ADDENDUM TO
HR-273

PILE DESIGN FOR SKEWED INTEGRAL ABUTMENT BRIDGES

DECEMBER 1987

ADDENDUM TO:

PILE DESIGN AND TESTS FOR INTEGRAL ABUTMENT BRIDGES

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TOPIC: PILE DESIGN FOR SKEW INTEGRAL ABUTMENT BRIDGES

1.0 Biaxial Bending - Stress Criteria

Piles in skewed integral abutment bridges may be bent about both the strong and weak axis as the bridge expands and contracts. The design criteria presented in the report can be generalized to biaxial bending for Case A (capacity of the pile as a structural member). Case B and Case C criteria require no generalization.

When the pile undergoes biaxial bending, two separate equivalent cantilevers must be developed. Each can be defined as described in the report, Sec. 5.2.1. For bending about the strong or x axis, the length of the equivalent cantilever will be denoted by L_x and the pile head displacement as Δ_x. For y axis bending, L_y is the length of the equivalent cantilever and Δ_y is the displacement. The corresponding soil stiffnesses are k_{hx} and k_{hy}. The generalized equations for Service Load Design are:

$$\frac{f_a}{F_a} + \frac{C_{mx}\, f_{bx}}{(1 - \frac{f_a}{F_{ex}'})\, F_{bx}} + \frac{C_{my}\, f_{by}}{(1 - \frac{f_a}{F_{ey}'})\, F_{by}} \qquad (1)$$

$$\frac{f_a}{0.472\, F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1 \qquad (2)$$

in which

f_a = applied axial stress

f_{bx}, f_{by} = applied stress for bending about the x and y axes, respectively

F_y = yield stress

F_a = allowable axial stress

F_{bx}, F_{by} = allowable stress for bending about the x and y axes, respectively

F_{ex}', F_{ey}' = Euler buckling stress divided by a factor of safety for buckling about the x and y axes, respectively

C_{mx}, C_{my} = equivalent moment factor for x and y bending, respectively.

1.1. Alternative One

As described in Sec. 5.2.3.1. of the report, Alternative One accounts for first order thermal stresses. The moments for biaxial bending of the fixed-head pile are

$$M_x = \frac{6EI_x\Delta_x}{L_x^2} \tag{3}$$

$$M_y = \frac{6EI_y\Delta_y}{L_y^2} \tag{4}$$

where M_x and M_y are the bending moments and I_x and I_y are the moments of inertia about the strong and weak axis, respectively.

1.2. Alternative Two

In Alternative Two, the stresses in the pile caused by thermal displacement of the bridge are neglected but the PΔ effect is included. As in Sec. 5.2.3.2. of the report, the moments for the fixed-head pile for this alternative are:

$$M_x = P\Delta_x/2 \tag{5}$$

$$M_y = P\Delta_y/2 \tag{6}$$

1.3. Comparison with Finite Element

In previous work conducted for the Iowa Department of Transportation [1], the finite element programs IAB2D and the three-dimensional version, IAB3D, were used to analyze end bearing piles bent about the strong axis (Δ_y equal zero) and about a 45 degree axis (Δ_x equal Δ_y). Fig. 6.20 (a and b) and 6.22 (a and b) in [1] present the results as plots of P/P_o versus Δ_x, in which P is the pile capacity corresponding to Δ_x and P_o is the pile capacity for Δ_x equal zero. Six soil types are presented. These results are compared to Eq. (1) and (2) in the following Fig. 1 and 2, respectively. (For the comparison purposes, the factor of safety has been removed from Eq. (1) & (2)). The conclusions from this comparison are the same as in Sec. 5.2.3.3. of the report:

(1) Both Alternative One and Two are conservative and

(2) Alternative One is very conservative and would dictate relatively short integral abutment bridges.

Alternative Two is recommended if the pile has sufficient inelastic rotation capacity.

2.0. Inelastic Rotation Capacity

Alternative Two requires that the pile have sufficient inelastic rotation capacity to permit some redistribution of the pile forces during the thermal expansion. The following section generalizes the inelastic rotation capacity developments in Sec. 6.1.1.2. of the report to the biaxial case.

2.1. Uniaxial Bending

In this section, the conservative nature of the inelastic rotation capacity in Sec. 6.1.1.2. of the report is demonstrated and a more appropriate value is suggested. The inelastic rotation capacity of a plastic hinge is presented in Eq. (6.51) of the report as

$$\Theta_{iC} = 3\, C_i \;\; \Theta_p \tag{7}$$

in which C_i is an inelastic rotation capacity reduction factor given in Eq. (6.52) of the report and

$$\Theta_p = \frac{M_p \ell_p}{EI} \tag{8}$$

is the elastic rotation corresponding to M_p. In the report, ℓ_p was taken as the length of the plastic hinge [2].

Work by Lukey and Adams [3] indicated that Eq. (7) and (8) are quite conservative. They tested several simply-supported beams with a center concentrated load and plotted ductility ($3\,C_i$) versus b_f/t_f. Eq. (6.52) of the report fits this plot well. Lukey and Adams used the length ℓ_p equal to one-half the span length of the simply supported beams or, in general, the distance from the maximum moment to the inflection point. The resulting rotation Θ_p is the rotation between the tangents to the elastic curve on each side of the plastic hinge. For a fixed head equivalent cantilever, the length ℓ_p is L/2. The inelastic rotation capacity at a plastic hinge at a fixed support will be one-half of the inelastic rotation capacity at the plastic hinge in a simply supported beam, since a fixed beam corresponds to one-half of a symmetrically loaded simple beam. Hence, applying the results in Ref. [3] for a fixed head equivalent cantilever of length L, the inelastic rotation capacity becomes

$$\Theta_{iC} = 3\, C_i \frac{M_p L}{4EI} \tag{9}$$

Other authors [4,5,6] have developed a different expression for inelastic rotation capacity, but the study in Ref. [3] has demonstrated that this expression was non-conservative for large b_f/t_f values such as HP shapes.

As a numerical example, consider an HP10x42 as a fixed head equivalent cantilever with a 12 ft. length and F_y equal to 36 ksi. Eq. (7) and (8), which are from the report, predict an inelastic rotation capacity for strong axis bending of 0.0062 radians. Eq. (9) predicts 0.024 radians, whereas the work in Ref. [4] yields 0.039 radians. Eq. (9) is recommended.

The inelastic rotation demand given by Eq. (6.48) of the report does not account for removing the girder load after the pile has been displaced to Point D' in Fig. 6.6. This unloading will cause an additional rotational demand from Point D' to Point D by an amount Θ_w. Therefore, the total rotational demand can be expressed as

$$\Theta_{iD} = 2 \left(\frac{\Delta}{L} - \frac{M_p L}{6EI} \right) + \Theta_w \qquad (10)$$

Eq. (6.50) in the report is inappropriate. Also, Θ_w should be interpreted as the <u>live</u> load rotation only.

The inelastic rotation demand, Eq. (10), must be less than the inelastic rotation capacity, Eq. (9); therefore,

$$\Delta \leq \Delta_p \left(1 + \frac{9}{4} C_i - \frac{3M_w}{4M_p}\right) \qquad (11)$$

in which

$$\Delta_p = \frac{M_p L^2}{6EI} \qquad (12)$$

and M_w is the live load moment corresponding to Θ_w. Now, if the live load stress is conservatively assumed to be equal to the allowable bending stress of 0.55 F_y, Eq. (11) can be simplified by letting

$$\frac{3M_w}{4M_p} = \frac{3(0.55\ F_y)S}{4\ F_y\ Z} = 0.4 \tag{13}$$

where S is the section modulus. The shape factor (Z/S) has been conservatively taken as one. With this simplification, Eq. (11) can be rewritten with a factor of safety as

$$\Delta \leq \Delta_i = \Delta_b\ (0.6 + 2.25\ C_i) \tag{14}$$

in which, for a fixed-head pile,

$$\Delta_b = \frac{F_b S L^2}{6EI} \tag{15}$$

With appropriate subscripts, Eq. (14) and (15) apply to both the x and y axes.

2.2. Biaxial Bending

No published work could be found which described the inelastic rotation capacity for biaxial bending. In fact, all of the available publications related to strong axis bending only. The development in the previous section implicidly assumed that the published work [3] was a conservative bound of the weak axis case. For biaxial bending, the strain at the extreme fiber of the flange will be assumed to control flange buckling and, thereby, limit the inelastic rotation capacity. The inelastic strain demand at the extreme fiber can be written as

$$\varepsilon_{iD} = \phi_{ixD} \frac{d}{2} + \phi_{iyD} \frac{b}{2} \tag{16}$$

in which ϕ_{ixD} and ϕ_{iyD} are the inelastic curvature demands for the x and y axes, respectively. They are proportional to the inelastic rotation demands Θ_{ixD} and Θ_{iyD}, which are discussed in Sec. 2.1. The criteria that inelastic strain demand, ε_{iD}, be less than the inelastic strain capacity, ε_{iC}, gives

$$\frac{\phi_{ixD}\ (d/2)}{\varepsilon_{iC}} + \frac{\phi_{iyD}\ (b/2)}{\varepsilon_{iC}} \leq 1 \tag{17}$$

or,

$$\frac{\phi_{ixD}}{\phi_{ixC}} + \frac{\phi_{iyD}}{\phi_{iyC}} \leq 1 \tag{18}$$

in which the inelastic curvature capacities are ϕ_{ixC} and ϕ_{iyC} for the x and y axes, respectively. Since curvatures are proportional to rotations, Eq. (18) can also be written as

$$\frac{\Theta_{ixD}}{\Theta_{ixC}} + \frac{\Theta_{iyD}}{\Theta_{iyC}} \leq 1 \tag{19}$$

Substituting Eqs. (9) and (10), specialized for x and y axis bending, into Eq. (19) and applying the conditions expressed in Eqs. (11) and (13), gives

$$\left(\frac{\dfrac{\Delta_x}{\Delta_{px}} - 0.6}{\dfrac{\Delta_{ix}}{\Delta_{px}} - 0.6} \right) + \left(\frac{\dfrac{\Delta_y}{\Delta_{py}} - 0.6}{\dfrac{\Delta_{iy}}{\Delta_{py}} - 0.6} \right) \leq 1 \tag{20}$$

in which Δ_x and Δ_y represent the displacement for bending about the x and y axes, respectively. The quantities Δ_{px}, Δ_{py}, Δ_{ix} and Δ_{iy} are obtained from Eqs. (12) and (14), specialized to the x and y axes. Note that a factor of safety has been incorporated into Eq. (14). For design purposes, Eq. (20) can be conservatively bounded by the simple interaction equation.

$$\frac{\Delta x}{\Delta_{ix}} + \frac{\Delta y}{\Delta_{iy}} \leq 1 \tag{21}$$

which is appropriate for design.

3.0 Summary

In summary, the design criteria for stress in integral abutment piles under biaxial bending are summarized in Eq. (1) and (2). For Alternative One which includes thermal stresses, the bending moments are given by Eq. (3) and (4). Alternative Two which neglects thermal stresses has moments as given in Eq. (5) and (6). The ductility requirements of Alternative Two are satisfied by Eq. (21) where Δ_{ix} and Δ_{iy} are given by Eq. (14).

References

(1) L. F. Greimann, P. S. Yang, S. K. Edmunds, A. M. Wolde-Tinsae, "Design of Piles for Integral Abutment Bridges," Final Report, HR-252, Iowa Department of Transportation, August 1984.

(2) Beedle, L. S., Plastic Design of Steel Frames, John Wiley, 1959, pg. 203.

(3) Lukey, A. F. and Adams, P. F., "Rotation Capacity of Beams Under Moment Gradient," Journal of the Structural Division, ASCE, Vol. 95, ST6, June 1969, pp. 1173-1188.

(4) Lay, M. G. and Galambos, T. V., "Inelastic Beams Under Moment Gradient," Journal of the Structural Division, ASCE, Vol. 93, ST1, Feb. 1967, pp. 381-399.

(5) Galambos, T. V. and Lay, M. G., "Studies of the Ductility of Steel Structures," Journal of the Structural Division, ASCE, Vol. 91, ST4, Aug. 1965, pp. 125-151.

(6) Lay, M. G., "Flange Local Buckling in Wide-Flange Shapes," Journal of the Structural Division, ASCE, Vol. 91, ST6, Dec. 1965, pp. 95-116.

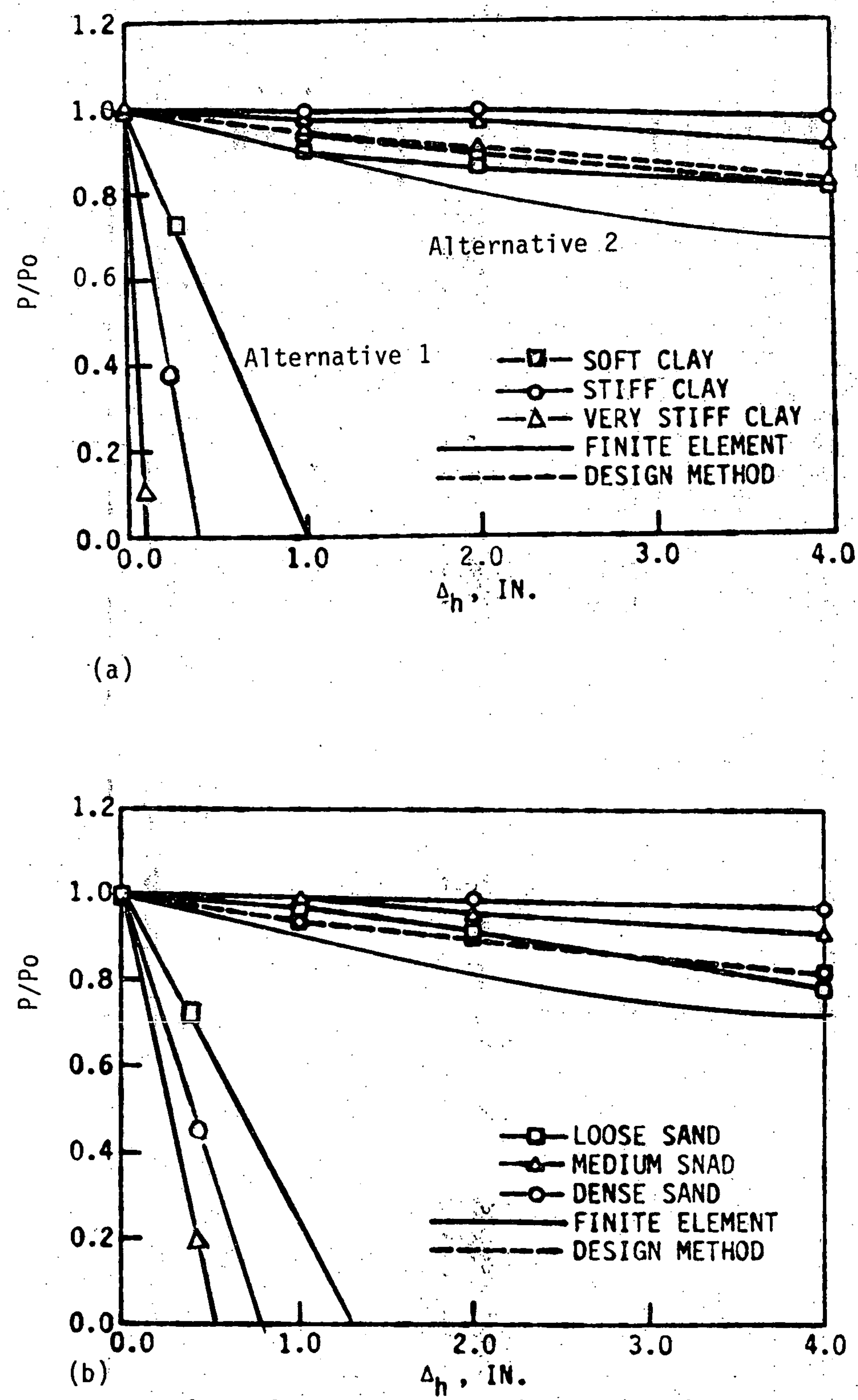


Fig. 1. **Ultimate vertical load ratio (end-bearing pile about strong axis).**

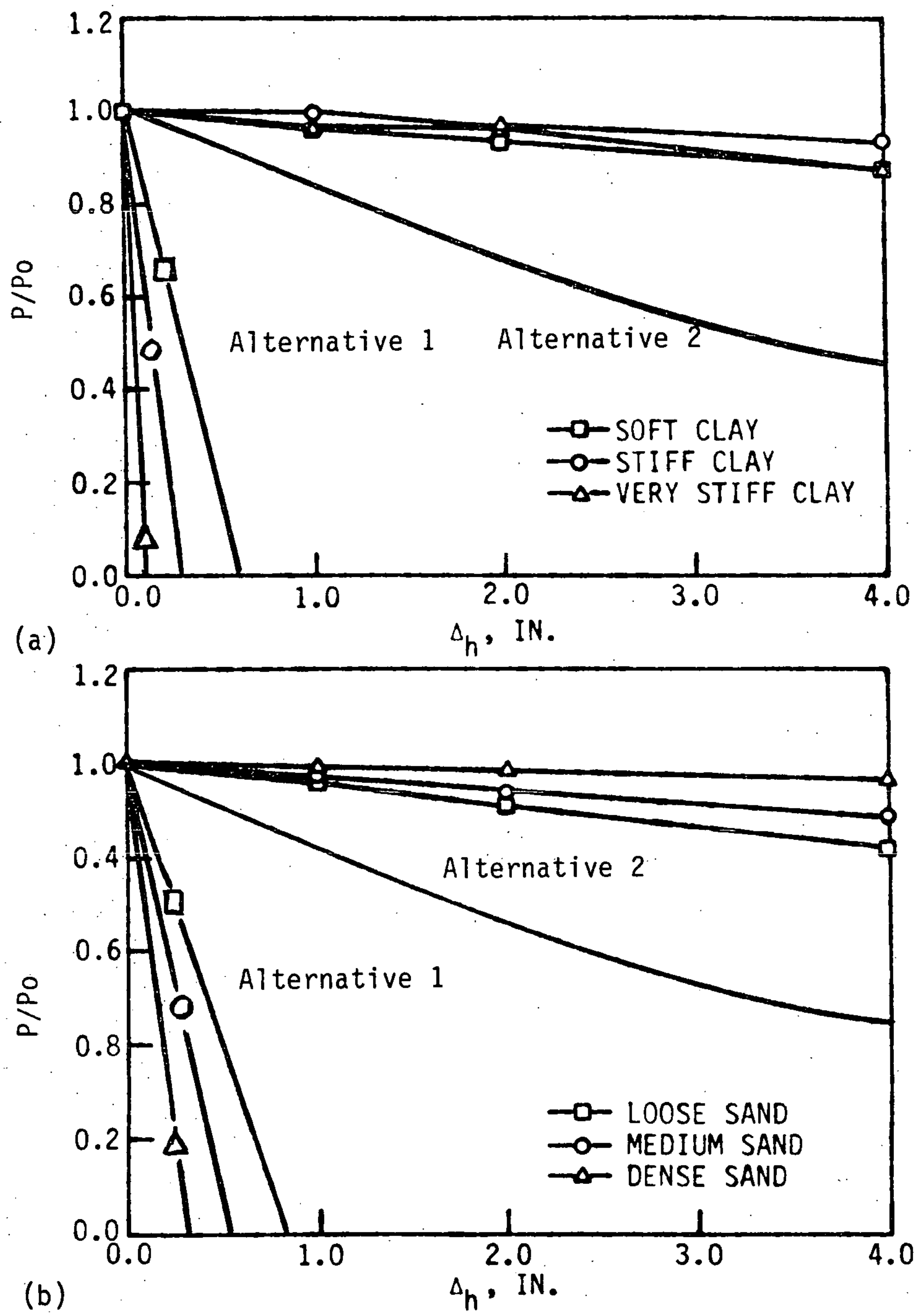


Fig. 2. Ultimate vertical load ratio (end-bearing piles about 45° axis) in Iowa soils.

BIAXIAL BENDING OF PILES

(see Addendum)

Stress

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right) F_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right) F_{by}} \leq 1$$

$$\frac{f_a}{0.472} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1$$

ALT #1

f_{bx} & f_{by} for temperature from elastic stress analysis & D+L+I

ALT #2

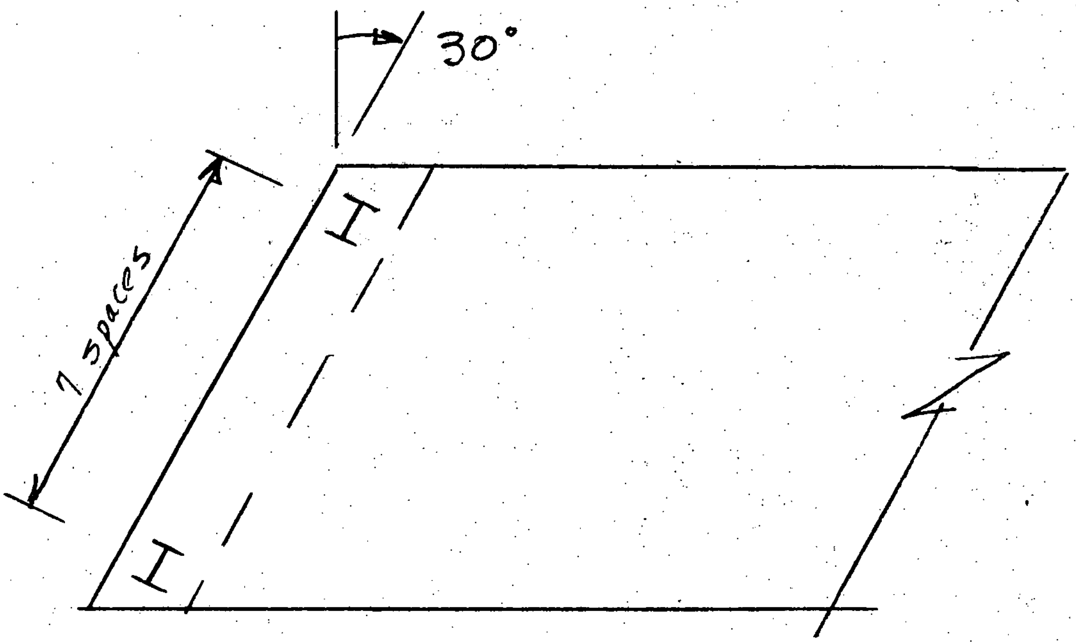
f_{bx} & f_{by} from PΔ moments & D+L+I

Ductility (ALT #2)

$$\frac{\Delta_x}{\Delta_{ix}} + \frac{\Delta_y}{\Delta_{iy}} \leq 1$$

$$\Delta_i = \Delta_b \left(0.6 + 2.25\, C_i\right)$$

SKEW BRIDGE EXAMPLE

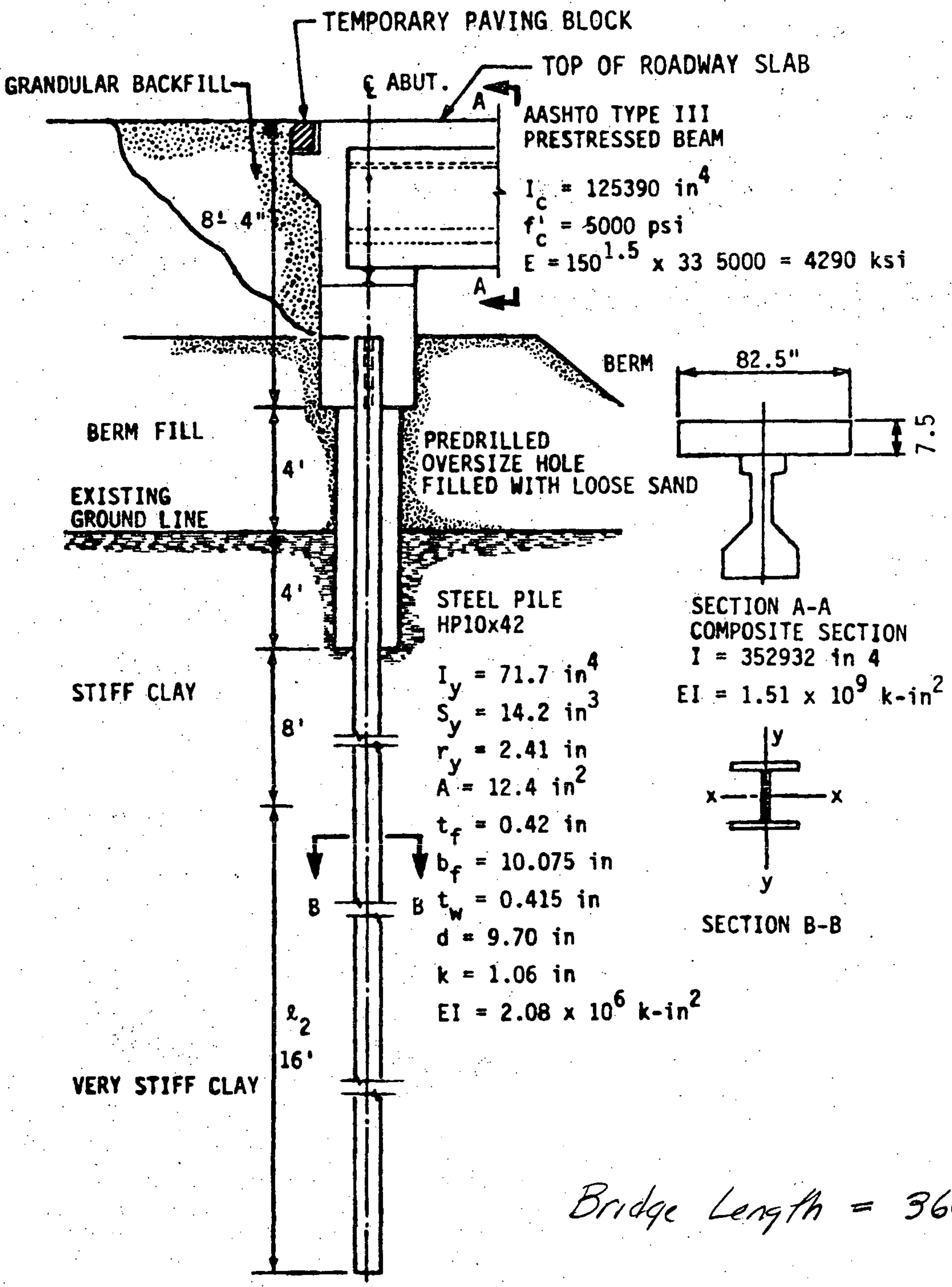


Reference: Pile Design & Tests for Integral Abutment Bridges, HR-273, Dec. 1987 and Addendum, March 1988

See Fig. 6.1 of report for soil conditions & pile, except pile orientation is shown above

CASE A ONLY (Pile as structural member)

4/13



Bridge Length = 360 ft

Figure 6.1. Section through abutment and soil profile.

- Equivalent soil stiffness

y axis

$$k_{ey} = 38.8 \text{ ksf}$$ (1)

$$l_{cy} = 17.6 \text{ ft}$$ (2)

Eq. 6.6, 6.7 of report

x axis

(1) guess $k_{ex} = 53$ ksf (see pg 285)

(2) find $l_{ox} = 2\sqrt[4]{EI_{x}/k_{ex}}$

$$= 2\sqrt[4]{\frac{(29000)(210)}{53(144)}} = 10.6 \text{ ft}$$

(3) find I_k (2^{nd} moment about l_o)

$$= 140\left[\frac{8^3}{36} + \frac{8}{2}\left(\frac{8}{3} + 2.6\right)^2\right] + 580\,\frac{(2.6)^3}{3}$$

$$= 20922 \text{ kft}$$

(4) establish new $k_e = 3I_k / l_o^3$

$$k_{ex} = \frac{3(20922)}{(10.6)^3} = 52.7 \text{ ksf}$$ (3)

use $k_{ex} = 52.7$ ksf

$$l_{cx} = 4\sqrt[4]{\frac{EI}{k_{ex}}}$$ (4)

$$= 4\sqrt[4]{\frac{(29000)(210)}{(52.7)(144)}}$$

$$= 21.3 \text{ ft}$$

• Length of equivalent cantilever

y axis

$$L_y = \begin{cases} 8.8 \text{ ft or } 106 \text{ in.} & \text{(stiffness)} \\ 10.6 \text{ ft or } 127 \text{ in.} & \text{(moment)} \\ 11.6 \text{ ft or } 139 \text{ in.} & \text{(buckling)} \end{cases} \quad (5)$$

(see report, pg 121)

x axis

From fig. 5.2, w/ $l_u/l_c = 0$ & $l_c = 21.3$ ft.

$$l_{ex} = \begin{cases} 0.5\, l_c = 10.7 \text{ ft} \\ 0.6\, l_c = 12.8 \text{ ft} & \text{(includes sand)} \\ 1.1\, l_c = 23.4 \text{ ft} \end{cases} \quad (6)$$

neglect sand in 8 ft. predrilled hole, $l_u = 8$ ft

$$l_{cx} = 4\sqrt[4]{\frac{(29000)(210)}{(580)(144)}} = 11.8 \text{ ft} \quad (7)$$

$$\frac{l_u}{l_{cx}} = \frac{8}{11.8} = 0.67$$

From Fig 5.2 w/ $l_u/l_c = 0.67$ & $l_c = 11.8$ ft

$$l_{ex} = \begin{cases} 0.4\, l_c = 4.7 \text{ ft} \\ 0.42\, l_c = 5.0 \text{ ft} \\ 0.55\, l_c = 6.5 \text{ ft} \end{cases} \quad (8)$$

add $l_u = 8$ ft to get L_x

Take equivalent cantilever length equal to minimum of: (1) length including sand and (2) length neglecting sand

$$L_x = \begin{cases} \text{min. } (10.7, 11.7) \\ \text{min. } (12.8, 13.0) \\ \text{min. } (23.4, 13.5) \end{cases}$$

$$= \begin{cases} 10.7 \text{ ft or } 128 \text{ in.} & (\text{stiffness}) \\ 12.8 \text{ ft or } 154 \text{ in.} & (\text{moment}) \\ 13.5 \text{ ft or } 162 \text{ in.} & (\text{buckling}) \end{cases} \quad (9)$$

- Structural Analysis of Bridge w/ Piles as Equivalent Cantilevers

Vertical Load (D + L + I) Analysis

A complete structural analysis of the bridge w/ piles, piers, etc. is required here to do a complete stress analysis for D+L+I loads. Several load combinations must be investigated. Suppose the following results are obtained for the pile stresses

Applied axial stress

$f_a = 4$ ksi $\quad (P = 49.6$ kip$)$

Applied bending stress (10)

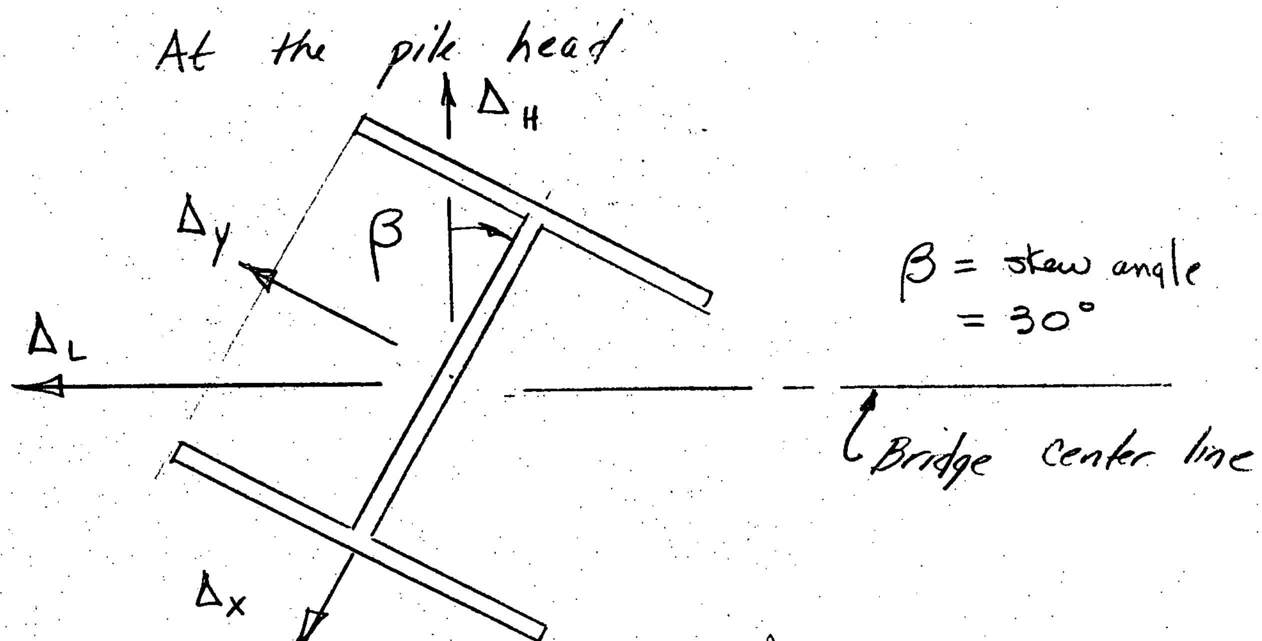
$f_{by} = 6$ ksi

$f_{bx} = 5$ ksi

Note: under usual methods of construction, the applied bending stresses will be zero for much of the dead load.

Temperature Load (T) Analysis

At the pile head



Δ_L = longitudinal expansion of bridge $= \alpha \Delta T L_b / 2$

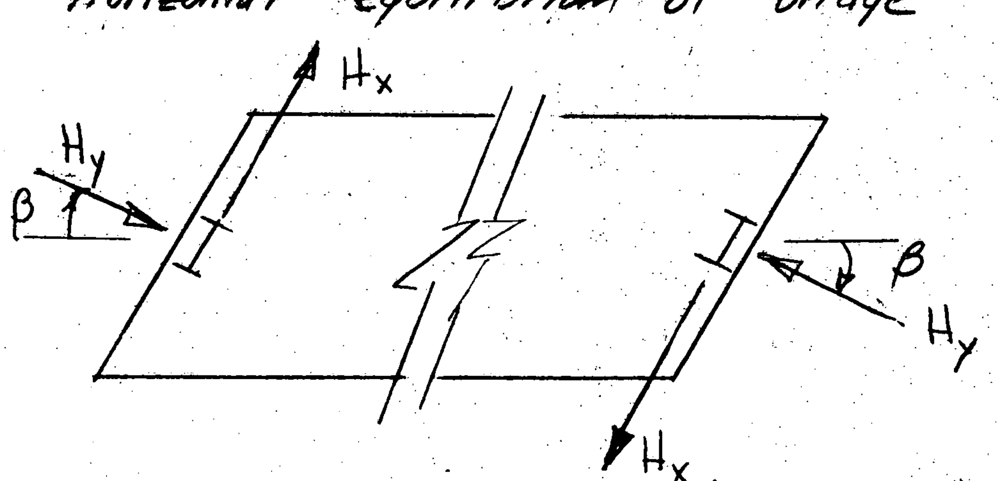
Δ_H = lateral motion of bridge @ top of pile

Δ_x, Δ_y = disp. @ pile head for x & y bending, resp.

geometry gives pile head displacements

$$\Delta_L = \Delta_x \sin\beta + \Delta_y \cos\beta$$
$$\Delta_H = -\Delta_x \cos\beta + \Delta_y \sin\beta \qquad (11)$$

Horizontal equilibrium of bridge



Pile head horizontal forces on bridge

$$\left.\begin{aligned} H_x &= \frac{12EI_x}{L_x^3}\Delta_x \\ H_y &= \frac{12EI_y}{L_y^3}\Delta_y \end{aligned}\right\} \text{for pile on left} \qquad (12)$$

ΣF_x & ΣF_y gives forces shown on right

ΣM about right pile gives (neglect soil passive pressure)

$$H_x = H_y \tan\beta \qquad (13)$$

Substitution of Eq. (12) into (13) gives

$$\Delta_x = \rho\,\Delta_y \tan\beta \qquad (14)$$

where the stiffness ratio ρ is

$$\rho = \frac{12EI_y}{L_y^3} \bigg/ \frac{12EI_x}{L_x^3} = \frac{I_y}{L_y^3} \bigg/ \frac{I_x}{L_x^3}$$

Substitution of (14) into the first of (11) gives

$$\Delta_y = \frac{\Delta_L \cos\beta}{\rho \sin^2\beta + \cos^2\beta} \qquad (15)$$

and, from (14)

$$\Delta_x = \frac{\Delta_L \rho \sin\beta}{\rho \sin^2\beta + \cos^2\beta} \qquad (16)$$

for this problem

$$\Delta_L = \alpha \, \Delta T \frac{L_b}{2} = \frac{(6.0)(10^{-6})(40)(360)(12)}{2} \qquad (17)$$

$$= 0.52 \text{ in.}$$

$$\rho = \frac{91.7}{(88)^3} \bigg/ \frac{210}{(10.7)^3} = 0.61 \qquad (18)$$

where the L corresponding to stiffness has been used (pg 4 & 5)

$$\beta = 30^\circ \qquad (19)$$

From Eq. (15) & (16)

$$\Delta_y = 0.50 \text{ in.}$$

$$\Delta_x = 0.18 \text{ in.} \qquad (20)$$

Admittedly, this structural analysis is quite crude. It is given here only for illustrative purpose. As discussed on page 5, in a real situation a more complete structural analysis of the entire bridge system with piles (equivalent cantilevers) and piers would be performed. The displacements in Eq. (20) are only representative. They will be used here to check the pile design, which is the primary purpose of this example.

• Allowable Stresses

Axial

y axis

$(Kl/r)_y = 37.5$ (see report, pg 130)

x axis ($K_x = 0.65$)

$(Kl/r)_x = 0.65(162)/4.13 = 25.5$

$\therefore$ y axis controls

$F_a = 20.3$ ksi (see report, pg 130) (21)

Bending

y axis

$F_{by} = 26.7$ ksi (see report, pg 133) (22)

x axis (Table 10.32.1A, AASHTO)

$F_{bx} = 0.55 F_y (1.25) = 24.8$ ksi (23)

(for temperature loading)

F_e'

y axis

$F_{ey}' = 120$ ksi (see report, pg 130) (24)

x axis

$$F_{ex}' = \frac{\pi^2(29000)(1.25)}{(25.5)^2(2.12)} = 260 \text{ ksi} \quad (25)$$

- Check Alternative #1

Bending Stresses for Temperature

Moments

$$M_y = \frac{6EI_y}{L_y^2}\Delta_y$$

$$= \frac{6(29000)(71.7)(6.50)}{(127)^2} = 387 \text{ k-in} \qquad (26)$$

$$M_x = \frac{6(29000)(210)(0.18)}{(154)^2} = 277 \text{ k-in}$$

Stresses

$$f_{by} = \frac{387}{14.2} = 27.2 \text{ ksi}$$

$$f_{bx} = \frac{277}{43.4} = 6.4 \text{ ksi} \qquad (27)$$

Stability equation

$$\frac{4}{20.3} + \frac{0.85(5+6.4)}{\left(1-\frac{4}{260}\right)24.9} + \frac{0.85(6+27.2)}{\left(1-\frac{4}{120}\right)26.7} \qquad (28)$$

since $\frac{C_m}{1-f_a/F_e'} < 1$, use 1

$$= 0.197 + 0.460 + 1.243 = 1.90 \quad \text{N.G.}$$

Yield equation

Also not satisfied

∴ Cannot use this design if Alternative #1 is adopted.

• Check Alternative #2

Bending Stresses for $P\Delta$ ($P = f_a A = 49.6\ k$)

$$f_{by} = \frac{P\Delta_y}{S_y 2} = \frac{49.6(0.50)}{2(14.2)} = 0.9\ ksi \quad (29)$$

$$f_{bx} = \frac{49.6(0.18)}{43.4(2)} = 0.10\ ksi$$

Stability equation

$$\frac{4}{20.3} + (1)\frac{(5+0.1)}{24.8} + (1)\frac{(6+0.9)}{26.7}$$

$$= 0.197 + 0.206 + 0.258 = 0.661 \quad \text{OK} \quad (30)$$

Yield equation

$$\frac{4}{0.472(36)(1.25)} + \frac{(5+0.1)}{24.8} + \frac{(6+0.9)}{26.7} = 0.652 \quad \text{OK} \quad (31)$$

Ductility (Plastic Hinge Rotation)

Eq. (14) from Addendum

$$\Delta_{iy} = \Delta_{by}(0.60 + 2.25\,C_i)$$

$$= 0.49(0.60 + 2.25(0.77)) = 1.14 \text{ in.} \quad (32)$$

$$\Delta_{ix} = \Delta_{bx}(0.60 + 2.25(0.77)) \quad (33)$$

$$\Delta_{bx} = \frac{F_{bx} S_x L_x^2}{6EI_x} = \frac{24.8(43.4)(154)^2}{6(29000)(210)} = 0.70$$

$\Delta_{ix} = 1.63$ in.

Check Eq. (21) from Addendum

$$\frac{\Delta_x}{\Delta_{ix}} + \frac{\Delta_y}{\Delta_{iy}}$$

$$\frac{0.18}{1.63} + \frac{0.50}{1.14} = 0.55 \quad \text{OK} \qquad (34)$$

∴ Design OK if Alternative #2 is acceptable

(Note: Case B and C have not been checked in this example)

DESIGN TABLES

Purpose: Specialize design equations to a common condition

Common Condition

(1) HP10x42 pile

(2) 8 ft predrilled hole w/ loose-to-medium sand

(3) 37 ton axial pile load

(4) No passive soil pressure on abutments

(5) Thermal exp. = $\Delta_L = \alpha(\Delta T) L_{bridge}/2$

Concrete: $\alpha = 6.0(10^{-6})/{}^\circ F$

$\Delta T = \pm 40^\circ F$

Steel: $\alpha = 6.5(10^{-6})/{}^\circ F$

$\Delta T = \pm 75^\circ F$

(6) D+L+I+T Load Case w/ 25% increase in allowables (D+L+I not checked)

(7) Approximate structural analysis of bridge (see report & skew bridge example)

(8) Lateral soil stiffness, k_h, less than 2000 ksf (very stiff clay)

- Equivalent soil stiffness (3 soils)

for k_h = 500, 1000 & 2000 ksf
(see page 120 & Fig. 6.3 of report)

k_{hy} = 38.8, 39.3, 39.5 ksf (1)

k_{hx} = 52.4, 57.4, 65.7 ksf

- Length of equivalent cantilever (3 soils)

for k_h = 500, 1000, 2000 ksf
(see page 121, take min. of L w/ & w/o sand)

$$L_y = \begin{cases} 8.8 \text{ ft or } 106 \text{ in. (stiffness)} \\ 10.6 \text{ ft or } 127 \text{ in. (moment)} \\ 11.6 \text{ ft or } 139 \text{ in. (buckling)} \end{cases}$$

For ALL 3 soils (2)

$$L_x = \begin{cases} 10.6 \text{ ft}, \; 10.5 \text{ ft}, \; 10.1 \text{ ft (stiffness)} \\ 12.8 \text{ ft}, \; 12.1 \text{ ft}, \; 11.4 \text{ ft (moment)} \\ 14.7 \text{ ft}, \; 12.6 \text{ ft}, \; 11.7 \text{ ft (buckling)} \end{cases}$$

• Structural Analysis

Vertical Load (D+L+I)

$$f_a = \frac{(37\ \text{Ton})(2)}{12.4} = 5.97\ \text{ksi}$$ for 37 Ton axial load (3)

$\bar{f}_{by}$ & $\bar{f}_{bx}$ – by structural analysis of bridge.

Temperature Load (T)

$$\Delta_L = \alpha \Delta T L_b / 2$$ (see page 1) (4)

From Eq.(15) & (16) of skew bridge example

$$\left.\begin{aligned} \Delta_y &= \frac{\Delta_L \cos\beta}{\rho \sin^2\beta + \cos^2\beta} \\ \Delta_x &= \frac{\Delta_L \rho \sin\beta}{\rho \sin^2\beta + \cos^2\beta} \end{aligned}\right\} \text{Oversimplified!} \quad (5)$$

β = skew angle

$$\rho = \frac{I_y}{L_y^3} \Big/ \frac{I_x}{L_x^3} = 0.60, 0.58, 0.52$$ for 3 soils (5b)

neglect passive soil resistance

REMINDER: This is a crude bridge analysis (see Skew bridge example, pg 9)

• Allowable Stress

Axial

y axis controls (pg 130 in report)

$F_a = 20.3$ ksi (6)

Bending

$F_{by} = 26.7$ ksi (pg 133, report)

$F_{bx} = 0.55 F_y (1.25) = 24.8$ ksi (7)

F_e'

$F_{ey}' = 120$ ksi (all three soils)

$F_{ex}' = 218, 298, 346$ ksi for 3 soils (8)

- Check Alternative #1

Bending Stresses due to Temperature

$$\bar{\bar{f}}_b = \frac{M_T}{S} = \frac{6EI}{SL^2}\Delta \quad \text{(Eq. 6.16 of report)}$$

$$\bar{\bar{f}}_{by} = 54.5\,\Delta_y \quad \text{for all three soils}$$

$$\bar{\bar{f}}_{bx} = 75.8\,\Delta_x,\ 84.8\,\Delta_x,\ 95.5\,\Delta_x \quad \text{for 3 soils} \qquad (9)$$

Stability equation

$$\frac{5.97}{20.3} + \frac{\bar{f}_{bx} + \bar{\bar{f}}_{bx}}{24.75} + \frac{\bar{f}_{by} + \bar{\bar{f}}_{by}}{26.7} = 1 \qquad (10)$$

$\frac{C_m}{1 - f_a/F_e'}$ is less than 1, therefore it is taken as 1

stability equation controls, need not check yield equation

let

$$\frac{\bar{f}_{bx}}{24.15} + \frac{\bar{f}_{by}}{26.7} = \epsilon \qquad \left(\text{represents bending stress caused by } D+L+I\right) \quad (11)$$

substitution of Eq. (5) into (9) and then into (10) and using (11) gives

$$\left(\begin{Bmatrix}48.8\\53.1\\53.6\end{Bmatrix}\sin\beta + 54.5\cos\beta\right)\frac{\Delta_L}{\begin{Bmatrix}0.60\\0.58\\0.52\end{Bmatrix}\sin^2\beta+\cos^2\beta} = 18.8-26.7\,\epsilon \qquad (12)$$

where the three numbers in the braces { } correspond to the three soils

$$k_h = \begin{Bmatrix}500 \text{ ksf}\\1000 \text{ ksf}\\2000 \text{ ksf}\end{Bmatrix}$$

Solve Eq. (12) for Δ_L (allowable bridge expansion).

<u>$\epsilon = 0$</u> (bending stresses for D+L+I are zero)

Δ_L values in inches (13)

k_h =	500	1000	2000
β =			
0°	0.344	0.344	0.344
30°	0.236	0.228	0.222
45°	0.205	0.195	0.187

<u>$\epsilon = 0.25$</u>

Δ_L values in inches (14)

k_h =	500	1000	2000
β =			
0°	0.222	0.222	0.222
30°	0.152	0.147	0.142
[illegible]	[illegible]	[illegible]	[illegible]

• Check Alternative #2

Bending stress due to temperature

$$\bar{\bar{f}}_b = P\Delta/2S \quad \text{(Eq. 6.40 of report)}$$

$P = 37$ Ton

$$\bar{\bar{f}}_{by} = 2.61\,\Delta_y \quad \text{for all three soils}$$

$$\bar{\bar{f}}_{bx} = 0.845\,\Delta_x \quad \text{for all three soils} \qquad (15)$$

Stability & yield equation

Does not control

Ductility

$$\Delta_{iy} = 1.14 \text{ in.}$$

(see Eq.(32) of Skew bridge example, same for 3 soils)

$$\Delta_{ix} = \Delta_{bx}\left(0.60 + 2.25(0.77)\right) \qquad (16)$$

$$\Delta_{bx} = \frac{f_{bx} S_x L_x^2}{6EI_x}$$

$= 1.66,\ 1.46,\ 1.29$ in. for 3 soils

Ductility criteria (Eq(21) from Addendum)

$$\frac{\Delta_x}{\Delta_{ix}} + \frac{\Delta_y}{\Delta_{iy}} = 1 \qquad (17)$$

Substitution of Eq(5) into Eq(17) and solving for Δ_L (the allowable thermal expansion) gives

$$\Delta_L = \frac{\rho \sin^2\beta + \cos^2\beta}{\dfrac{\rho \sin\beta}{\Delta_{ix}} + \dfrac{\cos\beta}{\Delta_{iy}}} \qquad (18)$$

Substitution of Eq.(5b) & Eq.(16) into Eq.(18) gives

Δ_L values in inches (19)

k_x =	500	1000	2000 ksf
β =			
0°	1.14	1.14	1.14
30°	0.96	0.93	0.92
45°	0.91	0.88	0.84

- Limiting Bridge Lengths
  (see page 1, Item (5))

  Concrete: $L_b = \Delta_L / 0.00012$

  steel : $L_b = \Delta_L / 0.00024$ (20)

  (steel length = ½ concrete length)

  divide by 12 to convert to ft.

For length limits, use minimum values for all soils in Eq. (13) & (14) (ALT #1) and Eq (19) (ALT #2), that is $k_h = 2000$ ksf. Using Eq. (20) gives the following length limits

Concrete Length Limit (ft)			
	ALT #1		ALT. #2
Skew	$\epsilon = 0$	$\epsilon = 0.25$	
0	240	154	792
30°	155	100	636
45°	130	84	583

Steel Length Limit
(1/2 Concrete Length Limit)

CONCLUSION

(1) Alternative #1 permits only very short bridges - (Differs from current DOT criteria because it includes D+L+I stresses)

(2) In skew bridges, orient weak axis perpendicular to bridge axis