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Nonlinear Finite Element Study of Piles in Integral Abutment Bridges

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Final Report

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ABSTRACT

The highway departments of the states which use integral abutments in bridge design were contacted in order to study the extent of integral abutment use in skewed bridges and to survey the different guidelines used for analysis and design of integral abutments in skewed bridges. The variation in design assumptions and pile orientations among the various states in their approach to the use of integral abutments on skewed bridges is discussed. The problems associated with the treatment of the approach slab, backfill, and pile cap, and the reason for using different pile orientations are summarized in the report.

An algorithm based on a state-of-the-art nonlinear finite element procedure previously developed by the authors was modified and used to study the influence of different factors on behavior of piles in integral abutment bridges. An idealized integral abutment was introduced by assuming that the pile is rigidly cast into the pile cap and that the approach slab offers no resistance to lateral thermal expansion. Passive soil and shear resistance of the cap are neglected in design.

A 40-foot H pile (HP 10 × 42) in six typical Iowa soils was analyzed for fully restrained pile head and pinned pile head. According to numerical results, the maximum safe length for fully restrained pile head is one-half the maximum safe length for pinned pile head. If the pile head is partially restrained, the maximum safe length will lie between the two limits.

The numerical results from an investigation of the effect of predrilled oversized holes indicate that if the length of the pre-

drilled oversized hole is at least 4 feet below the ground, the vertical load-carrying capacity of the H pile is only reduced by 10 percent for 4 inches of lateral displacement in very stiff clay. With no predrilled oversized hole, the pile failed before the 4-inch lateral displacement was reached. Thus, the maximum safe lengths for integral abutment bridges may be increased by predrilling.

Four different typical Iowa layered soils were selected and used in this investigation. In certain situations, compacted soil (> 50 blow count in standard penetration tests) is used as fill on top of natural soil. The numerical results showed that the critical conditions will depend on the length of the compacted soil. If the length of the compacted soil exceeds 4 feet, the failure mechanism for the pile is similar to one in a layer of very stiff clay. That is, the vertical load-carrying capacity of the H pile will be greatly reduced as the specified lateral displacement increases.

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1. INTRODUCTION

1.1. Statement of the Problem

The increasing popularity of integral abutment design for bridges has been recognized by many state highway agencies. In Iowa, the first integral abutment on a concrete bridge was built in 1965 [1]. The current length limitations in Iowa are based on a simple theoretical analysis of the effects of thermal expansion and contraction of bridges on piling stresses [2]. After yearly inspections over a period of several years, it has been found that no serious damage or distress is evident in bridges with integral abutments. At the present time, only short and moderate length bridges without joints have been adopted in Iowa. Iowa Department of Transportation (DOT) bridge engineers suspect that the use of long bridges without joints may result in damage to abutments because of the relatively large displacements associated with annual temperature changes. The use of expansion devices and rockers usually eliminates the problems caused by thermal expansion and contraction in long bridges. However, providing expansion devices on bridges not only increases the initial cost of construction, but also increases associated maintenance costs. An integral abutment design eliminates the use of expansion devices and rockers; the elimination of these items reduces both the initial cost of construction and the high cost of maintenance.

The safe length of integral abutment bridges is believed to be controlled by the piling ultimate vertical load-carrying capacity. The vertical load capacity can be reduced by large lateral movements resulting from thermal expansion and contraction [3]. If the safe length of integral abutment bridges can be increased on the basis of theoretical analysis, the economic advantage of integral abutments may be realized for longer bridges.

Integral abutment bridges fall into two categories: non-skewed and skewed. Site conditions determine whether or not a skewed bridge is necessary. Anyone who has been involved in the design and detailing of a skewed bridge rapidly realizes that skews present serious detailing and analysis problems. Also, for a sharply skewed bridge, design problems such as load distribution to longitudinal members and design of diaphrams are encountered. Load distribution factors furnished by the American Association of State Highway and Transportation Officials (AASHTO) do not necessarily give conservative values [4].

Bridge engineers have discovered several peculiarities in skewed concrete box-girder bridge behavior. Shear cracks develop in exterior girders near obtuse support corners, and transverse cracks develop at midspan of certain highly skewed, prestressed bridge decks. Tendencies toward uplift have been noted at acute support corners of long span bridges [5]. For skewed bridges with integral abutments, the movements caused by thermal expansion and contraction are more complex than they are for non-skewed bridges with integral abutments. These thermal-induced movements involve not only lateral direction, but transverse and diagonal direction as well.

Although skewness introduces analysis and design complications, it also affords design advantages. Since support reactions tend to concentrate at obtuse corners, reductions occur in effective structural spans. The thick end diaphrams act to reduce longitudinal stresses in the superstructure. Hence, possibilities exist for material savings by refining structural analysis [5].

1.2. Background

Prior to World War II, most bridges with an overall length of 50 feet or more were constructed with some form of expansion joints. Periodic inspection of these bridges revealed that expansion joints tended to freeze and close, and did not operate as intended. After observing the successful performance of many older bridges either constructed without joints or performing with inoperative joints, several states have elected to design and construct short and moderate length bridges without expansion joints.

Today more than half of the state highway agencies have developed design criteria for bridges without expansion joints. These design criteria are based on years of experience in integral abutment bridge design. This development led to wide variations in design criteria for integral abutments bridges from state to state. In July 1972, South Dakota State University issued a report summarizing the results of an investigation on stresses induced by thermal movements in the girder and upper portion of steel bearing piles of integral abutment-type bridges [1]. In November 1981, North Dakota State University issued a report on a study which was conducted to observe lateral movements resulting from annual temperature variations and monitor the temperature-induced piling stresses [6]. In February 1982, Iowa State

University published a report entitled "Nonlinear Pile Behavior in Integral Abutment Bridges" by A. M. Wolde-Tinsae, L. F. Greimann, and P. S. Yang. This report summarized the variation in design assumptions and length limitations among the various states in their approach to the use of integral abutments. Also, an algorithm based on a state-of-the-art nonlinear finite element procedure was developed and used to study piling stresses and pile-soil interaction in integral abutment bridges.

1.3. Objective and Scope

The present research is part of an ongoing research project on nonlinear pile behavior in integral abutment bridges [3]. As part of this investigation, the highway departments of different states which use integral abutments in bridge design were contacted in order to study the extent of integral abutment use in skewed bridges, and to survey the different guidelines used for analysis and design of integral abutments for skewed bridges. The influence of different factors on behavior of integral abutment bridges, e.g., restraint of the pile head, predrilled oversized hole effect, and pile behavior in different soil profiles, have been investigated by means of a state-of-the-art nonlinear finite element model. The results and conclusions are summarized in this report.

2. LITERATURE REVIEW

2.1. Analytical Studies

The piles of integral abutment bridges subjected to lateral movements caused by thermal expansion and contraction can be treated as piles subjected to lateral loads. The problem of a pile subjected to lateral loading is one of a class of problems concerning the interaction of soils and structures. The solution of such problems generally involves the use of iterative techniques since soil response is a nonlinear function of the structure's deflection.

In the past, analysis and design of laterally loaded piles were primarily empirical, based on data from full-scale tests of laterally loaded piles [7,8]. However, in recent years, extensive research and development have been undertaken to predict theoretically the behavior of a laterally loaded pile [9-11].

In a previous study by the authors [3], an algorithm based on a state-of-the-art nonlinear finite element procedure was developed and used to study piling stresses and pile-soil interaction in integral abutment bridges. The finite element idealization consists of a one-dimensional idealization for the pile and nonlinear springs for the soil. Important parameters for analysis are pile and soil characteristics. On the basis of a literature review, it was decided to represent pile characteristics by beam-column elements with geometric and material nonlinearities, and soil characteristics by lateral resistance-displacement (p-y), load-slip (f-z), and load-settlement (q-z) curves. An idealized soil model (modified Romberg-Osgood Model) was introduced to obtain the tangent stiffness of the nonlinear spring elements. The report includes a discussion of details pertaining to the derivation of the incremental finite element updated Langrangian formulation, including material and geometric nonlinearities, and the solution algorithm, involving incremental and iterative techniques used in the study [3]. A computer program developed for this study (Yang 5) is provided in Appendix 8.2.

2.2. Experimental Studies

Numerous experimental research projects on piles subjected to vertical and/or lateral loading in the laboratory or field have been performed in recent years. Seed and Reese studied small, displacement-type friction pile which was driven into a nonsensitive clay. In that study, the load-distribution curves and load-slip curves (f-z curves) for a friction pile were first defined [12]. Matlock [13], Reese and Welch [14], and Reese, Cox, and Koop [15] also performed experimental work on soft clay, stiff clay, and sand, respectively, to predict lateral resistance-displacement curves (p-y curves) for laterally loaded piles. A curve describing the load-settlement behavior of the pile's tip was given by Vijayvergiya [16]. Numerous methods exist for predicting these curves for different soil types. A brief discussion over some of these methods is given in our previous report [3].

Many tests on instrumented piles subjected to vertical and/or lateral loading in the field or laboratory have been performed [17-20]. In March 1973, a full-scale model representing the end portion of a typical highway bridge was constructed and tested in four construction stages by South Dakota State University [1]. During each stage, the test specimen was subjected to a series of predetermined longitudinal movements via hydraulic jacks to simulate expansion and contraction caused by temperature changes. In August of 1979, an operational county road bridge near Fargo, North Dakota was instrumented and monitored for temperature-induced stresses by North Dakota State University [6]. This study is being conducted by J. Jorgenson, Chairman of the Civil Engineering Department, and is sponsored by the State Highway Department. During one year of observation, monthly readings were taken on the length of the bridge, the gap between backfill and backside of abutment, etc. A preliminary report was published in November 1981. A brief summary of these two tests is given below.

2.2.1. South Dakota Tests

A full-scale model representing the end portion of a typical highway bridge was constructed and tested in four stages:

Stage I: The girders were erected and welded to the bearing piles (HP 10 × 42). See Fig. 1.

Stage II: The integral abutment was built and the concrete attained its design strength.

Stage III: The deck slab was poured and the concrete attained its design strength.

Stage IV: The granular backfill was placed.

For each stage the test specimen was subjected to a series of predetermined longitudinal movements simulating thermal movements of

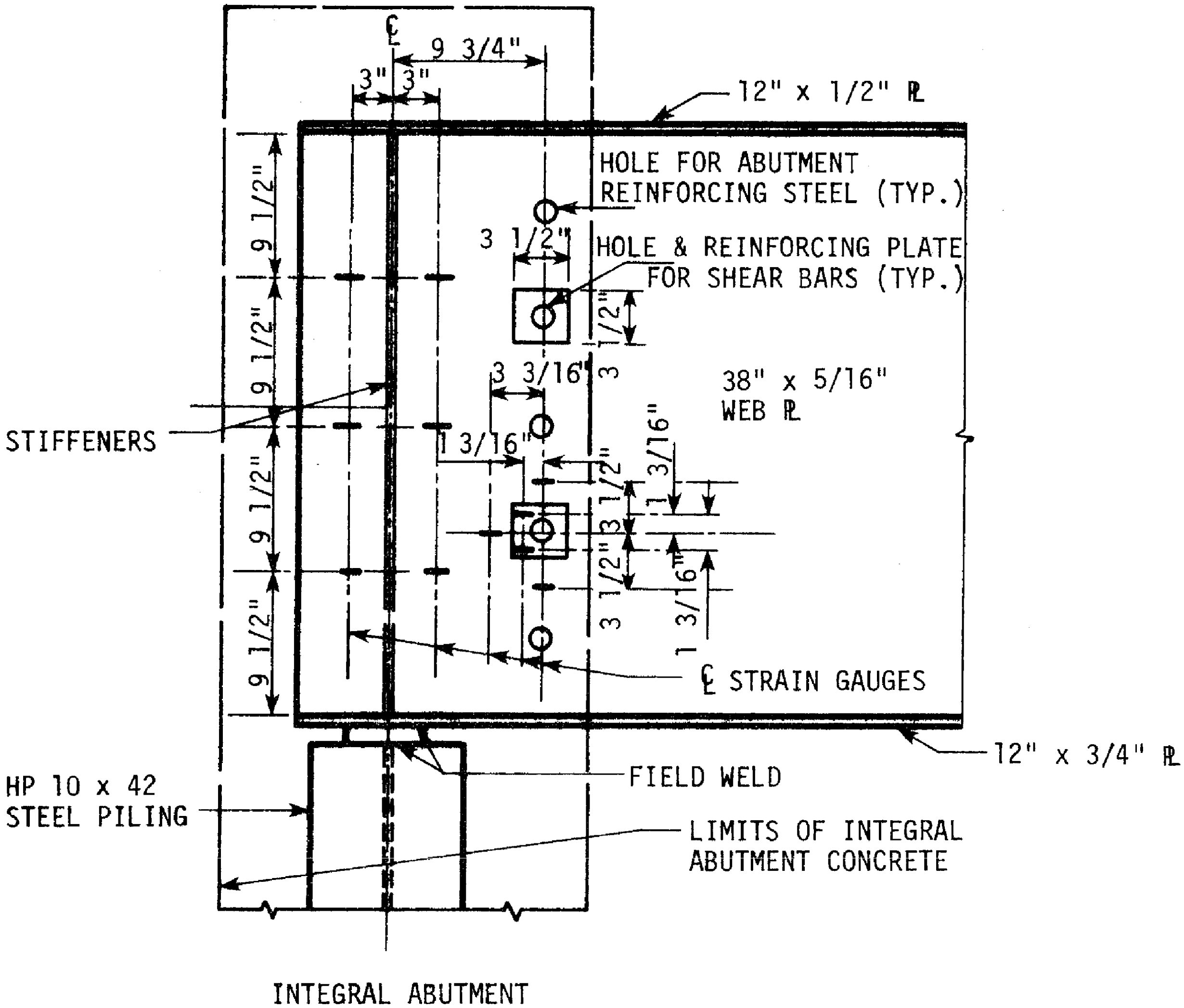

Fig. 1. Girder and piling details at integral abutment end.

expansion and contraction caused by temperature changes. Figure 2 shows the elevation and part plane view of the test specimen.

The main objective of this investigation was to study and evaluate the effects of thermal movements on integral abutment bridges during early and final construction stages. Particular emphasis was placed on evaluation of the resultant state of stress for the upper portion of the steel piling and portions of the girders immediately adjoining the integral abutment.

The details of specimen instrumentation and the arrangement of data acquisition are given in the report [1]. In order to simulate, within practical limits, possible extreme field temperature conditions and variations, two main test cycles were used for all stages, namely an expansion cycle and a contraction cycle. In both cycles the rate of applied movements at the jacking end of the girders relative to the fixed jacking abutment was established at 0.250 inch per hour. The maximum predetermined longitudinal movements were in the range of ±1.000 inch. The movements used in this study may be applied to an infinite number of span length combinations and initial temperature starting points.

The following results have been obtained from the observed data of this study:

1. Stage I: High stresses were not apparent on any part of the test specimen. The induced bending moment, however, was very small and produced only negligible flexural stresses in the girder. The flexural stresses in the steel piling were much more pronounced than the flexural stresses in the girder

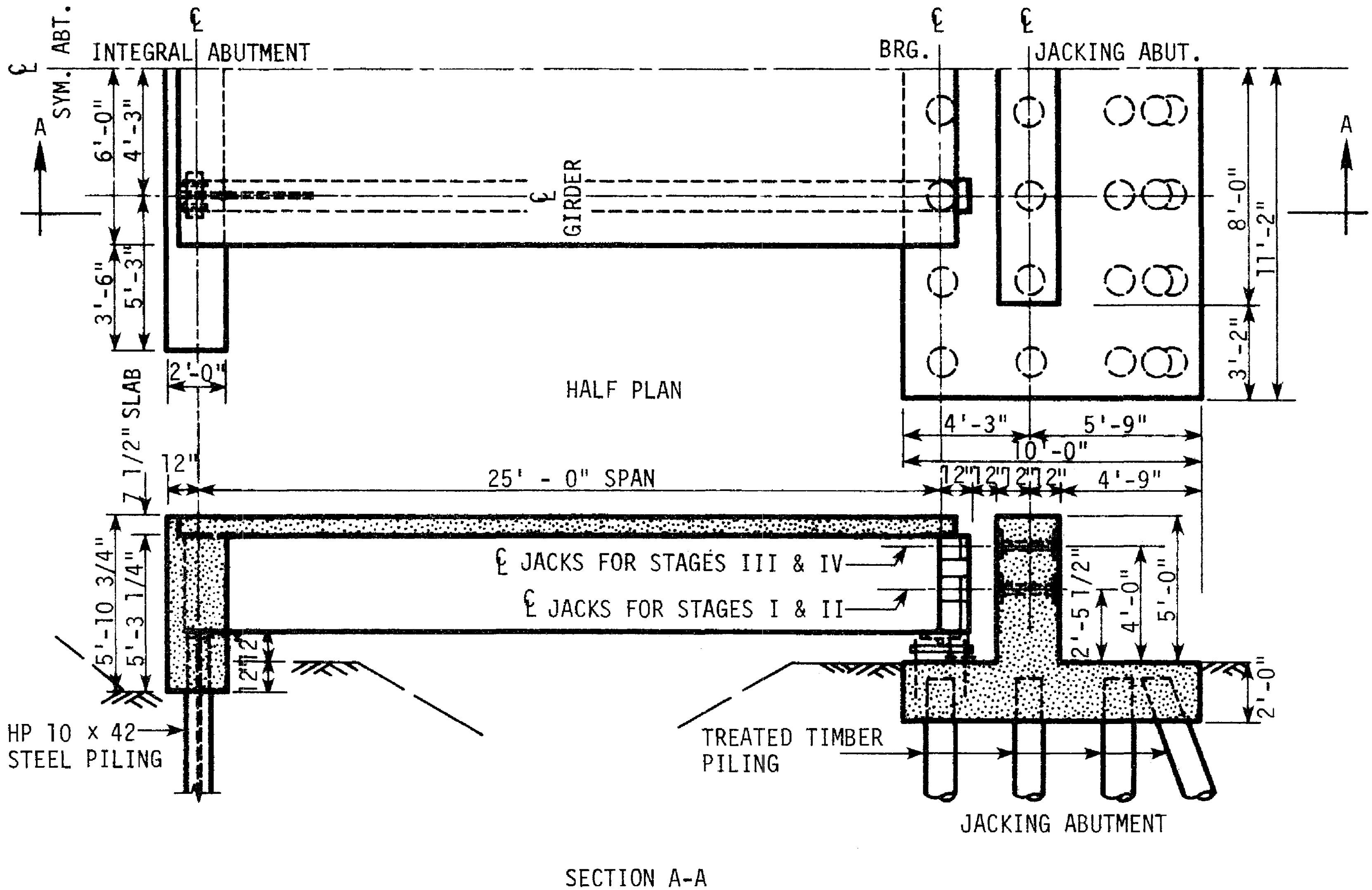

Fig. 2. General drawing of test model.

because of small section modulus about the weak axis of the pile. The maximum piling stresses were much less than the yielding stress of the steel pile.

2. Stage II: A much larger induced bending moment was observed in the girder near the integral abutment, and the induced flexural stresses by this moment were no longer negligible. The stresses in the steel piling were so large that definite yielding at the edges of the pile flange was observed. It was also noted that the recorded largest stress was only three inches below the bottom of the concrete abutment.
3. Stage III: The addition of a concrete deck slab with the girder and the integral abutment greatly reduced the rotation of the integral abutment. This result is attributable to the greater rigidity resulting from the composite action of the girder and the deck slab. The piling stresses were also larger than those of Stage II, and yielding was indicated at the edges of the steel pile flanges. Because of the increased rigidity, the induced bending moment in the girder was sizable and would have to be taken into account if the true girder stress was desired.
4. Stage IV: The structural response of the test specimen depends heavily upon the physical conditions of the backfill. As indicated by the test results, the stresses at various parts of the test specimen in this stage were of greater magnitude during the expansion cycle than during the contraction cycle. This result is attributed to the passive soil

resistance of the backfill to expansion, and to the fact that active soil pressure actually helps contraction. The induced bending moments and end shears were appreciable and could no longer be ignored; the flanges of the steel piling were yielding; and composite actions between the concrete slab and the steel girder did occur as indicated by the girder stresses.

The following conclusions were made from the test results. It can be seen that pile yielding would occur for a one-inch movement in all stages except Stage I. It is known that full plastic moment can be developed in the pile about its weak axis if the axial stress does not exceed 40% of the yielding stress. Stability of the pile and design safety factor must be considered at this point. A conservative 1/2-inch allowable lateral movement is required in integral abutment steel bridges to avoid pile yielding. The frozen and unfrozen condition of the backfill will cause a totally different stress resultant to develop in the girder. This problem, however, even if it cannot be eliminated, can at least be minimized by providing a proper drainage system in the backfill.

Special attention must be paid to the selection of an approach slab system. Out of many possible choices, the most favorable is the one which has little or no restriction to rotation or translation of the integral abutment and does not tend to compact the backfill material as a result of moving traffic loads. Compaction of the backfill by moving traffic increases earth resistance to expansion movement of the integral abutment and results in higher stresses in the girder [1].

2.2.2. North Dakota Tests

A 450-foot prestressed concrete box beam bridge with integral abutments and piers on a 0 degree skew was built in 1979 near Fargo, North Dakota and was instrumented to monitor temperature induced stresses [6]. There are no expansion joints on the bridge; however, expansion joints are located in the approach slab about twenty feet from each end of the bridge. A section through the abutment is shown in Fig. 3. As is the case with the piers, the pile cap, diaphram, concrete girder, and concrete deck are reinforced to act as a single monolithic unit. The pile is oriented with its weak axis in the longitudinal plane of the bridge and is reinforced within the abutment cap and diaphram to transmit the full plastic moment of the pile. In anticipation of thermal induced changes in the length of the bridge, a pressure relief system was set up between the back side of the abutment and the backfill soil. Corrugated metal was used to retain the granular backfill behind the pressure relief strips (also shown in Fig. 3).

In the design of the bridge, an effort was made to permit lateral movement of the abutment piles without causing any significant resistance to movement. The soil was predrilled with a 16-inch diameter hole to a depth of about 20 feet. Prior to driving the pile, a 2-inch thick layer of compressible material was placed on each side of the web of the pile. After driving, the remaining space around the pile was filled with sand. A detail of the pile and compressible material is shown in Fig. 4.

Figure 5 shows the detail of the expansion joint in the approach slab. One end of the approach slab is tied into the bridge abutment

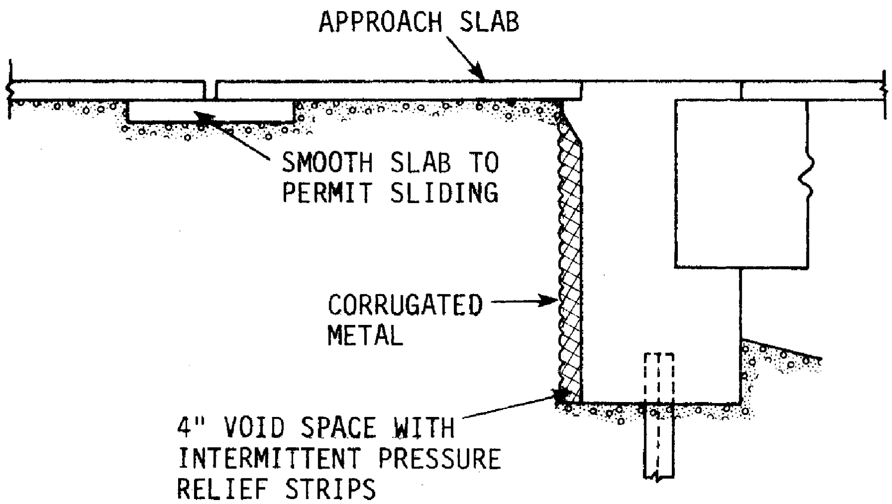

Fig. 3. Integral abutment system with pressure relief strips.

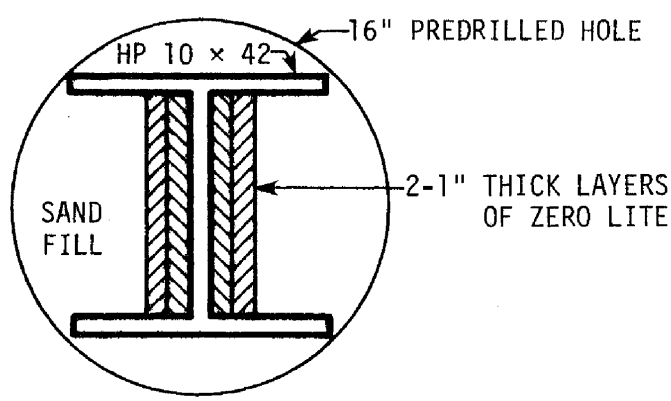

Fig. 4. Detail for abutment piles.

with the other end set on the smooth surface of a supporting slab. As the bridge changes in length, the expansion joint will open and close.

Bridge measurements were taken during the period from August 8, 1979 through September 7, 1980. The bridge measurements included (1) change in bridge length, (2) movements between abutments and soil backfill, (3) vertical movement of abutments and piers, (4) displacement of abutment piles, (5) measurement of stresses in piles, and (6) measurement of concrete temperature. Stable readings were observed in the laboratory check of gages and again in the fall after the abutment was poured. During the following spring the area was flooded to a level above all of the strain gages. Following the flood the readings for most gages would not stabilize. Due to these erratic readings, measurement of stresses in the piles could not be obtained from the electrical resistance gage data. An alternative way to estimate the pile stresses was based on pile and abutment displacement.

After an analysis of the data obtained for a one year period of observation, the following conclusions were made:

- The maximum change in the bridge's length resulting from thermal change can be calculated by using a temperature change equal to $\Delta T = T_1 - T_2 + (T_3 - T_1)/3$ where

  T_1 = at dawn air temperature on the hottest day.

  T_2 = at dawn air temperature on the coldest day.

  T_3 = maximum air temperature on the hottest day.
- The above change in bridge length agrees well with changes in length measured by tape measurement and measurements of openings in expansion joints.

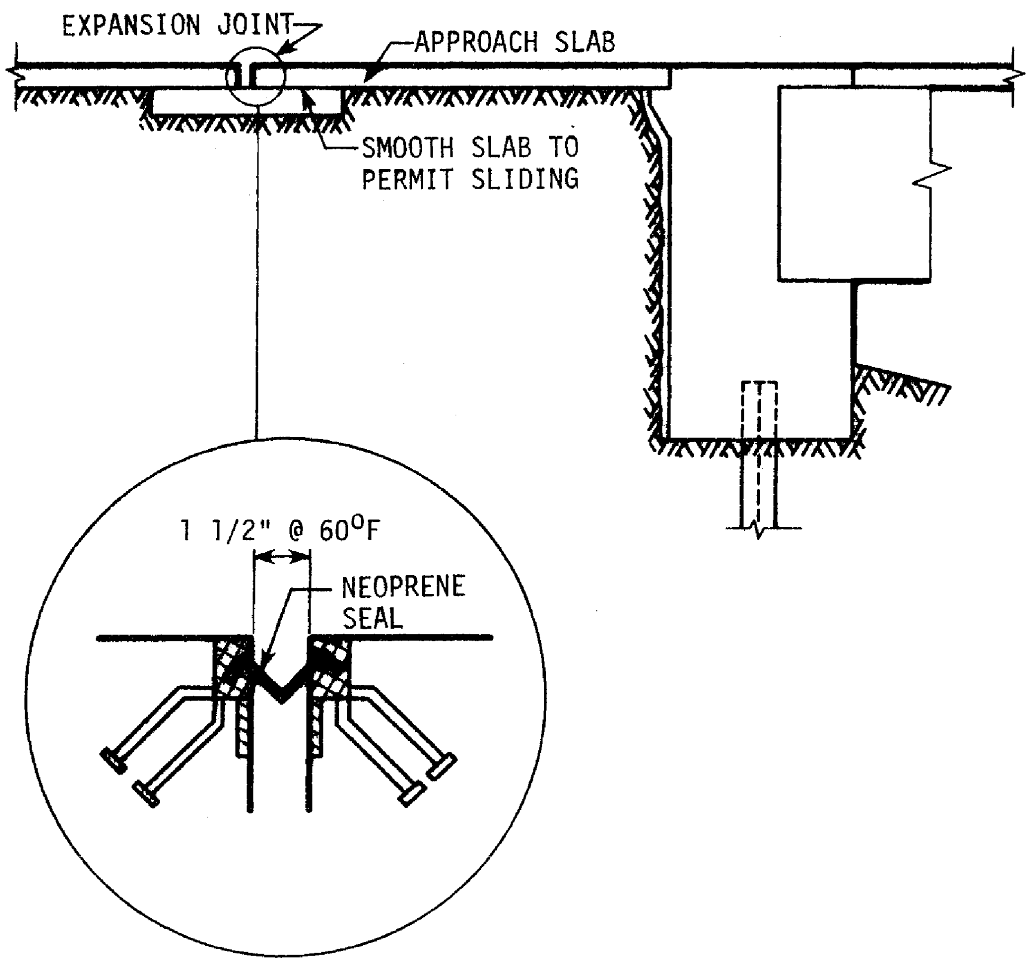

Fig. 5. Expansion joint in approach slab.

- The change in bridge length did not result in equal movement at the two ends of the bridge. At the point of maximum bridge shortening, the south abutment moved in 1.96 inches and the north abutment moved in 0.76 inches from their initial August positions.
- In one year the gap between the abutment and backfill closed about one-half inch on the north abutment and three-fourths of an inch on the south abutment.
- The vertical movements of the abutment and piers were nearly zero.
- The method of measuring pile stresses failed; however, an analytical model was developed to predict stresses in the piles caused by movements of the abutments.

The model developed to predict piling stresses caused by abutment movement in this investigation (North Dakota research) is similar to the model used in the previous report (Iowa State research). The main difference between the two models is that the one used by Iowa State researchers assumes nonlinear soil behavior while the one used by North Dakota researchers assumes linear soil behavior [6].

3. SURVEY OF CURRENT PRACTICE FOR SKEWED BRIDGES WITH INTEGRAL ABUTMENTS

As background for a theoretical investigation to establish tentative recommendations on maximum safe lengths and skew angles for concrete and steel skewed bridges with integral abutments, a survey of the different states was made to obtain information on the design and performance of integral abutment skewed bridges. This chapter summarizes the findings of the survey including

- Various design criteria and limitations used.
- Typical pile orientations used in bridge design by the different states, and types of analysis used for thermal expansion and contraction.
- Assumptions made regarding selected design parameters.
- Specific construction details used such as approach slab, backfill, and pile cap.
- Long-term performance of skewed bridges with integral abutments.

3.1. Method of Investigation

Surveys concerning the use of integral abutments have previously been conducted [3,6]. Responses indicate that most highway department agencies establish their own limitations and criteria in designing integral abutments. The bases of these limitations and criteria are primarily empirical.

Today, the use of integral abutments in design has been accepted by 28 state highway departments and the Direct Construction Office, Region 15, Federal Highway Administration [3]. A survey questionnaire was prepared in cooperation with the Office of Bridge Design, Highway Division, Iowa Department of Transportation (DOT) to obtain information concerning the use and design of skewed bridges with integral abutments. A copy of the questionnaire is shown in Appendix 8.1.

The survey questions were directed at pile orientations in the integral abutments. The states were asked to state what structural assumptions were being made in determining fixity conditions on pile-head and directions of thermal expansion and contraction of the integral abutments of skewed bridges. In addition, questions included the treatments of approach slab, backfill, and pile cap. Sketches of different types of pile orientations in integral abutments were also included in the questionnaire.

3.2. Trends of Response

Of the 28 responses received, 26 indicated the use of integral-type abutments on skewed bridges. Among these states, Virginia has designed its first integral abutment skewed bridge with a small skew (10°) and a relatively small anticipated movement at each abutment (± 3/8"). The states of Connecticut and Oklahoma indicated that they do not use integral abutments on skewed bridges. While Connecticut has not constructed any integral abutments on a skew, it has constructed one non-skewed integral abutment bridge. Oklahoma, in contrast, considered integral abutments on skews inappropriate because of integral displacement.

One of the purposes of this study is to present methods of analysis and design details of integral abutments on skewed bridges. Many of the states using integral abutments on skewed bridges provide useful empirical experience to shed some light on this problem.

Summary of Responses

Following is a discussion, keyed to survey question numbers, of the responses received from states using integral abutments on skewed bridges. (Also see Appendix 8.1.)

Question No. 1. The pile orientations in the integral abutments on skewed bridges shown in the first survey question can be classified into two parts: (1) the web of the pile perpendicular or parallel to roadway center line, e.g., Type (1a) and (1b), respectively; (2) the web of the pile parallel and perpendicular to center line of the abutment, e.g., Type (2a) and (2b), respectively. The responses indicated that six states use Type (1a) orientation, one state uses Type (1b), ten states use Type (2a), and fifteen states use Type (2b). In addition, three states use circular piles, Type (3), in integral abutment on skewed bridges.

One major difference between skewed and non-skewed integral abutment bridges is that when both are subjected to thermal expansion and contraction, the former will experience thermal-induced biaxial bending stresses on piles if the pile orientation specified is of Type (2a) or (2b). This becomes a 3-D analysis problem. For Types (1a), (1b), and (3), pile orientations will have the same thermal effects as those for non-skewed integral abutment bridges [3]. The responses showed that

15 of 26 states have adopted the pile orientations so that bending will be primarily about the strong axis.

A second questionnaire was sent out to investigate whether any theoretical, experimental, or empirical bases exist for the orientation of the piles and to find out if any distresses or problems associated with orientation of the piles have occurred. The responses received from the second questionnaire indicated that most states do not have any clear theoretical, experimental, or empirical bases. Idaho officials assumed some creep in the soils surrounding the piles and also assumed that a redistribution of stresses will occur since thermal forces are generally applied gradually. Also, the restraint provided by the integral abutment was assumed to reduce the magnitude of the thermal movement; orienting the piles with the strong axis parallel to the center line of the bearings was assumed to give more rigidity for earthquake loads when liquification of embankment is anticipated. Vermont oriented the piles to resist the force of earth pressure from the abutment backfill rather than the force of thermal expansion. California explained its policy of orienting the web of piles perpendicular to the center line of the abutment (see Appendix 8.1) as follows: for a square bridge, such orientation of piles results in bending about the strong axis of the piles because of both thermal forces and active soil pressure. When the bridge is skewed, however, temperature forces act along the center line of the roadway, not parallel to the pile web, and active soil pressure acts against the strong axis of the pile. A particular concern is rotational action caused by the active soil pressure on skewed bridges. Temperature effects are somewhat compensated for by predrilling for driven piles and filling the voids with pea gravel or sand [21].

Colorado replied that they were unaware of any distress in the piling. In a few cases, with cast-in-place post-tensioned bridges with integral abutments, cracks have been detected in the abutment wall at the intersection of the superstructure and the abutment. The state suspected that the cracks resulted primarily from movements of the superstructure caused by elastic shortening and creep from the post-tensioning forces. North Dakota has been building bridges for about 18 years using this method and so far is unaware of any problems. According to Iowa bridge engineer H. Gee [2], pile orientation Type (1a) is not considered in design because of construction work difficulty in arranging the reinforcement in the integral abutments. Thermal-induced biaxial bending stresses on piles can be avoided by using Type (3) circular pipe piles. The major disadvantages are that the vertical bearing capacities of these piles are usually less than those of the steel H piles, and they are stiffer than H piles about the weak axis.

Question No. 2. The second survey question revealed the following. (1): Two states indicated that a roller assumption was made at the pile top; eight reported a pinned assumption; one assumed partial fixity; and eight states assumed a totally fixed pile top. These assumptions were actually based on the restraint conditions on the pile top. In Iowa, the pile top is completely restrained by spiral reinforcement in the pile cap, and total fixity is assumed . For a pinned assumption, the top portion of piling is enclosed with a flexible material before casting in the concrete abutment [2]. (2) and (3): Only a few states consider thermal, shrinking and soil pressure forces when calculating pile loads. For a long integral abutment skewed bridge, temperature-induced stresses become very critical to the piling load capacities. If pile orientations (2a) and (2b) are adopted, the thermal expansion or contraction along the roadway center can be divided into two components, one parallel to pile web (transverse), the other perpendicular to pile web (longitudinal). Thus, the piles in integral abutment skewed bridges will be subjected to biaxial bending resulting from thermal movement. It is also possible that in the diagonal direction of long skewed bridges, diagonal thermal expansion and contraction will cause serious problems. However, none of the states indicated concern about this. Following are some of the remarks made regarding thermal effects on integral abutments on skewed bridges:

- Assume that the pile is fixed a certain depth below the bottom of the pile cap and any thermal movement is accomplished by bending in the pile.
- Thermal expansion parallel to the pile cap can be resisted by the friction force between the backfill and the end wall.
- The batter selected piles are adopted in the integral abutments to resist thermal movement.
- Shear keys are used on the bottom of the pile cap to prevent lateral movement of the pile cap on extreme skews ($40° \pm$).
- If the bridge design has a small skew ($\leq 10°$) and a relatively small anticipated movement at each abutment ($\pm$ 3/8"), no

special consideration need be given beyond that of a 0° skew condition.

Question No.3. Most states indicated that a free-draining granular material is used as backfill behind the abutment. One state uses 1-1/2 feet of porous backfill from subgrade to the bottom of the integral abutment along with 6-inch diameter pipe underdrain. Beyond that, normal job site available material is used. Some responses, however, indicated that backfill compaction has always been something of a problem with settlement just off the end of the bridge. Otherwise, no special treatment has been used. Several states indicated that rigid pile cap has been used, and pile was cast into a pile cap 1 to 2 feet long. Two states indicated that the pile cap is designed as a reinforced continuous beam over the piling.

The survey responses show, in general, that the approach slab can be tied to the abutment with dowels and moved back and forth with the superstructure if a construction joint is provided between the approach slab and the bridge slab. South Dakota stated that at least one approach slab panel with curb and gutter section attached to the bridge end is necessary to prevent erosion of the shoulder behind the abutment wing. One state pointed out that while an expansion joint is specified between rigid pavement and the approach slab, no special treatment is specified for flexible pavement. In Colorado, the approach slab was used if the bridge length was over 200 feet.

Question No. 4 Following are some additional comments on skewed bridges with integral abutments:

- Some of the piles in the abutment have to be battered to resist the active earth pressure acting behind the abutment.
- Rotational forces from the lateral earth pressure on the end walls cause a failure of the pier anchor bolts on the exterior girders.
- For a cast-in-place bridge, the ends of steel beams may be cast into the abutment concrete, reinforcing to the extent that they are considered essentially integral.
- Piles may be prebored for a distance of 5 to 20 feet below the bottom of the pile cap.
- Since the piles are oriented to allow bending about the weak axis, any stresses caused by rotation will only stress the outmost flange fibers and not the web and center portions of the flanges. When the abutment is skewed, some twisting may be induced in the piles when the structure deflects, but this problem can be considered negligible.

A comprehensive summary of the responses by the different states is given in Appendix 8.1, in which N and Y represent no and yes responses, respectively.

3.3. Summary and Conclusions

Previous research work in the area of integral abutments includes surveys of detailing and design criteria used by the state highway agencies, full-scale model tests, and monitoring of performance at actual bridge installations. The present survey responses indicated that 26 states use integral-type abutments on skewed bridges. Most states design integral abutments on skewed bridges on the basis of empirical experience, and no theoretical analysis is introduced in design.

For integral abutments on skewed bridges, 15 states orient their piles with the web of the piles perpendicular to center line of the abutment [Type (2b)] so that bending will be primarily about the strong axis. Thus, thermally-induced biaxial bending stresses will be introduced into the piles. But the survey responses show that most states ignore the thermally-induced bending stress caused by transverse thermal movement. Kansas indicated that transverse thermal movement can be eliminated by using shear keys on the bottom of the pile cap. The major reasons given for using Type (2b) pile orientation are

- The restraint provided by the integral abutment reduces the magnitude of the thermal movement. Orienting the pile with the strong axis parallel to the center line of the bearings gives more rigidity for earthquake loads when liquification of embankment is anticipated.
- Thermal expansion is actually very small, and the backfill material around the abutment and piling seems to yield sufficiently so that no distress is apparent. The piling is oriented to resist the force of earth pressure from the abutment backfill rather than the force of thermal expansion.
- Temperature forces act along the center line of the roadway, not parallel to the pile web, and active soil pressure acts against the strong axis of the pile. Temperature effects are

partially compensated for by predrilling for driven piles and filling the voids with pea gravel or sand.

Usually no special treatments are given to backfill and pile cap on skewed bridges, and they might be constructed as they are for non-skewed bridges. As for the approach slab, it may be tied to the abutment with dowels, or an expansion joint may be provided between the approach slab and the bridge slab. Some states put an expansion joint a certain distance behind the approach slab. In this case, the approach slab will act integrally with the abutment.

It has been over 15 years since the first integral abutments on skewed bridges were constructed. No serious problems or distresses have been discovered as yet. Given the lack of theoretical and experimental research in this area, it is hoped that this survey will provide some useful empirical experience and information on the design of skewed bridges with integral abutments.

4. INFLUENCE OF DIFFERENT FACTORS ON BEHAVIOR OF INTEGRAL ABUTMENT BRIDGES

4.1. Integral Abutment Idealization

In an integral abutment bridge with flexible piling, thermal stresses are transferred to the substructure via a rigid connection. In Iowa, the construction details shown in Fig. 6 have been developed to accomplish the transfer. The abutment contains sufficient bulk to be considered a rigid mass. A positive connection to the girder ends is generally provided by vertical and transverse reinforcing steel. Also, the pilings can be field welded to the bottom girder flange or completely reinforced into the pile cap. This provides for full transfer of temperature variation and live load rotational displacement to the abutment piling.

The approach slab, which is tied to the abutment with dowels, can move back and forth with the superstructure since an expansion joint at certain distance from the approach slab or a construction joint between the approach slab and the bridge slab is provided. In both cases, it may be assumed that the approach slab offers no resistance to the lateral thermal expansion or contraction. A new schematic representation of the loads acting on a pile foundation is shown in Fig. 7.

The pile cap may be subjected to moment, M, and lateral load, H, in addition to the usual vertical load. The vertical load is resisted by the axial capacity of the pile. The applied moment and lateral load are resisted to varying degrees by (a) passive soil resistance on the face of the cap, (b) shear along the base of the cap, and (c) moment

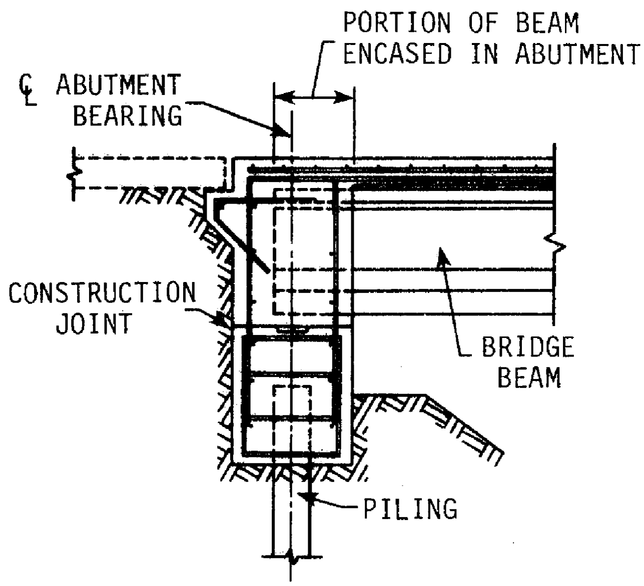

Fig. 6. Typical integral abutment used by Iowa DOT.

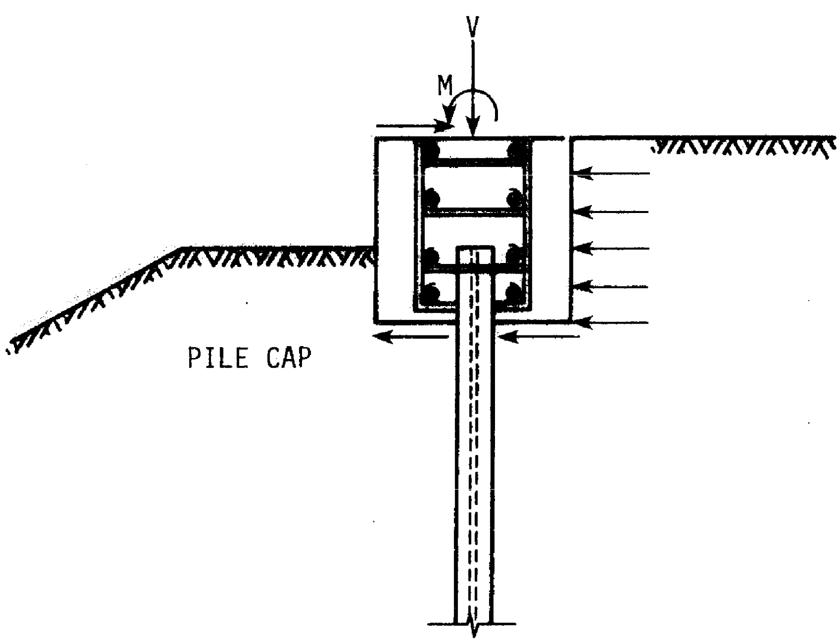

Fig. 7. A schematic representation of loads acting on a pile foundation.

and shear resistance of the pile at the junction to the cap. Clearly, the moment and shear resistance of the piles are functions of the strength and stiffness of both soil and pile.

Passive soil resistance can be very effective in resisting lateral loads, but consideration must be given to the fact that it may not be permanent. Repairs, alterations, or other projects may be cause for removal of the soil; therefore, passive soil resistance is usually discounted or ignored. Shear along the base of the cap also can be very effective in resisting lateral loads. However, a slight settlement of the soil beneath the cap can essentially eliminate this resistance, and it is usually ignored for design purposes. The moment and shear resistance of the pile is usually the major factor considered sufficiently permanent for use in design. According to Iowa bridge engineers, it would be more conservative in design to ignore the passive soil and shear resistance of the cap [2]. The loads transferred from the abutment to the pile are shown in Fig. 8.

4.2. Influence of Rotation at the Pile Head

The piling top embedded in the concrete abutment footing (pile cap) can be assumed to be (a) fully restrained without rotation, (b) partially restrained, allowing some degree of rotation, or (c) pinned, allowing rotation but not translation. For a small lateral displacement of superstructure, such conditions as (b) and (c) can be achieved to a large degree at the top of the piling by enclosing the top portion of

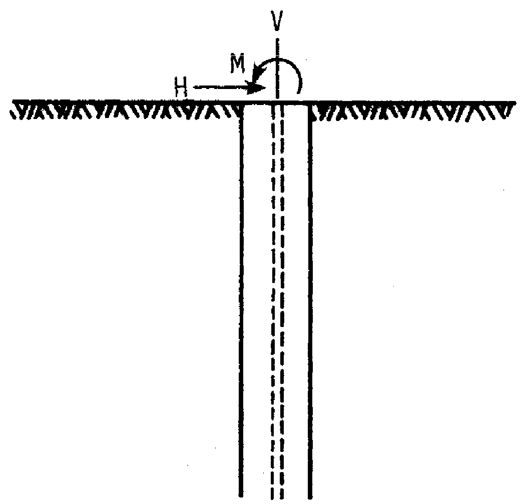

Fig. 8. Load transfer from abutment to pile.

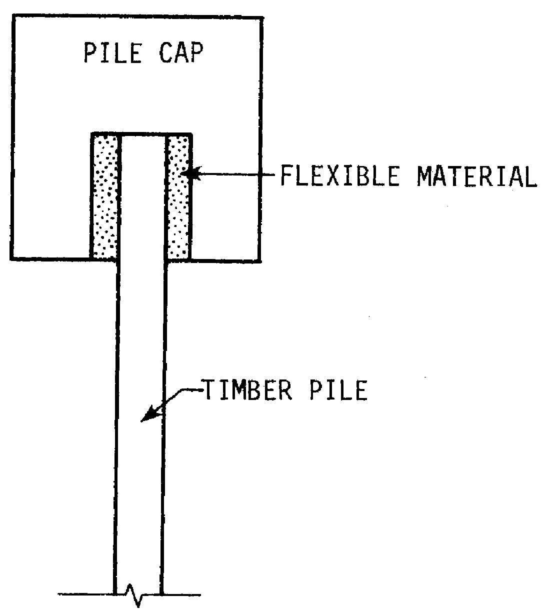

Fig. 9. Detail of a timber pile head designed to act as a pinned connection.

the piling with a flexible material before being cast in the concrete abutment footing, as shown in Fig. 9.

Two extreme cases which represent the upper bound and lower bound of the ultimate pile load capacities will be discussed in this investigation, that is, for fully restrained or pinned pile heads. Six typical Iowa soils and two different pile top boundary conditions are input into computer program Yang 5 to find out their ultimate pile load capacities. The case of fully restrained pile head was investigated in our previous report [3]. For pinned pile head, the loading pattern used in the previous study has been followed by applying a horizontal diplacement Δ_H (to simulate the induced thermal expansion or contraction) and free rotation at the top, and then applying a vertical load (to simulate the bridge load) until failure occurs. Results obtained by running the Yang 5 program will be presented here to show the behavior of steel piles embedded in Iowa soils with two different boundary conditions (fully restrained and pinned). A set of vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), are shown in Figs. 10 through 15 in which the total load is plotted as a function of the settlement of the pile head. In these figures, the vertical load-settlement curve for Δ_H (1, 2, or 4-in.) has been shifted to the origin in an amount Δ_{VH}, a second order vertical displacement, which is caused by the specified lateral displacement Δ_H before loading. The determination of the ultimate pile load follows the procedure used in the previous report [3]. Thus, the ultimate pile load V_{ult} is obtained from vertical load versus displacement curves for various specified lateral displacements (bridge motions).

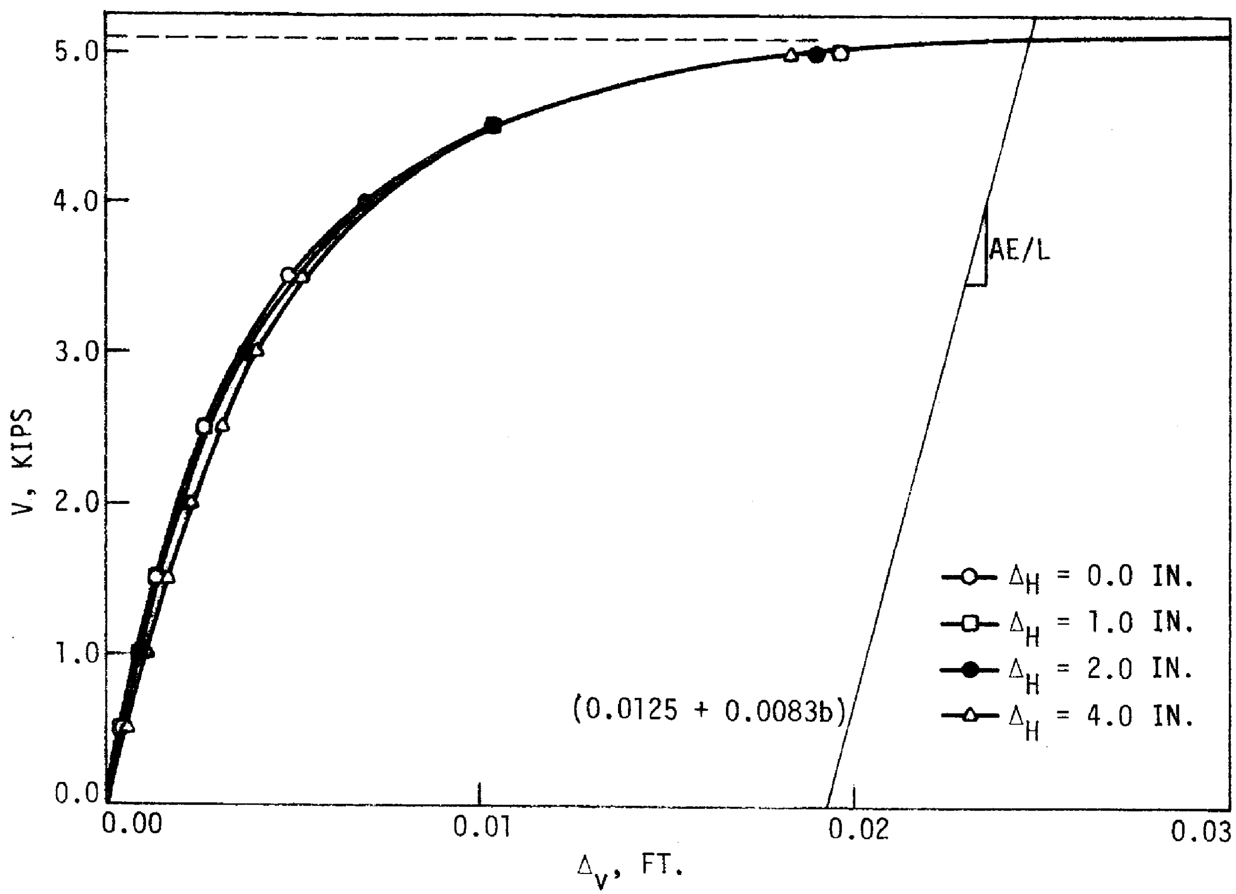

Fig. 10. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for soft clay with pinned condition at the pile head.

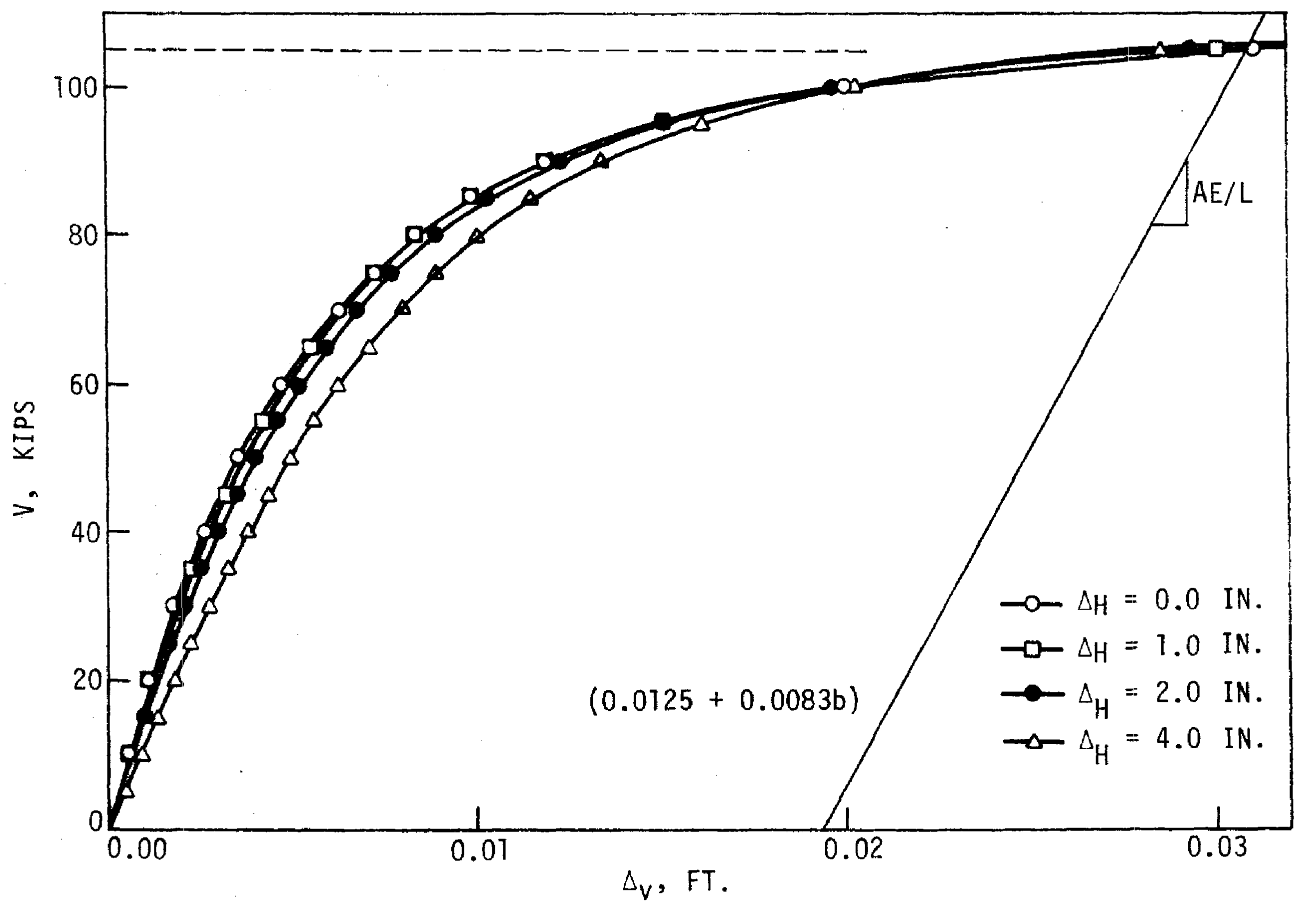

Fig. 11. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for stiff clay with pinned condition at the pile head.

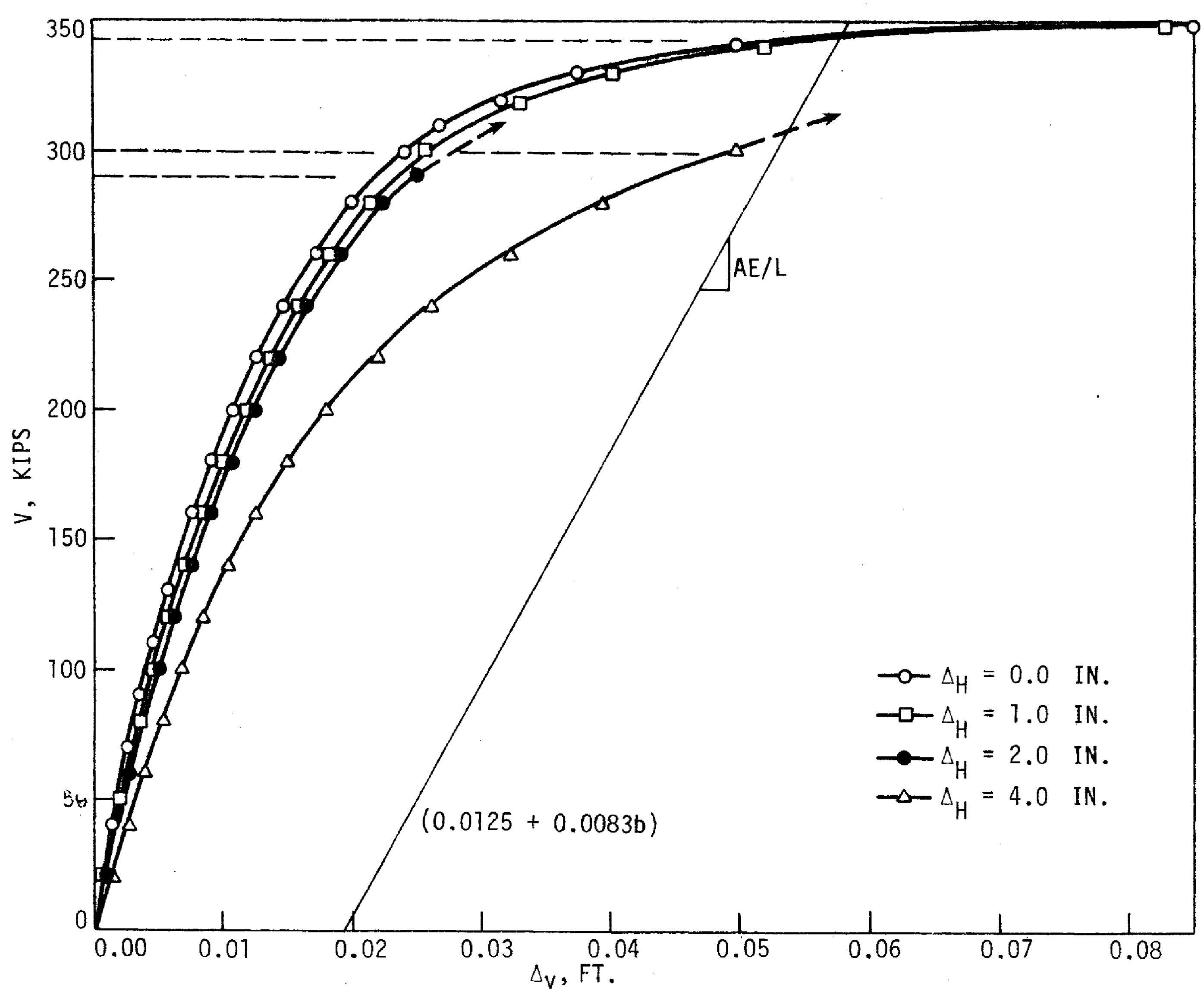

Fig. 12. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for very stiff clay with pinned condition at the pile head.

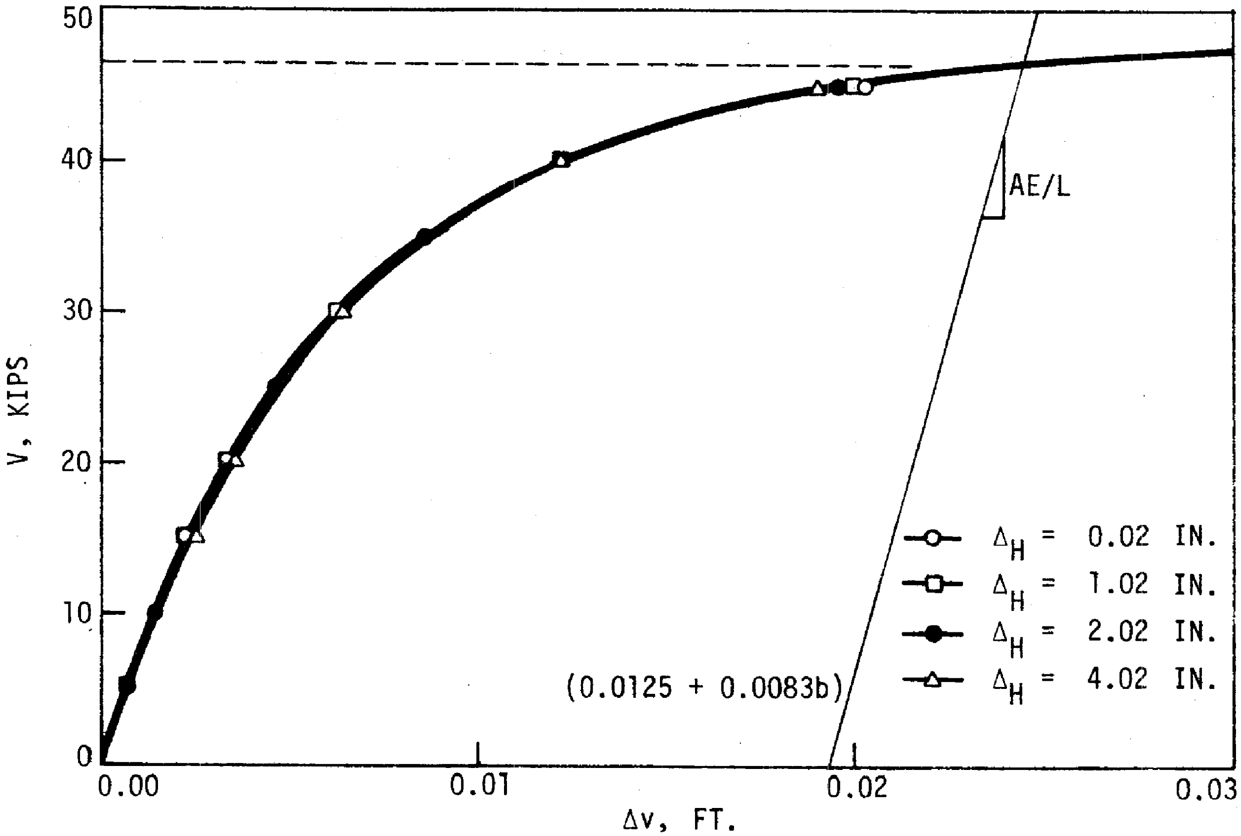

Fig. 13. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for loose sand with pinned condition at the pile head.

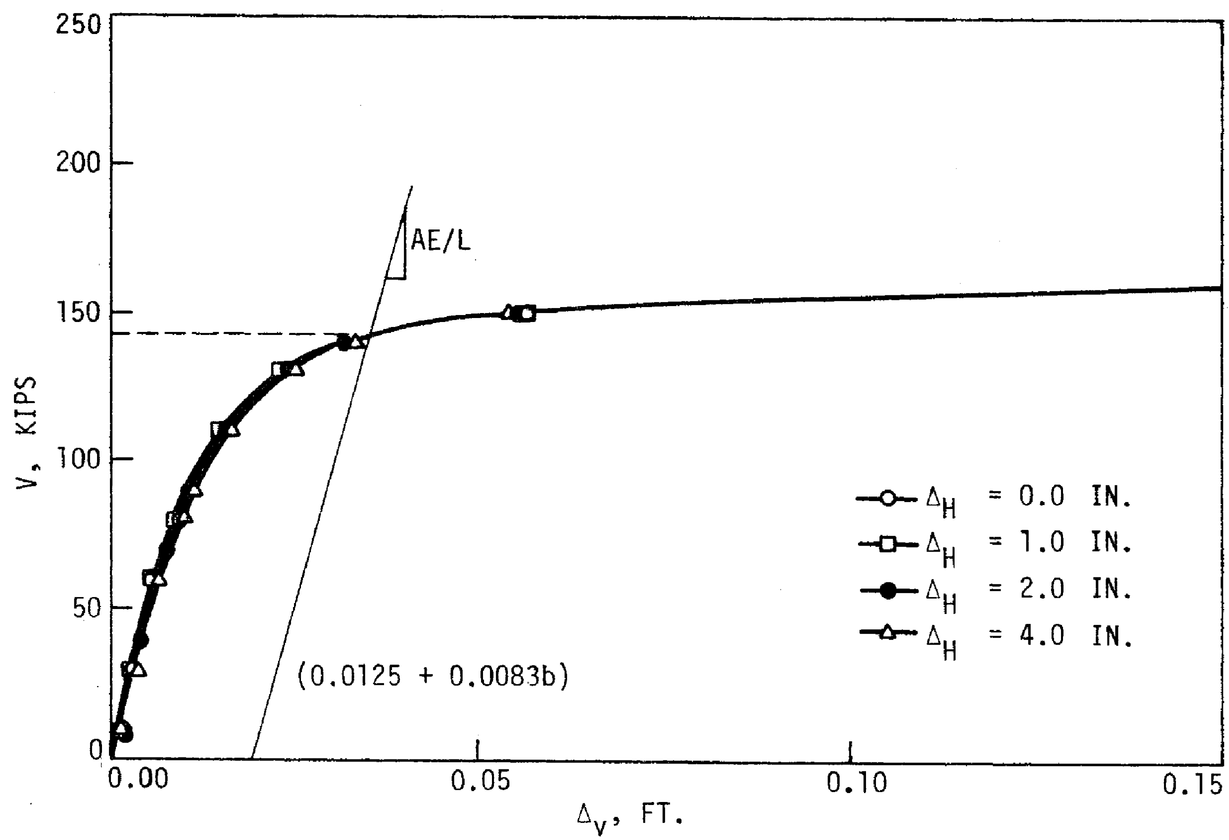

Fig. 14. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for medium sand with pinned condition at the pile head.

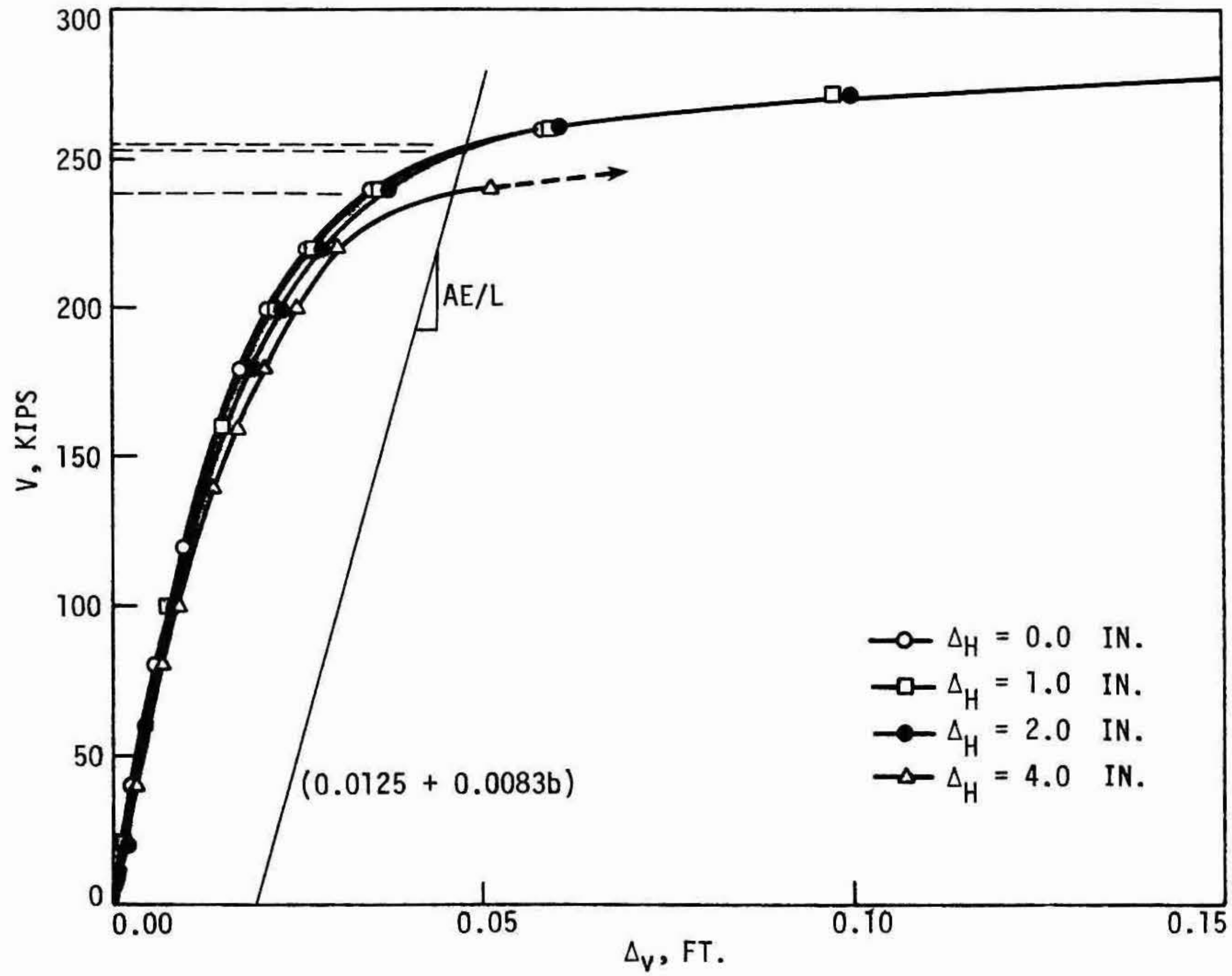

Fig. 15. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for dense sand with pinned condition at the pile head.

A set of curves showing ultimate pile load (V_{ult}) versus specified lateral displacements, Δ_H, for different Iowa soils is shown in Figs. 16 and 17. Nondimensional forms are given in Fig. 18.

From the previous report [3], failure mechanisms are generally of two types: (a) lateral type failure, and (b) vertical type failure. In the case of fully restrained piles, most of the analyzed pile failures are of the second type (vertical type failure). That is, the applied load reached the ultimate soil frictional resistance (for sands, soft clay and stiff clay). For the soils in very stiff clay, lateral failure occurs before vertical failure. In these cases a plastic hinge which forms in the pile significantly increases the number of iterations required for convergence. In this study, the last converged solution was taken as the ultimate load. A set of curves showing ultimate vertical load (V_{ult}) versus specified lateral displacements, Δ_H, for different Iowa soils is shown in Figs. 19 and 20. Nondimensional forms are given in Fig. 21.

If the pile head is allowed to rotate after a specified lateral displacement is reached for sands, soft clay and stiff clay, there is no significant difference. The results were similar to those of the fully restrained pile, and the vertical type failure happened in both the fully restrained and pinned cases. For very stiff clay, the applied load was terminated at V = 290 kips for both Δ_H = 2.0 in. and Δ_H = 4.0 in. under pinned conditions, and at V = 180 kips for Δ_H = 2.0 in. in the fully restrained condition. These two cases show a significant difference in ultimate vertical load carrying capacity because the plastic hinge forms early in the fully restrained pile as

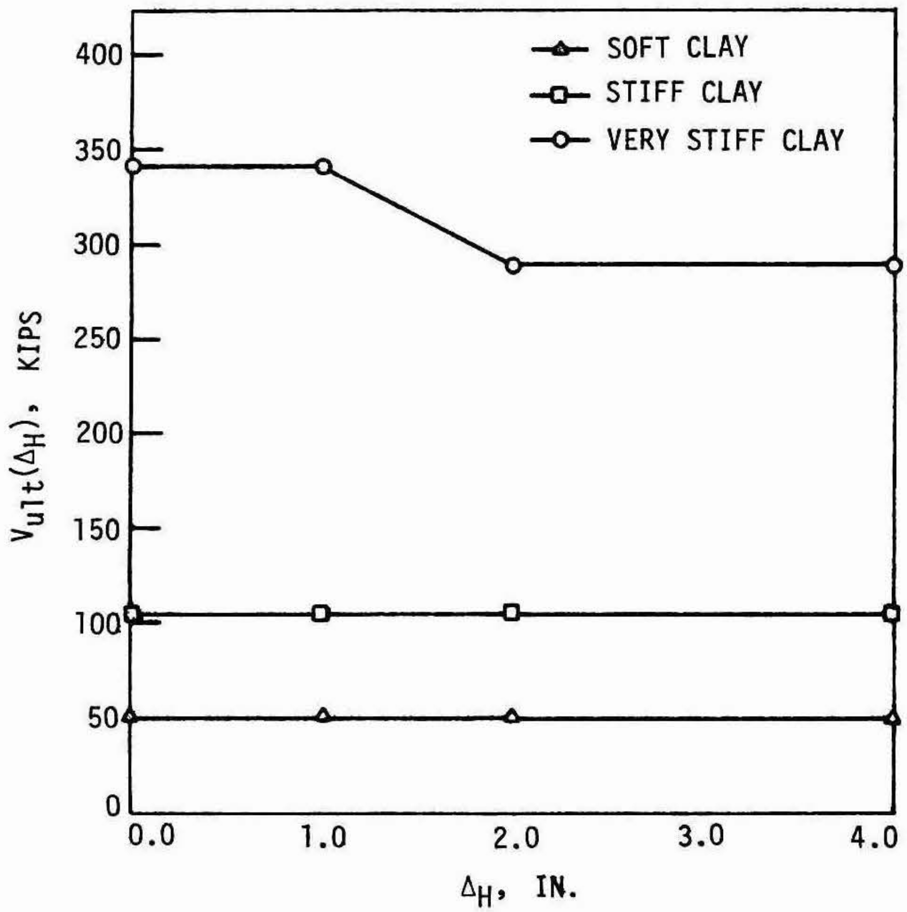

Fig. 16. Ultimate vertical load versus specified lateral displacements in clay with pinned condition at the pile head.

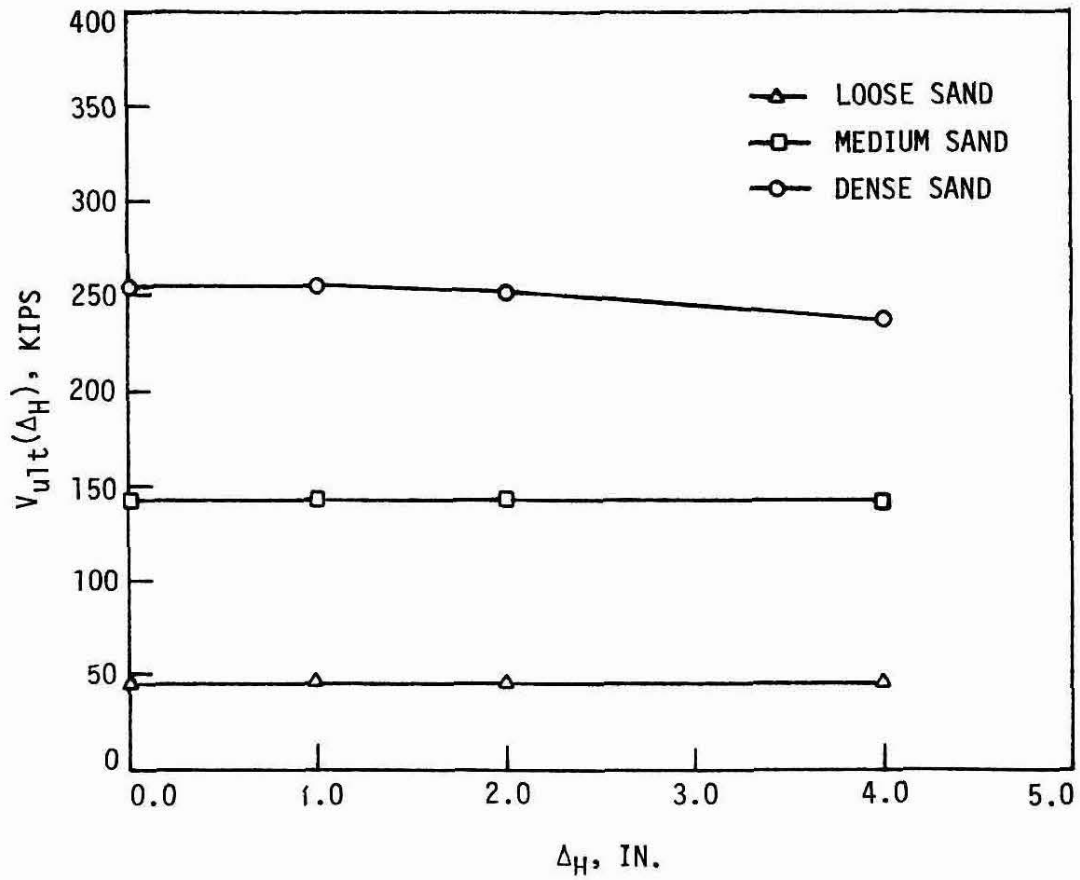

Fig. 17. Ultimate vertical load versus specified lateral displacements in sand with pinned condition at the pile head.

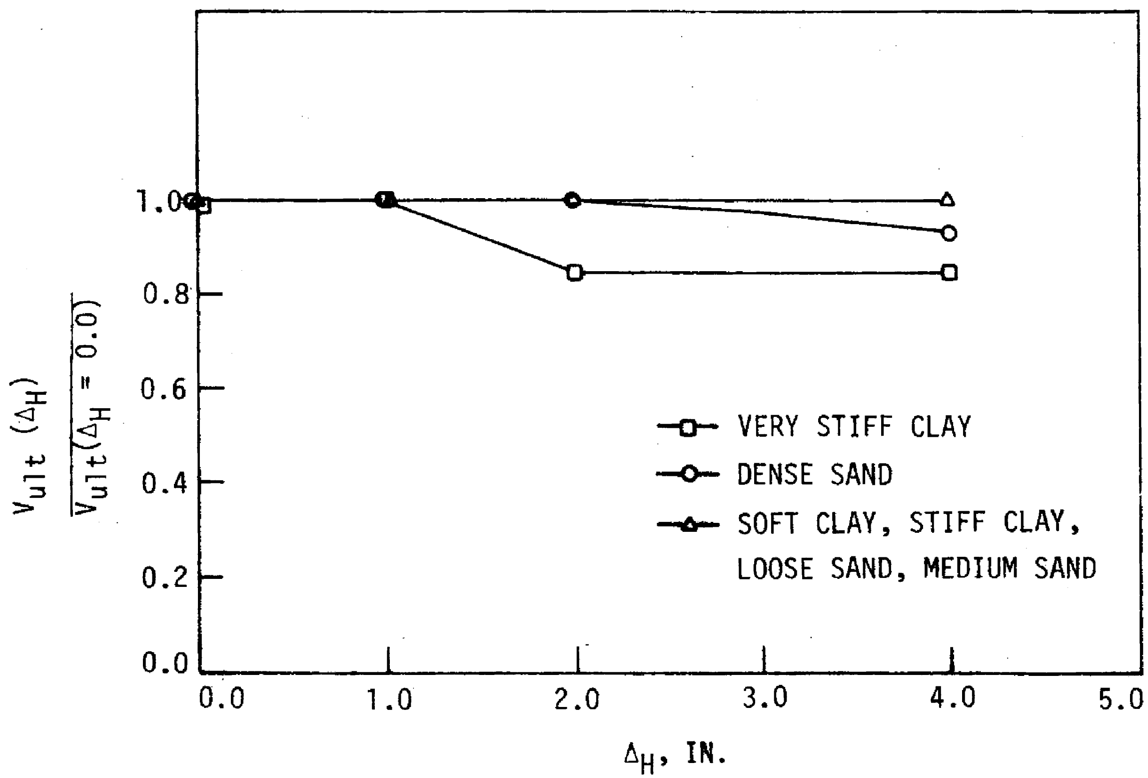

Fig. 18. Non-dimensional form of ultimate vertical load versus specified lateral displacements, Δ_H, in Iowa soils (pinned).

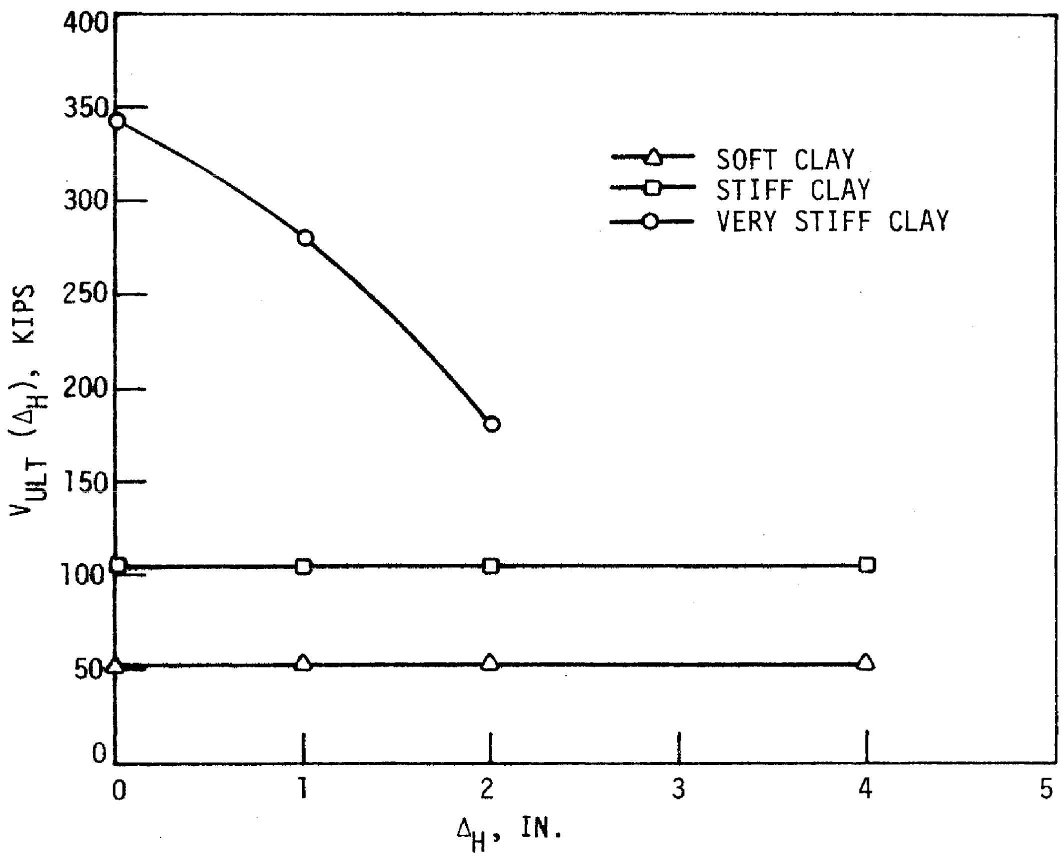

Fig. 19. Ultimate vertical load versus lateral specified displacements with fully restrained pile head (clay).

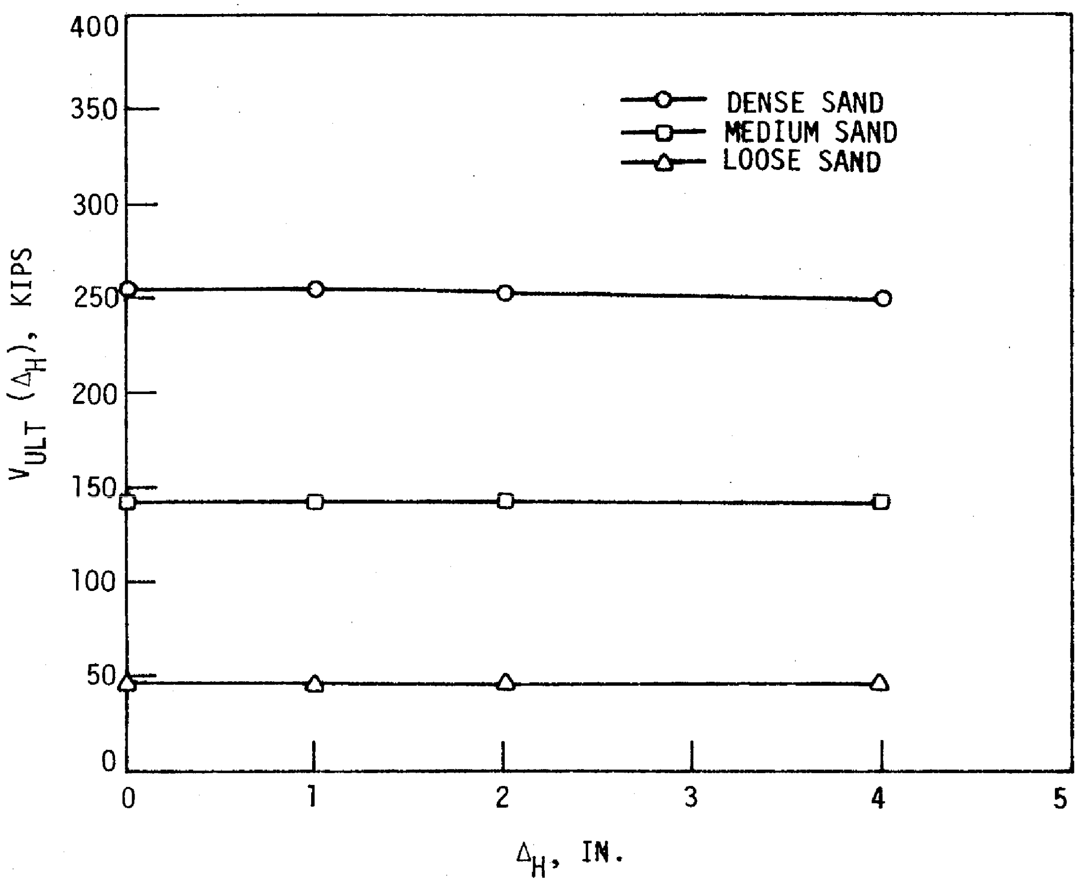

Fig. 20. Ultimate vertical load versus lateral specified displacements with fully restrained pile head (sand).

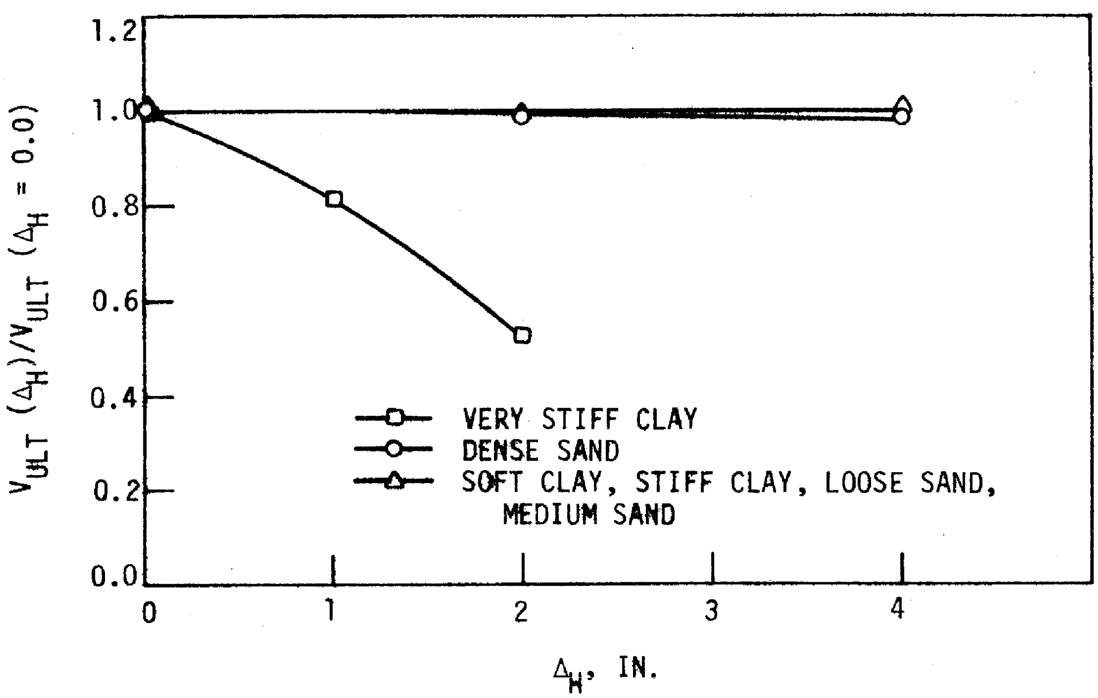

Fig. 21. Non-dimensional forms of ultimate vertical load versus lateral specified displacements, Δ_H, in Iowas soils (with fully restrained pile head).

opposed to the pinned pile. The previous report has shown that the critical location for lateral type failure occurs in the top 10 feet of the pile. The stresses induced by lateral movement and the applied load are much higher in this region for fully restrained piles. Since the pinned pile allows free rotation, the pile is more flexible to adjust its position in order to reduce the induced stresses caused by lateral movement.

The above discussion shows that for an integral abutment, if the piling is cast into the pile cap and a detailed design is used to eliminate moment constraint at the joint, the ultimate vertical load carrying capacity of the H pile after thermal expansion or contraction can be highly increased.

4.3. Effect of Predrilled Oversized Hole

Occasionally, during integral abutment construction, predrilled oversized hole is considered in the following situations: (a) where compressible soils are under a fill, the entire fill should be predrilled to prevent drag, (b) when steel H piles are used, the fill should be predrilled to prevent premature bearing from the resistance of the fill, (c) predrilled oversized hole is used to make piles that are more flexible to sustain thermal induced movement. Piling is driven through a predrilled oversized hole in the ground as shown in Fig. 22. The depth of predrilled oversized holes varies from state to state. In Iowa, all integral abutment piling of bridges with overall length greater than 130 feet must be driven through oversized holes

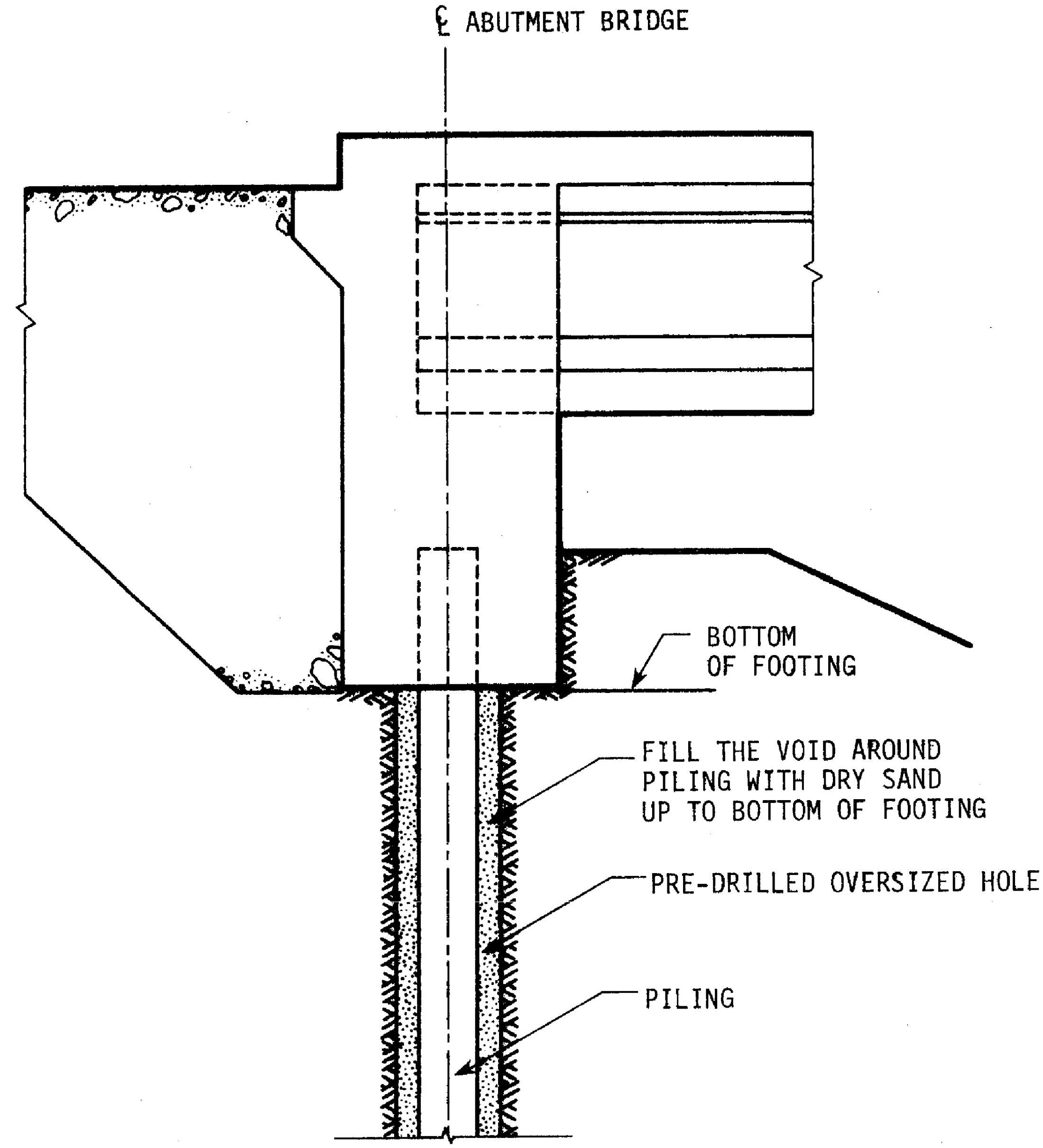

Fig. 22. A detailed design for predrilled oversized hole.

predrilled to a minimum of 8 feet below the bottom of the pile cap [2]. In other states, the depth may be up to 20 feet or more. After a pile has been driven through the predrilled oversized hole, the voids around the pile may be filled with dry sand to prevent the backfill from falling into the hole.

In this report the effect of predrilled oversized holes on the ultimate pile load capacity has been investigated. According to a previous study by the authors [3], the most critical type of soil (very stiff clay) and its loading pattern are used in this study. The cases studied may be classified into three categories: (a) without pre-drilled oversized holes, (b) with predrilled oversized hole from 0 to 10 feet, (c) with predrilled oversized hole with loose sand filling from 0 to 10 feet.

Results obtained from running the Yang 5 program will be presented here to show the influence of predrilled oversized holes to ultimate vertical load carrying capacity. A set of vertical load-settlement curves with specified lateral displacement, Δ_H (0, 1, 2, 4 in.), are shown in Figs. 23 through 27 and Figs. 28 through 32 for cases (b) and (c), respectively. A set of curves showing ultimate pile load (V_{ult}) versus specified lateral displacements, Δ_H, for the very stiff clay with predrilled oversized hole is shown in Figs. 33 and 34 for case (b) and (c), respectively. Also, non-dimensional forms are given in Figs. 35 and 36.

The results show that the ultimate pile load capacity is highly influenced by the effect of predrilled oversized hole. In both cases, cases (b) and (c), if the length of predrilled oversized hole falls in

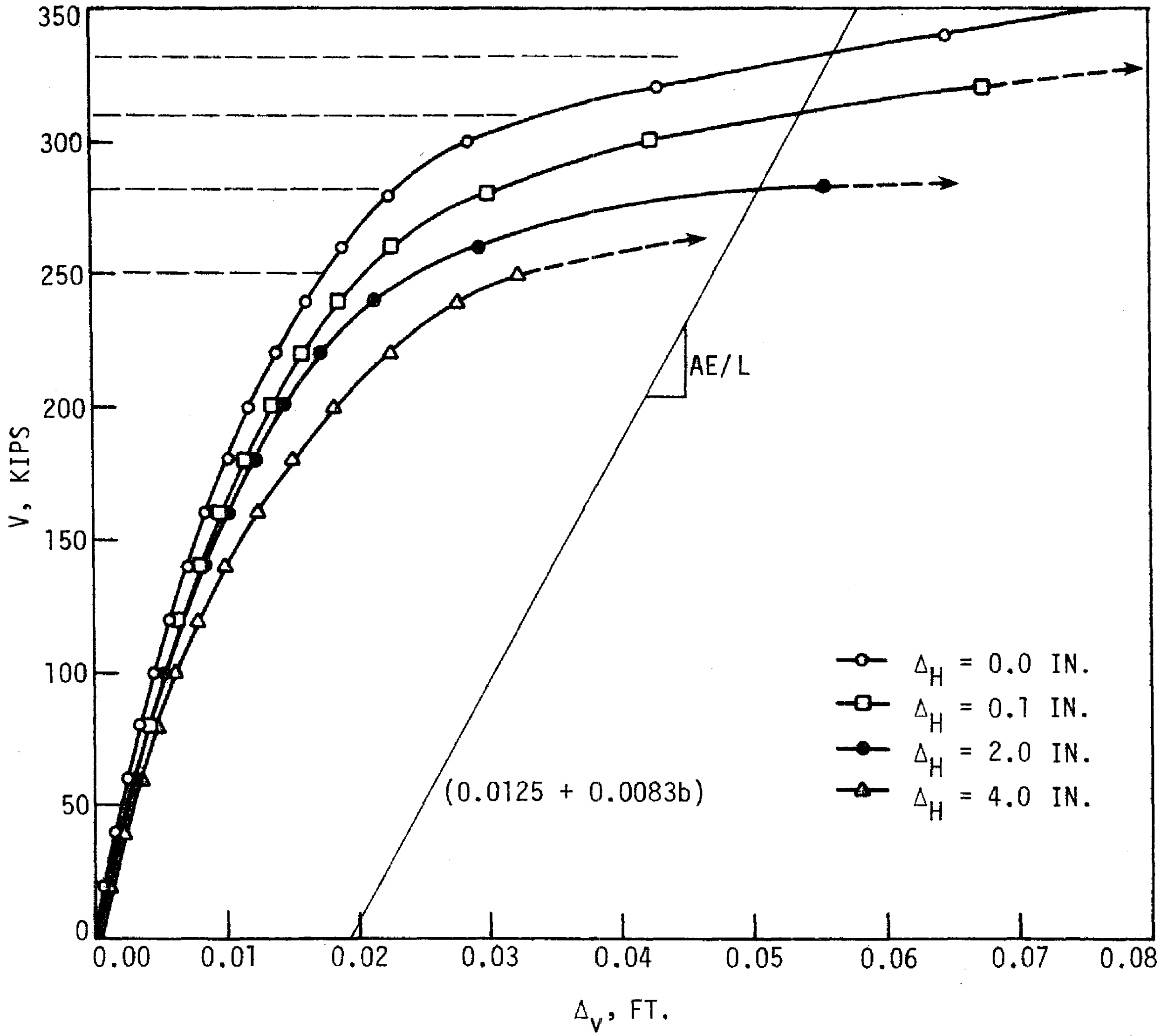

Fig. 23. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for very stiff clay with 2 foot long predrilled oversized hole without sand fill.

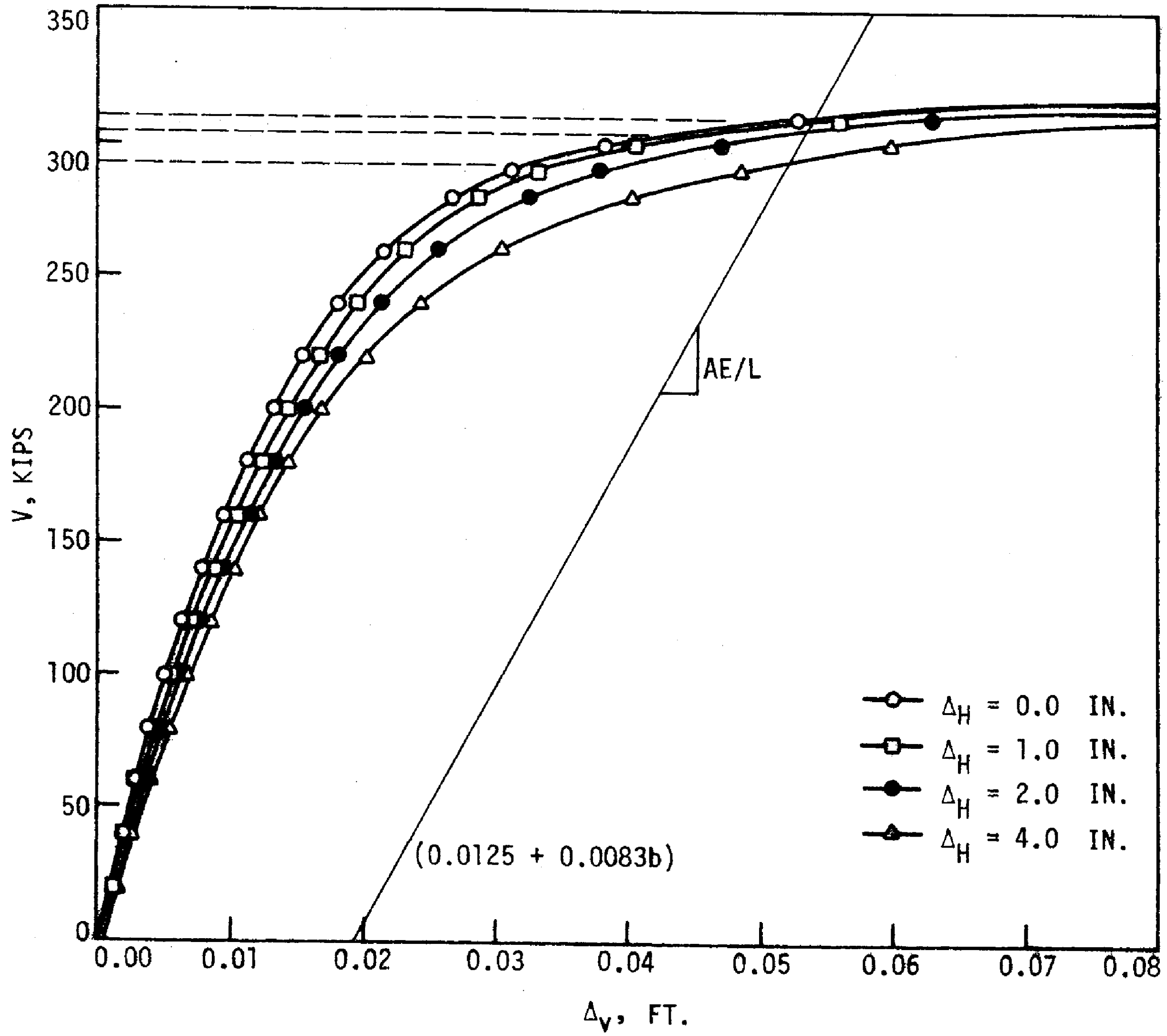

Fig. 24. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for very stiff clay with 4 foot long predrilled oversized hole without sand fill.

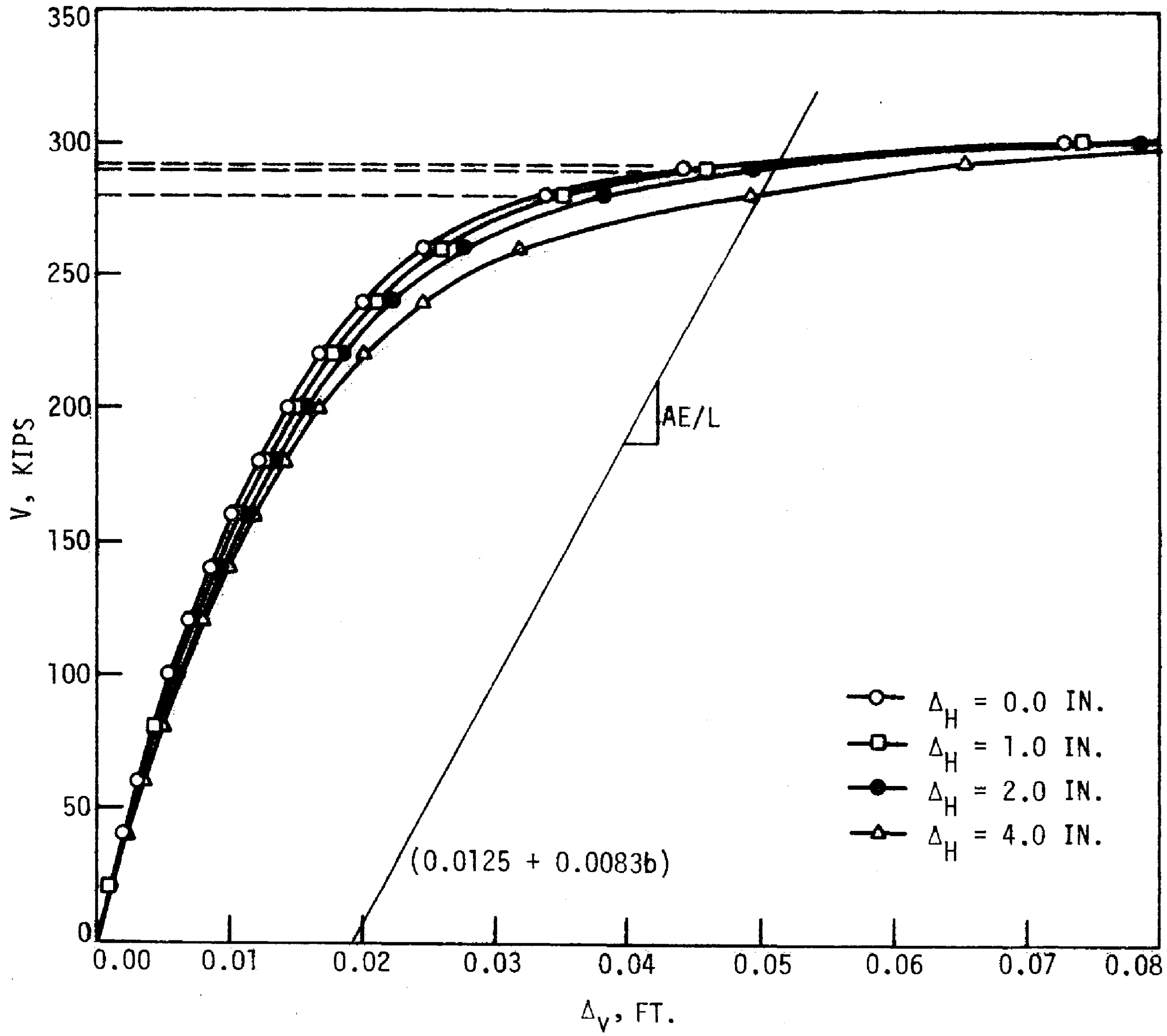

Fig. 25. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for very stiff clay with 6 foot long predrilled oversized hole without sand fill.

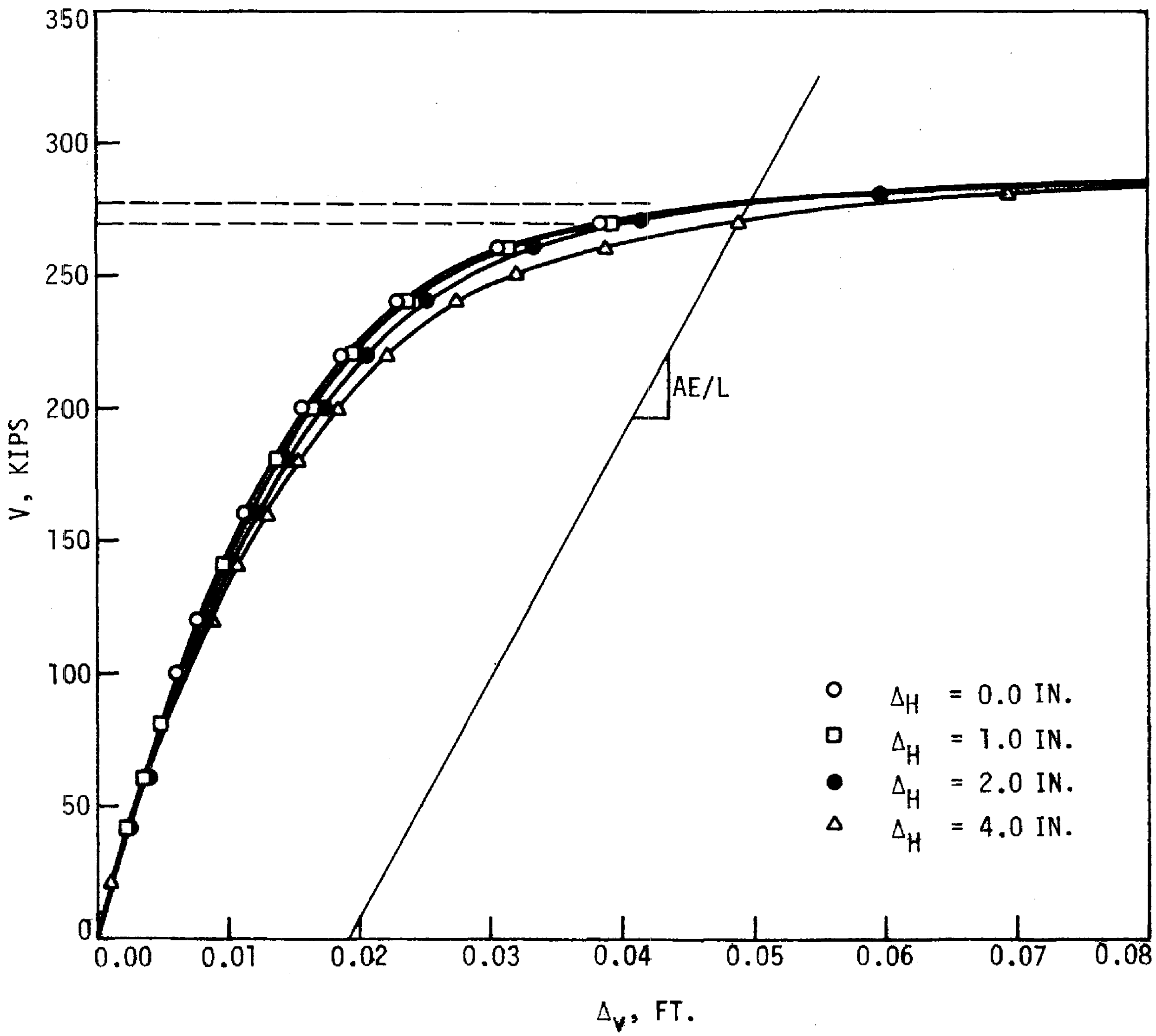

Fig. 26. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for very stiff clay with 8 foot long predrilled oversized hole without sand fill.

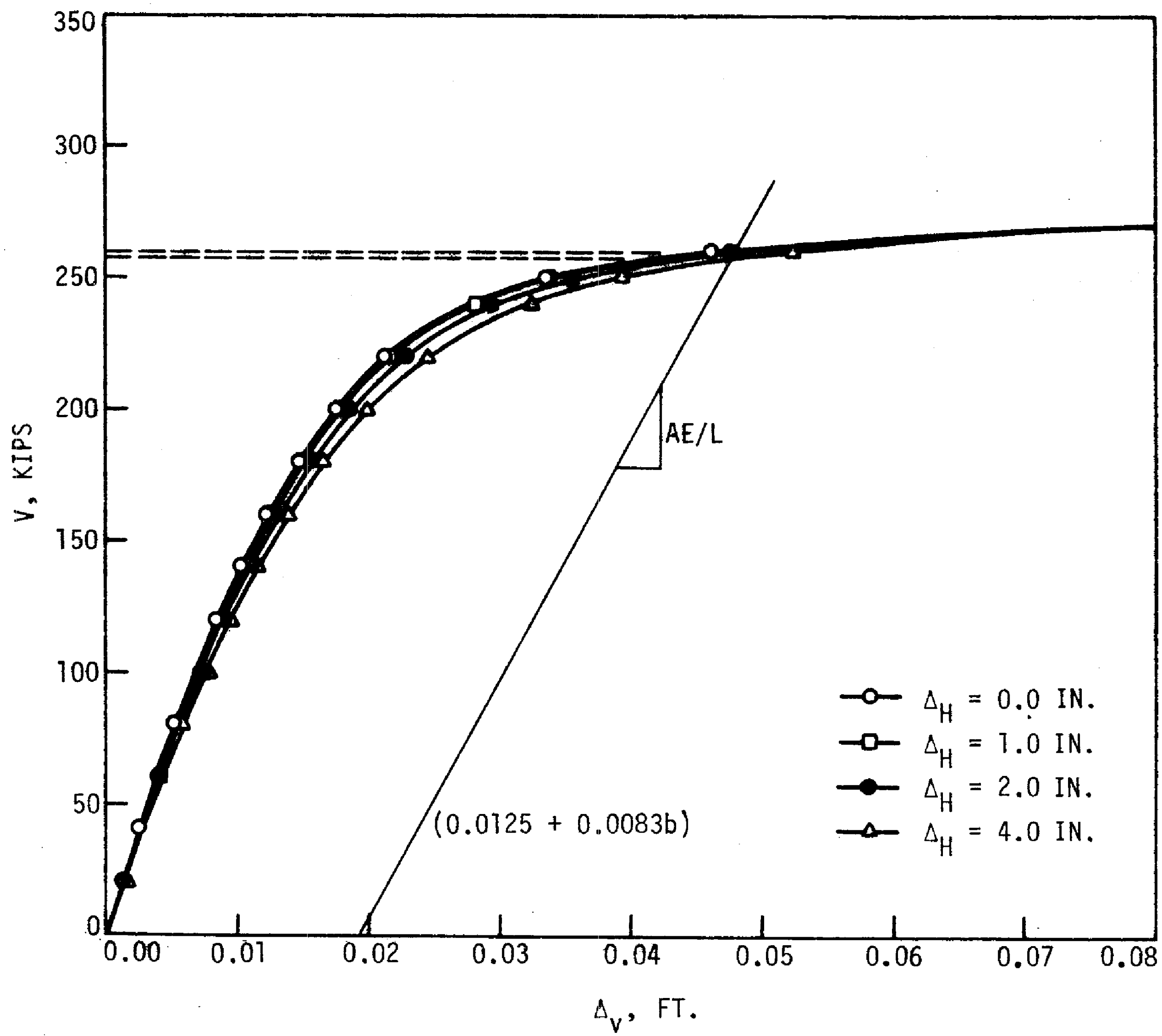

Fig. 27. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for very stiff clay with 10 foot long predrilled oversized hole without sand fill.

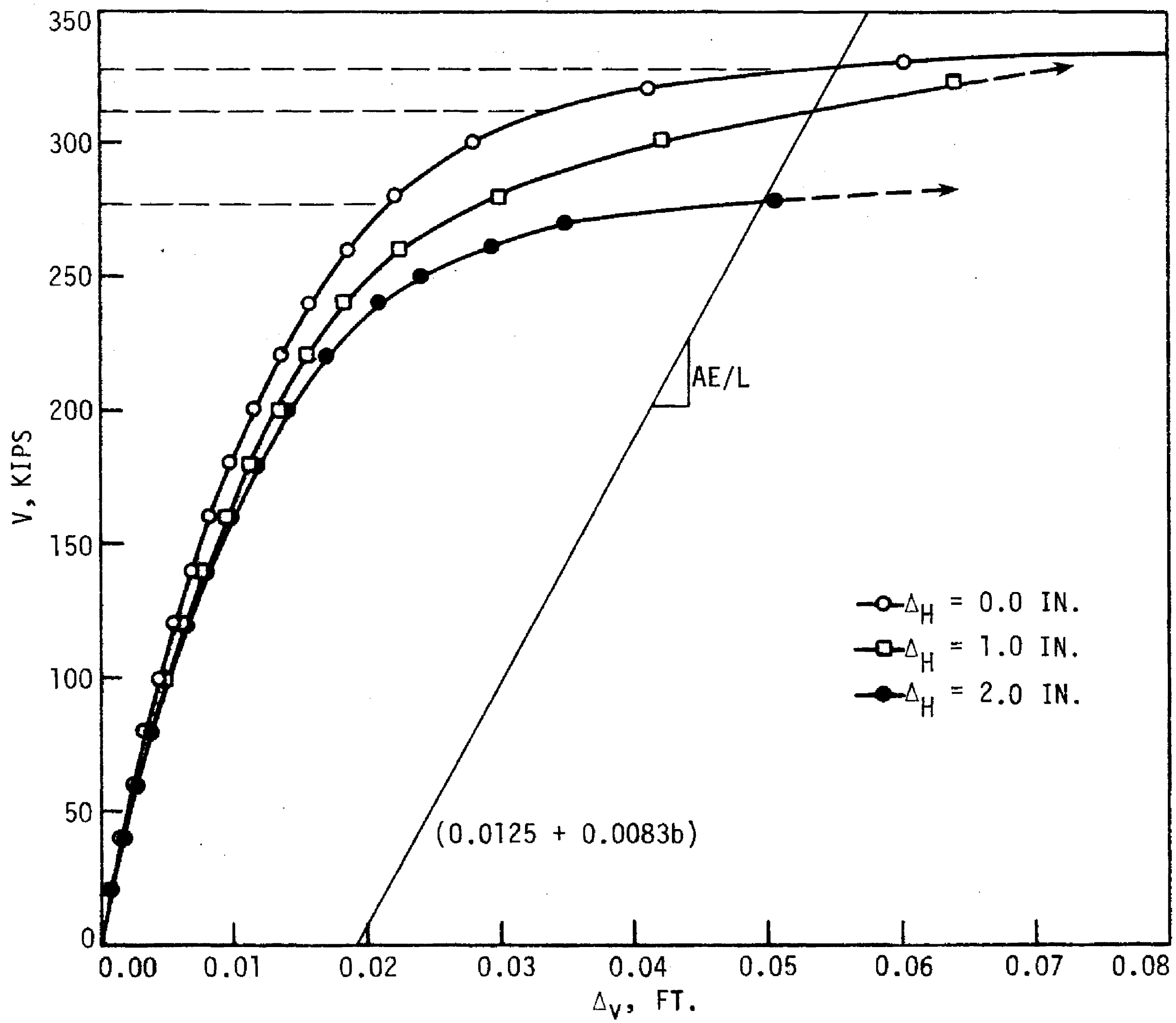

Fig. 28. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for very stiff clay with 2 foot long predrilled oversized hole filled with loose sand.

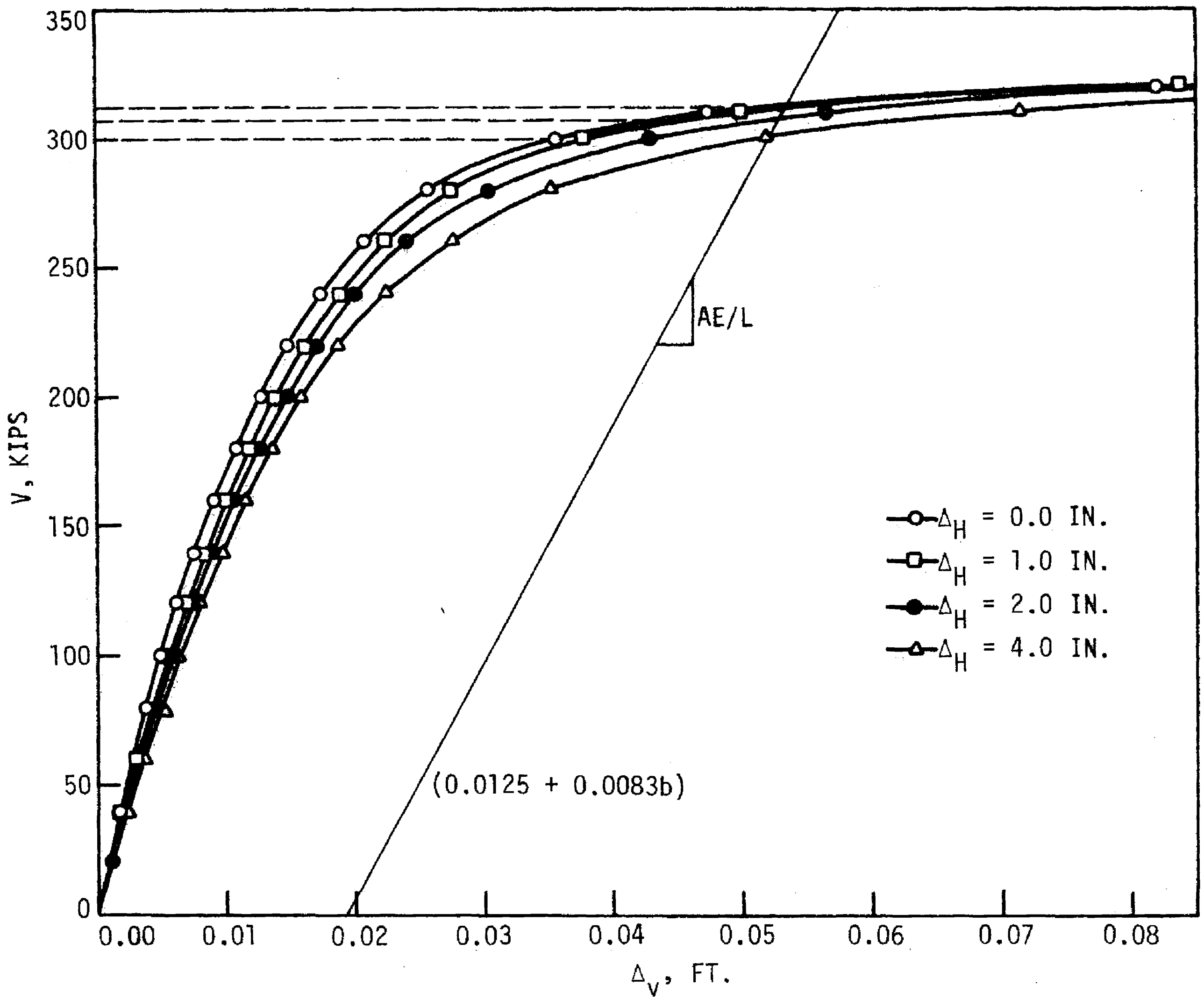

Fig. 29. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for very stiff clay with 4 foot long predrilled oversized hole filled with loose sand.

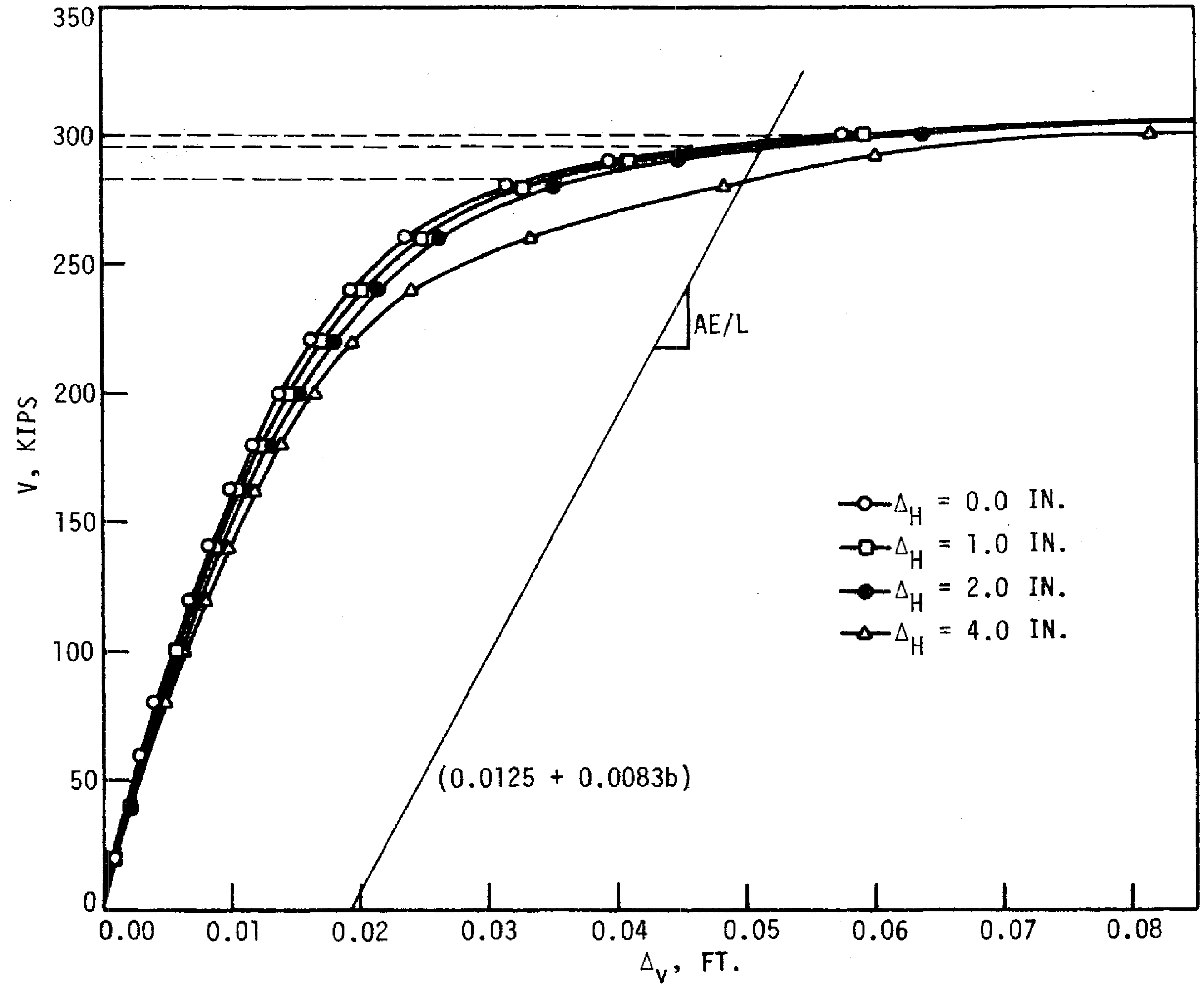

Fig. 30. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for very stiff clay with 6 foot long predrilled oversized hole filled with loose sand.

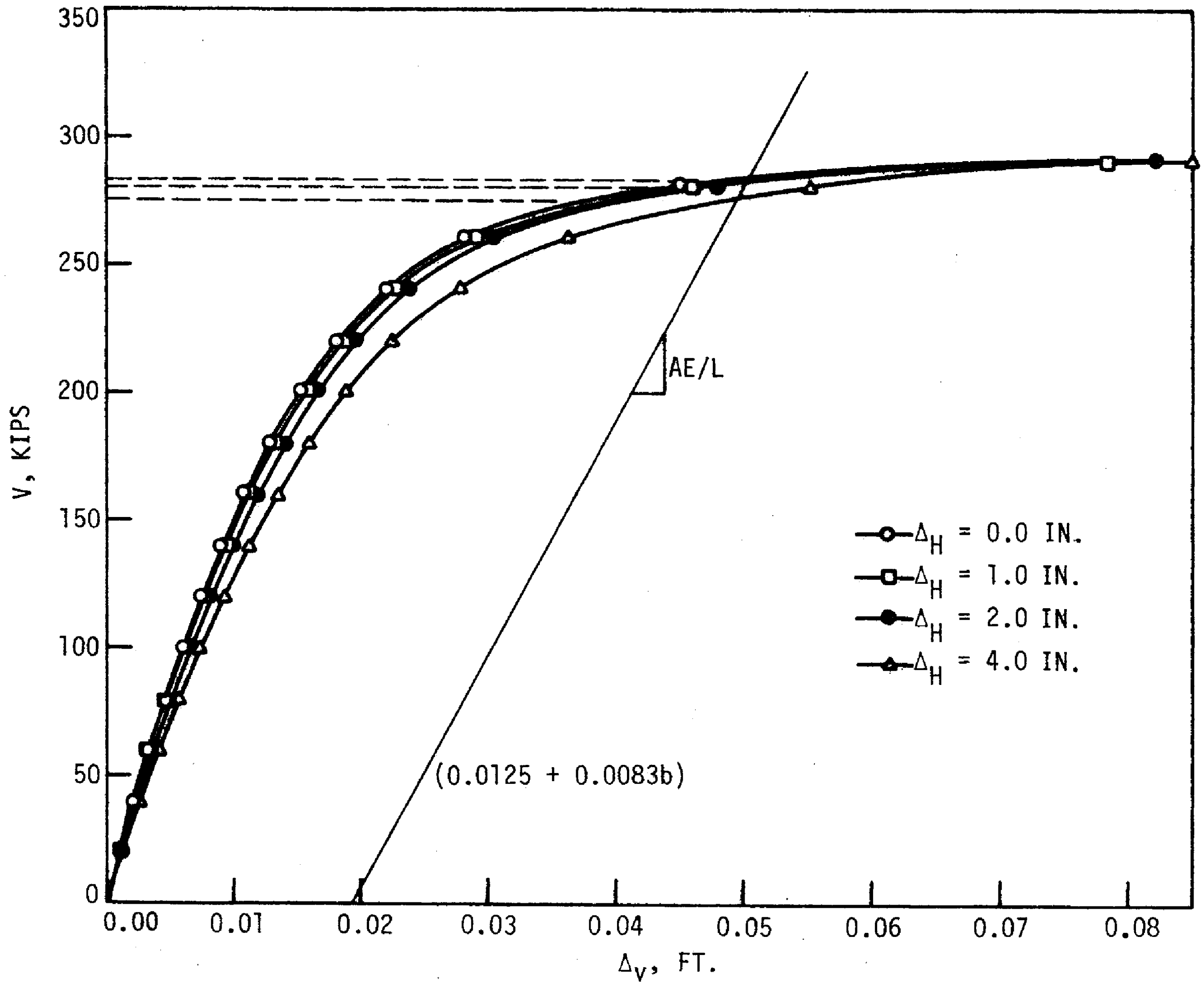

Fig. 31. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for very stiff clay with 8 foot long predrilled oversized hole filled with loose sand.

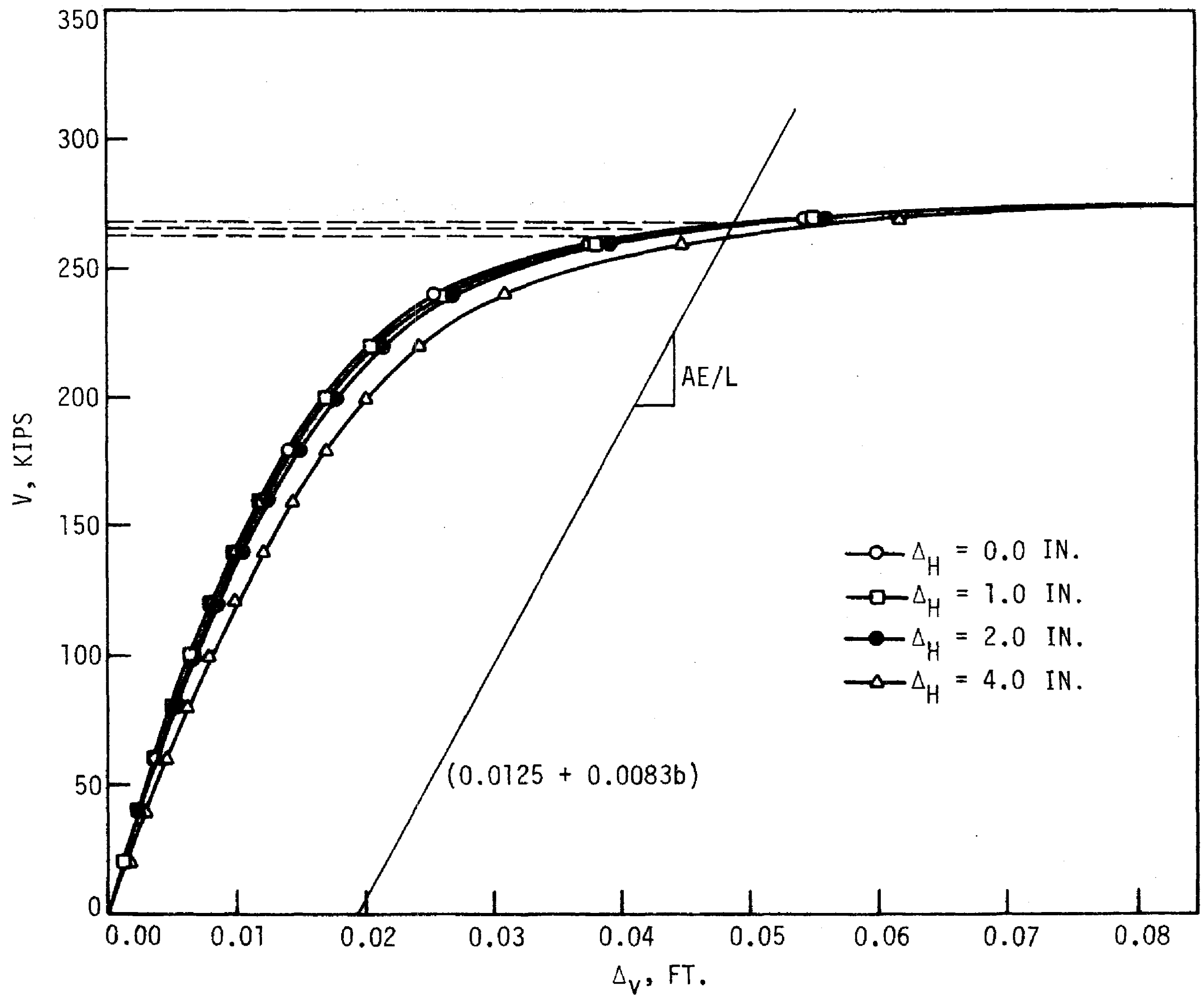

Fig. 32. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for very stiff clay with 10 foot long predrilled oversized hole filled with loose sand.

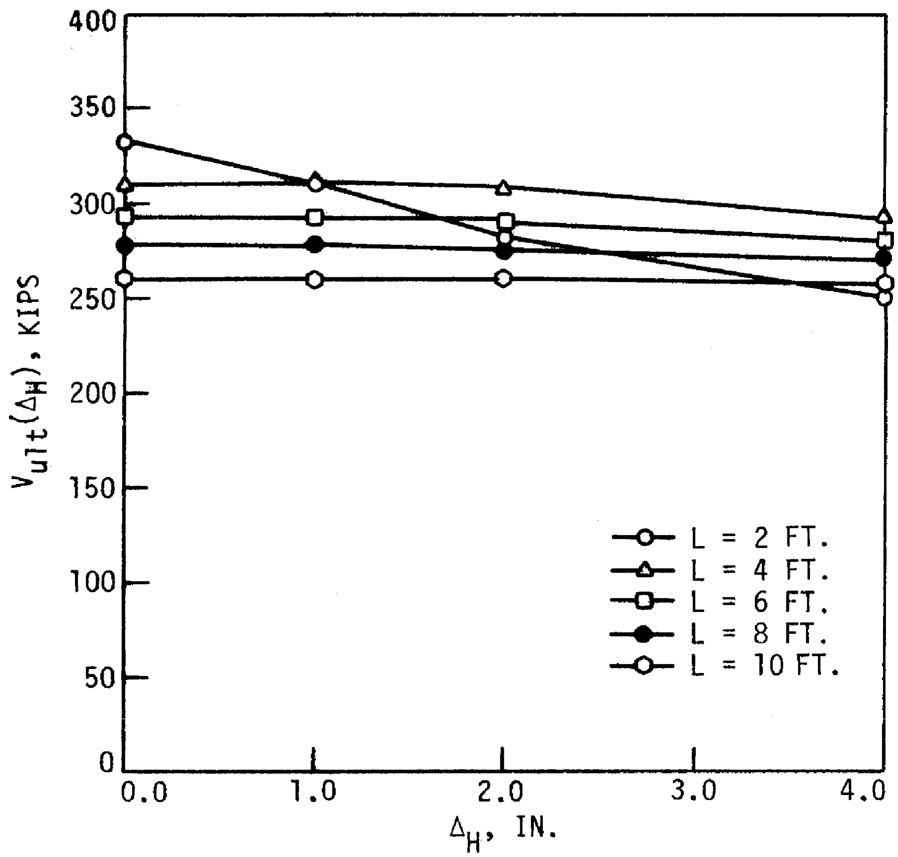

Fig. 33. Ultimate vertical load versus specified lateral displacements for very stiff clay with 10 foot long predrilled oversized hole (without sand fill).

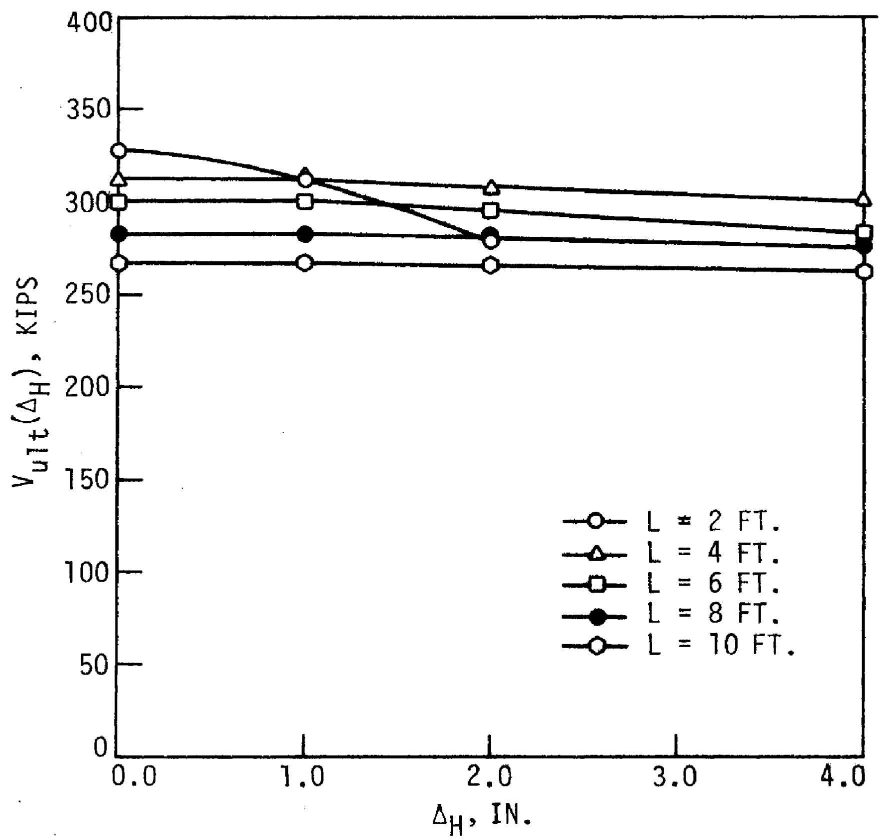

Fig. 34. Ultimate vertical load versus specified lateral displacements for very stiff clay with 10 foot long predrilled oversized hole (with loose sand).

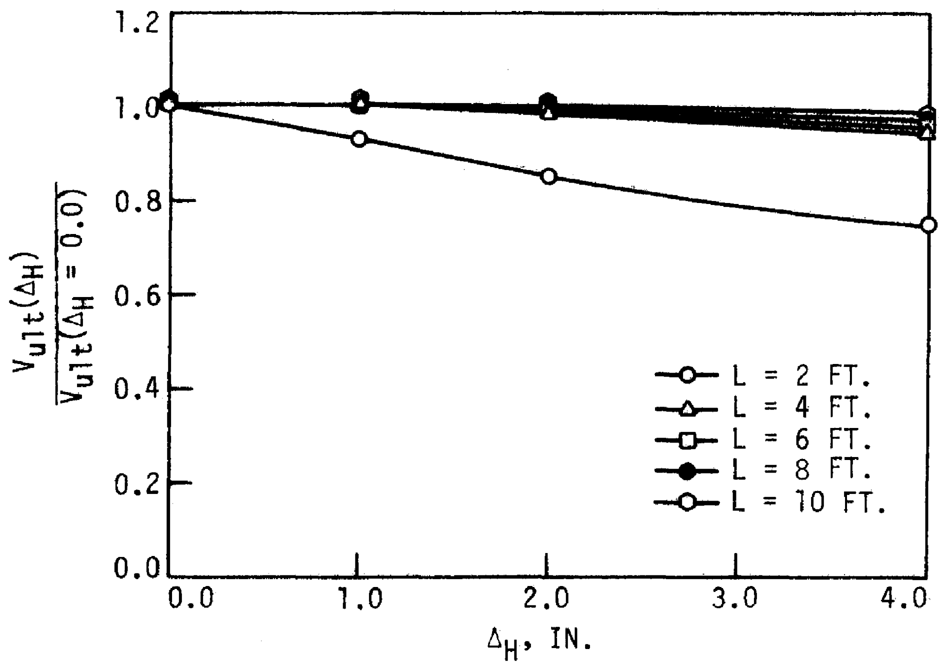

Fig. 35. Non-dimensional forms of ultimate vertical load versus specified lateral displacements, Δ_H, for very stiff clay with 10 foot long predrilled oversized hole (without sand fill).

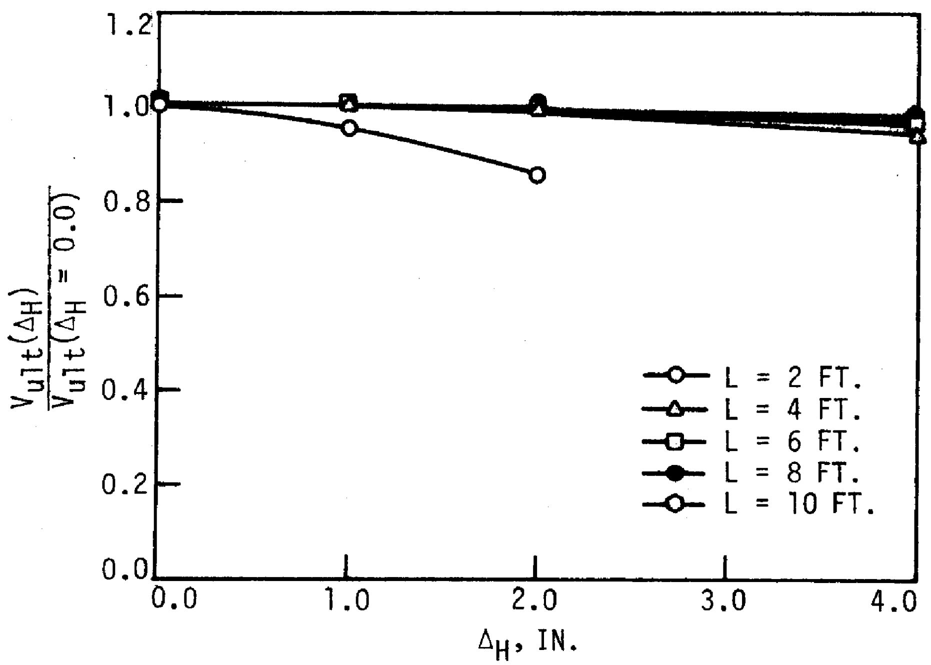

Fig. 36. Non-dimensional forms of ultimate vertical load versus specified lateral displacements, Δ_H, for very stiff clay with 10 foot long predrilled oversized hole (with loose sand).

the range of 4 ft to 10 ft below the ground, there is no significant reduction for ultimate load carrying capacity under specified lateral displacements. But the major significant difference from case (a) is that in cases (b) and (c), the ultimate load carrying capacity of the H piles is only reduced by about 10 percent for 4 inches of specified lateral displacement; in case (a), the ultimate load carrying capacity of H pile is reduced 100 percent for 4 inches of specified lateral displacement. The results obtained also indicated that the plastic hinge forms in the top 4 ft section of the pile. If the length of predrilled oversized hole falls in the range of 0 ft to 4 ft, the ultimate load carrying capacity of the H piles will be reduced in case (b) and highly reduced in case (c). The reduced ultimate load carrying capacity of H piles for cases (a), (b), and (c) under specified lateral displacements is summarized in Table 1.

As shown in Table 1, the ultimate load carrying capacity is highly increased in cases (b) and (c) since in both cases the piles are more flexible than in case (a). Lateral displacements induced by thermal expansion or contraction are easily adjusted by the flexible pile. That is, the lateral soil stiffness in case (a) is much higher than in cases (b) and (c), so pile stresses induced by specified lateral displacement (Δ_H) in case (a) are larger than in cases (b) and (c), and they reach the yield stress, thus causing plastic hinge to form. This explanation also applies in case (c) for 2 foot long predrilled oversized hole with loose sand; the lateral soil stiffness is so much greater that plastic hinge forms at Δ_H = 4.0 in. before being subjected to any vertical loading.

Table 1. The reduced ultimate vertical load carrying capacity in cases (a), (b) and (c) under specified lateral displacements.

Case	Length of Predrilled Oversized Hole, ft.	Reduced Ultimate Load Capacity, %			
		Specified Lateral Displacement, Δ_H, in.			
		$\Delta_H = 0.0$	$\Delta_H = 1.0$	$\Delta_H = 2.0$	$\Delta_H = 4.0$
a	0	0	20	50	100
b	2	0	7	15	25
	4	0	0	2	6
	6	0	0	1	4
	8	0	0	1	3
	10	0	0	0	1
c	2	0	5	15	100
	4	0	0	2	4
	6	0	0	2	6
	8	0	0	1	3
	10	0	0	1	2

The study on the effect of predrilled oversized holes assumes zero soil resistance in the predrilled oversized holes when Δ_H is less than 4 inches. In the real case, the diameter of predrilled oversized hole in Iowa is specified as 16 inches. If the induced thermal movement will cause pile head movement to exceed 4 inches, the soil outside the predrilled oversized hole will resist this movement. Thus, a gap element must be introduced into the present model because it is an important consideration not covered in the previous model.

4.4. Layered Soils

The behavior of a loaded pile is a classic example of soil structure interaction; the properties of the soil control the behavior of the embedded structure. Recent experimental investigations and advances in analytical techniques have added greatly to the general understanding of the problem. In a previous report by the authors [3], only homogenous soils (one layer soils) were investigated. In this study pile behavior in non-homogenous soils (multilayer soils) is investigated.

Solutions for piles in a two-layer system have been presented by Davisson and Gill [8]. A modulus of subgrade reaction is used to define soil stiffness; the stiffness of the surface layer is defined in terms of the underlying layer's stiffness. The complete range of relative stiffness and relative thickness of the two layers is investigated. Davisson and Gill have shown that the surface layer exerts an overwhelming influence on the behavior of a soil-pile system. This conclusion agrees with the findings reported in our previous work [3] which showed that the critical location where lateral type failure forms usually occurs from ground level to 10 ft below ground.

In this investigation, soil-pile interaction has been considered in developing the finite element model. The properties of soil and pile elements were investigated to obtain the parameters used in the Yang 5 Program [3]. Since the behavior of soil at a particular depth is independent of soil behavior at all other depths [22], the soil parameters for layered soils may be obtained with no added difficulty.

In consultation with Iowa Department of Transportation soil engineers, several typical Iowa layered soils have been selected. These soils and their properties are shown in Table 2. Actual soil properties are based on laboratory or field tests. In certain situations, because of geometry requirements for the lay of the land and traffic clearance, cut and fill in road alignment are necessary. Usually the fill is highly compacted above the ground, and the top portion of the pile is embedded in this compacted soil [2]. The depth of the compacted soil is specified in the range of 0 to 10 feet. Three different cases of layered soils are discussed in this investigation: (a) natural soil without compacted soil, (b) natural soil with 4 foot long compacted soil, (c) natural soil with 10 foot long compacted soil. Based on results obtained from the previous report [3] and from Section 4.3, the choice of the length of compacted soil can be understood. Tables 3 and 4 show the typical Iowa layered soils in cases (a) and (b), respectively.

A set of vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for case (a) is shown in Figs. 37

Table 2. Typical Iowa layered soils.

Soil Site	Layer Number	Soil Type	Thickness ft	Blow Count N	Cohesion lb/ft^2	Angle of Friction ϕ	Unit Weight lb/ft^3
Western	1	silt clay	30-90	4-6	200-800	8°	60-90
	2	sand	10-15	10-15	0	32°	110^+
	3	bed rock or glacial clay	17-25	17-25	1200^+	12°	120^+
Eastern	1	sandy silt clay or fine salty sand	5-15	2-8	500-700	12°	70-90
	2	medium sand	15-40	8-12	0	30°	100
	3	gravely sand	15-25	15-25	0	36°	115
	4	bed rock, glacial clay or pre-consolidated foundation layer	---	$15\text{-}100^+$	---	---	---
Southern	1	silty clay	10-35	2-5	300-600	8°	90
	2	glacial clay	15-50	12-18	700-1000	12°	110
	3	glacial clay	15-20	18-30	1200^+	12°	125
	4	bed rock	---	---	---	---	---
Northern	1	surface till sandy clay loam	5-15	5-9	500-700	12°-18°	70-90
	2	glacial clay foundation layer	30^+	12-16	70-1200	12°	110^+

Table 3. Modified Iowa layered soils without compacted soil.

Soil Site	Layer Number	Thickness, ft	Soil Type
Western	1	40	soft clay
Eastern	1	10	soft clay
	2	30	medium sand
Southern	1	20	soft clay
	2	20	stiff clay
Northern	1	10	soft clay
	2	18	stiff clay
	3	12	very stiff clay

Table 4. Modified Iowa layered soils with 4 foot long compacted soil.

Soil Site	Layer Number	Thickness, ft	Soil Type
Western	1	4	very stiff clay
	2	36	soft clay
Eastern	1	4	very stiff clay
	2	10	soft clay
	3	26	medium sand
Southern	1	4	very stiff clay
	2	20	soft clay
	3	16	stiff clay
Northern	1	4	very stiff clay
	2	10	soft clay
	3	18	stiff clay
	4	8	very stiff clay

through 40. Also, Fig. 41 presents a set of ultimate pile load (V_{ult}) versus specified lateral displacements (Δ_H) curves for typical Iowa layered soils. Non-dimensional forms are given in Fig. 42. Results indicate that no reduction on the ultimate load carrying capacity for typical Iowa layered soils occurs, because the lateral soil stiffness of typical Iowa layered soils in the first 10 ft below the ground line is less than for very stiff clay so that yield stress is not reached. The failure mechanism is controlled by vertical type failure, that is, the applied load reaches the ultimate soil frictional resistance.

For the case of compacted soil 4 feet above the ground surface, soil parameters can be divided into two parts. The very stiff clay can be used to represent compacted soil of the first layer. Soil parameters for this layer can be obtained from soil properties of the very stiff clay (0 to 4 ft below the ground line) [3]. The second division is natural ground soil. The soil parameters for this part depend on the type of soils below the natural ground line (0 to 36 ft below the natural ground line) [3].

Results obtained by running the Yang 5 program are plotted in Figs. 43 through 48. Again, the results indicate that the behavior of typical Iowa layered soils with 4 foot long compacted soil (very stiff clay) is similar to the behavior of very stiff clay. A plastic hinge forms before the specified lateral displacement reaches 4 inches. The evidence shown here indicates that the critical location for the plastic hinge is 0 to 4 ft below the ground line. Lateral type failure occurs before vertical failure in any case where the top portion of the pile is surrounded by soils with high lateral soil stiffness.

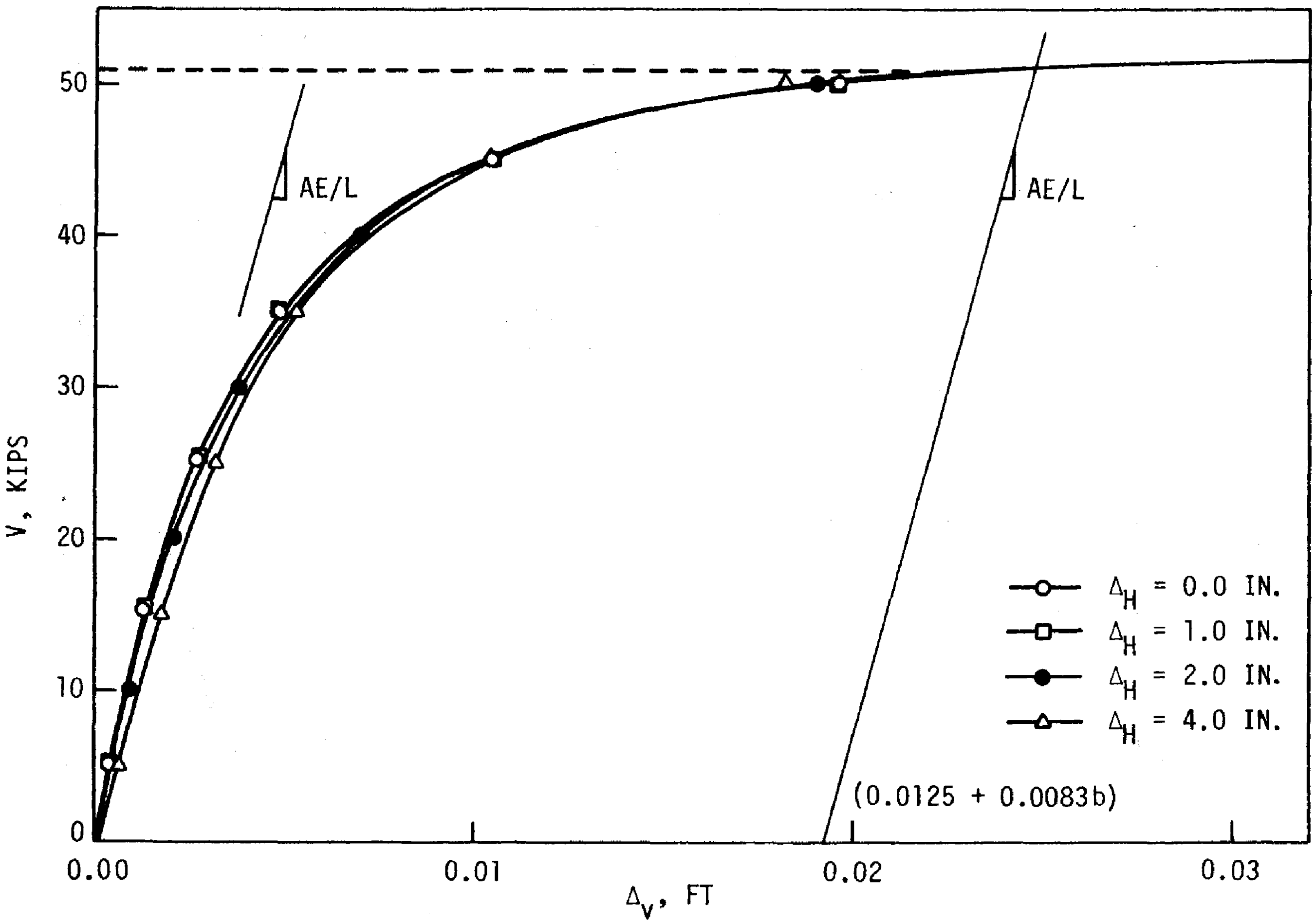

Fig. 37. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for western Iowa soil.

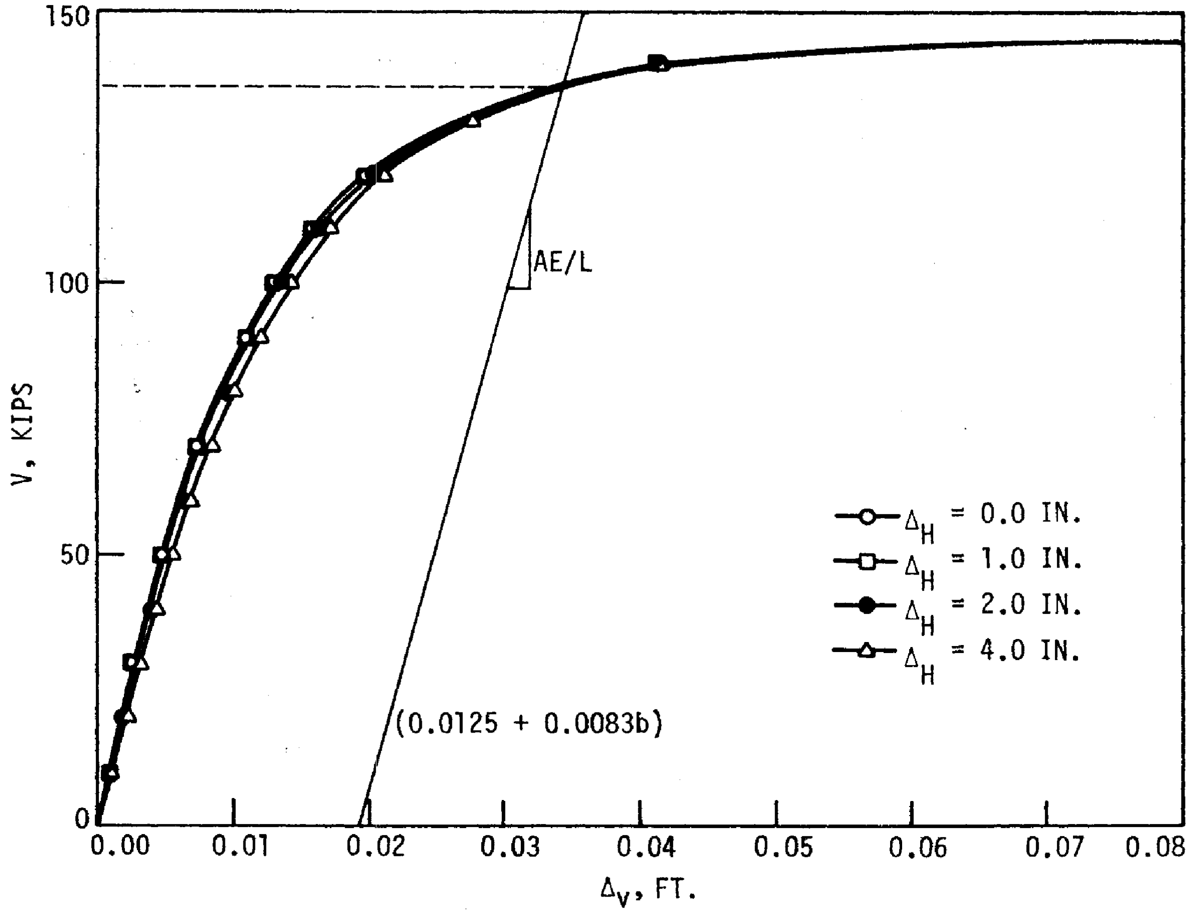

Fig. 38. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for eastern Iowa soil.

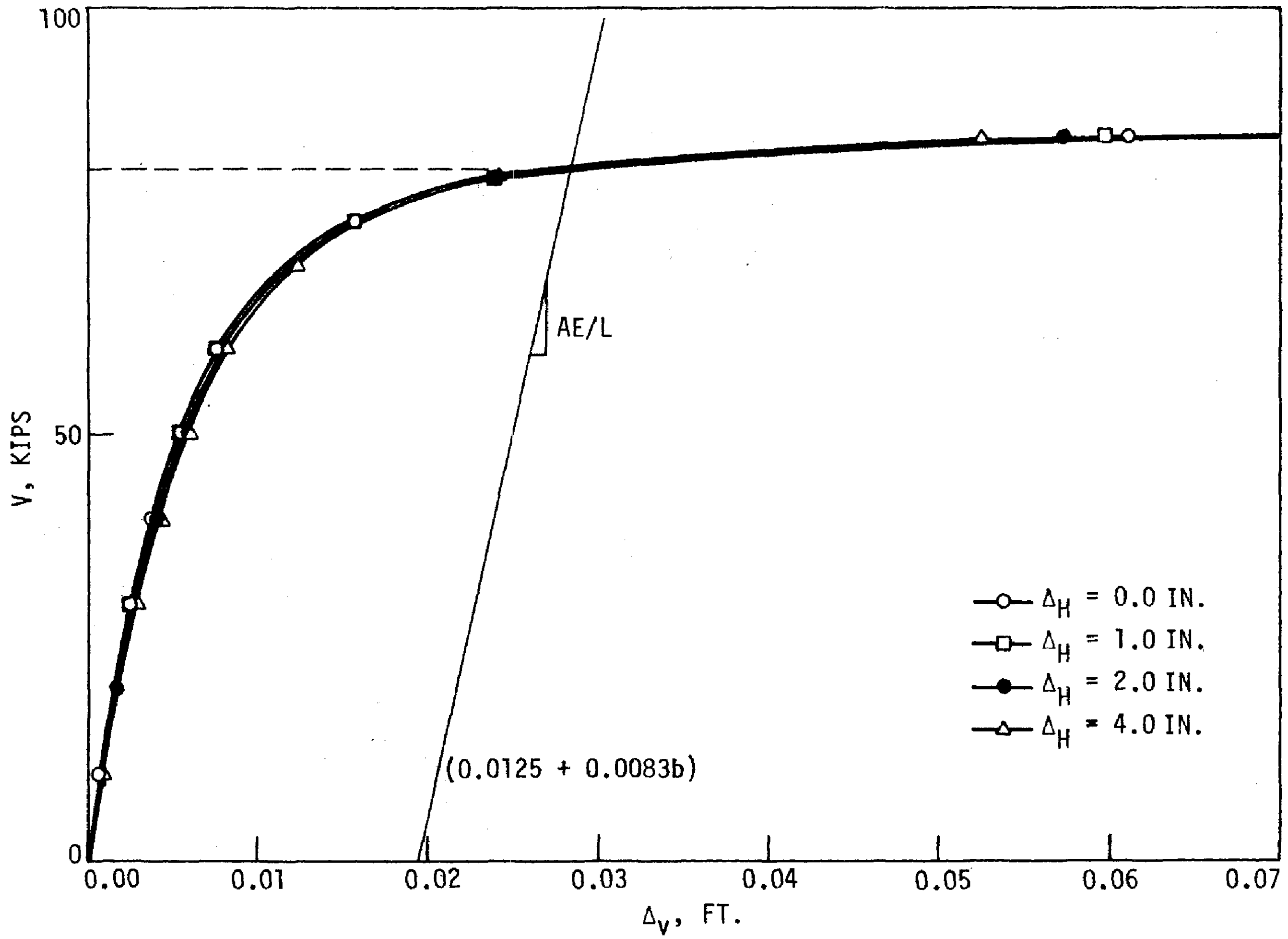

Fig. 39. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for southern Iowa soil.

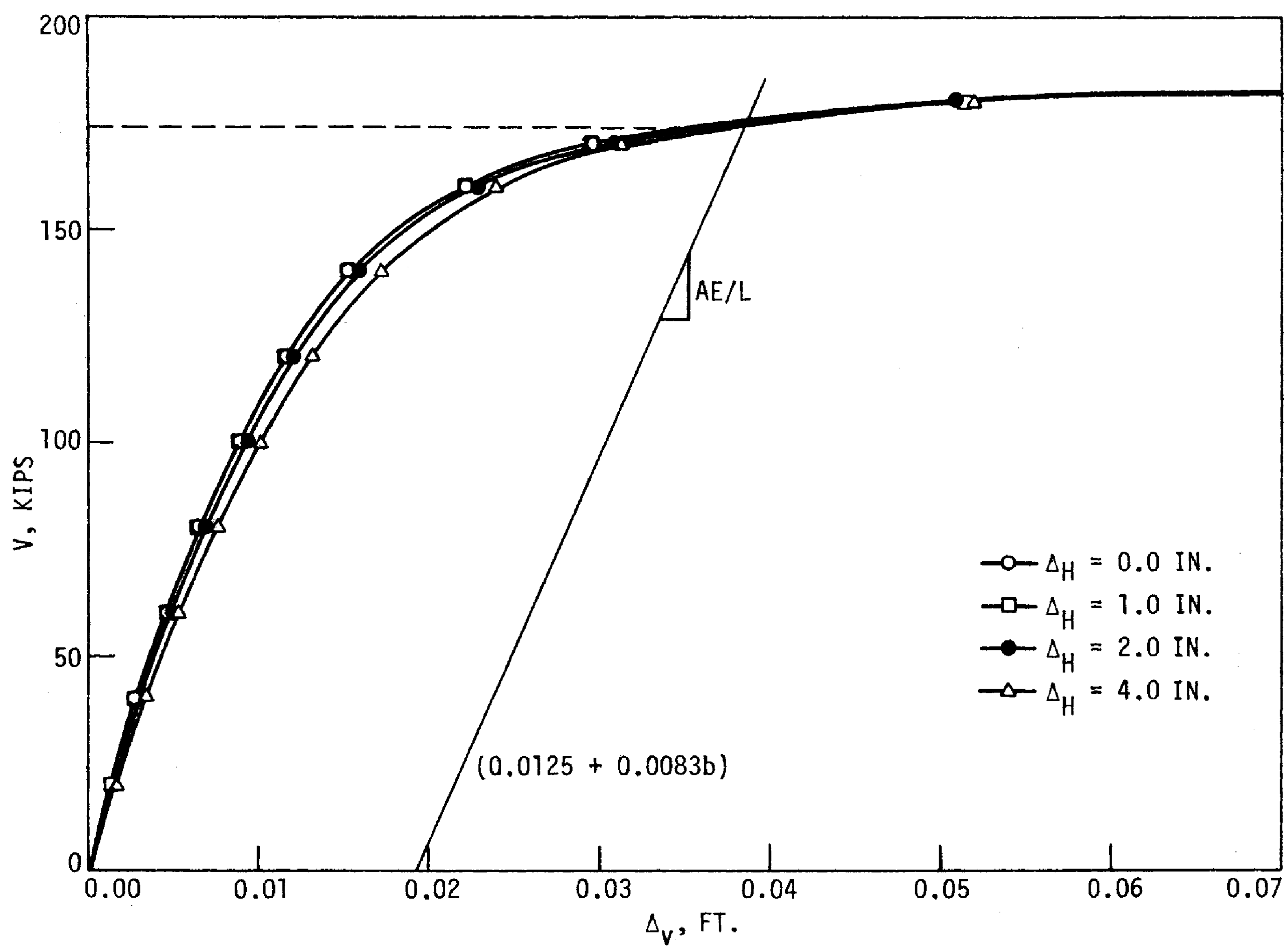

Fig. 40. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for northern Iowa soil.

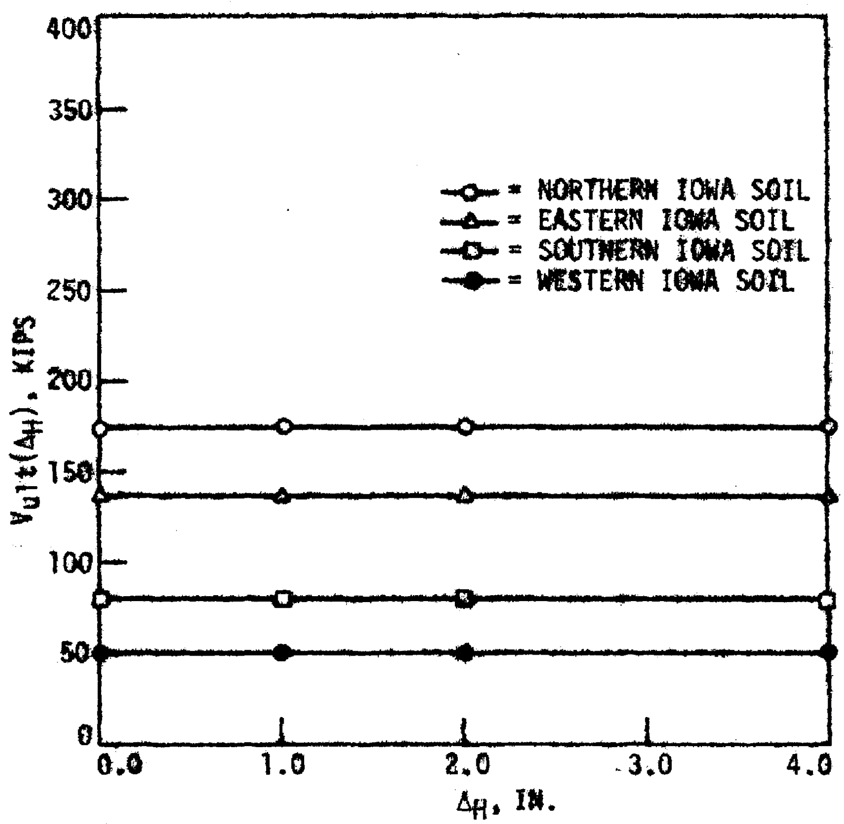

Fig. 41. Ultimate vertical load versus specified lateral displacements for typical Iowa layered soils.

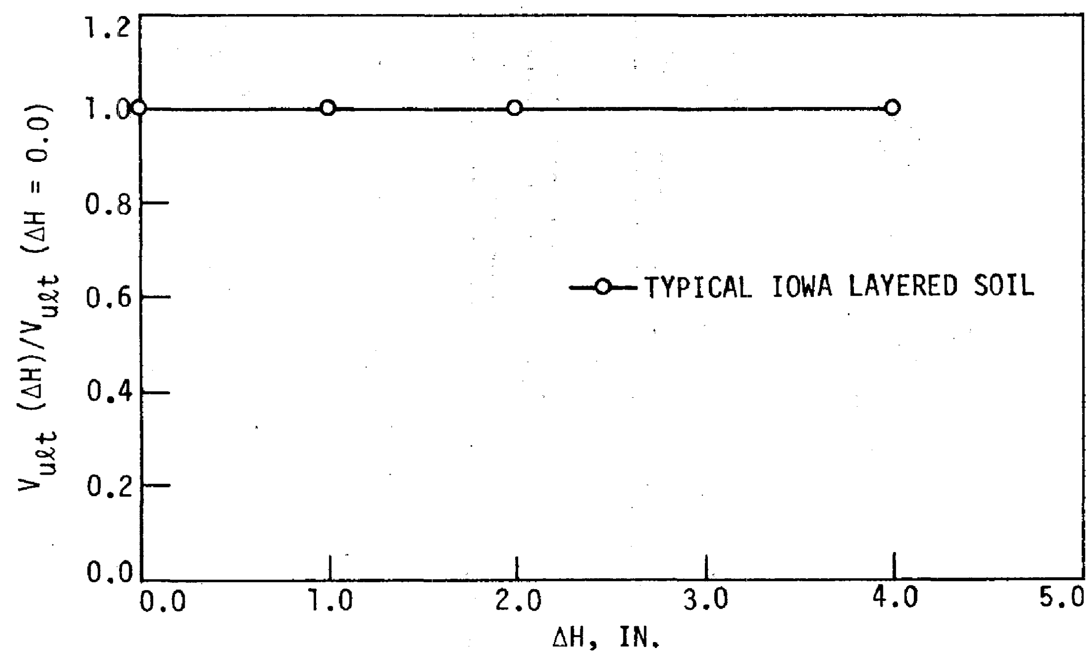

Fig. 42. Non-dimensional forms of ultimate vertical load versus specified lateral displacements, Δ_H, in typical Iowa layered soil.

Case (c) can be neglected in this investigation since the behavior of case (c) with 10 foot long compacted soil is the same as it is for case (b) or one layered type soil (very stiff clay).

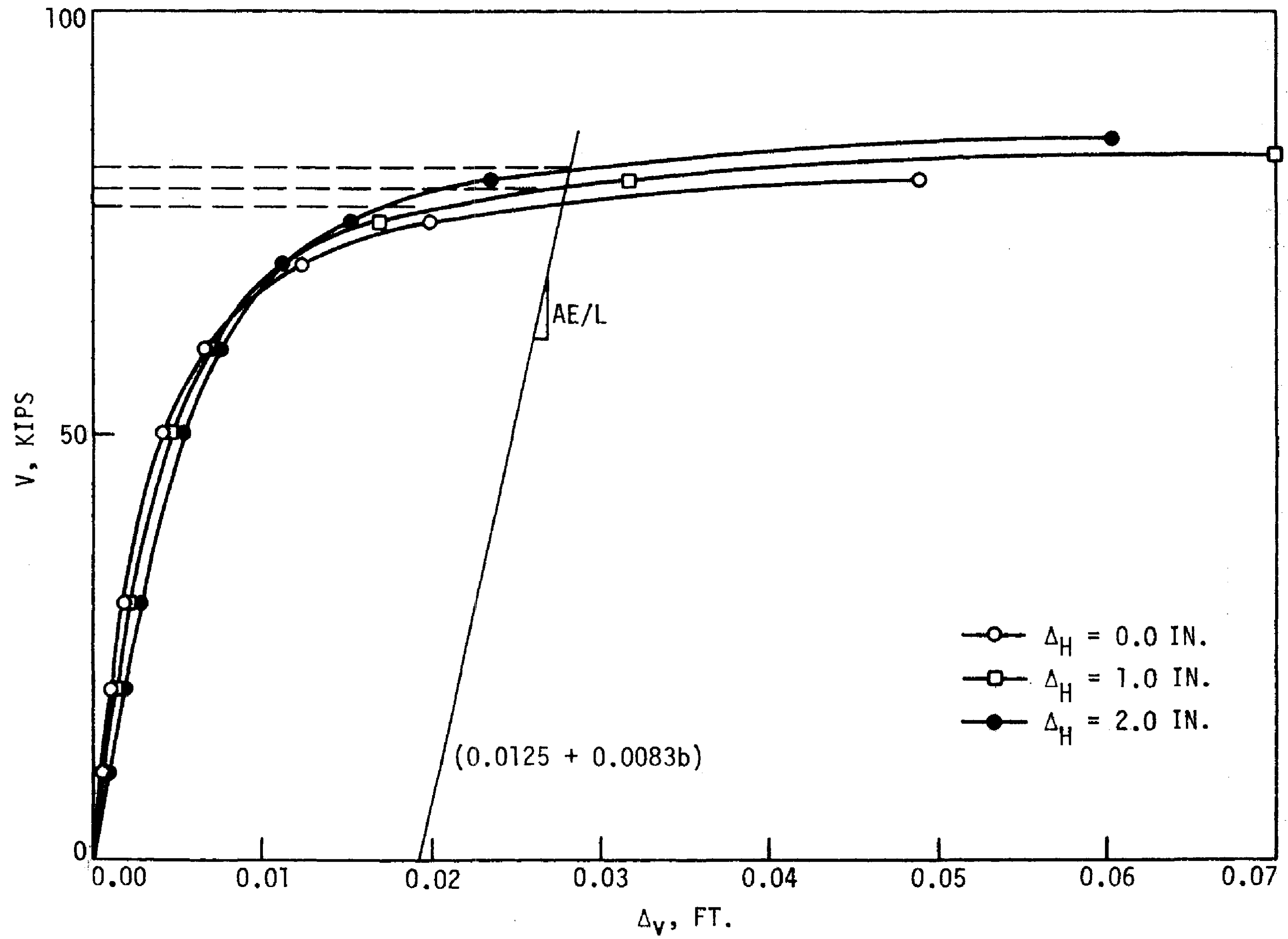

Fig. 43. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for western Iowa soil with 4 foot long compacted soil.

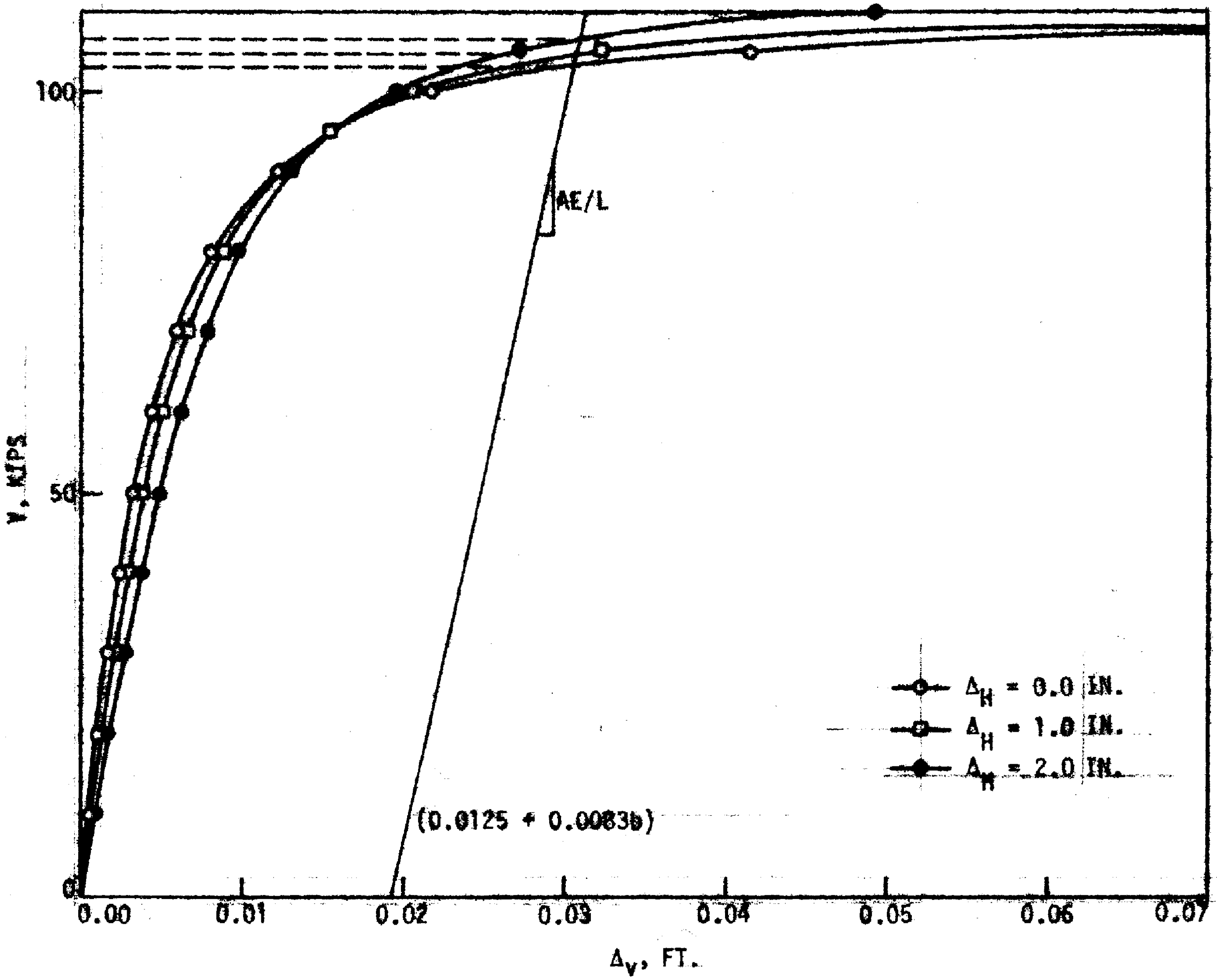

Fig. 44. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for southern Iowa soil with 4 foot long compacted soil.

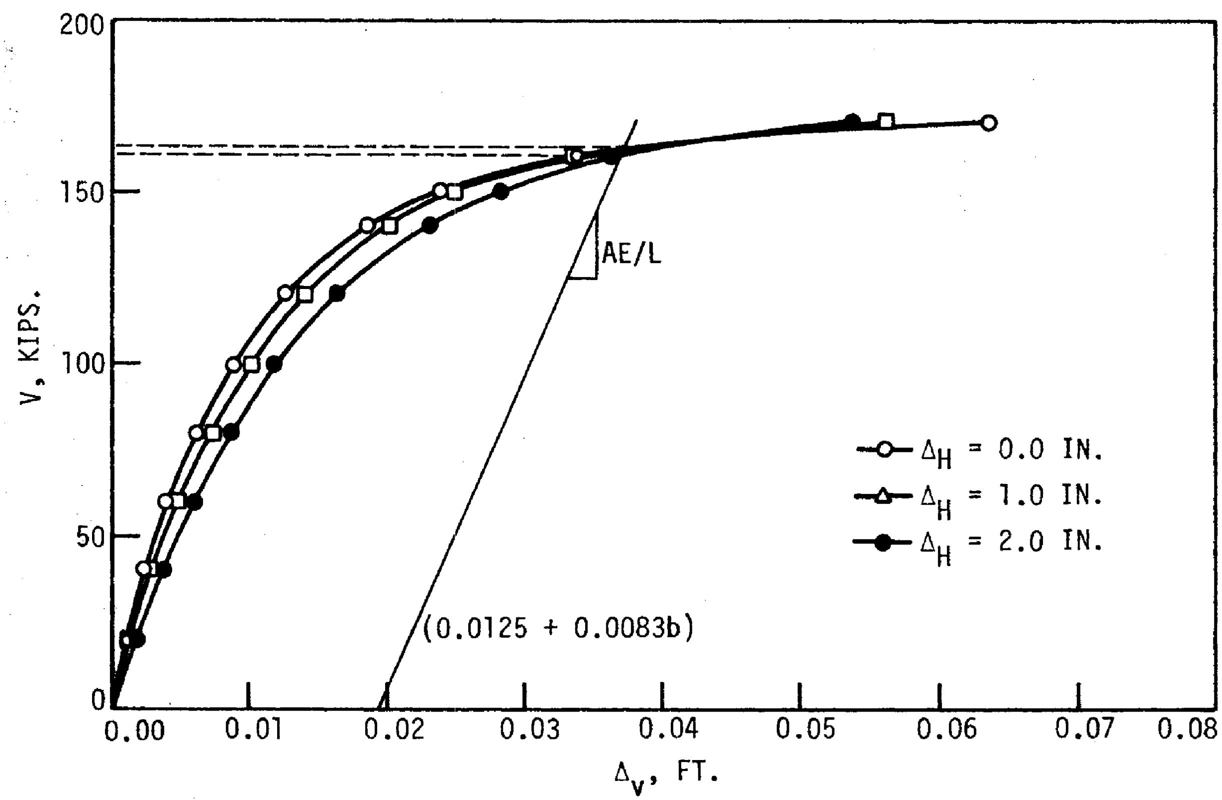

Fig. 45. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for eastern Iowa soil with 4 foot long compacted soil.

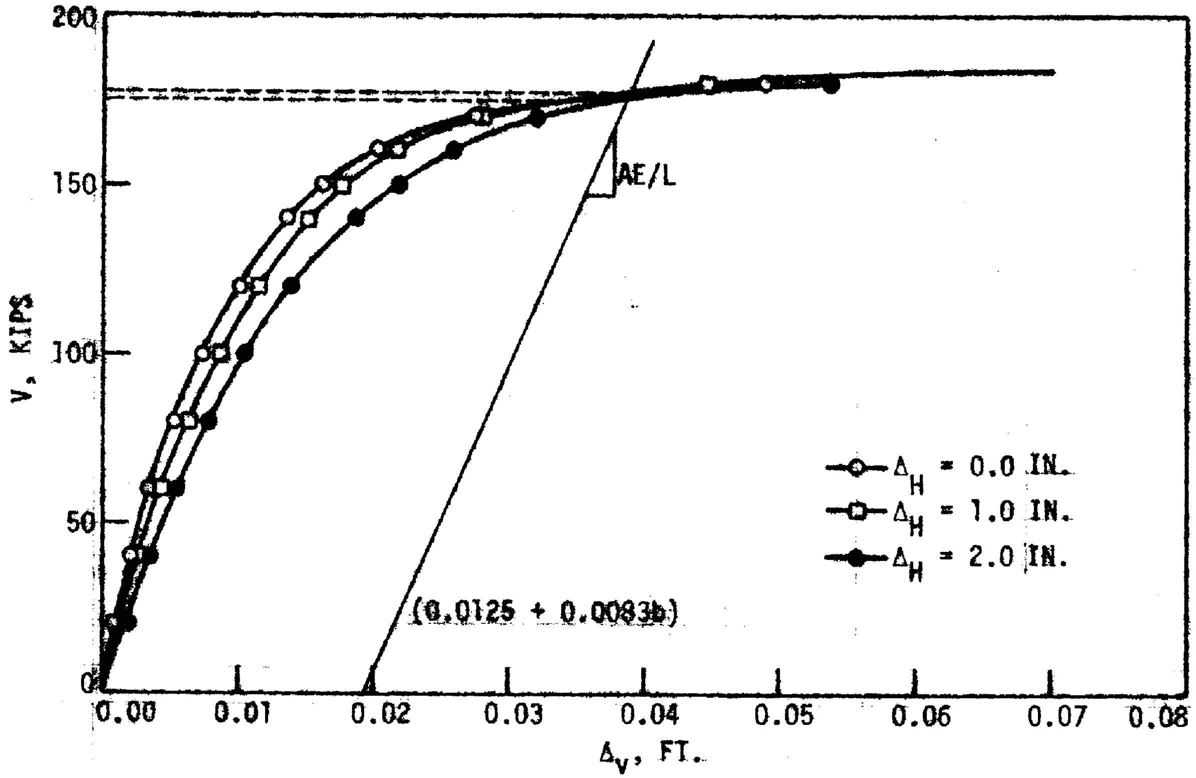

Fig. 46. Vertical load-settlement curves with specified lateral displacements, Δ_H (0, 1, 2, 4 in.), for northern Iowa soil with 4 foot long compacted soil.

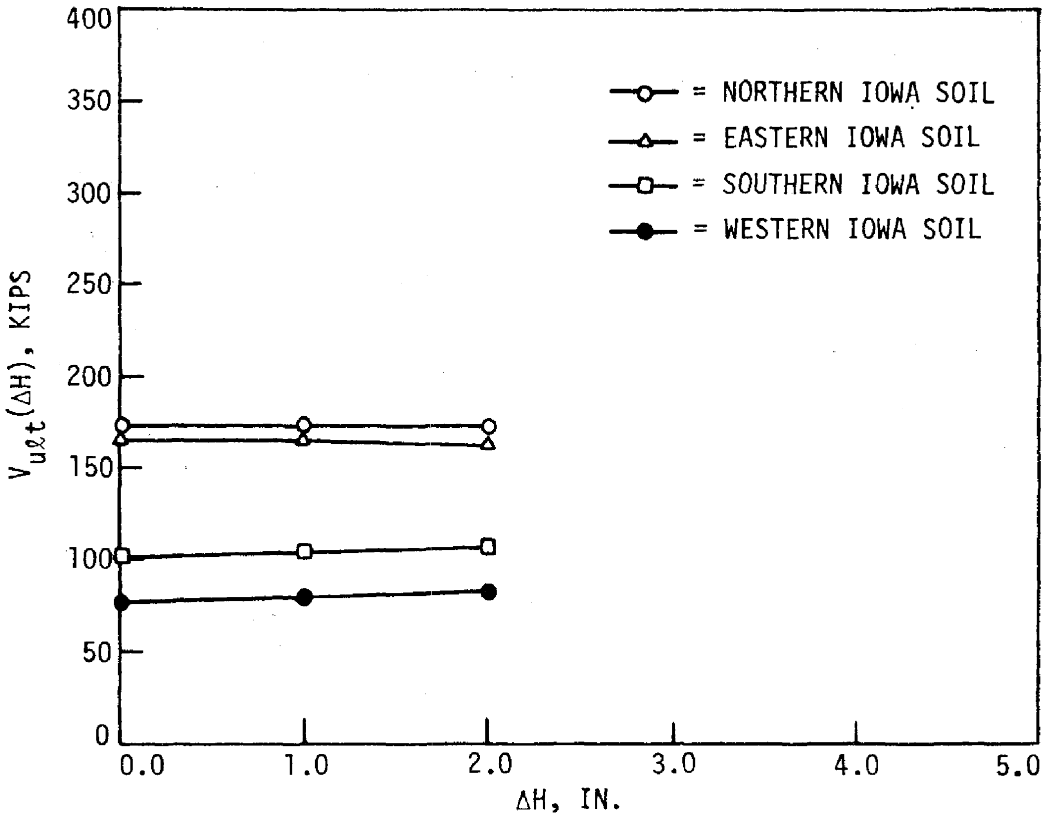

Fig. 47. Ultimate vertical load versus specified lateral displacements for typical Iowa layered soils with 4 foot long compacted soil.

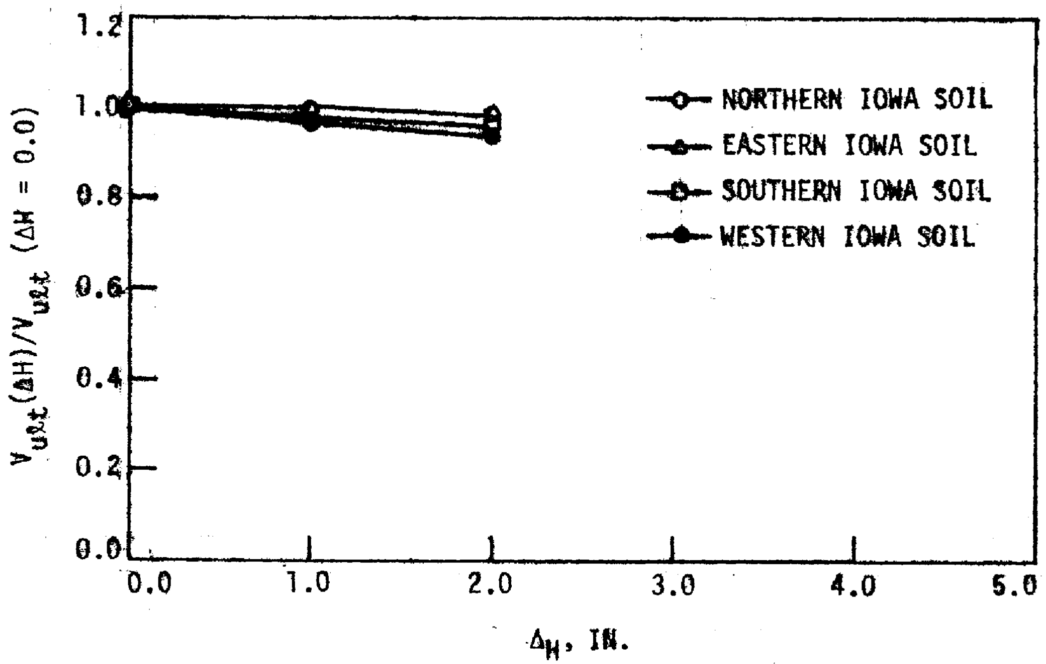

Fig. 48. Non-dimensional forms of ultimate vertical load versus specified lateral displacements, Δ_H, for typical Iowa layered soils with 4 foot long compacted soil.

5. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR FURTHER STUDY

5.1. Summary

The increasing popularity of integral abutment design for bridges has been recognized by many state highway agencies. The current length limitations in Iowa are based on a simple theoretical analysis of the effects of thermal expansion and contraction of bridges on piling stresses. At the present time, only short and moderate length bridges without joints have been built with integral abutments.

Integral abutment bridges fall into two categories: non-skewed and skewed. Site conditions determine whether or not a skewed bridge is necessary. For skewed bridges with integral abutments, the movements caused by thermal expansion and contraction are more complex than they are for non-skewed bridges with integral abutments. These thermal-induced movements involve not only longitudinal direction, but transverse direction and diagonal direction as well.

In July 1972, South Dakota State University issued a report on a study conducted to investigate the stresses induced by thermal movements in the girder and upper portion of steel bearing piles of integral abutment-type bridges. In November 1981, North Dakota State University issued a report based on tests conducted to observe the lateral movement caused by annual temperature variations and to monitor temperature-induced piling stresses. In February, 1982, Iowa State University published a study which summarized the variation in design assumptions and length limitations among the various states in their approach to the use of integral abutments. Also, an algorithm based on a state-of-the-art nonlinear finite element procedure was developed and used to study piling stresses and pile-soil interaction in integral abutment non-skewed bridges.

In this study the highway departments of the states which use integral abutments in bridge design were contacted in order to study the extent of integral abutment use in skewed bridges and to survey the different guidelines used for analysis and design of integral abutments for skewed bridges. Survey responses indicated that 26 states use integral type abutments on skewed bridges. Most states design integral abutments on skewed bridges on the basis of empirical experience, and no theoretical analysis is introduced in design.

For integral abutments on skewed bridges, fifteen states orient their piles with the web of the piles perpendicular to center line of the abutment (Type 2b) so that bending will be primarily about the strong axis. Thus, thermally-induced biaxial bending stresses are introduced into the piles. However, the survey responses show that most states ignore thermally-induced bending stresses caused by transverse movement. Kansas indicated that transverse thermal movement can be eliminated by using shear keys on the bottom of the pile cap. The major reasons given for selecting Type (2b) pile orientation are

- The restraint provided by the integral abutment reduces the magnitude of the thermal movement. Orienting the pile with its strong axis parallel to center line of the bearings gives more rigidity for earthquake loads when liquification of embankment is anticipated.

- Thermal expansion is actually very small, and the backfill material, the abutment, and the piling seem to yield sufficiently so that no distress is apparent. The piling is oriented to resist the force of earth pressure from the abutment backfill rather than the force of thermal expansion.
- Temperature forces act along the center line of the roadway, not parallel to the pile web, and active soil pressure acts against the strong axis of the pile. Temperature effects are partially compensated for by predrilling for driven piles and filling the voids with pea gravel or sand.

No special treatments are given to backfill and pile cap on skewed bridges, and they might be constructed in the same way as non-skewed bridges. As for the approach slab, it might be tied to the abutment with dowels, or an expansion joint may be provided between the approach slab and the bridge slab.

An idealized integral abutment was introduced by assuming that the pile was rigidly cast into the pile cap and that the approach slab offers no resistance to lateral thermal expansion. Also, it is more conservative in design to ignore the passive soil and shear resistance of the cap.

The influence of rotation at the pile head can be classified into three categories: (a) fully restrained without rotation, (b) partially restrained with some degree of rotation, (c) pinned, allowing rotation but not translation. Two extreme cases which represent the upper bound and lower bound of the ultimate pile load capacity will be investigated. The results obtained from the theoretical analysis show that if the piling is cast into the pile cap and a detailed design is used to eliminate moment constraint at the joint, the ultimate vertical load carrying capacity of the H pile after thermal movements can be highly increased.

The effect of predrilled oversized holes was also discussed. According to the survey responses, some states believe that the use of predrilled oversized holes tends to make piles more flexible when they are subject to thermal movements. Piling stresses are thus reduced, and yield stresses are scarcely reached. Results obtained with the Yang 5 program show that the above discussion is correct. If the lengths of the predrilled oversized holes are at least 4 feet in the ground, the ultimate vertical load carrying capacity of the H pile is reduced by only 10 percent for 4 inches of specified lateral displacement in very stiff clay.

The behavior of layered soils is also studied in this investigation. In consultation with Iowa DOT soil engineers, we considered four typical Iowa layered soils. In certain situations, pile is embedded in highly compacted fill above the ground. Three different cases of layered soils of varying length are discussed: (a) L = 0 ft, (b) L = 4 ft and (c) L = 10 ft. The results obtained from case (a) indicated that there is no reduction of ultimate vertical load capacity of the H pile, because the lateral soil stiffness of case (a) in the top 10 feet below the ground line is less than that of very stiff clay. Hence, the yield stress of the pile is not reached. Indeed, in case (b), which involves compacted soil 4 feet above the ground surface, the results show that the failure mechanism is similar to that of very stiff clay; a plastic hinge forms before the specified lateral displacement reaches 4 inches. Thus, if the top portion of the pile is surrounded by soils which possess high lateral soil stiffness, the lateral type failure occurs before vertical type failure. Case (c) can be neglected in this investigation, since the behavior of case (c) with 10 foot long compacted soil is the same as it is for case (b) or one layered type soil (the very stiff clay).

5.2. Conclusions

Most states design integral abutment bridges on the basis of many years of empirical experience, and no theoretical analysis is introduced in design. The survey responses on integral abutment skewed bridges received from state highway departments indicate that no serious problems or distresses have been discovered as yet. Because of their economic benefit and good serviceability, integral abutment bridges have become very popular.

Pile orientations in integral abutments on skewed bridges will cause thermally-induced biaxial bending stresses if the pile orientations specified are of Type (2a) or (2b). One way to avoid this thermally-induced biaxial bending stress is to use the pile orientations specified as Type (1a), (1b) and (3). The survey responses indicated that 15 out of 26 states have adopted these pile orientations so that bending will be primarily about the strong axis. Most states do not consider the thermally-induced bending stress caused by transverse movement. The major reasons given for selecting the Type (2b) pile orientation are given in Section 3.

A previous report by the authors [3] showed that two types of failure mechanism are possible for the pile-soil system: (a) lateral type failure and (b) vertical type failure. Vertical type failure usually occurs when the applied vertical load exceeds the ultimate soil frictional resistance. Lateral type failure usually occurs because of plastic hinge formation in the pile.

Preliminary results from this investigation showed that the vertical load-carrying capacity of H piles with pinned pile heads is not significantly affected by lateral displacements of 2 inches in soft clay, stiff clay, very stiff clay, loose sand, medium sand, and dense sand. However, in very stiff clay (average blow count of 50 from standard penetration tests), it was found that the vertical load-carrying capacity of the H pile is reduced by about 20 percent for 2 inches and 4 inches of lateral displacement. The vertical load-carrying capacity is increased if the pile head is free to rotate. The average AASHTO specified temperature change for Iowa is 40° F for concrete bridges and 75° F for steel bridges. In this case, if a thermal-induced movement of 2 inches was permitted, it would translate into allowable lengths of 1400 feet for concrete bridges and 700 feet for steel bridges without expansion joints. The allowable length for fully restrained pile head is one-half of the allowable length for pinned pile head. If the pile head is partially restrained, the allowable length lies between these two limits. If a detailed design can be achieved to eliminate moment constraint, the use of long integral abutment bridges may be realized.

The results obtained from the investigation into the effect of predrilled oversized hole indicated that the critical location for plastic hinges falls in the range of 0 to 4 feet below the ground. If the length of the predrilled oversized hole is 4 feet or more below the ground, the vertical load-carrying capacity of the H pile is reduced by 10 percent for 4 inches of specified lateral displacement in very stiff clay. If the length of the predrilled oversized hole falls in the range of 0 to 4 feet below the ground, the vertical load-carrying capacity of the H pile will decrease as lateral displacement increases. The results shown here indicate that the predrilled oversized hole affects the vertical load-carrying capacity of the H pile.

Four different typical Iowa layered soils were selected and used in this investigation. In certain situations, compacted soil (> 50 blow count in standard penetration tests) is used as fill on top of natural soil. The critical conditions depend on the length of the compacted soil. If the length of the compacted soil exceeds 4 feet, the failure mechanism for the pile is similar to one in a very stiff clay soil. That is, the vertical load-carrying capacity of the H pile will be highly reduced as the specified lateral displacement increases.

5.3. Recommendations for Further Study

1. Because of annual temperature changes, a bridge superstructure undergoes expansion and contraction, which in turn causes the piles in integral abutment bridges to move back and forth. Thus, the mathematical model used in this investigation must be modified to accommodate loading and unloading of the pile during cyclic loading.
2. Preliminary studies from this investigation showed that the fixity condition at pile head has some influence on the vertical load-carrying capacity of H piles. Additional analytical studies should focus on partially restrained pile head by taking into account the stiffness of the superstructure.
3. When the effect of predrilled oversized holes is considered, a gap element should be introduced into the soil model. This refined model will represent the situation more realistically when lateral displacement is such that the pile starts to push on the soil.
4. In skewed bridges with integral abutments, thermal movements caused by temperature changes in most cases induce biaxial bending in the pile. Thus, the mathematical model developed earlier must be modified to cover problems in three-dimensions.
5. Maximum safe lengths for integral abutment bridges with concrete and timber piles must be determined.
6. A scale model of a pile which represents the behavior of piles in integral abutment bridges should be set up and tested in the

laboratory. The obtained experimental results can be used to verify and refine the analytical model.

7. An actual bridge should be instrumented to monitor thermal movements and piling stresses during several cycles of temperature change.

6. ACKNOWLEDGMENT

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The authors wish to express their gratitude to the bridge engineers of all the fifty states for the 100 percent return rate on the survey questionnaire. In particular, special thanks go to those who provided additional information concerning their design and detailing procedures. The authors also wish to extend sincere appreciation to the engineers of the Iowa DOT for their support, cooperation, and counseling. Special thanks go to Charles A. Pestotnik, Henry Gee, Vernon Marks, and Kermet L. Dirks from the Iowa DOT, Bruce Johnson from the Federal Highway Administration, and Wallace W. Sanders, Jr. from Iowa State University.

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8. APPENDICES

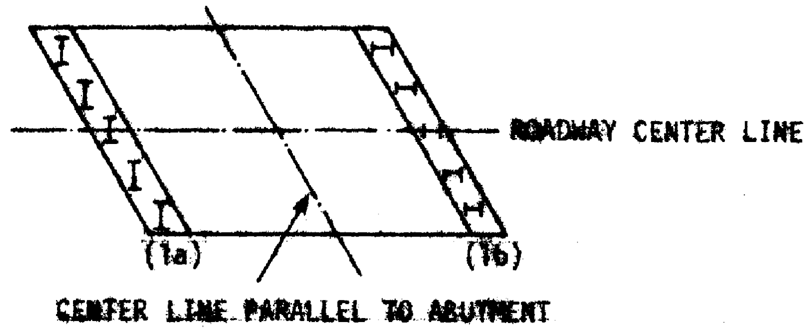
8.1. Questionnaire for Survey of Current Practice on Skewed Bridges with Integral Abutments and Summary of Responses

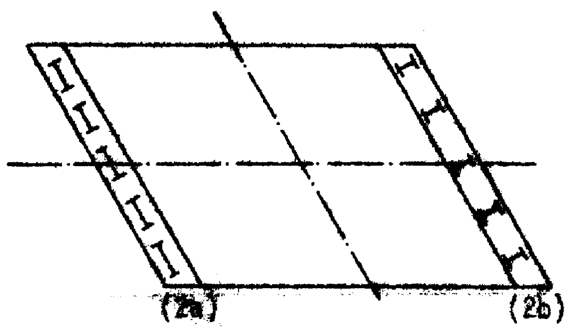
- Part 1. Questionnaire for skewed bridges with integral abutments
- Part 2. Summary of responses by the different states
- Part 3. Summary of additional comments made by some of the states

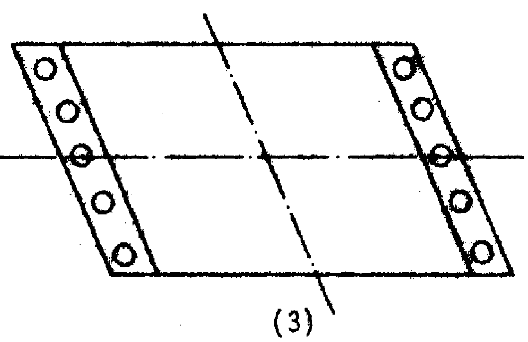
8.2. Listing of Program

Part One. Questionnaire for Skewed Bridges with Integral Abutments.

1. If you design skewed bridges with integral abutments, which of the kinds of pile orientations shown below do you use in the integral abutments? If neither, please sketch the type of pile orientation you use.







2. If you use either orientation, what structural assumptions are made for (1) the top of the pile, (2) thermal expansion or contraction (one direction or both directions) and (3) diagonal thermal expansion or contraction?

3. When you design skewed bridges with integral abutments, how do you treat the approach slab, backfill, and pile cap?

4. Any additional comments on skewed bridges with integral abutments?

Part 2. Summary of responses by the different states.

State	Pile Orientation 1(a)	1(b)	2(a)	2(b)	3	Structural Assumption: Pile Head	Thermal Exp. & Cont.: Long.	Trans.	Diag.	Design Consideration: Approach Slab	Backfill	Pile Cap	Comment
AK			---		---		---	---	---	---	---	---	---
AZ	N	N	N	Y	N	Roller	Y (due to roller)	Re-strained by abutment cap	N	Tied to abutment with dowels and moves back and forth with the superstructure	N	N	---
CA	N	N	N	Y	N	Hinge	N	N	N	---	---	---	Battered piles are used to resist the active earth pressure
CO	Y	Y	Y	Y	N	---	---	---	---	Bridge length >200' use approach slab	N	N	1) Steel bridge ≤250'; concrete bridge ≤350' 2) No problem in skew 3) Use predrilled oversize hole
CT	N	N	N	N	N	N	N	N	N	N	N	N	N
GA	Y	N	N	N	N	Free translation; free rotation; roller	Y	Y	N	Expansion joint between the approach slab and bridge slab	---	---	---
IA	N	N	Y	N	N	Fixed	Y	N	N	Neglect	Neglect	Neglect	Conservative design

Part 2 continued.

State	Pile Orientation 1(a)	1(b)	2(a)	2(b)	3	Structural Assumption: Pile Head	Thermal Exp. & Cont.: Long.	Trans.	Diag.	Design Consideration: Approach Slab	Backfill	Pile Cap	Comment
ID	N	N	Y	Y	N	Fixed	Y	Y	---	1) Expansion joint is specified between rigid pavement & approach slab 2) No special treatment is specified for flexible pavement	Use free draining granular material as backfill	Rigid pile cap	A skewed three span steel girder bridge with integral abutment was built. Rotational forces from the lateral earth pressure on the end wall caused a failure in the pier anchor bolts on the exterior girder
IN	N	N	N	Y	N	Hinge	N	N	N	20' approach slab integrally attached to bridges	Use select granular fill	Pile is cast in pile cap 1 ft	150' maximum
KS	N	N	Y	Y	N	Hinge	Y	Y	N	Uses slab support at backwall and pavement rests on slab w/ approx. 30' from end of wearing surface	Backfill compaction has settlement just off end of bridge	Pile caps are not used	Cast-in-plate bridges w/ the end of steel beams into abutment concrete, reinforcing to make them essentially integral
KY	Y	N	N	N	N	Partially restrained	Y	N	---	No special treatment with flexible pavement	Special granular backfill specified	---	Bridge length 300', max skews $\leq 30°$, pile prebored for distance of 8' before bottom of pile cap

Part 2 continued.

	Structural Assumption									Design Consideration			
	Pile Orientation						Thermal Exp. & Cont.						
State	1(a)	1(b)	2(a)	2(b)	3	Pile Head	Long.	Trans.	Diag.	Approach Slab	Backfill	Pile Cap	Comment
MO	N	N	N	Y	N	Fixed	N	N	N	---	---	Use shear key on bottom of pile cap to prevent lateral movement of pile cap on extreme skews (±40°)	Piles designed for direct load only: <500' for prestressed bridges, <400' for steel bridges
MT	N	N	N	Y	N	N	N	N	N	Not fixed to abutment	Granular material as backfill	N	≤30° skews
ND	N	N	N	Y	N	Fixed	Y	Y	N	Assume approach slab has no effect	Select granular material	Abutment wall is pile cap and is reinforced to resist bending below super structure	Hold skew to a max of 30°
NE	Y	N	N	N	N	---	Y	N	N	Same as square bridges with integral abutments	"	"	15° skew for integral abutment
NM	N	N	Y	N	N	Fixed	Y	N	N	Used on some bridges, not used on others	Do not use specified backfill anymore	---	Have built bridges with 15° skew; skew angle neglected

Part 2 continued.

State	Pile Orientation					Structural Assumption				Design Consideration			Comment
							Thermal Exp. & Cont.						
	1(a)	1(b)	2(a)	2(b)	3	Pile Head	Long.	Trans.	Diag.	Approach Slab	Backfill	Pile Cap	
NY	N	N	Y	N	N	---	N	N	N	Construction joint is provided between approach slab and bridge slab	Granular fill behind backwall and wing walls	N	1) Neglect caused by rotation; designed to take vertical load only 2) In skewed bridges, neglect some twisting induced in piles when structure deflects. Use pre-drilled oversize hole
OH	N	N	Y	N	N	---	N	N	N	Tie the approach slab to abutment	Same as non integral abutments for usual short bridge	Pile is cast in pile cap 2 ft	Oil country pipe lines are not used in integral abutments, because they are stiffer than H-piles about weak axis
OK	N	N	N	N	N	---	---	---	---	---	---	---	Integral abutments only with zero skews
OR	N	N	N	Y	N	Hinge	---	---	---	Approach slab was tied to pile cap	---	Pile is cast in pile cap 1 ft	---
SD	Y	N	Y	N	N	Fixed	Y	N	N	Tied w/bridge to prevent erosion of shoulder	---	N	---

Part 2 continued.

	Pile Orientation					Structural Assumption				Design Consideration			
							Thermal Exp. & Cont.						
State	1(a)	1(b)	2(a)	2(b)	3	Pile Head	Long.	Trans.	Diag.	Approach Slab	Backfill	Pile Cap	Comment
TN	N	N	N	Y	N	---	Y	N	N	A construction joint between the abutment backwall and approach slab	N	N	---
UT	N	N	N	N	Y	Hinge	---	---	---	Expansion joint between approach slab and bridge slab	96% of optimum	N	1) Steel piles used primarily thru granular material over bed rock 2) No problem in thermal movements
VA	N	N	Y	N	N	Fixed	N	N	N	No approach slab	Used 1'-6" of porous backfill w/ 6" dia. pipe underdrain	Uniform width and parallel to bridge skew	Max. skew 10°; relatively small movement at each abutment (±3/8")
VT	N	N	N	Y	N	Fixed	Y	N	N	The approach slab is anchored to the abutment	No special treatment	Rigid pile cap	$\leq$30° skew
WA	Y	N	N	Y	Y	Hinge	Y	N	N	Approach slab is attached to abutment with allowance for expansion	Backfill earth pressure is applied normal to abutment	Pile cap is designed as cross beam on simple supports	Calculate moments of inertia along roadway center
WS	N	N	N	Y	Y	---	N	N	N	Designed for vertical load only	---	Designed as reinforced continuous beam over piling	Piles designed for vertical loads $\leq$30° for slabs; $\leq$15° for prestressed or steel girders

Part 2 concluded.

	Pile Orientation					Structural Assumption				Design Consideration			
							Thermal Exp. & Cont.						
tate	1(a)	1(b)	2(a)	2(b)	3	Pile Head	Long.	Trans.	Diag.	Approach Slab	Backfill	Pile Cap	Comment
WY	N	N	N	Y	N	Plastic Hinge	Y	Y	N	Neglect	Neglect	Assumed to be a mass attached to end of girder	Max. length $\leq$300'
R15	N	N	Y	N	N	Hinge	N	N	N	---	---	Pile was cast in pile cap 1 ft	---

Part 3. Summary of Additional Comments Made by Some of the States

Wisconsin

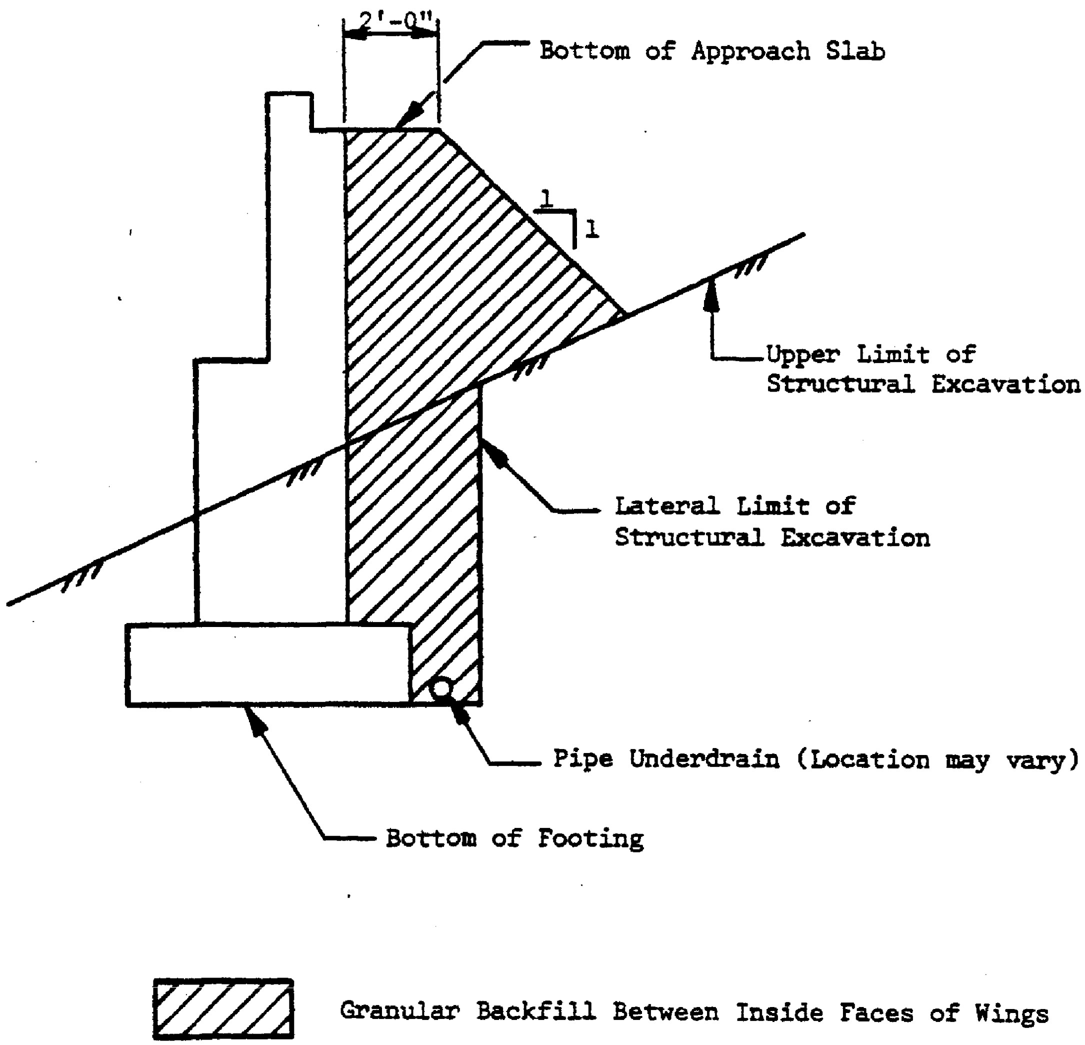
The pile orientation used in Wisconsin is based on past experience and site soil conditions.

In 1971, a study of abutments supported on piles driven through soft compressible soil showed that they have a tendency to tilt toward the backfill when the shearing strength of the soil is exceeded because of super-imposed embankment loads. To help prevent tilting of the abutments, the study recommended nondisplacement type piles with strong-axis orientation to resist bending forces.

The use of Type (1a) and (2a) abutments dates back some 20 years, and in general performance has been good. On larger skews (greater than 30 degrees), occasional cracking or spalling of the end diaphrams around the girders and/or at the construction joint has been observed. At present it is debatable whether this results from temperature movements, pile orientation, or the restraint of the integral abutments.

Selection of standard abutment types is described in the attached sheets (Bridge Design Manual). Type A1 with fixed seat, which is listed in the manual, appears to be equivalent to the Type (2b) integral abutment discussed earlier. At this time the change of pile orientation for added flexibility to accommodate thermal expansion is not considered [23].

Manual	Subject	Number
Bridge Design	Chapter 12 - Abutments	



12.7 SELECTION OF STANDARD ABUTMENT TYPES

From past experience and investigations the following types of abutments are generally most suitable and economical for the given conditions. Although piles are shown for each abutment type, spread footings may be utilized. The following chart is a guide only and need not be rigidly followed.

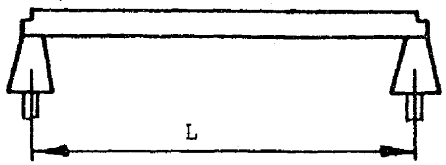
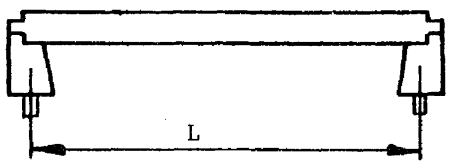
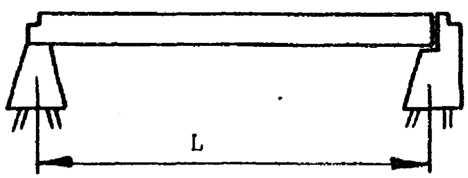
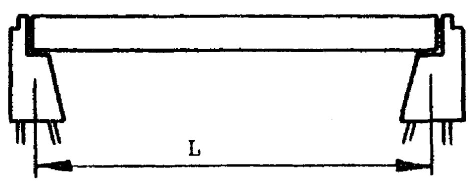
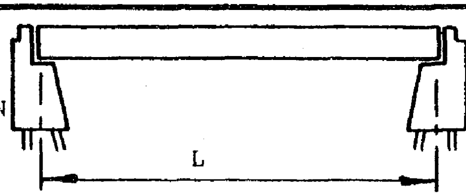
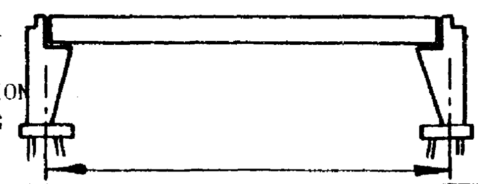
ABUTMENT ARRANGEMENTS L = Length of continuous superstructure between abutments				SUPERSTRUCTURES CONCRETE SLAB SPANS	 PRESTRESSED GIRDERS	 STEEL GIRDERS
				L = Length and S = Skew , AL = Abutment Length		
(1)	TYPE A1 WITH FIXED SEAT		TYPE A1 WITH FIXED SEAT	L $\leq$ 300' S $\leq$ 30° AL $\leq$ 50'	L $\leq$ 300' S $\leq$ 15° AL $\leq$ 50'	L $\leq$ 150' S $\leq$ 15° AL $\leq$ 50'
(2)	TYPE A1 WITH SEMI-EXP. SEAT		TYPE A1 WITH SEMI-EXP. SEAT	L $\leq$ 300' S $\leq$ 30° AL > 50'	L $\leq$ 300' S $\leq$ 40°	L $\leq$ 200' S $\leq$ 40°
(3)	TYPE A2 WITH FIXED SEAT		TYPE A3 WITH EXPANSION BEARING	L > 300' with flexible Piers and S $\leq$ 30° AL $\leq$ 50'	300' < L $\leq$ 400' AL $\leq$ 50' and with flexible Piers	NOT USED
(4)	TYPE A3 WITH FIXED BEARING		TYPE A3 WITH EXPANSION BEARING	NOT USED	NOT USED	L $\leq$ 200' S > 40°
(5)	TYPE A3 WITH EXPANSION BEARING		TYPE A3 WITH EXPANSION BEARING	a. L > 300' with rigid Piers and S $\leq$ 30°	Exceeds above Criteria for (1) and (2) and (3)	L > 200'
(6)	TYPE A4 WITH EXPANSION BEARING		TYPE A4 WITH EXPANSION BEARING	NOT USED	b. Based on Geometry and Economics	c. Based on Geometry and Economics

TABLE 12.1 ABUTMENT TYPES

TABLE 12.1 - Abutment Types (Footnotes)

a. Consider the flexibility of the piers when choosing these abutment types. Only one expansion bearing is needed if the structure is capable of expanding easily in one direction. With rigid piers, symmetry is important for getting equal expansion movements and minimizing the forces on substructures.

b. For two-span prestressed girder bridges, it may be necessary to investigate both sill type and semi-retaining type abutments. For example, semi-retaining abutments with 45-inch girders may be more economical than sill abutments with 54-inch girders. The sill abutment is usually more economical than a semi-retaining, if both are feasible for a given girder depth.

c. For two-span steel structures with long spans, the semi-retaining abutments are usually more economical than sill abutments due to the shorter bridge lengths.

New York

The piles are oriented so that the web of the pile will be parallel to the abutment stem. When there is no skew or a small skew (20 degrees or less), the bending caused by thermal expansion will occur about the weak axis of the pile.

A 119-ft span integral abutment structure with a 30 degree skew is presently being designed. The piles will be oriented as Type (1a), with the web of the pile perpendicular to the centerline of the roadway. This orientation simplifies the pile stringer connection and should minimize the twisting effect caused by the skew.

The only other skewed integral abutment bridge in New York state is supported on cast-in-place piles and contains a 25 degree skew. Since the piles are round, pile stiffness was not affected by the skew. The 125' span structure has been in service about 3 years and appears to be functioning as designed [24].

South Dakota

For Types (1a) and (1b) pile orientations no distinction is made since the skew angle is limited to approximately 30°. The majority of skewed bridges in South Dakota have been built with the piles oriented as Type (2a). Realizing that a tendency for undesirable lateral movement exists, it seemed logical to place the piles so that the strongest section can help to resist this movement [25].

Casual observation of integral abutment bridges in South Dakota seems to indicate that they do not get the full range of calculated temperature movement. In an effort to substantiate this observation, the highway department has asked its field bridge inspection personnel to place reference points on structures of this type and take measurements at temperature extremes. However, this effort was only initiated during the past spring, and it might be several years before any assumptions can be made.

```
C        THIS COMPUTER PROGRAM,YANG5,IS BASED ON THE STATE-OF-THE-ART
C        IN THE DESIGN OF PILE SUBJECTED TO STATIC LOADING, LINEAR AND
C        NONLINEAR BEHAVIOR OF SOIL AND PILE PROPERTIES HAVE BEEN
C        CONSIDERED IN THIS TWO-DIMENSIONAL PROBLEMS. A FINITE ELEMENT
C        MODEL IS BEING USED IN THIS PROGRAM. A NEWTON-RAPHSON SOLUTIO
C        ALGORITHM HAS BEEN ADOPTED BY USING INCREMENT AND ITERATION
C        APPROACHES IN ORDER TO OBTAIN THE NONLINEAR BEHAVIOR RESPONSE
C        OF THE PILE-SOIL INTERACTION.
C               IOWA STATE UNIVERSITY- - - DEPT. OF CIVIL ENGINEERING
C                      PE-SHEN YANG
         REAL LX,LY,LX1,LY1
         DIMENSION TITLE(20)
         COMMON/B/ASAT(60,6),ES(6,6),EA(6,6),EAT(6,6),ESAT(6,6),EASA(6
         COMMON/BB1/WT1(20,20),WD1(20,20),Y(20,20)
         COMMON/BB2/AR(20,20),XR(20,20)
         COMMON/BB3/TLOAD(60),DELTAP(60)
         COMMON/BB4/ACTP(60),ACTSF(6,20),ACTFOR(6,20)
         COMMON/BB5/FT(20),WT(20),WD(20),FW(20)
         COMMON/BB6/DFORCE(3,20),FORCE(6,20)
         COMMON/B1/X1(21),Y1(21),X2(21),Y2(21),X3(21),Y3(21)
         COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
         COMMON/B3/SSI(20),SSF(20),CH(20),PH(20)
         COMMON/B4/SF(6,20),SP(6,20),SS(6,20)
         COMMON/B5/SKI(20),SKF(20),CV(20),PV(20)
         COMMON/B6/SPI(20),SPF(20),CP(20),PP(20)
         COMMON/B7/DSTRA1(20,21,20),DSTRA2(20,21,20),DELSTR(20,21,20)
         COMMON/B8/ET(20,21,20),EK(20,21,20),EI(20,21,20)
         COMMON/B9/ET1(21,20),EK1(21,20),EI1(21,20)
         COMMON/B10/ELFOR(4,20)
         COMMON/B11/ES1(3,3),ES2(6,3),ES3(6,6)
         COMMON/B12/ER(6,3),ERT(3,6)
         COMMON/B13/X(60,2),DISPLA(60),DISTP(6,20),DIST(3,20)
         COMMON/B14/DELP(60),NPE(20,6)
         COMMON/B15/DISP(6,20),DIS(6,20)
         COMMON/B16/DELSTS(20,21,20),DSTRSS(20,21,20)
         COMMON/B17/ALN(3,4),ANLN(3,4)
         COMMON/B18/KODE(21),ULX(21),VLY(21),WLZ(21)
         COMMON/B19/CONST1(21),CONST2(21),CONST3(21)
         COMMON/B20/B(20),PER(20)
         COMMON/B21/XCOS,XSIN
         COMMON/B22/SSPF1,SSPF2,SSPR1,SSPR2,SSPP2,AREAP
         COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
         COMMON/B24/EY,SIGY,ETE,ETP
         COMMON/B25/UT1,UT2,UT3,UT4,UT5,UT6,UT7,UT8
         COMMON/B26/NJTER,NJICR,JJICR,JICR,JTER,DCOV
         COMMON/B27/NBAND,NSIZE
         COMMON/B28/IAZ
         CHARACTER*80 TITLE
         CHARACTER*4 UT1,UT2,UT3,UT4
         CHARACTER*8 UT5,UT6,UT7,UT8
         READ(5,1)TITLE
     1   FORMAT(20A4)
         READ(5,2)UT1,UT2,UT3,UT4,UT5,UT6,UT7,UT8
     2   FORMAT(A2,A2,A4,A4,A7,A7,A4,A7)
         PRINT 4,TITLE
     4   FORMAT(' ',5X,20A4)
C
C        INPUT PILE DATA
```

```
      CALL DATA1
C
C     INPUT SOIL DATA
C
      CALL DATA2
C
C     INPUT NODAL INFORMATION,CRITERIA OF CONVERGENCE
C
      CALL DATA3
C
C     START DO-LOOP TO APPLY INCREMENTAL LOADING ON TANGENTIAL
C     STRUCTURE STIFFNESS MATRIX
C
C     INITIALIZE THE NUMBER OF CYCLES
C
      NJICR=0
      NJTER=0
  230 NJICR=NJICR+1
      IAZ=0
      NJTER=0
  235 NJTER=NJTER+1
C
C     ZERO OUT BANDED STRUCTURAL STIFFNESS MATRIX,ASAT;AND
C     THE INCREASED DISPLACEMENT MATRIX,DISPLA;THE UNBALANCED
C     NODAL FORCE,DELP.
C
      NSIZE=NP
      NBAND=6
      DO 240 IR=1,NSIZE
      DISPLA(IR)=0.0
      DELP(IR)=0.0
      DO 240 IC=1,NBAND
  240 ASAT(IR,IC)=0.0
C
C     START DO-LOOP TO BUILD UP TANGENTIAL STRUCTURE STIFFNESS
C     MATRIX
C
      DO 245 N=1,NM
      CALL UPDATE(1,N)
      CALL ELSPRF(1,N)
      CALL ELSPFF(1,N)
      IF(N.EQ.NM)THEN
      CALL ELSPPF(1,N)
      END IF
      CALL ELDEDP(N)
      CALL STRAIN(N)
      CALL STRESS(N)
      CALL ELDEFR(N)
      CALL ELEAKI(N)
      CALL ELTRAM(N)
      CALL ELDEST(N)
      CALL ELSTIF(4,N)
      CALL ELTEMP(N)
      CALL ELGLOS(N)
      CALL ELASSM(N)
  245 CONTINUE
C
C     DURING THE ITERATION,COMPUTE THE ACTUAL INCREMENT FORCE
C     TO BE APPLIED
```

```
      DO 250 I=1,NP
  250 DELP(I)=DELTAP(I)-ACTP(I)
C
C     INTRODUCE KINEMATIC CONSTRANTS(GEOMETRIC BOUNDARY CONDITIONS)
C
      CALL CONSTR
C
C     SOLVE FOR INCREASED DISPLACEMENT : DELP=ASAT*DISPLA
C
      CALL BANSOL(1,ASAT,DELP,NSIZE,NBAND)
      CALL BANSOL(2,ASAT,DELP,NSIZE,NBAND)
      DO 255 I=1,NSIZE
  255 DISPLA(I)=DELP(I)
      DO 260 I=1,NMP
      X2(I)=X1(I)
      Y2(I)=Y1(I)
      IA1=3*I-2
      IA2=3*I-1
      IA3=3*I
      X(IA1,1)=X(IA1,1)+DISPLA(IA1)
      X(IA2,1)=X(IA2,1)+DISPLA(IA2)
      X(IA3,1)=X(IA3,1)+DISPLA(IA3)
      X1(I)=X1(I)+DISPLA(IA1)
      Y1(I)=Y1(I)+DISPLA(IA2)
  260 CONTINUE
      DO 262 I=1,NP
  262 ACTP(I)=0.0
C
C     CALCULATE ELEMENT FORCES IN TERMS OF LOCAL COORDINATE
C
      DO 265 N=1,NM
      CALL UPDATE(1,N)
      CALL ELTEMP(N)
      CALL ELDISP(N)
      CALL ELSPFR(N)
      CALL UPDATE(1,N)
      CALL ELDEDP(N)
      CALL STRAIN(N)
      CALL STRESS(N)
      CALL ELDEFR(N)
      CALL ELTRF(N)
      CALL ELTRAM(N)
      CALL ELMFOR(N)
C
C     ASSEMBLE EQUILIBRIUM ACTUAL FORCES AT EACH NODE IN GLOBAL
C
      CALL UPDATE(1,N)
      CALL ELTEMP(N)
      CALL ELACTP(N)
  265 CONTINUE
      IF(NJTER.LE.2) GO TO 310
C     TEST FOR DISPLACEMENT CONVERGENCE
      DO 315 I=1,NP
      DISCOV=X(I,1)-X(I,2)
  315 IF(ABS(DISCOV).GT.ABS(DCOV)) GO TO 310
C     PRINT NODAL DISPLACEMENT IN TERMS OF GLOBAL COORDINATE
      PRINT 35,NJTER
   35 FORMAT('0',5X,'NODAL DISPLACEMENT IN TERMS OF GLOBAL AXIS'
```

```
     1   'X-COORD','Y-COORD'
    62   FORMAT(//,5X,A9,10X,A8,10X,A8,10X,A10,10X,A7,10X,A7)
         DO 317 I=1,NMP
         IB1=3*I-2
         IB2=3*I-1
         IB3=3*I
   317   PRINT 64,I,X(IB1,1),X(IB2,1),X(IB3,1),X1(I),Y1(I)
    64   FORMAT(/,10X,I4,8X,F10.5,8X,F10.5,10X,F10.5,2(7X,F10.5))
         PRINT 68,'ELEMENT FORCES IN TERMS OF LOCAL AXIS'
    68   FORMAT(' ',5X,A37)
         PRINT 70,'MEMNO','AXAIL FORCE(KIPS)','SHEAR FORCE(KIPS)',
     1   'END MONENT(FT-KIPS)','AXIAL FORCE(KIPS)','SHEAR FORCE
     2   (KIPS)','END MONENT(FT-KIPS)'
    70   FORMAT(///5X,A5,3X,A17,3X,A17,1X,A19,3X,A17,1X,A19,1X,A19)
         DO 318 I=1,NM
         PRINT 72,I,(FORCE(L,I),L=1,6)
    72   FORMAT(' ',4X,I5,6(2X,F18.5))
         PRINT 74,(SF(L,I),L=1,6)
    74   FORMAT(' ',9X,6(2X,F18.5))
         PRINT 75,(SS(L,I),L=1,6)
    75   FORMAT(' ',9X,6(2X,F18.5))
   318   CONTINUE
         GO TO 320
   310   DO 325 I=1,NP
   325   X(I,2)=X(I,1)
         IF(NJTER.LT.JTER) GO TO 235
         PRINT 76,'THE CONVERGENCE IS NOT REACHED'
    76   FORMAT(' ',A30)
         GO TO 6000
C
C        COMPUTE FORCE INCREMENT TO BE APPLIED
C
   320   CALL FEDEIC
         IF(IAZ.EQ.1) GO TO 230
         IF(NJICR.LT.JJICR) GO TO 230
         PRINT 78,'ACTUAL NODAL LOADS IN TERMS OF STRUCTURAL AXIS'
    78   FORMAT(' ',5X,A46)
         PRINT 80,'JOINT','X-LOAD,KIPS','Y-LOAD,KIPS','Z-LOAD,FT-KIPS
    80   FORMAT(' ',5X,A5,5X,A11,5X,A11,5X,A14)
         DO 335 I=1,NMP
         JJ1=3*I-2
         JJ2=3*I-1
         JJ3=3*I
   335   PRINT 82,I,ACTP(JJ1),ACTP(JJ2),ACTP(JJ3)
    82   FORMAT(' ',5X,I5,5X,F11.5,5X,F11.5,7X,F11.5)
         DELV=X(1,1)*12.
         DELH=X(2,1)*12.
         ROT=X(3,1)
         PRINT 56,DELV,UT2,DELH,UT2,ROT
    56   FORMAT(///5X,'PILE TOP DEFLECTION: VERT =',F12.5,1X,A2/
     1   40X,'HORZ =',F12.5,1X,A2/41X,'ROT =',F12.5,' RAD ')
  6000   STOP
         END
C
C        THIS SUBROUTINE IS USED TO READ PROBLEM IDENTIFICATION AND
C        DESCRIPTION,IE,PILE PROPERTIES
C
         SUBROUTINE DATA1
C
```

```
      COMMON/BB1/WT1(20,20),WD1(20,20),Y(20,20)
      COMMON/BB2/AR(20,20),XR(20,20)
      COMMON/BB5/FT(20),WT(20),WD(20),FW(20)
      COMMON/B1/X1(21),Y1(21),X2(21),Y2(21),X3(21),Y3(21)
      COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
      COMMON/B14/DELP(60),NPE(20,6)
      COMMON/B20/B(20),PER(20)
      COMMON/B22/SSPF1,SSPF2,SSPR1,SSPR2,SSPP2,AREAP
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
      COMMON/B24/EY,SIGY,ETE,ETP
      COMMON/B25/UT1,UT2,UT3,UT4,UT5,UT6,UT7,UT8
      CHARACTER*4 UT1,UT2,UT3,UT4
      CHARACTER*8 UT5,UT6,UT7,UT8
      READ(5,*)NM,JTSOIL,LIST,IAX,IAY
      NMP=NM+1
      NP=3*NMP
      READ(5,*)PIL,AREAP,ALPHA
      PRINT 6,PIL,UT1,AREAP,UT1,NM,JTSOIL
    6 FORMAT('0',5X,'PILE LENGTH =',F7.3,1X,A2/5X,'PILE POINT AREA :
     1F8.3,' SQ ',A2/15X,'NO. OF PILE SEGMENTS =',I3/20X,'JOINT SOI|
     2STARTS =',I3)
      ALPHAR=ALPHA/57.2957795
      PRINT 7,UT1,UT1,UT1,UT1
    7 FORMAT('0',5X,'MEMNO',2X,'NP1',2X,'NP2',2X,'NP3',2X,'NP4',
     12X,'NP5',2X,'NP6',2X,'FL.TH.,',1X,A2,2X,'WB.TH.,',1X,A2,2X,
     2'WF.DH.,',1X,A2,2X,'FL.WH.,',1X,A2)
      K=1
      DO 300 I=1,NM
      MNO(I)=I
      NPE(I,1)=K
      NPE(I,2)=K+1
      NPE(I,3)=K+2
      NPE(I,4)=K+3
      NPE(I,5)=K+4
      NPE(I,6)=K+5
      K=K+3
      READ(5,*)FT(I),WT(I),WD(I),FW(I)
C
C     BUILD UP SECTION AREA,A, SECTION MODULUS,ZIN, MOMENT OF
C     INERTIA XIN IN DIFFERENT KINDS OF CROSS SECTION
C
      CALL ARSMMI(I)
  300 PRINT 8,MNO(I),(NPE(I,IZ),IZ=1,6),FT(I),WT(I),WD(I),FW(I)
    8 FORMAT(' ',5X,I5,6(2X,I3),4(2X,F9.6))
      PRINT 9,UT1,UT1,UT1,UT1,UT1,UT1,UT1,UT1
    9 FORMAT(' ',5X,'MEMNO',2X,'ALPHA',2X,'L,',5X,A2,2X,'I,',5X,A2,
     1'**4',2X,'WIDTH',5X,A2,2X,'PERMI,',5X,A2,2X,'A    SQ',2X,A2,2X
     2'Z,',5X,A2,'**3',2X,'X-COORD',4X,A2,2X,'Y-COORD',4X,A2)
      X3(1)=0.0
      Y3(1)=0.0
      DO 309 I=1,NM
      READ(5,*)XL(I),B(I),PER(I)
      XL1(I)=XL(I)
      A(I)=0.0
      XIN(I)=0.0
      ZIN(I)=0.0
      DO 311 M=1,7
      A(I)=A(I)+AR(I,M)*2.
```

```
      X3(I+1)=X3(I)+XL1(I)*COS(ALPHAR)
      Y3(I+1)=Y3(I)+XL1(I)*SIN(ALPHAR)
  309 PRINT 3,MNO(I),ALPHA,XL1(I),XIN(I),B(I),PER(I),A(I),ZIN(I),
     1 X3(I),Y3(I)
    3 FORMAT(' ',5X,I5,2X,F5.2,2X,F9.4,2X,F12.6,2X,F12.6,2X,F12.6,
     1 2X,F10.6,2X,F12.6,2(2X,F13.5))
      PRINT 13,X3(NMP),Y3(NMP)
   13 FORMAT(' ',96X,2(2X,F13.5))
      PRINT 14,'MEMBER NO','NODE I','NODE J'
   14 FORMAT('0',5X,A9,2(3X,A6))
      DO 110 K=1,NM
      NODI(K)=K
      NODJ(K)=K+1
      PRINT 16,MNO(K),NODI(K),NODJ(K)
   16 FORMAT(' ',11X,I3,6X,I3,6X,I3)
  110 CONTINUE
      DO 115 K=1,NMP
      X1(K)=X3(K)
      X2(K)=X3(K)
      Y1(K)=Y3(K)
  115 Y2(K)=Y3(K)
C
C     READ AND PRINT MATERIAL ELASTIC-PLASTIC PROPERTIES
C
      READ(5,*)EY,SIGY,ETE,ETP
      PRINT 19,EY,SIGY,ETE,ETP
   19 FORMAT(' ',5X,'THE YIELD STRAIN =',E15.2,'THE YIELD STRESS =
     1 E15.2/5X,'THE ELASTIC TANGENT MODULUS =',E15.5,'THE PLASTIC
     2 TANGENT MODULUS =',E15.5)
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO CALCULATE THE AREA AR, MOMENT OF
C     INTERIA XR, DISTANCE FROM NEUTRAL AXIS TO CENTROID OF EACH
C     LAYER Y.
C     IAX=1; H-PILE(BENDING ABOUT STRONG AXIS)
C     IAX=2; H-PILE(BENDING ABOUT WEAK AXIS)
C     IAX=3; RECTANGULAR CROSS SECTION
C     IAX=4; ANY SHAPE OF CROSS SECTION(SYMMETRY ON THE NEUTRAL AX
C     IAX=5; CONTROL KEY TO GENERATE INPUT AUTOMATICAIIY FOR IAX=4
C
      SUBROUTINE ARSMMI(I)
      COMMON/BB1/WT1(20,20),WD1(20,20),Y(20,20)
      COMMON/BB2/AR(20,20),XR(20,20)
      COMMON/BB5/FT(20),WT(20),WD(20),FW(20)
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
      IF(IAX.EQ.5)THEN
      DO 386 M=1,14
      AR(I,M)=AR(I-1,M)
      Y(I,M)=Y(I-1,M)
  386 XR(I,M)=XR(I-1,M)
      END IF
      IF(IAX.EQ.1)THEN
      DO 302 M=1,5
      WT1(I,M)=WT(I)
  302 WD1(I,M)=(WD(I)-2.*FT(I))/2./5.
      DO 303 M=1,2
      WT1(I,5+M)=FW(I)
  303 WD1(I,5+M)=FT(I)/2.
```

```
 304    Y(I,M)=(2.*M-1.)/2.*WD1(I,M)
        Y(I,6)=(WD(I)-2.*FT(I))/2.+WD1(I,6)/2.
        Y(I,7)=Y(I,6)+WD1(I,6)
        DO 301 M=8,14
        Y(I,M)=-Y(I,M-7)
        WT1(I,M)=WT1(I,M-7)
 301    WD1(I,M)=WD1(I,M-7)
        DO 381 M=1,14
        AR(I,M)=WT1(I,M)*WD1(I,M)
 381    XR(I,M)=WT1(I,M)*WD1(I,M)**3/12.+AR(I,M)*Y(I,M)**2
        END IF
        IF(IAX.EQ.2)THEN
        WT1(I,1)=WD(I)
        WD1(I,1)=WT(I)/2.
        DO 306 M=1,6
        WT1(I,M+1)=2.*FT(I)
 306    WD1(I,M+1)=(FW(I)-WT(I))/2./6.
        Y(I,1)=WD1(I,1)/2.
        DO 307 M=1,6
 307    Y(I,M+1)=WD1(I,1)+(2.*M-1.)/2.*WD1(I,M+1)
        DO 308 M=8,14
        Y(I,M)=-Y(I,M-7)
        WT1(I,M)=WT1(I,M-7)
 308    WD1(I,M)=WD1(I,M-7)
        DO 382 M=1,14
        AR(I,M)=WT1(I,M)*WD1(I,M)
 382    XR(I,M)=WT1(I,M)*WD1(I,M)**3/12.+AR(I,M)*Y(I,M)**2
        END IF
        IF(IAX.EQ.3)THEN
        DO 356 M=1,7
        WT1(I,M)=FW(I)
        WD1(I,M)=WD(I)/2./7.
 356    Y(I,M)=(2.*M-1.)/2.*WD1(I,M)
        DO 357 M=8,14
        WT1(I,M)=WT1(I,M-7)
        WD1(I,M)=WD1(I,M-7)
 357    Y(I,M)=-Y(I,M-7)
        DO 383 M=1,14
        AR(I,M)=WT1(I,M)*WD1(I,M)
 383    XR(I,M)=WT1(I,M)*WD1(I,M)**3/12.+AR(I,M)*Y(I,M)**2
        END IF
        IF(IAX.EQ.4)THEN
        DO 384 M=1,7
 384    READ(5,*)AR(I,M),Y(I,M),XR(I,M)
        DO 385 M=8,14
        AR(I,M)=AR(I,M-7)
        Y(I,M)=-Y(I,M-7)
 385    XR(I,M)=XR(I,M-7)
        IAX=IAX+1
        END IF
        RETURN
        END
C
C       THIS SUBROUTINE IS USED TO ESTIBLISH SOIL PARAMETERS WHICH
C       ARE CORRESPONDING TO P-Y,F-Z,AND Q-Z CURVES.
C
        SUBROUTINE DATA2
        COMMON/B3/SSI(20),SSF(20),CH(20),PH(20)
        COMMON/B5/SKI(20),SKF(20),CV(20),PV(20)
```

```
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
C
C     READ LATERAL,VERTICAL,POINT SOIL SPRING PROPERTIES
C
      PRINT 22,'NO OF PILE ELEMENTS','THE INITIAL MODULUS',
     1'THE FINAL MODULUS','SHAPE PARAMETER','THE ULTIMATE SOIL
     2RESISTANCE'
   22 FORMAT(' ',1X,A19,1X,A19,3X,A17,5X,A15,2X,A30)
      DO 220 JJ=1,NM
      READ(5,*)SSI(JJ),SSF(JJ),CH(JJ),PH(JJ)
  220 PRINT 24,JJ,SSI(JJ),SSF(JJ),CH(JJ),PH(JJ)
   24 FORMAT(' ',15X,I5,2(5X,F15.5),10X,F10.5,17X,F15.5)
      PRINT 26,'NO OF PILE ELEMENTS','THE INITIAL MODULUS',
     1'THE FINAL MODULUS','SHAPE PARAMETER','THE ULTIMATE SOIL
     2RESISTANCE'
   26 FORMAT(' ',1X,A19,1X,A19,3X,A17,5X,A15,2X,A30)
      DO 225 JJ=1,NM
      READ(5,*)SKI(JJ),SKF(JJ),CV(JJ),PV(JJ)
  225 PRINT 28,JJ,SKI(JJ),SKF(JJ),CV(JJ),PV(JJ)
   28 FORMAT(' ',15X,I5,2(5X,F15.5),10X,F10.5,17X,F15.5)
      PRINT 27,'NO OF PILE ELEMENT','THE INITIAL MODULUS',
     1'THE FINAL MODULUS','SHAPE PARAMETER','THE ULTIMATE SOIL
     2RESISTANCE'
   27 FORMAT(' ',1X,A18,2X,A19,3X,A17,5X,A15,2X,A30)
      READ(5,*)SPI(NM),SPF(NM),CP(NM),PP(NM)
      PRINT 29,NM,SPI(NM),SPF(NM),CP(NM),PP(NM)
   29 FORMAT(' ',14X,I5,6X,F15.5,5X,F15.5,10X,F10.5,17X,F15.5)
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO INPUT THE NODAL INFORMATION,
C     BOUNDARY CONDITION,CRITERIA OF CONVERGENCE,LIMITATION OF
C     CYCLES FOR INCREMENT AND ITERATION
C
      SUBROUTINE DATA3
      COMMON/BB3/TLOAD(60),DELTAP(60)
      COMMON/BB4/ACTP(60),ACTSF(6,20),ACTFOR(6,20)
      COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
      COMMON/B7/DSTRA1(20,21,20),DSTRA2(20,21,20),DELSTR(20,21,20
      COMMON/B13/X(60,2),DISPLA(60),DISTP(6,20),DIST(3,20)
      COMMON/B15/DISP(6,20),DIS(6,20)
      COMMON/B16/DELSTS(20,21,20),DSTRSS(20,21,20)
      COMMON/B18/KODE(21),ULX(21),VLY(21),WLZ(21)
      COMMON/B19/CONST1(21),CONST2(21),CONST3(21)
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
      COMMON/B26/NJTER,NJICR,JJICR,JICR,JTER,DCOV
C
C     SET UP INITIAL BOUNDARY CONDITIONS.
C     ZERO OUT TOTAL LOAD MATRIX,TLOAD;TOTAL DISPLACEMENT MATRIX,
C     THE LOAD INCREMENT MATRIX,DELTAP;THE ACTUAL LOAD MATRIX,ACTI
C     THE TOTAL DISPLACEMENT MATRIX AT LOCAL,DIS;THE LAYER STRAIN
C     DSTRA1;THE LAYER STRESS,DSTRSS
C
      DO 200 I=1,NP
      TLOAD(I)=0.0
      X(I,1)=0.0
      ACTP(I)=0.0
  200 DELTAP(I)=0.0
      DO 205 I=1,6
```

```
  205   DIS(I,J)=0.0
        DO 206 I=1,14
        DO 206 J=1,NM
        DSTRA1(I,NODI(J),J)=0.0
        DSTRA1(I,NODJ(J),J)=0.0
        DSTRA2(I,NODI(J),J)=0.0
        DSTRA2(I,NODJ(J),J)=0.0
        DSTRSS(I,NODI(J),J)=0.0
  206   DSTRSS(I,NODJ(J),J)=0.0
C
C       READ AND PRINT NODAL INFORMATION,APPLIED LOAD,BOUNDARY
C       CONDITIONS
C
        PRINT 12,'JOINT INFORMATION'
   12   FORMAT(' ',5X,A17)
        PRINT 15,'NODAL POINT DATA','NODAL POINT','TYPE','X-DISP,
     1  OR LOAD','Y-DISP,OR LOAD','Z-DISP,OR LOAD','X-ICR','Y-ICR',
     2  'Z-ICR'
   15   FORMAT(//5X,A16///5X,A11,1X,A4,5X,A16,2(5X,A14),3(5X,A5))
        DO 210 IM=1,NMP
        READ(5,*)I,KODE(I),ULX(I),VLY(I),WLZ(I),CONST1(I),CONST2(I),
     1  CONST3(I)
  210   PRINT 17,I,KODE(I),ULX(I),VLY(I),WLZ(I),CONST1(I),CONST2(I),
     1  CONST3(I)
   17   FORMAT(' ',11X,I4,1X,I4,5X,F16.5,2(5X,F14.5),3(2X,F8.2))
C
C       READ THE CRITERIA FOR CONVERGENCE
C
        READ(5,*)DCOV
        PRINT 10,DCOV
   10   FORMAT(' ','THE CRITERIA FOR CONVERGENCE IS =',F10.8)
C
C       READ AND PRINT NUMBER OF CYCLES FOR INCREMENT AND ITERATION
C
        READ(5,*)JICR,JTER,JJICR
        PRINT 11,JICR,JTER,JJICR
   11   FORMAT('0',5X,'NUMBER OF INCREMENTS =',I3//5X,'NUMBER OF
     1  ITERATIONS =',I3//5X,'THE MAXIMUM NUMBER OF LOAD INCREMENTS ='
     2  I4)
C
C       ADD EXTERNAL APPLIED CONCENTRATED NODAL LOAD TO TLOAD
C
        DO 215 I=1,NMP
        K=3*I-2
        L=3*I-1
        M=3*I
        TLOAD(K)=TLOAD(K)+ULX(I)
        TLCAD(L)=TLOAD(L)+VLY(I)
        TLOAD(M)=TLOAD(M)+WLZ(I)
        DELTAP(K)=TLOAD(K)/JICR
        DELTAP(L)=TLOAD(L)/JICR
        DELTAP(M)=TLOAD(M)/JICR
        IF(KODE(I).EQ.1.AND.IAY.EQ.1)THEN
        DELTAP(K)=TLOAD(K)
        END IF
        IF(KODE(I).EQ.2.AND.IAY.EQ.1)THEN
        DELTAP(L)=TLOAD(L)
        END IF
        IF(KODE(I).EQ.3.AND.IAY.EQ.1)THEN
```

```
      END IF
      IF(KODE(I).EQ.4.AND.IAY.EQ.1)THEN
      DELTAP(K)=TLOAD(K)
      DELTAP(L)=TLOAD(L)
      END IF
      IF(KODE(I).EQ.5.AND.IAY.EQ.1)THEN
      DELTAP(L)=TLOAD(L)
      DELTAP(M)=TLOAD(M)
      END IF
      IF(KODE(I).EQ.6.AND.IAY.EQ.1)THEN
      DELTAP(K)=TLOAD(K)
      DELTAP(M)=TLOAD(M)
      END IF
  215 CONTINUE
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO PRESCRIBE FORCES AND/OR
C     DISPLACEMENTS AT A NODE BY KODE(I)
C
      SUBROUTINE CONSTR
C
      COMMON/BB3/TLOAD(60),DELTAP(60)
      COMMON/B13/X(60,2),DISPLA(60),DISTP(6,20),DIST(3,20)
      COMMON/B14/DELP(60),NPE(20,6)
      COMMON/B18/KODE(21),ULX(21),VLY(21),WLZ(21)
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
C
      DO 251 M=1,NMP
      IF(KODE(M).EQ.0) GO TO 251
      IF(KODE(M).EQ.1)THEN
      DELP(3*M-2)=DELTAP(3*M-2)-X(3*M-2,1)
      CALL GEOMBC(DELP(3*M-2),3*M-2)
      END IF
      IF(KODE(M).EQ.2)THEN
      DELP(3*M-1)=DELTAP(3*M-1)-X(3*M-1,1)
      CALL GEOMBC(DELP(3*M-1),3*M-1)
      END IF
      IF(KODE(M).EQ.3)THEN
      DELP(3*M)=DELTAP(3*M)-X(3*M,1)
      CALL GEOMBC(DELP(3*M),3*M)
      END IF
      IF(KODE(M).EQ.4)THEN
      DELP(3*M-2)=DELTAP(3*M-2)-X(3*M-2,1)
      CALL GEOMBC(DELP(3*M-2),3*M-2)
      DELP(3*M-1)=DELTAP(3*M-1)-X(3*M-1,1)
      CALL GEOMBC(DELP(3*M-1),3*M-1)
      END IF
      IF(KODE(M).EQ.5)THEN
      DELP(3*M-1)=DELTAP(3*M-1)-X(3*M-1,1)
      CALL GEOMBC(DELP(3*M-1),3*M-1)
      DELP(3*M)=DELTAP(3*M)-X(3*M,1)
      CALL GEOMBC(DELP(3*M),3*M)
      END IF
      IF(KODE(M).EQ.6)THEN
      DELP(3*M-2)=DELTAP(3*M-2)-X(3*M-2,1)
      CALL GEOMBC(DELP(3*M-2),3*M-2)
      DELP(3*M)=DELTAP(3*M)-X(3*M,1)
      CALL GEOMBC(DELP(3*M),3*M)
```

```
      IF(KODE(M).EQ.7)THEN
      DELP(3*M-2)=DELTAP(3*M-2)-X(3*M-2,1)
      CALL GEOMBC(DELP(3*M-2),3*M-2)
      DELP(3*M-1)=DELTAP(3*M-1)-X(3*M-1,1)
      CALL GEOMBC(DELP(3*M-1),3*M-1)
      DELP(3*M)=DELTAP(3*M)-X(3*M,1)
      CALL GEOMBC(DELP(3*M),3*M)
      END IF
  251 CONTINUE
      RETURN
      END
C
C     THIS SUBROUTNE IS USED TO CALCULATE THE ELEMENT LOCAL
C     DISPLACEMENT BY ACCUMULATING INCREASED GLOBAL DISPLACEMENT
C
      SUBROUTINE ELDISP(N)
C
      COMMON/B/ASAT(60,6),ES(6,6),EA(6,6),EAT(6,6),ESAT(6,6),EASA(6,(
      COMMON/B13/X(60,2),DISPLA(60),DISTP(6,20),DIST(3,20)
      COMMON/B14/DELP(60),NPE(20,6)
      COMMON/B15/DISP(6,20),DIS(6,20)
C
      DO 266 I=1,6
      DISP(I,N)=0.0
      DO 266 K=1,6
      J=NPE(N,K)
  266 DISP(I,N)=DISP(I,N)+EA(I,K)*DISPLA(J)
      DO 267 I=1,6
  267 DIS(I,N)=DIS(I,N)+DISP(I,N)
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO CALCULATE LATERAL SPRING FORCES
C     VERTICAL SPRING FORCES, POINT SPRING FORCE IN LOCAL
C     COORDINATE. ALSO THE STRESS DISTRIBUTION OF THE SOIL SPRING
C     FORCES IN THE ELEMENT
C
      SUBROUTINE ELSPFR(N)
C
      COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
      COMMON/B3/SSI(20),SSF(20),CH(20),PH(20)
      COMMON/B4/SF(6,20),SP(6,20),SS(6,20)
      COMMON/B5/SKI(20),SKF(20),CV(20),PV(20)
      COMMON/B6/SPI(20),SPF(20),CP(20),PP(20)
      COMMON/B15/DISP(6,20),DIS(6,20)
      COMMON/B20/B(20),PER(20)
      COMMON/B22/SSPF1,SSPF2,SSPR1,SSPR2,SSPP2,AREAP
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
C
      DO 270 I=1,6
      SP(I,N)=0.0
  270 SF(I,N)=0.0
      CALL ELSPRF(2,N)
      CALL ELSPFF(2,N)
      SS(1,N)=SF(1,N)/XL(N)/PER(N)*2.
      SS(2,N)=SF(2,N)/XL(N)*2.
      SS(3,N)=SF(3,N)
      SS(4,N)=SF(4,N)/XL(N)/PER(N)*2.
      SS(5,N)=SF(5,N)/XL(N)*2.
      SS(6,N)=SF(6,N)
```

```
CALL ELSPPF(2,N)
SF(4,N)=SF(4,N)+SP(4,N)
```

```
      SS(4,N)=SS(4,N)+SP(4,N)/AREAP
      END IF
      RETURN
      END
C
C     THIS SUBROUTNE IS USED TO CALCULATE THE MEMBER FORCES IN LOCAL
C     COORDINATE
C
      SUBROUTINE ELMFOR(N)
C
      COMMON/BB6/DFORCE(3,20),FORCE(6,20)
      COMMON/B10/ELFOR(4,20)
      COMMON/B12/ER(6,3),ERT(3,6)
      COMMON/B17/ALN(3,4),ANLN(3,4)
C
      DO 275 I=1,3
      DFORCE(I,N)=0.0
      DO 275 J=1,4
  275 DFORCE(I,N)=DFORCE(I,N)+ALN(I,J)*ELFOR(J,N)+ANLN(I,J)*
     1 ELFOR(J,N)
      DO 280 I=1,6
      FORCE(I,N)=0.0
      DO 280 J=1,3
  280 FORCE(I,N)=FORCE(I,N)+ER(I,J)*DFORCE(J,N)
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO EVALUATE EQUILIBRIUM ACTUAL NODAL
C     FORCES AND ASSEMBLE FORCES IN GLOBAL COORDINATE
C
      SUBROUTINE ELACTP(N)
      COMMON/B/ASAT(60,6),ES(6,6),EA(6,6),EAT(6,6),ESAT(6,6),EASA(6,
      COMMON/BB4/ACTP(60),ACTSF(6,20),ACTFOR(6,20)
      COMMON/BB6/DFORCE(3,20),FORCE(6,20)
      COMMON/B4/SF(6,20),SP(6,20),SS(6,20)
      COMMON/B14/DELP(60),NPE(20,6)
C
      DO 290 I=1,6
      ACTFOR(I,N)=0.0
      ACTSF(I,N)=0.0
      DO 290 K=1,6
      ACTSF(I,N)=ACTSF(I,N)+EA(I,K)*SF(K,N)
  290 ACTFOR(I,N)=ACTFOR(I,N)+EA(I,K)*FORCE(K,N)
      DO 295 I=1,6
      K=NPE(N,I)
  295 ACTP(K)=ACTP(K)+ACTSF(I,N)+ACTFOR(I,N)
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO CALCULATE THE LOAD/DISPLACEMENT
C     INCREMENT IN DIFFERENT LOADING PATTERNS.
C     IAY=0  LOAD AND DISPLACEMENT INCREASE AT THE SAME RATE
C            UNTIL THE SPECIFIED VALUES ARE OBTAINED(JICR=JJICR)
C     IAY=1  LOAD INCREASES BUT SPECIFIED DISPLACEMENT KEEPS
C            CONSTANT(JICR=JJICR)
C     IAY=2  DISPLACEMENT INCREASES BY KEEPING LOAD CONSTANT
C            UNTIL THE SPECIFIED DISPLACEMENT IS REACHED THEN
C            THE LOAD WILL BE INCREASED
C
```

```
C
      COMMON/BB3/TLOAD(60),DELTAP(60)
      COMMON/B18/KODE(21),ULX(21),VLY(21),WLZ(21)
      COMMON/B19/CONST1(21),CONST2(21),CONST3(21)
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
      COMMON/B26/NJTER,NJICR,JJICR,JICR,JTER,DCOV
      COMMON/B28/IAZ
C
      IAZ=0
      DO 330 I=1,NMP
      K=3*I-2
      L=3*I-1
      M=3*I
      IF(IAY.EQ.2) GO TO 331
      DELTAP(K)=DELTAP(K)+TLOAD(K)/JICR
      DELTAP(L)=DELTAP(L)+TLOAD(L)/JICR
      DELTAP(M)=DELTAP(M)+TLOAD(M)/JICR
      IF(KODE(I).EQ.1.AND.IAY.EQ.1)THEN
      DELTAP(K)=TLOAD(K)
      END IF
      IF(KODE(I).EQ.2.AND.IAY.EQ.1)THEN
      DELTAP(L)=TLOAD(L)
      END IF
      IF(KODE(I).EQ.3.AND.IAY.EQ.1)THEN
      DELTAP(M)=TLOAD(M)
      END IF
      IF(KODE(I).EQ.4.AND.IAY.EQ.1)THEN
      DELTAP(K)=TLOAD(K)
      DELTAP(L)=TLOAD(L)
      END IF
      IF(KODE(I).EQ.5.AND.IAY.EQ.1)THEN
      DELTAP(L)=TLOAD(L)
      DELTAP(M)=TLOAD(M)
      END IF
      IF(KODE(I).EQ.6.AND.IAY.EQ.1)THEN
      DELTAP(K)=TLOAD(K)
      DELTAP(M)=TLOAD(M)
      END IF
  330 CONTINUE
      IF(NJICR.LT.JICR) GO TO 231
  331 IAY=2
      DO 332 I=1,NMP
      K=3*I-2
      L=3*I-1
      M=3*I
      IF(KODE(I).EQ.1.AND.IAY.EQ.2)THEN
      DELTAP(K)=TLOAD(K)
      DELTAP(L)=DELTAP(L)+CONST2(I)
      DELTAP(M)=DELTAP(M)+CONST3(I)
      END IF
      IF(KODE(I).EQ.2.AND.IAY.EQ.2)THEN
      DELTAP(K)=DELTAP(K)+CONST1(I)
      DELTAP(L)=TLOAD(L)
      DELTAP(M)=DELTAP(M)+CONST3(I)
      END IF
      IF(KODE(I).EQ.3.AND.IAY.EQ.2)THEN
      DELTAP(K)=DELTAP(K)+CONST1(I)
      DELTAP(L)=DELTAP(L)+CONST2(I)
      DELTAP(M)=TLOAD(M)
```

```
      IF(KODE(I).EQ.4.AND.IAY.EQ.2)THEN
      DELTAP(K)=TLOAD(K)
      DELTAP(L)=TLOAD(L)
      DELTAP(M)=DELTAP(M)+CONST3(I)
      END IF
      IF(KODE(I).EQ.5.AND.IAY.EQ.2)THEN
      DELTAP(K)=DELTAP(K)+CONST1(I)
      DELTAP(L)=TLOAD(L)
      DELTAP(M)=TLOAD(M)
      END IF
      IF(KODE(I).EQ.6.AND.IAY.EQ.2)THEN
      DELTAP(K)=TLOAD(K)
      DELTAP(L)=DELTAP(L)+CONST2(I)
      DELTAP(M)=TLOAD(M)
      END IF
  332 CONTINUE
      GO TO 232
  231 IAZ=1
  232 RETURN
      END
C
C     THIS SUBROUTINE IS USED TO FORM MEMBER GEOMETRIES
C     COMPUTE X-COORD,Y-COORD,LENGTH,SLOPE
C
      SUBROUTINE UPDATE(KK,N)
C
      REAL LX,LY,LX1,LY1
      COMMON/B1/X1(21),Y1(21),X2(21),Y2(21),X3(21),Y3(21)
      COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
      COMMON/B21/XCOS,XSIN
      IF(KK.EQ.2) GO TO 327
      LX=X1(NODJ(N))-X1(NODI(N))
      LY=Y1(NODJ(N))-Y1(NODI(N))
      GO TO 328
  327 LX=X2(NODJ(N))-X2(NODI(N))
      LY=Y2(NODJ(N))-Y2(NODI(N))
  328 ALH=SQRT(LX**2+LY**2)
      XCOS=LX/ALH
      XSIN=LY/ALH
      XL(N)=ALH
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO CALCULATE THE LATERAL SPRING
C     ELEMENT STIFFNESS IN LOCAL FOR EACH ELEMENT
C
      SUBROUTINE ELSPRF(KK,N)
C
      COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
      COMMON/B3/SSI(20),SSF(20),CH(20),PH(20)
      COMMON/B4/SF(6,20),SP(6,20),SS(6,20)
      COMMON/B15/DISP(6,20),DIS(6,20)
      COMMON/B20/B(20),PER(20)
      COMMON/B22/SSPF1,SSPF2,SSPR1,SSPR2,SSPP2,AREAP
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
C
      IF(KK.EQ.2) GO TO 405
      SSPR1=0.0
      SSPR2=0.0
```

```
      DAVE=DIS(2,N)
      EAVE=DIS(5,N)
      DSS1=SSI(N)-SSF(N)
      DSS2=(ABS(DSS1*DAVE/PH(N)))**CH(N)
      DSS3=((CH(N)+1.)/CH(N))
      DSS4=(1.+DSS2)**DSS3
      DSS5=DSS1/DSS4+SSF(N)
      SSPR1=DSS5*XL(N)/2.
      DSS6=(ABS(DSS1*EAVE/PH(N)))**CH(N)
      DSS7=(1.+DSS6)**DSS3
      DSS8=DSS1/DSS7+SSF(N)
      SSPR2=DSS8*XL(N)/2.
      GO TO 410
  405 SF(2,N)=0.0
      SF(5,N)=0.0
      IF(N.LT.JTSOIL) GO TO 410
      DAVE=DIS(2,N)
      EAVE=DIS(5,N)
      DSS1=SSI(N)-SSF(N)
      DSS2=(ABS(DSS1*DAVE/PH(N)))**CH(N)
      DSS3=1./CH(N)
      DSS4=(1.+DSS2)**DSS3
      DSS5=DSS1*DAVE/DSS4+SSF(N)*DAVE
      SF(2,N)=DSS5*XL(N)/2.
      DSS6=(ABS(DSS1*EAVE/PH(N)))**CH(N)
      DSS7=(1.+DSS6)**DSS3
      DSS8=DSS1*EAVE/DSS7+SSF(N)*EAVE
      SF(5,N)=DSS8*XL(N)/2.
  410 RETURN
      END
C
C     THIS SUROUTINE IS USED TO CALCULATE THE VERTICAL SPRING
C     ELEMENT STIFFNESS IN LOCAL FOR EACH ELEMENT
C
      SUBROUTINE ELSPFF(KK,N)
C
      COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
      COMMON/B4/SF(6,20),SP(6,20),SS(6,20)
      COMMON/B5/SKI(20),SKF(20),CV(20),PV(20)
      COMMON/B15/DISP(6,20),DIS(6,20)
      COMMON/B20/B(20),PER(20)
      COMMON/B22/SSPF1,SSPF2,SSPR1,SSPR2,SSPP2,AREAP
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
C
      IF(KK.EQ.2) GO TO 415
      SSPF1=0.0
      SSPF2=0.0
      IF(N.LT.JTSOIL) GO TO 420
      FAVE=DIS(1,N)
      GAVE=DIS(4,N)
      FSK1=SKI(N)-SKF(N)
      FSK2=(ABS(FSK1*FAVE/PV(N)))**CV(N)
      FSK3=((CV(N)+1.)/CV(N))
      FSK4=(1.+FSK2)**FSK3
      FSK5=FSK1/FSK4+SKF(N)
      SSPF1=FSK5*XL(N)*PER(N)/2.
      FSK6=(ABS(FSK1*GAVE/PV(N)))**CV(N)
      FSK7=(1.+FSK6)**FSK3
      FSK8=FSK1/FSK7+SKF(N)
```

```
      GO TO 420
  415 SF(1,N)=0.0
      SF(4,N)=0.0
      IF(N.LT.JTSOIL) GO TO 420
      FAVE=DIS(1,N)
      GAVE=DIS(4,N)
      FSK1=SKI(N)-SKF(N)
      FSK2=(ABS(FSK1*FAVE/PV(N)))**CV(N)
      FSK3=1./CV(N)
      FSK4=(1.+FSK2)**FSK3
      FSK5=FSK1*FAVE/FSK4+SKF(N)*FAVE
      SF(1,N)=FSK5*XL(N)*PER(N)/2.
      FSK6=(ABS(FSK1*GAVE/PV(N)))**CV(N)
      FSK7=(1.+FSK6)**FSK3
      FSK8=FSK1*GAVE/FSK7+SKF(N)*GAVE
      SF(4,N)=FSK8*XL(N)*PER(N)/2.
  420 RETURN
      END
C
C     THIS SUBROUTINE IS USED TO CALCULATE THE POINT SPRING
C     ELEMENT STIFFNESS IN LOCAL FOR EACH ELEMENT
C
      SUBROUTINE ELSPPF(KK,N)
C
      COMMON/B4/SF(6,20),SP(6,20),SS(6,20)
      COMMON/B6/SPI(20),SPF(20),CP(20),PP(20)
      COMMON/B15/DISP(6,20),DIS(6,20)
      COMMON/B22/SSPF1,SSPF2,SSPR1,SSPR2,SSPP2,AREAP
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
C
      IF(KK.EQ.2) GO TO 425
      SSPP2=0.0
      GAVE=DIS(4,N)
      GSP1=SPI(N)-SPF(N)
      GSP2=(ABS(GSP1*GAVE/PP(N)))**CP(N)
      GSP3=((CP(N)+1.)/CP(N))
      GSP4=(1.+GSP2)**GSP3
      GSP5=GSP1/GSP4+SPF(N)
      SSPP2=GSP5*AREAP
      GO TO 430
  425 SP(4,N)=0.0
      GAVE=DIS(4,N)
      GSP1=SPI(N)-SPF(N)
      GSP2=(ABS(GSP1*GAVE/PP(N)))**CP(N)
      GSP3=1./CP(N)
      GSP4=(1.+GSP2)**GSP3
      GSP5=GSP1*GAVE/GSP4+SPF(N)*GAVE
      SP(4,N)=GSP5*AREAP
  430 RETURN
      END
C
C     THIS SUBROUTIN IS USED TO EVALUATE (EA)T,(EK)T,(EI)T
C     AT THE END POINTS THROUGH THE LAYERS
C
      SUBROUTINE ELEAKI(N)
C
      COMMON/BB1/WT1(20,20),WD1(20,20),Y(20,20)
      COMMON/BB2/AR(20,20),XR(20,20)
```

```
      COMMON/B8/ET(20,21,20),EK(20,21,20),EI(20,21,20)
      COMMON/B9/ET1(21,20),EK1(21,20),EI1(21,20)
      COMMON/B24/EY,SIGY,ETE,ETP
C
      DO 760 I=1,14
      IF(ABS(DSTRA1(I,NODI(N),N)).GE.EY)THEN
      ET(I,NODI(N),N)=ETP*AR(N,I)
      EK(I,NODI(N),N)=ETP*Y(N,I)*AR(N,I)
      EI(I,NODI(N),N)=ETP*XR(N,I)
      ELSE
      ET(I,NODI(N),N)=ETE*AR(N,I)
      EK(I,NODI(N),N)=ETE*Y(N,I)*AR(N,I)
      EI(I,NODI(N),N)=ETE*XR(N,I)
      END IF
      IF(ABS(DSTRA1(I,NODJ(N),N)).GE.EY)THEN
      ET(I,NODJ(N),N)=ETP*AR(N,I)
      EK(I,NODJ(N),N)=ETP*Y(N,I)*AR(N,I)
      EI(I,NODJ(N),N)=ETP*XR(N,I)
      ELSE
      ET(I,NODJ(N),N)=ETE*AR(N,I)
      EK(I,NODJ(N),N)=ETE*Y(N,I)*AR(N,I)
      EI(I,NODJ(N),N)=ETE*XR(N,I)
      END IF
  760 CONTINUE
      ET1(NODI(N),N)=0.0
      ET1(NODJ(N),N)=0.0
      EK1(NODI(N),N)=0.0
      EK1(NODJ(N),N)=0.0
      EI1(NODI(N),N)=0.0
      EI1(NODJ(N),N)=0.0
      DO 765 I=1,14
      ET1(NODI(N),N)=ET1(NODI(N),N)+ET(I,NODI(N),N)
      ET1(NODJ(N),N)=ET1(NODJ(N),N)+ET(I,NODJ(N),N)
      EK1(NODI(N),N)=EK1(NODI(N),N)+EK(I,NODI(N),N)
      EK1(NODJ(N),N)=EK1(NODJ(N),N)+EK(I,NODJ(N),N)
      EI1(NODI(N),N)=EI1(NODI(N),N)+EI(I,NODI(N),N)
  765 EI1(NODJ(N),N)=EI1(NODJ(N),N)+EI(I,NODJ(N),N)
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO FORM BEAM ELEMENT DEFORMED
C     TANGENTIAL STIFFNESS MATRIX (3*3),AND CONVERT INTO (6*6)
C
      SUBROUTINE ELDEST(N)
C
      COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
      COMMON/B9/ET1(21,20),EK1(21,20),EI1(21,20)
      COMMON/B10/ELFOR(4,20)
      COMMON/B11/ES1(3,3),ES2(6,3),ES3(6,6)
      COMMON/B12/ER(6,3),ERT(3,6)
C
      ETT1=ET1(NODI(N),N)
      ETT2=ET1(NODJ(N),N)
      EKT1=EK1(NODI(N),N)
      EKT2=EK1(NODJ(N),N)
      EIT1=EI1(NODI(N),N)
      EIT2=EI1(NODJ(N),N)
      P1=ELFOR(1,N)
      P2=ELFOR(2,N)
```

```
      ES1(1,2)=EKT1/XL(N)
      ES1(1,3)=-EKT2/XL(N)
      ES1(2,2)=(3.*EIT1+EIT2)/XL(N)+(6.*P1+2.*P2)*XL(N)/60.
      ES1(2,3)=(EIT1+EIT2)/XL(N)-(P1+P2)*XL(N)/60.
      ES1(3,3)=(EIT1+3.*EIT2)/XL(N)+(2.*P1+6.*P2)*XL(N)/60.
      DO 800 I=1,3
      DO 800 J=1,3
  800 ES1(J,I)=ES1(I,J)
      DO 805 I=1,6
      DO 805 J=1,3
      ES2(I,J)=0.0
      DO 805 K=1,3
  805 ES2(I,J)=ES2(I,J)+ER(I,K)*ES1(K,J)
      DO 810 I=1,6
      DO 810 J=1,6
      ES3(I,J)=0.0
      DO 810 K=1,3
  810 ES3(I,J)=ES3(I,J)+ES2(I,K)*ERT(K,J)
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO FORM THE COMBINED ELEMENT
C     TANGENTIAL STIFFNESS MATRIX
C
      SUBROUTINE ELSTIF(KK,N)
C
      COMMON/B/ASAT(60,6),ES(6,6),EA(6,6),EAT(6,6),ESAT(6,6),EASA(6
      COMMON/B11/ES1(3,3),ES2(6,3),ES3(6,6)
      COMMON/B22/SSPF1,SSPF2,SSPR1,SSPR2,SSPP2,AREAP
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
C
      IF(KK.EQ.2) GO TO 440
      IF(KK.EQ.3) GO TO 461
      IF(KK.EQ.4) GO TO 466
C
C     FORM LOCAL SPRING ELEMENT STIFFNESS MATRIX
C
      DO 435 I=1,6
      DO 435 J=1,6
  435 ES(I,J)=0.0
      ES(1,1)=SSPF1
      ES(2,2)=SSPR1
      ES(4,4)=SSPF2
      ES(5,5)=SSPR2
      GO TO 467
C
C     FORM LOCAL POINT SPRING STIFFNESS MATRIX
C
  440 DO 450 I=1,6
      DO 450 J=1,6
  450 ES(I,J)=0.0
      IF(N.EQ.NM) ES(4,4)=SSPP2
      GO TO 467
C
C     FORM LOCAL BEAM ELEMENT STIFFNESS MATRIX
C
  461 DO 462 I=1,6
      DO 462 J=1,6
  462 ES(I,J)=0.0
```

```
      DO 463 J=1,6
  463 ES(I,J)=ES3(I,J)
      GO TO 467
C
C     FORM SPRING & BEAM ELEMENT TANGENTIAL STIFFNESS MATRIX
C
  466 DO 468 I=1,6
      DO 468 J=1,6
  468 ES(I,J)=0.0
      DO 469 I=1,6
      DO 469 J=1,6
  469 ES(I,J)=ES3(I,J)
      ES(1,1)=ES(1,1)+SSPF1
      ES(2,2)=ES(2,2)+SSPR1
      ES(4,4)=ES(4,4)+SSPF2
      IF(N.EQ.NM)ES(4,4)=ES(4,4)+SSPP2
      ES(5,5)=ES(5,5)+SSPR2
  467 RETURN
      END
C
C     THIS SUBROUTINE IS USED TO CALCULATE THE DEFORMED
C     DISPLACEMENTS (3*1) FROM GLOBAL NODAL DISPLACEMENTS
C
      SUBROUTINE ELDEDP(N)
C
      REAL LX,LY,LX1,LY1
      COMMON/B1/X1(21),Y1(21),X2(21),Y2(21),X3(21),Y3(21)
      COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
      COMMON/B13/X(60,2),DISPLA(60),DISTP(6,20),DIST(3,20)
      COMMON/B14/DELP(60),NPE(20,6)
C
      DO 272 K=1,6
      J=NPE(N,K)
  272 DISTP(K,N)=X(J,1)
      XL2=XL1(N)+XL(N)
      DISTP1=DISTP(4,N)-DISTP(1,N)
      DISTP2=DISTP(5,N)-DISTP(2,N)
      LX1=X3(NODJ(N))-X3(NODI(N))
      LY1=Y3(NODJ(N))-Y3(NODI(N))
      DIST(1,N)=(1./XL2)*((2.*LX1+DISTP1)*DISTP1+(2.*LY1+DISTP2)*
     1DISTP2)
      ANG1=ASIN((DISTP2*LX1-DISTP1*LY1)/XL1(N)/XL(N))
      DIST(2,N)=DISTP(3,N)-ANG1
      DIST(3,N)=DISTP(6,N)-ANG1
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO TRANSFORM LOCAL STIFFNESS INTO
C     GLOBAL STIFFNESS
C
      SUBROUTINE ELTEMP(N)
C
      COMMON/B/ASAT(60,6),ES(6,6),EA(6,6),EAT(6,6),ESAT(6,6),EASA(
      COMMON/B21/XCOS,XSIN
C
      DO 470 I=1,6
      DO 470 J=1,6
  470 EA(I,J)=0.0
      EA(1,1)=XCOS
      EA(1,2)=-XSIN
```

```
      EA(2,1)=XSIN
      EA(2,2)=XCOS
      EA(3,3)=1.
      EA(4,4)=XCOS
      EA(4,5)=-XSIN
      EA(5,4)=XSIN
      EA(5,5)=XCOS
      EA(6,6)=1.
      DO 471 I=1,6
      DO 471 J=1,6
  471 EAT(J,I)=EA(I,J)
C
C     FORM ELEMENT TEMPORARY STIFFNESS MATRIX
C
      DO 475 I=1,6
      DO 475 J=1,6
      ESAT(I,J)=0.0
      DO 475 K=1,6
  475 ESAT(I,J)=ESAT(I,J)+ES(I,K)*EAT(K,J)
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO CALCULATE THE ELEMENT STIFFNESS
C     MATRIX IN GLOBAL FOR EACH ELEMENT
C
      SUBROUTINE ELGLOS(N)
C
      COMMON/B/ASAT(60,6),ES(6,6),EA(6,6),EAT(6,6),ESAT(6,6),EASA(6
      COMMON/B23/LIST,JTSOIL,NM,NMP,NP,IAX,IAY
      COMMON/B26/NJTER,NJICR,JJICR,JICR,JTER,DCOV
C
      DO 480 I=1,6
      DO 480 J=1,6
      EASA(I,J)=0.0
      DO 480 K=1,6
  480 EASA(I,J)=EASA(I,J)+EA(I,K)*ESAT(K,J)
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO ASSEMBLE ELEMENT GLOBAL STIFFNESS
C     MATRIX INTO BANDED STRUCTURAL STIFFNESS MATRIX
C
      SUBROUTINE ELASSM(N)
C
      COMMON/B/ASAT(60,6),ES(6,6),EA(6,6),EAT(6,6),ESAT(6,6),EASA(6
C
      KK=3*N-3
      DO 500 I=1,6
      IR=KK+I
      DO 510 J=1,6
      IC=KK+J
      IF(IC.LT.IR) GO TO 510
      IC=IC-IR+1
      ASAT(IR,IC)=ASAT(IR,IC)+EASA(I,J)
  510 CONTINUE
  500 CONTINUE
      RETURN
      END
C
```

```
C         IN ORDER TO FIND NODAL DISPLACEMENT
C
          SUBROUTINE BANSOL(KKK,U,V,NSIZE,NBAND)
C
C         SYMMETRIC BAND MATRIX EQUATION SOLVER
C         KKK=1 TRANGULARIZE THE BAND MATRIX
C         KKK=2 SOLVE FOR RIGHT HAND SIDE V, SOLUTION RETURNS TO V
C
          DIMENSION U(60,6),V(60)
C
          NRS=NSIZE-1
          NR=NSIZE
          IF(KKK.EQ.2) GO TO 600
          DO 610 N=1,NRS
          M=N-1
          MR=MINO(NBAND,NR-M)
          PIVOT=U(N,1)
          DO 610 L=2,MR
          CP=U(N,L)/PIVOT
          I=M+L
          J=0
          DO 620 K=L,MR
          J=J+1
  620     U(I,J)=U(I,J)-CP*U(N,K)
  610     U(N,L)=CP
          GO TO 630
  600     DO 640 N=1,NRS
          M=N-1
          MR=MINO(NBAND,NR-M)
          CP=V(N)
          V(N)=CP/U(N,1)
          DO 640 L=2,MR
          I=M+L
  640     V(I)=V(I)-U(N,L)*CP
          V(NR)=V(NR)/U(NR,1)
          DO 650 I=1,NRS
          N=NR-I
          M=N-1
          MR=MINO(NBAND,NR-M)
          DO 650 K=2,MR
          L=M+K
C         STORE COMPUTED DISPLACEMENTS IN LOAD VECTOR V
  650     V(N)=V(N)-U(N,K)*V(L)
  630     RETURN
          END
C
C         THIS SUBROUTINE IS USED TO MODIFY THE ASSEMBLAGE BANDED
C         STIFFNESS AND LOAD FOR THE PRESCRIBED DISPLACEMENT U AT
C         DEGREE OF FREEDOM
C
          SUBROUTINE GEOMBC(U,N)
          COMMON/B/ASAT(60,6),ES(6,6),EA(6,6),EAT(6,6),ESAT(6,6),EASA(
          COMMON/B14/DELP(60),NPE(20,6)
          COMMON/B18/KODE(21),ULX(21),VLY(21),WLZ(21)
          COMMON/B27/NBAND,NSIZE
C
          DO 252 J=2,NBAND
          K=N-J+1
```

```
      ASAT(K,J)=0.0
  253 K=N+J-1
      IF(KK.GT.NSIZE) GO TO 252
      DELP(K)=DELP(K)-ASAT(N,J)*U
      ASAT(N,J)=0.0
  252 CONTINUE
      ASAT(N,1)=1.0
      DELP(N)=U
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO TRANSFER LOCAL COORDINATE
C     (3*3) INTO LOCAL COORDINATE (6*6), RIGID BODY MOTION IS
C     TAKING INTO ACCOUNT.
C
      SUBROUTINE ELTRAM(N)
C
      COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
      COMMON/B12/ER(6,3),ERT(3,6)
C
      DO 700 I=1,6
      DO 700 J=1,3
  700 ER(I,J)=0.0
      ER(1,1)=-1.
      ER(2,2)=1./XL(N)
      ER(2,3)=1./XL(N)
      ER(3,2)=1.
      ER(4,1)=1.
      ER(5,2)=-1./XL(N)
      ER(5,3)=-1./XL(N)
      ER(6,3)=1.
      DO 705 I=1,6
      DO 705 J=1,3
  705 ERT(J,I)=ER(I,J)
      RETURN
      END
```

```
C
C       THIS SUBROUTINE IS USED TO CALCULATE TOTAL STRAIN AND
C       EVALUATE INCREMENT STRAIN AND STORE THE PREVIOUS TOTAL
C       STRAIN
C
        SUBROUTINE STRAIN(N)
C
        COMMON/BB1/WT1(20,20),WD1(20,20),Y(20,20)
        COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
        COMMON/B7/DSTRA1(20,21,20),DSTRA2(20,21,20),DELSTR(20,21,20)
        COMMON/B13/X(60,2),DISPLA(60),DISTP(6,20),DIST(3,20)
C
        DO 730 I=1,14
        DSTRA1(I,NODI(N),N)=1./XL(N)*DIST(1,N)+4.*Y(N,I)*DIST(2,N)/
     1  XL(N)+2.*Y(N,I)*DIST(3,N)/XL(N)+0.5*DIST(2,N)*DIST(2,N)
        DSTRA1(I,NODJ(N),N)=1./XL(N)*DIST(1,N)+(-2.)*Y(N,I)*DIST(2,N
     1  XL(N)+(-4.)*Y(N,I)*DIST(3,N)/XL(N)+0.5*DIST(3,N)*DIST(3,N)
        DELSTR(I,NODI(N),N)=DSTRA1(I,NODI(N),N)-DSTRA2(I,NODI(N),N)
  730   DELSTR(I,NODJ(N),N)=DSTRA1(I,NODJ(N),N)-DSTRA2(I,NODJ(N),N)
        DO 735 I=1,14
        DSTRA2(I,NODI(N),N)=DSTRA1(I,NODI(N),N)
  735   DSTRA2(I,NODJ(N),N)=DSTRA1(I,NODJ(N),N)
        RETURN
        END
C
C       THIS SUBROUTINE IS USED TO CALCULATE INCREMENT STRESS AND
C       TOTAL STRESS THROUGH THE LAYER
C
        SUBROUTINE STRESS(N)
C
        COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
        COMMON/B7/DSTRA1(20,21,20),DSTRA2(20,21,20),DELSTR(20,21,20)
        COMMON/B16/DELSTS(20,21,20),DSTRSS(20,21,20)
        COMMON/B24/EY,SIGY,ETE,ETP
C
        DO 750 I=1,14
        IF(ABS(DSTRA1(I,NODI(N),N)).GE.EY)THEN
        DELSTS(I,NODI(N),N)=ETP*DELSTR(I,NODI(N),N)
        IF(DSTRA1(I,NODI(N),N).GE.0.)THEN
        DSTRSS(I,NODI(N),N)=SIGY+DELSTS(I,NODI(N),N)
        ELSE
        DSTRSS(I,NODI(N),N)=-SIGY+DELSTS(I,NODI(N),N)
        END IF
        ELSE
        DELSTS(I,NODI(N),N)=ETE*DELSTR(I,NODI(N),N)
        DSTRSS(I,NODI(N),N)=DSTRSS(I,NODI(N),N)+DELSTS(I,NODI(N),N)
        IF(DSTRSS(I,NODI(N),N).GT.0..AND.DSTRSS(I,NODI(N),N).GT.SIGY
     1  THEN
        DSTRSS(I,NODI(N),N)=SIGY
        END IF
        IF(DSTRSS(I,NODI(N),N).LT.0..AND.DSTRSS(I,NODI(N),N).LT.-SIG
     1  THEN
        DSTRSS(I,NODI(N),N)=-SIGY
        END IF
        END IF
        IF(ABS(DSTRA1(I,NODJ(N),N)).GE.EY)THEN
        DELSTS(I,NODJ(N),N)=ETP*DELSTR(I,NODJ(N),N)
        IF(DSTRA1(I,NODJ(N),N).GE.0.)THEN
        DSTRSS(I,NODJ(N),N)=SIGY+DELSTS(I,NODJ(N),N)
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      DSTRSS(I,NODJ(N),N)=-SIGY+DELSTS(I,NODJ(N),N)
      END IF
      ELSE
      DELSTS(I,NODJ(N),N)=ETE*DELSTR(I,NODJ(N),N)
      DSTRSS(I,NODJ(N),N)=DSTRSS(I,NODJ(N),N)+DELSTS(I,NODJ(N),N)
      IF(DSTRSS(I,NODJ(N),N).GT.0..AND.DSTRSS(I,NODJ(N),N).GT.SIGY)
     1 THEN
      DSTRSS(I,NODJ(N),N)=SIGY
      END IF
      IF(DSTRSS(I,NODJ(N),N).LT.0..AND.DSTRSS(I,NODJ(N),N).LT.-SIGY
     1 THEN
      DSTRSS(I,NODJ(N),N)=-SIGY
      END IF
      END IF
  750 CONTINUE
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO CONSTRUCT THE MATRIX ALN
C     AND ANLN WHICH ARE USED TO EVALUATE THE MEMBER FORCES
C
      SUBROUTINE ELTRF(N)
C
      COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
      COMMON/B13/X(60,2),DISPLA(60),DISTP(6,20),DIST(3,20)
      COMMON/B17/ALN(3,4),ANLN(3,4)
C
      CONS=XL(N)/60.
      DO 790 I=1,3
      DO 790 J=1,4
      ALN(I,J)=0.0
  790 ANLN(I,J)=0.0
      ALN(1,1)=0.5
      ALN(1,2)=0.5
      ALN(2,3)=-1.
      ALN(3,4)=1.
      ANLN(2,1)=CONS*(6.*DIST(2,N)-DIST(3,N))
      ANLN(2,2)=CONS*(2.*DIST(2,N)-DIST(3,N))
      ANLN(3,1)=CONS*(-1.*DIST(2,N)+2.*DIST(3,N))
      ANLN(3,2)=CONS*(-1.*DIST(2,N)+6.*DIST(3,N))
      RETURN
      END
C
C     THIS SUBROUTINE IS USED TO CALCULATE ELEMENT END FORCES
C     THROUGH THE LAYERS
C
      SUBROUTINE ELDEFR(N)
C
      COMMON/BB1/WT1(20,20),WD1(20,20),Y(20,20)
      COMMON/BB2/AR(20,20),XR(20,20)
      COMMON/B2/XL(20),XL1(20),NODI(20),NODJ(20)
      COMMON/B10/ELFOR(4,20)
      COMMON/B16/DELSTS(20,21,20),DSTRSS(20,21,20)
C
      DO 770 I=1,4
  770 ELFOR(I,N)=0.0
      DO 775 L=1,14
      ELFOR(1,N)=ELFOR(1,N)+DSTRSS(L,NODI(N),N)*AR(N,L)
      ELFOR(2,N)=ELFOR(2,N)+DSTRSS(L,NODJ(N),N)*AR(N,L)
```

```
775   ELFOR(4,N)=ELFOR(4,N)+(-1.)*DSTRSS(L,NODJ(N),N)*AR(N,L)*Y(N,I
      RETURN
      END
```