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Metric Short Course for the Office of Bridges and Structures

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Sponsored by the
Iowa Department of Transportation Highway Division
and the Iowa Highway Research Board

Iowa DOT Project HR-378

Iowa Department of Transportation

Department of Civil and
Construction Engineering
Iowa State University

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engineering
research institute
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Abstract

This metric short course was developed in response to a request from the Office of Bridges and Structures to assist in the training of engineers in the use of metric units of measure which will be required in all highway designs and construction after Sept. 30, 1996 (CFR Presidential Executive Order No. 12770).

The course notes which are contained in this report, were developed for an one-half day course. The course contains a brief review of metrication in the U.S., metric units, prefixes, symbols, basic conversions, etc. The unique part of the course is that it presents several typical bridge calculations (such as capacity of reinforced concrete compression members, strength of pile caps, etc.) worked two ways: inch-pound units throughout with end conversion to metric and initial hard conversion to metric with metric units throughout. Comparisons of partial results and final results (obtained by working the problems the two ways) are made for each of the example problems.

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1. HISTORICAL REVIEW OF THE METRIC SYSTEM

Before the metric system, every nation used measurement units that had grown from local customs. For example, England once used 3 barleycorns, round and dry as their standard for an inch. Grains of barley varied in size...and so did the inch.

1600 Some people recognized the need for a single, accurate, worldwide measurement system.

1670 Gabriel Mouton - A French clergyman - proposed a decimal system of measurement based on a fraction of the earth's circumference. The fraction was 1/10,000,000 of the distance from the North Pole to the equator. French scientists named this unit of length the meter, from the Greek word metron, meaning a measure.

1790 National Assembly of France requested the French Academy of Sciences to develop a standard system of weights and measures. The system developed became known as the Metric system.

Thomas Jefferson (U.S. Secretary of State) recommended that the U.S. use a decimal system of measurement; Congress rejected the idea.

1795 France adopted the metric system but allowed people to continue using other measurement units.

1821 John Quincy Adams (U.S. Secretary of State) proposed conversion to the metric system; Congress again rejected the proposal.

1837 France passed a law requiring all Frenchmen to begin using the metric system Jan. 1, 1840.

1866 Congress legalized the use of the metric system in the U.S. but did not require that it be used.

1870-1875 An international conference was held to update the metric system and to adopt new measurement standards for the kilogram and meter. Seventeen nations (including the U.S.) participated and in 1875 they signed the 'Treaty of the Meter' which established a permanent organization to change the metric system as necessary. This organization, International Bureau of Weights and Measures, is headquartered near Paris.

1893 U.S. began defining all its measurement units as fractions of the standard meter and kilogram.

1890's Several attempts in Congress to change U.S. measurements to metric - none were successful. Many people, especially those in industry, opposed any changes.

1957 U.S. Army and Marine Corps adopted the metric system as the basis for their weapons and equipment.

1960 An international conference of the Bureau of Weights and Measures met and adopted the current Systeme International d'Unite's.

1960's NASA began using metric units.

1965 Great Britain began the changeover to the metric system.

1968-1971 U.S. Congress explored the costs and benefits of converting and recommended U.S. make a planned conversion.

1970 Australia began a scheduled 10-year conversion to the metric system.

1975 Canada began a gradual changeover to the metric system. U.S. Congress passed the Metric Conversion act which called for a voluntary changeover to the metric system.

1988 Trade and Competitiveness Act (Sec 5164b of Public Law 100-418) amended the 1975 Metric Conversion Act declaring that the metric system is the preferred system of weights and measures for the U.S. trade and commerce. It required each Federal agency to convert to the metric system to the extent feasible by the end of fiscal year 1992. A qualifier was included which noted conversion may not be required if it is impractical, causes significant inefficiencies, or causes loss of markets.

1991 President Bush signed the Executive Order 12770 - Metric Usage in Federal Government Programs. The intent of this order was that Federal agencies are to convert to metric, under the Secretary of Commerce within a fixed period of time.

2. CONVERSION AND ROUNDING

The conversion of inch-pound units to metric is an important part of the metrication process. However, conversion can seem deceptively simple because most measurements have implied, not expressed, tolerance and many products are identified in easy-to-use nominal sizes, rather than actual sizes.

People working in a particular area have an intuitive feel for allowable tolerances in measurements they specify and know the difference between nominal and actual sizes. This knowledge must be relied upon when converting to metric.

For example, a given guardrail detail notes that anchor bolts are to be imbedded on concrete 8 in. What should this depth be in millimeters?

A strict conversion results in an exact dimension of 203.2 mm which implies an accuracy of 0.1 mm (1/254 in.) and a tolerance of ± 0.05 mm (1/508 in.) which is not possible to achieve (or needed) in the field. Likewise, 203 mm (accuracy 1 mm, tolerance of ± .5 mm) is overly precise.

An acceptable practical tolerance for setting anchor bolts is at least ± 1/4 in. (6 mm). Applying this to the 203.2 mm, the converted 8 in. requirement is in the 197-209 mm range. Actually, the range is 197 mm and larger since 8 in. was a minimum depth.

As metric measuring devices emphasize 10 mm increments, converting the 8 in. requirement to 200 mm would be a convenient depth for use in the field. In this case then, a reasonable metric conversion for 8 in. is 200 mm.

This example illustrates the need for experience, common sense, and consideration on how the measurement will be used.

Basic points to remember in conversion and rounding are the following:

- Conversion should be performed by experienced professionals. Any automated conversion program should be used with care.
- Understand the allowable tolerance for the measurements you are converting.
- Always convert with the end application or use in mind. Remember, dimensional tolerance on the job are rarely less than a few millimeters and that it is considerably easier for field personnel to measure in 10 mm increments.
- The most common conversion error is under-rounding which implies more precision than is inherent in the inch-pound number. If your linear conversions are accurate to 0.1 mm or even 1 mm, you are probably doing them incorrectly. Any dimension over a few inches, can usually be rounded to the nearest 5 mm and any dimension over a few feet, can be rounded to the nearest 10 mm.

- Practice helps improve speed and confidence!

2.1. Example 1

In the metric system, switching to larger (or/smaller) units is fast and easy and calculations are more efficient.

For example, determine the volume of concrete in a given concrete floor:

Floor: 200 ft long
180 ft wide
5 1/2 in. thick

a.) Inch - units

First calculation: $5\ 1/2 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = .458 \text{ ft}$

Second calculation: Volume = 200 x 180 x .458 = 16,500 cu ft

Third calculation: $16{,}500 \text{ cu ft} \times \frac{1 \text{ cu yd}}{27 \text{ cu ft}} = 611 \text{ cu yd}$

Therefore three calculations were required.

b.) Metric-units

Floor: 61 m long
55 m wide
140 mm thick

Volume = 61 x 55 x .14 = 470 m^3

Therefore, only one calculation (140 mm → 0.14 m was mentally converted) was required.

2.2. Example 2

Significant digits...a simple rule is to retain the number of significant digits that neither sacrifices nor exaggerates accuracy.

For example:

564 lbs. of cement/cu yd = ? kg/m^3

$$\frac{564\ \text{lb}}{\text{cu yd}} \times \frac{.4536\ \text{kg}}{\text{lb}} \times \frac{1\ \text{cu yd}}{.7646\ \text{m}^3} = 334.6212\ \text{kg/m}^3$$

which is rounded to 335 kg/m^3.

The value, 334.6212 kg/m^3, implies that cement is batched to the nearest 0.0001 kg.

Since cement batching tolerance is 1%, 335 kg/m^3 (or even 340 kg/m^3) is the appropriate rounded values.

Note: Neither original units nor conversion factors are rounded before multiplying. Only the product is rounded.

2.3. Rationalization and Pitfalls of Hard Conversion

Some agencies converted 4000 psi concrete to 30 MPa concrete. A 30 MPa concrete is actually a 4350 psi concrete - nearly 9% higher than the old design strength.

Concretes proportioned for a 4000 psi design strength may not have an adequate overdesign to meet acceptance requirement for a 30 MPa requirement.

Many mixes with established performance histories may have to be reproportioned thus requiring development of new data on strengthen variability. Also mix cost will increase.

ACI requires taking steps to increase the average strength of a concrete mix if a strength test falls below the design strength by more than 500 psi. This 500 psi converts to 3.45 MPa. Hard converting this value to 3 MPa produces a more stringent requirement and would result in more failing tests. Hard converting to 4 MPa would relax the requirement and have structural safety implication! A compromise is to retain two significant digits - 3.5 MPa. In the metric version of the ACI code, this has been done.

3. CONVERSION EXPERIENCES OF OTHER NATIONS

(Great Britain, Canada, South Africa, Australia, and New Zealand)

1. There was no appreciable increase in either building design or construction costs. Conversion costs for most sectors of the construction industry were minimal or offset by later savings.

2. Engineering/Architecture community liked metric dimensioning since it was less prone to error and easier to use.

3. Metric offered a one-time chance to reduce many product sizes and shapes.

4. Engineering/Architecture firms in these countries found that it took a week or less to learn to think and produce in metric.

Based on worldwide experience, the two areas that will require the highest investment of funds will be

- converting existing standards, specifications and computer programs to the metric system.
- converting traffic signs.

4. OVERVIEW OF SI UNITS

3 classes: base units, derived units, and supplementary units

4.1. Basic Units

Basic Units

Quantity	Unit	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin	K
luminous intensity	candela	cd
amount of substance	mole	mol

Meter - length equal to 1 650 763.73 wave lengths in a vacuum of the orange-red line of the spectrum of the krypton - 86 atom.

Football field goal line to goal line = slightly more than 91 m.
Height of the average male = a little less than 1.8 m
Distance between the bases in baseball = ≈ 27 m.
Length of full-size bed = ≈ 2 m.

Kilogram - standard for the unit of mass is a cylinder of platinum - iridium alloy kept by the International Bureau of Weights and Measures in Sevres, France.

1 kilogram ≈ 2.2 x the pound (mass)
5-pound bag of flour = ≈ 2-kg.
Professional football defensive lineman = ≈ 115kg.

Temperature - Celsius (°C) is more common than kelvin (K), however both have the same temperature gradients.

°C = K + 273.15

°C = 5/9(°F - 32)

Equivalent Temperatures

Event	°C	°F	K
Water Freezes	0°	32°	273.15
Water Boils	100°	212°	373.15
Body Temperature	37°	98.6°	310
Very Cold Day	-18°	0°	255

Second - the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium - 133 atom.

4.2 Supplementary Units

Supplementary Units*

Quantity	Unit	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

*11th General Conference on Weights and Measures declined to designate these as base or derived units, thus the third category.

Radian - the plane angle with its vertex at the center of a circle that is subtended by an arc equal in length to the radius.

1 rad = 57.2958°
2π rad = the angle of a complete circle (360°)

Steradian - the solid angle with the vertex at the center of a sphere that is subtended by an area of the spherical surface equal to that of a square having sides equal in length to the radius.

A sphere = 4π sterdians

4.3. Derived Units

The largest class of SI units, derived units, is formed by combining base, supplementary and other derived units according to the algebraic relations linking the corresponding quantities.

When two or more units expressed in base or supplementary units are multiplied or divided as required to obtain derived quantities, the result is a unit value. No numerical constant is introduced, thus a coherent system is maintained.

Derived Units

Quantity*	Name	Symbol	Expression
frequency	hertz	Hz	$Hz = s^{-1}$
force	newton	N	$N = kg \cdot m/s^2$
pressure, stress	pascal	Pa	$Pa = N/m^2$
energy, work, quantity of heat	joule	J	$J = N \cdot m$
power, radiant flux	watt	W	W = J/s
electric charge, quantity	coulomb	C	$C = A \cdot s$
electric potential	volt	V	V = W/A or J/C
capacitance	farad	F	F = C/V
electric resistance	ohm	Ω	Ω = V/A
electric conductance	siemens	S	S = A/V or Ω^{-1}
magnetic flux	weber	Wb	$Wb = V \cdot s$
magnetic flux density	tesla	T	$T = Wb/m^2$
inductance	henry	H	H = Wb/A
luminous flux	lumen	lm	$lm = cd \cdot sr$
illuminance	lux	lx	$lx = lm/m^2$

*Fifteen derived units with special names are used in engineering calculations.

5. SI STYLE AND EDITORIAL GUIDELINES

5.1. Unit Names

The SI unit names, including prefixes, are not capitalized when used within a sentence, except the first letter is capitalized when used as the first word of a sentence. An exception to this is when the name of the unit is derived from the name of a person.

Examples of lower cased and capitalized symbols

Lowercase symbols		Capitalized symbols		
meter	m	ampere	A	A.M. Ampere
gram	g	kelvin	K	Wm. Thompson 1st Baron Kelvin
second	s			
area	m^2	pascal	Pa	Blaise Pascal
volume	m^3	newton	N	Isaac Newton
radian	rad	degree Celsius	°C	Anders Celsius
		hertz	Hz	Heinrich R. Hertz

5.2. Prefixes

All prefix names (and SI names) when fully written out in a sentence are written in lowercase letters. Prefixes are used to form decimal multiples and submultiples of SI units. The most common prefixes are shown in the following table:

Prefix	Multiplication	Symbol*	Pronunciation
mega	10^6	M	megg-ah
kilo	10^3	K	kill-oh
milli	10^{-3}	m	mill-ee
micro	10^{-6}	μ	mike-ro

*Note: Short forms of prefixes as well as the short forms of SI unit names are called symbols. It is incorrect to refer to them as abbreviations or acronyms.

When symbols are used, some exceptions to the lowercase rules are noted in the following table:

Capital and lower case symbols

G for giga	g for gram
K for Kelvin	k for kilo
M for mega	m for milli or meter
N for Newton	n for nano
T for tera	t for metric ton

5.3. Punctuation

Period - a period is not used after a symbol, except at the end of a sentence.

	Preferred	Acceptable
newton meter	N·m	N.m
pascal second	Pa·s	Pa.s

Decimal marker - a period is used as a decimal marker. A zero is written before the decimal marker to prevent the possibililty that a faint decimal marker will be overlooked.

For example: 0.5 kg
0.65 N
0.27 KPa

5.4. Plurals

When written in full, the names of metric units are made plural (i.e., adding a 's') when appropriate. Symbols of SI units are never plural.

For example:		
	1.7 m	-30° C
	1 m	0° C
	0.75 m	100° C

5.5. Grouping Digits

All numbers are separated into groups of three on each side of the decimal marker. Do not use a comma to separate the group of three digits.

For example: 1 234.567 89
99 123.765 5
5 432 987.210 9

The only exception to the writing of numbers in groups of three is when one has a four-digit number. Four-digit numbers are given special consideration and are treated differently depending upon the context in which they are used - text or tabular format.

In text material, numbers having four or less digits on either side of the decimal marker are to be written with no spaces (4321.5678). In tabular listing, it is acceptable to leave columns of numbers - none of which have more than four digits on either side of the decimal marker - written without space. However, the three-digit grouping and spaces format is preferred.

Miscellaneous Numbers

There are certain numbers to which the previous grouping rules do not apply.

Social security numbers	505-42-7612
Part numbers	16P76AC-123477/231
Currency	$21,263.21

The symbol for angular degree (°) and degree Celsius (°C) should always be used when giving a measurement. When describing the measuring scale and not a specific measurement, use the full name.

6. FEATURED UNITS

Area

The U.S. customary acre is replaced with the hectare (ha) in the SI system. Hectare, a special name for the square hectometer (hm^2) equals 10 000m^2. The hectare is the preferred measure of land and water areas. However, the square meter (m^2) remains the preferred SI unit for other measures of area.

1 hectare	≈	2.5 acres
0.5 hectare	≈	a football field including end zones
1 hectare	≈	the whole playing area (fare and foul) of a major league baseball field.

Force, Weight, and Mass

In SI, there is one basic unit for force - newton (N)- and one basic unit for mass - kilogram (kg).

Kilogram - the kilogram is a measure of an object's mass; the mass of an item is constant and does not change with the gravitational field or its rate of acceleration. The term 'kilogram mass' is redundant and should not be avoided.

Newton - the newton is the SI unit of force. Using Newton's equation ($F = m \times a$), a newton is 1 kilogram times 1 meter per second squared. Acting under Earth's gravitational pull, a mass of 1 kg exerts a force of 9.806 65 newtons.

The use of weigh and weight should be avoided. They should not be used to indicate the measuring process or the measure of mass. Rather than say "it weighs" say "it has a mass of". Rather than "weighing the object" say "measuring the object's mass".

Pressure and Stress

Pressure - a pressure is a force per unit area; the SI unit of force, Newton, divided by the SI unit of area, the square meter, results in N/m^2. A newton per square meter is given the special name pascal (Pa). A dollar bill lying flat on a surface exerts a pressure of 1 Pa. Since the pascal is so small, all pressures should be given in kilopascals (kPa).

The pascal is also used to express stress levels and material modulus of elasticity values. Because of the magnitude of these values, stress levels should be given in megapascals (MPa) and the modules of elasticity values in gigapascals (GPa).

Use of GPa for the modulus of elasticity, MPa for stress and kPa for pressure provides a quick indication of the physical quantity being referenced.

7. CONVERSION FACTORS

In "soft" conversion, an inch-pound measurement is mathematically converted to its exact (or nearly exact) metric equivalent. With "hard" conversion, a new rounded, rationalized metric number is created that is convenient to work with and remembered.

7.1. Length, Area and Volume

One metric unit - the meter - is used to measure length, area, and volume in most design and construction works.

7.1.1. Rules for Linear Measurement

- Use only the meter and millimeter in design and construction.
- Use the kilometer for long distances and the micrometer for precision measurements.
- Avoid use of the centimeter.
- For survey measurement, use the meter and the kilometer.

7.1.2. Rules for Area

- The square meter is preferred.
- Very large areas may be expressed in square kilometers and very small areas in square millimeters.
- Use the hectare (10 000 square meters) for land and water measurement only.
- Avoid use of the square centimeter.
- Linear dimensions such as 40 x 90 mm may be used; if so, indicate width first and height second.

7.1.3. Rules for Volume and Fluid Capacity

- Cubic meter is preferred for volumes in construction and for large storage tanks.
- Use liter (L) and milliliter (mL) for fluid capacity (liquid volume). One liter is 1/1000 of a cubic meter or 1000 cubic centimeters.

Area, Length, and Volume Conversion Factors

Quantity	From Inch-Pound Units	To Metric Units	Multiply by
Length	mile yard foot inch	km m m mm mm	<u>1.609344</u> <u>0.914 4</u> <u>0.304 8</u> <u>304.8</u> <u>25.4</u>
Area	square mile acre square yard square foot square inch	km^2 m^2 ha (10 000 m^2) m^2 m^2 mm^2	2.590 00 4 046.856 0.404 685 6 <u>0.835 127 36</u> <u>0.092 903 04</u> <u>645.16</u>
Volume	acre foot cubic yard cubic foot cubic foot cubic foot 100 board feet gallon cubic inch cubic inch	m^3 m^3 m^3 cm^3 L (1000 cm^3) m^3 L (1000 cm^3) cm^3 mm^3	1 233.49 0.764 555 0.028 316 8 28 316.85 28.316.85 0.235 974 3.785 41 <u>16.387 064</u> <u>16 387.064</u>

NOTE: Underline denotes exact number.

7.2. Civil and Structural Engineering

The metric units used in civil and structural engineering are meter, kilograms, second, newton, and pascal.

7.2.1. Rules for Civil and Structural Engineering

- There are separate units for mass and force.
- The kilogram (kg) is the base unit for mass, which is the unit quantity of matter independent of gravity.
- The newton (N) is the derived unit force (mass times acceleration, kg·m/s^2).

- Do not use the joule to designate moment, which is always designated newton meter (N·m).
- The pascal (Pa) is the unit for pressure and stress (Pa = N/m^2).
- Structural calculations should be shown in MPa or kPa.
- Plane angles in surveying (cartography) will continue to be measured in degrees (either decimal degrees or degrees, minutes, and seconds) rather than the metric radian.
- Slope is expressed in nondimensional ratios. The vertical component is shown first and then the horizontal.
- For slopes less than 45°, the vertical component should be unitary (for example, 1:20). For slopes over 45°, the horizontal component should be unitary (for example, 5:1).

Civil and Structural Engineering Conversion Factors

Quantity	From Inch-Pound Units	To Metric Units	Multiply by
Mass	lb kip (1000 lb)	kg metric ton (1000 kg)	0.453 592 0.453 592
Mass/unit length	plf	kg/m	1.488 16
Mass/unit area	psf	kg/m^2	4.882 43
Mass density	pcf	kg/m^3	16.018 5
Force	lb kip	N kN	4.448 22 4.448 22
Force/unit length	plf klf	N/m kN/m	14.593 9 14.593 9
Pressure, stress, modulus of elasticity	psf ksf psi ksi	Pa kPa kPa Mpa	47.880 3 47.880 3 6.894 76 6.894 76
Bending moment, torque, moment of force	ft-lb ft-kip	N·m kN·m	1.355 82 1.355 82
Moment of mass	lb·ft	kg·m	0.138 255

First moment of area	lb · ft^2	kg · m^2	0.042 140 1
Second moment of area	in^4	mm^4	416 231
Section modulus	in^3	mm^3	16 387.064

Note: Underline denotes exact number.

8. WORKED EXAMPLES

8.1 Examples of Conversion:

8.1.1 Integral Wearing Surface

$$1/2 \text{ in.} \times \frac{25.4 \text{ mm}}{1 \text{ in.}} = 12.70 \text{ mm} \rightarrow 13 \text{ mm}$$

8.1.2 Construction Load

$$50 \text{ lb/ft}^2 \times \frac{4.448 \text{ N}}{1 \text{ lb}} \times \frac{1 \text{ ft}^2}{.0929 \text{ m}^2} = 2394.0 \text{ N/m}^2 = 2394 \text{ Pa} \rightarrow 2400 \text{ Pa}$$

$$50 \text{ lb/ft}^2 \times \frac{47.880 \text{ Pa}}{\text{lb/ft}^2} = 2394.0 \text{ Pa} \rightarrow 2400 \text{ Pa}$$

8.1.3 Temperature

$10°F \rightarrow °C = 5/9\,(10 - 32) = -12.22\ °C \rightarrow -10°C$

8.1.4 Bridge Length

3 spans: 30'-6, 39'-0, 30'-6, Total length = 100'

$$30'\text{-}6 = 30.5 \text{ ft} \times \frac{12 \text{ in.}}{\text{ft}} \times \frac{25.4 \text{ mm}}{1 \text{ in.}} = 9296.4 \text{ mm} \rightarrow 9100 \text{ mm}$$

$$39'\text{-}0 = 39 \text{ ft} \times \frac{12 \text{ in.}}{\text{ft}} \times \frac{25.4 \text{ mm}}{\text{in.}} = 11\ 887.2 \text{ mm} \rightarrow 11\ 800 \text{ mm}$$

$$\text{Total Bridge Length}_1 = 100 \text{ ft} \times \frac{12 \text{ in.}}{\text{ft}} \times \frac{25.4 \text{ mm}}{1 \text{ in.}} = 30\ 480 \text{ mm}$$

$\text{Total Bridge Length}_2 = 2(9100) + 11\ 800 = 30\ 000 \text{ mm}$

480 mm (18.9 in) shorter

8.1.5 Concrete Strength

$f'_c = 4000$ psi

$$4000 \text{ psi} \times \frac{4.448 \text{ N}}{\text{lb}} \times \frac{1 \text{ in}^2}{645.16 \text{ mm}^2} \times \frac{10^6 \text{ mm}^2}{1 \text{ m}^2}$$

$= 27.58 \times 10^6\ N/m^2 = 27.58 \times 10^6\ Pa$

$= 27.58$ MPa → 28 MPa

28 MPa (4060 psi) 1.5% 'stronger'

8.1.6 Grade of Steel

$f_y = 60,000$ psi

$$60,000 \text{ psi} \times \frac{6.895 \text{ kPa}}{1 \text{ psi}} = 413\ 700 \text{ kPa} = 414 \text{ MPa} \rightarrow 400 \text{ MPa}$$

400 MPa (58 ksi) 3.3% 'weaker'

8.2 Reinforcement

A given reinforced concrete girder requires 6 - #8 bars. What metric reinforcement is required to provide the girder with the same capacity? Remember, not only have the bar cross-sectional areas changed but also the yield strength of the reinforcement. Thus, metric replacement reinforcement is a function of bar size and grade of steel. In slabs, where reinforcement is given as a particular bar at a certain spacing, the conversion to metric will involve three variables: bar size, bar spacing, and grade of steel.

On the following page, information is provided on the metric bars as well as a comparison between the metric bars and the inch-pound bars. Several example problems for determining required metric reinforcement follow.

Table 1. Design Force, F_e, for in.-lb Bars (kips)

Bar Size	Grade 40	Grade 50	Grade 60	Grade 75
#3	4.400	5.500	6.600	-
#4	8.000	10.000	12.000	-
#5	12.400	15.500	18.600	-
#6	17.600	22.000	26.400	-
#7	24.000	30.000	36.000	-
#8	31.600	39.500	47.400	-
#9	40.000	50.000	60.000	-
#10	50.800	63.500	76.200	-
#11	62.400	78.000	93.600	117.000
#14	90.000	112.500	135.000	168.750
#18	160.000	200.000	240.000	300.000

ASTM A615 CHART FOR REINFORCING STEEL BARS

Inch-Pound Bar Size Designation	Nominal Weight lb./ft. (kg/m)	Nominal Dimensions: Diameter in. (mm)	Nominal Dimensions: Cross Sectional Area in² (mm²)
#3	0.376 (.560)	0.375 (9.5)	0.11 (71)
#4	0.668 (.994)	0.500 (12.7)	0.20 (129)
#5	1.043 (1.552)	0.625 (15.9)	0.31 (200)
#6	1.502 (2.235)	0.750 (19.1)	0.44 (284)
#7	2.044 (3.042)	0.875 (22.2)	0.60 (387)
#8	2.670 (3.974)	1.000 (25.4)	0.79 (510)
#9	3.400 (5.060)	1.128 (28.7)	1.00 (645)
#10	4.303 (6.404)	1.270 (32.3)	1.27 (819)
#11	5.313 (7.907)	1.410 (35.8)	1.56 (1006)
#14	7.65 (11.39)	1.693 (43.0)	2.25 (1452)
#18	13.60 (20.24)	2.257 (57.3)	4.00 (2581)

ASTM A615M CHART FOR REINFORCING STEEL BARS

Metric Bar Size Designation	Nominal Mass kg/m	Nominal Dimensions: Diameter mm	Nominal Dimensions: Cross Sectional Area mm²	Comparison To A615
10M	0.785	11.3	100	20% < #4
15M	1.570	16.0	200	SAME AS #5
20M	2.355	19.5	300	6.8% > #6
25M	3.925	25.2	500	1.3% < #8
30M	5.495	29.9	700	9% > #9
35M	7.850	35.7	1000	0.6% < #11
45M	11.775	43.7	1500	3.5% > #14
55M	19.625	56.4	2500	3% < #18

Table 2. Design Force, F_m, for Metric Bars (kips)

Bar Size	Grade 300	Grade 350	Grade 400	Grade 500
#10M	6.744	7.868	8.992	-
#15M	13.488	15.736	17.984	-
#20M	20.232	23.604	26.976	-
#25M	33.720	39.340	44.960	-
#30M	47.208	55.076	62.944	-
#35M	67.440	78.680	89.920	112.401
#45M	101.160	118.021	134.881	168.601
#55M	168.601	196.701	224.801	281.001

8.2.1 Converting required number of inch-pound bars to number of metric bars required.

N_m = number of metric bars
N_e = number of in-lb bars
F_e = force provided by in-lb bars (Table 1)
F_m = force provided by metric bars (Table 2)

$$N_m F_m = N_e F_e$$

$$N_m = N_e (F_e/F_m)$$

8.2.1.1 Example 1

Required 8 - #6 bars, Grade 60
Replace with ? #25 M bars, Grade 400
Replace with ? #30 M bars, Grade 400

$$N_{\#25M} = 8\ (26.4/44.960) = 4.7$$

∴ Use 5 - #25M bars, Grade 400

$$N_{\#30M} = 8\ (26.4/62.944) = 3.35$$

∴ Use 4 - #30M bars, Grade 400

8.2.1.2 Example 2

Required 5 - #8 bars, Grade 60
Replace with ? #25M bars, Grade 400
Replace with #35M bars, Grade 350

$N_{\#25} = 5\ (47.4/44.960) = 5.3$

∴ Use 6 - #25M bars, Grade 400

$N_{\#35} = 5\ (47.4/78.680) = 3.0$

∴ Use 3 - #35M bars, Grade 350

8.2.2 Converting inch-pound bars spaced at 'x' inches to metric bars spaced at 'y' mm.

S_e = spacing of in-lb bars, in.
S_m = spacing of metric bars, mm
F_e = force provided by in-lb bars (Table 1)
F_m = force provided by metric bars (Table 2)

$$F_m/S_m = F_e/S_e \times \frac{1 \text{ in.}}{25.44 \text{ mm}}$$

$S_m = S_e \times 25.4\ (F_m/F_e)$

8.2.2.1 Example 1

#9 (Grade 60) at 10 in equals #30M (Grade 400) at 'y' spacing

$y = S_m = 10 \times 25.4\ (62.944/60) = 266.5$ mm

∴ Use #30 (Grade 400) at 270 mm

8.2.2.2 Example 2

#6 (Grade 60) at 8 in.

Equals #20M (Grade 400) at 'x' spacing.

Equals #15M (Grade 400) at 'z' spacing.

$x = 8 \times 25.4\ (26.4/26.976) = 198.9$ mm

∴ Use #20M at 200 mm

z = 8 x 25.4 (26.4/17.984) = 298.3 mm

∴ Use #15M at 300 mm

8.2.3 Calculations have determined that a given slab requires 1.86 mm^2/mm (Grade 400). Reinforcement at what spacing will satisfy this requirement?

Bar Designation	Bar Area (mm^2)	Required Spacing	Area Steel Provided $(mm)^2/mm$
20 M	300	161.3 mm* → 160 mm (6.3 in.)	1.88
25 M	500	268.8 mm → 270 mm (10.6 in.)	1.85
30 M	700	376.3 mm → (375 mm) (14.8 in.)	1.87

*300 mm^2/(1.86 mm^2/mm) = 161.3 mm

Problem 1: U.S. Customary: Simple Span Composite Stringer

Required:

Design composite section

Given:

H.S. 20 Loading

Roadway width = 40 ft

Bridge span = 54 ft

Beam spacing = 9.25 ft (5 beams)

Deck thickness = 8 in. Haunch = 1 in.

$F_y = 50$ ksi

$f'_c = 3.5$ ksi

$$\text{Barrier rail} = 2.47 \frac{\text{ft}^3}{\text{ft length of rail}}$$

FWS = 20 psf

Density of concrete = 150 lb/ft^3

Solution:

Assume steel section:

U.S. Customary W36x135

W36 x 135:

$D = 35.55$ in.
$A = 39.7\ in^2$
$b_f = 11.95$ in.
weight/unit length = 135 plf
$I_x = 7800\ in^4$
$S_x = 439\ in^3$

Composite section:

$t_{slab} = 8$ in.

deduction for wear = 1/2 in.

$\therefore$ reduced $t_{slab} = 8 - 0.5 = 7.5$ in.

effective width = 12 x 7.5 in. = 90 in. ($\frac{L}{4}$ or 12 t_{slab} or S)

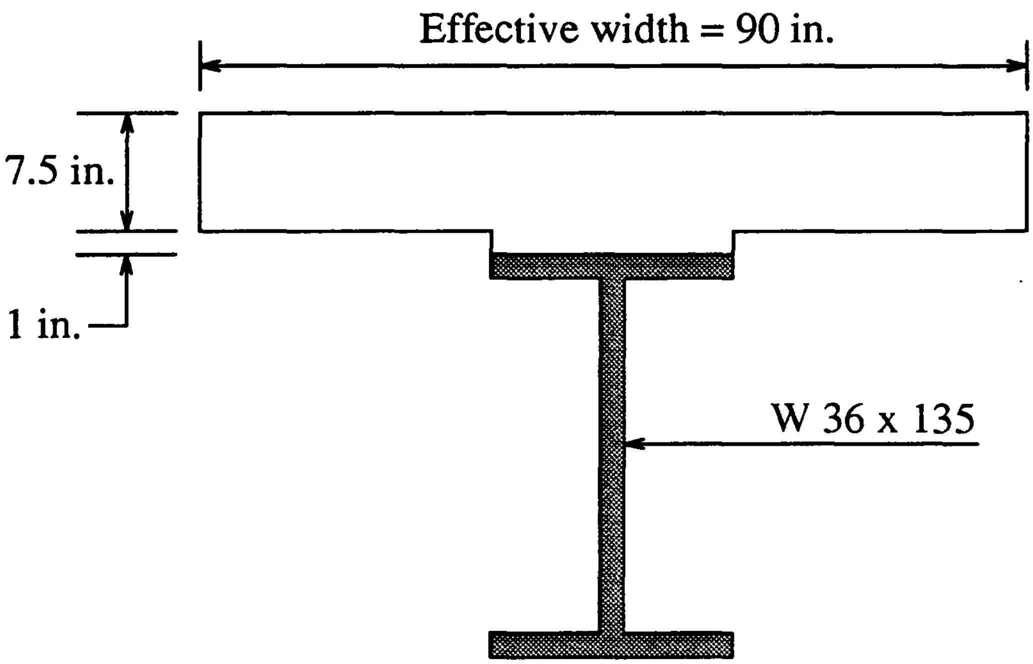
Haunch Dimensions:

$b = b_f$ of W-section = 11.95 in.

$t = 1$ in.

Diaphragm:

Assume wt. = 10 lb/ft



COMPOSITE SECTION DIMENSIONS

Composite Section Properties (N = 9 and 3N = 27 for long term)

	A (in^2)	Y (in.)	AY (in^3)	AY^2 (in^4)	I_o (in^4)
Beam	39.7	17.775	705.7	12543	7800
Haunch (N)	1.3	36.05	46.9	1689	--
Haunch (3N)	[0.4]	36.05	[14.4]	[520]	--
Slab (N)	75.0	40.3	3022.5	121807	352
Slab (3N)	[25.0]	40.3	[1007.5]	[40602]	[117]
Total (N)	116.0		3775.1		
Total (3N)	[65.1]		[1727.6]		

N = 9: $\overline{Y} = 32.54$ in $I = 21339\ in^4$ $S_{BF} = 656\ in^3$

3N = 27: [$\overline{Y} = 26.54$ in]   [$I = 15737\ in^4$]   [$S_{BF} = 593\ in^3$]

Computation of Section Modulus:

Steel section:

$S = 439\ in^3$ (from tables)

Composite section: N = 9:

$$Y = \frac{3775.1\ in^3}{116.0\ in^2} = 32.54\ in.$$

$$I = 7800 + 39.7\,(32.54 - 17.775)^2 + 1.3\,(36.05 - 32.54)^2 + 352 +$$

$$75\,(40.3 - 32.54)^2 = 21{,}339\ in^4$$

$$S_{BF} = \frac{21{,}339\ in^4}{32.54\ in.} = 656\ in^3$$

Composite section: 3N = 27:

$$Y = \frac{1727.6\ in^3}{65.1\ in^2} = 26.54\ in.$$

$$I = 7800 + 39.7\,(26.54 - 17.775)^2 + 0.4\,(36.05 - 26.54)^2 + 117 +$$

$$25\,(40.3 - 26.54)^2 = 15{,}737\ in^4$$

$$S_{BF} = \frac{15{,}737\ in^4}{26.54\ in.} = 593\ in^3$$

Loads and Moments for Interior Beam Line:

Non-Composite DL:

Deck: $9.25 \text{ ft} \times 8 \text{ in.} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right) \times 150 \frac{\text{lb}}{\text{ft}^3} = 925 \text{ plf}$

Beam + Diaph: (Assume Diaph: 10 plf) = 135 + 10 = 145 plf

Composite DL:

Rail: $2.47 \frac{\text{ft}^3}{\text{ft length of rail}} \times 150 \frac{\text{lb}}{\text{ft}^3}$ x 2 rails/5 beams = 148 plf

FWS: 20 psf x 40 ft/5 beams = 160 plf

DL Mom. (non comp.) = 1/8 x (0.925 + 0.145) kip/ft x $(54)^2$ ft^2 = 390.0 k-ft

DL Mom. (comp.) = 1/8 x (0.148 + 0.160) kip/ft x $(54)^2$ ft^2 = 112.3 k-ft

LL + I Mom. = 1.279 x 1.682 x 349.7 k-ft = 752.2 k-ft

where:

LL Dist. Factor $\frac{S}{5.5} = \frac{9.25}{5.5}$ = 1.682 wheel lines per beam

$$I = \frac{50}{L+125} = \frac{50}{54+125} = 0.279$$

$LL_{Moment} = \frac{699.3 \text{ k-ft}}{2}$ = 349.7 k-ft (AASHTO Appendix)

F_{all} = 27 ksi

$$f_{BF} = \frac{(390\ k\text{-}ft)\ \frac{12\ in.}{1\ ft}}{439\ in^3} + \frac{(112.3\ k\text{-}ft)\ \frac{12\ in.}{1\ ft}}{593\ in^3} + \frac{(752.2\ k\text{-}ft)\ \frac{12\ in.}{1\ ft}}{656\ in^3}$$

$= 10.7 + 2.3 + 13.8 = 26.8$ ksi < 27 ksi O.K.

LOAD FACTOR DESIGN: MAX. MOMENT CHECK ONLY

Check for compact section:

AASHTO [10.50.1.1.2] $\frac{2\ D_{cp}}{t_w} < \frac{19230}{\sqrt{F_y}}$ Eq. 10-93

Plastic N.A. position:

$C_{slab} = 0.85 \times 7.5$ in. x 90 in. x 3.5 ksi = 2008 k

(Disregard slab reinforcement)

$C_{beam} = 50$ ksi x 39.7 in^2 = 1985 k

$< 2008\ k = C_{slab}$

∴ Plastic N.A. is in the slab and Eq. 10-93 is satisfied.

$$D_p = \frac{1985\ k}{.85\ x\ 90\ in.\ x\ 3.5\ ksi} = 7.41\ in.$$

Check eqn. 10-128a:

$$D_p = 7.41\ in. < \frac{d + t_s + t_h}{7.5} = \frac{35.55+7.5+1}{7.5} = 5.87\ in.$$

(Check not satisfied)

Check maximum stress in flange:

$$f_{BF} = \frac{M}{S} = \frac{1.3 \text{ x } 390 \text{ k-ft x } \frac{12 \text{ in.}}{1 \text{ ft}}}{439 \text{ in}^3} + \frac{1.3 \text{ x } 112.3 \text{ k-ft x } \frac{12 \text{ in.}}{1 \text{ ft}}}{593 \text{ in}^3} +$$

$$\frac{1.3 \text{ x } 1.67 \text{ x } 752.2 \text{ k-ft x } \frac{12 \text{ in.}}{1 \text{ ft}}}{656 \text{ in}^3}$$

$= 13.9 + 3.0 + 29.9 = 46.8$ ksi $< F_y = 50$ ksi

∴ O.K. and Eq. 10-128a does not need to be checked.

$M_u = M_y =$ Mom. at first yielding

L + I mom. capacity $= (f_y - f_{DL1} - f_{DL2})\,(S_c)$

$$= (50 - 13.9 - 3.0) \text{ ksi } (656 \text{ in}^3)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 1809 \text{ k-ft}$$

$M_y = 1809 + (390 + 112.3)(1.3) = 2462$ k-ft

$M = (1.3)(390 + 112.3 + (1.67)(752.2)) = 2286$ k-ft $< M_y$

∴ O.K.

<u>Problem 1: Metric:</u> Simple Span Composite Stringer

<u>Required:</u>

Design composite section

<u>Given:</u>

HS-20 Loading (MS 18 AASHTO 1977, Sec. 1.2.5)

$$\text{Roadway width} = 40\ \text{ft x}\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right) = 12.192\ \text{m}$$

Use 12.0 m Iowa DOT Handbook pg. 11

$$\text{Bridge span} = 54\ \text{ft x}\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right) = 16.46\ \text{m}$$

Use 16.5 m

$$\text{Beam spacing} = 9.25\ \text{ft x}\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right) = 2.819\ \text{m}$$

Use 2.7 m Iowa DOT Handbook pg. 11

$$\text{Deck thickness} = 8\ \text{in. x}\left(\frac{25.4\ \text{mm}}{1\ \text{in.}}\right) = 203.2\ \text{mm}$$

Use 200 mm Iowa DOT Handbook pg. 19

$$F_y = 50\ \text{ksi x}\left(\frac{6.894\ 76\ \text{MPa}}{1\ \text{ksi}}\right) = 344.7\ \text{MPa}$$

Use 345 Mpa Iowa DOT Handbook pg. 9

$$f'_c = 3.5 \text{ ksi} \times \left(\frac{6.894\ 76 \text{ MPa}}{1 \text{ ksi}} \right) = 24.13 \text{ Mpa}$$

Use f'_c = 24 MPa Iowa DOT Handbook pg. 22

$$\text{Barrier rail} = 2.47 \ \frac{\text{ft}^3}{\text{ft}} \times \left(\frac{0.3048\text{m}}{1 \text{ ft}} \right)^2 = 0.229 \text{ m}^3/\text{m length of rail}$$

$$\text{FWS} = 20 \text{ psf} \times \left(\frac{47.8803 \text{ Pa}}{1 \text{ psf}} \right) = 957.6 \text{ Pa}$$

Use 960 Pa Iowa DOT Handbook pg. 10a

$$\text{Density of concrete} = 150 \text{ lb/ft}^3 \times 0.157\ 087 \ \frac{\text{kN/m}^3}{\text{lb/ft}^3} = 23.56 \text{ kN/m}^3$$

Use 24 kN/m^3 Iowa DOT Handbook pg. 22

Solution:

<u>Assume steel section:</u>

U.S. Customary W36x135 → SI W920 x 201

(Metric properties of structural shapes)

<u>W920 x 201:</u>

D = 903 mm
A = 25 600 mm^2
b_f = 304 mm
mass/unit length = 201 kg/m
$I_x = 3250 \times 10^6$ mm^4
$S_x = 7200 \times 10^3$ mm^3

QUICK CHECK

W36 x 135 (U.S. Customary) → W920 x 201 (Metric)

Remember: 1 in → 25 mm (approx.)
1 plf → 1.5 kg/m (aprox.)

W36	x	135
x 25		x 1.5
W900	x	202 (approx.)

Soft Conversion:

U.S. Customary W36 x 135:	METRIC W920 x 201:
$d = 35.55 \text{ in} \times \left(\frac{25.4 \text{ mm}}{1 \text{ in.}}\right) = 903.0 \text{ mm}$	903 mm
$b_f = 11.95 \text{ in} \times \left(\frac{25.4 \text{ mm}}{1 \text{ in.}}\right) = 303.5 \text{ mm}$	304 mm
$t_f = 0.79 \text{ in} \times \left(\frac{25.4 \text{ mm}}{1 \text{ in.}}\right) = 20.07 \text{ mm}$	20.1 mm
$t_w = 0.6 \text{ in} \times \left(\frac{25.4 \text{ mm}}{1 \text{ in.}}\right) = 15.24 \text{ mm}$	15.2 mm
$A = 39.7 \text{ in}^2 \times \left(\frac{25.4 \text{ mm}}{1 \text{ in.}}\right)^2 = 25\,613 \text{ mm}^2$	$25\,600 \text{ mm}^2$
$I_x = 7800 \text{ in}^4 \times \left(\frac{25.4 \text{ mm}}{1 \text{ in.}}\right)^4 = 3246.6\text{x}10^6 \text{ mm}^4$	$3247 \times 10^6 \text{ mm}^4$
$S_x = 439 \text{ in}^3 \times \left(\frac{25.4 \text{ mm}}{1 \text{ in.}}\right)^3 = 7194 \times 10^3 \text{ mm}^3$	$7200 \times 10^3 \text{ mm}^3$

Note: Steel sections are essentially the same.

Composite section:

$t_{slab} = 200$ mm

$$\text{deduction for wear} = 1/2 \text{ in. x} \left(\frac{25.4 \text{ mm}}{1 \text{ in.}} \right) = 12.7 \text{ mm}$$

Use 13 mm Iowa DOT Handbook pg. 19

$\therefore$ reduced $t_{slab} = 200 - 13 = 187$ mm

effective width = 12 x 187 = 2244 mm

Haunch Dimensions:

$b = b_f$ of W-section = 304 mm

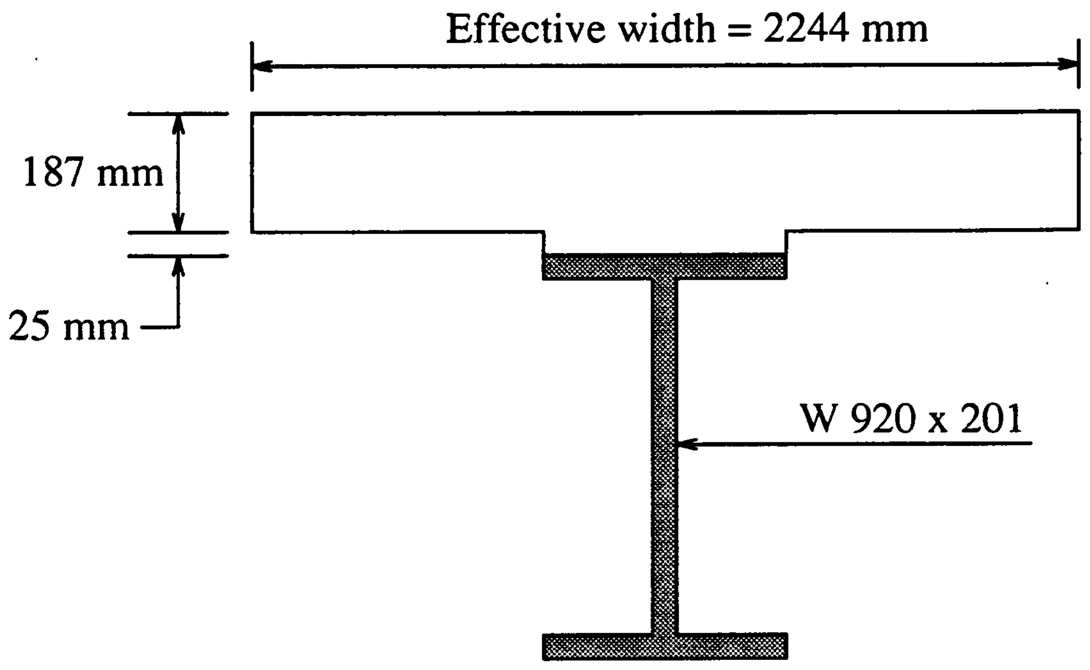
$$t = 1 \text{ in. x} \left(\frac{25.4 \text{ mm}}{1 \text{ in.}} \right) = 25.4 \text{ mm}$$

Use 25 mm

Diaphragm:

$$\text{Assumed mass} = 10 \text{ lb/ft x} \left(\frac{1.488\ 16 \text{ kg/m}}{1 \text{ lb/ft}} \right) = 14.88 \text{ kg/m}$$

Use 15 kg/m



COMPOSITE SECTION DIMENSIONS

Composite Section Properties (N = 9 and 3N = 27 for long term)

	A (mm^2)	Y (mm)	AY (mm^3)	AY^2 (mm^4)	I_o (mm^4)
Beam	25 600	451	1.156×10^7	5.22×10^9	3.250×10^9
Haunch (N)	844	915	7.731×10^5	7.080×10^8	
Haunch (3N)	[281]	915	[2.577×10^5]	[2.359×10^8]	
Slab (N)	46 625	1021	4.763×10^7	4.865×10^{10}	1.359×10^8
Slab (3N)	[15 542]	1021	[1.588×10^7]	[1.622×10^{10}]	[4.529×10^7]

N = 9: $\bar{Y} = 821$ mm — $I = 8760 \times 10^6\ mm^4$ — $S_{BF} = 10\,676 \times 10^3\ mm^3$

3N = 27: [$\bar{Y} = 668$ mm] — [$I = 6451 \times 10^6\ mm^4$] — [$S_{BF} = 9651 \times 10^3\ mm^3$]

Note: $S = 656\ in^3 \times \left[\frac{25.4\ mm}{1\ in}\right]^3$

$= 10\,750 \times 10^3\ mm^3$ **(0.7% difference)**

$S = 593\ in^3 \times \left[\frac{25.4\ mm}{1\ in}\right]^3$

$= 9718 \times 10^3\ mm^3$ **(0.7% difference)**

HELPFUL HINT

If S to the nearest 1 in^3 is sufficient accuracy, then corresponding accuracy in metric is:

$$1\ \text{in}^3 \times \left[\frac{25.4\ \text{mm}}{1\ \text{in.}}\right]^3 = 16\,387\ \text{mm}^3$$

see Iowa DOT Handbook pg. 7 for general recommendation

For example

Rounding S = 10 676 x 10^3 mm^3 to S = 10 700 x 10^3 mm^3 would imply an equivalent accuracy in in^3 of:

$$24 \times 10^3\ \text{mm}^3 \times \left[\frac{1\ \text{in.}}{25.4\ \text{mm}}\right]^3 = 1.5\ \text{in}^3$$

Loads and Moments for Interior Beam Line:

Non-Composite DL:

Deck: 2.7 m x 200 mm x 24 kN/m^3
= 2.7 m x 0.2 m x 24 kN/m^3
= 12.96 kN/m

Beam + Diaph: (Assume Diaph: 15 kg/m)
= 201 kg/m + 15 kg/m = 216 kg/m
To convert to (force/length) units, multiply by acceleration of gravity
= 216 kg/m x 9.806 m/sec^2 = 2118 N/m
= 2.12 kN/m

Composite DL:

Rail: $0.229 \dfrac{m^3}{\text{m length of rail}} \times 24\ kN/m^3 \times 2$ rails/5 beams
= 2.20 kN/m

FWS: 960 Pa x 12.0 m/5 beams = 960 N/m^2 x 12.0 m/5 = 2304 N/m
= 2.30 kN/m

DL Mom. (non comp.) = 1/8 x (12.96 + 2.12) kN/m x $(16.5)^2$ m^2
= 513.2 kN·m

DL Mom. (comp.) = 1/8 x (2.20 + 2.30) kN/m x $(16.5)^2$ m^2 =153.1 kN·m

LL + I Mom. = 1.28 x 1.611 x (474.06) kN·m = 977.6 kN·m.

where:

LL Dist. Factor = $\dfrac{2.7}{1.676}$ = 1.611 wheel lines per beam (AASHTO 1977, sec. 1.3.1)

If a formula you need is not available in Metric,

make a soft conversion.

$$\text{LL Distribution Factor} = \frac{S}{5.5}$$

where S is measured in ft
1 ft = 0.3048 m

$$\therefore S\ (\text{in ft}) = \frac{S(\text{in m})}{0.3048}$$

$$\therefore D.F = \frac{S}{0.3048 \times 5.5}$$ where S is measured in m.

$$D.F = \frac{S}{1.676}$$

Note: 1.682 U.S. Customary vs. 1.611 Metric (4.4% difference)

$$I = \frac{15.24}{L+38} = \frac{15.24}{16.5+38} = 0.28 \quad \text{(AASHTO Sec. 3.8.2, Appendix E)}$$

$$LL_{moment} = \frac{948.12 \text{ kN·m}}{2} = 474.06 \text{ kN·m (AASHTO Appendix A 1977)}$$

$$F_{all} = 27 \text{ ksi x} \left(\frac{6.894\ 76 \text{ MPa}}{1 \text{ ksi}} \right) = 186.2 \text{ MPa}$$

Use 185 MPa

$$f_{BF} = M/S = \left(\frac{513.2 \text{ kN·m}}{7200 \text{ x } 10^3 \text{ mm}^3} + \frac{153.1 \text{ kN·m}}{9651 \text{ x } 10^3 \text{ mm}^3} + \frac{977.6 \text{ kN·m}}{10\ 676 \text{ x } 10^3 \text{ mm}^3} \right)$$

$$\text{x} \left(\frac{10^9 \text{ mm}^3}{1 \text{ m}^3} \right)$$

$$= 178.7 \text{ x } 10^3 \text{ kN/m}^2 = 178.7 \text{ MN/m}^2$$

$$= 178.7 \text{ MPa} < 185 \text{ MPa}$$

∴ O.K.

Note: 26.8 ksi x $\left(\frac{6.894\ 76 \text{ MPa}}{1 \text{ ksi}} \right)$ **= 184.8 MPa (3.4% difference)**

Check for compact section:

AASHTO [10.50.1.1.2] $\frac{2\ D_{cp}}{t_w} \leq \frac{1597}{\sqrt{F_y}}$ Eq. 10-93

Plastic N.A. position:

$$C_{slab} = 0.85 \text{ x } 0.187 \text{ m x } 2.244 \text{ m x } 24 \text{ MPa x } \left(\frac{1\ \text{MN/m}^2}{1\ \text{MPa}}\right) = 8.56 \text{ MN}$$

(Disregard slab reinforcement)

$$C_{beam} = 345 \text{ MPa x } \left(\frac{1\ \text{MN/m}^2}{1\ \text{MPa}}\right) \text{ x } 25\ 600 \text{ x } 10^{-6} \text{ m}^2 = 8.83 \text{ MN}$$

$$= 8.83 \text{ MN} > 8.56 \text{ MN} = C_{slab}$$

∴ Plastic N.A. is below compression flange - using plastic stress block N.A. located 1 mm below top of top flange (D_{cp} = 1 mm) and Eq. 10-93 is satisfied.

D_p = 187 mm + 1 mm
= 188 mm

Check eqn. 10-128a:

$$D_p = 188 \text{ mm} < \frac{d + t_s + t_h}{7.5} = \frac{903 + 187 + 25}{7.5} = 149 \text{ mm}$$

(Check not satisfied)

Check maximum stress in flange:

$$f_{BF} = \frac{M}{S} = \frac{1.3 \text{ x } 513.2 \text{ kNm}}{7200 \text{ x } 10^{-6} \text{ m}^3} + \frac{1.3 \text{ x } 153.1 \text{ kNm}}{9651 \text{ x } 10^{-6} \text{ m}^3} +$$

$$\frac{1.3 \text{ x } 1.67 \text{ x } 977.6 \text{ kNm}}{10\ 676 \text{ x } 10^{-6} \text{ m}^3}$$

$= 92\ 661\ kN/m^2 + 20\ 623\ kN/m^2 + 198\ 798\ kN/m^2$

$= 312\ 082\ kN/m^2 = 312\ MN/m^2$

$= 312\ MPa < F_y = 345\ MPa$

∴ O.K. and AASHTO Eqn. 10-128a does not need to be checked

Note: $46.8\ ksi \times \left[\frac{6.894\ 76\ MPa}{1\ ksi}\right] = 322.7\ MPa$ **(3.4% difference)**

$M_u = M_y$ = Mom. at first yielding

L + I Mom. Capacity $= (f_y - f_{DL1} - f_{DL2})\ (S_c)$

$= (345\ MPa - 92.6\ MPa - 20.6\ Mpa) \times \frac{MN/m^2}{MPa} \times 10\ 676 \times 10^{-6}\ m^3$

$= 2474.7\ kN{\cdot}m$

$M_y = 2474.7\ kN{\cdot}m + (513.2\ kN{\cdot}m + 153.1\ kN{\cdot}m)\ 1.3 = 3340.9\ kN{\cdot}m$

$M = 1.3\ (513.2\ kN{\cdot}m + 153.1\ kN{\cdot}m + (1.67)(977.6\ kN{\cdot}m))$

$= 2988.6\ kN{\cdot}m < 3340.9\ kN{\cdot}m$ ok

Note: $2286\ k\text{-}ft \times \left(\frac{1.355\ 82}{1\ k\text{-}ft}\right) = 3099.4\ kN{\cdot}m$ **(3.7% difference)**

Problem 2: U.S. Customary: Continuous Three-Span Bridge

Required:

Maximum moment @ Pier and @ 0.4 point of Span 1 of interior beam line.

Given:

HS-20 Loading

Constant I

Beam spacing = 7.401 ft

Span lengths: $L_1 = 68.625$ ft

$L_2 = 87.75$ ft

Total length = S = 225 ft

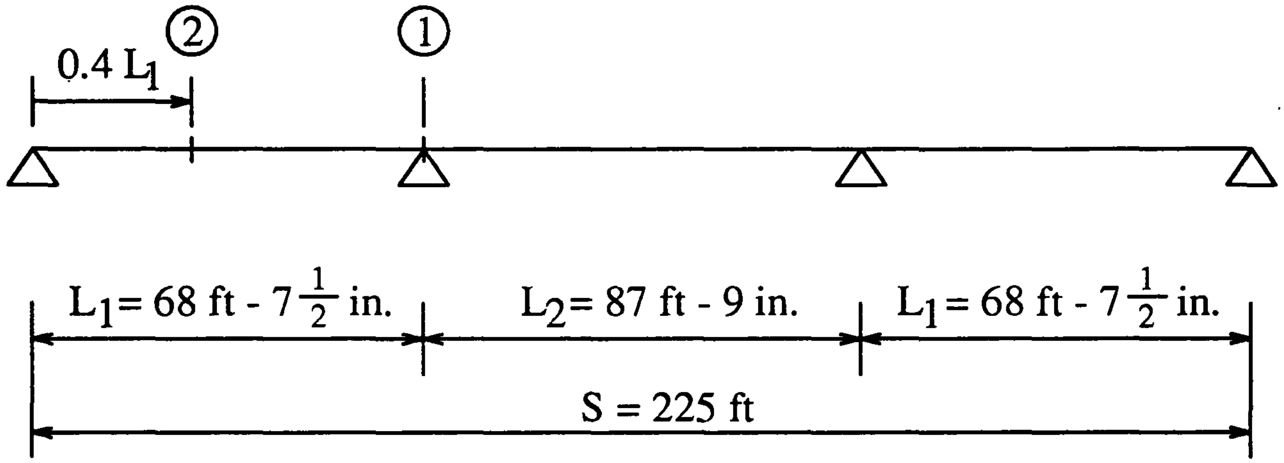
Uniform DL = 1.0 kip/ft

Solution:

Impact: $I = \dfrac{50}{L+125}$

@ Pier: $I = \dfrac{50}{\left(\dfrac{68.625 + 87.75}{2}\right) + 125} = 0.246$

@ 0.4 point of Span 1: $I = \dfrac{50}{68.625 + 125} = 0.258$



POSITIONS OF SECTIONS ALONG THE BRIDGE LENGTH

Moment at Pier on Beam Line:

- Uniform DL Moment:

$$M_{DL} = 1.0 \text{ k/ft} \times (0.005545\ S^2 + 0.008332\ S^2 - 0.00155\ S^2)$$

where S = 225 ft

M_{DL} = - 624.1 k-ft

- HS20 Loading, LL Mom:

LANE:

$$M_{uniform} = \frac{0.64 \text{ klf}}{2} \times \frac{7.401}{5.5} (0.005545 + 0.008332)(225 \text{ ft})^2$$

= - 302.5 k-ft

$$M_{concentrated} = \frac{18 \text{ k}}{2} \times \frac{7.401}{5.5} (0.02788 + 0.03448)(225 \text{ ft})$$

= - 169.9 k-ft

$M_{lane} = M_{uniform} + M_{concentrated}$ = - 472.4 k-ft

TRUCK:

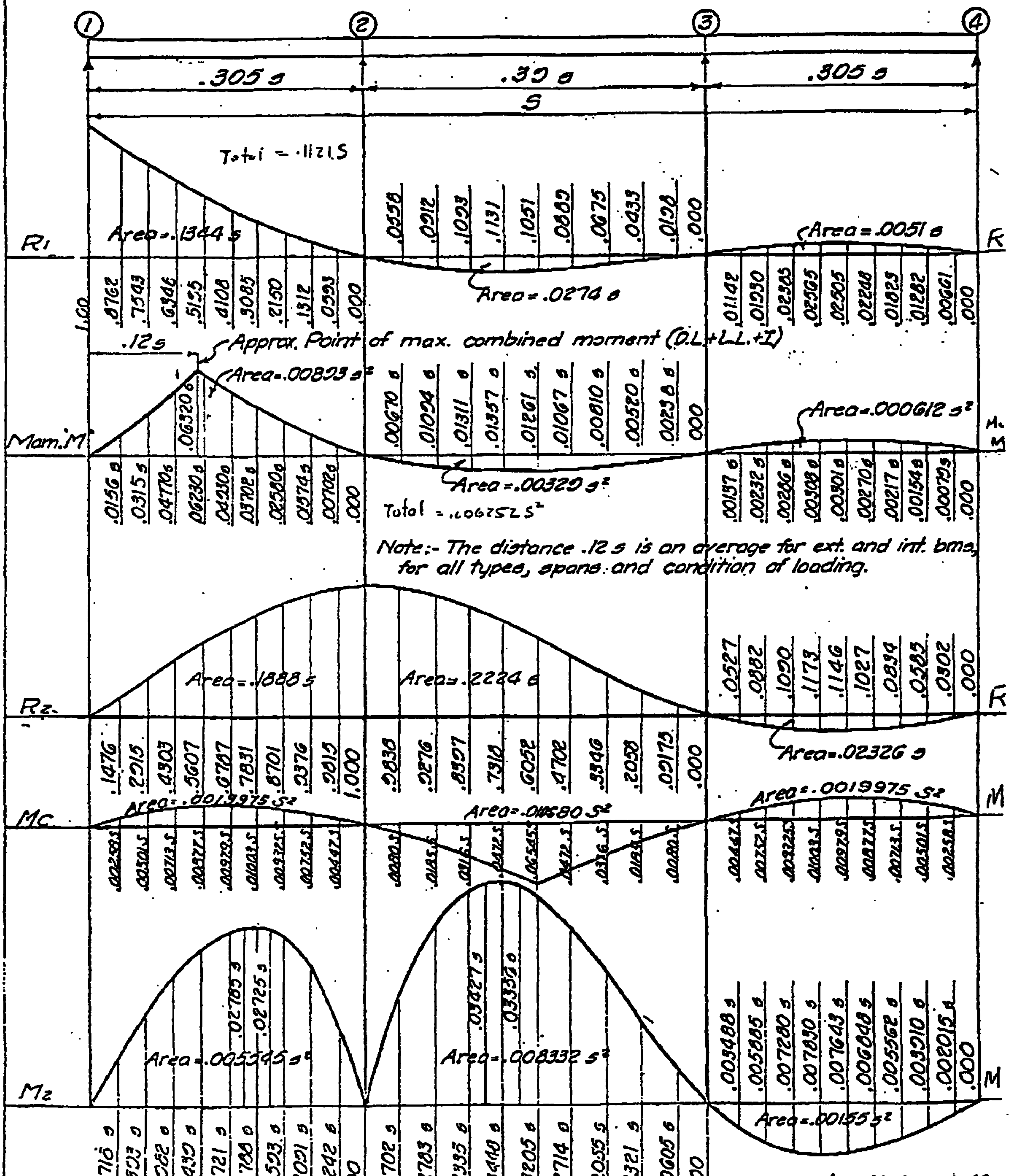
Place truck in Span 2 at high pt. of IFL

$$M_{truck} = \frac{7.401}{5.5} \left(\left[\frac{32 \text{ k}}{2} \times (0.03448 + 0.030038) \right] + \left[\frac{8 \text{ k}}{2} \times 0.029104 \right] \right) \times 225 \text{ ft} = -347.8 \text{ k-ft}$$

CENTER SPAN 0.39 OF TOTAL LENGTH

Iowa State Highway Commission

Uniform Moment of Inertia



Mc added 10-12-46
P.F.B. 12-14-37

$\therefore$ Total DL and LL moment @ Pier = - 624.1 + (- 472.4) (1.246)

= - 1213 k-ft

<u>Moment @ 0.4 point of Span 1 on Beam Line:</u>

- Uniform DL Moment:

M_{DL} = 1.0 k/ft x (0.00893 - 0.00329 + 0.000612) $(225 \text{ ft})^2$

= + 316.5 k-ft

- HS20 Loading, LL Mom:

LANE:

$$M_{uniform} = \frac{0.64 \text{ k/ft}}{2} \times \frac{7.401}{5.5} (0.00893 + 0.000612)(225 \text{ ft})^2$$

= + 208.0 k-ft

$$M_{concentrated} = \frac{18 \text{ k}}{2} \times \frac{7.401}{5.5} (0.06320)\ (225 \text{ ft}) = +172.2 \text{ k-ft}$$

$M_{lane} = M_{uniform} + M_{concentrated}$ = + 380.2 k-ft

TRUCK:

Place truck in Span 1 at high point of IFL

$$M_{truck} = \frac{7.401}{5.5}\left[\frac{32 \text{ k}}{2} \times (0.06320 + 0.03733) + \frac{8 \text{ k}}{2} \times 0.0298\right]$$

x 225 ft = 523.1 k-ft

∴ Total DL and LL moment @ 0.4 point of Span 1 = + 316.5 + 523.1 x 1.258 = 974.6 k-ft

<u>Problem 2: Metric:</u> Continuous Three-Span Bridge

<u>Required:</u>

Maximum moment @ Pier and @ 0.4 point of Span 1 of interior beam line.

<u>Given:</u>

HS-20 Loading (MS 18 AASHTO 1977 Sec. 1.2.5)

Constant I

$$\text{Beam spacing} = 7.401 \text{ ft} \times \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 2.256 \text{ m}$$

Use 2.2m   Iowa DOT Handbook pg. 11

$$\text{Span lengths: } L_1 = 68.625 \text{ ft} \times \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 20.917 \text{ m}$$

Use 20.9 m Iowa DOT Handbook pg. 10a

$$L_2 = 87.75 \text{ ft} \times \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 26.746 \text{ m}$$

Use 26.7 m

$$\text{Total length} = S = 225 \text{ ft} \times \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 68.58 \text{ m}$$

Use 68.6 m

Check for ripple effect:

$$\text{Total length} = 2 \times 20.9 \text{ m} + 26.7 \text{ m} = 68.5 \text{ m} \neq 68.6 \text{ m}$$

(0.1 m difference)

∴ Center-span length is adjusted.

$L_2 = 26.7 + 0.1 = 26.8$ m

$$\text{Uniform DL} = 1.0 \text{ kip/ft} \times \left(\frac{14.5939 \text{ kN/m}}{1 \text{ kip/ft}} \right) = 14.6 \text{ kN/m}$$

Solution:

$$\text{Impact:}\quad I = \frac{15.24}{L+38}$$

$$\text{@ Pier:}\quad I = \frac{15.24}{\left(\frac{20.9 + 26.8}{2} \right) + 38} = 0.246$$

$$\text{@ 0.4 point of Span 1:}\quad I = \frac{15.24}{20.9 + 38} = 0.259$$

Moment at Pier on Beam Line:

- Uniform DL Moment:

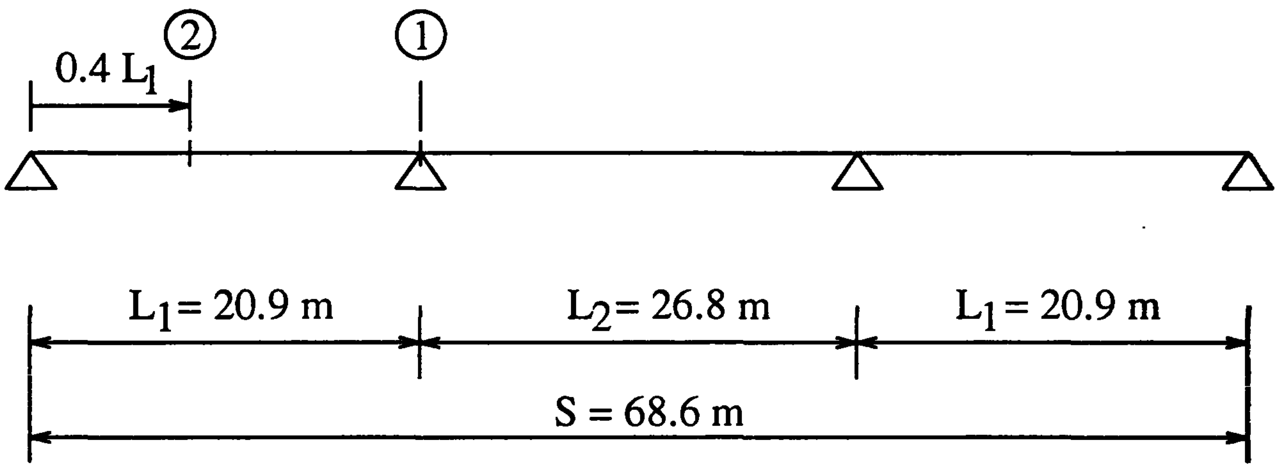
$$M_{DL} = 14.6 \text{ kN/m} \times (0.005545\ S^2 + 0.008332\ S^2 - 0.00155\ S^2)$$

where S = 68.6 m

$M_{DL} = -\ 847.0$ kN·m

Note: $624.1 \text{ k-ft} \times \left(\frac{1.355\ 82 \text{ kN·m}}{1 \text{ k-ft}} \right) = 846.2$ **kN·m (0.1% difference)**

- MS18 Loading, LL Mom:



POSITIONS OF SECTIONS ALONG THE BRIDGE LENGTH

LANE:

$$M_{uniform} = \frac{9.4\ \text{kN/m}}{2} \times \frac{2.2}{1.676} (0.005545 + 0.008332)(68.6\text{m})^2$$

$$= -402.9\ \text{kN} \cdot \text{m}$$

$$M_{concentrated} = \frac{80\ \text{kN}}{2} \times \frac{2.2}{1.676} (0.02788 + 0.03448)(68.6\ \text{m})$$

$$= -224.6\ \text{kN} \cdot \text{m}$$

$$M_{lane} = M_{uniform} + M_{concentrated} = -627.5\ \text{kN} \cdot \text{m}$$

Note: $472.4\ \text{k-ft} \times \left(\frac{1.355\ 82}{1\ \text{k-ft}} \right) = 640.5\ \text{kN} \cdot \text{m}$ (2.1% difference)

TRUCK:

Place truck in Span 2 at high pt. of IFL

$$M_{truck} = \frac{2.2}{1.676} \left(\left[\frac{144\ \text{kN}}{2} \times (0.03448 + 0.030038) \right] + \left[\frac{36\ \text{kN}}{2} \times 0.029104 \right] \right) \times 68.6\ \text{m} = -465.5\ \text{kN} \cdot \text{m}$$

Note: $347.8\ \text{k-ft} \times \left(\frac{1.355\ 82\ \text{kN m}}{1\ \text{k-ft}} \right) = 471.6\ \text{kN} \cdot \text{m}$ (1.3% difference)

LANE CONTROLS

$\therefore$ Total DL and LL moment @ Pier = $-847.0 + (-627.5)(1.246) =$
$-1628.9\ \text{kN} \cdot \text{m}$

NOTE

The MS 18 vehicle dimensions are exact soft values of HS 20 vehicle dimension. Therefore, IFL ordinates will be the same as used for HS 20 vehicle.

$$14 \text{ ft x} \left[\frac{.3048 \text{ m}}{1 \text{ ft}}\right] = 4.267 \text{ m vs. } 4.267 \text{ m}$$

$$6 \text{ ft x} \left[\frac{.3048 \text{ m}}{1 \text{ ft}}\right] = 1.829 \text{ m vs. } 1.830 \text{ m}$$

However, vehicle loads are not exact soft values.

$$32 \text{ kips x} \left[\frac{4.448\ 22 \text{ kN}}{1 \text{ kip}}\right] = 142.3 \text{ kN vs. } 144 \text{ kN}$$

$$.640 \text{ k/ft x} \left[\frac{14.5939 \text{ kN/m}}{\text{k/ft}}\right] = 9.3 \text{ kN/m vs. } 9.4 \text{ kN/m}$$

$$18 \text{ kips x} \left[\frac{4.448\ 22 \text{ kN}}{1 \text{ kip}}\right] = 80.1 \text{ kN vs. } 80 \text{ kN}$$

Note: $\mathbf{1213\ k\text{-}ft \times \left(\dfrac{1.355\ 82\ kN \cdot m}{1\ k\text{-}ft}\right) = 1644.6\ kN \cdot m}$ **(1.0% difference)**

Moment @ 0.4 point of Span 1 on Beam Line:

- Uniform DL Moment:

$$M_{DL} = 14.6\ \text{kN/m} \times (0.00893 - 0.00329 + 0.000612)\ (68.6\ \text{m})^2$$

$$= +\ 429.6\ \text{kN}\cdot\text{m}$$

Note: $\mathbf{316.5\ k\text{-}ft \times \left(\dfrac{1.355\ 82\ kN \cdot m}{1\ k\text{-}ft}\right) = +\ 429.1\ kN \cdot m}$ **(0.1% difference)**

- MS18 Loading, LL Mom:

LANE:

$$M_{uniform} = \frac{9.4\ \text{kN/m}}{2} \times \frac{2.2}{1.676}\ (0.00893 + 0.000612)(68.6\ \text{m})^2$$

$$= +\ 277.0\ \text{kN}\cdot\text{m}$$

$$M_{concentrated} = \frac{80\ \text{kN}}{2} \times \frac{2.2}{1.676}\ (0.06320)\ (68.6\ \text{m}) = +\ 227.6\ \text{kN}\cdot\text{m}$$

$$M_{lane} = M_{uniform} + M_{concentrated} = +\ 504.6\ \text{kN}\cdot\text{m}$$

Note: $\mathbf{380.2\ k\text{-}ft \times \left(\dfrac{1.355\ 82\ kN \cdot m}{1\ k\text{-}ft}\right) = 515.5\ kN \cdot m}$ **(2.2% difference)**

TRUCK:

Place truck in Span 1 at high point of IFL

$$M_{truck} = \frac{2.2}{1.676}\left[\frac{144\ kN}{2} \times (0.06320 + 0.03733) + \frac{36\ kN}{2} \times 0.0298\right]$$

$$\times\ 68.6\ m = +\ 700.1\ kN\cdot m$$

Note: $523.1\ \text{k-ft} \times \left(\frac{1.355\ 82\ kN \cdot m}{1\ \text{k-ft}}\right) = +\ 709.2\ kN\cdot m$ **(1.3% difference)**

TRUCK CONTROLS

∴ Total DL and LL moment @ 0.4 point of Span 1 = + 429.6

$$+\ 700.1 \times 1.259 = 1311.0\ kN\cdot m$$

Note: $974.6\ \text{k-ft} \times \left(\frac{1.355\ 82\ kN \cdot m}{1\ \text{k-ft}}\right) = 1321.4\ kN\cdot m$ **(0.8% difference)**

THINK IN METRIC

In the future, no need to convert any of the following values:

F_y = 345 MPa
f'_c = 24 MPa
Density of concrete = 24 kN/m^3
FWS = 960 MPa

Get these values from the
Iowa DOT Handbook right away.

Problem 3: U.S. Customary: Reinforced Concrete Compression Member

Required:

Check the stresses in reinforced concrete compression member under the effects of axial load only and combined axial load and moment.

Given:

$f'_c = 3.5$ ksi

F_y (reinforcement) = 60 ksi

Column size = 12 in. x 12 in.

Reinforcement: 4 - #8 bars

Concrete cover = 2.0 in.

Lateral ties

$$K \frac{\ell_u}{r} < 34 - (12\ M_{1b}/M_{2b})$$

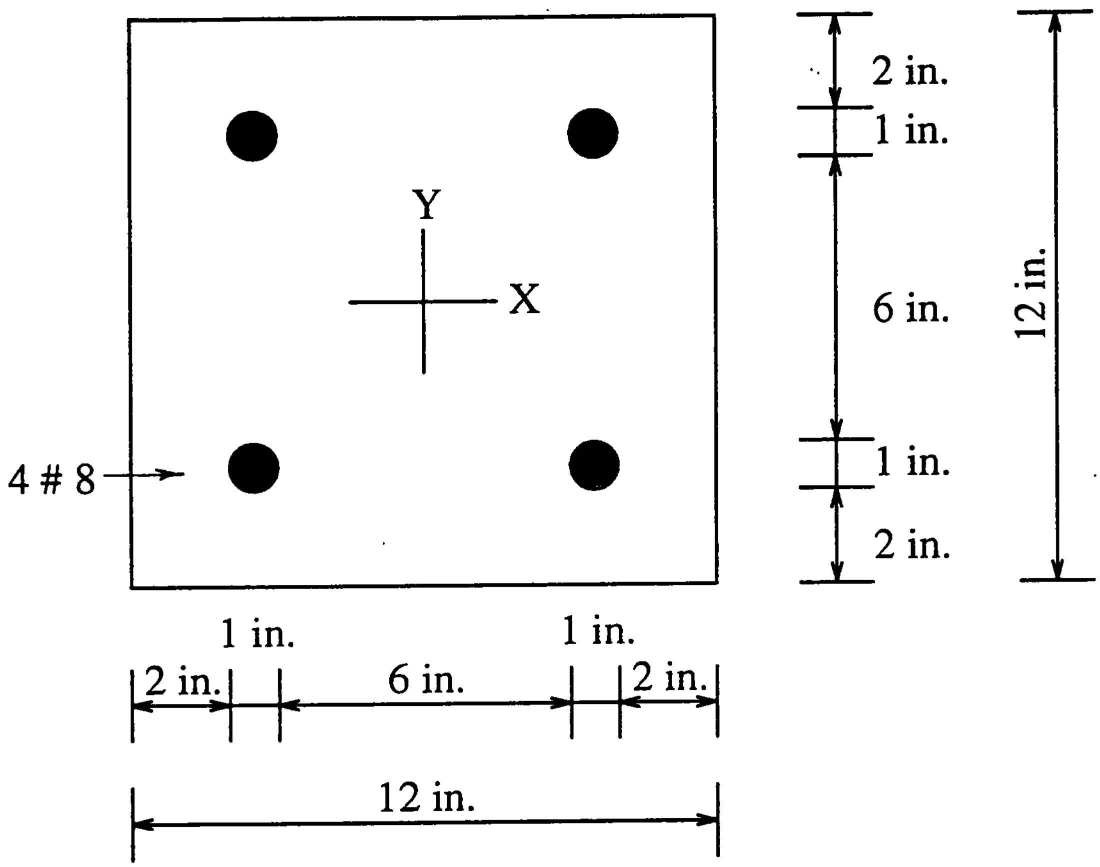
∴ the column is non-slender and minimum load eccentricities are not applicable. (AASHTO 8.16.5.2.4)

Load Case A: Axial Load Only

Factored axial load = 340 kips

Load Case B: Axial load + flexure

Factored axial load = 300 kips



COLUMN CROSS-SECTION

Factored moment about x-axis = 40 ft-kips

Solution:

Case A: $P_u = 340$ kips

$\phi P_n = \phi(0.80)\,[0.85\, f'_c\,(A_g - A_{st}) + (A_{st})(f_y)]$ (AASHTO 8.16.4.1.2b)

where P_n = nominal axial load strength of the section.

$A_g = 12 \text{ in.} \times 12 \text{ in.} = 144 \text{ in}^2$

$A_{st} = 4 \times 0.79 \text{ in}^2 = 3.16 \text{ in}^2$

$\phi\, P_n = 0.70 \times 0.80 \times [0.85 \times 3.5 \text{ ksi} \times (144 \text{ in}^2 - 3.16 \text{ in}^2)$

$+ 60 \text{ ksi} \times 3.16 \text{ in}^2] = 340.8 \text{ kips} > P_u = 340 \text{ kips}$

$\therefore$ OK

Case B: $P_u = 300$ kips, $M_u = 40$ ft-kips

$\phi P_n = 340.8$ kips (from Case A).

$$\text{Check if} \left(\frac{A_s - A_s'}{bd}\right) \geq 0.85\, \beta_1 \left(\frac{f_c'\, d'}{f_y\, d}\right)\left(\frac{87{,}000}{87{,}000 - f_y}\right)$$

(AASHTO 8.16.3.4.1 Eq. 8-24)

$A_s = A_s' = 2 \times 0.79 \text{ in}^2 = 1.58 \text{ in}^2$

$$\left(\frac{A_s - A_s'}{bd}\right) = 0,$$

which is less than that required by (AASTHO Eq. 8-24)

∴ Use Eq. 8-16 for M_N

$\phi M_n = \phi[A_s f_y (d-a/2)]$ (AASHTO 8.16.3.2.1. Eq. 8-16)

where a is the depth of the equivalent rectangular stress block

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{1.58\ in^2\ x\ 60\ ksi}{0.85\ x\ 3.5\ ksi\ x\ 12\ in.} = 2.655\ in.$$

$$d = 12\ in. - 2\ in.\ (cover) - \frac{1\ in.}{2} = 9.5\ in.$$

$$d' = 2\ in. + \frac{1\ in.}{2} = 2.5\ in.$$

$$\phi M_n = 0.9\ x\ 1.58\ in^2\ x\ 60\ ksi\ x \left(9.5\ in. - \frac{2.655\ in.}{2}\right) x \left(\frac{1\ ft}{12\ in.}\right)$$

$= 58.1$ k-ft

$$\phi P_o = \phi P_n/0.8 = \frac{340.8\ kips}{0.8} = 426\ kips$$

where P_o is the nominal axial load strength of the section at zero eccentricity

$\phi P_b = \phi[0.85\ f'_c b\ a_b + A_s'\ f'_s - A_s f_y]$ (AASHTO 8.16.4.2.3. Eq. 8-32)

where a_b = depth of equivalent rectangular stress block for balanced strain conditions

$$a_b = \left(\frac{87{,}000}{87{,}000 + f_y} \right) \beta_1 d \qquad \text{where } \beta_1 = 0.85$$

(AASHTO 8.16.4.2.3. Eq. 8-34)

$$= \left(\frac{87{,}000}{87{,}000 + 60{,}000 \text{ psi}} \right) \text{ x } 0.85 \text{ x } 9.5 \text{ in.} = 4.779 \text{ in.}$$

$$f_s' = 87{,}000 \left[1 - \left(\frac{d'}{d} \right) \left(\frac{87{,}000 + f_y}{87{,}000} \right) \right]$$

(AASHTO 8.16.4.2.3. Eq. 8-35)

$$= 87{,}000 \left[1 - \left(\frac{2.5 \text{ in.}}{9.5 \text{ in.}} \right) \left(\frac{87{,}000 + 60{,}000 \text{ psi}}{87{,}000} \right) \right]$$

$$= 48{,}316 \text{ psi} < 60{,}000 \text{ psi}$$

$\phi P_b = 0.7$ [0.85 x 3.5 ksi x 12 in. x 4.779 in. + 1.58 in^2 x 48.32 ksi

- 1.58 in^2 x 60 ksi] = 106.5 kips

$\phi M_b = \phi[0.85 f'_c b a_b (d - d'' - a_b/2) + A'_s f'_s (d - d' - d'') + A_s f_y d'']$

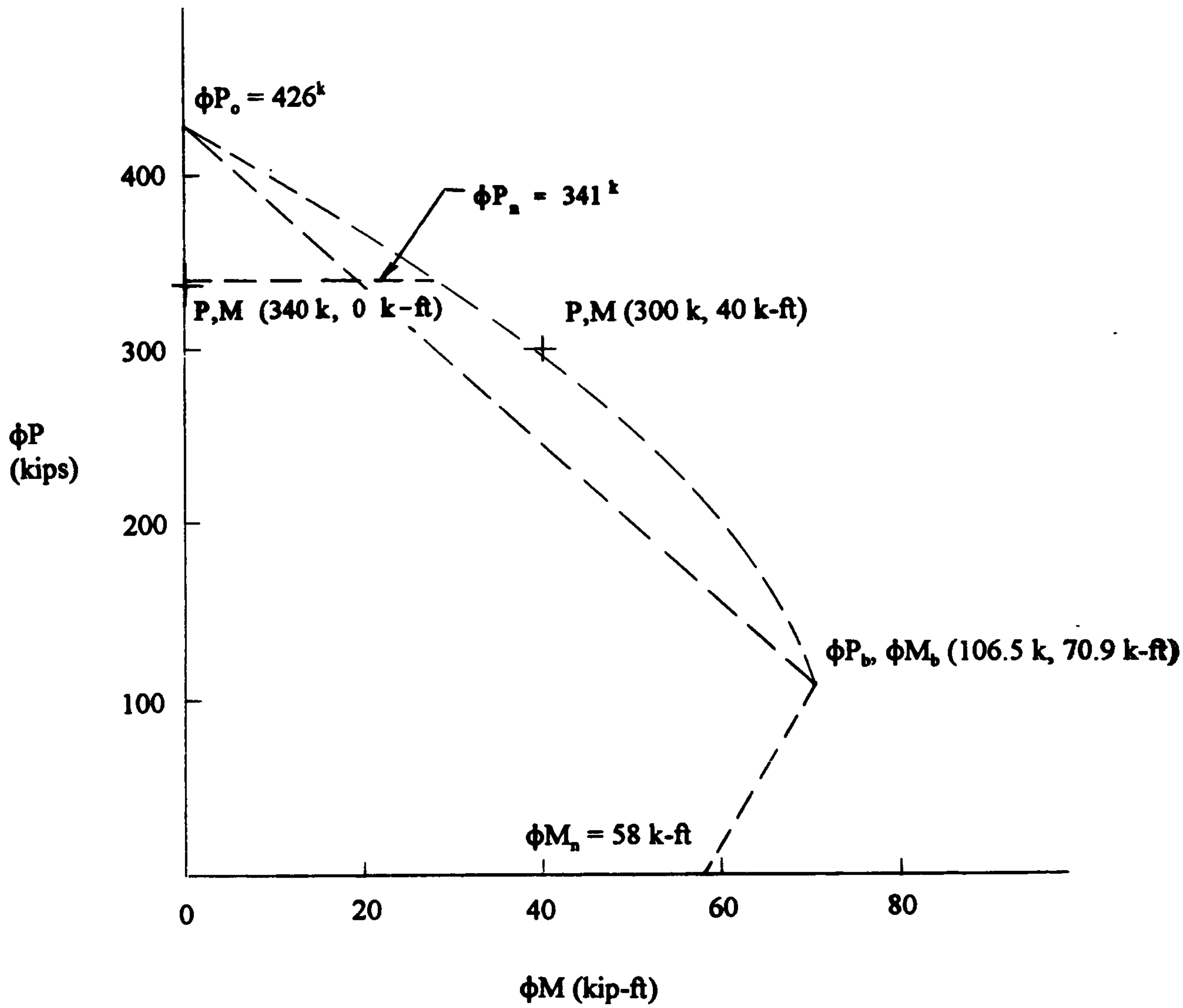
(AASHTO 8.16.4.2.3. Eq. 8-33)

d" = distance from centroid of gross section, neglecting the reinforcement, to centroid of tension reinforcement

= 6 - 2 - 1/2 = 3.5 in.

$$\phi M_b = 0.7\ [0.85 \times 3.5 \text{ ksi} \times 12 \text{ in.} \times 4.779 \text{ in.} \times \left(9.5 \text{ in.} - 3.5 \text{ in.} - \frac{4.779 \text{ in.}}{2}\right) + 1.58 \text{ in}^2 \times 48.32 \text{ ksi} \times (9.5 \text{ in.} - 2.5 \text{ in.} - 3.5 \text{ in.}) + 1.58 \text{ in}^2 \times 60 \text{ ksi} \times 3.5 \text{ in.}] \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)$$

$$= 70.9 \text{ k-ft}$$



To obtain an accurate plot of the interaction diagram, "PCACOL" program is used.

General Information:
====================
File Name: A:\PCA1.COL
Project: Code: ACI 318-89
Column: Units: US in-lbs
Engineer: Date: 02/12/95 Time: 15:46:30

Run Option: Investigation Short (nonslender) column
Run Axis: X-axis Column Type: Structural

Material Properties:
====================
f'c = 3.5 ksi fy = 60 ksi
Ec = 3586.62 ksi Es = 29000 ksi
fc = 2.975 ksi erup = 0 in/in
eu = 0.003 in/in
Stress Profile: Block Beta1 = 0.85

Geometry:
=========
Rectangular: Width = 12 in Depth = 12 in

Gross section area, Ag = 144 in^2
Ix = 1728 in^4 Xo = 0 in
Iy = 1728 in^4 Yo = 0 in

Reinforcement:
==============
Rebar Database: ASTM

Size	Diam	Area	Size	Diam	Area	Size	Diam	Area
3	0.38	0.11	4	0.50	0.20	5	0.63	0.31
6	0.75	0.44	7	0.88	0.60	8	1.00	0.79
9	1.13	1.00	10	1.27	1.27	11	1.41	1.56
14	1.69	2.25	18	2.26	4.00			

Confinement: Tied; phi(c) = 0.7, phi(b) = 0.9, a = 0.8
#3 ties with #10 bars, #4 with larger bars.

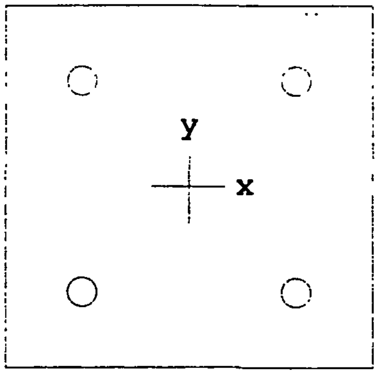
Layout: Rectangular
Pattern: All Sides Equal [Cover to longitudinal reinforcement]

Total steel area, As = 3.16 in^2 at 2.19%

4-#8 Cover = 2 in

Pt.	Applied Loads P (kips)	Applied Loads Mx (ft-k)	Computed Strength P (kips)	Computed Strength Mx (ft-k)	Computed/ Applied Ray length
1	340	0	341	-0	1.002
2	300	40	305	42	1.018

Program completed as requested!



12.0 x 12.0 inch

f'c = 3.5 ksi

fy = 60.0 ksi

Confinement: Tied

clr cover = 2.00 in

spacing = 6.00 in

4-#8 at 2.19%

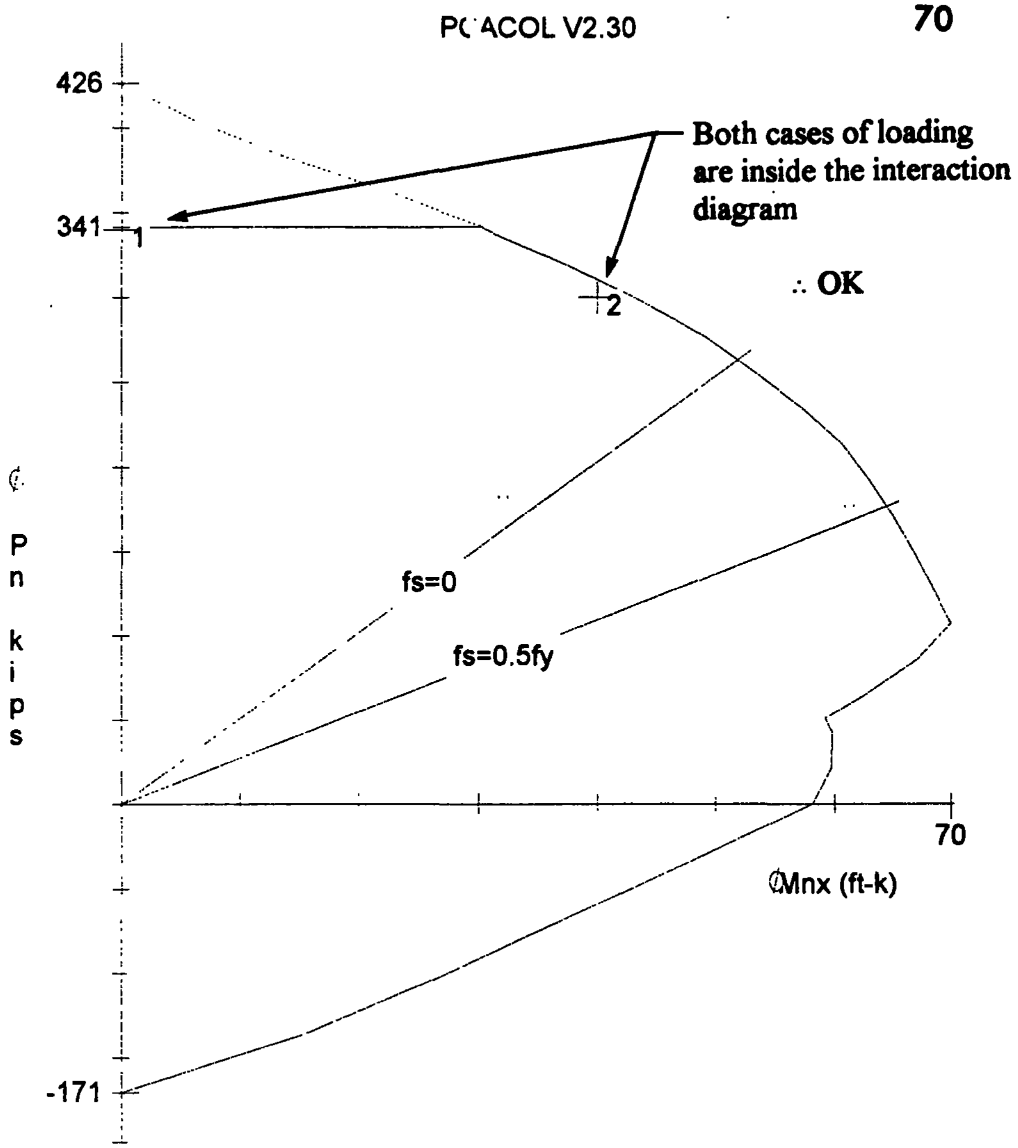
As = 3 in^2

Ix = 1728 in^4

Iy = 1728 in^4

Xo = 0.00 in

Yo = 0.00 in



File name: A:\PCA1.COL

Project:

Column Id:

Engineer:

Date: 02/12/95 Time: 15:46:30

Code: ACI 318-89

Units: in-lb

Material Properties:

Ec = 3587 ksi eu = 0.003 in/in

fc = 2.97 ksi Es = 29000 ksi

Beta1 = 0.85

Stress Profile: Block

phi(c) = 0.70, phi(b) = 0.90

X-axis slenderness is not considered.

Problem 3: Metric: Reinforced Concrete Compression Member

Required:

Check the stresses in reinforced concrete compression member under the effects of axial load only and combined axial load and moment.

Given:

$f'_c = 3.5$ ksi Use 24 MPa

F_y (reinforcement) = 60 ksi Use 400 MPa

Column size = 12 in. x 12 in. Use 300 mm x 300 mm

Reinforcement: 4 - #8 bars

Area = 4 x 0.79 in^2 = 3.16 in^2

$$F_{max} = 3.16 \text{ in}^2 \times 60 \text{ ksi} = 189.6 \text{ kips} \times \left(\frac{4.448\ 22 \text{ kN}}{1 \text{ kip}}\right) = 843.4 \text{ kN}$$

$$\text{Area of equivalent metric bars} = \frac{0.8434 \text{ MN}}{400 \text{ MPa}}$$

$$= 0.002\ 108 \text{ m}^2 = 2108 \text{ mm}^2$$

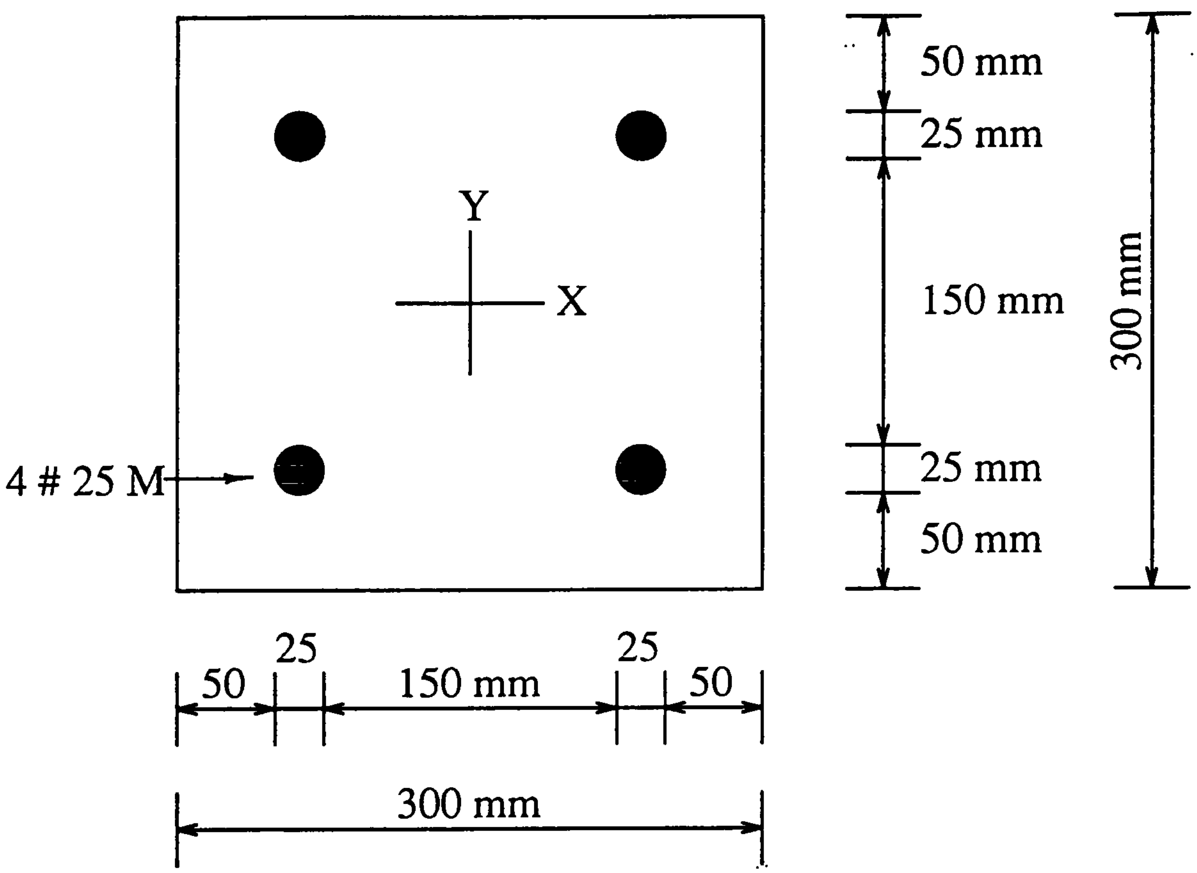
4 #25M bars: 4 x 500 mm^2 = 2000 mm^2 (Difference: - 5%)

4 #30M bars: 4 x 700 mm^2 = 2800 mm^2 (Difference + 33%)

Try 4 #25M

$$\text{Concrete cover} = 2.0 \text{ in.} \times \left(\frac{25.4 \text{ mm}}{1 \text{ in.}}\right) = 50.8 \text{ mm}$$

Use 50 mm



COLUMN CROSS-SECTION

Lateral ties

$$K \frac{\ell_u}{r} < 34 - (12\ M_{1b}/M_{2b})$$

∴ the column is non-slender and minimum load eccentricities are not applicable. (AASHTO 8.16.5.2.4)

Loading:

Case A: Axial Load Only

$$\text{Factored axial load} = 340 \text{ kips x} \left(\frac{4.448\ 22\ \text{kN}}{1\ \text{kip}}\right) = 1512\ \text{kN}$$

Case B: Axial load + flexure:

$$P_u = 300\ \text{kips x} \left(\frac{4.448\ 22\ \text{kN}}{1\ \text{kip}}\right) = 1334\ \text{kN}$$

$$M_{ux} = 40\ \text{ft-kips x} \left(\frac{1.355\ 82\ \text{kN}\cdot\text{m}}{1\ \text{ft-kip}}\right) = 54\ \text{kN}\cdot\text{m}$$

Solution:

Case A: $P_u = 1512$ kN, $M_u = 0$

$$\phi P_n = \phi(0.80)\ [0.85\ f'_c\ (A_g - A_{st}) + (A_{st})(f_y)]$$

(AASHTO 8.16.4.1.2b)

$Ag = 0.3$ m x 0.3 m $= 0.09$ m^2

$A_{st} = 4 \times 500 = 2000$ mm^2 $= 0.002$ m^2

$\phi P_n = 0.70 \times 0.80 \times [0.85 \times 24 \text{ MPa} \times (0.09 \text{ m}^2 - 0.002 \text{ m}^2)$

$+ 400 \text{ MPa} \times 0.002 \text{ m}^2] = 1.453 \text{ MN} = 1453 \text{ kN} < P_u = 1512 \text{ kN}$

∴ not O.K.

∴ Either increase the column dimensions or steel area.

Use Column 0.35 m x 0.30 m

$\phi P_n = 0.70 \times 0.80 \times [0.85 \times 24 \text{ Mpa} \times (0.35\text{m} \times 0.3\text{m} - 0.002 \text{ m}^2) + 400$

$\text{MPa} \times 0.002 \text{ m}^2] = 1.625 \text{ MN} = 1625 \text{ kN} > P_u = 1512 \text{ kN}$

∴O.K.

Case B: $P_u = 1334$ kN, $M_u = 54$ kN·m

$\phi P_N = 1453$ kN (from Case A).

Check if $\left(\dfrac{A_s - A_s'}{bd}\right) \geq 0.85\, \beta_1 \left(\dfrac{f_c'\, d'}{f_y\, d}\right)\left(\dfrac{599.843}{599.843 - f_y}\right)$

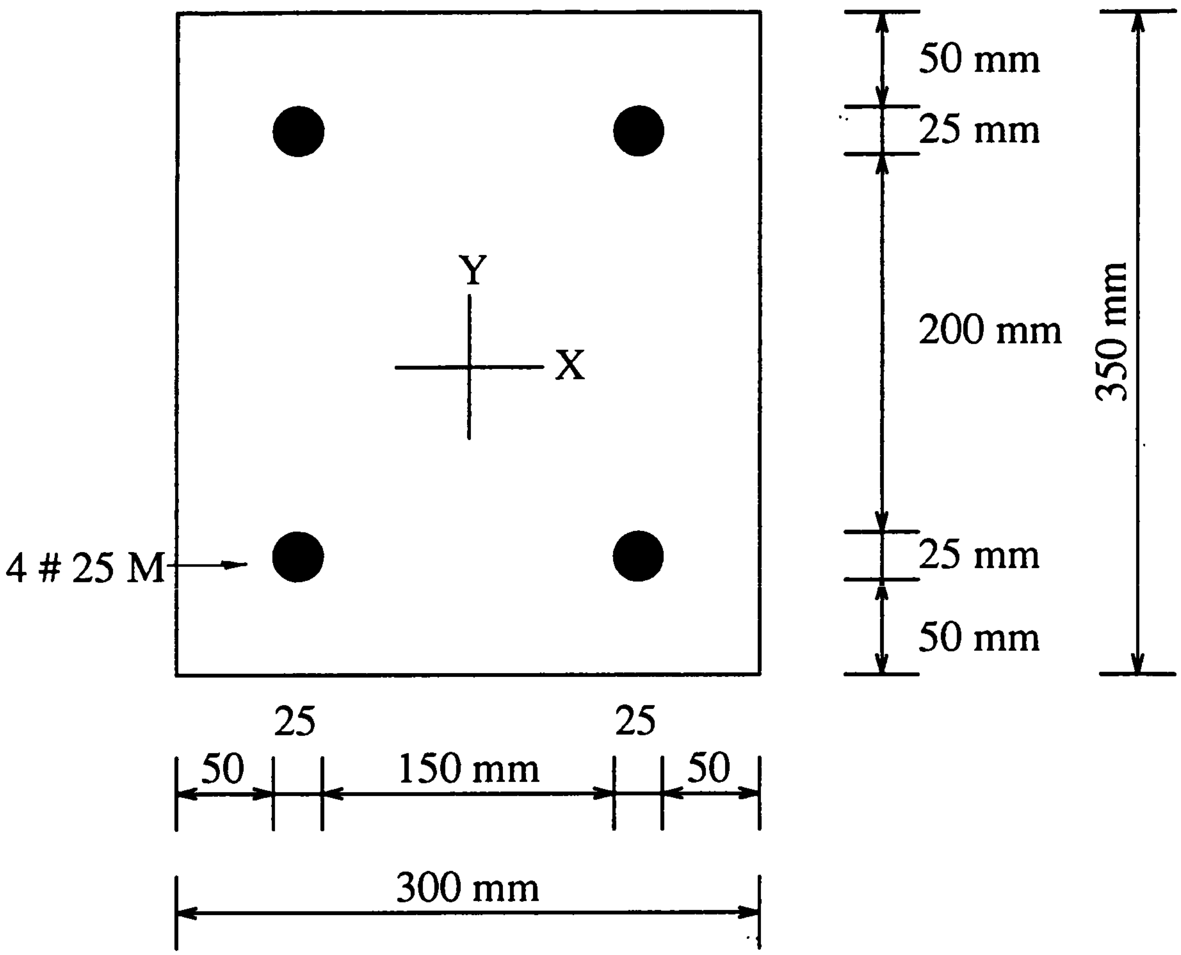
(AASHTO 8.16.3.4.1 eq. 8-24)

$A_s = A_s' = 2 \times 500 \times 10^{-6} \text{ m}^2 = 0.001 \text{ m}^2$

$\left(\dfrac{A_s - A_s'}{bd}\right) = 0,$

which is less than that required by (AASTHO Eq. 8-24)

∴ Use Eq. 8-16 for M_N



COLUMN CROSS-SECTION
(MODIFIED)

	U.S. Customary	Metric	Difference %
A_g A_{st} f_y f'_c	144 in^2 = 92 903 mm^2 3.16 in^2 = 2039 mm^2 60 ksi = 413.7 MPa 3.5 ksi = 24.1 MPa	90 000 mm^2 2000 mm^2 400 Mpa 24 MPa	-3.1% -1.9% -3.3% -0.5%
ϕP_n	340.8 kips = 1516 kN	1453 kN	-4.2%
	$\phi P_n = 1.002\ P_u$ O.K.	$\phi P_n = 0.961\ P_u$ not O.K.	

$\phi M_n = \phi[A_s f_y (d\text{-}a/2)]$ (AASHTO 8.16.3.2.1. Eq. 8-16)

$$a = \frac{A_s f_y}{0.85\ f_c' b} = \frac{0.001\ m^2 \times 400\ MPa}{0.85 \times 24\ MPa \times 0.30\ m}$$

$= 0.0654$ m $= 65.4$ mm

$$d = 350\ mm - 50\ mm\ (cover) - \frac{25\ mm}{2} = 287.5\ mm$$

$$d' = 50\ mm + \frac{25\ mm}{2} = 62.5\ mm$$

$$\phi M_n = 0.9\ [0.001\ m^2 \times 400\ MPa \times \left(0.2875\ m - \frac{0.0654\ m}{2}\right)$$

$= 0.092$ MN·m $= 92$ kN·m

$$\phi P_o = \phi P_n/0.8 = \frac{1625\ kN}{0.8} = 2031\ kN$$

$\phi P_b = \phi[0.85\ f'_c b\ a_b + A_s'\ f_s' - A_s f_y]$

$$a_b = \left(\frac{599.843}{599.843 + f_y}\right) \beta_1 d, \qquad \beta_1 = 0.85$$

$$a_b = \left(\frac{599.843}{599.843 + 400}\right) \times 0.85 \times 287.5\ mm = 146.6\ mm$$

(AASHTO 8.16.4.2.3. Eq. 8-34) (AASHTO App. E)

$$f_s' = 599.843 \left[1 - \left(\frac{d'}{d}\right) \left(\frac{599.843 + f_y}{599.843}\right)\right]$$

AASHTO FORMULA NOT AVAILABLE IN APPENDIX E

$$f'_s \text{ (in psi)} = 87{,}000 \left[1 - \left(\frac{d' \text{ (in inch)}}{d \text{ (in inch)}}\right)\left(\frac{87{,}000 + f_y \text{ (in psi)}}{87{,}000}\right)\right]$$

1 psi = 6.894 76 kPa = 6.894 76 x 10^{-3} MPa
1 MPa = 145.038 psi

f_y (measured in psi) = 145.038 f_y (measured in MPa)

1 in. = 25.4 mm

$$d \text{ (measured in inch)} = \frac{d \text{ (measured in mm)}}{25.4}$$

$$\therefore 145.038\, f'_s \text{(in MPa)} = 87{,}000 \times$$

$$\left[1 - \left(\frac{d' \text{ (in mm)}/25.4}{d \text{ (in mm)}/25.4}\right)\left(\frac{87{,}000 + 145.038\, f_y \text{ (in MPa)}}{87{,}000}\right)\right]$$

$$f'_s = \frac{87{,}000}{145.038}\left[1 - \left(\frac{d'}{d}\right)\left(\frac{87{,}000/145.038 + f_y}{87{,}000/145.038}\right)\right]$$

$$= 599.843\left[1 - \left(\frac{d'}{d}\right)\left(\frac{599.843 + f_y}{599.843}\right)\right]$$

where f_y, f'_s are in MPa
d, d' are in mm

(AASHTO 8.16.4.2.3. Eq. 8-35)

$$= 599.843 \left[1 - \left(\frac{62.5 \text{ mm}}{287.5 \text{ mm}}\right)\left(\frac{599.843 + 400 \text{ MPa}}{599.843}\right)\right]$$

$= 382 \text{ MPa} < 400 \text{ MPa} = f_y$

$\phi P_b = 0.7$ [0.85 x 24 MPa x 0.30 m x 0.1466 m + 0.001 m^2

x 382 MPa - 0.001 m^2 x 400 MPa] = 0.615 MN = 615 kN

$\phi M_b = \phi[0.85 f'_c b a_b (d - d'' - a_b/2) + A'_s f'_s (d - d' - d'') + A_s f_y d'']$

$$d'' = 175 - 50 - \frac{25}{2} = 112.5 \text{ mm}$$

$\phi M_b = 0.7$ [0.85 x 24 MPa x 0.30 m x 0.1466 m x

$\left(0.2875 \text{ m} - 0.1125 \text{ m} - \frac{0.1466 \text{ m}}{2}\right)$ + 0.001 m^2 x 382 MPa x

(0.2875 m - 0.0625 m - 0.1125 m) + 0.001m^2 x 400 MPa x 0.1125 m] =

0.1254 MN·m = 125.4 kN·m

First Trial

Column: 300 mm x 300 mm

General Information:
====================
File Name: A:\PCA1M.COL
Project:
Column:
Engineer:

Code: ACI 318-89
Units: SI Metric
Date: 02/12/95 Time: 15:46:30

Run Option: Investigation
Run Axis: X-axis

Short (nonslender) column
Column Type: Structural

Material Properties:
====================
f'c = 24 MPa
Ec = 24768 MPa
fc = 20.4 MPa
eu = 0.003 mm/mm
Stress Profile: Block

fy = 400 MPa
Es = 199955 MPa
erup = 0 mm/mm

Beta1 = 0.85

Geometry:
=========
Rectangular: Width = 300 mm   Depth = 300 mm

Gross section area, Ag = 90000 mm^2
Ix = 6.75e+008 mm^4
Iy = 6.75e+008 mm^4

Xo = 0 mm
Yo = 0 mm

Reinforcement:
==============
Rebar Database: ASTM

Size	Diam	Area	Size	Diam	Area	Size	Diam	Area
10	11	100	15	16	200	20	20	300
25	25	500	30	30	700	35	36	1000
45	44	1500	55	56	2500			

Confinement: Tied; phi(c) = 0.7, phi(b) = 0.9, a = 0.8
N-10 ties with N-30 bars, N-10 with larger bars.

Layout: Rectangular
Pattern: All Sides Equal [Cover to longitudinal reinforcement]

Total steel area, As = 2000 mm^2 at 2.22%

4N-25 Cover = 50 mm

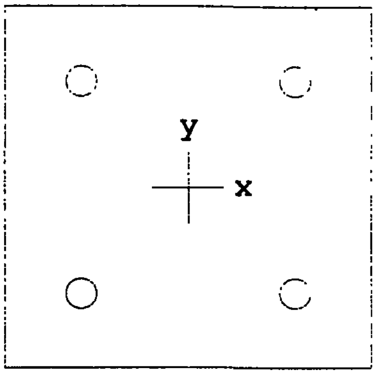
Pt.	Applied Loads P (kN)	Applied Loads Mx (kN-m)	Computed Strength P (kN)	Computed Strength Mx (kN-m)	Computed/Applied Ray length
1	1512	0	1453	0	0.961
2	1335	54	1297	54	0.971

< 1.0

∴ **not OK**

Program completed as requested!

PCACOL V2.30



300 x 300 mm

f'c = 24 MPa

fy = 400 MPa

Confinement: Tied

clr cover = 50 mm

spacing = 150 mm

4 N-25 at 2.22%

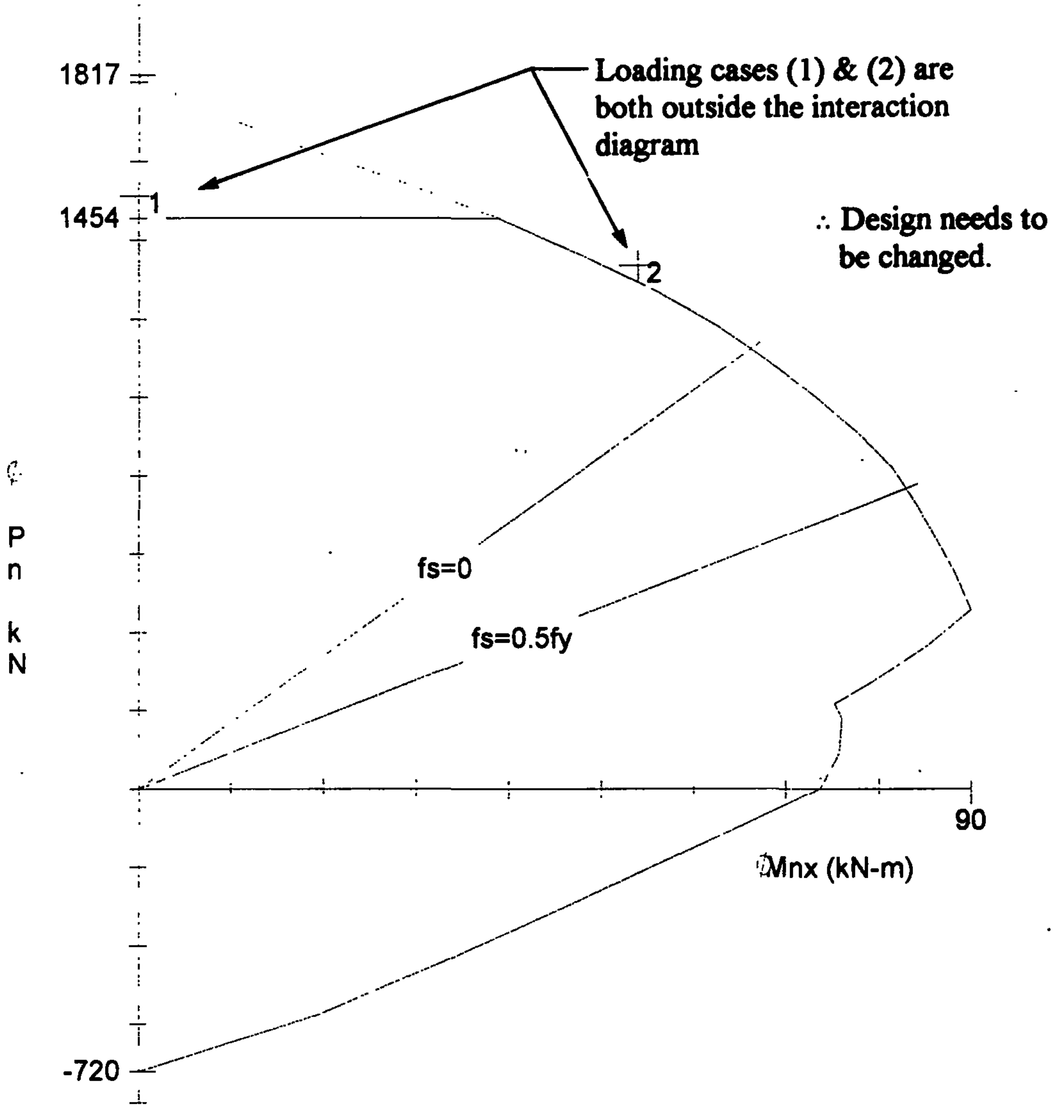
As = 2000 mm^2

Ix = 6.750e+008 mm^4

Iy = 6.750e+008 mm^4

Xo = 0 mm

Yo = 0 mm



Licensed To: Iowa State University, Ames, Iowa

File name: A:\PCA1M.COL

Project:

Column Id:

Engineer:

Date: 02/12/95 Time: 15:46:30

Code: ACI 318-89

Units: Metric

X-axis slenderness is not considered.

Material Properties:

Ec = 24768 MPa eu = 0.003 mm/mm

fc = 20.40 MPa Es = 199955 MPa

Beta1 = 0.85

Stress Profile: Block

phi(c) = 0.70, phi(b) = 0.90

New design (2nd trial)

Column: 300 mm x 350 mm


```
General Information:
====================
  File Name:  A:\PCA2M.COL
  Project:                                        Code: ACI 318-89
  Column:                                         Units: SI Metric
  Engineer:                                       Date: 02/12/95  Time: 15:46:30

  Run Option: Investigation                       Short (nonslender) column
  Run Axis:   X-axis                              Column Type: Structural

Material Properties:
====================
  f'c    = 24 MPa                                      fy    = 400 MPa
  Ec     = 24768 MPa                                   Es    = 199955 MPa
  fc     = 20.4 MPa                                    erup  = 0 mm/mm
  eu     = 0.003 mm/mm
  Stress Profile: Block                                Beta1 = 0.85

Geometry:
=========
  Rectangular: Width = 300 mm                          Depth = 350 mm  *

  Gross section area, Ag =  105000 mm^2
  Ix =  1.07188e+009 mm^4                              Xo =  0 mm
  Iy =  7.875e+008 mm^4                                Yo =  0 mm

Reinforcement:
==============
  Rebar Database: ASTM
```

Size	Diam	Area	Size	Diam	Area	Size	Diam	Area
10	11	100	15	16	200	20	20	300
25	25	500	30	30	700	35	36	1000
45	44	1500	55	56	2500			

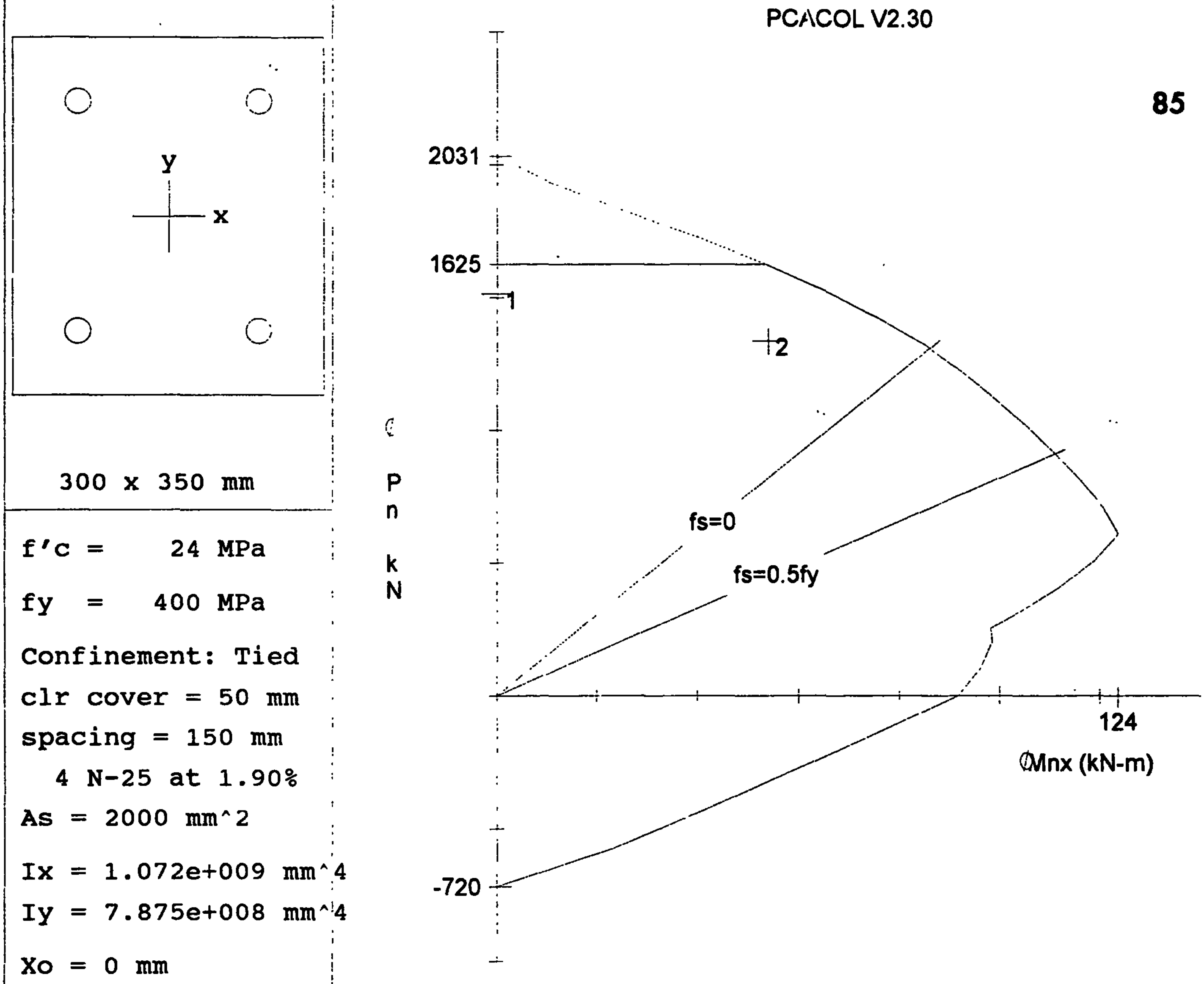
```
  Confinement: Tied; phi(c) = 0.7,  phi(b) = 0.9,  a = 0.8
  N-10 ties with N-30 bars, N-10 with larger bars.

  Layout: Rectangular
  Pattern: All Sides Equal  [Cover to longitudinal reinforcement]

  Total steel area, As = 2000 mm^2 at 1.90%

  4N-25   Cover = 50 mm
```

Pt.	Applied Loads P (kN)	Applied Loads Mx (kN-m)	Computed Strength P (kN)	Computed Strength Mx (kN-m)	Computed/ Applied Ray length
1	1512	0	1625	0	1.075
2	1335	54	1540	64	1.153

> 1.0

∴ O.K.

Program completed as requested!



f'c = 24 MPa

fy = 400 MPa

Confinement: Tied

clr cover = 50 mm

spacing = 150 mm

4 N-25 at 1.90%

As = 2000 mm^2

Ix = 1.072e+009 mm^4

Iy = 7.875e+008 mm^4

Xo = 0 mm

Yo = 0 mm

File name: A:\PCA2M.COL

Project:

Column Id:

Engineer:

Date: 02/12/95   Time: 15:46:30

Code: ACI 318-89

Units: Metric

Material Properties:

Ec = 24768 MPa   eu = 0.003 mm/mm

fc = 20.40 MPa   Es = 199955 MPa

Beta1 = 0.85

Stress Profile: Block

phi(c) = 0.70, phi(b) = 0.90

X-axis slenderness is not considered.

End of Problem 3

Problem 4: U.S. Customary: Concrete Footing on Piles

Required:

Check footing on piles for shear and moment.

Given:

Piles: HP 10 x 42, each 37 T

Equivalent square column 2.66 ft x 2.66 ft

All bars Grade 60

Reinforcement: 18 - #7 bars

Soil cover = 1.577 ft

Concrete density = 150 pcf

Soil density = 120 pcf

Solution:

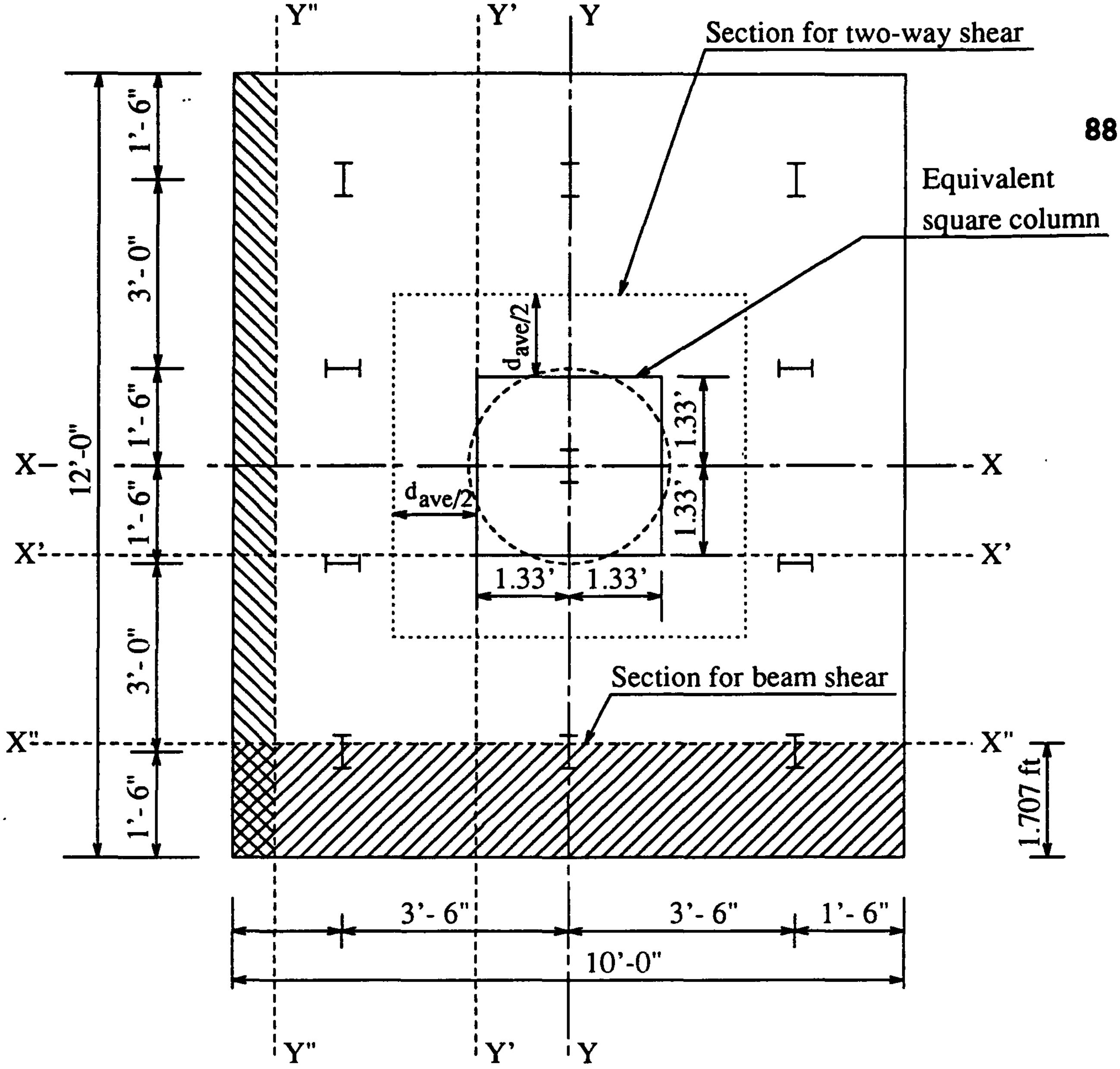
Beam Shear:

Check shear on Sec. X"-X" @ a distance (d) from the face of the equivalent square column

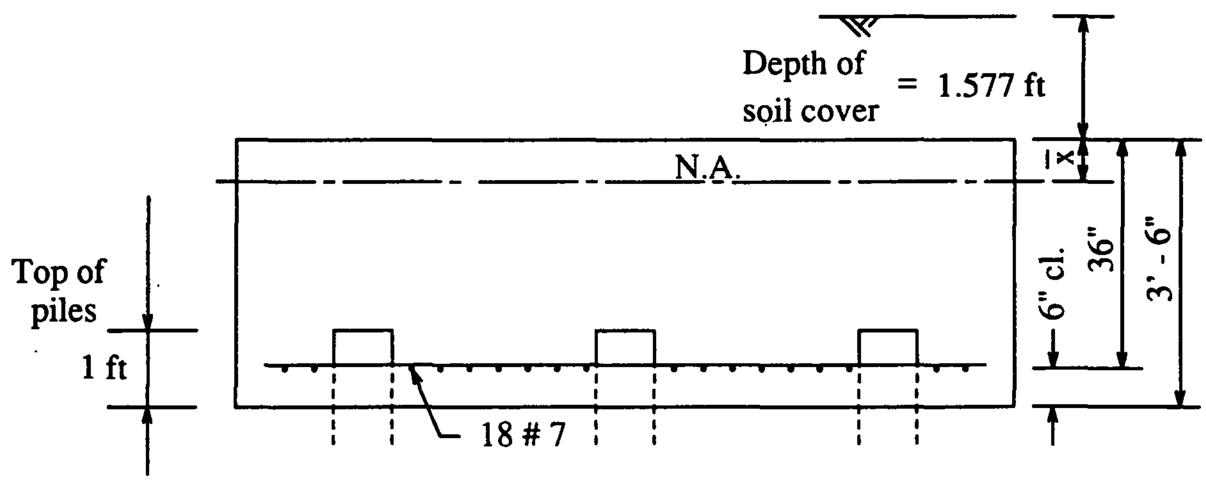
$$d_{x\text{-}x} = 42\text{ in.} - 6\text{ in.} - \frac{0.875\text{ in.}}{2} = 35.56\text{ in.}$$

$$d_{ave} = 42\text{ in.} - 6\text{ in.} - 0.875\text{ in.} = 35.13\text{ in.}$$

$$\text{Total pile load} = 3 \times 37\text{ T} \times \left(\frac{2\text{ kips}}{1\text{ T}}\right) = 222\text{ kips}$$



PLAN



ELEVATION

According to AASHTO 4.4.11.3.2, the fraction of the pile loads to be considered on Sec X"-X" is to be computed as follows:

d_p = depth of H-pile section = 9.75 in.
$d_p/2$ = 4.875 in.

Distance between center of H-piles and Sec X"-X" =

$$1.707 \text{ ft} \times \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) - 18 \text{ in.} = 2.48 \text{ in.}$$

$$\text{Load fraction to be considered} = \frac{(1.00) \times (2.48 + 4.875) \text{ in.}}{9.75 \text{ in.}}$$

= 0.754

V = 0.754 x 220 k - 1.707 ft x 10 ft x

(3.5 ft x 0.15 k/ft^3 + 1.577 ft x 0.12 k/ft^3) = 153.7 kips

$$v = \frac{V}{b_o d} = \frac{153.7 \text{ k}}{10 \text{ ft} \times \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) \times 35.56 \text{ in.}}$$

$$= 0.036 \text{ ksi} < 0.9\sqrt{f_c'} = 0.9\sqrt{3500 \text{ psi}} = 53 \text{ psi} = 0.053 \text{ ksi}$$

(AASHTO 8.15.5.6.4c) ∴ O.K.

<u>Beam Shear about Y"-Y":</u>

Similar procedure

Two-Way Shear:

Shear plane perimeter $= b_o = \left(\frac{35.13 \text{ in.}}{2} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right) + 1.33 \text{ ft} \right) \times 2 \times 4$

$= 22.35$ ft

Area outside shear plane $= (12 \times 10)\ \text{ft}^2 - (2 \times 2.79\ \text{ft})^2 = 88.86\ \text{ft}^2$

$V = 10 \times 74\ \text{k} - 88.86\ \text{ft}^2 \times (3.5\ \text{ft} \times 0.15\ \text{k/ft}^3 + 1.577\ \text{ft} \times 0.12\ \text{k/ft}^3)$

$= 676.5$ k

$$v = \frac{V}{b_o d} = \frac{676.5\ \text{k}}{22.35\ \text{ft} \times \left(\frac{12\ \text{in.}}{1\ \text{ft}} \right) \times 2.927\ \text{ft} \times \left(\frac{12\ \text{in.}}{1\ \text{ft}} \right)}$$

$= 0.072$ ksi

$< 1.8 \sqrt{f_c'} = 1.8 \sqrt{3500\ \text{psi}} = 106\ \text{psi} = 0.106\ \text{ksi}$

(AASHTO 8.15.6.3. Eq. 8-13) $\therefore$ O.K.

Flexural Check:

$M_{x'-x'} = 74\ \text{k}\ [2 \times 0.17\ \text{ft} + 3 \times 3.17\ \text{ft}]$

$- 10\ \text{ft} \times (3.5\ \text{ft} \times 0.15\ \text{k/ft}^3 + 1.577\ \text{ft} \times 0.12\ \text{k/ft}^3) \times \left(\frac{4.67^2\ \text{ft}^2}{2} \right) = 651.0\ \text{k-ft}$

Location of N.A.:

$$\frac{b\overline{x}^2}{2n} = A_s\ (d - \overline{x})$$

$A_s = 18$ - #7 bars

$= 18 \times 0.6\ \text{in}^2 = 10.8\ \text{in}^2$

$b = 10\ \text{ft} = 120$ in.

$$\therefore \frac{120 \text{ in. } (\bar{x})^2}{2(9)} = 10.8 \text{ in}^2 \, (35.56 \text{ in.} - \bar{x})$$

$$6.67 \, (\bar{x})^2 + 10.8 \, \bar{x} - 384.05 = 0$$

$$\bar{x} = \frac{-10.8 \pm \sqrt{(10.8)^2 - 4(6.67)(-384.05)}}{2(6.67)} = 6.82 \text{ in.}$$

$$I = A_s(d-\bar{x})^2 + 1/3 \, \frac{b \, (\bar{x})^3}{n}$$

$$= 10.8 \text{ in}^2 \, (35.56 \text{ in.} - 6.82 \text{ in.})^2 + \frac{120 \text{ in. x } (6.82 \text{ in.})^3}{3 \text{ x } 9}$$

$$= 10331 \text{ in}^4$$

<u>Check of Stresses:</u>

$$f_s = \frac{M \, c}{I} = \frac{651.0 \text{ k-ft x } \frac{12 \text{ in.}}{1 \text{ ft}} \text{ x } (35.56 \text{ in.} - 6.82 \text{ in.})}{10331 \text{ in}^4}$$

$= 21.73$ ksi < 24 ksi (allowable) $\therefore$ O.K.

$$f_c = \frac{651.0 \text{ k-ft x } \left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) \text{ x } 6.82 \text{ in.}}{9 \text{ x } 10331 \text{ in}^4}$$

$= 0.573$ ksi < 1.4 ksi (allowable) $\therefore$ O.K.

<u>Flexural check about Y'-Y'</u>

Similar procedure

<u>Problem 4: Metric:</u> Concrete Footing on Piles

<u>Required:</u>

Check footing on piles for shear and moment.

Given:

Piles: HP 250x62, each 330 kN

Equivalent square column 800 mm x 800 mm

All bars Grade 400

Reinforcement: 24 #20M bars (+ 3% difference in area)

Soil cover = 0.50 m

Concrete density = 24 kN/m^3

Soil density = 19 kN/m^3 (see Iowa DOT Handbook p. 10a)

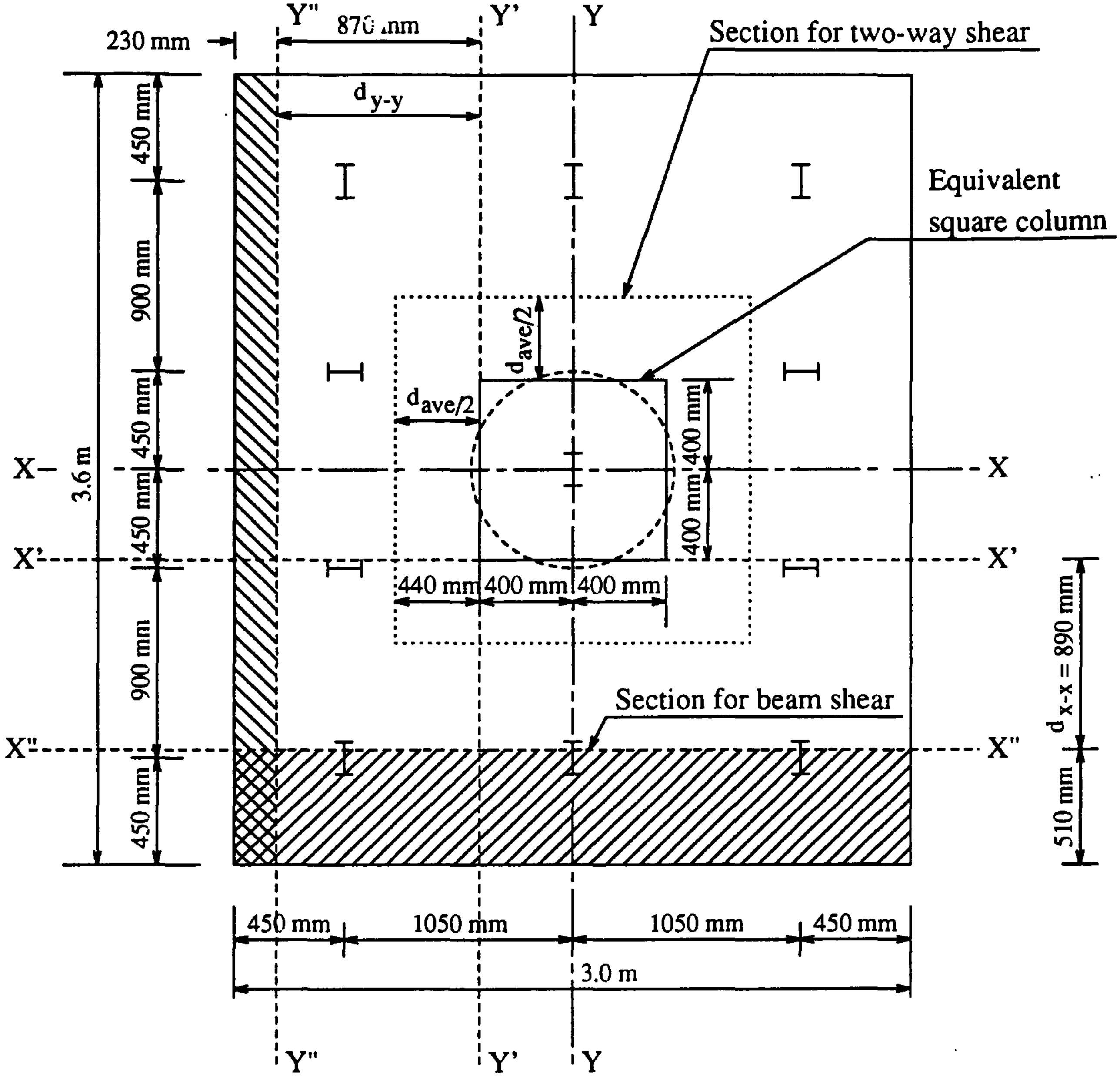
<u>Beam Shear:</u>

Check shear on Sec. X"-X" @ a distance (d) from the face of the column

$$d_{x\text{-}x} = 1050 \text{ mm} - 150 \text{ mm} - \frac{20 \text{ mm}}{2} = 890 \text{ mm}$$

$$d_{ave} = 1050 \text{ mm} - 150 \text{ mm} - 20 \text{ mm} = 880 \text{ mm}$$

Total pile load = 3 x 330 kN = 990 kN



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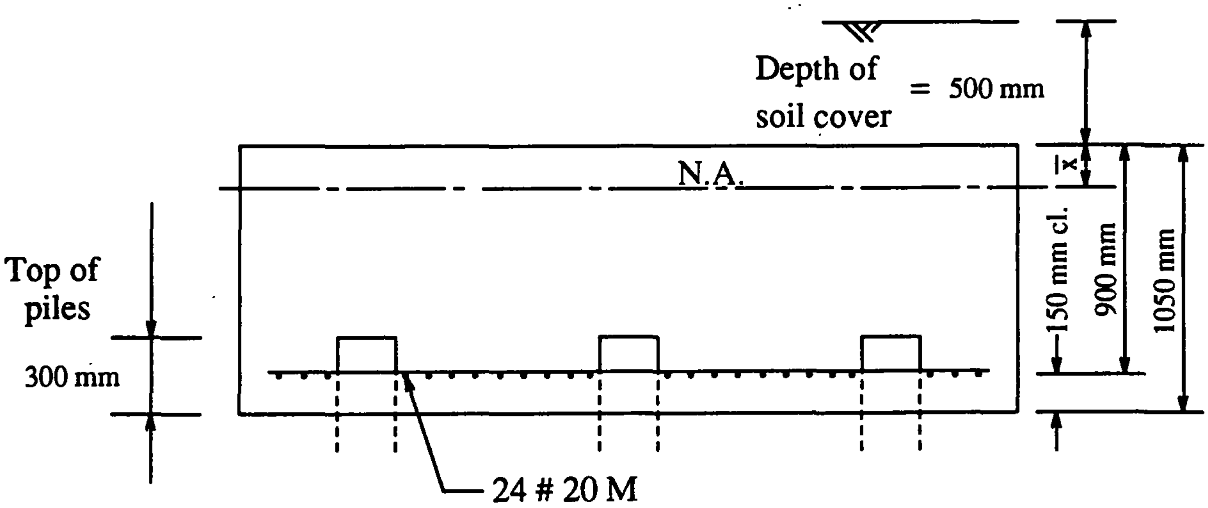

ELEVATION

According to AASHTO 4.4.11.3.2, the fraction of the pile loads to be considered on Sec X"-X" is to be computed as follows:

d_p = depth of H-pile section = 246 mm

$d_p/2$ = 123 mm

Distance between center of H-piles and Sec X"-X" = 510 - 450

= 60 mm

Load fraction to be considered = $\frac{(1.00)\ x\ (60+123)\ mm}{246\ mm}$ = 0.744

V = 0.744 x 990 kN - 0.51 m x 3.0 m x

$$\left(1.05\ m\ x\ 24\ \frac{kN}{m^3} + 0.5\ m\ x\ 19\ \frac{kN}{m^3}\right) = 683.5\ kN$$

$$v = \frac{V}{b_o d} = \frac{683.5\ kN}{3.0\ m\ x\ 0.89\ m}$$

= 256 kPa < $0.075\ \sqrt{f_c'} = 0.075\ \sqrt{24\ MPa} = 0.367\ MPa$=

367 kPa

(AASHTO 8.15.5.6.4c & AASHTO App. E)

Beam Shear about Y"-Y":

Similar procedure

IF YOU CANNOT REMEMBER WHAT UNITS TO USE

Allowable shear stress = $0.075\sqrt{f_c'}$ (SI) $0.9\sqrt{f_c'}$ (U.S. Customary)

where f'_c is measured in: ? psi

<u>1st method: try an example:</u>

$$v_{all} = 0.9\sqrt{3500\ \text{psi}}$$

$$= 53\ \text{psi} \times \left(\frac{6.894\ 76\ \text{kPa}}{1\ \text{psi}}\right)$$

$$= \underline{365\ \text{kPa}}$$

Pa: $0.075\sqrt{24\text{x}10^6\ \text{Pa}} = 367\ \text{Pa}$

$= 0.367$ kPa (incorrect)

kPa: $0.075\sqrt{24\ 000\ \text{kPa}} = 1800$ kPa (incorrect)

MPa: $0.075\sqrt{24\ \text{MPa}} = 0.367$ MPa = 367 kPa (correct)

$\therefore$ <u>Use MPa</u>

<u>2nd Method:</u> Convert the equation.

v_{all} (in psi) = $0.9\sqrt{f_c' \text{ (in psi)}}$

1 ksi = 6.894 76 MPa

1 MPa = 0.145 ksi = 145.0 psi

$145\, v_{all} \text{ (in MPa)} = 0.9\sqrt{145\ f_c' \text{ (in MPa)}}$

$$v_{all} \text{ (in MPa)} = \frac{0.9\sqrt{145}}{145}\sqrt{f_c' \text{ (in MPa)}}$$

$$= 0.075\sqrt{f_c' \text{ (in MPa)}}$$

∴ Use MPa in this equation.

Two-Way Shear:

Shear plane perimeter = b_o = (400 + 440) mm x 2 x 4

= 6720 mm = 6.72 m

Area outside shear plane = (3.6 x 3.0) m^2 - $(1.68)^2$ m^2 = 7.98 m^2

$$V = 10 \times 330 \text{ kN} - 7.98 \text{ m}^2 \times \left(1.05\text{m} \times 24 \frac{\text{kN}}{\text{m}^3} + 0.5\text{m} \times 19 \frac{\text{kN}}{\text{m}^3}\right) = 3023 \text{ kN}$$

$$v = \frac{V}{b_o d}$$

$$= \frac{3023 \text{ kN}}{6.72 \text{ m} \times 0.88 \text{ m}} = 511 \text{ kPa}$$

$$< 0.149 \sqrt{f_c'} = 0.149 \sqrt{24 \text{ MPa}} = 0.730 \text{ MPa} = 730 \text{ kPa}$$

(AASHTO 8.1.5.6.3. Eq. 8.13)

O.K.

Flexural Check:

$M_{x'-x'}$ = 330 kN [2 x 0.05 m + 3 x 0.95 m]

$$- 3.0 \text{ m} \left(1.05 \text{ m} \times 24 \frac{\text{kN}}{\text{m}^3} + 0.5 \text{ m} \times 19 \frac{\text{kN}}{\text{m}^3}\right) \times \frac{(1.4 \text{ m})^2}{2}$$

$= 871.5 \text{ kN} \cdot \text{m}$

Location of N.A.:

$$\frac{b\overline{x}^2}{2n} = A_s\ (d-\overline{x})$$

$$A_s = 24\ \#20M\ \text{bars}$$

$$= 24 \times 300\ mm^2$$

$$= 7200\ mm^2$$

$$b = 3.0\ m = 3000\ mm$$

$$\therefore \frac{3000\ mm\ (\overline{x})^2}{2(9)} = 7200\ mm^2\ (890\ mm - \overline{x})$$

$$166.67\ (\overline{x})^2 + 7200\ \overline{x} - 6\ 408\ 000 = 0$$

$$\overline{x} = \frac{-7200 \pm \sqrt{(7200)^2 - 4(166.67)(-6\ 408\ 000)}}{2(166.67)} = 176\ mm$$

$$I = A_s(d-\overline{x})^2 + 1/3\ \frac{b\ (\overline{x})^3}{n}$$

$$= 7200\ mm^2\ (890 - 176)^2\ mm^2 + \frac{3000\ mm \times (176)^3\ mm^3}{3 \times 9}\ mm^3$$

$$= 4276 \times 10^6\ mm^4$$

Check of Stresses:

$$f_s = \frac{M\ c}{I} = \frac{871.5\ \text{kN·m} \times (0.89-0.176)\ \text{m}}{4276 \times 10^{-6}\ \text{m}^4}$$

$= 145\ 522$ kPa = 146 MPa < 165 MPa (allow.)

∴ O.K.

$$f_c = \frac{871.5\ \text{kN·m} \times 0.176\ \text{m}}{9 \times 4276 \times 10^{-6}\ \text{m}^4}$$

$= 3986$ kPa = 4.00 MPa < 9.65 MPa (allow.)

Flexural check about Y'-Y' is similar

	U.S. Customary	Metric	% Difference
Beam Shear			
Load (V)	153.7 k = 683.7 kN	683.5 kN	0.0%
Stress (v)	36 psi = 248 kPa	256 kPa	-3.1%
Two-way Shear			
Load (V)	676.5 k = 3009 kN	3023 kN	-0.5%
Stress (v)	72 psi = 496 kPa	511 kPa	-2.9%
Flexure			
Moment (M)	651.0 k-ft = 882.6 kN·m	871.5 kN·m	1.3%
Stress (f_s)	21.73 ksi = 150 MPa	146 MPa	2.7%
Stress (f_c)	0.573 ksi = 3.95 MPa	4.0 MPa	-1.2%

<u>Basic Conversions:</u>

Mass: 1 lb mass = 0.4536 kg
Length: 1 ft = 0.3048 m
Time: 1 second = 1 second
$g = 9.806\ m/s^2$

All other conversion factors can be derived from these.

<u>Example:</u>

$$1\ \text{ksi} = \frac{1\ \text{kip}}{\text{in}^2} = 10^3\ \frac{\text{lb}}{\text{in}^2}$$

$$= 10^3\ \frac{\text{lb}}{\text{in}^2} \times \left(\frac{0.4536\ \text{kg}}{1\ \text{lb}}\right) \times \left(\frac{1\ \text{in.}}{25.4\ \text{mm}}\right)^2$$

$$= 0.7031\ \frac{\text{kg}}{\text{mm}^2}$$

$$= 703\ 100\ \frac{\text{kg}}{\text{m}^2} \times 9.806\ \frac{\text{m}}{\text{sec}^2}\ \text{(to change from mass to force)}$$

$$= 6.895 \times 10^6\ \frac{\text{N}}{\text{m}^2} = 6.895\ \text{MPa}$$

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