

Hydraulics of Box Culverts

By

DONALD E. METZLER

and

HUNTER ROUSE

Prepared by the Iowa Institute of Hydraulic Research
in co-operation with the Iowa State Highway Commission and the
U.S. Department of Commerce, Bureau of Public Roads

State University of Iowa

Studies in Engineering

Bulletin 38

1959

TABLE OF CONTENTS

INTRODUCTION	1
ELEMENTARY PRINCIPLES OF HYDRAULICS	2
Equations of Continuity and Energy	2
Flow Through Orifices and Nozzles	4
Pipe Resistance	6
Flow over Weirs	8
Uniform and Critical Open-Channel Flow	9
Non-Uniform Open-Channel Flow	11
Hydraulic Similarity	13
Methods of Application	14
CULVERTS WITH BARRELS OF MILD SLOPE	15
Flow with Inlet Unsubmerged	15
Flow with Partially Effective Inlet Submergence	16
Flow with Fully Effective Inlet Submergence	18
CULVERTS WITH BARRELS OF STEEP SLOPE	20
Flow with Inlet Unsubmerged	20
Flow with Partially Effective Inlet Submergence	21
Flow with Fully Effective Inlet Submergence	24
CULVERTS WITH BARRELS OF DISCONTINUOUS SLOPE	25
Design Characteristics	25
Characteristics of Operation	26
CONCLUSION	27
ACKNOWLEDGMENTS	30
REFERENCES	30
SYMBOLS	30

HYDRAULICS OF BOX CULVERTS

INTRODUCTION

Three decades ago Yarnell, Nagler, and Woodward published as Bulletin 1 of this series [1] a description of their experiments on "The Flow of Water through Culverts." Based upon more than 3000 large-scale tests on horizontal pipe and box culverts of various materials and shapes, the report dealt in detail with factors that were found to influence the culvert discharge capacity and with practical means of obtaining the maximum capacity at minimum cost. So great was the demand for this bulletin that it was out of print within a decade.

In the intervening years, not only has the information presented in this initial work become too limited in scope for present-day requirements, but methods of experimentation, interpretation, and application of results have been considerably advanced. Additional culvert studies have hence been conducted in many laboratories, sponsored in large part by State and Federal organizations. Some of these have dealt with the literature on the subject [2], some with hydrologic requirements, and many with detailed aspects of culvert performance. The most recent publication of research results is the culvert design manual of the Bureau of Public Roads [3], which embodies a reduction of experimental information to a series of rules for obtaining safely and economically the required flow capacity.

For a large organization that is able to standardize its product, preparation of such a design manual can effect a vast saving in time and cost, as well as a great reduction of failures resulting from the choice of improper flow characteristics. Its use by smaller organizations may well ensure the same advantages. On the other hand, conditions are often encountered that do not duplicate those for which standard design rules apply, and the designer must then depend upon his own knowledge of hydraulics to meet the situation. Even in the routine use of the design manual, moreover, it would be desirable to check the indicated results against elementary hydraulic principles. However, not all engineers have been able to retain through practice their university grounding in hydraulics, and many engineering draftsmen actually engaged in design have never had such training.

In order to obtain material for an up-to-date publication on culverts that would effectively replace Bulletin 1, a co-operative investigation was organized in 1952 between the Iowa Highway Research Board, the Bureau of Public Roads of the U.S. Department of Commerce, and the Iowa Institute

of Hydraulic Research. Emphasis given elsewhere to pipe culverts led the Iowa experiments to be restricted to culverts of the box type. Similarly, the decision by the Bureau of Public Roads to publish a manual of culvert design logically guided the present effort in the direction of an explanatory text on culvert hydraulics, in which the "why" rather than the "what," "how," or "how much" would be stressed.

As with any engineering structure, however complex its design may be, one can control the capacity of a culvert far more accurately than one can estimate the capacity that it will eventually need to have. Hydraulic measurements can be made to within 1% in the laboratory and perhaps 5% in the field, and a scatter of 10% either side of an empirical model-prototype curve is considered large for most types of flow prediction. Hydrologic estimates, on the other hand, involve uncertainties of another order of magnitude, and errors of as much as 200% in required capacity must sometimes be expected. For this reason it is pointless to seek undue precision in culvert design. Rather, the designer should understand from a relative point of view the advantages or disadvantages of various construction details, so that not only will the estimated capacity be realized at minimum expenditure but reasonably satisfactory performance will be assured even under overload conditions.

To accomplish this end, the following pages take the form of a refresher (or even beginning) course in culvert hydraulics, the experimental data being used to illustrate and substantiate the discussion rather than forming the principal exhibit as in the customary report. Even before speaking of culverts themselves, the writers have hence inserted basic material on the various flow phenomena involved in culvert operation, so that the subsequent treatment will not presume more than the reader already has in mind. In this connection simple use is made of the very convenient non-dimensional notation that typifies present-day hydraulics. Although the various graphs of experimental evidence are based entirely on box-culvert characteristics, the generality of the approach makes much of what is said applicable at least qualitatively to pipe culverts as well.

ELEMENTARY PRINCIPLES OF HYDRAULICS

Equations of Continuity and Energy.

The simplest type of hydraulic analysis is based upon the one-dimensional approximation of what are actually three-dimensional conditions. Herein the flow is considered to occur in a single gross filament such that average values of velocity, pressure, and elevation can be considered to represent the flow at any cross section. The principle of continuity for steady conditions thus states that the product of mean velocity V (in feet per second) and cross-sectional area A (in square feet) must indicate the

same rate of discharge Q (in cubic feet per second) at all successive cross sections:

$$V_1 A_1 = V_2 A_2 = Q \quad (1)$$

This relationship is an exact one, but its one-dimensional aspects sometimes become rather far-fetched. That is, whereas in a uniform pipe or even a somewhat non-uniform stream the concept is easily visualized as such, flow at the juncture between two branches of a ditch and a culvert can become decidedly three-dimensional. Nevertheless, the principle of continuity must still apply, in that the sum of the contributions of the two branches must equal the discharge through the culvert (plus or minus local pondage [4] if the flow is unsteady).

The principle of energy (or Bernoulli equation) for steady one-dimensional conditions states that the total head H consisting of the sum of velocity head $V^2/2g$, pressure head p/γ , and elevation z , at any section must equal that at any successive section plus the intervening losses H_L :

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + H_L \quad (2)$$

Herein g is the acceleration of gravity (in feet per second squared), p is the pressure (in pounds per square foot), and γ the unit weight (in pounds per cubic foot). Each term thus has the dimension of length, or head.

The energy relationship is particularly significant when plotted on a profile diagram of the flow system (see Fig. 1), for z represents the elevation of any point under consideration, p/γ is the height above that point to which water would rise in a glass manometer column connected to a longitudinal tube with a side opening at the point, and $V^2/2g$ is the additional distance that water would rise if the manometer column were connected instead to a tube with open end pointed upstream. A line having the elevation H above the same datum as z would thus show the sum of the three heads, and its fall with distance along the flow would show the magnitude of the intervening losses.

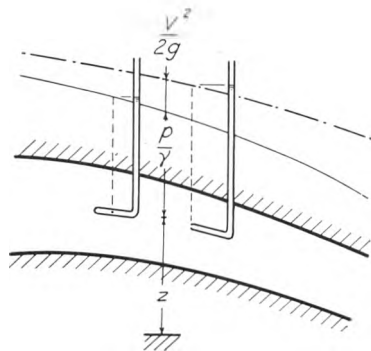


Fig. 1. GRAPHICAL REPRESENTATION OF VELOCITY HEAD, PRESSURE HEAD, AND ELEVATION.

As in the case of the continuity principle, the energy principle for one-dimensional flow represents a very great (and very convenient) simplification. Even if the flow is uniform, the velocity will generally vary from

point to point across the section. It is well known, however, that the square of the mean does not equal the mean of the squares, so that use of V , already a mean, to compute the mean velocity head must introduce some error. Moreover, the derivation of the energy equation actually proceeds on the basis of the power flux or rate at which energy is carried past a section by the flow, $QV^2/2g = AV^3/2g$. As a result, a numerical correction factor α of the form

$$\alpha = \frac{1}{A} \int \left(\frac{v^2}{V^2} \right) dA \quad (3)$$

must be used as a multiplier of the velocity head $V^2/2g$ to compensate for the variation of the local velocity v across the section. If the velocity does not vary, this factor is unity. For normal turbulent flow in rough conduits α can be as high as 1.1. In zones of reverse flow with standing eddies, however, it can increase without limit.

Again for uniform flow, the pressure head is distributed hydrostatically over the cross section just as in still water; that is, the sum $p/\gamma + z$ equals h , a constant known as the piezometric head. This evidently corresponds to the level of water in the manometer column connected to a side opening in either a tube (as in Fig. 1) or the wall of the conduit. If the flow is not under pressure, as in a pipe, but has a free surface, h will represent the elevation of this surface. Unfortunately, if the flow is not uniform, the pressure distribution will not be hydrostatic. Extreme non-uniformity is illustrated by rapid curvature of the flow lines, and it is known that the pressure must always be higher at the outside of the curve and lower at the inside—as, for example, at a pipe bend. Needless to say, use of the Bernoulli equation for one-dimensional flow in zones where the flow is markedly two- or even three-dimensional can lead to considerable error. In this event it is customary to compare sections in essentially uniform zones upstream and downstream from the non-uniformity under consideration—for instance, some distance either side of a partially open valve or gate.

Flow Through Orifices and Nozzles

Orifice flow is probably the simplest to analyze, which no doubt accounts for the fact that it was historically the first to be understood. The rate of flow through an orifice of diameter d under a differential head h is usually written in terms of a discharge coefficient as follows:

$$Q = C_d \frac{\pi d^2}{4} \sqrt{2gh} \quad (4)$$

The coefficient was once accepted as a somewhat mysterious correction factor introduced to compensate for one's lack of exact knowledge of the phenomenon. Actually, such a coefficient is simply a symbol for the non-

dimensional ratio of the dimensional terms involved in the equation. Thus,

$$\frac{Q}{\frac{1}{4} \pi d^2 \sqrt{2gh}} = C_d \quad (5)$$

It is specifically this dimensional equivalence of numerator and denominator which prescribes that—other things remaining the same— Q must vary as the $1/2$ power of h .

The actual magnitude of the discharge ratio, represented in shorthand form by the discharge coefficient, depends upon a series of geometric and dynamic characteristics of boundary and fluid. It will vary if the orifice is changed in form, if the approach is confined or obstructed, and if the orifice size or the head is very small or the fluid is very viscous. For the present, only geometric effects at large scale need be considered.

How the discharge ratio and the ratio of the areas of the jet cross section and the outlet (indicated by the contraction coefficient C_c) vary with the ratio of the outlet diameter to the diameter of the approach is shown in Fig. 2 for a sharp-edged orifice at the center of a normal end plate. The contraction ratio evidently changes from a minimum of about 0.6 to a maximum of unity as the discharge ratio ranges from the same minimum to a maximum of infinity (the only limit compatible with a zero end constriction and a zero pressure drop). For the limiting condition of a relatively large approach section, the identical discharge and contraction ratios vary as indicated in the schematic plot of Fig. 3. It is seen that what is known as a reentrant outlet

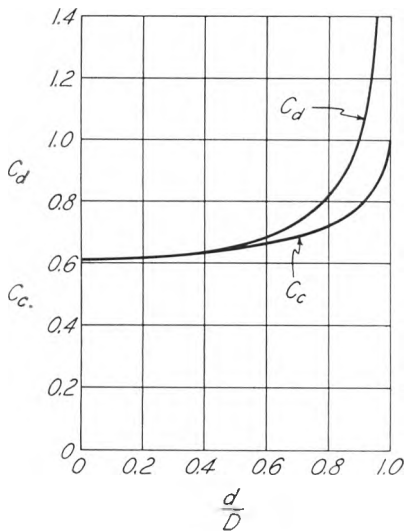


Fig. 2. VARIATION OF DISCHARGE AND CONTRACTION COEFFICIENTS WITH RATIO OF ORIFICE AND PIPE DIAMETERS.

produces a maximum reduction in jet cross-sectional area and hence a minimum discharge ratio, whereas the opposite form of outlet—practically a nozzle—yields no contraction and a discharge ratio of unity. Actually, relatively little rounding is necessary, for even the free jet from a normal sharp-edged orifice plate is only 20% smaller in diameter than the orifice. As a result, simply chamfering the inside edge of the orifice plate will produce a considerable increase in both contraction and discharge ratios.

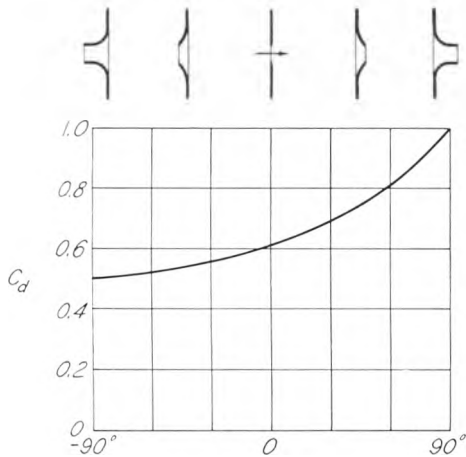


Fig. 3. VARIATION OF DISCHARGE COEFFICIENT WITH SHAPE OF OPENING.

orifice (in accordance with the Bernoulli equation) to the point at which vapor cavities form and disrupt the flow. On the other hand, even a tube of constant diameter used in conjunction with a sharp-edged orifice of the same diameter (Fig. 5) will increase the discharge coefficient to about 0.8, provided that the tube is long enough for the jet to expand and fill it beyond the contracted section. However, turbulence is generated at the zone of contact between the contracted jet and the surrounding eddy, and this leads to a loss of head H_L equal to about half the velocity head:

$$\frac{H_L}{V^2/2g} = C_i = 0.5 \quad (6)$$

Pipe Resistance

Increasing the length of the uniform tube completes the transition from orifice flow to pipe flow. If the head on the system is still measured to the centerline of the orifice, then the pipe can be used like a siphon to increase or decrease the discharge coefficient by lowering or raising the outlet end. If, on the contrary, h represents the differential head between inlet and

If rounding the orifice can raise the coefficient as high as unity, one wonders why it would not be possible to increase its magnitude still further by flaring the outlet. As a matter of fact, even if a well-rounded orifice is followed by a divergent tube (Fig. 4), the free jet can be made to expand only slightly beyond the final outlet. Based upon the area of the orifice itself, however, the only limit to the increase in discharge thus produced is the reduction of pressure at the

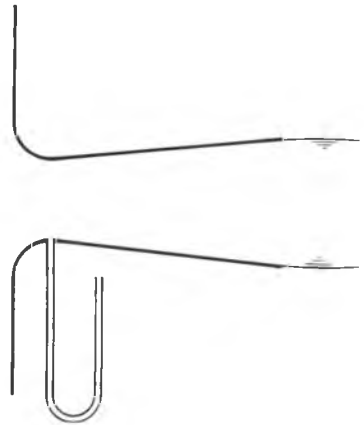


Fig. 4. EFFLUX FROM A DIVERGENT TUBE WITH ROUNDED INLET.

outlet, the sole effect of the pipe is to increase the resistance to flow as compared to that of a rounded outlet of the same diameter but negligible length. This resistance gives rise to a relative loss in head which, per diameter of pipe length, is indicated by a coefficient commonly given the symbol f :

$$\frac{H_L}{V^2/2g} \frac{D}{L} = f \quad (7)$$

The loss ratio represented by the coefficient f varies with the relative viscosity of the fluid, the relative roughness of the pipe surface, and the relative length (and shape) of the conduit. In fact, so complex is this variation, and so small is the influence of surface resistance in most culverts, that it will suffice simply to assume the coefficient to lie between 0.015 and 0.06, the former magnitude applying to the smooth and the latter to the rougher surfaces.

The end of such a pipe could conceivably be contracted or enlarged relative to its uniform intermediate diameter, with an accompanying reduction or increase in the relative discharge. Now, however, the effect will be less than before, because the outlet represents a proportionately smaller part of the system. If the outlet is submerged, not only must the effective differential head be measured between the two surfaces (see Fig. 6) but an outlet loss equal to the entire velocity head must be assumed to occur as the kinetic energy of the efflux stream is dissipated:

$$\frac{H_L}{V^2/2g} = C_o = 1 \quad (8)$$

The discharge for the whole system can be represented by the sum of the individual coefficients:

$$\frac{h}{Q^2/2gA^2} = \frac{h}{V^2/2g} = C_i + f \frac{L}{D} + C_o \quad (9)$$

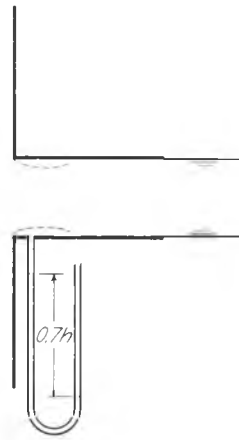


Fig. 5. EFFLUX FROM A UNIFORM TUBE WITH SHARP-EDGED INLET.

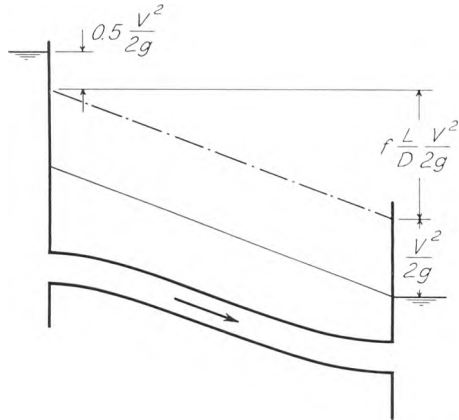


Fig. 6. GRAPHICAL REPRESENTATION OF INLET, PIPE, AND OUTLET LOSSES.

Flow over Weirs

Discharge over a weir (Fig. 7) differs from that through an orifice primarily in that the increase in differential head (measured with respect to the crest) results in an increase in both the velocity and the depth of the flow section. Therefore, the discharge ratio involves h to the $3/2$ power rather than the $1/2$ power:

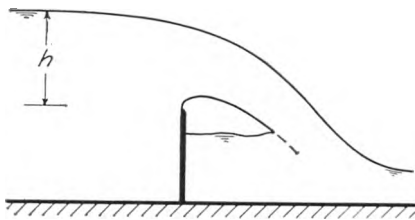


Fig. 7. PROFILE OF FLOW OVER A SHARP-CRESTED WEIR.

$$\frac{Q}{\sqrt{2g} L h^{3/2}} = C_d \quad (10)$$

Herein the length L is that of the weir crest. The ratio represented by the discharge coefficient of a weir, like that of a large orifice, varies primarily with the geometry of the overflow section. For a very wide, horizontal, sharp-crested weir of considerable height, it will have the magnitude of about 0.4. For a narrow vertical slot extending to the floor of the approach channel this ratio will also obtain. Smaller values will characterize weirs of rectangular proportions with both side and bottom contractions. Triangular, circular, and other shapes will require reformulation of the discharge coefficient, because the width as well as the depth of the flow section varies with the head.

A weir can readily be sloped or rounded in profile to produce changes in discharge coefficient similar to those of the orifice. The extreme case of rounding is that of the ogee spillway, the crest of which is in fact based in form upon that of the weir nappe. The normal spillway discharge coefficient is about 0.5, but this can be increased by accentuating the curvature of the crest. The counterpart of the short tube of uniform section is illustrated by what is called the broad-crested weir, the discharge over which is said to be at critical stage—i.e., a maximum for the given total head. Under conditions of uniform critical flow, the velocity head is equal to half the depth, so that

$$0.5 = \frac{V_c^2/2g}{d_c} = \frac{Q^2/2gL^2d_c^2}{d_c} = \frac{Q^2}{2gL^2(\frac{2}{3}h)^3}$$

and

$$\frac{Q}{\sqrt{2g} L h^{3/2}} = C_d = 0.27 \quad (11)$$

Although this situation is a fair approximation to such flow, in actuality uniform conditions are never fully realized. If the upstream edge is not rounded, separation will occur, and the flow will pass over the eddy (Fig. 8a) as though it were a low spillway, the depth downstream then being less than the critical and the discharge coefficient less than 0.27. If the upstream edge is rounded (Fig. 8b), flow over the crest will be characterized by the series of waves that appear whenever the depth is slightly greater than the critical; the discharge coefficient will again be somewhat less than 0.27.

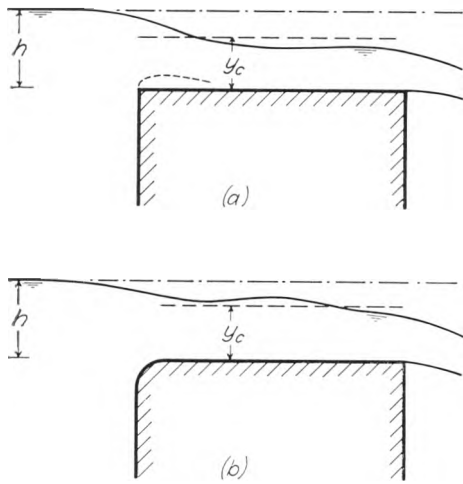


Fig. 8. PROFILES OF FLOW OVER BROAD-CRESTED WEIRS.

As any type of weir becomes partially submerged by the pool downstream (Fig. 9), it will begin to behave—as a rough approximation—in part as a weir and in part as an orifice. This means that the combined discharge coefficient will involve a power of the differential head somewhere between $3/2$ and $1/2$ depending upon the ratio between the differential head and the depth of submergence.

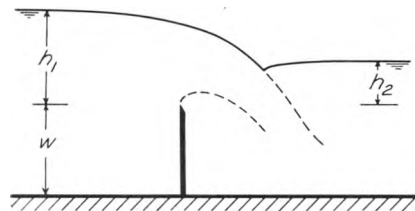


Fig. 9. PROFILE OF FLOW OVER A SUBMERGED WEIR.

In the case of the broad-crested weir, it should be noted, the surface of the downstream pool can rise a distance above the crest about equal to the critical depth before the submergence effect becomes apparent.

Uniform and Critical Open-Channel Flow

The counterpart of the pipe as an extension of the orifice and short tube is found in a smooth continuation of the broad-crested weir as the floor of an open channel. Like the pipe, of course, the channel must eventually reach an end. But unlike pipe flow, which generally occurs under pressure, open-channel flow is invariably characterized by a free surface which can vary in elevation and configuration. What form the surface profile will actually have depends upon many factors: the geometry of the channel profile, cross section, and boundary roughness; the rate of flow; and the

depth imposed at one end or the other. However, unless the geometry is more complex than usual, knowledge of the depth at any control section and of the depths at which the given flow in the given channel would be uniform and critical, respectively, will permit the surface profile to be determined, as described in the following paragraphs.

A relationship for the uniform depth of flow in a channel of any cross-sectional form can be written as

$$\frac{Q}{A\sqrt{RS}} = C = \sqrt{\frac{8g}{f}} = 1.5 \frac{R^{1/6}}{n} \quad (12)$$

which corresponds to what is called the Chezy formula. Herein the hydraulic radius R is the ratio of the cross-sectional area A to the wetted perimeter P , and S is the channel slope. Unlike the resistance coefficient f , the Chezy discharge coefficient C is not non-dimensional, for it is seen to involve the square root of g . However, it is rather simply evaluated in terms of the Manning roughness factor n , which can be considered to vary from about 0.012 for smooth concrete to about 0.025 for rough masonry and corrugated metal. Knowledge of the channel geometry and rate of flow thus permits the corresponding hydraulic radius to be computed, from which the centerline depth of uniform flow can readily be ascertained.

Whether the depth of flow in a channel of arbitrary cross section is greater or less than the critical depends upon the magnitude of a ratio known as the Froude number:

$$\frac{Q}{\sqrt{gA^3/W}} = F \quad (13)$$

Herein W is the width of the free surface. If this ratio is smaller than unity, the velocity of flow is less than that of a small wave, and the flow is said to be subcritical; the mean depth A/W is then greater than the critical. If the ratio is greater than unity, the velocity is greater than that of a wave, and the flow is called supercritical; the mean depth is then less than the critical. Knowledge of the cross-sectional geometry permits either the average depth or the actual centerline depth for critical conditions to be calculated. If the section is rectangular, the Froude number of unity will be found to correspond to a velocity head equal to half the depth, as previously stated to be true for critical flow.

If the centerline depth y is plotted against the elevation $H - z_0$ of the line of total head above the channel bottom for a constant value of Q , according to the following form of the definition equation for total head,

$$H - z_0 = \frac{Q^2}{2gA^2} + y \quad (14)$$

the result will be as shown in Fig. 10. The curve is seen to have two branches, which meet at the common minimum value of $H - z_0$ for the critical stage just discussed. The upper branch corresponds to Froude numbers less than 1 and the lower branch to those greater than 1. What is important about this diagram is that it shows whether the depth of flow will increase or decrease as the line of total head and the channel bottom converge or diverge (i.e., as $H - z_0$ decreases or increases) for any reason. One reason might be a local rise or fall in channel bottom, as at a low weir or sill. (A lateral contraction or enlargement, it should be noted, would produce the same change.) Another reason could be the loss of head due to surface resistance, as will be discussed directly.

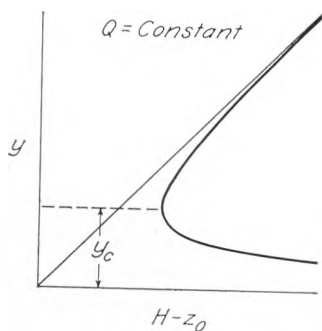


Fig. 10. DIAGRAM OF DEPTH VERSUS RELATIVE TOTAL HEAD FOR CONSTANT DISCHARGE.

Non-Uniform Open-Channel Flow

Several classes of channel and stages of flow remain to be distinguished. If the channel slope is less than that at which the given discharge would be both uniform and critical (i.e., if the computed uniform depth is greater than the computed critical depth), the channel is classed as mild (*M*). If the slope is greater than that at which the given discharge would be uniform and critical (i.e., if the computed uniform depth is less than the computed critical), then the channel is classed as steep (*S*). In either case, the flow can take place (1) at a depth greater than both the computed normal and critical depths, (2) at a depth between the normal and critical, and (3) at a depth less than both the normal and critical.

Consider, for example, flow of the *M2* category. The depth is less and the velocity hence greater than for uniform flow, and the line of total head (the slope of which is proportional to the rate of loss of head) must therefore be steeper than the channel itself. The quantity $H - z_0$ thus decreases in the direction of flow, and reference to the upper branch of the curve in Fig. 10 will show that the depth has to decrease accordingly. In fact, the free surface must slope even more steeply than the line of total head, so that the velocity head will properly increase as the depth decreases. The general form of the free surface will be as sketched in Fig. 11a. If, on the other hand, the flow is of the *S2* category, the line of total head will slope less steeply than the channel, yet the depth will again decrease in the direction of flow—now in accordance with the trend of the lower branch of the curve in Fig. 10. The general result is indicated in Fig. 11b.

Comparison of the two profiles in Fig. 11 will indicate that each ap-

proaches (albeit in opposite directions) the line of computed normal depth asymptotically, whereas each approaches the line of computed critical depth at an ever-increasing angle. The limit of the latter trend is 90° , but then the two-dimensional curvilinearity of the flow is so great that the assumed one-dimensionality is no longer applicable and the indicated limit is meaningless. However, if the same simplified reasoning is applied to conditions of the M1, M3, S1, and S3 categories, the schematic indications combined in Fig. 12 will be obtained. The special cases of horizontal and critical slopes can be visualized therefrom.

Several characteristics of these curves must be emphasized. First, their horizontal scale is greatly foreshortened, particularly in the upper stages, for backwater curves in canals and rivers sometimes display measurable variations in depth over distances of many thousands of feet. Second, though for present purposes only the general proportions of the curves are significant, their coordinates can be computed by step processes with satisfactory approximation. Third, whereas any of these curves is defined over its entire length by the two reference depths, only a portion of it will apply to a given situation. This is determined by the actual channel length, and by the depth at what is known as the control section. Since at depths greater than the critical even the smallest waves can travel upstream, the controls for subcritical flow must lie at the downstream end of the channel—for example, forebays, gates, weirs, and overfalls. At depths less than the critical, however, the velocity of flow is so great that elementary waves can travel only downstream, whence for

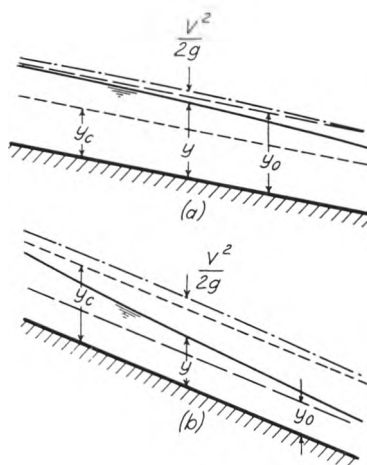


Fig. 11. VARIATION IN TOTAL HEAD AND DEPTH OF FLOW WITH RELATION TO LINES OF UNIFORM AND CRITICAL DEPTH.

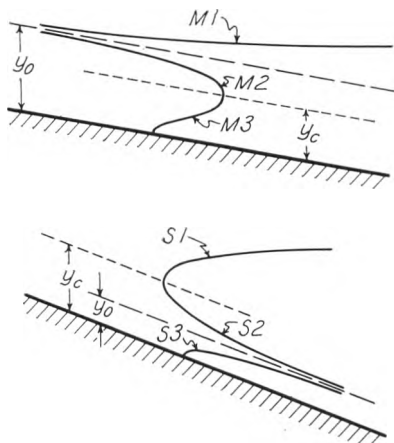


Fig. 12. COMPOSITE DIAGRAM OF SURFACE PROFILES FOR MILD AND STEEP CHANNELS.

supercritical flow the controls invariably lie upstream—such, for instance, as sluices and spillways or the inlets to steep channels. In some instances controls exist at both ends (i.e., a steep channel that is partially submerged by backwater from a downstream pool), in which instance a hydraulic jump from supercritical to subcritical flow will occur at some intermediate section (Fig. 13).

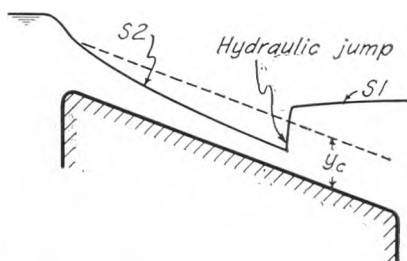


Fig. 13. SCHEMATIC REPRESENTATION OF FLOW WITH UPSTREAM AND DOWNSTREAM CONTROLS.

Hydraulic Similarity

Whereas the tests described in Bulletin 1 were performed at as nearly full scale as possible, it is modern practice to conduct laboratory investigations with considerable scale reduction. This is not because small-scale tests are more accurate, but because the cost in materials and labor varies approximately as the cube of the scale. For the same outlay, therefore, many more conditions can be investigated at small scale than at large. However, to ensure that the small-scale occurrence correctly simulates its prototype, certain principles of similitude must be followed.

If two states of flow are to be similar to each other, it is necessary first of all that the corresponding linear dimensions of the solid boundaries bear a constant ratio to each other—in other words, the boundaries must be geometrically similar. Sometimes this is not feasible (small-scale river models, for example, might become excessively shallow), and the vertical and horizontal scales are reduced in different ratios—i.e., the model is distorted. Perfect similarity can then no longer be realized, though extreme departure in one regard may well have been avoided by introducing a lesser degree of departure in another.

Even though geometric similarity of the boundaries prevails, dynamic similarity of the flows can obtain only if corresponding forces in the two systems also bear a constant ratio to each other. When the forces involved are those of inertia and gravity, this condition will be fulfilled if the Froude number already introduced,

$$F = \frac{V}{\sqrt{gL}} \quad (15)$$

in which L is some characteristic length, has the same magnitude for both systems. This criterion evidently requires that the velocities of flow vary with the $1/2$ -power of the scale ratio; since rates of flow are proportional to

velocities times cross-sectional areas, such rates should vary with the 5/2-power of the scale ratio. When the forces involved are those of inertia and viscosity, on the other hand, similarity of the flow requires instead that a quantity known as the Reynolds number be the same for the two systems:

$$R = \frac{VL}{\nu} \quad (16)$$

the factor ν is the kinematic viscosity in square feet per second. Likewise, when inertial and capillary forces prevail, it is the Weber number that must be constant:

$$W = \frac{V}{\sqrt{\sigma/\rho L}} \quad (17)$$

herein σ is the surface tension, in pounds per foot, and ρ the mass density, in slugs per cubic foot.

Comparison of these three similarity parameters will reveal the fact that each requires a different variation in velocity as the scale is changed. As a result, it is very difficult to produce flow in a model that is dynamically similar to the flow in the prototype if more than one similarity criterion is involved. On the other hand, one effect or another will generally predominate to the extent that one parameter can be used to determine the corresponding rates of flow, and the effect of the others can then be approximated by a different procedure. Flow with a free surface is thus primarily gravitational if the scale is sufficiently large, for viscous and capillary action is ordinarily involved only at small scales or low velocities. However, both types of action, small as they may be, are sometimes sufficient to play a determining role under certain critical conditions. In this event it must be remembered that full similarity between model and prototype does not obtain.

Methods of Application

Numerous as the foregoing principles of hydraulics may be, they are still so limited in both accuracy and completeness that they will permit the solution of none but the simplest problems. Even the method of model simulation is seen to have its limitations, however great the confidence that is now placed in this once-belittled laboratory tool. Nevertheless, in the introduction the fact was stressed that the highway engineer does not need the ability to design a specific structure with precision so much as a general understanding of culvert performance. As will be shown in the following pages, the principles discussed are quite adequate for this purpose.

For the most complex cases, a model study is definitely a sound investment. Impossible as it may be to reproduce every detail of the flow pre-

cisely, the overall picture is readily brought to light, and various combinations of extremes can be reproduced rapidly and instructively. A model of every installation, however, would be neither economically sound nor even necessary. On the contrary, most culvert types have already been tested experimentally to such an extent that their basic characteristics are well understood; in fact, some of these experiments have provided material for the foregoing discussion of elementary flow principles. It is hence now in order to utilize the principles, singly and in combination, to assess the effect of each essential culvert feature under various conditions of operation.

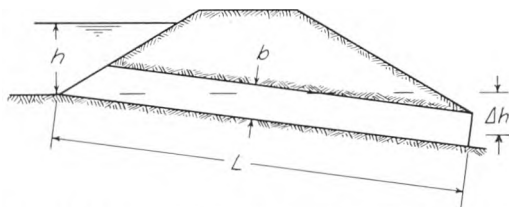


Fig. 14. DEFINITION SKETCH FOR CULVERT NOTATION.

The most logical means of organizing the following analysis is primarily on the basis of the barrel slope and secondarily on the basis of upstream depth. The simplest (and least effective) case of barrels on a mild slope is thus considered first, followed by that of barrels on a steep slope, the sequence culminating in the recommended compound type of profile. Each aspect of the resulting flow is discussed in terms of the elementary principles already presented and, where desirable, is illustrated by experimental data taken from special laboratory tests conducted on models with barrels of square cross section. For purposes of generality, all linear dimensions are referred to the barrel dimension, and rates of flow are made non-dimensional through the further incorporation of the gravitational acceleration. The numerical values—while by no means precise—are sufficiently representative of quantitative behavior to serve for general design purposes.

CULVERTS WITH BARRELS OF MILD SLOPE

Flow with Inlet Unsubmerged

Whether the slope of a culvert barrel is classed as mild or steep depends upon whether the depth just before the culvert begins to flow full is greater or less than the critical. A culvert that does not flow full, of course, differs little from any open channel. If the barrel slope is mild, the section that controls the flow will lie at the downstream end—either in the form of the free overfall at critical depth, if the outlet is unsubmerged, or as the beginning of a backwater curve if the tailwater is of sufficient depth.

To say that the discharge control is located downstream does not signify

that no other factors have any influence upon the discharge. In actuality, flow in such a culvert depends to some degree upon each of the following: (a) inlet head; (b) inlet shape; (c) shape of barrel cross section; (d) barrel roughness; (e) barrel length; (f) outlet shape; (g) outlet depth. For a given set of conditions, the surface profile could be evaluated in much the same manner as any backwater curve: if the flow were known, by step computation; if not, by assuming successive flow rates, carrying out the step computations, and interpolating for the profile that agreed with the given inlet depth.

As has previously been indicated, such complexities are not often warranted in culvert design, partly for lack of even approximate knowledge of probable requirements, and partly because the final proportions are seldom based upon the conditions encountered before the inlet has become submerged. A sufficient degree of accuracy is hence obtainable by assuming uniform flow to take place through the barrel at a slope equal to the average between that of the barrel and that indicated by the inlet and outlet depths adjusted for inlet loss and velocity head.

Even a rough calculation such as this would show that there is little to gain at this stage from streamlining the inlet unless the barrel is very short. Similarly, the expense of reducing roughness would not be justified unless the barrel were very long. In fact, the few variables over which the designer has control seldom affect the culvert performance appreciably in this range of operation. Model tests, moreover, must be regarded as yielding relative rather than absolute indications, since gravitational and viscous effects of comparable magnitude are involved at small scale. Hence the plot of measured data shown in Fig. 15 merely demonstrates the lack at model scale of any appreciable effect of doubling either the barrel length or the barrel resistance. Generally noteworthy, however, is the type of discharge function that is representative of this range:

$$\frac{Q}{\sqrt{g} b^{5/2}} = K \left(\frac{h}{b} \right)^{3/2} \quad (18)$$

in which K varies with the factors already itemized—including to some degree the ratio h/b itself. (In this and subsequent pages, it should be noted, h represents the depth of the inlet pool above the barrel invert, and b the vertical or horizontal dimension of the square cross section, as shown in the definition sketch of Fig. 14.)

Flow with Partially Effective Inlet Submergence

Once the headwater depth exceeds the height of the inlet by more than the inlet loss and velocity head, the flow can no longer be considered wholly like that of an open channel. However, neither can the barrel be presumed

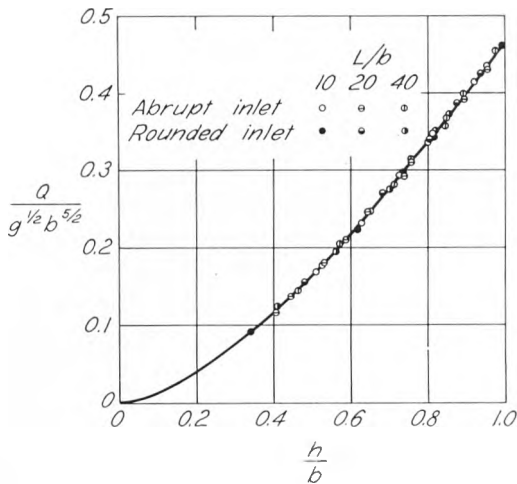


Fig. 15. PERFORMANCE CURVE OF BOX CULVERTS ON MILD SLOPES WITH INLET UNSUBMERGED.

to function like a pipe flowing full, for there will continue to be a free water surface over part of its length for at least part of the time until the head-water rises to a level considerably above the inlet. This is the range of indeterminate operation. Not only is it difficult to predict the behavior of a given design, but it is often impossible to obtain consistent laboratory indications on a specific structure.

The factors that are involved in determining whether or not a culvert barrel will flow full are many and varied. Principal among these are the depths of the inlet and outlet pools, and the form of the inlet; at model scale, moreover, such extraneous influences as viscosity and surface tension can wholly change the flow regime. Since the problem does not become acute till the barrel slope exceeds the critical, its detailed discussion will be reserved for the next section, and present comments will concern solely those aspects associated with mild slopes.

If the inlet is sharp-edged, heads that are not much greater than the barrel height will be marked by inflow very similar to discharge from a gate. A free surface will exist beyond the point of contact, and the flow will be supercritical. If the barrel is sufficiently short and the outlet is unsubmerged, the inlet will be the sole control and the flow will reach the outlet still in a supercritical state. Barrels of moderate length, however, will probably display another control at the outlet, and a jump must then form at some intermediate section. At such low Froude numbers the jump will be of the undular type, and in all probability the surface waves will intermittently touch the top of the barrel, thereupon causing the culvert to tend to

flow full. So long as the head is small, the irregularity will not be serious; and as the head is increased, the tendency to fill will become steadily greater.

If, on the other hand, the inlet is sufficiently well rounded, the contraction (and hence separation) tendency will not exist locally, and the barrel will flow full—at least at its upstream end. How far downstream separation occurs will depend upon the headwater and tailwater depths and the barrel slope. The greater the two depths, the farther downstream it will be; the greater the slope, the farther upstream. Because viscous and capillary effects increase as the scale is reduced, the point of separation cannot be determined by model test. Fortunately, this matters little in barrels of mild slope, and for general purposes a barrel with rounded inlet can be assumed to flow full when $h/d > 1.2$, and a barrel with square-edged inlet when $h/d > 1.5$.

Flow with Fully Effective Inlet Submergence

When the inlet depth is sufficiently great to make the culvert barrel flow full, the discharge relationship can be computed with fair accuracy by writing the energy equation between the upstream pool and the outlet. The result is expressible in the form

$$\frac{Q}{\sqrt{g} b^{5/2}} = K \left(\frac{h + \Delta h}{b} \right)^{1/2} \quad (19)$$

in which Δh is the further change in head over the length of the barrel, and

$$K = \frac{1}{\sqrt{1/2 \left(1 + C_i + \frac{fL}{4R} + C_o \right)}}$$

What head to assume at the outlet depends in part upon the Froude number and in part on the tailwater elevation. If the outlet is completely unsubmerged, the line of piezometric head at the outlet section may be considered to lie slightly below the centerline for high Froude numbers, gradually increasing to a level about half way between centerline and top of barrel for a Froude number approaching unity. (The exact elevation depends not only on the Froude number but also upon the resistance, and the interdependence is not yet fully understood.) If, on the other hand, the level of the outlet pool is greater than that of free outflow, the piezometric head will be controlled accordingly.

Remarks made in the foregoing pages as to the relative effects of inlet form, resistance, and length of barrel continue to be applicable in this advanced stage. Typical experimental results obtained at model scale are

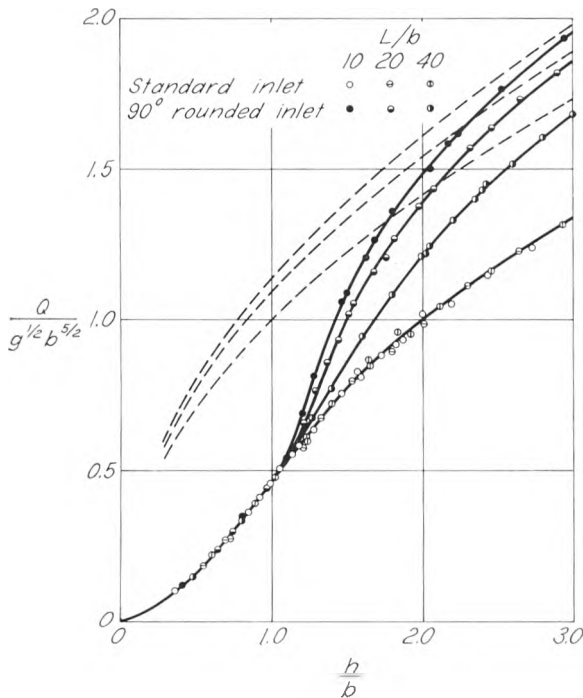


Fig. 16. PERFORMANCE CURVES OF BOX CULVERTS ON MILD SLOPES WITH INLET PARTIALLY AND COMPLETELY SUBMERGED.

shown in Fig. 16. Since the barrel surface was smooth, rounding of the inlet played a considerable role for flow through the shorter barrel. The absolute effect of rounding is, of course, of a dual nature, for it reduces both the entrance loss and the tendency toward part-full flow. The frequent belief that inlet and outlet should be of similar shape is obviously without hydraulic justification.

Figure 16 is seen to combine in one plot the three stages of inlet submergence for unsubmerged outlet. It is possible to include curves for the submerged outlet in the same diagram if one then let the abscissa scale represent the differential head $h + \Delta h$ between headwater and tailwater (Δh , of course, will become negative with sufficient outlet submergence). As indicated by the broken lines in the figure, these curves approach the others asymptotically with increasing head, but unlike the others they follow the relationship $Q \propto (h + \Delta h)^{1/2}$ from the outset.

Which of the several stages to use for design purposes is a moot question. However, the fact that the discharge increases only as the square root of the head once the final stage is reached does not make culverts with barrels on

mild slopes particularly appropriate for conditions in which severe overload can occur. Probably the proper design stage should be that for which the pool level is about at the top of the barrel; conditions of incomplete and complete inlet submergence are thus held in reserve for such factor of safety as they can offer.

CULVERTS WITH BARRELS OF STEEP SLOPE

Flow with Inlet Unsubmerged

So long as both inlet and outlet of a culvert barrel on a steep slope remain unsubmerged, the control section will lie at the upstream end and the flow over the entire length will be supercritical. In accordance with the critical-flow relationship given under elementary principles, the depth just within the inlet can be taken as two-thirds the depth of the upstream pool and the corresponding velocity head as half the critical depth. The rate of discharge will then depend upon these two values in combination with the effective width of the flow section, and the factor K in Eq. (18) is determined accordingly.

If the inlet is sharp-edged, the flow section just within the barrel can be taken as about 20% less than the barrel width because of separation and lateral contraction. If the inlet is well rounded, the critical-flow section will have the full width of the barrel. To extend this line of reasoning, it would seem possible to increase the rate of flow by further widening the inlet. This is in fact perfectly feasible, provided only that the remainder of the barrel can carry the increased flow. In other words, whereas the control section of a uniform barrel would lie at the inlet, that of a convergent barrel might lie at any intermediate section, depending upon rate of taper, length, slope, roughness, and inlet head. As a simple check on its location, the variation in depth can be approximated by ignoring resistance and solving the open-channel form of the one-dimensional energy equation (much as was done in determining the specific-head diagram of Fig. 11). The influence of boundary resistance would be to accentuate somewhat the effect of the convergence, the extent of which could be ascertained by giving the line of total head a reasonable slope rather than assuming it horizontal.

Since gravitational action predominates in the neighborhood of an upstream control, model studies can then be expected to simulate prototype performance with good approximation. Barrels of 4-inch, 6-inch, and 12-inch section and lengths from $10b$ to $40b$ thus gave identical performance curves, as shown in Fig. 17. Identical results were also obtained regardless of whether the bottom slope was continuous or displayed a break in grade at the inlet. On the other hand, the difference between sharp-edged and rounded inlet is seen to be appreciable.

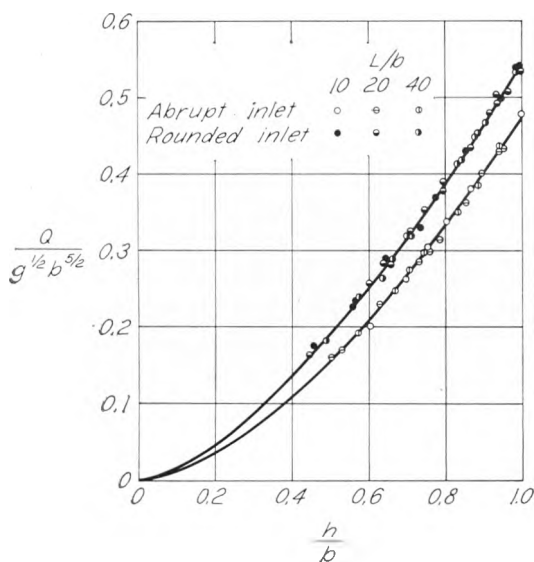


Fig. 17. PERFORMANCE CURVES OF BOX CULVERTS ON STEEP SLOPES WITH INLET UNSUBMERGED.

Flow with Partially Effective Inlet Submergence

Sharp-edged inlets only slightly submerged are also readily studied at model scale, since—as for those on mild slope—the inlet functions like a gate, and the steep slope now maintains the flow in its supercritical state until the outlet—or the backwater from the outlet pool—is reached. As a matter of fact, the only way to make a culvert with a steep barrel and a sharp-edged inlet flow full is by raising the level of the outlet pool sufficiently far to shift the control from the upstream to the downstream end. However, the performance is then still poor compared to that for other conditions of flow.

Rounded inlets only slightly submerged represent those for which the performance is least susceptible to model simulation, since their effect upon the discharge varies not only with the scale but with many extraneous factors. Basically, the flow tends to follow the same type of S2 curve as in the foregoing case. However, the initial cause of separation has been eliminated, and the separation point hence moves downstream along the top of the conduit an indeterminate distance. A surface irregularity like an expansion joint might be the factor that controls its location, or the degree of turbulence of the incoming flow. In the model it can be shifted either upstream or downstream by the application of either grease or a wetting agent, respectively, to the barrel surface. For any given culvert, moreover, it will vary with the rate of flow.

Even assuming that a culvert is so constructed as to begin flowing full at relatively small inlet submergence, the rate of flow will then increase because of the increase in effective head, the upstream pool level will drop, partial ventilation will occur, and the cycle will begin again. Increased submergence may eventually prevent a fall in pool level below the top of the inlet, but then the vortex that invariably forms in such regions will probably grow to the extent that it introduces air at an appreciable rate. In any event this will reduce the rate of flow because of the reduction in flow section. However, if the barrel flows full only by virtue of capillary or similar effects, the vortex will very likely cause intermittent separation and ventilation. In the latter event, the discharge function will display a severe discontinuity (Fig. 18), and its mean form and temporal deviation therefrom will be impossible to predict.

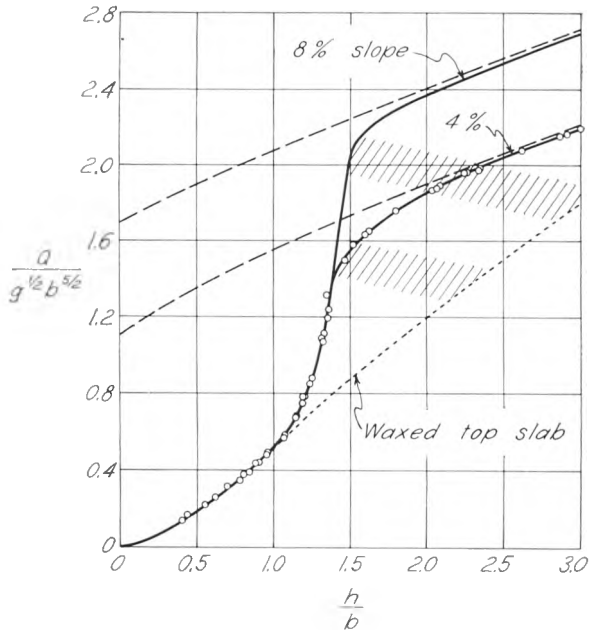


Fig. 18. PERFORMANCE CURVES OF BOX CULVERTS ON STEEP SLOPES UNDER VARIOUS CONDITIONS OF FLOW.

Various methods have been devised to prevent such a vortex from forming—or at least from developing an air-filled core. Sometimes rafts are floated at the water surface over the inlet to inhibit ventilation, but this would not be a very realistic solution in the case of a culvert. Splitter walls would serve to prevent the circulation from beginning or becoming very

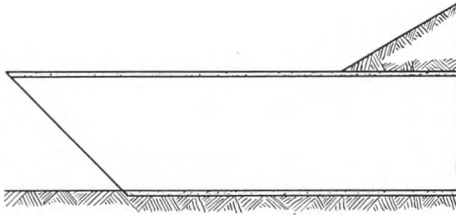


Fig. 19. SIMPLE HOODED INLET.

intense, but these are always debris collectors. Use of a reentrant pipe simply cut off at 45° , as in Fig. 19, has proved effective, although it is subject to considerable capillary bias at small scale. Extension of the top of the barrel to meet a vertical rather than backward-sloping headwall is preferable. Still better is the combination of a vertical headwall with a lateral enlargement of the inlet to perhaps twice the width of the barrel (Fig. 20), so that the velocity of flow in the inlet vicinity will not be sufficiently great to produce more than desultory circulation and vortex formation. Such an

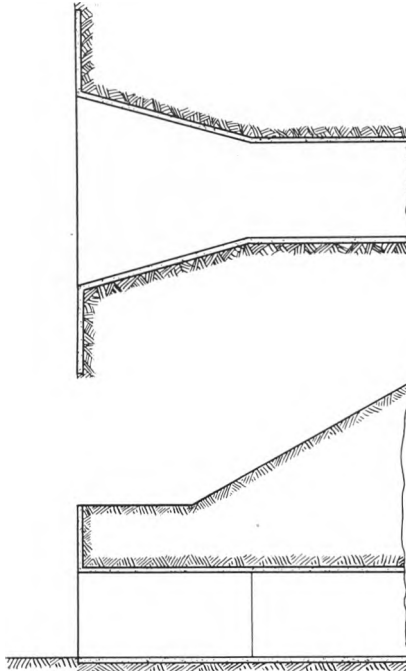


Fig. 20. PLAN AND ELEVATION OF Laterally Convergent Inlet.

inlet form could well be designed to satisfy the requirements of both free inflow at critical depth and initial inlet submergence.

Flow with Fully Effective Inlet Submergence

Full flow of a culvert barrel with steep slope is much more important than that of a barrel with mild slope for two reasons: first, for a mild slope the control section is usually already at the downstream end; second, for a steep slope the difference in elevation between inlet and outlet is appreciably greater. In other words, when a steep barrel begins to flow full, there is not only a change in the discharge function from Eq. (18) to Eq. (19) but also a comparatively great initial increase in the discharge itself. This is evidenced in the discharge-head diagram of Fig. 18 by a curve that abruptly becomes very steep and then gradually flattens out. Note should be made of the fact that it is the steep portion of which particular advantage must be taken, because the subsequent variation of the discharge with the square root of the head, as in all closed-conduit flow, requires an ever-increasing change in head to produce a given change in discharge.

As has already been emphasized, the head at which such a culvert will continuously flow full depends upon so many factors as to be rather unpredictable. Probably the only certain criterion for full flow is the combination of a rounded inlet with such conditions beyond it that the computed normal depth is equal to at least 90% of the barrel height. The free-surface part of the flow would then tend to follow an S2 curve, which is everywhere above its asymptotic limit of uniformity. Since the actual water surface is seldom smooth, waves touching the top slab of the barrel serve to maintain full flow from the inlet on, even though the barrel height is actually greater than the normal depth. Although model studies produced full flow below this stage by means of wetting agents, at and above this stage the use of an anti-wetting agent would no longer result in ventilation. The fact should be emphasized, however, that these remarks apply only to well-rounded, separation-free inlets.

Culvert barrels on steep slopes can also be made to flow full, of course, by submerging the outlet to a sufficient depth. The same discharge diagram (Fig. 17) can still be used for such conditions if it is recalled that the sum $h + \Delta h$ represents the differential between headwater and tailwater pools (i.e., the vertex of the parabola lies the distance $-\Delta h$ from the origin). A curve of this nature is shown as a broken line in Fig. 18. Lest the reader assume that the asymptotic approach to the other curve indicates comparable performance, it must be remarked that the degree of submergence of a steep-barrel outlet necessary to provide full flow must usually be so great as to restrict the differential head to a small fraction of that for free outflow. For conditions of outlet submergence there is evidently little difference in performance between barrels of mild and steep slope.

CULVERTS WITH BARRELS OF DISCONTINUOUS SLOPE

Design Characteristics

At least two disadvantages of culverts with uniform barrels on either mild or steep slopes should be apparent from the foregoing discussion. If the barrel is on a mild slope, the full differential head between headwater and tailwater pools is seldom utilized; and even if it is on a steep slope, advantage can be taken of the available differential head only after there is sufficient inlet submergence to make the barrel flow full.

Two possible remedies suggest themselves. One would be to taper the barrel over its entire length and place it on a steep slope. Tapering (convergent in the downstream direction) would yield a large enough inlet to produce critical inflow at low inlet heads and freedom from serious vortex formation at moderate to high heads. The compensating effects of taper and slope would permit optimum utilization of the differential head. A somewhat different solution is found in the drop-inlet type of structure: a short vertical shaft at the bottom of the upstream channel connecting with a horizontal barrel at the level of the outlet channel. This would likewise promote critical inflow—very likely to a considerably higher inlet head—and the abrupt drop would have much the same effect as the steep slope. However, the tapered barrel would be less economical of construction than the uniform type, and the inlet at ground level would in addition attract the accumulation of debris.

A type of culvert that seems to combine the advantages of all discussed, and at the same time to eliminate many of their disadvantages, is the following. As indicated in Fig. 21, the inlet proper is placed in the face of a

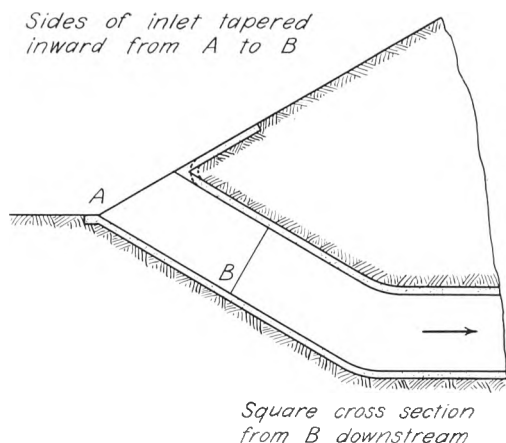


Fig. 21. ELEVATION DETAILS OF INLET TO BROKEN-BACK TYPE OF CULVERT.

slab or riprap having the same slope as the embankment. The opening is flared laterally to perhaps twice the width of the subsequent barrel. A convergent passage leads downward at an angle of about 30° to the horizontal, connecting—at a level several barrel heights below the inlet—with the uniform part of the barrel, which can be set on a mild or even zero slope. The juncture between the two parts of the barrel should be rounded (or broken in profile into a series of chords) both top and bottom on a radius of the same order as the barrel height. It is desirable that the juncture between the sloping face of the embankment and the top slab of the inlet passage be rounded as shown on a radius of about $0.2b$, and that the inclined edges be rounded or chamfered; however, these features are not mandatory.

Characteristics of Operation

Operation of such a culvert with discontinuous barrel slope—sometimes called the broken-back type—can be discussed effectively in terms of the same three stages as those with continuous and uniform barrels: unsubmerged inlet, partially effective inlet submergence, and fully effective inlet submergence. However, the discharge-head relationship now displays a smooth, rather than unpredictably discontinuous, transition from each stage to the next. The rate of flow, moreover, can be simply computed with acceptable approximation.

So long as the inlet is unsubmerged, the flow will pass through the critical stage at the inlet brink. If the side edges are not rounded or chamfered, some allowance may be made for lateral contraction of the inflow, but this is actually an unnecessary refinement because of the initial widening of the opening. In fact, as the head increases, inflow over the sides more than compensates for the reduction in width due to separation. Even though the subsequent 2:1 reduction in the width of the convergent passage in itself tends to obstruct the flow, the accompanying 30° downward slope completely offsets this. In addition, the obstruction resulting from the break in grade at the juncture with the uniform barrel—already reduced in comparison with the 90° drop-inlet type of structure—is further minimized by rounding. As a result, the control remains at the inlet section and the discharge follows Eq. (18) so long as both inlet and outlet are unsubmerged.

Partial submergence of the inlet begins as the level of the pool rises above the top of the opening. Little change then occurs in the discharge function, however, for inflow simply takes place over all edges of the opening, with a residual zone of ventilation somewhat above center. The influx of air with the water does not disturb the flow, since the control is still in the vicinity of the inlet. In fact, the combination of convergence, initial steepness, and break in grade is such as to effect a gradual shift in control section from the inlet toward the break as the head increases, even though continued—or even irregular—ventilation occurs. Measurements on scale models (see Fig.

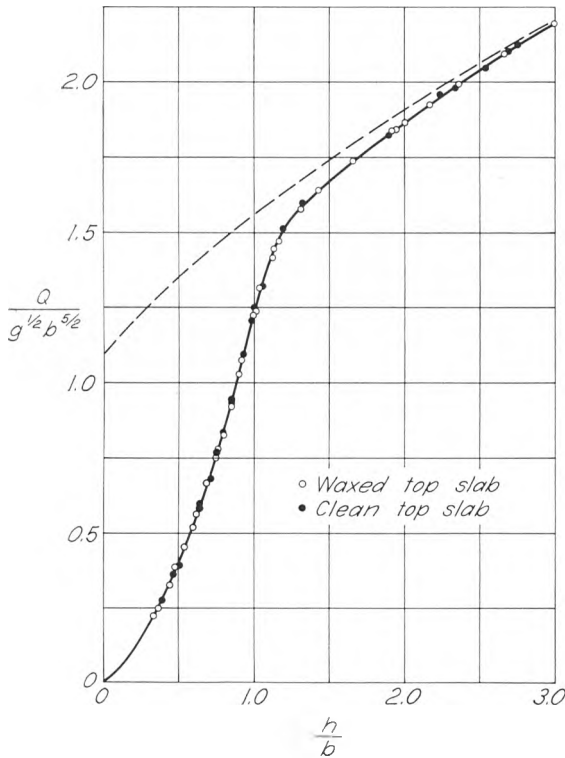


Fig. 22. PERFORMANCE CURVES OF BOX CULVERT WITH DISCONTINUOUS BARREL ALIGNMENT FOR ALL STATES OF SUBMERGENCE.

22) indicate that the discharge-head curve, while not so simply computable as that for the initial stage, still follows the relationship $Q \propto h^{3/2}$.

Because of the mild slope of the uniform barrel, the transition to full-barrel flow occurs without abrupt effect upon the rate of flow; in other words, the slope and resistance of the barrel are self-compensating, so that the discharge is neither augmented nor retarded as the barrel fills. Once full-flow conditions are realized, the culvert behaves like any other closed conduit (see Fig. 22) and the discharge is as readily computable through Eq. (19). It is hence in the first two stages that the advantages of the broken-back type of culvert are to be found. Like other culverts, its behavior with both inlet and outlet submerged can be included on the same discharge diagram with $h + \Delta h$ representing the differential head between inlet and outlet pools; such a curve is shown in the figure as a broken line.

CONCLUSION

In the foregoing pages considerable use has been made of the elementary

principles of orifice, weir, pipe, and channel flow to clarify the various aspects of culvert hydraulics. Performance curves for the primary culvert forms and flow conditions have been examined in basic detail. Certain practices in design have been shown to be of dubious value and others have been suggested in their place. Laboratory measurements have been presented in illustration of the statements made, but the fact has been stressed that model studies can sometimes mislead if not properly interpreted. For purposes of emphasis, the major points that have been made are summarized in the concluding remarks that follow.

Orifice flow and weir flow differ primarily in the exponent of the discharge-head relationship. The discharge rate for orifice flow varies—like the velocity itself—with only the one-half power of the head, because the cross section remains constant. The discharge rate for weir flow, however, varies with the three-halves power of the head, because the head controls both the velocity and the size of cross section. Orifice flow is thus one of diminishing returns, for the ratio dQ/dh (i.e., the change in discharge for a given change in head) decreases as h increases; just the opposite is true of weir flow, and hence a culvert with the attributes of a weir is—safety-wise—preferable to one with those of an orifice.

Whether a culvert behaves like an orifice or a pipe, on the one hand, or like a weir or open channel, on the other, depends on a combination of factors such as relative length, roughness, slope, inlet form, and degrees of inlet and outlet submergence. Barrels on mild slopes generally have little to recommend them; suffice it to say that the inlet shape is then important only if the barrel is relatively short, and the surface roughness only if the barrel is relatively long. Barrels on steep slopes have definite advantages, but also definite drawbacks. When their inlets are unsubmerged, the control is necessarily at the upstream end—i.e., they function like weirs. When their inlets are sufficiently submerged, moreover, the full effect of the drop in level from inlet to outlet comes into play; although the discharge then varies only with the square root of the head, the initially rapid increase in effective head represents a net gain of appreciable magnitude. However, the point at which the inlet submergence becomes sufficient to produce this condition depends upon so many factors as to be generally indeterminate, even from model tests. In fact, in many instances the stage of partial inlet submergence may be one of marked instability, the flow alternately filling the barrel and then separating and ventilating. For this reason it is presently considered unwise to design a culvert for flow with submerged inlet; rather, the possibility of increased effectiveness at higher inlet heads is regarded simply as an additional safety factor.

Use of a laterally convergent inlet, together with a break in the vertical alignment of the barrel, tends to emphasize the weir-type performance of a

culvert when the inlet is unsubmerged, to eliminate the uncertainty of the intermediate range, and yet to sacrifice none of the ultimate full-flow capacity. For purposes of comparison, curves for the so-called "standard" type of culvert with vertical headwall, sharp-edged inlet, and barrel on a mild slope (taken from Fig. 16), for the steep barrel with rounded inlet (from Fig. 18), and for the broken-back type (from Fig. 22) are superposed in Fig. 23.

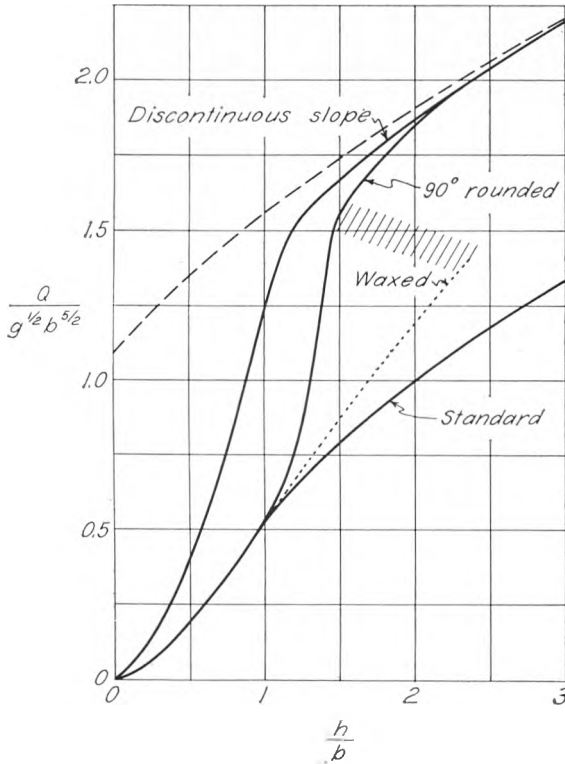


Fig. 23. COMPARISON OF PERFORMANCE CURVES FOR CULVERTS WITH BARRELS OF MILD, STEEP, AND DISCONTINUOUS SLOPES.

The relative advantages of the discontinuous barrel with convergent inlet are evident. Not only is its capacity at least twice that of the standard type with inlet unsubmerged, but its transition from unsubmerged to submerged operation is smooth and predictable. Problems of intermittent ventilation are thus eliminated.

Brief mention has been made of the custom of repeating the inlet shape at the outlet. Hydraulically this is of no use whatever, and it is doubtful whether more than a very gentle outlet flare would effectively reduce the

erosive effect of the outflow. Nothing has been said about the protection of the outlet against damage due to scour, in part because this problem is as much structural as hydraulic, and in part because of the many factors (channel alignment, shape, material, grade, and degree of outlet submergence) that are actually involved. If the structure is a large and important one, surely model studies of erosion control will be warranted. If it is small, probably the best safeguard is the protection of areas adjacent to the structure with rubblework, or at least with riprap that is coarse enough not to be dislodged by the flow. To be noted is the fact that it is not only the main stream that causes erosion damage, but also the secondary currents represented by the rollers or eddies which the primary flow maintains in motion.

ACKNOWLEDGMENTS

For the six years of its duration, the investigation culminating in the present bulletin was jointly supported by the Iowa State Highway Commission and the Bureau of Public Roads. The original project was planned by John S. McNown, then Associate Director of the Institute, Mark Morris, Director of Highway Research of the Commission, Carl F. Izzard, Chief of the Hydraulics Research Branch of the Bureau, and the second author. Essentially all of the experimental work was conducted by, or under the supervision of, the first author. Graduate theses on various aspects of the investigation were written by Henry Maksoud, J. E. Flack, Peter M. Smith, and Narain R. Rijhwani. The manuscript of the bulletin was illustrated by the first author and written by the second.

REFERENCES

- [1] Yarnell, D. L., Nagler, F. A., and Woodward, S. M., "The Flow of Water through Culverts," University of Iowa Studies in Engineering, Bulletin 1, 1926.
- [2] Larson, C. L., and Morris, H. M., "Hydraulics of Flow in Culverts," Project Report No. 6, St. Anthony Falls Hydraulic Laboratory, University of Minnesota, 1948.
- [3] Bureau of Public Roads, U. S. Department of Commerce, "Hydraulic Design of Highway Culverts," to be published.
- [4] Howe, J. W., and Metzler, D. E., "Culvert Diameters in Relation to Upstream Channel Storage for Small Watersheds in Iowa," Bulletin No. 2, Iowa Highway Research Board, 1954.

SYMBOLS

A	cross sectional area, ft^2
α	correction factor for non uniform velocity distribution
b	height or width of square culvert, ft
γ	specific weight, lb/ft^3
C	Chezy discharge coefficient
C_c	contraction coefficient
C_d	discharge coefficient
C_i	inlet loss coefficient
C_o	outlet loss coefficient

d	diameter of orifice, ft
d_c	critical depth, ft
D	diameter of circular pipe, ft
f	pipe resistance coefficient
F	Froude number
g	acceleration of gravity, ft/sec ²
h	piezometric head $p/\gamma + z$, ft
	differential head, ft
Δh	difference in level of invert at entrance and effective tailwater level
H	total head, $p/\gamma + z + V^2/2g$, ft
H_L	lost head, ft
K	proportionality factor
L	length of weir crest, ft
	a characteristic length
	length of barrel, ft
n	Manning roughness factor
ν	kinematic viscosity, ft ² /sec
p	pressure intensity, lb/ft ²
P	wetted perimeter, ft
π	ratio: circumference of circle to diameter
Q	discharge rate, ft ³ /sec
R	hydraulic radius, A/P
ρ	mass density, slugs/ft ³
R	Reynolds number
σ	surface tension, lb/ft
S	slope
v	velocity at a point, ft/sec
V	mean velocity at section, ft/sec
V_c	critical velocity, ft/sec
W	width of free surface, ft
W	Weber number
y	depth at centerline, ft
z	elevation, ft
z_0	elevation of channel bed

COLLEGE OF ENGINEERING STATE UNIVERSITY OF IOWA

Complete undergraduate courses are offered in Chemical, Civil, Commercial, Electrical, and Mechanical Engineering, as well as graduate courses in these fields and in Mechanics and Hydraulics. For detailed information, application may be made to T. H. McCarrel, Registrar.

IOWA INSTITUTE OF HYDRAULIC RESEARCH

The Institute was organized to co-ordinate the talents and facilities available at the State University of Iowa for the investigation of problems in the fields of fluid mechanics, hydrology, and hydraulic engineering. Through this medium the University has co-operated with government agencies, technical societies, and industrial concerns throughout the country. Correspondence regarding the services of the Institute should be addressed to Hunter Rouse, Director.

STUDIES IN ENGINEERING

Bulletin 1. "The Flow of Water Through Culverts," by D. L. Yarnell, F. A. Nagler, and S. M. Woodward, 1926. 128 pages, 26 figures, 23 plates. Out of print.

Bulletin 2. "Laboratory Tests on Hydraulic Models of the Hastings Dam," by Martin E. Nelson, 1932. 72 pages, 40 figures. Out of print.

Bulletin 3. "Tests of Anchorages for Reinforcing Bars," by Chesley J. Posey, 1933. 32 pages, 18 figures, price \$0.50.

Bulletin 4. "The Physical and Anti-Knock Properties of Gasoline Blends," by Theodore R. Thoren, 1934. 32 pages, 13 figures, price \$0.35.

Bulletin 5. "The Transportation of Detritus by Flowing Water — I," by F. T. Mavis, Chitty Ho, and Yun-Cheng Tu, 1935. 56 pages, 15 figures, price \$0.50.

Bulletin 6. "An Investigation of Some Hand Motions Used in Factory Work," by Ralph M. Barnes, 1936. 60 pages, 22 figures. Out of print.

Bulletin 7. "A Study of the Permeability of Sand," by F. T. Mavis and Edward F. Wilsey, 1936. 32 pages, 12 figures, price \$0.35.

Bulletin 8. "Radiation Intensities and Heat-Transfer in Boiler Furnaces," by Huber O. Croft and C. F. Schmarje, 1936. 32 pages, 17 figures, price \$0.35.

Bulletin 9. "A Summary of Hydrologic Data, Ralston Creek Watershed, 1924-35," by F. T. Mavis and Edward Soucek, 1936. 72 pages, 25 figures, price \$0.50.

Bulletin 10. "Report on Hydraulics and Pneumatics of Plumbing Drainage Systems — I," by F. M. Dawson and A. A. Kalinske, 1937. 32 pages, 5 figures, price \$0.35.

Bulletin 11. "The Transportation of Detritus by Flowing Water — II," by F. T. Mavis, Te-Yun Liu, and Edward Soucek, 1937. 32 pages, 8 figures, price \$0.35.

Bulletin 12. "Studies of Hand Motions and Rhythm Appearing in Factory Work," by Ralph M. Barnes and Marvin E. Mundel, 1938. 64 pages, 24 figures. Out of print.

Bulletin 13. "Hydraulic Tests of Small Diffusers," by F. T. Mavis, Andreas Luksch, and Hsi-Hou Chang, 1938. 32 pages, 16 figures, price \$0.25.

Bulletin 14. "A Study in Flood Waves," by Elmer E. Moots, 1938. 32 pages, 7 figures, price \$0.25.

Bulletin 15. "The Road Map of Hydraulic Engineering in Iowa," by E. W. Lane and Edward Soucek, 1938. 16 pages, 4 figures, price \$0.25.

Bulletin 16. "A Study of Hand Motions Used in Small Assembly Work," by Ralph M. Barnes and Marvin E. Mundel, 1939. 68 pages, 33 figures, price \$0.50.

Bulletin 17. "A Study of Simultaneous Symmetrical Hand Motions," by Ralph M. Barnes and Marvin E. Mundel, 1939. 40 pages, 15 figures, price \$0.40.

Bulletin 18. "Percolation and Capillary Movements of Water Through Sand Prisms," by F. T. Mavis and Tsung-Pei Tsui, 1939. 32 pages, 13 figures, price \$0.25.

Bulletin 19. "Two Decades of Hydraulics at the University of Iowa, Abstracts of Theses, Publications, and Research Reports, 1919-1938," edited by F. T. Mavis, 1939. 84 pages, price \$0.50.

Bulletin 20. "Proceedings of Hydraulics Conference," edited by J. W. Howe, 1940. 260 pages, 84 figures. Out of print.

Bulletin 21. "Studies of One- and Two-Handed Work," by Ralph M. Barnes, Marvin E. Mundel, and John M. MacKenzie, 1940. 68 pages, 31 figures, price \$0.50.

Bulletin 22. "The Study of the Effect of Practice on the Elements of a Factory Operation," by Ralph M. Barnes and James S. Perkins with the assistance and collaboration of J. M. Juran, 1940. 96 pages, 34 figures. Out of print.

Bulletin 23. "An Annotated Bibliography of Fishways," by Paul Nemenyi, 1941. 72 pages, 12 figures. Out of print.

Bulletin 24. "An Investigation of Fishways," by A. M. McLeod and Paul Nemenyi, 1941. 64 pages, 15 figures, 6 plates, price \$0.50.

- Bulletin 25. "The Electrostatic Effect and the Heat Transmission of a Tube," by Melvin R. Wahlert and Huber O. Croft, 1941. 40 pages, 10 figures, price \$0.40.
- Bulletin 26. "Investigations of the Iowa Institute of Hydraulic Research," 1939-1940, edited by J. W. Howe, 1941. 64 pages, 15 figures, price \$0.40.
- Bulletin 27. "Proceedings of the Second Hydraulics Conference," edited by J. W. Howe and Hunter Rouse, 1943. 352 pages, 167 figures. Out of print.
- Bulletin 28. "The Preparation of Stoker Coals from Iowa Screenings," by H. L. Olin, 1942. 64 pages, 22 figures, price \$0.50.
- Bulletin 29. "Study of Transportation of Fine Sediments by Flowing Water," by A. A. Kalinske and C. H. Hsia, 1945. 40 pages, 18 figures, price \$0.40.
- Bulletin 30. "The Iowa Institute of Hydraulic Research," 1946. 80 pages, 36 figures, price \$0.50.
- Bulletin 31. "Proceedings of the Third Hydraulics Conference," edited by J. W. Howe and John S. McNown, 1947. 332 pages, 163 figures, price \$2.50.
- Bulletin 32. "Cavitation and Pressure Distribution—Head Forms at Zero Angle of Yaw," by Hunter Rouse and John S. McNown, 1948. 70 pages, 53 figures, price \$1.00.
- Bulletin 33. "Third Decade of Hydraulics at the State University of Iowa," edited by M. C. Boyer, 1949. 84 pages, 8 figures, price \$0.50.
- Bulletin 34. "Proceedings of the Fifth Hydraulics Conference," edited by John S. McNown and M. C. Boyer, 1953. 302 pages, 139 figures, price \$3.50.
- Bulletin 35. "Free-Streamline Analyses of Transition Flow and Jet Deflection," edited by John S. McNown and Chia-Shun Yih, 1953. 82 pages, 37 figures, price \$1.00.
- Bulletin 36. "Proceedings of the Sixth Hydraulics Conference," edited by Louis Landweber and Philip G. Hubbard, 1956. 276 pages, 131 figures, price \$3.50.
- Bulletin 37. "Operating Manual for the IHR Hot-Wire and Hot-Film Anemometers," by Philip G. Hubbard, 1958. 40 pages, 16 figures, price \$1.00.
- Bulletin 38. "Hydraulics of Box Culverts," edited by Donald E. Metzler and Hunter Rouse, 1959. 34 pages, 23 figures, price \$1.00.