 = **the whole** 189 = **the part**

$\frac{189 \times 100}{225} = \mathbf{84\%}$ **(the percent accepted)**

100% - 84% = **16%** **(the percent rejected)**

III.

Formula: $\frac{\text{The part} \times 100}{\text{The percent}} = \text{the whole}$

Example: A shovel moved 95 Mg of earth in an hour. But this is only 85% of what it moved in the preceding hour. How many megagrams (Mg) were moved in the preceding hour?

95 Mg = **the part** 85% = **the percent**

to find: the whole

$\frac{95(100)}{85\%} = \mathbf{112\ Mg}$ **(the whole)**

EXERCISE NO. 1

Solve the following problems:

1. 30.9 Mg of mix contains 22% of Aggregate A. How much does the aggregate weigh? ________________

2. A project calls for total excavation of 247,600 cubic meters of earth. To date 49,600 cubic meters have been moved. What percentage remains to be moved? ________________

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Exercise NO. 1 Continued

3. Following are percentages of materials in a 0.680 Mg mix:

   (a) 0.2% (b) 0.5% (c) 18.5% (d) 8.5% How much does each weigh?

   (a) ________ (b) ________ (c) ________ (d) ________

4. 8700 cubic meters of earth has been moved. This represents 73% of total excavation. What is the total excavation to the nearest cubic meter? ______________

5. 1,475 reinforcing bars have been delivered to a project. Only 1,260 bars are required by the specifications. What is the percentage of the extra bars? ______________

(Refer to answers on Page 114)

Averages

An average is computed by (1) adding the series of numbers involved, and (2) dividing the answer by the number of items in the series.

Example 1: **Add:** **and Divide:**

```
     12            9.4
     27        5)97.0
     18          5
     21          47
     19          45
     97           2 0
                  2 0
                    0
```

One - Add the series of numbers.
Two - Divide the answer by the number of items added -- in this case, 5.
Three - The average of the series is the answer obtained by dividing.

Averages are used to make it easier to understand a series of measurement and to calculate values from a series of measurements. It is hard to think of 12 cubic meters, 27 cubic meters, 18 cubic meters, 21 cubic meters and 19 cubic meters, or to work such a series of numbers. If you add them and then divide by 5--the total of numbers in the series--you get the **average**. The average is one number that can be used to represent a whole series of measurements.

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Example 2: An asphalt paving job was divided up into seven sections of work. The first section required 431 hours to complete; the second required 500 hours; the third took 414 hours; the fourth took 439 hours; the fifth took 2011 hours; the sixth took 1991 hours, and the seventh required 2180 hours to complete.

What was the average time required to complete a section of paving?

One: Add the numbers

```
 431
 500
 414
 439
2011
1991
2180
----
7966 hours
```

Two: Divide by the number of measurements

```
  1138
7)7966
  7
   9
   7
   26
   21
    56
    56
     0
```

Three: The average was 1138 hours per section.

EXERCISE NO. 2

Solve the following problems:

1. What is the average of 16, 27, 19, 23, and 15? ____________

2. If five different survey crews used these numbers of hours per unit of work -- 8.00, 8.25, 7.75, 8.50, and 8.75 -- what was the average number of hours used per unit of work?

____

(Refer to answers on Page 114)

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Conversion of Percentage - Decimals - Fractions

Many times in highway work it will be necessary for you to change fractions to percents, percents to decimals, and vice versa. This could occur in calculations of slopes and grades; reports of construction completed or unfinished; and the establishing of amounts of various materials which make up a "**mix**"; as examples. At this point we will study how decimals° fractions, and percents may be converted one to the other, and so used interchangeably:

Rule 1: To convert a decimal to a percent.

Example: Show 0.25 as a percent.

Step 1: Multiply the decimal number by 100:

0.25 x 100 = 25.0

Step 2: Write the % sign instead of the decimal point: **25%**

Thus a quarter of a dollar can be shown as $0.25; or 25¢: or 25% of a dollar.

Example: (Follow the above two steps)

Write 0.4 as a percent:

Solution: 0.4 x 100 = **40%**

Rule 2: To convert a fraction to a percent:

Example: Convert 1/5 to a percent.

Step 1: Convert the fraction to a decimal by dividing the top number by the bottom one:

$$1 \div 5 = 1.00 \div 5 = 0.20$$

Step 2: Convert the decimal to a percent (according to Rule 1, above):

Solution:

$$1 \div 5 = 1.00 \div 5 = 0.20$$
$$= \mathbf{20\%}$$

Example: Write 5/8 as a percent (Following the above two steps)

Solution: 5 ÷ 8 = 5.00 ÷ 8 = 0.625 = **62.5%**

Example: (An improper fraction) Write 11/10 as a percent:

Solution: 11 ÷ 10 = 11.00 ÷ 10 = 1.10 = **110%**

Rule 3: To convert a percent to a decimal:

Example: Convert 35% to a decimal.

Step 1: Drop the % sign.

Step 2: Divide the number by 100:

35 ÷ 100 = 0.35

Note: Go back and read **Rule 1.** Observe that the operation of **Rule 3** is just the reverse of **Rule 1.**

Rule 4: To convert a percent to a fraction:

Example: Convert 60% to a fraction.

Step 1: Convert a decimal (Rule 3) 60% = 0.60 or 0.6

Step 2: Write six tenths as a fraction 6/10

Step 3: Reduce (if possible) the fraction 6/10 = 3/5

Note: In step 2, always "read" the decimal 0.60 as "sixty hundredths", or 60/100.

0.6 is read "six tenths", or 6/10

0.625 is read "six hundred twenty-five thousandths", or 625/1000.

EXERCISE NO. 3

Using the above rules, work the following problems:
Convert the following to percents:

1. 0.6 = ______________ 2. 0.49 = ______________

3. 0.491 = ______________ 4. 3.00 = ______________

5. 6 = ______________ 6. 2.096 = ______________

Exercise No. 3 Continued

7. 0.0019 = ________________ 8. 1.69 = ________________

9. 0.009 = ________________ 10. 19 = ________________

EXERCISE NO. 4

Convert the following to percents:

1. 3/4 = ________________ 6. 51/250 = ________________

2. 7/8 = ________________ 7. 19/1000 = ________________

3. 8/50 = ________________ 8. 8/9 = ________________

4. 4/25 = ________________ 9. 11/12 = ________________

5. 7/40 = ________________ 10. 1/100 = ________________

EXERCISE NO. 5

Convert the following to decimals:

1. 39% = ________________ 6. 4% = ________________

2. 17.4% = ________________ 7. 40% = ________________

3. 135% = ________________ 8. 400% = ________________

4. 1.08% = ________________ 9. 0.04% = ________________

5. 55% = ________________ 10. 0.004% = ________________

EXERCISE NO. 6

Convert the following to fractions:

1. 50% = ________________ 6. 83.3% = ________________

2. 5% = ________________ 7. 125% = ________________

3. 0.05% = ________________ 8. 3.6% = ________________

4. 0.005% = ________________ 9. 65% = ________________

5. 19% = ________________ 10. 0.004% = ________________

(Refer to answers on Page 114)

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SECTION I

Chapter 3 Quiz

1. (a) 80% of 800 = ________________

   (b) 180% of 800 = ________________

   (c) 0.80 is what percent of 3? ________________

   (d) 4/5 is what percent of 1? ________________

2. If the costs of bituminous mix materials increases from \$120 to \$135 per 1000 cubic meters, what is the percent increase in cost (over original cost)? ________________

3. A sample of dried aggregate weighs 1350.25 grams. 20% does not pass the 4.75 mm sieve. 46% of what passes the 4.75 mm sieve does not pass the 600 μm sieve? What is the mass of the aggregate that passes the 600 μm sieve? **(hint: what percent is retained by the 600 μm sieve).**

   ____

4. If the percent moisture, based on dry mass is 5.6%--how much would 467.20 grams of dry aggregate weigh wet?

   ____

5. What is the cost per cubic meter of aggregate placed for each contractor? (To the nearest cent)

Cubic Meters of Aggregate Placed:		**Total In-Place Costs**	**Individual Averages**
Contractor A	67.7	\$182	________
Contractor B	40.0	\$ 83	________
Contractor C	74.0	\$188	________

What is the weighted* average cost for the 3 contractors?

____

What percent is the highest cost per cubic meter greater than the weighted average cost?

____

***Note: Weighted average is** $\dfrac{\textbf{Sum of Numerators}}{\textbf{Sum of Denominators}}$

(Refer to answers on Page 115)

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SECTION I

CHAPTER 4 Fractions

FRACTIONS

Quantities may be expressed by either whole numbers or fractions. For example, 1, 2, 3, etc., are whole numbers; 1/2, 1/3, 1/4, etc., are fractions, or parts of whole numbers. In this chapter, we will discuss the fraction and its uses.

A fraction is a quantity that is a part of a whole unit. If you were to slice an apple into four equal parts, each slice would represent a fraction of the apple. To show the value of each part, you may use the fraction 1/4 (read as one fourth). The number below the line would indicate that the apple was divided into four equal parts and the number above the line would indicate that one part of the same apple is being used.

REMEMBER:

In any fraction, the number below the line is called the **DENOMINATOR** and represents the number of equal parts into which an object has been divided. The number above the line is called the **NUMERATOR** and represents the number of these equal parts which is being used.

NUMERATOR	1
DENOMINATOR	4

The two most common types of fractions are **PROPER** and **IMPROPER**. If the numerator is less than the denominator, as 4/5, 3/10, 7/8, it is a **PROPER** fraction. If the numerator is equal to, or greater than the denominator, as 4/4, 6/4, 7/2, it is an **IMPROPER** fraction.

EXERCISE NO. 1

Write each of the following as a fraction. Indicate by a check mark whether it is a proper or improper fraction.

	Fraction	Proper	Improper
I. Three fifths	______	______	______
2. Eight thirds	______	______	______
3. Thirteen sixths	______	______	______
4. Fourteen thirty-ninths	______	______	______
5. Nineteen sixty-fifths	______	______	______

(Refer to answers on Page 116)

If a whole number and a fraction are used together, as 1 3/4, 2 5/7, 3/5, the combination is called a **MIXED NUMBER**.

To change an improper fraction to a whole number, divide the number above the line (numerator) by the number below the line (denominator). For example, the improper fraction 8/2 can be converted to the whole number 4 simply by dividing 8 by 2. An improper fraction whose numerator cannot be divided evenly by the denominator can be changed to a mixed number by using the above method. To illustrate this, the improper fraction 9/2 can be converted to the mixed number 4 1/2 by dividing 9 by 2.

NOTE:

	$\underline{4}$	**(whole number)**
(DENOMINATOR)	2)9	**(NUMERATOR)**
	$\underline{8}$	
	1	**(remainder)**

First write the whole number (4) and then write the remainder over the denominator (1/2). When these two are combined, we have the mixed number 4 1/2.

To change a mixed number (4 1/2) back to an improper fraction (9/2) multiply the whole number (4) by the denominator of the fraction (2) and add the numerator of the fraction (1) to the product, keeping the denominator of the mixed number (2) as the denominator of the improper fraction.

$$\frac{4 \times 2 + 1}{2} = \frac{9}{2}$$

EXERCISE NO. 2

Write the following as mixed numbers and as improper fractions:

	Mixed number	Improper fraction
1. Two and three eighths	________	________
2. Seven and six sevenths	________	________
3. Nine and one half	________	________
4. Four and three thirty-seconds	________	________
5. Fourteen and one fifth	________	________

(Refer to answers on Page 116)

The form of a fraction can be changed without changing its value. This operation is known as the **REDUCTION OF FRACTIONS** either to higher terms or to lower terms. A fraction may be changed to higher terms by multiplying **both** the numerator and denominator by the same number. Let us assume that you were asked to change the fraction 3/8 to a fraction having a denominator of 16. By inspection, we know that 2 x 8 = 16, therefore, multiply the numerator (3) and the denominator (8) by the number c. This would be shown as follows:

$$\frac{3 \times 2}{8 \times 2} = \frac{6}{16} \quad \textbf{(Answer)}$$

Fractions can be added, subtracted, multiplied and divided. The first step before starting any of these operations is to have a thorough knowledge of the following:

1. Fractions expressed in their lowest terms

2. Factoring

3. Least Common Denominator (abbreviated as L.C.D.)

4. Cancellation

Fractions expressed in their lowest terms

Whenever possible, fractions should be written in their **lowest** terms. To reduce a fraction to its **lowest** terms, both the numerator and denominator must be divided evenly by the largest possible number.

Example: Reduce 6/16 to its lowest terms.

As you can see, 2 is the largest number which will divide evenly into the numerator 6 and the denominator 16.

$$\frac{6 \div 2}{16 \div 2} = \frac{3}{8} \quad \textbf{(Answer)}$$

Looking at the fraction 3/8, it is seen that the only number which will evenly divide the numerator 3 and the denominator 8 is 1. **If both the numerator and the denominator cannot be divided evenly by any number other than 1, then the fraction is written in its lowest terms.**

Sometimes it is difficult to determine the largest number which will divide evenly into both terms of a fraction. To save time, you may continue to divide the terms of the fraction by smaller numbers until the fraction has been reduced to its lowest terms.

Example: Reduce 32/64 to its lowest terms.

Both terms of the fraction are even numbers and can be divided even by 4.

$$\frac{32 \div 4}{64 \div 4} = \frac{8}{16}$$

Again, both terms of the fraction 8/16 are even numbers and can be divided by 8.

$$\frac{8 \div 8}{16 \div 8} = \frac{1}{2}$$ **(Answer) Lowest terms of the fraction 32/64**

EXERCISE NO. 3

Change the following to higher terms by multiplying the terms of each fraction by three:

1. One half __________
2. Two thirds __________
3. Seven eights __________
4. Nine Sixteenths __________
5. Five eights __________

(Refer to answers on Page 116)

A fraction may be changed to lower terms by dividing both the numerator and the denominator by the same number. For example, the fraction 32/64 may be written as a fraction whose denominator is 16. In this case, the denominator 64 can be divided by 4 to produce the required denominator 16. Therefore, dividing both the numerator (32) and the denominator (64) by 4, we will obtain the fraction 8/16.

$$\frac{32 \div 4}{64 \div 4} = \frac{8}{16}$$ **(Answer)**

EXERCISE NO. 4

Change the following to lower terms by dividing the terms of each fraction by two:

1. Four eighths __________
2. Six thirty-seconds __________
3. Ten sixty-fourths __________

4. Fourteen sixteenths ________

5. Eight thirty-seconds ________

(Refer to answers on Page 116)

EXERCISE NO. 5

Write the following fractions and their lowest form:

	Fraction	Lowest Form
1. Two fourths	________	________
2. Six eighths	________	________
3. Eight sixteenths	________	________
4. Fifty hundredths	________	________
5. Twenty-one thirty-fifths	________	________

(Refer to answers on Page 116)

Factoring

Most numbers are the product of a group of smaller numbers. A value may be written as a single number or a group of smaller numbers which, when multiplied together, are equal to the single number. To illustrate this, multiply 2 x 2 x 3. The answer to this problem is 12. This value may be written as the single number 12 or as the group of smaller numbers 2 x 2 x 3. In this case, the group of numbers are commonly called the factors of the single number.

Factoring is the process of finding the group of smallest numbers (**PRIME FACTORS**) which, when multiplied together, are equal to a given number. For examPle, 2 and 3 are the prime factors of 6 because they are the smallest numbers which, when multiplied together equal 6; 2 and 2 and 2 are prime factors of 8; 2 and 2 and 3 are the prime factors of 12, etc.

NOTE: If a whole number cannot be evenly divided by any other whole number other than 1 and itself, the number is a prime number.

(Examples, 7, 11, 13)

The following rules will be found helpful in reducing fractions to their lowest terms and in factoring:

1. Any even number can be divided evenly by 2.

2. If the sum of the digits in any number can be evenly divided by 3, then the number can be divided evenly by 3.

3. Four will divide evenly into any number whose last two digits on the right can be evenly divided by 4.

4. Any number ending in 0 or 5 can be divided evenly by 5.

5. If the sum of the digits in any **even** number can be divided evenly by 3, then the number can be divided evenly by 6.

   Example: The number 2616 can be evenly divided by 6 because the digits 2 + 6 + 1 + 6 are equal to 15 which can be divided evenly by 3.

6. Eight will divide evenly into any number whose last three digits on the right can be evenly divided by 8.

7. If the sum of the digits in any number can be evenly divided by 9, then the number can be divided evenly by 9.

   Example: The number 32, 193 can be evenly divided by 9 because the digits 3 + 2 + 1 + 9 + 3 are equal to 18 which can be evenly divided by 9.

EXERCISE NO. 6

Factor the following numbers:

1. Eight __________
2. Nine __________
3. Twenty-one __________
4. Thirty-two __________
5. Sixty-four __________

(Refer to answers on Page 116)

Least Common Denominator

The smallest number that can be divided evenly by each of the denominators of two or more fractions is the Least Common Denominator (L.C.D.) for those fractions. The L.C.D. of 1/3 and 1/4 is 12, since 12 is the smallest number that can be divided evenly by both 3 and 4. Similarly the L.C.D. for 1/4, 2/3, and 1/5 is 60, since 60 is the smallest number divisible by the denominators 4, 3, and 5.

There are occasions when the L.C.D. cannot be determined by inspection, therefore, the following method may be used:

Find the L.C.D. of 3/4, 1/6, 5/12, 7/18

First, factor each of the denominators of the given fractions.

DENOMINATORS		PRIME FACTORS
4	=	2 x 2
6	=	2 x 3
12	=	2 x 2 x 3
18	=	2 x 3 x 3

The greatest number of times that 2 appears in the above number as a factor is twice (for the denominators 4 & 12). The greatest number of times that 3 appears as a factor is twice in the denominator 18. To obtain the L.C.D., each different factor is taken the greatest number of times it appears in any one denominator and the product of these factors is determined. In this case, the different factors are 2 and 3. Therefore, the L.C.D. in this problem is equal to (2) x (2) x (3) x (3) which is equal to 36.

EXERCISE NO. 7

Find the least common denominator for each of the following:

1. 1/2 + 2/3 + 3/4 = ____________

2. 1/6 + 2/3 + 1/8 = ____________

3. 1/15 + 1/3 + 1/60 = ____________

4. 1/5 + 1/7 + 1/2 = ____________

5. 1/6 + 3/8 + 1/2 = ____________

(Refer to answers on Page 116)

Cancellation

Cancellation is the process of eliminating a common factor in both the numerator and denominator of a fraction. It is most useful in multiplying and dividing fractions, and in reducing a fraction to its lowest terms. For instance, in the fraction $\frac{52 \times 18}{12}$, the number 6 can be evenly divided into 18 and 12.

Therefore, 6 is a common factor of these two numbers. On the other hand, 2 is a factor of 52 because 2 can be divided evenly into 52. So we have the following:

$$\frac{52 \times 18}{12} = \frac{52 \times 3}{2} = \frac{26 \times 3}{1} = \frac{78}{} = \mathbf{78} \quad \textbf{(Answer)}$$

(For additional examples of cancellation, refer to the section of this chapter covering the multiplication and division of fractions.)

WORK PAGE

Addition of Fractions

In the addition of whole numbers, you may recall that things cannot be added unless they are alike. It would be impossible to add 10 dollars to 6 milliliters of butter or 3 pieces of pie to 5 liters of oil because their sums would be meaningless. This general rule applies to the addition of fractions. Fractions cannot be added unless they have the same denominator (number below the line).

To add fractions having the same (therefore common) denominators, simply add the numerators, then write the sum of the numerators, over the common denominator.

Example: $\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$ **(Sum of the numerators 2 & 4)** **(Common denominator)**

The following steps must be followed before adding fractions with different denominators:

1. Find the least common denominator of the fractions to be added.

2. Change both terms of each fraction to a fraction having the least common denominator (L.C.D.) as its denominator.

3. Add the numerators together, place their sum over the L.C.D., and reduce the fraction to its lowest terms.

We shall now trace through the above steps with the following example:

ADD: $\frac{2}{3} + \frac{3}{4} + \frac{5}{6} + \frac{7}{8}$

1. Find the L.C.D. of 3, 4, 6, and 8. We do this by finding the prime factors of each denominator by the following procedure:

$$3 = 3 \times 1$$
$$4 = 2 \times 2$$
$$6 = 2 \times 3$$
$$8 = 2 \times 2 \times 2$$

L.C.D. (2) (2) (2) (3) = 2 x 2 x 2 x 3 = 24

2. We now change the fractions 2/3, 3/4, 5/6, and 7/8 to fractions having the L.C.D. as their denominator. To do this, we divide the L.C.D. (24) by each of the given denominators.

$24 \div 3 = 8 \quad 24 \div 4 = 6 \quad 24 \div 6 = 4 \quad 24 \div 8 = 3$

Each of the previous answers (8, 6, 4, and 3) will be used to multiply both the numerator and denominator which, in this case, produce a fraction whose denominator is 24.

$$24 \div 3 = 8; \text{ therefore } \frac{2}{3} \times \frac{8}{8} = \frac{16}{24}$$

$$24 \div 4 = 6; \text{ therefore } \frac{3}{4} \times \frac{6}{6} = \frac{18}{24}$$

$$24 \div 6 = 4; \text{ therefore } \frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$$

$$24 \div 8 = 3; \text{ therefore } \frac{7}{8} \times \frac{3}{3} = \frac{21}{24}$$

3. We now add the numerators of the fractions:

$$16 + 18 + 20 + 21 = 75$$

We then place the sum of the numerators over the Least Common Denominator and reduce to the lowest terms:

$$\frac{75}{24} \div \frac{3}{3} = \frac{25}{8}$$

As you can readily see, the answer 25/8 is an improper fraction which should be changed to a mixed number. As a final step we have the following:

$$\frac{25}{8} = 25 \div 8 = \mathbf{3\ 1/8} \quad \textbf{(Answer)}$$

To add mixed numbers, we must first convert the mixed numbers to improper fractions. We can then proceed in the manner prescribed for the addition of proper fractions. If we were to add the mixed numbers 7 1/3, 2 2/5, 3 1/2, we would first convert them to 22/3, 12/5, 7/2.

We are now ready to find the L.C.D. of the fractions whose denominators are 3, 5, 2. Thus:

$$\begin{aligned} 3 &= 3 \times 1 \\ 5 &= 5 \times 1 \\ 2 &= 2 \times 1 \end{aligned}$$

L.C.D. (3) x (5) x (2) = 30

To convert the fractions 22/3, 12/5, and 7/2 to the L.C.D. of these fractions, we proceed as follows:

$30 \div 3 = 10$ $\qquad$ $30 \div 5 = 6$ $\qquad$ $30 \div 2 = 15$

Each of these answers (10, 6, and 15) will be used to multiply both the numerator and denominator to produce a denominator of 30. Thus we have:

$$\frac{22}{3} \times \frac{10}{10} = \frac{220}{30}$$

$$\frac{12}{5} \times \frac{6}{6} = \frac{72}{30}$$

$$\frac{7}{2} \times \frac{15}{15} = \frac{105}{30}$$

We now add the numerators: 220 + 72 + 105 = 397

Placing the sum of the numerators over the least common denominator we have:

$$\frac{397}{30} = \mathbf{13\ 7/30} \quad \textbf{(Answer)}$$

EXERCISE NO. 8

Add the following problems and reduce the answers to the lowest terms:

1. 7/6 + 4/6 + 5/6 + 1/6 = ____________________
2. 2/3 + 3/4 + 5/6 + 7/8 = ____________________
3. 3/8 + 1/4 + 11/32 + 7/16 = ________________
4. 1/3 + 3/5 + 4/15 + 9/30 = _________________
5. 4/5 + 5/6 + 9/10 + 6/14 = _________________
6. 3 7/8 + 6 7/8 + 4 3/8 + 7 5/8 = ____________
7. 2 1/3 + 6 2/3 + 4 1/3 + 5 = ________________
8. 4 1/3 + 3/4 + 3 5/6 + 7/8 = ________________
9. 1 3/4 + 7/8 + 15/32 + 8 = _________________
10. 2/3 + 1/2 + 5/6 + 2/9 + 4/7 = _____________

(Refer to answers on Page 116)

WORK PAGE

Subtraction of Fractions

To subtract fractions having the same denominators, subtract the numerators, write this remainder over the common denominator, and reduce to lowest terms if necessary.

Example: $\frac{5}{6} - \frac{1}{6}$

$\frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \mathbf{\frac{2}{3}}$ **(Answer)**

If the fractions to be subtracted do not have the same denominators, the steps followed are the same as in addition except that the new numerators are subtracted instead of added.

Example: $\frac{5}{6} - \frac{1}{4}$

L.C.D. for this problem is 12

$\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$

Minus: $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$

7/12 (Answer)

In the subtraction of mixed numbers, first change them to improper fractions, find their least common denominator, then subtract the numerators, write this remainder over the L.C.D., and reduce to lowest terms if necessary.

Example: 4 5/8 - 1 3/4 (L.C.D. for this problem is 8)

4 5/8 = 37/8

Minus: 1 3/4 - $\frac{7 \times 2}{4 \times 2}$ or 14/8

Therefore:

37/8 - 14/8 = 23/8
= **2 7/8 (Answer)**

WORK PAGE

EXERCISE NO. 9

Subtract the following problems and reduce the answer to lowest terms:

1. 7/8 - 3/8 = ____________________
2. 6/13 - 2/13 = ____________________
3. 5/6 - 3/8 = ____________________
4. 15/16 - 3/8 = ____________________
5. 6 1/4 - 3 7/16 = ________________
6. 15 3/16 - 4 3/4 = _______________
7. 12 1/2 - 4 7/8 = ________________
8. 28 16/63 - 3 9/14 = _____________
9. 14/39 - 19/65 = _________________
10. 75 - 4 3/7 = ____________________

(Refer to answers on Page 116)

Multiplication of Fractions

To multiply one fraction by one or more fractions, **first cancel where possible, then write the product of the numerators over the product of the denominator's, and reduce to lowest terms if necessary.**

Example: Multiply $\frac{2}{3} \times \frac{5}{6} \times \frac{9}{15} \times \frac{7}{8} = \frac{7}{24}$

$\frac{7}{24}$ **(Answer)**

If mixed numbers are to be multiplied, change the mixed numbers to improper fractions, cancel where possible, then write the product of the numerators over the product of the denominators, and reduce to lowest terms if necessary.

Example: Multiply 8 3/5 x 6 1/4 =

$53\frac{3}{4}$ **(Answer)**

WORK PAGE

If a whole number and a fraction are to be multiplied, cancel where possible, multiply the whole number by the numerator, and divide by the denominator.

Example: Multiply 250 x 3/4

250/1 x 3/4 = 375/2 = **187 1/2 (Answer)**

EXERCISE NO. 10

Multiply the following problems and reduce the answers to lowest terms:

1. 2/3 x 5/7 x 4/5 x 3/4 = ____________________
2. 2/3 x 5/6 x 9/15 x 7/8 = ____________________
3. 7/8 x 15/28 x 4/11 x 44/75 = ____________________
4. 9/10 x 7/8 x 5/7 x 4/5 = ____________________
5. 4/5 x 10/25 x 4/3 x 12/8 = ____________________
6. 3/7 x 15/16 x 4/5 x 21/24 = ____________________
7. 9 7/8 x 2 4/5 x 21/24 = ____________________
8. 11/2 x 12 x 3/4 x 5/8 = ____________________
9. 2/3 x 11/12 x 9/64 x 8 = ____________________
10. 4 2/4 x 7 5/8 x 2 20/65 x 8/61 = ____________________

(Refer to answers on Page 116)

WORK PAGE

Division of Fractions

To divide fractions, first invert the second fraction (interchange its numerator and denominator), and multiply the first fraction by this inverted form, then reduce to lowest terms, if necessary.

Example: Divide 5/8 ÷ 3/7

5/8 ÷ 3/7 = 5/8 x 7/3 = 35/24 = **1 11/24** **(Answer)**

Note: The inverted form of 3/7 is 7/3

If the mixed numbers are to be divided, change these to improper fractions, invert the second expression, cancel where possible, and proceed as in multiplication.

Example: Divide 3 5/7 ÷ 8 2/5

3 5/7 ÷ 8 2/5 = 26/7 ÷ 42/5 = 26/7 x 5/42 = **65/147** **(Answer)**

If a fraction is to be divided by a whole number, invert the whole number, cancel where possible, and proceed as in multiplication.

Example: Divide 8/3 ÷ 2

3/8 ÷ 2 = 3/8 ÷ 2/1 = 3/8 x 1/2 = **3/16** **(Answer)**

Note: The whole number 2 is first written as an improper fraction $\left(\frac{2}{1}\right)$ and then inverted to the proper fraction of $\frac{1}{2}$.

EXERCISE NO. 11

Divide the following and reduce the answer to lowest terms:

1. 15/24 ÷ 5/6 = ______________________
2. 12/55 ÷ 9/77 = ______________________
3. 26/27 ÷ 8/9 = ______________________

Exercise No. 11 Continued

4. 15/17 ÷ 3 = ______________________________

5. 4 ÷ 1/3 = ______________________________

6. 10 ÷ 2/7 = ______________________________

7. 1 7/8 ÷ 1 1/8 = ______________________________

8. 1 14/91 ÷ 35/52 = ______________________________

9. 12/35 ÷ 4 = ______________________________

10. $\dfrac{9/16 \times 2/3}{11/12 \times 1/2}$ = ______________________________

(Refer to answers on Page 117)

WORK SPACE

SECTION I

Chapter 4 Quiz

1. A plan calls for 1,400 sacks of cement to be used. If yesterday the contractor used 100 sacks and today he used 600 sacks, what fraction of the plan should be complete?

   ____

2. 1000 mm^2 is what fraction of a hectare? (Use Appendix A, page A-2)

   ____

3. Alcoa Aluminum Company just discovered a cheaper material to use for drain pipes. The pipes are made of plastic and though they weigh about 1.5 as much as the old pipe, they cost only 0.80 as much. If the old pipes weigh 45.35 kg each, cost $1.25 per meter, and are about 6.1 meters long, how much does the new pipe weigh and cost?

   ____

4. From a stockpile containing 500 m^3 of material, a volume is being backfilled which will require 340.91 m^3. The material being used has a 10% shrink. What fraction of the stockpile will have to be used to backfill the volume?

   ____

5. During a bridge deck pour which required 633 m^3 of concrete, the inspector determined that 422 m^3 had been placed. What fraction of the total concrete to be placed in the structure remained to be placed at this time?

   ____

(Refer to answers on Page 117)

WORK PAGE

SECTION I

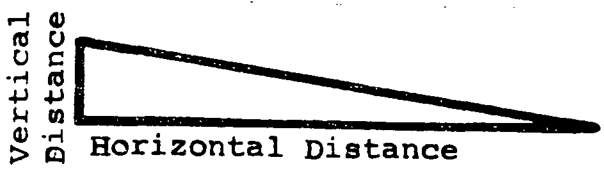
CHAPTER 5 Ratios and Proportions

Ratios and Proportions

Ratios represent relationships between values. A ratio of 1 engineer to 4 technicians implies that one engineer is hired to four technicians. The ratio is thus 1:4 -- read one to four. A ratio of ten to one (written 10:1 (H:V)) can be used to mean a ten meter horizontal distance to a one meter vertical distance.

Ratios are used in a variety of problems. Solving for slopes vertical distances, horizontal distances, or proportions of mixes are just a few ways.

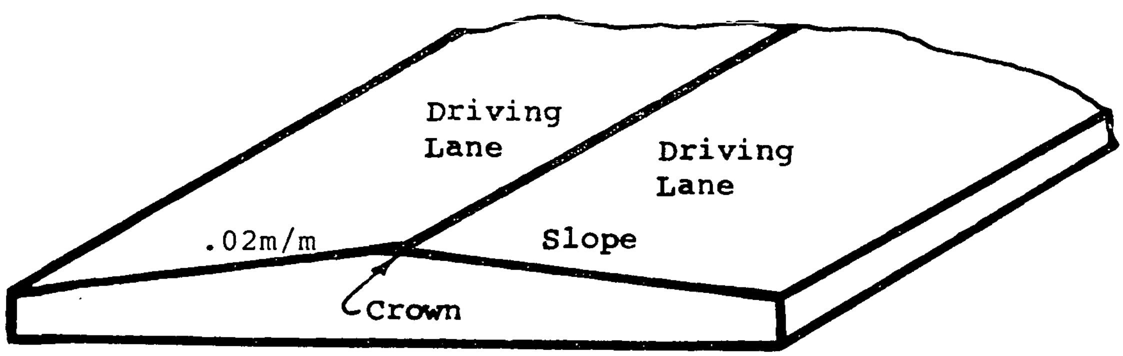
Slope ratios are expressed as 3:1 (H:V), 4:1 (H:V), 5:1 (H:V), meaning three meter to one meter, four meter to one meter, and five meter to one meter. The first number represents the horizontal distance and the second represents the vertical distance. The symbol (H:V) is used to clarify the designer's intent.



Vertical means straight up and down. Other terms used are rise, fall and drop meaning distance up or down from the crown of the road, top of the slope or highest point on a ditch bottom.

Horizontal means straight out. It is the distance away from the crown, the top of the slope or the highest point on a ditch bottom.

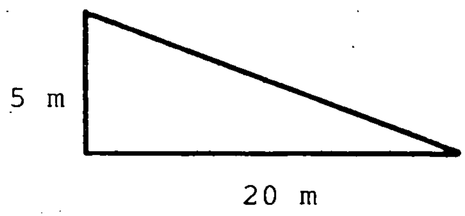
In the diagram below, the slope is shown as .02 m/m. This is spoken of as 0.02 meter drop per meter. It means that every meter away from the crown, the roadway will drop 0.02 meter.



Note: the slope ratio has nothing to do with the profile grade of the highway.

Ratios can be expressed in several ways. 3:1 (H:V) means 3 in the horizontal to 1 in the vertical. If both numbers represent meters, as they usually do in slope ratios, the ratio means three meters in the horizontal to one meter in the vertical. It can also be written as 3 to 1, 3/1.

Slope ratios can be found if both the horizontal and vertical distances are known. Slope ratios are found by dividing both distances by the vertical distance. If the horizontal distance is 20 meters and the vertical distance is 5 meters,



<u>**first**</u>, write the horizontal distance in front of the two dots that stand for "**to**" like this:

20 m:

<u>**second**</u>, write the vertical distance behind the dots like this:

20 m : 5 m

<u>**third**</u>, divide both numbers by the value of the vertical distance, like this:

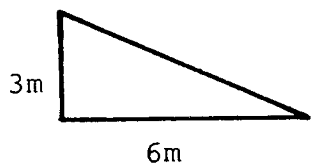
20 m ÷ 5 m = 4

5 m ÷ 5 m = 1

thus, the slope ratio is **4:1 (H:V)**

A slope ratio represents the number of times the vertical distance can be divided by the horizontal distance. Another illustration of a slope ratio calculation is shown below. Study the steps in the example and then do the practice problem.

Example:



<u>**first**</u>, write the horizontal distance in front of the dots: **6 m:**

<u>**second**</u>, write the vertical distance behind the dots: **6 m : 3 m**

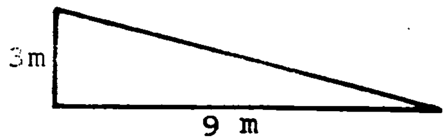
third, divide both distances by the vertical distance:

$6 \text{ m} \div 3 \text{ m} = 2$

$3 \text{ m} \div 3 \text{ m} = 1$

the slope ratio is **2:1 (H:V)**

Practice Problem:



first,

second,

third,

the slope ratio is: ____________

(Refer to answers on Page 118)

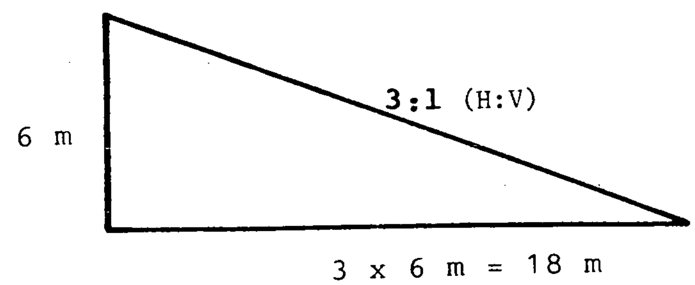
EXERCISE NO. 1

Calculate these slope ratios:

Horizontal Distance	Vertical Distance	Slope ratio
60 m	15 m	______
25 m	10 m	______
22 m	7 m	______
35.5 m	5 m	______
33 m	1.1 m	______

(Refer to answers on Page 118)

An unknown horizontal distance can be calculated if the vertical distance and slope ratio is known. Multiply the value for the horizontal distance in the slope ratio by the actual vertical distance to find horizontal distances. If, for example, the slope ratio is 3:1 (H:V) and the vertical distance is six meters, then multiply the value for the horizontal distance in the slope ratio, 3, by the actual vertical distance, 6 meters. In this case the ratio value for the horizontal distance is 3 and the vertical distance is 6 m, so 3 x 6 m = 18 m. The horizontal distance is 18 m. In diagram form the problem looks like this:



WORK PAGE

EXERCISE NO. 2

Compute the horizontal distance in each of these problems:

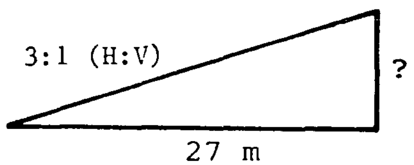
Slope ratios	Vertical distance	Horizontal distance
3.5:1 (H:V)	7.2	________
2.01:1 (H:V)	5.7	________

(Refer to answers on Page 118)

To find horizontal distances using vertical distances and slope ratios, **multiply** the vertical distance by the value in the slope ratio that represents the horizontal distance. In the slope ratio 3.5:1 (H:V), the number 3.5 represents the horizontal distance--distance out. The ratio means 3.5 meters out to one meter down. So, for every one meter down, the slope must extend 3.5 meters out. Since the drop down will be 7.2 meters, the slope must extend out 3.5 times 7.2 meters. 7.2 meters x 3.5 = 25.20 meters. And, since the drop in the second calculation will be 5.7 meters - and the slope will extend 2.01 meters for every meter of drop, multiply 5.7 meters by 2.01.

$$5.7 \text{ m} \times 2.01 = 11.46 \text{ m}$$

To calculate vertical distances, rises and falls, when horizontal distances and slope ratios are known, divide the horizontal distances by the ratio value for the horizontal distances. If the slope ratio is 3:1 (H:V) and a horizontal distance is 27 m divide the horizontal distance, 27 m by the ratio value for horizontal dis- tance, 3. 27 m divided by 3 = 9 m, 9 m is the vertical distance. In diagram form, the problem looks like this:



$$\begin{array}{r} 9 \text{ m} \\ 3\overline{)\,27 \text{ m}} \end{array} = \text{Vertical Distance}$$

WORK PAGE

EXERCISE NO. 3

1. Calculate these vertical distances:

Slope Ratio	Horizontal Distance	Vertical Distance
3.4:1 (H:V)	22.8 m	______________
4.2:1 (H:V)	27.7 m	______________
2.5:1 (H:V)	75.5 m	______________

2. If a ratio is 0.06 m/m, how much lower will the outside edge of the roadway be than the crown if the roadway is 7.2 m wide?

______________ m
(Hundredths)

(Refer to answers on Page 118)

Ratios can express the relationship between two or more ingredients in a mixture. As an example, two parts of coarse aggregate for one part of fine aggregate could be expressed as 2:1. A mix ratio of 1:2:4 means that there are three ingredients in the mix. If the ingredients were say, water, cement, and gravel, then for every part of water there would be two parts cement and four parts gravel.

EXERCISE NO. 4

Show the following as mix ratios:

3 parts to 2 parts = ______________
4 parts to 2 parts to 2 parts = ______________
1 parts to 3 parts to 5 parts = ______________
5 parts to 5 parts to 2 parts = ______________

Mix ratios tell the combination of parts. Parts can stand for anything. As an example, 3:1 can mean 3 cubic meters of coarse aggregate to 1 cubic meter of sand. 3:2:1 can mean three parts aggregate, two parts sand and one part cement.

Note that when you are mixing such things as coarse aggregate and fine aggregate, or gravel, sand and cement, the total volume of the mix can be less than the total volume of the separate parts in the mix. This is because the finer material will fill the voids in the coarse material and give you less volume than expected. The ratios tell you the numbers of parts of each ingredient to use in a mix, but not always the total volume of the final mix.

Suppose that a mix contained sand and salt at a ratio of 30:1. If there was 100 cubic meters of sand in the mix, how many cubic meters of salt are needed? The problem can be solved in a similar fashion to slope ratios. Divide 100 cubic meters by 30 cubic meters of sand per cubic meter of salt. This equals 3.333 cubic meter of salt.

$$100 \div 30 = 3.333$$

Suppose 230 kg of salt were in the mix. How much sand was there? In this case multiply:

> 230 kg of salt times 30 parts of sand
> to one part salt = 6900 kg sand.

The dry mix ratio for portland cement is 3:2:1 which means 3 parts gravel to 2 parts sand to 1 part cement. Suppose a mix containing 1.5 cubic meters of gravel was needed:

One - add the parts: = 3 parts + 2 parts + 1 part
= 6 parts in the mix.

Two - since 1.5 cubic meters of gravel represents 3 parts, each part represents 0.5 cubic meters.

Three - multiply 2 parts sand by 0.5 cubic meters per part. 2 parts x 0.5 cubic meters = 1 cubic meter.

Four - one part cement = 0.5 cubic meters cement.

(Refer to answers on Page 118)

WORK PAGE

SECTION I

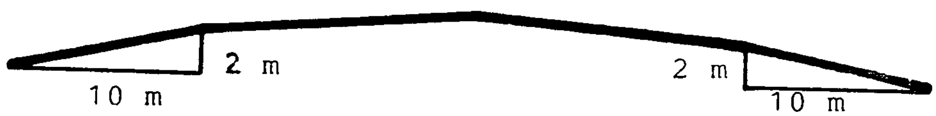
Chapter 5 Quiz

1. You want to use one cubic meter of gravel in 3:2:1 mix of gravel, sand, and cement. How many cubic meters of sand and cement will you need?

_______________ m^3 sand
_______________ m^3 cement

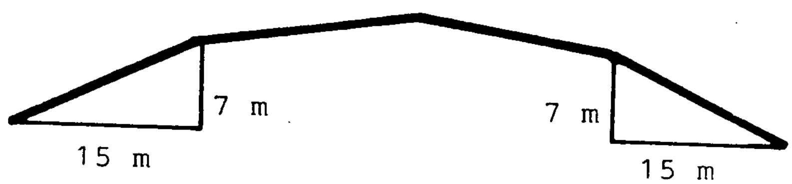
2. What are the slope ratios for these in slopes?

(a)



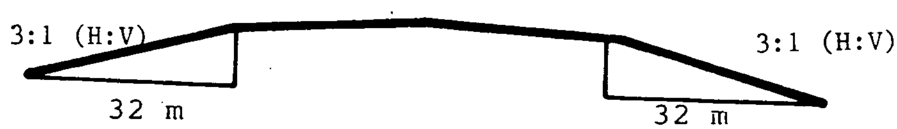
Slope ratio = ____________________

(b)



Slope ratio = ____________________

3. Determine the vertical distance for the following slope:



Vertical distance = ____________________

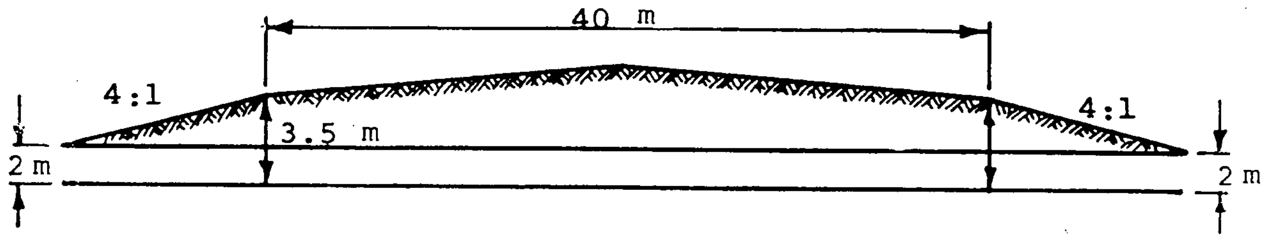
WORK PAGE

Chapter 5 Quiz Continued

4. According to the contract plans, the following information is known:

   (a) the pipe must be level with the ditch bottom
   (b) the surface and shoulder width = 12 meters
   (c) the vertical distance from the road surface to the ditch bottom = 1.1 meters
   (d) the slope ratio = 4:1
   (e) the diameter of the new pipe = 600 mm

   What is the length of the culvert?



(Refer to answers on Page 118)

WORK PAGE

WORK PAGE

SECTION I

CHAPTER 6 Rounding, Precision, and Estimating

Rounding

Numbers are rounded for at least two reasons:

(1) To make numbers easier to use. You need to know asphalt application rates to hundredths of a liter-not to thousandths or ten thousandths.

(2) To simplify calculations. It is much easier to calculate to tenths than to hundredths.

In either case, to round a number decreases the number of digits.

The number 21.6666 can be rounded to three decimal places, two decimal places, one decimal place or the nearest whole number as follows:

Original Number	Decimal Places Needed	Rounded Number
21.6666	three decimal places	21.667
21.6666	two decimal places	21.67
21.6666	one decimal place	21.7
21.6666	whole number	22

Note that the digits following the last digit used are dropped. In the first three examples, the last digit used was increased to seven.

Before discussing the rules of rounding, we had best name the digits that make up numbers again. Digits are named according to where they stand in relation to the decimal point.

The digits in the number 3475.182 have these names:

3 thousands
4 hundreds
7 tens
5 units
. decimal
1 tens
8 hundredths
2 thousandths

The digits in the number 927.65 have these names:

9 hundreds
2 tens
7 units
. decimal
6 tenths
5 hundredths

Notice that digit names do not change - just the digits used in numbers change. The **"tens"** are always two places left of the decimal point. The **"hundreds"** are always three places left. The tenths, different than tens, are one place right of the decimal point. Hundredths, not hundreds, are two places right.

To round a number is to reduce the number of digits used in it, or to change the digits used.

If you found, by calculation, that the average man can cut 3.41678 hectares of grass per day, you wouldn't know any more than that he can cut 3.4 hectares. The digits 1, 6, 7, and 8 don't tell you very much. Even the **"1"** tells you only that he can cut one hundredth more than 3.4 hectares--and who can use the information? The **"1"** means that he can cut 100 square meters more than 3.4 hectares--a piece about 5 meters wide and 20 meters long.

If you know, by calculation, that there are 12, 499.83 cubic meters of aggregate in a stockpile, you wouldn't even tell anybody. Such a number should be rounded to 12,500 cubic meters first. Use of the 0.83 is of little value to anyone. In this case, the number of digits used has been reduced and some of the digits are changed.

There are four rules of rounding:

Rule 1: Determine the **LAST DIGIT TO BE USED**--the last digit needed for precision.

In the number 624.371, the last digit to be used might be the thousandth, hundredth, tenth or the unit - - the last whole-value digit.

624.371	thousandths
624.37	hundredths
624.3	tenths
624.	last whole-value digit

If the number represents dollars and cents--$624.371--it can be rounded to hundredths, $624.37. If it represents cubic meters of earth, it can usually be rounded to tenths, and sometimes to the last whole unit.

Rule 2: If the digit **FOLLOWING** the last digit used is **less than 5**--do not change the last digit used.

2.384 when rounded to **hundredths** is 2.38.
4 is the digit following the last digit used. 8 is the last digit used, because only hundredths are needed. Since 8 is followed by a digit less than 5, don't change the 8 and the number then becomes 2.38.

The number 5.33 rounded to tenths is 5.3. The number rounded to the last whole digit is 5.

EXERCISE NO. 1

Round the following numbers:

From	To	
23.444	Tenths	______
3.9418	Hundredths	______
92.4	Nearest whole-unit digit	______
57.39949	Thousandths	______

(Refer to answers on Page 119)

Rule 3: If the digit **FOLLOWING** the last one used is **more than 5**, raise the last digit one number.

2.3851 when rounded to **hundredths** is 2.39. The digit following the 8 is more than 5, so the 8 must be raised one number to 9. Therefore, the number becomes 2.39 instead of 2.3851.

The number 5.79 rounded to tenths is 5.8.
The number 27.74 rounded to the nearest whole-unit digit is 28.
The number 337.675504 rounded to thousandths is 337.676.

Rule 4: If the digit **FOLLOWING** the last one used is exactly 5, round the number used to an even number.

12.375 when rounded to hundredths would be 12.38. so the 7 is rounded to the even number or 12.38.

12.365 when rounded to **hundredths** would be 12.36. so the number is rounded to the even number or 12.36.

EXERCISE NO. 2

1. Round the following numbers:

From	To	
23.699	Tenths	______
3.94818	Hundredths	______
92.9	Nearest whole-unit digit	______
57.39969	Thousandths	______

2. Round 137.5685 to thousandths: ______
Round 0.1975 to thousandths: ______
Round 981.0949 to hundredths: ______
Round 5.8850 to hundredths: ______
Round 28.7651 to hundredths: ______
Round 9.9950 to hundredths: ______
Round 0.8500 to tenths: ______
Round 66.6650 to tenths: ______
Round 8.8466 to tenths: ______
Round 1.2500 to tenths: ______

(Refer to answers on Page 119)

A summary of the rules covered so far is:

Rule One: Decide on the digits to be dropped:

If you need an answer in tenths, drop the digits that represent hundredths and thousandths.

If you need an answer to hundredths, drop the digits that represent thousandths and ten thousandths

Rule Two: Don't change the last digit to be kept:

If it is followed by less than 5.

Rule Three: Change the last digit to be kept:

If it is followed by more than 5, increase it by "**1**".

Rule Four: If the number following the last digit is exactly 5, round to the closest even number.

EXERCISE NO. 3

Round these numbers:

271.547 to tenths ______
271.547 to whole units ______
271.547 to tens ______
271.547 to hundreds ______

(Refer to answers on Page 119)

Rounding makes calculating faster and easier. If you want to calculate to tenths, there is no reason to have the calculating numbers more accurate than hundredths. As an example:

Cubic meters of concrete placed to date:

6.25 cubic meters	These calculations are
4.50 cubic meters	rounded to **hundredths.**
+ 4.72 cubic meters	
15.47 cubic meters	
15.5 cubic meters	**Final answer is rounded to tenths**

The rules of rounding are shown below.

1. Find the last digit to be used

2. If the digit following the last one to be used is 0,1,2,3, or 4, do not change the last digit to be used.

3. If the digit following the last one to be used is 5 followed by no digit or a zero and the last digit is even, don't change the last digit to be used. If the digit following the 5 is odd or any digit greater than zero, add 1 to the last digit to be used.

4. If the digit following the last one to be used is greater than 5, add 1 to the last digit to be used.

Always round the same way and in the same places. If you don't, you will get different answers with the same data--on the same problems and on different problems.

Always round the way the Department wants you to round. If you don't, other persons will get different answers when they check your work.

Accuracy - Precision Relationship

In dealing with measurements and calculations it is important to distinguish between "**accuracy**" and "**precision**". As defined by the American Society of Civil Engineers, accuracy is "**nearness to the truth**" whereas precision is "**degree of fineness of reading in a measurement, or, the number of places to which a computation is carried.**" It follows that a measurement may be accurate without being precise, and vice versa. For example, a line is in fact 62.00 meters long. It is measured as 63.00 meters due to an error in reading the chain. The 63.00 meters measurement had a degree of precision to the nearest hundreth of a meter, but the accuracy was one (1) meter off of the true length.

Precision

Generally, only three degrees of decimal precision are required by Iowa Department of Transportation. They are:

1. Calculating to units (1)
2. Calculating to tenths (0.1)
3. Calculating to hundredths (0.01)

DOT reports and calculations generally specify rounding of final calculations to certain degrees of precision, that is, fixed numbers of decimal places. The degree of precision, needed in final answers depends on (1) the nature of each report, the physical limitations of measurements.

Operation	Degree of precision in Final Answer
Excavation	1 cubic meter
Standard Paving	0.1 square meters
Structural Concrete	0.01 cubic meters

Note: For other examples of degrees of precision, consult Appendix B, page B-1.

Most calculations are carried out one decimal place more than is needed in the final answer. The answer is then rounded to the desired accuracy.

Example 1: You want the final answer in tenths and your measurements are in hundredths:

510.37 square meters
270.13 square meters
<u>121.89 square meters</u>
902.39 square meters - Total square meters

902.4 square meters - Final answer recorded

Don't carry calculations to decimal places beyond those needed--even when greater degrees of precision are easy to obtain. The workload is reduced for everybody if every original and checking calculation results in the same answer.

When units are required, calculations should generally be rounded to tenths. Final answers are then rounded to units.

Example 2: Multiplying

<u>**Calculation A**</u>	<u>**Calculation B**</u>
25.5 x 13.1 = 334.0	9.4 x 3.2 = 30.1

<u>**Final Calculation**</u>

334.0 x 30.1 = 10,053.4

Rounded to 10,053

When the final answer is required to tenths, carry the calculations out to hundredths. Final answers are rounded to tenths (0.1).

Example 3: Dividing

<u>**Calculation A**</u>	<u>**Calculation B**</u>
542.15 ÷ 8.47 = 19.04	138.37 ÷ 2.12 = 11.42

<u>**Final Calculation**</u>

19.04 ÷ 1.42 = 1.67

Rounded to 1.7

If hundredths are required, calculations should be carried out to thousandths. Final answers are then rounded to hundredths.

Example 4: Adding

<u>**Calculation A**</u>	<u>**Calculation B**</u>
4.469	3.972
2.157	1.059
1.120	2.183
<u>0.019</u>	<u>1.506</u>
7.765	8.720

<u>**Final Calculation**</u>

8.720
<u>7.765</u>
16.485

Rounded to 16.48

EXERCISE NO. 4

1. Round the final answer to units:

    Calculation A = 12.3 x 7.6 = 93.5

    Calculation B = 7.1 x 1.1 = 7.8

    Final calculation A x B = ____________________

    Final answer = ____________________

2. Round the final answer to tenths:

    Calculation A = 137.29 ÷ 25.16 = 5.46

    Calculation B = 547.15 ÷ 89.28 = 6.13

    Final calculation B ÷ A = ____________________

    Final answer = ____________________

3. Round the final answer to hundredths:

    Calculation A = 15.448 - 12.396 = 3.052

    Calculation B = 10.523 - 8.328 = 2.195

    Final calculation A - B = ____________________

    Final answer = ____________________

(Refer to answers on Page 119)

Estimating Reasonable Answers

A reasonable answer is an answer known to the individual to be approximately right. Estimating is a process of guessing or quickly working out an approximate answer. A general way to estimate is:

1. Round the numbers to be calculated to simplify the calculation.

2. Estimate the probable answer by calculating either in your head or on paper.

WORK PAGE

Example 1: Divide 1550 by 325

1. Round each number - 1550 and 325 - to the nearest 100; 1500 and 300.

2. Divide 1500 by 300 - either in your head or on paper. The answer is 5.

   1550 ÷ 325 = 4.77 or 4.8 - the estimate of 5 is reasonably close.

Estimated answers usually are slightly greater or slightly less than calculated answers. The idea is that estimated answers provide a basis of quick comparison with calculated answers.

Estimating is particularly helpful in checking the placement of decimal points. In the previous example, if the calculated answer had been 0.477, a comparison with the estimated answer would show a mistake had been made.

EXERCISE NO. 5

Estimate the answers to these calculations

1. 79.3 ÷ 19.8 = ____________________
   7.1 x 29.93 = ____________________
   1242.57 + 138.92 + 17.67 = ____________________
   182.17 - 13.87 = ____________________
   22000 x 0.33 = ____________________

2. Compare your estimates to those below. Were you close?

Estimated Answers	Calculated Answers	Reasonable
4	4.005	__________
210	212.503	__________
1400	1399.16	__________
165 to 170	168.3092	__________
7000 to 8000	7260	__________

(Refer to answers on Page 119)

Estimating answers takes practice. Skill is not developed as a result of a few problems given in this training course. In fact, many persons have real difficulty with estimating even after extensive practice.

Generally, estimating can save time. When a problem needs calculation, first, try and estimate the answer; second, do the calculation; third, compare answers. If the two answers are not within 10% of each other, then recheck the estimate. If the estimate seems correct, then the calculated answer must be rechecked.

Example 2:

A survey crew estimated that a box-shaped trench 50.12 meters by 12.10 meters by 3.02 meters needed 1800 - 1900 cubic meters of compacted dirt to fill it. But they found in their calculation that they needed 18,315.0 cubic meters. Which answer was more correct?

First: **estimate:** 50.12 meters is about 50 meters
12.10 meters is about 12 meters
3.02 meters is about 3 meters

$50 \text{ m} \times 12 \text{ m} \times 3 \text{ m} = 1800 \text{ m}^3$

Second: **calculate:** (calculation in this case should be carried out to hundredths and the answer rounded to tenths.)

50.12 m x 12.10 m = 606.4520 square meters, which rounded to hundredths is 606.45 square meters.

606.45 square meters x 3.02 meters = 1831.4790 cubic meters, which rounded to tenths of a cubic meter = 1831.5 cubic meters.

Third: **compare:** 1800 cubic meters as an estimate 1831.5 cubic meters as a calculated answer agree very closely. Obviously, the crew's calculation of 18,315.0 meters was a simple decimal error.

EXERCISE NO. 6

1. Estimate answers for each of the following calculations. Compare your estimates to the calculated answers shown. Decide whether or not your answer is reasonable.

Calculation	Your Estimate	Reasonable?
21.5 - 13.87 = 7.63	________	________
157.5 x 12.1 = 1905.75	________	________
6247.12 x 24.8 = 154928.58	________	________

WORK PAGE

EXERCISE NO. 6 Continued

2. A highway project consists of the following lengths of road:

87.35 km
22.85 km
6.95 km
13.84 km
1.09 km

95.41 km of the project has been completed. A Tech III claimed that about 72% of the project was complete. How close was his estimate to the actual kilometers complete? (Answer in tenths of a kilometer) ____________
Was his answer reasonable? ______________

(Refer to answers on Page 119)

WORK PAGE

SECTION I

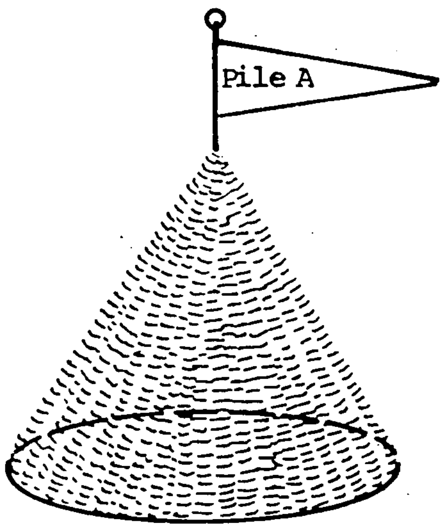
Chapter 6 Quiz

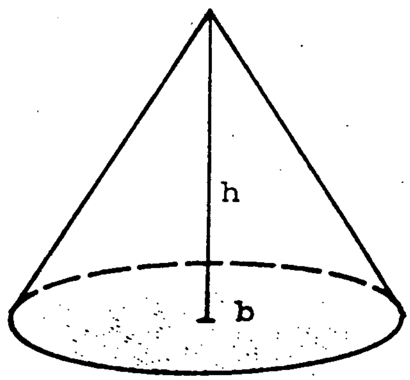
1. (a) A contract plan called for a structure excavation 33.2 m x 100.2 m x 2 m. What was the volume of excavation?

______________ m^3
(units)

(b) After excavating, a concrete structure is built. If the dimensions of the structure were 33 m x 100 m x 1.8 m, how many cubic meters of back fill are required to backfill the trench assuming 6% shrink on the backfill?

______________ m^3
(units)





2. Pile A had the dimensions of a cone with the base equal to 27.017 square meters and a height of 1.83 meters. Approximately how many cubic meters (to the nearest meter) is there of Pile A?

The formula for a cone = (1/3)bh.

______________ m^3

(Refer to answers on page 120)

WORK PAGE

WORK PAGE

Answers

OPERATIONS IN BASIC ARITHMETIC

(For Exercises on Pages 1 thru 3)

Addition:

(1) 2809	(2) 2471	(3) 20,980
(4) 31,729	(5) 13,684	(6) 56,547
(7) 98,357		

Subtraction:

(8) 2786	(9) 1382	(10) 1462
(11) 15,899	(12) 78,380	(13) 6824
(14) 7418	(15) 41,889	

Multiplication:

(16) 3654	(17) 26,132
(18) 3,262,900	(19) 9,917,814
(20) 83,935,146	(21) 186,373,283
(22) 172,382,696	

Division:

(23) 2872	(24) 16,018	(25) 4046.5
(26) 5968		

Answers

PRE-TEST EXERCISE
For Exercise on Pages 5 thru 7

(1) 5565.537, Decimals
(2) 67,502.20332, Decimals
(3) 619.0129, Decimals
(4) 0.11270, Decimals
(5) 279° 59′ 51", Numbers and Measurement
(6) 0.0138 cubic meters, Numbers and Measurement
(7) 93° 22′ 36", Numbers and Measurement
(8) 17 hrs. 20 min. 40 sec., Numbers and Measurement
(9) 9.006 m, Numbers and Measurement
(10) 37.5%, 3/8, Conversion of Decimals (In chapter on percentages)
(11) 0.125, 12.5%, Conversion of fractions (In chapter on percentages)
(12) 37.8, Percentage
(13) 45, Percentage
(14) 1 11/72, Fractions
(15) 4.40625, Fractions
(16) 7/36, Fractions
(17) -85/192, Fractions
(18) 1 5/21, Fractions
(19) 5.88%, Percentage
(20) 18.3303, 5.61235, 8.9443, Squares, Square Roots, Cubes and Cube Roots
(21) 157.5 square meters, Areas
(22) 53.4072 m, Areas
(23) 91.6091 m^3, Volumes
(24) 100 m, Trigonometry
(25) 5.950 m, Vertical Curves

NOTE: Answers are taken out far more decimal places than necessary, so your answer may differ by some slight amount.

SECTION I ANSWERS

Chapter 1

Exercise No. 1, Page 11:

(1) 52,017.800 m
(2) 160° 48′ 43"
(3) 12.700 L
(4) 25,026.216 km^2
(5) 18,044.519 km^3

Exercise No. 2, Page 11:

(1) 8° 55′ 50"
(2) 3000.800 km
(3) 84° 55′ 32"
(4) 2998.330 km^2
(5) 23,797.663 km^2

Exercise No. 3, Page 13:

(1) 154° 01′ 36"
(2) 462,059.400 km
(3) 13,000.500 L
(4) 132,307.560 km^2
(5) 1,119,637.532 km^3

Exercise No. 4, Page 16:

(1) 15.100 L
(2) 47° 38′ 42"
(3) 4.822 m
(4) 47° 34′ 42"
(5) 24.370 m

Chapter 1 Quiz, Page 17:

(1) 24,558.800 km
(2) 24,007 kg
(3) The Sum = 354° 15′ 13"
(4) 100 liters of water, 5 liters per sack of cement - so needs 20 sacks. Each sack wighs xxx kg according to Appendix A, so need xxx 20 or xxx kg.

SECTION I ANSWERS

Chapter 2

Exercise No. 1, Page 21:

(1)	2,671,986	(2)	9,000,201	(3)	807,090
(4)	0.184130	(5)	0.1007	(6)	0.20006

Exercise No. 2, Pages 23:

(1) 0.181, one hundred eighty-one thousandths
(2) 0.0061, sixty-one ten thousandths
(3) 0.17, seventeen hundredths
(4) 0.89576 eighty-nine thousand five hundred seventy-six hundred thousandths
(5) 0.989, nine hundred eighty-nine thousandths

Exercise No. 3, Page 23:

(1)	1.61	(2)	8.762	(3)	1.4
(4)	17.674	(5)	8.76241		

Exercise No. 4, Page 25:

(1)	270.7011	(2)	32.02	(3)	120.0042
(4)	89.962	(5)	71,406.66376		

Exercise No. 5, Page 25:

(1)	1.09	(2)	11.616	(3)	13.0099
(4)	23,229.7104	(5)	847.079		

Exercise No. 6, Page 27:

(1)	44.7102	(2)	647.2904	(3)	900.2672
(4)	1,032.39521	(5)	76,148.814		

Exercise No. 7, Page 31:

(1)	7.5068	(2)	21.2958	(3)	44.2913
(4)	3.6101	(5)	1.1728		

Exercise No. 8, Page 33:

(1)	yes	(2)	no	(3)	yes
(4)	yes	(5)	no	(6)	no
(7)	yes	(8)	no	(9)	no
(10)	yes				

Chapter 2 Quiz, Page 35:

(1) 30,257.00 m^2
(2) 60 x 305 m = 18,300 m^2
Since 10,000 square meters = one hectare

$$\frac{18,300}{10,000} = \#\text{ of hectares} = 1.83\text{ ha}$$

Chapter 2 Quiz Continued

(3) Since the slope drops 0.02 m/m out, the total distance drop in:

2.4 m is 2.4 m x 0.02 = 0.048 m or 48 mm

(4) The volume of fill is 2000 m by 7.2 m by 240 mm. The fraction form is most accurate, therefore,

(2000 m) x (7.2 m) x (240/1000 m) = 3456 m^3

$$\text{Weight needed} = (3456 \text{ cubic meters}) \times \frac{1724 \text{ kg}}{1000 \text{ kg/Mg}}$$

$$= 5958.14 \text{ Mg}$$

SECTION I ANSWERS

Chapter 3

Exercise No. 1, Page 41 :

(1) 6.798 Mg
(2) 79.96%
(3) a. 0.00136 Mg
b. 0.0034 Mg
c. 0.1258 Mg
d. 0.0578 Mg
(4) 11,918 m^3
(5) 17.06%

Exercise No. 2, Page 43:

(1) 20	(2) 8.25

Exercise No. 3, Page 47:

(1) 60%	(2) 49%	(3) 49.1%
(4) 300%	(5) 600%	(6) 209.6%
(7) 0.19%	(8) 169%	(9) 0.9%
(10) 1900%		

Exercise No. 4, Page 47:

(1) 75%	(2) 87.5%	(3) 16%
(4) 16%	(5) 17.5%	(6) 20.4%
(7) 1.9%	(8) 88.9%	(9) 91.7%
(10) 1%		

Exercise No. 5, Page 47:

(1) 0.39	(2) 0.174	(3) 1.35
(4) 0.0108	(5) 0.55	(6) 0.04
(7) 0.4	(8) 4.0	(9) 0.0004
(10) 0.00004		

Exercise No. 6, Page 47:

(1) 1/2	(2) 1/20	(3) 1/2,000
(4) 1/20000	(5) 19/100	(6) 833/1000
(7) 1 1/4	(8) 9/250	(9) 13/20
(10) 1/25000		

SECTION I ANSWERS

Chapter 3 Quiz: Page 49

(1)
a. 80% of 800 = .8 x 800 = 640.0
b. 180% of 800 = 1.8 x 800 = 1440
c. 0.8/3 x 100 = 26.66 .....%
d. 4/5 = 0.80 so 0.80/1 x 100 = 80%

(2) Percent increase = $\frac{135 - 120}{120} \times 100 = \frac{15}{120} \times 100 = 12.5\%$

(3) Since 20% doesn't pass the 4.75 mm sieve, then 0.2 x 1350.25 g or 270.05 g of material is held. 46% of the material that passes the 4.75 mm sieve sticks in the 600 μm sieve - this is 46% of 1350.25 - 270.05 = 0.46 x 1080.20 g = 496.8220 g. Thus, 1080.02 g - 496.8220 g = 583.3080 g pass the 600 μm sieve.

(4) 5.6% of 467.20 g = 26.1632 g
Total wet weight = 467.20 g + 26.1632 g = 493.3632 g

(5) Individual Averages:

A: 182 ÷ 67.7 = $2.69

B: 83 ÷ 40.0 = $2.08

C: 188 ÷ 74.0 = $2.54

Weighted Average Cost = $\frac{\$182 + \$83 + \$188}{67.7 + 40.0 + 74.0} = \frac{\$453}{181.7}$

= $2.49 per m^3

Since the highest cost is $2.69 and the weighted average is $2.49 the percent difference from the weighted average cost is

$$\left(\frac{2.69 - 2.49}{2.49}\right) \times 100 = 8.03\%$$

SECTION I ANSWERS

Chapter 4

Exercise No. 1, Page 52:

(1) 3/5 proper	(2) 8/3 improper	(3) 13/6 improper
(4) 14/39 proper	(5) 19/65 proper	

Exercise No. 2, Page 53:

(1) 2 3/8, 19/8	(2) 7 6/7, 55/7	(3) 9 1/2, 19/2
(4) 4 3/32, 131/32	(5) 14 1/5, 71/5	

Exercise No. 3, Page 55:

(1) 3/6	(2) 6/9	(3) 21/24
(4) 27/48	(5) 15/24	

Exercise No. 4, Page 56:

(1) 2/4	(2) 3/16	(3) 5/32
(4) 7/8	(5) 8/32, 4/16, 1/4	

Exercise No. 5, Page 56:

(1) 2/4, 1/2	(2) 6/8, 3/4	(3) 8/16, 1/2
(4) 50/100, 1/2	(5) 21/35, 3/5	

Exercise No. 6, Page 57:

(1) 2 x 2 x 2	(2) 3 x 3	(3) 3 x 7
(4) 2 x 2 x 2 x 2 x 2	(5) 2 x 2 x 2 x 2 x 2 x 2	

Exercise No. 7, Page 58:

(1) 12	(2) 24	(3) 60
(4) 70	(5) 24	

Exercise No. 8, Page 63:

(1) 2 5/6	(2) 3 1/8	(3) 1 13/32
(4) 1 1/2	(5) 2 101/105	(6) 22 3/4
(7) 18 1/3	(8) 9 19/24	(9) 11 3/32
(10) 2 50/63		

Exercise No. 9, Pages 67:

(1) 1/2	(2) 4/13	(3) 11/24
(4) 9/16	(5) 2 13/16	(6) 10 7/16
(7) 7 5/8	(8) 24 11/18	(9) 1/15
(10) 70 4/7		

Exercise No. 10, Page 69:

(1) 2/7	(2) 7/24	(3) 1/10
(4) 9/20	(5) 16/25	(6) 9/32
(7) 86 13/32	(8) 30 15/16	(9) 11/16
(10) 10 5/13		

SECTION I ANSWERS

Exercise No. 11, Page 72 :

(1) 3/4	(2) 1 13/15	(3) 1 1/12
(4) 5/17	(5) 12	(6) 35
(7) 1 2/3	(8) 1 5/7	(9) 3/35
(10) 9/11		

Chapter 4 Quiz, Page 73:

(1) 1/2 (2) 1/10 of an hectare

(3) New pipes weigh 68 kg and cost $6.10 for 6.1 m of pipe

(4) 3/4 (5) 1/3

SECTION I ANSWERS

Chapter 5

Practice Problem, Page 78:

(1) 9: (2) 9:3

(3) 9 ÷ 3 = 3 and 3 ÷ 3 = 1, therefore, the slope ratio is 3:1

Exercise No. 1, Page 78:

(1) 4:1 (H:V) (2) 2.5:1 (H:V) (3) 3.14:1 (H:V)

(4) 7.1:1 (H:V) (5) 30:1 (H:V)

Exercise No 2, Page 81:

(1) 25.2 (2) 11.5 m

Exercise No. 3, Page 83:

(1) 6.706 m 6.595 m 30.2 m

(2) 0.22 m. On this last problem, since the roadway is 7.2 m wide, each lane is 3.6 m wide. The outside edge should be 0.06 m below the crown for each meter of **lane width**.

3.6 m x 0.06 = 0.22 m

Exercise No. 4, Pages 83:

(1) 3:2 (2) 4:2:2 (3) 1:3:5

(4) 5:5:2

Chapter 5 Quiz, Page 86:

(1) 0.667 or 2/3, 0.333 or 1/3

(2) (a) 5:1 (H:V) (b) 2.14:1 (H:V)

(3) 10.667 m

(4) 16 m

SECTION I ANSWERS

Chapter 6

Exercise No. 1, Page 94:

(1)	23.4	(2)	3.94	(3)	92
(4)	57.400				

Exercise No. 2, Page 95:

(1)
- a. 23.7
- b. 3.95
- c. 93
- d. 57.400

(2)

a.	137.568	f.	10.00
b.	0.198	g.	0.8
c.	981.09	h.	66.7
d.	5.88	i.	8.8
e.	28.77	j.	1.2

Exercise No. 3, Page 95:

(1) 271.5 (2) 272 (3) 270 (4) 300

Exercise No. 4, Page 99:

(1)	729.30	(2)	1.12	(3)	0.857
	729		1.1		0.86

Exercise No. 5, Page 101:

(1)
- a. 4
- b. 210
- c. 1400
- d. 170
- e. 7000

(2)

a.	yes	d.	yes
b.	yes	e.	yes
c.	yes	f.	yes

Exercise No. 6, Page 104:

(1)
- a. 8 yes
- b. 1900 yes
- c. 150,000 yes

(2) By his estimate, 95.1 km is complete, so he was only 0.3 km off. His answer was reasonable.

SECTION I ANSWERS

<u>Chapter 6 Quiz, Page 106</u>:

1. a. Volume = 2 m x 33.2 m x 100.2 m = 6653.28 m^3
   Rounded = 6653 m^3

   b. Volume of excavation = 6653.3 m^3
   Volume of structure = 33 x 100 x 1.8 = 5940 m^3
   Volume to fill = 6653.3 - 5940.0 = 713.3 m^3

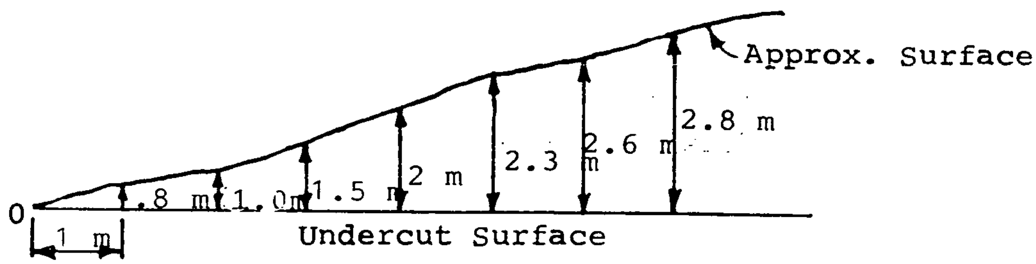
   With a 6% shrink multiply 1.06 times 713.3.
   Answer: 756 m^3

2. (27.017 m^2 x 1.83 m) 1/3 = (27 m^2 x 1.83 m) (1/3)
   = (9 x 1.83) m^3 = 18 m^3

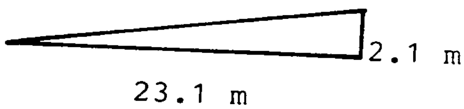
SECTION I

COMPREHENSIVE TEST

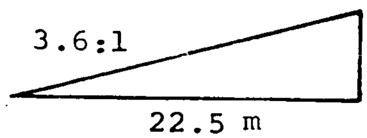
1. Determine the average undercut depth from the information shown in the diagram. (tenths).



2. (a) Compute the slope ratio:

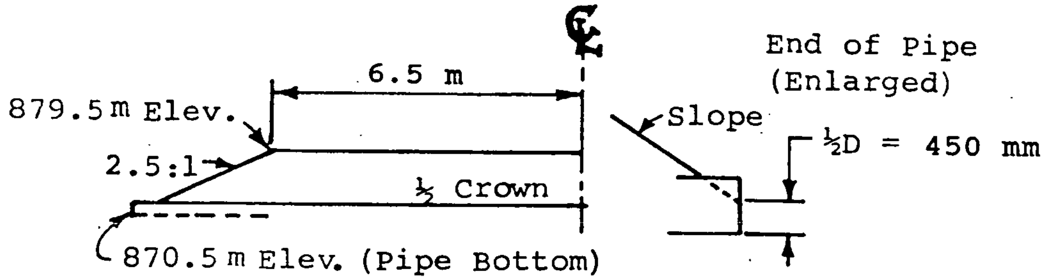


(b) Compute the vertical distance:



WORK PAGE

3. What length of 900 mm--diameter pipe is needed for the culvert below? The slope crosses the pipe halfway up the pipe. Disregard the slope of the pipe itself.



Pipe length = ________________ m
(tenths)

Note: 1/2 crown means **"half the roadway width."**

4. A sample of wet sand has a mass of 5 kg. The sample contains 0.55 kg of water. How much water will 725 kg of wet sand contain?

____

5. A sample of asphalt mix weighs 1000 grams. When the asphalt is removed, the aggregate weighs 934.6 grams. What is the % asphalt contained in the mix?

$$\% \text{ A.C.} = \frac{\text{mass of mix} - \text{mass of aggregate}}{\text{mass of mix}} \times 100$$

6. A sample was taken from a soil cement job. The mass of wet material was 621.50 grams and the mass of dry material was 565.00 grams. What was the moisture content (%) of this sample dry based on mass?

$$\% \text{ M.C.} = \frac{\text{wet mass} - \text{dry mass}}{\text{dry mass}} \times 100$$

Moisture Content = ________________ %
(Tenths)

7. Percent grade = $\frac{\text{vertical rise or fall}}{\text{horizontal distance}} \times 100$

A centerline elevation rises 2.3 m in a 77 m section of roadway. What is the percent grade?

____

8. It takes 66.6 cubic meters of P.C. concrete per 100 m section of highway. How many cubic meters of concrete will be needed for a 4550 m project?

_____________________ m^3

9. How much concrete mix should be ordered for a driveway entrance if the volume for the nominal dimensions within the forms is 18 cubic meters and 5% is to be added for irregular subgrade? Round answer to the nearest tenth of a cubic meter.

_____________________ m^3

WORK PAGE

Section I Comprehensive Test Continued

10. If granular backfill shrinks 6%, how many cubic meters will have to be ordered to fill a 2.55 cubic meter excavation?

____

11. If sand contains 17 percent water based on dry mass, how much wet sand is needed to provide 1565 kg of dry sand?

____

12. If coarse aggregate and fine aggregate are mixed in a ratio of 2.5:1 part fine, and you have 2000 cubic meters of fine aggregate. How many cubic meters of coarse aggregate will be needed?

____

13. If a certain aggregate asphalt mix weighs 1474.6 kg/m^3, how many megameters will 9.51 cubic meters weigh?

________________ Mg
(nearest Mg)

14. Suppose a tank contains 30 cubic meters of capacity. If 1 liter of asphalt is equivalent to 1000 cubic meters, how many liter of asphalt can be placed in the tank? Round to whole liters.

____

15. A highway structure contract contained the following items:

1. Removal of Existing Structures--Lump Sum			$4,000.00
2. Conxcrete Structural	244.6	m^3 @$125	30,575.00
3. Steel, Reinforcing	19915	kg @$0.16	3186.40
4. Excavation, Class 20	110	m^3 @$20.00	2200.00
Total			$39,961.40

If Item 1 is 90% complete, Item 2 & 3 are each 65% complete and Item 4 is 55% complete, what percent of the contract is complete?

________________ %
(hundreths)

SECTION I

COMPREHENSIVE TEST ANSWERS

Question 1 - 1.86 m

Question 2 - (a) 11:1 H:V (b) 6.25 m

Question 3 - 65.75 m

Question 4 - 79.75 kg

Question 5 - 6.54%

Question 6 - 10%

Question 7 - 2.98%

Question 8 - 2963.7 cubic meters

Question 9 - 18.9 cubic meters

Question 10 - 2.703 cubic meters

Question 11 - 1885.54 kg

Question 12 - 5000 kg

Question 13 - 14 Mg

Question 14 - 0.03 liters

Question 15 - 66.95%

SECTION II

SECTION II

Square and Square Roots

In working highway problems, squares, cubes, square roots, and cube roots* appear in many calculations. The purpose of this chapter is to acquaint you with these operations.

To square a number means to multiply that number by itself. As an example, 5 squared means 5 x 5 or $5^2 = 25$.

A square root of a number is a number which, when multiplied by itself, will result in the original number. Essentially, a square root undoes what squaring does. As an example, the square root of 25 means

$$\sqrt{25} = 5$$

To find the square root of 123481.96, $\sqrt{123481.96}$, first group the radicand in intervals of two digits, left and right from the decimal point.

$$\sqrt{12'\,34'\,81'.96}$$

Next determine the largest perfect square in the first interval, **'12'**. This is found to be 9. Place the square root of 9, which is 3, in the answer above the **'12'** and subtract the 9 from the 12.

```
  3
√12'34'81'.96
  9
 ---
  3
```

Bring down the second interval, **'34'**, to begin the next step. A trial divisor is found by doubling the answer at this point, which is 3, and multiplying by 10.

```
     3
   √12'34'81'.96
     9
60 | 334
```

Divide 60 into 334. The largest whole number quotient, which is 5, is placed as the second digit in the answer and added to the divisor.

```
     3  5
   √12'34'81'.96
     9
60 | 334
65 |
```

***Cube roots seldom, appear, but it is instructional to include them in this course.**

The new divisor, 65, is then multiplied by the quotient, 5, and subtracted from 334.

```
       3   5
     √12'34'81'.96
       9
  60 | 334
  65 | 325
         9
```

The third interval, **'81'**, is brought down and the trial divisor is found by doubling the answer at this point, which is 35, and multiplying by 10.

```
       3   5
     √12'34'81'.96
       9
  60 | 334
  65 | 325
   700 |981
```

The 700 is divided into 981 and the whole number quotient, 1, is placed in the answer, and added to 700. 701 is multiplied by 1, and subtracted from 981

```
       3   5   1 .
     √12'34'81'.96
       9
  60 | 334
  65 | 325
   700|  981
   701|  701
         280
```

The process is continued until the final answer is found.

```
       3   5   1 . 4
     √12'34'81'.96
       9
  60 | 334
  65 | 325
   700|  981
   701|  701
  7020|  28096
  7024|  28096
             0
```

To check the answer multiply the answer by itself.

```
     351.4
     351.4
     -----
     14056
     3514
    17570
   10542
   ---------
   123481.96
```

EXERCISE NO. 1

Find the square roots of the following:

1. $\sqrt{1866.24}$ = ______________

2. $\sqrt{205.0624}$ = ______________

3. $\sqrt{1298552.96}$ = ______________

(Refer to answers on Page 206)

WORK PAGE

For convenience a table is included in Appendix C for determining the squares, square roots, cubes, and cube roots of numbers.

Took at the table in Appendix C. At the top of column **two** is the word '**square**'. This refers to the square of the number in column one As an example, look at the number 3 in column one. The square of 3 or $3^2 = 9$, as confirmed by the answer in column two. Note likewise that column four has the word '**square root**'. This column contains the square roots of the numbers in column one.

EXERCISE NO. 2

Use the table to perform the following:

1. $(59)^2$ = ____________
2. $(1)^2$ = ____________
3. $\sqrt{3}$ = ____________ (round to tenths)
4. $\sqrt{47}$ = ____________ (round to hundredths)

(Refer to answers on Page 206)

To cube a number means to multiply that number twice by itself. As an example, 5 cubed means 5 x 5 x 5 or $5^3 = 125$.

The cube root of a number is a number which, when cubed will result in the original number. As an example the cube root of 125 or

$$\sqrt[3]{125} = 5.$$

EXERCISE NO. 3

Use the table to perform the following:

1. $3\sqrt{5}$ ____________
2. $\sqrt{7^2}$ ____________
3. $(82)^3$ ____________
4. $8^3 \times \sqrt[3]{1}$ ____________
5. $\sqrt{16} \times \sqrt{16}$ ____________

(Refer to answers on Page 206)

WORK PAGE

Suppose you had to find the square root of 98.5? The square root tables indicate that the square root of 98 is 9.8995 and the square root of 99 is 9.9499. So, now what? By interpolating, the approximate answer can be found. This is how to interpolate.

One - Set up the numbers which come immediately before or after the number under consideration and their square roots.

Number	Square Root
98	9.8995
98.5	?
99	9.9499

Two - Subtract the smaller root value from the larger one.
$9.9499 - 9.8995 = 0.0504$

Three - 98.5 is larger than 98, and so, the square root value of 98.5 must be larger than the value of 98. Since 98.5 is 5 tenths of the value between 98 and 99, the square root of 98.5 is approximately $9.8995 + (.5)(0.0504) - 9.8995 + 0.0252 = 9.9247$.

Example: What is the square root of 74.3?

One -

Number	Square Root
74	8.6023
74.3	?
75	8.6603

Two - $8.6603 - 8.6023 = 0.0580$

Three - $\sqrt{74.3} = 8.6023 + .3(0.0580) = 8.6023 + 0.0174 = 8.6197$

EXERCISE NO. 4

Interpolate to find the following:

1. $\sqrt{89.5}$ = ________________
2. $\sqrt{71.2}$ = ________________
3. $\sqrt{12.6}$ = ________________

If you have a good understanding of how to find squares, cubes, and square roots in the tables, a lot of time may be saved in the calculations of the following chapters. If you still don't understand how to use the tables, then study the material again before you take the quiz.

(Refer to answers on Page 206)

WORK PAGE

SECTION II

Chapter 1 Quiz

1. (a) $\sqrt{27}$ = ______________

   (b) 18^3 = ______________

   (c) 17^2 = ______________

2. (a) $\sqrt{8^3}$ = ______________

   (b) $\sqrt{4 \times 16}$ = ______________

   (c) $12^2 \times 13 \times \sqrt{7}$ = ______________ (round to tenths)

   (d) $\sqrt{4 + 16}$ = ______________ (round to tenths)

3. The volume of this concrete cone is $(H/3)(B + b + \sqrt{Bb})$.

   Where B is the area of the bottom base and b is the area of the top base. If $B = 15\ m^2$, $b = 7\ m^2$, and $H = 9.7$ m, how many cubic meters of capacity does the cement cone contain?

   ____
   (Tenths)

(Refer to answers on Page 206)

WORK PAGE

WORK PAGE

SECTION II

CHAPTER 2 Areas

AREA

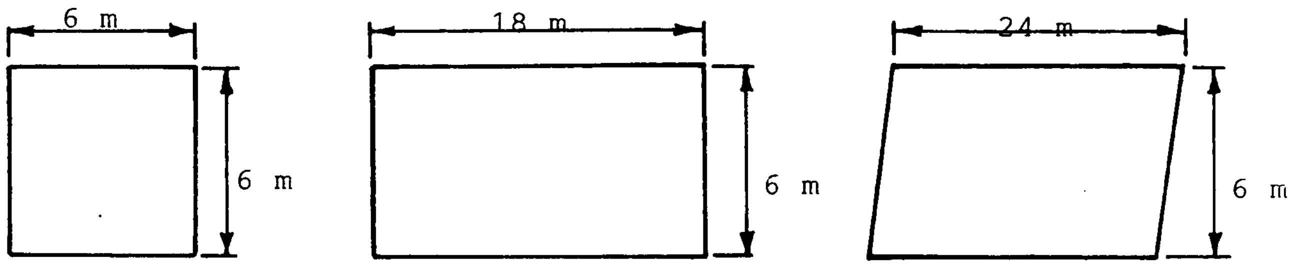
This chapter begins with simple areas, but don't be misled. The calculation of these simple areas will later lead to the solution of complex volumes.

"Squares and Rectangles"

Calculating the areas of squares and rectangles is easy. Calculating the areas of parallelograms is almost as easy.

Multiply the length of one side by the length of a perpendicular side to get the areas of squares and rectangles, and multiply the length by the height to get the areas of parallelograms.

Example: Formulas: Area = LW and A = LH

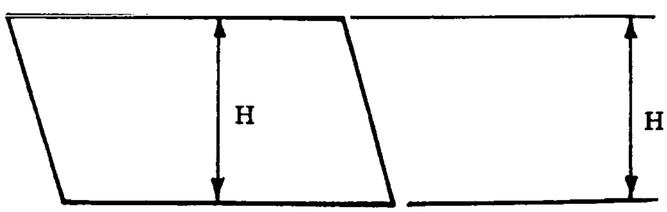


square — $6 \times 6 = 36\ m^2$

rectangle — $6 \times 18 = 108\ m^2$

parallelogram — $6 \times 24 = 144\ m^2$

Be careful when measuring the length of parallelograms. Since the opposite sides are always parallel, measure the perpendicular distance from the side of the length to the opposite side.



Both H's are perpendicular distances.

EXERCISE NO. 1

Calculate the following areas. No diagrams are shown.

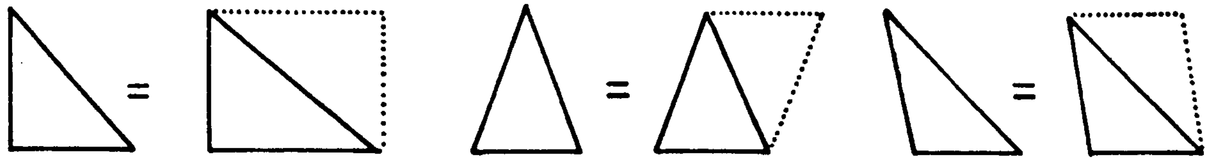
1.	L = 21 m	2.	W = 9.1 m	3.	L = 7.6 m
	W = 27 m		L = 10.0 m		H = 7.2 m
	A =		A =		A =

(Refer to answers on Page 207)

WORK SPACE

'Triangles'

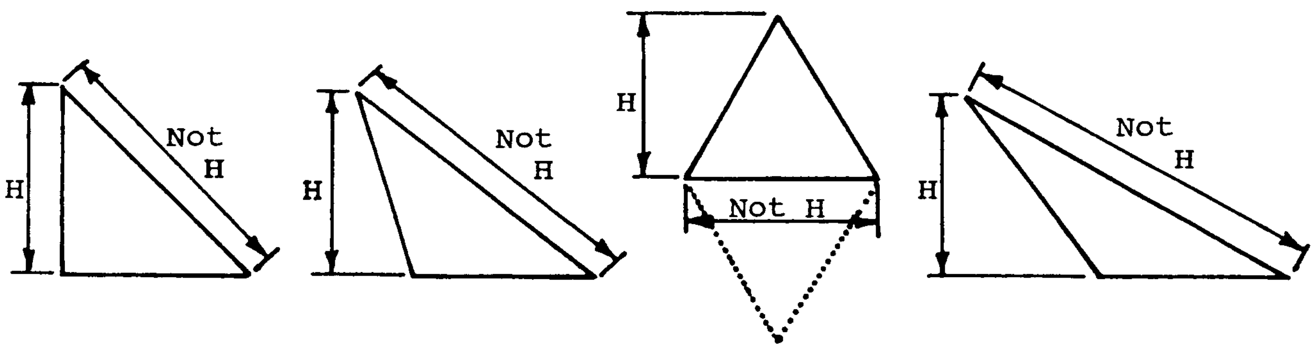
Any triangle is half of a parallelogram!



Right Triangle	Half a Rectangle	Acute Triangle	Half of a Parallelogram	Obtuse Triangle	Half of a Parallelogram

There are three types of triangles: right, which has a 90 degree angle; acute, which has all three angles less than 90 degrees each; and obtuse, which has one angle larger than 90 degrees. Note that the interior angles of any triangle must add to 180 degrees.

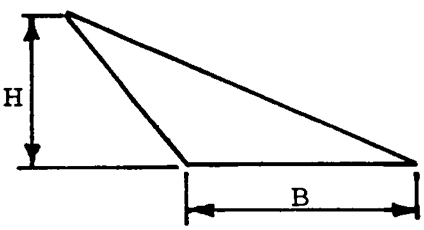
The area of any triangle is half of the area of the parallelogram it would form.



$A = \frac{BH}{2}$ Area equals Base times Height, divided by 2

Be sure to use the right dimensions for the base and the height volumes: As can be seen in the diagrams, any line can be selected as a base line, but visualize the parallelogram accordingly so that you use the proper height value.

EXERCISE NO. 2

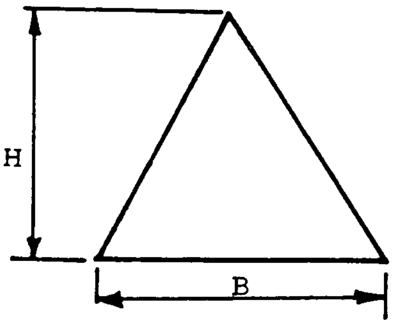


1. B = 7.8 m
   H = 11.3 m
   A = __________

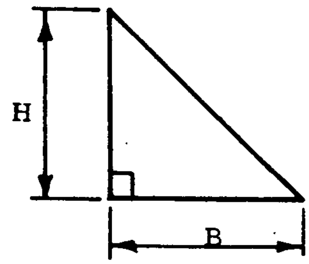
WORK PAGE

Exercise No. 2 Continued

2. B = 8.3
   H = 5
   A = ________

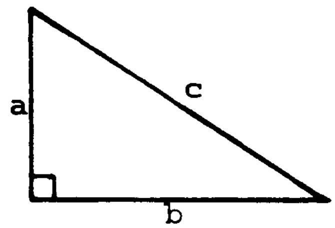


3. B = 18.9
   H = 15.2
   A = ________



(Refer to answers on Page 207)

The length of any one side of a right triangle can be found if the other two lengths are known,



The length of the hypotenuse (long side) is equal to the $\sqrt{A^2 + b^2}$

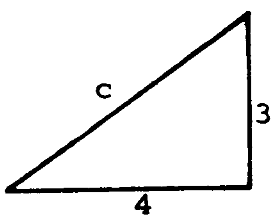
The length of Side a is equal to the $\sqrt{c^2 - b^2}$.

The length of Side b is equal to the $\sqrt{c^2 - a^2}$.

EXERCISE NO. 3

Find the unknown side:

1.

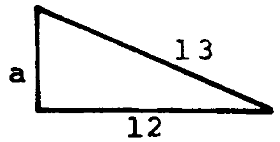


c = ________

WORK PAGE

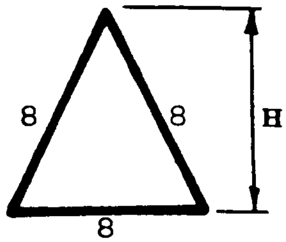
Exercise No. 3 Continued

2.



a = ____________

3.

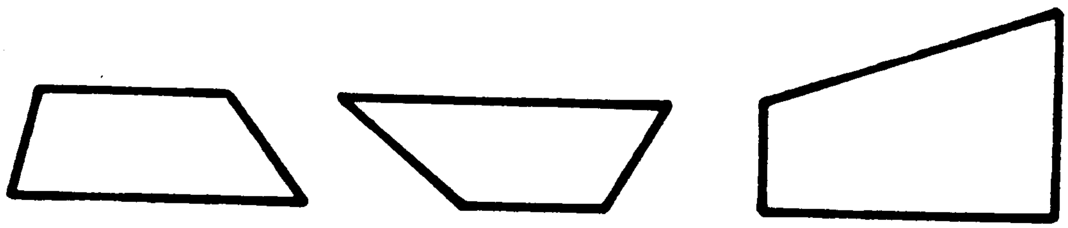


H = ____________

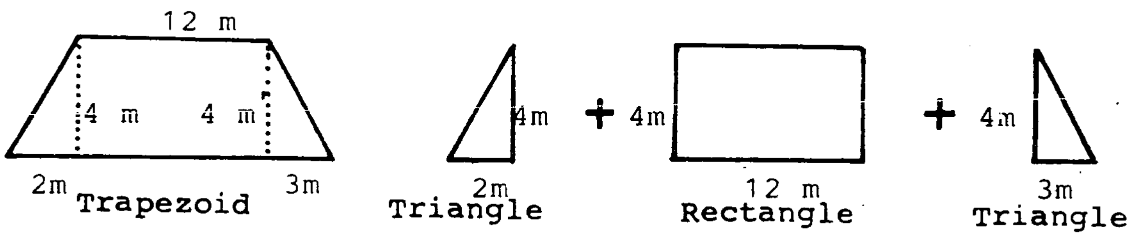
(Refer to answers on Page 207)

'Trapezoids'

Trapezoids are four-sided figures having two parallel sides and two nonparallel sides.



Trapezoids can be broken up into two triangles and a rectangle - and solved by parts.



The area of each of the three parts can be calculated separately and added.

A = Area of triangle + Area of rectangle + Area of triangle

$$= \frac{(4\ m \times 2\ m)}{2} + (4\ m \times 12\ m) + \frac{(4\ m \times 3\ m)}{2}$$

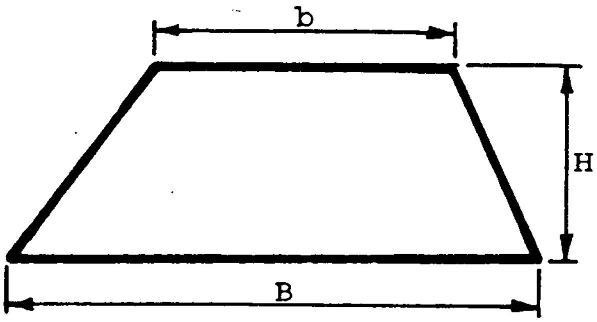
$$= 4\ m^2 + 48\ m^2 + 6\ m^2 = \mathbf{58\ m^2}$$

WORK PAGE

The formula for calculating the areas of trapezoids is:

$A = \frac{B + b}{2} \times H$ 'B' and 'b' are the base lines and H is the height.

Multiply the height times the average of the two base line lengths.

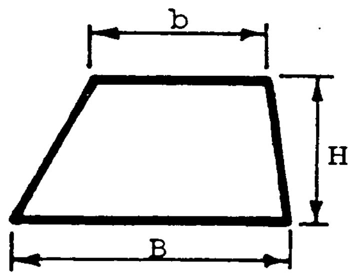


Example: B = 30 m
b = 20 m
H = 10 m

$$A = \frac{B + b}{2} \times H = \frac{30\ m + 20\ m}{2} \times 10\ m = 25\ m \times 10\ m = 250\ m^2$$

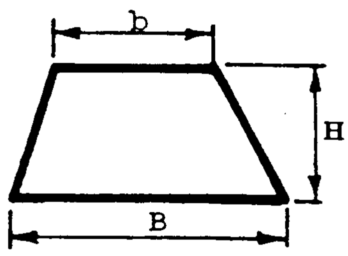
EXERCISE NO. 4

Calculate the area of the following:



1. b = 26.2 m
B = 38.5 m
H = 18.7 m

A = __________ m^2 (round to tenths)



2. b = 439.4 mm
B = 1018.5 mm
H = 546.1 mm

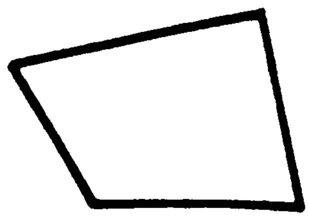
A = __________ mm^2 (round to tenths)

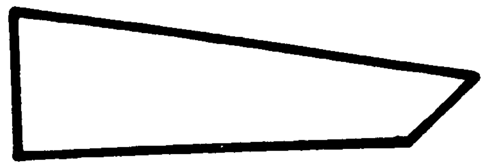
(Refer to answers on Page 207)

WORK PAGE

'Trapeziums'

Trapeziums are four-sided figures with no sides parallel.

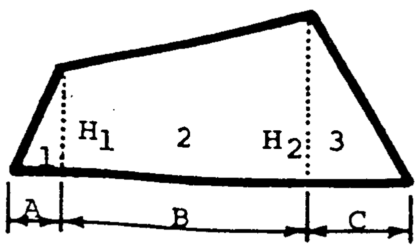




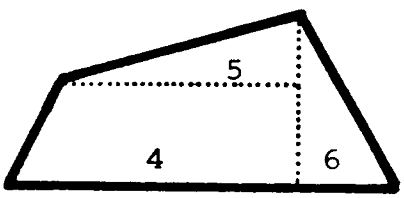
The areas of trapeziums are found by dividing them into triangles and trapezoids.

Example: Two different ways to divide the same trapezium.

A.



B.



You already know how to calculate the areas of triangles, rectangles, and trapezoids. All you need to know to calculate areas 1, 2, and 3, and areas 4 and 5 are a few dimensions.

EXERCISE NO. 5

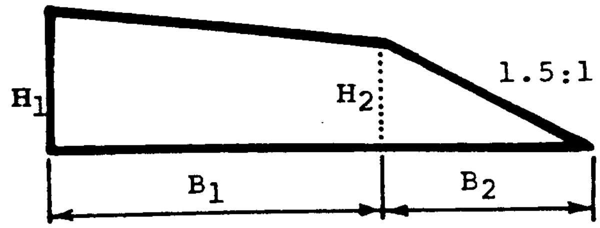
1. Calculate the area of figure A if: A = 5 m B = 20 m C = 10 m H_1 = 8 m H_2 = 12 m

   Area = __________ m^2

WORK PAGE

2. Calculate the area of the following trapezium:

$H_1 = 10$ m
$H_2 = 9$ m
$B_1 = 13$ m



Area = __________ m^2 (Round to hundredths)

(Refer to answers on Page 207)

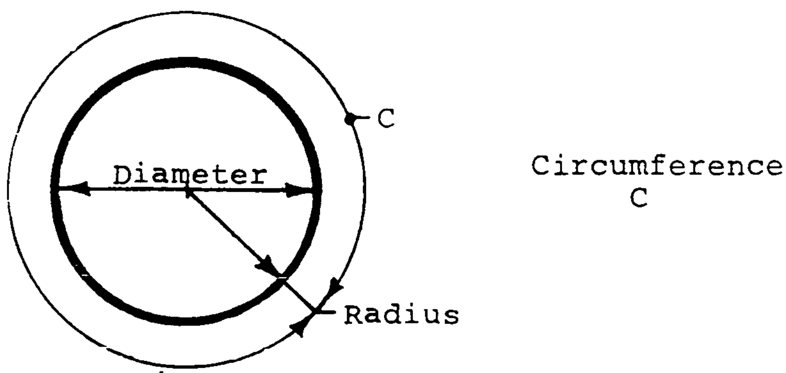
WORK SPACE

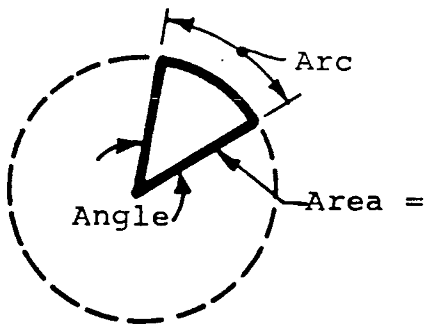
'Circles'

Before proceeding into a discussion of the area of a circle, a few definitions need stating:

1. The **circumference** of a circle is the length of the line that makes the circle.
2. The **diameter** of a circle is the straight-line distance across the center of the circle.
3. The **radius** of a circle equals half its diameter.
4. The **circumference** of a circle is equal to the diameter times Pi (π).
5. The area of a circle is equal to Pi times the squared length of the radius: $A = \pi r^2$
6. The **inside diameter** of a pipe culvert is the longest measurement across the pipe opening.
7. The **outside diameter** is the longest measurement across the pipe opening plus twice the thickness of the culvert wall.
8. The **inside circumference** is equal to the inside diameter times Pi.
9. The **outside circumference** is equal to the outside diameter times Pi.

Pi or π = 3.1416
Circum. $= \pi D$
Dia. $= 2r$
radius $= D/2$
Area $= A = \pi r^2$
$= \pi (D^2/4)$





$\text{Area} = \dfrac{\text{angle}}{360°} (\pi r^2)$

$\text{Arc} = C \dfrac{\text{angle}}{360°}$

10. The arc length for a segment is equal to the circle circumference times the segment angle divided by 360°
11. The segment area is equal to the full circle area times the segment angle divided by 360°.

Example: If the diameter of a circle is 250 mm, what are the area and circumference?

$A = \pi r^2 = (D/2 \times D/2) \times 3.1416$
$= (250/2 \times 250/2) \times 3.1416$
$=$ **49,087.500 mm**

$C = \pi D = 3.1416 \times 250 =$ **785.400 mm (Answer)**

Example: If the circumference of a circle is 1570.800 mm, what are its diameter and area?

$C = \pi D$ so $D = C/\pi = 1570.800/3.1416 =$ **500 mm**

$A = \pi r^2$ and $r = D/2$ so,
$A = (500/2 \times 500/2) \times 3.1416 =$ **196,350.000 mm²**
$=$ **0.200 m² (Answer)**

Example: If the radius is 1.3 m, what are the diameter, circumference and area?

$r = 1.3$ m
$D = 2r = 2 \times 1.3$ m $= 2.6$ m
$C = \pi D = 2.6 \times 3.1416 = 8.16820$ m or **8.17 m**
$A = \pi r^2 = 3.1416 \times 1.3^2$ m $= 3.1416 \times 1.69$ m
$=$ **5.3093 m² or 5.31 m² (Answer)**

EXERCISE NO. 6

Round all numbers to hundredths:

1. C = 157.08 m

   D = ________ m

   R = ________ m

   A = ________ m^2

2. R = 400 mm

   D = ________ mm

   C = ________ mm

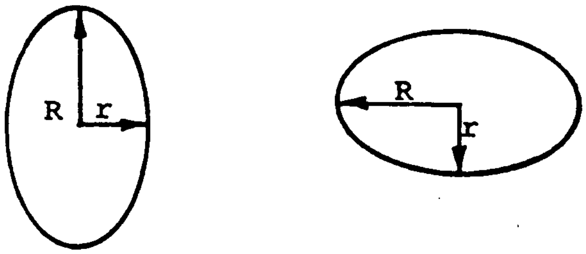
   A = ________ mm^2

(Refer to answers on Page 207)

WORK PAGE

'Ellipses'

Ellipses are oblong circles (egg shaped circles). The radii of ellipses vary depending on the measurements made:

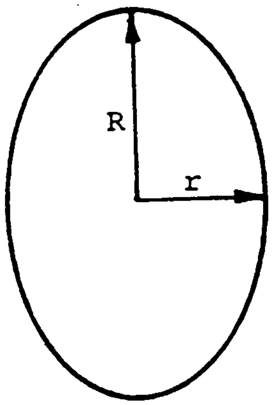


Note: The term "**radii**" is the plural of radius.

Elliptical areas are calculated by using the formula $A = \pi(Rr)$ where R represents the long radius and r represents the short one.

Example:

r = 250 mm
R = 375 mm



$$\begin{aligned} A &= \pi(Rr) \\ &= 3.1416 \times (375 \times 250) \\ &= 3.1416 \times 93{,}750\ \text{mm}^2 \\ &= 294{,}524.31\ \text{mm}^2 \\ &= 0.29\ \text{m}^2 \end{aligned}$$

EXERCISE NO. 7

Round to tenths:

1. R = 775.5 mm

   r = 350.3 mm

   A = __________ mm^2

WORK PAGE

Exercise No. 7 Continued

2. $R = 4.8$ m

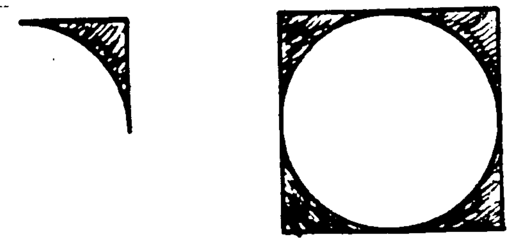
$r = 2.8$ m

$A =$ ________ mm^2

(Refer to answers on Page 207)

'Fillets'

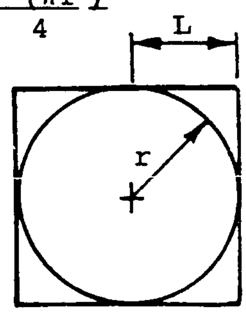
"Fillets" are pronounced **"fill its"**. The shaded areas below represent fillet areas.



As you see, fillet areas are leftovers of squares when maximum circle areas have been deleted.

There are several characteristics of fillets:

1. The length of the side of a 90° fillet is equal to the radius of the circle that could be formed with it.
2. The length of the side of a fillet is also equal to half the length of the square that could be formed.
3. The area of a fillet = $\frac{(2L)^2 - (\pi r^2)}{4}$ or $\frac{(2r)^2 - (\pi r^2)}{4}$



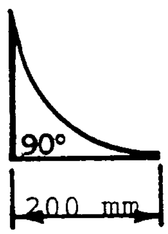
L = Length of the side of the fillet

r = radius of the circle or length of the 90° fillet side.

EXERCISE NO. 8

Round the following to hundredths.

1.

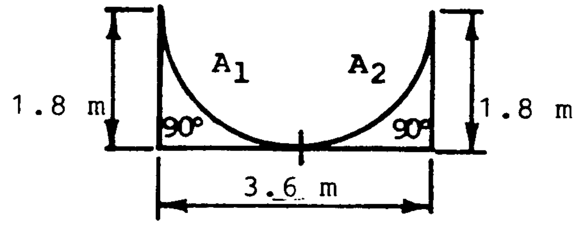


$A =$ ________ mm^2

WORK PAGE

Exercise No. 8 Continued

2.



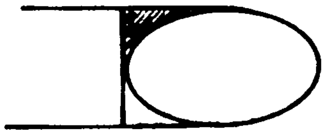
A_1 = ________ m^2

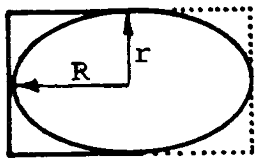
A_2 = ________ m^2

(Refer to answers on Page 207)

WORK SPACE

Suppose you run into an elliptical fillet?



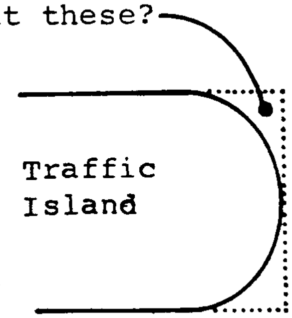


The shaded area at left has one long side and one short one. You already know the answer! Calculate the area of the rectangle that can be formed, subtract the area of the ellipse, and divide by four.

R = length of long side

r = length of short side

How about these?

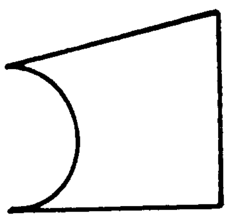


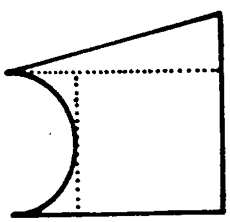
They are done the same way, of course.

R = length long side
r = length of short side

One - calculate the area of the of the total area
Two - deduct the areas of the two simulated fillets

What about this one?

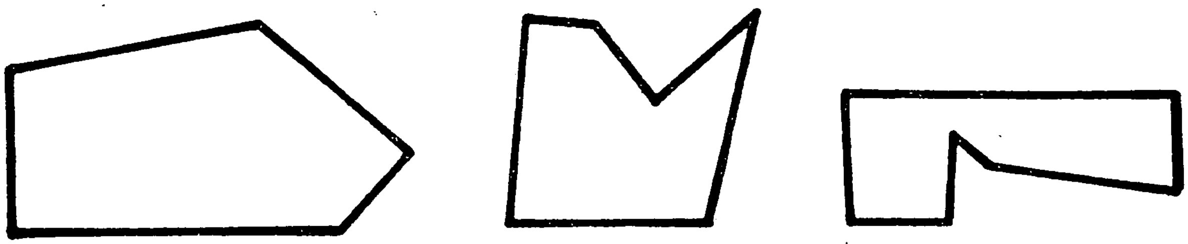




Divide the area as below, of course. Calculate the four areas separately. If you run into any fillet areas that involve other than 90° angles, work out ways of making them 90° angles by using rectangles and triangles.

'Irregular Survey Areas'

Irregular figures have more than four sides:

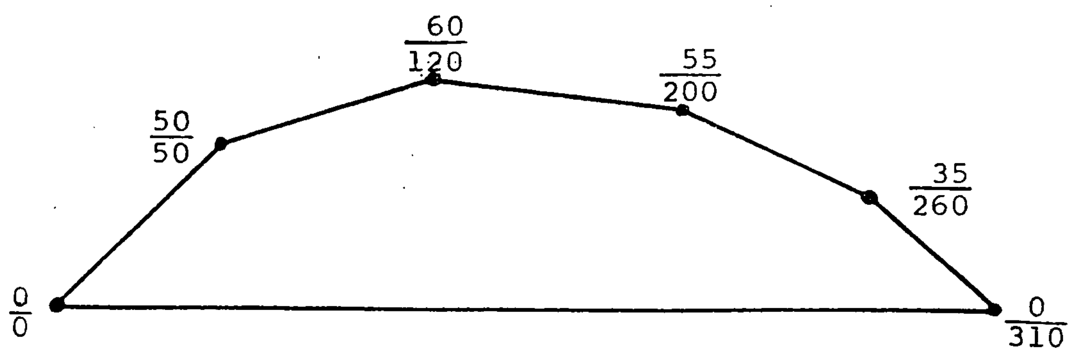


The areas of irregular figures can be calculated by the parts method used for trapeziums.

Example: Field notes for a typical area to be cleared might look like:

$\frac{0}{0}$ $\quad$ $\frac{50}{50}$ $\quad$ $\frac{60}{120}$ $\quad$ $\frac{55}{200}$ $\quad$ $\frac{53}{260}$ $\quad$ $\frac{0}{310}$

For instructional purposes, $\frac{\text{zero (0)}}{\text{zero (0)}}$ is always shown in the lower left hand corner of each figure. Point $\frac{0}{0}$ is the starting point of all measurements and all measurements are taken from it. Field notes are written like fractions. In the diagram below, you have a **"set"** of six field notes - $\frac{0}{0}$, $\frac{50}{50}$, etc. Each note expresses the numerical location of each point in relation to the $\frac{0}{0}$ starting point.



For example, the expression, $\frac{50}{50}$ tells you that point 50 is +50 meters out from point $\frac{0}{0}$ and 50 meter to the right of point $\frac{0}{0}$. It could also be 50 meters above point $\frac{0}{0}$, and 50 meter to the right of point $\frac{0}{0}$. In the one case, the figure represents a flat land area; in the other, the vertical cross-section of an area or a stock pile.

The top number refers to the vertical change from the baseline. The bottom numbers refer to horizontal change. All numbers represent "**meters**".

50-----------Vertical change
50-----------Horizontal change

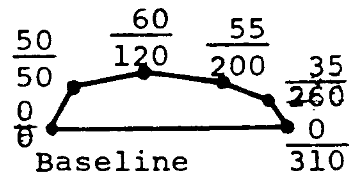


Diagram one, below, represents plotted field notes. $\frac{0}{0}$ is the starting point.

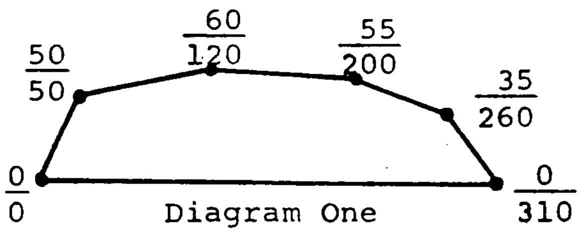
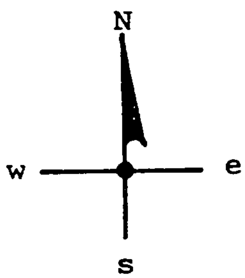


Diagram One



$\frac{50}{50}$ represents a point 50 m north and 50 m east.

$\frac{60}{120}$ represents a point 60 m north and 120 m east.

$\frac{55}{200}$ represents a point 55 m north and 200 m east.

Example: Diagram two, below, represents Diagram one divided into a series of triangles for area calculation purposes -- **this is one method of working the problem.**

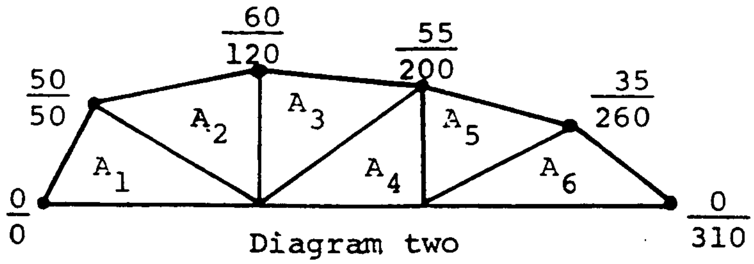


Diagram two

$$A = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$$

$A_1 = \dfrac{120\text{ m} \times 50\text{ m}}{2}$ $\quad A_2 = \dfrac{70\text{ m} \times 60\text{ m}}{2}$ $\quad A_3 = \dfrac{60\text{ m} \times 80\text{ m}}{2}$

$A_4 = \dfrac{80\text{ m} \times 55\text{ m}}{2}$ $\quad A_5 = \dfrac{55\text{ m} \times 60\text{ m}}{2}$ $\quad A_6 = \dfrac{110\text{ m} \times 35\text{ m}}{2}$

Another method of calculating the areas of irregular figures is the **"Station and Offset Method"**.

Study Diagram three carefully.

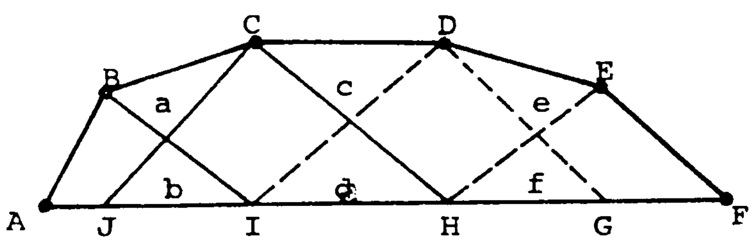


Diagram three

Note particularly that:

- Triangles a and b, c and d, e and f are equal.
- By calculating the areas of triangles ABI, JCH, IDG, and HEF, you will calculate areas b, d, and f, twice -- **but you won't calculate the areas of a, c, and e, at all.**
- The areas ABI, JCH, IDG and HEF are equal to the areas ABCJ, JCDI, IDEH and HEF.

Example: Diagram four is the same as Diagrams 1, 2, and 3 -- with the triangular areas to be calculated identified as 1, 2, 3, and 4.

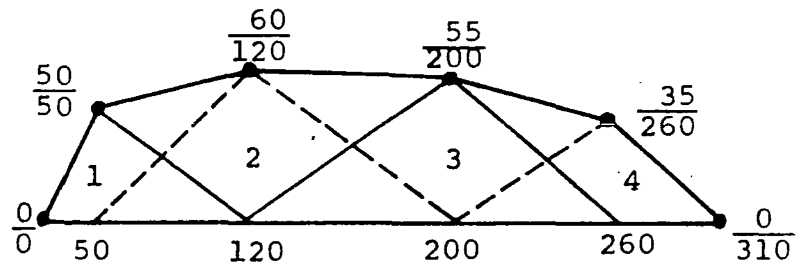


Diagram four

The Area of the total diagram = $\dfrac{A_1 + A_2 + A_3 + A_4}{2}$

$$A = \frac{120\text{ m} \times 50\text{ m}}{2} + \frac{150\text{ m} \times 60\text{ m}}{2} + \frac{140\text{ m} \times 55\text{ m}}{2} + \frac{110\text{ m} \times 35\text{ m}}{2}$$

OR

$$A = \frac{(120\text{ m} \times 50\text{ m}) + (150\text{ m} \times 60\text{ m}) + (140\text{ m} \times 55\text{ m}) + (110\text{ m} \times 35\text{ m})}{2}$$

$= 6000\text{ m}^2 + 9000\text{ m}^2 + 7700\text{ m}^2 + 3850\text{ m}^2$

$=$ **13,275 m^2 (Answer)**

Actually, you don't need the diagram. The readings $\frac{0}{0}$, $\frac{50}{50}$, $\frac{60}{120}$, $\frac{55}{200}$, $\frac{35}{260}$, $\frac{0}{310}$, can be changed to

$$\frac{(120\text{ m} \times 50\text{ m}) + (150\text{ m} \times 60\text{ m}) + (140\text{ m} \times 55\text{ m}) + (110\text{ m} \times 35\text{ m})}{2}$$

without drawing a series of overlapping triangles. You need only to figure the base lengths and heights in a series.

- the **first** base length has to be 120 m, and the first height has to be 50 m.
- the **second** base length has to be 200 m - 50 m, and the second height has to be 60 m.
- the **third** base length has to be 260 m - 120 m, and the third height has to be 55 m.

- the **fourth**, fifth and following base lengths follow in series.

Note: Most persons set up the calculations by showing height x base lengths. The formula above written with the heights first, base lengths second is shown below.

$$A = \frac{(50\text{ m} \times 120\text{ m})+(60\text{ m} \times 150\text{ m})+(55\text{ m} \times 140\text{ m})+(35 \times 110\text{ m})}{2}$$

Example: Calculate the area for the following:
The answer is shown on the next page.

$$\frac{0}{0} \qquad \frac{50}{10} \qquad \frac{110}{90} \qquad \frac{110}{300} \qquad \frac{70}{360} \qquad \frac{0}{400}$$

$$A = (50\text{ m} \times 90\text{ m})+(110\text{ m} \times 290\text{ m})+(110\text{ m} \times 270\text{ m})+(70\text{ m} \times 100\text{ m})$$

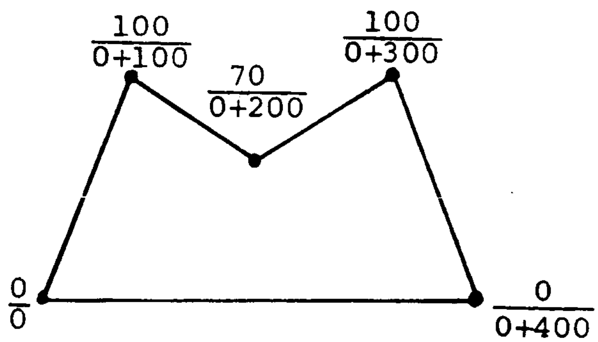
$$= \frac{4500\text{ m}^2 + 31{,}900\text{ m}^2 + 29{,}700\text{ m}^2 + 7000\text{ m}^2}{2}$$

= **36,550 square meters (Answer)**

If the field notes had been

$$\frac{0}{0} \qquad \frac{100}{0+100} \qquad \frac{70}{0+200} \qquad \frac{100}{0+300} \quad \text{and} \quad \frac{0}{0+400}$$

they would result in Diagram five below.



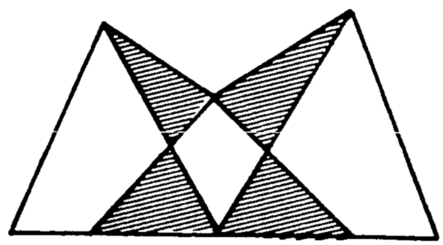
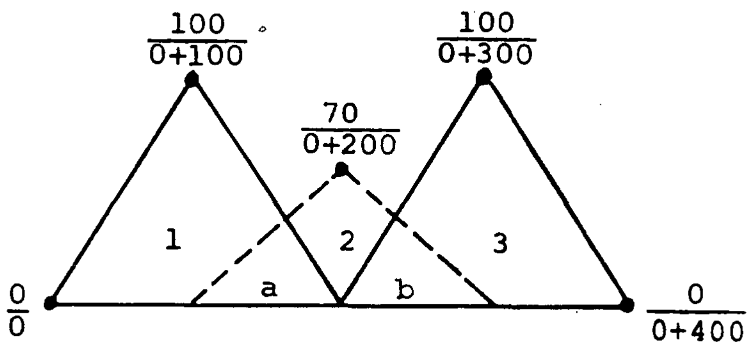


Diagram six is a copy of Diagram five. Study the four triangles that have been shaded. Note how each pair is identical to the other. The area of any one is equal to the area of any other.

Study Diagram seven:



Triangle 1, 2, and 3 in Diagram seven represent the areas that will be calculated to find the total area of Diagram five. Notice how the areas of a and b will be calculated twice: (I) a is part of both 1 and 2; (2) b is part of both 2 and 3. Notice, too, how two triangular areas in Diagram six are not shown in Diagram seven. Those two areas will not be included in the total area calculation of Diagram seven. But the two areas omitted are **offset** by the two areas that are included twice!

The total area of Diagram five is equal to the sum of the areas 1, 2, and 3 in Diagram seven. The field notes for Diagram five can be set up this way -- without the diagram:

B_1 = (0 + 200) - 0, or 200 $\quad$ H_1 = 100 m

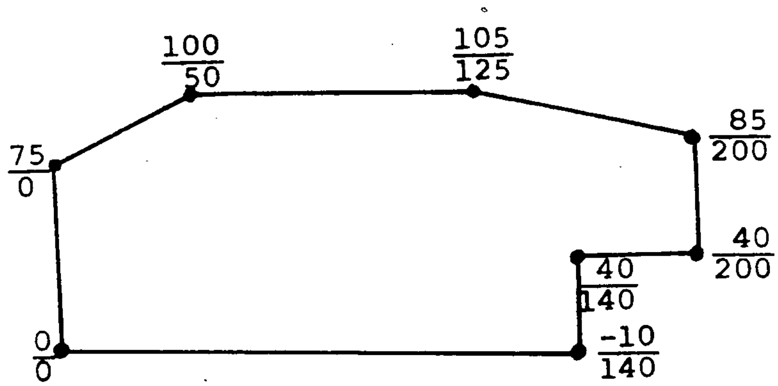
B_2 = (0 + 300) - 100, or 200 m $\quad$ H_2 = 70 m

B_3 = (0 + 400) - 200, or 200 m $\quad$ H_3 = 100 m

So, $\frac{B_1H_1}{2} + \frac{B_2H_2}{2} + \frac{B_3H_3}{2}$ = Total Area

Whatever the shape of the irregular area, the Station and Offset Method can be used.

Example: If the irregular area is shaped this way, set up your field notes and formula accordingly:



$$\frac{0}{0} \quad \frac{75}{0} \quad \frac{100}{50} \quad \frac{105}{125} \quad \frac{85}{200} \quad \frac{40}{200} \quad \frac{40}{140} \quad \frac{-10}{140} \quad \frac{0}{0}*$$

$$A = \frac{(75\text{ m} \times 50\text{ m}) + (100\text{ m} \times 125\text{ m}) + (105\text{ m} \times 150\text{ m})}{2}$$

$$\frac{+(85\text{m} \times 75\text{ m}) + (40\text{ m} \times -60\text{ m}) + (40\text{ m} \times -60\text{ m}) + (-10\text{ m} \times -140\text{ m})}{2}$$

$$A = \frac{3750\text{ m}^2 + 12{,}500\text{ m}^2 + 15{,}750\text{ m}^2 + 6375\text{ m}^2}{2}$$

$$\frac{-\ 2400\text{ m}^2 - 2400\text{ m}^2 + 1400\text{ m}^2}{2}$$

$$= \frac{34{,}975}{2}\text{ m}^2$$

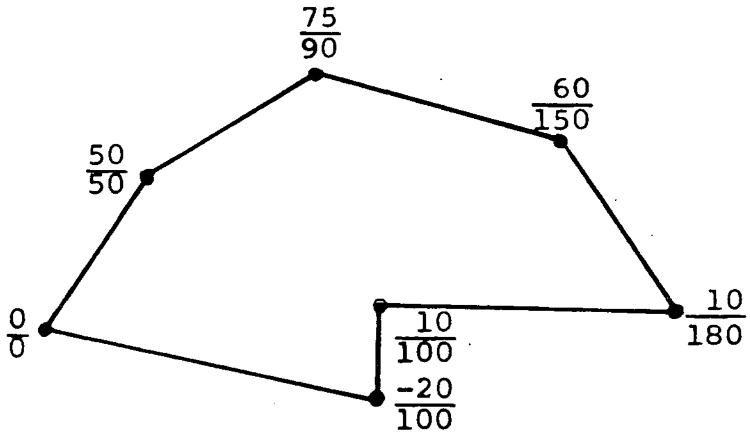
$= \mathbf{17{,}487.5\ m^2}$ **(Answer)**

Note the minus signs.
Note also that a minus value multiplied by a minus value results in a plus value.

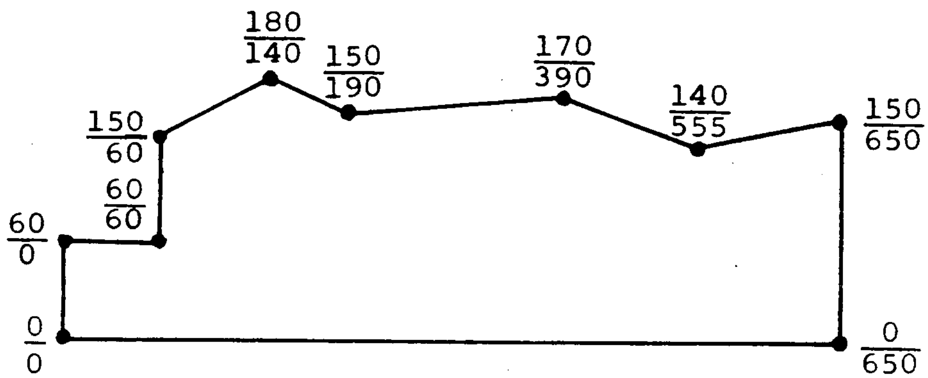
* **Note:** $\frac{0}{0}$ must be included at the beginning and at the end.

EXERCISE NO. 9: Calculate the areas:

1.



2.



(Refer to answers on Page 207)

WORK PAGE

SECTION II

Chapter 2 Quiz

1. Calculate Area:

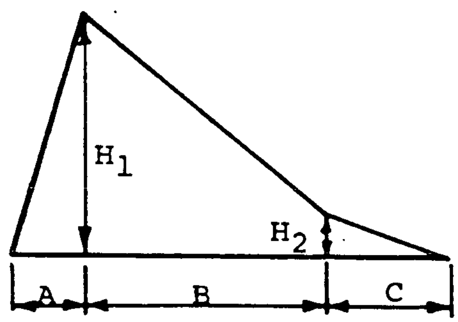
H_1 = 40 m

H_2 = 8 m

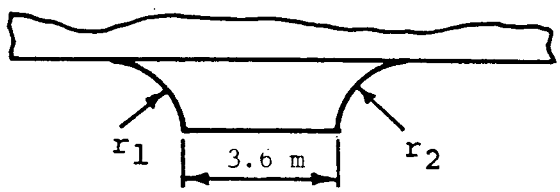
A = 10 m

B = 35 m

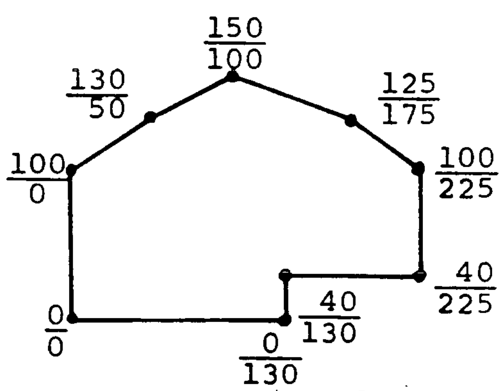
C = 20 m



2. Calculate the area of the following driveway entrance in square meters. The fillets are 90°. r_1 = 3 m r_2 = 3 m



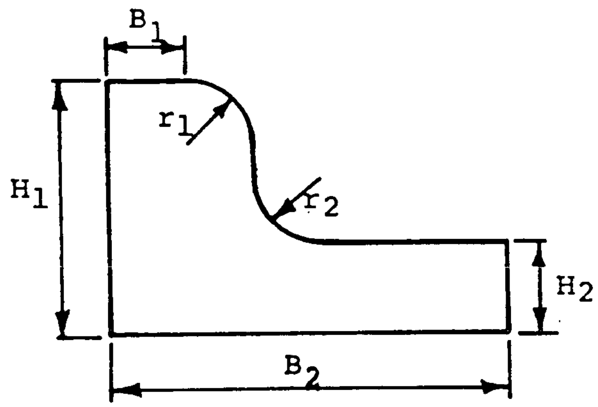
3. Determine the number of square meters to be cleared--based on the following field notes:



(Refer to answers on Page on 208)

WORK PAGE

4. Calculate the end area of this curb and gutter section in square meters.



$H_1 = 600$ mm

$H_2 = 200$ mm

$r_1 = 150$ mm

$r_2 = 150$ mm

$B_1 = 150$ mm

$B_2 = 750$ mm

(Refer to answers on Page 209)

WORK SPACE

WORK PAGE

SECTION II

CHAPTER 3 Volumes

VOLUMES

Volume problems are more time consuming, but not any more complicated than area problems. Generally there are only three formulas for volumes:

$V = AH$ Volume = Area times Height

$V = (\bar{x}A)H$ Volume = Average Area times Height*

$V = A(H/3)$ Volume = Area times one third of the Height

"Length" is substituted for **"height"** in many calculations.

V = AH or V = AL applies to all objects having equal end areas.

Figure One

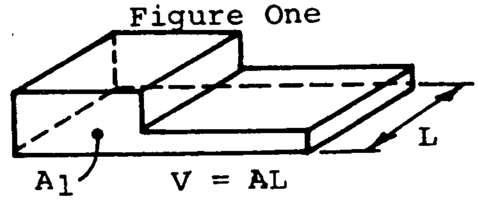
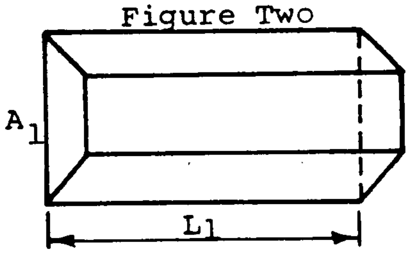
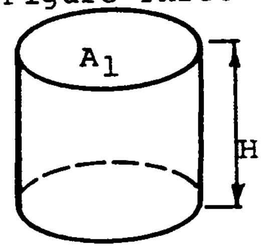


Figure Two



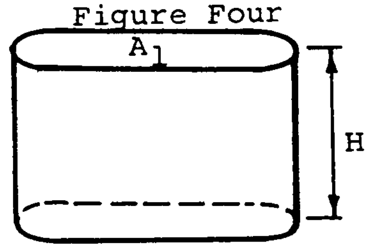
V = AL

Figure Three



V = AH

Figure Four



V = AH

As with areas that have to be converted from square millimeters to square meters, volumes often have to be converted to cubic millimeters to cubic meters. Sometimes cubic meters must be converted to liters. As a reminder:

One cubic meter = 1,000,000,000 mm^3
One cubic meter = 1×10^9 mm^3
One cubic meter = 1000 liters

Example: Suppose you have a cube with one face having 100 square millimeters area.

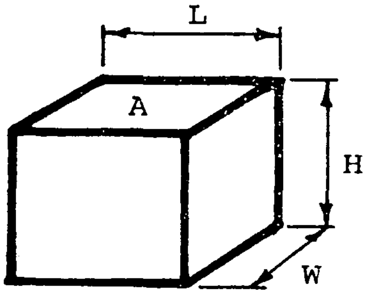
What is the volume?

Recall that the volumes of cubic and rectangular solids are calculated by using either of two formulas:

V = L W H or V = Volume; L = Length; W = Width
H = Height; A = Area

*Average area, $\bar{x}A$, will be discussed later on.

Figure Five



Since $A = 100\ mm^2$ and since $V = AH$, you need only to know the value of H! Since, in a cube, L, W, and H are equal, H must be the square root of $100\ mm^2$! The square root of $100\ mm^2$ is 10, so H = 10 mm.

$$V = 100\ mm^2 \times 10\ mm = \mathbf{1000\ mm^3}$$ **(Answer)**

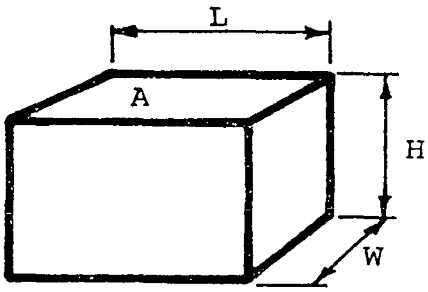
That's one cubic meter and the equivalent of 1000 liters.

Example: Suppose you had a rectangular solid where area of one end equals 400 square meters. Height equals 30 meters. V = ?

Since V = AH, and $A = 400\ m^2$

$$V = 400\ m^2 \times 30\ m$$
$$= \mathbf{12{,}000\ m^3}$$ **(Answer)**

Figure Six



You don't have to know the L and W values of rectangular solids to calculate their volumes. You need only know (1) that it is a rectangular solids so that the opposite surface areas are equal, (2) the area of one surface and (3) the length of the dimension not used in calculating the area.

In Figure Six, for instance, you can calculate the area of LH, LW, or WH first. If the area is LH, the volume has to be LH x W. If the area is LW, the volume has to be LW x H, or if the area is WH, the volume has to be WH x L.

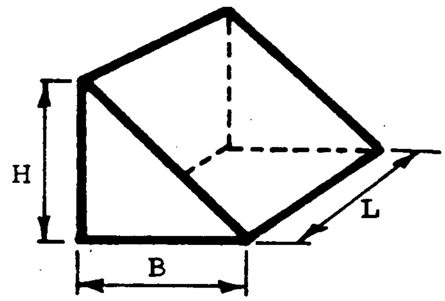
EXERCISE NO. 1

1. Cube $H = 3$ m

   $A =$ __________ m^2

   $V =$ __________ m^3

2. Triangle Solid



Shaded Area = 72 mm^2

$L = 12$ mm

$V =$ __________ mm^3

(Refer to answers on Page 210)

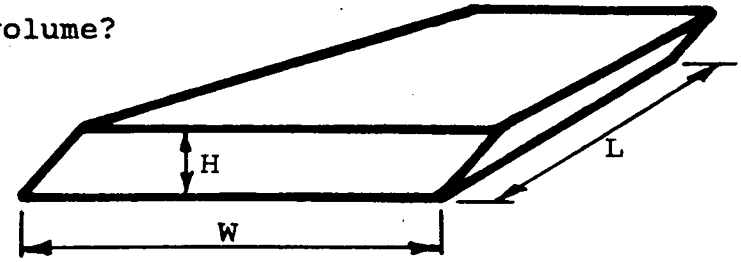
WORK PAGE

Parallelogram Solids

The volumes of solids having equal parallelogram end areas are calculated the same way as other sides: V = AL or V = AH. Be careful and watch end area calculations.

Example: What is the volume?

H must be vertical!

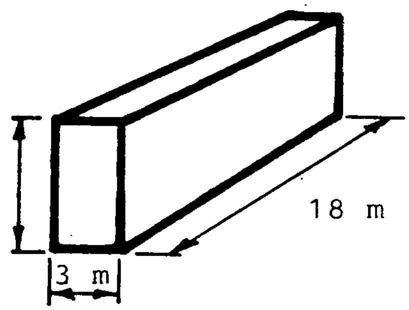


L = 12 m W = 10 m H = 1.5 m V = ________ m^3

V = AL
A = WH = 10 m x 1.5 m = 15 m^2
V = 15 m^2 x 12 m = **180 m^3 (Answer)**

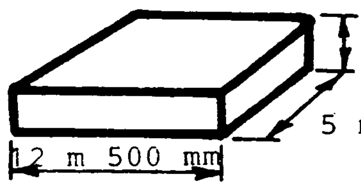
EXERCISE NO. 2

1. Calculate these volumes in cubic meters.



V = ______________ m^3
tenths

2.

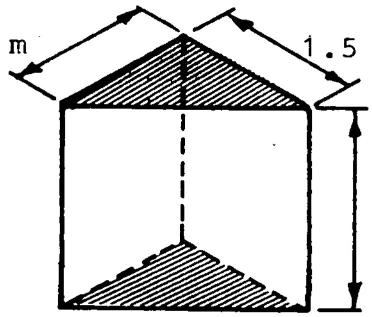


V = ______________ m^3
tenths

WORK PAGE

Exercise No. 2 Continued

3.



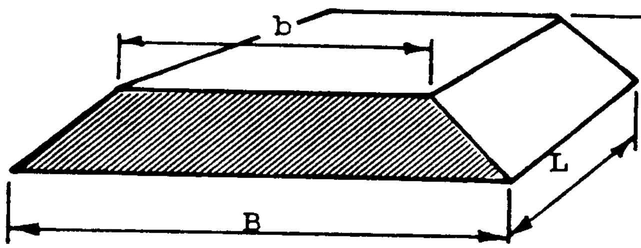
V = ______________ m^3
tenths

(Refer to answers on Page 210)

'Trapezoidal Solids'

Trapezoidal solids are three-dimensional trapezoidal figures.

$A - \frac{B + b}{2}(H)$ $\qquad$ $V = A \times L$



Earlier, you calculated the area of a trapezoid. If both end areas are equal, the volume calculation is simply end area times length.

Example: If B = 50 m b = 25 m H = 10 m L = 30 m

End area = $\frac{B + b}{2}(H)$ and V = A x L

$A = \frac{(50\text{ m} + 25\text{ m})}{2} 10\text{ m}$

$A = 375\text{ m}^2$

$V = A \times L$
$= 375\text{ m}^2 \times 30\text{ m}$

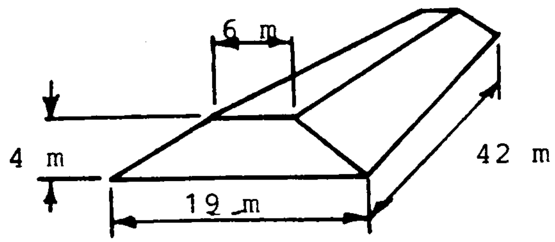
$=$ **11,250 m^3 (Answer)**

WORK PAGE

EXERCISE NO. 3

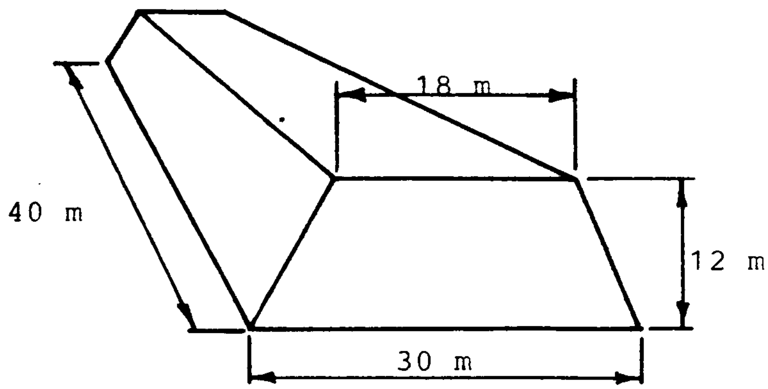
Compute the volumes of each of the following trapezoidal solids--in cubic meters. Assume the opposing end areas are equal.

1.



V = ________ m^3
tenths

2.



V = ________ m^3
tenths

(Refer to answers on Page 210)

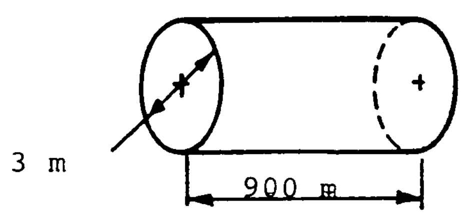
'Cylinders- Round and Elliptical'

The volumes of circular and elliptical cylinders having equal end areas are based on the formula: **V = AH or V = AL**

Example: Determine the volumes in cubic meters and liters of these cylinders.

V ________ m^3
tenths

V = ________ L
tenths



WORK PAGE

The available data: D = 3 m, L = 900 m

The formula: V = AL, so V = A x 900 m.

What is the value of A?

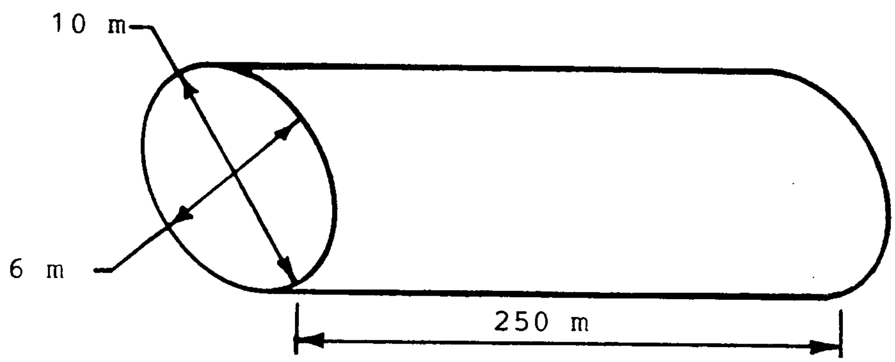
$A = \pi r^2$, and r = D/2 = 3/2 = 1.5 m

So, A = 3.1416 (1.5 m x 1.5 m) = 3.1416 x 2.25 m^2

= 7.07 m^2

V = 7.07 m^2 x 900 m
= **6363.00 m^3**
or V = **6,363,000 L (Answer)**

Example:



V = __________ m^3

V = __________ L

The available data: D = 10 m d = 6 m L = 250 m

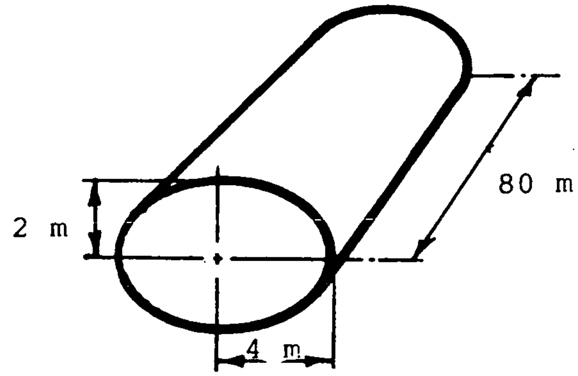
The formula: V = A x L A = π(Rr)
R = Diameter/2 r = diameter/2

So, R = 5 m r = 3 m and
A = (3 m x 5 m)3.1416 = 47.124 m^2
V = 47.12 m^2 x 250 m = 11,781.00 m^3
= 11,781 m^3 x 1000 L/1 m^3
= **11,781,000 L (Answer)**

EXERCISE NO. 4

Find the following volumes:

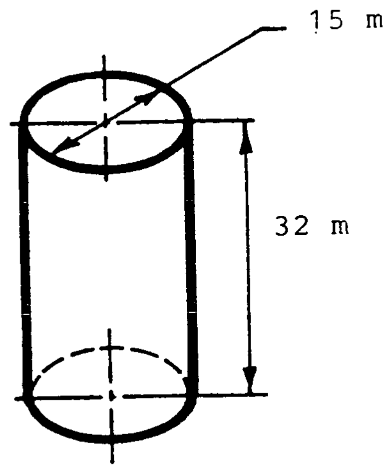
1.



V = ________ m^3
tenths

V = ________ L
tenths

2.



V = ________ m^3
tenths

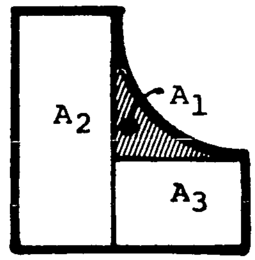
V = ________ L
tenths

(Refer to answers on Page 210)

WORK PAGE

'Fillet Area Solids'

As with cylinders, the volumes of fillet areas are calculated by multiplying the end area by the height. The volume a fillet area solid includes the volume of the fillet area plus the volumes of the rectangular solids.



The shaded area is the fillet area. It is identified here as A Areas 2 and 3 are rectangles.

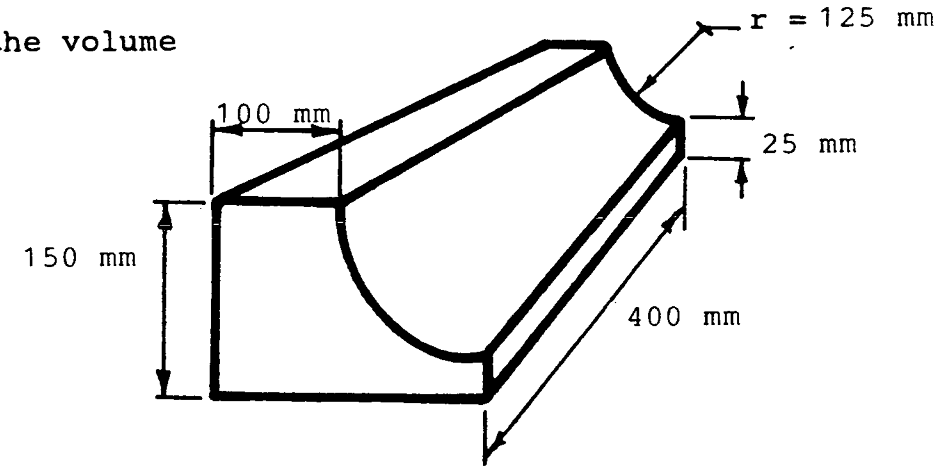
The formula is $V = (A_1 + A_2 + A_3)L$.

Remember the characteristics of fillets:

1. The **length of the side** equals the length of the **radius** of circle that would be formed.
2. The length of the side equals half the length of the square that would be formed.
3. The area equals one fourth the **"area of the square minus the area of the circle."**

EXERCISE NO. 5

Calculate the volume



V = ______________ mm^3 (hundredths)

(Refer to answer on Page 210)

WORK PAGE

EXERCISE NO. 6

All solids having equal end areas can be thought of as simple solids -- because their volumes are equal to one area times height or length.

The Quiz below is meant to summarize certain area and volume characteristics of simple solids. See if you can answer all correctly without looking back into the section.

1. Volume of cube = 729 m^3 A = 81 m^2 H = ______________

2. Area of parallelogram-shape solid = 200 m^2 and the Volume = 20,000 m^3. Length = ______________

3. Area and R of ellipse are known. What is the formula for finding r? ___________

4. One end area and the length of a uniform cylinder are known. What is the formula for volume? ___________________

5. What is the formula for the area of an ellipse? __________

6. What are the formulae for the volume of a cube? Show two: ______________________; ________________________

7. A cubic meter (m^3) = _____________ cubic millimeters (mm^3)

8. A cubic meter (m^3) = _____________ liters (L)

9. A square meter (m^2) = _____________ square millimeters (mm^2)

10. A square kilometer (km^2) = _____________ square meters (m^2)

(Refer to answers on Page 210)

WORK PAGE

Discussion on Studying

If you are having difficulty following the volume calculations in this chapter, the difficulty most likely is attributable to one of the following:

1. The rounding of values. If the final answer is to be in tenths, that answer has to be in hundredths before rounding. Therefore, the values used in the calculations that result in the final answers are rounded to hundredths.

   If the final answer is to be in hundredths, the values used to get that answer are rounded to thousandths.

   If your problem is this one, review **"Rounding and Accuracy," Section I.**

2. The lack of detailed calculations in this chapter. All calculations involve addition, subtraction, multiplication and division -- either of whole numbers, decimal numbers, or fractions.

   If each calculation was shown in this book, only a very few persons would find them useful. If you would start checking your calculations by working backwards. You will find your errors -- and get the same answers as are shown in the answer book.

3. The use of formulas -- particularly letters, symbols and constant values. The more you work with formulas, the more familiar and useful they become to you. When working a problem, try and imagine what formulas could simplify the problem, and then, begin to write on paper. Often times the solution of a tough problem is just a matter of thinking it through.

'Average End Area Calculations'

The volumes of solids having different-sized end areas are calculated by multiplying the average of the two end areas by the length. Volume = $(\bar{x}A)L$. The volume of Figure eight is calculated with the formula:

$$V = \frac{(A_1 + A_2)}{2} L$$

or

$$V = (\bar{x}A)L$$

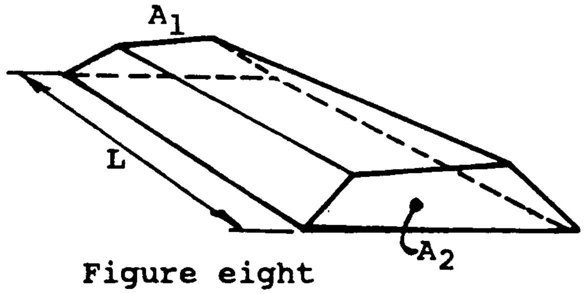


Figure eight

Example: Assume the data below applies to Figure eight. What is the volume?

$A_1 = 50\ m^2$

$A_2 = 150\ m^2$

$L = 42\ m$

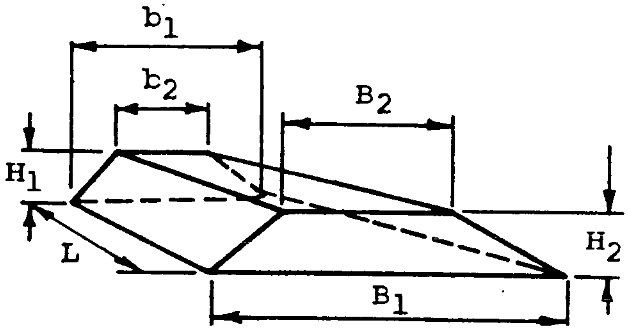
$V =$ ______________ m^3

$$V = \frac{(A_1 + A_2)}{2} \times L = \left(\frac{50\ m^2 + 150\ m^2}{2}\right) \times 42\ m$$

$= 100\ m^2 \times 42\ m =$ **$4200\ m^3$** **(Answer)**

EXERCISE NO. 7

1. Calculate the average end area and the volume:



b_1 = 25 m b_2 = 15 m H_1 = 8 m B_1 = 37.5 m B_2 = 23.9 m
H_2 = 9.6 m L = 43.5 m

Average end area = ____________________ m^2 (hundredths)

Volume = ____________________ m^3 (tenths)

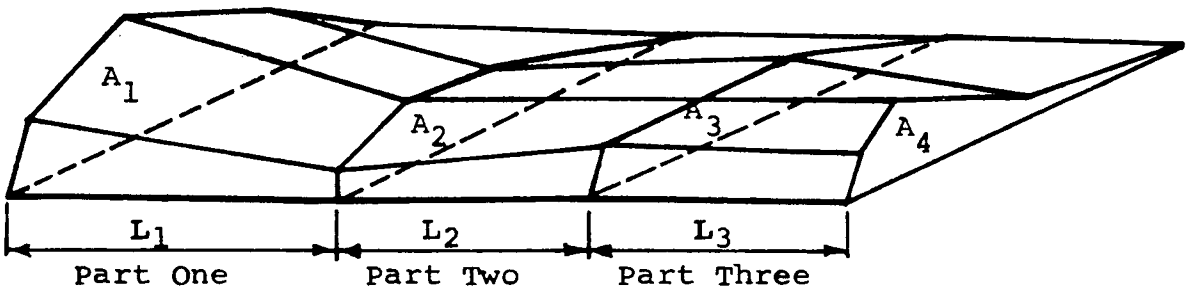
(Refer to answers on Page 210)

WORK SPACE

'Cross-Section Calculations'

Figure nine represents an irregularly-shaped solid. The shaded areas are cross-sectional areas -- cuts that provide end areas.

To calculate the volumes of solids as irregularly shaped as this one, it is necessary to (1) calculate the area of a cross-sectional surface face wherever the topography changes, (2) use the average-end-area-times-length approach to find the volume of each part, and (3) add the volumes of the parts to find the volume of the whole.



Example: Calculate the volume.

Assume the following data apply to Figure nine:

Area, cross-section 1 = 360 m^2 L_1 = 120 m
Area, cross-section 2 = 230 m^2
Area, cross-section 3 = 180 m^2 L_2 = 90 m
Area, cross-section 4 = 220 m^2
L_3 = 100 m

Calculate the volume.

$$V = \left(\frac{360\ m^2 + 230\ m^2}{2}\right) \times 120\ m + \left(\frac{230\ m^2 + 180\ m^2}{2}\right) \times 90\ m$$

$$+ \left(\frac{180\ m^2 + 220\ m^2}{2}\right) \times 100\ m$$

$$= (295\ m^2 \times 120\ m) + (205\ m^2 \times 90\ m) + (200\ m^2 \times 100\ m)$$

$$= 35{,}400\ m^3 + 18{,}450\ m^3 + 20{,}000\ m^3$$

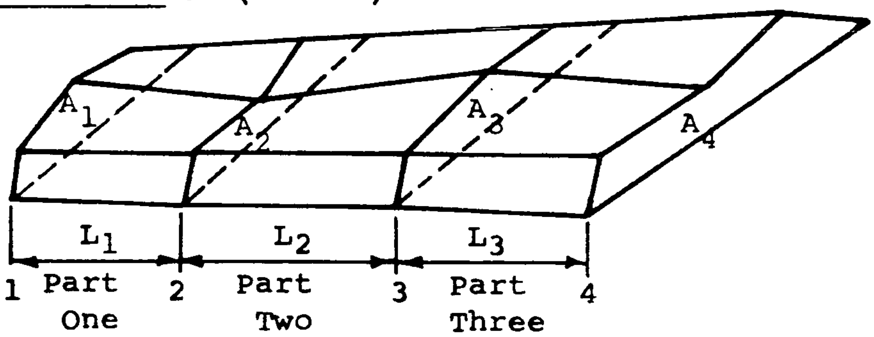
= **73,850 m^3 (Answer)**

EXERCISE NO. 8

1. Compute the total volume of the following figure:

Cross-section 1 = 130 m^2 — L_1 = 40 m
Cross-section 2 = 150 m^2 — L_2 = 30 m
Cross-section 3 = 170 m^2 — L_3 = 20 m
Cross-section 4 = 200 m^2

V = ________________ m^3 (tenths)



(Refer to answer on Page 210)

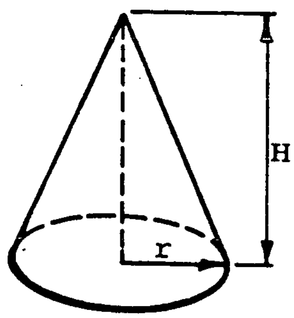
'Cones'

All cones are one third the size of a cylinder they would make. V = (H/3) A. Volume equals area times one-third of the height.

Example: Using the formula V =(H/3)A, calculate the volume of this cone:

H = 12 m
r = 5 m

V = **314.16 m^3 (hundredths)**

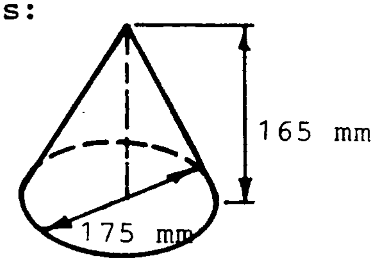


$V = (H/3)A$
$= 12/3(\pi(5)^2)$
$= 4(3.1416 \times 25)$
$= 314.16\ m^3$

EXERCISE NO. 9

Calculate the volumes:

1.

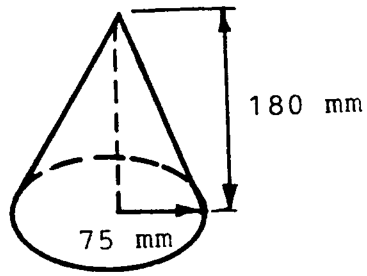


V = ____________ mm^3
(tenths)

WORK PAGE

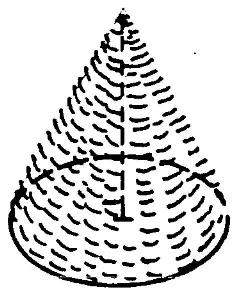
Exercise No. 9 Continued

2. Calculate the volume:



V = ______________ mm^3
(tenths)

3. Calculate the volume of this stockpile. (use π = 3.14)



Circumference = 31.4 m
Height = 9 m

Volume = ____________ m^3
(tenths)

(Refer to answers on Page 210)

'Frustrums of Cones'

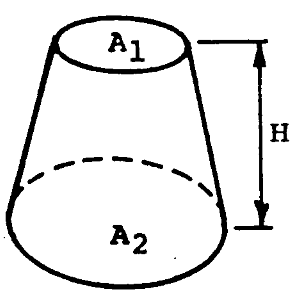
A frustrum of a cone is part of a cone. The formula for the volume is:

$$V = \frac{H}{3}\left[(A_1 + A_2) + \sqrt{A_1 \times A_2}\,\right]$$

Volume equals one-third of the height times the total two end areas and the square root of one end area times another.

Example:

A_1 = 5.5 m^2
A_2 = 8.6 m^2
H = 6.3 m
V = ______________ m^3



WORK PAGE

$V = [(A_1 + A_2) + \sqrt{A_1 \times A_2}](H/3)$

$V = [(5.5\ m^2 + 8.6\ m^2) + \sqrt{5.5 m^2 \times 8.6\ m^2}]\ 6.3/3$

$= [14.1\ m^2 + \sqrt{47.30 m^2}]2.1\ m$

$= (14.1\ m^2 + 6.88\ m^2)2.1\ m$

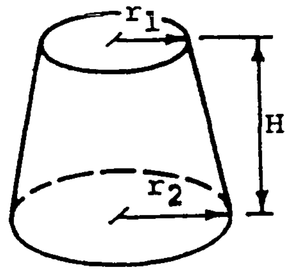
$= 20.98\ m^2 \times 2.1\ m = 44.058\ m^3 =$ **$44.1\ m^3$ (Answer)**

Note: A_1 is parallel to A_2 in all frustrums.

EXERCISE NO. 10

1. $r_1 = 2.3$ m
   $r_2 = 3.6$ m
   $H = 7.1$ m

   V = ________ m^3
   (tenths)

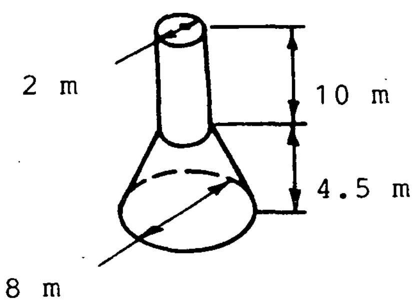


(Refer to answer on Page 210)

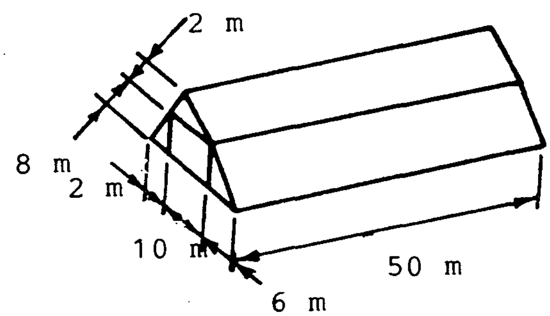
'Compound Volumes -- Structures'

Some compound volumes are sketched below:

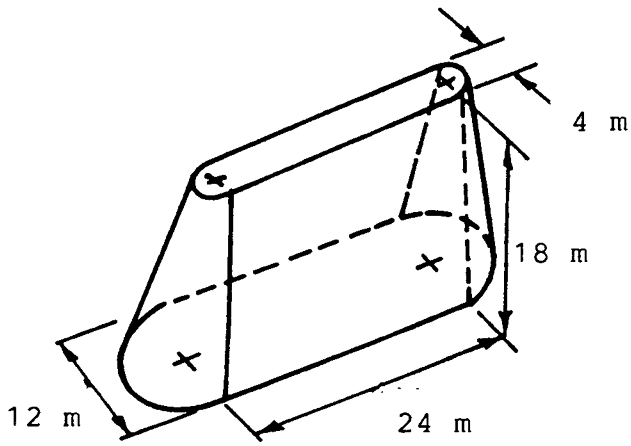
Bridge bent

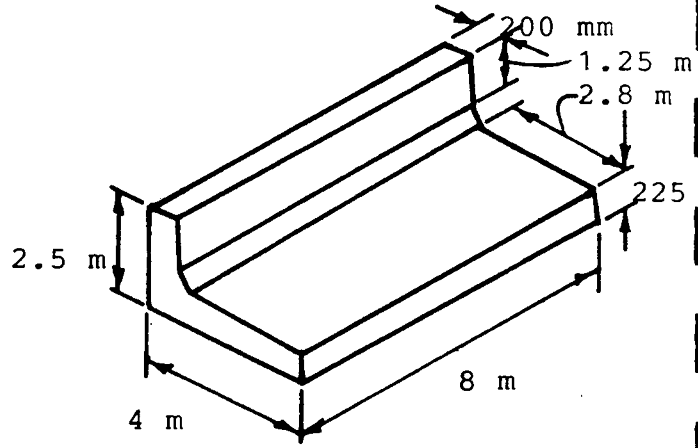


Portland cement concrete structures



Bridge bent

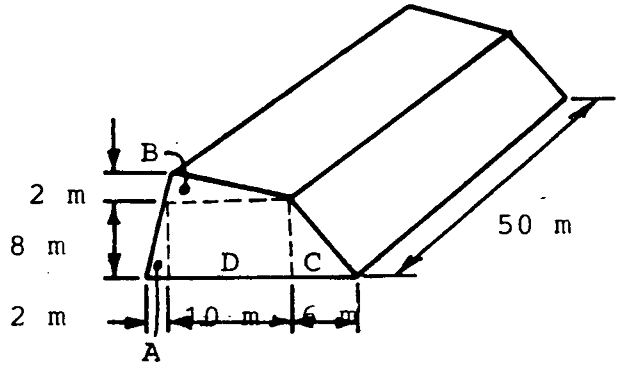




WORK PAGE

Compound volume calculations can be divided into different series of separate calculations. Total volumes are the result of adding all the part volumes.

Example: The compound figure below can be divided into four solids: three triangular solids--A, B, and C, and one rectangular solid--D. Calculate the volume:



V = ______________ m³ (tenths)

Volume A = Area of A x Length

$= \frac{bh}{2} \times 50\ m$

$= \frac{2\ m \times 8\ m}{2} \times 50\ m =$ **400 m³**

Volume B = Area of B x Length

$= \frac{2\ m \times 10\ m}{2} \times 50\ m =$ **500 m³**

Volume C = Area of C x Length

$= 6\ m \times 8\ m \times 50\ m =$ **1200 m³**

Volume D = Length x Width x Height

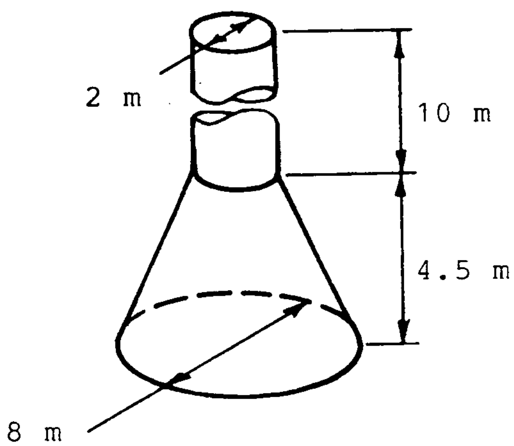
$= 10\ m \times 8\ m \times 50\ m =$ **4000 m³**

Total Volume = Volumes of **A** + **B** + **C** + **D**

$= 400\ m^3 + 500\ m^3 + 1200\ m^3 + 4000\ m^3$

= **6100 m³ (Answer)**

<u>EXERCISE NO. 11</u> Calculate the following volume:



SECTION II ANSWERS

Chapter 1:

Exercise No. 1, Page 132:

(1) 43.20 (2) 14.32 (3) 1139.54

Exercise No. 2, Page 134:

(1) 3481 (2) 1 (3) 1.7 (4) 6.86

Exercise No. 3, page 134:

(1) 6.7082 (2) 7 (3) 551,368 (4) 512
(5) 16

Exercise No. 4, Page 136:

(1) 9.4604 (2) 8.4379 (3) 3.5490

Chapter 1 Quiz, Page 138:

(1) (a) $\sqrt{27} = 5.1962$ (b) $18^3 = 5832$ (c) $17^2 = 289$

(2) (a) $\sqrt{8^3} = 512 = 22.6274$ (b) $\sqrt{4 \times 16} = 8$

(c) $12^2 \times 13 \times \sqrt{7} = 144 \times 13 \times 2.6458 = 4952.9376$ which rounded is 4952.9

(d) $\sqrt{4 + 16} = \sqrt{20} = 4.4721$ which rounded is 4.5

(3) Volume $= 1/3\ H(B + b + \sqrt{(Bb)})$

$= 1/3\ (9.7) \times (15 + 7 + \sqrt{15 \times 7})$
$= 3.23 \times 32.25 = 104.2651$, which rounded is $104.3\ m^3$

SECTION II ANSWERS

Chapter 2:

Exercise No. 1, Page 143:
(1) 567 m^2 (2) 91.0 m^2 (3) 54.72 m^2

Exercise No. 2, Page 146:
(1) 44.07 m^2 (2) 20.75 m^2 (3) 143.64 m^2

Exercise No. 3, Page 148:
(1) 5 m (2) 5 m (3) $\sqrt{48} = 6.9282$

Exercise No. 4, Page 150:
(1) 604.9 m^2 (2) 398,079.6 mm^2 = 0.4 m^2

Exercise No. 5, Page 154:
(1) 280 m^2 (2) 184.25 m^2

Exercise No. 6, Page 156:
(1) D = 50.00 m r = 25.00 m A = 1963.50 m^2
(2) D = 800 mm C = 2513.28 mm
A = 502,656.00 mm^2 = 0.50 m^2

Exercise No. 7, Page 160:
(1) 853,437.7 mm^2 (2) 42.2 mm^2

Exercise No. 8, Page 162:
(1) 8584.07 mm^2 (2) 0.70 m^2 0.70 m^2

Exercise No. 9, Page 171:
(1) 9050 m^2 (2) 78.92 m^2

Chapter 2 Quiz: Page 175:

(1) $$A = \frac{(AH_1)}{2} + \frac{(H_1 + H_2)B}{2} + \frac{CH_2}{2}$$
$$= \frac{(10\ m \times 40\ m)}{2} + \frac{(40\ m + 8\ m \times 35\ m)}{2} + \frac{(20\ m \times 8\ m)}{2}$$
$$= 200\ m^2 + 840\ m^2 + 80\ m = 1120\ m^2$$

(2) $$A = (LW) + \frac{2(2r_1 \times 2r_1 - \pi r_1^2)}{4}$$
$$= (3.6\ m \times 3\ m) + \frac{2(6\ m \times 6\ m - (3.1416 \times 9\ m))}{4}$$
$$= 10.8\ m + \frac{2(7.73\ m^2)}{4} = 14.66\ m^2$$

(3)
Field notes $\frac{0}{0}$ $\frac{100}{0}$ $\frac{130}{50}$ $\frac{150}{100}$ $\frac{125}{175}$ $\frac{100}{225}$ $\frac{40}{225}$ $\frac{40}{130}$ $\frac{0}{130}$

SECTION II ANSWERS

Chapter 2 Quiz: Page 175 No. 3 Continued

Area formula:

$$A = \frac{(100\ m \times 50\ m) + (130\ m \times 100\ m) + (150\ m \times 125\ m)}{2}$$

$$\frac{+ (125\ m \times 125\ m) + (100\ m \times 50\ m) + (40 \times -95\ m)}{2}$$

$$\frac{+ (40\ m \times -95\ m)}{2}$$

$$= \frac{500\ m^2 + 13000\ m^2 + 18750\ m^2 + 15625\ m^2 + 5000\ m^2}{2}$$

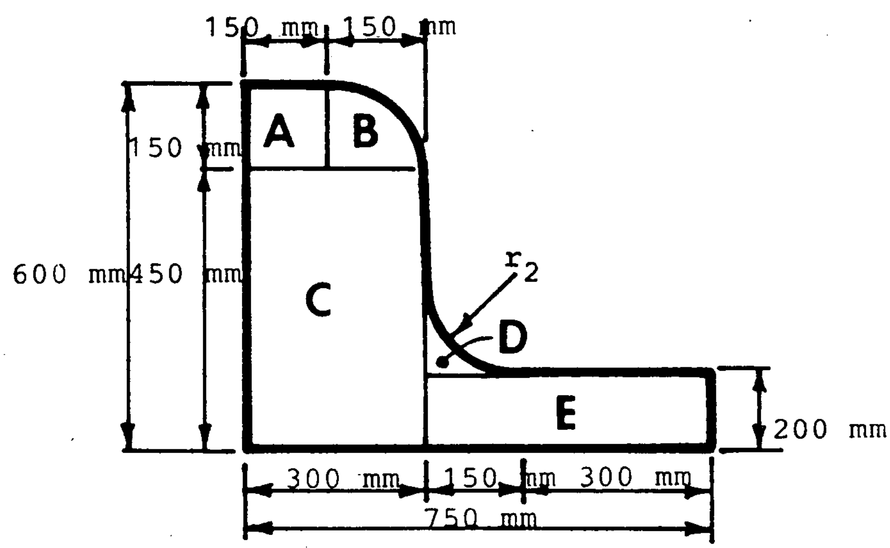
$$\frac{- 3800\ m^2 - 3800\ m^2}{2}$$

$$= 24887.5\ m^2$$

Chapter 2 Quiz, Page 175 Continued

(4) The end area can be broken up a number of ways. The figure shown below will be used in the calculations.

The unknown distances are determined by extending the radius lengths----as shown below.



Area A = 150 mm x 150 mm = **22,500.00 mm^2**

Area B = $1/4(\pi r^2)$ = 1/4 x 3.1416 x $(150)^2$
= 1/4 x 70,686.00 mm^2
= **17,671.50 mm^2**

Area C = 300 mm x 450 mm = **135,000 mm^2**

Area D = (Area of a square -- having a 300 mm side -- minus the area of a circle -- having a 150 mm radius)*1/4
$= \dfrac{90{,}000 \text{ mm}^2 - 70{,}685.8347 \text{ mm}^2}{4}$
= **4828.50 mm^2**

Area E = 450 x 200 m = **90,000 mm^2**

End Area = Areas **A** + **B** + **C** + **D** + **E** = **270,000 mm^2 = 0.3 m^2**

SECTION II ANSWERS

Chapter 3:

Exercise No. 1, Page 180:

(1) $A = 9\ m^2$ $V = 27\ m^3$
(2) $V = 864\ m^3$

Exercise No. 2, Page 184:

(1) 12.0 m (2) 125.9 m^3 (3) 3.4 m^3

Exercise No. 3, Page 186:

(1) 2100.0 m^3 (2) 11,520 m^3

Exercise No. 4, Page 189:

(1) $V = 2010.6\ m^3$ $V = 2.0$ L
(2) $V = 5654.9\ m^3$ $V = 5.7$ L*

Exercise No. 5, Page 191:

(1) $V = 8,591,261.48\ mm^3 = 0.01\ m^3$

Exercise No. 6, Page 193:

(1) 9 m (2) 100 m (3) $r = A/\pi R$
(4) $V = AL$ (5) $A = \pi(Rr)$
(6) $V = H^3$ or $V = AH$ or
$V = L^3$ or $V = AL$ or
$V = W^3$ or $V = AW$ or
$V = LWH$
(7) $1x(10^9)\ mm^3$ (8) $1x(10^6)$ L (9) $1x(10^6)\ mm^2$
(10) $1x(10^6)\ m^2$

Exercise No. 7, Page 197:

(1) Average end area = 227.36 m^2
Volume = 9890.2 m^3

Exercise No. 8, Page 199:

(1) Volume = 14,100.0 m^3

Exercise No. 9, Page 201:

(1) $V = 1,322,905.0\ mm^3$ (2) $V = 1,060,287.5\ mm^2$
(3) 235.5 m^3

Exercise No. 10, Page 202:

(1) Volume 197.3 m^3

Exercise No. 11, Page 205:

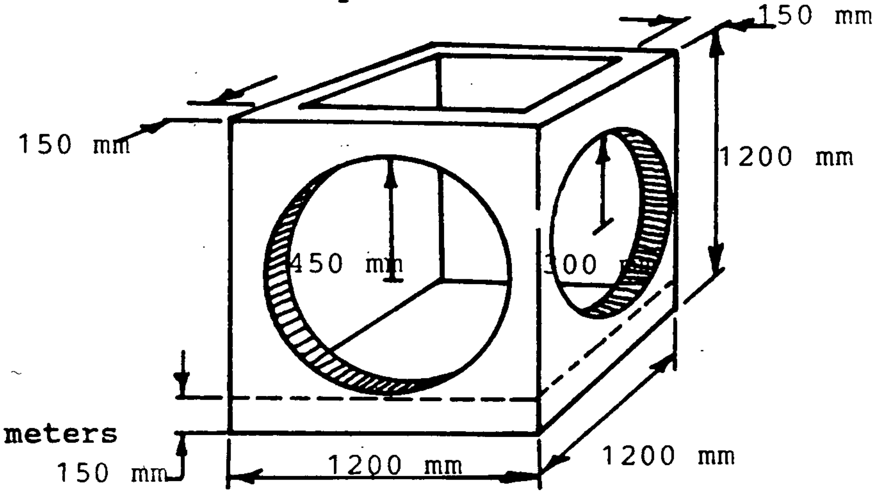
(1) Volume = 130.37 m^3

***Do not round the answer until the last step.**

SECTION II

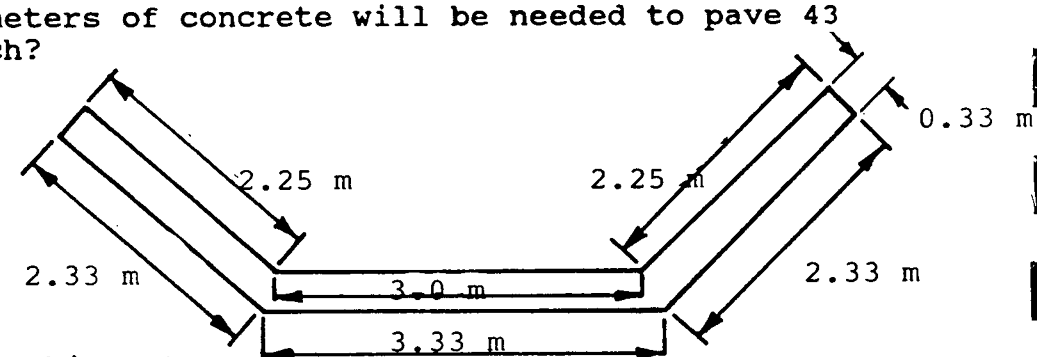
Chapter 3 Quiz

1. Compute the cubic meters of concrete required for this structure:



V = ___________ cubic meters
(tenths)

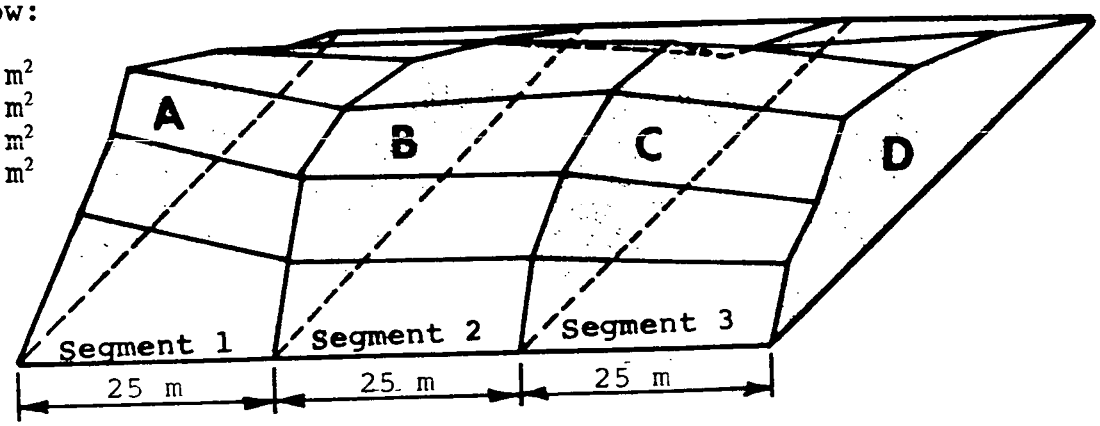
2. How many cubic meters of concrete will be needed to pave 43 meters of this ditch?



V = __________ cubic meters

3. Compute the total cubic meters of material contained in the stockpile below:

A = 98.3 m^2
B = 103.8 m^2
C = 105.4 m^2
D = 92.9 m^2



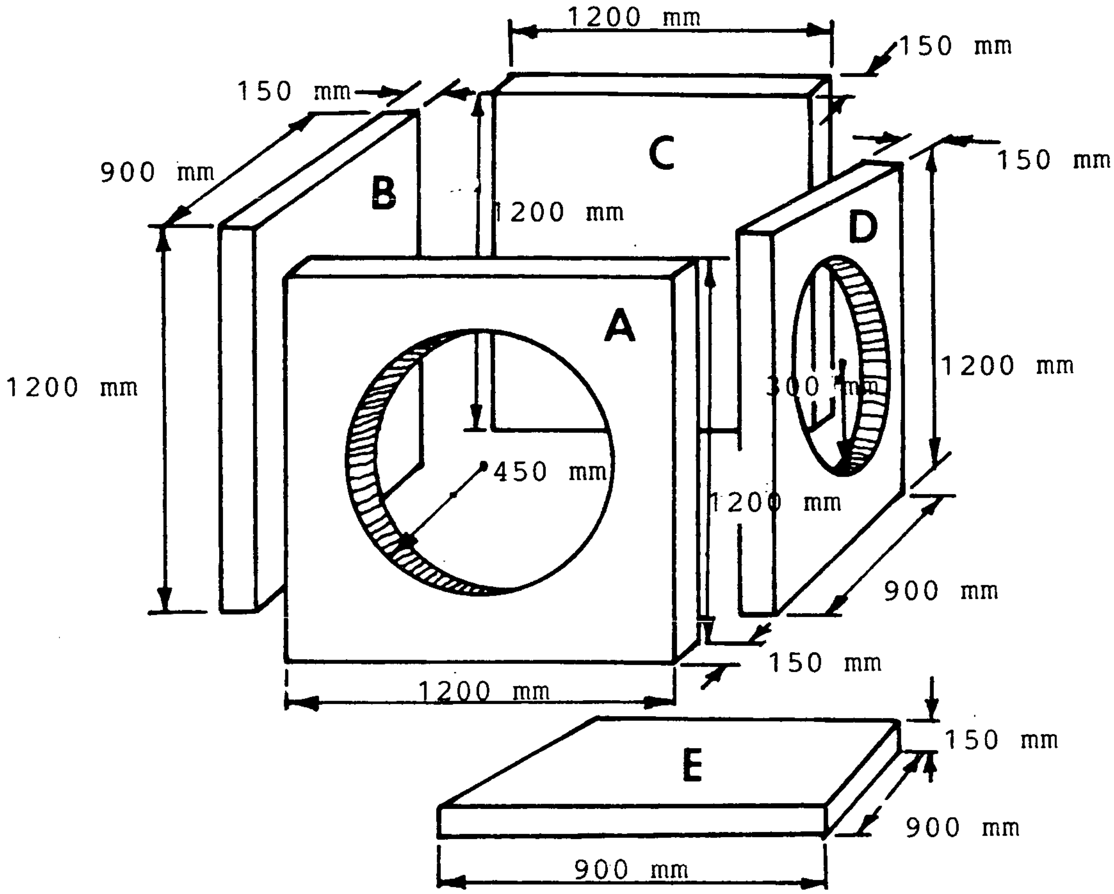
V = ______________ cubic meters
(whole number)

(Refer to answers on Page 213)

SECTION II ANSWERS

Chapter 3 Quiz: Page 219:

(1)



V = VA + VB + VC + VD + VE

VA = LWH - [(πr^2)(W)] = 1200 mm x 1200 m x 150 mm - [3.1416(450 mm x 450 mm)(150 mm)]

= 216,000,000 mm^3 - [(3.1416 x 202,500 mm^2)(150 mm)]
= 216,000,000 mm^3 - (636,174 mm^2 x 150 mm)
= 216,000,000 mm^3 - 95,426,100.00 mm^3
= **120,573,900.00 mm^3**

VB = LWH = 1200 mm x 900 mm x 150 mm = **162,000,000 mm^3**

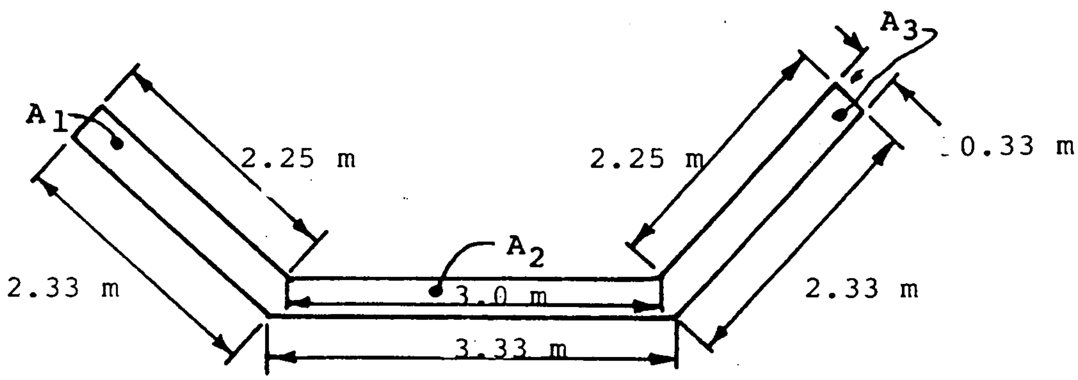
VC = 1200 mm 1200 mm x 150 mm
= **216,000,000 mm^3**

$$
\begin{aligned}
\mathbf{VD} &= (LWH) - (\pi r^2 \times W) \\
&= [(900\ mm \times 1200\ mm \times 150\ mm) - (3.1416 \times 300^2)150\ mm] \\
&= 162{,}000{,}000\ mm^3 - 282{,}744\ m^2 \times 150\ mm \\
&= 162{,}000{,}000\ mm^3 - 42{,}411{,}600\ mm^3 = \mathbf{119{,}588{,}400.00\ mm^3}
\end{aligned}
$$

$$\mathbf{VE} = LWH = 900\ mm \times 900\ mm \times 150\ mm = \mathbf{121{,}500{,}000\ mm^3}$$

$$
\begin{aligned}
\mathbf{V} &= 120{,}573{,}900.00 + 162{,}000{,}000.00 + 216{,}000{,}000.00 + \\
&\quad 119{,}588{,}400.00 + 121{,}500{,}000.00 \\
&= \mathbf{739{,}662{,}300.00\ mm^3\ or\ 0.7\ m^3}
\end{aligned}
$$

(2)



$$A_1 = \frac{(B + b)H}{2} = \frac{2.33\ m + 2.25\ m}{2} \times 0.33\ A_1 = 2.29\ m \times 0.33\ m$$
$$= \mathbf{0.76\ m^2}$$

$$A_2 = \frac{(B + b)H}{2} = \frac{(3.33\ m + 3\ m)}{2} \times 0.33\ m = 3.16\ m \times 0.33\ m$$
$$= \mathbf{1.04\ m^2}$$

A_3 = $\mathbf{0.76\ m^2}$ (same as A_1)

Total end area = $A_1 + A_2 + A_3$
$= 0.76\ m^2 + 1.04\ m^2 + 0.76\ m^2 = \mathbf{2.56\ m^2}$

V = End area x length = $2.56\ m^2 \times 43\ m = \mathbf{110.08\ m^3}$

(3)

Average end area, segment 1 = $\dfrac{98.3\ m^2 + 103.8\ m^2}{2} = 101.0\ m^2$

Average end area, segment 2 = $104.6\ m^2$

Average end area, segment 3 = $99.2\ m^2$

Volume, segment 1 = Average end area x Length
= 101.0 m^2 x 25 m = **2525 m^3**

Volume, segment 2 = **2615 m^3**

Volume, segment 3 = **2480 m^3**

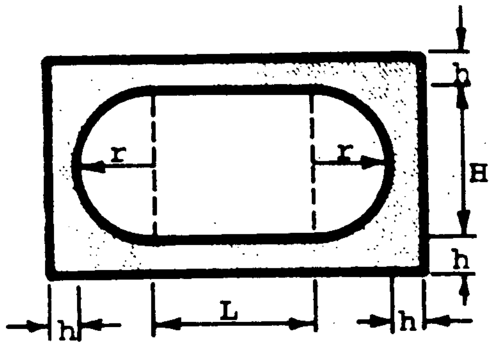
Total volume = **7620 m^3**

SECTION II

Comprehensive Test

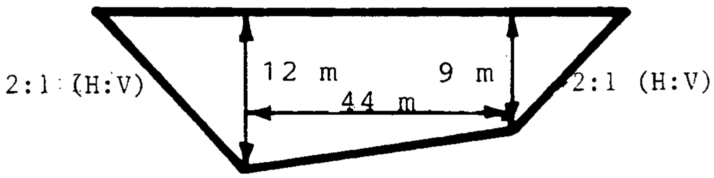
1. Calculate the shaded end area of the concrete structure.

r = 5 m
L = 10 m
h = 2 m
H = 10 m



A = ______________ m^2
(tenths)

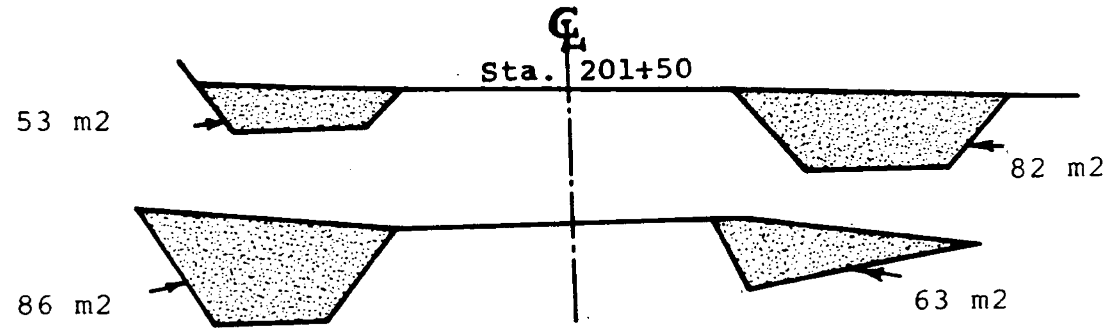
2. Find the cross-sectional area of this cut.



A = ______________ m^2
(tenths)

WORK PAGE

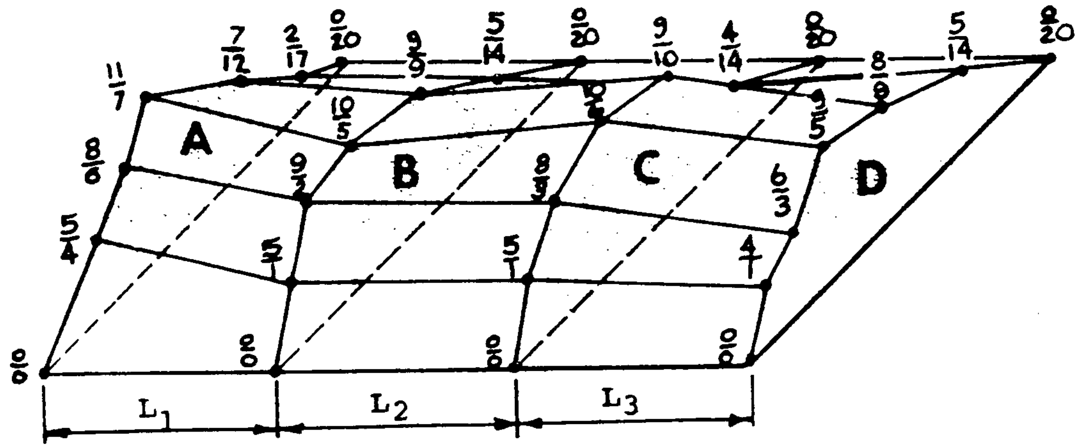
3. Using the excavation cross-sections below, compute the cubic meters of earth to be removed between the two stations.
Hint: A station = 100 m



Sta. 201 + 10

V = ____________ m^3
(tenths)

4. Calculate the volume:

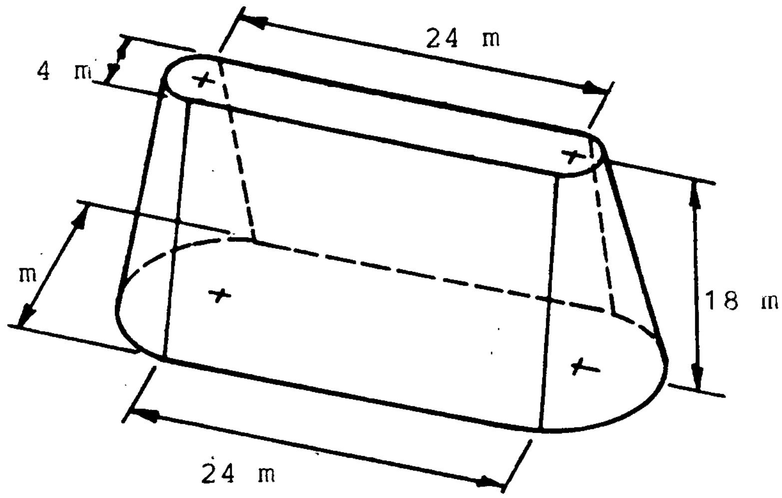


L_1 = 25 m
L_2 = 25 m
L_3 = 25 m

V = ____________ m^3
(whole numbers)

WORK PAGE

5. Compute the volume of this bridge bent.



$V = \underline{\hspace{4cm}}$ m^3
(tenths)

WORK PAGE

SECTION II

COMPREHENSIVE EXAM ANSWERS

Question 1

Area total = (14 x 14) - $3.14(5^2)$ - (10 x 10)
= 336 - 78.5 - 100 = 157.5 m^2

Question 2

Area one = 1/2(24 x 12) = 144
Area two = 44 ((9+12)/2) = 462
Area three = 1/2(18 x 9) = 81
Total area = 687 m^2

Question 3

Area one = 53 + 82 = 135
Area two = 86 + 63 = 284
Total area + 419
Volume = (419/2)(20) = 8380 m^3

Question 4

Area A = 103 Area B = 126 Area C = 118.5 Area D = 108.5

Volume = {(103+126)/2} + {(126+118.5)/2} + {(118.5+108.5)/2} x 25
= (114.5 + 122.25 + 113.5)(25)
= 8756.25 m^3

Question 5

Volume of both ends of the bent = 764.81 m^3
Volume of middle section of bent = 3456.00 m^3
Total volume of the solid = 4220.81 or 4220.8 m^3

SECTION III

SECTION III

CHAPTER 1 Trigonometry

FUNDAMENTALS OF APPLIED TRIGONOMETRY

Unit 1

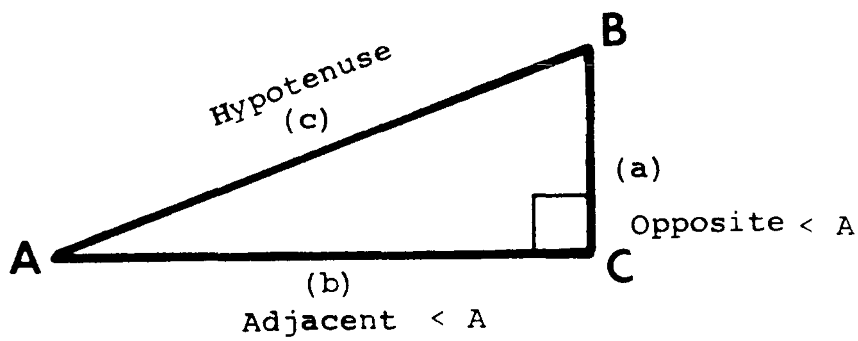
The Concept of Right Triangle Trigonometry

The word trigonometry, as derived from the Greek, means two things: **triangle** and **measurement**. Trigonometry is another branch of mathematics which deals with the measurement of angles, triangles and distances. A working knowledge of trigonometry requires an understanding of fundamental principles and constructions in geometry. Also, since symbols are used and algebraic equations are constructed, problems in trigonometry are solved by using algebra.

Trigonometry is preferred by many craftsman to other branches of mathematics because it conserves time and effort, and simplifies the solution of common trade and industrial problems.

A. PARTS OF THE RIGHT TRIANGLE

While trigonometry includes both the right triangle and the oblique triangle, the first named type is covered in this unit. A **right triangle**, as the name implies, is a triangle with one angle a right angle (90°). In the illustration of the right triangle ABC, the side opposite the right angle C is the hypotenuse (c). The side opposite the acute angle A is called the side opposite (a). Adjacent to angle A is the side adjacent (b). Note on the drawing and in the explanation that the angles are indicated by capital letters and the sides by either lower case letters or the words opposite, adjacent or hypotenuse.

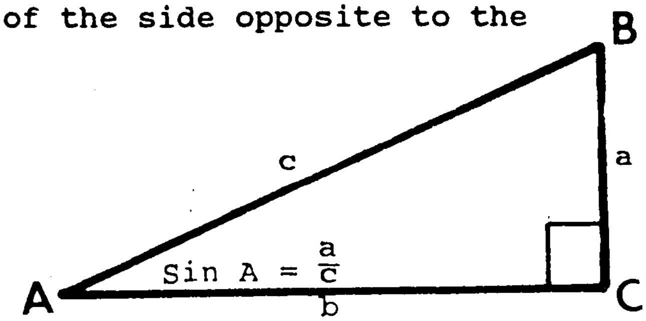


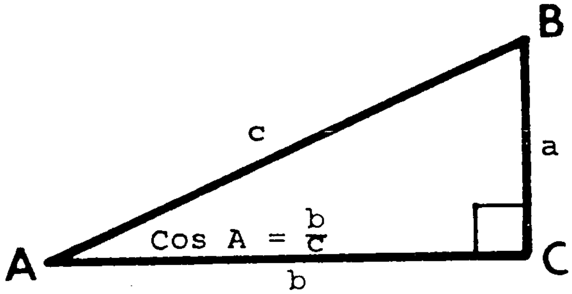
B. TRIGONOMETRIC FUNCTIONS

There are six terms that are widely used in trigonometry to express the ratios between the sides. The terms and the abbreviation of each are: **sine (sin)**, **cosine (cos)**, **tangent (tan)**, **cotangent (cot)**, **secant (sec)** and **cosecant (csc)**.

The sine (sin) is the ratio of the side opposite to the hypotenuse. In triangle ABC, the

$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{a}{c}$$



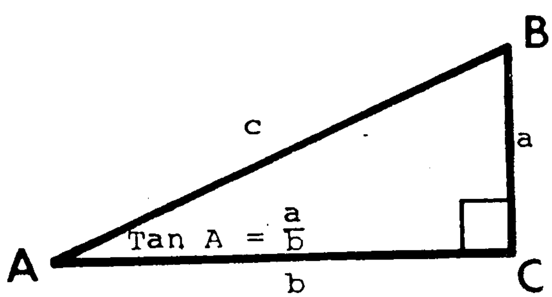


The cosine (cos) is the ratio of the side adjacent to the hypotenuse.

$$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

The tangent (tan) is the ratio of the side opposite to the side adjacent.

$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{a}{b}$$



The cotangent (cot) is the ratio of the side adjacent to the side opposite.

$$\cot A = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{b}{a}$$

form.

$$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{c}{b} \qquad \csc A = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{c}{a}$$

A comparison of the values of the six trigonometric functions shows two things. First, the sine and cosine, tangent and cotangent, and secant and cosecant may be grouped as pairs. Second, three of the trigonometric functions are reciprocals of the other three, starting with the sine.

sin A = a/c and the cosecant reciprocal, csc A = c/a

cos A = b/c and the secant reciprocal, sec A = c/b

tan A = a/b and the cotangent reciprocal, cot A = b/a

C. FUNCTIONS AND COFUNCTIONS

In right triangle ABC, angle A and angle B are **complementary** to each other and the sum of the two angles is 90°. The term **cofunction** refers to the function of the complementary angle.

* The sine of one acute angle is equal to the cosine of its complement.
* The tangent of one acute angle is equal to the cotangent of its complement.
* The secant of an acute angle is equal to the cosecant of its complement.

With the knowledge that the function of one acute angle in a right triangle is equal to the cofunction of the complement, it is possible to work with either acute angle. For instance, the tan of 30° = cot of 60°; the sin of 25° = cos 65°; the sec 75° = csc 15°.

D. INTERPOLATION OF TABLES OF TRIGONOMETRIC FUNCTIONS

Angles that are given in degrees or trigonometric ratios of whole degrees may be read directly on all Tables of Trigonometric Functions. The longer tables make it possible to read degrees and minutes directly from the table.

When such tables are not available, or when degrees, minutes and seconds are needed and direct values are not accessible in table form, the values are interpolated. **Interpolation** is the process of finding an exact value between two consecutive values in a series.

For instance, if the cosine of an angle of 45°30′25" is needed, and a table is available reading in minutes, the desired reading falls between cos 45°30′ and cos 45°31′. To be exact, the numerical value is 25/60ths of the difference between 30′ and 31′. This difference is subtracted from cos 45°30′ because the cos 45°30′ is larger than the cos of 45°31′.

RULE FOR INTERPOLATING TABLES OF TRIGONOMETRIC FUNCTIONS

* Find the numerical difference in a Table of Trigonometric Functions between the function of the given number of minutes and the next larger number.

* Place the given number of seconds over 60 making the number of seconds a fraction of a minute.

* Multiply the numerical difference by the fraction.

1. **Sine, Tangent** and **Secant Functions**

   * Add the product to the function of the given number of minutes.

   The sum is the value of the sine, tangent or secant function of any angle in degrees, minutes and seconds.

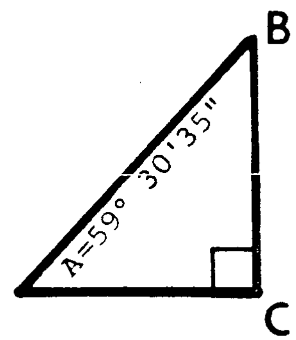
2. **Cosine, Cotangent** and **Cosecant**

   * Subtract the product from the function of the given number of minutes.

   The difference is the value of the cosine, cotangent, and cosecant functions.

EXAMPLE: Determine the sine value of angle A.

Angle A = 59° 30′ 35"



STEP 1. Locate two numbers in the sine column of a Table of Trigonometric Functions, one smaller and one 1′ larger (Refer to Appendix E)

sin 59° 31′	=	0.8617768
sin 59° 30′	=	<u>0.8616292</u>
Difference	=	0.0001476

Step 2. Subtract. Then take 35/60ths of the differences (0.0001476).
35/60(.0001476) = 0.0000861

Step 3. Add the difference to the function of the smaller angle.

$$\begin{array}{r} 0.0000861 \\ +\underline{0.8616292} \\ 0.8617153 \end{array}$$

Solution:

The sum 0.8617153 is the numerical value of the sin 59° 30′ 35″.

Unit 2

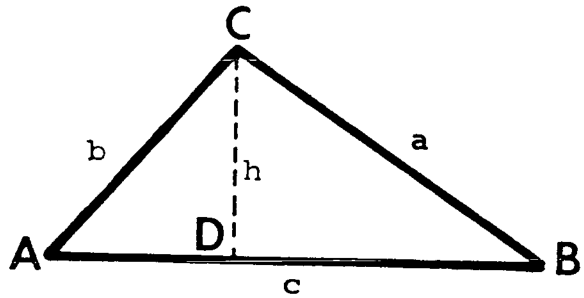
Oblique Triangles

Oblique triangles may be solved by drawing perpendiculars to make right triangles and using any one or combination of the six basic ratios of sides and angles. However, such a practice is time consuming and there are easier ways to solve such problems. Two new laws, the Sine Law and The Cosine Law, simplify the solution of measurements of sides and angles in oblique triangles.

A. THE LAW OF SINES

The Sine Law is a short way of saying that the sides of any triangle are proportional to the sines of the opposite angles. For instance, in triangle ABC the acute angles are A and B, the altitude (h) and the sides (a, b and c). The triangle ADC,

$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{h}{b}$$



In triangle BCD,

$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{h}{a}$$

If the values of the two sines are written as a proportion, then

$$\sin A : \sin B = \frac{h}{b} : \frac{h}{a}, \quad \frac{\text{Sin } A}{\text{Sin } B} = \frac{h/b}{h/a} = \frac{a}{b}$$

$$\frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B} = \frac{c}{\text{Sin } C}$$

The Law of Sines, when written as an equation, has four unknowns. In order to solve for any one of the four values, the other three values must be given.

RULE FOR SOLVING OBLIQUE TRIANGLES WITH THE LAW OF SINES

* Draw an oblique triangle and label the given parts.

* Write the Law of Sines and substitute known values in the equation.

* Locate the value of each given angle in the sine column of the Table of Trigonometric Functions.

* Determine the number of degrees, minutes and/or seconds in the computed value by locating this value in the sine column (or by interpolating) and its angular equivalent in the angle column.

* Solve the problem for the required value. Use tables when needed.

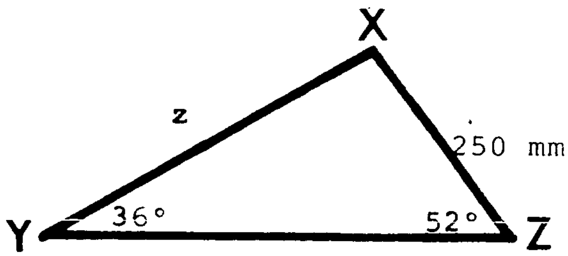
Two common types of problems may be solved readily with the Law of Sines.

* When two sides and the angle opposite one of the sides are given.

* When two angles and one side are given.

An example of the first type is given to show how the Law of Sines is applied. Similar steps may be used with other types of problems.

EXAMPLE: Find the missing sides and the third angle in triangle XYZ.



Step 1. Find angle X by subtracting angles Y and Z from 180°.

$$\angle X = (180° + Z)$$
$$= 92°$$

STEP 2. Write the Law of Sines in terms of the given sides and angles.

$$\frac{\sin Y}{\sin Z} = \frac{y}{z}$$

STEP 3. Determine the value of angles Y and Z in the sine column of a table.

$$Z = 52°$$
$$\sin Z = 0.7880108$$

STEP 4. Substitute known values in the Law of Sines and solve.

$$Y = 36°$$
$$\sin Y = 0.5877853$$

$$\frac{\sin Y}{\sin Z} = \frac{(0.5877853)}{(0.7880108)} = \frac{Y}{z} = \frac{250}{z} = 335.161 \text{ mm}$$

STEP 5. Solve for the third side (x) by again using the Law of Sines and three known values.

$$\frac{\sin X}{\sin Y} = \frac{x}{y}; \quad x = \frac{\sin X(y)}{\sin Y}$$

STEP 6. Determine the value of angle X from the sine column of a table.

$$X = 92°$$
$$\sin 92° = 0.9993908$$

STEP 7. Substitute known values in the Law of Sines.

The values computed by the Law of Sines for side z = 335.2 mm, and x = 425.1 mm and angle X = 92°.

B. SUPPLEMENTS OF ANGLES

Problems in trigonometry are simplified by using **supplements of angles**. The supplement of an angle, between 90° and 180° is simply the difference between the angle and 180°. Thus, the sine of 100° is equal to its supplement (180° - 100° = sin 80°).

The cosine is equal to the cosine of the supplement. For example, the cosine of 115° - (180° - 115°) = cos 65°.

C. THE LAW OF COSINES

When three sides of a triangle are given, the Law of Sines cannot be used as two of the four values are unknown. In such cases, the **Law of Cosines** applies. This law states that in any triangle the square of any side is equal to the sum of the squares of the other two sides minus twice the produce of the two sides and the cosine of their included angle.

$$a^2 = b^2 + c^2 - 2(bc \cos A)$$

$$b^2 = a^2 + c^2 - 2(ac \cos B)$$

$$c^2 = a^2 + b^2 - 2(ab \cos C)$$

The cosine values of angles A, B and C are:

$\cos A = \frac{b^2 + c^2 + a^2}{2bc}$ $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

RULE FOR APPLYING THE LAW OF COSINES

* Make a sketch of the triangle and show all given dimensions.
* Write the Law of Cosines in terms of the unknown angles.
* Substitute the values of the known sides in the equation.
* Solve for the cosine of one unknown angle.
* Locate and/or interpolate the angle corresponding to the cosine value.
* Repeat the same processes for the second angle.
* Find the third angle by adding the two computed angles and subtracting the sum from 180°.

EXAMPLE: Find the number of degrees in angle B.

Step 1. Write the Law of Cosines for the required angle.

Given: $a = 8$, $b = 6$, $c = 4$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

Step 2. Substitute the known values of the sides and solve for cos B.

$$\cos B = \frac{8^2 + 4^2 - 6^2}{(2)\ (8)\ (4)} = \frac{44}{64} = 0.6875000$$

Step 3. Determine the angle whose cosine = 0.6875000, by using a table and interpolating, if necessary.

cos 0.6825000 = 46° 34′ 03″

Step 4. Use either the Law of Sines or the Law of Cosines to find the other two angles.

Step 5. Check the angles by addition. The sum of the three angles equals 180°.

SOLUTION OF TRIANGLES

KIND OF TRIANGLE	GIVEN		WANTED	TOOL
Right triangle (one 90° angle)	One side, one angle	Hypotenuse	Leg opp. given angle	Sine function
			Leg adj. given angle	Cosine function
		Leg adj.	Hypotenuse	Cosine function
			Leg opp. given angle	Tangent function
		Leg opp.	Hypotenuse	Sine function
			Leg adj. given angle	Cotangent function
	Two sides, no angles given	Hypotenuse and leg opp.	Angle	Sine function
			Leg adjacent	Tan or cosine
Study the data and see what is known		Hypotenuse and leg adj.	Angle	Cosine function
			Leg opposite	Sine or tangent
		Two legs	Angle	Tangent function
			Hypotenuse	Sine or cosine
Oblique triangle (no 90° angles)		Two angles one side	Sides and third angle	Sine law Subt. for angle
		Two sides angle opp.	Two angles one side	Sine law-side and Subt. for third angle
		Two sides and included angle	Third side and two angles	Cut the figure into right triangles. See above. Cosine law. Use for three significant figures only.
		Three sides	All three angles	Half-angle law for logarithms. Segment law for computer or longhand K formula, when area is also wanted

AREAS OF TRIANGLES

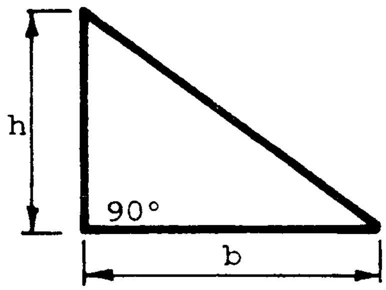
I. Areas of Right Triangles

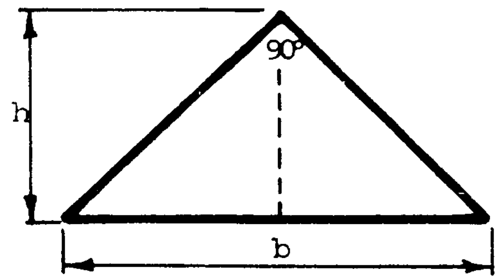
A =(1/2) bh

A = area in square units

b = length of base of triangle

h = perpendicular height of triangle





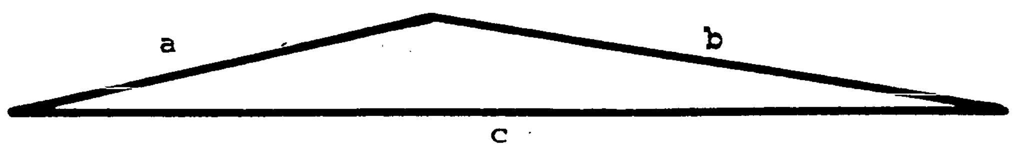
EXAMPLE:

1. b = 125 mm h = 150 mm
   A = (1/2)bh = (1/2)(125)(150)
   = **9375 mm^2**

2. b = 10 m h = 2.6 m
   A = (1/2)(10)(2.6 m)
   = **13 m^2**

II. Areas of Oblique Triangles

$A = \sqrt{s(s-a)(s-b)(s-c)}$ $s = (1/2)(a + b + c)$



EXAMPLE:

a = 27.5 m b = 39.3 m c = 64.7 m

s = (1/2)(27.5 + 39.3 + 64.7) = 65.75

$A = \sqrt{65.75(38.25)(26.45)(1.05)}$

$= \sqrt{69{,}846.1017}$

= **264.284 square meters (Answer)**

EXERCISE NO. 1

1. Find the natural function of each of the following angles:

   a. sin 48°17′40″ Answer ________

   b. tan 31°45′45″ Answer ________

   c. cos 18°36′20″ Answer ________

2. Find the value of angle A in the following problems:

   a. sin A = 0.7497904 Answer ________

   b. tan A = 0.7848050 Answer ________

   c. cot A = 1.4108999 Answer ________

EXERCISE NO. 2

1. Find the area of a right triangle given:

   h = 10 m b = 20 m

   A = ____________ m^2

2. Find the area of an oblique triangle given:
   a = 30 m b = 40 m c = 60 m

   A = ____________ m^2

(Refer to answers on Page 277

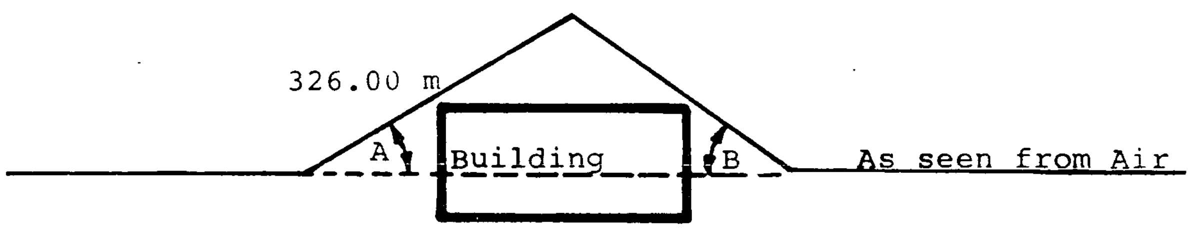
WORK PAGE

EXERCISE NO. 3

Solve the following triangles by the use of natural functions. Compute the sides to the nearest tenth and the angles to the nearest minute.

1.	b = 86.4 c = 98.7 C = 90°00'00"	A = ____________ B = ____________ a = ____________	Answer Answer Answer
2.	c = 51 A = 72°14'00" C = 90°00'00"	a = ____________ b = ____________ B = ____________	Answer Answer Answer
3.	a = 80.4 A = 99°55'00" B = 45°01'00"	b = ____________ c = ____________ C = ____________	Answer Answer Answer
4.	b = 87.9 c = 49.8 A = 33°10'00"	a = ____________ B = ____________ C = ____________	Answer Answer Answer

5. A building is in the path of a survey crew. In order to continue the survey, the crew chief triangulates around the building in the following fashion:



Angle A = 33°15'00"
Angle B = 35°00'00"

What is the length of the third side of the triangle? (hundredths)

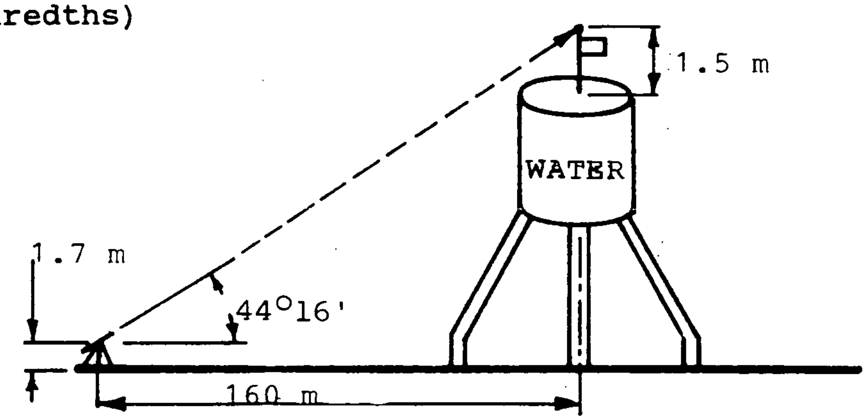
(Refer to answers on Page 277)

WORK PAGE

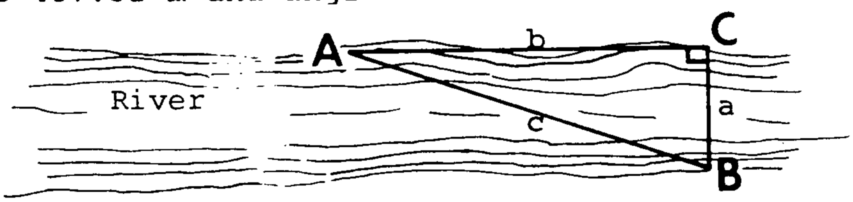
SECTION III

Chapter 1 Quiz

1. What is the height of the tower, including the flag? (hundredths)

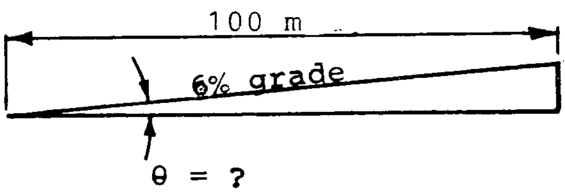


2. A survey crew sets points A and C. The distance b is measured and also the angle A. What is the distance across the river if b is 487.32 m and angle 35° 17′ 00"?



a = __________ m

3. A 100 m length of highway has a +6% grade. At what angle does the highway rise?



Recall that percent grade = $\frac{\text{vertical rise}}{\text{horizontal distance}}$ x 100

(Refer to answers on Page 277)

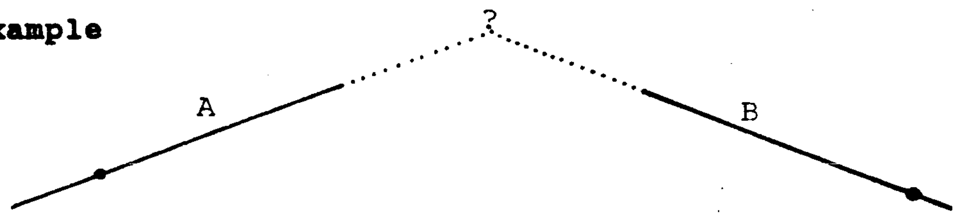
SECTION III

CHAPTER 2 Vertical Curves

Vertical Curves

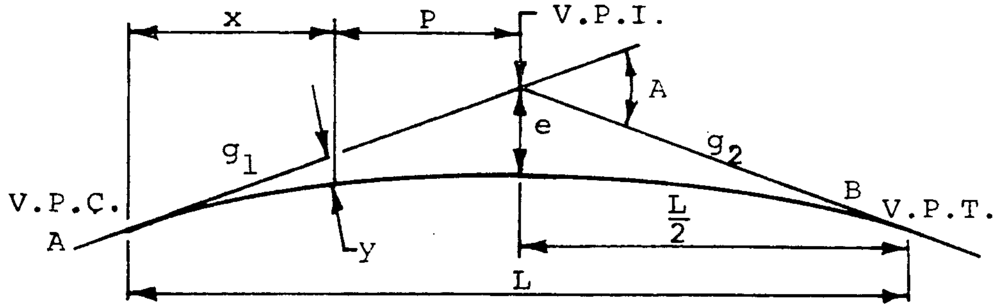
On highways, it is impossible for moving vehicles to change from one grade to another without the use of some sort of curve. The curve that provides this transition is called the "V" curve or vertical curve.

Example



Two construction crews are building a roadway on side A and side B of a hill. How should the crews complete the roadway - what curve is needed to give a smooth transition between grades?

In order to work this problem, it is necessary that you first understand the terms and equations used in the calculations. Study Figure One.



Definitions: g_1 and g_2 are gradients (or tangent grades) of the slope in %. Recall that the gradient

$$= \frac{\text{vertical rise or fall}}{\text{horizontal distance}} \times 100$$

A = Algebraic difference in gradients ($g_2 - g_1$)
L = Length of vertical curve in **stations**. Where one station is equivalent to 100 meters.
V.P.C. = The beginning of vertical curve
V.P.T. = The end of vertical curve
V.P.I. = The vertical point of intersection
x = Horizontal distance to any point on the curve from the V.P.C. or V.P.T. in **stations**.

p = Horizontal distance to any point on the curve from the V.P.I. **in stations.**

e = Vertical distance from the point of intersection to the curve V.P.I. in **meters.***

y = Vertical distance at any point in the curve to the tangent grade in meters.

k = A constant for any particular grade K = A/2L

L_1 = Length of a vertical curve which will pass pass through a given point.

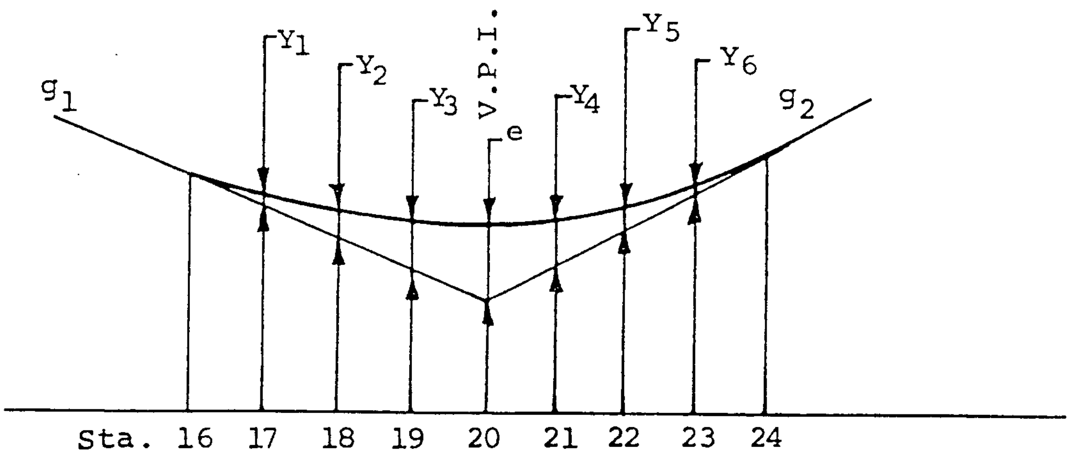
***Also known as middle ordinate = M.O.**

Formulas: $A = (g_2) - (g_1)$ $\quad y = \frac{4ex^2}{L^2} = Kx^2 = \frac{Ax^2}{2L}$

$$e = \frac{AL}{8} \qquad L_1 = \frac{2(AP + 2y + 2\sqrt{APy + y^2})}{A}$$

Study the following examples

Example:



Elevation Sta. 16 = 348.52
L = 8 stations
g_1 = -6.00%
g_2 = +5.90% e = ____________

Y_1, Y_2, Y_3, Y_4, Y_5, Y_6 = ____________

Solution: $e = \frac{AL}{8} = \frac{(g_2 - g_1)L}{8} = \frac{[+5.90 - (-600)]8}{8} = 11.90$ m

$$y_1 = y_6 = \frac{4ex^2}{L^2} = \frac{4(11.9\text{ m})(1)^2}{(8)^2} = \frac{4(11.9)}{64} = 0.74\text{ m}$$

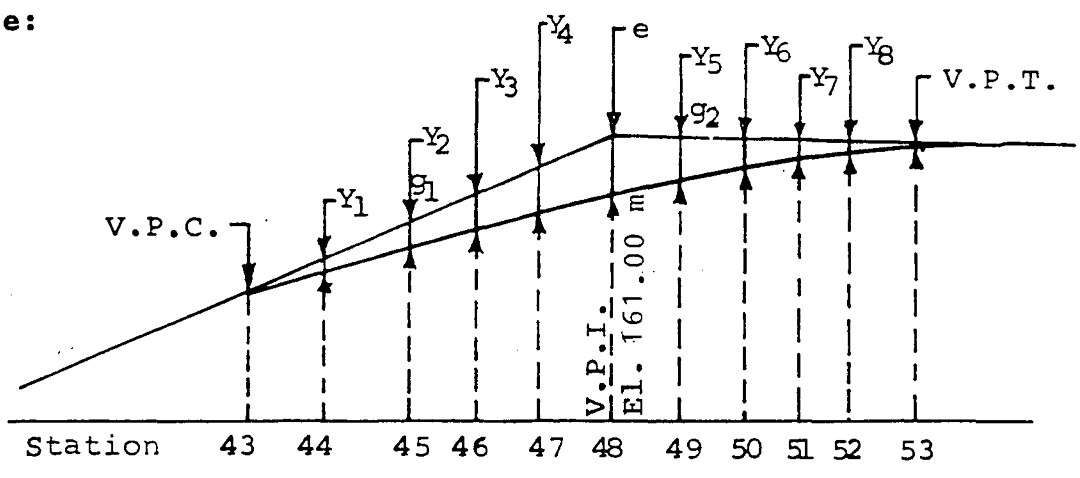
$$y_2 = y_5 = \frac{4(11.90)(2)^2}{8^2} = \frac{16(11.90)}{64} = \frac{11.90}{4} = 2.98\text{ m}$$

$$y_3 = y_4 = \frac{4(11.90)(3)^2}{82} = \frac{36(11.90)}{64} = 6.69\text{ m}$$

To find the elevation of the curve we must first calculate the height of the tangent grade at each station. Since station 17 is 100 meters from V.P.C. and since g_1 (percent grade) is -6%, the tangent grade must have an elevation of 348.52 m - 6.00 m or 342.52 m. Since y_1 = 0.74 m, the elevation of the curve at station 17 = 342.52 + 0.74 = 343.26 m. Since e is the vertical distance from the vertex to the middle of the curve, it is measured downward in a negative direction for all crest curves. Hence, the algebraic sign of e indicates whether elevations along the curve are to be obtained by adding or subtracting offsets from the corresponding tangent elevations.

Station	Straight Grade Elevation	Offsets	Grade Elevation Curve
V.P.C. 16	348.52 m	0.00 m	348.52 m
17	342.52 m	y_1 = 0.74 m	343.26 m
18	336.52 m	y_2 = 2.98 m	339.50 m
19	330.52 m	y_3 = 6.69 m	337.21 m
V.P.I. 20	324.52 m	e = 11.90 m	336.42 m
21	330.42 m	y_4 = 6.69 m	337.11 m
22	336.32 m	y_5 = 2.98 m	339.30 m
23	342.22 m	y_6 = 0.74 m	342.96 m
V.P.T. 24	348.12 m	0.00 m	348.12 m

Example:



$g_1 = +5.6\%$ $g_2 = +0.6\%$ V.P.I. El = 161.00 m L = 10

What is the elevation of the curve at each station?

First - We must calculate the offsets or height of tangent grade.

$$e = \frac{AL}{8} = \frac{(g_2 - g_1)10}{8} = \frac{[+0.6 - (+5.6)]10}{8} = -6.25 \text{ m}$$

$$y_1 = y_8 = \frac{4ex^2}{L^2} = \frac{4(6.25)(1)}{100} = -0.25 \text{ m}$$

$$y_2 = y_7 = \frac{100}{100} = -1.00 \text{ m}$$

$$y_3 = y_6 = \frac{225}{100} = -2.25 \text{ m}$$

$$y_4 = y_5 = \frac{400}{100} = -4.00 \text{ m}$$

Second - Construct a Table:

Station	Straight Grade Elevation	Offsets	Grade Elevation Curve
V.P.C. 43	133.00 m	0.00 m	133.00 m
44	138.60 m	-0.25 m	138.35 m
45	144.20 m	-1.00 m	143.20 m
46	149.80 m	-2.25 m	147.55 m
47	155.40 m	-4.00 m	151.40 m
V.P.I. 48	161.00 m	-6.25 m	154.75 m
49	161.60 m	-4.00 m	157.60 m
50	162.20 m	-2.25 m	159.95 m
51	162.80 m	-1.00 m	161.80 m
52	163.40 m	-0.25 m	163.15 m
V.P.T. 53	164.00 m	0.00 m	164.00 m

For roadway construction, there is usually a maximum allowable change in grade per station. This maximum change must be considered when choosing the length of a vertical curve connecting two grades.

$$\frac{g_2 - g_1}{\text{max. allowable change/sta.}} = \text{length in sta.}$$

Example: On a roadway a +0.8 percent grade meets a -0.4 percent grade. If the maximum allowable change per station is 0.2 percent, how long must the curve be?

Solution: +0.8 - (-0.4) = 1.2%
Minimum length or curve is then = 1.2/2 = 6 stations or 600 m total length

EXERCISE NO. 1

1. On a highway a -6.0 percent grade meets a +4.0 percent grade at station 67 + 00 and at elevation 516.00 m. The maximum allowable change in grade per station is 1.25%. Compute the elevation of stations at 100 m intervals along a vertical curve connecting the two grades.

(Refer to answers on Page 277)

WORK SPACE

High or Low Point on a Curve

The high points of crest curves or low points of sag curves are not usually determinable except by special calculation.

Study Figure two and three:

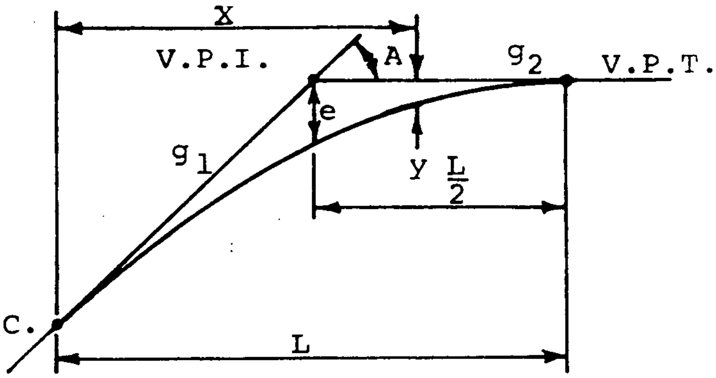


Figure 2
Crest Curve

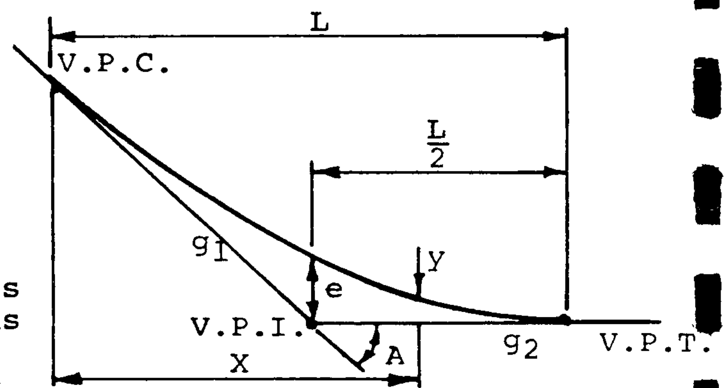


Figure 3
Sag Curve

A = Algebraic difference in gradients
L = Length of vertical curve stations

V.P.C. = Beginning of Vertical Curve
V.P.I. = Vertical Point of Intersection
V.P.T. = End of Vertical Curve
X = Distance from the V.P.C. to the low or high point in stations.
$= g_1 \frac{(L)}{(A)}$

Example: g_1 = -3.00% g_2 = +2.00% L = 4.00 stations A = 5.00

What is the length of X?

Solution:

$$X = 3.00 \times \frac{4.0}{5} = 2.4 \text{ stations} = 240 \text{ m}$$

EXERCISE NO. 2

1. A +6% grade meets a -0.8% grade. If the length of the vertical curve is 26 stations, how far from the V.P.C. is the high point of the curve? (The V.P.C. is on the +6% grade.)

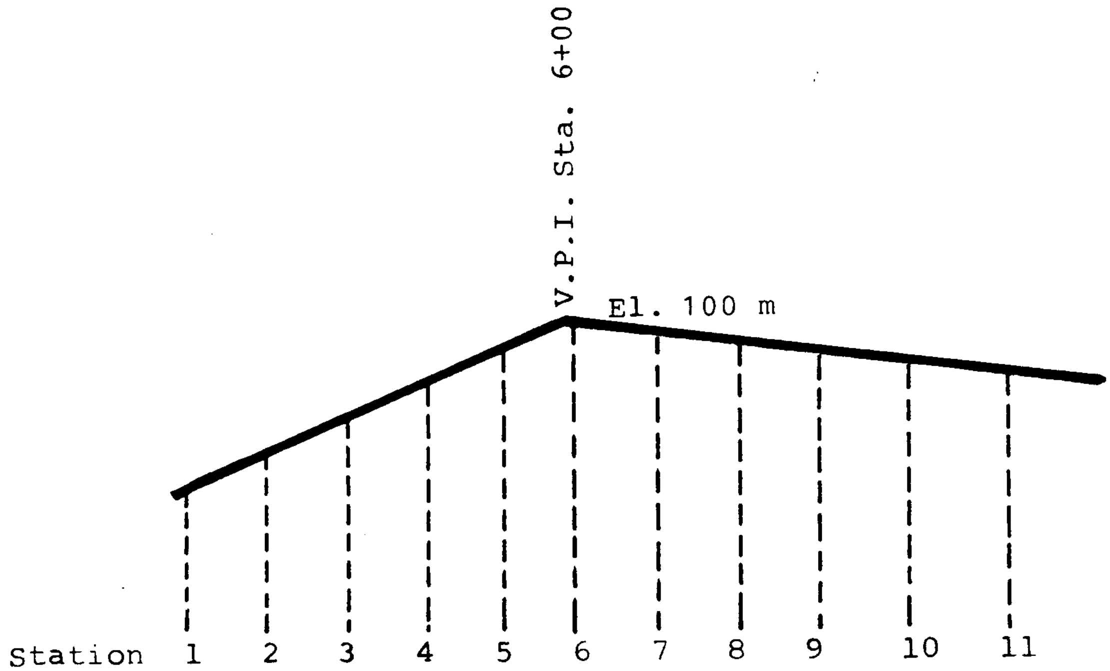
(Refer to answer on Page 277)

WORK PAGE

SECTION III

Chapter 2 Quiz

1. On a roadway a +3.2 percent grade meets a -0.8 percent grade at elevation 100.00 m. The maximum allowable change in grade per station is 0.5%. Compute the elevation of stations at 100 meters intervals along a vertical curve connecting the two grades, and find the high point on the curve.



(Refer to answers on Page 277)

WORK PAGE

SECTION III

Chapter 3	Conversion of Degrees, Minutes, Seconds to Decimal Degrees
	Conversion of Decimal Degrees to Degrees, Minutes, Seconds
	Horizontal Circular Curves

Conversion of Degrees, Minutes, & Seconds to Decimal Degrees

Example: Convert 96° 41′ 19″ to Decimal Degrees.

Divide 19″ by 60 (60 seconds = 1 minute) to convert seconds to decimal minutes,

(19″ ÷ 60 = 0.31667)

Add 41′ to the converted decimal minutes.

(41′ + 0.31667 = 41.31667′)

Divide 41.31667′ by 60 (60 minutes = 1 degree) to convert minutes to decimal degrees,

(41.31667 ÷ 60 = 0.6886°)

Add 96° to the converted decimal degrees.

(96° + 0.6886° = 96.6886°)

Answer: **96.6886° = 96° 41° 19″**

Conversion of Decimal Degrees to Degrees, Minutes, & Seconds

Example: Convert 59.255960 to Degrees, Minutes, & Seconds.

Subtract 59 to determine the decimal part of degrees,

$$(59.25596° - 59° = 0.255960°)$$

Multiply remainder by 60 (60 minutes = 1 degree) to calculate number of minutes.

$$(0.25596° \times 60° = 15.3576')$$

Subtract 15 to determine the decimal part of minutes. 15 is the number of whole minutes.

$$(15.3576' - 15' = 0.3576')$$

Multiply the remainder by 60 (60 seconds = 1 minute) to calculate the number of seconds.

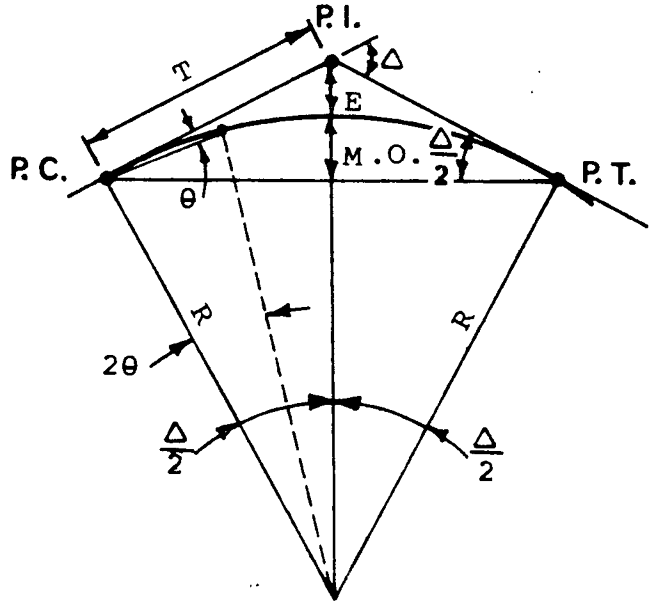
$$(0.3536' \times 60° = 21.474")$$

Answer: **= 59.25596 = 59° 15' 21"**

Horizontal Circular Curves

It would be impossible for a fast moving car to change its direction abruptly at each of the stations on a traverse where a deflection angle has been measured. For this reason straight lines or tangents are connected by simple circular curves. The curves provide gradual change in direction. Study Figure one.

Figure One



DEFINITIONS:

P.I. = point of intersection of back tangent and forward tangent

P.C. = Point of Curvature - Point of change from back tangent to circular curve.

P.T. = Point of Tangency - Point of change from circular curve to forward tangent.

L = Total length of any circular curve measured along its arc in meters.

R = Radius of circular curve in meters.

Δ = Total intersection (or central) angle between back and forward tangents.

$\Delta/2$ = Deflection angle for full circular curve measured from tangent at P.C. or P.T.

θ = Deflection angle required from tangent to a circular curve to any other point on a circular curve.

L.C. = Total chord length, or long chord, for a circular curve in meters.

C = Chord length between any two points on a circular curve in meters.

T = Tangent distance. The distance between the PC and PI or the PI and the PT (in meters).

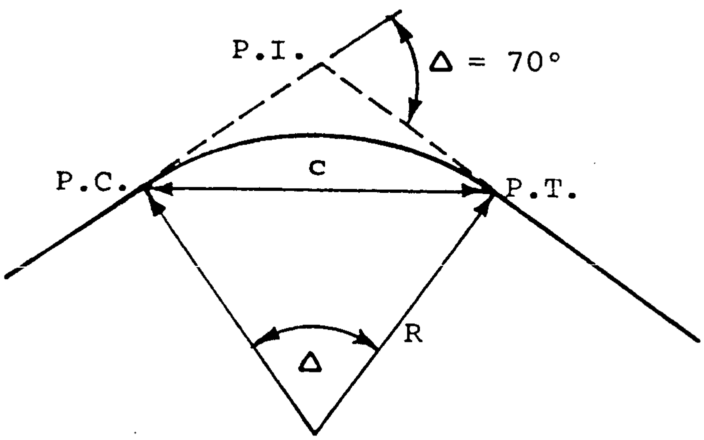
E = External distance (radial distance) in meter from P.I. to mid-point on a simple circular curve.

M.O. = Middle Ordinate. Length of the ordinate from the middle of the long chord to the middle of the curve.

Formulas:

Equation	**Unit**
$T = R\ [\tan(\Delta/2)]$	Meters
$E = (R/\cos(\Delta/2))-R = T \tan\ (1/4)\Delta$	Meters
$L.C. = 2\ R \sin\ (\Delta/2)$	Meters
$M = R\ (1-\cos\ \Delta/2)$	Meters
$C = 2\ R \sin\ \theta$	Meters

Example: A circular curve must be built to connect two principle roadways (speed limit 100 km/h).



What length should the curve be if the Radius (R) = 500 m
What is the length of the long chord (LC)?

Solution: $L = \dfrac{R\pi\Delta}{180}$

$= 500(3.1416)(70°)/180$

$= 610.90$ m

$LC = 2 R \sin (\Delta/2)$

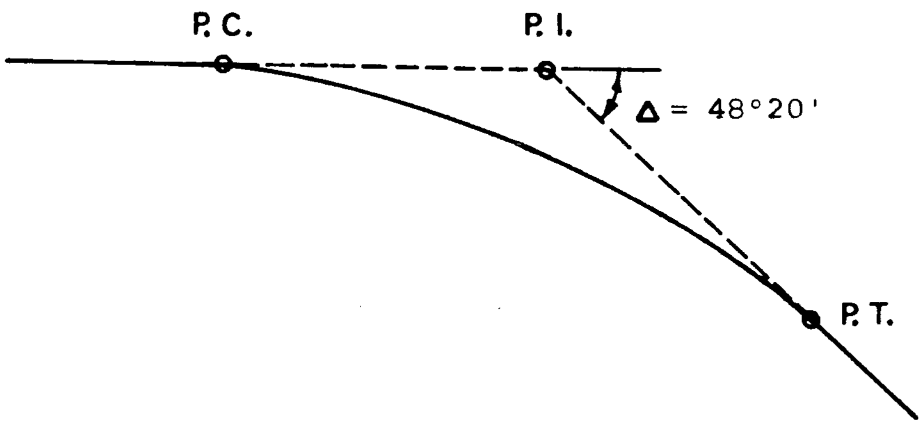
$= 2(500) \sin (70°/2)$

$= 1000 (0.5735764)$ (From Appendix E)

= 573.58 m

Example: A major multilane highway is being designed (speed limit 100 km/h). According to **AASHTO's 'A Policy on Geometric design of Highways and Streets'**, the desirable radius for any curves on that highway is at least 400 m.

A section of highway under construction looks like:



P.C. is at Station 28 + 10.50
P.I. is at Station 33 + 00.00
$\Delta = 48° 20' 00''$

(a) What is the length of curve (L)?
(b) Where is point P.T.? (in stations)
(c) What is the length of the external distance from P.I. to mid-point on a simple curve? (in meters)

Solution:

(a) In order to find L we need to know the radius of the curve (R). Recall that the distance from P.C. to P.I. is called the tangent distance T, and $T = R \tan \Delta/2$. The distance between the P.I. and P.C. is 149 m so T = 149 m.

Since $T = R \tan \Delta/2$ this implies

$$R = T/\tan(\Delta/2)$$

$$R = \frac{489.50 \text{ m}}{\left[\tan\left(\frac{48°20'}{2}\right)\right]}$$

$$R = \frac{489.50 \text{ m}}{\tan 24°10'}$$

$$R = \frac{489.50 \text{ m}}{0.4487187} = 1090.88 \text{ m}$$

Since the radius is under 1500 m, the district engineer decides to lower the speed. The speed limit is lowered to 80 km/h which only requires a desirable minimum radius of 230 m.

Now since the length of the curve is equal to $(R\,\pi\,\Delta)$, it follows that:

$$L = (1090.88 \text{ m})(48.33°)(0.017453293)$$

$$= 920.176535 \text{ m}$$

$$= \mathbf{920.18 \text{ m}} \quad \textbf{(Rounded)}$$

Note: that 48°20′ was changed to 48.33° for convenience of calculation.

(b) Station P.T. is determined by the length of the curve. Station P.C. is at Station 28 + 10.50 and the length of the curve is 920.18 m, P.C. comes at Station **2810.50 + 920.18** or **37 + 30.68.**

(c) The length of the external distance E (refer to Figure One for exact location) is determined by:

$$E = [R/\cos(\Delta/2)] - R$$

$$= \frac{1090.88}{[\cos \frac{48°20'}{2}]} - 1090.88$$

$$= 1195.67 - 1090.88$$

$$E = \mathbf{104.79\ m}$$

EXERCISE NO. 1

1. A design for a new multilane interstate highway will have a speed limit of 130 kph (which by design specification will require that all curves on that highway have a radius of at least 2500 m). What is the minimum length of a curve if the angle between forward and backward tangents 15°15'?

   If P.C. is at Station 27 + 00.00, where is Station P.T. and P.I. if R = 2500 m exactly?

   How long is the external distance?

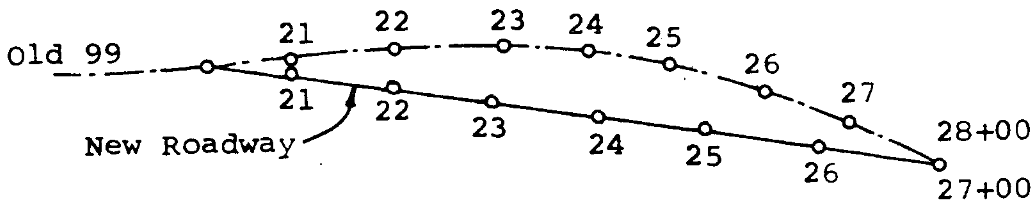
(Refer to answers on Page 278)

WORK PAGE

Equations

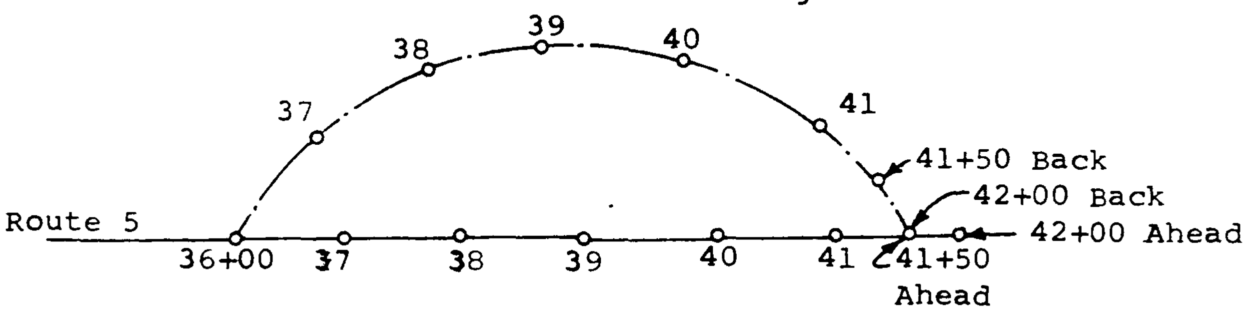
Up to this point we have been dealing with almost ideal conditions with horizontal curves. It happens that in most cases determining stations along the curve requires use of '**equations**'.

An equation develops because the length of a section of highway is changed. As an example: it is decided that a new section of highway will be built to take the curve (shown below) out of old Highway 99.



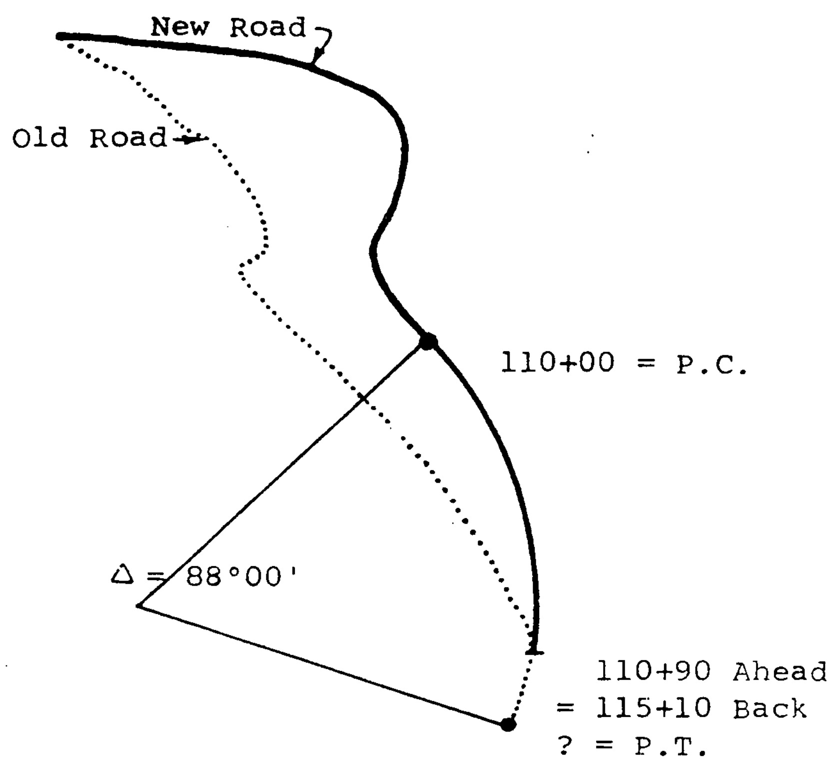
The question is what can be done about the obvious difference in the stationing at point of intersection of the old and new route? Is the intersection point station 28 + 00 or 27 + 00? The problem is solved by equations. According to that procedure, the old highways stationing is relabeled 28 + 00 **ahead** while the new station number is 27 + 00 **back**, when referring to that intersection point. It is called 28 + 00 ahead (which equals 27 + 00 back).

A similar situation occurs in the following:



In this situation, route 5 is being changed in order to bypass a section of highway that keeps washing out. What is the point of intersection? The answer is 41 + 50 ahead which is equal to 42 + 00 back. Note that when the new section of roadway is longer than the old, a duplication of stations exist. The duplication refers only to the stations on the new highway that are the same as the stations on the old highway. The stations on the old highway refer, in this case, to station 41 + 50 and beyond.

Practical problems are more involved than our previous examples. In a real case many more unknowns must be accounted. Situation: A roadway must be built that allows for a faster speed limit. The roadway proposed is shown. If the speed limit is set for 80 km/h and will have a radius of 1000 m, what will be the station number at P.T.? The curve begins at station 110 + 00.000 and Δ = 88°.



First: Figure out the length of the curve.

$$
\begin{aligned}
L &= R\,\Delta\,(0.017453293) \\
&= 1000\text{ m}(88.00°)(0.017453293) \\
&= 1535.89\text{ m}
\end{aligned}
$$

Second: Calculate difference between ahead and back stations. Always subtract the back from the ahead.

110 + 90 - 115 + 10 = -4 + 20

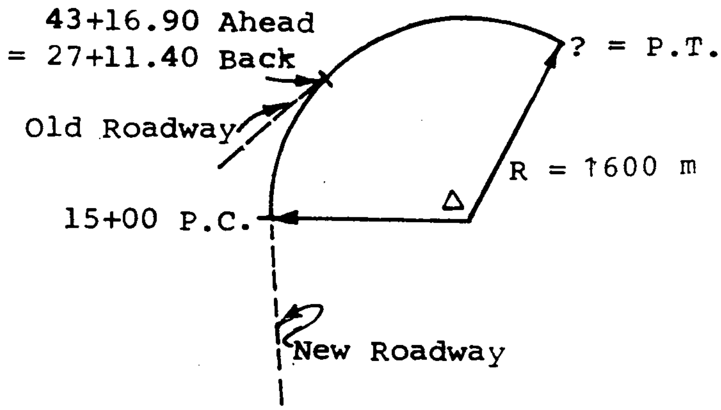
Third: Add the length of the curve to 110 + 00
Then add -4 + 20.

110 + 00 + 15 + 35.89 m + -4 + 20 = 121 + 15.89 ahead

This number is equal to the station at point of Tangency.

EXERCISE NO. 2

1.



The new roadway intersects with the old roadway at station 43 + 16.90 ahead. The new road has a speed limit of 100 km/h and a radius of 1600 m. $\Delta = 115.00°$. What is the station at P.T.?

(Refer to answer on Page 278)

WORK PAGE

Example:

Find the curve deflections to the nearest tenth of a minute on the following curve from the PC to PT for each full station and end points on the curve.

Given: Δ = 44°02'00" Rt

R = 955.0 m E = 75.06 m

L = 733.9 m T = 386.2 m

Solution: P.I. Sta = 283 + 35.5

PI	283 + 35.5
- T	386.2
PC	279 + 49.3
+ L	733.9
	286 + 83.2

θ = 1/2 D per 100 m

= 1/2(6)

= 3°00'

θ_{PC} to Sta. 280 + 00
= 50.7/100 (6/2)
= 1.521°

Station	Deflection	Total Deflection
PC 279 + 49.3	0	0°00.0'
280 + 00	1.521	1°31.3'
281 + 00	1.521 + 3.000	4°31.3'
282 + 00	4.521 + 3.000	7°31.3'
283 + 00	7.521 + 3.000	10°31.3'
284 + 00	10.521 + 3.000	13°31.3'
285 + 00	13.521 + 3.000	16°31.3'
286 + 00	16.521 + 3.000	19°31.3'
PT 286 + 83.2	19.521 + 2.496	22°01'*

***22°01' = (1/2)Δ checks, Total Deflection is equal to (1/2)Δ**

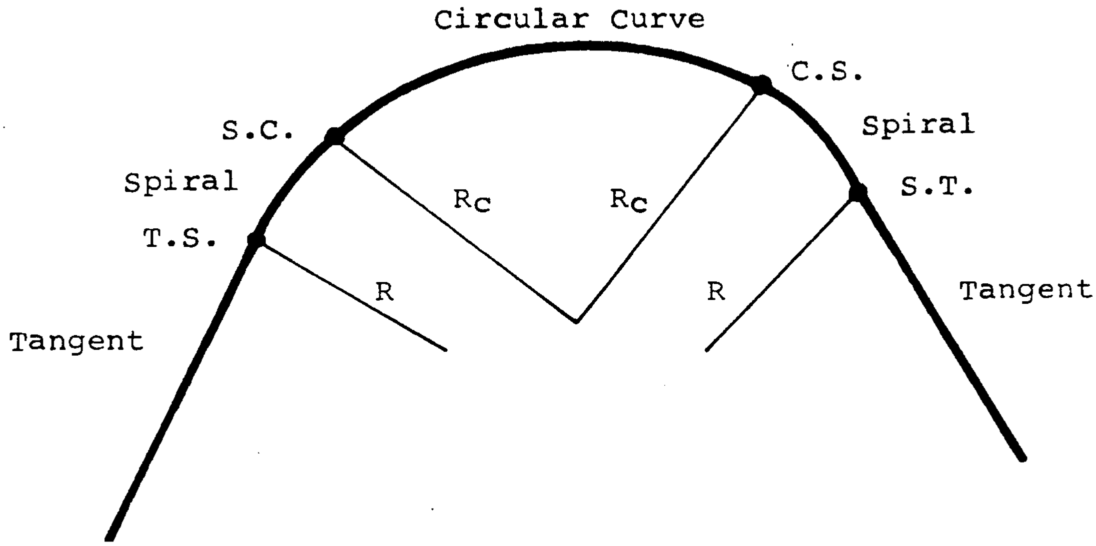
WORK PAGE

SECTION III

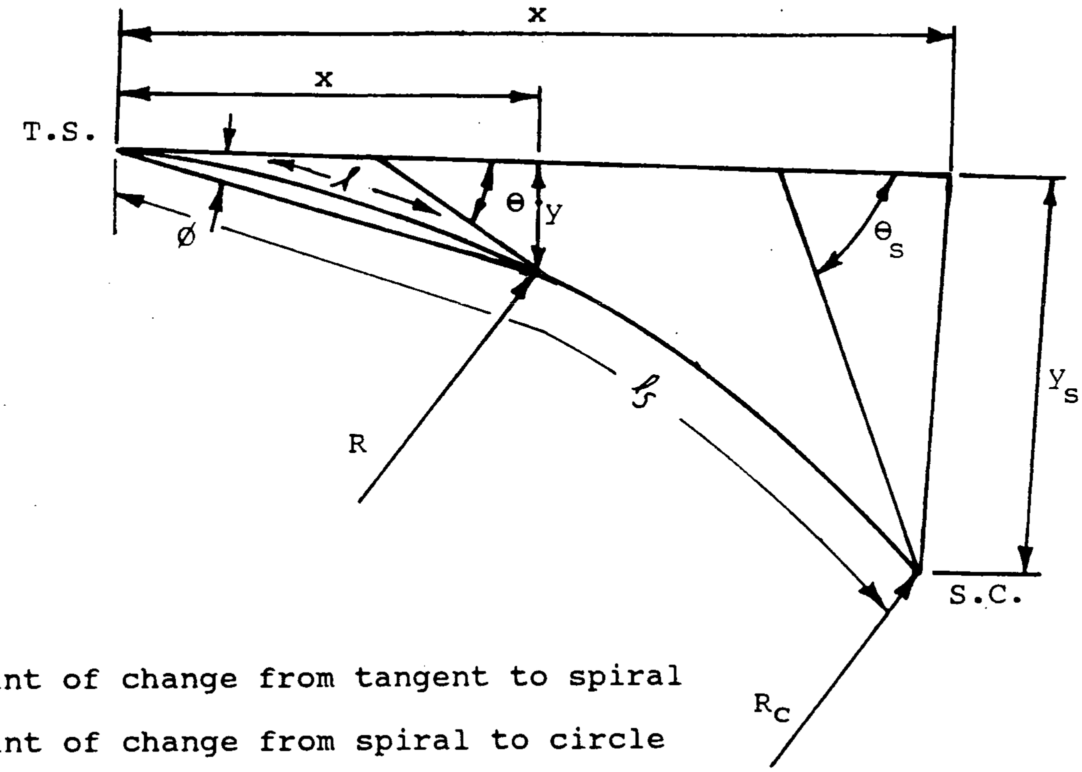
CHAPTER 4 Spiral Curves

SPIRAL CURVES

A spiral curve is generally used to lessen the sudden change in curvature at the junction of a tangent and a circular curve. With a spiral, the radius is decreasing from infinity at the tangent to that of the curve it meets.



The notation used on a spiral are as follows:



TS = point of change from tangent to spiral

SC = point of change from spiral to circle

CS = point of change from circle to spiral

ST = point of change from spiral to tangent

ℓ = spiral arc from the TS to any point on spiral

ℓ_s = total length of spiral from TS to SC

θ = central angle of spiral arc ℓ

θ_s = central angle of spiral arc ℓ_s called **"spiral angle"**

ϕ = spiral deflection angle at the TS from initial tangent to any point on spiral

R = radius

R_c = radius of circular curve

y = tangent offset of any point on spiral with reference to TS and initial tangent

x = tangent distance of any point on spiral with references to TS and initial tangent

y_s = tangent offset at the SC

x_s = tangent distance at the SC

Δ = total central angle of the circular curve

Δ_c = central angle of circular arc of length LC from SC to CS

LT = long tangent

ST = short tangent

LC = long chord

E_s = total external distance

p = offset from the initial tangent to the PC of the shifted circle

T_s = total tangent distance: PI to TS or PI to ST

Formulae

$$\ell = 200 \frac{\theta_s}{D_c}$$

$$\theta = \frac{[\ell]^2 \theta_s}{[\ell_s]^2}$$

$$\phi = \frac{\theta}{3} - C_s$$

C_s = Correction. For values of θ less than 15°, C_s is negligible.

Example

Given ℓ_s = 400 m. θ_s = 15° and the TS at Sta. 37 + 10.2 required location of the spiral by deflections from the TS and the initial tangent. Also note C_s is neglected.

Solution

$$\phi = \frac{1}{3}\theta = \frac{1}{3}\left[\frac{\ell}{\ell_s}\right]^2\theta_s$$

express $\frac{1}{3}\theta_s$ in minutes thus:

$$\frac{1}{3}(15°) = 300 \text{ minutes}$$

$$\phi = \left[\frac{\ell}{\ell_s}\right]^2 \times 300$$

Station	ℓ	$\frac{\ell}{\ell_s}$	$\left[\frac{\ell}{\ell_s}\right]^2$	Deflection
37 + 10.2TS	0	0	0	0
+ 50	39.8	0.100	0.010	0°03′
38 + 00	89.8	0.224	0.050	0°15′
+ 50	139.8	0.350	0.122	0°37′
39 + 00	189.8	0.474	0.225	1°07.5′
+ 50	239.8	0.600	0.360	1°48′
40 + 00	289.8	0.724	0.524	2°37′
+ 50	339.8	0.850	0.722	3°36.5′
41 + 00	389.8	0.974	0.949	4°45′
41 + 10.2ST	400.0	1.000	1.000	5°00′

EXERCISE NO. 1

1. Solve for the deflections from the TS and the initial tangent given the following θ_s = 8°00′ ℓ_s = 250 m. TS = Sta. 54 + 25.5

(Refer to answers on Page 278)

WORK PAGE

WORK PAGE

SECTION III

CHAPTER 5 Superelevation

SUPERELEVATION

The effect of centrifugal force on a vehicle passing through a curve is counteracted by "**superelevation**" of the outer edge of a highway pavement. Proper superelevation insures smooth and safe riding through the curve. The transition to full superelevation is shown below for a spiral and a normal curve.

R (meters)	V_d = 30 km/h e (%)	L (meters) Two Lanes	Four Lanes	V_d = 40 km/h e (%)	L (meters) Two Lanes	Four Lanes	V_d = 50 km/h e (%)	L (meters) Two Lanes	Four Lanes	V_d = 60 km/h e (%)	L (meters) Two Lanes	Four Lanes	V_d = 70 km/h e (%)	L (meters) Two Lanes	Four Lanes
7000	NC	0	0	NC	0	0	NC	0	0	NC	0	0	NC	0	0
5000	NC	0	0	NC	0	0	NC	0	0	NC	0	0	NC	0	0
3000	NC	0	0	NC	0	0	NC	0	0	NC	0	0	NC	0	0
2500	NC	0	0	NC	0	0	NC	0	0	NC	0	0	NC	0	0
2000	NC	0	0	NC	0	0	NC	0	0	NC	0	0	NC	0	0
1500	NC	0	0	NC	0	0	NC	0	0	NC	0	0	RC	39	59
1400	NC	0	0	NC	0	0	NC	0	0	RC	33	50	2.1	39	59
1300	NC	0	0	NC	0	0	NC	0	0	RC	33	50	2.2	39	59
1200	NC	0	0	NC	0	0	NC	0	0	RC	33	50	2.4	39	59
1000	NC	0	0	NC	0	0	RC	28	42	2.2	33	50	2.8	39	59
900	NC	0	0	NC	0	0	RC	28	42	2.4	33	50	3.1	39	59
800	NC	0	0	NC	0	0	RC	28	42	2.7	33	50	3.4	39	59
700	NC	0	0	RC	22	33	2.2	28	42	3.0	33	50	3.8	39	59
600	NC	0	0	RC	22	33	2.6	28	42	3.4	33	50	4.3	39	59
500	NC	0	0	2.2	22	33	3.0	28	42	3.9	33	50	4.9	39	59
400	RC	17	26	2.7	22	33	3.6	28	42	4.7	33	50	5.7	39	59
300	2.1	17	26	3.4	22	33	4.5	28	42	5.6	34	51	6.7	44	66
250	2.5	17	26	4.0	22	33	5.1	28	42	6.2	37	56	7.3	48	72
200	3.0	17	26	4.6	24	36	5.8	31	47	7.0	42	63	7.9	52	78
175	3.4	17	26	5.0	26	39	6.2	33	50	7.4	44	67	8.0	52	79
150	3.8	18	27	5.4	28	42	6.7	36	54	7.8	47	70	R_{min} = 175		
140	4.0	19	29	5.6	29	43	6.9	37	56	7.9	47	71			
130	4.2	20	30	5.8	30	45	7.1	38	58	8.0	48	72			
120	4.4	21	32	6.0	31	46	7.3	39	59	R_{min} = 125					
110	4.7	23	34	6.3	32	49	7.6	41	62						
100	4.9	23	35	6.5	33	50	7.8	42	63						
90	5.2	25	37	6.9	36	53	7.9	43	64						
80	5.5	26	40	7.2	37	56	8.0	43	65						
70	5.9	28	42	7.5	39	58	R_{min} = 80								
60	6.4	31	46	7.8	40	60									
50	6.9	33	50	8.0	41	62									
40	7.5	36	54	R_{min} = 50											
30	8.0	38	57												
	R_{min} = 30														

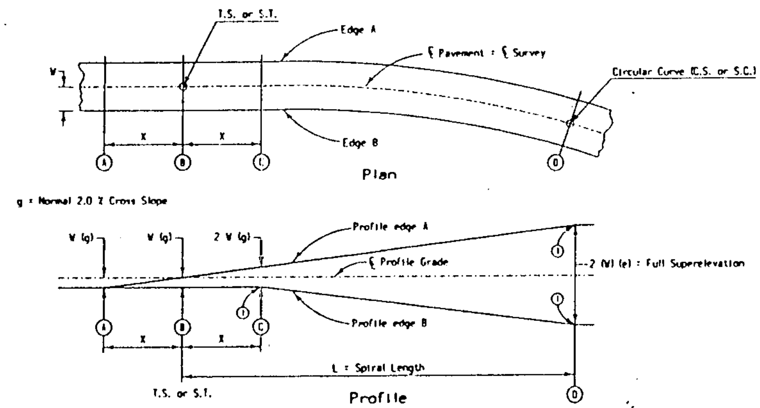
R (meters)	V_d = 80 km/h e (%)	L (meters) Two Lanes	Four Lanes	V_d = 90 km/h e (%)	L (meters) Two Lanes	Four Lanes	V_d = 100 km/h e (%)	L (meters) Two Lanes	Four Lanes	V_d = 110 km/h e (%)	L (meters) Two Lanes	Four Lanes	V_d = 120 km/h e (%)	L (meters) Two Lanes	Four Lanes
7000	NC	0	0	NC	0	0	NC	0	0	NC	0	0	NC	0	0
5000	NC	0	0	NC	0	0	NC	0	0	NC	0	0	NC	0	0
3000	NC	0	0	NC	0	0	RC	56	84	2.1	61	92	2.4	67	101
2500	NC	0	0	RC	50	75	2.1	56	84	2.4	61	92	2.9	67	101
2000	RC	44	66	2.2	50	75	2.6	56	84	3.0	61	92	3.5	67	101
1500	2.4	44	66	2.8	50	75	3.4	56	84	3.9	61	92	4.6	67	101
1400	2.5	44	66	3.0	50	75	3.6	56	84	4.1	61	92	4.9	67	101
1300	2.7	44	66	3.2	50	75	3.8	56	84	4.4	61	92	5.2	67	101
1200	2.9	44	66	3.4	50	75	4.1	56	84	4.7	61	92	5.6	67	101
1000	3.4	44	66	4.0	50	75	4.8	56	84	5.5	61	92	6.5	67	101
900	3.7	44	66	4.4	50	75	5.2	56	84	6.0	61	92	7.1	67	101
800	4.1	44	66	4.8	50	75	5.7	56	84	6.5	61	92	7.6	68	103
700	4.5	44	66	5.3	50	75	6.3	56	84	7.2	62	93	8.0	72	108
600	5.1	44	66	6.0	50	75	6.9	56	84	7.7	66	99	R_{min} = 665		
500	5.8	44	66	6.7	51	76	7.6	61	91	8.0	69	103			
400	6.6	48	71	7.5	57	85	8.0	64	96	R_{min} = 500					
300	7.6	55	82	R_{min} = 305			R_{min} = 395								
250	7.9	57	85												
	R_{min} = 230														

e_{max} = 8.0%

R = radius of curve
V = assumed design speed
e = rate of superelevation (%)
L = minimum length of runoff (does not include tangent runout)

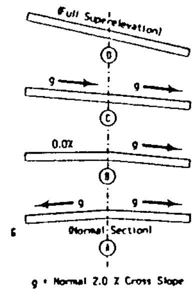
NC = normal cross slope
RC = remove adverse cross slope, superelevate at normal cross slope

Note: Lengths rounded in multiples of 10 m permit simpler calculations.

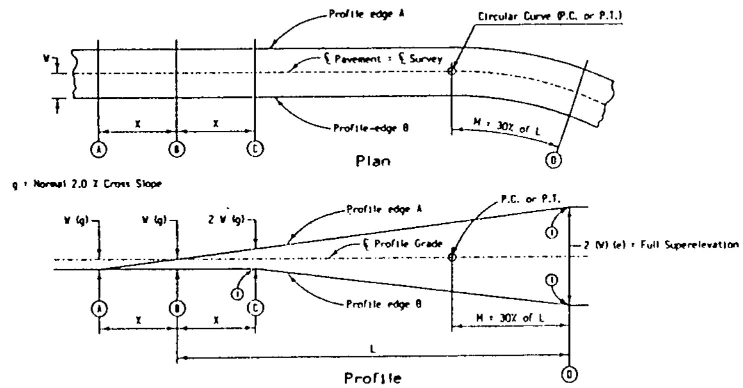


TRANSITION DETAILS WHEN SPIRAL IS USED

Pavement Crown shall be removed at point (C)



AXIS OF ROTATION
AT CENTER LINE



TRANSITION DETAILS WHEN SPIRAL IS NOT USED

Pavement Crown shall be removed at point (C)

GENERAL NOTES:

Details hereon cover construction details for superelevating a horizontal curve on a two lane roadway where the axis of rotation is about the centerline of the roadway.

Materials and methods of construction shall be in accordance with current Standard and Supplemental Specifications.

Refer to specific curve data contained in detail project plans for transition lengths and rate of superelevation.

Design edge slope is the ratio of the longitudinal profiles between axis of rotation and outer edge of slab. The exact edge slope ratio for a given radius and design speed may be determined by the following formula:

$$\text{Edge Slope Ratio} = \frac{L}{(e)\,(W)}$$

When spiral transitions are not required, normal practice shall be to place 30% of the transition length 'L' within the curve. The remaining transition length shall be placed on the tangent. 70% of full super shall be placed at the P.C. and P.T.

For details of shoulder treatment in areas of superelevated curves, refer to appropriate Typical Cross Sections.

Unless otherwise specified, the transition lengths are measured at the centerline of the roadway.

(1) Smooth curve established at time of construction.

MAXIMUM SUPERELEVATION
RATE (e) = 0.08 m/m (8.0%)

All dimensions given in millimeters unless noted.

M — METRIC VERSION

Iowa Department of Transportation
Project Development Division

STANDARD ROAD PLAN **RP-1**

REVISION: Metric conversion of Standard Road Plan RP-1 no. 3 ... 5-13-88

05-20-94
APPROVED BY DESIGN METHODS ENGINEER

REVISION NO. 3
REVISION DATE 09-27-94

SUPERELEVATED DETAILS
TWO LANE ROADWAYS

Definitions

g = Normal cross slope rate.

e = Superelevation rate - max. superelevation rate = 0.08 m/m. or 8%

w = Pavement width.

x = Distance from the beginning of the transition to a point where the crown is no longer in the outside lane of the road.

L = Transition length.

Formula

L = ESR(e)(w)

L = Transition length

ESR = Edge Slope Ratio

e = Rate of Superelevation (m/m)

w = Pavement width (axis of rotation to edge of slab)

Design Speed	Pavement Width (W)				
	3.6m	4.8m	5.5m	7.2m	11.0m
50	1:150	1:138	1:131	1:113	1.100
60	1:160	1:147	1:140	1:120	1:107
70	1:175	1:160	1:153	1:131	1:117
80	1:185	1:170	1:162	1:139	1:123
90	1:200	1:183	1:175	1:150	1:133
100	1:211	1:193	1:185	1:158	1:141
110	1:222	1:204	1:195	1:167	1:148
120	1:244	1:224	1:214	1:183	1:163
130	1:250	1:229	1:219	1:188	1:167

MINIMUM DESIGN EDGE SLOPE RATIO

Example:

Given R = xxx and a 100 km/h design speed, and a spiral with the T.S. station 550 + 10. Solve for the cross-slope rate at Sta. 551 + 00 with normal slope crown of 2.00% in the road.

Solution:

From Table A we find the values e = 0.078, L = 208 m, x = 53. Solve for beginning of transition

$$(550 + 10) - 53 = (549 + 57)$$

Point A or beginning of transition is at Sta. 549 + 57. The end of the transition is therefore at 549 + 57 + 208 m + 53 = 552 + 18 or Sta 552 + 18. And at Point C the Station is 549 + 57 + 106 = 550 + 63 or Sta. 550 + 63 where the outer edge is Parallel to inside edge which is at the normal cross- slope rate or 0.020 m/m. At Sta. 552 + 18 the cross slope is 0.078 m/m.

Sta. Diff.

$$155.0\left[\;118\begin{cases}\text{Sta. } 550+63 = 0.020' \text{ per ft} \\ \text{Sta. } 551 + 00 = ? \\ \text{Sta.} 552+18 = 0.078' \text{ per ft.}\end{cases}\right] 0.078-0.020=0.058 \text{ per ft.}$$

$$\frac{118}{155} \times 0.058 = 0.044 \text{ m/m}$$

Therefore, at Sta. 551 + 00 the cross-slope rate is 0.078 - 0.044 = 0.034 m/m

EXERCISE NO. 1

1. Solve for the cross-slope rate at Sta. 73 + 00. Given a design speed of 110 km/h and R = xxx and a spiral with T.S. at Sta. 71 + 50. The road has a normal crown of 2.00%.

EXERCISE NO. 2

1. Solve for the cross-slope rate at Sta. 52 + 00. Given the design speed of 110 km/h. and R = xxx and a normal curve with P.C. at Sta. 51 + 50. The road has a normal crown of 2.00%.

(Refer to answers on Page 279)

WORK PAGE

SECTION III ANSWERS

Chapter 1:

Exercise No. 1, Page 234:

(1) (a) 0.7465736 (b) 0.6191205 (c) 0.9477375
(2) (a) 48°34′ 20" (b) 38°07′30" (c) 35°19′40"

Exercise No 2, Page 234:

(1) Area = 100 m^2
(2) Area = 533 m^2

Exercise No. 3, Page 236:

(1) A = 28°55′ B = 61°05′ a = 47.7
(2) a = 48.6 b = 15.6 B = 17°46′
(3) b = 57.7 c = 46.9 C = 35°04′
(4) a = 53.7 B = 116°18′ C = 30°31′
(5) 527.89 m

Chapter 1 Quiz: Page 238:

(1) Height = 157.36 m
(2) a = 344.83 m
(3) Angle = 3°26′01"

Chapter 2:

Exercise No. 1, Page 244:

(1)

Station	Straight Grade Elevation	Offsets	Grade Elevation Curve
VPC 63	540.00 m	0.00	540.00
64	534.00 m	0.63	534.63
65	528.00 m	2.50	530.50
66	522.00 m	5.63	527.63
VPI 67	516.00 m	10.00	526.00
68	520.00 m	5.63	525.63
69	524.00 m	2.50	526.50
70	528.00 m	0.63	528.63
VPT 71	532.00 m	0.00	532.00

Exercise No. 2, Page 245:

(1) The high point of the curve is 2294.12 m from the VPC.

Chapter 2 Quiz, Page 246:

(1) The minimum number of stations is $\frac{+3.2 - (-0.8)}{0.5} = 8$

So, L = 8 stations $e = \frac{AL}{8} = \frac{(-0.8 - 3.2)8}{8} = -4.0$ m

Chapter 2 Quiz, Page 247 Continued

$$y_1 = y_6 = \frac{4et^2}{L^2} = \frac{4(-4)(1)^2}{8^2} = \frac{16}{64} = -0.25 \text{ m}$$

$$y_2 = y_5 = \frac{4(-4)(2)^2}{64} = \frac{64}{64} = -1.0 \text{ m}$$

$$y_3 = y_4 = \frac{4(-4)(3)^2}{64} = -2.25 \text{ m}$$

Station	Straight Grade Elevation	Offsets	Grade Elevation Curve
2	87.2 m	0.00	87.2 m
3	90.4 m	-0.25	90.15 m
4	93.6 m	-1.00	92.6 m
5	96.8 m	-2.25	94.55 m
6	100.0 m	-4.00	96.00 m
7	99.2 m	-2.25	96.95 m
8	98.4 m	-1.00	97.4 m
9	97.6 m	-0.25	97.35 m
10	96.8 m	0.00	96.8 m

The high point is at Station X = 3.2 x (8/4) = 6.4 or the high point comes at Station 8 + 40.00.

Chapter 3:

Exercise No. 1, Page 256:

(1) Length of curve = 665.41 m. P.T. is at station 33 + 65.41 and P.I. is at Station 30 + 34.68. The external distance is 22.30 m.

Exercise No. 2, Page 260:

(1) P.T. = 63 + 16.91

Chapter 4:

Exercise No. 1, Page 268:

(1) Sta. 54 + 25.5 = 0° 00′ 00"
+ 50 = 0° 01′ 32"
55 + 00 = 0° 14′ 12"
+ 50 = 0° 39′ 41"
56 + 00 = 1° 17′ 57"
+ 50 = 2° 09′ 01"
+ 75.5 = 2° 40′ 00"

Chapter 5:

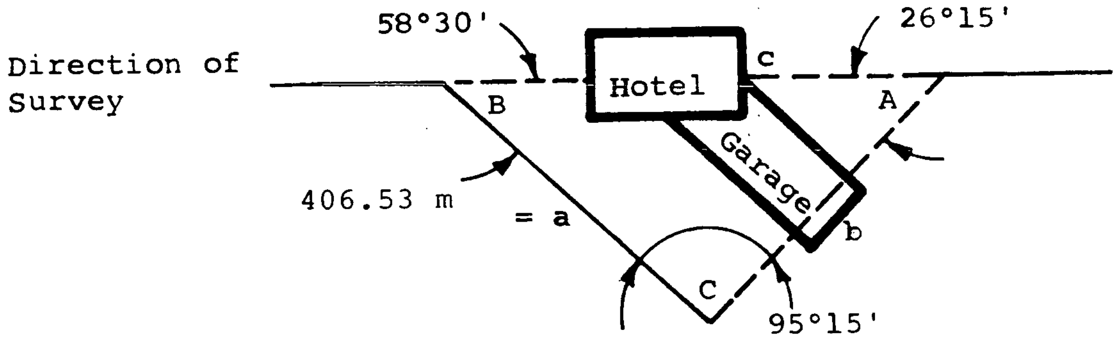
Exercise No. 1, Page 293:
(1) 0.050 m/m

Exercise No. 2, Page 293:
(1) 0.027 m/m

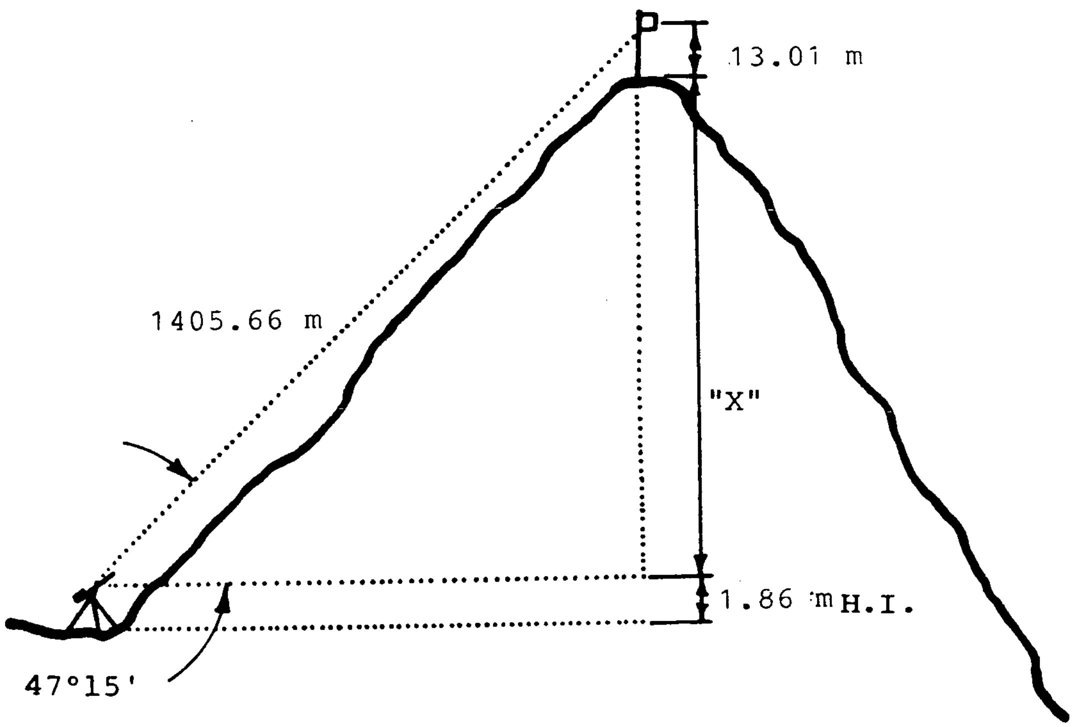
SECTION III

COMPREHENSIVE TEST

1. While making a survey, the transit man found it necessary to triangulate around several obstructions. Find the lengths he couldn't measure. (hundredths)

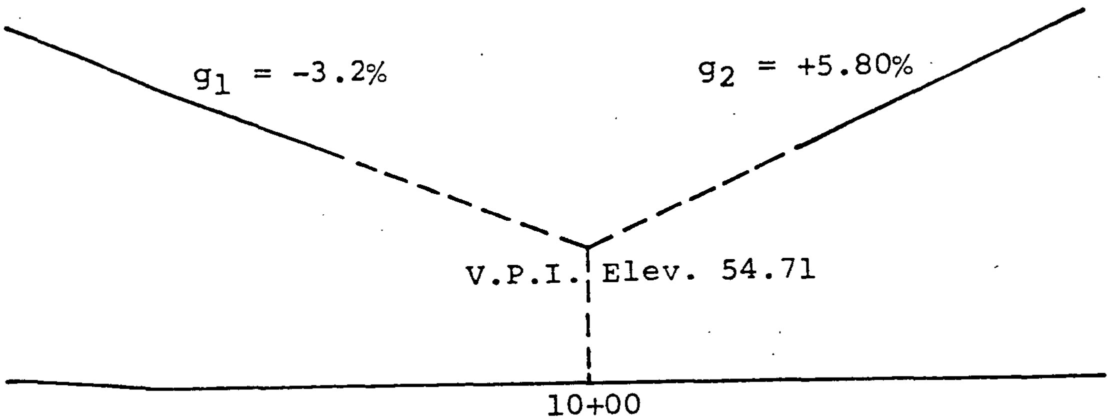


2. What is the height (hundredths) of hill above the ground that the instrument stands on?



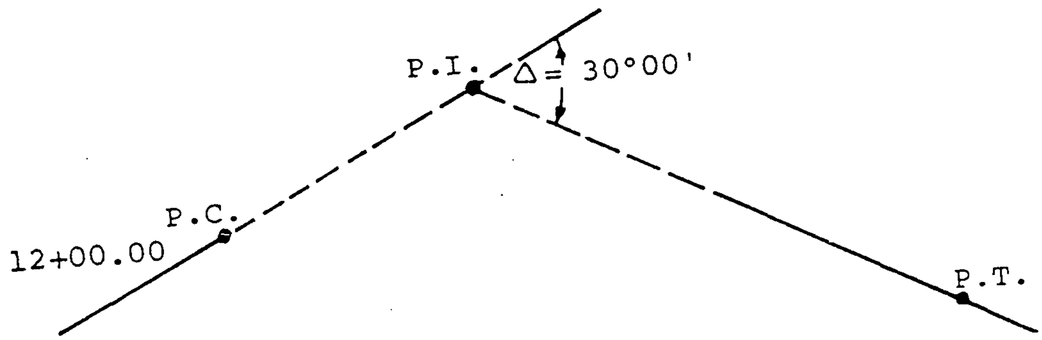
WORK PAGE

3. Vertical Curve:



The maximum allowable change in grade per station is 1.5 percent. Calculate the elevation of stations at 100 m. intervals along the vertical curve connecting the two grades. At what station does the lowest point lie? (Answers should all be in hundredths).

4. Horizontal Curve:



This highway has a 100 km/h speed limit requiring a minimum radius for any curve to be 395 m. If the curve is to begin at Station 12 + 00.00 and is to have a radius of 395 m, how long will the curve be? At what station will the P.I. and P.T. go? How long will E be? (Answers should all be in hundredths).

WORK PAGE

5. Spiral Curve:

Given R = 3000 m ℓ_s = 61 m θ_s = 12° 00′

The TS is at Sta = 74 + 50.5. Solve deflection from T.S. and Initial Tangent for Sta 75 + 00.00.

6. Solve for the cross-slope rate at Sta. 30 + 00. Given the Design speed of 90 km/h and a curve with R = 2000 m and a spiral with the T.S. at 28 + 75.5 assume a natural crown at 2.00% and roadway width of 7.2 m.

SECTION THREE COMPREHENSIVE TEST ANSWERS

Question 1: c1 = 90 - 58.5 = 31.5 degrees
c2 = 95.25 - 31.50 = 63.75 degrees
Using the sine law and the given angle a and side a, solve for b and c sides. b = 783.70 m c = 915.29 m

Question 2: Using the sine relationship, given angle and hypotenuse, calculate the height to the top of the flag.
H = 1032.21 m Hill height = 1032.21 -13.01 + 1.86 = 1021.06 m

Question 3: A = 1.5 by problem Min Length = (g2 - g1)/A = 6 stations. e = AL/8 = 6.75 sta
y1 = y4 = $4ex^2/L^2$ = 0.75 m y2 = y3 = 3.00 m

Station	St. Gr. Elev	Offset	Curve Elev
VPC 7	64.31	0	64.31
8	61.11	0.75	61.86
9	57.91	3.00	60.91
VPI 10	54.71	6.75	61.46
11	60.51	3.00	63.51
12	66.31	0.75	67.06
VPT 13	72.11	0	72.11

Low point = g_1 {$L/(g_2- g_1)$ = 3.2(6/9) = 2.13333 stations
= (7+00) + (2+13.33) = Sta. 9+13.33

Question 4: Using the R = 395 m find the tangent length
T = 105.84 m Find the length of curve LC = 20.68 stations
PI sta = PC + T = 13+05.84 PT = PC + LC = 14+06.80
E = [R/Cos(delta/2)] - R = 13.93 m

Question 5: R = 3000 m l_s = 61 m Q_s =12 degrees
At station 75+00, deflection = 2.688 degrees or 2 degrees, 41 minutes

Question 6: Station 30+00 is in the circular curve and from the table, the suggested rate of superelevation is 2.2% for any station in the circular curve.

GLOSSARY

AASHTO - American Association of State Highway Transportation Officials.

acceleration - The rate of change of velocity with respect to time.

ampere - The unit of current in two straight parallel wires of a long length separated by one meter in free space, which produces a magnetic force between the two wires of 2×10^{-7} newtons per meter length.

ANMC - American National Metric Council.

ASTM - American Society of Testing Materials.

base units - Seven **SI** units which by choice are regarded as dimensionally independent: **meter, kilogram, second, ampere, kelvin, mole,** and **candela.**

candela - Unit of luminous intensity in a given direction of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and has a radiant intensity in that direction of 1/683 watt per steradian.

Celsius - A temperature scale that registers the freezing point and boiling point of water at - mospheric pressure as 0 and 100 degrees, respectively.

coherent - A system of measurement units mutually related by rules of system multiplication and division without any conversion factor, except for unity.

conversion, hard - Conversion from one measurement system to another using numerical conversion factor to calculate quantities in a new system and then rounding to a convenient dimension.

conversion, soft - Conversion from one measurement system another using the numerical conversion factor to calculate quantities in a new system.

derived units - Units that can be formed by combining base units according to the algebraic relations linking the corresponding quantities.

DOC - Department of Commerce.

FHWA - Federal Highway Administration.

force - A vector quantity that tends to produce an acceleration of a body in the direction of application.

gram 1/1000 of a kilogram.

GSA - General Services Administration.

hectare - The derived unit of area equal to 10,000 m^2.

ISO - International Organization for Standardizat-ion.

joule - The derived unit of energy equal to one newton meter.

kelvin - Unit of temperature that is the fraction 1/273.16 of the thermodynamic triple point of water.

kilogram - The mass of a cylinder of platinum-iridium alloy kept by the **International Bureau of Weights and Measures** near Paris.

liter - The unit of volume occupied by a mass of 1 kilogram of pure water at its maximum density and at standard atmospheric pressure. Also 1 dm^3.

mass - The measure of inertia that an object has, or the measure of the ability of an object to resist acceleration.

meter - The length of path traveled by light in a vacuum during a time interval of 1/299,792,458 of a second.

Metric Conversion Act of 1975 - Established for voluntary conversion to metric by industries in the United States.

mole - The unit of the amount of substance of a system that contains many elementary entities as there are atoms in 0.012 kilograms carbon-12.

newton - The derived unit of force that is equal to one kilogram meter per squared second.

NIST - National Institute of Standards and Technology.

Omnibus Trade and and Competitiveness of Act of 1988 - Establishes the policy of the United States that metric is preferred system of measurement and each Federal agency use the metric system by a date economically feasible.

pascal - The derived unit of pressure equal to one newton per square meter.

plane angle - An angle formed by two straight lines.

radian - The supplementary unit of plane angle with its vertex at the center of a circle that is subtended by arc equal in length to the radius

second - Duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium-133 atom.

SI - International System of Units.

solid angle - An angle subtended at a point by a surface measured in steradians.

standard - A criterion or level of performance required to be met.

specification - A criterion or measurement found on a design drawing.

steradian - The solid angle with its vertex at the center of a sphere such that the surface area is equal to radius of the sphere squared.

supplementary units - 2 SI units that are dimensionless. They are the radian and units steradian.

USMA - United States Metric Association.

watt - The derived SI unit of power equal to one newton•meter/second, or one joule per second.

weight - The force of an object due to gravity derived as mass multiplied by gravitational acceleration.

APPENDICES

APPENDIX A

MEASURES AND EQUIVALENTS

Conversion factors: When converting a quantity from U.S. customary units to metric units, multiply the quantity by a conversion factor that is more accurate than required. Do not round either the conversion factor or the quantity before multiplication, as accuracy may be reduced. After multiplication, round the result so that the converted value does not imply a greater degree of precision (i.e., more decimal places or more significant digits) than existed in the original value.

Below is a table of simple conversion factors. Others may be found in ASTM E 380-92.

CONVERSION FACTORS

Quantity	From U.S. Customary Units	To Metric Units	Multiply By
Length	mile (U.S. Statute)′	km	1.609347
	mile (international)	km	1.609344*
	yard	m	0.9144*
	foot (U.S. Survey)′	m	0.3048006
	foot (international)	m	0.3048*
	inch	mm	25.4*
Area	square mile (U.S. Statute)′	km^2	2.589998
	square mile (international)	km^2	2.589988
	acre (U.S. Survey)′	m^2	4046.873
		ha (10,000 m^2)	0.4046873
	square yard	m^2	0.83612736*
	square foot	m^2	0.09290304*
	square inch	mm^2	645.16*
Volume	acre foot (U.S. Survey)′	m^3	1233.489
	cubic yard	m^3	0.76455486
	cubic foot	m^3	0.02831685
		cm^3	28316.85
		L (1000 cm^3)	28.31685
	100 board feet	m^3	0.2359737
	gallon	L	3.785412
	cubic inch	cm^3	16.387064*
		mm^3	16387.064*
Mass	pound	kg	0.45359237*
	kip (1000 pounds)	Mg (1000 kg)	0.45359237*
Temperature	°F	°C	Formula: °C=(°F - 32)/1.8*

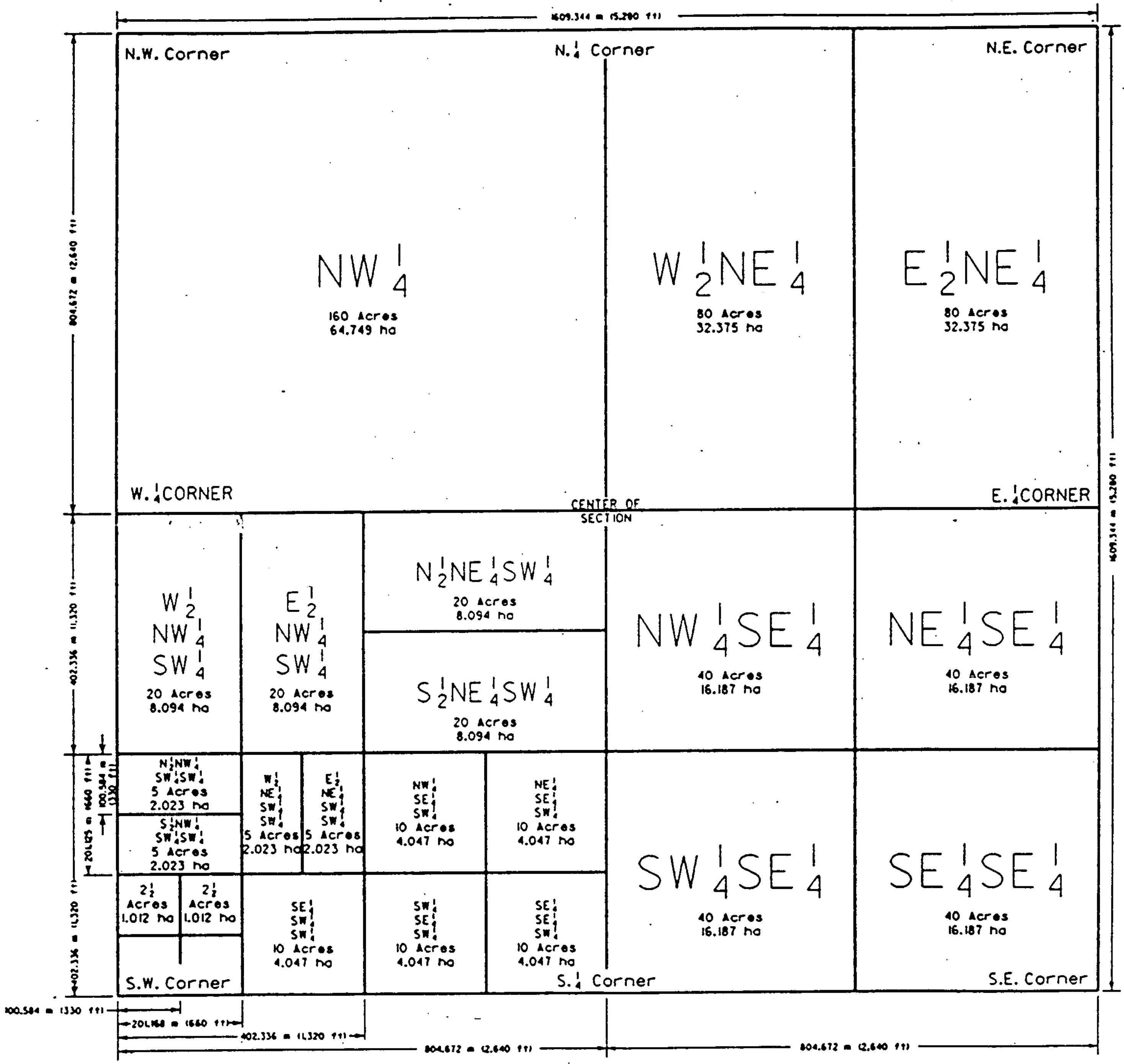
* exact conversion

′ Conversion factors that are referenced to this footnote are based on the U.S. survey foot. The U.S. survey foot equals 1200/3937 meter.

Metric Conversion Factors
(approximate conversion from metric units)

Quantity	From Metric Units	Symbol	To English Units	Multiply by
Length	millimeters	mm	inches	.04
	meters	m	feet	3.3
	kilometers	km	miles	.6
Area	square meters	m^2	square yards	1.2
	square kilometers	km^2	square miles	.4
	hectares	ha	acres	2.5
Mass	grams	g	ounces	.035
	kilograms	kg	pounds	2.2
Volume	milliliters	mL	fliud ounces	.03
	milliliters	mL	cubic inches	.06
	liters	L	gallons	.26
	cubic meters	m^3	cubic feet	35.0
	cubic meters	m^3	cubic yards	1.3
Temperature	degrees Celsius	°C	degrees Fahrenheit	9/5 and add 32

METRIC AND ENGLISH UNITS
QUARTER SECTIONS AND SUBDIVISIONS



1 acre = 0.405 hectares	1 ft = 0.305 m
1 acre = 43,560 sq ft	1 rod = 16.5 ft
1 sq ft = 0.093 sq m	

USEFUL CONVERSION FACTORS

LENGTH

inches	mm	inches	mm	inches	mm
0.00	0.00	8.00	203.20	16.00	406.40
0.20	5.08	8.20	208.28	16.20	411.48
0.40	10.16	8.40	213.36	16.40	416.56
0.60	15.24	8.60	218.44	16.60	421.64
0.80	20.32	8.80	223.52	16.80	426.72
1.00	25.40	9.00	228.60	17.00	431.80
1.20	30.48	9.20	233.68	17.20	436.88
1.40	35.56	9.40	238.76	17.40	441.96
1.60	40.64	9.60	243.84	17.60	447.04
1.80	45.72	9.80	248.92	17.80	452.12
2.00	50.80	10.00	254.00	18.00	457.20
2.20	55.88	10.20	259.08	18.20	462.28
2.40	60.96	10.40	264.16	18.40	467.36
2.60	66.04	10.60	269.24	18.60	472.44
2.80	71.12	10.80	274.32	18.80	477.52
3.00	76.20	11.00	279.40	19.00	482.60
3.20	81.28	11.20	284.48	19.20	487.68
3.40	86.36	11.40	289.56	19.40	492.76
3.60	91.44	11.60	294.64	19.60	497.84
3.80	96.52	11.80	299.72	19.80	502.92
4.00	101.60	12.00	304.80	20.00	508.00
4.20	106.68	12.20	309.88	20.20	513.08
4.40	111.76	12.40	314.96	20.40	518.16
4.60	116.84	12.60	320.04	20.60	523.24
4.80	121.92	12.80	325.12	20.80	528.32
5.00	127.00	13.00	330.20	21.00	533.40
5.20	132.08	13.20	335.28	21.20	538.48
5.40	137.16	13.40	340.36	21.40	543.56
5.60	142.24	13.60	345.44	21.60	548.64
5.80	147.32	13.80	350.52	21.80	553.72
6.00	152.40	14.00	355.60	22.00	558.80
6.20	157.48	14.20	360.68	22.20	563.88
6.40	162.56	14.40	365.76	22.40	568.96
6.60	167.64	14.60	370.84	22.60	574.04
6.80	172.72	14.80	375.92	22.80	579.12
7.00	177.80	15.00	381.00	23.00	584.20
7.20	182.88	15.20	386.08	23.20	589.28
7.40	187.96	15.40	391.16	23.40	594.36
7.60	193.04	15.60	396.24	23.60	599.44
7.80	198.12	15.80	401.32	23.80	604.52

feet	meters	feet	meters	feet	meters
0.0	0.0	8.0	2.4	16.0	4.9
0.2	0.1	8.2	2.5	16.2	4.9
0.4	0.1	8.4	2.6	16.4	5.0
0.6	0.2	8.6	2.6	16.6	5.1
0.8	0.2	8.8	2.7	16.8	5.1
1.0	0.3	9.0	2.7	17.0	5.2
1.2	0.4	9.2	2.8	17.2	5.2
1.4	0.4	9.4	2.9	17.4	5.3
1.6	0.5	9.6	2.9	17.6	5.4
1.8	0.5	9.8	3.0	17.8	5.4
2.0	0.6	10.0	3.0	18.0	5.5
2.2	0.7	10.2	3.1	18.2	5.5
2.4	0.7	10.4	3.2	18.4	5.6
2.6	0.8	10.6	3.2	18.6	5.7
2.8	0.9	10.8	3.3	18.8	5.7
3.0	0.9	11.0	3.4	19.0	5.8
3.2	1.0	11.2	3.4	19.2	5.9
3.4	1.0	11.4	3.5	19.4	5.9
3.6	1.1	11.6	3.5	19.6	6.0
3.8	1.2	11.8	3.6	19.8	6.0
4.0	1.2	12.0	3.7	20.0	6.1
4.2	1.3	12.2	3.7	20.2	6.2
4.4	1.3	12.4	3.8	20.4	6.2
4.6	1.4	12.6	3.8	20.6	6.3
4.8	1.5	12.8	3.9	20.8	6.3
5.0	1.5	13.0	4.0	21.0	6.4
5.2	1.6	13.2	4.0	21.2	6.5
5.4	1.6	13.4	4.1	21.4	6.5
5.6	1.7	13.6	4.1	21.6	6.6
5.8	1.8	13.8	4.2	21.8	6.6
6.0	1.8	14.0	4.3	22.0	6.7
6.2	1.9	14.2	4.3	22.2	6.8
6.4	2.0	14.4	4.4	22.4	6.8
6.6	2.0	14.6	4.5	22.6	6.9
6.8	2.1	14.8	4.5	22.8	6.9
7.0	2.1	15.0	4.6	23.0	7.0
7.2	2.2	15.2	4.6	23.2	7.1
7.4	2.3	15.4	4.7	23.4	7.1
7.6	2.3	15.6	4.8	23.6	7.2
7.8	2.4	15.8	4.8	23.8	7.3

feet	km	feet	km	feet	km
0.0	0.0	8,000.0	2.4	15,000.0	4.6
200.0	0.1	8,200.0	2.5	15,200.0	4.6
400.0	0.1	8,400.0	2.6	15,400.0	4.7
600.0	0.2	8,600.0	2.6	15,600.0	4.8
800.0	0.2	8,800.0	2.7	15,800.0	4.8
1,000.0	0.3	9,000.0	2.7	16,000.0	4.9
1,200.0	0.4	9,200.0	2.8	16,200.0	4.9
1,400.0	0.4	9,400.0	2.9	16,400.0	5.0
1,600.0	0.5	9,500.0	2.9	16,600.0	5.1
1,800.0	0.5	9,600.0	2.9	16,800.0	5.1
2,000.0	0.6	10,000.0	3.0	17,000.0	5.2
2,200.0	0.7	10,200.0	3.1	17,200.0	5.2
2,400.0	0.7	10,400.0	3.2	17,400.0	5.3
2,600.0	0.8	10,600.0	3.2	17,600.0	5.4
2,800.0	0.9	10,800.0	3.3	17,800.0	5.4
3,000.0	0.9	11,000.0	3.4	18,000.0	5.5
3,200.0	1.0	11,200.0	3.4	18,200.0	5.5
3,400.0	1.0	11,400.0	3.5	18,400.0	5.6
3,600.0	1.1	11,600.0	3.5	18,600.0	5.7
3,800.0	1.2	11,800.0	3.6	18,800.0	5.7
4,000.0	1.2	11,000.0	3.4	19,000.0	5.8
4,200.0	1.3	11,200.0	3.4	19,200.0	5.9
4,400.0	1.3	11,400.0	3.5	19,400.0	5.9
4,600.0	1.4	11,600.0	3.5	19,600.0	6.0
4,800.0	1.5	11,800.0	3.6	19,800.0	6.0
5,000.0	1.5	12,000.0	3.7	20,000.0	6.1
5,200.0	1.6	12,200.0	3.7	20,200.0	6.2
5,400.0	1.6	12,400.0	3.8	20,400.0	6.2
5,600.0	1.7	12,600.0	3.8	20,600.0	6.3
5,800.0	1.8	12,800.0	3.9	20,800.0	6.3
6,000.0	1.8	13,000.0	4.0	21,000.0	6.4
6,200.0	1.9	13,200.0	4.0	21,200.0	6.5
6,400.0	2.0	13,400.0	4.1	21,400.0	6.5
6,600.0	2.0	13,600.0	4.1	21,600.0	6.6
6,800.0	2.1	13,800.0	4.2	21,800.0	6.6
7,000.0	2.1	14,000.0	4.3	22,000.0	6.7
7,200.0	2.2	14,200.0	4.3	22,200.0	6.8
7,400.0	2.3	14,400.0	4.4	22,400.0	6.8
7,600.0	2.3	14,600.0	4.5	22,600.0	6.9
7,800.0	2.4	14,800.0	4.5	22,800.0	6.9

miles	km	miles	km	miles	km
0.0	0.0	8.0	12.9	16.0	25.7
0.2	0.3	8.2	13.2	16.2	26.1
0.4	0.6	8.4	13.5	16.4	26.4
0.6	1.0	8.6	13.8	16.6	26.7
0.8	1.3	8.8	14.2	16.8	27.0
1.0	1.6	9.0	14.5	17.0	27.4
1.2	1.9	9.2	14.8	17.2	27.7
1.4	2.3	9.4	15.1	17.4	28.0
1.6	2.6	9.6	15.4	17.6	28.3
1.8	2.9	9.8	15.8	17.8	28.6
2.0	3.2	10.0	16.1	18.0	29.0
2.2	3.5	10.2	16.4	18.2	29.3
2.4	3.9	10.4	16.7	18.4	29.6
2.6	4.2	10.6	17.1	18.6	29.9
2.8	4.5	10.8	17.4	18.8	30.3
3.0	4.8	11.0	17.7	19.0	30.6
3.2	5.1	11.2	18.0	19.2	30.9
3.4	5.5	11.4	18.3	19.4	31.2
3.6	5.8	11.6	18.7	19.6	31.5
3.8	6.1	11.8	19.0	19.8	31.9
4.0	6.4	12.0	19.3	20.0	32.2
4.2	6.8	12.2	19.6	20.2	32.5
4.4	7.1	12.4	20.0	20.4	32.8
4.6	7.4	12.6	20.3	20.6	33.2
4.8	7.7	12.8	20.6	20.8	33.5
5.0	8.0	13.0	20.9	21.0	33.8
5.2	8.4	13.2	21.2	21.2	34.1
5.4	8.7	13.4	21.6	21.4	34.4
5.6	9.0	13.6	21.9	21.6	34.8
5.8	9.3	13.8	22.2	21.8	35.1
6.0	9.7	14.0	22.5	22.0	35.4
6.2	10.0	14.2	22.9	22.2	35.7
6.4	10.3	14.4	23.2	22.4	36.0
6.6	10.6	14.6	23.5	22.6	36.4
6.8	10.9	14.8	23.8	22.8	36.7
7.0	11.3	15.0	24.1	23.0	37.0
7.2	11.6	15.2	24.5	23.2	37.3
7.4	11.9	15.4	24.8	23.4	37.7
7.6	12.2	15.6	25.1	23.6	38.0
7.8	12.6	15.8	25.4	23.8	38.3

AREA

sq ft	sq m	sq ft	sq m	sq ft	sq m
0.000	0.000	8.000	0.743	16.000	1.486
0.200	0.019	8.200	0.762	16.200	1.505
0.400	0.037	8.400	0.780	16.400	1.524
0.600	0.056	8.600	0.799	16.600	1.542
0.800	0.074	8.800	0.818	16.800	1.561
1.000	0.093	9.000	0.836	17.000	1.579
1.200	0.111	9.200	0.855	17.200	1.598
1.400	0.130	9.400	0.873	17.400	1.617
1.600	0.149	9.600	0.892	17.600	1.635
1.800	0.167	9.800	0.910	17.800	1.654
2.000	0.186	10.000	0.929	18.000	1.672
2.200	0.204	10.200	0.948	18.200	1.691
2.400	0.223	10.400	0.966	18.400	1.709
2.600	0.242	10.600	0.985	18.600	1.728
2.800	0.260	10.800	1.003	18.800	1.747
3.000	0.279	11.000	1.022	19.000	1.765
3.200	0.297	11.200	1.041	19.200	1.784
3.400	0.316	11.400	1.059	19.400	1.802
3.600	0.334	11.600	1.078	19.600	1.821
3.800	0.353	11.800	1.096	19.800	1.839
4.000	0.372	12.000	1.115	20.000	1.858
4.200	0.390	12.200	1.133	20.200	1.877
4.400	0.409	12.400	1.152	20.400	1.895
4.600	0.427	12.600	1.171	20.600	1.914
4.800	0.446	12.800	1.189	20.800	1.932
5.000	0.465	13.000	1.208	21.000	1.951
5.200	0.483	13.200	1.226	21.200	1.970
5.400	0.502	13.400	1.245	21.400	1.988
5.600	0.520	13.600	1.263	21.600	2.007
5.800	0.539	13.800	1.282	21.800	2.025
6.000	0.557	14.000	1.301	22.000	2.044
6.200	0.576	14.200	1.319	22.200	2.062
6.400	0.595	14.400	1.338	22.400	2.081
6.600	0.613	14.600	1.356	22.600	2.100
6.800	0.632	14.800	1.375	22.800	2.118
7.000	0.650	15.000	1.394	23.000	2.137
7.200	0.669	15.200	1.412	23.200	2.155
7.400	0.687	15.400	1.431	23.400	2.174
7.600	0.706	15.600	1.449	23.600	2.193
7.800	0.725	15.800	1.468	23.800	2.211

sq yd	sq m	sq yd	sq m	sq yd	sq m
0.0	0.0	8.0	6.7	16.0	13.4
0.2	0.2	8.2	6.9	16.2	13.5
0.4	0.3	8.4	7.0	16.4	13.7
0.6	0.5	8.6	7.2	16.6	13.9
0.8	0.7	8.8	7.4	16.8	14.0
1.0	0.8	9.0	7.5	17.0	14.2
1.2	1.0	9.2	7.7	17.2	14.4
1.4	1.2	9.4	7.9	17.4	14.5
1.6	1.3	9.6	8.0	17.6	14.7
1.8	1.5	9.8	8.2	17.8	14.9
2.0	1.7	10.0	8.4	18.0	15.1
2.2	1.8	10.2	8.5	18.2	15.2
2.4	2.0	10.4	8.7	18.4	15.4
2.6	2.2	10.6	8.9	18.6	15.6
2.8	2.3	10.8	9.0	18.8	15.7
3.0	2.5	11.0	9.2	19.0	15.9
3.2	2.7	11.2	9.4	19.2	16.1
3.4	2.8	11.4	9.5	19.4	16.2
3.6	3.0	11.6	9.7	19.6	16.4
3.8	3.2	11.8	9.9	19.8	16.6
4.0	3.3	12.0	10.0	20.0	16.7
4.2	3.5	12.2	10.2	20.2	16.9
4.4	3.7	12.4	10.4	20.4	17.1
4.6	3.8	12.6	10.5	20.6	17.2
4.8	4.0	12.8	10.7	20.8	17.4
5.0	4.2	13.0	10.9	21.0	17.6
5.2	4.3	13.2	11.0	21.2	17.7
5.4	4.5	13.4	11.2	21.4	17.9
5.6	4.7	13.6	11.4	21.6	18.1
5.8	4.8	13.8	11.5	21.8	18.2
6.0	5.0	14.0	11.7	22.0	18.4
6.2	5.2	14.2	11.9	22.2	18.6
6.4	5.4	14.4	12.0	22.4	18.7
6.6	5.5	14.6	12.2	22.6	18.9
6.8	5.7	14.8	12.4	22.8	19.1
7.0	5.9	15.0	12.5	23.0	19.2
7.2	6.0	15.2	12.7	23.2	19.4
7.4	6.2	15.4	12.9	23.4	19.6
7.6	6.4	15.6	13.0	23.6	19.7
7.8	6.5	15.8	13.2	23.8	19.9

sq miles	sq km	sq miles	sq km	sq miles	sq km
0.0	0.0	8.0	20.7	16.0	41.4
0.2	0.5	8.2	21.2	16.2	42.0
0.4	1.0	8.4	21.8	16.4	42.5
0.6	1.6	8.6	22.3	16.6	43.0
0.8	2.1	8.8	22.8	16.8	43.5
1.0	2.6	9.0	23.3	17.0	44.0
1.2	3.1	9.2	23.8	17.2	44.5
1.4	3.6	9.4	24.3	17.4	45.1
1.6	4.1	9.6	24.9	17.6	45.6
1.8	4.7	9.8	25.4	17.8	46.1
2.0	5.2	10.0	25.9	18.0	46.6
2.2	5.7	10.2	26.4	18.2	47.1
2.4	6.2	10.4	26.9	18.4	47.7
2.6	6.7	10.6	27.5	18.6	48.2
2.8	7.3	10.8	28.0	18.8	48.7
3.0	7.8	11.0	28.5	19.0	49.2
3.2	8.3	11.2	29.0	19.2	49.7
3.4	8.8	11.4	29.5	19.4	50.2
3.6	9.3	11.6	30.0	19.6	50.8
3.8	9.8	11.8	30.6	19.8	51.3
4.0	10.4	12.0	31.1	20.0	51.8
4.2	10.9	12.2	31.6	20.2	52.3
4.4	11.4	12.4	32.1	20.4	52.8
4.6	11.9	12.6	32.6	20.6	53.4
4.8	12.4	12.8	33.2	20.8	53.9
5.0	12.9	13.0	33.7	21.0	54.4
5.2	13.5	13.2	34.2	21.2	54.9
5.4	14.0	13.4	34.7	21.4	55.4
5.6	14.5	13.6	35.2	21.6	55.9
5.8	15.0	13.8	35.7	21.8	56.5
6.0	15.5	14.0	36.3	22.0	57.0
6.2	16.1	14.2	36.8	22.2	57.5
6.4	16.6	14.4	37.3	22.4	58.0
6.6	17.1	14.6	37.8	22.6	58.5
6.8	17.6	14.8	38.3	22.8	59.1
7.0	18.1	15.0	38.8	23.0	59.6
7.2	18.6	15.2	39.4	23.2	60.1
7.4	19.2	15.4	39.9	23.4	60.6
7.6	19.7	15.6	40.4	23.6	61.1
7.8	20.2	15.8	40.9	23.8	61.6

acres	sq m	acres	sq m	acres	sq m
0.0	0.0	8.0	32,376.0	16.0	64,752.0
0.2	809.4	8.2	33,185.4	16.2	65,561.4
0.4	1,618.8	8.4	33,994.8	16.4	66,370.8
0.6	2,428.2	8.6	34,804.2	16.6	67,180.2
0.8	3,237.6	8.8	35,613.6	16.8	67,989.6
1.0	4,047.0	9.0	36,423.0	17.0	68,799.0
1.2	4,856.4	9.2	37,232.4	17.2	69,608.4
1.4	5,665.8	9.4	38,041.8	17.4	70,417.8
1.6	6,475.2	9.6	38,851.2	17.6	71,227.2
1.8	7,284.6	9.8	39,660.6	17.8	72,036.6
2.0	8,094.0	10.0	40,470.0	18.0	72,846.0
2.2	8,903.4	10.2	41,279.4	18.2	73,655.4
2.4	9,712.8	10.4	42,088.8	18.4	74,464.8
2.6	10,522.2	10.6	42,898.2	18.6	75,274.2
2.8	11,331.6	10.8	43,707.6	18.8	76,083.6
3.0	12,141.0	11.0	44,517.0	19.0	76,893.0
3.2	12,950.4	11.2	45,326.4	19.2	77,702.4
3.4	13,759.8	11.4	46,135.8	19.4	78,511.8
3.6	14,569.2	11.6	46,945.2	19.6	79,321.2
3.8	15,378.6	11.8	47,754.6	19.8	80,130.6
4.0	16,188.0	12.0	48,564.0	20.0	80,940.0
4.2	16,997.4	12.2	49,373.4	20.2	81,749.4
4.4	17,806.8	12.4	50,182.8	20.4	82,558.8
4.6	18,616.2	12.6	50,992.2	20.6	83,368.2
4.8	19,425.6	12.8	51,801.6	20.8	84,177.6
5.0	20,235.0	13.0	52,611.0	21.0	84,987.0
5.2	21,044.4	13.2	53,420.4	21.2	85,796.4
5.4	21,853.8	13.4	54,229.8	21.4	86,605.8
5.6	22,663.2	13.6	55,039.2	21.6	87,415.2
5.8	23,472.6	13.8	55,848.6	21.8	88,224.6
6.0	24,282.0	14.0	56,658.0	22.0	89,034.0
6.2	25,091.4	14.2	57,467.4	22.2	89,843.4
6.4	25,900.8	14.4	58,276.8	22.4	90,652.8
6.6	26,710.2	14.6	59,086.2	22.6	91,462.2
6.8	27,519.6	14.8	59,895.6	22.8	92,271.6
7.0	28,329.0	15.0	60,705.0	23.0	93,081.0
7.2	29,138.4	15.2	61,514.4	23.2	93,890.4
7.4	29,947.8	15.4	62,323.8	23.4	94,699.8
7.6	30,757.2	15.6	63,133.2	23.6	95,509.2
7.8	31,566.6	15.8	63,942.6	23.8	96,318.6

acres	hectares	acres	hectares	acres	hectares
0.0	0.0	8.0	3.2	16.0	6.5
0.2	0.1	8.2	3.3	16.2	6.6
0.4	0.2	8.4	3.4	16.4	6.6
0.6	0.2	8.6	3.5	16.6	6.7
0.8	0.3	8.8	3.6	16.8	6.8
1.0	0.4	9.0	3.6	17.0	6.9
1.2	0.5	9.2	3.7	17.2	7.0
1.4	0.6	9.4	3.8	17.4	7.0
1.6	0.6	9.6	3.9	17.6	7.1
1.8	0.7	9.8	4.0	17.8	7.2
2.0	0.8	10.0	4.0	18.0	7.3
2.2	0.9	10.2	4.1	18.2	7.4
2.4	1.0	10.4	4.2	18.4	7.4
2.6	1.1	10.6	4.3	18.6	7.5
2.8	1.1	10.8	4.4	18.8	7.6
3.0	1.2	11.0	4.5	19.0	7.7
3.2	1.3	11.2	4.5	19.2	7.8
3.4	1.4	11.4	4.6	19.4	7.9
3.6	1.5	11.6	4.7	19.6	7.9
3.8	1.5	11.8	4.8	19.8	8.0
4.0	1.6	12.0	4.9	20.0	8.1
4.2	1.7	12.2	4.9	20.2	8.2
4.4	1.8	12.4	5.0	20.4	8.3
4.6	1.9	12.6	5.1	20.6	8.3
4.8	1.9	12.8	5.2	20.8	8.4
5.0	2.0	13.0	5.3	21.0	8.5
5.2	2.1	13.2	5.3	21.2	8.6
5.4	2.2	13.4	5.4	21.4	8.7
5.6	2.3	13.6	5.5	21.6	8.7
5.8	2.3	13.8	5.6	21.8	8.8
6.0	2.4	14.0	5.7	22.0	8.9
6.2	2.5	14.2	5.7	22.2	9.0
6.4	2.6	14.4	5.8	22.4	9.1
6.6	2.7	14.6	5.9	22.6	9.1
6.8	2.8	14.8	6.0	22.8	9.2
7.0	2.8	15.0	6.1	23.0	9.3
7.2	2.9	15.2	6.2	23.2	9.4
7.4	3.0	15.4	6.2	23.4	9.5
7.6	3.1	15.6	6.3	23.6	9.6
7.8	3.2	15.8	6.4	23.8	9.6

VOLUME

cu ft	cubic meters	cu ft	cubic meters	cu ft	cubic me
0.000	0.000	8.000	0.227	16.000	0.453
0.200	0.006	8.200	0.232	16.200	0.459
0.400	0.011	8.400	0.238	16.400	0.464
0.600	0.017	8.600	0.244	16.600	0.470
0.800	0.023	8.800	0.249	16.800	0.476
1.000	0.028	9.000	0.255	17.000	0.481
1.200	0.034	9.200	0.261	17.200	0.487
1.400	0.040	9.400	0.266	17.400	0.493
1.600	0.045	9.600	0.272	17.600	0.498
1.800	0.051	9.800	0.278	17.800	0.504
2.000	0.057	10.000	0.283	18.000	0.510
2.200	0.062	10.200	0.289	18.200	0.515
2.400	0.068	10.400	0.294	18.400	0.521
2.600	0.074	10.600	0.300	18.600	0.527
2.800	0.079	10.800	0.306	18.800	0.532
3.000	0.085	11.000	0.311	19.000	0.538
3.200	0.091	11.200	0.317	19.200	0.544
3.400	0.096	11.400	0.323	19.400	0.549
3.600	0.102	11.600	0.328	19.600	0.555
3.800	0.108	11.800	0.334	19.800	0.561
4.000	0.113	12.000	0.340	20.000	0.566
4.200	0.119	12.200	0.345	20.200	0.572
4.400	0.125	12.400	0.351	20.400	0.578
4.600	0.130	12.600	0.357	20.600	0.583
4.800	0.136	12.800	0.362	20.800	0.589
5.000	0.142	13.000	0.368	21.000	0.595
5.200	0.147	13.200	0.374	21.200	0.600
5.400	0.153	13.400	0.379	21.400	0.606
5.600	0.159	13.600	0.385	21.600	0.612
5.800	0.164	13.800	0.391	21.800	0.617
6.000	0.170	14.000	0.396	22.000	0.623
6.200	0.176	14.200	0.402	22.200	0.629
6.400	0.181	14.400	0.408	22.400	0.634
6.600	0.187	14.600	0.413	22.600	0.640
6.800	0.193	14.800	0.419	22.800	0.646
7.000	0.198	15.000	0.425	23.000	0.651
7.200	0.204	15.200	0.430	23.200	0.657
7.400	0.210	15.400	0.436	23.400	0.663
7.600	0.215	15.600	0.442	23.600	0.668
7.800	0.221	15.800	0.447	23.800	0.674

cyds	cubic meters	cyds	cubic meters	cyds	cubic meters
0.000	0.000	8.000	6.116	16.000	12.233
0.200	0.153	8.200	6.269	16.200	12.386
0.400	0.306	8.400	6.422	16.400	12.539
0.600	0.459	8.600	6.575	16.600	12.692
0.800	0.612	8.800	6.728	16.800	12.845
1.000	0.765	9.000	6.881	17.000	12.997
1.200	0.917	9.200	7.034	17.200	13.150
1.400	1.070	9.400	7.187	17.400	13.303
1.600	1.223	9.600	7.340	17.600	13.456
1.800	1.376	9.800	7.493	17.800	13.609
2.000	1.529	10.000	7.646	18.000	13.762
2.200	1.682	10.200	7.798	18.200	13.915
2.400	1.835	10.400	7.951	18.400	14.068
2.600	1.988	10.600	8.104	18.600	14.221
2.800	2.141	10.800	8.257	18.800	14.374
3.000	2.294	11.000	8.410	19.000	14.527
3.200	2.447	11.200	8.563	19.200	14.679
3.400	2.599	11.400	8.716	19.400	14.832
3.600	2.752	11.600	8.869	19.600	14.985
3.800	2.905	11.800	9.022	19.800	15.138
4.000	3.058	12.000	9.175	20.000	15.291
4.200	3.211	12.200	9.328	20.200	15.444
4.400	3.364	12.400	9.480	20.400	15.597
4.600	3.517	12.600	9.633	20.600	15.750
4.800	3.670	12.800	9.786	20.800	15.903
5.000	3.823	13.000	9.939	21.000	16.056
5.200	3.976	13.200	10.092	21.200	16.209
5.400	4.129	13.400	10.245	21.400	16.361
5.600	4.282	13.600	10.398	21.600	16.514
5.800	4.434	13.800	10.551	21.800	16.667
6.000	4.587	14.000	10.704	22.000	16.820
6.200	4.740	14.200	10.857	22.200	16.973
6.400	4.893	14.400	11.010	22.400	17.126
6.600	5.046	14.600	11.163	22.600	17.279
6.800	5.199	14.800	11.315	22.800	17.432
7.000	5.352	15.000	11.468	23.000	17.585
7.200	5.505	15.200	11.621	23.200	17.738
7.400	5.658	15.400	11.774	23.400	17.891
7.600	5.811	15.600	11.927	23.600	18.043
7.800	5.964	15.800	12.080	23.800	18.196

cyds	cubic meters	cyds	cubic meters	cyds	cubic meter
0.0	0.0	800.0	611.6	7,000.0	5,351.9
20.0	15.3	820.0	626.9	7,200.0	5,504.8
40.0	30.6	840.0	642.2	7,400.0	5,657.7
60.0	45.9	860.0	657.5	7,600.0	5,810.6
80.0	61.2	880.0	672.8	7,800.0	5,963.5
100.0	76.5	900.0	688.1	8,000.0	6,116.4
120.0	91.7	920.0	703.4	8,200.0	6,269.4
140.0	107.0	940.0	718.7	8,400.0	6,422.3
160.0	122.3	960.0	734.0	8,600.0	6,575.2
180.0	137.6	980.0	749.3	8,800.0	6,728.1
200.0	152.9	1,000.0	764.6	9,000.0	6,881.0
220.0	168.2	1,200.0	917.5	9,200.0	7,033.9
240.0	183.5	1,400.0	1,070.4	9,400.0	7,186.8
260.0	198.8	1,600.0	1,223.3	9,600.0	7,339.7
280.0	214.1	1,800.0	1,376.2	9,800.0	7,492.6
300.0	229.4	2,000.0	1,529.1	10,000.0	7,645.5
320.0	244.7	2,200.0	1,682.0	10,200.0	7,798.5
340.0	259.9	2,400.0	1,834.9	10,400.0	7,951.4
360.0	275.2	2,600.0	1,987.8	10,600.0	8,104.3
380.0	290.5	2,800.0	2,140.8	10,800.0	8,257.2
400.0	305.8	3,000.0	2,293.7	11,000.0	8,410.1
420.0	321.1	3,200.0	2,446.6	11,200.0	8,563.0
440.0	336.4	3,400.0	2,599.5	11,400.0	8,715.9
460.0	351.7	3,600.0	2,752.4	11,600.0	8,868.8
480.0	367.0	3,800.0	2,905.3	11,800.0	9,021.7
500.0	382.3	4,000.0	3,058.2	12,000.0	9,174.7
520.0	397.6	4,200.0	3,211.1	12,200.0	9,327.6
540.0	412.9	4,400.0	3,364.0	12,400.0	9,480.5
560.0	428.2	4,600.0	3,517.0	12,600.0	9,633.4
580.0	443.4	4,800.0	3,669.9	12,800.0	9,786.3
600.0	458.7	5,000.0	3,822.8	13,000.0	9,939.2
620.0	474.0	5,200.0	3,975.7	13,200.0	10,092.1
640.0	489.3	5,400.0	4,128.6	13,400.0	10,245.0
660.0	504.6	5,600.0	4,281.5	13,600.0	10,397.9
680.0	519.9	5,800.0	4,434.4	13,800.0	10,550.9
700.0	535.2	6,000.0	4,587.3	14,000.0	10,703.8
720.0	550.5	6,200.0	4,740.2	14,200.0	10,856.7
740.0	565.8	6,400.0	4,893.2	14,400.0	11,009.6
760.0	581.1	6,600.0	5,046.1	14,600.0	11,162.5
780.0	596.4	6,800.0	5,199.0	14,800.0	11,315.4

MASS

lbs	kg	lbs	kg	lbs	kg
0.0	0.0	8.0	3.6	16.0	7.3
0.2	0.1	8.2	3.7	16.2	7.3
0.4	0.2	8.4	3.8	16.4	7.4
0.6	0.3	8.6	3.9	16.6	7.5
0.8	0.4	8.8	4.0	16.8	7.6
1.0	0.5	9.0	4.1	17.0	7.7
1.2	0.5	9.2	4.2	17.2	7.8
1.4	0.6	9.4	4.3	17.4	7.9
1.6	0.7	9.6	4.4	17.6	8.0
1.8	0.8	9.8	4.4	17.8	8.1
2.0	0.9	10.0	4.5	18.0	8.2
2.2	1.0	10.2	4.6	18.2	8.3
2.4	1.1	10.4	4.7	18.4	8.3
2.6	1.2	10.6	4.8	18.6	8.4
2.8	1.3	10.8	4.9	18.8	8.5
3.0	1.4	11.0	5.0	19.0	8.6
3.2	1.5	11.2	5.1	19.2	8.7
3.4	1.5	11.4	5.2	19.4	8.8
3.6	1.6	11.6	5.3	19.6	8.9
3.8	1.7	11.8	5.4	19.8	9.0
4.0	1.8	12.0	5.4	20.0	9.1
4.2	1.9	12.2	5.5	20.2	9.2
4.4	2.0	12.4	5.6	20.4	9.3
4.6	2.1	12.6	5.7	20.6	9.3
4.8	2.2	12.8	5.8	20.8	9.4
5.0	2.3	13.0	5.9	21.0	9.5
5.2	2.4	13.2	6.0	21.2	9.6
5.4	2.4	13.4	6.1	21.4	9.7
5.6	2.5	13.6	6.2	21.6	9.8
5.8	2.6	13.8	6.3	21.8	9.9
6.0	2.7	14.0	6.4	22.0	10.0
6.2	2.8	14.2	6.4	22.2	10.1
6.4	2.9	14.4	6.5	22.4	10.2
6.6	3.0	14.6	6.6	22.6	10.3
6.8	3.1	14.8	6.7	22.8	10.3
7.0	3.2	15.0	6.8	23.0	10.4
7.2	3.3	15.2	6.9	23.2	10.5
7.4	3.4	15.4	7.0	23.4	10.6
7.6	3.4	15.6	7.1	23.6	10.7
7.8	3.5	15.8	7.2	23.8	10.8

lbs	kg	lbs	kg	lbs	kg	lbs	kg
0.0	0.0	800.0	362.9	7,000.0	3,175.2	15,000.0	6,804.0
20.0	9.1	820.0	372.0	7,200.0	3,265.9	15,200.0	6,894.7
40.0	18.1	840.0	381.0	7,400.0	3,356.6	15,400.0	6,985.4
60.0	27.2	860.0	390.1	7,600.0	3,447.4	15,600.0	7,076.2
80.0	36.3	880.0	399.2	7,800.0	3,538.1	15,800.0	7,166.9
100.0	45.4	900.0	408.2	8,000.0	3,628.8	16,000.0	7,257.6
120.0	54.4	920.0	417.3	8,200.0	3,719.5	16,200.0	7,348.3
140.0	63.5	940.0	426.4	8,400.0	3,810.2	16,400.0	7,439.0
160.0	72.6	960.0	435.5	8,600.0	3,901.0	16,600.0	7,529.8
180.0	81.6	980.0	444.5	8,800.0	3,991.7	16,800.0	7,620.5
200.0	90.7	1,000.0	453.6	9,000.0	4,082.4	17,000.0	7,711.2
220.0	99.8	1,200.0	544.3	9,200.0	4,173.1	17,200.0	7,801.9
240.0	108.9	1,400.0	635.0	9,400.0	4,263.8	17,400.0	7,892.6
260.0	117.9	1,600.0	725.8	9,600.0	4,354.6	17,600.0	7,983.4
280.0	127.0	1,800.0	816.5	9,800.0	4,445.3	17,800.0	8,074.1
300.0	136.1	2,000.0	907.2	10,000.0	4,536.0	18,000.0	8,164.8
320.0	145.2	2,200.0	997.9	10,200.0	4,626.7	18,200.0	8,255.5
340.0	154.2	2,400.0	1,088.6	10,400.0	4,717.4	18,400.0	8,346.2
360.0	163.3	2,600.0	1,179.4	10,600.0	4,808.2	18,600.0	8,437.0
380.0	172.4	2,800.0	1,270.1	10,800.0	4,898.9	18,800.0	8,527.7
400.0	181.4	3,000.0	1,360.8	11,000.0	4,989.6	19,000.0	8,618.4
420.0	190.5	3,200.0	1,451.5	11,200.0	5,080.3	19,200.0	8,709.1
440.0	199.6	3,400.0	1,542.2	11,400.0	5,171.0	19,400.0	8,799.8
460.0	208.7	3,600.0	1,633.0	11,600.0	5,261.8	19,600.0	8,890.6
480.0	217.7	3,800.0	1,723.7	11,800.0	5,352.5	19,800.0	8,981.3
500.0	226.8	4,000.0	1,814.4	12,000.0	5,443.2	20,000.0	9,072.0
520.0	235.9	4,200.0	1,905.1	12,200.0	5,533.9	20,200.0	9,162.7
540.0	244.9	4,400.0	1,995.8	12,400.0	5,624.6	20,400.0	9,253.4
560.0	254.0	4,600.0	2,086.6	12,600.0	5,715.4	20,600.0	9,344.2
580.0	263.1	4,800.0	2,177.3	12,800.0	5,806.1	20,800.0	9,434.9
600.0	272.2	5,000.0	2,268.0	13,000.0	5,896.8	21,000.0	9,525.6
620.0	281.2	5,200.0	2,358.7	13,200.0	5,987.5	21,200.0	9,616.3
640.0	290.3	5,400.0	2,449.4	13,400.0	6,078.2	21,400.0	9,707.0
660.0	299.4	5,600.0	2,540.2	13,600.0	6,169.0	21,600.0	9,797.8
680.0	308.4	5,800.0	2,630.9	13,800.0	6,259.7	21,800.0	9,888.5
700.0	317.5	6,000.0	2,721.6	14,000.0	6,350.4	22,000.0	9,979.2
720.0	326.6	6,200.0	2,812.3	14,200.0	6,441.1	22,200.0	10,069.9
740.0	335.7	6,400.0	2,903.0	14,400.0	6,531.8	22,400.0	10,160.6
760.0	344.7	6,600.0	2,993.8	14,600.0	6,622.6	22,600.0	10,251.4
780.0	353.8	6,800.0	3,084.5	14,800.0	6,713.3	22,800.0	10,342.1

lbs	kg	lbs	kg	lbs	kg
10,000.0	4,536.0	18,000.0	8,164.8	26,000.0	11,793.6
10,200.0	4,626.7	18,200.0	8,255.5	26,200.0	11,884.3
10,400.0	4,717.4	18,400.0	8,346.2	26,400.0	11,975.0
10,600.0	4,808.2	18,600.0	8,437.0	26,600.0	12,065.8
10,800.0	4,898.9	18,800.0	8,527.7	26,800.0	12,156.5
11,000.0	4,989.6	19,000.0	8,618.4	27,000.0	12,247.2
11,200.0	5,080.3	19,200.0	8,709.1	27,200.0	12,337.9
11,400.0	5,171.0	19,400.0	8,799.8	27,400.0	12,428.6
11,600.0	5,261.8	19,600.0	8,890.6	27,600.0	12,519.4
11,800.0	5,352.5	19,800.0	8,981.3	27,800.0	12,610.1
12,000.0	5,443.2	20,000.0	9,072.0	28,000.0	12,700.8
12,200.0	5,533.9	20,200.0	9,162.7	28,200.0	12,791.5
12,400.0	5,624.6	20,400.0	9,253.4	28,400.0	12,882.2
12,600.0	5,715.4	20,600.0	9,344.2	28,600.0	12,973.0
12,800.0	5,806.1	20,800.0	9,434.9	28,800.0	13,063.7
13,000.0	5,896.8	21,000.0	9,525.6	29,000.0	13,154.4
13,200.0	5,987.5	21,200.0	9,616.3	29,200.0	13,245.1
13,400.0	6,078.2	21,400.0	9,707.0	29,400.0	13,335.8
13,600.0	6,169.0	21,600.0	9,797.8	29,600.0	13,426.6
13,800.0	6,259.7	21,800.0	9,888.5	29,800.0	13,517.3
14,000.0	6,350.4	22,000.0	9,979.2	30,000.0	13,608.0
14,200.0	6,441.1	22,200.0	10,069.9	30,200.0	13,698.7
14,400.0	6,531.8	22,400.0	10,160.6	30,400.0	13,789.4
14,600.0	6,622.6	22,600.0	10,251.4	30,600.0	13,880.2
14,800.0	6,713.3	22,800.0	10,342.1	30,800.0	13,970.9
15,000.0	6,804.0	23,000.0	10,432.8	31,000.0	14,061.6
15,200.0	6,894.7	23,200.0	10,523.5	31,200.0	14,152.3
15,400.0	6,985.4	23,400.0	10,614.2	31,400.0	14,243.0
15,600.0	7,076.2	23,600.0	10,705.0	31,600.0	14,333.8
15,800.0	7,166.9	23,800.0	10,795.7	31,800.0	14,424.5
16,000.0	7,257.6	24,000.0	10,886.4	32,000.0	14,515.2
16,200.0	7,348.3	24,200.0	10,977.1	32,200.0	14,605.9
16,400.0	7,439.0	24,400.0	11,067.8	32,400.0	14,696.6
16,600.0	7,529.8	24,600.0	11,158.6	32,600.0	14,787.4
16,800.0	7,620.5	24,800.0	11,249.3	32,800.0	14,878.1
17,000.0	7,711.2	25,000.0	11,340.0	33,000.0	14,968.8
17,200.0	7,801.9	25,200.0	11,430.7	33,200.0	15,059.5
17,400.0	7,892.6	25,400.0	11,521.4	33,400.0	15,150.2
17,600.0	7,983.4	25,600.0	11,612.2	33,600.0	15,241.0
17,800.0	8,074.1	25,800.0	11,702.9	33,800.0	15,331.7

lbs	Mg	lbs	Mg	lbs	Mg
0.0	0.0	200,000.0	9.1	400,000.0	18.1
5,000.0	0.2	205,000.0	9.3	405,000.0	18.4
10,000.0	0.5	210,000.0	9.5	410,000.0	18.6
15,000.0	0.7	215,000.0	9.8	415,000.0	18.8
20,000.0	0.9	220,000.0	10.0	420,000.0	19.1
25,000.0	1.1	225,000.0	10.2	425,000.0	19.3
30,000.0	1.4	230,000.0	10.4	430,000.0	19.5
35,000.0	1.6	235,000.0	10.7	435,000.0	19.7
40,000.0	1.8	240,000.0	10.9	440,000.0	20.0
45,000.0	2.0	245,000.0	11.1	445,000.0	20.2
50,000.0	2.3	250,000.0	11.3	450,000.0	20.4
55,000.0	2.5	255,000.0	11.6	455,000.0	20.6
60,000.0	2.7	260,000.0	11.8	460,000.0	20.9
65,000.0	2.9	265,000.0	12.0	465,000.0	21.1
70,000.0	3.2	270,000.0	12.2	470,000.0	21.3
75,000.0	3.4	275,000.0	12.5	475,000.0	21.5
80,000.0	3.6	280,000.0	12.7	480,000.0	21.8
85,000.0	3.9	285,000.0	12.9	485,000.0	22.0
90,000.0	4.1	290,000.0	13.2	490,000.0	22.2
95,000.0	4.3	295,000.0	13.4	495,000.0	22.5
100,000.0	4.5	300,000.0	13.6	500,000.0	22.7
105,000.0	4.8	305,000.0	13.8	505,000.0	22.9
110,000.0	5.0	310,000.0	14.1	510,000.0	23.1
115,000.0	5.2	315,000.0	14.3	515,000.0	23.4
120,000.0	5.4	320,000.0	14.5	520,000.0	23.6
125,000.0	5.7	325,000.0	14.7	525,000.0	23.8
130,000.0	5.9	330,000.0	15.0	530,000.0	24.0
135,000.0	6.1	335,000.0	15.2	535,000.0	24.3
140,000.0	6.4	340,000.0	15.4	540,000.0	24.5
145,000.0	6.6	345,000.0	15.6	545,000.0	24.7
150,000.0	6.8	350,000.0	15.9	550,000.0	24.9
155,000.0	7.0	355,000.0	16.1	555,000.0	25.2
160,000.0	7.3	360,000.0	16.3	560,000.0	25.4
165,000.0	7.5	365,000.0	16.6	565,000.0	25.6
170,000.0	7.7	370,000.0	16.8	570,000.0	25.9
175,000.0	7.9	375,000.0	17.0	575,000.0	26.1
180,000.0	8.2	380,000.0	17.2	580,000.0	26.3
185,000.0	8.4	385,000.0	17.5	585,000.0	26.5
190,000.0	8.6	390,000.0	17.7	590,000.0	26.8
195,000.0	8.8	395,000.0	17.9	595,000.0	27.0

TEMPERATURE

F	C	F	C	F	C
-10.0	-23.3	30.0	-1.1	70.0	21.1
-9.0	-22.8	31.0	-0.6	71.0	21.7
-8.0	-22.2	32.0	0.0	72.0	22.2
-7.0	-21.7	33.0	0.6	73.0	22.8
-6.0	-21.1	34.0	1.1	74.0	23.3
-5.0	-20.6	35.0	1.7	75.0	23.9
-4.0	-20.0	36.0	2.2	76.0	24.4
-3.0	-19.4	37.0	2.8	77.0	25.0
-2.0	-18.9	38.0	3.3	78.0	25.6
-1.0	-18.3	39.0	3.9	79.0	26.1
0.0	-17.8	40.0	4.4	80.0	26.7
1.0	-17.2	41.0	5.0	81.0	27.2
2.0	-16.7	42.0	5.6	82.0	27.8
3.0	-16.1	43.0	6.1	83.0	28.3
4.0	-15.6	44.0	6.7	84.0	28.9
5.0	-15.0	45.0	7.2	85.0	29.4
6.0	-14.4	46.0	7.8	86.0	30.0
7.0	-13.9	47.0	8.3	87.0	30.6
8.0	-13.3	48.0	8.9	88.0	31.1
9.0	-12.8	49.0	9.4	89.0	31.7
10.0	-12.2	50.0	10.0	90.0	32.2
11.0	-11.7	51.0	10.6	91.0	32.8
12.0	-11.1	52.0	11.1	92.0	33.3
13.0	-10.6	53.0	11.7	93.0	33.9
14.0	-10.0	54.0	12.2	94.0	34.4
15.0	-9.4	55.0	12.8	95.0	35.0
16.0	-8.9	56.0	13.3	96.0	35.6
17.0	-8.3	57.0	13.9	97.0	36.1
18.0	-7.8	58.0	14.4	98.0	36.7
19.0	-7.2	59.0	15.0	99.0	37.2
20.0	-6.7	60.0	15.6	100.0	37.8
21.0	-6.1	61.0	16.1	101.0	38.3
22.0	-5.6	62.0	16.7	102.0	38.9
23.0	-5.0	63.0	17.2	103.0	39.4
24.0	-4.4	64.0	17.8	104.0	40.0
25.0	-3.9	65.0	18.3	105.0	40.6
26.0	-3.3	66.0	18.9	106.0	41.1
27.0	-2.8	67.0	19.4	107.0	41.7
28.0	-2.2	68.0	20.0	108.0	42.2
29.0	-1.7	69.0	20.6	109.0	42.8

APPENDIX B

PRECISION OF CALCULATIONS

APPENDIX B
PRECISION OF CALCULATIONS

Item	Calculate to	Round Calc. Answer to
Granular Materials, Placing	0.01 mg	0.1 mg
Granluar Materials, Reclaiming	0.1 m^3	1 m^3
Revetment, Class A, B, C	0.1 m^2	1 m^2
Revetment, Class D	0.1 mg	1 mg
Excavation or Embankment Construction	1 m^3	1 m^3
Overhaul	0.01 sta. m	1 sta m
M & D	1 m^3	1 m^3
Compacting Trench Bottom	0.01 sta	0.01 sta
Intercepting Ditch	0.1 m	1 m
Drians - Footing, Teench, etc	0.1 m	1 m
Water	0.1 l	1 l
Shoulders, Earth-Finish or Construct.	0.01 sta	0.01 sta
Subbase construction	0.001 km	0.001 km
Concrete Base, Drives, Pav't., Shldr. Slopes	0.01 m^2	1 m^2
Concrete-Remove or Break	0.01 m^2	1 m^2
Concrete-Structural	0.01 m^3	1 m^3
Side Walk	0.01 m^2	0.1 m^2
Concrete-Other	0.01 m^3	0.1 m^3
Cement	0.01 kg	0.1 kg
Curb &/or Gutter-Remove/Construct	0.01 m	0.1 m
Saw Cut	0.01 m	0.1 m
A.C. Base, Pav't., Shldr	0.01 m^2	0.1 m^2
Asphlat Cement	0.01 kg	1 kg
Cleaning Old Base	0.001 km	0.01 km
Primer or Bitumen	0.001 l	0.01 l
Patches, A.C.-Full Depth or Surface	0.01 m^2	0.1 m^2
Bridge Seat Sealer	0.001 m^2	0.1 m^2
Erosion Control Items, Rural/Urban	0.1 h	0.5 h
Erosion Control Items, Rural or Urban	0.5 m^2	1 m^2
Conduit, Cable or Wire	0.01 m	0.1 m
Pipes, Flumes, or Sewers	0.01 m	0.1 m
Fence	0.01 sta	0.1 sta
Guard Rail	0.001 m	0.01 m
Bridge Railing	0.001 m	0.01 m
Piling	0.01 m	0.1 m
Drilling	0.01 m	0.1 m
Water Proofing	0.01 m^2	0.1 m^2
Joints, Preformed Elastic Neoprene	0.001 m	0.01 m
Walls-Crib	0.01 m^2	0.1 m^2
Walls-Concrete Retaining	0.001 m^3	0.01 m^3
Signs	0.01 m^2	0.1 m^2
Sign Posts, Wood/Steel	0.001 m	0.01 m

Note: The above should be used when preparing estimated quantities and when checking quantities given on the plans. Refer to the Standard Specifications, "Basis of Payment" Sections to determine the degree of precision required for Final Pay Quantities.

APPENDIX C

FUNCTIONS OF NUMBERS

SQUARES, CUBES AND ROOTS

Roots of numbers other than those given directly may be found by the following relations: $\sqrt{100n} = 10\sqrt{n}$; $\sqrt{1000n} = 10\sqrt{10n}$; $\sqrt{\frac{1}{10}n} = \frac{1}{10}\sqrt{10n}$; $\sqrt{\frac{1}{100}n} = \frac{1}{10}\sqrt{n}$; $\sqrt{\frac{1}{1000}n} = \frac{1}{100}\sqrt{10n}$; $\sqrt[3]{1000n} = 10\sqrt[3]{n}$; $\sqrt[3]{10{,}000n} = 10\sqrt[3]{10n}$; $\sqrt[3]{100{,}000n} = 10\sqrt[3]{100n}$; $\sqrt[3]{\frac{1}{10}n} = \frac{1}{10}\sqrt[3]{100n}$; $\sqrt[3]{\frac{1}{100}n} = \frac{1}{10}\sqrt[3]{10n}$; $\sqrt[3]{\frac{1}{1000}n} = \frac{1}{10}\sqrt[3]{n}$.

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
1	1	1.000 000	3.162 278	1	1.000 000	2.154 435	4.641 589
2	4	1.414 214	4.472 136	8	1.259 921	2.714 418	5.848 035
3	9	1.732 051	5.477 226	27	1.442 250	3.107 233	6.694 330
4	16	2.000 000	6.324 555	64	1.587 401	3.419 952	7.368 063
5	25	2.236 068	7.071 068	125	1.709 976	3.684 031	7.937 005
6	36	2.449 490	7.745 967	216	1.817 121	3.914 868	8.434 327
7	49	2.645 751	8.366 600	343	1.912 931	4.121 285	8.879 040
8	64	2.828 427	8.944 272	512	2.000 000	4.308 869	9.283 178
9	81	3.000 000	9.486 833	729	2.080 084	4.481 405	9.654 894
10	100	3.162 278	10.00000	1 000	2.154 435	4.641 589	10.00000
11	121	3.316 625	10.48809	1 331	2.223 980	4.791 420	10.32280
12	144	3.464 102	10.95445	1 728	2.289 428	4.932 424	10.62659
13	169	3.605 551	11.40175	2 197	2.351 335	5.065 797	10.91393
14	196	3.741 657	11.83216	2 744	2.410 142	5.192 494	11.18689
15	225	3.872 983	12.24745	3 375	2.466 212	5.313 293	11.44714
16	256	4.000 000	12.64911	4 096	2.519 842	5.428 835	11.69607
17	289	4.123 106	13.03840	4 913	2.571 282	5.539 658	11.93483
18	324	4.242 641	13.41641	5 832	2.620 741	5.646 216	12.16440
19	361	4.358 899	13.78405	6 859	2.668 402	5.748 897	12.38562
20	400	4.472 136	14.14214	8 000	2.714 418	5.848 035	12.59921
21	441	4.582 576	14.49138	9 261	2.758 924	5.943 922	12.80579
22	484	4.690 416	14.83240	10 648	2.802 039	6.036 811	13.00591
23	529	4.795 832	15.16575	12 167	2.843 867	6.126 926	13.20006
24	576	4.898 979	15.49193	13 824	2.884 499	6.214 465	13.38866
25	625	5.000 000	15.81139	15 625	2.924 018	6.299 605	13.57209
26	676	5.099 020	16.12452	17 576	2.962 496	6.382 504	13.75069
27	729	5.196 152	16.43168	19 683	3.000 000	6.463 304	13.92477
28	784	5.291 503	16.73320	21 952	3.036 589	6.542 133	14.09460
29	841	5.385 165	17.02939	24 389	3.072 317	6.619 106	14.26043
30	900	5.477 226	17.32051	27 000	3.107 233	6.694 330	14.42250
31	961	5.567 764	17.60682	29 791	3.141 381	6.767 899	14.58100
32	1 024	5.656 854	17.88854	32 768	3.174 802	6.839 904	14.73613
33	1 089	5.744 563	18.16590	35 937	3.207 534	6.910 423	14.88806
34	1 156	5.830 952	18.43909	39 304	3.239 612	6.979 532	15.03695
35	1 225	5.916 080	18.70829	42 875	3.271 066	7.047 299	15.18294
36	1 296	6.000 000	18.97367	46 656	3.301 927	7.113 787	15.32619
37	1 369	6.082 763	19.23538	50 653	3.332 222	7.179 054	15.46680
38	1 444	6.164 414	19.49359	54 872	3.361 975	7.243 156	15.60491
39	1 521	6.244 998	19.74842	59 319	3.391 211	7.306 144	15.74061
40	1 600	6.324 555	20.00000	64 000	3.419 952	7.368 063	15.87401
41	1 681	6.403 124	20.24846	68 921	3.448 217	7.428 959	16.00521
42	1 764	6.480 741	20.49390	74 088	3.476 027	7.488 872	16.13429
43	1 849	6.557 439	20.73644	79 507	3.503 398	7.547 842	16.26133
44	1 936	6.633 250	20.97618	85 184	3.530 348	7.605 905	16.38643
45	2 025	6.708 204	21.21320	91 125	3.556 893	7.663 094	16.50964
46	2 116	6.782 330	21.44761	97 336	3.583 048	7.719 443	16.63103
47	2 209	6.855 655	21.67948	103 823	3.608 826	7.774 980	16.75069
48	2 304	6.928 203	21.90890	110 592	3.634 241	7.829 735	16.86865
49	2 401	7.000 000	22.13594	117 649	3.659 306	7.883 735	16.98499
50	2 500	7.071 068	22.36068	125 000	3.684 031	7.937 005	17.09976

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
50	2 500	7.071 068	22.36068	125 000	3.684 031	7.937 005	17.09976
51	2 601	7.141 428	22.58318	132 651	3.708 430	7.989 570	17.21301
52	2 704	7.211 103	22.80351	140 608	3.732 511	8.041 452	17.32478
53	2 809	7.280 110	23.02173	148 877	3.756 286	8.092 672	17.43513
54	2 916	7.348 469	23.23790	157 464	3.779 763	8.143 253	17.54411
55	3 025	7.416 198	23.45208	166 375	3.802 952	8.193 213	17.65174
56	3 136	7.483 315	23.66432	175 616	3.825 862	8.242 571	17.75808
57	3 249	7.549 834	23.87467	185 193	3.848 501	8.291 344	17.86316
58	3 364	7.615 773	24.08319	195 112	3.870 877	8.339 551	17.96702
59	3 481	7.681 146	24.28992	205 379	3.892 996	8.387 207	18.06969
60	3 600	7.745 967	24.49490	216 000	3.914 868	8.434 327	18.17121
61	3 721	7.810 250	24.69818	226 981	3.936 497	8.480 926	18.27160
62	3 844	7.874 008	24.89980	238 328	3.957 892	8.527 019	18.37091
63	3 969	7.937 254	25.09980	250 047	3.979 057	8.572 619	18.46915
64	4 096	8.000 000	25.29822	262 144	4.000 000	8.617 739	18.56636
65	4 225	8.062 258	25.49510	274 625	4.020 726	8.662 391	18.66256
66	4 356	8.124 038	25.69047	287 496	4.041 240	8.706 588	18.75777
67	4 489	8.185 353	25.88436	300 763	4.061 548	8.750 340	18.85204
68	4 624	8.246 211	26.07681	314 432	4.081 655	8.793 659	18.94536
69	4 761	8.306 624	26.26785	328 509	4.101 566	8.836 556	19.03778
70	4 900	8.366 600	26.45751	343 000	4.121 285	8.879 040	19.12931
71	5 041	8.426 150	26.64583	357 911	4.140 818	8.921 121	19.21997
72	5 184	8.485 281	26.83282	373 248	4.160 168	8.962 809	19.30979
73	5 329	8.544 004	27.01851	389 017	4.179 339	9.004 113	19.39877
74	5 476	8.602 325	27.20294	405 224	4.198 336	9.045 042	19.48695
75	5 625	8.660 254	27.38613	421 875	4.217 163	9.085 603	19.57434
76	5 776	8.717 798	27.56810	438 976	4.235 824	9.125 805	19.66095
77	5 929	8.774 964	27.74887	456 533	4.254 321	9.165 656	19.74681
78	6 084	8.831 761	27.92848	474 552	4.272 659	9.205 164	19.83192
79	6 241	8.888 194	28.10694	493 039	4.290 840	9.244 335	19.91632
80	6 400	8.944 272	28.28427	512 000	4.308 869	9.283 178	20.00000
81	6 561	9.000 000	28.46050	531 441	4.326 749	9.321 698	20.08299
82	6 724	9.055 385	28.63564	551 368	4.344 481	9.359 902	20.16530
83	6 889	9.110 434	28.80972	571 787	4.362 071	9.397 796	20.24694
84	7 056	9.165 151	28.98275	592 704	4.379 519	9.435 388	20.32793
85	7 225	9.219 544	29.15476	614 125	4.396 830	9.472 682	20.40828
86	7 396	9.273 618	29.32576	636 056	4.414 005	9.509 685	20.48800
87	7 569	9.327 379	29.49576	658 503	4.431 048	9.546 403	20.56710
88	7 744	9.380 832	29.66479	681 472	4.447 960	9.582 840	20.64560
89	7 921	9.433 981	29.83287	704 969	4.464 745	9.619 002	20.72351
90	8 100	9.486 833	30.00000	729 000	4.481 405	9.654 894	20.80084
91	8 281	9.539 392	30.16621	753 571	4.497 941	9.690 521	20.87759
92	8 464	9.591 663	30.33150	778 688	4.514 357	9.725 888	20.95379
93	8 649	9.643 651	30.49590	804 357	4.530 655	9.761 000	21.02944
94	8 836	9.695 360	30.65942	830 584	4.546 836	9.795 861	21.10454
95	9 025	9.746 794	30.82207	857 375	4.562 903	9.830 476	21.17912
96	9 216	9.797 959	30.98387	884 736	4.578 857	9.864 848	21.25317
97	9 409	9.848 858	31.14482	912 673	4.594 701	9.898 983	21.32671
98	9 604	9.899 495	31.30495	941 192	4.610 436	9.932 884	21.39975
99	9 801	9.949 874	31.46427	970 299	4.626 065	9.966 555	21.47229
100	10 000	10.00000	31.62278	1 000 000	4.641 589	10.00000	21.54435

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
100	10 000	10.00000	31.62278	1 000 000	4.641 589	10.00000	21.54435
101	10 201	10.04988	31.78050	1 030 301	4.657 010	10.03322	21.61592
102	10 404	10.09950	31.93744	1 061 208	4.672 329	10.06623	21.68703
103	10 609	10.14889	32.09361	1 092 727	4.687 548	10.09902	21.75767
104	10 816	10.19804	32.24903	1 124 864	4.702 669	10.13159	21.82786
105	11 025	10.24695	32.40370	1 157 625	4.717 694	10.16396	21.89760
106	11 236	10.29563	32.55764	1 191 016	4.732 623	10.19613	21.96689
107	11 449	10.34408	32.71085	1 225 043	4.747 459	10.22809	22.03575
108	11 664	10.39230	32.86335	1 259 712	4.762 203	10.25986	22.10419
109	11 881	10.44031	33.01515	1 295 029	4.776 856	10.29142	22.17220
110	12 100	10.48809	33.16625	1 331 000	4.791 420	10.32280	22.23980
111	12 321	10.53565	33.31666	1 367 631	4.805 896	10.35399	22.30699
112	12 544	10.58301	33.46640	1 404 928	4.820 285	10.38499	22.37378
113	12 769	10.63015	33.61547	1 442 897	4.834 588	10.41580	22.44017
114	12 996	10.67708	33.76389	1 481 544	4.848 808	10.44644	22.50617
115	13 225	10.72381	33.91165	1 520 875	4.862 944	10.47690	22.57179
116	13 456	10.77033	34.05877	1 560 896	4.876 999	10.50718	22.63702
117	13 689	10.81665	34.20526	1 601 613	4.890 973	10.53728	22.70189
118	13 924	10.86278	34.35113	1 643 032	4.904 868	10.56722	22.76638
119	14 161	10.90871	34.49638	1 685 159	4.918 685	10.59699	22.83051
120	14 400	10.95445	34.64102	1 728 000	4.932 424	10.62659	22.89428
121	14 641	11.00000	34.78505	1 771 561	4.946 087	10.65602	22.95770
122	14 884	11.04536	34.92850	1 815 848	4.959 676	10.68530	23.02078
123	15 129	11.09054	35 07136	1 860 867	4.973 190	10.71441	23.08350
124	15 376	11.13553	35 21363	1 906 624	4.986 631	10.74337	23.14589
125	15 625	11.18034	35.35534	1 953 125	5.000 000	10.77217	23.20794
126	15 876	11.22497	35.49648	2 000 376	5.013 298	10.80082	23.26967
127	16 129	11.26943	35.63706	2 048 383	5.026 526	10.82932	23.33107
128	16 384	11.31371	35.77709	2 097 152	5.039 684	10.85767	23.39214
129	16 641	11.35782	35.91657	2 146 689	5.052 774	10.88587	23.45290
130	16 900	11.40175	36.05551	2 197 000	5.065 797	10.91393	23.51335
131	17 161	11.44552	36.19392	2 248 091	5.078 753	10.94184	23.57348
132	17 424	11.48913	36.33180	2 299 968	5.091 643	10.96961	23.63332
133	17 689	11.53256	36.46917	2 352 637	5.104 469	10.99724	23.69285
134	17 956	11.57584	36.60601	2 406 104	5.117 230	11.02474	23.75208
135	18 225	11.61895	36.74235	2 460 375	5.129 928	11.05209	23.81102
136	18 496	11.66190	36.87818	2 515 456	5.142 563	11.07932	23.86966
137	18 769	11.70470	37.01351	2 571 353	5.155 137	11.10641	23.92803
138	19 044	11.74734	37.14835	2 628 072	5.167 649	11.13336	23.98610
139	19 321	11.78983	37.28270	2 685 619	5.180 101	11.16019	24.04390
140	19 600	11.83216	37.41657	2 744 000	5.192 494	11.18689	24.10142
141	19 881	11.87434	37.54997	2 803 221	5.204 828	11.21346	24.15867
142	20 164	11.91638	37.68289	2 863 288	5.217 103	11.23991	24.21565
143	20 449	11.95826	37.81534	2 924 207	5.229 322	11.26623	24.27236
144	20 736	12.00000	37.94733	2 985 984	5.241 483	11.29243	24.32881
145	21 025	12.04159	38.07887	3 048 625	5.253 588	11.31851	24.38499
146	21 316	12.08305	38.20995	3 112 136	5.265 637	11.34447	24.44092
147	21 609	12.12436	38.34058	3 176 523	5.277 632	11.37031	24.49660
148	21 904	12.16553	38.47077	3 241 792	5.289 572	11.39604	24.55202
149	22 201	12.20656	38.60052	3 307 949	5.301 459	11.42165	24.60719
150	22 500	12.24745	38.72983	3 375 000	5.313 293	11.44714	24.66212

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
150	22 500	12.24745	38.72983	3 375 000	5.313 293	11.44714	24.66212
151	22 801	12.28821	38.85872	3 442 951	5.325 074	11.47252	24.71680
152	23 104	12.32883	38.98718	3 511 808	5.336 803	11.49779	24.77125
153	23 409	12.36932	39.11521	3 581 577	5.348 481	11.52296	24.82545
154	23 716	12.40967	39.24283	3 652 264	5.360 108	11.54800	24.87942
155	24 025	12.44990	39.37004	3 723 875	5.371 685	11.57295	24.93315
156	24 336	12.49000	39.4[illegible]	3 796 416	5.383 213	11.59778	24.98666
157	24 649	12.52996	39.62323	3 869 893	5.394 691	11.62251	25.03994
158	24 964	12.56981	39.74921	3 944 312	5.406 120	11.64713	25.09299
159	25 281	12.60952	39.87480	4 019 679	5.417 502	11.67165	25.14581
160	25 600	12.64911	40.00000	4 096 000	5.428 835	11.69607	25.19842
161	25 921	12.68858	40.12481	4 173 281	5.440 122	11.72039	25.25081
162	26 244	12.72792	40.24922	4 251 528	5.451 362	11.74460	25.30298
163	26 569	12.76715	40.37326	4 330 747	5.462 556	11.76872	25.35494
164	26 896	12.80625	40.49691	4 410 944	5.473 704	11.79274	25.40668
165	27 225	12.84523	40.62019	4 492 125	5.484 807	11.81666	25.45822
166	27 556	12.88410	40.74310	4 574 296	5.495 865	11.84048	25.50954
167	27 889	12.92285	40.86563	4 657 463	5.506 878	11.86421	25.56067
168	28 224	12.96148	40.98780	4 741 632	5.517 848	11.88784	25.61158
169	28 561	13.00000	41.10961	4 826 809	5.528 775	11.91138	25.66230
170	28 900	13.03840	41.23106	4 913 000	5.539 658	11.93483	25.71282
171	29 241	13.07670	41.35215	5 000 211	5.550 499	11.95819	25.76313
172	29 584	13.11488	41.47288	5 088 448	5.561 298	11.98145	25.81326
173	29 929	13.15295	41.59327	5 177 717	5.572 055	12.00463	25.86319
174	30 276	13.19091	41.71331	5 268 024	5.582 770	12.02771	25.91292
175	30 625	13.22876	41.83300	5 359 375	5.593 445	12.05071	25.96247
176	30 976	13.26650	41.95235	5 451 776	5.604 079	12.07362	26.01183
177	31 329	13.30413	42.07137	5 545 233	5.614 672	12.09645	26.06100
178	31 684	13.34166	42.19005	5 639 752	5.625 226	12.11918	26.10999
179	32 041	13.37909	42.30839	5 735 339	5.635 741	12.14184	26.15879
180	32 400	13.41641	42.42641	5 832 000	5.646 216	12.16440	26.20741
181	32 761	13.45362	42.54409	5 929 741	5.656 653	12.18689	26.25586
182	33 124	13.49074	42.66146	6 028 568	5.667 051	12.20929	26.30412
183	33 489	13.52775	42.77850	6 128 487	5.677 411	12.23161	26.35221
184	33 856	13.56466	42.89522	6 229 504	5.687 734	12.25385	26.40012
185	34 225	13.60147	43.01163	6 331 625	5.698 019	12.27601	26.44786
186	34 596	13.63818	43.12772	6 434 856	5.708 267	12.29809	26.49543
187	34 969	13.67479	43.24350	6 539 203	5.718 479	12.32009	26.54283
188	35 344	13.71131	43.35897	6 644 672	5.728 654	12.34201	26.59006
189	35 721	13.74773	43.47413	6 751 269	5.738 794	12.36386	26.63712
190	36 100	13.78405	43.58899	6 859 000	5.748 897	12.38562	26.68402
191	36 481	13.82027	43.70355	6 967 871	5.758 965	12.40731	26 73075
192	36 864	13.85641	43.81780	7 077 888	5.768 998	12.42893	26.77732
193	37 249	13.89244	43.93177	7 189 057	5.778 997	12.45047	26.82372
194	37 636	13.92839	44.04543	7 301 384	5.788 960	12.47194	26.86997
195	38 025	13.96424	44.15880	7 414 875	5.798 890	12.49333	26.91606
196	38 416	14.00000	44.27189	7 529 536	5.808 786	12.51465	26.96199
197	38 809	14.03567	44.38468	7 645 373	5.818 648	12.53596	27.00777
198	39 204	14.07125	44.49719	7 762 392	5.828 477	12.55707	27.05339
199	39 601	14.10674	44.60942	7 880 599	5.838 272	12.57818	27.09886
200	40 000	14.14214	44.72136	8 000 000	5.848 035	12.59921	27.14418

(C-2)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
200	40 000	14.14214	44.72136	8 000 000	5.848 035	12.59921	27.14418
201	40 401	14.17745	44.83302	8 120 601	5.857 766	12.62017	27.18934
202	40 804	14.21267	44.94441	8 242 408	5.867 464	12.64107	27.23436
203	41 209	14.24781	45.05552	8 365 427	5.877 131	12.66189	27.27922
204	41 616	14.28286	45.16636	8 489 664	5.886 765	12.68265	27.32394
205	42 025	14.31782	45.27693	8 615 125	5.896 369	12.70334	27.36852
206	42 436	14.35270	45.38722	8 741 816	5.905 941	12.72396	27.41295
207	42 849	14.38749	45.49725	8 869 743	5.915 482	12.74452	27.45723
208	43 264	14.42221	45.60702	8 998 912	5.924 992	12.76501	27.50138
209	43 681	14.45683	45.71652	9 129 329	5.934 472	12.78543	27.54538
210	44 100	14.49138	45.82576	9 261 000	5.943 922	12.80579	27.58924
211	44 521	14.52584	45.93474	9 393 931	5.953 342	12.82609	27.63296
212	44 944	14.56022	46.04346	9 528 128	5.962 732	12.84632	27.67655
213	45 369	14.59452	46.15192	9 663 597	5.972 093	12.86648	27.72000
214	45 796	14.62874	46.26013	9 800 344	5.981 424	12.88659	27.76331
215	46 225	14.66288	46.36809	9 938 375	5.990 726	12.90663	27.80649
216	46 656	14.69694	46.47580	10 077 696	6.000 000	12.92661	27.84953
217	47 089	14.73092	46.58326	10 218 313	6.009 245	12.94653	27.89244
218	47 524	14.76482	46.69047	10 360 232	6.018 462	12.96638	27.93522
219	47 961	14.79865	46.79744	10 503 459	6.027 650	12.98618	27.97787
220	48 400	14.83240	46.90416	10 648 000	6.036 811	13.00591	28.02039
221	48 841	14.86607	47.01064	10 793 861	6.045 944	13.02559	28.06278
222	49 284	14.89966	47.11688	10 941 048	6.055 049	13.04521	28.10505
223	49 729	14.93318	47.22288	11 089 567	6.064 127	13.06477	28.14718
224	50 176	14.96663	47.32864	11 239 424	6.073 178	13.08427	28.18919
225	50 625	15.00000	47.43416	11 390 625	6.082 202	13.10371	28.23108
226	51 076	15.03330	47.53946	11 543 176	6.091 199	13.12309	28.27284
227	51 529	15.06652	47.64452	11 697 083	6.100 170	13.14242	28.31448
228	51 984	15.09967	47.74935	11 852 352	6.109 115	13.16169	28.35600
229	52 441	15.13275	47.85394	12 008 989	6.118 033	13.18090	28.39739
230	52 900	15.16575	47.95832	12 167 000	6.126 926	13.20006	28.43867
231	53 361	15.19868	48.06246	12 326 391	6.135 792	13.21916	28.47983
232	53 824	15.23155	48.16638	12 487 168	6.144 634	13.23821	28.52086
233	54 289	15.26434	48.27007	12 649 337	6.153 449	13.25721	28.56178
234	54 756	15.29706	48.37355	12 812 904	6.162 240	13.27614	28.60259
235	55 225	15.32971	48.47680	12 977 875	6.171 006	13.29503	28.64327
236	55 696	15.36229	48.57983	13 144 256	6.179 747	13.31386	28.68384
237	56 169	15.39480	48.68265	13 312 053	6.188 463	13.33264	28.72430
238	56 644	15.42725	48.78524	13 481 272	6.197 154	13.35136	28.76464
239	57 121	15.45962	48.88763	13 651 919	6.205 822	13.37004	28.80487
240	57 600	15.49193	48.98979	13 824 000	6.214 465	13.38866	28.84499
241	58 081	15.52417	49.09175	13 997 521	6.223 084	13.40723	28.88500
242	58 564	15.55635	49.19350	14 172 488	6.231 680	13.42575	28.92489
243	59 049	15.58846	49.29503	14 348 907	6.240 251	13.44421	28.96468
244	59 536	15.62050	49.39636	14 526 784	6.248 800	13.46263	29.00436
245	60 025	15.65248	49.49747	14 706 125	6.257 325	13.48100	29.04393
246	60 516	15.68439	49.59839	14 886 936	6.265 827	13.49931	29.08339
247	61 009	15.71623	49.69909	15 069 223	6.274 305	13.51758	29.12275
248	61 504	15.74802	49.79960	15 252 992	6.282 761	13.53580	29.16199
249	62 001	15.77973	49.89990	15 438 249	6.291 195	13.55397	29.20114
250	62 500	15.81139	50.00000	15 625 000	6.299 605	13.57209	29.24018

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
250	62 500	15.81139	50.00000	15 625 000	6.299 605	13.57209	29.24018
251	63 001	15.84298	50.09990	15 813 251	6.307 994	13.59016	29.27911
252	63 504	15.87451	50.19960	16 003 008	6.310 360	13.60818	29.31794
253	64 009	15.90597	50.29911	16 194 277	6.324 704	13.62616	29.35667
254	64 516	15.93738	50.39841	16 387 064	6.333 026	13.64409	29.39530
255	65 025	15.96872	50.49752	16 581 375	6.341 326	13.66197	29.43383
256	65 536	16.00000	50.59644	16 777 216	6.349 604	13.67981	29.47225
257	66 049	16.03122	50.69517	16 974 593	6.357 861	13.69760	29.51058
258	66 564	16.06238	50.79370	17 173 512	6.366 097	13.71534	29.54880
259	67 081	16.09348	50.89204	17 373 979	6.374 311	13.73304	29.58698
260	67 600	16.12452	50.99020	17 576 000	6.382 504	13.75069	29.62496
261	68 121	16.15549	51.08816	17 779 581	6.390 677	13.76830	29.66289
262	68 644	16.18641	51.18594	17 984 728	6.398 828	13.78586	29.70073
263	69 169	16.21727	51.28353	18 191 447	6.406 959	13.80337	29.73847
264	69 696	16.24808	51.38093	18 399 744	6.415 069	13.82085	29.77611
265	70 225	16.27882	51.47815	18 609 625	6.423 158	13.83828	29.81366
266	70 756	16.30951	51.57519	18 821 096	6.431 228	13.85566	29.85111
267	71 289	16.34013	51.67204	19 034 163	6.439 277	13.87300	29.88847
268	71 824	16.37071	51.76872	19 248 832	6.447 306	13.89030	29.92574
269	72 361	16.40122	51.86521	19 465 109	6.455 315	13.90755	29.96292
270	72 900	16.43168	51.96152	19 683 000	6.463 304	13.92477	30.00000
271	73 441	16.46208	52.05766	19 902 511	6.471 274	13.94194	30.03699
272	73 984	16.49242	52.15362	20 123 648	6.479 224	13.95906	30.07389
273	74 529	16.52271	52.24940	20 346 417	6.487 154	13.97615	30.11070
274	75 076	16.55295	52.34501	20 570 824	6.495 065	13.99319	30.14742
275	75 625	16.58312	52.44044	20 796 875	6.502 957	14.01020	30.18405
276	76 176	16.61325	52.53570	21 024 576	6.510 830	14.02716	30.22060
277	76 729	16.64332	52.63079	21 253 933	6.518 684	14.04408	30.25705
278	77 284	16.67333	52.72571	21 484 952	6.526 519	14.06096	30.29342
279	77 841	16.70329	52.82045	21 717 639	6.534 335	14.07780	30.32970
280	78 400	16.73320	52.91503	21 952 000	6.542 133	14.09460	30.36589
281	78 961	16.76305	53.00943	22 188 041	6.549 912	14.11136	30.40200
282	79 524	16.79286	53.10367	22 425 768	6.557 672	14.12808	30.43802
283	80 089	16.82260	53.19774	22 665 187	6.565 414	14.14476	30.47395
284	80 656	16.85230	53.29165	22 906 304	6.573 138	14.16140	30.50981
285	81 225	16.88194	53.38539	23 149 125	6.580 844	14.17800	30.54557
286	81 796	16.91153	53.47897	23 393 656	6.588 532	14.19456	30.58126
287	82 369	16.94107	53.57238	23 639 903	6.596 202	14.21109	30.61686
288	82 944	16.97056	53.66563	23 887 872	6.603 854	14.22757	30.65238
289	83 521	17.00000	53.75872	24 137 569	6.611 489	14.24402	30.68781
290	84 100	17.02939	53.85165	24 389 000	6.619 106	14.26043	30.72317
291	84 681	17.05872	53.94442	24 642 171	6.626 705	14.27680	30.75844
292	85 264	17.08801	54.03702	24 897 088	6.634 287	14.29314	30.79363
293	85 849	17.11724	54.12947	25 153 757	6.641 852	14.30944	30.82875
294	86 436	17.14643	54.22177	25 412 184	6.649 400	14.32570	30.86378
295	87 025	17.17556	54.31390	25 672 375	6.656 930	14.34192	30.89873
296	87 616	17.20465	54.40588	25 934 336	6.664 444	14.35811	30.93361
297	88 209	17.23369	54.49771	26 198 073	6.671 940	14.37426	30.96840
298	88 804	17.26268	54.58938	26 463 592	6.679 420	14.39037	31.00312
299	89 401	17.29162	54.68089	26 730 899	6.686 883	14.40645	31.03776
300	90 000	17.32051	54.77226	27 000 000	6.694 330	14.42250	31.07233

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
300	90 000	17.32051	54.77226	27 000 000	6.694 330	14.42250	31.07233
301	90 601	17.34935	54.86347	27 270 901	6.701 759	14.43850	31.10681
302	91 204	17.37815	54.95453	27 543 608	6.709 173	14.45447	31.14122
303	91 809	17.40690	55.04544	27 818 127	6.716 570	14.47041	31.17556
304	92 416	17.43560	55.13620	28 094 464	6.723 951	14.48631	31.20982
305	93 025	17.46425	55.22681	28 372 625	6.731 315	14.50218	31.24400
306	93 636	17.49286	55.31727	28 652 616	6.738 664	14.51801	31.27811
307	94 249	17.52142	55.40758	28 934 443	6.745 997	14.53381	31.31214
308	94 864	17.54993	55.49775	29 218 112	6.753 313	14.54957	31.34610
309	95 481	17.57840	55.58777	29 503 629	6.760 614	14.56530	31.37999
310	96 100	17.60682	55.67764	29 791 000	6.767 899	14.58100	31.41381
311	96 721	17.63519	55.76737	30 080 231	6.775 169	14.59666	31.44755
312	97 344	17.66352	55.85696	30 371 328	6.782 423	14.61229	31.48122
313	97 969	17.69181	55.94640	30 664 297	6.789 661	14.62788	31.51482
314	98 596	17.72005	56.03570	30 959 144	6.796 884	14.64344	31.54834
315	99 225	17.74824	56.12486	31 255 875	6.804 092	14.65897	31.58180
316	99 856	17.77639	56.21388	31 554 496	6.811 285	14.67447	31.61518
317	100 489	17.80449	56.30275	31 855 013	6.818 462	14.68993	31.64850
318	101 124	17.83255	56.39149	32 157 432	6.825 624	14.70536	31.68174
319	101 761	17.86057	56.48008	32 461 759	6.832 771	14.72076	31.71492
320	102 400	17.88854	56.56854	32 768 000	6.839 904	14.73613	31.74802
321	103 041	17.91647	56.65686	33 076 161	6.847 021	14.75146	31.78106
322	103 684	17.94436	56.74504	33 386 248	6.854 124	14.76676	31.81403
323	104 329	17.97220	56.83309	33 698 267	6.861 212	14.78203	31.84693
324	104 976	18.00000	56.92100	34 012 224	6.868 285	14.79727	31.87976
325	105 625	18.02776	57.00877	34 328 125	6.875 344	14.81248	31.91252
326	106 276	18.05547	57.09641	34 645 976	6.882 389	14.82766	31.94522
327	106 929	18.08314	57.18391	34 965 783	6.889 419	14.84280	31.97785
328	107 584	18.11077	57.27128	35 287 552	6.896 434	14.85792	32.01041
329	108 241	18.13836	57.35852	35 611 289	6.903 436	14.87300	32.04291
330	108 900	18.16590	57.44563	35 937 000	6.910 423	14.88806	32.07534
331	109 561	18.19341	57.53260	36 264 691	6.917 396	14.90308	32.10771
332	110 224	18.22087	57.61944	36 594 368	6.924 356	14.91807	32.14001
333	110 889	18.24829	57.70615	36 926 037	6.931 301	14.93303	32.17225
334	111 556	18.27567	57.79273	37 259 704	6.938 232	14.94797	32.20442
335	112 225	18.30301	57.87918	37 595 375	6.945 150	14.96287	32.23653
336	112 896	18.33030	57.96551	37 933 056	6.952 053	14.97774	32.26857
337	113 569	18.35756	58.05170	38 272 753	6.958 943	14.99259	32.30055
338	114 244	18.38478	58.13777	38 614 472	6.965 820	15.00740	32.33247
339	114 921	18.41195	58.22371	38 958 219	6.972 683	15.02219	32.36433
340	115 600	18.43909	58.30952	39 304 000	6.979 532	15.03695	32.39612
341	116 281	18.46619	58.39521	39 651 821	6.986 368	15.05167	32.42785
342	116 964	18.49324	58.48077	40 001 688	6.993 191	15.06637	32.45952
343	117 649	18.52026	58.56620	40 353 607	7.000 000	15.08104	32.49112
344	118 336	18.54724	58.65151	40 707 584	7.006 796	15.09568	32.52267
345	119 025	18.57418	58.73670	41 063 625	7.013 579	15.11030	32.55415
346	119 716	18.60108	58.82176	41 421 736	7.020 349	15.12488	32.58557
347	120 409	18.62794	58.90671	41 781 923	7.027 106	15.13944	32.61694
348	121 104	18.65476	58.99152	42 144 192	7.033 850	15.15397	32.64824
349	121 801	18.68154	59.07622	42 508 549	7.040 581	15.16847	32.67948
350	122 500	18.70829	59.16080	42 875 000	7.047 299	15.18294	32.71066

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
350	122 500	18.70829	59.16080	42 875 000	7.047 299	15.18294	32.71066
351	123 201	18.73499	59.24525	43 243 551	7.054 004	15.19739	32.74179
352	123 904	18.76166	59.32959	43 614 208	7.060 697	15.21181	32.77285
353	124 609	18.78829	59.41380	43 986 977	7.067 377	15.22620	32.80386
354	125 316	18.81489	59.49790	44 361 864	7.074 044	15.24057	32.83480
355	126 025	18.84144	59.58188	44 738 875	7.080 699	15.25490	32.86569
356	126 736	18.86796	59.66574	45 118 016	7.087 341	15.26921	32.89652
357	127 449	18.89444	59.74948	45 499 293	7.093 971	15.28350	32.92730
358	128 164	18.92089	59.83310	45 882 712	7.100 588	15.29775	32.95801
359	128 881	18.94730	59.91661	46 268 279	7.107 194	15.31198	32.98867
360	129 600	18.97367	60.00000	46 656 000	7.113 787	15.32619	33.01927
361	130 321	19.00000	60.08328	47 045 881	7.120 367	15.34037	33.04982
362	131 044	19.02630	60.16644	47 437 928	7.126 936	15.35452	33.08031
363	131 769	19.05256	60.24948	47 832 147	7.133 492	15.36864	33.11074
364	132 496	19.07878	60.33241	48 228 544	7.140 037	15.38274	33.14112
365	133 225	19.10497	60.41523	48 627 125	7.146 569	15.39682	33.17144
366	133 956	19.13113	60.49793	49 027 896	7.153 090	15.41087	33.20170
367	134 689	19.15724	60.58052	49 430 863	7.159 599	15.42489	33.23191
368	135 424	19.18333	60.66300	49 836 032	7.166 096	15.43889	33.26207
369	136 161	19.20937	60.74537	50 243 409	7.172 581	15.45286	33.29217
370	136 900	19.23538	60.82763	50 653 000	7.179 054	15.46680	33.32222
371	137 641	19.26136	60.90977	51 064 811	7.185 516	15.48073	33.35221
372	138 384	19.28730	60.99180	51 478 848	7.191 966	15.49462	33.38215
373	139 129	19.31321	61.07373	51 895 117	7.198 405	15.50849	33.41204
374	139 876	19.33908	61.15554	52 313 624	7.204 832	15.52234	33.44187
375	140 625	19.36492	61.23724	52 734 375	7.211 248	15.53616	33.47165
376	141 376	19.39072	61.31884	53 157 376	7.217 652	15.54996	33.50137
377	142 129	19.41649	61.40033	53 582 633	7.224 045	15.56373	33.53105
378	142 884	19.44222	61.48170	54 010 152	7.230 427	15.57748	33.56067
379	143 641	19.46792	61.56298	54 439 939	7.236 797	15.59121	33.59024
380	144 400	19.49359	61.64414	54 872 000	7.243 156	15.60491	33.61975
381	145 161	19.51922	61.72520	55 306 341	7.249 505	15.61858	33.64922
382	145 924	19.54482	61.80615	55 742 968	7.255 842	15.63224	33.67863
383	146 689	19.57039	61.88699	56 181 887	7.262 167	15.64587	33.70800
384	147 456	19.59592	61.96773	56 623 104	7.268 482	15.65947	33.73731
385	148 225	19.62142	62.04837	57 066 625	7.274 786	15.67305	33.76657
386	148 996	19.64688	62.12890	57 512 456	7.281 079	15.68661	33.79578
387	149 769	19.67232	62.20932	57 960 603	7.287 362	15.70014	33.82494
388	150 544	19.69772	62.28965	58 411 072	7.293 633	15.71366	33.85405
389	151 321	19.72308	62.36986	58 863 869	7.299 894	15.72714	33.88310
390	152 100	19.74842	62.44998	59 319 000	7.306 144	15.74061	33.91211
391	152 881	19.77372	62.52999	59 776 471	7.312 383	15.75405	33.94107
392	153 664	19.79899	62.60990	60 236 288	7.318 611	15.76747	33.96999
393	154 449	19.82423	62.68971	60 698 457	7.324 829	15.78087	33.99885
394	155 236	19.84943	62.76942	61 162 984	7.331 037	15.79424	34.02766
395	156 025	19.87461	62.84903	61 629 875	7.337 234	15.80759	34.05642
396	156 816	19.89975	62.92853	62 099 136	7.343 420	15.82092	34.08514
397	157 609	19.92486	63.00794	62 570 773	7.349 597	15.83423	34.11381
398	158 404	19.94994	63.08724	63 044 792	7.355 762	15.84751	34.14242
399	159 201	19.97498	63.16645	63 521 199	7.361 918	15.86077	34.17100
400	160 000	20.00000	63.24555	64 000 000	7.368 063	15.87401	34.19952

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
400	160 000	20.00000	63.24555	64 000 000	7.368 063	15.87401	34.19952
401	160 801	20.02498	63.32456	64 481 201	7.374 198	15.88723	34.22799
402	161 604	20.04994	63.40347	64 964 808	7.380 323	15.90042	34.25642
403	162 409	20.07486	63.48228	65 450 827	7.386 437	15.91360	34.28480
404	163 216	20.09975	63.56099	65 939 264	7.392 542	15.92675	34.31314
405	164 025	20.12461	63.63961	66 430 125	7.398 636	15.93988	34.34143
406	164 836	20.14944	63.71813	66 923 416	7.404 721	15.95299	34.36967
407	165 649	20.17424	63.79655	67 419 143	7.410 795	15.96607	34.39786
408	166 464	20.19901	63.87488	67 917 312	7.416 860	15.97914	34.42601
409	167 281	20.22375	63.95311	68 417 929	7.422 914	15.99218	34.45412
410	168 100	20.24846	64.03124	68 921 000	7.428 959	16.00521	34.48217
411	168 921	20.27313	64.10928	69 426 531	7.434 994	16.01821	34.51018
412	169 744	20.29778	64.18723	69 934 528	7.441 019	16.03119	34.53815
413	170 569	20.32240	64.26508	70 444 997	7.447 034	16.04415	34.56607
414	171 396	20.34699	64.34283	70 957 944	7.453 040	16.05709	34.59395
415	172 225	20.37155	64.42049	71 473 375	7.459 036	16.07001	34.62178
416	173 056	20.39608	64.49806	71 991 296	7.465 022	16.08290	34.64956
417	173 889	20.42058	64.57554	72 511 713	7.470 999	16.09578	34.67731
418	174 724	20.44505	64.65292	73 034 632	7.476 966	16.10864	34.70500
419	175 561	20.46949	64.73021	73 560 059	7.482 924	16.12147	34.73266
420	176 400	20.49390	64.80741	74 088 000	7.488 872	16.13429	34.76027
421	177 241	20.51828	64.88451	74 618 461	7.494 811	16.14708	34.78783
422	178 084	20.54264	64.96153	75 151 448	7.500 741	16.15986	34.81535
423	178 929	20.56696	65.03845	75 686 967	7.506 661	16.17261	34.84283
424	179 776	20.59126	65.11528	76 225 024	7.512 572	16.18534	34.87027
425	180 625	20.61553	65.19202	76 765 625	7.518 473	16.19806	34.89766
426	181 476	20.63977	65.26868	77 308 776	7.524 365	16.21075	34.92501
427	182 329	20.66398	65.34524	77 854 483	7.530 248	16.22343	34.95232
428	183 184	20.68816	65.42171	78 402 752	7.536 122	16.23608	34.97958
429	184 041	20.71232	65.49809	78 953 589	7.541 987	16.24872	35.00680
430	184 900	20.73644	65.57439	79 507 000	7.547 842	16.26133	35.03398
431	185 761	20.76054	65.65059	80 062 991	7.553 689	16.27393	35.06112
432	186 624	20.78461	65.72671	80 621 568	7.559 526	16.28651	35.08821
433	187 489	20.80865	65.80274	81 182 737	7.565 355	16.29906	35.11527
434	188 356	20.83267	65.87868	81 746 504	7.571 174	16.31160	35.14228
435	189 225	20.85665	65.95453	82 312 875	7.576 985	16.32412	35.16925
436	190 096	20.88061	66.03030	82 881 856	7.582 787	16.33662	35.19618
437	190 969	20.90454	66.10598	83 453 453	7.588 579	16.34910	35.22307
438	191 844	20.92845	66.18157	84 027 672	7.594 363	16.36156	35.24991
439	192 721	20.95233	66.25708	84 604 519	7.600 139	16.37400	35.27672
440	193 600	20.97618	66.33250	85 184 000	7.605 905	16.38643	35.30348
441	194 481	21.00000	66.40783	85 766 121	7.611 663	16.39883	35.33021
442	195 364	21.02380	66.48308	86 350 888	7.617 412	16.41122	35.35689
443	196 249	21.04757	66.55825	86 938 307	7.623 152	16.42358	35.38354
444	197 136	21.07131	66.63332	87 528 384	7.628 884	16.43593	35.41014
445	198 025	21.09502	66.70832	88 121 125	7.634 607	16.44826	35.43671
446	198 916	21.11871	66.78323	88 716 536	7.640 321	16.46057	35.46323
447	199 809	21.14237	66.85806	89 314 623	7.646 027	16.47287	35.48971
448	200 704	21.16601	66.93280	89 915 392	7.651 725	16.48514	35.51616
449	201 601	21.18962	67.00746	90 518 849	7.657 414	16.49740	35.54257
450	202 500	21.21320	67.08204	91 125 000	7.663 094	16.50964	35.56893

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
450	202 500	21.21320	67.08204	91 125 000	7.663 094	16.50964	35.56893
451	203 401	21.23676	67.15653	91 733 851	7.668 766	16.52186	35.59526
452	204 304	21.26029	67.23095	92 345 408	7.674 430	16.53406	35.62155
453	205 209	21.28380	67.30527	92 959 677	7.680 086	16.54624	35.64780
454	206 116	21.30728	67.37952	93 576 664	7.685 733	16.55841	35.67401
455	207 025	21.33073	67.45369	94 196 375	7.691 372	16.57056	35.70018
456	207 936	21.35416	67.52777	94 818 816	7.697 002	16.58269	35.72632
457	208 849	21.37756	67.60178	95 443 993	7.702 625	16.59480	35.75242
458	209 764	21.40093	67.67570	96 071 912	7.708 239	16.60690	35.77848
459	210 681	21.42429	67.74954	96 702 579	7.713 845	16.61897	35.80450
460	211 600	21.44761	67.82330	97 336 000	7.719 443	16.63103	35.83048
461	212 521	21.47091	67.89698	97 972 181	7.725 032	16.64308	35.85642
462	213 444	21.49419	67.97058	98 611 128	7.730 614	16.65510	35.88233
463	214 369	21.51743	68.04410	99 252 847	7.736 188	16.66711	35.90820
464	215 296	21.54066	68.11755	99 897 344	7.741 753	16.67910	35.93404
465	216 225	21.56386	68.19091	100 544 625	7.747 311	16.69108	35.95983
466	217 156	21.58703	68.26419	101 194 696	7.752 861	16.70303	35.98559
467	218 089	21.61018	68.33740	101 847 563	7.758 402	16.71497	36.01131
468	219 024	21.63331	68.41053	102 503 232	7.763 936	16.72689	36.03700
469	219 961	21.65641	68.48357	103 161 709	7.769 462	16.73880	36.06265
470	220 900	21.67948	68.55655	103 823 000	7.774 980	16.75069	36.08826
471	221 841	21.70253	68.62944	104 487 111	7.780 490	16.76256	36.11384
472	222 784	21.72556	68.70226	105 154 048	7.785 993	16.77441	36.13938
473	223 729	21.74856	68.77500	105 823 817	7.791 488	16.78625	36.16488
474	224 676	21.77154	68.84766	106 496 424	7.796 975	16.79807	36.19035
475	225 625	21.79449	68.92024	107 171 875	7.802 454	16.80988	36.21578
476	226 576	21.81742	68.99275	107 850 176	7.807 925	16.82167	36.24118
477	227 529	21.84033	69.06519	108 531 333	7.813 389	16.83344	36.26654
478	228 484	21.86321	69.13754	109 215 352	7.818 846	16.84519	36.29187
479	229 441	21.88607	69.20983	109 902 239	7.824 294	16.85693	36.31716
480	230 400	21.90890	69.28203	110 592 000	7.829 735	16.86865	36.34241
481	231 361	21.93171	69.35416	111 284 641	7.835 169	16.88036	36.36763
482	232 324	21.95450	69.42622	111 980 168	7.840 595	16.89205	36.39282
483	233 289	21.97726	69.49820	112 678 587	7.846 013	16.90372	36.41797
484	234 256	22.00000	69.57011	113 379 904	7.851 424	16.91538	36.44308
485	235 225	22.02272	69.64194	114 084 125	7.856 828	16.92702	36.46817
486	236 196	22.04541	69.71370	114 791 256	7.862 224	16.93865	36.49321
487	237 169	22.06808	69.78539	115 501 303	7.867 613	16.95026	36.51822
488	238 144	22.09072	69.85700	116 214 272	7.872 994	16.96185	36.54320
489	239 121	22.11334	69.92853	116 930 169	7.878 368	16.97343	36.56815
490	240 100	22.13594	70.00000	117 649 000	7.883 735	16.98499	36.59306
491	241 081	22.15852	70.07139	118 370 771	7.889 095	16.99654	36.61793
492	242 064	22.18107	70.14271	119 095 488	7.894 447	17.00807	36.64278
493	243 049	22.20360	70.21396	119 823 157	7.899 792	17.01959	36.66758
494	244 036	22.22611	70.28513	120 553 784	7.905 129	17.03108	36.69236
495	245 025	22.24860	70.35624	121 287 375	7.910 460	17.04257	36.71710
496	246 016	22.27106	70.42727	122 023 936	7.915 783	17.05404	36.74181
497	247 009	22.29350	70.49823	122 763 473	7.921 099	17.06549	36.76649
498	248 004	22.31591	70.56912	123 505 992	7.926 408	17.07693	36.79113
499	249 001	22.33831	70.63993	124 251 499	7.931 710	17.08835	36.81574
500	250 000	22.36068	70.71068	125 000 000	7.937 005	17.09976	36.84031

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
500	250 000	22.36068	70.71068	125 000 000	7.937 005	17.09976	36.84031
501	251 001	22.38303	70.78135	125 751 501	7.942 293	17.11115	36.86486
502	252 004	22.40536	70.85196	126 506 008	7.947 574	17.12253	36.88937
503	253 009	22.42766	70.92249	127 263 527	7.952 848	17.13389	36.91385
504	254 016	22.44994	70.99296	128 024 064	7.958 114	17.14524	36.93830
505	255 025	22.47221	71.06335	128 787 625	7.963 374	17.15657	36.96271
506	256 036	22.49444	71.13368	129 554 216	7.968 627	17.16789	36.98709
507	257 049	22.51666	71.20393	130 323 843	7.973 873	17.17919	37.01144
508	258 064	22.53886	71.27412	131 096 512	7.979 112	17.19048	37.03576
509	259 081	22.56103	71.34424	131 872 229	7.984 344	17.20175	37.06004
510	260 100	22.58318	71.41428	132 651 000	7.989 570	17.21301	37.08430
511	261 121	22.60531	71.48426	133 432 831	7.994 788	17.22425	37.10852
512	262 144	22.62742	71.55418	134 217 728	8.000 000	17.23548	37.13271
513	263 169	22.64950	71.62402	135 005 697	8.005 205	17.24669	37.15687
514	264 196	22.67157	71.69379	135 796 744	8.010 403	17.25789	37.18100
515	265 225	22.69361	71.76350	136 590 875	8.015 595	17.26908	37.20509
516	266 256	22.71563	71.83314	137 388 096	8.020 779	17.28025	37.22916
517	267 289	22.73763	71.90271	138 188 413	8.025 957	17.29140	37.25319
518	268 324	22.75961	71.97222	138 091 832	8.031 129	17.30254	37.27720
519	269 361	22.78157	72.04165	139 798 359	8.036 293	17.31367	37.30117
520	270 400	22.80351	72.11103	140 608 000	8.041 452	17.32478	37.32511
521	271 441	22.82542	72.18033	141 420 761	8.046 603	17.33588	37.34902
522	272 484	22.84732	72.24957	142 236 648	8.051 748	17.34696	37.37290
523	273 529	22.86919	72.31874	143 055 667	8.056 886	17.35804	37.39675
524	274 576	22.89105	72.38784	143 877 824	8.062 018	17.36909	37.42057
525	275 625	22.91288	72.45688	144 703 125	8.067 143	17.38013	37.44436
526	276 676	22.93469	72.52586	145 531 576	8.072 262	17.39116	37.46812
527	277 729	22.95648	72.59477	146 363 183	8.077 374	17.40218	37.49185
528	278 784	22.97825	72.66361	147 197 952	8.082 480	17.41318	37.51555
529	279 841	23.00000	72.73239	148 035 889	8.087 579	17.42416	37.53922
530	280 900	23.02173	72.80110	148 877 000	8.092 672	17.43513	37.56286
531	281 961	23.04344	72.86975	149 721 291	8.097 759	17.44609	37.58647
532	283 024	23.06513	72.93833	150 568 768	8.102 839	17.45704	37.61005
533	284 089	23.08679	73.00685	151 419 437	8.107 913	17.46797	37.63360
534	285 156	23.10844	73.07530	152 273 304	8.112 980	17.47889	37.65712
535	286 225	23.13007	73.14369	153 130 375	8.118 041	17.48979	37.68061
536	287 296	23.15167	73.21202	153 990 656	8.123 096	17.50068	37.70407
537	288 369	23.17326	73.28028	154 854 153	8.128 145	17.51156	37.72751
538	289 444	23.19483	73.34848	155 720 872	8.133 187	17.52242	37.75091
539	290 521	23.21637	73.41662	156 590 819	8.138 223	17.53327	37.77429
540	291 600	23.23790	73.48469	157 464 000	8.143 253	17.54411	37.79763
541	292 681	23.25941	73.55270	158 340 421	8.148 276	17.55493	37.82095
542	293 764	23.28089	73.62065	159 220 088	8.153 294	17.56574	37.84424
543	294 849	23.30236	73.68853	160 103 007	8.158 305	17.57654	37.86750
544	295 936	23.32381	73.75636	160 989 184	8.163 310	17.58732	37.89073
545	297 025	23.34524	73.82412	161 878 625	8.168 300	17.59809	37.91393
546	298 116	23.36664	73.89181	162 771 336	8.173 302	17.60885	37.93711
547	299 209	23.38803	73.95945	163 667 323	8.178 289	17.61959	37.96025
548	300 304	23.40940	74.02702	164 566 592	8.183 269	17.63032	37.98337
549	301 401	23.43075	74.09453	165 469 149	8.188 244	17.64104	38.00646
550	302 500	23.45208	74.16198	166 375 000	8.193 213	17.65174	38.02952

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
550	302 500	23.45208	74.16198	166 375 000	8.193 213	17.65174	38.02952
551	303 601	23.47339	74.22937	167 284 151	8.198 175	17.66243	38.05256
552	304 704	23.49468	74.29670	168 196 608	8.203 132	17.67311	38.07557
553	305 809	23.51595	74.36397	169 112 377	8.208 082	17.68378	38.09854
554	306 916	23.53720	74.43118	170 031 464	8.213 027	17.69443	38.12149
555	308 025	23.55844	74.49832	170 953 875	8.217 966	17.70507	38.14442
556	309 136	23.57965	74.56541	171 879 616	8.222 899	17.71570	38.16731
557	310 249	23.60085	74.63243	172 808 693	8.227 825	17.72631	38.19018
558	311 364	23.62202	74.69940	173 741 112	8.232 746	17.73691	38.21302
559	312 481	23.64318	74.76630	174 676 879	8.237 661	17.74750	38.23584
560	313 600	23.66432	74.83315	175 616 000	8.242 571	17.75808	38.25862
561	314 721	23.68544	74.89993	176 558 481	8.247 474	17.76864	38.28138
562	315 844	23.70654	74.96666	177 504 328	8.252 372	17.77920	38.30412
563	316 969	23.72762	75.03333	178 453 547	8.257 263	17.78973	38.32682
564	318 096	23.74868	75.09993	179 406 144	8.262 149	17.80026	38.34950
565	319 225	23.76973	75.16648	180 362 125	8.267 029	17.81077	38.37215
566	320 356	23.79075	75.23297	181 321 496	8.271 904	17.82128	38.39478
567	321 489	23.81176	75.29940	182 284 263	8.276 773	17.83177	38.41737
568	322 624	23.83275	75.36577	183 250 432	8.281 635	17.84224	38.43995
569	323 761	23.85372	75.43209	184 220 009	8.286 493	17.85271	38.46249
570	324 900	23.87467	75.49834	185 193 000	8.291 344	17.86316	38.48501
571	326 041	23.89561	75.56454	186 169 411	8.296 190	17.87360	38.50750
572	327 184	23.91652	75.63068	187 149 248	8.301 031	17.88403	38.52997
573	328 329	23.93742	75.69676	188 132 517	8.305 865	17.89444	38.55241
574	329 476	23.95830	75.76279	189 119 224	8.310 694	17.90485	38.57482
575	330 625	23.97916	75.82875	190 109 375	8.315 517	17.91524	38.59721
576	331 776	24.00000	75.89466	191 102 976	8.320 335	17.92562	38.61958
577	332 929	24.02082	75.96052	192 100 033	8 325 148	17.93599	38.64191
578	334 084	24.04163	76.02631	193 100 552	8.329 954	17.94634	38.66422
579	335 241	24.06242	76.09205	194 104 539	8.334 755	17.95669	38.68651
580	336 400	24.08319	76.15773	195 112 000	8.339 551	17.96702	38.70877
581	337 561	24.10394	76.22336	196 122 941	8.344 341	17.97734	38.73100
582	338 724	24.12468	76.28892	197 137 368	8.349 126	17.98765	38.75321
583	339 889	24.14539	76.35444	198 155 287	8.353 905	17.99794	38.77539
584	341 056	24.16609	76.41989	199 176 704	8.358 678	18.00823	38.79755
585	342 225	24.18677	76.48529	200 201 625	8.363 447	18.01850	38.81968
586	343 396	24.20744	76.55064	201 230 056	8.368 209	18.02876	38.84179
587	344 569	24.22808	76.61593	202 262 003	8.372 967	18.03901	38.86387
588	345 744	24.24871	76.68116	203 297 472	8.377 719	18.04925	38.88593
589	346 921	24.26932	76.74634	204 336 469	8.382 465	18.05947	38.90796
590	348 100	24.28992	76.81146	205 379 000	8.387 207	18.06969	38.92996
591	349 281	24.31049	76.87652	206 425 071	8.391 942	18.07989	38.95195
592	350 464	24.33105	76.94154	207 474 688	8.396 673	18.09008	38.97390
593	351 649	24.35159	77.00649	208 527 857	8.401 398	18.10026	38.99584
594	352 836	24.37212	77.07140	209 584 584	8.406 118	18.11043	39.01774
595	354 025	24.39262	77.13624	210 644 875	8.410 833	18.12059	39.03963
596	355 216	24.41311	77.20104	211 708 736	8.415 542	18.13074	39.06149
597	356 409	24.43358	77.26578	212 776 173	8.420 246	18.14087	39.08332
598	357 604	24.45404	77.33046	213 847 192	8.424 945	18.15099	39.10513
599	358 801	24.47448	77.39509	214 921 799	8.429 638	18.16111	39.12692
600	360 000	24.49490	77.45967	216 000 000	8.434 327	18.17121	39.14868

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
600	360 000	24.49490	77.45967	216 000 000	8.434 327	18.17121	39.14868
601	361 201	24.51530	77.52419	217 081 801	8.439 010	18.18130	39.17041
602	362 404	24.53569	77.58866	218 167 208	8.443 688	18.19137	39.19213
603	363 609	24.55606	77.65307	219 256 227	8.448 361	18.20144	39.21382
604	364 816	24.57641	77.71744	220 348 864	8.453 028	18.21150	39.23548
605	366 025	24.59675	77.78175	221 445 125	8.457 691	18.22154	39.25712
606	367 236	24.61707	77.84600	222 545 016	8.462 348	18.23158	39.27874
607	368 449	24.63737	77.91020	223 648 543	8.467 000	18.24160	39.30033
608	369 664	24.65766	77.97435	224 755 712	8.471 647	18.25161	39.32190
609	370 881	24.67793	78.03845	225 866 529	8.476 289	18.26161	39.34345
610	372 100	24.69818	78.10250	226 981 000	8.480 926	18.27160	39.36497
611	373 321	24.71841	78.16649	228 099 131	8.485 558	18.28158	39.38647
612	374 544	24.73863	78.23043	229 220 928	8.490 185	18.29155	39.40795
613	375 769	24.75884	78.29432	230 346 397	8.494 807	18.30151	39.42940
614	376 996	24.77902	78.35815	231 475 544	8.499 423	18.31145	39.45083
615	378 225	24.79919	78.42194	232 608 375	8.504 035	18.32139	39.47223
616	379 456	24.81935	78.48567	233 744 896	8.508 642	18.33131	39.49362
617	380 689	24.83948	78.54935	234 885 113	8.513 243	18.34123	39.51498
618	381 924	24.85961	78.61298	236 029 032	8.517 840	18.35113	39.53631
619	383 161	24.87971	78.67655	237 176 659	8.522 432	18.36102	39.55763
620	384 400	24.89980	78.74008	238 328 000	8.527 019	18.37091	39.57892
621	385 641	24.91987	78.80355	239 483 061	8.531 601	18.38078	39.60018
622	386 884	24.93993	78.86698	240 641 848	8.536 178	18.39064	39.62143
623	388 129	24.95997	78.93035	241 804 367	8.540 750	18.40049	39.64265
624	389 376	24.97999	78.99367	242 970 624	8.545 317	18.41033	39.66385
625	390 625	25.00000	79.05694	244 140 625	8.549 880	18.42016	39.68503
626	391 876	25.01999	79.12016	245 314 376	8.554 437	18.42998	39.70618
627	393 129	25.03997	79.18333	246 491 883	8.558 990	18.43978	39.72731
628	394 384	25.05993	79.24645	247 673 152	8.563 538	18.44958	39.74842
629	395 641	25.07987	79.30952	248 858 189	8.568 081	18.45937	39.76951
630	396 900	25.09980	79.37254	250 047 000	8.572 619	18.46915	39.79057
631	398 161	25.11971	79.43551	251 239 591	8.577 152	18.47891	39.81161
632	399 424	25.13961	79.49843	252 435 968	8.581 681	18.48867	39.83263
633	400 689	25.15949	79.56130	253 636 137	8.586 205	18.49842	39.85363
634	401 956	25.17936	79.62412	254 840 104	8.590 724	18.50815	39.87461
635	403 225	25.19921	79.68689	256 047 875	8.595 238	18.51788	39.89556
636	404 496	25.21904	79.74961	257 259 456	8.599 748	18.52759	39.91649
637	405 769	25.23886	79.81228	258 474 853	8.604 252	18.53730	39.93740
638	407 044	25.25866	79.87490	259 694 072	8.608 753	18.54700	39.95829
639	408 321	25.27845	79.93748	260 917 119	8.613 248	18.55668	39.97916
640	409 600	25.29822	80.00000	262 144 000	8.617 739	18.56636	40.00000
641	410 881	25.31798	80.06248	263 374 721	8.622 225	18.57602	40.02082
642	412 164	25.33772	80.12490	264 609 288	8.626 706	18.58568	40.04162
643	413 449	25.35744	80.18728	265 847 707	8.631 183	18.59532	40.06240
644	414 736	25.37716	80.24961	267 089 984	8.635 655	18.60495	40.08316
645	416 025	25.39685	80.31189	268 336 125	8.640 123	18.61458	40.10390
646	417 316	25.41653	80.37413	269 586 136	8.644 585	18.62419	40.12461
647	418 609	25.43619	80.43631	270 840 023	8.649 044	18.63380	40.14530
648	419 904	25.45584	80.49845	272 097 792	8.653 497	18.64340	40.16598
649	421 201	25.47548	80.56054	273 359 449	8.657 947	18.65298	40.18663
650	422 500	25.49510	80.62258	274 625 000	8.662 391	18.66256	40.20726

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
650	422 500	25.49510	80.62258	274 625 000	8.662 391	18.66256	40.20726
651	423 801	25.51470	80.68457	275 894 451	8.666 831	18.67212	40.22787
652	425 104	25.53429	80.74652	277 167 808	8.671 266	18.68168	40.24845
653	426 409	25.55386	80.80842	278 445 077	8.675 697	18.69122	40.26902
654	427 716	25.57342	80.87027	279 726 264	8.680 124	18.70076	40.28957
655	429 025	25.59297	80.93207	281 011 375	8.684 546	18.71029	40.31009
656	430 336	25.61250	80.99383	282 300 416	8.688 963	18.71980	40.33059
657	431 649	25.63201	81.05554	283 593 393	8.693 376	18.72931	40.35108
658	432 964	25.65151	81.11720	284 890 312	8.697 784	18.73881	40.37154
659	434 281	25.67100	81.17881	286 191 179	8.702 188	18.74830	40.39198
660	435 600	25.69047	81.24038	287 496 000	8.706 588	18.75777	40.41240
661	436 921	25.70992	81.30191	288 804 781	8.710 983	18.76724	40.43280
662	438 244	25.72936	81.36338	290 117 528	8.715 373	18.77670	40.45318
663	439 569	25.74879	81.42481	291 434 247	8.719 760	18.78615	40.47354
664	440 896	25.76820	81.48620	292 754 944	8.724 141	18.79559	40.49388
665	442 225	25.78759	81.54753	294 079 625	8.728 519	18.80502	40.51420
666	443 556	25.80698	81.60882	295 408 296	8.732 892	18.81444	40.53449
667	444 889	25.82634	81.67007	296 740 963	8.737 260	18.82386	40.55477
668	446 224	25.84570	81.73127	298 077 632	8.741 625	18.83326	40.57503
669	447 561	25.86503	81.79242	299 418 309	8.745 985	18.84265	40.59526
670	448 900	25.88436	81.85353	300 763 000	8.750 340	18.85204	40.61548
671	450 241	25.90367	81.91459	302 111 711	8.754 691	18.86141	40.63568
672	451 584	25.92296	81.97561	303 464 448	8.759 038	18.87078	40.65585
673	452 929	25.94224	82.03658	304 821 217	8.763 381	18.88013	40.67601
674	454 276	25.96151	82.09750	306 182 024	8.767 719	18.88948	40.69615
675	455 625	25.98076	82.15838	307 546 875	8.772 053	18.89882	40.71626
676	456 976	26.00000	82.21922	308 915 776	8.776 383	18.90814	40.73636
677	458 329	26.01922	82.28001	310 288 733	8.780 708	18.91746	40.75644
678	459 684	26.03843	82.34076	311 665 752	8.785 030	18.92677	40.77650
679	461 041	26.05763	82.40146	313 046 839	8.789 347	18.93607	40.79653
680	462 400	26.07681	82.46211	314 432 000	8.793 659	18.94536	40.81655
681	463 761	26.09598	82.52272	315 821 241	8.797 968	18.95465	40.83655
682	465 124	26.11513	82.58329	317 214 568	8.802 272	18.96392	40.85653
683	466 489	26.13427	82.64381	318 611 987	8.806 572	18.97318	40.87649
684	467 856	26.15339	82.70429	320 013 504	8.810 868	18.98244	40.89643
685	469 225	26.17250	82.76473	321 419 125	8.815 160	18.99169	40.91635
686	470 596	26.19160	82.82512	322 828 856	8.819 447	19.00092	40.93625
687	471 969	26.21068	82.88546	324 242 703	8.823 731	19.01015	40.95613
688	473 344	26.22975	82.94577	325 660 672	8.828 010	19.01937	40.97599
689	474 721	26.24881	83.00602	327 082 769	8.832 285	19.02858	40.99584
690	476 100	26.26785	83.06624	328 509 000	8.836 556	19.03778	41.01566
691	477 481	26.28688	83.12641	329 939 371	8.840 823	19.04698	41.03546
692	478 864	26.30589	83.18654	331 373 888	8.845 085	19.05616	41.05525
693	480 249	26.32489	83.24662	332 812 557	8.849 344	19.06533	41.07502
694	481 636	26.34388	83.30666	334 255 384	8.853 599	19.07450	41.09476
695	483 025	26.36285	83.36666	335 702 375	8.857 849	19.08366	41.11449
696	484 416	26.38181	83.42661	337 153 536	8.862 095	19.09281	41.13420
697	485 809	26.40076	83.48653	338 608 873	8.866 338	19.10195	41.15389
698	487 204	26.41969	83.54639	340 068 392	8.870 576	19.11108	41.17357
699	488 601	26.43861	83.60622	341 532 099	8.874 810	19.12020	41.19322
700	490 000	26.45751	83.66600	343 000 000	8.879 040	19.12931	41.21285

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
700	490 000	26.45751	83.66600	343 000 000	8.879 040	19.12931	41.21285
701	491 401	26.47640	83.72574	344 472 101	8.883 266	19.13842	41.23247
702	402 804	26.49528	83.78544	345 948 408	8.887 488	19.14751	41.25207
703	494 209	26.51415	83.84510	347 428 927	8.891 706	19.15660	41.27164
704	495 616	26.53300	83.90471	348 913 664	8.895 920	19.16568	41.29120
705	497 025	26.55184	83.96428	350 402 625	8.900 130	19.17475	41.31075
706	498 436	26.57066	84.02381	351 895 816	8.904 337	19.18381	41.33027
707	499 849	26.58947	84.08329	353 393 243	8.908 539	19.19286	41.34977
708	501 264	26.60827	84.14274	354 894 912	8.912 737	19.20191	41.36926
709	502 681	26.62705	84.20214	356 400 829	8.916 931	19.21095	41.38873
710	504 100	26.64583	84.26150	357 911 000	8.921 121	19.21997	41.40818
711	505 521	26.66458	84.32082	359 425 431	8.925 308	19.22899	41.42761
712	506 944	26.68333	84.38009	360 944 128	8.929 490	19.23800	41.44702
713	508 369	26.70206	84.43933	362 467 097	8.933 669	19.24701	41.46642
714	509 796	26.72078	84.49852	363 994 344	8.937 843	19.25600	41.48579
715	511 225	26.73948	84.55767	365 525 875	8.942 014	19.26499	41.50515
716	512 656	26.75818	84.61678	367 061 696	8.946 181	19.27396	41.52449
717	514 089	26.77686	84.67585	368 601 813	8.950 344	19.28293	41.54382
718	515 524	26.79552	84.73488	370 146 232	8.954 503	19.29189	41.56312
719	516 961	26.81418	84.79387	371 694 959	8.958 658	19.30084	41.58241
720	518 400	26.83282	84.85281	373 248 000	8.962 809	19.30979	41.60168
721	519 841	26.85144	84.91172	374 805 361	8.966 957	19.31872	41.62093
722	521 284	26.87006	84.97058	376 367 048	8 971 101	19.32765	41.64016
723	522 729	26.88866	85.02941	377 933 067	8.975 241	19.33657	41.65938
724	524 176	26.90725	85.08819	379 503 424	8.979 377	19.34548	41.67857
725	525 625	26.92582	85.14693	381 078 125	8.983 509	19.35438	41.69775
726	527 076	26.94439	85.20563	382 657 176	8.987 637	19.36328	41.71692
727	528 529	26.96294	85.26429	384 240 583	8.991 762	19.37216	41.73606
728	529 984	26.98148	85.32292	385 828 352	8.995 883	19.38104	41.75519
729	531 441	27.00000	85.38150	387 420 489	9.000 000	19.38991	41.77430
730	532 900	27.01851	85.44004	389 017 000	9.004 113	19.39877	41.79339
731	534 361	27.03701	85.49854	300 617 891	9.008 223	19.40763	41.81247
732	535 824	27.05550	85 55700	392 223 168	9.012 329	19.41647	41.83152
733	537 289	27.07307	85.61542	393 832 837	9.016 431	19.42531	41.85056
734	538 756	27.09243	85.67380	395 446 904	9.020 529	19.43414	41.86959
735	540 225	27.11088	85.73214	397 065 375	9.024 624	19.44296	41.88859
736	541 696	27.12932	85.79044	398 688 256	9.028 715	19.45178	41.90758
737	543 169	27.14774	85.84870	400 315 553	9.032 802	19.46058	41.92655
738	544 644	27.16616	85.90693	401 947 272	9.036 886	19.46938	41.94551
739	546 121	27.18455	85.96511	403 583 419	9.040 966	19.47817	41.96444
740	547 600	27.20294	86.02325	405 224 000	9.045 042	19.48695	41.98336
741	549 081	27.22132	86.08136	406 869 021	9.049 114	19.49573	42.00227
742	550 564	27.23968	86.13942	408 518 488	9.053 183	19.50449	42.02115
743	552 049	27.25803	86.19745	410 172 407	9.057 248	19.51325	42.04002
744	553 536	27.27636	86.25543	411 830 784	9.061 310	19.52200	42.05887
745	555 025	27 29469	86.31338	413 493 625	9.065 368	19.53074	42.07771
746	556 516	27.31300	86.37129	415 160 936	9.069 422	19.53948	42.09653
747	558 009	27.33130	86.42916	416 832 723	9.073 473	19.54820	42.11533
748	559 504	27.34959	86.48699	418 508 992	9.077 520	19 55692	42.13411
749	561 001	27.36786	86.54479	420 189 749	9.081 563	19.56563	42.15288
750	562 500	27.38613	86.60254	421 875 000	9.085 603	19.57434	42.17163

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
750	562 500	27.38613	86.60254	421 875 000	9.085 603	19.57434	42.17163
751	564 001	27.40438	86.66026	423 564 751	9.089 639	19.58303	42.19037
752	565 504	27.42262	86.71793	425 259 008	9.093 672	19.59172	42.20909
753	567 009	27.44085	86.77557	426 957 777	9.097 701	19.60040	42.22770
754	568 516	27.45906	86.83317	428 661 064	9.101 727	19.60908	42.24047
755	570 025	27.47726	86.89074	430 368 875	9.105 748	19.61774	42.26514
756	571 536	27.49545	86.94826	432 081 216	9.109 767	19.62640	42.28379
757	573 049	27.51363	87.00575	433 798 093	9.113 782	19.63505	42.30243
758	574 564	27.53180	87.06320	435 519 512	9.117 793	19.64369	42.32105
759	576 081	27.54995	87.12061	437 245 479	9.121 801	19.65232	42.33965
760	577 600	27.56810	87.17798	438 976 000	9.125 805	19.66095	42.35824
761	579 121	27.58623	87.23531	440 711 081	9.129 806	19.66957	42.37681
762	580 644	27.60435	87.29261	442 450 728	9.133 803	19.67818	42.39536
763	582 169	27.62245	87.34987	444 194 947	9.137 797	19.68079	42.41390
764	583 696	27.64055	87.40709	445 943 744	9.141 787	19.69538	42.43242
765	585 225	27.65863	87.46428	447 697 125	9.145 774	19.70397	42.45092
766	586 756	27.67671	87.52143	449 455 096	9.149 758	19.71256	42.46941
767	588 289	27.69476	87.57854	451 217 663	9.153 738	19.72113	42.48789
768	589 824	27.71281	87.63561	452 984 832	9.157 714	19.72970	42.50634
769	591 361	27.73085	87.69265	454 756 609	9.161 687	19.73826	42.52478
770	592 900	27.74887	87.74964	456 533 000	9.165 656	19.74681	42.54321
771	594 441	27.76689	87.80661	458 314 011	9.169 623	19.75535	42.56162
772	595 984	27.78489	87.86353	460 099 648	9.173 585	19.76389	42.58001
773	597 529	27.80288	87.92042	461 889 917	9.177 544	19.77242	42.50839
774	599 076	27.82086	87.97727	463 684 824	9.181 500	19.78094	42.61675
775	600 625	27.83882	88.03408	465 484 375	9.185 453	19.78946	42.63509
776	602 176	27.85678	88.09086	467 288 576	9.189 402	19.79797	42.65342
777	603 729	27.87472	88.14760	469 097 433	9.193 347	19.80647	42.67174
778	605 284	27.89265	88.20431	470 910 952	9.197 290	19.81496	42.69004
779	606 841	27.91057	88.26098	472 729 139	9.201 229	19.82345	42.70832
780	608 400	27.92848	88.31761	474 552 000	9.205 164	19.83192	42.72659
781	609 961	27.94638	88.37420	476 379 541	9.209 096	19.84040	42.74484
782	611 524	27.96426	88.43076	478 211 768	9.213 025	19.84886	42.76307
783	613 089	27.98214	88.48729	480 048 687	9.216 950	19.85732	42.78129
784	614 656	28.00000	88.54377	481 890 304	9.220 873	19.86577	42.79950
785	616 225	28.01785	88.60023	483 736 625	9.224 791	19.87421	42.81769
786	617 796	28.03569	88.65664	485 587 650	9.228 707	19.88265	42.83586
787	619 369	28.05352	88.71302	487 443 403	9.232 619	19.89107	42.85402
788	620 944	28.07134	88.76936	480 303 872	9.236 528	19.89950	42.87216
789	622 521	28.08914	88.82567	491 169 069	9.240 433	19.90791	42.89029
790	624 100	28.10694	88.88194	493 039 000	9.244 335	19.91632	42.90840
791	625 681	28.12472	88.93818	494 913 671	9.248 234	19.92472	42.92650
792	627 264	28.14249	88.99438	496 793 088	9.252 130	19.93311	42.94458
793	628 849	28.16026	89.05055	498 677 257	9.256 022	19.94150	42.96265
794	630 436	28.17801	89.10668	500 566 184	9.259 911	19.94987	42.98070
795	632 025	28.19574	89.16277	502 459 875	9.263 797	19.95825	42.99874
796	633 616	28.21347	89.21883	504 358 336	9.267 680	19.96661	43.01676
797	635 209	28.23119	89.27486	506 261 573	9.271 559	19.97497	43 03477
798	636 804	28.24889	89.33085	508 169 592	9.275 435	19.98332	43.05276
799	638 401	28.26659	89.38680	510 082 399	9.279 308	19.99166	43.07073
800	640 000	28.28427	89.44272	512 000 000	9.283 178	20.00000	43.08869

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
800	640 000	28.28427	89.44272	512 000 000	9.283 178	20.00000	43.08869
801	641 601	28.30194	89.49860	513 922 401	9.287 044	20.00833	43.10664
802	643 204	28.31960	89.55445	515 849 608	9.290 907	20.01665	43.12457
803	644 809	28.33725	89.61027	517 781 627	9.294 767	20.02497	43.14249
804	646 416	28.35489	89.66605	519 718 464	9.298 624	20.03328	43.16039
805	648 025	28.37252	89.72179	521 660 125	9.302 477	20.04158	43.17828
806	649 636	28.39014	89.77750	523 606 616	9.306 328	20.04988	43.19615
807	651 249	28.40775	89.83318	525 557 943	9.310 175	20.05816	43.21400
808	652 864	28.42534	89.88882	527 514 112	9.314 019	20.06645	43.23185
809	654 481	28.44293	89.94443	529 475 129	9.317 860	20.07472	43.24967
810	656 100	28.46050	90.00000	531 441 000	9.321 698	20.08299	43.26749
811	657 721	28.47806	90.05554	533 411 731	9.325 532	20.09125	43.28529
812	659 344	28.49561	90.11104	535 387 328	9.329 363	20.09950	43.30307
813	660 969	28.51315	90.16651	537 367 797	9.333 192	20.10775	43.32084
814	662 596	28.53069	90.22195	539 353 144	9.337 017	20.11599	43.33859
815	664 225	28.54820	90.27735	541 343 375	9.340 839	20.12423	43.35633
816	665 856	28.56571	90.33272	543 338 496	9.344 657	20.13245	43.37406
817	667 489	28.58321	90.38805	545 338 513	9.348 473	20.14067	43.39177
818	669 124	28.60070	90.44335	547 343 432	9.352 286	20.14889	43.40947
819	670 761	28.61818	90.49862	549 353 259	9.356 095	20.15710	43.42715
820	672 400	28.63564	90.55385	551 368 000	9.359 902	20.16530	43.44481
821	674 041	28.65310	90.60905	553 387 661	9.363 705	20.17349	43.46247
822	675 684	28.67054	90.66422	555 412 248	9.367 505	20.18168	43.48011
823	677 329	28.68798	90.71935	557 441 767	9.371 302	20.18986	43.49773
824	678 976	28.70540	90.77445	559 476 224	9.375 096	20.19803	43.51534
825	680 625	28.72281	90.82951	561 515 625	9.378 887	20.20620	43.53294
826	682 276	28.74022	90.88454	563 559 976	9.382 675	20.21436	43.55052
827	683 929	28.75761	90.93954	565 609 283	9.386 460	20.22252	43.56809
828	685 584	28.77499	90.99451	567 663 552	9.390 242	20.23066	43.58564
829	687 241	28.79236	91.04944	569 722 789	9.394 021	20.23880	43.60318
830	688 900	28.80972	91.10434	571 787 000	9.397 796	20.24694	43.62071
831	690 561	28.82707	91.15920	573 856 191	9.401 569	20.25507	43.63822
832	692 224	28.84441	91.21403	575 930 368	9.405 339	20.26319	43.65572
833	693 889	28.86174	91.26883	578 009 537	9.409 105	20.27130	43.67320
834	695 556	28.87906	91.32360	580 093 704	9.412 869	20.27941	43.69067
835	697 225	28.89637	91.37833	582 182 875	9.416 630	20.28751	43.70812
836	698 896	28.91366	91.43304	584 277 056	9.420 387	20.29561	43.72556
837	700 569	28.93095	91.48770	586 376 253	9.424 142	20.30370	43.74299
838	702 244	28.94823	91.54234	588 480 472	9.427 894	20.31178	43.76041
839	703 921	28.96550	91.59694	590 589 719	9.431 642	20.31986	43.77781
840	705 600	28.98275	91.65151	592 704 000	9.435 388	20.32793	43.79519
841	707 281	29.00000	91.70605	594 823 321	9.439 131	20.33599	43.81256
842	708 964	29.01724	91.76056	596 947 688	9.442 870	20.34405	43.82992
843	710 649	29.03446	91.81503	599 077 107	9.446 607	20.35210	43.84727
844	712 336	29.05168	91.86947	601 211 584	9.450 341	20.36014	43.86460
845	714 025	29.06888	91.92388	603 351 125	9.454 072	20.36818	43.88191
846	715 716	29.08608	91.97826	605 495 736	9.457 800	20.37621	43.89922
847	717 409	29.10326	92.03260	607 645 423	9.461 525	20.38424	43.91651
848	719 104	29.12044	92.08692	609 800 192	9.465 247	20.39226	43.93378
849	720 801	29.13760	92.14120	611 960 049	9.468 966	20.40027	43.95105
850	722 500	29.15476	92.19544	614 125 000	9.472 682	20.40828	43.96830

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
850	722 500	29.15476	92.19544	614 125 000	9.472 682	20.40828	43.96830
851	724 201	29.17190	92.24966	616 295 051	9.476 396	20.41628	43.98553
852	725 904	29.18904	92.30385	618 470 208	9.480 106	20.42427	44.00275
853	727 609	29.20616	92.35800	620 650 477	9.483 814	20.43226	44.01996
854	729 316	29.22328	92.41212	622 835 864	9.487 518	20.44024	44.03716
855	731 025	29.24038	92.46621	625 026 375	9.491 220	20.44821	44.05434
856	732 736	29.25748	92.52027	627 222 016	9.494 919	20.45618	44.07151
857	734 449	29.27456	92.57429	629 422 793	9.498 615	20.46415	44.08866
858	736 164	29.29164	92.62829	631 628 712	9.502 308	20.47210	44.10581
859	737 881	29.30870	92.68225	633 839 779	9.505 998	20.48005	44.12293
860	739 600	29.32576	92.73618	636 056 000	9.509 685	20.48800	44.14005
861	741 321	29.34280	92.79009	638 277 381	9.513 370	20.49593	44.15715
862	743 044	29.35984	92.84396	640 503 928	9.517 052	20.50387	44.17424
863	744 769	29.37686	92.89779	642 735 647	9.520 730	20.51179	44.19132
864	746 496	29.39388	92.95160	644 972 544	9.524 406	20.51971	44.20838
865	748 225	29.41088	93.00538	647 214 625	9.528 079	20.52762	44.22543
866	749 956	29.42788	93.05912	649 461 896	9.531 750	20.53553	44.24246
867	751 689	29.44486	93.11283	651 714 363	9.535 417	20.54343	44.25949
868	753 424	29.46184	93.16652	653 972 032	9.539 082	20.55133	44.27650
869	755 161	29.47881	93.22017	656 234 909	9.542 744	20.55922	44.29349
870	756 900	29.49576	93.27379	658 503 000	9.546 403	20.56710	44.31048
871	758 641	29.51271	93.32738	660 776 311	9.550 059	20.57498	44.32745
872	760 384	29.52965	93.38094	663 054 848	9.553 712	20.58285	44.34440
873	762 129	29.54657	93.43447	665 338 617	9.557 363	20.59071	44.36135
874	763 876	29.56349	93.48797	667 627 624	9.561 011	20.59857	44.37828
875	765 625	29.58040	93.54143	669 921 875	9.564 656	20.60643	44.39520
876	767 376	29.59730	93.59487	672 221 376	9.568 298	20.61427	44.41211
877	769 129	29.61419	93.64828	674 526 133	9.571 938	20.62211	44.42900
878	770 884	29.63106	93.70165	676 836 152	9.575 574	20.62995	44.44588
879	772 641	29.64793	93.75500	679 151 439	9.579 208	20.63778	44.46275
880	774 400	29.66479	93.80832	681 472 000	9.582 840	20.64560	44.47960
881	776 161	29.68164	93.86160	683 797 841	9.586 468	20.65342	44.49644
882	777 924	29.69848	93.91486	686 128 968	9.590 094	20.66123	44.51327
883	779 689	29.71532	93.96808	688 465 387	9.593 717	20.66904	44.53009
884	781 456	29.73214	94.02127	690 807 104	9.597 337	20.67684	44.54689
885	783 225	29.74895	94.07444	693 154 125	9.600 955	20.68463	44.56368
886	784 996	29.76575	94.12757	695 506 456	9.604 570	20.69242	44.58046
887	786 769	29.78255	94.18068	697 864 103	9.608 182	20.70020	44.59723
888	788 544	29.79933	94.23375	700 227 072	9.611 791	20.70798	44.61398
889	790 321	29.81610	94.28680	702 595 369	9.615 398	20.71575	44.63072
890	792 100	29.83287	94.33981	704 969 000	9.619 002	20.72351	44.64745
891	793 881	29.84962	94.39280	707 347 971	9.622 603	20.73127	44.66417
892	795 664	29.86637	94.44575	709 732 288	9.626 202	20.73902	44.68087
893	797 449	29.88311	94.49868	712 121 957	9.629 797	20.74677	44.69756
894	799 236	29.89983	94.55157	714 516 984	9.633 391	20.75451	44.71424
895	801 025	29.91655	94.60444	716 917 375	9.636 981	20.76225	44.73090
896	802 816	29.93326	94.65728	719 323 136	9.640 569	20.76998	44.74756
897	804 609	29.94996	94.71008	721 734 273	9.644 154	20.77770	44.76420
898	806 404	29.96665	94.76286	724 150 792	9.647 737	20.78542	44.78083
899	808 201	29.98333	94.81561	726 572 699	9.651 317	20.79313	44.79744
900	810 000	30.00000	94.86833	729 000 000	9.654 894	20.80084	44.81405

SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
900	810 000	30.00000	94.86833	729 000 000	9.654 894	20.80084	44.81405
901	811 801	30.01666	94.92102	731 432 701	9.658 468	20.80854	44.83064
902	813 604	30.03331	94.97368	733 870 808	9.662 040	20.81623	44.84722
903	815 409	30.04996	95.02631	736 314 327	9.665 610	20.82392	44.86379
904	817 216	30.06659	95.07891	738 763 264	9.669 176	20.83161	44.88034
905	819 025	30.08322	95.13149	741 217 625	9.672 740	20.83929	44.89688
906	820 836	30.09983	95.18403	743 677 416	9.676 302	20.84696	44.91341
907	822 649	30.11644	95.23655	746 142 643	9.679 860	20.85463	44.92993
908	824 464	30.13304	95.28903	748 613 312	9.683 417	20.86229	44.94644
909	826 281	30.14963	95.34149	751 089 429	9.686 970	20.86994	44.96293
910	828 100	30.16621	95.39392	753 571 000	9.690 521	20.87759	44.97941
911	829 921	30.18278	95.44632	756 058 031	9.694 069	20.88524	44.99588
912	831 744	30.19934	95.49869	758 550 528	9.697 615	20.89288	45.01234
913	833 569	30.21589	95.55103	761 048 497	9.701 158	20.90051	45.02879
914	835 396	30.23243	95.60335	763 551 944	9.704 699	20.90814	45.04522
915	837 225	30.24897	95.65563	766 060 875	9.708 237	20.91576	45.06164
916	839 056	30.26549	95.70789	768 575 296	9.711 772	20.92338	45.07805
917	840 889	30.28201	95.76012	771 095 213	9.715 305	20.93099	45.09445
918	842 724	30.29851	95.81232	773 620 632	9.718 835	20.93860	45.11084
919	844 561	30.31501	95.86449	776 151 559	9.722 363	20.94620	45.12721
920	846 400	30.33150	95.91663	778 688 000	9.725 888	20.95379	45.14357
921	848 241	30.34798	95.96874	781 229 961	9.729 411	20.96138	45.15992
922	850 084	30.36445	96.02083	783 777 448	9.732 931	20.96896	45.17626
923	851 929	30.38092	96.07289	786 330 467	9.736 448	20.97654	45.19259
924	853 776	30.39737	96.12492	788 889 024	9.739 963	20.98411	45.20891
925	855 625	30.41381	96.17692	791 453 125	9.743 476	20.99168	45.22521
926	857 476	30.43025	96.22889	794 022 776	9.746 986	20.99924	45.24150
927	859 329	30.44667	96.28084	796 597 983	9.750 493	21.00680	45.25778
928	861 184	30.46309	96.33276	799 178 752	9.753 998	21.01435	45.27405
929	863 041	30.47950	96.38465	801 765 089	9.757 500	21.02190	45.29030
930	864 900	30.49590	96.43651	804 357 000	9.761 000	21.02944	45.30655
931	866 761	30.51229	96.48834	806 954 491	9.764 497	21.03697	45.32278
932	868 624	30.52868	96.54015	809 557 568	9.767 992	21.04450	45.33900
933	870 489	30.54505	96.59193	812 166 237	9.771 485	21.05203	45.35521
934	872 356	30.56141	96.64368	814 780 504	9.774 974	21.05954	45.37141
935	874 225	30.57777	96.69540	817 400 375	9.778 462	21.06706	45.38760
936	876 096	30.59412	96.74709	820 025 856	9.781 946	21.07456	45.40377
937	877 969	30.61046	96.79876	822 656 953	9.785 429	21.08207	45.41994
938	879 844	30.62679	96.85040	825 293 672	9.788 909	21.08956	45.43609
939	881 721	30.64311	96.90201	827 936 019	9.792 386	21.09706	45.45223
940	883 600	30.65942	96.95360	830 584 000	9.795 861	21.10454	45.46836
941	885 481	30.67572	97.00515	833 237 621	9.799 334	21.11202	45.48448
942	887 364	30.69202	97.05668	835 896 888	9.802 804	21.11950	45.50058
943	889 249	30.70831	97.10819	838 561 807	9.806 271	21.12697	45.51668
944	891 136	30.72458	97.15966	841 232 384	9.809 736	21.13444	45.53276
945	893 025	30.74085	97.21111	843 908 625	9.813 199	21.14190	45.54883
946	894 916	30.75711	97.26253	846 590 536	9.816 659	21.14935	45.56490
947	896 809	30.77337	97.31393	849 278 123	9.820 117	21.15680	45.58095
948	898 704	30.78961	97.36529	851 971 392	9.823 572	21.16424	45.59698
949	900 601	30.80584	97.41663	854 670 349	9.827 025	21.17168	45.61301
950	902 500	30.82207	97.46794	857 375 000	9.830 476	21.17912	45.62903

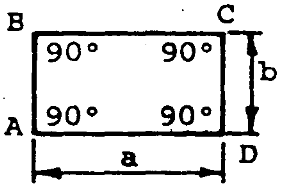
SQUARES, CUBES AND ROOTS (Continued)

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
950	902 500	30.82207	97.46794	857 375 000	9.830 476	21.17912	45.62903
951	904 401	30.83829	97.51923	860 085 351	9.833 924	21.18655	45.64503
952	906 304	30.85450	97.57049	862 801 408	9.837 369	21.19397	45.66102
953	908 209	30.87070	97.62172	865 523 177	9.840 813	21.20139	45.67701
954	910 116	30.88689	97.67292	868 250 664	9.844 254	21.20880	45.69298
955	912 025	30.90307	97.72410	870 983 875	9.847 692	21.21621	45.70894
956	913 936	30.91925	97.77525	873 722 816	9.851 128	21.22361	45.72489
957	915 849	30.93542	97.82638	876 467 493	9.854 562	21.23101	45.74082
958	917 764	30.95158	97.87747	879 217 912	9.857 993	21.23840	45.75675
959	919 681	30.96773	97.92855	881 974 079	9.861 422	21.24579	45.77267
960	921 600	30.98387	97.97959	884 736 000	9.864 848	21.25317	45.78857
961	923 521	31.00000	98.03061	887 503 681	9.868 272	21.26055	45.80446
962	925 444	31.01612	98.08160	890 277 128	9.871 694	21.26792	45.82035
963	927 369	31.03224	98.13256	893 056 347	9.875 113	21.27529	45.83622
964	929 296	31.04835	98.18350	895 841 344	9.878 530	21.28265	45.85208
965	931 225	31.06445	98.23441	898 632 125	9.881 945	21.29001	45.86793
966	933 156	31.08054	98.28530	901 428 696	9.885 357	21.29736	45.88376
967	935 089	31.09662	98.33616	904 231 063	9.888 767	21.30470	45.89959
968	937 024	31.11270	98.38699	907 039 232	9.892 175	21.31204	45.91541
969	938 961	31.12876	98.43780	909 853 209	9.895 580	21.31938	45.93121
970	940 900	31.14482	98.48858	912 673 000	9.898 983	21.32671	45.94701
971	942 841	31.16087	98.53933	915 498 611	9.902 384	21.33404	45.96279
972	944 784	31.17691	98.59006	918 330 048	9.905 782	21.34136	45.97857
973	946 729	31.19295	98.64076	921 167 317	9.909 178	21.34868	45.99433
974	948 676	31.20897	98.69144	924 010 424	9.912 571	21.35599	46.01008
975	950 625	31.22499	98.74209	926 859 375	9.915 962	21.36329	46.02582
976	952 576	31.24100	98.79271	929 714 176	9.919 351	21.37059	46.04155
977	954 529	31.25700	98.84331	932 574 833	9.922 738	21.37789	46.05727
978	956 484	31.27299	98.89388	935 441 352	9.926 122	21.38518	46.07298
979	958 441	31.28898	98.94443	938 313 739	9.929 504	21.39247	46.08868
980	960 400	31.30495	98.99495	941 192 000	9.932 884	21.39975	46.10436
981	962 361	31.32092	99.04544	944 076 141	9.936 261	21.40703	46.12004
982	964 324	31.33688	99.09591	946 966 168	9.939 636	21.41430	46.13571
983	966 289	31.35283	99.14636	949 862 087	9.943 009	21.42156	46.15136
984	968 256	31.36877	99.19677	952 763 904	9.946 380	21.42883	46.16700
985	970 225	31.38471	99.24717	955 671 625	9.949 748	21.43608	46.18264
986	972 196	31.40064	99.29753	958 585 256	9.953 114	21.44333	46.19826
987	974 169	31.41656	99.34787	961 504 803	9.956 478	21.45058	46.21387
988	976 144	31.43247	99.39819	964 430 272	9.959 839	21.45782	46.22948
989	978 121	31.44837	99.44848	967 361 669	9.963 198	21.46506	46.24507
990	980 100	31.46427	99.49874	970 299 000	9.966 555	21.47229	46.26065
991	982 081	31.48015	99.54898	973 242 271	9.969 910	21.47952	46.27622
992	984 064	31.49603	99.59920	976 191 488	9.973 262	21.48674	46.29178
993	986 049	31.51190	99.64939	979 146 657	9.976 612	21.49396	46.30733
994	988 036	31.52777	99.69955	982 107 784	9.979 960	21.50117	46.32287
995	990 025	31.54362	99.74969	985 074 875	9.983 305	21.50838	46.33840
996	992 016	31.55947	99.79980	988 047 936	9.986 649	21.51558	46.35392
997	994 009	31.57531	99.84989	991 026 973	9.989 990	21.52278	46.36943
998	996 004	31.59114	99.89995	994 011 992	9.993 329	21.52997	46.38492
999	998 001	31.60696	99.94999	997 002 999	9.996 666	21.53716	46.40041
1000	1 000 000	31.62278	100.00000	1 000 000 000	10.000 000	21.54435	46.41589

APPENDIX D

AREAS AND VOLUMES

APPENDIX D
AREAS OF PLANE FIGURES



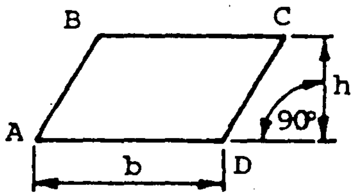
RECTANGLE

Area = (a)(b)

Side AB is parallel and equal to Side CD

Side AD is parallel and equal to Side BC

Sum of interior angles = 360°



PARALLELOGRAM

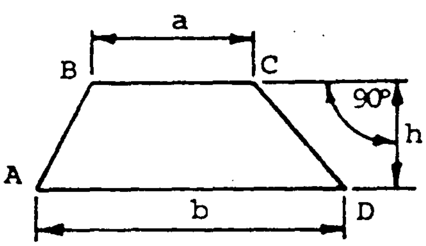
Area = (b)(h)

Side AB is parallel and equal to Side CD

Side AD is parallel and equal to Side BC

The height, h, is perpendicular to Side AD and Side BC

Sum of interior angles = 360°



TRAPEZOID

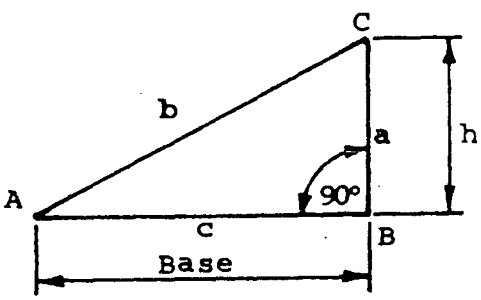
Area = $\frac{1}{2}$(h)(a+b)

Side AD is parallel to Side BC.

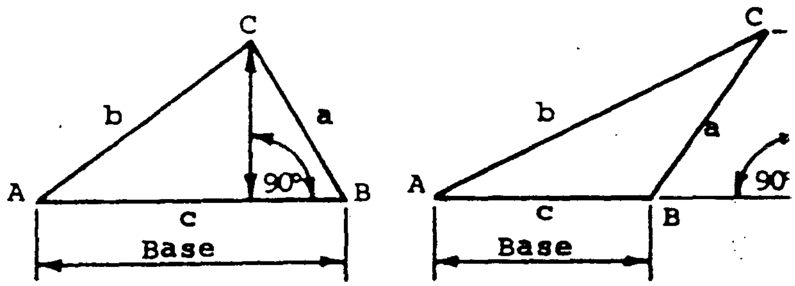
Side AB is <u>not</u> parallel to Side CD

The height, h, is perpendicular to Side AD and Side BC

Sum of interior angles = 360°



RIGHT TRIANGLE



OBLIQUE TRIANGLES

AREA OF ANY TRIANGLE: Area = $\frac{1}{2}$(Base)(Height) The height, h, is perpendicular to the base

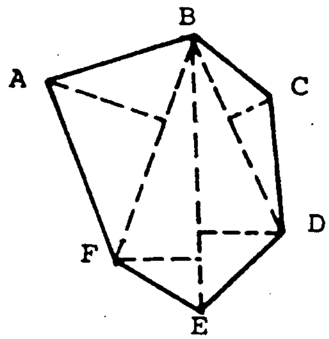
also,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{when } s = \frac{a+b+c}{2}$$

Sum of interior angles = 180°

APPENDIX D
AREAS OF PLANE FIGURES



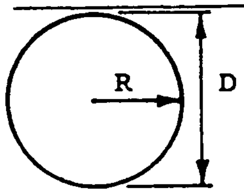
POLYGON

Divide the polygon into separate triangles and calculate the areas of the individual triangles. In this case, triangles ABF, BEF, BDE and BCD. Sum the triangle areas to obtain the total area of the polygon.

The sum of the interior angles of any polygon is determined by the following formula:

$$(N-2)(180°)$$

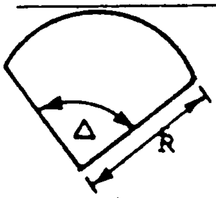
Where N = the number of sides in the polygon. In this case, N = 6. Therefore, the interior angles should total (6-2)(180°) = (4)(180° = 720°



CIRCLE

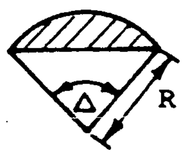
$$\text{Area} = \pi R^2 \qquad \pi = 3.1416$$

$$\text{Circumference} = 2\pi R = \pi D$$



SECTOR

$$\text{Area} = \frac{\Delta}{360} R^2\pi, \quad \Delta \text{ in degrees}, \quad \pi = 3.1416$$

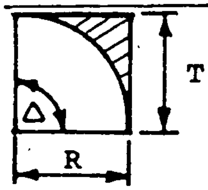


SEGMENT

Crosshatched

$$\text{Area} = \left[\frac{\Delta}{360}\pi R^2\right] - \left[\frac{R^2 \sin \Delta}{2}\right]$$

$\pi = 3.1416$, Δ in degrees



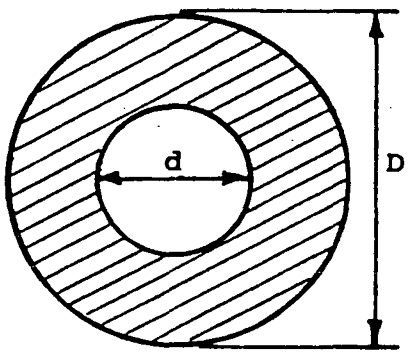
FILLET

Crosshatched

$$\text{Area} = [(R)(T)] - \left[\frac{\Delta}{360}\pi R^2\right]$$

Δ in degrees , $\pi = 3.1416$

APPENDIX D
AREAS OF PLANE FIGURES



Crosshatched

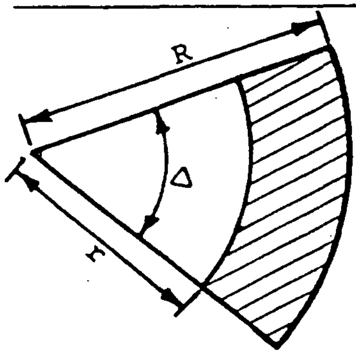
$$\text{Area} = \frac{\pi}{4}(D^2 - d^2)$$

$$\text{also} = \frac{\pi}{4}(D + d)(D - d)$$

$\pi = 3.1416$

D & d must be in the same units

CIRCULAR RING



Crosshatched

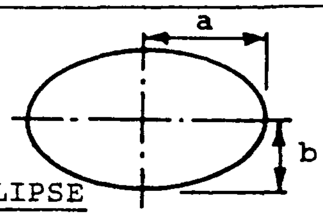
$$\text{Area} = \frac{\Delta}{360}(\pi)(R^2 - r^2)$$

Δ in degrees

$\pi = 3.1416$

R & r in same units

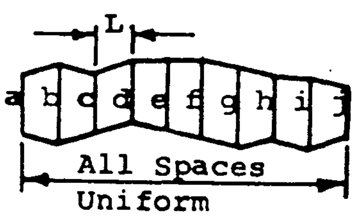
PORTION OF CIRCULAR RING



$$\text{Area} = \pi(a)(b)$$

$\pi = 3.1416$

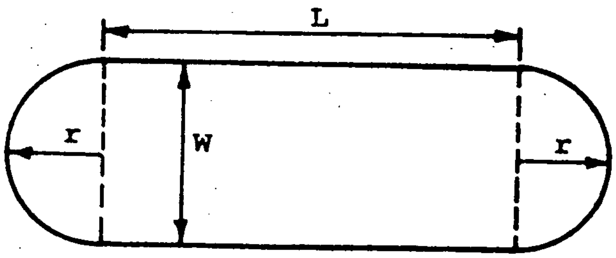
ELLIPSE



$$\text{Area} = \left(\frac{a+j}{2}+b+c+d+e+f+g+h+i\right)(L)$$

IRREGULAR FIGURE

COMBINATION AREAS

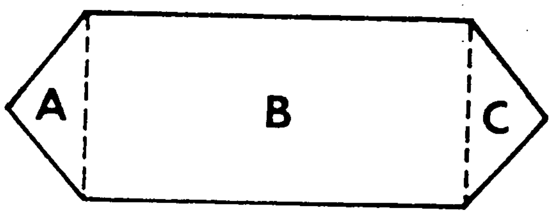


When one of the previous area formulii does not fit the situation, break the area into sections where the previous formulii can be used.

The cross sectional area of a bridge pier stem is an example.

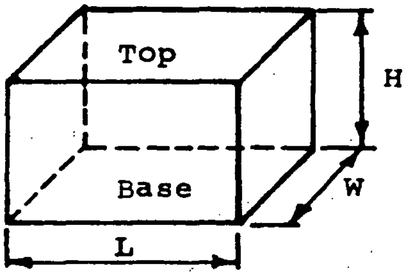
$$\text{Area} = (L)(W) + \pi r^2$$

$\pi = 3.1416$
L, W & r in same units



$$\text{Area} = \text{Area A} + \text{Area B} + \text{Area C}$$

APPENDIX D
VOLUMES OF SOLIDS

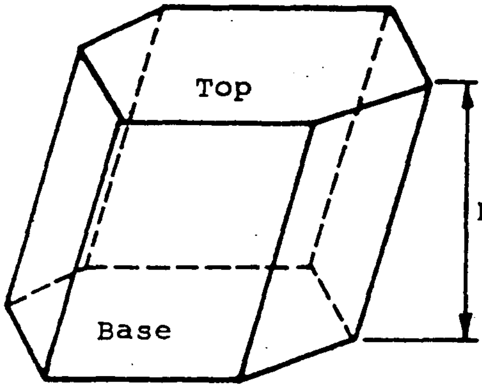


Volume = (area of base) (height)
= (L) (W) (H)

The top is parallel to the base and sides are perpendicular to the base and top.

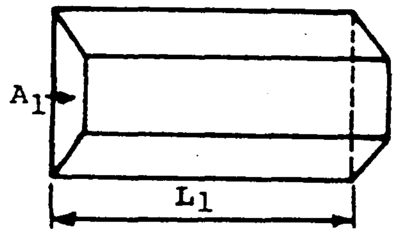
RECTANGULAR PRISM

Volume = (area of base) (perpendicular height)

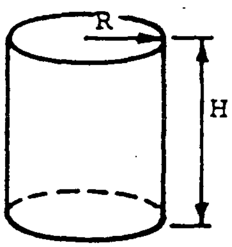


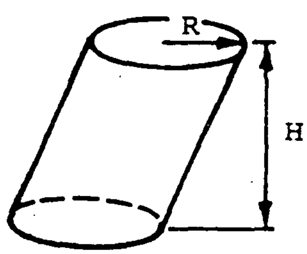
The top is parallel to the base and both have identical shapes and areas. The height is perpendicular to both the top and base, not measured along the side of the prism.

also,
$V = (A_1)(L_1)$



ANY PRISM



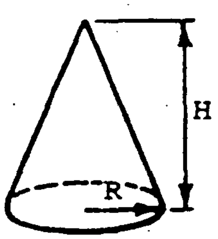


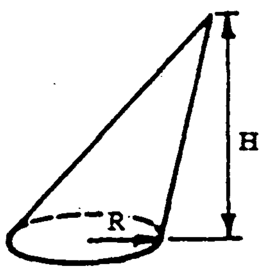
For both,
Volume = $(\pi R^2)(H)$

Where H is the perpendicular height. The top and base are parallel and have the same radius.

$\pi = 3.1416$

RIGHT CYLINDER   OBLIQUE CYLINDER





For both,
Volume = $\frac{1}{3}$(base area) (height)

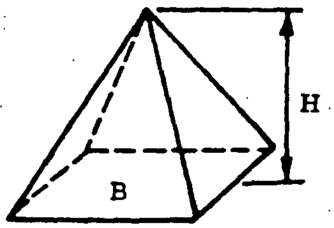
$V = \frac{1}{3}(\pi R^2)(H)$

Where H is the perpendicular height from the base.

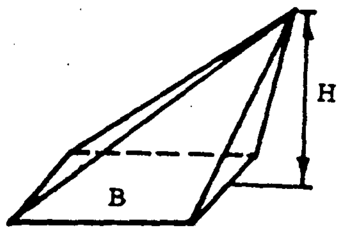
$\pi = 3.1416$

RIGHT CONE   OBLIQUE CONE

VOLUMES OF SOLIDS



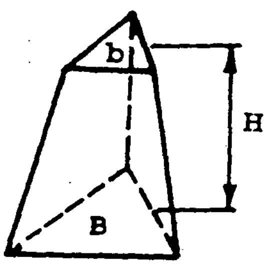
RIGHT PYRAMID



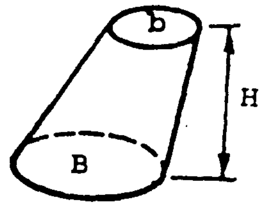
OBLIQUE PYRAMID

For both,
Volume = $\frac{1}{3}$(base area) (height)

Where H is the perpendicular height from the base



FRUSTRUM OF PYRAMID

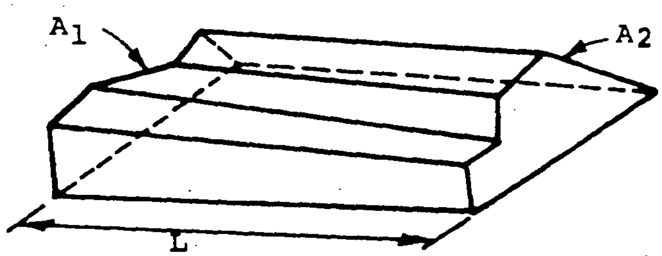


FRUSTRUM OF CONE

Frustrum of any pyramid or cone, with base and top parallel to each other

$$\text{Volume} = \frac{1}{3}(H)(B + b + \sqrt{Bb})$$

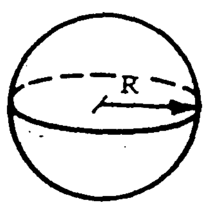
Where H is the perpendicular height between the base and the top. B is the area of the base and b is the area of the top.



EARTHWORK VOLUME BY AVERAGE END AREA METHOD

$$\text{Volume} = \left(\frac{A_1 + A_2}{2}\right)(L)$$

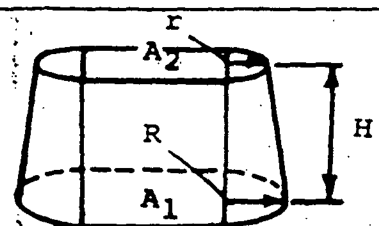
A_1 & A_2 areas are usually determined by the cross section method and L is the distance between cross section locations. This formula is only approximate when A_1 & A_2 are not equal, but is sufficiently precise for ordinary earthwork.



SPHERE

Volume = $\frac{4}{3}\pi R^3$

π= 3.1416



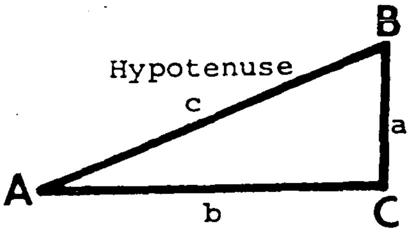
$$\text{Volume} = \left(\frac{A_1+A_2}{2}\right)(H) + \left(\frac{H}{3}\right)(\pi r^2+\pi R^2+\sqrt{\pi^2 r^2 R^2})$$

Where H is the perpendicular height between the base and the top.

APPENDIX E

TRIGONOMETRY

APPENDIX E
TRIGONOMETRIC RELATIONSHIPS



RIGHT TRIANGLE

PYTHAGOREAN THEOREM:
In a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. This theorem is expressed by the following formula:

$$c^2 = a^2 + b^2$$

Trigonometric Function	Abbreviation	Definition	Angle A Terms	Angle B Terms
sine	sin	$\frac{\text{opposite side}}{\text{hypotenuse}}$	$\frac{a}{c}$	$\frac{b}{c}$
cosine	cos	$\frac{\text{adjacent side}}{\text{hypotenuse}}$	$\frac{b}{c}$	$\frac{a}{c}$
tangent	tan	$\frac{\text{opposite side}}{\text{adjacent side}}$	$\frac{a}{b}$	$\frac{b}{a}$
cotangent	cot	$\frac{\text{adjacent side}}{\text{opposite side}}$	$\frac{b}{a}$	$\frac{a}{b}$
secant	sec	$\frac{\text{hypotenuse}}{\text{adjacent side}}$	$\frac{c}{b}$	$\frac{c}{a}$
cosecant	csc	$\frac{\text{hypotenuse}}{\text{opposite side}}$	$\frac{c}{a}$	$\frac{c}{b}$

In any triangle the SINE LAW and COSINE LAW relationships apply. For angles between 90° and 180° the cosine function is negative. Be sure to watch the algebraic signs when using the cosine law.

SINE LAW: $$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

COSINE LAW: $$a^2 = b^2 + c^2 - 2bc\text{Cos A}$$

$$b^2 = a^2 + c^2 - 2ac\text{Cos B}$$

$$c^2 = a^2 + b^2 - 2ab\text{Cos C}$$

Algebraic signs & value ranges for trigonometric functions for angles between 0° and 360° are given in the adjacent table.

Quadrant	I	II	III	IV	Angle		
Angles Measured	0 to 90	90 to 180	180 to 270	270 to 360	30°	45°	60°
Functions	Values vary from				Equivalent values		
sin	+0 to +1	+1 to +0	−0 to −1	−1 to −0	½	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$
cos	+1 to +0	−0 to −1	−1 to −0	+0 to +1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	½
tan	+0 to +∞	−∞ to −0	+0 to +∞	−∞ to −0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$
cot	+∞ to +0	−0 to −∞	+∞ to +0	−0 to −∞	$\sqrt{3}$	1	$\frac{1}{3}\sqrt{3}$

When the trig. function for an angle over 90° is needed the adjacent chart may be used.
Example: The cosine function for 107° is needed & your tables only go to 90°. Use, the 90° + θ column & look up sine function for 17°. The algebraic sign is minus (-).

Function \ Angle	90°−θ	90°+θ	180°−θ	180°+θ	270°−θ	270°+θ	360°−θ	360°+θ
Sine	cos θ	cos θ	sin θ	−sin θ	−cos θ	−cos θ	−sin θ	sin θ
Cosine	sin θ	−sin θ	−cos θ	−cos θ	−sin θ	sin θ	cos θ	cos θ
Tangent	cot θ	−cot θ	−tan θ	tan θ	cot θ	−cot θ	−tan θ	tan θ

NATURAL TRIGONOMETRIC FUNCTIONS

0° (180°) (359°) 179°

′	Sin	Tan	Cot	Cos	′
0	.00000	.00000		1.0000	60
1	.00029	.00029	3437.7	1.0000	59
2	.00058	.00058	1718.9	1.0000	58
3	.00087	.00087	1145.9	1.0000	57
4	.00116	.00116	859.44	1.0000	56
5	.00145	.00145	687.55	1.0000	55
6	.00175	.00175	572.96	1.0000	54
7	.00204	.00204	491.11	1.0000	53
8	.00233	.00233	429.72	1.0000	52
9	.00262	.00262	381.97	1.0000	51
10	.00291	.00291	343.77	1.0000	50
11	.00320	.00320	312.52	.99999	49
12	.00349	.00349	286.48	.99999	48
13	.00378	.00378	264.44	.99999	47
14	.00407	.00407	245.55	.99999	46
15	.00436	.00436	229.18	.99999	45
16	.00465	.00465	214.86	.99999	44
17	.00495	.00495	202.22	.99999	43
18	.00524	.00524	190.98	.99999	42
19	.00553	.00553	180.93	.99998	41
20	.00582	.00582	171.89	.99998	40
21	.00611	.00611	163.70	.99998	39
22	.00640	.00640	156.26	.99998	38
23	.00669	.00669	149.47	.99998	37
24	.00698	.00698	143.24	.99998	36
25	.00727	.00727	137.51	.99997	35
26	.00756	.00756	132.22	.99997	34
27	.00785	.00785	127.32	.99997	33
28	.00814	.00815	122.77	.99997	32
29	.00844	.00844	118.54	.99996	31
30	.00873	.00873	114.59	.99996	30
31	.00902	.00902	110.89	.99996	29
32	.00931	.00931	107.43	.99996	28
33	.00960	.00960	104.17	.99995	27
34	.00989	.00989	101.11	.99995	26
35	.01018	.01018	98.218	.99995	25
36	.01047	.01047	95.489	.99995	24
37	.01076	.01076	92.908	.99994	23
38	.01105	.01105	90.463	.99994	22
39	.01134	.01135	88.144	.99994	21
40	.01164	.01164	85.940	.99993	20
41	.01193	.01193	83.844	.99993	19
42	.01222	.01222	81.847	.99993	18
43	.01251	.01251	79.943	.99992	17
44	.01280	.01280	78.126	.99992	16
45	.01309	.01309	76.390	.99991	15
46	.01338	.01338	74.729	.99991	14
47	.01367	.01367	73.139	.99991	13
48	.01396	.01396	71.615	.99990	12
49	.01425	.01425	70.153	.99990	11
50	.01454	.01455	68.750	.99989	10
51	.01483	.01484	67.402	.99989	9
52	.01513	.01513	66.105	.99989	8
53	.01542	.01542	64.858	.99988	7
54	.01571	.01571	63.657	.99988	6
55	.01600	.01600	62.499	.99987	5
56	.01629	.01629	61.383	.99987	4
57	.01658	.01658	60.306	.99986	3
58	.01687	.01687	59.266	.99986	2
59	.01716	.01716	58.261	.99985	1
60	.01745	.01746	57.290	.99985	0
′	Cos	Cot	Tan	Sin	′

90° (270°) (269°) 89°

1° (181°) (358°) 178°

′	Sin	Tan	Cot	Cos	′
0	.01745	.01746	57.290	.99985	60
1	.01774	.01775	56.351	.99984	59
2	.01803	.01804	55.442	.99984	58
3	.01832	.01833	54.561	.99983	57
4	.01862	.01862	53.709	.99983	56
5	.01891	.01891	52.882	.99982	55
6	.01920	.01920	52.081	.99982	54
7	.01949	.01949	51.303	.99981	53
8	.01978	.01978	50.549	.99980	52
9	.02007	.02007	49.816	.99980	51
10	.02036	.02036	49.104	.99979	50
11	.02065	.02066	48.412	.99979	49
12	.02094	.02095	47.740	.99978	48
13	.02123	.02124	47.085	.99977	47
14	.02152	.02153	46.449	.99977	46
15	.02181	.02182	45.829	.99976	45
16	.02211	.02211	45.226	.99976	44
17	.02240	.02240	44.639	.99975	43
18	.02269	.02269	44.066	.99974	42
19	.02298	.02298	43.508	.99974	41
20	.02327	.02328	42.964	.99973	40
21	.02356	.02357	42.433	.99972	39
22	.02385	.02386	41.916	.99972	38
23	.02414	.02415	41.411	.99971	37
24	.02443	.02444	40.917	.99970	36
25	.02472	.02473	40.436	.99969	35
26	.02501	.02502	39.965	.99969	34
27	.02530	.02531	39.506	.99968	33
28	.02560	.02560	39.057	.99967	32
29	.02589	.02589	38.618	.99966	31
30	.02618	.02619	38.188	.99966	30
31	.02647	.02648	37.769	.99965	29
32	.02676	.02677	37.358	.99964	28
33	.02705	.02706	36.956	.99963	27
34	.02734	.02735	36.563	.99963	26
35	.02763	.02764	36.178	.99962	25
36	.02792	.02793	35.801	.99961	24
37	.02821	.02822	35.431	.99960	23
38	.02850	.02851	35.070	.99959	22
39	.02879	.02881	34.715	.99959	21
40	.02908	.02910	34.368	.99958	20
41	.02938	.02939	34.027	.99957	19
42	.02967	.02968	33.694	.99956	18
43	.02996	.02997	33.366	.99955	17
44	.03025	.03026	33.045	.99954	16
45	.03054	.03055	32.730	.99953	15
46	.03083	.03084	32.421	.99952	14
47	.03112	.03114	32.118	.99952	13
48	.03141	.03143	31.821	.99951	12
49	.03170	.03172	31.528	.99950	11
50	.03199	.03201	31.242	.99949	10
51	.03228	.03230	30.960	.99948	9
52	.03257	.03259	30.683	.99947	8
53	.03286	.03288	30.412	.99946	7
54	.03316	.03317	30.145	.99945	6
55	.03345	.03346	29.882	.99944	5
56	.03374	.03376	29.624	.99943	4
57	.03403	.03405	29.371	.99942	3
58	.03432	.03434	29.122	.99941	2
59	.03461	.03463	28.877	.99940	1
60	.03490	.03492	28.636	.99939	0
′	Cos	Cot	Tan	Sin	′

91° (271°) (268°) 88°

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

2° (182°) (357°) 177°

′	Sin	Tan	Cot	Cos	′
0	.03490	.03492	28.636	.99939	60
1	.03519	.03521	28.399	.99938	59
2	.03548	.03550	28.166	.99937	58
3	.03577	.03579	27.937	.99936	57
4	.03606	.03609	27.712	.99935	56
5	.03635	.03638	27.490	.99934	55
6	.03664	.03667	27.271	.99933	54
7	.03693	.03696	27.057	.99932	53
8	.03723	.03725	26.845	.99931	52
9	.03752	.03754	26.637	.99930	51
10	.03781	.03783	26.432	.99929	50
11	.03810	.03812	26.230	.99927	49
12	.03839	.03842	26.031	.99926	48
13	.03868	.03871	25.835	.99925	47
14	.03897	.03900	25.642	.99924	46
15	.03926	.03929	25.452	.99923	45
16	.03955	.03958	25.264	.99922	44
17	.03984	.03987	25.080	.99921	43
18	.04013	.04016	24.898	.99919	42
19	.04042	.04046	24.719	.99918	41
20	.04071	.04075	24.542	.99917	40
21	.04100	.04104	24.368	.99916	39
22	.04129	.04133	24.196	.99915	38
23	.04159	.04162	24.026	.99913	37
24	.04188	.04191	23.859	.99912	36
25	.04217	.04220	23.695	.99911	35
26	.04246	.04250	23.532	.99910	34
27	.04275	.04279	23.372	.99909	33
28	.04304	.04308	23.214	.99907	32
29	.04333	.04337	23.058	.99906	31
30	.04362	.04366	22.904	.99905	30
31	.04391	.04395	22.752	.99904	29
32	.04420	.04424	22.602	.99902	28
33	.04449	.04454	22.454	.99901	27
34	.04478	.04483	22.308	.99900	26
35	.04507	.04512	22.164	.99898	25
36	.04536	.04541	22.022	.99897	24
37	.04565	.04570	21.881	.99896	23
38	.04594	.04599	21.743	.99894	22
39	.04623	.04628	21.606	.99893	21
40	.04653	.04658	21.470	.99892	20
41	.04682	.04687	21.337	.99890	19
42	.04711	.04716	21.205	.99889	18
43	.04740	.04745	21.075	.99888	17
44	.04769	.04774	20.946	.99886	16
45	.04798	.04803	20.819	.99885	15
46	.04827	.04833	20.693	.99883	14
47	.04856	.04862	20.569	.99882	13
48	.04885	.04891	20.446	.99881	12
49	.04914	.04920	20.325	.99879	11
50	.04943	.04949	20.206	.99878	10
51	.04972	.04978	20.087	.99876	9
52	.05001	.05007	19.970	.99875	8
53	.05030	.05037	19.855	.99873	7
54	.05059	.05066	19.740	.99872	6
55	.05088	.05095	19.627	.99870	5
56	.05117	.05124	19.516	.99869	4
57	.05146	.05153	19.405	.99867	3
58	.05175	.05182	19.296	.99866	2
59	.05205	.05212	19.188	.99864	1
60	.05234	.05241	19.081	.99863	0
′	Cos	Cot	Tan	Sin	′

92° (272°) (267°) 87°

3° (183°) (356°) 176°

′	Sin	Tan	Cot	Cos	′
0	.05234	.05241	19.081	.99863	60
1	.05263	.05270	18.976	.99861	59
2	.05292	.05299	18.871	.99860	58
3	.05321	.05328	18.768	.99858	57
4	.05350	.05357	18.666	.99857	56
5	.05379	.05387	18.564	.99855	55
6	.05408	.05416	18.464	.99854	54
7	.05437	.05445	18.366	.99852	53
8	.05466	.05474	18.268	.99851	52
9	.05495	.05503	18.171	.99849	51
10	.05524	.05533	18.075	.99847	50
11	.05553	.05562	17.980	.99846	49
12	.05582	.05591	17.886	.99844	48
13	.05611	.05620	17.793	.99842	47
14	.05640	.05649	17.702	.99841	46
15	.05669	.05678	17.611	.99839	45
16	.05698	.05708	17.521	.99838	44
17	.05727	.05737	17.431	.99836	43
18	.05756	.05766	17.343	.99834	42
19	.05785	.05795	17.256	.99833	41
20	.05814	.05824	17.169	.99831	40
21	.05844	.05854	17.084	.99829	39
22	.05873	.05883	16.999	.99827	38
23	.05902	.05912	16.915	.99826	37
24	.05931	.05941	16.832	.99824	36
25	.05960	.05970	16.750	.99822	35
26	.05989	.05999	16.668	.99821	34
27	.06018	.06029	16.587	.99819	33
28	.06047	.06058	16.507	.99817	32
29	.06076	.06087	16.428	.99815	31
30	.06105	.06116	16.350	.99813	30
31	.06134	.06145	16.272	.99812	29
32	.06163	.06175	16.195	.99810	28
33	.06192	.06204	16.119	.99808	27
34	.06221	.06233	16.043	.99806	26
35	.06250	.06262	15.969	.99804	25
36	.06279	.06291	15.895	.99803	24
37	.06308	.06321	15.821	.99801	23
38	.06337	.06350	15.748	.99799	22
39	.06366	.06379	15.676	.99797	21
40	.06395	.06408	15.605	.99795	20
41	.06424	.06438	15.534	.99793	19
42	.06453	.06467	15.464	.99792	18
43	.06482	.06496	15.394	.99790	17
44	.06511	.06525	15.325	.99788	16
45	.06540	.06554	15.257	.99786	15
46	.06569	.06584	15.189	.99784	14
47	.06598	.06613	15.122	.99782	13
48	.06627	.06642	15.056	.99780	12
49	.06656	.06671	14.990	.99778	11
50	.06685	.06700	14.924	.99776	10
51	.06714	.06730	14.860	.99774	9
52	.06743	.06759	14.795	.99772	8
53	.06773	.06788	14.732	.99770	7
54	.06802	.06817	14.669	.99768	6
55	.06831	.06847	14.606	.99766	5
56	.06860	.06876	14.544	.99764	4
57	.06889	.06905	14.482	.99762	3
58	.06918	.06934	14.421	.99760	2
59	.06947	.06963	14.361	.99758	1
60	.06976	.06993	14.301	.99756	0
′	Cos	Cot	Tan	Sin	′

93° (273°) (266°) 86°

4° (184°) (355°) **175°**

′	Sin	Tan	Cot	Cos	′
0	.06976	.06993	14.301	.99756	60
1	.07005	.07022	14.241	.99754	59
2	.07034	.07051	14.182	.99752	58
3	.07063	.07080	14.124	.99750	57
4	.07092	.07110	14.065	.99748	56
5	.07121	.07139	14.008	.99746	55
6	.07150	.07168	13.951	.99744	54
7	.07179	.07197	13.894	.99742	53
8	.07208	.07227	13.838	.99740	52
9	.07237	.07256	13.782	.99738	51
10	.07266	.07285	13.727	.99736	50
11	.07295	.07314	13.672	.99734	49
12	.07324	.07344	13.617	.99731	48
13	.07353	.07373	13.563	.99729	47
14	.07382	.07402	13.510	.99727	46
15	.07411	.07431	13.457	.99725	45
16	.07440	.07461	13.404	.99723	44
17	.07469	.07490	13.352	.99721	43
18	.07498	.07519	13.300	.99719	42
19	.07527	.07548	13.248	.99716	41
20	.07556	.07578	13.197	.99714	40
21	.07585	.07607	13.146	.99712	39
22	.07614	.07636	13.096	.99710	38
23	.07643	.07665	13.046	.99708	37
24	.07672	.07695	12.996	.99705	36
25	.07701	.07724	12.947	.99703	35
26	.07730	.07753	12.898	.99701	34
27	.07759	.07782	12.850	.99699	33
28	.07788	.07812	12.801	.99696	32
29	.07817	.07841	12.754	.99694	31
30	.07846	.07870	12.706	.99692	30
31	.07875	.07899	12.659	.99689	29
32	.07904	.07929	12.612	.99687	28
33	.07933	.07958	12.566	.99685	27
34	.07962	.07987	12.520	.99683	26
35	.07991	.08017	12.474	.99680	25
36	.08020	.08046	12.429	.99678	24
37	.08049	.08075	12.384	.99676	23
38	.08078	.08104	12.339	.99673	22
39	.08107	.08134	12.295	.99671	21
40	.08136	.08163	12.251	.99668	20
41	.08165	.08192	12.207	.99666	19
42	.08194	.08221	12.163	.99664	18
43	.08223	.08251	12.120	.99661	17
44	.08252	.08280	12.077	.99659	16
45	.08281	.08309	12.035	.99657	15
46	.08310	.08339	11.992	.99654	14
47	.08339	.08368	11.950	.99652	13
48	.08368	.08397	11.909	.99649	12
49	.08397	.08427	11.867	.99647	11
50	.08426	.08456	11.826	.99644	10
51	.08455	.08485	11.785	.99642	9
52	.08484	.08514	11.745	.99639	8
53	.08513	.08544	11.705	.99637	7
54	.08542	.08573	11.664	.99635	6
55	.08571	.08602	11.625	.99632	5
56	.08600	.08632	11.585	.99630	4
57	.08629	.08661	11.546	.99627	3
58	.08658	.08690	11.507	.99625	2
59	.08687	.08720	11.468	.99622	1
60	.08716	.08749	11.430	.99619	0
′	Cos	Cot	Tan	Sin	′

94° (274°) (265°) **85°**

5° (185°) (354°) **174°**

′	Sin	Tan	Cot	Cos	′
0	.08716	.08749	11.430	.99619	60
1	.08745	.08778	11.392	.99617	59
2	.08774	.08807	11.354	.99614	58
3	.08803	.08837	11.316	.99612	57
4	.08831	.08866	11.279	.99609	56
5	.08860	.08895	11.242	.99607	55
6	.08889	.08925	11.205	.99604	54
7	.08918	.08954	11.168	.99602	53
8	.08947	.08983	11.132	.99599	52
9	.08976	.09013	11.095	.99596	51
10	.09005	.09042	11.059	.99594	50
11	.09034	.09071	11.024	.99591	49
12	.09063	.09101	10.988	.99588	48
13	.09092	.09130	10.953	.99586	47
14	.09121	.09159	10.918	.99583	46
15	.09150	.09189	10.883	.99580	45
16	.09179	.09218	10.848	.99578	44
17	.09208	.09247	10.814	.99575	43
18	.09237	.09277	10.780	.99572	42
19	.09266	.09306	10.746	.99570	41
20	.09295	.09335	10.712	.99567	40
21	.09324	.09365	10.678	.99564	39
22	.09353	.09394	10.645	.99562	38
23	.09382	.09423	10.612	.99559	37
24	.09411	.09453	10.579	.99556	36
25	.09440	.09482	10.546	.99553	35
26	.09469	.09511	10.514	.99551	34
27	.09498	.09541	10.481	.99548	33
28	.09527	.09570	10.449	.99545	32
29	.09556	.09600	10.417	.99542	31
30	.09585	.09629	10.385	.99540	30
31	.09614	.09658	10.354	.99537	29
32	.09642	.09688	10.322	.99534	28
33	.09671	.09717	10.291	.99531	27
34	.09700	.09746	10.260	.99528	26
35	.09729	.09776	10.229	.99526	25
36	.09758	.09805	10.199	.99523	24
37	.09787	.09834	10.168	.99520	23
38	.09816	.09864	10.138	.99517	22
39	.09845	.09893	10.108	.99514	21
40	.09874	.09923	10.078	.99511	20
41	.09903	.09952	10.048	.99508	19
42	.09932	.09981	10.019	.99506	18
43	.09961	.10011	9.9893	.99503	17
44	.09990	.10040	9.9601	.99500	16
45	.10019	.10069	9.9310	.99497	15
46	.10048	.10099	9.9021	.99494	14
47	.10077	.10128	9.8734	.99491	13
48	.10106	.10158	9.8448	.99488	12
49	.10135	.10187	9.8164	.99485	11
50	.10164	.10216	9.7882	.99482	10
51	.10192	.10246	9.7601	.99479	9
52	.10221	.10275	9.7322	.99476	8
53	.10250	.10305	9.7044	.99473	7
54	.10279	.10334	9.6768	.99470	6
55	.10308	.10363	9.6493	.99467	5
56	.10337	.10393	9.6220	.99464	4
57	.10366	.10422	9.5949	.99461	3
58	.10395	.10452	9.5679	.99458	2
59	.10424	.10481	9.5411	.99455	1
60	.10453	.10510	9.5144	.99452	0
′	Cos	Cot	Tan	Sin	′

95° (275°) (264°) **84°**

6° (186°) (353°) **173°**

′	Sin	Tan	Cot	Cos	′
0	.10453	.10510	9.5144	.99452	60
1	.10482	.10540	9.4878	.99449	59
2	.10511	.10569	9.4614	.99446	58
3	.10540	.10599	9.4352	.99443	57
4	.10569	.10628	9.4090	.99440	56
5	.10597	.10657	9.3831	.99437	55
6	.10626	.10687	9.3572	.99434	54
7	.10655	.10716	9.3315	.99431	53
8	.10684	.10746	9.3060	.99428	52
9	.10713	.10775	9.2806	.99424	51
10	.10742	.10805	9.2553	.99421	50
11	.10771	.10834	9.2302	.99418	49
12	.10800	.10863	9.2052	.99415	48
13	.10829	.10893	9.1803	.99412	47
14	.10858	.10922	9.1555	.99409	46
15	.10887	.10952	9.1309	.99406	45
16	.10916	.10981	9.1065	.99402	44
17	.10945	.11011	9.0821	.99399	43
18	.10973	.11040	9.0579	.99396	42
19	.11002	.11070	9.0338	.99393	41
20	.11031	.11099	9.0098	.99390	40
21	.11060	.11128	8.9860	.99386	39
22	.11089	.11158	8.9623	.99383	38
23	.11118	.11187	8.9387	.99380	37
24	.11147	.11217	8.9152	.99377	36
25	.11176	.11246	8.8919	.99374	35
26	.11205	.11276	8.8686	.99370	34
27	.11234	.11305	8.8455	.99367	33
28	.11263	.11335	8.8225	.99364	32
29	.11291	.11364	8.7996	.99360	31
30	.11320	.11394	8.7769	.99357	30
31	.11349	.11423	8.7542	.99354	29
32	.11378	.11452	8.7317	.99351	28
33	.11407	.11482	8.7093	.99347	27
34	.11436	.11511	8.6870	.99344	26
35	.11465	.11541	8.6648	.99341	25
36	.11494	.11570	8.6427	.99337	24
37	.11523	.11600	8.6208	.99334	23
38	.11552	.11629	8.5989	.99331	22
39	.11580	.11659	8.5772	.99327	21
40	.11609	.11688	8.5555	.99324	20
41	.11638	.11718	8.5340	.99320	19
42	.11667	.11747	8.5126	.99317	18
43	.11696	.11777	8.4913	.99314	17
44	.11725	.11806	8.4701	.99310	16
45	.11754	.11836	8.4490	.99307	15
46	.11783	.11865	8.4280	.99303	14
47	.11812	.11895	8.4071	.99300	13
48	.11840	.11924	8.3863	.99297	12
49	.11869	.11954	8.3656	.99293	11
50	.11898	.11983	8.3450	.99290	10
51	.11927	.12013	8.3245	.99286	9
52	.11956	.12042	8.3041	.99283	8
53	.11985	.12072	8.2838	.99279	7
54	.12014	.12101	8.2636	.99276	6
55	.12043	.12131	8.2434	.99272	5
56	.12071	.12160	8.2234	.99269	4
57	.12100	.12190	8.2035	.99265	3
58	.12129	.12219	8.1837	.99262	2
59	.12158	.12249	8.1640	.99258	1
60	.12187	.12278	8.1443	.99255	0
′	Cos	Cot	Tan	Sin	′

96° (276°) (263°) **83°**

7° (187°) (352°) **172°**

′	Sin	Tan	Cot	Cos	′
0	.12187	.12278	8.1443	.99255	60
1	.12216	.12308	8.1248	.99251	59
2	.12245	.12338	8.1054	.99248	58
3	.12274	.12367	8.0860	.99244	57
4	.12302	.12397	8.0667	.99240	56
5	.12331	.12426	8.0476	.99237	55
6	.12360	.12456	8.0285	.99233	54
7	.12389	.12485	8.0095	.99230	53
8	.12418	.12515	7.9906	.99226	52
9	.12447	.12544	7.9718	.99222	51
10	.12476	.12574	7.9530	.99219	50
11	.12504	.12603	7.9344	.99215	49
12	.12533	.12633	7.9158	.99211	48
13	.12562	.12662	7.8973	.99208	47
14	.12591	.12692	7.8789	.99204	46
15	.12620	.12722	7.8606	.99200	45
16	.12649	.12751	7.8424	.99197	44
17	.12678	.12781	7.8243	.99193	43
18	.12706	.12810	7.8062	.99189	42
19	.12735	.12840	7.7882	.99186	41
20	.12764	.12869	7.7704	.99182	40
21	.12793	.12899	7.7525	.99178	39
22	.12822	.12929	7.7348	.99175	38
23	.12851	.12958	7.7171	.99171	37
24	.12880	.12988	7.6996	.99167	36
25	.12908	.13017	7.6821	.99163	35
26	.12937	.13047	7.6647	.99160	34
27	.12966	.13076	7.6473	.99156	33
28	.12995	.13106	7.6301	.99152	32
29	.13024	.13136	7.6129	.99148	31
30	.13053	.13165	7.5958	.99144	30
31	.13081	.13195	7.5787	.99141	29
32	.13110	.13224	7.5618	.99137	28
33	.13139	.13254	7.5449	.99133	27
34	.13168	.13284	7.5281	.99129	26
35	.13197	.13313	7.5113	.99125	25
36	.13226	.13343	7.4947	.99122	24
37	.13254	.13372	7.4781	.99118	23
38	.13283	.13402	7.4615	.99114	22
39	.13312	.13432	7.4451	.99110	21
40	.13341	.13461	7.4287	.99106	20
41	.13370	.13491	7.4124	.99102	19
42	.13399	.13521	7.3962	.99098	18
43	.13427	.13550	7.3800	.99094	17
44	.13456	.13580	7.3639	.99091	16
45	.13485	.13609	7.3479	.99087	15
46	.13514	.13639	7.3319	.99083	14
47	.13543	.13669	7.3160	.99079	13
48	.13572	.13698	7.3002	.99075	12
49	.13600	.13728	7.2844	.99071	11
50	.13629	.13758	7.2687	.99067	10
51	.13658	.13787	7.2531	.99063	9
52	.13687	.13817	7.2375	.99059	8
53	.13716	.13846	7.2220	.99055	7
54	.13744	.13876	7.2066	.99051	6
55	.13773	.13906	7.1912	.99047	5
56	.13802	.13935	7.1759	.99043	4
57	.13831	.13965	7.1607	.99039	3
58	.13860	.13995	7.1455	.99035	2
59	.13889	.14024	7.1304	.99031	1
60	.13917	.14054	7.1154	.99027	0
′	Cos	Cot	Tan	Sin	′

97° (277°) (262°) **82°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

8° (188°) (351°) **171°**

′	Sin	Tan	Cot	Cos	′
0	.13917	.14054	7.1154	.99027	60
1	.13946	.14084	7.1004	.99023	59
2	.13975	.14113	7.0855	.99019	58
3	.14004	.14143	7.0706	.99015	57
4	.14033	.14173	7.0558	.99011	56
5	.14061	.14202	7.0410	.99006	55
6	.14090	.14232	7.0264	.99002	54
7	.14119	.14262	7.0117	.98998	53
8	.14148	.14291	6.9972	.98994	52
9	.14177	.14321	6.9827	.98990	51
10	.14205	.14351	6.9682	.98986	50
11	.14234	.14381	6.9538	.98982	49
12	.14263	.14410	6.9395	.98978	48
13	.14292	.14440	6.9252	.98973	47
14	.14320	.14470	6.9110	.98969	46
15	.14349	.14499	6.8969	.98965	45
16	.14378	.14529	6.8828	.98961	44
17	.14407	.14559	6.8687	.98957	43
18	.14436	.14588	6.8548	.98953	42
19	.14464	.14618	6.8408	.98948	41
20	.14493	.14648	6.8269	.98944	40
21	.14522	.14678	6.8131	.98940	39
22	.14551	.14707	6.7994	.98936	38
23	.14580	.14737	6.7856	.98931	37
24	.14608	.14767	6.7720	.98927	36
25	.14637	.14796	6.7584	.98923	35
26	.14666	.14826	6.7448	.98919	34
27	.14695	.14856	6.7313	.98914	33
28	.14723	.14886	6.7179	.98910	32
29	.14752	.14915	6.7045	.98906	31
30	.14781	.14945	6.6912	.98902	30
31	.14810	.14975	6.6779	.98897	29
32	.14838	.15005	6.6646	.98893	28
33	.14867	.15034	6.6514	.98889	27
34	.14896	.15064	6.6383	.98884	26
35	.14925	.15094	6.6252	.98880	25
36	.14954	.15124	6.6122	.98876	24
37	.14982	.15153	6.5992	.98871	23
38	.15011	.15183	6.5863	.98867	22
39	.15040	.15213	6.5734	.98863	21
40	.15069	.15243	6.5606	.98858	20
41	.15097	.15272	6.5478	.98854	19
42	.15126	.15302	6.5350	.98849	18
43	.15155	.15332	6.5223	.98845	17
44	.15184	.15362	6.5097	.98841	16
45	.15212	.15391	6.4971	.98836	15
46	.15241	.15421	6.4846	.98832	14
47	.15270	.15451	6.4721	.98827	13
48	.15299	.15481	6.4596	.98823	12
49	.15327	.15511	6.4472	.98818	11
50	.15356	.15540	6.4348	.98814	10
51	.15385	.15570	6.4225	.98809	9
52	.15414	.15600	6.4103	.98805	8
53	.15442	.15630	6.3980	.98800	7
54	.15471	.15660	6.3859	.98796	6
55	.15500	.15689	6.3737	.98791	5
56	.15529	.15719	6.3617	.98787	4
57	.15557	.15749	6.3496	.98782	3
58	.15586	.15779	6.3376	.98778	2
59	.15615	.15809	6.3257	.98773	1
60	.15643	.15838	6.3138	.98769	0
′	Cos	Cot	Tan	Sin	′

98° (278°) (261°) **81°**

9° (189°) (350°) **170°**

′	Sin	Tan	Cot	Cos	′
0	.15643	.15838	6.3138	.98769	60
1	.15672	.15868	6.3019	.98764	59
2	.15701	.15898	6.2901	.98760	58
3	.15730	.15928	6.2783	.98755	57
4	.15758	.15958	6.2666	.98751	56
5	.15787	.15988	6.2549	.98746	55
6	.15816	.16017	6.2432	.98741	54
7	.15845	.16047	6.2316	.98737	53
8	.15873	.16077	6.2200	.98732	52
9	.15902	.16107	6.2085	.98728	51
10	.15931	.16137	6.1970	.98723	50
11	.15959	.16167	6.1856	.98718	49
12	.15988	.16196	6.1742	.98714	48
13	.16017	.16226	6.1628	.98709	47
14	.16046	.16256	6.1515	.98704	46
15	.16074	.16286	6.1402	.98700	45
16	.16103	.16316	6.1290	.98695	44
17	.16132	.16346	6.1178	.98690	43
18	.16160	.16376	6.1066	.98686	42
19	.16189	.16405	6.0955	.98681	41
20	.16218	.16435	6.0844	.98676	40
21	.16246	.16465	6.0734	.98671	39
22	.16275	.16495	6.0624	.98667	38
23	.16304	.16525	6.0514	.98662	37
24	.16333	.16555	6.0405	.98657	36
25	.16361	.16585	6.0296	.98652	35
26	.16390	.16615	6.0188	.98648	34
27	.16419	.16645	6.0080	.98643	33
28	.16447	.16674	5.9972	.98638	32
29	.16476	.16704	5.9865	.98633	31
30	.16505	.16734	5.9758	.98629	30
31	.16533	.16764	5.9651	.98624	29
32	.16562	.16794	5.9545	.98619	28
33	.16591	.16824	5.9439	.98614	27
34	.16620	.16854	5.9333	.98609	26
35	.16648	.16884	5.9228	.98604	25
36	.16677	.16914	5.9124	.98600	24
37	.16706	.16944	5.9019	.98595	23
38	.16734	.16974	5.8915	.98590	22
39	.16763	.17004	5.8811	.98585	21
40	.16792	.17033	5.8708	.98580	20
41	.16820	.17063	5.8605	.98575	19
42	.16849	.17093	5.8502	.98570	18
43	.16878	.17123	5.8400	.98565	17
44	.16906	.17153	5.8298	.98561	16
45	.16935	.17183	5.8197	.98556	15
46	.16964	.17213	5.8095	.98551	14
47	.16992	.17243	5.7994	.98546	13
48	.17021	.17273	5.7894	.98541	12
49	.17050	.17303	5.7794	.98536	11
50	.17078	.17333	5.7694	.98531	10
51	.17107	.17363	5.7594	.98526	9
52	.17136	.17393	5.7495	.98521	8
53	.17164	.17423	5.7396	.98516	7
54	.17193	.17453	5.7297	.98511	6
55	.17222	.17483	5.7199	.98506	5
56	.17250	.17513	5.7101	.98501	4
57	.17279	.17543	5.7004	.98496	3
58	.17308	.17573	5.6906	.98491	2
59	.17336	.17603	5.6809	.98486	1
60	.17365	.17633	5.6713	.98481	0
′	Cos	Cot	Tan	Sin	′

99° (279°) (260°) **80°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

10° (190°) (349°) **169°**

′	Sin	Tan	Cot	Cos	′
0	.17365	.17633	5.6713	.98481	60
1	.17393	.17663	5.6617	.98476	59
2	.17422	.17693	5.6521	.98471	58
3	.17451	.17723	5.6425	.98466	57
4	.17479	.17753	5.6329	.98461	56
5	.17508	.17783	5.6234	.98455	55
6	.17537	.17813	5.6140	.98450	54
7	.17565	.17843	5.6045	.98445	53
8	.17594	.17873	5.5951	.98440	52
9	.17623	.17903	5.5857	.98435	51
10	.17651	.17933	5.5764	.98430	50
11	.17680	.17963	5.5671	.98425	49
12	.17708	.17993	5.5578	.98420	48
13	.17737	.18023	5.5485	.98414	47
14	.17766	.18053	5.5393	.98409	46
15	.17794	.18083	5.5301	.98404	45
16	.17823	.18113	5.5209	.98399	44
17	.17852	.18143	5.5118	.98394	43
18	.17880	.18173	5.5026	.98389	42
19	.17909	.18203	5.4936	.98383	41
20	.17937	.18233	5.4845	.98378	40
21	.17966	.18263	5.4755	.98373	39
22	.17995	.18293	5.4665	.98368	38
23	.18023	.18323	5.4575	.98362	37
24	.18052	.18353	5.4486	.98357	36
25	.18081	.18384	5.4397	.98352	35
26	.18109	.18414	5.4308	.98347	34
27	.18138	.18444	5.4219	.98341	33
28	.18166	.18474	5.4131	.98336	32
29	.18195	.18504	5.4043	.98331	31
30	.18224	.18534	5.3955	.98325	30
31	.18252	.18564	5.3868	.98320	29
32	.18281	.18594	5.3781	.98315	28
33	.18309	.18624	5.3694	.98310	27
34	.18338	.18654	5.3607	.98304	26
35	.18367	.18684	5.3521	.98299	25
36	.18395	.18714	5.3435	.98294	24
37	.18424	.18745	5.3349	.98288	23
38	.18452	.18775	5.3263	.98283	22
39	.18481	.18805	5.3178	.98277	21
40	.18509	.18835	5.3093	.98272	20
41	.18538	.18865	5.3008	.98267	19
42	.18567	.18895	5.2924	.98261	18
43	.18595	.18925	5.2839	.98256	17
44	.18624	.18955	5.2755	.98250	16
45	.18652	.18986	5.2672	.98245	15
46	.18681	.19016	5.2588	.98240	14
47	.18710	.19046	5.2505	.98234	13
48	.18738	.19076	5.2422	.98229	12
49	.18767	.19106	5.2339	.98223	11
50	.18795	.19136	5.2257	.98218	10
51	.18824	.19166	5.2174	.98212	9
52	.18852	.19197	5.2092	.98207	8
53	.18881	.19227	5.2011	.98201	7
54	.18910	.19257	5.1929	.98196	6
55	.18938	.19287	5.1848	.98190	5
56	.18967	.19317	5.1767	.98185	4
57	.18995	.19347	5.1686	.98179	3
58	.19024	.19378	5.1606	.98174	2
59	.19052	.19408	5.1526	.98168	1
60	.19081	.19438	5.1446	.98163	0
′	Cos	Cot	Tan	Sin	′

100° (280°) (259°) **79°**

11° (191°) (348°) **168°**

′	Sin	Tan	Cot	Cos	′
0	.19081	.19438	5.1446	.98163	60
1	.19109	.19468	5.1366	.98157	59
2	.19138	.19498	5.1286	.98152	58
3	.19167	.19529	5.1207	.98146	57
4	.19195	.19559	5.1128	.98140	56
5	.19224	.19589	5.1049	.98135	55
6	.19252	.19619	5.0970	.98129	54
7	.19281	.19649	5.0892	.98124	53
8	.19309	.19680	5.0814	.98118	52
9	.19338	.19710	5.0736	.98112	51
10	.19366	.19740	5.0658	.98107	50
11	.19395	.19770	5.0581	.98101	49
12	.19423	.19801	5.0504	.98096	48
13	.19452	.19831	5.0427	.98090	47
14	.19481	.19861	5.0350	.98084	46
15	.19509	.19891	5.0273	.98079	45
16	.19538	.19921	5.0197	.98073	44
17	.19566	.19952	5.0121	.98067	43
18	.19595	.19982	5.0045	.98061	42
19	.19623	.20012	4.9969	.98056	41
20	.19652	.20042	4.9894	.98050	40
21	.19680	.20073	4.9819	.98044	39
22	.19709	.20103	4.9744	.98039	38
23	.19737	.20133	4.9669	.98033	37
24	.19766	.20164	4.9594	.98027	36
25	.19794	.20194	4.9520	.98021	35
26	.19823	.20224	4.9446	.98016	34
27	.19851	.20254	4.9372	.98010	33
28	.19880	.20285	4.9298	.98004	32
29	.19908	.20315	4.9225	.97998	31
30	.19937	.20345	4.9152	.97992	30
31	.19965	.20376	4.9078	.97987	29
32	.19994	.20406	4.9006	.97981	28
33	.20022	.20436	4.8933	.97975	27
34	.20051	.20466	4.8860	.97969	26
35	.20079	.20497	4.8788	.97963	25
36	.20108	.20527	4.8716	.97958	24
37	.20136	.20557	4.8644	.97952	23
38	.20165	.20588	4.8573	.97946	22
39	.20193	.20618	4.8501	.97940	21
40	.20222	.20648	4.8430	.97934	20
41	.20250	.20679	4.8359	.97928	19
42	.20279	.20709	4.8288	.97922	18
43	.20307	.20739	4.8218	.97916	17
44	.20336	.20770	4.8147	.97910	16
45	.20364	.20800	4.8077	.97905	15
46	.20393	.20830	4.8007	.97899	14
47	.20421	.20861	4.7937	.97893	13
48	.20450	.20891	4.7867	.97887	12
49	.20478	.20921	4.7798	.97881	11
50	.20507	.20952	4.7729	.97875	10
51	.20535	.20982	4.7659	.97869	9
52	.20563	.21013	4.7591	.97863	8
53	.20592	.21043	4.7522	.97857	7
54	.20620	.21073	4.7453	.97851	6
55	.20649	.21104	4.7385	.97845	5
56	.20677	.21134	4.7317	.97839	4
57	.20706	.21164	4.7249	.97833	3
58	.20734	.21195	4.7181	.97827	2
59	.20763	.21225	4.7114	.97821	1
60	.20791	.21256	4.7046	.97815	0
′	Cos	Cot	Tan	Sin	′

101° (281°) (258°) **78°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

12° (192°) (347°) **167°**

′	Sin	Tan	Cot	Cos	′
0	.20791	.21256	4.7046	.97815	60
1	.20820	.21286	4.6979	.97809	59
2	.20848	.21316	4.6912	.97803	58
3	.20877	.21347	4.6845	.97797	57
4	.20905	.21377	4.6779	.97791	56
5	.20933	.21408	4.6712	.97784	55
6	.20962	.21438	4.6646	.97778	54
7	.20990	.21469	4.6580	.97772	53
8	.21019	.21499	4.6514	.97766	52
9	.21047	.21529	4.6448	.97760	51
10	.21076	.21560	4.6382	.97754	50
11	.21104	.21590	4.6317	.97748	49
12	.21132	.21621	4.6252	.97742	48
13	.21161	.21651	4.6187	.97735	47
14	.21189	.21682	4.6122	.97729	46
15	.21218	.21712	4.6057	.97723	45
16	.21246	.21743	4.5993	.97717	44
17	.21275	.21773	4.5928	.97711	43
18	.21303	.21804	4.5864	.97705	42
19	.21331	.21834	4.5800	.97698	41
20	.21360	.21864	4.5736	.97692	40
21	.21388	.21895	4.5673	.97686	39
22	.21417	.21925	4.5609	.97680	38
23	.21445	.21956	4.5546	.97673	37
24	.21474	.21986	4.5483	.97667	36
25	.21502	.22017	4.5420	.97661	35
26	.21530	.22047	4.5357	.97655	34
27	.21559	.22078	4.5294	.97648	33
28	.21587	.22108	4.5232	.97642	32
29	.21616	.22139	4.5169	.97636	31
30	.21644	.22169	4.5107	.97630	30
31	.21672	.22200	4.5045	.97623	29
32	.21701	.22231	4.4983	.97617	28
33	.21729	.22261	4.4922	.97611	27
34	.21758	.22292	4.4860	.97604	26
35	.21786	.22322	4.4799	.97598	25
36	.21814	.22353	4.4737	.97592	24
37	.21843	.22383	4.4676	.97585	23
38	.21871	.22414	4.4615	.97579	22
39	.21899	.22444	4.4555	.97573	21
40	.21928	.22475	4.4494	.97566	20
41	.21956	.22505	4.4434	.97560	19
42	.21985	.22536	4.4373	.97553	18
43	.22013	.22567	4.4313	.97547	17
44	.22041	.22597	4.4253	.97541	16
45	.22070	.22628	4.4194	.97534	15
46	.22098	.22658	4.4134	.97528	14
47	.22126	.22689	4.4075	.97521	13
48	.22155	.22719	4.4015	.97515	12
49	.22183	.22750	4.3956	.97508	11
50	.22212	.22781	4.3897	.97502	10
51	.22240	.22811	4.3838	.97496	9
52	.22268	.22842	4.3779	.97489	8
53	.22297	.22872	4.3721	.97483	7
54	.22325	.22903	4.3662	.97476	6
55	.22353	.22934	4.3604	.97470	5
56	.22382	.22964	4.3546	.97463	4
57	.22410	.22995	4.3488	.97457	3
58	.22438	.23026	4.3430	.97450	2
59	.22467	.23056	4.3372	.97444	1
60	.22495	.23087	4.3315	.97437	0
′	Cos	Cot	Tan	Sin	′

102° (282°) (257°) **77°**

13° (193°) (346°) **166°**

′	Sin	Tan	Cot	Cos	′
0	.22495	.23087	4.3315	.97437	60
1	.22523	.23117	4.3257	.97430	59
2	.22552	.23148	4.3200	.97424	58
3	.22580	.23179	4.3143	.97417	57
4	.22608	.23209	4.3086	.97411	56
5	.22637	.23240	4.3029	.97404	55
6	.22665	.23271	4.2972	.97398	54
7	.22693	.23301	4.2916	.97391	53
8	.22722	.23332	4.2859	.97384	52
9	.22750	.23363	4.2803	.97378	51
10	.22778	.23393	4.2747	.97371	50
11	.22807	.23424	4.2691	.97365	49
12	.22835	.23455	4.2635	.97358	48
13	.22863	.23485	4.2580	.97351	47
14	.22892	.23516	4.2524	.97345	46
15	.22920	.23547	4.2468	.97338	45
16	.22948	.23578	4.2413	.97331	44
17	.22977	.23608	4.2358	.97325	43
18	.23005	.23639	4.2303	.97318	42
19	.23033	.23670	4.2248	.97311	41
20	.23062	.23700	4.2193	.97304	40
21	.23090	.23731	4.2139	.97298	39
22	.23118	.23762	4.2084	.97291	38
23	.23146	.23793	4.2030	.97284	37
24	.23175	.23823	4.1976	.97278	36
25	.23203	.23854	4.1922	.97271	35
26	.23231	.23885	4.1868	.97264	34
27	.23260	.23916	4.1814	.97257	33
28	.23288	.23946	4.1760	.97251	32
29	.23316	.23977	4.1706	.97244	31
30	.23345	.24008	4.1653	.97237	30
31	.23373	.24039	4.1600	.97230	29
32	.23401	.24069	4.1547	.97223	28
33	.23429	.24100	4.1493	.97217	27
34	.23458	.24131	4.1441	.97210	26
35	.23486	.24162	4.1388	.97203	25
36	.23514	.24193	4.1335	.97196	24
37	.23542	.24223	4.1282	.97189	23
38	.23571	.24254	4.1230	.97182	22
39	.23599	.24285	4.1178	.97176	21
40	.23627	.24316	4.1126	.97169	20
41	.23656	.24347	4.1074	.97162	19
42	.23684	.24377	4.1022	.97155	18
43	.23712	.24408	4.0970	.97148	17
44	.23740	.24439	4.0918	.97141	16
45	.23769	.24470	4.0867	.97134	15
46	.23797	.24501	4.0815	.97127	14
47	.23825	.24532	4.0764	.97120	13
48	.23853	.24562	4.0713	.97113	12
49	.23882	.24593	4.0662	.97106	11
50	.23910	.24624	4.0611	.97100	10
51	.23938	.24655	4.0560	.97093	9
52	.23966	.24686	4.0509	.97086	8
53	.23995	.24717	4.0459	.97079	7
54	.24023	.24747	4.0408	.97072	6
55	.24051	.24778	4.0358	.97065	5
56	.24079	.24809	4.0308	.97058	4
57	.24108	.24840	4.0257	.97051	3
58	.24136	.24871	4.0207	.97044	2
59	.24164	.24902	4.0158	.97037	1
60	.24192	.24933	4.0108	.97030	0
′	Cos	Cot	Tan	Sin	′

103° (283°) (256°) **76°**

14° (194°) (345°) **165°**

′	Sin	Tan	Cot	Cos	′
0	.24192	.24933	4.0108	.97030	60
1	.24220	.24964	4.0058	.97023	59
2	.24249	.24995	4.0009	.97015	58
3	.24277	.25026	3.9959	.97008	57
4	.24305	.25056	3.9910	.97001	56
5	.24333	.25087	3.9861	.96994	55
6	.24362	.25118	3.9812	.96987	54
7	.24390	.25149	3.9763	.96980	53
8	.24418	.25180	3.9714	.96973	52
9	.24446	.25211	3.9665	.96966	51
10	.24474	.25242	3.9617	.96959	50
11	.24503	.25273	3.9568	.96952	49
12	.24531	.25304	3.9520	.96945	48
13	.24559	.25335	3.9471	.96937	47
14	.24587	.25366	3.9423	.96930	46
15	.24615	.25397	3.9375	.96923	45
16	.24644	.25428	3.9327	.96916	44
17	.24672	.25459	3.9279	.96909	43
18	.24700	.25490	3.9232	.96902	42
19	.24728	.25521	3.9184	.96894	41
20	.24756	.25552	3.9136	.96887	40
21	.24784	.25583	3.9089	.96880	39
22	.24813	.25614	3.9042	.96873	38
23	.24841	.25645	3.8995	.96866	37
24	.24869	.25676	3.8947	.96858	36
25	.24897	.25707	3.8900	.96851	35
26	.24925	.25738	3.8854	.96844	34
27	.24954	.25769	3.8807	.96837	33
28	.24982	.25800	3.8760	.96829	32
29	.25010	.25831	3.8714	.96822	31
30	.25038	.25862	3.8667	.96815	30
31	.25066	.25893	3.8621	.96807	29
32	.25094	.25924	3.8575	.96800	28
33	.25122	.25955	3.8528	.96793	27
34	.25151	.25986	3.8482	.96786	26
35	.25179	.26017	3.8436	.96778	25
36	.25207	.26048	3.8391	.96771	24
37	.25235	.26079	3.8345	.96764	23
38	.25263	.26110	3.8299	.96756	22
39	.25291	.26141	3.8254	.96749	21
40	.25320	.26172	3.8208	.96742	20
41	.25348	.26203	3.8163	.96734	19
42	.25376	.26235	3.8118	.96727	18
43	.25404	.26266	3.8073	.96719	17
44	.25432	.26297	3.8028	.96712	16
45	.25460	.26328	3.7983	.96705	15
46	.25488	.26359	3.7938	.96697	14
47	.25516	.26390	3.7893	.96690	13
48	.25545	.26421	3.7848	.96682	12
49	.25573	.26452	3.7804	.96675	11
50	.25601	.26483	3.7760	.96667	10
51	.25629	.26515	3.7715	.96660	9
52	.25657	.26546	3.7671	.96653	8
53	.25685	.26577	3.7627	.96645	7
54	.25713	.26608	3.7583	.96638	6
55	.25741	.26639	3.7539	.96630	5
56	.25769	.26670	3.7495	.96623	4
57	.25798	.26701	3.7451	.96615	3
58	.25826	.26733	3.7408	.96608	2
59	.25854	.26764	3.7364	.96600	1
60	.25882	.26795	3.7321	.96593	0
′	Cos	Cot	Tan	Sin	′

104° (284°) (255°) **75°**

15° (195°) (344°) **164°**

′	Sin	Tan	Cot	Cos	′
0	.25882	.26795	3.7321	.96593	60
1	.25910	.26826	3.7277	.96585	59
2	.25938	.26857	3.7234	.96578	58
3	.25966	.26888	3.7191	.96570	57
4	.25994	.26920	3.7148	.96562	56
5	.26022	.26951	3.7105	.96555	55
6	.26050	.26982	3.7062	.96547	54
7	.26079	.27013	3.7019	.96540	53
8	.26107	.27044	3.6976	.96532	52
9	.26135	.27076	3.6933	.96524	51
10	.26163	.27107	3.6891	.96517	50
11	.26191	.27138	3.6848	.96509	49
12	.26219	.27169	3.6806	.96502	48
13	.26247	.27201	3.6764	.96494	47
14	.26275	.27232	3.6722	.96486	46
15	.26303	.27263	3.6680	.96479	45
16	.26331	.27294	3.6638	.96471	44
17	.26359	.27326	3.6596	.96463	43
18	.26387	.27357	3.6554	.96456	42
19	.26415	.27388	3.6512	.96448	41
20	.26443	.27419	3.6470	.96440	40
21	.26471	.27451	3.6429	.96433	39
22	.26500	.27482	3.6387	.96425	38
23	.26528	.27513	3.6346	.96417	37
24	.26556	.27545	3.6305	.96410	36
25	.26584	.27576	3.6264	.96402	35
26	.26612	.27607	3.6222	.96394	34
27	.26640	.27638	3.6181	.96386	33
28	.26668	.27670	3.6140	.96379	32
29	.26696	.27701	3.6100	.96371	31
30	.26724	.27732	3.6059	.96363	30
31	.26752	.27764	3.6018	.96355	29
32	.26780	.27795	3.5978	.96347	28
33	.26808	.27826	3.5937	.96340	27
34	.26836	.27858	3.5897	.96332	26
35	.26864	.27889	3.5856	.96324	25
36	.26892	.27921	3.5816	.96316	24
37	.26920	.27952	3.5776	.96308	23
38	.26948	.27983	3.5736	.96301	22
39	.26976	.28015	3.5696	.96293	21
40	.27004	.28046	3.5656	.96285	20
41	.27032	.28077	3.5616	.96277	19
42	.27060	.28109	3.5576	.96269	18
43	.27088	.28140	3.5536	.96261	17
44	.27116	.28172	3.5497	.96253	16
45	.27144	.28203	3.5457	.96246	15
46	.27172	.28234	3.5418	.96238	14
47	.27200	.28266	3.5379	.96230	13
48	.27228	.28297	3.5339	.96222	12
49	.27256	.28329	3.5300	.96214	11
50	.27284	.28360	3.5261	.96206	10
51	.27312	.28391	3.5222	.96198	9
52	.27340	.28423	3.5183	.96190	8
53	.27368	.28454	3.5144	.96182	7
54	.27396	.28486	3.5105	.96174	6
55	.27424	.28517	3.5067	.96166	5
56	.27452	.28549	3.5028	.96158	4
57	.27480	.28580	3.4989	.96150	3
58	.27508	.28612	3.4951	.96142	2
59	.27536	.28643	3.4912	.96134	1
60	.27564	.28675	3.4874	.96126	0
′	Cos	Cot	Tan	Sin	′

105° (285°) (254°) **74°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

16° (196°) (343°) **163°**

′	Sin	Tan	Cot	Cos	′
0	.27564	.28675	3.4874	.96126	**60**
1	.27592	.28706	3.4836	.96118	59
2	.27620	.28738	3.4798	.96110	58
3	.27648	.28769	3.4760	.96102	57
4	.27676	.28801	3.4722	.96094	56
5	.27704	.28832	3.4684	.96086	**55**
6	.27731	.28864	3.4646	.96078	54
7	.27759	.28895	3.4608	.96070	53
8	.27787	.28927	3.4570	.96062	52
9	.27815	.28958	3.4533	.96054	51
10	.27843	.28990	3.4495	.96046	**50**
11	.27871	.29021	3.4458	.96037	49
12	.27899	.29053	3.4420	.96029	48
13	.27927	.29084	3.4383	.96021	47
14	.27955	.29116	3.4346	.96013	46
15	.27983	.29147	3.4308	.96005	**45**
16	.28011	.29179	3.4271	.95997	44
17	.28039	.29210	3.4234	.95989	43
18	.28067	.29242	3.4197	.95981	42
19	.28095	.29274	3.4160	.95972	41
20	.28123	.29305	3.4124	.95964	**40**
21	.28150	.29337	3.4087	.95956	39
22	.28178	.29368	3.4050	.95948	38
23	.28206	.29400	3.4014	.95940	37
24	.28234	.29432	3.3977	.95931	36
25	.28262	.29463	3.3941	.95923	**35**
26	.28290	.29495	3.3904	.95915	34
27	.28318	.29526	3.3868	.95907	33
28	.28346	.29558	3.3832	.95898	32
29	.28374	.29590	3.3796	.95890	31
30	.28402	.29621	3.3759	.95882	**30**
31	.28429	.29653	3.3723	.95874	29
32	.28457	.29685	3.3687	.95865	28
33	.28485	.29716	3.3652	.95857	27
34	.28513	.29748	3.3616	.95849	26
35	.28541	.29780	3.3580	.95841	**25**
36	.28569	.29811	3.3544	.05832	24
37	.28597	.29843	3.3509	.95824	23
38	.28625	.29875	3.3473	.95816	22
39	.28652	.29906	3.3438	.95807	21
40	.28680	.29938	3.3402	.95799	**20**
41	.28708	.29970	3.3367	.95791	19
42	.28736	.30001	3.3332	.95782	18
43	.28764	.30033	3.3297	.95774	17
44	.28792	.30065	3.3261	.95766	16
45	.28820	.30097	3.3226	.95757	**15**
46	.28847	.30128	3.3191	.95749	14
47	.28875	.30160	3.3156	.95740	13
48	.28903	.30192	3.3122	.95732	12
49	.28931	.30224	3.3087	.95724	11
50	.28959	.30255	3.3052	.95715	**10**
51	.28987	.30287	3.3017	.95707	9
52	.29015	.30319	3.2983	.95698	8
53	.29042	.30351	3.2948	.95690	7
54	.29070	.30382	3.2914	.95681	6
55	.29098	.30414	3.2879	.95673	**5**
56	.29126	.30446	3.2845	.95664	4
57	.29154	.30478	3.2811	.95656	3
58	.29182	.30509	3.2777	.95647	2
59	.29209	.30541	3.2743	.95639	1
60	.29237	.30573	3.2709	.95630	**0**
′	Cos	Cot	Tan	Sin	′

106° (286°) (253°) **73°**

17° (197°) (342°) **162°**

′	Sin	Tan	Cot	Cos	′
0	.29237	.30573	3.2709	.95630	**60**
1	.29265	.30605	3.2675	.95622	59
2	.29293	.30637	3.2641	.95613	58
3	.29321	.30669	3.2607	.95605	57
4	.29348	.30700	3.2573	.95596	56
5	.29376	.30732	3.2539	.95588	**55**
6	.29404	.30764	3.2506	.95579	54
7	.29432	.30796	3.2472	.95571	53
8	.29460	.30828	3.2438	.95562	52
9	.29487	.30860	3.2405	.95554	51
10	.29515	.30891	3.2371	.95545	**50**
11	.29543	.30923	3.2338	.95536	49
12	.29571	.30955	3.2305	.95528	48
13	.29599	.30987	3.2272	.95519	47
14	.29626	.31019	3.2238	.95511	46
15	.29654	.31051	3.2205	.95502	**45**
16	.29682	.31083	3.2172	.95493	44
17	.29710	.31115	3.2139	.95485	43
18	.29737	.31147	3.2106	.95476	42
19	.29765	.31178	3.2073	.95467	41
20	.29793	.31210	3.2041	.95459	**40**
21	.29821	.31242	3.2008	.95450	39
22	.29849	.31274	3.1975	.95441	38
23	.29876	.31306	3.1943	.95433	37
24	.29904	.31338	3.1910	.95424	36
25	.29932	.31370	3.1878	.95415	**35**
26	.29960	.31402	3.1845	.95407	34
27	.29987	.31434	3.1813	.95398	33
28	.30015	.31466	3.1780	.95389	32
29	.30043	.31498	3.1748	.95380	31
30	.30071	.31530	3.1716	.95372	**30**
31	.30098	.31562	3.1684	.95363	29
32	.30126	.31594	3.1652	.95354	28
33	.30154	.31626	3.1620	.95345	27
34	.30182	.31658	3.1588	.95337	26
35	.30209	.31690	3.1556	.95328	**25**
36	.30237	.31722	3.1524	.95319	24
37	.30265	.31754	3.1492	.95310	23
38	.30292	.31786	3.1460	.95301	22
39	.30320	.31818	3.1429	.95293	21
40	.30348	.31850	3.1397	.95284	**20**
41	.30376	.31882	3.1366	.95275	19
42	.30403	.31914	3.1334	.95266	18
43	.30431	.31946	3.1303	.95257	17
44	.30459	.31978	3.1271	.95248	16
45	.30486	.32010	3.1240	.95240	**15**
46	.30514	.32042	3.1209	.95231	14
47	.30542	.32074	3.1178	.95222	13
48	.30570	.32106	3.1146	.95213	12
49	.30597	.32139	3.1115	.95204	11
50	.30625	.32171	3.1084	.95195	**10**
51	.30653	.32203	3.1053	.95186	9
52	.30680	.32235	3.1022	.95177	8
53	.30708	.32267	3.0991	.95168	7
54	.30736	.32299	3.0961	.95159	6
55	.30763	.32331	3.0930	.95150	**5**
56	.30791	.32363	3.0899	.95142	4
57	.30819	.32396	3.0868	.95133	3
58	.30846	.32428	3.0838	.95124	2
59	.30874	.32460	3.0807	.95115	1
60	.30902	.32492	3.0777	.95106	**0**
′	Cos	Cot	Tan	Sin	′

107° (287°) (252°) **72°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

18° (198°) (341°) **161°**

′	Sin	Tan	Cot	Cos	′
0	.30902	.32492	3.0777	.95106	**60**
1	.30929	.32524	3.0746	.95097	59
2	.30957	.32556	3.0716	.95088	58
3	.30985	.32588	3.0686	.95079	57
4	.31012	.32621	3.0655	.95070	56
5	.31040	.32653	3.0625	.95061	**55**
6	.31068	.32685	3.0595	.95052	54
7	.31095	.32717	3.0565	.95043	53
8	.31123	.32749	3.0535	.95033	52
9	.31151	.32782	3.0505	.95024	51
10	.31178	.32814	3.0475	.95015	**50**
11	.31206	.32846	3.0445	.95006	49
12	.31233	.32878	3.0415	.94997	48
13	.31261	.32911	3.0385	.94988	47
14	.31289	.32943	3.0356	.94979	46
15	.31316	.32975	3.0326	.94970	**45**
16	.31344	.33007	3.0296	.94961	44
17	.31372	.33040	3.0267	.94952	43
18	.31399	.33072	3.0237	.94943	42
19	.31427	.33104	3.0208	.94933	41
20	.31454	.33136	3.0178	.94924	**40**
21	.31482	.33169	3.0149	.94915	29
22	.31510	.33201	3.0120	.94906	38
23	.31537	.33233	3.0090	.94897	37
24	.31565	.33266	3.0061	.94888	36
25	.31593	.33298	3.0032	.94878	**35**
26	.31620	.33330	3.0003	.94869	34
27	.31648	.33363	2.9974	.94860	33
28	.31675	.33395	2.9945	.94851	32
29	.31703	.33427	2.9916	.94842	31
30	.31730	.33460	2.9887	.94832	**30**
31	.31758	.33492	2.9858	.94823	29
32	.31786	.33524	2.9829	.94814	28
33	.31813	.33557	2.9800	.94805	27
34	.31841	.33589	2.9772	.94795	26
35	.31868	.33621	2.9743	.94786	**25**
36	.31896	.33654	2.9714	.94777	24
37	.31923	.33686	2.9686	.94768	23
38	.31951	.33718	2.9657	.94758	22
39	.31979	.33751	2.9629	.94749	21
40	.32006	.33783	2.9600	.94740	**20**
41	.32034	.33816	2.9572	.94730	19
42	.32061	.33848	2.9544	.94721	18
43	.32089	.33881	2.9515	.94712	17
44	.32116	.33913	2.9487	.94702	16
45	.32144	.33945	2.9459	.94693	**15**
46	.32171	.33978	2.9431	.94684	14
47	.32199	.34010	2.9403	.94674	13
48	.32227	.34043	2.9375	.94665	12
49	.32254	.34075	2.9347	.94656	11
50	.32282	.34108	2.9319	.94646	**10**
51	.32309	.34140	2.9291	.94637	9
52	.32337	.34173	2.9263	.94627	8
53	.32364	.34205	2.9235	.94618	7
54	.32392	.34238	2.9208	.94609	6
55	.32419	.34270	2.9180	.94599	**5**
56	.32447	.34303	2.9152	.94590	4
57	.32474	.34335	2.9125	.94580	3
58	.32502	.34368	2.9097	.94571	2
59	.32529	.34400	2.9070	.94561	1
60	.32557	.34433	2.9042	.94552	**0**
′	Cos	Cot	Tan	Sin	′

108° (288°) (251°) **71°**

19° (199°) (340°) **160°**

′	Sin	Tan	Cot	Cos	′
0	.32557	.34433	2.9042	.94552	**60**
1	.32584	.34465	2.9015	.94542	59
2	.32612	.34498	2.8987	.94533	58
3	.32639	.34530	2.8960	.94523	57
4	.32667	.34563	2.8933	.94514	56
5	.32694	.34596	2.8905	.94504	**55**
6	.32722	.34628	2.8878	.94495	54
7	.32749	.34661	2.8851	.94485	53
8	.32777	.34693	2.8824	.94476	52
9	.32804	.34726	2.8797	.94466	51
10	.32832	.34758	2.8770	.94457	**50**
11	.32859	.34791	2.8743	.94447	49
12	.32887	.34824	2.8716	.94438	48
13	.32914	.34856	2.8689	.94428	47
14	.32942	.34889	2.8662	.94418	46
15	.32969	.34922	2.8636	.94409	**45**
16	.32997	.34954	2.8609	.94399	44
17	.33024	.34987	2.8582	.94390	43
18	.33051	.35020	2.8556	.94380	42
19	.33079	.35052	2.8529	.94370	41
20	.33106	.35085	2.8502	.94361	**40**
21	.33134	.35118	2.8476	.94351	39
22	.33161	.35150	2.8449	.94342	38
23	.33189	.35183	2.8423	.94332	37
24	.33216	.35216	2.8397	.94322	36
25	.33244	.35248	2.8370	.94313	**35**
26	.33271	.35281	2.8344	.94303	34
27	.33298	.35314	2.8318	.94293	33
28	.33326	.35346	2.8291	.94284	32
29	.33353	.35379	2.8265	.94274	31
30	.33381	.35412	2.8239	.94264	**30**
31	.33408	.35445	2.8213	.94254	29
32	.33436	.35477	2.8187	.94245	28
33	.33463	.35510	2.8161	.94235	27
34	.33490	.35543	2.8135	.94225	26
35	.33518	.35576	2.8109	.94215	**25**
36	.33545	.35608	2.8083	.94206	24
37	.33573	.35641	2.8057	.94196	23
38	.33600	.35674	2.8032	.94186	22
39	.33627	.35707	2.8006	.94176	21
40	.33655	.35740	2.7980	.94167	**20**
41	.33682	.35772	2.7955	.94157	19
42	.33710	.35805	2.7929	.94147	18
43	.33737	.35838	2.7903	.94137	17
44	.33764	.35871	2.7878	.94127	16
45	.33792	.35904	2.7852	.94118	**15**
46	.33810	.35937	2.7827	.94108	14
47	.33846	.35969	2.7801	.94098	13
48	.33874	.36002	2.7776	.94088	12
49	.33901	.36035	2.7751	.94078	11
50	.33929	.36068	2.7725	.94068	**10**
51	.33956	.36101	2.7700	.94058	9
52	.33983	.36134	2.7675	.94049	8
53	.34011	.36167	2.7650	.94039	7
54	.34038	.36199	2.7625	.94029	6
55	.34065	.36232	2.7600	.94019	**5**
56	.34093	.36265	2.7575	.94009	4
57	.34120	.36298	2.7550	.93999	3
58	.34147	.36331	2.7525	.93989	2
59	.34175	.36364	2.7500	.93979	1
60	.34202	.36397	2.7475	.93969	**0**
′	Cos	Cot	Tan	Sin	′

109° (289°) (250°) **70°**

20° (200°) (339°) **159°**

′	Sin	Tan	Cot	Cos	′
0	.34202	.36397	2.7475	.93969	**60**
1	.34229	.36430	2.7450	.93959	59
2	.34257	.36463	2.7425	.93949	58
3	.34284	.36496	2.7400	.93939	57
4	.34311	.36529	2.7376	.93929	56
5	.34339	.36562	2.7351	.93919	**55**
6	.34366	.36595	2.7326	.93909	54
7	.34393	.36628	2.7302	.93899	53
8	.34421	.36661	2.7277	.93889	52
9	.34448	.36694	2.7253	.93879	51
10	.34475	.36727	2.7228	.93869	**50**
11	.34503	.36760	2.7204	.93859	49
12	.34530	.36793	2.7179	.93849	48
13	.34557	.36826	2.7155	.93839	47
14	.34584	.36859	2.7130	.93829	46
15	.34612	.36892	2.7106	.93819	**45**
16	.34639	.36925	2.7082	.93809	44
17	.34666	.36958	2.7058	.93799	43
18	.34694	.36991	2.7034	.93789	42
19	.34721	.37024	2.7009	.93779	41
20	.34748	.37057	2.6985	.93769	**40**
21	.34775	.37090	2.6961	.93759	39
22	.34803	.37123	2.6937	.93748	38
23	.34830	.37157	2.6913	.93738	37
24	.34857	.37190	2.6889	.93728	36
25	.34884	.37223	2.6865	.93718	**35**
26	.34912	.37256	2.6841	.93708	34
27	.34939	.37289	2.6818	.93698	33
28	.34966	.37322	2.6794	.93688	32
29	.34993	.37355	2.6770	.93677	31
30	.35021	.37388	2.6746	.93667	**30**
31	.35048	.37422	2.6723	.93657	29
32	.35075	.37455	2.6699	.93647	28
33	.35102	.37488	2.6675	.93637	27
34	.35130	.37521	2.6652	.93626	26
35	.35157	.37554	2.6628	.93616	**25**
36	.35184	.37588	2.6605	.93606	24
37	.35211	.37621	2.6581	.93596	23
38	.35239	.37654	2.6558	.93585	22
39	.35266	.37687	2.6534	.93575	21
40	.35293	.37720	2.6511	.93565	**20**
41	.35320	.37754	2.6488	.93555	19
42	.35347	.37787	2.6464	.93544	18
43	.35375	.37820	2.6441	.93534	17
44	.35402	.37853	2.6418	.93524	16
45	.35429	.37887	2.6395	.93514	**15**
46	.35456	.37920	2.6371	.93503	14
47	.35484	.37953	2.6348	.93493	13
48	.35511	.37986	2.6325	.93483	12
49	.35538	.38020	2.6302	.93472	11
50	.35565	.38053	2.6279	.93462	**10**
51	.35592	.38086	2.6256	.93452	9
52	.35619	.38120	2.6233	.93441	8
53	.35647	.38153	2.6210	.93431	7
54	.35674	.38186	2.6187	.93420	6
55	.35701	.38220	2.6165	.93410	**5**
56	.35728	.38253	2.6142	.93400	4
57	.35755	.38286	2.6119	.93389	3
58	.35782	.38320	2.6096	.93379	2
59	.35810	.38353	2.6074	.93368	1
60	.35837	.38386	2.6051	.93358	**0**
′	Cos	Cot	Tan	Sin	′

110° (290°) (249°) **69°**

21° (201°) (338°) **158°**

′	Sin	Tan	Cot	Cos	′
0	.35837	.38386	2.6051	.93358	**60**
1	.35864	.38420	2.6028	.93348	59
2	.35891	.38453	2.6006	.93337	58
3	.35918	.38487	2.5983	.93327	57
4	.35945	.38520	2.5961	.93316	56
5	.35973	.38553	2.5938	.93306	**55**
6	.36000	.38587	2.5916	.93295	54
7	.36027	.38620	2.5893	.93285	53
8	.36054	.38654	2.5871	.93274	52
9	.36081	.38687	2.5848	.93264	51
10	.36108	.38721	2.5826	.93253	**50**
11	.36135	.38754	2.5804	.93243	49
12	.36162	.38787	2.5782	.93232	48
13	.36190	.38821	2.5759	.93222	47
14	.36217	.38854	2.5737	.93211	46
15	.36244	.38888	2.5715	.93201	**45**
16	.36271	.38921	2.5693	.93190	44
17	.36298	.38955	2.5671	.93180	43
18	.36325	.38988	2.5649	.93169	42
19	.36352	.39022	2.5627	.93159	41
20	.36379	.39055	2.5605	.93148	**40**
21	.36406	.39089	2.5583	.93137	39
22	.36434	.39122	2.5561	.93127	38
23	.36461	.39156	2.5539	.93116	37
24	.36488	.39190	2.5517	.93106	36
25	.36515	.39223	2.5495	.93095	**35**
26	.36542	.39257	2.5473	.93084	34
27	.36569	.39290	2.5452	.93074	33
28	.36596	.39324	2.5430	.93063	32
29	.36623	.39357	2.5408	.93052	31
30	.36650	.39391	2.5386	.93042	**30**
31	.36677	.39425	2.5365	.93031	29
32	.36704	.39458	2.5343	.93020	28
33	.36731	.39492	2.5322	.93010	27
34	.36758	.39526	2.5300	.92999	26
35	.36785	.39559	2.5279	.92988	**25**
36	.36812	.39593	2.5257	.92978	24
37	.36839	.39626	2.5236	.92967	23
38	.36867	.39660	2.5214	.92956	22
39	.36894	.39694	2.5193	.92945	21
40	.36921	.39727	2.5172	.92935	**20**
41	.36948	.39761	2.5150	.92924	19
42	.36975	.39795	2.5129	.92913	18
43	.37002	.39829	2.5108	.92902	17
44	.37029	.39862	2.5086	.92892	16
45	.37056	.39896	2.5065	.92881	**15**
46	.37083	.39930	2.5044	.92870	14
47	.37110	.39963	2.5023	.92859	13
48	.37137	.39997	2.5002	.92849	12
49	.37164	.40031	2.4981	.92838	11
50	.37191	.40065	2.4960	.92827	**10**
51	.37218	.40098	2.4939	.92816	9
52	.37245	.40132	2.4918	.92805	8
53	.37272	.40166	2.4897	.92794	7
54	.37299	.40200	2.4876	.92784	6
55	.37326	.40234	2.4855	.92773	**5**
56	.37353	.40267	2.4834	.92762	4
57	.37380	.40301	2.4813	.92751	3
58	.37407	.40335	2.4792	.92740	2
59	.37434	.40369	2.4772	.92729	1
60	.37461	.40403	2.4751	.92718	**0**
′	Cos	Cot	Tan	Sin	′

111° (291°) (248°) **68°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

22° (202°) (337°) **157°**

′	Sin	Tan	Cot	Cos	′
0	.37461	.40403	2.4751	.92718	**60**
1	.37488	.40436	2.4730	.92707	59
2	.37515	.40470	2.4709	.92697	58
3	.37542	.40504	2.4689	.92686	57
4	.37569	.40538	2.4668	.92675	56
5	.37595	.40572	2.4648	.92664	**55**
6	.37622	.40606	2.4627	.92653	54
7	.37649	.40640	2.4606	.92642	53
8	.37676	.40674	2.4586	.92631	52
9	.37703	.40707	2.4566	.92620	51
10	.37730	.40741	2.4545	.92609	**50**
11	.37757	.40775	2.4525	.92598	49
12	.37784	.40809	2.4504	.92587	48
13	.37811	.40843	2.4484	.92576	47
14	.37838	.40877	2.4464	.92565	46
15	.37865	.40911	2.4443	.92554	**45**
16	.37892	.40945	2.4423	.92543	44
17	.37919	.40979	2.4403	.92532	43
18	.37946	.41013	2.4383	.92521	42
19	.37973	.41047	2.4362	.92510	41
20	.37999	.41081	2.4342	.92499	**40**
21	.38026	.41115	2.4322	.92488	39
22	.38053	.41149	2.4302	.92477	38
23	.38080	.41183	2.4282	.92466	37
24	.38107	.41217	2.4262	.92455	36
25	.38134	.41251	2.4242	.92444	**35**
26	.38161	.41285	2.4222	.92432	34
27	.38188	.41319	2.4202	.92421	33
28	.38215	.41353	2.4182	.92410	32
29	.38241	.41387	2.4162	.92399	31
30	.38268	.41421	2.4142	.92388	**30**
31	.38295	.41455	2.4122	.92377	29
32	.38322	.41490	2.4102	.92366	28
33	.38349	.41524	2.4083	.92355	27
34	.38376	.41558	2.4063	.92343	26
35	.38403	.41592	2.4043	.92332	**25**
36	.38430	.41626	2.4023	.92321	24
37	.38456	.41660	2.4004	.92310	23
38	.38483	.41694	2.3984	.92299	22
39	.38510	.41728	2.3964	.92287	21
40	.38537	.41763	2.3945	.92276	**20**
41	.38564	.41797	2.3925	.92265	19
42	.38591	.41831	2.3906	.92254	18
43	.38617	.41865	2.3886	.92243	17
44	.38644	.41899	2.3867	.92231	16
45	.38671	.41933	2.3847	.92220	**15**
46	.38698	.41968	2.3828	.92209	14
47	.38725	.42002	2.3808	.92198	13
48	.38752	.42036	2.3789	.92186	12
49	.38778	.42070	2.3770	.92175	11
50	.38805	.42105	2.3750	.92164	**10**
51	.38832	.42139	2.3731	.92152	9
52	.38859	.42173	2.3712	.92141	8
53	.38886	.42207	2.3693	.92130	7
54	.38912	.42242	2.3673	.92119	6
55	.38939	.42276	2.3654	.92107	**5**
56	.38966	.42310	2.3635	.92096	4
57	.38993	.42345	2.3616	.92085	3
58	.39020	.42379	2.3597	.92073	2
59	.39046	.42413	2.3578	.92062	1
60	.39073	.42447	2.3559	.92050	**0**
′	Cos	Cot	Tan	Sin	′

112° (292°) (247°) **67°**

23° (203°) (336°) **156°**

′	Sin	Tan	Cot	Cos	′
0	.39073	.42447	2.3559	.92050	**60**
1	.39100	.42482	2.3539	.92039	59
2	.39127	.42516	2.3520	.92028	58
3	.39153	.42551	2.3501	.92016	57
4	.39180	.42585	2.3483	.92005	56
5	.39207	.42619	2.3464	.91994	**55**
6	.39234	.42654	2.3445	.91982	54
7	.39260	.42688	2.3426	.91971	53
8	.39287	.42722	2.3407	.91959	52
9	.39314	.42757	2.3388	.91948	51
10	.39341	.42791	2.3369	.91936	**50**
11	.39367	.42826	2.3351	.91925	49
12	.39394	.42860	2.3332	.91914	48
13	.39421	.42894	2.3313	.91902	47
14	.39448	.42929	2.3294	.91891	46
15	.39474	.42963	2.3276	.91879	**45**
16	.39501	.42998	2.3257	.91868	44
17	.39528	.43032	2.3238	.91856	43
18	.39555	.43067	2.3220	.91845	42
19	.39581	.43101	2.3201	.91833	41
20	.39608	.43136	2.3183	.91822	**40**
21	.39635	.43170	2.3164	.91810	39
22	.39661	.43205	2.3146	.91799	38
23	.39688	.43239	2.3127	.91787	37
24	.39715	.43274	2.3109	.91775	36
25	.39741	.43308	2.3090	.91764	**35**
26	.39768	.43343	2.3072	.91752	34
27	.39795	.43378	2.3053	.91741	33
28	.39822	.43412	2.3035	.91729	32
29	.39848	.43447	2.3017	.91718	31
30	.39875	.43481	2.2998	.91706	**30**
31	.39902	.43516	2.2980	.91694	29
32	.39928	.43550	2.2962	.91683	28
33	.39955	.43585	2.2944	.91671	27
34	.30982	.43620	2.2925	.91660	26
35	.40008	.43654	2.2907	.91648	**25**
36	.40035	.43689	2.2889	.91636	24
37	.40062	.43724	2.2871	.91625	23
38	.40088	.43758	2.2853	.91613	22
39	.40115	.43793	2.2835	.91601	21
40	.40141	.43828	2.2817	.91590	**20**
41	.40168	.43862	2.2799	.91578	19
42	.40195	.43897	2.2781	.91566	18
43	.40221	.43932	2.2763	.91555	17
44	.40248	.43966	2.2745	.91543	16
45	.40275	.44001	2.2727	.91531	**15**
46	.40301	.44036	2.2709	.91519	14
47	.40328	.44071	2.2691	.91508	13
48	.40355	.44105	2.2673	.91496	12
49	.40381	.44140	2.2655	.91484	11
50	.40408	.44175	2.2637	.91472	**10**
51	.40434	.44210	2.2620	.91461	9
52	.40461	.44244	2.2602	.91449	8
53	.40488	.44279	2.2584	.91437	7
54	.40514	.44314	2.2566	.91425	6
55	.40541	.44349	2.2549	.91414	**5**
56	.40567	.44384	2.2531	.91402	4
57	.40594	.44418	2.2513	.91390	3
58	.40621	.44453	2.2496	.91378	2
59	.40647	.44488	2.2478	.91366	1
60	.40674	.44523	2.2460	.91355	**0**
′	Cos	Cot	Tan	Sin	′

113° (293°) (246°) **66°**

24° (204°) (335°) **155°**

′	Sin	Tan	Cot	Cos	′
0	.40674	.44523	2.2460	.91355	60
1	.40700	.44558	2.2443	.91343	59
2	.40727	.44593	2.2425	.91331	58
3	.40753	.44627	2.2408	.91319	57
4	.40780	.44662	2.2390	.91307	56
5	.40806	.44697	2.2373	.91295	55
6	.40833	.44732	2.2355	.91283	54
7	.40860	.44767	2.2338	.91272	53
8	.40886	.44802	2.2320	.91260	52
9	.40913	.44837	2.2303	.91248	51
10	.40939	.44872	2.2286	.91236	50
11	.40966	.44907	2.2268	.91224	49
12	.40992	.44942	2.2251	.91212	48
13	.41019	.44977	2.2234	.91200	47
14	.41045	.45012	2.2216	.91188	46
15	.41072	.45047	2.2199	.91176	45
16	.41098	.45082	2.2182	.91164	44
17	.41125	.45117	2.2165	.91152	43
18	.41151	.45152	2.2148	.91140	42
19	.41178	.45187	2.2130	.91128	41
20	.41204	.45222	2.2113	.91116	40
21	.41231	.45257	2.2096	.91104	39
22	.41257	.45292	2.2079	.91092	38
23	.41284	.45327	2.2062	.91080	37
24	.41310	.45362	2.2045	.91068	36
25	.41337	.45397	2.2028	.91056	35
26	.41363	.45432	2.2011	.91044	34
27	.41390	.45467	2.1994	.91032	33
28	.41416	.45502	2.1977	.91020	32
29	.41443	.45538	2.1960	.91008	31
30	.41469	.45573	2.1943	.90996	30
31	.41496	.45608	2.1926	.90984	29
32	.41522	.45643	2.1909	.90972	28
33	.41549	.45678	2.1892	.90960	27
34	.41575	.45713	2.1876	.90948	26
35	.41602	.45748	2.1859	.90936	25
36	.41628	.45784	2.1842	.90924	24
37	.41655	.45819	2.1825	.90911	23
38	.41681	.45854	2.1808	.90899	22
39	.41707	.45889	2.1792	.90887	21
40	.41734	.45924	2.1775	.90875	20
41	.41760	.45960	2.1758	.90863	19
42	.41787	.45995	2.1742	.90851	18
43	.41813	.46030	2.1725	.90839	17
44	.41840	.46065	2.1708	.90826	16
45	.41866	.46101	2.1692	.90814	15
46	.41892	.46136	2.1675	.90802	14
47	.41919	.46171	2.1659	.90790	13
48	.41945	.46206	2.1642	.90778	12
49	.41972	.46242	2.1625	.90766	11
50	.41998	.46277	2.1609	.90753	10
51	.42024	.46312	2.1592	.90741	9
52	.42051	.46348	2.1576	.90729	8
53	.42077	.46383	2.1560	.90717	7
54	.42104	.46418	2.1543	.90704	6
55	.42130	.46454	2.1527	.90692	5
56	.42156	.46489	2.1510	.90680	4
57	.42183	.46525	2.1494	.90668	3
58	.42209	.46560	2.1478	.90655	2
59	.42235	.46595	2.1461	.90643	1
60	.42262	.46631	2.1445	.90631	0
′	Cos	Cot	Tan	Sin	′

114° (294°) (245°) **65°**

25° (205°) (334°) **154°**

′	Sin	Tan	Cot	Cos	′
0	.42262	.46631	2.1445	.90631	60
1	.42288	.46666	2.1429	.90618	59
2	.42315	.46702	2.1413	.90606	58
3	.42341	.46737	2.1396	.90594	57
4	.42367	.46772	2.1380	.90582	56
5	.42394	.46808	2.1364	.90569	55
6	.42420	.46843	2.1348	.90557	54
7	.42446	.46879	2.1332	.90545	53
8	.42473	.46914	2.1315	.90532	52
9	.42499	.46950	2.1299	.90520	51
10	.42525	.46985	2.1283	.90507	50
11	.42552	.47021	2.1267	.90495	49
12	.42578	.47056	2.1251	.90483	48
13	.42604	.47092	2.1235	.90470	47
14	.42631	.47128	2.1219	.90458	46
15	.42657	.47163	2.1203	.90446	45
16	.42683	.47199	2.1187	.90433	44
17	.42709	.47234	2.1171	.90421	43
18	.42736	.47270	2.1155	.90408	42
19	.42762	.47305	2.1139	.90396	41
20	.42788	.47341	2.1123	.90383	40
21	.42815	.47377	2.1107	.90371	39
22	.42841	.47412	2.1092	.90358	38
23	.42867	.47448	2.1076	.90346	37
24	.42894	.47483	2.1060	.90334	36
25	.42920	.47519	2.1044	.90321	35
26	.42946	.47555	2.1028	.90309	34
27	.42972	.47590	2.1013	.90296	33
28	.42999	.47626	2.0997	.90284	32
29	.43025	.47662	2.0981	.90271	31
30	.43051	.47698	2.0965	.90259	30
31	.43077	.47733	2.0950	.90246	29
32	.43104	.47769	2.0934	.90233	28
33	.43130	.47805	2.0918	.90221	27
34	.43156	.47840	2.0903	.90208	26
35	.43182	.47876	2.0887	.90196	25
36	.43209	.47912	2.0872	.90183	24
37	.43235	.47948	2.0856	.90171	23
38	.43261	.47984	2.0840	.90158	22
39	.43287	.48019	2.0825	.90146	21
40	.43313	.48055	2.0809	.90133	20
41	.43340	.48091	2.0794	.90120	19
42	.43366	.48127	2.0778	.90108	18
43	.43392	.48163	2.0763	.90095	17
44	.43418	.48198	2.0748	.90082	16
45	.43445	.48234	2.0732	.90070	15
46	.43471	.48270	2.0717	.90057	14
47	.43497	.48306	2.0701	.90045	13
48	.43523	.48342	2.0686	.90032	12
49	.43549	.48378	2.0671	.90019	11
50	.43575	.48414	2.0655	.90007	10
51	.43602	.48450	2.0640	.89994	9
52	.43628	.48486	2.0625	.89981	8
53	.43654	.48521	2.0609	.89968	7
54	.43680	.48557	2.0594	.89956	6
55	.43706	.48593	2.0579	.89943	5
56	.43733	.48629	2.0564	.89930	4
57	.43759	.48665	2.0549	.89918	3
58	.43785	.48701	2.0533	.89905	2
59	.43811	.48737	2.0518	.89892	1
60	.43837	.48773	2.0503	.89879	0
′	Cos	Cot	Tan	Sin	′

115° (295°) (244°) **64°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

26° (206°) (333°) **153°**

′	Sin	Tan	Cot	Cos	′
0	.43837	.48773	2.0503	.89879	60
1	.43863	.48809	2.0488	.89867	59
2	.43889	.48845	2.0473	.89854	58
3	.43916	.48881	2.0458	.89841	57
4	.43942	.48917	2.0443	.89828	56
5	.43968	.48953	2.0428	.89816	55
6	.43994	.48989	2.0413	.89803	54
7	.44020	.49026	2.0398	.89790	53
8	.44046	.49062	2.0383	.89777	52
9	.44072	.49098	2.0368	.89764	51
10	.44098	.49134	2.0353	.89752	50
11	.44124	.49170	2.0338	.89739	49
12	.44151	.49206	2.0323	.89726	48
13	.44177	.49242	2.0308	.89713	47
14	.44203	.49278	2.0293	.89700	46
15	.44229	.49315	2.0278	.89687	45
16	.44255	.49351	2.0263	.89674	44
17	.44281	.49387	2.0248	.89662	43
18	.44307	.49423	2.0233	.89649	42
19	.44333	.49459	2.0219	.89636	41
20	.44359	.49495	2.0204	.89623	40
21	.44385	.49532	2.0189	.89610	39
22	.44411	.49568	2.0174	.89597	38
23	.44437	.49604	2.0160	.89584	37
24	.44464	.49640	2.0145	.89571	36
25	.44490	.49677	2.0130	.89558	35
26	.44516	.49713	2.0115	.89545	34
27	.44542	.49749	2.0101	.89532	33
28	.44568	.49786	2.0086	.89519	32
29	.44594	.49822	2.0072	.89506	31
30	.44620	.49858	2.0057	.89493	30
31	.44646	.49894	2.0042	.89480	29
32	.44672	.49931	2.0028	.89467	28
33	.44698	.49967	2.0013	.89454	27
34	.44724	.50004	1.9999	.89441	26
35	.44750	.50040	1.9984	.89428	25
36	.44776	.50076	1.9970	.89415	24
37	.44802	.50113	1.9955	.89402	23
38	.44828	.50149	1.9941	.89389	22
39	.44854	.50185	1.9926	.89376	21
40	.44880	.50222	1.9912	.89363	20
41	.44906	.50258	1.9897	.89350	19
42	.44932	.50295	1.9883	.89337	18
43	.44958	.50331	1.9868	.89324	17
44	.44984	.50368	1.9854	.89311	16
45	.45010	.50404	1.9840	.89298	15
46	.45036	.50441	1.9825	.89285	14
47	.45062	.50477	1.9811	.89272	13
48	.45088	.50514	1.9797	.89259	12
49	.45114	.50550	1.9782	.89245	11
50	.45140	.50587	1.9768	.89232	10
51	.45166	.50623	1.9754	.89219	9
52	.45192	.50660	1.9740	.89206	8
53	.45218	.50696	1.9725	.89193	7
54	.45243	.50733	1.9711	.89180	6
55	.45269	.50769	1.9697	.89167	5
56	.45295	.50806	1.9683	.89153	4
57	.45321	.50843	1.9669	.89140	3
58	.45347	.50879	1.9654	.89127	2
59	.45373	.50916	1.9640	.89114	1
60	.45399	.50953	1.9626	.89101	0
′	Cos	Cot	Tan	Sin	′

116° (296°) (243°) **63°**

27° (207°) (332°) **152°**

′	Sin	Tan	Cot	Cos	′
0	.45399	.50953	1.9626	.89101	60
1	.45425	.50989	1.9612	.89087	59
2	.45451	.51026	1.9598	.89074	58
3	.45477	.51063	1.9584	.89061	57
4	.45503	.51099	1.9570	.89048	56
5	.45529	.51136	1.9556	.89035	55
6	.45554	.51173	1.9542	.89021	54
7	.45580	.51209	1.9528	.89008	53
8	.45606	.51246	1.9514	.88995	52
9	.45632	.51283	1.9500	.88981	51
10	.45658	.51319	1.9486	.88968	50
11	.45684	.51356	1.9472	.88955	49
12	.45710	.51393	1.9458	.88942	48
13	.45736	.51430	1.9444	.88928	47
14	.45762	.51467	1.9430	.88915	46
15	.45787	.51503	1.9416	.88902	45
16	.45813	.51540	1.9402	.88888	44
17	.45839	.51577	1.9388	.88875	43
18	.45865	.51614	1.9375	.88862	42
19	.45891	.51651	1.9361	.88848	41
20	.45917	.51688	1.9347	.88835	40
21	.45942	.51724	1.9333	.88822	39
22	.45968	.51761	1.9319	.88808	38
23	.45994	.51798	1.9306	.88795	37
24	.46020	.51835	1.9292	.88782	36
25	.46046	.51872	1.9278	.88768	35
26	.46072	.51909	1.9265	.88755	34
27	.46097	.51946	1.9251	.88741	33
28	.46123	.51983	1.9237	.88728	32
29	.46149	.52020	1.9223	.88715	31
30	.46175	.52057	1.9210	.88701	30
31	.46201	.52094	1.9196	.88688	29
32	.46226	.52131	1.9183	.88674	28
33	.46252	.52168	1.9169	.88661	27
34	.46278	.52205	1.9155	.88647	26
35	.46304	.52242	1.9142	.88634	25
36	.46330	.52279	1.9128	.88620	24
37	.46355	.52316	1.9115	.88607	23
38	.46381	.52353	1.9101	.88593	22
39	.46407	.52390	1.9088	.88580	21
40	.46433	.52427	1.9074	.88566	20
41	.46458	.52464	1.9061	.88553	19
42	.46484	.52501	1.9047	.88539	18
43	.46510	.52538	1.9034	.88526	17
44	.46536	.52575	1.9020	.88512	16
45	.46561	.52613	1.9007	.88499	15
46	.46587	.52650	1.8993	.88485	14
47	.46613	.52687	1.8980	.88472	13
48	.46639	.52724	1.8967	.88458	12
49	.46664	.52761	1.8953	.88445	11
50	.46690	.52798	1.8940	.88431	10
51	.46716	.52836	1.8927	.88417	9
52	.46742	.52873	1.8913	.88404	8
53	.46767	.52910	1.8900	.88390	7
54	.46793	.52947	1.8887	.88377	6
55	.46819	.52985	1.8873	.88363	5
56	.46844	.53022	1.8860	.88349	4
57	.46870	.53059	1.8847	.88336	3
58	.46896	.53096	1.8834	.88322	2
59	.46921	.53134	1.8820	.88308	1
60	.46947	.53171	1.8807	.88295	0
′	Cos	Cot	Tan	Sin	′

117° (297°) (242°) **62°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

28° (208°) — (331°) **151°**

′	Sin	Tan	Cot	Cos	′
0	.46947	.53171	1.8807	.88295	**60**
1	.46973	.53208	1.8794	.88281	59
2	.46999	.53246	1.8781	.88267	58
3	.47024	.53283	1.8768	.88254	57
4	.47050	.53320	1.8755	.88240	56
5	.47076	.53358	1.8741	.88226	**55**
6	.47101	.53395	1.8728	.88213	54
7	.47127	.53432	1.8715	.88199	53
8	.47153	.53470	1.8702	.88185	52
9	.47178	.53507	1.8689	.88172	51
10	.47204	.53545	1.8676	.88158	**50**
11	.47229	.53582	1.8663	.88144	49
12	.47255	.53620	1.8650	.88130	48
13	.47281	.53657	1.8637	.88117	47
14	.47306	.53694	1.8624	.88103	46
15	.47332	.53732	1.8611	.88089	**45**
16	.47358	.53769	1.8598	.88075	44
17	.47383	.53807	1.8585	.88062	43
18	.47409	.53844	1.8572	.88048	42
19	.47434	.53882	1.8559	.88034	41
20	.47460	.53920	1.8546	.88020	**40**
21	.47486	.53957	1.8533	.88006	39
22	.47511	.53995	1.8520	.87993	38
23	.47537	.54032	1.8507	.87979	37
24	.47562	.54070	1.8495	.87965	36
25	.47588	.54107	1.8482	.87951	**35**
26	.47614	.54145	1.8469	.87937	34
27	.47639	.54183	1.8456	.87923	33
28	.47665	.54220	1.8443	.87909	32
29	.47690	.54258	1.8430	.87896	31
30	.47716	.54296	1.8418	.87882	**30**
31	.47741	.54333	1.8405	.87868	29
32	.47767	.54371	1.8392	.87854	28
33	.47793	.54409	1.8379	.87840	27
34	.47818	.54446	1.8367	.87826	26
35	.47844	.54484	1.8354	.87812	**25**
36	.47869	.54522	1.8341	.87798	24
37	.47895	.54560	1.8329	.87784	23
38	.47920	.54597	1.8316	.87770	22
39	.47946	.54635	1.8303	.87756	21
40	.47971	.54673	1.8291	.87743	**20**
41	.47997	.54711	1.8278	.87729	19
42	.48022	.54748	1.8265	.87715	18
43	.48048	.54786	1.8253	.87701	17
44	.48073	.54824	1.8240	.87687	16
45	.48099	.54862	1.8228	.87673	**15**
46	.48124	.54900	1.8215	.87659	14
47	.48150	.54938	1.8202	.87645	13
48	.48175	.54975	1.8190	.87631	12
49	.48201	.55013	1.8177	.87617	11
50	.48226	.55051	1.8165	.87603	**10**
51	.48252	.55089	1.8152	.87589	9
52	.48277	.55127	1.8140	.87575	8
53	.48303	.55165	1.8127	.87561	7
54	.48328	.55203	1.8115	.87546	6
55	.48354	.55241	1.8103	.87532	**5**
56	.48379	.55279	1.8090	.87518	4
57	.48405	.55317	1.8078	.87504	3
58	.48430	.55355	1.8065	.87490	2
59	.48456	.55393	1.8053	.87476	1
60	.48481	.55431	1.8040	.87462	**0**
′	Cos	Cot	Tan	Sin	′

118° (298°) — (241°) **61°**

29° (209°) — (330°) **150°**

′	Sin	Tan	Cot	Cos	′
0	.48481	.55431	1.8040	.87462	**60**
1	.48506	.55469	1.8028	.87448	59
2	.48532	.55507	1.8016	.87434	58
3	.48557	.55545	1.8003	.87420	57
4	.48583	.55583	1.7991	.87406	56
5	.48608	.55621	1.7979	.87391	**55**
6	.48634	.55659	1.7966	.87377	54
7	.48659	.55697	1.7954	.87363	53
8	.48684	.55736	1.7942	.87349	52
9	.48710	.55774	1.7930	.87335	51
10	.48735	.55812	1.7917	.87321	**50**
11	.48761	.55850	1.7905	.87306	49
12	.48786	.55888	1.7893	.87292	48
13	.48811	.55926	1.7881	.87278	47
14	.48837	.55964	1.7868	.87264	46
15	.48862	.56003	1.7856	.87250	**45**
16	.48888	.56041	1.7844	.87235	44
17	.48913	.56079	1.7832	.87221	43
18	.48938	.56117	1.7820	.87207	42
19	.48964	.56156	1.7808	.87193	41
20	.48989	.56194	1.7796	.87178	**40**
21	.49014	.56232	1.7783	.87164	39
22	.49040	.56270	1.7771	.87150	38
23	.49065	.56309	1.7759	.87136	37
24	.49090	.56347	1.7747	.87121	36
25	.49116	.56385	1.7735	.87107	**35**
26	.49141	.56424	1.7723	.87093	34
27	.49166	.56462	1.7711	.87079	33
28	.49192	.56501	1.7699	.87064	32
29	.49217	.56539	1.7687	.87050	31
30	.49242	.56577	1.7675	.87036	**30**
31	.49268	.56616	1.7663	.87021	29
32	.49293	.56654	1.7651	.87007	28
33	.49318	.56693	1.7639	.86993	27
34	.49344	.56731	1.7627	.86978	26
35	.49369	.56769	1.7615	.86964	**25**
36	.49394	.56808	1.7603	.86949	24
37	.49419	.56846	1.7591	.86935	23
38	.49445	.56885	1.7579	.86921	22
39	.49470	.56923	1.7567	.86906	21
40	.49495	.56962	1.7556	.86892	**20**
41	.49521	.57000	1.7544	.86878	19
42	.49546	.57039	1.7532	.86863	18
43	.49571	.57078	1.7520	.86849	17
44	.49596	.57116	1.7508	.86834	16
45	.49622	.57155	1.7496	.86820	**15**
46	.49647	.57193	1.7485	.86805	14
47	.49672	.57232	1.7473	.86791	13
48	.49697	.57271	1.7461	.86777	12
49	.49723	.57309	1.7449	.86762	11
50	.49748	.57348	1.7437	.86748	**10**
51	.49773	.57386	1.7426	.86733	9
52	.49798	.57425	1.7414	.86719	8
53	.49824	.57464	1.7402	.86704	7
54	.49849	.57503	1.7391	.86690	6
55	.49874	.57541	1.7379	.86675	**5**
56	.49899	.57580	1.7367	.86661	4
57	.49924	.57619	1.7355	.86646	3
58	.49950	.57657	1.7344	.86632	2
59	.49975	.57696	1.7332	.86617	1
60	.50000	.57735	1.7321	.86603	**0**
′	Cos	Cot	Tan	Sin	′

119° (299°) — (240°) **60°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

30° (210°) — (329°) **149°**

′	Sin	Tan	Cot	Cos	′
0	.50000	.57735	1.7321	.86603	**60**
1	.50025	.57774	1.7309	.86588	59
2	.50050	.57813	1.7297	.86573	58
3	.50076	.57851	1.7286	.86559	57
4	.50101	.57890	1.7274	.86544	56
5	.50126	.57929	1.7262	.86530	**55**
6	.50151	.57968	1.7251	.86515	54
7	.50176	.58007	1.7239	.86501	53
8	.50201	.58046	1.7228	.86486	52
9	.50227	.58085	1.7216	.86471	51
10	.50252	.58124	1.7205	.86457	**50**
11	.50277	.58162	1.7193	.86442	49
12	.50302	.58201	1.7182	.86427	48
13	.50327	.58240	1.7170	.86413	47
14	.50352	.58279	1.7159	.86398	46
15	.50377	.58318	1.7147	.86384	**45**
16	.50403	.58357	1.7136	.86369	44
17	.50428	.58396	1.7124	.86354	43
18	.50453	.58435	1.7113	.86340	42
19	.50478	.58474	1.7102	.86325	41
20	.50503	.58513	1.7090	.86310	**40**
21	.50528	.58552	1.7079	.86295	39
22	.50553	.58591	1.7067	.86281	38
23	.50578	.58631	1.7056	.86266	37
24	.50603	.58670	1.7045	.86251	36
25	.50628	.58709	1.7033	.86237	**35**
26	.50654	.58748	1.7022	.86222	34
27	.50679	.58787	1.7011	.86207	33
28	.50704	.58826	1.6999	.86192	32
29	.50729	.58865	1.6988	.86178	31
30	.50754	.58905	1.6977	.86163	**30**
31	.50779	.58944	1.6965	.86148	29
32	.50804	.58983	1.6954	.86133	28
33	.50829	.59022	1.6943	.86119	27
34	.50854	.59061	1.6932	.86104	26
35	.50879	.59101	1.6920	.86089	**25**
36	.50904	.59140	1.6909	.86074	24
37	.50929	.59179	1.6898	.86059	23
38	.50954	.59218	1.6887	.86045	22
39	.50979	.59258	1.6875	.86030	21
40	.51004	.59297	1.6864	.86015	**20**
41	.51029	.59336	1.6853	.86000	19
42	.51054	.59376	1.6842	.85985	18
43	.51079	.59415	1.6831	.85970	17
44	.51104	.59454	1.6820	.85956	16
45	.51129	.59494	1.6808	.85941	**15**
46	.51154	.59533	1.6797	.85926	14
47	.51179	.59573	1.6786	.85911	13
48	.51204	.59612	1.6775	.85896	12
49	.51229	.59651	1.6764	.85881	11
50	.51254	.59691	1.6753	.85866	**10**
51	.51279	.59730	1.6742	.85851	9
52	.51304	.59770	1.6731	.85836	8
53	.51329	.59809	1.6720	.85821	7
54	.51354	.59849	1.6709	.85806	6
55	.51379	.59888	1.6698	.85792	**5**
56	.51404	.59928	1.6687	.85777	4
57	.51429	.59967	1.6676	.85762	3
58	.51454	.60007	1.6665	.85747	2
59	.51479	.60046	1.6654	.85732	1
60	.51504	.60086	1.6643	.85717	**0**
′	Cos	Cot	Tan	Sin	′

120° (300°) — (239°) **59°**

31° (211°) — (328°) **148°**

′	Sin	Tan	Cot	Cos	′
0	.51504	.60086	1.6643	.85717	**60**
1	.51529	.60126	1.6632	.85702	59
2	.51554	.60165	1.6621	.85687	58
3	.51579	.60205	1.6610	.85672	57
4	.51604	.60245	1.6599	.85657	56
5	.51628	.60284	1.6588	.85642	**55**
6	.51653	.60324	1.6577	.85627	54
7	.51678	.60364	1.6566	.85612	53
8	.51703	.60403	1.6555	.85597	52
9	.51728	.60443	1.6545	.85582	51
10	.51753	.60483	1.6534	.85567	**50**
11	.51778	.60522	1.6523	.85551	49
12	.51803	.60562	1.6512	.85536	48
13	.51828	.60602	1.6501	.85521	47
14	.51852	.60642	1.6490	.85506	46
15	.51877	.60681	1.6479	.85491	**45**
16	.51902	.60721	1.6469	.85476	44
17	.51927	.60761	1.6458	.85461	43
18	.51952	.60801	1.6447	.85446	42
19	.51977	.60841	1.6436	.85431	41
20	.52002	.60881	1.6426	.85416	**40**
21	.52026	.60921	1.6415	.85401	39
22	.52051	.60960	1.6404	.85385	38
23	.52076	.61000	1.6393	.85370	37
24	.52101	.61040	1.6383	.85355	36
25	.52126	.61080	1.6372	.85340	**35**
26	.52151	.61120	1.6361	.85325	34
27	.52175	.61160	1.6351	.85310	33
28	.52200	.61200	1.6340	.85294	32
29	.52225	.61240	1.6329	.85279	31
30	.52250	.61280	1.6319	.85264	**30**
31	.52275	.61320	1.6308	.85249	29
32	.52299	.61360	1.6297	.85234	28
33	.52324	.61400	1.6287	.85218	27
34	.52349	.61440	1.6276	.85203	26
35	.52374	.61480	1.6265	.85188	**25**
36	.52399	.61520	1.6255	.85173	24
37	.52423	.61561	1.6244	.85157	23
38	.52448	.61601	1.6234	.85142	22
39	.52473	.61641	1.6223	.85127	21
40	.52498	.61681	1.6212	.85112	**20**
41	.52522	.61721	1.6202	.85096	19
42	.52547	.61761	1.6191	.85081	18
43	.52572	.61801	1.6181	.85066	17
44	.52597	.61842	1.6170	.85051	16
45	.52621	.61882	1.6160	.85035	**15**
46	.52646	.61922	1.6149	.85020	14
47	.52671	.61962	1.6139	.85005	13
48	.52696	.62003	1.6128	.84989	12
49	.52720	.62043	1.6118	.84974	11
50	.52745	.62083	1.6107	.84959	**10**
51	.52770	.62124	1.6097	.84943	9
52	.52794	.62164	1.6087	.84928	8
53	.52819	.62204	1.6076	.84913	7
54	.52844	.62245	1.6066	.84897	6
55	.52869	.62285	1.6055	.84882	**5**
56	.52893	.62325	1.6045	.84866	4
57	.52918	.62366	1.6034	.84851	3
58	.52943	.62406	1.6024	.84836	2
59	.52967	.62446	1.6014	.84820	1
60	.52992	.62487	1.6003	.84805	**0**
′	Cos	Cot	Tan	Sin	′

121° (301°) — (238°) **58°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

32° (212°) (327°) **147°**

′	Sin	Tan	Cot	Cos	′
0	.52992	.62487	1.6003	.84805	60
1	.53017	.62527	1.5993	.84789	59
2	.53041	.62568	1.5983	.84774	58
3	.53066	.62608	1.5972	.84759	57
4	.53091	.62649	1.5962	.84743	56
5	.53115	.62689	1.5952	.84728	55
6	.53140	.62730	1.5941	.84712	54
7	.53164	.62770	1.5931	.84697	53
8	.53189	.62811	1.5921	.84681	52
9	.53214	.62852	1.5911	.84666	51
10	.53238	.62892	1.5900	.84650	50
11	.53263	.62933	1.5890	.84635	49
12	.53288	.62973	1.5880	.84619	48
13	.53312	.63014	1.5869	.84604	47
14	.53337	.63055	1.5859	.84588	46
15	.53361	.63095	1.5849	.84573	45
16	.53386	.63136	1.5839	.84557	44
17	.53411	.63177	1.5829	.84542	43
18	.53435	.63217	1.5818	.84526	42
19	.53460	.63258	1.5808	.84511	41
20	.53484	.63299	1.5798	.84495	40
21	.53509	.63340	1.5788	.84480	39
22	.53534	.63380	1.5778	.84464	38
23	.53558	.63421	1.5768	.84448	37
24	.53583	.63462	1.5757	.84433	36
25	.53607	.63503	1.5747	.84417	35
26	.53632	.63544	1.5737	.84402	34
27	.53656	.63584	1.5727	.84386	33
28	.53681	.63625	1.5717	.84370	32
29	.53705	.63666	1.5707	.84355	31
30	.53730	.63707	1.5697	.84339	30
31	.53754	.63748	1.5687	.84324	29
32	.53779	.63789	1.5677	.84308	28
33	.53804	.63830	1.5667	.84292	27
34	.53828	.63871	1.5657	.84277	26
35	.53853	.63912	1.5647	.84261	25
36	.53877	.63953	1.5637	.84245	24
37	.53902	.63994	1.5627	.84230	23
38	.53926	.64035	1.5617	.84214	22
39	.53951	.64076	1.5607	.84198	21
40	.53975	.64117	1.5597	.84182	20
41	.54000	.64158	1.5587	.84167	19
42	.54024	.64199	1.5577	.84151	18
43	.54049	.64240	1.5567	.84135	17
44	.54073	.64281	1.5557	.84120	16
45	.54097	.64322	1.5547	.84104	15
46	.54122	.64363	1.5537	.84088	14
47	.54146	.64404	1.5527	.84072	13
48	.54171	.64446	1.5517	.84057	12
49	.54195	.64487	1.5507	.84041	11
50	.54220	.64528	1.5497	.84025	10
51	.54244	.64569	1.5487	.84009	9
52	.54269	.64610	1.5477	.83994	8
53	.54293	.64652	1.5468	.83978	7
54	.54317	.64693	1.5458	.83962	6
55	.54342	.64734	1.5448	.83946	5
56	.54366	.64775	1.5438	.83930	4
57	.54391	.64817	1.5428	.83915	3
58	.54415	.64858	1.5418	.83899	2
59	.54440	.64899	1.5408	.83883	1
60	.54464	.64941	1.5399	.83867	0
′	Cos	Cot	Tan	Sin	′

122° (302°) (237°) **57°**

33° (213°) (326°) **146°**

′	Sin	Tan	Cot	Cos	′
0	.54464	.64941	1.5399	.83867	60
1	.54488	.64982	1.5389	.83851	59
2	.54513	.65024	1.5379	.83835	58
3	.54537	.65065	1.5369	.83819	57
4	.54561	.65106	1.5359	.83804	56
5	.54586	.65148	1.5350	.83788	55
6	.54610	.65189	1.5340	.83772	54
7	.54635	.65231	1.5330	.83756	53
8	.54659	.65272	1.5320	.83740	52
9	.54683	.65314	1.5311	.83724	51
10	.54708	.65355	1.5301	.83708	50
11	.54732	.65397	1.5291	.83692	49
12	.54756	.65438	1.5282	.83676	48
13	.54781	.65480	1.5272	.83660	47
14	.54805	.65521	1.5262	.83645	46
15	.54829	.65563	1.5253	.83629	45
16	.54854	.65604	1.5243	.83613	44
17	.54878	.65646	1.5233	.83597	43
18	.54902	.65688	1.5224	.83581	42
19	.54927	.65729	1.5214	.83565	41
20	.54951	.65771	1.5204	.83549	40
21	.54975	.65813	1.5195	.83533	39
22	.54999	.65854	1.5185	.83517	38
23	.55024	.65896	1.5175	.83501	37
24	.55048	.65938	1.5166	.83485	36
25	.55072	.65980	1.5156	.83469	35
26	.55097	.66021	1.5147	.83453	34
27	.55121	.66063	1.5137	.83437	33
28	.55145	.66105	1.5127	.83421	32
29	.55169	.66147	1.5118	.83405	31
30	.55194	.66189	1.5108	.83389	30
31	.55218	.66230	1.5099	.83373	29
32	.55242	.66272	1.5089	.83356	28
33	.55266	.66314	1.5080	.83340	27
34	.55291	.66356	1.5070	.83324	26
35	.55315	.66398	1.5061	.83308	25
36	.55339	.66440	1.5051	.83292	24
37	.55363	.66482	1.5042	.83276	23
38	.55388	.66524	1.5032	.83260	22
39	.55412	.66566	1.5023	.83244	21
40	.55436	.66608	1.5013	.83228	20
41	.55460	.66650	1.5004	.83212	19
42	.55484	.66692	1.4994	.83195	18
43	.55509	.66734	1.4985	.83179	17
44	.55533	.66776	1.4975	.83163	16
45	.55557	.66818	1.4966	.83147	15
46	.55581	.66860	1.4957	.83131	14
47	.55605	.66902	1.4947	.83115	13
48	.55630	.66944	1.4938	.83098	12
49	.55654	.66986	1.4928	.83082	11
50	.55678	.67028	1.4919	.83066	10
51	.55702	.67071	1.4910	.83050	9
52	.55726	.67113	1.4900	.83034	8
53	.55750	.67155	1.4891	.83017	7
54	.55775	.67197	1.4882	.83001	6
55	.55799	.67239	1.4872	.82985	5
56	.55823	.67282	1.4863	.82969	4
57	.55847	.67324	1.4854	.82953	3
58	.55871	.67366	1.4844	.82936	2
59	.55895	.67409	1.4835	.82920	1
60	.55919	.67451	1.4826	.82904	0
′	Cos	Cot	Tan	Sin	′

123° (303°) (236°) **56°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

34° (214°) (325°) **145°**

′	Sin	Tan	Cot	Cos	′
0	.55919	.67451	1.4826	.82904	60
1	.55943	.67493	1.4816	.82887	59
2	.55968	.67536	1.4807	.82871	58
3	.55992	.67578	1.4798	.82855	57
4	.56016	.67620	1.4788	.82839	56
5	.56040	.67663	1.4779	.82822	55
6	.56064	.67705	1.4770	.82806	54
7	.56088	.67748	1.4761	.82790	53
8	.56112	.67790	1.4751	.82773	52
9	.56136	.67832	1.4742	.82757	51
10	.56160	.67875	1.4733	.82741	50
11	.56184	.67917	1.4724	.82724	49
12	.56208	.67960	1.4715	.82708	48
13	.56232	.68002	1.4705	.82692	47
14	.56256	.68045	1.4696	.82675	46
15	.56280	.68088	1.4687	.82659	45
16	.56305	.68130	1.4678	.82643	44
17	.56329	.68173	1.4669	.82626	43
18	.56353	.68215	1.4659	.82610	42
19	.56377	.68258	1.4650	.82593	41
20	.56401	.68301	1.4641	.82577	40
21	.56425	.68343	1.4632	.82561	39
22	.56449	.68386	1.4623	.82544	38
23	.56473	.68429	1.4614	.82528	37
24	.56497	.68471	1.4605	.82511	36
25	.56521	.68514	1.4596	.82495	35
26	.56545	.68557	1.4586	.82478	34
27	.56569	.68600	1.4577	.82462	33
28	.56593	.68642	1.4568	.82446	32
29	.56617	.68685	1.4559	.82429	31
30	.56641	.68728	1.4550	.82413	30
31	.56665	.68771	1.4541	.82396	29
32	.56689	.68814	1.4532	.82380	28
33	.56713	.68857	1.4523	.82363	27
34	.56736	.68900	1.4514	.82347	26
35	.56760	.68942	1.4505	.82330	25
36	.56784	.68985	1.4496	.82314	24
37	.56808	.69028	1.4487	.82297	23
38	.56832	.69071	1.4478	.82281	22
39	.56856	.69114	1.4469	.82264	21
40	.56880	.69157	1.4460	.82248	20
41	.56904	.69200	1.4451	.82231	19
42	.56928	.69243	1.4442	.82214	18
43	.56952	.69286	1.4433	.82198	17
44	.56976	.69329	1.4424	.82181	16
45	.57000	.69372	1.4415	.82165	15
46	.57024	.69416	1.4406	.82148	14
47	.57047	.69459	1.4397	.82132	13
48	.57071	.69502	1.4388	.82115	12
49	.57095	.69545	1.4379	.82098	11
50	.57119	.69588	1.4370	.82082	10
51	.57143	.69631	1.4361	.82065	9
52	.57167	.69675	1.4352	.82048	8
53	.57191	.69718	1.4344	.82032	7
54	.57215	.69761	1.4335	.82015	6
55	.57238	.69804	1.4326	.81999	5
56	.57262	.69847	1.4317	.81982	4
57	.57286	.69891	1.4308	.81965	3
58	.57310	.69934	1.4299	.81949	2
59	.57334	.69977	1.4290	.81932	1
60	.57358	.70021	1.4281	.81915	0
′	Cos	Cot	Tan	Sin	′

124° (304°) (235°) **55°**

35° (215°) (324°) **144°**

′	Sin	Tan	Cot	Cos	′
0	.57358	.70021	1.4281	.81915	60
1	.57381	.70064	1.4273	.81899	59
2	.57405	.70107	1.4264	.81882	58
3	.57429	.70151	1.4255	.81865	57
4	.57453	.70194	1.4246	.81848	56
5	.57477	.70238	1.4237	.81832	55
6	.57501	.70281	1.4229	.81815	54
7	.57524	.70325	1.4220	.81798	53
8	.57548	.70368	1.4211	.81782	52
9	.57572	.70412	1.4202	.81765	51
10	.57596	.70455	1.4193	.81748	50
11	.57619	.70499	1.4185	.81731	49
12	.57643	.70542	1.4176	.81714	48
13	.57667	.70586	1.4167	.81698	47
14	.57691	.70629	1.4158	.81681	46
15	.57715	.70673	1.4150	.81664	45
16	.57738	.70717	1.4141	.81647	44
17	.57762	.70760	1.4132	.81631	43
18	.57786	.70804	1.4124	.81614	42
19	.57810	.70848	1.4115	.81597	41
20	.57833	.70891	1.4106	.81580	40
21	.57857	.70935	1.4097	.81563	39
22	.57881	.70979	1.4089	.81546	38
23	.57904	.71023	1.4080	.81530	37
24	.57928	.71066	1.4071	.81513	36
25	.57952	.71110	1.4063	.81496	35
26	.57976	.71154	1.4054	.81479	34
27	.57999	.71198	1.4045	.81462	33
28	.58023	.71242	1.4037	.81445	32
29	.58047	.71285	1.4028	.81428	31
30	.58070	.71329	1.4019	.81412	30
31	.58094	.71373	1.4011	.81395	29
32	.58118	.71417	1.4002	.81378	28
33	.58141	.71461	1.3994	.81361	27
34	.58165	.71505	1.3985	.81344	26
35	.58189	.71549	1.3976	.81327	25
36	.58212	.71593	1.3968	.81310	24
37	.58236	.71637	1.3959	.81293	23
38	.58260	.71681	1.3951	.81276	22
39	.58283	.71725	1.3942	.81259	21
40	.58307	.71769	1.3934	.81242	20
41	.58330	.71813	1.3925	.81225	19
42	.58354	.71857	1.3916	.81208	18
43	.58378	.71901	1.3908	.81191	17
44	.58401	.71946	1.3899	.81174	16
45	.58425	.71990	1.3891	.81157	15
46	.58449	.72034	1.3882	.81140	14
47	.58472	.72078	1.3874	.81123	13
48	.58496	.72122	1.3865	.81106	12
49	.58519	.72167	1.3857	.81089	11
50	.58543	.72211	1.3848	.81072	10
51	.58567	.72255	1.3840	.81055	9
52	.58590	.72299	1.3831	.81038	8
53	.58614	.72344	1.3823	.81021	7
54	.58637	.72388	1.3814	.81004	6
55	.58661	.72432	1.3806	.80987	5
56	.58684	.72477	1.3798	.80970	4
57	.58708	.72521	1.3789	.80953	3
58	.58731	.72565	1.3781	.80936	2
59	.58755	.72610	1.3772	.80919	1
60	.58779	.72654	1.3764	.80902	0
′	Cos	Cot	Tan	Sin	′

125° (305°) (234°) **54°**

36° (216°) (323°) **143°**

′	Sin	Tan	Cot	Cos	′
0	.58779	.72654	1.3764	.80902	60
1	.58802	.72699	1.3755	.80885	59
2	.58826	.72743	1.3747	.80867	58
3	.58849	.72788	1.3739	.80850	57
4	.58873	.72832	1.3730	.80833	56
5	.58896	.72877	1.3722	.80816	55
6	.58920	.72921	1.3713	.80799	54
7	.58943	.72966	1.3705	.80782	53
8	.58967	.73010	1.3697	.80765	52
9	.58990	.73055	1.3688	.80748	51
10	.59014	.73100	1.3680	.80730	50
11	.59037	.73144	1.3672	.80713	49
12	.59061	.73189	1.3663	.80696	48
13	.59084	.73234	1.3655	.80679	47
14	.59108	.73278	1.3647	.80662	46
15	.59131	.73323	1.3638	.80644	45
16	.59154	.73368	1.3630	.80627	44
17	.59178	.73413	1.3622	.80610	43
18	.59201	.73457	1.3613	.80593	42
19	.59225	.73502	1.3605	.80576	41
20	.59248	.73547	1.3597	.80558	40
21	.59272	.73592	1.3588	.80541	39
22	.59295	.73637	1.3580	.80524	38
23	.59318	.73681	1.3572	.80507	37
24	.59342	.73726	1.3564	.80489	36
25	.59365	.73771	1.3555	.80472	35
26	.59389	.73816	1.3547	.80455	34
27	.59412	.73861	1.3539	.80438	33
28	.59436	.73906	1.3531	.80420	32
29	.59459	.73951	1.3522	.80403	31
30	.59482	.73996	1.3514	.80386	30
31	.59506	.74041	1.3506	.80368	29
32	.59529	.74086	1.3498	.80351	28
33	.59552	.74131	1.3490	.80334	27
34	.59576	.74176	1.3481	.80316	26
35	.59599	.74221	1.3473	.80299	25
36	.59622	.74267	1.3465	.80282	24
37	.59646	.74312	1.3457	.80264	23
38	.59669	.74357	1.3449	.80247	22
39	.59693	.74402	1.3440	.80230	21
40	.59716	.74447	1.3432	.80212	20
41	.59739	.74492	1.3424	.80195	19
42	.59763	.74538	1.3416	.80178	18
43	.59786	.74583	1.3408	.80160	17
44	.59809	.74628	1.3400	.80143	16
45	.59832	.74674	1.3392	.80125	15
46	.59856	.74719	1.3384	.80108	14
47	.59879	.74764	1.3375	.80091	13
48	.59902	.74810	1.3367	.80073	12
49	.59926	.74855	1.3359	.80056	11
50	.59949	.74900	1.3351	.80038	10
51	.59972	.74946	1.3343	.80021	9
52	.59995	.74991	1.3335	.80003	8
53	.60019	.75037	1.3327	.79986	7
54	.60042	.75082	1.3319	.79968	6
55	.60065	.75128	1.3311	.79951	5
56	.60089	.75173	1.3303	.79934	4
57	.60112	.75219	1.3295	.79916	3
58	.60135	.75264	1.3287	.79899	2
59	.60158	.75310	1.3278	.79881	1
60	.60182	.75355	1.3270	.79864	0
′	Cos	Cot	Tan	Sin	′

126° (306°) (233°) **53°**

37° (217°) (322°) **142°**

′	Sin	Tan	Cot	Cos	′
0	.60182	.75355	1.3270	.79864	60
1	.60205	.75401	1.3262	.79846	59
2	.60228	.75447	1.3254	.79829	58
3	.60251	.75492	1.3246	.79811	57
4	.60274	.75538	1.3238	.79793	56
5	.60298	.75584	1.3230	.79776	55
6	.60321	.75629	1.3222	.79758	54
7	.60344	.75675	1.3214	.79741	53
8	.60367	.75721	1.3206	.79723	52
9	.60390	.75767	1.3198	.79706	51
10	.60414	.75812	1.3190	.79688	50
11	.60437	.75858	1.3182	.79671	49
12	.60460	.75904	1.3175	.79653	48
13	.60483	.75950	1.3167	.79635	47
14	.60506	.75996	1.3159	.79618	46
15	.60529	.76042	1.3151	.79600	45
16	.60553	.76088	1.3143	.79583	44
17	.60576	.76134	1.3135	.79565	43
18	.60599	.76180	1.3127	.79547	42
19	.60622	.76226	1.3119	.79530	41
20	.60645	.76272	1.3111	.79512	40
21	.60668	.76318	1.3103	.79494	39
22	.60691	.76364	1.3095	.79477	38
23	.60714	.76410	1.3087	.79459	37
24	.60738	.76456	1.3079	.79441	36
25	.60761	.76502	1.3072	.79424	35
26	.60784	.76548	1.3064	.79406	34
27	.60807	.76594	1.3056	.79388	33
28	.60830	.76640	1.3048	.79371	32
29	.60853	.76686	1.3040	.79353	31
30	.60876	.76733	1.3032	.79335	30
31	.60899	.76779	1.3024	.79318	29
32	.60922	.76825	1.3017	.79300	28
33	.60945	.76871	1.3009	.79282	27
34	.60968	.76918	1.3001	.79264	26
35	.60991	.76964	1.2993	.79247	25
36	.61015	.77010	1.2985	.79229	24
37	.61038	.77057	1.2977	.79211	23
38	.61061	.77103	1.2970	.79193	22
39	.61084	.77149	1.2962	.79176	21
40	.61107	.77196	1.2954	.79158	20
41	.61130	.77242	1.2946	.79140	19
42	.61153	.77289	1.2938	.79122	18
43	.61176	.77335	1.2931	.79105	17
44	.61199	.77382	1.2923	.79087	16
45	.61222	.77428	1.2915	.79069	15
46	.61245	.77475	1.2907	.79051	14
47	.61268	.77521	1.2900	.79033	13
48	.61291	.77568	1.2892	.79016	12
49	.61314	.77615	1.2884	.78998	11
50	.61337	.77661	1.2876	.78980	10
51	.61360	.77708	1.2869	.78962	9
52	.61383	.77754	1.2861	.78944	8
53	.61406	.77801	1.2853	.78926	7
54	.61429	.77848	1.2846	.78908	6
55	.61451	.77895	1.2838	.78891	5
56	.61474	.77941	1.2830	.78873	4
57	.61497	.77988	1.2822	.78855	3
58	.61520	.78035	1.2815	.78837	2
59	.61543	.78082	1.2807	.78819	1
60	.61566	.78129	1.2799	.78801	0
′	Cos	Cot	Tan	Sin	′

127° (307°) (232°) **52°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

38° (218°) (321°) **141°**

′	Sin	Tan	Cot	Cos	′
0	.61566	.78129	1.2799	.78801	60
1	.61589	.78175	1.2792	.78783	59
2	.61612	.78222	1.2784	.78765	58
3	.61635	.78269	1.2776	.78747	57
4	.61658	.78316	1.2769	.78729	56
5	.61681	.78363	1.2761	.78711	55
6	.61704	.78410	1.2753	.78694	54
7	.61726	.78457	1.2746	.78676	53
8	.61749	.78504	1.2738	.78658	52
9	.61772	.78551	1.2731	.78640	51
10	.61795	.78598	1.2723	.78622	50
11	.61818	.78645	1.2715	.78604	49
12	.61841	.78692	1.2708	.78586	48
13	.61864	.78739	1.2700	.78568	47
14	.61887	.78786	1.2693	.78550	46
15	.61909	.78834	1.2685	.78532	45
16	.61932	.78881	1.2677	.78514	44
17	.61955	.78928	1.2670	.78496	43
18	.61978	.78975	1.2662	.78478	42
19	.62001	.79022	1.2655	.78460	41
20	.62024	.79070	1.2647	.78442	40
21	.62046	.79117	1.2640	.78424	39
22	.62069	.79164	1.2632	.78405	38
23	.62092	.79212	1.2624	.78387	37
24	.62115	.79259	1.2617	.78369	36
25	.62138	.79306	1.2609	.78351	35
26	.62160	.79354	1.2602	.78333	34
27	.62183	.79401	1.2594	.78315	33
28	.62206	.79449	1.2587	.78297	32
29	.62229	.79496	1.2579	.78279	31
30	.62251	.79544	1.2572	.78261	30
31	.62274	.79591	1.2564	.78243	29
32	.62297	.79639	1.2557	.78225	28
33	.62320	.79686	1.2549	.78206	27
34	.62342	.79734	1.2542	.78188	26
35	.62365	.79781	1.2534	.78170	25
36	.62388	.79829	1.2527	.78152	24
37	.62411	.79877	1.2519	.78134	23
38	.62433	.79924	1.2512	.78116	22
39	.62456	.79972	1.2504	.78098	21
40	.62479	.80020	1.2497	.78079	20
41	.62502	.80067	1.2489	.78061	19
42	.62524	.80115	1.2482	.78043	18
43	.62547	.80163	1.2475	.78025	17
44	.62570	.80211	1.2467	.78007	16
45	.62592	.80258	1.2460	.77988	15
46	.62615	.80306	1.2452	.77970	14
47	.62638	.80354	1.2445	.77952	13
48	.62660	.80402	1.2437	.77934	12
49	.62683	.80450	1.2430	.77916	11
50	.62706	.80498	1.2423	.77897	10
51	.62728	.80546	1.2415	.77879	9
52	.62751	.80594	1.2408	.77861	8
53	.62774	.80642	1.2401	.77843	7
54	.62796	.80690	1.2393	.77824	6
55	.62819	.80738	1.2386	.77806	5
56	.62842	.80786	1.2378	.77788	4
57	.62864	.80834	1.2371	.77769	3
58	.62887	.80882	1.2364	.77751	2
59	.62909	.80930	1.2356	.77733	1
60	.62932	.80978	1.2349	.77715	0
′	Cos	Cot	Tan	Sin	′

128° (308°) (231°) **51°**

39° (219°) (320°) **140°**

′	Sin	Tan	Cot	Cos	′
0	.62932	.80978	1.2349	.77715	60
1	.62955	.81027	1.2342	.77696	59
2	.62977	.81075	1.2334	.77678	58
3	.63000	.81123	1.2327	.77660	57
4	.63022	.81171	1.2320	.77641	56
5	.63045	.81220	1.2312	.77623	55
6	.63068	.81268	1.2305	.77605	54
7	.63090	.81316	1.2298	.77586	53
8	.63113	.81364	1.2290	.77568	52
9	.63135	.81413	1.2283	.77550	51
10	.63158	.81461	1.2276	.77531	50
11	.63180	.81510	1.2268	.77513	49
12	.63203	.81558	1.2261	.77494	48
13	.63225	.81606	1.2254	.77476	47
14	.63248	.81655	1.2247	.77458	46
15	.63271	.81703	1.2239	.77439	45
16	.63293	.81752	1.2232	.77421	44
17	.63316	.81800	1.2225	.77402	43
18	.63338	.81849	1.2218	.77384	42
19	.63361	.81898	1.2210	.77366	41
20	.63383	.81946	1.2203	.77347	40
21	.63406	.81995	1.2196	.77329	39
22	.63428	.82044	1.2189	.77310	38
23	.63451	.82092	1.2181	.77292	37
24	.63473	.82141	1.2174	.77273	36
25	.63496	.82190	1.2167	.77255	35
26	.63518	.82238	1.2160	.77236	34
27	.63540	.82287	1.2153	.77218	33
28	.63563	.82336	1.2145	.77199	32
29	.63585	.82385	1.2138	.77181	31
30	.63608	.82434	1.2131	.77162	30
31	.63630	.82483	1.2124	.77144	29
32	.63653	.82531	1.2117	.77125	28
33	.63675	.82580	1.2109	.77107	27
34	.63698	.82629	1.2102	.77088	26
35	.63720	.82678	1.2095	.77070	25
36	.63742	.82727	1.2088	.77051	24
37	.63765	.82776	1.2081	.77033	23
38	.63787	.82825	1.2074	.77014	22
39	.63810	.82874	1.2066	.76996	21
40	.63832	.82923	1.2059	.76977	20
41	.63854	.82972	1.2052	.76959	19
42	.63877	.83022	1.2045	.76940	18
43	.63899	.83071	1.2038	.76921	17
44	.63922	.83120	1.2031	.76903	16
45	.63944	.83169	1.2024	.76884	15
46	.63966	.83218	1.2017	.76866	14
47	.63989	.83268	1.2009	.76847	13
48	.64011	.83317	1.2002	.76828	12
49	.64033	.83366	1.1995	.76810	11
50	.64056	.83415	1.1988	.76791	10
51	.64078	.83465	1.1981	.76772	9
52	.64100	.83514	1.1974	.76754	8
53	.64123	.83564	1.1967	.76735	7
54	.64145	.83613	1.1960	.76717	6
55	.64167	.83662	1.1953	.76698	5
56	.64190	.83712	1.1946	.76679	4
57	.64212	.83761	1.1939	.76661	3
58	.64234	.83811	1.1932	.76642	2
59	.64256	.83860	1.1925	.76623	1
60	.64279	.83910	1.1918	.76604	0
′	Cos	Cot	Tan	Sin	′

129° (309°) (230°) **50°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

40° (220°) (319°) **139°**

′	Sin	Tan	Cot	Cos	′
0	.64279	.83910	1.1918	.76604	**60**
1	.64301	.83960	1.1910	.76586	59
2	.64323	.84009	1.1903	.76567	58
3	.64346	.84059	1.1896	.76548	57
4	.64368	.84108	1.1889	.76530	56
5	.64390	.84158	1.1882	.76511	**55**
6	.64412	.84208	1.1875	.76492	54
7	.64435	.84258	1.1868	.76473	53
8	.64457	.84307	1.1861	.76455	52
9	.64479	.84357	1.1854	.76436	51
10	.64501	.84407	1.1847	.76417	**50**
11	.64524	.84457	1.1840	.76398	49
12	.64546	.84507	1.1833	.76380	48
13	.64568	.84556	1.1826	.76361	47
14	.64590	.84606	1.1819	.76342	46
15	.64612	.84656	1.1812	.76323	**45**
16	.64635	.84706	1.1806	.76304	44
17	.64657	.84756	1.1799	.76286	43
18	.64679	.84806	1.1792	.76267	42
19	.64701	.84856	1.1785	.76248	41
20	.64723	.84906	1.1778	.76229	**40**
21	.64746	.84956	1.1771	.76210	39
22	.64768	.85006	1.1764	.76192	38
23	.64790	.85057	1.1757	.76173	37
24	.64812	.85107	1.1750	.76154	36
25	.64834	.85157	1.1743	.76135	**35**
26	.64856	.85207	1.1736	.76116	34
27	.64878	.85257	1.1729	.76097	33
28	.64901	.85308	1.1722	.76078	32
29	.64923	.85358	1.1715	.76059	31
30	.64945	.85408	1.1708	.76041	**30**
31	.64967	.85458	1.1702	.76022	29
32	.64989	.85509	1.1695	.76003	28
33	.65011	.85559	1.1688	.75984	27
34	.65033	.85609	1.1681	.75965	26
35	.65055	.85660	1.1674	.75946	**25**
36	.65077	.85710	1.1667	.75927	24
37	.65100	.85761	1.1660	.75908	23
38	.65122	.85811	1.1653	.75889	22
39	.65144	.85862	1.1647	.75870	21
40	.65166	.85912	1.1640	.75851	**20**
41	.65188	.85963	1.1633	.75832	19
42	.65210	.86014	1.1626	.75813	18
43	.65232	.86064	1.1619	.75794	17
44	.65254	.86115	1.1612	.75775	16
45	.65276	.86166	1.1606	.75756	**15**
46	.65298	.86216	1.1599	.75738	14
47	.65320	.86267	1.1592	.75719	13
48	.65342	.86318	1.1585	.75700	12
49	.65364	.86368	1.1578	.75680	11
50	.65386	.86419	1.1571	.75661	**10**
51	.65408	.86470	1.1565	.75642	9
52	.65430	.86521	1.1558	.75623	8
53	.65452	.86572	1.1551	.75604	7
54	.65474	.86623	1.1544	.75585	6
55	.65496	.86674	1.1538	.75566	**5**
56	.65518	.86725	1.1531	.75547	4
57	.65540	.86776	1.1524	.75528	3
58	.65562	.86827	1.1517	.75509	2
59	.65584	.86878	1.1510	.75490	1
60	.65606	.86929	1.1504	.75471	**0**
′	Cos	Cot	Tan	Sin	′

130° (310°) (229°) **49°**

41° (221°) (318°) **138°**

′	Sin	Tan	Cot	Cos	′
0	.65606	.86929	1.1504	.75471	**60**
1	.65628	.86980	1.1497	.75452	59
2	.65650	.87031	1.1490	.75433	58
3	.65672	.87082	1.1483	.75414	57
4	.65694	.87133	1.1477	.75395	56
5	.65716	.87184	1.1470	.75375	**55**
6	.65738	.87236	1.1463	.75356	54
7	.65759	.87287	1.1456	.75337	53
8	.65781	.87338	1.1450	.75318	52
9	.65803	.87389	1.1443	.75299	51
10	.65825	.87441	1.1436	.75280	**50**
11	.65847	.87492	1.1430	.75261	49
12	.65869	.87543	1.1423	.75241	48
13	.65891	.87595	1.1416	.75222	47
14	.65913	.87646	1.1410	.75203	46
15	.65935	.87698	1.1403	.75184	**45**
16	.65956	.87749	1.1396	.75165	44
17	.65978	.87801	1.1389	.75146	43
18	.66000	.87852	1.1383	.75126	42
19	.66022	.87904	1.1376	.75107	41
20	.66044	.87955	1.1369	.75088	**40**
21	.66066	.88007	1.1363	.75069	39
22	.66088	.88059	1.1356	.75050	38
23	.66109	.88110	1.1349	.75030	37
24	.66131	.88162	1.1343	.75011	36
25	.66153	.88214	1.1336	.74992	**35**
26	.66175	.88265	1.1329	.74973	34
27	.66197	.88317	1.1323	.74953	33
28	.66218	.88369	1.1316	.74934	32
29	.66240	.88421	1.1310	.74915	31
30	.66262	.88473	1.1303	.74896	**30**
31	.66284	.88524	1.1296	.74876	29
32	.66306	.88576	1.1290	.74857	28
33	.66327	.88628	1.1283	.74838	27
34	.66349	.88680	1.1276	.74818	26
35	.66371	.88732	1.1270	.74799	**25**
36	.66393	.88784	1.1263	.74780	24
37	.66414	.88836	1.1257	.74760	23
38	.66436	.88888	1.1250	.74741	22
39	.66458	.88940	1.1243	.74722	21
40	.66480	.88992	1.1237	.74703	**20**
41	.66501	.80045	1.1230	.74683	19
42	.66523	.89097	1.1224	.74664	18
43	.66545	.89149	1.1217	.74644	17
44	.66566	.89201	1.1211	.74625	16
45	.66588	.89253	1.1204	.74606	**15**
46	.66610	.89306	1.1197	.74586	14
47	.66632	.89358	1.1191	.74567	13
48	.66653	.89410	1.1184	.74548	12
49	.66675	.89463	1.1178	.74528	11
50	.66697	.89515	1.1171	.74509	**10**
51	.66718	.89567	1.1165	.74489	9
52	.66740	.89620	1.1158	.74470	8
53	.66762	.89672	1.1152	.74451	7
54	.66783	.89725	1.1145	.74431	6
55	.66805	.89777	1.1139	.74412	**5**
56	.66827	.89830	1.1132	.74392	4
57	.66848	.89883	1.1126	.74373	3
58	.66870	.89935	1.1119	.74353	2
59	.66891	.89988	1.1113	.74334	1
60	.66913	.90040	1.1106	.74314	**0**
′	Cos	Cot	Tan	Sin	′

131° (311°) (228°) **48°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

42° (222°) (317°) **137°**

′	Sin	Tan	Cot	Cos	′
0	.66913	.90040	1.1106	.74314	**60**
1	.66935	.90093	1.1100	.74295	59
2	.66956	.90146	1.1093	.74276	58
3	.66978	.90199	1.1087	.74256	57
4	.66999	.90251	1.1080	.74237	56
5	.67021	.90304	1.1074	.74217	**55**
6	.67043	.90357	1.1067	.74198	54
7	.67064	.90410	1.1061	.74178	53
8	.67086	.90463	1.1054	.74159	52
9	.67107	.90516	1.1048	.74139	51
10	.67129	.90569	1.1041	.74120	**50**
11	.67151	.90621	1.1035	.74100	49
12	.67172	.90674	1.1028	.74080	48
13	.67194	.90727	1.1022	.74061	47
14	.67215	.90781	1.1016	.74041	46
15	.67237	.90834	1.1009	.74022	**45**
16	.67258	.90887	1.1003	.74002	44
17	.67280	.90940	1.0996	.73983	43
18	.67301	.90993	1.0990	.73963	42
19	.67323	.91046	1.0983	.73944	41
20	.67344	.91099	1.0977	.73924	**40**
21	.67366	.91153	1.0971	.73904	39
22	.67387	.91206	1.0964	.73885	38
23	.67409	.91259	1.0958	.73865	37
24	.67430	.91313	1.0951	.73846	36
25	.67452	.91366	1.0945	.73826	**35**
26	.67473	.91419	1.0939	.73806	34
27	.67495	.91473	1.0932	.73787	33
28	.67516	.91526	1.0926	.73767	32
29	.67538	.91580	1.0919	.73747	31
30	.67559	.91633	1.0913	.73728	**30**
31	.67580	.91687	1.0907	.73708	29
32	.67602	.91740	1.0900	.73688	28
33	.67623	.91794	1.0894	.73669	27
34	.67645	.91847	1.0888	.73649	26
35	.67666	.91901	1.0881	.73629	**25**
36	.67688	.91955	1.0875	.73610	24
37	.67709	.92008	1.0869	.73590	23
38	.67730	.92062	1.0862	.73570	22
39	.67752	.92116	1.0856	.73551	21
40	.67773	.92170	1.0850	.73531	**20**
41	.67795	.92224	1.0843	.73511	19
42	.67816	.92277	1.0837	.73491	18
43	.67837	.92331	1.0831	.73472	17
44	.67859	.92385	1.0824	.73452	16
45	.67880	.92439	1.0818	.73432	**15**
46	.67901	.92493	1.0812	.73413	14
47	.67923	.92547	1.0805	.73393	13
48	.67944	.92601	1.0799	.73373	12
49	.67965	.92655	1.0793	.73353	11
50	.67987	.92709	1.0786	.73333	**10**
51	.68008	.92763	1.0780	.73314	9
52	.68029	.92817	1.0774	.73294	8
53	.68051	.92872	1.0768	.73274	7
54	.68072	.92926	1.0761	.73254	6
55	.68093	.92980	1.0755	.73234	**5**
56	.68115	.93034	1.0749	.73215	4
57	.68136	.93088	1.0742	.73195	3
58	.68157	.93143	1.0736	.73175	2
59	.68179	.93197	1.0730	.73155	1
60	.68200	.93252	1.0724	.73135	**0**
′	Cos	Cot	Tan	Sin	′

132° (312°) (227°) **47°**

43° (223°) (316°) **136°**

′	Sin	Tan	Cot	Cos	′
0	.68200	.93252	1.0724	.73135	**60**
1	.68221	.93306	1.0717	.73116	59
2	.68242	.93360	1.0711	.73096	58
3	.68264	.93415	1.0705	.73076	57
4	.68285	.93469	1.0699	.73056	56
5	.68306	.93524	1.0692	.73036	**55**
6	.68327	.93578	1.0686	.73016	54
7	.68349	.93633	1.0680	.72996	53
8	.68370	.93688	1.0674	.72976	52
9	.68391	.93742	1.0668	.72957	51
10	.68412	.93797	1.0661	.72937	**50**
11	.68434	.93852	1.0655	.72917	49
12	.68455	.93906	1.0649	.72897	48
13	.68476	.93961	1.0643	.72877	47
14	.68497	.94016	1.0637	.72857	46
15	.68518	.94071	1.0630	.72837	**45**
16	.68539	.94125	1.0624	.72817	44
17	.68561	.94180	1.0618	.72797	43
18	.68582	.94235	1.0612	.72777	42
19	.68603	.94290	1.0606	.72757	41
20	.68624	.94345	1.0599	.72737	**40**
21	.68645	.94400	1.0593	.72717	39
22	.68666	.94455	1.0587	.72697	38
23	.68688	.94510	1.0581	.72677	37
24	.68709	.94565	1.0575	.72657	36
25	.68730	.94620	1.0569	.72637	**35**
26	.68751	.94676	1.0562	.72617	34
27	.68772	.94731	1.0556	.72597	33
28	.68793	.94786	1.0550	.72577	32
29	.68814	.94841	1.0544	.72557	31
30	.68835	.94896	1.0538	.72537	**30**
31	.68857	.94952	1.0532	.72517	29
32	.68878	.95007	1.0526	.72497	28
33	.68899	.95062	1.0519	.72477	27
34	.68920	.95118	1.0513	.72457	26
35	.68941	.95173	1.0507	.72437	**25**
36	.68962	.95229	1.0501	.72417	24
37	.68983	.95284	1.0495	.72397	23
38	.69004	.95340	1.0489	.72377	22
39	.69025	.95395	1.0483	.72357	21
40	.69046	.95451	1.0477	.72337	**20**
41	.69067	.95506	1.0470	.72317	19
42	.69088	.95562	1.0464	.72297	18
43	.69109	.95618	1.0458	.72277	17
44	.69130	.95673	1.0452	.72257	16
45	.69151	.95729	1.0446	.72236	**15**
46	.69172	.95785	1.0440	.72216	14
47	.69193	.95841	1.0434	.72196	13
48	.69214	.95897	1.0428	.72176	12
49	.69235	.95952	1.0422	.72156	11
50	.69256	.96008	1.0416	.72136	**10**
51	.69277	.96064	1.0410	.72116	9
52	.69298	.96120	1.0404	.72095	8
53	.69319	.96176	1.0398	.72075	7
54	.69340	.96232	1.0392	.72055	6
55	.69361	.96288	1.0385	.72035	**5**
56	.69382	.96344	1.0379	.72015	4
57	.69403	.96400	1.0373	.71995	3
58	.69424	.96457	1.0367	.71974	2
59	.69445	.96513	1.0361	.71954	1
60	.69466	.96569	1.0355	.71934	**0**
′	Cos	Cot	Tan	Sin	′

133° (313°) (226°) **46°**

NATURAL TRIGONOMETRIC FUNCTIONS (Continued)

44° (224°) (315°) **135°**

′	Sin	Tan	Cot	Cos	′
0	.69466	.96569	1.0355	.71934	**60**
1	.69487	.96625	1.0349	.71914	59
2	.69508	.96681	1.0343	.71894	58
3	.69529	.96738	1.0337	.71873	57
4	.69549	.96794	1.0331	.71853	56
5	.69570	.96850	1.0325	.71833	**55**
6	.69591	.96907	1.0319	.71813	54
7	.69612	.96963	1.0313	.71792	53
8	.69633	.97020	1.0307	.71772	52
9	.69654	.97076	1.0301	.71752	51
10	.69675	.97133	1.0295	.71732	**50**
11	.69696	.97189	1.0289	.71711	49
12	.69717	.97246	1.0283	.71691	48
13	.69737	.97302	1.0277	.71671	47
14	.69758	.97359	1.0271	.71650	46
15	.69779	.97416	1.0265	.71630	**45**
16	.69800	.97472	1.0259	.71610	44
17	.69821	.97529	1.0253	.71590	43
18	.69842	.97586	1.0247	.71569	42
19	.69862	.97643	1.0241	.71549	41
20	.69883	.97700	1.0235	.71529	**40**
21	.69904	.97756	1.0230	.71508	39
22	.69925	.97813	1.0224	.71488	38
23	.69946	.97870	1.0218	.71468	37
24	.69966	.97927	1.0212	.71447	36
25	.69987	.97984	1.0206	.71427	**35**
26	.70008	.98041	1.0200	.71407	34
27	.70029	.98098	1.0194	.71386	33
28	.70049	.98155	1.0188	.71366	32
29	.70070	.98213	1.0182	.71345	31
30	.70091	.98270	1.0176	.71325	**30**
31	.70112	.98327	1.0170	.71305	29
32	.70132	.98384	1.0164	.71284	28
33	.70153	.98441	1.0158	.71264	27
34	.70174	.98499	1.0152	.71243	26
35	.70195	.98556	1.0147	.71223	**25**
36	.70215	.98613	1.0141	.71203	24
37	.70236	.98671	1.0135	.71182	23
38	.70257	.98728	1.0129	.71162	22
39	.70277	.98786	1.0123	.71141	21
40	.70298	.98843	1.0117	.71121	**20**
41	.70319	.98901	1.0111	.71100	19
42	.70339	.98958	1.0105	.71080	18
43	.70360	.99016	1.0099	.71059	17
44	.70381	.99073	1.0094	.71039	16
45	.70401	.99131	1.0088	.71019	**15**
46	.70422	.99189	1.0082	.70998	14
47	.70443	.99247	1.0076	.70978	13
48	.70463	.99304	1.0070	.70957	12
49	.70484	.99362	1.0064	.70937	11
50	.70505	.99420	1.0058	.70916	**10**
51	.70525	.99478	1.0052	.70896	9
52	.70546	.99536	1.0047	.70875	8
53	.70567	.99594	1.0041	.70855	7
54	.70587	.99652	1.0035	.70834	6
55	.70608	.99710	1.0029	.70813	**5**
56	.70628	.99768	1.0023	.70793	4
57	.70649	.99826	1.0017	.70772	3
58	.70670	.99884	1.0012	.70752	2
59	.70690	.99942	1.0006	.70731	1
60	.70711	1.0000	1.0000	.70711	**0**
′	Cos	Cot	Tan	Sin	′

134° (314°) (225°) **45°**