Development of Asphalt Dynamic Modulus Master Curve Using Falling Weight Deflectometer Measurements

Final Report
June 2014

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16. Abstract

The asphalt concrete (AC) dynamic modulus ($|E*|$) is a key design parameter in mechanistic-based pavement design methodologies such as the American Association of State Highway and Transportation Officials (AASHTO) MEPDG/Pavement-ME Design. The objective of this feasibility study was to develop frameworks for predicting the AC $|E*|$ master curve from falling weight deflectometer (FWD) deflection-time history data collected by the Iowa Department of Transportation (Iowa DOT). A neural networks (NN) methodology was developed based on a synthetically generated viscoelastic forward solutions database to predict AC relaxation modulus (E(t)) master curve coefficients from FWD deflection-time history data. According to the theory of viscoelasticity, if AC relaxation modulus, E(t), is known, $|E*|$ can be calculated (and vice versa) through numerical inter-conversion procedures. Several case studies focusing on full-depth AC pavements were conducted to isolate potential backcalculation issues that are only related to the modulus master curve of the AC layer. For the proof-of-concept demonstration, a comprehensive full-depth AC analysis was carried out through 10,000 batch simulations using a viscoelastic forward analysis program. Anomalies were detected in the comprehensive raw synthetic database and were eliminated through imposition of certain constraints involving the sigmoid master curve coefficients.

The surrogate forward modeling results showed that NNs are able to predict deflection-time histories from E(t) master curve coefficients and other layer properties very well. The NN inverse modeling results demonstrated the potential of NNs to backcalculate the E(t) master curve coefficients from single-drop FWD deflection-time history data, although the current prediction accuracies are not sufficient to recommend these models for practical implementation. Considering the complex nature of the problem investigated with many uncertainties involved, including the possible presence of dynamics during FWD testing (related to the presence and depth of stiff layer, inertial and wave propagation effects, etc.), the limitations of current FWD technology (integration errors, truncation issues, etc.), and the need for a rapid and simplified approach for routine implementation, future research recommendations have been provided making a strong case for an expanded research study.

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DEVELOPMENT OF ASPHALT DYNAMIC MODULUS MASTER CURVE USING FALLING WEIGHT DEFLECTOMETER MEASUREMENTS

Final Report
June 2014

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EXECUTIVE SUMMARY

The asphalt concrete (AC) dynamic modulus ($|E^*|$) is a key design parameter in the American Association of State Highway and Transportation Officials (AASHTO) Mechanistic-Empirical Pavement Design Guide (MEPDG)/AASHTOWare Pavement-ME Design. The standard laboratory procedures for AC dynamic modulus testing and development of a master curve require time and considerable resources. The objective of this feasibility study was to develop frameworks for predicting the AC relaxation modulus ($E(t)$) or dynamic modulus master curve from routinely collected falling weight deflectometer (FWD) time history data. According to the theory of viscoelasticity, if the AC relaxation modulus, $E(t)$, is known, $|E^*|$ can be calculated (and vice versa) through numerical inter-conversion procedures.

The overall research approach involved the following steps:

- Conduct numerous viscoelastic (VE) forward analysis simulations by varying $E(t)$ master curve coefficients, shift factors, pavement temperatures, and other layer properties
- Extract simulation inputs and outputs and assemble a synthetic database
- Train, validate, and test neural network (NN) inverse mapping models to predict $E(t)$ master curve coefficients from single-drop FWD deflection-time histories

A computationally efficient VE forward analysis program developed by Michigan State University (MSU) researchers was adopted in this study to generate the synthetic database. The VE forward analysis program accepts pavement temperature and layer properties (AC $E(t)$ master curve, $E_{b/sub}$, h, μ,) and outputs surface deflection-time histories. Several case studies were conducted to establish detailed frameworks for predicting the AC $E(t)$ master curve from single-drop FWD time history data. Case studies focused on full-depth AC pavements as a first step to isolate potential backcalculation issues that are only related to the modulus master curve of the AC layer. For the proof-of-concept demonstration, a comprehensive full-depth AC analysis was carried out through 10,000 batch simulations of a VE forward analysis program. Anomalies were detected in the comprehensive raw synthetic database and were eliminated through imposition of certain constraints on the sum of $E(t)$ sigmoid coefficients, $c1 + c2$.

Except for the first two or three time intervals, deflection-time histories at all other time intervals considered in the analysis were predicted by NNs with very high accuracy (R-values greater than 0.97). The NN inverse modeling results demonstrated the potential of NNs to predict the $E(t)$ master curve coefficients from single-drop FWD deflection-time history data. However, the current prediction accuracies are not sufficient to recommend these models for practical implementation.

Considering the complex nature of the problem with many uncertainties involved, including the possible presence of dynamics during FWD testing (related to the presence and depth of stiff layer, inertial and wave propagation effects, etc.), the limitations of current FWD technology (integration errors, truncation issues, etc.), and the need for a rapid and simplified approach for

routine implementation, future research recommendations have been provided that make a strong case for an expanded research study.

INTRODUCTION

Background

The new American Association of State Highway and Transportation Officials (AASHTO) pavement design guide (Mechanistic-Empirical Pavement Design Guide [MEPDG]) and the associated software (AASHTOWare Pavement ME Design, formerly known as DARWin ME) represents a major advancement in pavement design and analysis. The MEPDG employs the principle of a master curve based on time-temperature superposition principles to characterize the viscoelastic-plastic property of asphalt materials. The MEPDG recommends the use of asphalt dynamic modulus, |E*|, as the design parameter. The dynamic modulus master curve is constructed from multiple values of measured dynamic modulus at different temperature and frequency conditions. The standard laboratory procedure for dynamic modulus testing requires time and considerable resources.

State agencies, faced with the challenge of implementing the MEPDG/Pavement ME Design, are looking to field testing as a possibility for obtaining values for use in new design. The laboratory testing requirements are extensive, and the idea of obtaining default regional properties for specific materials and structures in the field is attractive. Falling weight deflectometer (FWD) testing has become the predominant method for characterizing in situ material properties for rehabilitation design. The state of the practice in FWD analysis involves static backcalculation of pavement layer moduli, although FWD measurements capture the entire time history of deflections under dynamic loading conditions.

In the MEPDG/Pavement ME Design flexible pavement rehabilitation analysis (NCHRP 2004, ASHTO 2008, AASHTO 2012), the pre-overlay damaged master curve of the existing asphalt concrete (AC) layer is determined by first calculating an "undamaged" modulus and then adjusting this modulus for damage using the pre-overlay condition. The undamaged AC master curve is derived from its aggregate gradation and laboratory-tested asphalt binder properties/asphalt binder grade using Witczak's dynamic modulus predictive equation. Both aggregate gradation and asphalt binder properties/asphalt binder grade may be obtained from construction records or testing of field-cored samples. To characterize the damage in the existing pavement at the time of overlay, MEPDG/Pavement ME Design allows the input of backcalculated moduli from nondestructive testing (NDT) with frequency and temperature under the Level 1 rehabilitation input option. The process is shown schematically in Figure 1.

If the damaged |E*| master curve of the AC in an in-service pavement can be derived from the time histories of routinely collected FWD deflection data, it would not only save lab time and resources, but it could also lead to a more accurate prediction of the pavement's remaining service life.
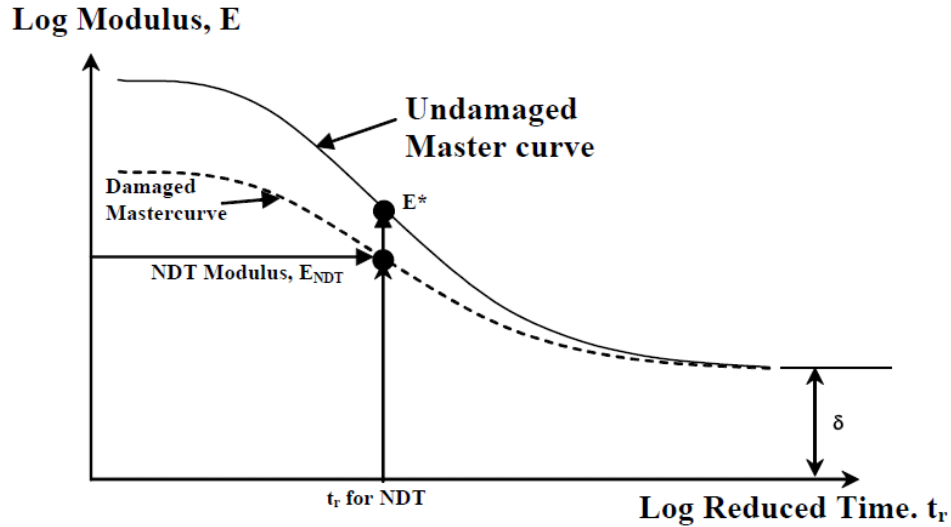
Figure 1. AC layer damage master curve computation in MEPDG/Pavement ME Design Level 1 (NCHRP 2004)

Objectives and Scope

The objective of this study is to develop the asphalt dynamic modulus master curve directly from time histories of routinely collected FWD test data. For this project, the Iowa Department of Transportation (Iowa DOT) is primarily interested in delivering a proof-of-concept methodology for documenting the Iowa AC mix damaged master curve shape parameters (C_1, C_2, C_3, C_4) relative to the mix IDs/Station Nos., if possible, in the Pavement Management Information System (PMIS). This would be of significant use to the city, county, and state engineers because the outcome of this research would enable them to look up the damaged master curve shape parameters from the PMIS while running a flexible pavement rehabilitation analysis and design using MEPDG/Pavement ME Design.

OVERVIEW OF ASPHALT MASTER CURVE AND FWD BACKCALCULATION

Dynamic Modulus (|E*|) Master Curve of Asphalt Mixtures

The |E*| value is one of the asphalt mixture stiffness measures that determines the strains and displacements in a flexible pavement structure as it is loaded or unloaded. The asphalt mixture stiffness can alternatively be characterized via the flexural stiffness, creep compliance, relaxation modulus, and resilient modulus. The |E*| value is one of the primary material property inputs required in the MEPDG/Pavement ME Design procedure (NCHRP 2004, ASHTO 2008, AASHTO 2012).

Definition of AC Dynamic Modulus (|E|)*

The definition of |E*| comes from the complex modulus (E*), consisting of both a real and imaginary component, as shown in the following equation:

$$E^* = E_1 + iE_2 \tag{1}$$

Here, $i = \sqrt{-1}$, E_1 is the storage modulus part of the complex modulus, and E_2 is the loss modulus part of the complex modulus. The |E*| value can be mathematically defined as the magnitude of the complex modulus, as shown in the following equation:

$$\left| E^* \right| = \sqrt{E_1^2 + E_2^2} \tag{2}$$

|E*| is also determined experimentally as the ratio of the applied stress amplitude to the strain response amplitude under a sinusoidal loading, as shown in the following equation:

$$\left| E^* \right| = \frac{\sigma_o}{\varepsilon_o} \tag{3}$$

Here, σ_0 is the average stress amplitude and ε_0 is the average recoverable strain. The |E*| value of the asphalt mixture is strongly dependent upon temperature (T) and loading rate, defined either in terms of frequency (*f*) or load time (t).

Figure 2 illustrates how the dynamic modulus can be determined in the laboratory. The peak points of applied load and strain response at each of the test frequencies and temperatures are utilized to determine the dynamic modulus under given conditions. The measured dynamic moduli at different frequencies and temperatures are utilized to construct the AC master curve.

$$|E| = \frac{\sigma_o}{\varepsilon_o}$$
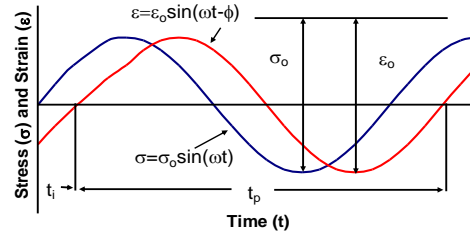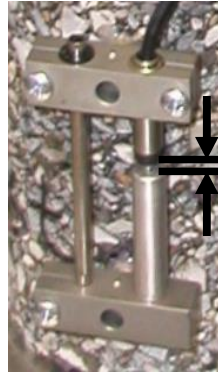
$$\sigma = \text{P/A}, \quad \varepsilon = \text{I/L}$$

$$\phi = \frac{t_i}{t_p} \times 360$$

Test Temp. (°F)	Freq. (Hz)	Cycles	Rest Period (Sec)	Cycles to Compute E*
14, 40, 70, 100, 130 (Unless otherwise specified)	25	200	-	196 to 200
	10	100	60	196 to 200
	5	50	60	96 to 100
	1	20	60	16 to 20
	0.5	15	60	11 to 15
	0.1	15	60	11 to 15

|E*| Test
(*www.training.ce.washington.edu*)

|E*| Test conditions
(*Witczak, 2005*)
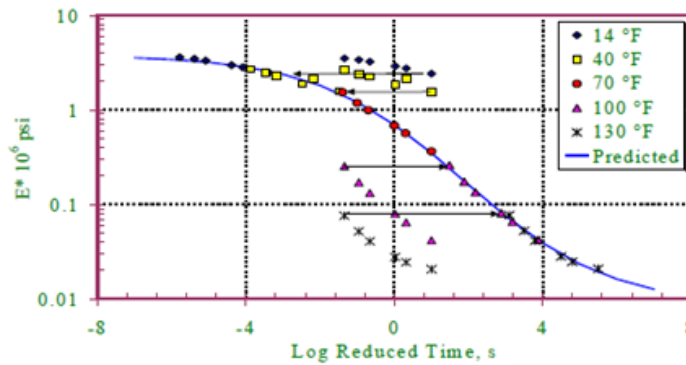
Figure 2. Laboratory AC dynamic modulus (|E*|) test protocol

AC Dynamic Modulus (|E|) Master Curve*

Pavement ME Design (AASHTO 2012) builds the |E*| master curve at a reference temperature by using it to determine |E*| at all levels of temperature and time rate of load.

The |E*| master curves are constructed using frequency-temperature (or time-temperature) superposition concepts represented by shift factors. The combined effects of temperature and loading rate can be represented in the form of a master curve relating |E*| to a reduced frequency (f_r) or a reduced time (t_r) by a sigmoidal function. Each of the parameters (i.e., a reduced frequency [f_r] or a reduced time [t_r]) utilizes a sigmoidal function equation of the |E*| master curve. However, the various equations of a sigmoidal function for the |E*| master curve have been reported in the literature (Pellinen et al. 2004, Schwartz 2005, Witczak 2005, Kutay et al. 2011). For clarification, in this study, the sigmoidal function equation using a reduced frequency (f_r) is defined as the dynamic modulus |E*| master curve equation, while the sigmoidal function equation using a reduced time (t_r) is defined as the relaxation modulus E(t) master curve equation. From the theory of viscoelasticity, |E*| and E(t) can be converted from each into the other through numerical procedures (Park and Schapery 1999). The dynamic modulus |E*| master curve equation using a reduced frequency (f_r) in this study is described as follows:

$$Log\left|E*\right| = C_1 + \frac{C_2}{1 + e^{(-C_3 - C_4 \log(f_r))}}$$

(4)

Where,

f_r = reduced frequency of loading at reference temperature
C_1 = minimum value of |E*|
$C_1 + C_2$ = maximum value of |E*|
C_3 and C_4 = parameters describing the shape of the sigmoidal function

The function parameters C_1 and C_2 in general depend on the aggregate gradation and mixture volumetrics, while the parameters C_3 and C_4 depend primarily on the characteristics of the asphalt binder (Schwartz 2005). The reduced frequency (f_r) can be shown in the following form:

$$f_r = (f)\,a_T(T)$$

(5)

Where,

f = frequency of loading at desired temperature
T = temperature of interest
$a_T(T)$ = shift factor as a function of temperature

The equations widely used to express the temperature-shift factor of $a_T(T)$ include Williams-Landel-Ferry equations, the Arrhenius equations, and the second-order polynomial equations (Pellinen et al. 2004, Witczak 2005, Kutay et al. 2011, Varma et al. 2013b). The shift factor utilized in this study is the logarithm of the shift factor computed by using a second-order polynomial (Kutay et al. 2011, Varma et al. 2013b), described as follows:

$$\log(a_T(T)) = a_1(T^2 - T_{ref}^2) + a_2(T - T_{ref})$$

(6)

Where,

> T_{ref} = reference temperature, 19°C (or 66.2°F)
> a_1 and a_2 = the shift factor polynomial coefficients

The values for C_1, C_2, C_3, C_4, and $a_T(T)$ in a sigmoidal function of master curve are all simultaneously determined from test data using nonlinear optimization techniques, e.g., the Solver function in Excel software. The relaxation modulus E(t) using a reduced time (t_r) can be converted from the dynamic modulus |E*| using a reduced frequency (f_r) through numerical procedures (Park and Schapery 1999) and described as follows:

$$Log(E(\text{t})) = c_1 + \frac{c_2}{1 + e^{(-c_3 - c_4 \log(t_r))}}$$

(7)

Where,

> t_r = reduced time at reference temperature
> c_1, c_2, c_3, and c_4 = the relaxation modulus E(t) coefficients

FWD Backcalculation

Static FWD Backcalculation Approaches

The FWD backcalculation procedure involves two calculation directions, namely forward and inverse. In the forward direction of analysis, theoretical deflections are computed under the applied load and the given pavement structure using assumed pavement layer moduli. In the inverse direction of analysis, these theoretical deflections are compared with measured deflections, and the assumed moduli are then adjusted in an iterative procedure until the theoretical and measured deflection basins match acceptably well. The moduli derived in this way are considered representative of the pavement response to load and can be used to calculate stresses or strains in the pavement structure for analysis purposes. This is an iterative method of solving the inverse problem and will not have a unique solution in most cases.

In the FWD test, an impulse load within the range of 6.7 to 156 kN is impacted on the pavement surface, and associated surface deflection values in the time domain are measured at different locations (usually at six or seven locations) by geophones. Figure 3 illustrates the typical result of an FWD test. In general, deflection-time history curves for each geophone exhibit Haversine behavior, and peak values of these curves for each geophone are used to plot the deflection basin curve. Static FWD backcalculation methods utilize only peak values of deflection-time history curves to compute moduli values.
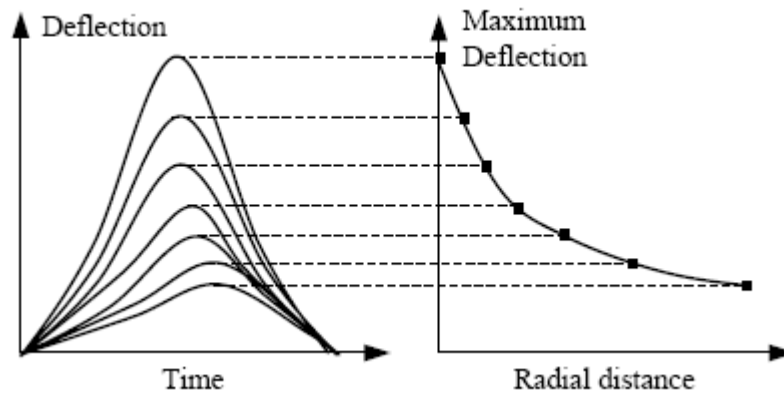
Figure 3. Illustration of typical FWD deflection measurements (Goktepe et al. 2006)

Some of the major factors that can lead to erroneous results in backcalculation, and some cautions for avoiding them, are as follows (Irwin 2002, Von Quintus and Killingsworth 1998, Ullidtz and Coetzee 1998):

- There must be a good match between the assumptions that underlie backcalculation and the realities of the pavement.
- The loading is assumed to be static in backcalculation programs while, in reality, FWD loading is dynamic.
- Major cracks in the pavement, or testing near a pavement edge or joint, can cause the deflection data to depart drastically from the assumed conditions.
- Pavements with cracks or various discontinuities and other such features, which are the main focus of maintenance and rehabilitation efforts, are ill-suited for any backcalculation analysis or moduli determination that is based on elastic layered theory.
- FWD deflection data have seating, random, and systematic errors.
- It is seldom clear just how to set up the pavement model. Layer thicknesses are often not known, and subsurface layers can be overlooked. A trial-and-error approach is often used.
- Layer thicknesses are not uniform in the field, nor are materials in the layers completely homogeneous.
- There are vertical changes in the pavement materials and subgrade soils at each site. This change in the vertical profile is minor at some sites, whereas at other sites the change is substantial.
- Some pavement layers are too thin to be backcalculated in the pavement model. Thin layers contribute only a small portion to the overall deflection, and, as a result, the accuracy of their backcalculated values is reduced.
- Moisture contents and depth to hard bottom can vary widely along the road.
- The presence of a shallow water table and related hard layer effects can influence the backcalculation results.
- Temperature gradients exist in the pavement, which can lead to modulus variation in asphalt layers and warping in concrete layers.

- Most unbound pavement materials are stress-dependent, and most backcalculation programs do not have the capability to handle that.
- Spatial and seasonal variations of pavement layer properties exist in the field.
- Input data effects are a factor. These include seed moduli, modulus limits, and layer thicknesses, as well as program controls such as number of iterations and convergence criteria.

The viscoelastic (VE) pavement properties and dynamic effects such as inertia and damping under FWD testing with dynamic loads can affect the pavement response. However, static backcalculation neglects these effects and therefore less reflects the actual situation. In addition, while the MEPDG uses elastic, plastic, viscous, and creep properties of materials to predict pavement performance over the design life, current static backcalculation methods cannot capture all of these properties. Further, FWD deflection-time history curves contain richer information that have the potential to reduce erroneous results.

Dynamic Backcalculation Approaches

Dynamic pavement response and backcalculation models have been studied by a number of researchers (Al-Khoury et al. 2001a, Al-Khoury et al. 2001b, Al-Khoury et al. 2002a, Al-Khoury et al. 2002b, Callop and Cebon 1996, Chang et al. 1992, Dong et al. 2002, Foinquinos et al. 1995, Goktepe et al. 2006, Grenier and Konrad 2009, Grenier et al. 2009, Hardy and Cebon 1993, Kausel and Roesset 1981, Liang and Zhu 1998, Liang and Zeng 2002, Lytton 1989, Lytton et al. 1993, Magnuson 1998, Magnuson et al. 1991, Maina et al. 2000, Mamlouk and Davies 1984, Mamlouk 1985, Nilsson et al. 1996, Roesset 1980, Roesset and Shao 1985, Shoukry and William 2000, Sousa and Monismith 1987, Stubbs et al. 1994, Ullidtz 2000, Uzan 1994a, Uzan 1994b, Zaghloul and White 1993).

Most of the developed methods employ dynamic pavement response models in the forward calculations of backcalculation procedures. Most of the forward methods adapted analytical or semi-analytical approaches in the solution methodologies, whereas some utilized finite element (FE) or numerical methods.

The material properties affecting the dynamic response of a pavement are Young's modulus (E), complex modulus (G^* or E^*), Poisson's ratio (υ), mass densities (ρ), and damping ratio (β). The complex modulus is related to the viscoelastic property of asphalt materials and internal damping as a function of inertia is considered for unbound and subgrade layers in electrodynamic analyses (Nilsson et al. 1996, Maina et al. 2000, Stubbs et al. 1994, Ullidtz 2000, Uzan 1994a, Uzan 1994b). Among material properties, the complex modulus has been considered as an unknown parameter in dynamic backcalculation analysis. The other material properties have been generally assumed to be known because these properties have only slight influence on the dynamic response of the pavement (Goktepe et al. 2006).

In the developed dynamic backcalculation methods, the FWD time history data may be fitted into the frequency domain or the time domain (Uzan 1994a, Uzan 1994b). Figure 4 and Figure 5

8

present schematic representation of both fitting approaches for dynamic load backcalculation. Fourier analyses and inverse Fourier analyses can be conducted for the transformation of the domain of data.
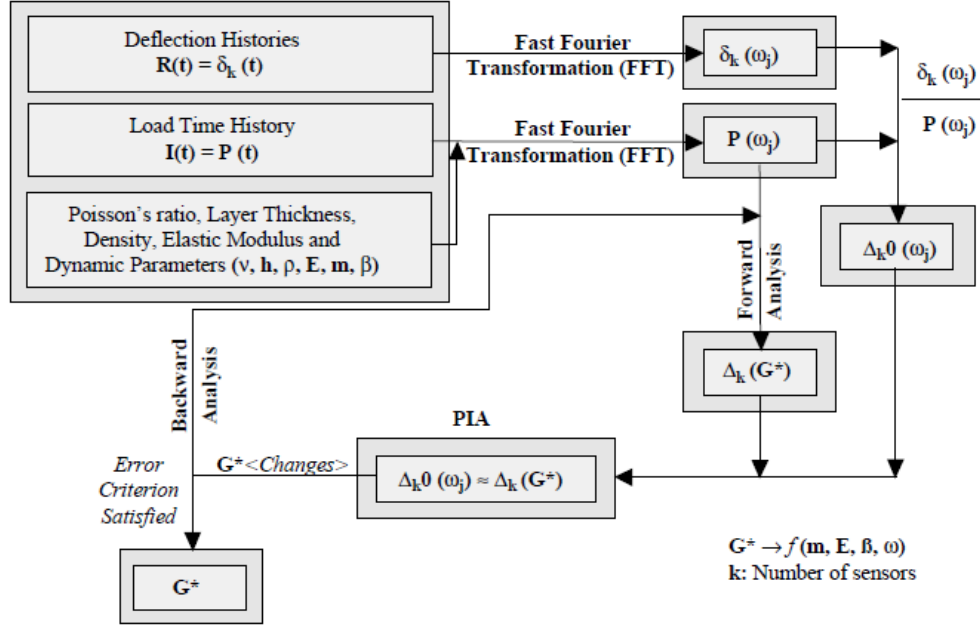
Figure 4. Schematic representation of the frequency domain fitting for dynamic load backcalculation (Goktepe et al. 2006)
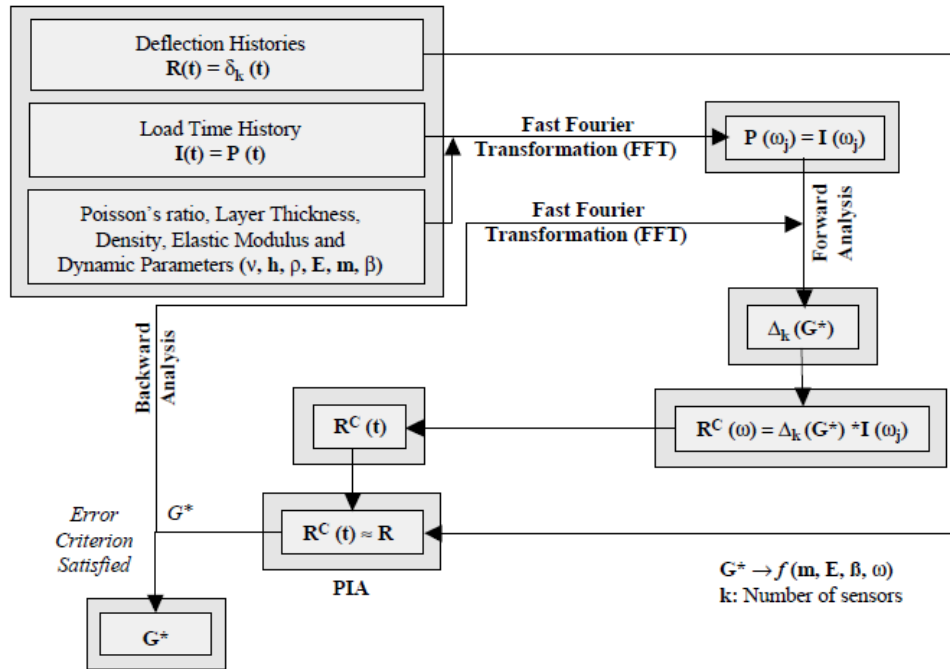
Figure 5. Schematic representation of dynamic time domain fitting for dynamic load backcalculation (Goktepe et al. 2006)

In frequency domain fitting, the applied load and deflection response time histories are transformed into the frequency domain by using a Fourier transformation. The compliance function of the complex deflection function divided by the complex load function is the measured complex unit response of the pavement at each frequency. Similarly to static backcalculation methods, an iterative procedure is carried out to find the set of complex moduli that will generate the calculated complex unit response close to the measured one. In time domain fitting, the impulse load time histories should be transformed into the frequency domain data in order to input the available forward model into the frequency domain. Inverse Fourier transformations should be carried out to compare calculated and measured deflections in the time domain.

Some advantages of developed dynamic backcalculation procedures include considering asphalt viscoelastic properties and obtaining more precise results than static procedures. However, some of the limitations of current dynamic backcalculation procedures include the following (Goktepe et al. 2006, Grenier and Konrad 2009):

- The current dynamic backcalculation procedures have more complexity and greater computational expense.
- The error minimization scheme can fall into a local minimum (which may not be the absolute minimum), depending on the complexity of the error function.
- The uniqueness of the solution is not always guaranteed and depends on the number of unknown parameters and the correlation between these parameters.
- Because many observations are used in the dynamic approach, correlations between unknown parameters are usually low, which is not the case in the static approach that uses only the deflection basin.

Viscoelastic Backcalculation Approach

Although many static and dynamic backcalculation approaches have been proposed in the past, only fewer recent studies have attempted to develop dynamic backcalculation approaches to derive the AC |E*| master curve from FWD defection-time history data.

Kutay et al. (2011) developed a methodology that backcalculates the damaged dynamic modulus |E*| master curve of asphalt concrete by utilizing the time histories of FWD surface deflections. A computationally efficient layered viscoelastic forward solution, referred to as LAVA, was employed iteratively to backcalculate the AC E(t) master curve (which can then be converted to the |E*| master curve using numerical inter-conversion procedures) based on FWD deflection-time history data. Certain information such as the thickness of each layer (AC, base, subbase, etc.), the modulus, and the Poisson's ratio of the layers under the AC layer are required for backcalculation of E(t). Specifically, the number of layers and thickness of each layer, modulus and Poisson's ratio of unbound layers, and Poisson's ratio of the AC layer are required as inputs.

Using simulated examples of two pavement structures, Kutay et al. (2011) demonstrated that it is possible to backcalculate certain portions of the E(t) master curve using deflection-time histories

from a typical FWD test. The study noted that the proposed backcalculation algorithm is independent of layer geometry because the layer structure and the thickness of the asphalt layer, for the cases analyzed, did not have an influence on the backcalculated E(t) or |E*| master curves. The authors also proposed modifications to the current FWD technology to enable longer FWD pulses and to ensure more reliable readings in the tail regions of the FWD deflection-time histories.

As a follow-up to the work by Kutay et al. (2011), Varma et al. (2013a) estimated and proposed a set of temperatures at which FWD tests should be conducted to be able to maximize the portion of the E(t) curve that can be accurately backcalculated. A genetic algorithm–based (GA-based) viscoelastic backcalculation algorithm was proposed that is capable of predicting E(t) and |E*| master curves as well as time-temperature superposition shift factors from a set of FWD deflection-time histories at different temperatures. The study concluded that deflection-time histories from FWD tests conducted between 68–104 °F (20–40 °C) are useful in accurately estimating the entire E(t) or |E*| master curve.

Varma et al. (2013b) considered FWD deflection-time history data from a single FWD drop combined with the temperature gradient across the AC layer at the time of FWD testing in their GA-based viscoelastic backcalculation approach, referred to as BACKLAVA. The study concluded that, unless a stiff layer (bedrock) exists close to the pavement surface that can contribute to the dynamics in the FWD test, BACKLAVA is capable of inferring E(t)/|E*| master curve coefficients (including shift factors) as well as the linear elastic moduli of the base and subgrade layers.

DEVELOPMENET OF FRAMEWORK FOR DERIVING AC MASTER CURVE FROM FWD DATA

Based on the research team's discussions with the Iowa DOT's Office of Special Investigations and Bituminous Materials Office regarding the specific objectives of this project, the Iowa DOT is eventually interested in documenting the Iowa AC mix damaged master curve coefficients relative to the mix IDs/Station Nos., if possible, in the Pavement Management Information System (PMIS). This would be of significant use to the city, county, and state engineers because the outcome of this research would enable them to look up the damaged master curve shape parameters from the PMIS while running a flexible pavement rehabilitation analysis and design using MEPDG/Pavement ME Design. As a first and foundational step, this feasibility research study focused on establishing frameworks for predicting the AC E(t) master curve coefficients from FWD time history data.

Based on a comprehensive literature review, the existing direct, indirect, and derivative approaches to damaged master curve determination using FWD time history data were synthesized in the previous section. This section describes the development of a detailed framework as a first step in a proof-of-concept demonstration for deriving the AC |E*| master curve coefficients from single-drop FWD time history data.

In the proposed approach, a layered viscoelastic forward analysis tool is first used to generate a database of AC master curve (input)–pavement surface deflection time history (output) scenarios for a variety of pavement layer thicknesses and pavement temperatures (see Figure 6). In the second step, the neural network (NN) methodology is employed to map the AC surface deflection time history data (generated through a forward-layered viscoelastic analysis model) to (damaged) AC relaxation modulus master curve shape parameters (see Figure 7).
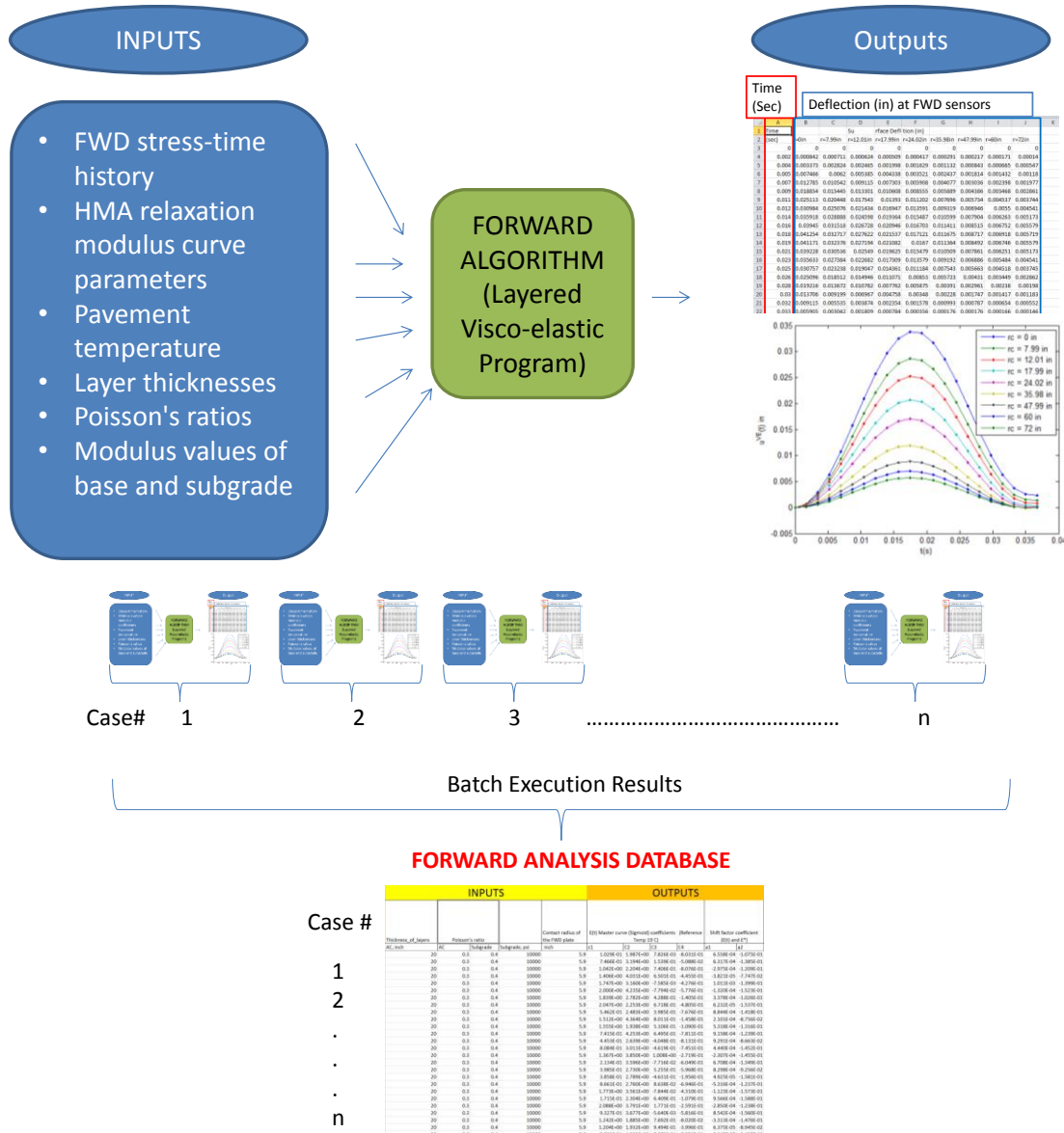
Figure 6. Schematic of synthetic database development approach using the viscoelastic forward analysis tool
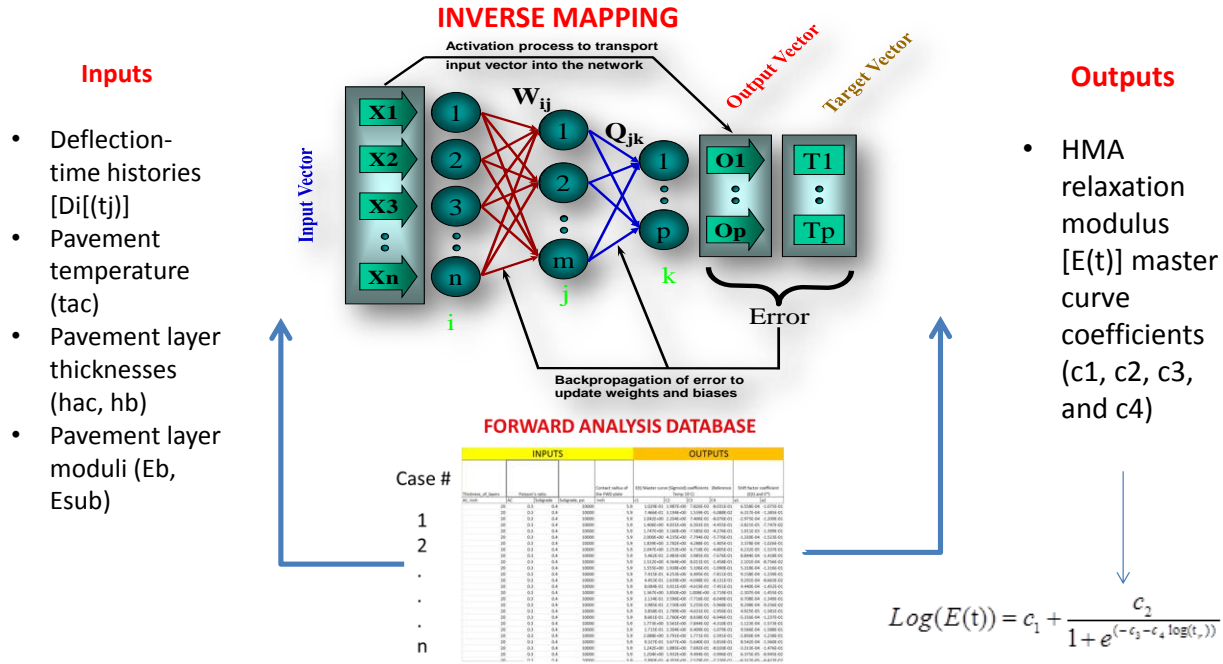
Figure 7. Schematic of neural networks approach to predict AC E(t) master curve from FWD deflection-time history data

This simplified approach necessitates the use of a computationally efficient layered VE forward analysis algorithm to generate a database of master curve–dynamic deflection case scenarios for a variety of pavement layer thicknesses and pavement temperatures. The VE forward analysis tool developed by Kutay et al. (2011) was employed for generating the forward synthetic database. This VE forward analysis program attempts to simulate more realistic FWD test conditions with respect to the existence of a nonuniform temperature profile across the depth of the AC layer (Varma et al. 2013a). This enables usage of a single FWD drop to characterize the VE properties, such as time function and time-temperature shifting, to get the AC relaxation modulus master curve, which can subsequently be converted to provide the |E*| master curve.

As mentioned previously, according to the theory of viscoelasticity, if the AC relaxation modulus (E(t)) master curve is known, the AC dynamic modulus (|E*|) master curve can be calculated from it (and vice versa) using well-established numerical inter-conversion procedures (Park and Shapery 1999, Kutay et al. 2011). Since the VE forward analysis program used in this study outputs E(t) master curve parameters, the rest of the analysis and discussion focuses on deriving E(t) master curve coefficients based on single-drop FWD deflection-time history data. In-service AC pavements are typically composed of multiple layers of different AC mixture types, and therefore the E(t) or |E*| master curve backcalculated from FWD deflection-time history data through this approach represents a single equivalent E(t) or |E*| master curve for all the AC sublayers.

The backcalculation of shift factors for the master curve requires knowledge of the temperature profile at the time of the FWD testing, which translates into additional dimensions of complexity in the forward analysis and database generation using the current approach. Therefore, considering the limited project duration and lack of development time, the current approach is restricted to the backcaclulation of AC E(t) master curve coefficients. However, it is recommended that future research efforts include backcalculation of shift factors for the master curve from FWD deflection-time history data.

Case Studies

First, a preliminary (screening) analysis was carried out for full-depth AC through various case studies to verify the feasibility of the NN approach, identify the promising input features and NN parameters for inverse modeling, and identify associated modeling challenges. The primary goal of these case studies was to answer the question: Are NNs capable of learning/mapping the complex, nonlinear relationship between AC E(t) master curve coefficients and FWD time history data? These case studies (as well as the rest of the report) focus on full-depth AC pavements as a first step to isolate potential backcalculation issues that are only related to the modulus master curve of the AC layer.

Among the several case studies conducted by the research team, three case studies are reported here that systematically varied the inputs for the NN inverse modeling: (1) consider only FWD D0 time history data; (2) consider FWD D0, D8, and D12 time history data; (3) consider only FWD Surface Curvature Index (D0–D12). Here D0, D8, and D12 refer to deflection-time history data recorded at an offset of 0, 8, and 12 inches, respectively, from the center of the FWD loading plate. It is expected that the effect of viscoelasticity will be more pronounced in the sensors closest to the load plate. Further, some studies have reported that the addition of further sensors in the backcalculation process tends to increase the error in E(t) predictions (Varma et al. 2013a). However, future research should consider all sensors in the standard FWD configuration to elaborately investigate their influence on the accuracy of the backcalculated E(t) master curve and unbound layers.

Development of Synthetic Database

As mentioned previously, the VE forward analysis program outputs pavement surface deflection-time histories based on the following inputs: FWD stress-time history, AC E(t) master curve coefficients and shift factors, pavement temperature, pavement layer thicknesses and Poisson's ratios, and unbound layer moduli. The FWD stress-time history for a standard 9 kip loading was used for these case studies and for other analyses discussed in the rest of the report. For full-depth AC analysis, the inputs were reduced to AC E(t) master curve coefficients ($c1$, $c2$, $c3$, and $c4$) and shift factors ($a1$ and $a2$), pavement temperature (T_{ac}), AC layer thickness (H_{ac}), subgrade layer modulus (E_{sub}), AC Poisson's ratio (μ_{ac}), and subgrade Poisson's ratio (μ_{sub}).

Because the goal of this exercise was to quickly verify the feasibility of the NN approach, certain inputs were blocked out from the modeling by assigning them constant values for all simulations. This enabled a more focused evaluation of the ability of the NNs in modeling the relationship

between the E(t) master curve and deflection-time histories. A synthetic database consisting of 100 scenarios was generated through batch simulations of the VE forward analysis program using the input ranges summarized in Table 1. The min-max ranges of E(t) master curve coefficients and shift factors are based on the Michigan State University (MSU) E(t) database of 100+ hot-mix asphalt (HMA) mixtures (Varma et al. 2013a).

Table 1. Summary of input ranges used in the generation of 100 VE forward analysis scenarios for case studies

Input Parameter	Min Value	Max Value
Pavement temperature (T_{ac})	32 deg-F (0 deg-C)	113 deg-F (45 deg-C)
AC layer thickness (H_{ac}):	20 in. (constant)	
Subgrade modulus (E_{sub}):	10,000 psi (constant)	
AC Poisson's ratio (μ_{ac}):	0.3 (constant)	
Subgrade Poisson's ratio (μ_{sub}):	0.4 (constant)	
c1	0.045	2.155
c2	1.8	4.4
c3	-0.523	1.025
c4	-0.845	-0.38
a1	-5.380E-4	1.136E-3
a2	-1.598E-1	-0.770E-1

Descriptive statistics (such as the mean, standard deviation, and data about the shape of the distribution) were calculated separately for each of the variables in the synthetic datasets. To visually see the distribution of generated synthetic datasets, a compound graph consisting of a histogram (with the normal distribution curve superimposed over the observed frequencies), normal probability plot, box plot, and descriptive statistics was compiled for each of the variable inputs. These graphs are displayed in Figure 8.
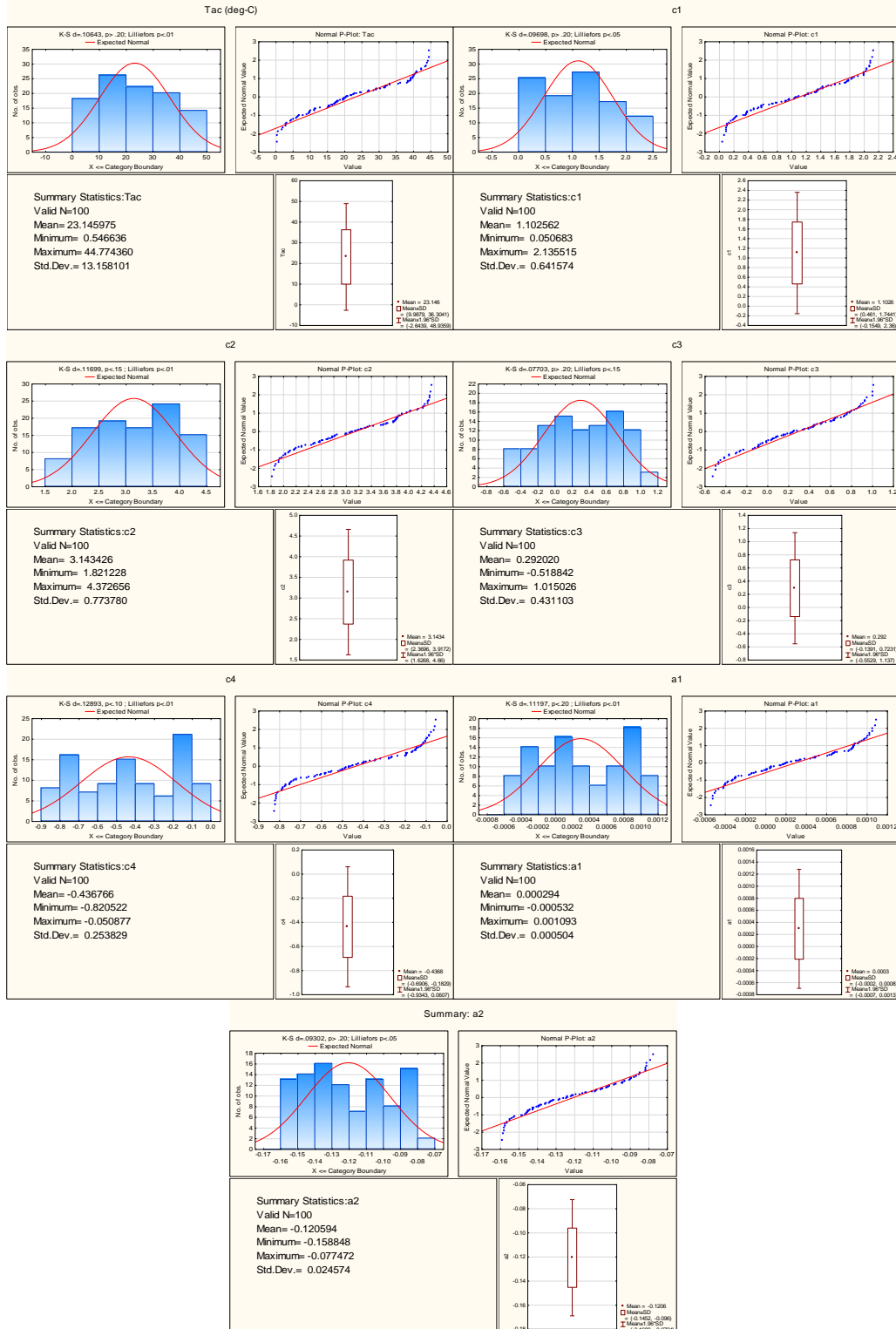
Figure 8. Compound graphs summarizing descriptive statistics, histograms, box plots, and normal probability plots for variable inputs in the synthetic datasets

The latter portion of the FWD deflection-time history curve typically includes noise and integration errors, and some recent studies have concluded that the current FWD technology needs modification to ensure reliable measurements in the tail regions of the FWD deflection-time histories (Kutay et al. 2011). Consequently, it was decided to use the left half of the deflection-time history data (i.e., up to peak deflections) in the NN inverse modeling. This corresponds to deflection-time histories at the first 20 discrete time intervals, as shown in Figure 9. Box and whisker plots for D0, D8, and D12 deflection-time histories (outputs) are displayed in Figure 10. In these plots, the central square indicates the mean, the box indicates the mean plus/minus the standard deviation, and whiskers around the box indicate the mean plus/minus 1.96×standard deviation.
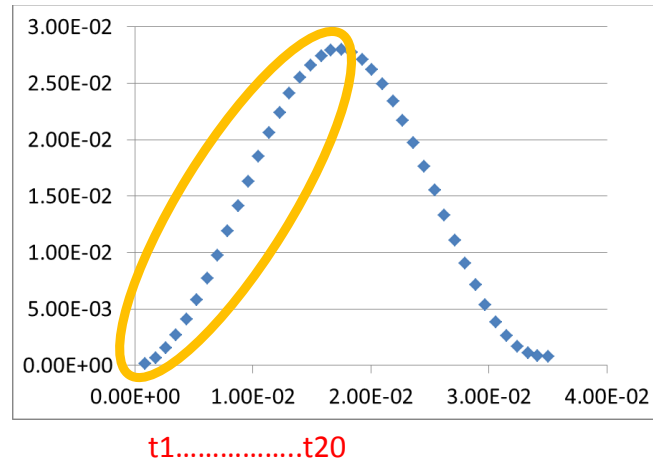
Figure 9. Typical deflection-time history generated by the VE forward analysis program at one location. Only the left half of the deflection-time history data was considered in NN inverse modeling in this study.
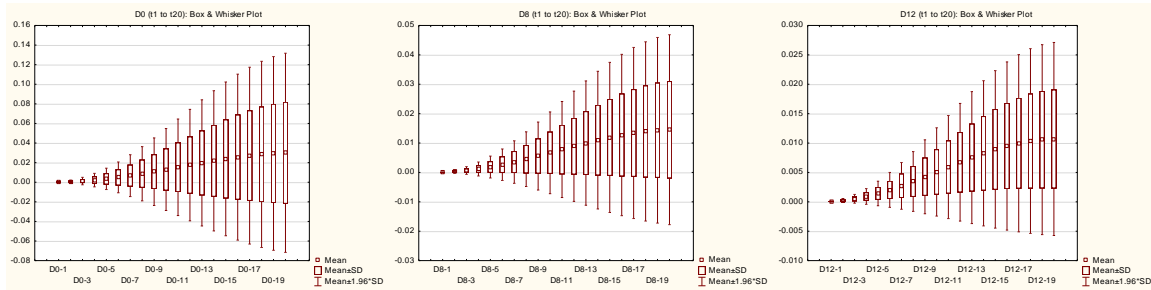
Figure 10. Box and whisker plots of D0, D8, and D12 deflection-time histories generated by VE forward analysis simulations for the case studies

Neural Networks Inverse Modeling

A literature review (Dougherity 1995, TR Circular 1999, Adeli 2001, Gopalakrishnan et al. 2009) suggested that artificial neural networks (ANNs, more recently referred to as simply neural networks [NNs]) and other soft computing techniques like fuzzy mathematical programming and evolutionary computing (including genetic algorithms) are increasingly used instead of the

traditional methods in civil and transportation engineering applications (Flintsch 2003). They have become standard data fitting tools, especially for problems that are too complex, poorly understood, or resource intensive to tackle using more traditional numerical and/or statistical techniques. They can, in one sense, be viewed as similar to nonlinear regression, except that the functional form of the fitting equation does not need to be specified a priori. The adoption and use of NN modeling techniques in the MEPDG/Pavement-ME Design (NCHRP 2004) has especially placed emphasis on the successful use of neural nets in geomechanical and pavement systems.

Given the successful utilization of NN modeling techniques in the previous IHRB projects focusing on nondestructive evaluation of Iowa pavements and static backcalculation of pavement layer moduli from routine FWD test data (Ceylan et al. 2007, Ceylan et al. 2009, Ceylan et al. 2013), the research team's first choice was to employ NN for this study. The ability to "learn" the mapping between inputs and outputs is one of the main advantages that make the NNs so attractive. Efficient learning algorithms have been developed and proposed to determine the weights of the network, according to the data of the computational task to be performed. The learning ability of the NNs makes them suitable for unknown and nonlinear problem structures such as pattern recognition, medical diagnosis, time series prediction, and other applications (Haykin 1999).

The NNs in this study were designed, trained, validated, and tested using the MATLAB Neural Network toolbox (Beale et al. 2011). All of the NNs were conventional two-layer (one hidden layer and one output layer) feed-forward networks. Sigmoid transfer functions were used for all hidden layer neurons, while linear transfer functions were employed for the output neurons. Training was accomplished using the Levenberg-Marquardt (LM) backpropagation algorithm. Considerable research has been carried out to accelerate the convergence of learning/training algorithms, which can be broadly classified into two categories: (1) development of ad hoc heuristic techniques that include such ideas as varying the learning rate, using momentum, and rescaling variables; and (2) development of standard numerical optimization techniques.

The three types of numerical optimization techniques commonly used for NN training include the conjugate gradient algorithms, quasi-Newton algorithms, and the LM algorithm. The LM algorithm used in this study is a second-order numerical optimization technique that combines the advantages of Gauss–Newton and steepest descent algorithms. While this method has better convergence properties than the conventional backpropagation method, it requires $O(N^2)$ storage and calculations of order $O(N^2)$, where N is the total number of weights in a multi-layer perceptron (MLP) backpropagation. The LM training algorithm is considered to be very efficient when training networks have up to a few hundred weights. Although the computational requirements are much higher for each iteration of the LM training algorithm, this is more than made up for by the increased efficiency. This is especially true when high precision is required (Beale et al. 2011).

Separate NN models were developed for each of the four E(t) master curve coefficients, c1, c2, c3, and c4. Seventy percent of the 100 datasets were used for training, 15% were used for validation (to halt training when generalization stops improving), and 15% were used for

independent testing of the trained model. As mentioned previously, the highlighted case studies considered three input scenarios in the prediction of E(t) master curve coefficients: (1) use only D0 time history data as inputs; (2) use D0, D8, and D12 time history data as inputs; and (3) use only the differences in magnitudes between D0 and D12 time history data (i.e., Surface Curvature Index (SCI) = D0–D12) as inputs. Because the SCI is known to be strongly correlated to (static) backcalculated Eac, its usefulness (i.e., differences between D0 and D12 time history data) in the backcalculation of E(t) master curve coefficients was evaluated in case study #3. Graphical summaries of the NN inverse modeling training curves and correlations between observed and predicted data are displayed for all three case studies in Figure 11, Figure 12, and Figure 13.
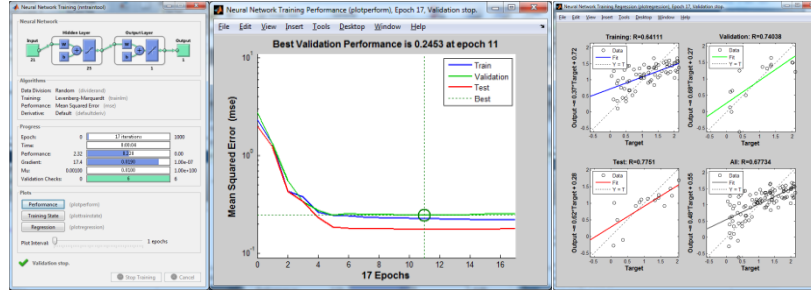
(a)
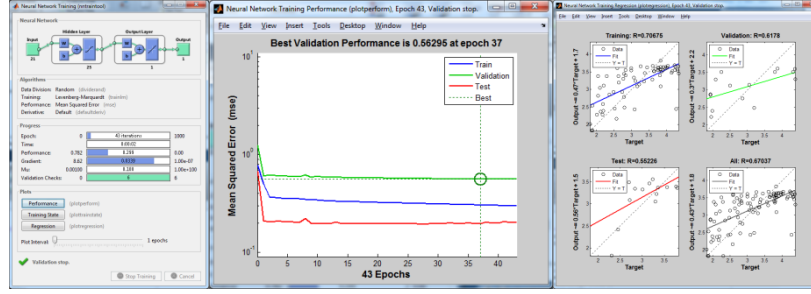
(b)
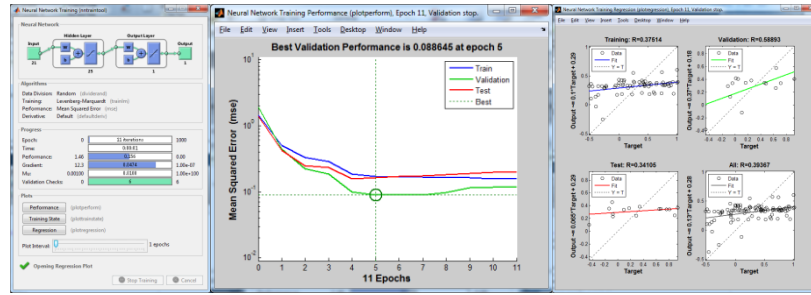
(c)
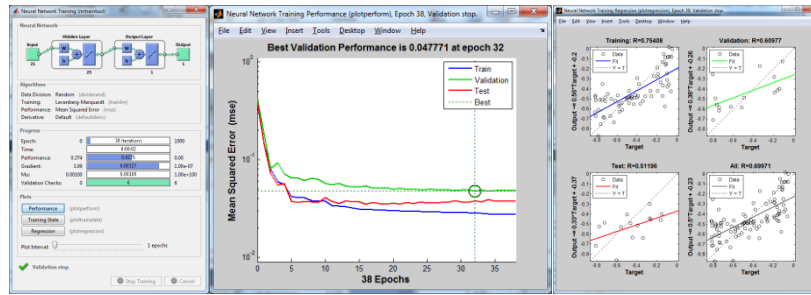
(d)

Figure 11. NN prediction of E(t) master curve coefficients from D0 time history data using 100 datasets (case study #1): (a) c1, (b) c2, (c) c3, (d) c4
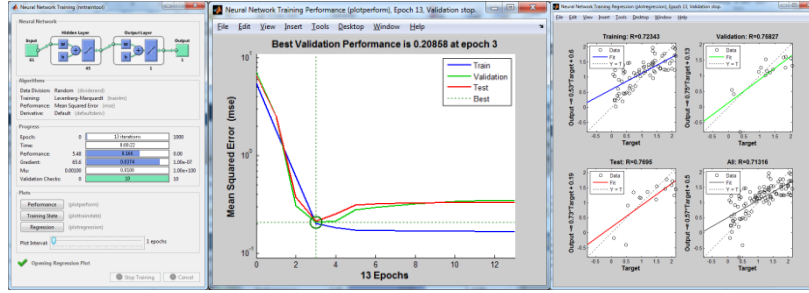
(a)
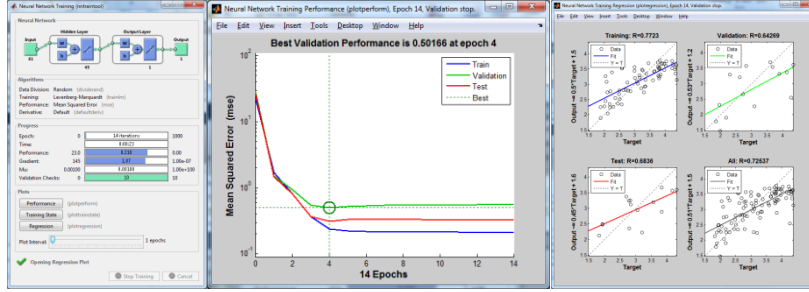
(b)
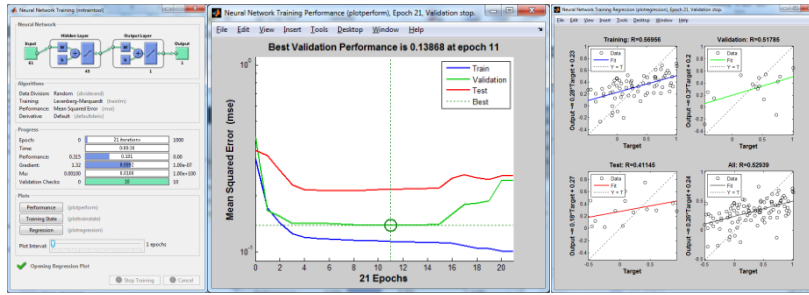
(c)
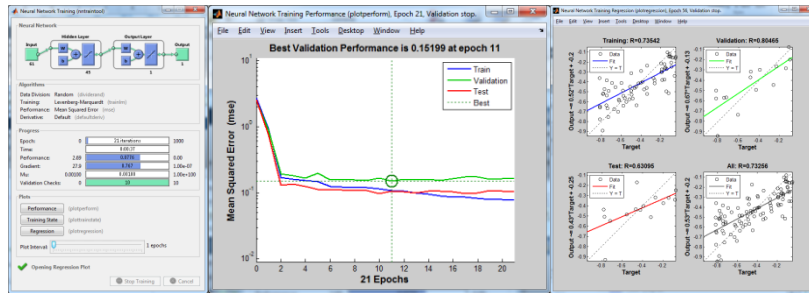
(d)

Figure 12. NN prediction of E(t) master curve coefficients from D0, D8, and D12 time history data using 100 datasets (case study #2): (a) c1, (b) c2, (c) c3, (d) c4
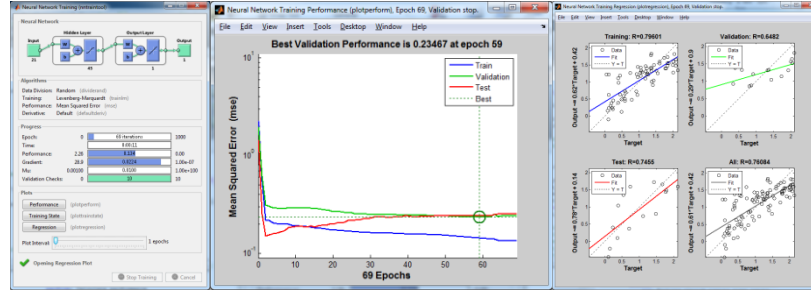
(a)
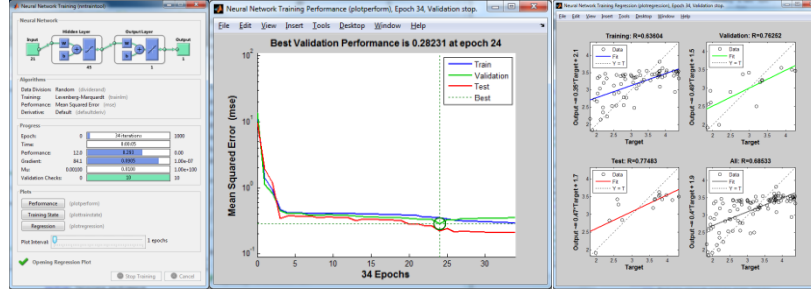
(b)
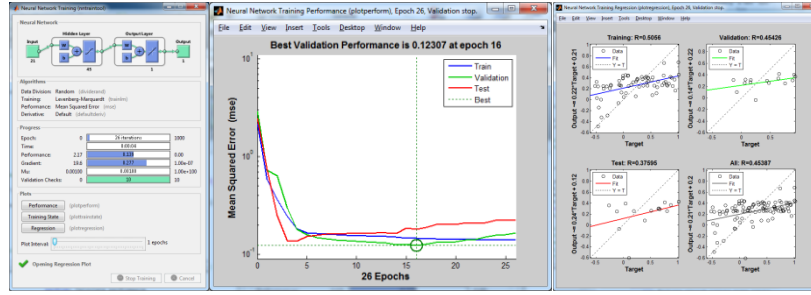
(c)
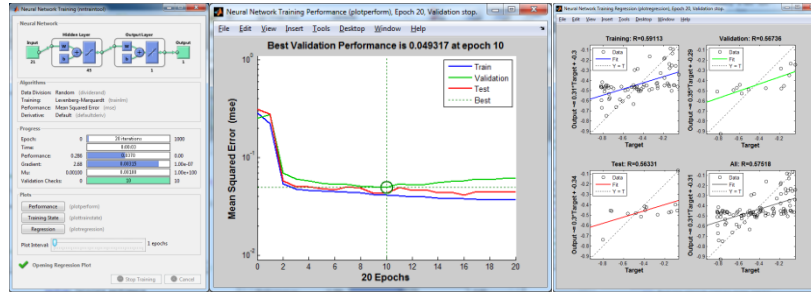
(d)

Figure 13. NN prediction of E(t) master curve coefficients from differences in magnitudes between D0 and D12 time history data (SCI) using 100 datasets (case study #3): (a) c1, (b) c2, (c) c3, (d) c4

In general, the results from the case studies demonstrate that the NNs have the potential to model the complex relationships between E(t) master curve coefficients and surface deflection-time histories. The use of D0, D8, and D12 time history data, as opposed to the use of D0 time history alone, seems to result in relatively higher prediction accuracies of E(t) master curve coefficients. The use of differences in magnitudes between D0 and D12 time history data (i.e., SCI) alone as

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an input in NN inverse modeling did not increase the prediction accuracies and is therefore not recommended for future analysis. Future analysis should also consider the effect of including deflection-time history data from all sensors in the standard FWD configuration (D0, D8, D12, D18, D24, D36, D48, D60, and D72) on the prediction accuracies.

PROOF-OF-CONCEPT DEMONSTRATION:COMPREHENSIVE FULL-DEPTH AC PAVEMENT ANALYSIS

Development of Comprehensive Synthetic Database

The focused case studies carried out and discussed in the previous section established the framework for deriving the AC E(t) or |E*| master curve based on a single FWD test performed at a single temperature, thereby fulfilling the main objective of this study. In this section, the proposed methodology is further explored through a comprehensive forward and inverse analysis of full-depth AC. A comprehensive synthetic database consisting of 10,000 datasets was generated through batch simulations of the VE forward analysis program by randomly varying the inputs within the min-max ranges, summarized in Table 2.

Table 2. Summary of input ranges used in the generation of 10,000 VE forward analysis scenarios for comprehensive full-depth AC analysis

Input Parameter	Min Value	Max Value
Pavement temperature (T_{ac})	32 deg-F (0 deg-C)	113 deg-F (45 deg-C)
AC layer thickness (H_{ac}):	5 in.	45 in.
Subgrade modulus (E_{sub}):	5,000 psi	20,000 psi
AC Poisson's ratio (μ_{ac}):	0.3 (constant)	
Subgrade Poisson's ratio (μ_{sub}):	0.4 (constant)	
c1	0.045	2.155
c2	1.8	4.4
c3	-0.523	1.025
c4	-0.845	-0.38
a1	-5.380E-4	1.136E-3
a2	-1.598E-1	-0.770E-1

Graphical comparative summaries (a histogram, box plot, and descriptive statistics) for each of the input variables in the synthetic database are displayed in Figure 14 and Figure 15. The case studies discussed in the previous section tend to indicate that the deflection-time history data at D0, D8, and D12 sensors are necessary inputs for backcalculating the E(t) master curve coefficients. As discussed previously, only the first half of the deflection-time history data (i.e., corresponding to time intervals 1 to 20 in the x-axis) were considered in the analysis.
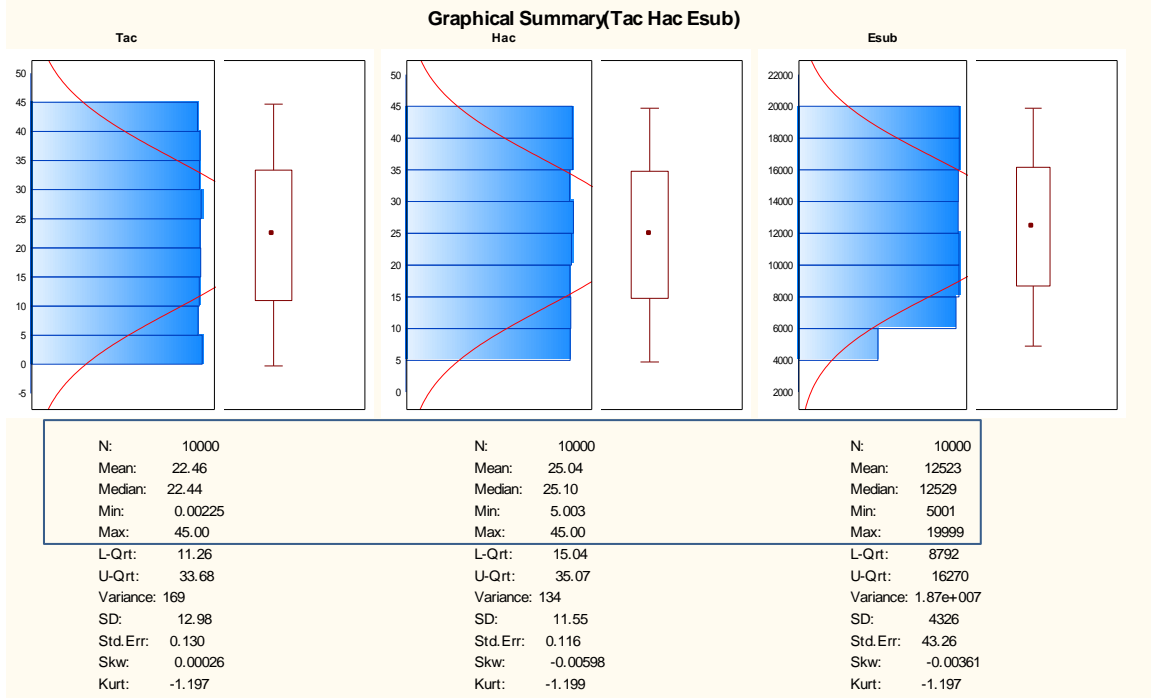
Figure 14. Graphical statistical summaries of Tac, Hac, and Esub in the comprehensive raw synthetic database
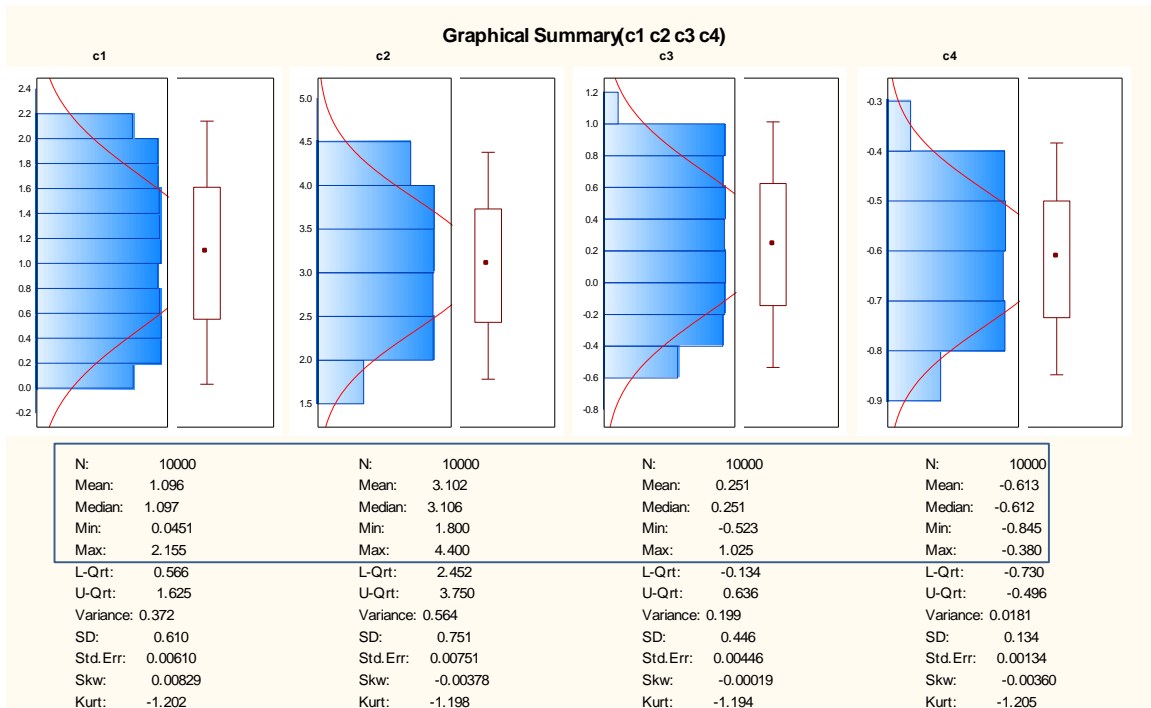
Figure 15. Graphical statistical summaries of c1, c2, c3, and c4 in the comprehensive raw synthetic database

Synthetic Database: Data Pre-processing

Some anomalies were discovered in the outputs extracted from the results of 10,000 VE forward analysis simulations. It was discovered that the E(t) curves generated by considering the upper and lower limits of c1, c2, c3, and c4 based on the MSU E(t) database of 100+ AC mixtures can result in master curves well outside the database. This can lead to unexpectedly high deflection magnitudes, unreasonable deflection time-histories, and sometimes negative deflections. Some of these anomalies are captured in the D0, D8, and D12 deflection-time histories (outputs) resulting from the 10,000 VE forward runs, depicted in the form of 3-D plots in Figure 16, Figure 17, and Figure 18. To overcome these issues, it was decided to include only those scenarios in the database where the sum of E(t) sigmoid coefficients c1 and c2 was within certain limits. Varma et al. (2013a) used a lower limit of 3.239 and an upper limit of 4.535 based on the MSU database of 100+ AC mixtures. These limits are 4.000 and 5.880, respectively, based on the E(t) curves generated by the ISU research team using the FHWA mobile lab asphalt mixture database.
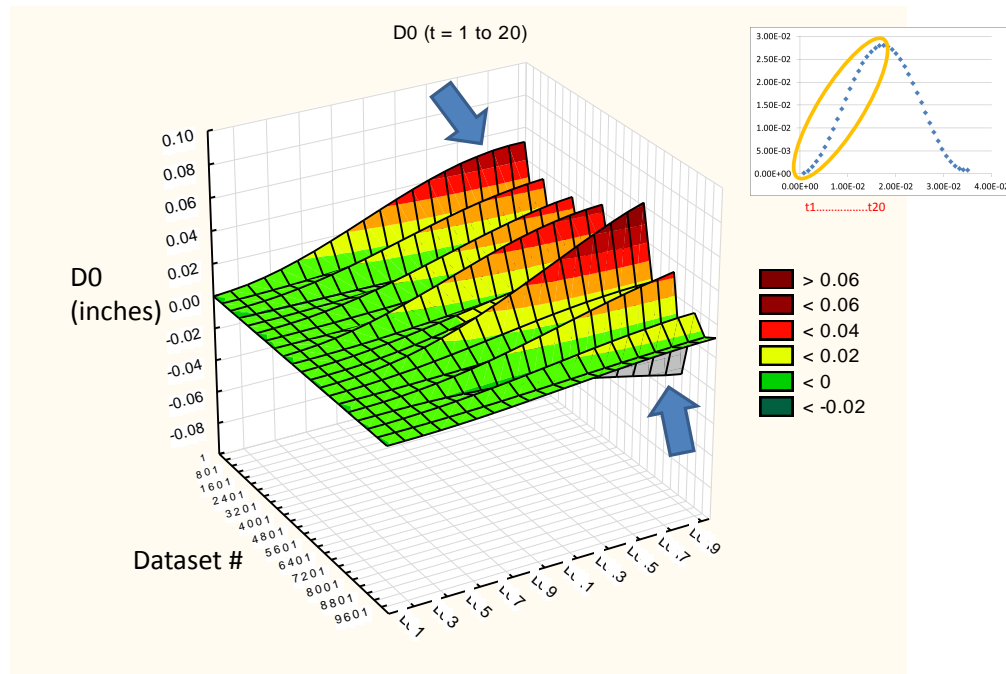
Figure 16. D0 deflection-time history (from time interval 1 to 20) outputs from 10,000 VE forward analysis simulations
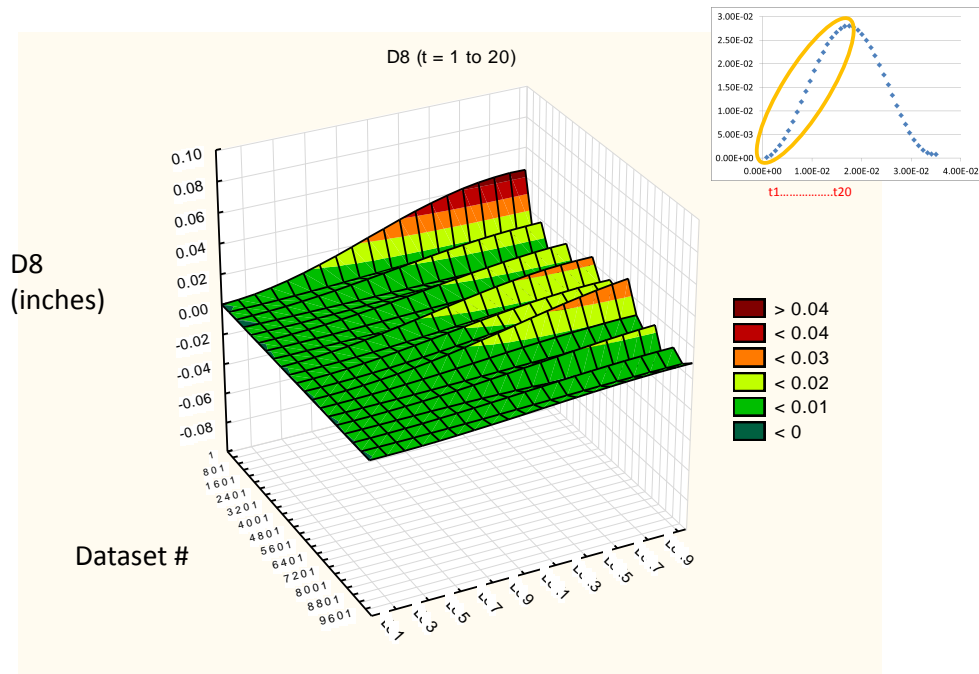
Figure 17. D8 deflection-time history (from time interval 1 to 20) outputs from 10,000 VE forward analysis simulations
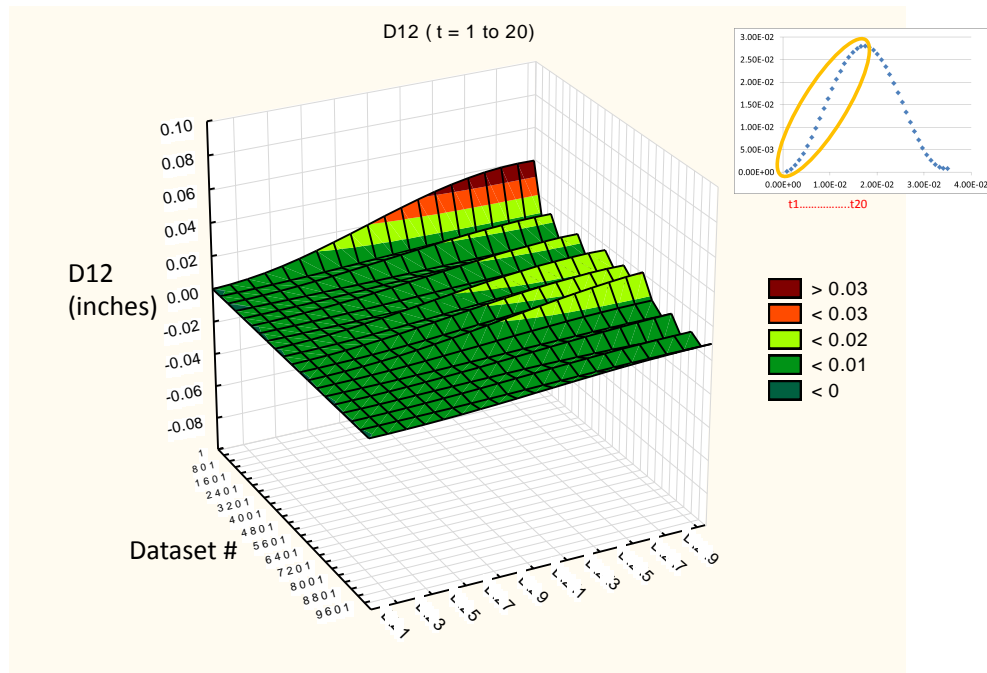
Figure 18. D12 deflection-time history (from time interval 1 to 20) outputs from 10,000 VE forward analysis simulations

After imposing the limits on the sum of sigmoid coefficients, $c_1 + c_2$, and eliminating unreasonable deflection-time histories, the processed database size decreased from 10,000 to

3,338. Graphical comparative summaries for each of the input variables in the processed synthetic database are displayed in Figure 19 and Figure 20. The D0, D8, and D12 deflection-time histories (outputs) from the processed synthetic database captured in Figure 21, Figure 22, and Figure 23 appear reasonable.
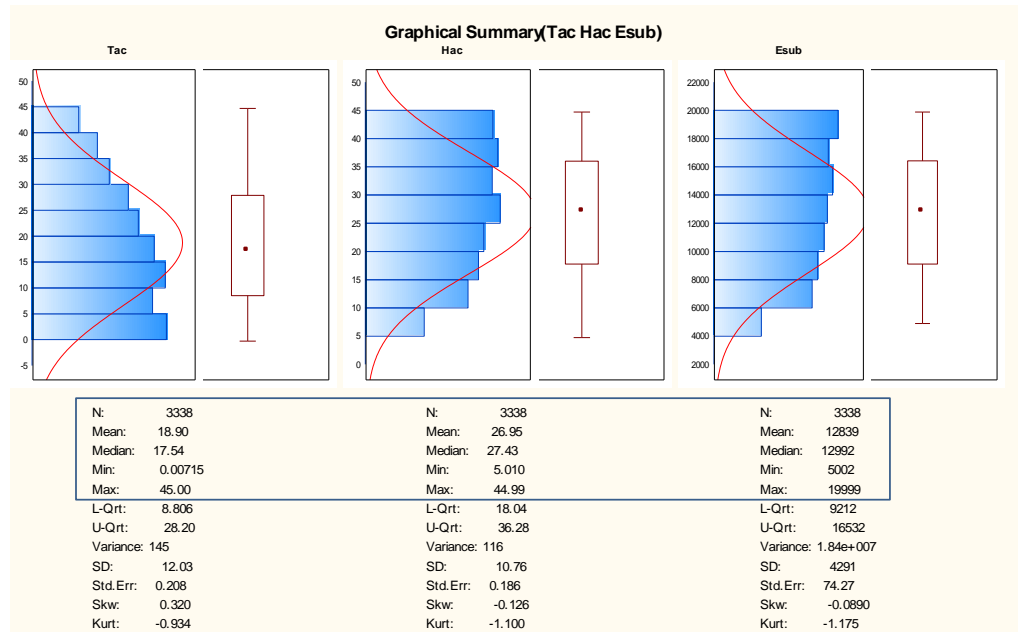
Figure 19. Graphical statistical summaries of Tac, Hac, and Esub in the processed synthetic database
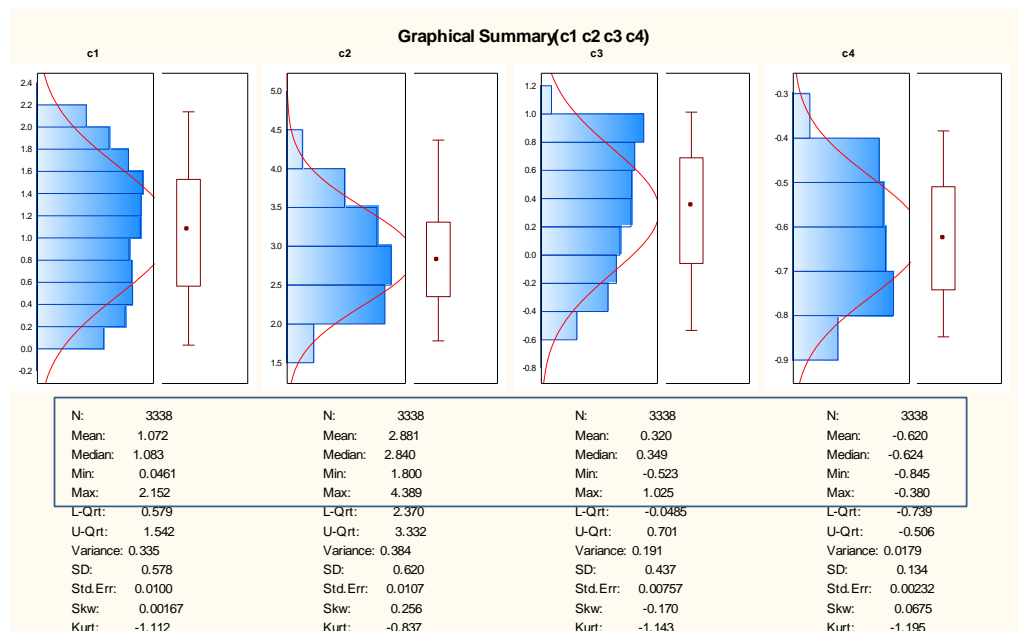
Figure 20. Graphical statistical summaries of c1, c2, c3, and c4 in the processed synthetic database

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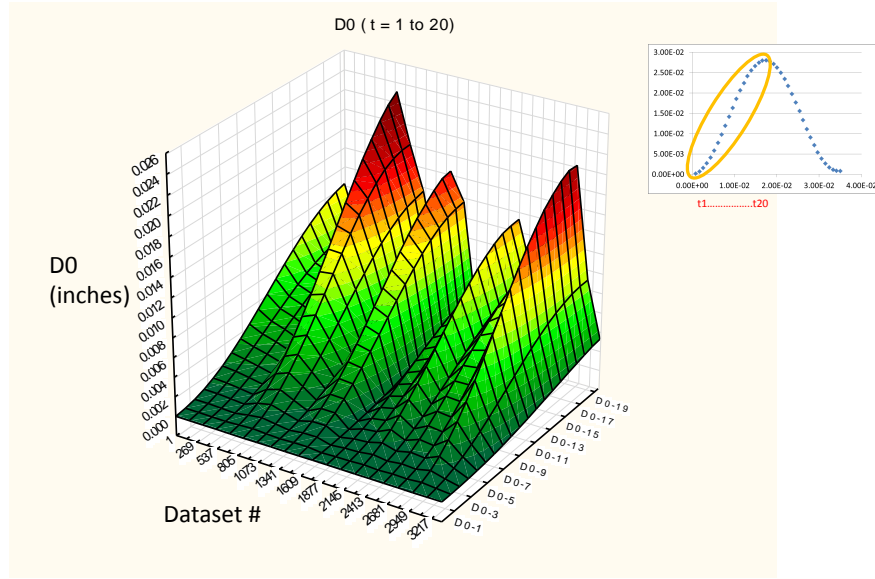
Figure 21. D0 deflection-time history (from time interval 1 to 20) outputs in the processed synthetic database
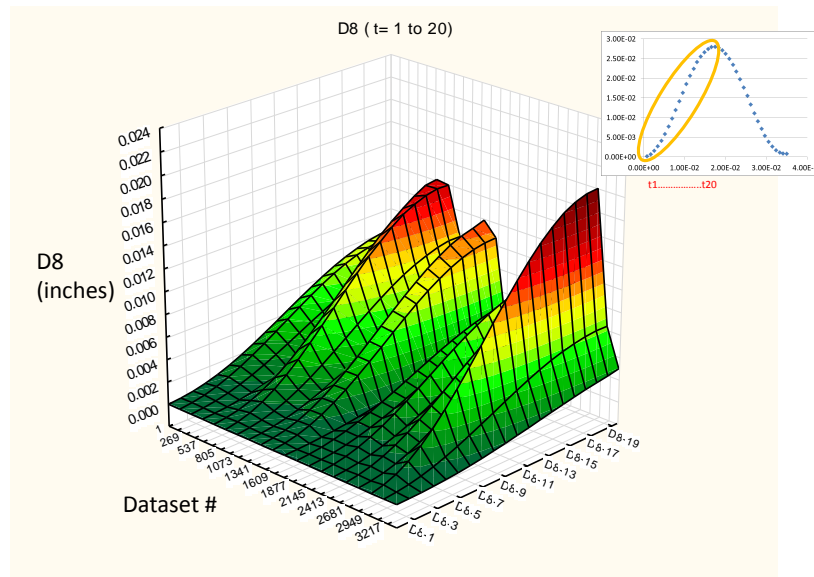
Figure 22. D8 deflection-time history (from time interval 1 to 20) outputs in the processed synthetic database
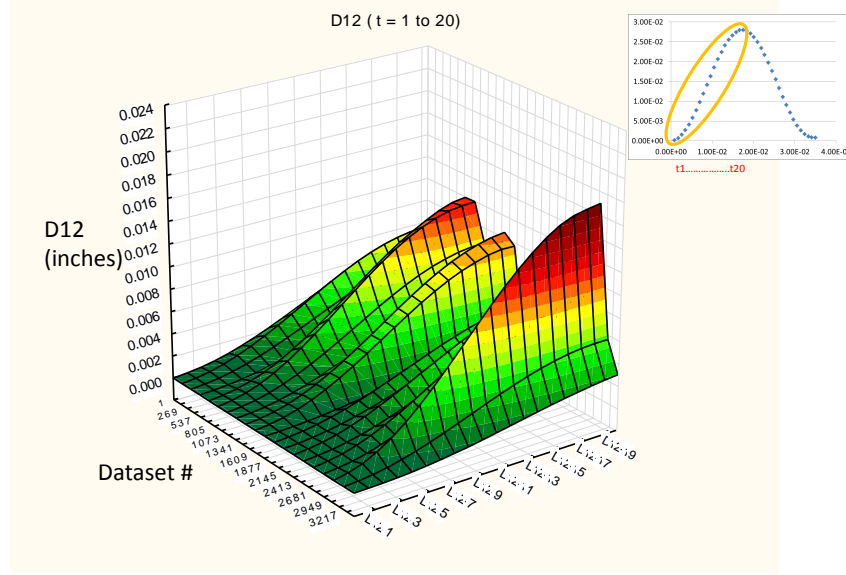
Figure 23. D12 deflection-time history (from time interval 1 to 20) outputs in the processed synthetic database

Neural Networks Forward Modeling

Before carrying out NN inverse modeling to map E(t) master curve coefficients from FWD deflection-time histories, NN forward modeling was carried out to see how accurately NNs can predict the individual deflection-time histories (D0, D8, and D12) based on NN E(t) master curve coefficients and other inputs in the processed synthetic database. If successful, the NN forward model could also serve as a surrogate model (within the specified input ranges) that could replace the VE forward analysis runs.

Based on a parametric sensitivity analysis, a conventional two-layer (1 hidden layer with 25 neurons and 1 output layer) feed-forward network was deemed sufficient for forward modeling. Sigmoid transfer functions were used for all hidden layer neurons, while linear transfer functions were employed for the output neurons. Training was accomplished using the LM backpropagation algorithm implemented in the MATLAB NN Toolbox. Separate NN models were developed for each of the deflections (3 sensor locations and 20 time intervals). Thus, the model inputs were E(t) master curve coefficients (c_1, c_2, c_3, and c_4), Tac, Hac, and Esub. Seventy percent of the 3,338 datasets were used for training, 15% were used for validation (to halt training when generalization stops improving), and 15% were used for independent testing of the trained model.

The NN forward modeling regression results for predicting D0, D8, and D12 deflection-time histories are summarized in Figure 24, Figure 25, and Figure 26, respectively. As seen in these plots, except for the first two or three time intervals, deflection-time histories at all other time intervals are predicted by NN analysis with very high accuracy (R-values greater than 0.97). Consequently, it was decided to eliminate these (D0-1, D0-2, D0-3, D8-1, D8-2, D8-3, D12-1, D12-2, and D12-3) from the input set for the NN inverse modeling discussed in the next section.

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D0-1 D0-2 D0-3 D0-4 D0-5 D0-6 D0-7
D0-8 D0-9 D0-10 D0-11 D0-12 D0-13 D0-14
D0-15 D0-16 D0-17 D0-18 D0-19 D0-20
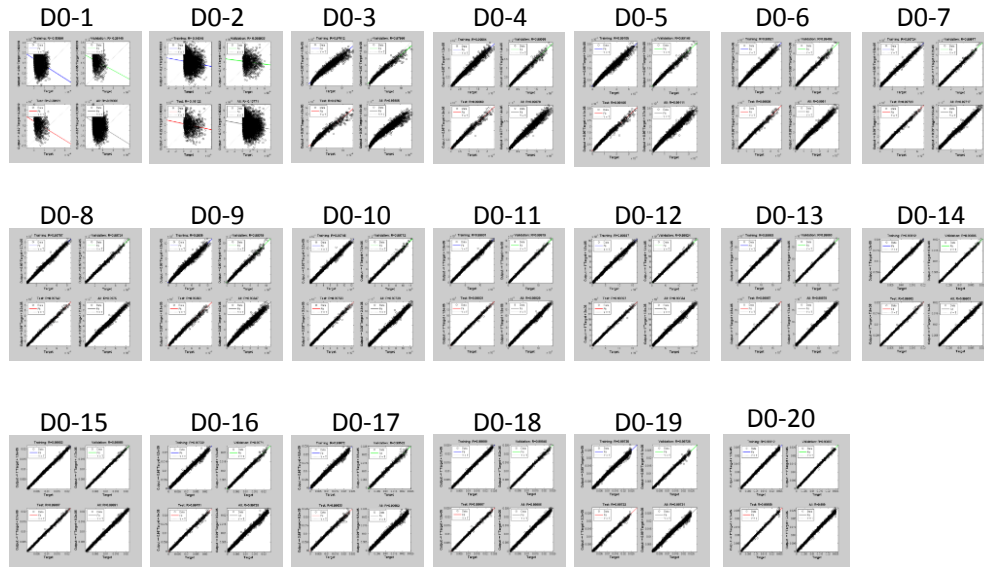
Figure 24. NN forward modeling regression results for predicting D0 deflection-time history data from E(t) master curve coefficients

D8-1 D8-2 D8-3 D8-4 D8-5 D8-6 D8-7
D8-8 D8-9 D8-10 D8-11 D8-12 D8-13 D8-14
D8-15 D8-16 D8-17 D8-18 D8-19 D8-20
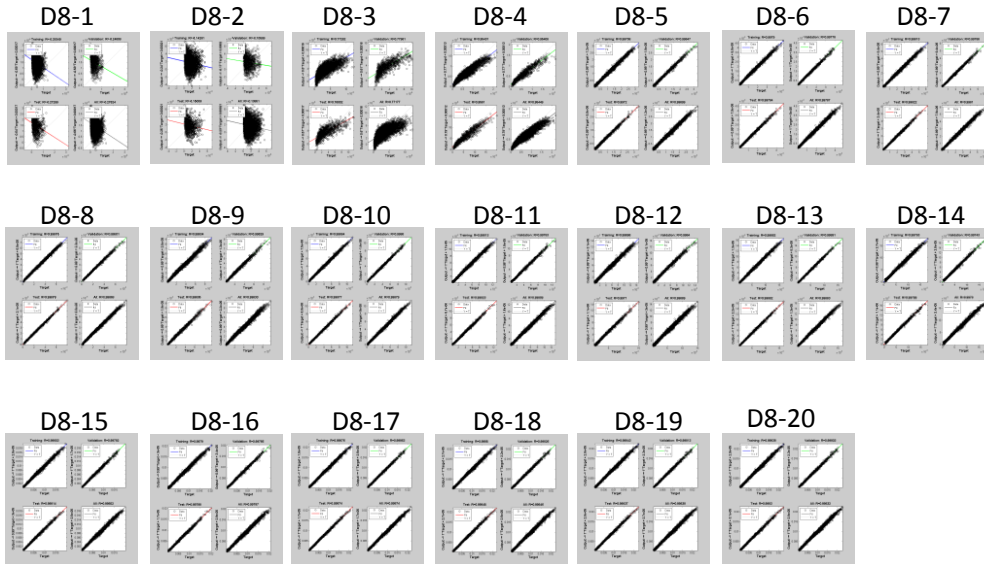
Figure 25. NN forward modeling regression results for predicting D8 deflection-time history data from E(t) master curve coefficients
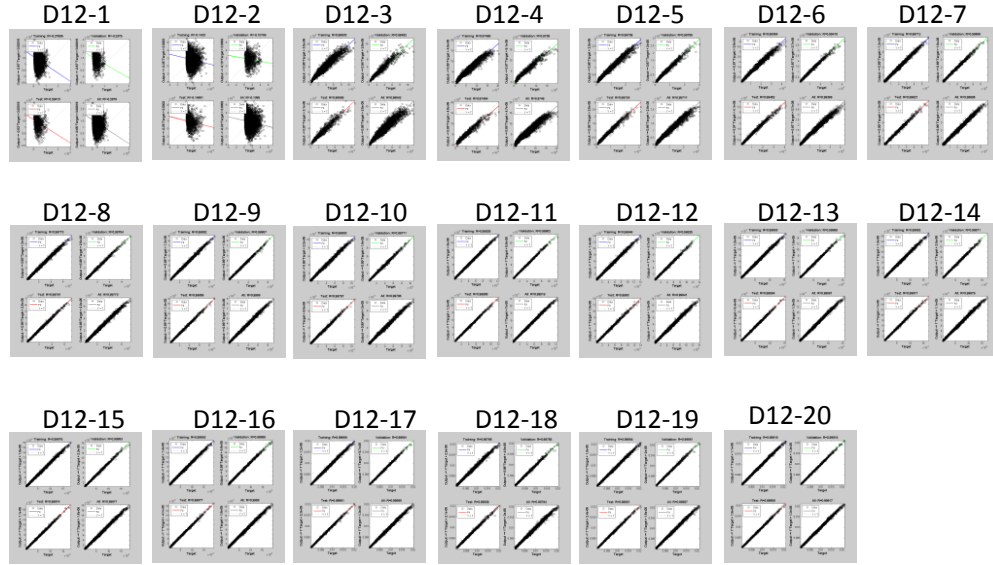
Figure 26. NN forward modeling regression results for predicting D12 deflection-time history data from E(t) master curve coefficients

Neural Networks Inverse Modeling Considering only D0, D8, and D12

The processed synthetic database consisting of 3,338 input-output scenarios was utilized in developing NN inverse models for predicting E(t) master curve coefficients from FWD deflection-time histories by considering only D0, D8, and D12 sensors. The inputs, outputs, and the generic network architecture details for the NN inverse mapping models are summarized in Figure 27.

Inputs

- D0, D8, D12 Deflection-time histories [D0(tj)] (j = 4 to 20)
- Tac
- Hac
- Esub
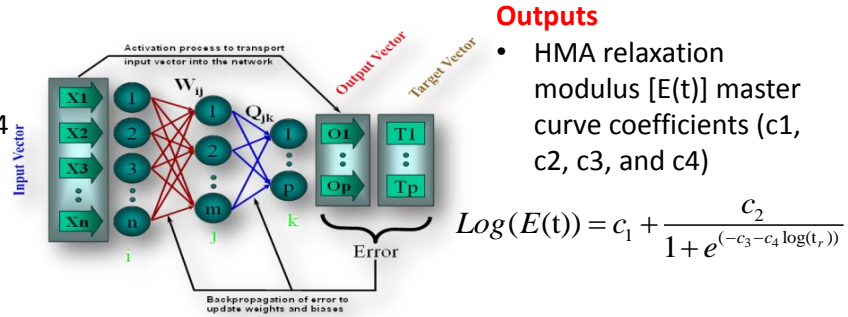
Outputs

- HMA relaxation modulus [E(t)] master curve coefficients (c_1, c_2, c_3, and c_4)

$$Log(E(\text{t})) = c_1 + \frac{c_2}{1+e^{(-c_3-c_4\log(t_r))}}$$

Generic network architecture: 54-h-1

54 inputs: Tac, Hac, Esub, D0(tj); D8(tj);D12(tj)[j= 4 to 20]
h hidden neurons
1 output: ci (each of the 4 coefficients predicted separately)

Figure 27. Inputs, outputs, and generic network architecture details for the NN inverse mapping models considering only D0, D8, and D12

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A conventional two-layer (1 hidden layer with 25 neurons and 1 output layer) feed-forward network was employed for inverse modeling. Sigmoid transfer functions were used for all hidden layer neurons, while linear transfer functions were employed for the output neurons. Training was accomplished using the LM backpropagation algorithm implemented in the MATLAB NN Toolbox. Separate NN models were developed for each of the E(t) master curve coefficients (c1, c2, c3, and c4). Seventy percent of the 3,338 datasets were used for training, 15% were used for validation (to halt training when generalization stops improving), and 15% were used for independent testing of the trained model. Graphical summaries of the NN inverse modeling training curves and correlations between observed and predicted data are displayed for each of the E(t) master curve coefficients, c1, c2, c3, and c4 in Figure 28, Figure 29, Figure 30, and Figure 31, respectively.
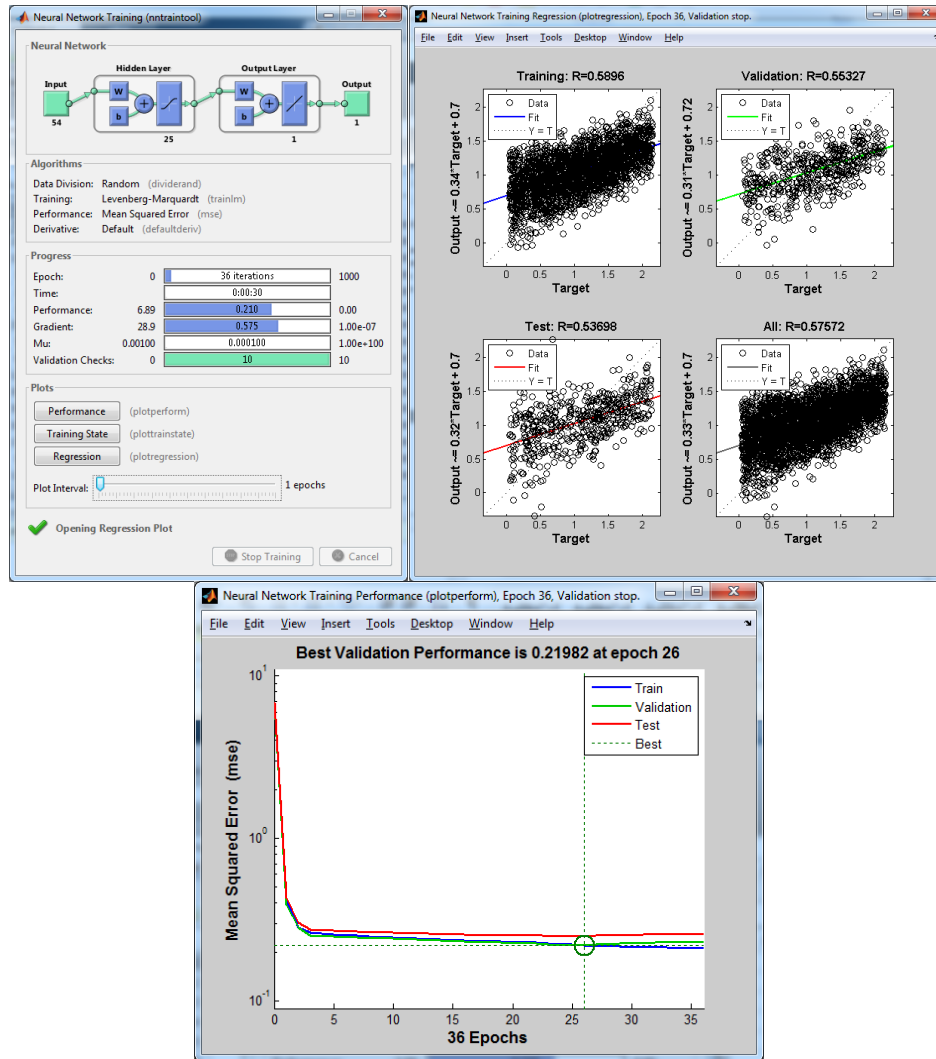
Figure 28. NN prediction of E(t) master curve coefficient, c1, from D0, D8, and D12 time history data using the processed synthetic database
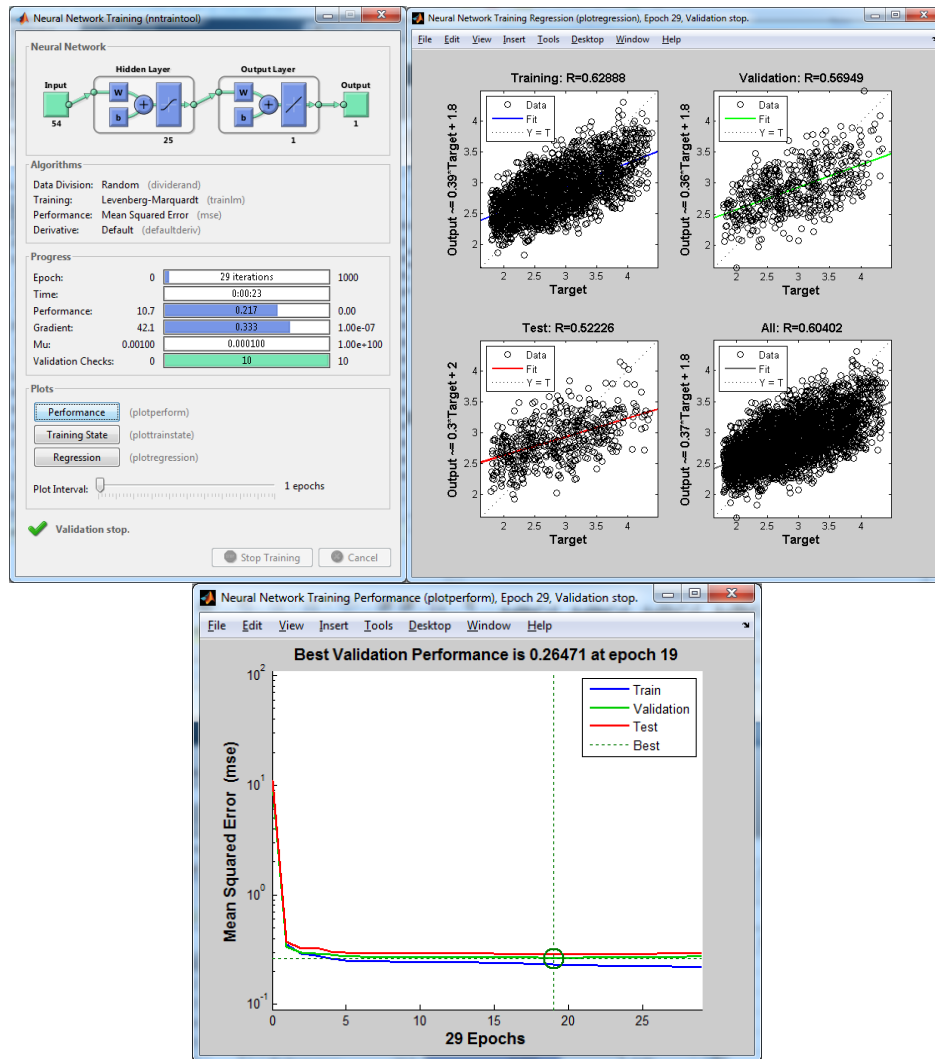
Figure 29. NN prediction of E(t) master curve coefficient, c2, from D0, D8, and D12 time history data using the processed synthetic database
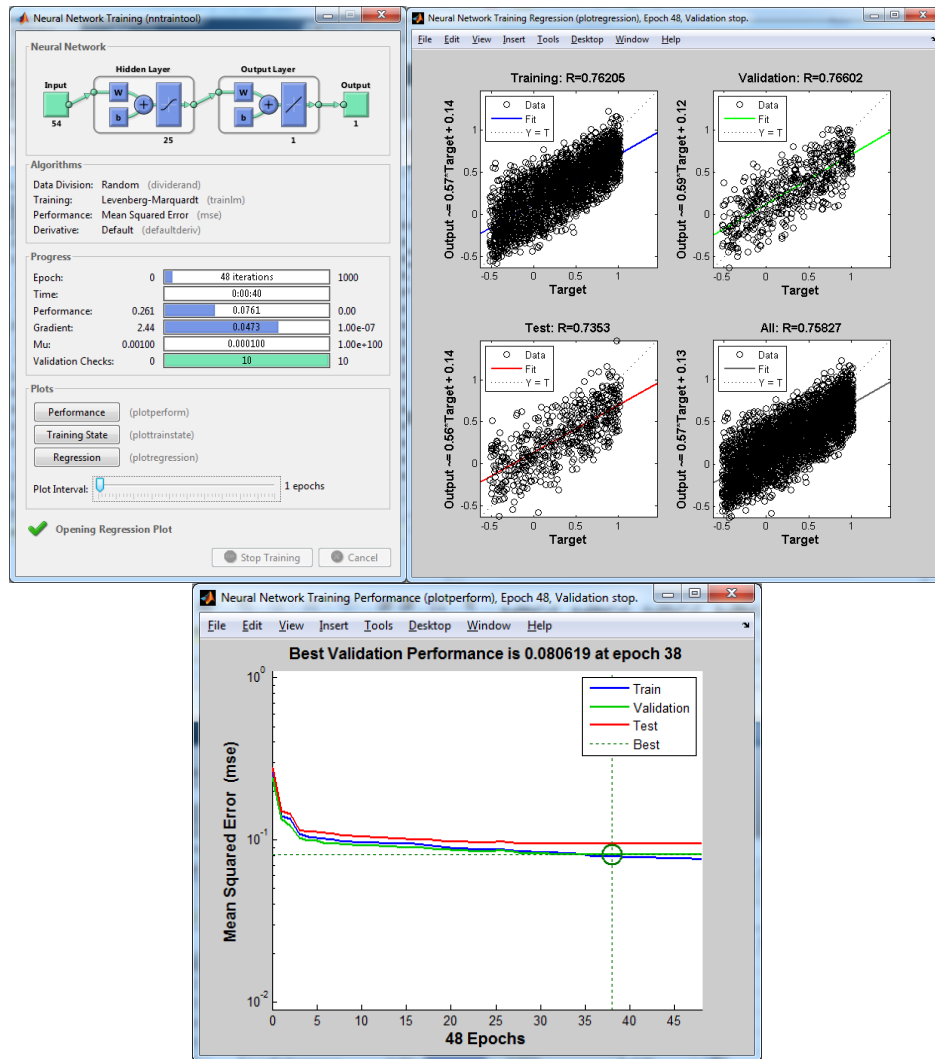
Figure 30. NN prediction of E(t) master curve coefficient, c3, from D0, D8, and D12 time history data using the processed synthetic database
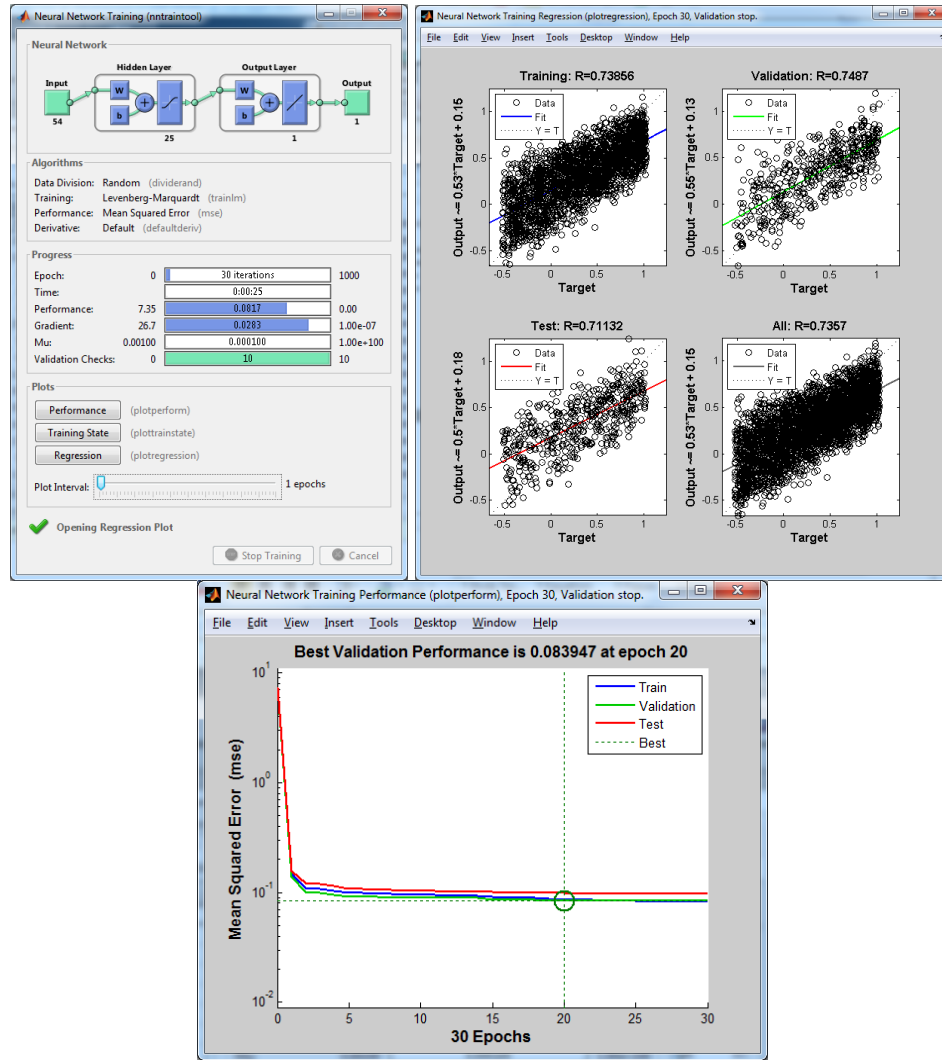
Figure 31. NN prediction of E(t) master curve coefficient, c4, from D0, D8, and D12 time history data using the processed synthetic database

Neural Networks Inverse Modeling Considering Data from all FWD Sensors: Partial Pulse (Pre-Peak) Time Histories

The previous analysis considered only time history data from the sensors close to the FWD loading plate: D0, D8, and D12. Further analyses were carried out to investigate whether the NN predictions could be improved by considering time history data from all the sensors of the standard FWD configuration: D0, D8, D12, D18, D24, D36, D48, D60, and D72. These analyses considered two scenarios: (1) using only partial (pre-peak) pulse deflection-time history data and (2) using full pulse deflection-time history data. This section focuses on the first scenario, while the next section presents the results for the second scenario, where the full deflection pulses from all FWD sensor time history data are included as inputs. The inputs, outputs, and the generic network architecture details for the NN inverse mapping models, considering pre-peak deflection pulse time history data (except the first three time steps), are summarized in Figure 32.

Inputs
- D0, D8, D12, D18, D24, D36, D48, D60, D72 Deflection-time histories [$D_i(t_j)$] (j = 4 to 20)
- Tac
- Hac
- Esub
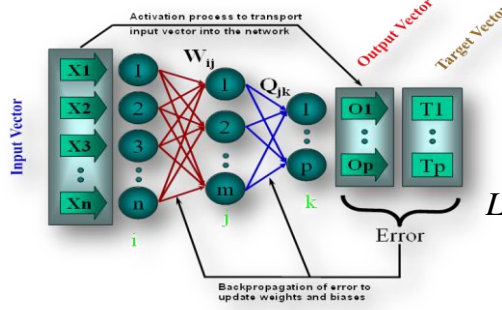
Outputs
- AC relaxation modulus [E(t)] master curve coefficients (c1, c2, c3, and c4)

$$Log(E(\text{t})) = c_1 + \frac{c_2}{1 + e^{(-c_3 - c_4 \log(t_r))}}$$

Generic network architecture: 156-h-1

156 inputs: Tac, Hac, Esub, $D_i(t_j)$ [i=0,8,12,18,24,36,48,60,72; j= 4 to 20]
h hidden neurons
1 output: c_i (each of the 4 coefficients predicted separately)

Figure 32. Inputs, outputs, and generic network architecture details for the NN inverse mapping models considering data from all FWD sensors and pre-peak time history data

The NN modeling approach remained essentially the same as that used for the previous analyses (i.e., feed-forward network using the LM backpropagation training algorithm implemented in the MATLAB NN Toolbox), with one difference. The hidden neurons were varied (25, 30, 45, and 60) to determine the best-performance NN architecture. Separate NN models were developed for each of the E(t) master curve coefficients (c1, c2, c3, and c4). Seventy percent of the 2,000 datasets (a subset of the 3,338 datasets used in the previous analyses), were used for training, 15% were used for validation (to halt training when generalization stops improving), and 15% were used for independent testing of the trained model.

The performances of various NN architectures in predicting E(t) master curve coefficients, c1, c2, c3, and c4, from D0, D8, D12, D18, D24, D36, D48, D60, and D72 pre-peak deflection-time history data are summarized in Table 3. While 25 or 30 hidden neurons were deemed sufficient to achieve best-performance models for three of the E(t) master curve coefficients (c1, c2, and c3), 60 hidden neurons were required to predict c4 with reasonable prediction accuracy. Compared to the previous NN analyses, which considered only information from D0, D8, and D12 sensors, the NN prediction accuracies from the current analyses, which considered pre-peak deflection-time history data from all the FWD sensors, have improved in general, especially for c1 and c2. The best-performance NN prediction models are highlighted in light blue.

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Table 3. NN prediction of E(t) master curve coefficients, c1, c2, c3, and c4, from D0, D8, D12, D18, D24, D36, D48, D60, and D72 pre-peak deflection-time history data

Output	NN Arch.	# Epochs	Training Perf. (MSE)	Gradient	Training R	Validation R	Testing R	All R
c1	156-25-1	29	0.190	0.063	0.688	0.689	0.628	0.680
	156-30-1	73	0.169	0.248	0.733	0.656	0.673	0.713
	156-45-1	23	0.175	0.229	0.710	0.692	0.651	0.699
	156-60-1	32	0.190	0.082	0.691	0.654	0.670	0.682
c2	156-25-1	50	0.170	0.417	0.817	0.776	0.789	0.807
	156-30-1	49	0.181	0.188	0.814	0.751	0.760	0.797
	156-45-1	21	0.185	0.246	0.799	0.721	0.768	0.781
	156-60-1	23	0.197	0.056	0.795	0.761	0.724	0.780
c3	156-25-1	44	0.065	0.171	0.758	0.674	0.648	0.727
	156-30-1	14	0.158	0.588	0.433	0.344	0.268	0.372
	156-45-1	83	0.079	0.925	0.751	0.639	0.640	0.717
	156-60-1	79	0.084	0.208	0.736	0.536	0.635	0.692
c4	156-25-1	14	0.016	0.001	0.258	0.106	0.207	0.227
	156-30-1	13	0.016	0.040	0.269	0.098	0.223	0.224
	156-45-1	12	0.017	0.071	0.238	0.218	0.220	0.231
	156-60-1	150	0.009	0.090	0.709	0.518	0.516	0.655

Neural Networks Inverse Modeling Considering Data from all FWD Sensors: Full Pulse Time Histories

In this analysis, the full pulse deflection-time history data from all the sensors in the standard FWD configuration (D0, D8, D12, D18, D24, D36, D48, D60, and D72) were considered in NN inverse modeling. The performances of various NN architectures in predicting E(t) master curve coefficients, c1, c2, c3, and c4, from D0, D8, D12, D18, D24, D36, D48, D60, and D72 full pulse deflection-time history data are summarized in Table 4. The best-performance NN prediction models are highlighted in light blue. The NN prediction accuracies for all four E(t) master curve coefficients (c1, c2, c3, and c4) have improved further when considering the full pulse deflection-time history data as opposed to considering only pre-peak deflection-time history data.

These results demonstrate the potential of NNs to predict the E(t) master curve coefficients from single-drop FWD deflection-time history data. However, the current prediction accuracies are not sufficient to recommend these models for practical implementation. This feasibility study has identified a number of challenging issues and future research areas that need to be investigated thoroughly through a Phase II study, as discussed in the next section.

Table 4. NN prediction of E(t) master curve coefficients, c1, c2, c3, and c4, from D0, D8, D12, D18, D24, D36, D48, D60, and D72 full pulse deflection-time history data

Output	NN Arch.	# Epochs	Training Perf. (MSE)	Gradient	Training R	Validation R	Testing R	All R
c1	332-25-1	33	0.115	0.129	0.815	0.731	0.714	0.788
	332-30-1	89	0.117	1.660	0.805	0.728	0.718	0.780
	332-45-1	36	0.108	1.160	0.819	0.724	0.664	0.780
	332-60-1	87	0.129	0.038	0.803	0.727	0.727	0.780
c2	332-25-1	32	0.145	4.210	0.825	0.800	0.782	0.815
	332-30-1	61	0.163	0.068	0.821	0.781	0.800	0.811
	332-45-1	24	0.183	0.087	0.804	0.773	0.759	0.793
	332-60-1	38	0.170	1.240	0.816	0.778	0.774	0.804
c3	332-25-1	40	0.068	1.150	0.759	0.651	0.742	0.739
	332-30-1	33	0.047	0.646	0.820	0.712	0.572	0.765
	332-45-1	36	0.085	0.717	0.734	0.571	0.603	0.689
	332-60-1	94	0.087	0.868	0.729	0.718	0.637	0.713
c4	332-25-1	154	0.005	0.060	0.814	0.746	0.737	0.792
	332-30-1	160	0.004	0.007	0.861	0.718	0.645	0.797
	332-45-1	161	0.008	0.209	0.737	0.639	0.681	0.713
	332-60-1	54	0.012	0.132	0.579	0.454	0.516	0.550

SUMMARY AND CONCLUSIONS

The AC dynamic modulus ($|E^*|$) is a key design parameter in the AASHTO MEPDG/Pavement-ME Design. The standard laboratory procedures for AC dynamic modulus testing and development of the master curve require time and considerable resources. The objective of this feasibility study was to develop frameworks for predicting the AC dynamic modulus master curve from routinely collected FWD time history data. The Iowa DOT is eventually interested in documenting the Iowa AC mix damaged master curve coefficients relative to the mix IDs/Station Nos., if possible, in the PMIS. This would be of significant use to the city, county, and state engineers because the outcome of this research would enable them to look up the damaged master curve shape parameters from the PMIS while running a flexible pavement rehabilitation analysis and design using MEPDG/Pavement ME Design. As a first and foundational step, this feasibility research study focused on establishing frameworks for predicting AC $E(t)$ master curve coefficients from FWD time history data. According to the theory of viscoelasticity, if the AC relaxation modulus, $E(t)$, is known, $|E^*|$ can be calculated (and vice versa) through numerical inter-conversion procedures.

The overall research approach involved the following steps:

- Conduct numerous VE forward analysis simulations by varying $E(t)$ master curve coefficients, shift factors, pavement temperatures, and other layer properties
- Extract simulation inputs and outputs and assemble a synthetic database
- Train, validate, and test NN inverse mapping models to predict $E(t)$ master curve coefficients from single-drop FWD deflection-time histories

A computationally efficient VE forward analysis program developed by MSU researchers was adopted in this study to generate the synthetic database. The VE forward analysis program accepts pavement temperature and layer properties (AC $E(t)$ master curve, $E_{b/sub}$, h, μ,) and outputs surface deflection-time histories. Several case studies were conducted to establish detailed frameworks for predicting the AC $E(t)$ master curve from single-drop FWD time history data. Case studies focused on full-depth AC pavements as a first step to isolate potential backcalculation issues that are only related to the modulus master curve of the AC layer. For the proof-of-concept demonstration, a comprehensive full-depth AC analysis was carried out through 10,000 batch simulations of a VE forward analysis program. Anomalies were detected in the comprehensive raw synthetic database and were eliminated through imposition of certain constraints on the sum of $E(t)$ sigmoid coefficients, c1 + c2.

NN forward modeling was carried out to see how accurately NNs can predict the individual deflection-time histories (D0, D8, and D12) based on NN $E(t)$ master curve coefficients and other inputs in the processed synthetic database. If successful, the NN forward model could also serve as a surrogate model (within the specified input ranges) that could replace the VE forward analysis runs. Except for the first two or three time intervals, deflection-time histories at all other time intervals were predicted by NN analysis with very high accuracy (R-values greater than 0.97). The NN inverse modeling results demonstrated the potential of NNs to predict the $E(t)$ master curve coefficients from single-drop FWD deflection-time history data. However, the

current prediction accuracies are not sufficient to recommend these models for practical implementation. Some recommendations are presented in the next section for an expanded Phase II research project to arrive at high-accuracy E(t) master curve backcalculation models.

FUTURE RESEARCH RECOMMENDATIONS

The present feasibility study established a basic NN-based framework for predicting E(t) master curve coefficients from single-drop FWD deflection-time history data. It also identified a number of limitations, modeling challenges, and additional data needs that need to be investigated through a future research study with an expanded scope:

1. The multilayered VE forward analysis program adopted for this study assumes the AC layer as a linear viscoelastic material and unbound layers as linear elastic and predicts the behavior of flexible pavement as a (massless) viscoeleastic damped structure (Kutay et al. 2011). Although it is very computationally efficient, thus facilitating a large number of simulations in a very short time, it has limitations in simulating real-world FWD deflection-time histories where dynamics resulting from inertial and wave propagation effects are often prevalent. According to Varma et al. (2013b), the presence and depth of a stiff layer (bedrock) has significant influence on the contributions of dynamics in the FWD test.

    An example of the manifestation of this dynamic behavior is the occurrence of time delays in the deflection histories with respect to the FWD load pulse (stress wave). In order to use the VE forward analysis approach, the deflection pulses need to be shifted to the left such that they all coincide with the beginning of the load pulse, as illustrated in Figure 33. The use of a time-domain–based dynamic viscoeleastic forward routine is an alternative approach to overcome this problem.
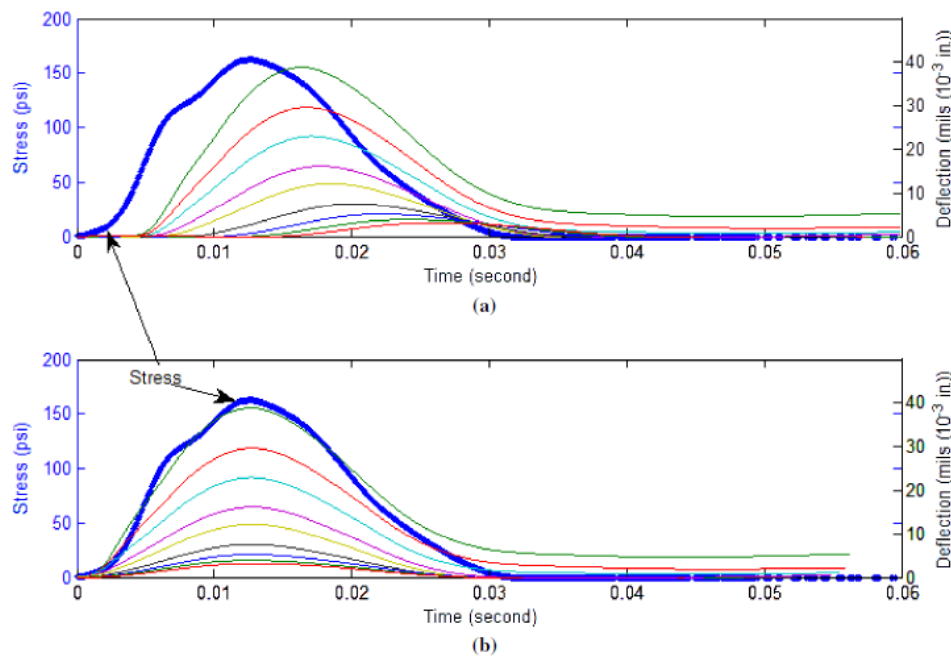
Figure 33. An example of time delay (dynamic behavior) in FWD deflection-time histories (top) and the shifting of deflection pulses to the left (bottom) (Kutay et al. 2011)

2. The proposed NN-based framework computes E(t) master curve coefficients based on single-drop FWD test data (i.e., single FWD test performed at a single temperature). Using this approach, it is only possible to accurately backcalculate certain portions of the E(t), i.e., at high frequencies (short times). Note that the post-peak portion of the deflection-time history curve is generally not considered reliable due to FWD integration errors, which leaves limited information for backcalculating the entire E(t) master curve.

   Kutay et al. (2011) and Varma et al. (2013a) noted that E(t) (|E*|) can be backcalculated accurately up to about t = 0.1 sec (f = 10^{-3} Hz) using the deflection-time histories from a typical single-drop FWD test. They further noted that, in order to accurately predict the entire E(t) master curve, longer pulse durations need to be employed in the FWD test (which will result in long-duration deflection-time history) or FWD tests need to be conducted at different pavement temperatures (during different times of the day or seasons) and use the concept of time-temperature superposition. Varma et al. (2013a) concluded that deflection-time histories from FWD tests conducted between 68–104 °F (20–40 °C) are useful in accurately estimating the entire E(t) or |E*| master curve.

   If the prediction of E(t) from single-drop FWD test data is desirable (which is most often the case), Varma et al. (2013b) recommends using deflection-time history data from FWD tests conducted under an AC layer temperature gradient of preferably 41 °F (5 °C) or more. Future research should consider the use of the enhanced VE forward analysis tool recommended by Varma et al. (2013b) for synthetic database generation because it attempts to simulate more realistic FWD test conditions with respect to the presence of an uneven temperature distribution across the depth of the AC layer.

3. This feasibility study was restricted to the prediction of E(t) master curve coefficients based on single-drop FWD test performed at a single temperature. Consequently, the prediction of time-temperature superposition shift factors (a1 and a2) was omitted. Again, by including the AC temperature profile information at the time of FWD testing in the generation of the synthetic database using the enhanced VE forward analysis program proposed by Varma et al. (2013b), it may be possible to backcalculate the entire E(t) master curve, including the shift factors, from FWD deflection-time histories.

4. Apart from the use of NNs for the inverse analysis of viscoelastic asphalt layer properties from FWD time history data, future research should also consider the use of an evolutionary global optimization technique in combination with the VE forward solver. For instance, Varma et al. (2013b) proposed the use of a GA-based optimization scheme in combination with a VE forward solver to backcalculate E(t) master curve coefficients and shift factors from FWD time history data. Such an approach involves minimizing the differences between the responses calculated from the forward analysis and those from the FWD test by varying the pavement layer properties until a best match is achieved. The researchers for the present project have successfully employed this approach for static backcalculation with different evolutionary optimization techniques such as GA (Gopalakrishnan 2012), particle swarm optimization (PSO) (Gopalakrishnan 2010), covariance matrix adaptation evolution strategy (CMA-ES) (Gopalakrishnan and Manik

2010), and shuffled complex evolution (SCE) (Gopalakrishnan and Kim 2010, Ceylan and Gopalakrishnan 2014).

The overall proposed approach for backcalculating the AC E(t) master curve from FWD deflection-time history data using an evolutionary optimization search scheme is illustrated in Figure 34. Note that the use of trained NN-based surrogate forward analysis models in place of actual forward calculations during the backcalculation can significantly speed up the process, especially when the forward solver is time intensive. Another promising approach, in terms of speeding up the convergence of the global optimizer, is to use the NN inverse mapping model solutions of E(t) master curve coefficients as seed moduli for the global optimization.
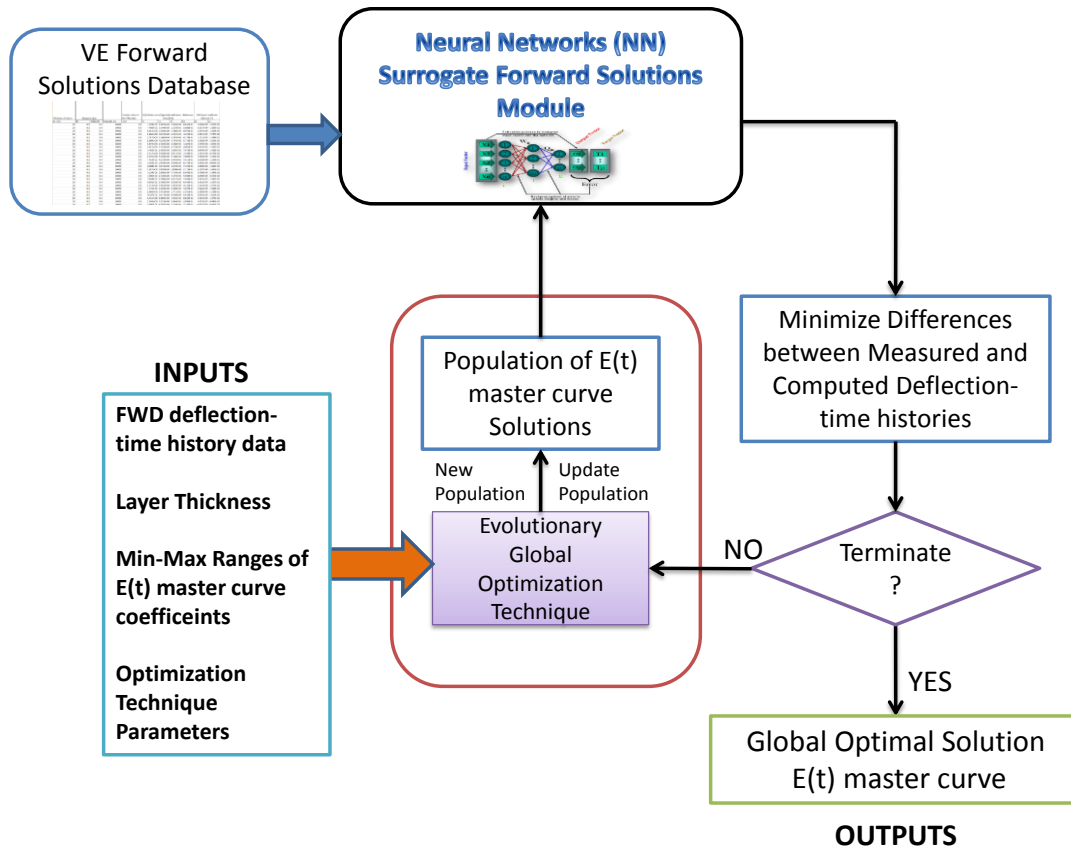
Figure 34. Overall proposed approach for backcalculating the AC E(t) master curve from FWD deflection-time history data using an evolutionary global optimization search scheme

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